Parity Space Residual Generator for Non-uniformly Sampled Systems: Direct Designs

Iman Izadi, Sirish L. Shah and Tongwen Chen

Abstract— Non-uniform sampled-data systems are widely found in industry. In these systems the process output is sampled and the control input is generated at non-uniformly distributed time instants. In this paper, an optimal parity based residual generator is developed for non-uniform sampled-data systems. In the direct approach used here, the intersample behavior of fault and unknown inputs is captured by introducing operators that map continuous-time signals to discrete-time signals.

I. Introduction

Modern industrial control systems are widely exposed to faults which can cause undesirable performance, instability, total failure of the system and even dangerous situations. In order to maintain quality, reliability and safety, faults should be promptly detected and identified so that appropriate remedies can be applied. The problem of fault detection and isolation (FDI) has widely been studied in the past decades and numerous design methods are available in the literature [?], [?].

Traditionally a sampled-data controller/FDI is designed using indirect approach. In the indirect approach, design is carried out through an intermediate step of discretizing a continuous-time system. The discretization usually involves some level of approximation which makes the design unacceptable if sampling intervals are large. As an alternative approach, direct methods have been used to design sampled-data controllers and more recently sampled-data FDI [?], [?], [?]. In direct approach, a discrete-time controller/FDI is directly designed for the continuous-time process taking into account the hybrid nature of the sampled-data system. Therefore, no approximation is involved in direct designs.

In this paper, we develop an optimal FDI for non-uniform sampled-data systems using direct design. The FDI which is based on the parity space approach is designed in three steps: First the relationship between the input and output of the process in a certain time frame is obtained. Then a residual generator is constructed based on the input-output relation. Finally the design parameter is calculated by minimizing a performance index. As the main assumption in direct approach, the fault and disturbance input can arbitrarily vary over time. This is in contrast to the assumption usually made in indirect approach that fault and disturbance signals are constant over the sampling intervals. The output sampling and control updating times can also be arbitrarily distributed over time and there is no need for them to follow a periodic pattern. This makes the residual generator applicable to general non-uniform sampled-data systems.

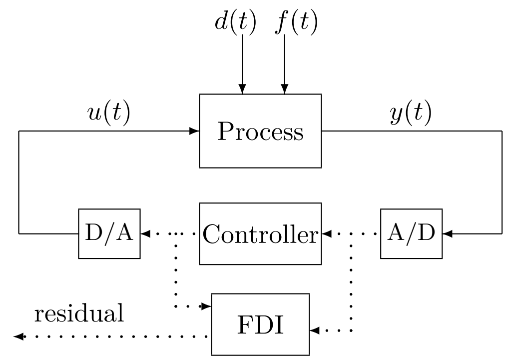

Fig. 1. FDI in a sampled-data scheme

The paper is organized as follows. In Section ??, the parity space approach of residual generation and other required preliminaries are reviewed, and also the discretion of the system is given. Final conclusions and remarks are given in Section ??.

II. Preliminaries

A. Parity-space Approach

Consider the discrete-time system

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Ed(k) + Ff(k) \\ y(k) = Cx(k) \end{cases}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector, $u(k) \in \mathbb{R}^{n_u}$ the vector of control input, $y(k) \in \mathbb{R}^{n_y}$ the vector of process output, $d(k) \in \mathbb{R}^{n_d}$ the vector of unknown inputs (e.g., disturbance, noise, model mismatch, etc.) and $f(k) \in \mathbb{R}^{n_f}$ the vector of faults to be detected. A, B, C, E, and F are known matrices of appropriate dimensions.

For a fixed number s, referred to as the order of parity relation, define $y_s(k)$ as

$$y_s(k) = \begin{bmatrix} y(k-s) \\ y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix}_{(s+1)n_y \times 1},$$

$u_s(k)$, $d_s(k)$ and $f_s(k)$ are also defined similarly. It can be easily shown that $y_s(k)$, $u_s(k)$, $d_s(k)$ and $f_s(k)$ are related through the following expression

$$y_s(k) = H_o x(k-s) + H_u u_s(k) + H_d d_s(k) + H_f f_s(k), \quad (1)$$

where

$$H_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix},$$

$$H_u = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ CB & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ CA^{s-1}B & CA^{s-2}B & \cdots & CB & 0 \end{bmatrix}.$$

H_d and H_f are defined similar to H_u. Based on (??), a parity space based residual generator can be formulated as

$$r(k) = v_s\big(y_s(k) - H_u u_s(k)\big),$$

where $r(k) \in \mathbb{R}$ is the residual.

B. Operator Norm and Adjoint Operator

Consider Hilbert spaces $\mathcal{X}$ and $\mathcal{Y}$ with inner products $\langle x_1, x_2\rangle_{\mathcal{X}}$, $x_1, x_2 \in \mathcal{X}$ and $\langle y_1, y_2\rangle_{\mathcal{Y}}$, $y_1, y_2 \in \mathcal{Y}$, respectively. $\mathcal{X}$ and $\mathcal{Y}$ are not necessarily the same space, and even if they are, the inner products can be different. The norms of members of $\mathcal{X}$ and $\mathcal{Y}$ are defined using the corresponding inner products as $\|x\|_{\mathcal{X}}^2 = \langle x, x\rangle_{\mathcal{X}}$, $x \in \mathcal{X}$ and $\|y\|_{\mathcal{Y}}^2 = \langle y, y\rangle_{\mathcal{Y}}$, $y \in \mathcal{Y}$. Also assume that $T : \mathcal{X} \to \mathcal{Y}$ is a bounded operator that maps $\mathcal{X}$ to $\mathcal{Y}$. The adjoint of T, denoted by T^*, is the unique bounded operator mapping $\mathcal{Y}$ to $\mathcal{X}$ that satisfies

$$\langle Tx, y\rangle_{\mathcal{Y}} = \langle x, T^* y\rangle_{\mathcal{X}}, \quad x \in \mathcal{X},\ y \in \mathcal{Y}.$$

It can be easily shown that the adjoint of a constant matrix is its transpose.

The induced norm of the operator T is defined by

$$\|T\| = \sup_{\|x\|_{\mathcal{X}} \le 1} \|Tx\|_{\mathcal{Y}}.$$

It is a well known fact that

$$\|T\|^2 = \|T^*\|^2 = \|T^*T\| = \|TT^*\|. \quad (2)$$

C. Process Description

Consider an LTI, strictly proper, continuous-time process with the following state-space representation

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ed(t) + Ff(t) \\ y(t) = Cx(t) \end{cases} \quad (3)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $u(t) \in \mathbb{R}^{n_u}$ the known vector of control input, $y(t) \in \mathbb{R}^{n_y}$ the vector of process output, $d(t) \in \mathbb{R}^{n_d}$ the vector of unknown input (to represent disturbance, noise, model mismatch and other uncertainties) and $f(t) \in \mathbb{R}^{n_f}$ the vector of fault to be detected. A, B, C, E and F are known matrices of appropriate dimensions. The assumption of strictly properness is standard in the sampled-data literature and necessary for boundedness of the sampling operator. In practice, because of antialiasing filters that are used before sampling, the systems are always strictly proper.

In the sampled-data framework, the measured output of the continuous-time process is sampled and transferred to a computer using an analog-to-digital converter (A/D). The control signal is then generated by the computer and applied to the process using a digital-to-analog converter (D/A). In a non-uniform sampled-data system, the output is sampled at non-uniformly spaced time instants and/or the control input is generated at non-uniformly spaced time instants. In this paper, for simplicity we assume that the input/output are generated/sampled synchronously at the same times.

Let $T = \{t_0, t_1, t_2, \cdots\}$ be the set of time instants when the output is sampled (or the input is updated). Let $\ell_T(\mathbb{Z})$ be the vector space of all discrete-time signals corresponding to the time instants in T. Notice that the discrete-time signals in $\ell_T(\mathbb{Z})$ have no practical meaning unless the corresponding time instants, given by T, are known. Let $\mathcal{L}(\mathbb{R})$ be the vector space of all continuous-time signals.

The D/A converter is modeled by non-uniform (zero-order) hold operator $H_T : \ell_T(\mathbb{Z}) \to \mathcal{L}(\mathbb{R})$ defined as

$$u(t) = H_T \upsilon_T(k) = \upsilon_T(k), \quad t_k \le t < t_{k+1}.$$

Here $\upsilon_T(k)$ represent the discrete-time input.

The A/D converter is also modeled by non-uniform sampling operator $S_T : \mathcal{L}(\mathbb{R}) \to \ell_T(\mathbb{Z})$ defined as

$$\psi_T(k) = S_T y(t) = y(t_k),$$

where $\psi_T(k)$ denotes the discrete-time output.

III. Conclusions

In this paper, we presented a direct method to design optimal parity based residual generator for non-uniform sampled-data systems. In direct design, no assumption is made on fault and disturbance input and hence there is no approximation in the solution. As a result, the relationship between fault/disturbance and residual is expressed in terms of an operator rather than a matrix. However, it was shown that the norm of the operator is equal to the norm of a certain matrix. Therefore, the optimization problem can be converted to a regular matrix problem whose solution is known.

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