(3) FOURIER TRANSFORM

$$X(\Omega) = \frac{e^{j\Omega}}{3 + e^{j\Omega}} = \frac{1}{1 + 3e^{-j\Omega}}$$

(a) FROM TABLE 6.2 ~~WEIGHTED~~ ~~EXPONENTIAL~~

EXPONENTIAL $a^n u[n]$ $\quad \frac{e^{j\Omega}}{e^{j\Omega}+3}\left(\frac{e^{-j\Omega}}{e^{-j\Omega}}\right)$

$x[n] = (3)^n u[n]$

(b) (i) $y[n] = x[n+3]$

TIME SHIFTING

$x[n-k] = e^{-j\Omega k} x(\Omega)$

$e^{+j\Omega(3)} x(\Omega)$

(ii) $y[n] = 2x[n]e^{j6n}$

$2x(\Omega - 6) = 2\left[\frac{e^{j\Omega - 6}}{3 + e^{j\Omega}}\right]$

(iii) $y[n] = x[n] * x[n]$

$x(\Omega)x(\Omega) = \left(\frac{e^{j\Omega}}{3 + e^{j\Omega}}\right)^2$

$x(j\omega) \quad |\omega| > \omega_0$

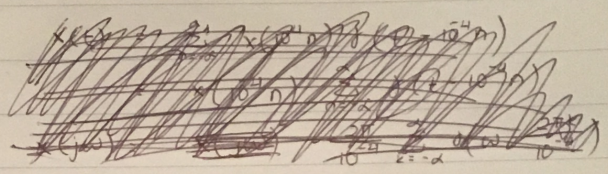
$x_1(j\omega) \quad |\omega| > \frac{\omega_0}{2}$

NYQUIST RATE $= 2\omega_0$

(b) $x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$

$T_s = 10^{-4}\,s$

$x(j\omega) * x(j\omega) = 0 \qquad |\omega| > 5000\pi$



$\omega_s > 2\omega_m$

$\frac{2\pi}{T_s} > 2\omega_m$

$T_s < \frac{2\pi}{2\omega_m} \approx \frac{\pi}{2\omega_m}$

$x(t)$ CAN BE RECOVERED IF $T_s < \frac{\pi}{\omega_m}$

$= \frac{\pi}{5000\pi} = 1.25 \times 10^{-4}\,s$

SINCE, $T_s < \frac{\pi}{5000\pi}$, ~~(a)~~ & IS TRUE, ~~(b) IS FALSE~~

THEN $x(t)$ CAN BE RECOVERED FROM $x_p(t)$.

(5)(a) SAMPLING

NYQUIST RATE $= \frac{\omega_0}{2}$

(i) $5x(t)\sin\omega_0 t + 6x^2(t)$

↳ FOURIER TRANSFORM

MODULATION: $5x(t)\sin\omega_0 t$

$\frac{j}{2}[5x(\omega+\omega_0) - 5x(\omega-\omega_0)]$

~~LINEARITY~~ LINEARITY: $6x^2(t)$

$6X^2(\omega)$

$= \underbrace{\frac{j}{2}[5x(\omega+\omega_0) - 5x(\omega-\omega_0)]}_{x(j\omega)} + \underbrace{6[x(j\omega) * x(j\omega)]}_{x_1(j\omega)}$

$x_1(j\omega) = 0$ FOR $|\omega| > \frac{\omega_0}{2}$

$x(j\omega) = 0$ FOR $|\omega| > \left(\frac{\omega_0}{2} + \omega_0\right) > \frac{3\omega_0}{2}$

NYQUIST RATE $= 2\left(\frac{3\omega_0}{2}\right) = 3\omega_0$

(ii) $\frac{3}{2}\,dx(t)/dt + \frac{1}{4}x(t)\cos\omega_0 t$

↳ FOURIER TRANSFORM

TIME DIFF: $\frac{3}{2}\frac{dx(t)}{dt}$

$j\omega\frac{3}{2}x(\omega)$

MODULATION: $\frac{1}{4}x(t)\cos\omega_0 t$

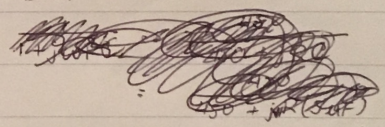
$\frac{1}{2}\left[\frac{1}{4}x(\omega+\omega_0) + \frac{1}{4}x(\omega-\omega_0)\right]$

$= \underbrace{j\omega\frac{3}{2}x(\omega)}_{x(j\omega)} + \underbrace{\frac{1}{2}\left[\frac{1}{4}x(\omega+\omega_0) + \frac{1}{4}x(\omega-\omega_0)\right]}_{x_1(j\omega)}$

(4) FILTERING

(a) $H(j\omega) = \frac{450}{450 + j\omega}$

$C = 5\,\mu F$



$\frac{1}{450} = RC = R(5\times10^{-6}\,F)$

$R = 444.4\,\Omega$

$f = \frac{1}{2\pi RC} = \frac{1}{2\pi(444.4\,\Omega)(5\times10^{-6}F)}$

$f = 71.6\,Hz$

(b) $C = 5\,\mu F \qquad R = 48\,k\Omega$

$H(j\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega(5\times10^{-6})(48\times10^{6})}{1 + j\omega(5\times10^{-6})(48\times10^{6})}$

$H(j\omega) = \frac{0.240\,j\omega}{1 + 0.240\,j\omega}$

$f = \frac{1}{2\pi RC} = \frac{1}{2\pi(0.240)} = 0.663\,Hz$

(c) $H_{BP} = H_{LP}(j\omega) \cdot H_{HP}(j\omega)$

$= \frac{1}{1 + \frac{1}{450}j\omega}\left[\frac{0.240\,j\omega}{1 + 0.240\,j\omega}\right]$

$H_{BP} = \frac{0.240\,j\omega}{(1 + \frac{1}{450}j\omega)(1 + 0.240\,j\omega)}$

(d) $C = 5\,\mu F \qquad f = 250\,Hz$

$f = \frac{1}{2\pi RC}$

$R = \frac{1}{2\pi fC} = \frac{1}{2\pi(250)(5\times10^{-6})} = 127.3\,\Omega$

$H_{HP}(j\omega) = \frac{j\omega(127.3)(5\times10^{-6})}{1 + j\omega(127.3)(5\times10^{-6})} = \frac{6.37\times10^{-4}\,j\omega}{1 + 6.37\times10^{-4}\,j\omega}$