(1)

الف) $x(t) = |t|\, e^{-2|t|}$

$$x(t) = \underbrace{t\, e^{-2t} u(t)}_{x_1(t)} \underbrace{- t\, e^{2t} u(-t)}_{x_2(t)}$$

$$x_1(t) = t\, \underbrace{e^{-2t} u(t)}_{\downarrow L \; \frac{1}{s+2},\; Re(s) > -2} \xrightarrow{L} -\left(\frac{1}{s+2}\right)' = -\left(\frac{-1}{(s+2)^2}\right) = \frac{1}{(s+2)^2} \quad , \; Re(s) > -2$$

$$x_2(t) = -t\, \underbrace{e^{2t} u(-t)}_{\downarrow L \; \frac{-1}{s-2},\; Re(s) < 2} \xrightarrow{L} \left(\frac{-1}{s-2}\right)' = \frac{1}{(s-2)^2} \quad , \; Re(s) < 2$$

$$\Rightarrow x(t) = x_1(t) + x_2(t) \xrightarrow{L} X(s) = X_1(s) + X_2(s)$$

$$X(s) = \frac{1}{(s+2)^2} + \frac{1}{(s-2)^2} \quad , \; -2 < Re(s) < 2$$

ب) $x(t) = \delta(2t-3) + e^{-|t|}\sin(t)$ ①

$$x(t) = \underbrace{\delta(2t-3)}_{x_1(t)} + \underbrace{e^{-t}\sin(t)\,u(t)}_{x_2(t)} + \underbrace{e^{t}\sin(t)\,u(-t)}_{x_3(t)}$$

$$x_1(t) = \delta\left(2\left(t-\tfrac{3}{2}\right)\right) \xrightarrow{L} X_1(s) = \frac{e^{-\frac{3s}{4}}}{2} \quad , \quad Re(s) = \text{کل S ها}$$

$$x_2(t) = e^{-t}\sin t\,u(t) \xrightarrow{L} X_2(s) = \frac{1}{(s+1)^2+1} \quad , Re(s) > 1$$

$$x_3(t) = e^{t}\left(\frac{1}{2j}e^{jt} - \frac{1}{2j}e^{-jt}\right)u(-t) = \frac{1}{2j}e^{(1+j)t}u(-t) - \frac{1}{2j}e^{(1-j)t}u(-t)$$

$$\xrightarrow{L} X_3(s) = \frac{-1}{2j}\underbrace{\frac{1}{s-(1+j)}}_{Re(s) < -1-j} + \frac{1}{2j}\underbrace{\frac{1}{s-(1-j)}}_{Re(s) < -1+j} \quad , \; Res < -1-j$$

$$\Rightarrow X(s) = \frac{1}{2}e^{-\frac{3s}{4}} + \frac{1}{s^2+2s+2} + \frac{1}{2j}\left[\frac{1}{s+j-1} - \frac{1}{s-j-1}\right] \quad , \quad 1 < Re(s)$$

$$\text{C) } x(t) = \begin{cases} 1 & ; \quad 0 \leq t \leq 1 \\ 0 & ; \quad \text{oth} \end{cases}$$

①

$$x(t) = u(t) - u(t-1) \xrightarrow{L} X(s) = \underbrace{\frac{1}{s}}_{Re(s)>0} - \underbrace{\frac{e^{-s}}{s}}_{Re(s)>0}$$

$$\xrightarrow{\cap} X(s) = \frac{1-e^{-s}}{s} \quad , \quad Re(s) > 0$$

الف) $X_1(s) = \dfrac{s+1}{s^2+5s+6}$, $-3 < Re(s) < -2$ (2)

$$X_1(s) = \frac{s+1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = \lim_{s \to -2} X_1(s)(s+2) = \frac{s+1}{s+3}\Big|_{s=-2} = -1$$

$$B = \lim_{s \to -3} X_1(s)(s+3) = \frac{s+1}{s+2}\Big|_{s=-3} = 2$$

$$\Longrightarrow X_1(s) = \underbrace{\frac{-1}{s+2}}_{Re(s) < -2} + \underbrace{\frac{2}{s+3}}_{Re(s) > -3}$$

$$\Longrightarrow x_1(t) = e^{-2t}u(-t) + 2e^{-3t}u(t)$$

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ب) $X_2(s) = \dfrac{s^2 - s + 1}{(s+1)^2}$, $Re(s) > -1$

$$X_2(s) = \frac{(s+1)^2 - 3s}{(s+1)^2} = 1 + \frac{-3s}{(s+1)^2} = 1 + \underbrace{\frac{A}{(s+1)^2}}_{Re(s) > -1} + \underbrace{\frac{B}{s+1}}_{Re(s) > -1}$$

$$A = \lim_{s \to -1} X_2(s)(s+1)^2 = s^2 - s + 1 \Big|_{s=-1} = 3$$

$$B = \lim_{s \to -1} \left[\frac{d}{ds} (s+1)^2 X_2(s) \right] = \lim_{s \to -1} \left[\frac{d}{ds} (s^2 - s + 1) \right] = 2s - 1 \Big|_{s=-1} = -3$$

$$\Longrightarrow X_2(s) = \frac{3}{(s+1)^2} - \frac{3}{s+1} + 1 \Rightarrow x_2(t) = 3te^{-t}u(t) - 3e^{-t}u(t) + \delta(t)$$

ج) $X_3(s) = e^s \underbrace{\ln(s)}_{= X_4(s)}$, $Re(s) > 0$

$X_4(s) = \ln(s) \longrightarrow \frac{d}{ds} X_4(s) = \frac{1}{s}$

$-t\, x_4(t) \xrightarrow{L} \frac{d}{ds} X_4(s)$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \Longrightarrow -t\, x_4(t) \xrightarrow{L} \frac{1}{s}$, $Re(s) > 0$

با توجه به اینکه : $u(t) \xrightarrow{L} \frac{1}{s}$ $Re(s) > 0$

پس : $-t\, x_4(t) = u(t) \Longrightarrow x_4(t) = \frac{-u(t)}{t}$

$X_3(s) = e^s X_4(s) \xrightarrow{L^{-1}} \frac{-u(t+1)}{(t+1)}$

$$H(s) = \frac{s^2+4s+6}{s^2+5s+6}$$ ③

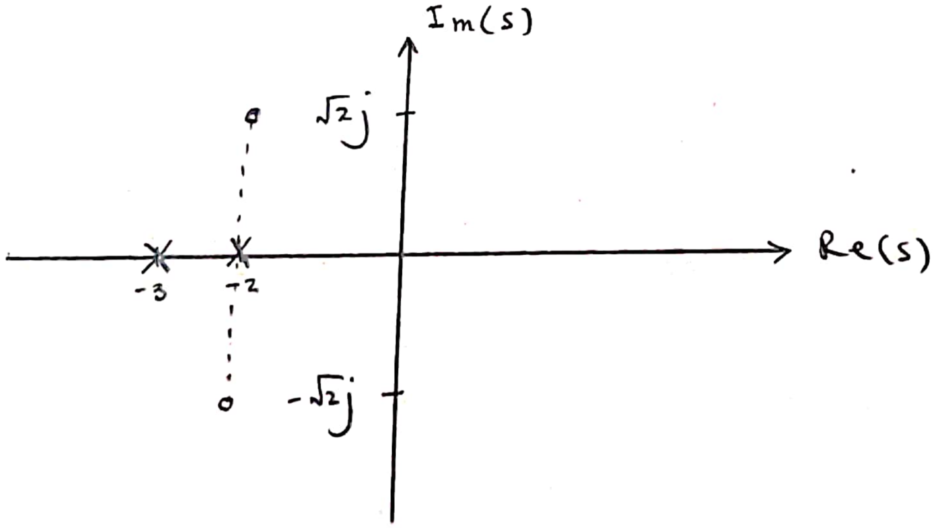
الف)

$$H(s) = \frac{s^2+4s+6}{(s+2)(s+3)}$$

$\longrightarrow$ قطب‌ها : -2 و -3

صفرها : $s^2+4s+6 \longrightarrow \Delta = (4)^2-(4)(6) = -8 \longrightarrow s = \frac{4 \pm \sqrt{-8}}{2} = -2 \pm j\sqrt{2}$

$\longrightarrow$ صفرها : $-2 \pm j\sqrt{2}$



③

ب)

علی و پایدار $\Rightarrow$ ROC رو به راست و شامل محور $j\omega$ $\Rightarrow$ $Re(s) > -2$

$$H(s) = \frac{s^2+4s+6}{(s+2)(s+3)} = \frac{(s+2)(s+3) - s}{(s+2)(s+3)} = 1 - \underbrace{\frac{s}{(s+2)(s+3)}}_{F(s)} = 1 - \left[\frac{A}{s+2} + \frac{B}{s+3}\right]$$

$$A = \lim_{s \to -2} (s+2)F(s) = \frac{s}{s+3}\bigg|_{s=-2} = -2$$

$$B = \lim_{s \to -3} (s+3)F(s) = \frac{s}{s+2}\bigg|_{s=-3} = 3$$

$$\Longrightarrow H(s) = 1 + \frac{2}{s+2} + \frac{-3}{s+3} \Longrightarrow h(t) = \delta(t) + 2e^{-2t}u(t) - 3e^{-3t}u(t)$$

$Re(s) > -2$ $\quad s \in \mathbb{C}$ $\quad Re(s) > -2$ $\quad Re(s) > -3$

(2 ③

$$x(t) = A\,u(t) \xrightarrow{L} X(s) = \frac{A}{s}$$

$$Y(s) = X(s)\,H(s) = \frac{A}{s}\left(1 + \frac{2}{s+2} + \frac{-3}{s+3}\right) = \frac{1}{s}\left[A + \frac{2A}{s+2} + \frac{-3A}{s+3}\right]$$

$$y(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y(s) = \lim_{s\to 0}\left[A + \frac{2A}{s+2} + \frac{-3A}{s+3}\right) = A + A - A = \boxed{A}$$

$$y(0^+) = \lim_{s\to\infty} s\,Y(s) = \lim_{s\to\infty}\left[A + \frac{2A}{s+2} + \frac{-3A}{s+3}\right] = A + 0 + 0 = \boxed{A}$$

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(3)

$x(t) = e^{-t}u(t) \xrightarrow{L} X(s) = \frac{1}{s+1}$, $Re(s) > -1$

$$Y(s) = X(s)\,H(s) = \frac{s^2+4s+6}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$, ROC $\supseteq R_X \cap R_H$ (✱)

$$A = \lim_{s \to -1} (s+1)Y(s) = \left.\frac{s^2+4s+6}{(s+2)(s+3)}\right|_{s=-1} = \frac{1-4+6}{(1)(2)} = \frac{3}{2}$$

$$B = \lim_{s \to -2} (s+2)Y(s) = \left.\frac{s^2+4s+6}{(s+1)(s+3)}\right|_{s=-2} = \frac{4-8+6}{(-1)(1)} = -2$$

$$C = \lim_{s \to -3} (s+3)Y(s) = \left.\frac{s^2+4s+6}{(s+1)(s+2)}\right|_{s=-3} = \frac{9-12+6}{(-2)(-1)} = \frac{3}{2}$$

$$\Rightarrow Y(s) = \frac{3/2}{s+1} + \frac{-2}{s+2} + \frac{3/2}{s+3}$$ و $\xrightarrow[\text{(✱)}]{\text{نطبق}}$ $Re(s) > -1$

$Re(s) > -1$ $\qquad$ $Re(s) > -2$ $\qquad$ $Re(s) > -3$: ج

$$\Rightarrow y(t) = \frac{3}{2}e^{-t}u(t) - 2e^{-2t} + \frac{3}{2}e^{-3t}$$

④ اختیاری :

$y(t) = e^{-2t} u(-t)$ و

$H(s) = \dfrac{s-1}{s+1}$ علی و $\Rightarrow Re(s) > -1$

الف)

$\int_{-\infty}^{+\infty} |x(t)|\, dt < \infty \Longrightarrow$ ورودی پایدار

$Y(s) = \dfrac{1}{s+2} \qquad Re(s) > -2$

$X(s) = \dfrac{Y(s)}{H(s)} = \dfrac{1}{s+2} \cdot \dfrac{s+1}{s-1} = \dfrac{A}{s+2} + \dfrac{B}{s-1}$

$A = \lim_{s \to -2} X(s)(s+2) = \dfrac{s+1}{s-1}\Big|_{s=-2} = \dfrac{-1}{-3} = \dfrac{1}{3}$

$B = \lim_{s \to 1} X(s)(s-1) = \dfrac{s+1}{s+2}\Big|_{s=1} = \dfrac{2}{3}$

$\Rightarrow X(s) = \dfrac{\frac{1}{3}}{s+2} + \dfrac{\frac{2}{3}}{s-1}$

پایدار $\downarrow$ $-2 < Re(s) < 1$ پس $Re(s) > -2$ $Re(s) < 1$

$x(t) = \frac{1}{3} e^{-2t} u(t) - \frac{3}{2} e^{t} u(-t)$

۴) اختیاری :

ب) $x(t)$ ای که در قسمت قبل داشتیم، در شرایط این مسئله صدق می‌کند.

که سیستم هم علی است.

$$H(s) = \frac{s-1}{s+1} = \underbrace{\frac{s}{s+1}}_{H_1(s)} - \frac{1}{s+1} \xrightarrow[\mathrm{I}]{L^{-1}} h(t) = -2e^{-t}u(t) + e^{-t}\delta(t)$$

$Re(s) > -1$

$$H_1(s) = s \cdot \frac{1}{s+1} \xrightarrow{L^{-1}} \left(e^{-t}u(t)\right)' = -e^{-t}u(t) + e^{-t}\delta(t) \quad \mathrm{I}$$

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۴) اختیاری :

ج) سیستم و ورودی بخش قبل، در این سوال هم صدق می‌کنند.

چون h، پایدار هست (ROC شامل $j\omega$)

⑤ 9-27 : (x حقیقی ← X گویا)

$$\xrightarrow{\text{طبق (1)}} X(s) = \frac{P(s)}{(s+a)(s+b)} \underset{\text{طبق (2)}}{=} \frac{c}{(s+a)(s+b)}$$

$\xrightarrow{\text{طبق (3)}}$ یک قطب $= -1+j$ $\xrightarrow{X(s)\ \text{حقیقی}}$ قطب‌ها مزدوج $\Longrightarrow$ قطب‌ها $= -1 \pm j$

$$\text{پس} \Longrightarrow X(s) = \frac{c}{(s+1+j)(s+1-j)} = \frac{c}{s^2+s-sj+s+1-j+js+j+1} = \frac{c}{s^2+2s+2}$$

$$\xrightarrow{\text{طبق (5)}} X(0) = 8 \Longrightarrow X(0) = \frac{c}{0+0+2} = 8 \Longrightarrow c = 16 \xrightarrow{\text{پس}} X(s) = \frac{16}{s^2+2s+2}$$

$\xrightarrow{\text{طبق (4)}}$ پایدار است. $\underbrace{e^{2t}x(t)}_{= x_1(t)}$ $\Longrightarrow$ ROC شامل محور $j\omega$ است

$x_1(t) = e^{2t}x(t) \Longrightarrow X_1(s) = X(s-2)$ ⟵ ناحیه‌ی همگرایی X_1، به اندازه‌ی 2 واحد، نسبت به X شیفت یافته به راست

$\xrightarrow{\text{پس}}$ $X(s)$ ناحیه همگرایی : $Re(s) > -1$ ، $Re(s) < -1$ $\xrightarrow[\text{باید شامل } j\omega \text{ شود}]{\text{وقتی 2 واحد به راست شیفت}}$ $\boxed{Re(s) > -1}$

$$\frac{d^3 y(t)}{dt^3} + 6\frac{d^2 y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = x(t)$$: 9-40 (6)

با توجه به شرایط اولیه ، تبدیل لاپلاس یک طرفه باید بگیریم :

$$s^3 Y(s) - s^2 y(0^-) - s y'(0^-) - y''(0^-) + 6s^2 Y(s) - 6s\, y(0^-)$$
$$- 6y'(0^-) + 11 s Y(s) - 11 y(0^-) + 6Y(s) = X(s)$$

a)

$$x(t) = e^{-4t} u(t) \xrightarrow{L} X(s) = \frac{1}{s+4}$$

فرض شرایط اولیه صفر $\Rightarrow$ $s^3 Y(s) + 6s^2 Y(s) + 11 s Y(s) + 6Y(s) = X(s)$

$$Y(s)\left[s^3 + 6s^2 + 11s + 6\right] = \frac{1}{s+4}$$

$$\Rightarrow Y(s) = \frac{1}{(s+4)(s+3)(s+2)(s+1)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} + \frac{D}{s+4}$$

(با استفاده از قضیه مانده ها حساب کردم): $A = \frac{1}{6}$, $B = -\frac{1}{2}$, $C = \frac{1}{2}$, $D = -\frac{1}{6}$

$$\Rightarrow Y(s) = \frac{1/6}{s+1} + \frac{-1/2}{s+2} + \frac{1/2}{s+3} + \frac{-1/6}{s+4}$$

$$\xrightarrow{L^{-1}} y(t) = \frac{1}{6} e^{-t} u(t) - \frac{1}{2} e^{-2t} u(t) + \frac{1}{2} e^{-3t} u(t) - \frac{1}{6} e^{-4t} u(t)$$

b) (6)

اعمال شرایط اولیه :

$$s^3Y(s) - s^2 + s - 1 + 6s^2Y(s) - 6s + 6 + 11sY(s) - 11 + 6Y(s) = 0$$

$$\rightarrow s^3Y(s) + 6s^2Y(s) + 11sY(s) + 6Y(s) - s^2 - 5s - 6 = 0$$

$$\Rightarrow Y(s)\left[s^3 + 6s^2 + 11s + 6\right] = s^2 + 5s + 6$$

$$\Rightarrow Y(s) = \frac{s^2+5s+6}{s^3+6s^2+11s+6} = \frac{(s^2+5s+6)}{(s+1)(s^2+5s+6)} = \frac{1}{s+1}$$

$$\rightarrow y(t) = e^{-t}u(t)$$

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c) (6)

$$\left(x(t) = e^{-4t}u(t) \xrightarrow{L} X(s) = \frac{1}{s+4} \right)$$

$$Y(s)\left[s^3 + 6s^2 + 11s + 6\right] = s^2 + 5s + 6 + \frac{1}{s+4}$$

$$\Rightarrow Y(s) = \frac{1}{s+1} + \frac{1}{(s+4)(s^3+6s^2+11s+6)}$$

$$= \frac{1}{s+1} + \frac{1}{6(s+1)} - \frac{1}{2(s+2)} + \frac{1}{2(s+3)} - \frac{1}{6(s+4)}$$

$$\rightarrow y(t) = \frac{7}{6}e^{-t}u(t) - \frac{1}{2}e^{-2t}u(t) + \frac{1}{2}e^{-3t}u(t) - \frac{1}{6}e^{-4t}u(t)$$
