

«به نام خدا»

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(سوال های 1 تا 4)

1.

a) $n = \frac{1}{2} + \sqrt{\frac{1}{4} + 2 * \ln(2) * 365} = 22.999 \approx 23$

b) $p(\text{at least two students}) = 1 - \prod_{i=1}^{k-1} (1 - \frac{i}{N})$

c) $t = 2^{n+1/2} \sqrt{\ln\left(\frac{1}{1-\lambda}\right)} = 2^{n+1/2} \sqrt{\ln\left(\frac{1}{1-0.5}\right)} = 2^{n+1/2} \sqrt{\ln(2)} = 0.83 * 2^{n+1/2}$

2. from this formula of birth-date problem: $k \approx \sqrt{n * m * \frac{1}{1-\epsilon}}$
we got:

n	$\epsilon = 0.5$	$\epsilon = 0.1$
2^{64}	$3.6 * 10^9$	$1.4 * 10^9$
2^{128}	$1.5 * 10^{19}$	$6 * 10^{18}$
2^{160}	$1.5 * 10^{24}$	$3.9 * 10^{23}$

3.

3.1. $c_i = z_i \oplus \{x_1 x_2 \dots x_n \parallel H_1(x) H_2(x) \dots H_m(x)\}; \quad (i = 1, 2, \dots, n + m)$
assume the length of x is n.

the attacker computes: $z_i = x_i \oplus c_i \quad (i = 1, 2, \dots, n)$

and computes $H(x)$ because he has x.

again computes: $z_{j+n} = H_j(x) \oplus c_{j+n} \quad (j = 1, 2, \dots, m)$

He computes $H(x')$

Then computes: $c'_i = z_i \oplus x'_i \quad (i = 1, 2, \dots, n)$

$c'_{j+n} = z_{j+n} \oplus H_j(x') \quad (j = 1, 2, \dots, m)$

3.2. No.

The attacker can still recover z_1, z_2, \dots, z_n .

But he can't recover the bit-stream portion $z_{n+1}, z_{n+2}, \dots, z_{n+m}$ which was used for encrypting $\text{MAC}_{k_2}(x)$.

Even if he would know the whole bit-stream, he wouldn't be able to compute a valid $\text{MAC}_{k_2}(x')$, since he doesn't know k_2 .

4.

4.1. $t = 10^6$ bits/sec

storage = $t * r = 2h * 10^6$ bits/sec = $2 * 3600 * 10^6$ bits/sec = 7.2 Gbits = 0.9 GByte

Storage of less than 1 GByte can be done at moderate costs, e.g., on hard disks or CDs.

4.2. We should first Compute the number of keys that an attacker can recover in 30 days:

$$\text{Number of keys} = \frac{30 \text{ days}}{10 \text{ mins}} = \frac{30 * 24 * 60}{10} = 4320$$

$$\text{Key derivation period} = \frac{2 \text{ h}}{4320} = 1.67 \text{ sec}$$

Because has functions are fast, a key derivation can easily be performed (in software) at such a rate.
