**NAME:** **ASU ID:**

# BME 350: Signals and Systems

# Final Exam

**05/07/2015 2:30-3:45 pm (Total 60 Points, 75 min)**

## Instructions:

1) **This exam is Closed Book, Closed Notes.**
2) **Use of any electronic device is prohibited.**
3) **Read each question carefully and answer all sub parts.**
4) **<u>Solve any 4 questions out of Questions 3-8. If you solve 5, top 4 scores will count towards the exam score, 5th will contribute towards 1% extra credit on final grade.</u> If you solve 6, top 4 scores will count towards the exam score, 5th highest score will contribute towards 1% extra credit on final grade and the $6^{th}$ (if correct) will get you invited to be a BME 350 UGTA for fall 2015.**
5) **Show all relevant steps for Questions 3-8. You will lose points if you cannot show how you arrived at the answer**
6) **Write clearly. If we cannot read your work, we cannot assign credit for it.**
7) **Write your name and ASU ID above.**
8) **Tables and formulae are provided at the end.**

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**Question 1 (10 points total, 1 point each)** Select if the following statements are True or False. Circle your answer.

1. Brain computer interfaces (BCIs) can be employed to restore communication and motor function to patients suffering from neurological disorders. **T F**
2. Analog filtering is commonly used as an anti-aliasing filter, prior to sampling. **T F**
3. For a finite duration x[n], the ROC is the entire z-plane, including z = 0 and infinity. **T F**
4. If $\omega_s$ is the sampling frequency and $\omega_M$ is the Nyquist frequency, aliasing will occur only when $\omega_s = 2\omega_M$. **T F**
5. BIBO stable causal systems will have all poles inside the unit circle. **T F**
6. The Z transform is an extension of the continuous-time Fourier transform. **T F**
7. Ideal frequency domain filters can be either causal or non-causal. **T F**
8. The Laplace transform is equivalent to the Fourier transform for points in the s plane along the imaginary axis. **T F**
9. In MRI, the k-space is the 2D Fourier transform of the MR image. **T F**
10. The Bode plot helps one to visualize the magnitude and phase of the frequency response of a system. **T F**

**Question 2. (10 points total, 2 points each)** Answer the following multiple choice questions by circling the best answer from the available choices:

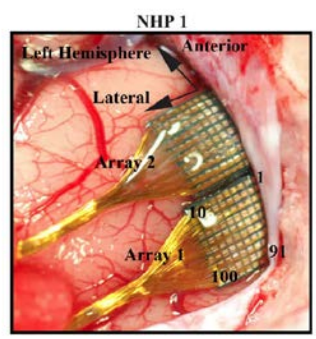
1. The given picture represents

a. MRI data acquisition

b. Microelectrodes for microstimulation/recording

c. ECG leads for measuring heart rate

d. A pacemaker




2. Given a right handed signal x(t), the ROC is

a. To the right of the left-most pole

b. To the right of the right-most zero

c. To the right of the right-most pole

d. To the left of the right-most pole

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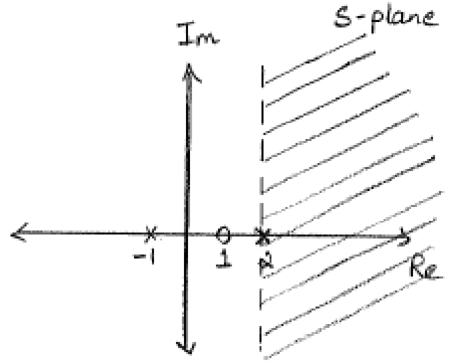
3. The ROC of the given pole-zero plot could represent



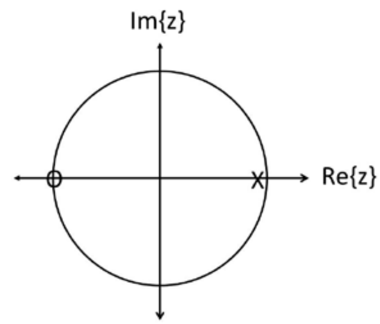

a. A causal, unstable system

b. A causal, stable system

c. A non causal, stable system

d. An anticausal, unstable system

4. The given pole-zero plot (the circle represents a unit circle) corresponds to a transfer function H(z) of a




a. High pass filter

b. Low pass filter

c. Band pass filter

d. Notch filter

5. For a causal system specified by $H(z) = \frac{1}{(1-\frac{1}{2}z^{-1})} + \frac{1}{(1-2z^{-1})}$, the ROC is

a. $|z| < \frac{1}{2}$

b. $|z| > 2$

c. $|z| < \frac{-1}{2}$

d. Either a or b

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**Note: Solve any 4 out of Questions 3-8. If you solve 5, top 4 scores will count towards the exam score, 5th will contribute towards 1% extra credit on final grade.**

**Question 3. (10 points)** Answer the following questions. Clearly show the procedure and highlight your final answer(s).

Consider the signal,

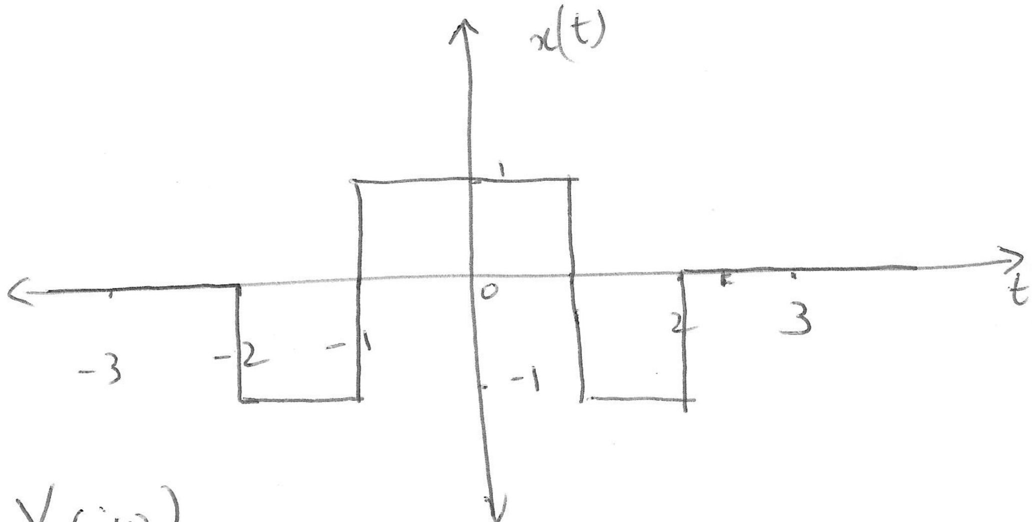
$$x(t) = \begin{cases} -1, & -2 \leq t < -1 \\ 1, & -1 \leq t < 1 \\ -1, & 1 \leq t \leq 2 \\ 0, & |t| > 2 \end{cases}$$

a) Plot x(t) b) Determine the Fourier transform $X(j\omega)$ of x(t) using either the Fourier transform analysis equation or the Tables 4.1 and 4.2. unnecessa

Q3

a)

$$x = \begin{cases} -1 & -2 \le t < -1 \\ 1 & -1 \le t < 1 \\ -1 & 1 \le t < 2 \\ 0 & |t| > 2 \end{cases}$$

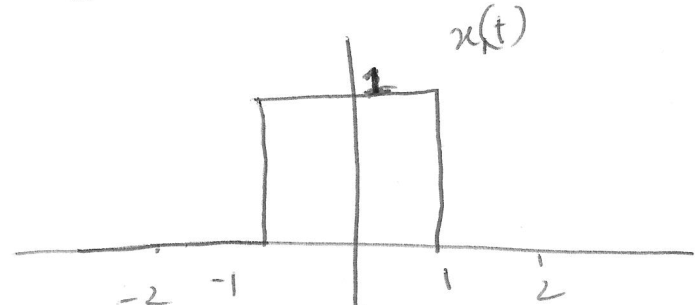



b) find $X(j\omega)$

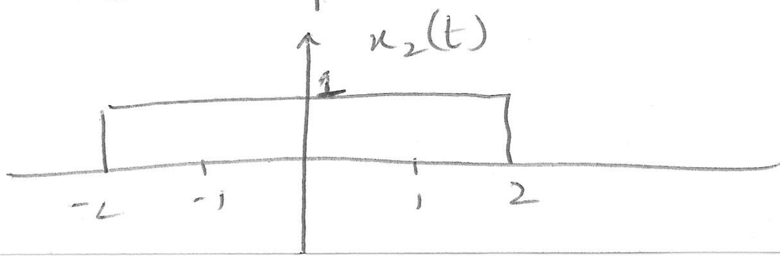
There are 2 ways of solving this

1) using tables: Note that $x(t)$ is the ~~sum of~~ linear combination of $x_1(t)$ and $x_2(t)$ where

$x_1(t) =$




and $x_2(t) =$



$$x(t) = 2x_1(t) - x_2(t)$$

$$\therefore \quad X(j\omega) = 2X_1(j\omega) - X_2(j\omega)$$

Using table 4.2 $X_1(j\omega) = \frac{2\sin\omega(1)}{\omega}$

$$= \frac{2\sin\omega}{\omega}$$

$$X_2(j\omega) = \frac{2\sin\omega(2)}{\omega}$$

$$= \frac{2\sin 2\omega}{\omega}$$

$$\therefore \quad X(j\omega) = 2 \cdot \frac{2\sin\omega}{\omega} - \frac{2\sin 2\omega}{\omega}$$

$$\boxed{X(j\omega) = \frac{2}{\omega}(2\sin\omega - \sin 2\omega)}$$

2] using integration (Analysis equation)

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-2}^{-1} (-1) e^{-j\omega t} \, dt$$

$$+ \int_{-1}^{1} (1) e^{-j\omega t} \, dt$$

$$+ \int_{1}^{2} (-1) e^{-j\omega t} \, dt$$

$$= - \left[ \frac{e^{-j\omega t}}{(-j\omega)} \right]_{-2}^{-1}$$

$$+ \left[ \frac{e^{-j\omega t}}{(-j\omega)} \right]_{-1}^{1}$$

$$- \left[ \frac{e^{-j\omega t}}{(-j\omega)} \right]_{1}^{2}$$

$$X(j\omega) = \frac{1}{j\omega} \left[ e^{j\omega} - e^{2j\omega} - e^{-j\omega} + e^{j\omega} + e^{-2j\omega} - e^{-j\omega} \right]$$

$$X(j\omega) = \frac{1}{j\omega} \left[ 2(e^{j\omega} - e^{-j\omega}) - (e^{2j\omega} - e^{-2j\omega}) \right]$$

$$X(j\omega) = \frac{1}{j\omega}\left[4j\sin\omega - 2j\sin 2\omega\right]$$

$$= \frac{2}{\omega}\left(2\sin\omega - \sin 2\omega\right)$$

[ Note: further simplification is possible but unnecessary e.g. $X(j\omega) = \frac{4}{\omega}\sin\omega(1-\cos\omega)$ ]

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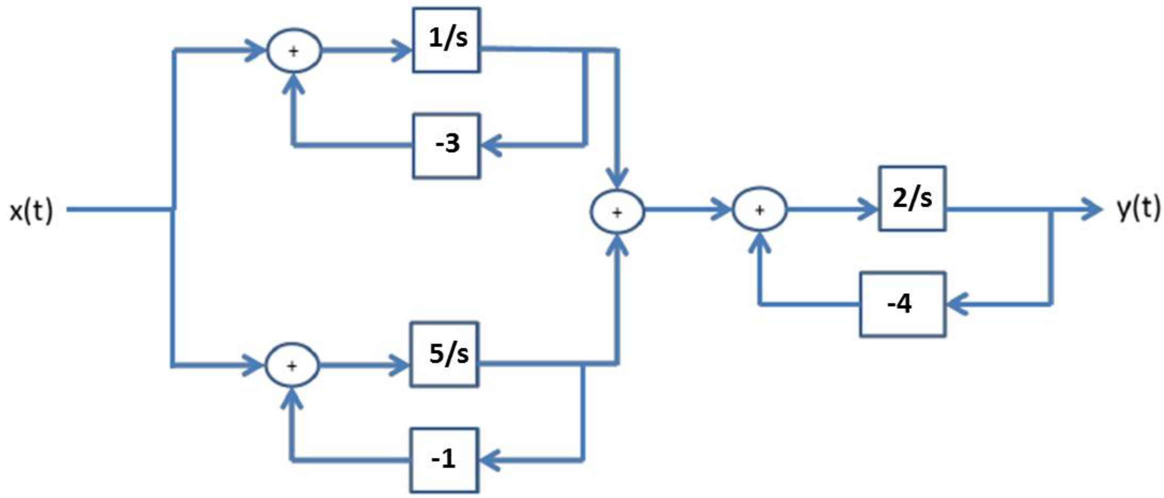
**Question 4. (10 points)**




Consider the block diagram shown above of a casual LTI system.

a) Find the H(s) of the system, report your answer in the rational form, and specify its ROC.
b) Find the differential equation that represents the above block diagram.
c) Draw the "parallel only" equivalent of this block diagram.

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Q4 a) $H(s) = (H_1 + H_2) \cdot H_3$

$$H_1 = \frac{1/s}{1-(-3)(\frac{1}{s})} = \frac{1/s}{1+\frac{3}{s}} = \frac{1}{s+3}$$

$$H_2 = \frac{5/s}{1-(-1)(\frac{5}{s})} = \frac{5/s}{1+\frac{5}{s}} = \frac{5}{s+5}$$

$$H_3 = \frac{2/s}{1-(-4)(\frac{2}{s})} = \frac{2/s}{1+\frac{8}{s}} = \frac{2}{s+8}$$

$$\therefore H(s) = \left(\frac{1}{s+3} + \frac{5}{s+5}\right)\left(\frac{2}{s+8}\right)$$

$$= \left(\frac{s+5+5s+15}{(s+3)(s+5)}\right)\left(\frac{2}{s+8}\right)$$

$$= \frac{(6s+20)\,2}{(s+3)(s+5)(s+8)}$$

In the rational form, $H(s) = \dfrac{12s+40}{s^3+16s^2+79s+120}$

$$\boxed{ROC \rightarrow Re\{s\} > -3}$$

b) $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{12s+40}{s^3+16s^2+79s+120}$.

$\Rightarrow s^3Y(s) + 16s^2Y(s) + 79sY(s) + 120Y(s) = 12sX(s) + 40X(s)$

$\Rightarrow$ Taking the inverse Laplace Transform,

$$\boxed{\frac{d^3y(t)}{dt^3} + 16\frac{d^2y(t)}{dt^2} + 79\frac{dy(t)}{dt} + 120y(t) = 12\frac{dx(t)}{dt} + 40x(t)}$$

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c) "Parallel only" equivalent

$$H(s) = \frac{A}{(s+3)} + \frac{B}{(s+5)} + \frac{C}{(s+8)}$$

$$12s+40 = A(s+5)(s+8) + B(s+3)(s+8) + C(s+3)(s+5)$$

If $s = -5$

$$-20 = 0 + B(-2)(3) + C(0)$$

$$\boxed{B = \frac{10}{3}}$$

If $s = -8$

$$-56 = A(0) + B(0) + C(-5)(-3)$$

$$\boxed{C = \frac{-56}{15}}$$

If $s = -3$

$$4 = A(2)(5) + B(0) + C(0)$$

$$\boxed{A = \frac{2}{5}}$$

$$\therefore\ H(s) = \frac{2/5}{(s+3)} + \frac{10/3}{(s+5)} + \frac{(-56/15)}{(s+8)}$$

$$\Rightarrow H(s) = \frac{2/5s}{1-\left(+\frac{2}{5s}\right)\left(-\frac{15}{2}\right)} + \frac{10/3s}{1-\left(\frac{10}{3s}\right)\left(-\frac{3}{2}\right)} + \frac{-\frac{56}{15s}}{1-\left(-\frac{56}{15s}\right)\left(\frac{15}{7}\right)}$$

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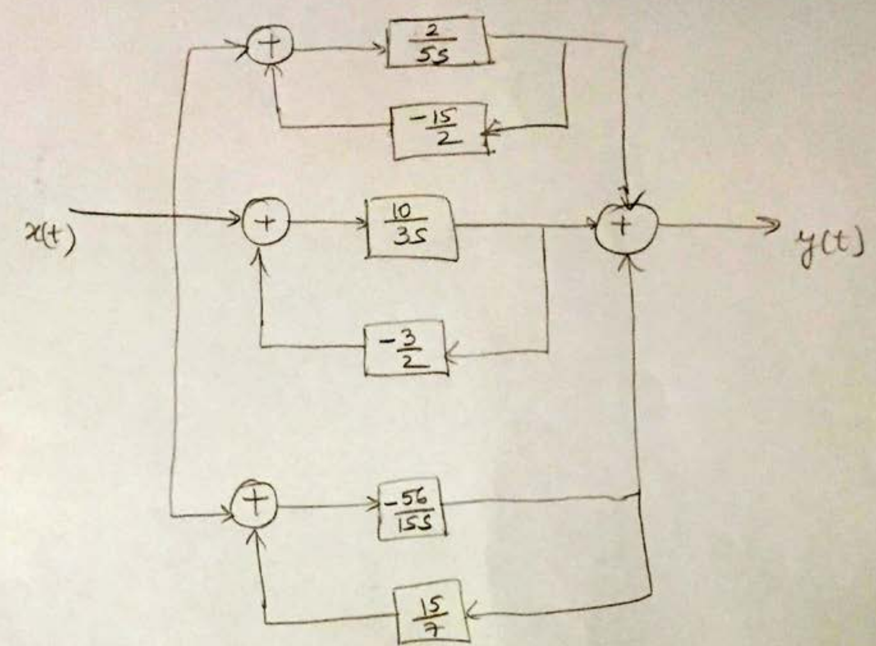

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**Question 5. (10 points)** Consider the LTI system, where the output $y$ is the convolution of two signals $x_1$ and $x_2$

$$y(t) = x_1(t) * x_2(t)$$

where:

$x_1(t) = e^{-t}u(t)$ and $x_2(t) = e^{-4t}\cos(2t)\,u(t)$

a) Find the expression for Y(s), its poles and zeros.

b) Specify and sketch the ROC for Y(s).

c) Specify if the system is stable and causal.

Q5

$y(t) = x_1(t) * x_2(t)$

$x_1(t) = e^{-t}u(t)$

$x_2(t) = e^{-4t}\cos(2t)\,u(t)$

a) $X_1(s) = \frac{1}{s+1}$

$X_2(s) = \frac{s+4}{(s+4)^2 + 2^2}$

$X_2(s) = \frac{s+4}{s^2 + 16 + 8s + 4}$

$X_2(s) = \frac{s+4}{s^2 + 8s + 20}$

$\therefore Y(s) = X_1(s)\,X_2(s)$

$$\boxed{Y(s) = \frac{s+4}{(s+1)(s^2+8s+20)}}$$

To find the poles,

$s = \frac{-8 \pm \sqrt{64 - 4(1)(20)}}{2} = \frac{-8 \pm \sqrt{64-80}}{2}$

$= \frac{-8 \pm \sqrt{-16}}{2}$

$s = -4 \pm j2$

$\therefore$ Zeros at $-4$

Poles at $-1, -4 \pm 2j$

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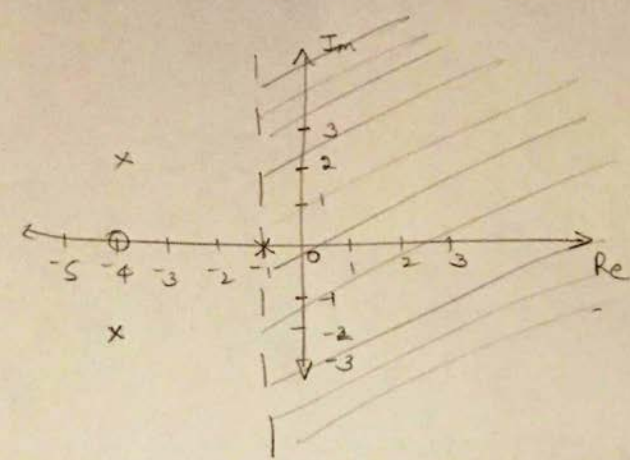
b)




ROC → $Re\{s\} > -1$

c) The system is stable and causal. The ROC is to the right of the right-most pole. The ROC includes the imaginary axis.

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**Question 6. (10 points)** Given the following differential equation of a causal LTI system:

$$y[n] - 10y[n-1] + 3y[n-2] = x[n] - x[n-1]$$

a) Find H(z), sketch the pole-zero plot and specify its ROC.

b) Determine the response y[n] if the input $x[n] = u[n]$.

$y[n] - 10y[n-1] + 3y[n-2] = x[n] - x[n-1]$

a) Find H(z) and its ROC

Applying the z-transform on both sides:-

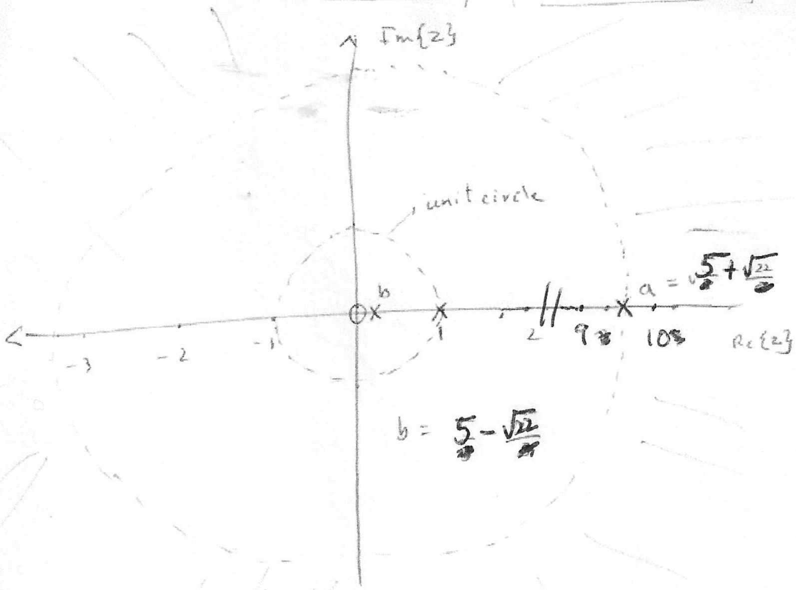
$Y(z) - 10z^{-1}Y(z) + 3z^{-2}Y(z) = X(z) - z^{-1}X(z)$

$Y(z)(1 - 10z^{-1} + 3z^{-2}) = X(z)(1 - z^{-1})$

$$\frac{Y(z)}{X(z)} = \frac{(1 - z^{-1})}{(1 - 10z^{-1} + 3z^{-2})}$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{(1 - z^{-1})}{(1 - 10z^{-1} + 3z^{-2})} = \frac{z(z-1)}{z^2 - 10z + 3}$$

$\boxed{\text{Zeros at } 0, 1}$ $\boxed{\text{poles at } 5 \pm \sqrt{22}}$




ROC: $|z| > 5 + \sqrt{22}$

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b) Find $y[n]$ if $x[n] = u[n]$

$$X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = X(z)\,H(z)$$

$$= \frac{1}{1-z^{-1}} \cdot \frac{1-z^{-1}}{1-10z^{-1}+3z^{-2}}$$

$$= \frac{1}{1-10z^{-1}+3z^{-2}}$$

$$= \frac{1}{(1-az^{-1})(1-bz^{-1})}$$

where $a = 5+\sqrt{22}$

$b = 5-\sqrt{22}$

Let $Y(z) = \frac{A}{1-az^{-1}} + \frac{B}{1-bz^{-1}}$

$$= \frac{A - Abz^{-1} + B - aBz^{-1}}{(1-az^{-1})(1-bz^{-1})}$$

$$= \frac{(A+B) - (Ab+aB)z^{-1}}{(1-az^{-1})(1-bz^{-1})}$$

comparing with $Y(z) = \frac{1}{(1-az^{-1})(1-bz^{-1})}$,

$$A+B = 1, \quad Ab + aB = 0$$

$$B(b-a) = b$$

$$B = \frac{b}{b-a}, \quad A = \frac{-a}{b-a}$$

$$B = \frac{\frac{5-\sqrt{22}}{\cancel{2}}}{-\frac{2\sqrt{22}}{\cancel{2}}} = -\frac{5-\sqrt{22}}{2\sqrt{22}}$$

$$A = \frac{5+\sqrt{22}}{2\sqrt{22}}$$

$$Y(z) = A\,u[n]\,a^n + B\,u[n]\,b^n$$

where $A = \frac{5+\sqrt{22}}{2\sqrt{22}}$, $B = -\frac{5-\sqrt{22}}{2\sqrt{22}}$

$a = 5+\sqrt{22}$, $b = 5-\sqrt{22}$

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**Question 7. (10 points)** Answer the following questions. Clearly show the procedure and highlight your final answer(s).

Consider a LTI system whose response to the input $x(t) = [2e^{-3t} + 2e^{-t}]u(t)$ is given by $y(t) = [5e^{-t} - 5e^{-5t}]u(t)$.

(a) Find the frequency response $H(j\omega)$ of this system.

(b) Determine the impulse response $h(t)$ of the system.

(c) Find the differential equation relating the input and output of this system.

Q7

Given $x(t) = [2e^{-3t} + 2e^{-t}]\, u(t)$

$y(t) = [5e^{-t} - 5e^{-5t}]\, u(t)$

(a) Frequency response of the system

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$Y(j\omega) = \frac{5}{(j\omega+1)} - \frac{5}{(j\omega+5)}$$

$$\Rightarrow Y(j\omega) = 5\left(\frac{j\omega+5-j\omega-1}{(j\omega+1)(j\omega+5)}\right)$$

$$\boxed{Y(j\omega) = \frac{20}{(j\omega+1)(j\omega+5)}}$$

$$X(j\omega) = \frac{2}{(j\omega+3)} + \frac{2}{(j\omega+1)}$$

$$\Rightarrow X(j\omega) = 2\left(\frac{j\omega+1+j\omega+3}{(j\omega+3)(j\omega+1)}\right)$$

$$\boxed{X(j\omega) = \frac{4j\omega+8}{(j\omega+3)(j\omega+1)}}$$

$$\Rightarrow H(j\omega) = \frac{\dfrac{20}{\cancel{(j\omega+1)}(j\omega+5)}}{\dfrac{4(j\omega+2)}{(j\omega+3)\cancel{(j\omega+1)}}} = \frac{5(j\omega+3)}{(j\omega+5)(j\omega+2)}$$

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$$\therefore \boxed{H(j\omega) = \frac{5(j\omega+3)}{(j\omega+5)(j\omega+2)}}$$

(b) Impulse response of the system

Using partial fraction decomposition,

$$\frac{j\omega+3}{(j\omega+2)(j\omega+5)} = \frac{A}{(j\omega+2)} + \frac{B}{(j\omega+5)}$$

$$j\omega+3 = A(j\omega+5) + B(j\omega+2)$$

If $j\omega = -5$

$$-2 = B(-5+2)$$

$$\Rightarrow B = \frac{2}{3}$$

If $j\omega = -2$

$$1 = A(-2+5)$$

$$\Rightarrow A = \frac{1}{3}$$

$$\therefore H(j\omega) = \frac{5/3}{(j\omega+2)} + \frac{10/3}{(j\omega+5)}$$

$$\Rightarrow h(t) = \left(\frac{5}{3}e^{-2t} + \frac{10}{3}e^{-5t}\right)u(t)$$

or $h(t) = 5u(t)\left(\frac{1}{3}e^{-2t} + \frac{2}{3}e^{-5t}\right)$

(c) $$\frac{Y(j\omega)}{X(j\omega)} = \frac{5j\omega+15}{(j\omega+5)(j\omega+2)}$$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{5j\omega+15}{(j\omega)^2+7j\omega+10}$$

Cross-multiplying:

$$(j\omega)^2Y(j\omega) + 7j\omega Y(j\omega) + 10Y(j\omega) = 5j\omega X(j\omega) + 15X(j\omega)$$

Taking the inverse Fourier transform,

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 10y(t) = 5\frac{dx(t)}{dt} + 15x(t)$$

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## Question 8. Sampling and Filtering (2 parts)

**8(a) Sampling (6 points)** Let x(t) be a signal with Nyquist rate $\frac{5}{19}\omega_0$. Determine the Nyquist frequency and rate of

$$y(t) = 3x(t)\sin(\omega_0 t) + \frac{1}{2}\frac{dx(t)}{dt}$$

Q8(a)

$x(t)$ is a signal with Nyquist rate $\frac{5}{19}\omega_0$

$$y(t) = 3x(t)\sin(\omega_0 t) + \frac{1}{2}\frac{d}{dt}x(t)$$

Taking the Fourier transform on both sides,

$$Y(j\omega) = \frac{3}{2\pi}\left[X(j\omega) * FT\{\sin\omega_0 t\}\right] + \frac{1}{2}j\omega X(j\omega)$$

$$Y(j\omega) = \frac{3}{2\pi} X(j\omega) * \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right] + \frac{1}{2}j\omega X(j\omega)$$

$$Y(j\omega) = \frac{3}{2j}\left[X(j(\omega-\omega_0)) - X(j(\omega+\omega_0))\right] + \frac{1}{2}j\omega X(j\omega)$$

Nyquist frequency $= \frac{5}{38}\omega_0$

$\therefore X(j\omega) = 0$ for $|\omega| > \frac{5}{38}\omega_0$

$$\therefore Y(j\omega) = \underbrace{\frac{3}{2j}\left[X(j(\omega-\omega_0)) - X(j(\omega+\omega_0))\right]}_{\left(\frac{5}{38}\omega_0 + \omega_0\right) \Rightarrow \boxed{\frac{43\omega_0}{38}}} + \underbrace{\frac{1}{2}j\omega X(j\omega)}_{\boxed{\frac{5}{38}\omega_0}}$$

$\Rightarrow Y(j\omega) = 0$ for $|\omega| > \frac{43}{38}\omega_0$

$\therefore$ The Nyquist rate $= 2\left(\frac{43}{38}\right)\omega_0 = \frac{43}{19}\omega_0$

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**<u>8 (b) Filtering (4 points)</u>** The frequency response of a causal LTI system is given by:

$$H(j\omega) = \frac{1}{(j\omega)^2 + 2j\omega + 1}$$

Determine the magnitude and phase of the frequency response. Also, specify the type of filter (low-pass, high-pass, or band-pass) that the system could represent.

Q8(b)

given $H(j\omega) = \dfrac{1}{(j\omega)^2 + 2j\omega + 1}$

$$|H(j\omega)| = \sqrt{Re(H(j\omega))^2 + Im(H(j\omega))^2}$$

$$= \sqrt{H(j\omega)\, H^*(j\omega)}$$

$$|H(j\omega)| = \sqrt{\frac{1}{(-\omega^2+2j\omega+1)}\,\frac{1}{(-\omega^2-2j\omega+1)}}$$

$$= \sqrt{\left(\frac{-1}{(\omega-j)^2}\right)\left(\frac{-1}{(\omega+j)^2}\right)}$$

$$= \sqrt{\frac{1}{(\omega^2+1)^2}}$$

$$\boxed{|H(j\omega)| = \frac{1}{\omega^2+1}}$$

Phase of $H(j\omega) = \tan^{-1}\left(\dfrac{Im(H(j\omega))}{Re(H(j\omega))}\right)$

$$= \tan^{-1}\left(\frac{\frac{-2\omega}{(\omega^2+1)}}{\frac{-\omega^2+1}{(\omega^2+1)}}\right)$$

$$= \tan^{-1}\left(\frac{-2\omega}{-\omega^2+1}\right)$$

The system represents a low-pass filter.

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# Relevant Formulae and Tables

| | Continuous time Fourier Transform | Discrete time Fourier Transform |
|---|---|---|
| Analysis equation | $X(\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$ | $X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\Omega n}$ |
| Synthesis equation | $x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega)\, e^{j\omega t}\, d\omega$ | $x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega)\, e^{j\Omega n}\, d\Omega$ |

Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, d$$

Z Transform

$$X = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

**TABLE 4.1** PROPERTIES OF THE FOURIER TRANSFORM

| Section | Property | Aperiodic signal | Fourier transform |
|---|---|---|---|
| | | $x(t)$<br>$y(t)$ | $X(j\omega)$<br>$Y(j\omega)$ |
| 4.3.1 | Linearity | $ax(t) + by(t)$ | $aX(j\omega) + bY(j\omega)$ |
| 4.3.2 | Time Shifting | $x(t - t_0)$ | $e^{-j\omega t_0} X(j\omega)$ |
| 4.3.6 | Frequency Shifting | $e^{j\omega_0 t} x(t)$ | $X(j(\omega - \omega_0))$ |
| 4.3.3 | Conjugation | $x^*(t)$ | $X^*(-j\omega)$ |
| 4.3.5 | Time Reversal | $x(-t)$ | $X(-j\omega)$ |
| 4.3.5 | Time and Frequency Scaling | $x(at)$ | $\frac{1}{\lvert a\rvert} X\left(\frac{j\omega}{a}\right)$ |
| 4.4 | Convolution | $x(t) * y(t)$ | $X(j\omega)Y(j\omega)$ |
| 4.5 | Multiplication | $x(t)y(t)$ | $\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega-\theta))d\theta$ |
| 4.3.4 | Differentiation in Time | $\frac{d}{dt}x(t)$ | $j\omega X(j\omega)$ |
| 4.3.4 | Integration | $\int_{-\infty}^{t} x(t)dt$ | $\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$ |
| 4.3.6 | Differentiation in Frequency | $tx(t)$ | $j\frac{d}{d\omega}X(j\omega)$ |
| 4.3.3 | Conjugate Symmetry for Real Signals | $x(t)$ real | $\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re e\{X(j\omega)\} = \Re e\{X(-j\omega)\} \\ \Im m\{X(j\omega)\} = -\Im m\{X(-j\omega)\} \\ \lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$ |
| 4.3.3 | Symmetry for Real and Even Signals | $x(t)$ real and even | $X(j\omega)$ real and even |
| 4.3.3 | Symmetry for Real and Odd Signals | $x(t)$ real and odd | $X(j\omega)$ purely imaginary and odd |
| 4.3.3 | Even-Odd Decomposition for Real Signals | $x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real]<br>$x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real] | $\Re e\{X(j\omega)\}$<br>$j\Im m\{X(j\omega)\}$ |

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \lvert X(j\omega)\rvert^2 d\omega$$

**TABLE 4.2** BASIC FOURIER TRANSFORM PAIRS

| Signal | Fourier transform |
|---|---|
| $\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$ | $2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$ |
| $e^{j\omega_0 t}$ | $2\pi\delta(\omega - \omega_0)$ |
| $\cos \omega_0 t$ | $\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$ |
| $\sin \omega_0 t$ | $\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$ |
| $x(t) = 1$ | $2\pi\,\delta(\omega)$ |
| Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \le \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$ | $\sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k}\delta(\omega - k\omega_0)$ |
| $\sum_{n=-\infty}^{+\infty} \delta(t - nT)$ | $\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\omega - \frac{2\pi k}{T}\right)$ |
| $x(t)\begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$ | $\frac{2\sin \omega T_1}{\omega}$ |
| $\frac{\sin Wt}{\pi t}$ | $X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$ |
| $\delta(t)$ | $1$ |
| $u(t)$ | $\frac{1}{j\omega} + \pi\,\delta(\omega)$ |
| $\delta(t - t_0)$ | $e^{-j\omega t_0}$ |
| $e^{-at}u(t), \Re e\{a\} > 0$ | $\frac{1}{a + j\omega}$ |
| $te^{-at}u(t), \Re e\{a\} > 0$ | $\frac{1}{(a + j\omega)^2}$ |

**NAME:** **ASU ID:**

**TABLE 9.1** PROPERTIES OF THE LAPLACE TRANSFORM

| Section | Property | Signal | Laplace Transform | ROC |
|---|---|---|---|---|
| | | $x(t)$ | $X(s)$ | $R$ |
| | | $x_1(t)$ | $X_1(s)$ | $R_1$ |
| | | $x_2(t)$ | $X_2(s)$ | $R_2$ |
| 9.5.1 | Linearity | $ax_1(t)+bx_2(t)$ | $aX_1(s)+bX_2(s)$ | At least $R_1 \cap R_2$ |
| 9.5.2 | Time shifting | $x(t-t_0)$ | $e^{-st_0}X(s)$ | $R$ |
| 9.5.3 | Shifting in the $s$-Domain | $e^{s_0t}x(t)$ | $X(s-s_0)$ | Shifted version of $R$ (i.e., $s$ is in the ROC if $s-s_0$ is in R) |
| 9.5.4 | Time scaling | $x(at)$ | $\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$ | Scaled ROC (i.e., $s$ is in the ROC if $s/a$ is in $R$) |
| 9.5.5 | Conjugation | $x^*(t)$ | $X^*(s^*)$ | $R$ |
| 9.5.6 | Convolution | $x_1(t)*x_2(t)$ | $X_1(s)X_2(s)$ | At least $R_1 \cap R_2$ |
| 9.5.7 | Differentiation in the Time Domain | $\frac{d}{dt}x(t)$ | $sX(s)$ | At least $R$ |

**TABLE 10.1** PROPERTIES OF THE $z$-TRANSFORM

| Section | Property | Signal | $z$-Transform | ROC |
|---|---|---|---|---|
| | | $x[n]$ | $X(z)$ | $R$ |
| | | $x_1[n]$ | $X_1(z)$ | $R_1$ |
| | | $x_2[n]$ | $X_2(z)$ | $R_2$ |
| 10.5.1 | Linearity | $ax_1[n]+bx_2[n]$ | $aX_1(z)+bX_2(z)$ | At least the intersection of $R_1$ and $R_2$ |
| 10.5.2 | Time shifting | $x[n-n_0]$ | $z^{-n_0}X(z)$ | $R$, except for the possible addition or deletion of the origin |
| 10.5.3 | Scaling in the $z$-domain | $e^{j\omega_0 n}x[n]$ | $X(e^{-j\omega_0}z)$ | $R$ |
| | | $z_0^n x[n]$ | $X\left(\frac{z}{z_0}\right)$ | $z_0R$ |
| | | $a^n x[n]$ | $X(a^{-1}z)$ | Scaled version of $R$ (i.e., $\lvert a\rvert R$ = the set of points $\{\lvert a\rvert z\}$ for $z$ in $R$) |
| 10.5.4 | Time reversal | $x[-n]$ | $X(z^{-1})$ | Inverted $R$ (i.e., $R^{-1}$ = the set of points $z^{-1}$, where $z$ is in $R$) |

**TABLE 9.2** LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

| Transform pair | Signal | Transform | ROC |
|---|---|---|---|
| 1 | $\delta(t)$ | $1$ | All $s$ |
| 2 | $u(t)$ | $\frac{1}{s}$ | $\mathcal{R}e\{s\}>0$ |
| 3 | $-u(-t)$ | $\frac{1}{s}$ | $\mathcal{R}e\{s\}<0$ |
| 4 | $\frac{t^{n-1}}{(n-1)!}u(t)$ | $\frac{1}{s^n}$ | $\mathcal{R}e\{s\}>0$ |
| 5 | $-\frac{t^{n-1}}{(n-1)!}u(-t)$ | $\frac{1}{s^n}$ | $\mathcal{R}e\{s\}<0$ |
| 6 | $e^{-\alpha t}u(t)$ | $\frac{1}{s+\alpha}$ | $\mathcal{R}e\{s\}>-\alpha$ |
| 7 | $-e^{-\alpha t}u(-t)$ | $\frac{1}{s+\alpha}$ | $\mathcal{R}e\{s\}<-\alpha$ |
| 8 | $\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$ | $\frac{1}{(s+\alpha)^n}$ | $\mathcal{R}e\{s\}>-\alpha$ |
| 9 | $-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$ | $\frac{1}{(s+\alpha)^n}$ | $\mathcal{R}e\{s\}<-\alpha$ |
| 10 | $\delta(t-T)$ | $e^{-sT}$ | All $s$ |
| 11 | $[\cos\omega_0 t]u(t)$ | $\frac{s}{s^2+\omega_0^2}$ | $\mathcal{R}e\{s\}>0$ |
| 12 | $[\sin\omega_0 t]u(t)$ | $\frac{\omega_0}{s^2+\omega_0^2}$ | $\mathcal{R}e\{s\}>0$ |
| 13 | $[e^{-\alpha t}\cos\omega_0 t]u(t)$ | $\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$ | $\mathcal{R}e\{s\}>-\alpha$ |
| 14 | $[e^{-\alpha t}\sin\omega_0 t]u(t)$ | $\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$ | $\mathcal{R}e\{s\}>-\alpha$ |

**TABLE 10.2** SOME COMMON $z$-TRANSFORM PAIRS

| Signal | Transform | ROC |
|---|---|---|
| 1. $\delta[n]$ | $1$ | All $z$ |
| 2. $u[n]$ | $\frac{1}{1-z^{-1}}$ | $\lvert z\rvert>1$ |
| 3. $-u[-n-1]$ | $\frac{1}{1-z^{-1}}$ | $\lvert z\rvert<1$ |
| 4. $\delta[n-m]$ | $z^{-m}$ | All $z$, except 0 (if $m>0$) or $\infty$ (if $m<0$) |
| 5. $\alpha^n u[n]$ | $\frac{1}{1-\alpha z^{-1}}$ | $\lvert z\rvert>\lvert\alpha\rvert$ |
| 6. $-\alpha^n u[-n-1]$ | $\frac{1}{1-\alpha z^{-1}}$ | $\lvert z\rvert<\lvert\alpha\rvert$ |
| 7. $n\alpha^n u[n]$ | $\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$ | $\lvert z\rvert>\lvert\alpha\rvert$ |
| 8. $-n\alpha^n u[-n-1]$ | $\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$ | $\lvert z\rvert<\lvert\alpha\rvert$ |