(4.) $3t \times(t)$

↳ DIFF IN FREQ.

$t x(t) \rightarrow j \frac{d}{dw} x(jw)$

$3 j \frac{4 \sin(5w)}{qw}$

$\frac{d}{dw} = \frac{60 j \cos(5w)(9w) - 12 j \sin(5w)(9)}{81 w^2}$

$= \frac{540 j \cos(5w) - 108 j \sin(5w)}{81 w^2}$

(5) **FILTERING**

(a) $2 \frac{dy(t)}{dt} + 8y(t) = 6x(t)$

↳ TAKE FOURIER TRANSFORM ON BOTH SIDES

$2jw\, Y(jw) + 8 Y(jw) = 6 x(jw)$

$(2jw + 8) Y(jw) = 6x(jw)$

$\frac{Y(jw)}{X(jw)} = \frac{6}{2jw + 8}$

$\frac{Y(jw)}{X(jw)} = \frac{6}{2(jw+4)}$

$H(jw) = \frac{3}{jw+4}$

$\Rightarrow h(t) = 3 e^{-4t} u(t)$

(b) (i) $x(t) = e^{-6t} u(t)$

↳ $x(jw) = \frac{1}{jw+6}$

$Y(jw) = x(jw) \cdot H(jw) = \frac{1}{jw+6} \cdot \frac{3}{jw+4}$

$= \frac{3}{(jw+6)(jw+4)}$

$\frac{3}{(jw+6)(jw+4)} = \frac{A}{jw+6} + \frac{B}{jw+4}$

$\frac{3}{(jw+6)(jw+4)} = \frac{Ajw + 4A + Bjw + 6B}{(jw+6)(jw+4)}$

$3 = 4A + 6B \quad ; \quad A+B = 0$

$2B = 3$

$B = \frac{3}{2} \qquad A = -\frac{3}{2}$

$Y(jw) = \frac{-3/2}{jw+6} + \frac{3/2}{jw+4}$

$y(t) = -\frac{3}{2} e^{-6t} u(t) + \frac{3}{2} e^{-4t} u(t)$

(ii) $x(jw) = \frac{jw+4}{jw+3}$

$Y(jw) = x(jw) \cdot H(jw) = \frac{jw+4}{jw+3} \cdot \frac{3}{jw+4}$

$Y(jw) = \frac{3}{jw+3}$

$Y(t) = 3 e^{-3t} u(t)$

(iii) $x(jw) = \frac{jw+2}{jw+4}$

$Y(jw) = x(jw) \cdot H(jw) = \frac{jw+2}{jw+4} \cdot \frac{3}{jw+4}$

$\frac{3jw+6}{(jw+4)^2} = \frac{A}{jw+4} + \frac{B}{(jw+4)^2}$

$3jw+6 = Ajw + 4A + B$

$A = 3$

$6 = 4A + B \quad \Rightarrow \quad B = -6$

$Y(jw) = \frac{3}{jw+4} - \frac{6}{(jw+4)^2}$

$y(t) = 3e^{-4t} u(t) - 6t e^{-4t} u(t)$

(iv) $x(jw) = \frac{1}{(jw+2)(jw+4)}$

$Y(jw) = x(jw) \cdot H(jw) = \frac{1}{(jw+2)(jw+4)} \cdot \frac{3}{(jw+4)}$

$= \frac{3}{(jw+2)(jw+4)(jw+4)}$

$\frac{3}{(jw+2)(jw+4)^2} = \frac{A}{jw+2} + \frac{B}{jw+4} + \frac{C}{(jw+4)^2}$

$3 = A(jw+4)^2 + B(jw+2)(jw+4) + Cjw + 2C$

$3 = Aj^2w^2 + 8Ajw + 16A + Bj^2w^2 + 6Bjw + 8B$
$\qquad + 2jw + 2C \qquad ; \quad A+B = 0$

$0 = 8A + 6B + C$

$0 = -8B + 6B + C \quad \Rightarrow \quad C = 2B$

$3 = 16A + 8B + 2C$

$3 = -16B + 8B + 4B$

$3 = -4B$

$B = -\frac{3}{4} \qquad A = \frac{3}{4} \qquad C = -\frac{3}{2}$

$Y(jw) = \frac{3/4}{jw+2} - \frac{3/4}{jw+4} - \frac{3/2}{(jw+4)^2}$

$y(t) = \frac{3}{4} e^{-2t} u(t) - \frac{3}{4} e^{-4t} u(t) - \frac{3}{2} e^{-4t} u(t)$