1

الف) $\left(\frac{1}{2}\right)^n \{u[n+4] - u[n-5]\}$

$$x[n] = \underbrace{\left(\frac{1}{2}\right)^n u[n+4]}_{x_1[n]} - \underbrace{\left(\frac{1}{2}\right)^n u[n-5]}_{x_2[n]}$$

$$X_1(z) = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{2}\right)^n u[n+4] z^{-n} = \sum_{n=-4}^{+\infty} \left(\frac{1}{2}\right)^n z^{-n} = \sum_{n=-4}^{+\infty} \left(\frac{1}{2} z^{-1}\right)^n$$

$$= \frac{\left(\frac{1}{2} z^{-1}\right)^{-4}}{1-\frac{1}{2} z^{-1}} \quad ; \left|\frac{1}{2} z^{-1}\right| < 1 \Rightarrow |z^{-1}| < 2 \Rightarrow |z| > \frac{1}{2}$$

$$X_2(z) = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{2}\right)^n u[n-5] z^{-n} = \sum_{n=5}^{+\infty} \left(\frac{1}{2} z^{-1}\right)^n = \frac{\left(\frac{1}{2} z^{-1}\right)^5}{1-\frac{1}{2} z^{-1}} \quad ; |z| > \frac{1}{2}$$

$$X(z) = X_1(z) - X_2(z) = \frac{\left(\frac{1}{2} z^{-1}\right)^{-4} - \left(\frac{1}{2} z^{-1}\right)^5}{1-\frac{1}{2} z^{-1}} = \frac{\left(\frac{1}{2} z^{-1}\right)^{-4}\left(1-\left(\frac{1}{2} z^{-1}\right)^9\right)}{1-\frac{1}{2} z^{-1}}$$

$\Rightarrow$ ROC : $|z| > \frac{1}{2}$

و شامل ∞ هم نمی‌شود

$\ominus$ $|n| \left(\frac{1}{2}\right)^{|n|} = x[n]$ ①

$$X(z) = \sum_{n=-\infty}^{+\infty} |n| \left(\frac{1}{2}\right)^{|n|} z^{-n} = \sum_{n=-\infty}^{-1} -n \left(\frac{1}{2}\right)^{-n} z^{-n} + \sum_{n=0}^{+\infty} n \left(\frac{1}{2}\right)^{n} z^{-n} =$$

$$= \underbrace{\sum_{n=-\infty}^{-1} -n \left(\frac{1}{2} z\right)^{-n}}_{\text{I}} + \underbrace{\sum_{n=0}^{+\infty} n \left(\frac{1}{2} z^{-1}\right)^{n}}_{\text{II}}$$

$$\text{I} \rightarrow \sum_{n=-\infty}^{-1} \underbrace{-n \left(\frac{1}{2}\right)^{-n}}_{n x_1[n]} z^{-n} = -z \frac{dX_1}{dz} = -z \frac{d}{dz} \sum_{n=-\infty}^{-1} -2^{n} z^{-n} = z \frac{d}{dz} \sum_{n=-\infty}^{-1} (2z^{-1})^{n}$$

$$\text{II} \rightarrow \sum_{n=0}^{+\infty} \underbrace{n \left(\frac{1}{2}\right)^{n}}_{n x_2[n]} z^{-n} = -z \frac{dX_2}{dz} = -z \frac{d}{dz} \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^{n} z^{-n} = -z \frac{d}{dz} \sum_{n=0}^{+\infty} \left(\frac{1}{2} z^{-1}\right)^{n}$$

$$\Rightarrow X(z) = z \frac{d}{dz} \left[\sum_{n=-\infty}^{-1} (2z^{-1})^{n} - \sum_{n=0}^{+\infty} \left(\frac{1}{2} z^{-1}\right)^{n} \right]$$

$$= z \frac{d}{dz} \left[\sum_{n=1}^{+\infty} (2z^{-1})^{-n} - \sum_{n=0}^{+\infty} \left(\frac{1}{2} z^{-1}\right)^{n} \right]$$

$$= z \frac{d}{dz} \left[\sum_{n=0}^{+\infty} \left(\frac{1}{2} z\right)^{n+1} - \sum_{n=0}^{+\infty} \left(\frac{1}{2} z^{-1}\right)^{n} \right]$$

$$= z\frac{d}{dz}\left[\frac{\frac{1}{2}z}{1-\frac{1}{2}z} - \frac{1}{1-\frac{1}{2}z^{-1}}\right]$$

$$= z\left[\frac{\frac{1}{2}(1-\frac{1}{2}z)-(-\frac{1}{2})(\frac{1}{2}z)}{(1-\frac{1}{2}z)^2} - \frac{(1-\frac{1}{2}z^{-1}) - 2z^{-2}}{(1-\frac{1}{2}z^{-1})^2}\right]$$

$$= z\left[\frac{\frac{1}{2}-\frac{1}{4}z+\frac{1}{2}-\frac{1}{2}z}{(1-\frac{1}{2}z)^2} - \frac{1-\frac{1}{2}z^{-1}-2z^{-2}}{(1-\frac{1}{2}z^{-1})^2}\right]$$

$$= z\left[\frac{1-\frac{3}{4}z}{(1-\frac{1}{2}z)^2} + \frac{2z^{-2}+\frac{1}{2}z^{-1}-1}{(1-\frac{1}{2}z^{-1})^2}\right]$$

$$= \frac{z(1-\frac{3}{4}z)}{(1-\frac{1}{2}z)^2} + \frac{2z^{-1}+\frac{1}{2}-z}{(1-\frac{1}{2}z^{-1})^2}$$

$$= \frac{z(1-\frac{3}{4}z)(1-\frac{1}{2}z^{-1})^2 + (1-\frac{1}{2}z)^2(2z^{-1}+\frac{1}{2}-z)}{(1-\frac{1}{2}z)^2(1-\frac{1}{2}z^{-1})^2}$$

$\to$ ROC: $\frac{1}{2} < |z| < 2$

(C) $4^n \cos\left(\frac{\pi}{3}n + \frac{\pi}{4}\right) u[-n-1] = x[n]$ ①

$$X(z) = \sum_{n=-\infty}^{-1} 4^n \cos\left(\frac{\pi}{3}n+\frac{\pi}{4}\right) z^{-n} = \sum_{n=-\infty}^{-1} (4z^{-1})^n \frac{e^{j(\frac{\pi}{3}n+\frac{\pi}{4})} + e^{-j(\frac{\pi}{3}n+\frac{\pi}{4})}}{2}$$

$$= \sum_{n=-\infty}^{-1} (4z^{-1})^n \frac{e^{j(\frac{\pi}{3}n+\frac{\pi}{4})}}{2} + \sum_{n=-\infty}^{-1} (4z^{-1})^n \frac{e^{-j(\frac{\pi}{3}n+\frac{\pi}{4})}}{2}$$

$$= \frac{e^{j\frac{\pi}{4}}}{2} \sum_{n=1}^{+\infty} (4z^{-1})^{-n} e^{-j\frac{\pi}{3}n} + \frac{e^{-j\frac{\pi}{4}}}{2} \sum_{n=1}^{+\infty} (4z^{-1})^{-n} e^{j\frac{\pi}{3}n}$$

$$= \frac{e^{j\frac{\pi}{4}}}{2} \sum_{n=1}^{+\infty} \left(\frac{1}{4} z e^{-j\frac{\pi}{3}}\right)^n + \frac{e^{-j\frac{\pi}{4}}}{2} \sum_{n=1}^{+\infty} \left(\frac{1}{4} z e^{j\frac{\pi}{3}}\right)^n$$

$$= \frac{e^{j\frac{\pi}{4}}}{2} \left(\frac{\frac{1}{4} z e^{-j\frac{\pi}{3}}}{1-\frac{1}{4} z e^{-j\frac{\pi}{3}}}\right) + \frac{e^{-j\frac{\pi}{4}}}{2} \left(\frac{\frac{1}{4} z e^{j\frac{\pi}{3}}}{1-\frac{1}{4} z e^{j\frac{\pi}{3}}}\right)$$

$$= \frac{z}{8} \left[\frac{e^{-j\frac{\pi}{12}}}{1-\frac{1}{4} z e^{-j\frac{\pi}{3}}} + \frac{e^{j\frac{\pi}{12}}}{1-\frac{1}{4} z e^{j\frac{\pi}{3}}}\right] =$$

$$= \frac{z}{8} \left[\frac{e^{-j\frac{\pi}{12}} - \frac{1}{4} z e^{\frac{\pi}{4}j} + e^{j\frac{\pi}{12}} - \frac{1}{4} z e^{-\frac{\pi}{4}j}}{(1-\frac{1}{4} z e^{-j\frac{\pi}{3}})(1-\frac{1}{4} z e^{j\frac{\pi}{3}})}\right] =$$

$$= \frac{z}{8} \left[\frac{2\cos(\frac{\pi}{12}) - \frac{1}{2} z \cos(\frac{\pi}{4})}{(1-\frac{1}{4} z e^{-j\frac{\pi}{3}})(1-\frac{1}{4} z e^{j\frac{\pi}{3}})}\right]$$

$$\left.\begin{array}{l} \left|\frac{1}{4} z e^{j\pi/3}\right| < 1 \\ \left|\frac{1}{4} z e^{-j\pi/3}\right| < 1 \end{array}\right\} \Rightarrow |z| < 4 \quad \text{ROC:}$$

الف) $X_1(z) = \dfrac{z^{-1} - \frac{1}{2}}{1-\frac{1}{2}z^{-1}}$ $\quad |z| > \frac{1}{2}$ (2)

$|z| >$ قطب بزرگتر

$\Downarrow$

سیگنال دست راستی است.

$$X_1(z) = \frac{-2\left(1-\frac{1}{2}z^{-1}\right) + \frac{3}{2}}{1-\frac{1}{2}z^{-1}} = \underbrace{-2}_{Y_1} + \underbrace{\frac{3/2}{1-\frac{1}{2}z^{-1}}}_{Y_2}$$

اشتراک ROC این دو باید $|z| > \frac{1}{2}$ باشد. پس:

$Y_1(z) = -2 \quad ; \quad \text{all } z \Longrightarrow y_1[n] = -2\,\delta[n]$

$Y_2(z) = \dfrac{3/2}{1-\frac{1}{2}z^{-1}} \quad ; \quad |z| > \frac{1}{2} \Longrightarrow y_2[n] = \frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$

$\Longrightarrow x[n] = y_1[n] + y_2[n] = -2\,\delta[n] + \frac{3}{2}\left(\frac{1}{2}\right)^n u[n]$

ب) $X_2(z) = \dfrac{z^{-1} - \frac{1}{2}}{(1-\frac{1}{2}z^{-1})^2} \quad ; \quad |z| > \frac{1}{2}$ ②

$$X_2(z) = \frac{-2}{1-\frac{1}{2}z^{-1}} + \frac{\frac{3}{2}}{(1-\frac{1}{2}z^{-1})^2} = \underbrace{\frac{-2}{1-\frac{1}{2}z^{-1}}}_{|z|>\frac{1}{2}} + 3z\underbrace{\frac{\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}}_{|z|>\frac{1}{2}}$$

طبق اینکه اشتراک ROC هایشان باید $|z|>\frac{1}{2}$ باشد:

$$\Rightarrow x_2[n] = -2\left(\tfrac{1}{2}\right)^n u[n] + \left[3\,\delta[n+1] * n\left(\tfrac{1}{2}\right)^n u[n]\right] =$$

$$\Rightarrow x_2[n] = -2\left(\tfrac{1}{2}\right)^n u[n] + 3\,(n+1)\left(\tfrac{1}{2}\right)^{n+1} u[n+1]$$

$$X(z) = \frac{z^{2} + z - 7 + 9z^{-2} + 3z^{-3}}{1 + 3z^{-1} + 2z^{-2}}$$

$$\begin{array}{l|l} z^{2} + z - 7 + 9z^{-2} + 3z^{-3} & 1 + 3z^{-1} + 2z^{-2} \\ \hline z^{2} + 3z + 2 & z^{2} - 2z - 3 + 13z^{-1} - 24z^{-2} + \dots \end{array}$$

$$\begin{array}{l} \hline -2z - 9 + 9z^{-2} + 3z^{-3} \\ \underset{+}{-}2z \ \underset{+}{-}6 \ \underset{+}{-}4z^{-1} \\ \hline -3 + 4z^{-1} + 9z^{-2} + 3z^{-3} \\ \mp 3 \mp 9z^{-1} \mp 6z^{-2} \\ \hline 13z^{-1} + 15z^{-2} + 3z^{-3} \\ 13z^{-1} \pm 39z^{-2} \pm 26z^{-3} \\ \hline -24z^{-2} - 23z^{-3} \end{array}$$

$$\sum_{n=-\infty}^{+\infty} x[n]\, z^{-n} = z^{2} - 2z - 3 + 13z^{-1} - 24z^{-2} + \dots \xRightarrow{n \leq 0} \begin{cases} x[n] = 0 & n \leq -3 \\ x[-2] = 1 \\ x[-1] = -2 \\ x[0] = -3 \end{cases}$$

الف) $y[n] - \frac{1}{2} y[n-1] + \frac{1}{4} y[n-2] = x[n]$ ④

$$\xrightarrow{Z} Y(Z) - \frac{1}{2} Z^{-1} Y(Z) + \frac{1}{4} Z^{-2} Y(Z) = X(Z)$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \frac{1}{1 - \frac{1}{2} Z^{-1} + \frac{1}{4} Z^{-2}} = \frac{1}{\left(1 - \frac{1+j\sqrt{3}}{4} Z^{-1}\right)\left(1 - \frac{1-j\sqrt{3}}{4} Z^{-1}\right)}$$

$$\boxed{\begin{array}{l} 1 - \frac{1}{2} Z^{-1} + \frac{1}{4} Z^{-2} = 0 \Rightarrow Z^2 - \frac{1}{2} Z + \frac{1}{4} = 0 \\ \Delta = \frac{1}{4} - 4(1)\left(\frac{1}{4}\right) = \frac{1}{4} - 1 = -\frac{3}{4} \\ Z = \frac{\frac{1}{2} \pm \sqrt{-\frac{3}{4}}}{2} \Rightarrow Z_1 = \frac{\frac{1}{2} + j\frac{\sqrt{3}}{2}}{2} = \frac{1+j\sqrt{3}}{4} \\ \qquad\qquad\quad\; Z_2 = \frac{\frac{1}{2} - j\frac{\sqrt{3}}{2}}{2} = \frac{1-j\sqrt{3}}{4} \end{array}}$$

سیستم علی $\Longrightarrow$ ROC : $|Z| >$ قطب بزرگتر $\Rightarrow |Z| > \left|\frac{1+j\sqrt{3}}{4}\right| \Rightarrow |Z| > \frac{1}{2}$

(7) $x[n] = \left(\frac{1}{2}\right)^n u[n] \longrightarrow X(Z) = \dfrac{1}{1-\frac{1}{2}Z^{-1}}$ (4)

$$Y(Z) = X(Z) \cdot H(Z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1+j\sqrt{3}}{4}z^{-1}\right)\left(1-\frac{1-j\sqrt{3}}{4}z^{-1}\right)}$$

$$= \frac{A}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1-\frac{1+j\sqrt{3}}{4}z^{-1}\right)} + \frac{C}{\left(1-\frac{1-j\sqrt{3}}{4}z^{-1}\right)}$$

$$A = \lim_{z \to \frac{1}{2}} \frac{1}{\left(1-\frac{1+j\sqrt{3}}{4}z^{-1}\right)\left(1-\frac{1-j\sqrt{3}}{4}z^{-1}\right)} = \frac{1}{\left(1-\frac{1+j\sqrt{3}}{2}\right)\left(1-\frac{1-j\sqrt{3}}{2}\right)} = \frac{\frac{2}{1+j\sqrt{3}}}{\frac{1-j\sqrt{3}}{2}} =$$

$$= \frac{4}{(1+j\sqrt{3})(1-j\sqrt{3})} = \frac{4}{1-3j^2} = 1$$

$$B = \lim_{z \to \frac{1+j\sqrt{3}}{4}} \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1-j\sqrt{3}}{4}z^{-1}\right)} = \frac{1}{\left(1-\frac{1}{2}\left(\frac{4}{1+j\sqrt{3}}\right)\right)\left(1-\frac{1-j\sqrt{3}}{1+j\sqrt{3}}\right)} =$$

$$= \frac{1}{\left(\frac{j\sqrt{3}-1}{1+j\sqrt{3}}\right)\left(\frac{1+j\sqrt{3}-1+j\sqrt{3}}{1+j\sqrt{3}}\right)} = \frac{1}{\left(\frac{j\sqrt{3}-1}{j\sqrt{3}+1}\right)\left(\frac{2j\sqrt{3}}{j\sqrt{3}+1}\right)} =$$

$$= \frac{(j\sqrt{3}+1)^2}{(j\sqrt{3}-1)\,2j\sqrt{3}} = \frac{-3+1+2j\sqrt{3}}{-2(3)-2j\sqrt{3}} = \frac{-2+2j\sqrt{3}}{-6-2j\sqrt{3}} = \frac{j\sqrt{3}-1}{-j\sqrt{3}-3} = \frac{-j\sqrt{3}}{3}$$

$$C = \lim_{z \to \frac{1-j\sqrt{3}}{4}} \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1+j\sqrt{3}}{4}z^{-1}\right)} = \frac{1}{\left(1-\frac{2}{1-j\sqrt{3}}\right)\left(1-\frac{1+j\sqrt{3}}{1-j\sqrt{3}}\right)} =$$

$$= \frac{1}{\left(\frac{1-j\sqrt{3}-2}{1-j\sqrt{3}}\right)\left(\frac{1-j\sqrt{3}-1-j\sqrt{3}}{1-j\sqrt{3}}\right)} = \frac{(1-j\sqrt{3})^2}{(-1-j\sqrt{3})(-2j\sqrt{3})} = \frac{1-3-2j\sqrt{3}}{2j\sqrt{3}-2(3)} =$$

$$= \frac{-j\sqrt{3}-1}{j\sqrt{3}-3} \times \frac{j\sqrt{3}+3}{j\sqrt{3}+3} = \frac{-4j\sqrt{3}}{-12} = \frac{j\sqrt{3}}{3}.$$

$$\Longrightarrow Y(z) = \underbrace{\frac{1}{\left(1-\frac{1}{2}z^{-1}\right)}} + \underbrace{\frac{-j\frac{\sqrt{3}}{3}}{\left(1-\frac{1+j\sqrt{3}}{4}z^{-1}\right)}}_{|z|>\frac{1}{2}} + \underbrace{\frac{j\frac{\sqrt{3}}{3}}{\left(1-\frac{1-j\sqrt{3}}{4}z^{-1}\right)}}_{|z|>\frac{1}{2}}$$

سیگنال x دست راستی $\Rightarrow |z|>\frac{1}{2}$

$$y[n] = \left(\frac{1}{2}\right)^n u[n] - j\frac{\sqrt{3}}{3}\left(\frac{1+j\sqrt{3}}{4}\right)^n u[n] + j\frac{\sqrt{3}}{3}\left(\frac{1-j\sqrt{3}}{4}\right)^n u[n]$$

$$= u[n]\left[\left(\frac{1}{2}\right)^n + j\frac{\sqrt{3}}{3}\left[\left(\frac{1-j\sqrt{3}}{4}\right)^n - \left(\frac{1+j\sqrt{3}}{4}\right)^n\right]\right]$$

الف $x[n] = u[n] - u[n-N]$ 5

$$y[n] = x[n] * h[n] = \sum_{K=-\infty}^{+\infty} x[K]\, h[n-K] = \sum_{K=-\infty}^{+\infty} \left[u[K] - u[K-N]\right] \cdot a^{n-K}\, u[n-K]$$

$$= \sum_{K=0}^{N-1} a^{n-K}\, u[n-K] = \begin{cases} 0 & n < 0 \\ \sum_{K=0}^{n} a^{n-K} & 0 \leqslant n \leqslant N-1 \quad \text{I} \\ \sum_{K=0}^{N-1} a^{n-K} & N-1 < n \quad \text{II} \end{cases}$$

$$\text{I} \rightarrow a^n \sum_{K=0}^{n} (a^{-1})^K = a^n \frac{1-a^{-n-1}}{1-a^{-1}} = \frac{a^n - a^{-1}}{1-a^{-1}}$$

$$\text{II} \rightarrow a^n \sum_{K=0}^{N-1} (a^{-1})^K = a^n \frac{1-a^{-N}}{1-a^{-1}} = \frac{a^n - a^{n-N}}{1-a^{-1}}$$

$$\Rightarrow y[n] = \begin{cases} 0 & n < 0 \\ \dfrac{a^n - a^{-1}}{1-a^{-1}} & 0 \leqslant n \leqslant N-1 \\ \dfrac{a^n - a^{n-N}}{1-a^{-1}} & N-1 < n \end{cases}$$

ب 5

$$H(z)=\frac{1}{1-az^{-1}} \quad ; |z|>|a|$$

$$X(z)=\frac{1}{1-z^{-1}}-\frac{z^{-N}}{1-z^{-1}}=\frac{1-z^{-N}}{1-z^{-1}} \quad ; \text{all } z$$

$$Y(z)=X(z)H(z)=\frac{1-z^{-N}}{(1-az^{-1})(1-z^{-1})} \quad ; |z|>a$$

$$Y(z)=(1-z^{-N})\cdot\frac{1}{(1-az^{-1})(1-z^{-1})}=(1-z^{-N})\left[\frac{A}{1-az^{-1}}+\frac{B}{1-z^{-1}}\right]$$

$$A=\lim_{z\to a}\frac{1}{1-z^{-1}}=\frac{1}{1-\frac{1}{a}}=\frac{1}{\frac{a-1}{a}}=\frac{a}{a-1}$$

$$B=\lim_{z\to 1}\frac{1}{1-az^{-1}}=\frac{1}{1-a}$$

$$\Longrightarrow Y(z)=(1-z^{-N})\left[\frac{\frac{a}{a-1}}{1-az^{-1}}+\frac{\frac{1}{1-a}}{1-z^{-1}}\right]=\frac{1-z^{-N}}{a-1}\left[\frac{a}{1-az^{-1}}-\frac{1}{1-z^{-1}}\right]$$

$$=\frac{1}{a-1}\left[\frac{a-az^{-N}}{1-az^{-1}}-\frac{1-z^{-N}}{1-z^{-1}}\right]=$$

$$=\frac{1}{a-1}\left[\frac{a}{1-az^{-1}}-\frac{az^{-N}}{1-az^{-1}}-\frac{1}{1-z^{-1}}+\frac{z^{-N}}{1-z^{-1}}\right]=$$

$$=\frac{1}{1-a^{-1}}\left[\frac{1}{1-az^{-1}}-\frac{z^{-N}}{1-az^{-1}}\right]+\frac{1}{1-a}\left[\frac{1}{1-z^{-1}}-\frac{z^{-N}}{1-z^{-1}}\right]$$

$$\Longrightarrow y[n]=\frac{1}{1-a^{-1}}\left[(a)^n u[n]-(a)^{n-N}u[n-N]\right]+\frac{1}{1-a}\left[u[n]-u[n-N]\right]$$

$y[n] \Longleftarrow$ در دو حالت ، یکسان می شود.

(6)

الف

شرط اول:

$$z_0^n \longrightarrow \boxed{LTI} \longrightarrow H(z_0)\, z_0^n$$

$x_1[n] = (-2)^n$

$$y_1[n] = 0 = H(-2)\,(-2)^n \Rightarrow H(-2) = 0 \quad (I)$$

شرط دوم:

$$H(z) = \frac{Y_2(z)}{X_2(z)} = \frac{1 + \dfrac{a}{1-\frac{1}{4}z^{-1}}}{\dfrac{1}{1-\frac{1}{2}z^{-1}}} = \left(1-\tfrac{1}{2}z^{-1}\right)\left[1 + \frac{a}{1-\frac{1}{4}z^{-1}}\right] \; ; |z| > \tfrac{1}{2} \quad (II)$$

$Y_2(z)$: ROC $|z| > \frac{1}{4}$

$X_2(z)$: ROC $|z| > \frac{1}{2}$

(I), (II).

$$\Longrightarrow H(-2) = 0 = \underbrace{\left(1 + \tfrac{1}{4}\right)}_{5/4}\left[1 + \frac{a}{1+\frac{1}{8}}\right] \Rightarrow \frac{a}{9/8} = -1 \Rightarrow \boxed{a = -9/8}$$

ب) $x[n] = 1 = z_0^n \Big|_{z_0=1} \Longrightarrow y[n] = z_0^n \, H(z_0) = H(1)\,(1)^n = H(1) \boxed{= -\frac{1}{4}}$ 6

$$H(1) = \left(1 - \frac{1}{2}\right)\left[1 + \frac{-9/8}{1 - \frac{1}{4}}\right] = \frac{1}{2}\left(\frac{\frac{3}{4} - \frac{9}{8}}{\frac{3}{4}}\right) = \frac{\frac{6-9}{8}}{\frac{2 \times 3}{4}} = \frac{-3 \times 4}{8 \times 2 \times 3} = -\frac{1}{4}$$

ج) بله؛ چون محدوده‌ی همگرایی دایره‌ی به شعاع واحد را هم شامل می‌شود. ← تبدیل فوریه وجود دارد ✓ 6

$|z| > \frac{1}{2}$