BME 350: Signals and Systems for Bioengineers
HW 5 (50 points)

Deadline: Nov 15th 2016 by 9AM
Late assignments: 5 points deducted per hour (delay rounded up to the nearest hour after deadline).
Note: For questions 3-5, show all your work to receive full credit

1. (7 points) Indicate whether the following statements are true or false:

(a) Frequency response of a system is the fourier transform of its impulse response	**T**	**F**
(b) An ideal sampling produces samples equivalent to the instantaneous value of the continuous signal at desired points	**T**	**F**
(c) Ideal frequency domain filters can be applied in real time	**T**	**F**
(d) Local field potentials are low frequency signals obtained from a population of neurons.	**T**	**F**
(e) Filtering allows recording of multiple signals at different frequencies	**T**	**F**
(f) A digital filter is commonly used as an anti-aliasing filter	**T**	**F**
(g) An image is a representation of a view of an object, where intensity varies with coordinates x and y	**T**	**F**

2. **(8 points)** Fill in the blanks for the following statements:

(a) K-space____ is the 2D fourier transform of an MR image.
(b) Sampling allows conversion of continuous_ signal into discrete___ signal
(c) A signal is band limited____ if it does not have any frequency components above a certain value.
(d) A shaping___ filter allows frequencies within a certain range.
(e) A signal can be uniquely recovered from its samples if ws>2wm_____.
(f) Digital______ filters are implemented using a computer.
(g) x(t) and X(ω) are referred to as the fourier transform for the signal.

Q3. **Fourier Transform (9 points)** A discrete time signal x[n] has Discrete Time Fourier Transform given as

$$X(\Omega) = \frac{e^{j\Omega}}{3 + e^{j\Omega}}$$

a) Find the inverse fourier transform of $X(\Omega)$ to determine x[n] using *Tables 6.1 (Properties of DTFT) and 6.2 (Common DTFT Pairs)* in textbook.

b) Find the DTFT of the following signals using *Table 6.1* in textbook:

i. $y[n] = x[n+3]$

ii. $y[n] = 2x[n]e^{j6n}$

iii. $y[n] = x[n] * x[n]$

Q4. **Filtering (12 points)** The following frequency response belongs to a low-pass filter implemented using an RC circuit:

$$H(j\omega) = \frac{450}{450 + j\omega}$$

(a) Given that the capacitor used in the circuit had a value of C = 5μF. What is the value of the resistor (R)? What is its cutoff frequency in hertz?

(b) Using the same capacitance value and a resistor of R = 48KΩ, what would be the transfer function of a high-pass passive analog filter? What would be its cutoff frequency in hertz? Using the transfer function, plot the magnitude and phase of its frequency response in decibels using MATLAB. *(Hint: Use 'tf' and 'bode' functions)*

(c) Calculate the transfer function (frequency response) of the band-pass filter resulting from the series connection of the given low-pass filter and your calculated high-pass filter in part (b). What are the cutoff frequencies for the band pass filter? Using the transfer function, plot the magnitude and phase of its frequency response in decibels using MATLAB.

(d) Using the same capacitance value (C = 5μF), what resistor value will you have to use to build a high-pass passive analog filter with a cutoff frequency of 250Hz? What would be its transfer function? Using the transfer function, plot the magnitude and phase of the frequency response in decibels using MATLAB.

Q5. **Sampling** (a) **(7 points)** Let x(t) be a signal with Nyquist rate, $\frac{w_0}{2}$. Determine the Nyquist rate for each of the following signals:

(i) $5x(t)\sin w_0 t + 6x^2(t)$

(ii) $\frac{3}{2}\frac{dx(t)}{dt} + \frac{1}{4}x(t)\cos w_0 t$

(b) **(3 points)** A signal x(t) with Fourier transform X(jω) undergoes impulse-train sampling to generate

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

Where T = 10^{-4} s. Given that $X(jw) * X(jw) = 0$ for $|w| > 8000\pi$, can x(t) be recovered exactly from $x_p(t)$?

Q6. **(4 points)** (a) Discuss the role of signal processing in magnetic resonance imaging (MRI).

(b) Give a brief description of at least of two applications of MRI. (Hint: refer to the lecture on MRI applications)

a. Signal processing is the core of MRI. It is used to non-invasively study a human or animal body or diagnose disease and monitor improvement of a patient.

b. Imaging gene activity - images from pre until post injection can be obtained and can show the effects of the gene to the tissue. It could potentially show the side effect of the injection to the tissue.

Imaging the effects of traumatic brain injury - images as time passes by after the injury can be obtained, this can help doctors assess the condition of the patient and how they are recovering from the injury as well as if there's any other complications.

(3) FOURIER TRANSFORM

$$X(\Omega) = \frac{e^{j\Omega}}{3 + e^{j\Omega}} = \frac{1}{1+3e^{-j\Omega}}$$

(a) FROM TABLE 6.2 ~~WEIGHTED~~ ~~EXPONENTIAL~~

EXPONENTIAL $\frac{e^{j\Omega}}{e^{j\Omega}+3}\left(\frac{e^{-j\Omega}}{e^{-j\Omega}}\right)$

$a^n u[n]$

$x[n] = (3)^n u[n]$

(b) (i) $y[n] = x[n+3]$

TIME SHIFTING

$x[n-k] = e^{-j\Omega k} x(\Omega)$

$e^{+j\Omega(3)} x(\Omega)$

(ii) $y[n] = 2x[n]e^{j6n}$

$$2x(\Omega - 6) = 2\left[\frac{e^{j\Omega - 6}}{3+e^{j\Omega}}\right]$$

(iii) $y[n] = x[n] * x[n]$

$$x(\Omega)x(\Omega) = \left(\frac{e^{j\Omega}}{3+e^{j\Omega}}\right)^2$$

(5) (a) SAMPLING

NYQUIST RATE $= \frac{\omega_0}{2}$

(i) $5x(t)\sin\omega_0 t + 6x^2(t)$

→ FOURIER TRANSFORM

MODULATION: $5x(t)\sin\omega_0 t$

$\frac{j}{2}[5X(\omega+\omega_0) - 5X(\omega-\omega_0)]$

~~SCALING~~ LINEARITY: $6x^2(t)$

$6X^2(\omega)$

$$= \underbrace{\frac{j}{2}[5X(\omega+\omega_0) - 5X(\omega-\omega_0)]}_{X(j\omega)} + \underbrace{6[X(j\omega) * X(j\omega)]}_{X_1(j\omega)}$$

$X_1(j\omega) = 0$ FOR $|\omega| > \frac{\omega_0}{2}$

$X(j\omega) = 0$ FOR $|\omega| > \left(\frac{\omega_0}{2} + \omega_0\right) > \frac{3\omega_0}{2}$

NYQUIST RATE $= 2\left(\frac{3\omega_0}{2}\right) = 3\omega_0$

(ii) $\frac{3}{2}dx(t)/dt + \frac{1}{4}x(t)\cos\omega_0 t$

→ FOURIER TRANSFORM

TIME ~~FREQ~~. DIFF: $\frac{3}{2}\frac{dx(t)}{dt}$

$j\omega\frac{3}{2}X(\omega)$

MODULATION: $\frac{1}{4}x(t)\cos\omega_0 t$

$\frac{1}{2}\left[\frac{1}{4}X(\omega+\omega_0) + \frac{1}{4}X(\omega-\omega_0)\right]$

$$= \underbrace{j\omega\frac{3}{2}X(\omega)}_{X(j\omega)} + \underbrace{\frac{1}{2}\left[\frac{1}{4}X(\omega+\omega_0) + \frac{1}{4}X(\omega-\omega_0)\right]}_{X_1(j\omega)}$$

$X(j\omega)$ $|\omega| > \omega_0$

$X_1(j\omega)$ $|\omega| > \frac{\omega_0}{2}$

NYQUIST RATE $= 2\omega_0$

(b) $x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)$

$T_s = 10^{-4}$ s

$x(j\omega) * x(j\omega) = 0$ $|\omega| > 8000\pi$

$\omega_s > 2\omega_m$

$\frac{2\pi}{T_s} > 2\omega_m$

$T_s < \frac{2\pi}{2\omega_m} \approx \frac{\pi}{\omega_m}$

$x(t)$ CAN BE RECOVERED IF $T_s < \frac{\pi}{\omega_m}$

$= \frac{\pi}{8000\pi} = 1.25 \times 10^{-4}$ s

SINCE, $T_s < \frac{\pi}{8000\pi}$, & IS TRUE,

THEN $x(t)$ CAN BE RECOVERED FROM $x_p(t)$.

(4) FILTERING

(a) $H(j\omega) = \frac{450}{450+j\omega}$

$C = 5\mu F$

$\frac{1}{450} = RC = R(5\times10^{-6}F)$

$R = 444.4\Omega$

$f = \frac{1}{2\pi RC} = \frac{1}{2\pi(444.4\Omega)(5\times10^{-6}F)}$

$f = 71.6$ Hz

(b) $C = 5\mu F$ $R = 48k\Omega$

$H(j\omega) = \frac{j\omega RC}{1+j\omega RC} = \frac{j\omega(5\times10^{-6})(48\times10^{6})}{1+j\omega(5\times10^{-6})(48\times10^{6})}$

$H(j\omega) = \frac{0.240j\omega}{1+0.240j\omega}$

$f = \frac{1}{2\pi RC} = \frac{1}{2\pi(0.240)} = 0.663$ Hz

(c) $H_{BP} = H_{LP}(j\omega) \cdot H_{HP}(j\omega)$

$= \frac{1}{1+\frac{1}{450}j\omega}\left[\frac{0.240j\omega}{1+0.240j\omega}\right]$

$H_{BP} = \frac{0.240j\omega}{(1+\frac{1}{450}j\omega)(1+0.240j\omega)}$

(d) $C = 5\mu F$ $f = 250$ Hz

$f = \frac{1}{2\pi RC}$

$R = \frac{1}{2\pi fC} = \frac{1}{2\pi(250)(5\times10^{-6})} = 127.3\Omega$

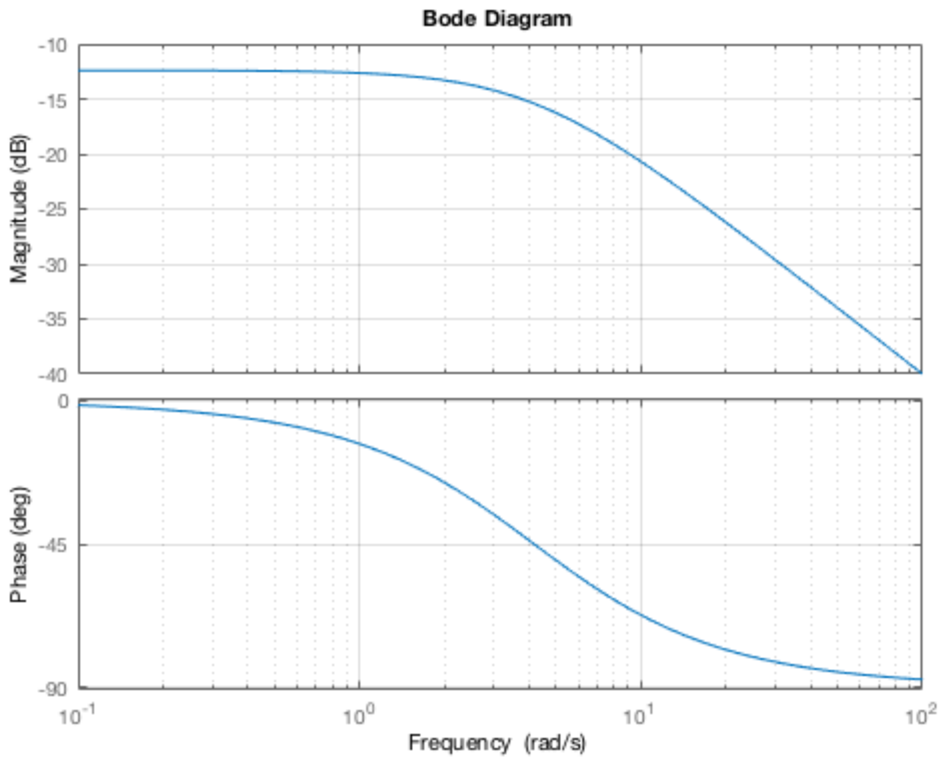
$H_{HP}(j\omega) = \frac{j\omega(127.3)(5\times10^{-6})}{1+j\omega(127.3)(5\times10^{-6})} = \frac{6.37\times10^{-4}j\omega}{1+6.37\times10^{-4}j\omega}$

Table of Contents

4b

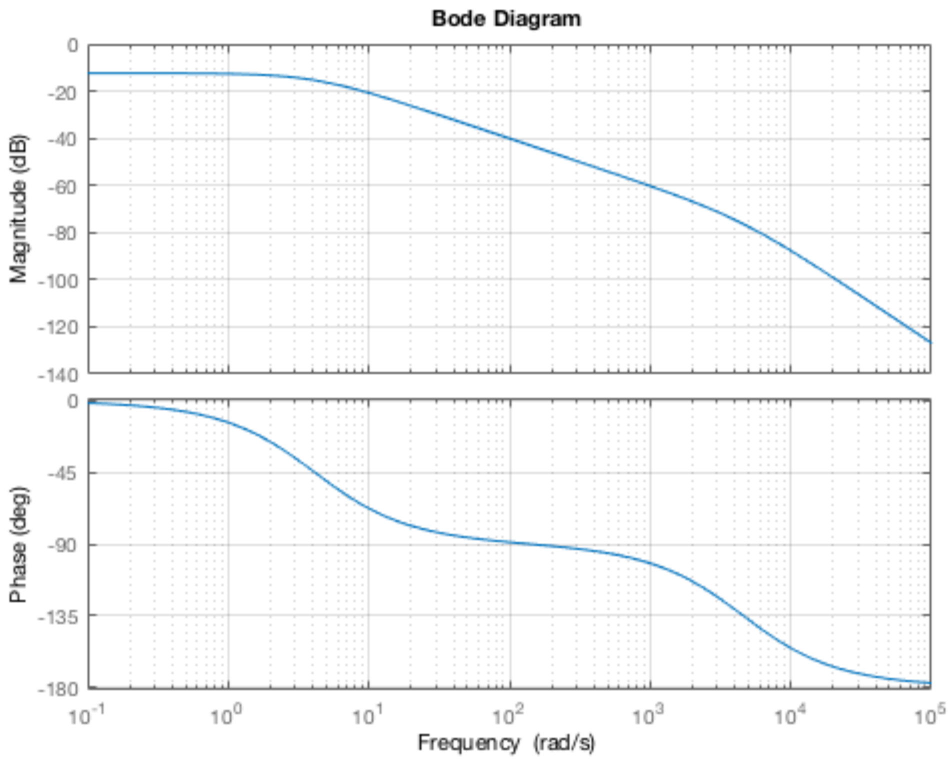
```
% C = 5uF
% R = 48 kOhm
% f = 6.63*10^-4 Hz

close all;
clear all;
clc;

% Transfer function: H(s)=0.240s/0.240s+1

% Numerator
num = [0.240];

% Denominator
den = [0.240 1];

% Transfer function
G = tf(num, den)

% Plot frequency response
figure (1)
bode(G), grid
```

```
G =

     0.24
  ----------
  0.24 s + 1

Continuous-time transfer function.
```



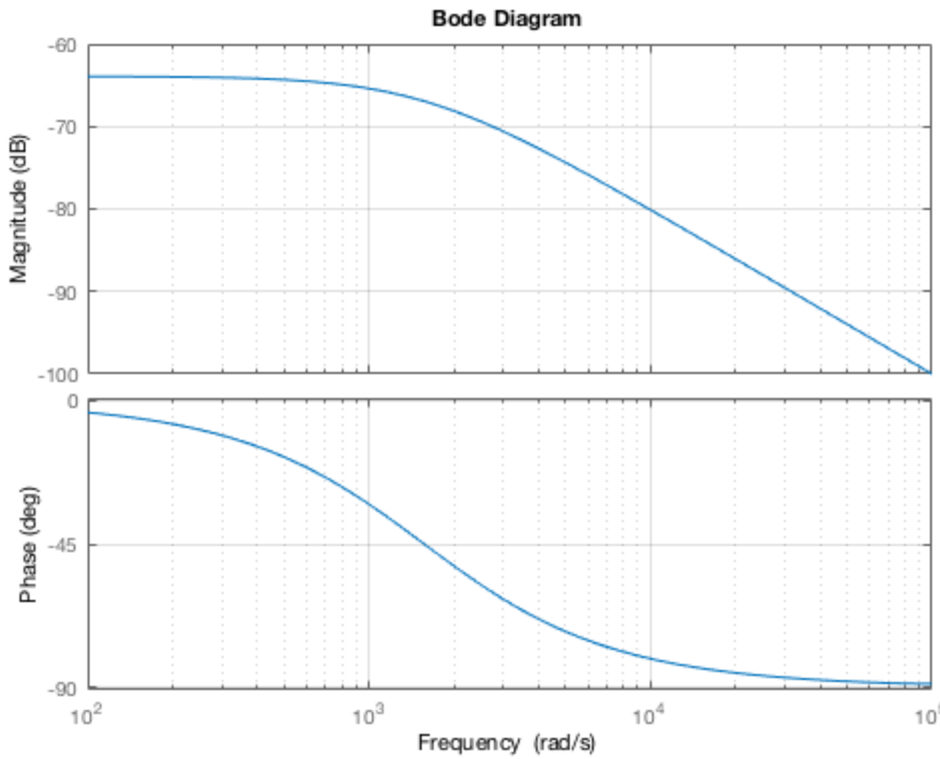
4c

```
% Transfer function: H(s)=0.240s/(0.53*10^-4s^2+0.242s+1)

% Numerator
num1 = [0.240];

% Denominator
den1 = [(0.53*10^-4) 0.242 1];

% Transfer function
G1 = tf(num1, den1)

% Plot frequency response
figure (2)
bode(G1), grid


G1 =

             0.24
  -------------------------
  5.3e-05 s^2 + 0.242 s + 1

Continuous-time transfer function.
```



4d

```
% Transfer function: H(s)=(6.37*10^-4)s/(6.37*10^-4s+1)

% Numerator
num2 = [(6.37*10^-4)];

% Denominator
den2 = [(6.37*10^-4) 1];

% Transfer function
G2 = tf(num2, den2)

% Plot frequency response
figure (3)
bode(G2), grid


G2 =

      0.000637
  ---------------
  0.000637 s + 1

Continuous-time transfer function.
```



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