

(3) LAPLACE TRANSFORM

$$(a) \quad x(t) = \frac{1}{5} |t| e^{-3t} u(-t)$$

$$x_1(t) = e^{-3t} u(-t)$$

$$LT \{x_1(t)\} = -\frac{1}{s+3} \quad \text{Re}\{s\} < -3$$

DIFFERENTIATION

$$-t x(t) = \frac{dX(s)}{ds}$$

$$\frac{d}{ds} \left[-\frac{1}{s+3} \right]$$

$$= \frac{1}{(s+3)^2} \quad \text{ROC: } \text{Re}\{s\} < -3$$

$$(b) \quad x(t) = e^{-4t} \sin(2t) u(t) + e^{-6t} u(t)$$

$$x_1(t) = e^{-4t} \sin(2t) u(t)$$

$$x_2(t) = e^{-6t} u(t)$$

$$LT \{x_1(t)\} = \frac{2}{(s+4)^2 + 4} \quad \text{Re}\{s\} > -4$$

$$LT \{x_2(t)\} = \frac{1}{s+6} \quad \text{Re}\{s\} > -6$$

LINEARITY:

$$\frac{2}{(s+4)^2 + 4} + \frac{1}{s+6}$$

$$\frac{2s+12}{s^2+14s^2+68s+120} + \frac{s^2+8s+20}{s^2+14s^2+68s+120}$$

$$\frac{s^2+20s+32}{s^2+14s^2+68s+120}$$

USING QUADRATIC [CALCULATOR]:

$$\text{ROOTS: } -4 \pm 2i$$

$$\text{ROC: } -6 < \sigma < -4$$

$$(c) \quad X(s) = \frac{s+4}{s^2+20s+75}$$

$$\frac{s+4}{(s+5)(s+15)}$$

$$\frac{A}{s+5} + \frac{B}{s+15}$$

$$A(s+15) + B(s+5) = s+4$$

$$s = -15: -15B + 5B = -15 + 4$$

$$-10B = -11$$

$$B = \frac{11}{10}$$

$$s = -5: -5A + 15A = -5 + 4$$

$$10A = -1$$

$$A = -\frac{1}{10}$$

$$-\frac{1}{10(s+5)} + \frac{11}{10(s+15)}$$

$$-\frac{1}{10} e^{-5t} u(t) + \frac{11}{10} e^{-15t} u(t)$$

(4) LAPLACE TRANSFORM

$$y(t) = x_1(3t-7) * x_2(-t+5)$$

$$LT \{x_1(t)\} = LT \{e^{-3t} u(t)\} = \frac{1}{s+3}$$

$$LT \{x_2(t)\} = LT \{e^{-4t} u(t)\} = \frac{1}{s+4}$$

$$LT \{x_1(3t)\} : \text{TIME SCALE } \frac{1}{3} \left(\frac{1}{3s+12} \right) = \frac{1}{9s+36}$$

$$LT \{x_1(3t-7)\} : \text{TIME SHIFT } e^{-7s} \frac{1}{9s+36}$$

$$LT \{x_2(-t)\} : \text{TIME SCALE } -\frac{1}{s+4}$$

$$LT \{x_2(-t+5)\} : \text{TIME SHIFT } -e^{-5s} \frac{1}{s+4}$$

$$e^{-7s} \left(\frac{1}{9s+36} \right) \left(e^{-5s} \right) \left(\frac{1}{s+4} \right)$$

$$= \frac{e^{-12s}}{(9s+36)(s+4)}$$

(5) Z-TRANSFORM

$$y[n] = \frac{9}{2} y[n-1] + 2y[n-2] = x[n]$$

APPLYING Z-TRANSFORM TO BOTH SIDES

$$Y(z) = \frac{9}{2} z^{-1} Y(z) + 2z^{-2} Y(z) = X(z)$$

$$Y(z) \left(1 - \frac{9}{2} z^{-1} + 2z^{-2} \right) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{9}{2} z^{-1} + 2z^{-2}}$$

$$= \frac{1}{1 - \frac{9}{2} z^{-1} + 2z^{-2} + \frac{1}{2}}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{2} z^{-1} \right)^2 + \left(\frac{1}{\sqrt{2}} \right)^2}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{2} z^{-1} \right)^2 + \left(\frac{1}{\sqrt{2}} \right)^2}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{2} z^{-1} + \frac{1}{\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{2} z^{-1} - \frac{1}{\sqrt{2}} \right)}$$