الف) $X(j\omega) = \dfrac{\sin^2(2\omega)\cos(\omega)}{\omega^2}$ 1

$$= \underbrace{\frac{-1}{(0+j\omega)^2}}_{\mathrm{I}} \cdot \underbrace{\left(\frac{e^{j2\omega}}{2} - \frac{e^{-j2\omega}}{2}\right)^2 \cdot \left(\frac{e^{j2\omega}}{2} + \frac{e^{-j2\omega}}{2}\right)}_{\mathrm{II}}$$

$$\mathrm{I} \xrightarrow{F^{-1}} -t\,u(t)$$

$$\mathrm{II} \xrightarrow{F^{-1}} \frac{1}{2}\big(\delta(t+2) - \delta(t-2)\big) * \frac{1}{2}\big(\delta(t+2) + \delta(t-2)\big)$$

$$= \frac{1}{4}\Big[\big(\delta(t+2) * \delta(t+2)\big) + \big(\delta(t+2) * \delta(t-2)\big) - \big(\delta(t-2) * \delta(t+2)\big) - \big(\delta(t-2) * \delta(t-2)\big)\Big] =$$

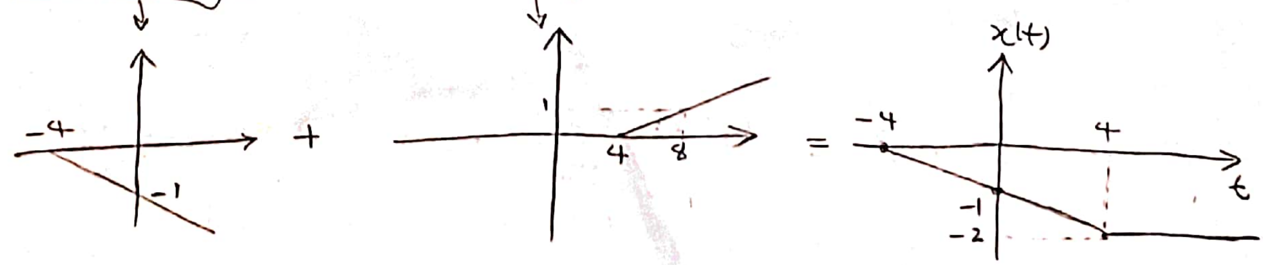
$$= \frac{1}{4}\Big[\delta(t+4) + \cancel{\delta(t)} - \cancel{\delta(t)} - \delta(t-4)\Big] = \frac{1}{4}\delta(t+4) - \frac{1}{4}\delta(t-4)$$

$$x(t) = \mathrm{I} * \mathrm{II} = -t\,u(t) * \left[\frac{1}{4}\delta(t+4) - \frac{1}{4}\delta(t-4)\right] =$$

$$= -\frac{1}{4}\big[t\,u(t) * \delta(t+4)\big] + \frac{1}{4}\big[t\,u(t) * \delta(t-4)\big] =$$

$$= -\frac{1}{4}(t+4)\,u(t+4) + \frac{1}{4}(t-4)\,u(t-4) =$$

$$= \underbrace{\left(-\frac{t}{4} - 1\right) u(t+4)} + \underbrace{\left(\frac{t}{4} - 1\right) u(t-4)}$$



ت) $X(j\omega) = \sum_{k=-\infty}^{+\infty} j^k \, \delta\left(\omega - \frac{k\pi}{2}\right)$ ①

طرح : $\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \xleftrightarrow{F} 2\pi \sum_{k=-\infty}^{+\infty} a_k \, \delta(\omega - k\omega_0)$

$\xrightarrow{\text{انطباق :}}$ $\omega_0 = \frac{\pi}{2}$ $\qquad 2\pi a_k = j^k \Rightarrow a_k = \frac{j^k}{2\pi}$

ن۱ : $x(t) = \sum_{k=-\infty}^{+\infty} \frac{j^k}{2\pi} e^{jk\frac{\pi}{2}t} = \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} j^k e^{j\frac{k\pi}{2}t} = \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} e^{j\frac{k\pi}{2}(t+1)}$

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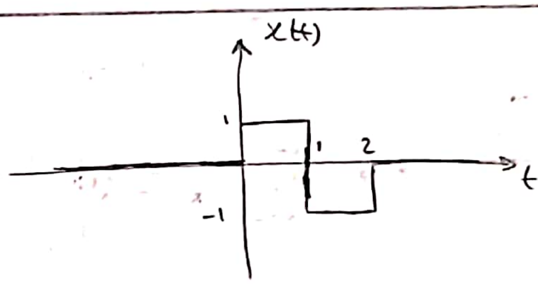
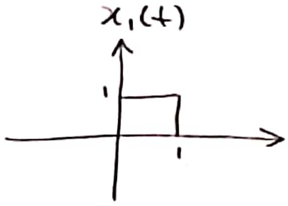
②

الف) $x(t) = \begin{cases} 0 & t<0 \\ 1 & 0<t<1 \\ -1 & 1<t<2 \\ 0 & 2<t \end{cases}$





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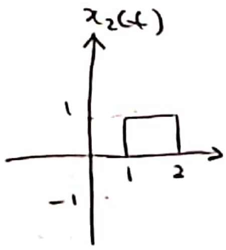
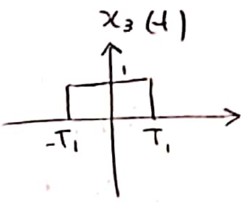


$\Rightarrow x(t) = x_1(t) - x_2(t)$

طرح :



$\xleftrightarrow{F} \frac{2\sin \omega T_1}{\omega} = X_3(j\omega)$

$\xrightarrow{\text{انطباق :}}$ $x_1(t) \overset{T_1=\frac{1}{2}}{=} x_3\left(t-\frac{1}{2}\right) \xleftrightarrow{F} X_1(j\omega) = e^{-j\omega\left(\frac{1}{2}\right)} X_3(j\omega) = \frac{2e^{-j\frac{\omega}{2}} \sin\left(\frac{\omega}{2}\right)}{\omega}$

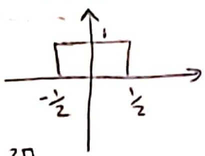
$\quad x_2(t) \overset{T_1=\frac{1}{2}}{=} x_3\left(t-\frac{3}{2}\right) \xleftrightarrow{F} X_2(j\omega) = e^{-j\frac{3\omega}{2}} X_3(j\omega) = \frac{2e^{-j\frac{3\omega}{2}} \sin\left(\frac{\omega}{2}\right)}{\omega}$

$x(t) = x_1(t) - x_2(t) \xleftrightarrow{F} X(j\omega) = X_1(j\omega) - X_2(j\omega)$

$\overset{\text{II}}{\Longrightarrow} X(j\omega) = \frac{2\sin\left(\frac{\omega}{2}\right)\left[e^{-j\frac{\omega}{2}} - e^{-j\frac{3\omega}{2}}\right]}{\omega}$

ب) $x(t) = \frac{2}{\pi} \sum_{n=-\infty}^{+\infty} p(t-4n)$, $p(t) = \begin{cases} 0 & t < -\frac{1}{2} \\ 1 & -\frac{1}{2} < t < \frac{1}{2} \\ 0 & \frac{1}{2} < t \end{cases}$

رسم :



برای قطار پالس داریم : $\sum_{n=-\infty}^{+\infty} p(t-4n)$

$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$T_1 = \frac{1}{2}$

$\Longrightarrow a_k = \frac{\sin k\omega_0 T_1}{\pi k} = \frac{\sin \frac{k\pi}{4}}{k\pi}$

داریم: $x_1(t) = \sum_{n=-\infty}^{+\infty} p(t-4n) = \sum_{k=-\infty}^{+\infty} a_k e^{+jk\omega_0 t}$

$\xrightarrow{F} X_1(j\omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{k\pi}{2}\right) = 2\pi \sum_{k=-\infty}^{+\infty} \frac{\sin \frac{k\pi}{4}}{k\pi} \delta\left(\omega - \frac{k\pi}{2}\right)$

$x(t) \longleftrightarrow X(j\omega) = 4 \sum_{k=-\infty}^{+\infty} \frac{\sin \frac{k\pi}{4}}{k\pi} \delta\left(\omega - \frac{k\pi}{2}\right)$

الف) $h(t) = ?$ (3)

$$H(jw) = \frac{3+jw}{2-w^2+3jw} \xrightarrow{F^{-1}} h(t)$$

$$H(jw) = \frac{(jw+3)}{(jw+2)(jw+1)} = \frac{A}{jw+2} + \frac{B}{jw+1}$$

$$jw = s \longrightarrow H(s) = \frac{(s+3)}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$$

$$A = \lim_{s \to -2} (s+2)H(s) = \frac{s+3}{s+1}\Big|_{s=-2} = \frac{1}{-1} = -1$$

$$B = \lim_{s \to -1} (s+1)H(s) = \frac{s+3}{s+2}\Big|_{s=-1} = \frac{2}{1} = 2$$

$$\Rightarrow H(jw) = \frac{-1}{jw+2} + \frac{2}{jw+1} \xrightarrow{F^{-1}} h(t) = -e^{-2t}u(t) + 2e^{-t}u(t)$$

$$\Rightarrow h(t) = u(t)\left[2e^{-t} - e^{-2t}\right]$$

ب) $\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$ (3)

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{3+jw}{2-w^2+3jw}$$

$$\Rightarrow 2Y(jw) - w^2 Y(jw) + 3jw Y(jw) = 3X(jw) + jwX(jw)$$

$$\Big\downarrow F^{-1} \qquad \left(\text{نکته:} \ \frac{d x^k(t)}{d^k t} \xleftrightarrow{F} (jw)^k X(jw) \right)$$

$$2y(t) + \frac{dy^2(t)}{dt^2} + 3\frac{dy(t)}{dt} = 3x(t) + \frac{dx(t)}{dt}$$

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پ) $x(t) = (1-t)e^{-3t}u(t) = e^{-3t}u(t) - te^{-3t}u(t)$ ③

$$\xrightarrow{F} X(j\omega) = \frac{1}{j\omega+3} - \frac{1}{(j\omega+3)^2} = \frac{j\omega+2}{(j\omega+3)^2}$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega) = \frac{j\omega+2}{(j\omega+3)^2} \cdot \frac{j\omega+3}{(j\omega+1)(j\omega+2)}$$

$$\Rightarrow Y(j\omega) = \frac{1}{(j\omega+3)(j\omega+1)} \xrightarrow{F^{-1}} y(t) = ?$$

$$j\omega = s \Longrightarrow Y(s) = \frac{1}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1}$$

$$B = \lim_{s\to -1}(s+1)Y(s) = \frac{1}{s+3}\Big|_{s=-1} = \frac{1}{2}$$

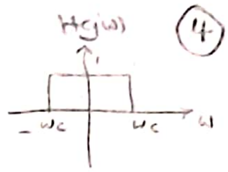
$$A = \lim_{s\to -3}(s+3)Y(s) = \frac{1}{s+1}\Big|_{s=-3} = -\frac{1}{2}$$

$$\Rightarrow Y(j\omega) = \frac{-\frac{1}{2}}{s+3} + \frac{\frac{1}{2}}{s+1}$$

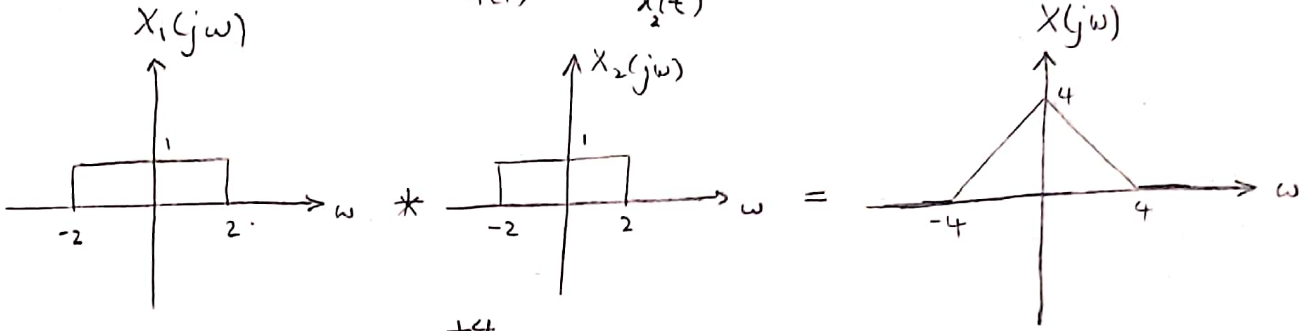
$$\Longrightarrow Y(j\omega) = \frac{-\frac{1}{2}}{j\omega+3} + \frac{\frac{1}{2}}{j\omega+1} \xrightarrow{F^{-1}} y(t) = u(t)\left[-\frac{1}{2}e^{-3t} + \frac{1}{2}e^{-t}\right]$$

(4)

$$h(t) = \frac{\sin(\omega_c t)}{\pi t} \xleftrightarrow{F} H(j\omega) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & |\omega| \geq \omega_c \end{cases}$$



$$x(t) = \left(\frac{\sin(2t)}{\pi t}\right)^2 = \underbrace{\frac{\sin(2t)}{\pi t}}_{x_1(t)} \cdot \underbrace{\frac{\sin(2t)}{\pi t}}_{x_2(t)} \xleftrightarrow{F} X(j\omega) = X_1(j\omega) * X_2(j\omega)$$



انرژی سیگنال ورودی : $\displaystyle \frac{1}{2\pi} \int_{\omega=-4}^{+4} |X(j\omega)|^2 \, d\omega =$

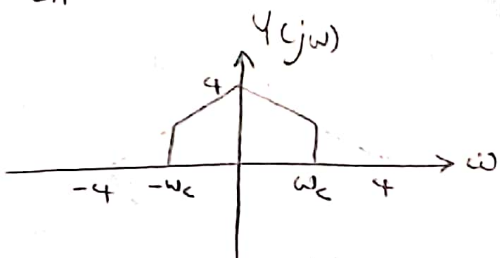
$$= \frac{1}{2\pi}\left[\int_{\omega=-4}^{0} (\omega+4)^2 \, d\omega + \int_{\omega=0}^{4} (4-\omega)^2 \, d\omega\right] =$$

$$= \frac{1}{2\pi}\left[\int_{\omega=-4}^{0} (\omega^2+8\omega+16) \, d\omega + \int_{\omega=0}^{4} (\omega^2-8\omega+16) \, d\omega\right] =$$

$$= \frac{1}{2\pi}\left[\left(\frac{\omega^3}{3}+4\omega^2+16\omega\right)\Big|_{-4}^{0} + \left(\frac{\omega^3}{3}-4\omega^2+16\omega\right)\Big|_{0}^{4}\right] =$$

$$= \frac{1}{2\pi}\left[\frac{64}{3}+\frac{64}{3}\right] = \frac{64}{3\pi}$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$



انرژی سیگنال خروجی : $\displaystyle \frac{1}{2\pi} \int_{\omega=-\omega_c}^{\omega_c} |Y(j\omega)|^2 \, d\omega = \frac{1}{2\pi} \int_{\omega=-\omega_c}^{\omega_c} |X(j\omega)|^2 \, d\omega =$

$$= \frac{1}{2\pi}\left[\left(\frac{\omega^3}{3}+4\omega^2+16\omega\right)\Big|_{-\omega_c}^{0} + \left(\frac{\omega^3}{3}-4\omega^2+16\omega\right)\Big|_{0}^{\omega_c}\right] =$$

$$= \frac{1}{2\pi}\left[+\frac{\omega_c^3}{3} - 4\omega_c^2 - \cancel{16\omega_c} + \frac{\omega_c^3}{3} - 4\omega_c^2 + \cancel{16\omega_c}\right] = \frac{\omega_c^3}{3\pi} - \frac{4\omega_c^2}{\pi}$$

رابطه‌ی انرژی سیگنال‌های ورودی و خروجی : $\frac{w_c^3}{3\pi} - \frac{4w_c^2}{\pi} = \frac{3}{4} \times \frac{64}{3\pi} = \frac{16}{\pi}$

$$\Rightarrow \frac{w_c^3}{3} - 4w_c^2 = 16 \Rightarrow w_c^3 - 12w_c^2 - 48 = 0 \Rightarrow w_c = -0,158 + 1,968j$$

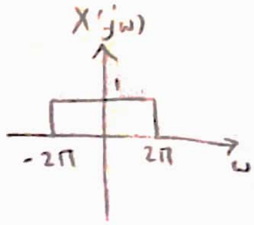
$$\pm\ 0,158 - 1,968j$$

← جواب های مقبول

الف)

5

$$x(t) = \frac{\sin(2\pi t)}{\pi t} \xleftrightarrow{F} X(j\omega) = \begin{cases} 1 & |\omega| < 2\pi \\ 0 & |\omega| > 2\pi \end{cases}$$

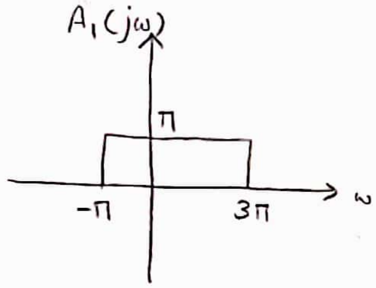


$$p(t) = \cos(\pi t) \xleftrightarrow{F} P(j\omega) = \pi[\delta(\omega-\pi) + \delta(\omega+\pi)]$$

$$a(t) = x(t) \times p(t) \Longrightarrow A(j\omega) = X(j\omega) * P(j\omega) =$$

$$= X(j\omega) * [\pi\delta(\omega-\pi) + \pi\delta(\omega+\pi)]$$

$$= \underbrace{\pi X(j(\omega-\pi))}_{A_1(j\omega)} + \underbrace{\pi X(j(\omega+\pi))}_{A_2(j\omega)}$$



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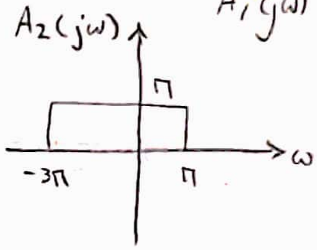

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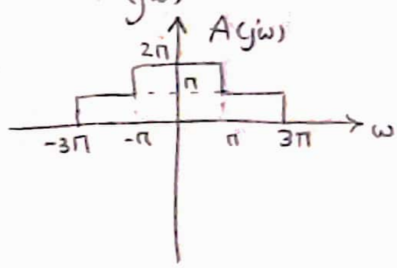
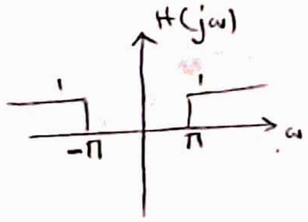


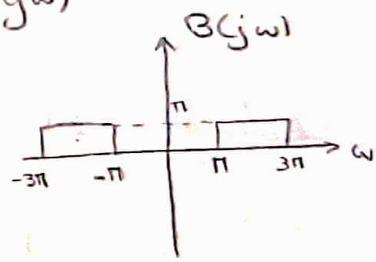
$$\Longrightarrow A(j\omega) = \begin{cases} 0 & \omega < -3\pi \\ \pi & -3\pi < \omega < -\pi \\ 2\pi & -\pi < \omega < \pi \\ \pi & \pi < \omega < 3\pi \\ 0 & 3\pi < \omega \end{cases}$$

$$b(t) = a(t) * h(t) \Longrightarrow B(j\omega) = A(j\omega) \cdot H(j\omega)$$

حل :

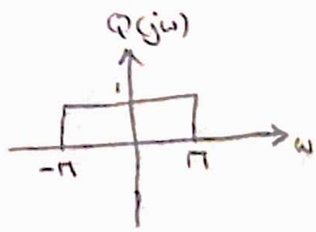


$\longrightarrow$



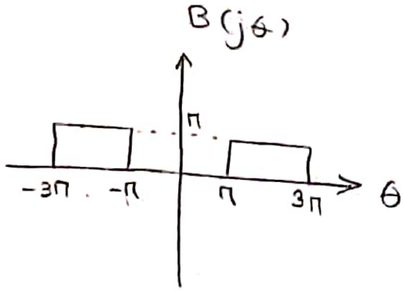
$$\Longrightarrow B(j\omega) = \begin{cases} 0 & \omega < -3\pi \\ \pi & -3\pi < \omega < -\pi \\ 0 & -\pi < \omega < \pi \\ \pi & \pi < \omega < 3\pi \\ 0 & 3\pi < \omega \end{cases}$$

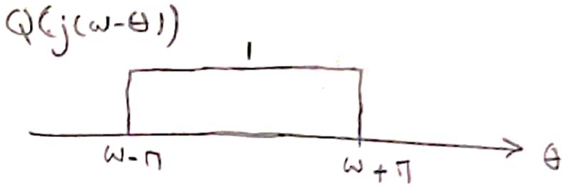
$$q(t) = \frac{\sin(\pi t)}{\pi t} \xleftrightarrow{F} Q(j\omega) = \begin{cases} 1 & |\omega| < \pi \\ 0 & |\omega| > \pi \end{cases}$$



$$c(t) = b(t) \cdot q(t) \Longrightarrow C(j\omega) = B(j\omega) * Q(j\omega)$$

$$= \int_{\theta=-\infty}^{+\infty} B(j\theta)\, Q(j(\omega-\theta))\, d\theta$$





if: $\omega+\pi < -3\pi \Longrightarrow \omega < -4\pi \Longrightarrow C(j\omega) = 0$

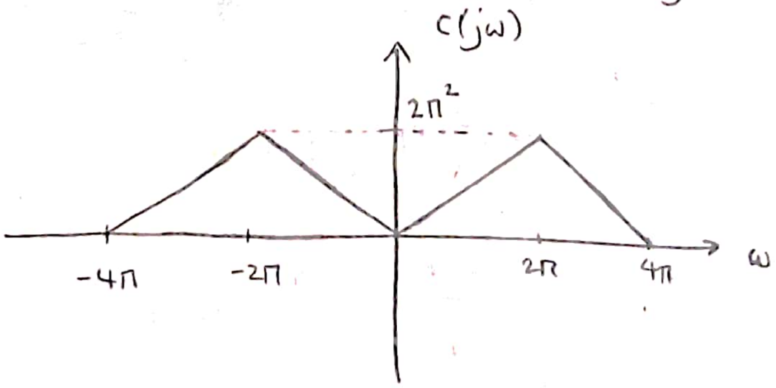
if: $-3\pi < \omega+\pi < -\pi \Longrightarrow -4\pi < \omega < -2\pi \Longrightarrow C(j\omega) = \pi \times (\overbrace{\pi+\omega+3\pi}^{\omega+4\pi}) = \pi\omega + 4\pi^2$

if: $-\pi < \omega+\pi < \pi \Longrightarrow -2\pi < \omega < 0 \Longrightarrow C(j\omega) = \pi \times (-\pi-\omega+\pi) = -\pi\omega$

if: $\pi < \omega+\pi < 3\pi \Longrightarrow 0 < \omega < 2\pi \Longrightarrow C(j\omega) = \pi \times (\omega+\pi-\pi) = \pi\omega$

if: $\pi < \omega-\pi < 3\pi \Longrightarrow 2\pi < \omega < 4\pi \Longrightarrow C(j\omega) = \pi \times (3\pi-\omega+\pi) = -\pi\omega + 4\pi^2$

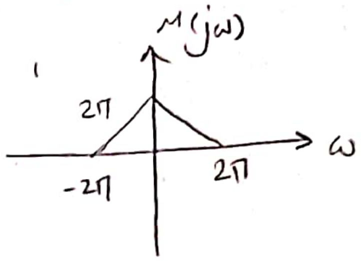
if: $3\pi < \omega-\pi \Longrightarrow 4\pi < \omega \Longrightarrow C(j\omega) = 0$



۵) ب)

مثل ″

داریم : $\Pi\left(\frac{\omega}{2\pi}\right) * \Pi\left(\frac{\omega}{2\pi}\right) =$



برای برای سیگنال $C(j\omega)$ داریم :

$$C(j\omega) = M(j(\omega - 2\pi)) + M(j(\omega + 2\pi))$$

$$= M(j\omega - 2\pi j) + M(j\omega + 2\pi j)$$

$$\xrightarrow{F^{-1}} c(t) = m(t)e^{-j2\pi t} + m(t)e^{j2\pi t} =$$

$$= \frac{\sin(\pi t)}{\pi t}\left(e^{j2\pi t} + e^{-j2\pi t}\right) =$$

$$= \frac{\sin(\pi t)}{\pi t} \times 2\cos(2\pi t)$$

$$= \frac{2\sin(\pi t)\cos(2\pi t)}{\pi t}$$