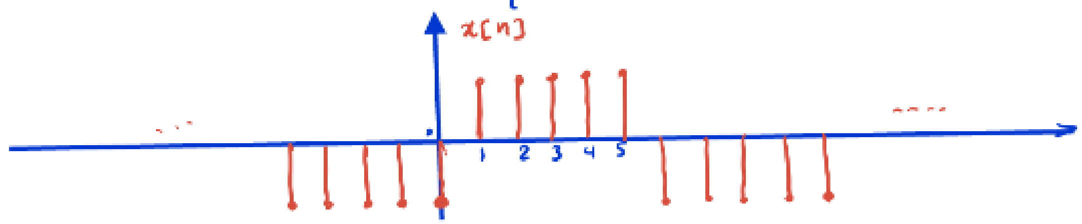
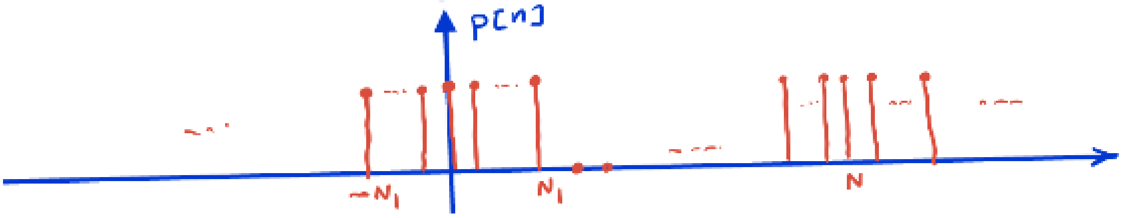
ادامه حل تکلیف شماره ۳ درس تجزیه و تحلیل
نیمسال اول ۱۴۰۰-۱۳۹۹

۶) الف -

$N = 10 \qquad x[n] = \begin{cases} -1 & -4 \le n \le 0 \\ 1 & 1 \le n \le 5 \end{cases}$





$$p[n] \overset{FS}{\longleftrightarrow} C_k = \begin{cases} \frac{1}{N} \cdot \frac{\sin[2\pi k (N_1 + 1/2)/N]}{\sin \pi k/N} & k \neq 0, \pm N, \dots \\ \frac{2N_1+1}{N} & k = 0, \pm N, \dots \end{cases}$$

$N_1 = 2$

$N = 10$

$$x[n] = \underbrace{p[n-3]}_{e^{-jk(\frac{2\pi}{N})3} C_k} - \underbrace{p[n+2]}_{e^{+jk(\frac{2\pi}{N})2} C_k}$$

$$x[n] \overset{FS}{\longleftrightarrow} a_k = \left[e^{-jk(\frac{2\pi}{10})3} - e^{jk(\frac{2\pi}{10})2} \right] C_k$$

ب) $N = 4 \quad , \quad x[n] = 1 - \sin(\frac{\pi n}{4}) \qquad 0 \le n \le 3$

$$a_k = \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-jk(\frac{2\pi}{4})n} = \frac{1}{4}\left[1 + (1 - \tfrac{\sqrt{2}}{2}) e^{-jk\pi/2} + 0 + (1 - \tfrac{\sqrt{2}}{2}) e^{-jk3\pi/2} \right]$$

$$a_k = \frac{1}{4}\left[1 + 2(1 - \tfrac{\sqrt{2}}{2})(-1)^k \cos k\pi/2 \right]$$

راه دیگر:

$$x_1[n] = 1 \longleftrightarrow \sum_{\ell} \delta(k - 4\ell)$$

$$x_2[n] = \sin \frac{\pi n}{4} = \underbrace{\frac{1}{2j} e^{j\pi n/4}}_{\frac{1}{4j[e^{-j(2k-1)\pi/4} - 1]}} - \underbrace{\frac{1}{2j} e^{-j\pi n/4}}_{\frac{-1}{4j[e^{-j(2k+1)\pi/4} - 1]}}$$

$$x[n] = x_1[n] + x_2[n] \overset{FS}{\longleftrightarrow} \sum_{\ell} \delta(k - 4\ell) + \frac{1}{4j[e^{-j(2k-1)\pi/4} - 1]} - \frac{1}{4j[e^{-j(2k+1)\pi/4} - 1]}$$

$$\begin{cases} a_0 = \dots \\ a_1 = 1/4 \\ a_2 = -1/4 + \sqrt{2}/4 \\ a_3 = 1/4 \end{cases}$$

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$$N = 8 \qquad a_k = \begin{cases} \sin k\pi/3 & 0 \le k \le 6 \\ 0 & k = 7 \end{cases}$$

$$x[n] = \sum_{k=0}^{7} a_k e^{jk(\frac{2\pi}{8})n} = 0 + \frac{\sqrt{3}}{2} e^{j\pi/4\, n} + \frac{\sqrt{3}}{2} e^{j\pi/2\, n} + 0 + \left(-\frac{\sqrt{3}}{2}\right) e^{j\pi n} - \frac{\sqrt{3}}{2} e^{j\frac{5\pi}{4} n} + 0 + 0$$

$$x[n] = -j\sqrt{3}\, e^{j\frac{3\pi}{4} n} \left[\sin\frac{\pi}{2} n + \sin\frac{\pi}{4} n \right] \qquad 0 \le n \le 7$$

- تغییر فرکانسی سری فوریه (توان سیگنال در مضارب ω_0)

- تغییر سیستمی

$x(t)$ با دوره تناوب T ضرایب a_k

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) \qquad b_k = a_k H(jk\omega_0)$$

$$H(j\omega) = \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} dt$$

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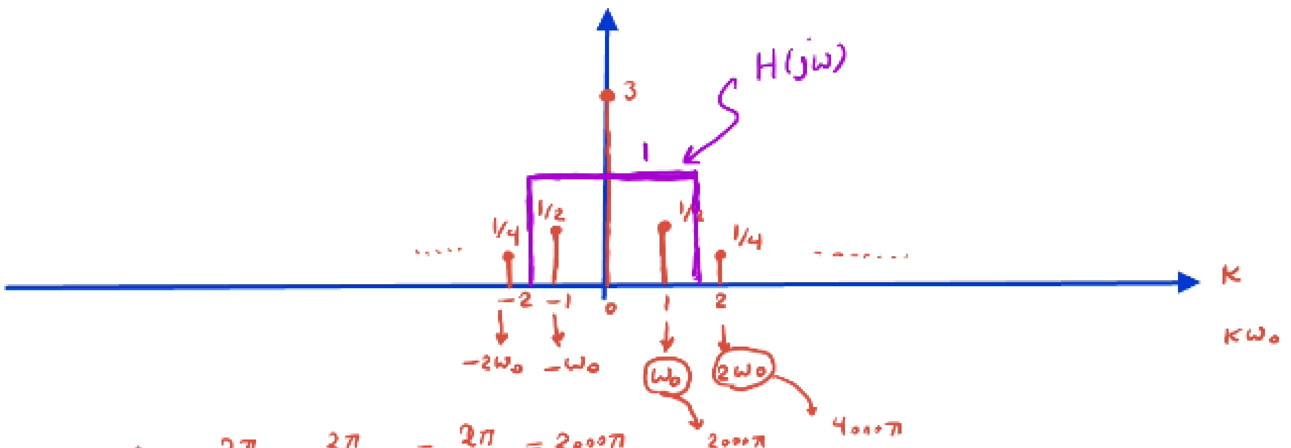
$$H(j\omega) = \begin{cases} 1 & |\omega| < 3000\pi \\ 0 & |\omega| > 3000\pi \end{cases}$$

$x(t)$ متناوب $\qquad T = 1\ \text{msec} \qquad a_k = \begin{cases} 3 & k = 0 \\ (1/2)^{|k|} & \text{oth.} \end{cases}$

$$b_k = a_k H(jk\omega_0)$$



$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1\,\text{mse}} = \frac{2\pi}{10^{-3}} = 2000\pi$$

$$b_0 = a_0 H(0) = a_0 = 3$$

$$b_1 = a_1 \underbrace{H(j\omega_0)}_{1} = \frac{1}{2} \times 1 = 1/2$$

$$b_{-1} = a_{-1} \underbrace{H(-j\omega_0)}_{1} = 1/2 \times 1 = 1/2$$

$$b_k = 0 \qquad k \ne 0, \pm 1$$

$$y(t) = \sum_k b_k e^{jk\omega_0 t} = 3 + \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t} \qquad \omega_0 = 200\pi$$

$$\boxed{y(t) = 3 + \cos(200\pi t)}$$

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19) LTI: $y[n] - 3y[n-1] = x[n]$

$$x[n] = \begin{cases} 1 & n = \pm 1, 0 \\ 0 & n = \pm 2, 3 \end{cases} \qquad N = 6$$

$$b_k = a_k H(e^{jk\omega_0})$$

$$a_k = \frac{1}{6} \sum_{n=-2}^{3} x[n] e^{-jk\omega_0 n} \underset{\omega_0 = \frac{2\pi}{6}}{=} \frac{1}{6}\left[e^{jk\pi/3} + 1 + e^{-jk\pi/3}\right] = \frac{1}{6}\left[1 + 2\cos\frac{k\pi}{3}\right]$$

$k = 0, 1, \dots, 5$

تابع ویژه z^n $\longrightarrow$ $\boxed{\begin{matrix}\text{LTI} \\ h[n]\end{matrix}}$ $\longrightarrow$ $H(z) z^n$ $\qquad H(z) = \sum_{n=-\infty}^{+\infty} h[n] z^{-n}$

$$\begin{cases} x[n] = z^n \\ y[n] = H(z) z^n \end{cases} \xrightarrow{\text{در معادله}} H(z).z^n - 3H(z) z^{n-1} = z^n$$

$$H(z).z^{n-1}\left[z - 3\right] = z^n$$

$$H(z) = \frac{z}{z-3} \longrightarrow H(e^{jk\omega_0}) = \frac{e^{jk\omega_0}}{e^{jk\omega_0} - 3}$$

$$b_k = a_k H(e^{jk\omega_0}) = \frac{1}{6}\left[1 + 2\cos k\pi/3\right] . \frac{e^{jk\pi/3}}{e^{jk\pi/3} - 3}$$

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10) $$H(j\omega) = \begin{cases} 1 & |\omega| \leq W \\ 0 & oth. \end{cases}$$

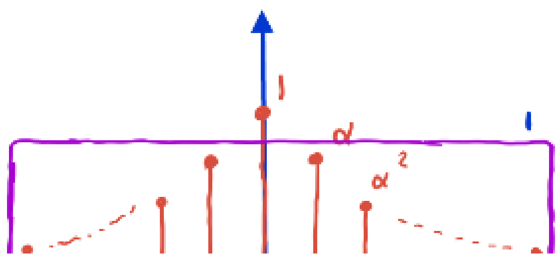
$$x(t) = \sum_{k=-\infty}^{+\infty} \alpha^{|k|} e^{jk(\frac{\pi}{4})t} \qquad 0 < \alpha < 1$$

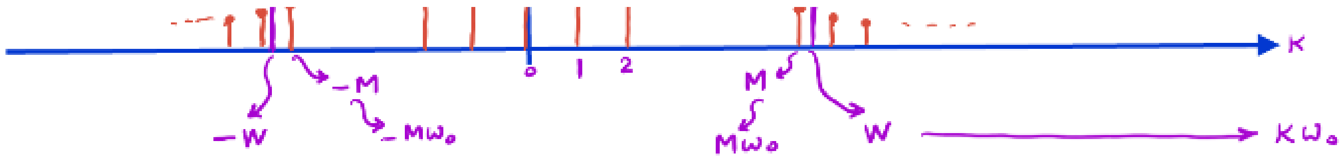
$W = ?$ $\leftarrow$ تا حداقل ۹۰٪ توان ورودی در خروجی قرار گیرد

توان متوسط سیگنال در یک دوره تناوب $= \frac{1}{T}\int_T |x(t)|^2 dt \underset{\text{از خواص سری فوریه}}{=} \sum_k |a_k|^2$

$$= \sum_k \left|\alpha^{|k|}\right|^2 = 2\sum_{k=0}^{\infty} \alpha^{2k} \underbrace{-1}_{\text{چون درصفر دوبار شمارش شده}} = \frac{2}{1-\alpha^2} - 1$$

$$P_{av} = \frac{1+\alpha^2}{1-\alpha^2}$$





$$b_k = a_k H(jk\omega_0) = \alpha^{|k|} \overbrace{H(jk\pi/4)}^{1} = \begin{cases} \alpha^{|k|} & -M \le k \le M \\ 0 & oth. \end{cases}$$

$$\text{توان خروجی سیستم} = \sum_{k=-M}^{M} |b_k|^2 = \sum_{k=-M}^{M} \left|\alpha^{|k|}\right|^2 = 2\sum_{k=0}^{M} \alpha^{2k} - 1 = 2\left[\frac{1-(\alpha^2)^{M+1}}{1-\alpha^2}\right] - 1$$

$$\text{توان خروجی} = 0.9 \,(\text{توان ورودی})$$

$$2\left[\frac{1-\alpha^{2(M+1)}}{1-\alpha^2}\right] - 1 = \frac{9}{10}\left(\frac{1+\alpha^2}{1-\alpha^2}\right)$$

$$10\left[2\,[1-\alpha^{2(M+1)}] - (1-\alpha^2)\right] = 9(1+\alpha^2)$$

$$10\left[1 - 2\alpha^{2(M+1)} + \alpha^2\right] = 9(1+\alpha^2)$$

$$10(1+\alpha^2) - 20\alpha^{2(M+1)} = 9(1+\alpha^2)$$

$$1+\alpha^2 = 20\,\alpha^{2(M+1)} \longrightarrow \ln\left(\frac{1+\alpha^2}{20}\right) = \ln\alpha^2 + \ln\alpha^{2M}$$

$$\ln\alpha^{2M} = \ln\left(\frac{1+\alpha^2}{20\alpha^2}\right) \qquad M = \left[\frac{\ln\left(\frac{1+\alpha^2}{20\alpha^2}\right)}{2\ln\alpha}\right]$$

$$M = [2.6] = 3 \longleftarrow \alpha = 0.7 \qquad \text{مثلاً)}$$

$$\boxed{W \geqslant M\omega_0 = M\pi/4 = \frac{\pi}{4}\left[\frac{\ln\left(\frac{1+\alpha^2}{20\alpha^2}\right)}{2\ln\alpha}\right]}$$

$$x[n] \underset{N}{\overset{FS}{\longleftrightarrow}} a_k \qquad y[n] = x[n] - x[n-1] \qquad \text{b)} \qquad \text{3-48)}$$

$$b_k = a_k - a_k e^{-jk\omega_0} = a_k\left[1 - e^{-jk\frac{2\pi}{N}}\right]$$

$$y[n] = x[n] - x[n - N/2] \qquad \text{c)}$$

N زوج

$$y[n+N] = x[n+N] - x[n+N-N/2]$$

$$= x[n] - x[n+N/2]$$

$$= x[n] - x[n-N/2] = y[n]$$

$$y[n] \overset{FS}{\longleftrightarrow} a_k - a_k e^{-jk\omega_0(N/2)} \qquad \omega_0 = \frac{2\pi}{N} \longrightarrow \omega_0\frac{N}{2} = \pi$$

$$b_k = a_k - a_k e^{-jk\pi} = a_k - a_k(-1)^k = a_k[1-(-1)^k]$$

$$b_k = \begin{cases} 0 & \text{زوج } k \\ 2a_k & \text{فرد } k \end{cases}$$

$$y[n] = x[n] + x[n+N/2]$$ تمرین :

h)
$$y[n] = \begin{cases} x[n] & \text{زوج } n \\ 0 & \text{فرد } n \end{cases}$$

$$y[n] = \left(\frac{1+(-1)^n}{2}\right)x[n] = \frac{1}{2}x[n] + \frac{1}{2}\underbrace{(-1)^n}_{e^{j\pi n}}x[n]$$

$$y[n] = \frac{1}{2}x[n] + \frac{1}{2}e^{j\left(\frac{2\pi}{N}\right)\frac{N}{2}n}x[n]$$

$$\frac{1}{2}x[n] \xrightarrow{FS} \frac{1}{2}a_k \qquad \frac{1}{2}e^{j\left(\frac{2\pi}{N}\right)\frac{N}{2}n}x[n] \xrightarrow{FS} \frac{1}{2}a_{k-N/2}$$

$$\underbrace{e^{j\left(\frac{2\pi}{N}\right)Mn}x[n] \longleftrightarrow a_{k-M}}$$

$$b_k = \frac{1}{2}a_k - \frac{1}{2}a_{k-N/2}$$