BME 350: Signals and Systems for Bioengineers
HW 4 (50 points)

Deadline: **Nov 1st 2016 (Tuesday) by 9AM**
Late assignments: **5 points deducted per hour** **(delay rounded up to the nearest hour after deadline).**
Note: For questions 3-6, show all your work to receive full credit

1. **(7 points)** Indicate whether the following statements are true or false:

a. Fourier transform is the extension of fourier series to aperiodic signals **T** **F**
b. The Fourier transform of $u(t)$ is 1 **T** **F**
c. The Fourier transform of a real and odd signal is complex and odd **T** **F**
d. Filtering allows enhancement and elimination of certain frequencies of a signal **T** **F**
e. Ideal frequency domain filters are non-causal **T** **F**
f. The response of a filter with transfer function H(jω) and input X(jω) is H(jω)*X(jω), where '*' represents convolution **T** **F**
g. Frequency shaping filters are commonly used for enhancement of edges in image processing **T** **F**

2. **(8 points)** Fill in the blanks for the following statements:

a. The Fourier transform of δ(t-t_0) is e^-jwto___.
b. The Inverse Fourier transform is computed using the synthesis___ equation.
c. Multiplication of two signals in the time domain corresponds to convolution___ of their frequency profiles.
d. A passive____ filter contains only R, L, and C components.
e. Sharp, finite edges in one domain correspond to ____________ in the other domain.
f. EMI_____ filters are used for elimination of interference from power line.
g. High-frequency oscillations (HFOs) are temporally _______ events and are markers for ________.

3. **Fourier Transform (8 points)** Use the analysis and synthesis equations to solve the following problems. (Do not use 'Fourier Transform properties' table)

a. Compute the Fourier transform of the following signal using the analysis equation:

$$x(t) = e^{(t+2)}[u(t+4) - u(t-5)]$$

b. Retrieve the following signal from its Fourier transform using the synthesis equation:

$$X(jw) = \begin{Bmatrix} \frac{w}{3} - 3 & -4 \le w \le 4 \\ 0 & |w| > 4 \end{Bmatrix}$$

(Hint: $\int we^{aw}dw = \frac{e^{aw}}{a}[w - \frac{1}{a}]$

4. **Fourier Transform Properties (8 points)** The Fourier transform of a signal x(t) is given by:

$$X(jw) = \frac{4\sin(5w)}{9w}$$

Use 'Fourier Transform properties' table to calculate the Fourier transform of:

a. $x(\frac{-2}{3}t+4)$

b. $\frac{7}{2}\frac{dx(t)}{dt}$

c. $e^{-j8t}x(t)$

d. $3tx(t)$

5. **Filtering (10 points)** The output of a causal LTI filter is related to the input x(t) by the following differential equation:

$$2\frac{dy(t)}{dt} + 8y(t) = 6x(t)$$

a. **(2 points)** Determine the impulse response (h(t)) of the filter.

b. **(8 points)** Determine the output of the filter (y(t)) for the following inputs:

(i) $x(t) = e^{-6t}u(t)$

(ii) $X(jw) = \frac{jw+4}{jw+3}$

(iii) $X(jw) = \frac{jw+2}{jw+4}$

(iv) $X(jw) = \frac{1}{(jw+2)(jw+4)}$

(Hint: Use partial fractions for parts (i), (iii) and (iv))

6. **(4 points)** Give a brief description of three biomedical applications of microelectrode array technology (Hint: Refer to the lecture on Neural engineering applications)

7. **MATLAB (5 points)** Consider the signals x(t), y(t) and z(t) given as,

$x(t) = \cos(30\pi t)$, $y(t) = \sin(56\pi t)$ and $z(t) = \cos(72\pi t)$

a. Compute (using MATLAB) and plot the fourier transforms, X(jω), Y(jω) and Z(jω) (Use 'subplot'). What do the peaks in each of the fourier transform plots represent?

b. Plot the signal $v(t) = 2x(t) + 4y(t) - 3z(t)$ and its fourier transform, V(jω) (Use 'subplot'). What do the peaks in this fourier transform plot represent?

(3.) FOURIER TRANSFORM

$$x(t) = e^{(t+2)}\left[u(t+4) - u(t-5)\right]$$

(a) $$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{(t+2)}\left[u(t+4) - u(t-5)\right]\cdot e^{-j\omega t}\, dt$$

$$= \int_{-4}^{5} e^{t+2-j\omega t}\, dt$$

$$= e^{2} \int_{-4}^{5} e^{t-j\omega t}\, dt$$

$$= e^{2} \int_{-4}^{5} e^{(1-j\omega)t}\, dt$$

$$= \frac{e^{(1-j\omega)t}}{1-j\omega}\Bigg|_{-4}^{5}$$

$$= e^{2}\left[\frac{e^{(1-j\omega)5}}{1-j\omega} - \frac{e^{-4(1-j\omega)}}{1-j\omega}\right]$$

$$= e^{2}\left[\frac{e^{5-5j\omega} - e^{-4+4j\omega}}{1-j\omega}\right]$$

$$= \frac{e^{2}e^{5-5j\omega} - e^{2}e^{-4+4j\omega}}{1-j\omega}$$

$$= \frac{e^{7-5j\omega} - e^{-2+4j\omega}}{1-j\omega}$$

(b) $$X(j\omega) = \begin{cases} \frac{\omega}{3} - 3 & -4 \le \omega \le 4 \\ 0 & |\omega| > 4 \end{cases}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-4}^{4}\left(\frac{\omega}{3} - 3\right) e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-4}^{4}\left(\frac{\omega}{3} e^{j\omega t} - 3e^{j\omega t}\right) d\omega$$

$$= \frac{1}{6\pi}\int_{-4}^{4} \omega e^{j\omega t}\, d\omega - \frac{1}{2\pi}\int_{-4}^{4} 3e^{j\omega t}\, d\omega$$

$$= \frac{1}{6\pi}\frac{e^{j\omega t}}{jt}\left[\omega - \frac{1}{jt}\right]\Bigg|_{-4}^{4} - \frac{3}{2\pi}\left[\frac{e^{j\omega t}}{jt}\right]_{-4}^{4}$$

$$= \left\{\frac{1}{6\pi}\frac{e^{j4t}}{jt}\left[4 - \frac{1}{jt}\right] - \frac{1}{6\pi}\frac{e^{-j4t}}{jt}\left[-4 - \frac{1}{jt}\right]\right\} - \left\{\frac{3}{2\pi}\frac{e^{j4t}}{jt} + \frac{3}{2\pi}\frac{e^{-4jt}}{jt}\right\}$$

$$= \cancel{\frac{2}{3\pi jt} e^{j4t}} - \cancel{\frac{1}{6\pi j^2 t^2} e^{j4t}} + \cancel{\frac{2}{3\pi jt} e^{-4jt}}$$

$$+ \cancel{\frac{1}{6\pi j^2 t^2} e^{-4jt}} - \frac{3}{2\pi jt} e^{j4t} - \frac{3}{2\pi jt} e^{-4jt}$$

$$= \frac{4}{3\pi jt}\cos(4t) - \frac{j}{3\pi t^2}\sin(4t) -$$

$$\frac{3}{2\pi jt}\cos(4t)$$

(4) FOURIER TRANSFORM PROPERTY

$$X(j\omega) = \frac{4\sin(5\omega)}{9\omega}$$

(a) $$x\left(-\frac{2}{3}t + 4\right)$$

→ TIME & FREQ. SCALING PROP:

$$x(at) \rightarrow \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

→ TIME SHIFTING PROP:

$$x(t - t_0) \rightarrow e^{-j\omega t_0} X(j\omega)$$

$t_0 = -4 \qquad a = -\frac{2}{3}$

$$= e^{-j6\omega}\cdot\left(-\frac{3}{2}\right)\cdot X\left(-\frac{3j\omega}{2}\right)$$

$$= -\frac{3}{2} e^{-j6\omega}\cdot\frac{4}{9}\frac{\sin\left[5\left(\frac{-3\omega}{2}\right)\right]}{-\frac{3\omega}{2}}$$

$$= \frac{4}{9\omega} e^{-j6\omega}\sin\left[\frac{-15\omega}{2}\right]$$

USING EULER'S FORMULA

$$F\left\{x\left(-\frac{2}{3}t + 4\right)\right\} = \frac{4}{9\omega} e^{-j6\omega}\left(\frac{e^{\frac{-15\omega}{2}} - e^{\frac{15\omega}{2}}}{2}\right)$$

$$= \frac{2}{9\omega}\left[e^{-\frac{27\omega}{2}j} - e^{\frac{3\omega}{2}j}\right]$$

(b) $$\frac{7}{2}\frac{dx(t)}{dt}$$

→ DIFF. IN TIME PROP.

$$\frac{dx(t)}{dt} \rightarrow j\omega X(j\omega)$$

$$= \frac{7}{2} j\omega\, F\{x(t)\}$$

$$= \frac{7}{2} j\omega\, \frac{4\sin(5\omega)}{9\omega}$$

$$= \frac{14}{9} j\left(\frac{e^{5j\omega} - e^{-5j\omega}}{2}\right)$$

$$= \frac{7j}{9}\left(e^{5j\omega} - e^{-5j\omega}\right)$$

(c) $$e^{-j8t} x(t)$$

→ FREQ. SHIFTING

$$e^{j\omega_0 t} x(t) \rightarrow X[j(\omega - \omega_0)]$$

$\omega_0 = -8$

$$= \frac{4\sin[5(\omega+8)]}{9(\omega+8)}$$

$$= \frac{4\sin(5\omega+40)}{9\omega+72}$$

(4.) $3t\,x(t)$

↳ DIFF IN FREQ.

$$t\,x(t) \rightarrow j\frac{d}{d\omega}X(j\omega)$$

$$3j\,\frac{4\sin(5\omega)}{9\omega}$$

$$\frac{d}{d\omega} = \frac{60j\cos(5\omega)(9\omega) - 12j\sin(5\omega)(9)}{81\omega^2}$$

$$= \frac{540j\cos(5\omega) - 108j\sin(5\omega)}{81\omega^2}$$

(5) FILTERING

(a) $2\frac{dy(t)}{dt} + 8y(t) = 6x(t)$

↳ TAKE FOURIER TRANSFORM ON BOTH SIDES

$$2j\omega\,Y(j\omega) + 8Y(j\omega) = 6X(j\omega)$$

$$(2j\omega + 8)\,Y(j\omega) = 6X(j\omega)$$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{6}{2j\omega+8}$$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{6}{2(j\omega+4)}$$

$$H(j\omega) = \frac{3}{j\omega+4}$$

$$\Rightarrow h(t) = 3e^{-4t}u(t)$$

(b) (i) $x(t) = e^{-6t}u(t)$

$$\Rightarrow X(j\omega) = \frac{1}{j\omega+6}$$

$$Y(j\omega) = X(j\omega)\cdot H(j\omega) = \frac{1}{j\omega+6}\cdot\frac{3}{j\omega+4}$$

$$= \frac{3}{(j\omega+6)(j\omega+4)}$$

$$\frac{3}{(j\omega+6)(j\omega+4)} = \frac{A}{j\omega+6} + \frac{B}{j\omega+4}$$

$$\frac{3}{(j\omega+6)(j\omega+4)} = \frac{Aj\omega + 4A + Bj\omega + 6B}{(j\omega+6)(j\omega+4)}$$

$$3 = 4A + 6B \;;\quad A + B = 0$$

$$2B = 3$$

$$B = 3/2 \qquad A = -3/2$$

$$Y(j\omega) = \frac{-3/2}{j\omega+6} + \frac{3/2}{j\omega+4}$$

$$y(t) = -3/2\,e^{-6t}u(t) + 3/2\,e^{-4t}u(t)$$

(ii) $X(j\omega) = \frac{j\omega+4}{j\omega+3}$

$$Y(j\omega) = X(j\omega)\cdot H(j\omega) = \frac{\cancel{j\omega+4}}{j\omega+3}\cdot\frac{3}{\cancel{j\omega+4}}$$

$$Y(j\omega) = \frac{3}{j\omega+3}$$

$$Y(t) = 3e^{-3t}u(t)$$

(iii) $X(j\omega) = \frac{j\omega+2}{j\omega+4}$

$$Y(j\omega) = X(j\omega)\cdot H(j\omega) = \frac{j\omega+2}{j\omega+4}\cdot\frac{3}{j\omega+4}$$

$$\frac{3j\omega+6}{(j\omega+4)^2} = \frac{A}{j\omega+4} + \frac{B}{(j\omega+4)^2}$$

$$3j\omega + 6 = Aj\omega + 4A + B$$

$$A = 3$$

$$6 = 4A + B \Rightarrow B = -6$$

$$Y(j\omega) = \frac{3}{j\omega+4} - \frac{6}{(j\omega+4)^2}$$

$$y(t) = 3e^{-4t}u(t) - 6te^{-4t}u(t)$$

(iv) $X(j\omega) = \frac{1}{(j\omega+2)(j\omega+4)}$

$$Y(j\omega) = X(j\omega)\cdot H(j\omega) = \frac{1}{(j\omega+2)(j\omega+4)}\cdot\frac{3}{(j\omega+4)}$$

$$= \frac{3}{(j\omega+2)(j\omega+4)(j\omega+4)}$$

$$\frac{3}{(j\omega+2)(j\omega+4)^2} = \frac{A}{j\omega+2} + \frac{B}{j\omega+4} + \frac{C}{(j\omega+4)^2}$$

$$3 = A(j\omega+4)^2 + B(j\omega+2)(j\omega+4) + Cj\omega + 2C$$

$$3 = \cancel{Aj^2\omega^2} + \cancel{8Aj\omega} + 16A + \cancel{Bj^2\omega^2} + \cancel{6Bj\omega} + 8B$$

$$+ \cancel{Cj\omega} + 2C \;;\quad A + B = 0$$

$$0 = 8A + 6B + C$$

$$0 = -8B + 6B + C \Rightarrow C = 2B$$

$$3 = 16A + 8B + 2C$$

$$3 = -16B + 8B + 4B$$

$$3 = -4B$$

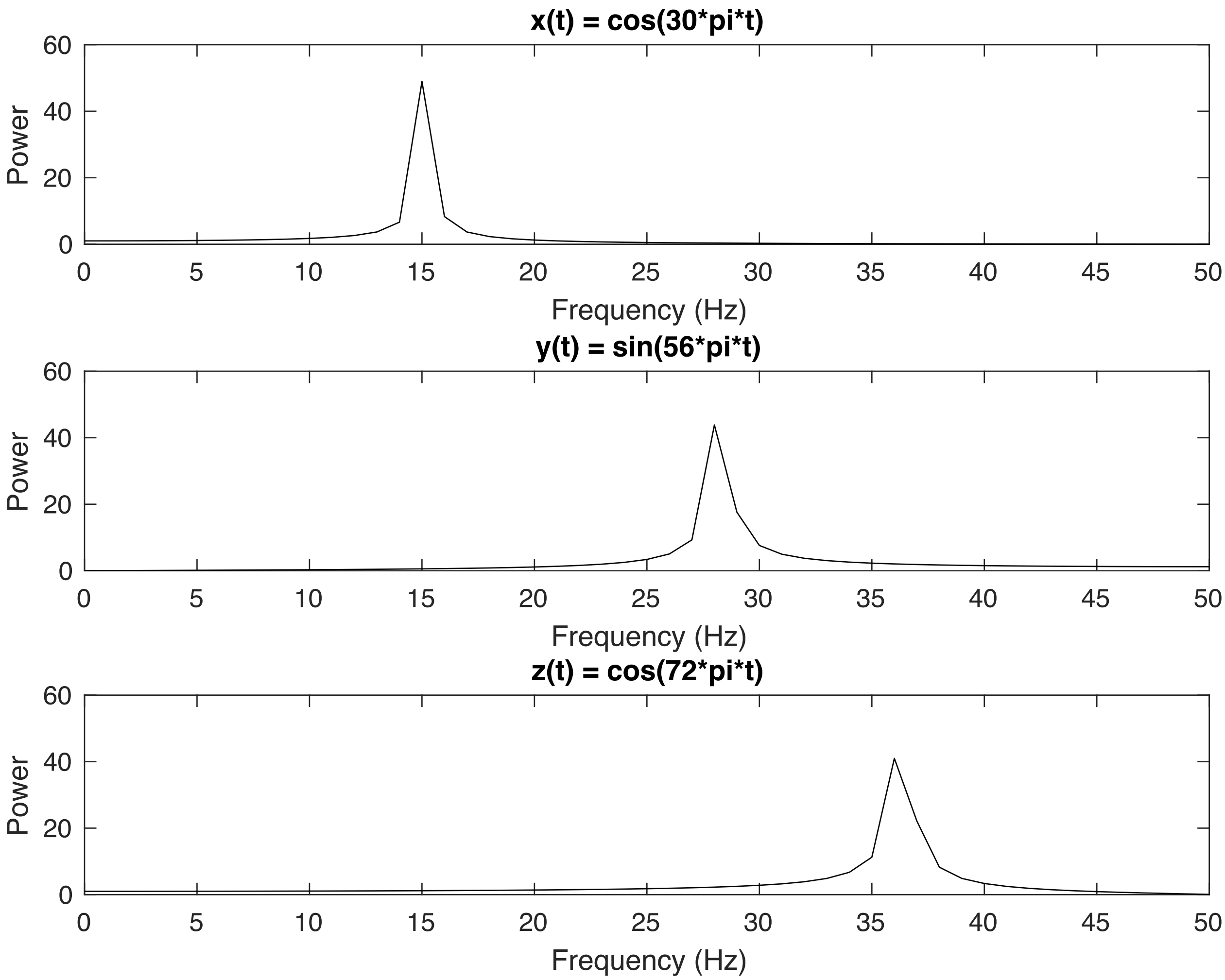
$$B = -3/4 \qquad A = 3/4 \qquad C = -3/2$$

$$Y(j\omega) = \frac{3/4}{j\omega+2} - \frac{3/4}{j\omega+4} - \frac{3/2}{(j\omega+4)^2}$$

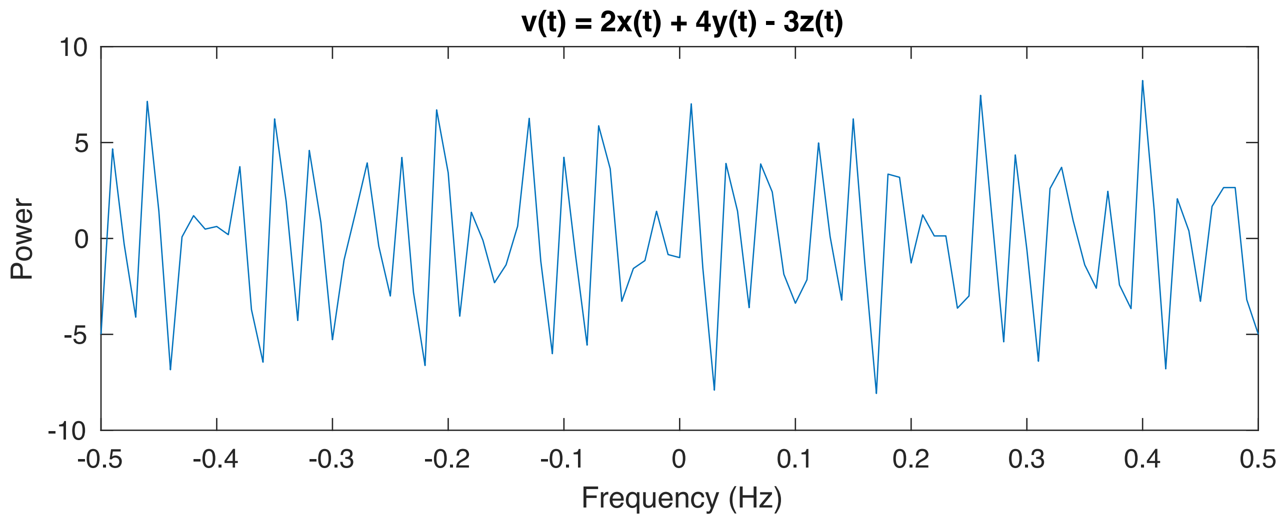
$$y(t) = 3/4\,e^{-2t}u(t) - 3/4\,e^{-4t}u(t) - 3/2\,e^{-4t}u(t)$$

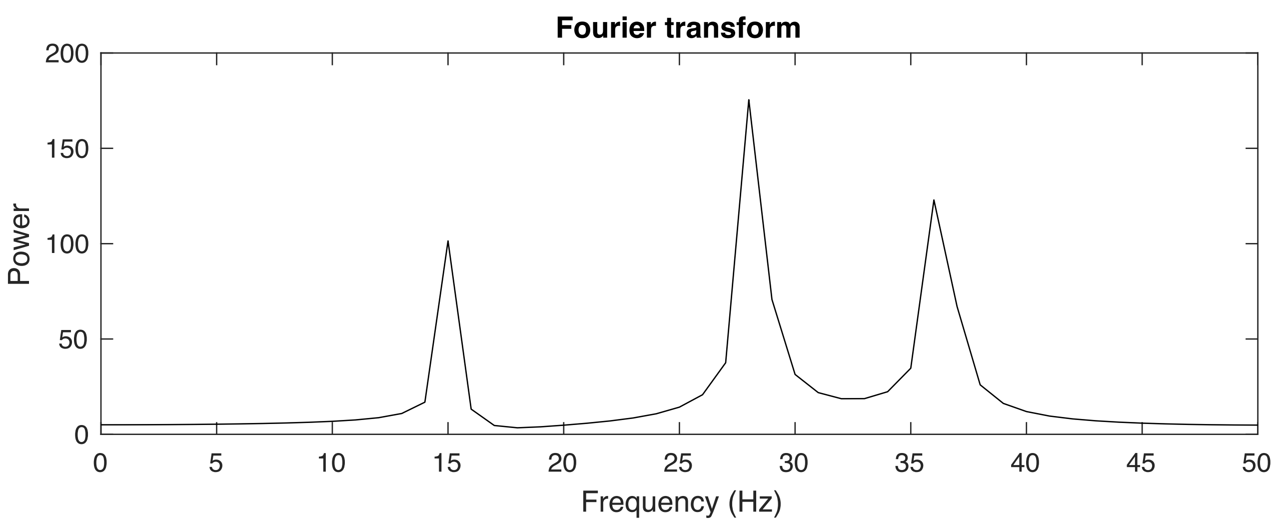
(6) BIOMEDICAL APPLICATION

- MICROELECTRODE ARRAY IN PERIPHERAL NERVE
  ↳ THIS CAN RECORD ACTION POTENTIALS FROM MULTIPLE AXONS IN THE NERVE, AS WELL AS PROVIDE MICRO-STIMULATION TO MULTIPLE LOCATIONS THROUGHOUT THE NERVE
- MICROELECTRODE ARRAY IN DRUG DISCOVERY & BASIC RESEARCH
  ↳ THIS CAN RECORD & MONITOR EFFECTS OF DRUGS & TOXINS THAT COULD HELP FUTURE STUDIES.
- MICROELECTRODE ARRAY IN MODULATION OF SENSATION
  ↳ THIS CAN RECORD INPUT SIGNALS & SENSITIVITY TO HELP MODULATE STIMULATION WITH INTACT HAND.



The peak signifies a periodic component in the input data.
For x(t), it signifies at 15Hz, the power is at strongest.
For y(t), it signifies at 28Hz, the power is at strongest.
For z(t), it signifies at 36Hz, the power is at strongest.





Fourier transform signifies at 15, 28, and 36Hz, the power is at strongest.