BME 350: Signals & Systems for Bioengineers

HW 3 (50 points)

Deadline: **Oct 4th (Tuesday) at 9AM**
Late assignments: **5 points deducted/hour** **(delay rounded up to the nearest hour after deadline).**
Note: **For questions 3-6, show all your work to receive full credit**

1. (7 points) Indicate whether the following statements are true or false:

(a) Systems cannot have feedback. T F

(b) Convolution is both commutative and associative. T F

(c) A LTI system has memory if h(t) = 0 for t $\neq$ 0. T F

(d) All causal systems are also memoryless. T F

(e) A discrete LTI system with impulse response 'h' is invertible if an inverse system with impulse response h1 exists such that h[n]*h1[n] = δ[n]. T F

(f) The Fourier coefficients a_k can be calculated using the analysis equation. T F

(g) Given x(t) with fourier series coefficients ak, αx(t), for α > 0 will result in same Fourier series coefficients ak. T F

2. (8 points) Fill in the blanks for the following statements:

(a) A LTI system is stable only if h(t) is absolutely integrable.

(b) Linearity and time invariance properties allow simplification of signal output prediction and analysis by use of the impulse response.

(c) In block diagrams, the outputs of systems connected in parallel are added.

(d) If a_k are the fourier series coefficients of x[n], the fourier series coefficients of x[-n] are -ak.

(e) The Fourier series for x(t) is periodic if it has finite number of maxima and minima in a finite time interval.

(f) If x(t) is a real and odd signal, then the Fourier series coefficients are a(-k) and -ak.

3. Block diagrams (7 points). Draw a block diagram representation of the following casual LTI systems:

a. $$\frac{6}{5}y(t)+\frac{dy(t)}{dt}=4x(t)+\frac{4}{5}\frac{dx(t)}{dt}+2\frac{d^2x(t)}{dt^2}$$

b. $$5y[n]-y[n-3]+\frac{3}{2}y[n-4]=x[n-2]+\frac{2}{7}x[n-5]+6x[n]$$

4. Continuous-time Fourier series (12 points). Given a periodic wave x(t) with period, T = 8 defined as:

$$x(t) = \begin{cases} 3t & 0 \le t \le 4 \\ 24 - 3t & 4 \le t \le 8 \end{cases}$$

Draw the signal over two periods, and calculate the a_k coefficients for the signal using the analysis equation (Eqs. 3.39 and 3.40, page 191). Show all your procedure and clearly state your final answer for a_0 and a_k (*Hint: Use the following formula:* $\int te^{at}dt = \frac{e^{at}}{a}[t - \frac{1}{a}]$ *,which is obtained using integration by parts; 'a' can be positive or negative)*.

5. Fourier series properties (11 points). x(t) is a periodic signal with fundamental period T = 4 and Fourier coefficients a_0 and a_k as depicted below :

$$a_0 = \frac{3}{4} \quad ; \quad a_k = \frac{2\cos(\frac{4\pi k}{3})}{9k\pi}$$

Using Fourier series properties (Table 3.1 from the textbook), calculate the new a_k coefficients for the following cases, for k = -4, -3, 0, 4. *(Note: Do not use the analysis equation)*.

a) $-2x(-t)$

b) $4\frac{dx(t)}{dt}$

c) $x(\frac{3}{5}t)$

d) $x(2t+5)$

e) $e^{-j3\pi t}x(t)$

6. MATLAB (5 points). Consider the signal x(t) with period, T = 4 and fourier series coefficients given as:

$a_0 = \frac{3}{4}$ and $a_k = \frac{2\cos(\frac{4\pi k}{3})}{9k\pi}$ for -3-ASUID ≤ k ≤ ASUID+3 (if your ASUID is between 0 and 4) **<u>or</u>** 2-ASUID ≤ k ≤ ASUID-2 (if your ASUID is between 5 and 9). Rest of the coefficients are zero. Construct the fourier series expansion of x(t) using the synthesis equation. Plot the real and imaginary parts of the signal x(t). Include the 10 digit ASU ID in the title of your plot. (ASUID is the last digit of your 10 digit ASU ID).

Ungraded additional practice problem (not extra credit):

Continuous-time convolution: Given the signals x(t) and h(t) and below, calculate the output signal,

y(t)= x(t) * h(t), where "*" denotes the convolution. Show all your procedure.

$$x(t) = \begin{cases} \dfrac{-3t-2}{2} & -2 \le t \le 2 \\ 0 & \text{everywhere else} \end{cases}$$

$$h(t) = u(t)e^{-5t/2}$$

(3) BLOCK DIAGRAM

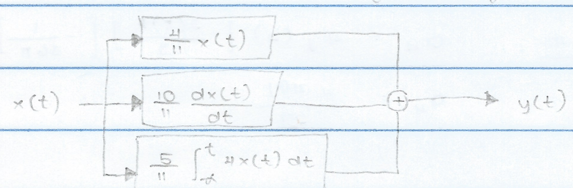
(a) $\frac{6}{5} y(t) + \frac{dy(t)}{dt} = 4x(t) + \frac{4}{5}\frac{dx(t)}{dt} + \frac{2d^2x(t)}{dt^2}$

$\frac{6}{5} y(t) + \int \frac{dy(t)}{dt} = \int \left[4x(t) + \frac{4}{5}\frac{dx(t)}{dt} + \frac{2d^2x(t)}{dt^2} \right]$

$\frac{6}{5} y(t) + y(t) = \int [4x(t)\,dt] + \frac{4}{5}x(t) + \frac{2dx(t)}{dt}$

$\frac{11}{5} y(t) = \frac{4}{5}x(t) + \frac{2dx(t)}{dt} + \int_{-\infty}^{t} 4x(t)\,dt$

$y(t) = \frac{4}{11}x(t) + \frac{10}{11}\frac{dx(t)}{dt} + \frac{5}{11}\int_{-\infty}^{t} 4x(t)\,dt$

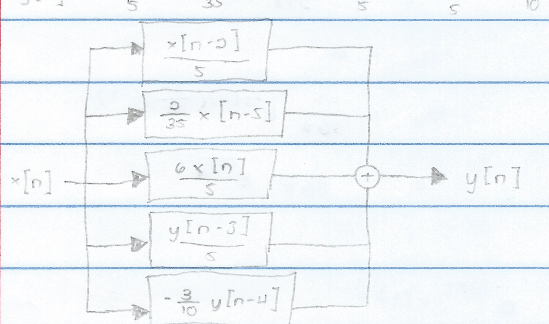


(b) $5y[n] - y[n-3] + \frac{3}{2}y[n-4] = x[n-2] + \frac{2}{7}x[n-5]$

$+ 6x[n]$

$5y[n] = x[n-2] + \frac{2}{7}x[n-5] + 6x[n] + y[n-3] - \frac{3}{2}y[n-4]$

$y[n] = \frac{x[n-2]}{5} + \frac{2}{35}x[n-5] + \frac{6x[n]}{5} + \frac{y[n-3]}{5} - \frac{3}{10}y[n-4]$

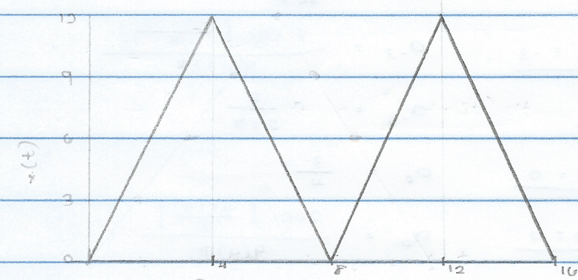


$$\boxed{= \frac{3 - 123e^{-\pi jk} + 180e^{-2\pi jk}}{32\left(-jk\,\pi/4\right)^2}}$$

(4) CONTINUOUS - TIME FOURIER SERIES

$$x(t) = \begin{cases} 3t & 0 \le t \le 4 \\ 24 - 3t & 4 \le t \le 8 \end{cases}$$

$T = 8$ [PERIOD]



$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$

$= \frac{1}{8}\left[\int_0^4 3t\,dt + \int_4^8 (24-3t)\,dt\right)$

$= \frac{1}{8}\left[\left(\frac{3t^2}{2}\right)_0^4 + \left(24t - \frac{3t^2}{2}\right)_4^8\right]$

$= \frac{1}{8}\left[\frac{3(4)^2}{2} + \left[24(8) - \frac{3(8)^2}{2}\right] - \left[24(4) - \frac{3(4)^2}{2}\right]\right]$

$= \frac{1}{8}\left[24 + \cancel{192} - 96 - \cancel{96} + 24\right]$

$\boxed{a_0 = 6}$

$T_0 = 8 \qquad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{8} = \pi/4$

LET: $a = -jk(\pi/4)$

$a_k = \frac{1}{T_0}\left[\int_0^4 3t\,e^{at}\,dt + \int_4^8 [24 + 3t]\,e^{at}\,dt\right]$

$= \frac{1}{8}\left[\frac{3e^{at}}{a}\left(t - \frac{1}{a}\right)\Big|_0^4 + \left[\frac{24e^{at}}{a} + \right.\right.$

$\left.\left.\frac{3e^{at}}{a}\left(t - \frac{1}{a}\right)\Big|_4^8\right]\right]$

$= \frac{1}{8}\left[\left[\frac{3e^{a(4)}}{4}\left(4 - \frac{1}{a}\right)\right] - \left[\frac{3e^{0a}}{4}\left(\cancel{0} - \frac{1}{a}\right)\right] + \right.$

$\left.\left[\frac{24e^{8a}}{a} + \frac{3e^{8a}}{a}\left(8 - \frac{1}{a}\right)\right] - \left[\frac{24e^{4a}}{a} + \frac{3e^{4a}}{a}\left(4 - \frac{1}{a}\right)\right]\right]$

$= \frac{1}{8}\left[\left(\cancel{3e^{4a}} - \frac{3e^{4a}}{\cancel{4a}}\right) + \frac{3}{4a} + \frac{24e^{8a}}{a} + \frac{24e^{8a}}{a}\right.$

$\left. - \frac{3e^{8a}}{a^2} - \frac{24e^{4a}}{\cancel{a}} - \frac{12e^{4a}}{\cancel{a}} + \frac{3e^{4a}}{a^2}\right]$

$= \frac{1}{8}\left[3e^{4a} + \frac{3(1-e^{4a})}{4a} + \frac{48e^{8a} - 36e^{4a}}{a}\right.$

$\left. + \frac{3(e^{4a} - e^{8a})}{a^2}\right]$

$= \frac{3e^{4a}}{8} + \frac{3(1-e^{4a})}{32a} + \frac{48e^{8a} - 36e^{4(a)}}{8a} + \frac{3(e^{4a} - e^{8a})}{8a^2}$

$= \frac{12e^{4a} + 3(1 - e^{4a}) + 48(4e^{8a} - 3e^{4a}) + 12(e^{4a} - e^{8a})}{32a^2}$

$= \frac{3 - 123e^{4a} + 180e^{8a}}{32a^2}$

FOURIER SERIES PROP

$a_0 = \frac{3}{4} \qquad a_k = \frac{2\cos\left(\frac{4\pi k}{3}\right)}{9k\pi}$

$: \quad a_{-4} = \frac{2\cos\left[\frac{4\pi(4)}{3}\right]}{9(-4)\pi}$

$a_{-4} = \frac{1}{36\pi}$

$: \quad a_{-3} = \frac{2\cos\left[\frac{4\pi(-3)}{3}\right]}{9(-3)\pi}$

$a_{-3} = -\frac{2}{27\pi}$

$a_0 = \frac{3}{4}$

$a_4 = \frac{2\cos\left[\frac{4\pi(4)}{3}\right]}{9(4)\pi}$

$a_4 = -\frac{1}{36\pi}$

$-2x(-t)$

$_k = -a_{-k}$

$-a_4 = \frac{2}{36\pi}$

$-a_3 = -\frac{4}{27\pi}$

$-a_0 = -\frac{3}{2}$

$-a_{-4} = -\frac{2}{36\pi}$

$4\frac{dx(t)}{dt}$

$_k = 4jk\left(\frac{2\pi}{T_0}\right)$

$a_{-4} = 4j(-4)\left(\frac{2\pi}{4}\right)\left(\frac{1}{36\pi}\right)$

$a_{-4} = -\frac{2j}{9}$

$a_{-3} = 4j(-3)\left(\frac{2\pi}{4}\right)\left(-\frac{2}{27\pi}\right)$

$a_{-3} = \frac{4j}{9}$

$a_0 = 4j(0)\left(\frac{2\pi}{4}\right)\left(\frac{3}{4}\right)$

$a_0 = 0$

$a_4 = 4j(4)\left(\frac{2\pi}{4}\right)\left(-\frac{1}{36\pi}\right)$

$a_4 = \frac{2j}{9}$

(c) $x\left(\frac{3}{5}t\right)$

$\underline{k=-4:}$ $a_{-4} = 4j(-4)\left[\frac{2(5)\pi}{3}\right]\left[\frac{1}{36\pi}\right]$

$a_{-4} = \frac{-40j}{27}$

$\underline{k=-3:}$ $a_{-3} = 4j(-3)\left[\frac{2(5)\pi}{3}\right]\left[-\frac{2}{27\pi}\right]$

$a_{-3} = \frac{-80j}{27}$

$\underline{k=0}$ $a_0 = 0$

$\underline{k=4:}$ $a_4 = 4j(4)\left[\frac{2(5)\pi}{3}\right]\left[-\frac{1}{36\pi}\right]$

$a_4 = \frac{-40j}{27}$

(d) $x(2t+5)$

$\underline{k=-4:}$ $a_{-4} = \frac{1}{36\pi} e^{-j(-4)\left(\frac{2\pi}{2}\right)(-5)}$

$a_{-4} = \frac{1}{36\pi} e^{-20\pi j}$

$\underline{k=-3:}$ $a_{-3} = -\frac{2}{27\pi} e^{-j(-3)\left(\frac{2\pi}{2}\right)(-5)}$

$a_{-3} = -\frac{2}{27\pi} e^{-15\pi j}$

$\underline{k=0}$ $a_0 = 0$

$\underline{k=4}$ $a_4 = \frac{-1}{36\pi} e^{-j(4)\left(\frac{2\pi}{2}\right)(-5)}$

$a_4 = -\frac{1}{36\pi} e^{20\pi j}$

(e) $e^{-j3\pi t} x(t)$

$e^{-j3\pi t} x(t) = e^{jm\left(\frac{2\pi}{T_0}\right)t} x(t)$

$jm\left(\frac{2\pi}{T_0}\right)t = -j3\pi t \qquad M = -4$

$\underline{k=-4:}$ $a_{k-m} = a_{-4+4} = a_0 = \frac{3}{4}$

$\underline{k=-3}$ $a_{k-m} = a_{-3+4} = a_1$

$a_1 = \frac{2\cos\left(\frac{4\pi}{3}\right)}{9\pi}$

$a_1 = -\frac{1}{9\pi}$

$\underline{k=0:}$ $a_{k-m} = a_{0+4} = a_4 = -\frac{1}{36\pi}$

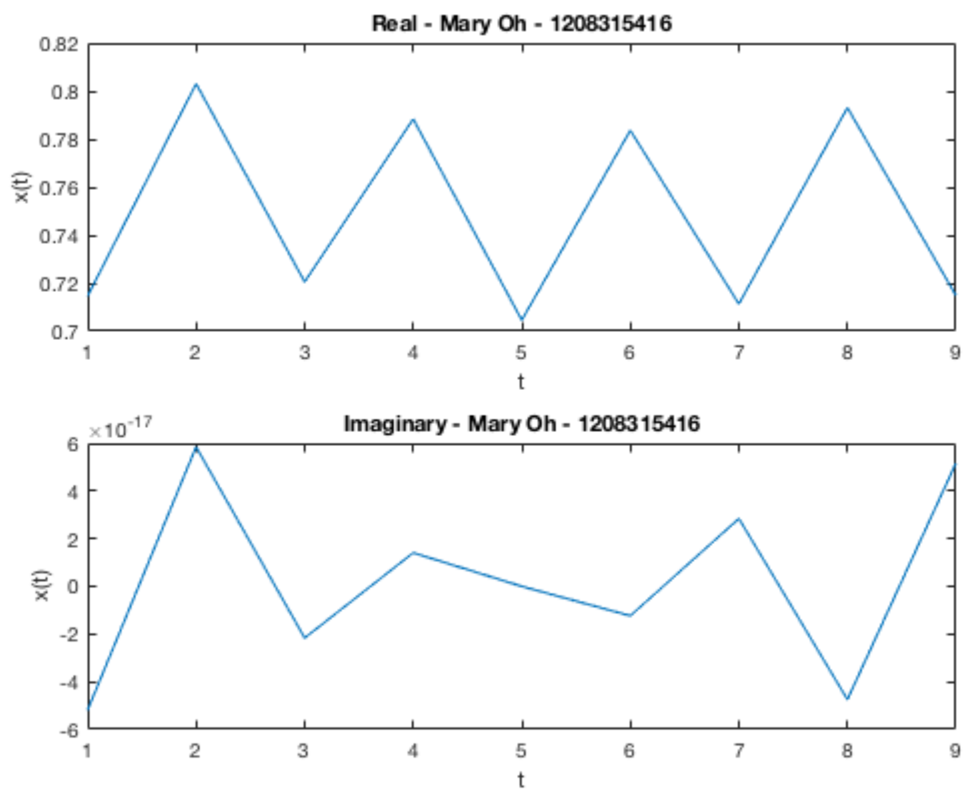
$\underline{k=4:}$ $a_{k-m} = a_{4+4} = a_8$

$a_8 = \frac{2\cos\left[\frac{4\pi(8)}{3}\right]}{9(8)\pi}$

$a_8 = -\frac{1}{72\pi}$

```
% BME 350 HW 3
% Mary Oh
% 1208315416
% Fourier series expansion of x(t)
% Using synthesis equation

clc;
clear all;

asuid = 6;

T = 4; %period
wo = (2*pi)/T; %frequency

ao = 3/4;
t = asuid:0.01:asuid; %define time vector
x_rec = zeros(1,length(t)); %initialize signal to be synthesized

% compute ak coefficients
kvals = 2-asuid:asuid-2;
for k = 1:numel(kvals)
    if k == 0;
        ak(k) = ao;
    else
        ak(k) = (2*cos((4*pi*k)/3))/(9*pi*k);
    end;

    x(k) = sum(ak*exp(1i*kvals(k)*wo*t)); % Taking only cosine terms
end;

x_rec = ao + x; % add a0

figure(1);

subplot(2,1,1);
real(x_rec)
plot(real(x_rec))
xlabel('t')
ylabel('x(t)')
title('Real - Mary Oh - 1208315416')

subplot(2,1,2)
imag(x_rec)
plot(imag(x_rec))
xlabel('t')
ylabel('x(t)')
title('Imaginary - Mary Oh - 1208315416')


ans =

  Columns 1 through 7
```

```
    0.7146    0.8031    0.7205    0.7883    0.7046    0.7836    0.7113

  Columns 8 through 9

    0.7931    0.7148


ans =

   1.0e-16 *

  Columns 1 through 7

   -0.5198    0.5847   -0.2166    0.1408         0   -0.1234    0.2840

  Columns 8 through 9

   -0.4747    0.5175
```



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