(3.) FOURIER TRANSFORM

$$x(t) = e^{(t+2)}\,[u(t+4) - u(t-5)]$$

(a)
$$X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$
$$= \int_{-\infty}^{\infty} e^{(t+2)}[u(t+4) - u(t-5)] \cdot e^{-jwt}\, dt$$
$$= \int_{-4}^{5} e^{t+2-jwt}\, dt$$
$$= e^{2} \int_{-4}^{5} e^{t-jwt}\, dt$$
$$= e^{2} \int_{-4}^{5} e^{(1-jw)t}\, dt$$
$$= \left. \frac{e^{(1-jw)t}}{1-jw} \right|_{-4}^{5}$$
$$= e^{2}\left[\frac{e^{(1-jw)5}}{1-jw} - \frac{e^{-4(1-jw)}}{1-jw} \right]$$
$$= e^{2}\left[\frac{e^{5-5jw} - e^{-4+4jw}}{1-jw} \right]$$
$$= \frac{e^{2} e^{5-5jw} - e^{2} e^{-4+4jw}}{1-jw}$$
$$= \frac{e^{7-5jw} - e^{-2+4jw}}{1-jw}$$

(b)
$$X(jw) = \begin{cases} \frac{w}{3} - 3 & -4 \le w \le 4 \\ 0 & |w| > 4 \end{cases}$$
$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(w)\, e^{jwt}\, dw$$
$$= \frac{1}{2\pi} \int_{-4}^{4} \left(\frac{w}{3} - 3\right) e^{jwt}\, dw$$
$$= \frac{1}{2\pi} \int_{-4}^{4} \left(\frac{w}{3} e^{jwt} - 3e^{jwt}\right) dw$$
$$= \frac{1}{6\pi} \int_{-4}^{4} w e^{jwt}\, dw - \frac{1}{2\pi} \int_{-4}^{4} 3e^{jwt}\, dw$$
$$= \frac{1}{6\pi} \frac{e^{jwt}}{jt}\left[w - \frac{1}{jt}\right]\Big|_{-4}^{4} - \frac{3}{2\pi}\left[\frac{e^{jwt}}{jt}\right]_{-4}^{4}$$
$$= \left\{ \frac{1}{6\pi} \frac{e^{j4t}}{jt}\left[4 - \frac{1}{jt}\right] - \frac{1}{6\pi} \frac{e^{-j4t}}{jt}\left[-4 - \frac{1}{jt}\right] \right\} - \left\{ \frac{3}{2\pi} \frac{e^{j4t}}{jt} + \frac{3}{2\pi} \frac{e^{-4jt}}{jt} \right\}$$
$$= \cancel{\frac{2}{3\pi jt} e^{j4t}} - \cancel{\frac{1}{6\pi j^2 t^2} e^{j4t}} + \cancel{\frac{2}{3\pi jt} e^{-4jt}}$$
$$+ \cancel{\frac{1}{6\pi j^2 t^2} e^{-4jt}} - \frac{3}{2\pi jt} e^{j4t} - \frac{3}{2\pi jt} e^{-4jt}$$
$$= \frac{4}{3\pi jt} \cos(4t) - \frac{j}{3\pi t^2} \sin(4t) -$$
$$\frac{3}{2\pi jt} \cos(4t)$$

(4) FOURIER TRANSFORM PROPERTY

$$X(jw) = \frac{4 \sin(5w)}{9w}$$

(a) $x\left(-\frac{2}{3}t + 4\right)$

→ TIME & FREQ. SCALING PROP:
$$x(at) \rightarrow \frac{1}{|a|} X\left(\frac{jw}{a}\right)$$

→ TIME SHIFTING PROP:
$$x(t - t_0) \rightarrow e^{-jwt_0} X(jw)$$

$t_0 = -4$ $\quad a = -\frac{2}{3}$

$$= e^{-j6w} \cdot \left(-\frac{3}{2}\right) \cdot X\left(-\frac{3jw}{2}\right)$$
$$= -\frac{3}{2} e^{-j6w} \cdot \frac{4}{9} \frac{\sin\left[5\left(\frac{-3w}{2}\right)\right]}{-\frac{3w}{2}}$$
$$= \frac{4}{9w} e^{-j6w} \sin\left[-\frac{15w}{2}\right]$$

USING EULER'S FORMULA

$$F\left\{x\left(-\frac{2}{3}t + 4\right)\right\} = \frac{4}{9w} e^{-j6w} \left(\frac{e^{-\frac{15w}{2}} - e^{\frac{15w}{2}}}{2}\right)$$
$$= \frac{2}{9w}\left[e^{-\frac{27w}{2}j} - e^{\frac{3w}{2}j}\right]$$

(b) $\frac{7}{2} \frac{dx(t)}{dt}$

→ DIFF. IN TIME PROP.
$$\frac{dx(t)}{dt} \rightarrow jw\, X(jw)$$
$$= \frac{7}{2} jw\, F\{x(t)\}$$
$$= \frac{7}{2} jw \frac{4 \sin(5w)}{9w}$$
$$= \frac{14}{9} j \left(\frac{e^{5jw} - e^{-5jw}}{2}\right)$$
$$= \frac{7j}{9}\left(e^{5jw} - e^{-5jw}\right)$$

(c) $e^{-j8t} x(t)$

→ FREQ. SHIFTING
$$e^{jw_0 t} x(t) \rightarrow X[j(w - w_0)]$$

$w_0 = -8$

$$= \frac{4 \sin[5(w+8)]}{9(w+8)}$$
$$= \frac{4 \sin(5w + 40)}{9w + 72}$$