

Mary Catherine Scott

HW 9

9.4 a,b,c,d, 9.13, 9.23

9.4 For each pair of atomic sentences, give the most general unifier if it exists:

a. $P(A, B, B), P(x, y, z)$.

$\{x/A, y/B, z/B\}$

b. $Q(y, G(A, B)), Q(G(x, x), y)$

Does not exist, x cannot be both A and B , while y has to be $G(A, B)$.

c. $Older(Father(y), y), Older(Father(x), John)$

$\{y/John, x/John\}$

d. $Knows(Father(y), y), Knows(x, x)$

Does not exist, x cannot be both y and $Father(y)$.

//Not assigned, but needed for next problem.

9.6 Write down the logical representations for the following sentences, suitable for use with Generalized Modus Ponens:

a. Horses, cows and pigs are mammals.

$(\forall x)Horse(x) \vee Cow(x) \vee Pig(x) \Rightarrow Mammal(x)$

b. An offspring of a horse is a horse.

$Offspring(x, y) \wedge Horse(y) \Rightarrow Horse(x)$

c. Bluebeard is a horse.

$Horse(Bluebeard)$

d. Bluebeard is Charlie's parent.

$Parent(Bluebeard, Charlie)$

e. Offspring and parent are inverse relations.

$Parent(x, y) \Rightarrow Offspring(y, x)$

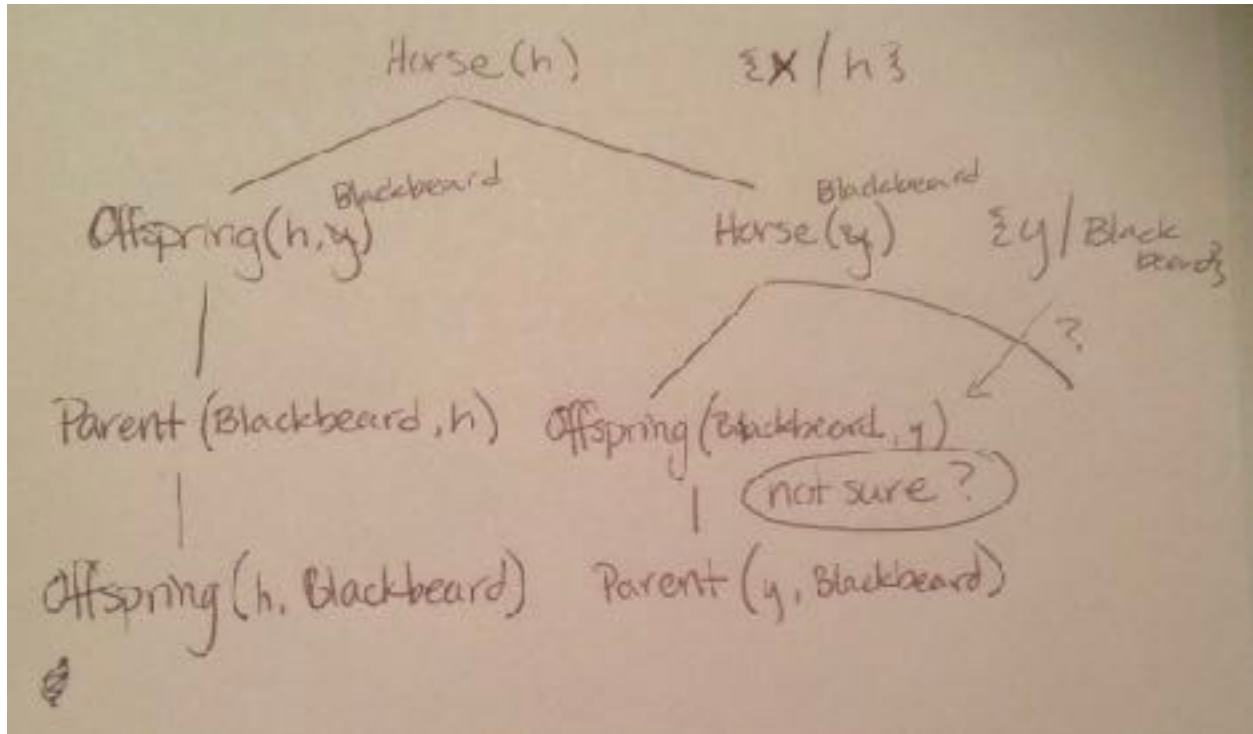
$Offspring(x, y) \Rightarrow Parent(y, x)$

f. Every mammal has a parent.

$Mammal(x) \Rightarrow Parent(G(x), x)$

9.13 In this exercise, use the sentences you wrote in Exercise 9.6 to answer a question by using a backward-chaining algorithm.

- a. Draw the proof tree generated by an exhaustive backward-chaining algorithm for the query $(\exists h)Horse(h)$, where clauses are matched in the order given.



- b. What do you notice about this domain?

We end up in some weird loops such that it looks like we will never find an answer.

- c. How many solutions for h actually follow from your sentences?

Looks like it can be proved that Blackbeard and Charlie are horses.

- d. Can you think of a way to find all of them?(Hint: See Smith et al (1986).)

9.23 From “Horses are animals” it follows that “The head of a horse is the head of an animal.” Demonstrate that this inference is valid by carrying out the following steps:

- a. Translate the premise and the conclusion into the language of first-order logic. Use three predicates: $\text{HeadOf}(h,x)$ (meaning “h is the head of x”), $\text{Horse}(x)$, and $\text{Animal}(x)$.

$$\text{Horse}(x) \Rightarrow \text{Animal}(x)$$

$$\text{HeadOf}(h,x) \wedge \text{Horse}(x) \Rightarrow \text{HeadOf}(h,x) \wedge \text{Animal}(x)$$

- b. Negate the conclusion, and convert the premise and the negated conclusion into conjunctive normal form.

$$\neg [\text{HeadOf}(h,x) \wedge \text{Horse}(x) \rightarrow \text{HeadOf}(h,x) \wedge \text{Animal}(x)]$$

$$\neg [\neg (\text{HeadOf}(h,x) \wedge \text{Horse}(x)) \vee (\text{HeadOf}(h,x) \wedge \text{Animal}(x))]$$

$$\neg [\neg \text{HeadOf}(h,x) \vee \neg \text{Horse}(x) \vee (\text{HeadOf}(h,x) \wedge \text{Animal}(x))]$$

$$\text{HeadOf}(h,x) \wedge \text{Horse}(x) \wedge \neg (\text{HeadOf}(h,x) \wedge \text{Animal}(x))$$

$$\begin{array}{ll} \textcircled{1} & \text{HeadOf}(h,x) \qquad \neg \text{HeadOf}(h,x) \vee \neg \text{Animal}(x) \\ \textcircled{2} & \text{Horse}(x) \\ \textcircled{3} & \neg \text{HeadOf}(h,x) \vee \neg \text{Animal}(x) \end{array}$$

$$\text{Horse}(x) \Rightarrow \text{Animal}(x)$$

$$\textcircled{4} \quad \neg \text{Horse}(x) \vee \text{Animal}(x)$$

- c. Use resolution to show that the conclusion follows from the premise.

Resolve equation 1 with equation 3 to get $\neg \text{Animal}(x)$ as equation 5

Resolve equation 5 with equation 4 to get $\neg \text{Horse}(x)$ as equation 6

Resolve equation 6 with equation 2 to get an empty set, a contradiction.