BME 350: Signals and Systems for Bioengineers
HW 6 (50 points)

Deadline: Dec 1st 2016 (Thursday) by 9AM
Late assignments: 5 points deducted per hour (delay rounded up to the nearest hour after deadline).
Note: For questions 3-6, show all your work to receive full credit

1. **(7 points)** Indicate whether the following statements are true or false

(a) The Z transform is an extension of the continuous Fourier transform.	**T**	**F**
(b) Poles and Zeros are always real numbers	**T**	**F**
(c) Given X(s) = N(s)/D(s), "zero" is the value of s for which N(s) = 0	**T**	**F**
(d) The region of convergence is the region of s for which the integral for laplace transform converges	**T**	**F**
(e) ROC of a signal X(z) consists of a circular strip around origin in Z plane	**T**	**F**
(f) The ROC of x[n] is the same as the ROC of x[-n]	**T**	**F**
(g) The laplace transform of δ(t) is 1/s	**T**	**F**

2. **(8 points)** Fill in the blanks for the following statements

(a) Given X(z) = N(z)/D(z), the value of z when D(z) is zero is referred to as pole___.

(b) Computation of the inverse Z transform requires contour integration

(c) For a signal $x(t) = e^{-at}u(t)$, the condition Re{s} = -Re{a} is not allowed for ROC.

(d) For a left handed signal x(t), the ROC is to the left of the left most pole.

(e) The Laplace transform of δ(t-1) is e^(-s) X(s)__ and its ROC is R_______.

(f) For a finite duration x[n], the ROC is z pole______ except z = 0 or z = ∞.

(g) In laplace transform, the ROC consists of a strip parallel to jw_____ in 's' plane.

3. **(12 points) Laplace Transform –** Use 'Laplace transform properties' and 'Laplace transform pairs' tables from lecture slides to determine:

(a) The Laplace transform of: $x(t) = \frac{1}{5}|t|e^{-3t}u(-t)$. Also find its ROC. (Hint: Think of values of x(t) for t < 0 and t > 0)

(b) The Laplace transform of: $x(t) = e^{-4t}(\sin 2t)u(t) + e^{-6t}u(t)$. Also find its ROC. (Hint: The roots of a quadratic equation $ax^2 + bx + c$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$)

(c) The time function x(t) with the Laplace transform: $X(s) = \frac{s+4}{s^2 + 20s + 75}$, with Region of Convergence $-15 < Re\{s\} < -5$. (Hint: Use partial fractions)

4. **(8 points) Laplace Transform -** Consider a signal y(t) that is related to two signals $x_1(t)$ and $x_2(t)$ by:
$y(t) = x_1(3t-7) * x_2(-t+5)$
Where, $x_1(t) = -e^{-2t}u(-t)$ and $x_2(t) = e^{-4t}u(t)$. Use the properties of the Laplace transform to determine Y(s) (the Laplace transform of y(t)).

5**. (10 points) Z transform** – Using the 'Z-Transform pairs' and 'Z transform properties' tables from lecture slides, determine the system function h[n] and the ROC of H(z) for the casual LTI system (causality implies h[n] is right handed) with difference equation:

$$y[n] - \frac{9}{2}y[n-1] + 2y[n-2] = x[n]$$

Also, determine y[n] if the input to the system is: $x[n] = \left(\frac{1}{6}\right)^n u[n]$

(Hint: Use partial fractions)

6. **(5 points) Matlab -** Given the transfer function of a system,

$$H(s) = \frac{s^2 + 10s + 5}{s^3 + 4s^2 + 10s + 6}$$

Using MATLAB,
a) Plot the magnitude and phase response of the system.
b) Compute and plot the poles and zeros of the system.
b) Compute and plot the step response of the system.
d) Compute and plot the impulse response of the system.

(3) LAPLACE TRANSFORM

(a) $x(t) = \frac{1}{5}|t|e^{-3t}u(-t)$

$x_1(t) = e^{-3t}u(-t)$

$LT\{x_1(t)\} = -\frac{1}{s+3}$ $Re\{s\} < -3$

DIFFERENTIATION

$-t\,x(t) = \frac{d\,x(s)}{ds}$

$\frac{d}{ds}\left[-\frac{1}{s+3}\right]$

$= \frac{1}{(s+3)^2}$ ROC R

(b) $x(t) = e^{-4}\sin(2t)u(t) + e^{-6t}u(t)$

$x_1(t) = e^{-4}\sin(2t)u(t)$

$x_2(t) = e^{-6t}u(t)$

$LT\{x_1(t)\} = \frac{2}{(s+4)^2+4}$ $Re\{s\} > -4$

$LT\{x_2(t)\} = \frac{1}{s+6}$ $Re\{s\} > -6$

LINEARITY:

$\frac{2}{(s+4)^2+4} + \frac{1}{s+6}$

$\frac{2s+12}{s^3+14s^2+68s+120} + \frac{s^2+8s+20}{s^3+14s^2+68s+120}$

$\frac{s^2+20s+32}{s^3+14s^2+68s+120}$

USING QUADRATIC [CALCULATOR]:

ROOTS $-4 \pm 2i$

ROC: $-6 \cap -4$

(c) $X(s) = \frac{s+4}{s^2+20s+75}$

$\frac{s+4}{(s+5)(s+15)}$

$\frac{A}{s+5} + \frac{B}{s+15}$

$A(s+15) + B(s+5) = s+4$

$s = -15$: $-15B + 5B = -15+4$

$-10B = -11$ $B = \frac{11}{10}$

$s = -5$ $-5A + 15A = -5+4$

$10A = -1$

$A = -1/10$

$-\frac{1}{10(s+5)} + \frac{11}{10(s+15)}$

$-\frac{1}{10}e^{-5t}u(t) + \frac{11}{10}e^{-15t}u(t)$

(4) LAPLACE TRANSFORM

$y(t) = x_1(3t-7) * x_2(-t+5)$

$LT\{x_1(t)\} = LT\{-e^{-2t}u(t)\} = \frac{1}{s+2}$

$LT\{x_2(t)\} = LT\{e^{-4}u(t)\} = \frac{1}{s+4}$

$LT\{x_1(3t)\}$: TIME SCALE $\frac{1}{3}\left(\frac{1}{3s+12}\right) = \frac{1}{9s+36}$

$LT\{x_1(3t-7)\}$ TIME SHIFT $e^{-7s}\frac{1}{9s+36}$

$LT\{x_2(-t)\}$. TIME SCALE $-\frac{1}{s+4}$

$LT\{x_2(-t+5)\}$ TIME SHIFT $-e^{-5s}\frac{1}{s+4}$

$e^{-7s}\left(\frac{1}{9s+36}\right)\left(e^{-5s}\right)\left(\frac{1}{s+4}\right)$

$-\frac{e^{-12s}}{(9s+36)(s+4)}$

(5) Z - TRANSFORM

$y[n] - \frac{9}{2}y[n-1] + 2y[n-2] = x[n]$

APPLYING Z - TRANSFORM TO BOTH SIDES

$Y(z) - \frac{9}{2}z^{-1}Y(z) + 2z^{-2}Y(z) = X(z)$

$Y(z)\left(1 - \frac{9}{2}z^{-1} + 2z^{-2}\right) = X(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{9}{2}z^{-1}+2z^{-2}}$

$= \frac{1}{\frac{1}{2}-\frac{9}{2}z^{-1}+2z^{-2}+\frac{1}{2}}$

$= \frac{1}{\left(\frac{1}{\sqrt{2}}-\frac{\sqrt{5}}{\sqrt{2}}z^{-1}\right)^2-\left(\frac{1}{\sqrt{2}}\right)^2}$

$= \frac{1}{\left(\frac{1}{\sqrt{2}}-\frac{\sqrt{5}}{\sqrt{2}}z^{-1}\right)^2-\left(\frac{i}{\sqrt{2}}\right)^2}$

$= \frac{1}{\left(\frac{1}{\sqrt{5}}-\frac{\sqrt{5}z^{-1}}{\sqrt{2}}+\frac{1}{j\sqrt{5}}\right)\left(\frac{1}{\sqrt{2}}-\frac{\sqrt{5}}{\sqrt{5}}z^{-1}-\frac{1}{j\sqrt{5}}\right)}$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}} - \frac{\sqrt{5}}{\sqrt{2}} z^{-1} + j\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}} - \frac{\sqrt{5}}{\sqrt{2}} z^{-1} - j\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{\sqrt{2}}{(1 - \sqrt{5} z^{-1} + j)(1 - \sqrt{5} z^{-1} - j)}$$

POLES AT $1 - \sqrt{5} z^{-1} + j = 0$ $\qquad 1 - \sqrt{5} z^{-1} - j = 0$

OR $z = \frac{\sqrt{5}}{1+j}$ $\qquad z = \frac{\sqrt{5}}{1-j}$

Table of Contents


```
% BME 350 HW 6
% Mary Oh
% 1208315416

clc;
close all;
clear all;
```

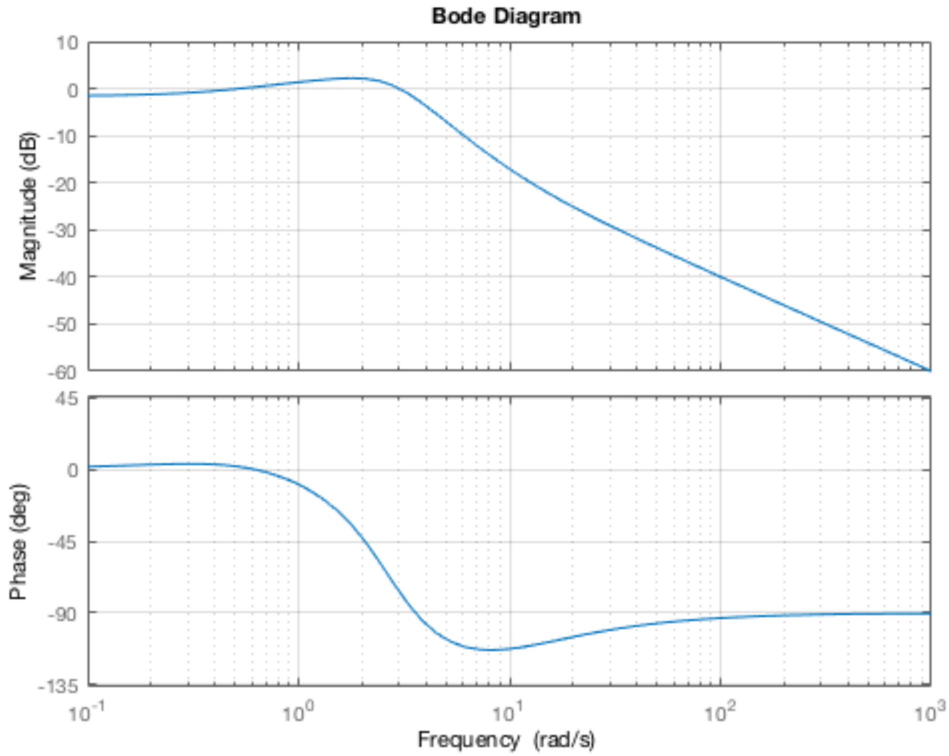
Q6

```
% Transfer Function: H(s) = (s^2+10s+5)/(s^3+4s^2+10s+6)

% Numerator
num = [1 10 5];

% Denominator
den = [1 4 10 6];

% Tranfer function
H = tf(num, den)
```

```
H =

      s^2 + 10 s + 5
  ----------------------
  s^3 + 4 s^2 + 10 s + 6

Continuous-time transfer function.
```

Q6a

```
% Plot frequency response
figure(1)
bode(H), grid
```

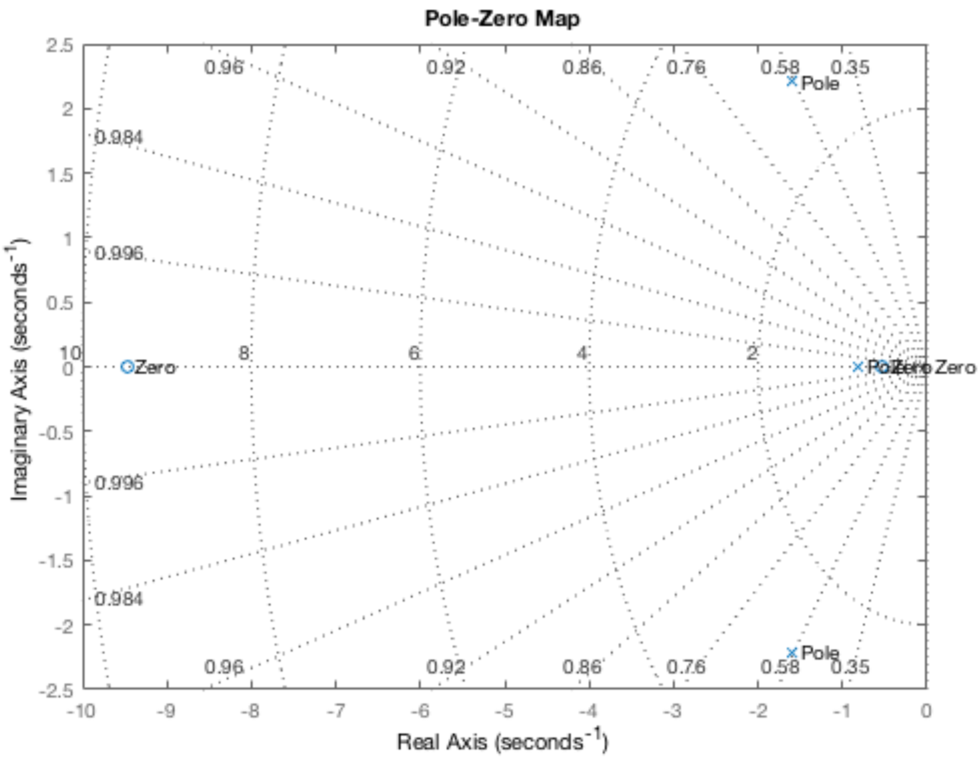


Q6b

```
% Compute and plot poles and zero
[num, den] = eqtflength(num, den);
[z,p,k] = tf2zp(num, den)

figure(2)
pzplot(H), grid
text(real(z)+.1,imag(z),'Zero')
text(real(p)+.1,imag(p),'Pole')
```

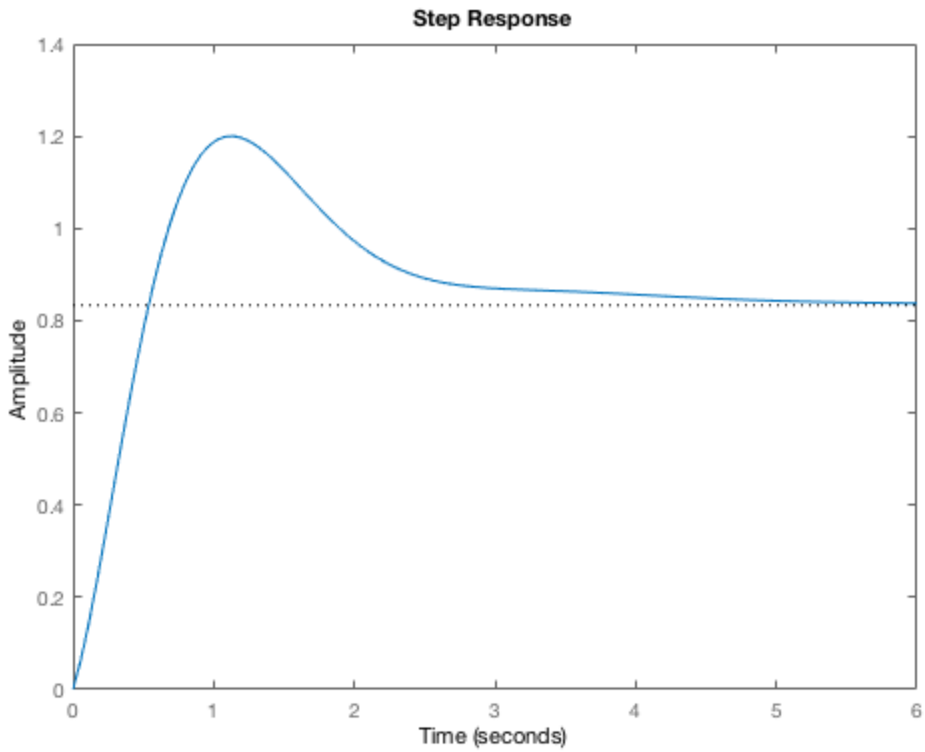
```
z =

         0
   -9.4721
   -0.5279


p =

  -1.5956 + 2.2075i
  -1.5956 - 2.2075i
  -0.8087 + 0.0000i
```

```
k =

    1
```



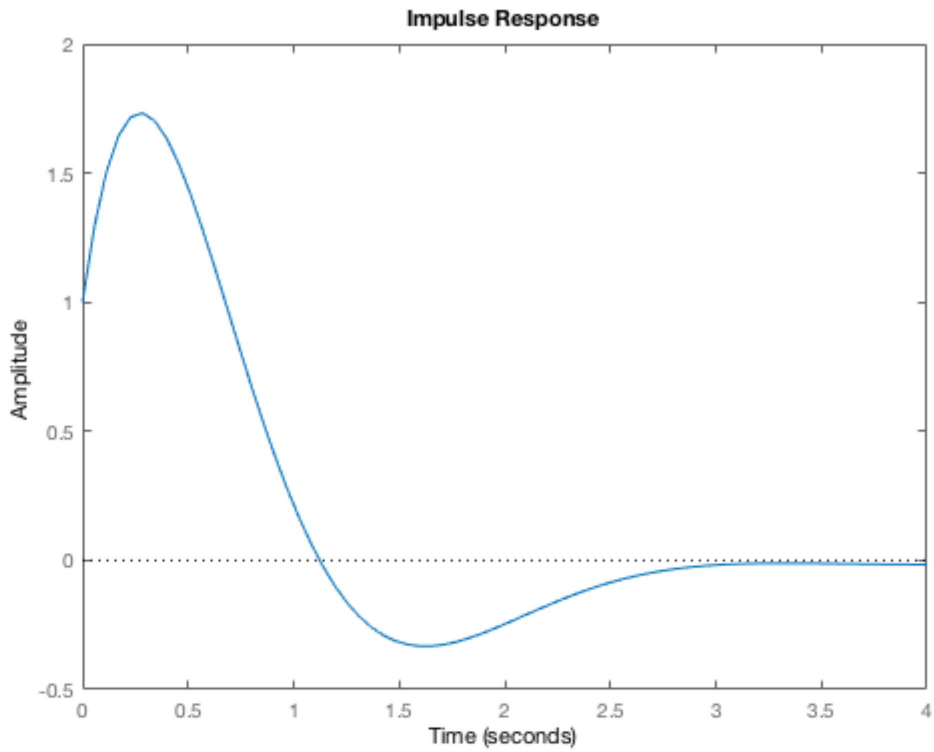
Q6c

```
% Compute and plot step response
figure(3)
step(H)
```



Q6d

```
% Compute and plot impulse response
figure(4)
impulse(H)
```



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