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Ch 13 HW

13.7 Consider the set of all possible five-card poker hands dealt fairly from a standard deck of fifty-two cards.

a. How many atomic events are there in the joint probability distribution (i.e., how many five-card hands are there)?

   52*51*50*49*48 = 311,875,200

b. What is the probability of each atomic event?

   1:311875200

c. What is the probability of being dealt a royal straight flush? Four of a kind?

   Royal Flush - 1:77968800
   Four of a kind - 13:23990400

13.8 Given the full joint distribution shown in Figure 13.3, calculate the following:
Known According to

a. P(toothache)

   0.108 + 0.012 + 0.016 + 0.064 = 0.2

b. P(Cavity)

   0.108 + 0.012 + 0.072 + 0.008 = 0.2

c. P(Toothache | cavity)

   P(toothache AND cavity) -> 0.108 + 0.012 = 0.12
   P(toothache AND cavity)/P(cavity) -> 0.12/0.2 = 0.6

d. P(Cavity | toothache V catch)

   P(toothache V catch) -> 0.108 + 0.012 + 0.016 + 0.064 + 0.072 + 0.144 = 0.416
   P(Cavity AND (toothache V catch)) -> 0.108 + 0.012 + 0.072 = 0.192
   P(Cavity AND (toothache V catch)) / P(toothache V catch) ->
   0.192 / 0.416 = 0.4615

13.13 Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present(P(+|virus)= 0.95), but has a 10% false positive rate(P(+|~virus) = 0.1) (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus(P(+|virsu) = 0.9), but has a 5% false positive rate(P(+|~virus) = 0.05). The two tests use independent methods of identify8ing the virus. The virus is carried by 1% of all people(P(virus) = 0.01). Say that a person is tested for the virus using only one of the tests, and that test comes back positive for carrying the virus. Which test returning positive is more indicative of someone really carrying the virus? Justify your answer mathematically.

Derived From Problem:
TestA:
    $P(p|v) = 0.95$
    $P(p|\sim v) = 0.1$
TestB:
    $P(p|v) = 0.9$
    $P(p|\sim v) = 0.05$
General
    $P(v) = 0.01$
    $P(\sim v) = 0.99$
Find for both A & B:
    $P(v|p) = (P(p|v) * P(v)) / P(p)$

Solution Test A:

$$
\begin{aligned}
P(p) &= P(p|v)*P(v) + P(p|\sim v)*P(\sim v) \\
&= (0.95)*(0.01) + (0.1)*(0.99) \\
&= 0.0095 + 0.099 \\
&= 0.1085
\end{aligned}
$$

$$
\begin{aligned}
P(v|p) &= (P(p|v) * P(v)) / P(p) \\
&= (0.95*0.01) / 0.1085 \\
&= 0.0875576
\end{aligned}
$$

Solution Test B:

$$
\begin{aligned}
P(p) &= P(p|v)*P(v) + P(p|\sim v)*P(\sim v) \\
&= (0.9)*(0.01) + (0.05)*(0.99) \\
&= 0.009 + 0.0495 \\
&= 0.0585
\end{aligned}
$$

$$
\begin{aligned}
P(v|p) &= (P(p|v) * P(v)) / P(p) \\
&= (0.9*0.01) / 0.0585 \\
&= 0.153846
\end{aligned}
$$

When looking strictly at probabilities we see that for test A $P(v|p) = 0.088$ and for test B $P(v|p) = 0.154$. This indicates that test B is more accurate. There is a 15% chance that if you get a positive test result you have the virus vs test A in which there is only an 8.8 % chance.

13.18 Suppose you are given a bag containing n unbiased coins. You are told that n-1 of these coins are normal, with head on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

a. Suppose you reach into the bag, pick out a coin at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?

    Find $P(f|H)$ ->
        $P(H|f)*P(f) / P(H)$
    $p(f) = 1/n$
    $p(H|f) = 1$
    $p(H|\sim f) = 1/2$

$P(H) = P(H|f)*P(f) + P(H|\sim f)*P(\sim f)$

$= 1*(1/n) + 1/2*(1-1/n)$

$= 1/n + 1/2 - 1/2n = 2/2n + 1n/2n - 1/2n = (n+1)/2n$

$P(H|f)*P(f) / P(H) = 1*(1/n) / ((n+1)/2n)$

$= (1/n) * (2n/(n+1))$

$= 2n / (n(n+1))$

$= 2 / (n+1)$

b. Suppose you continue flipping the coin for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin?

$P(f|H\text{^}k)$

Find $P(f|H\text{^}k)$ ->

$P(H\text{^}k|f)*P(f) / P(H\text{^}k)$

$p(f) = 1/n$

$p(H\text{^}k|f) = 1\text{^}k$

$p(H\text{^}k|\sim f) = 1/2\text{^}k$

$P(H\text{^}k) = P(H\text{^}k|f)*P(f) + P(H\text{^}k|\sim f)*P(\sim f)$

$= 1*(1/n) + 1/2*(1-1/n)$

$= 1/n + 1/2 - 1/2n = 2/2n + 1n/2n - 1/2n = (n+1)/2n$

$P(H\text{^}k|f)*P(f) / P(H\text{^}k) = 1*(1/n) / ((n+1)/2n)$

$= (1/n) * (2n/(n+1))$

$= 2n / (n(n+1))$

$= 2 / (n+1)$

c. Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns fake if all k flips come up heads; otherwise it returns normal. What is the (unconditional) probability that this procedure makes an error?