Advanced Microeconomic Theory

Chapter 7: Monopoly

Outline

- Barriers to Entry
- Profit Maximization under Monopoly
- Welfare Loss of Monopoly
- Multiplant Monopolist

Barriers to Entry

Barriers to Entry

- ***Entry barriers***: elements that make the entry of potential competitors either impossible or very costly.

- Three main categories:
  1) ***Legal***: the incumbent firm in an industry has the legal right to charge monopoly prices during the life of the patent
     - *Example*: newly discovered drugs

Barriers to Entry

2) ***Structural***: the incumbent firm has a cost or demand advantage relative to potential entrants.
   – superior technology
   – a loyal group of customers
     - positive network externalities (Facebook, eBay)

3) ***Strategic***: the incumbent monopolist has a reputation of fighting off newcomers, ultimately driving them off the market.
   – price wars

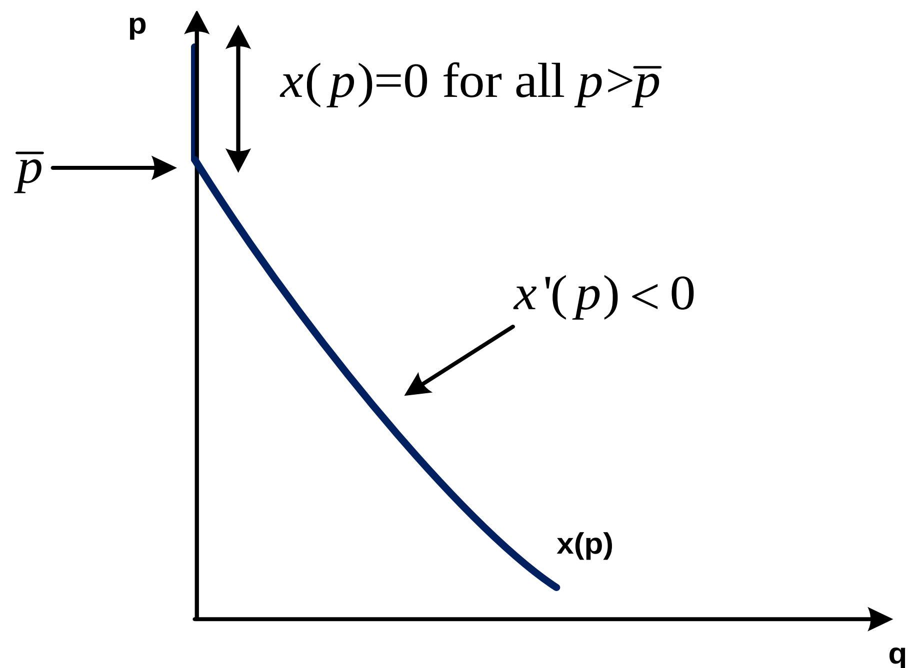
Profit Maximization under Monopoly

Profit Maximization

- Consider a demand function $x(p)$, which is continuous and strictly decreasing in p, i.e., $x'(p) < 0$.
- We assume that there is price $\bar{p} < \infty$ such that $x(p) = 0$ for all $p > \bar{p}$.
- Also, consider a general cost function $c(q)$, which is increasing and convex in q.

Profit Maximization

- $\bar{p}$ is a "choke price"
- No consumers buy positive amounts of the good for $p > \bar{p}$.



Profit Maximization

- Monopolist's decision problem is
$$\max_{p} \; px(p) - c(x(p))$$

- Alternatively, using $x(p) = q$, and taking the inverse demand function $p(q) = x^{-1}(p)$, we can rewrite the monopolist's problem as
$$\max_{q \geq 0} \; p(q)q \; - c(q)$$

Profit Maximization

- Differentiating with respect to q,

$$p(q^m) + p'(q^m)q^m - c'(q^m) \leq 0$$

- Rearranging,

$$\underbrace{p(q^m) + p'(q^m)q^m}_{MR=\frac{d[p(q)q]}{dq}} \leq \underbrace{c'(q^m)}_{MC}$$

with equality if $q^m > 0$.

- Recall that total revenue is $TR(q) = p(q)q$

Profit Maximization

- In addition, we assume that $p(0) \geq c'(0)$.
  - That is, the inverse demand curve originates above the marginal cost curve.
  - Hence, the consumer with the highest willingness to pay for the good is willing to pay more than the variable costs of producing the first unit.

- Then, we must be at an interior solution $q^m > 0$, implying

$$\underbrace{p(q^m) + p'(q^m)q^m}_{MR} = \underbrace{c'(q^m)}_{MC}$$

Profit Maximization

- Note that

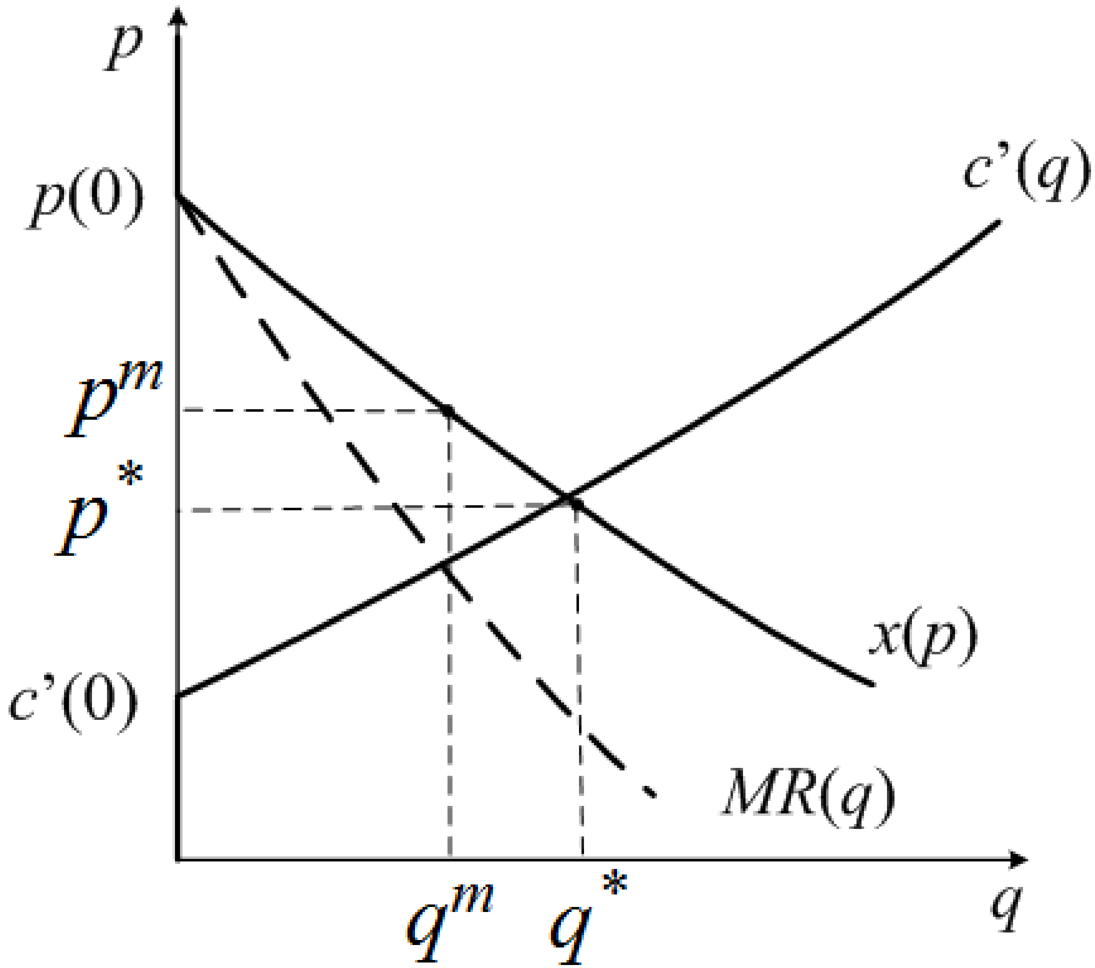
$$p(q^m) + \underbrace{p'(q^m)q^m}_{-} = c'(q^m)$$

- Then, $p(q^m) > c'(q^m)$, i.e.,

monopoly price $> MC$

- Moreover, we know that in competitive equilibrium $p(q^*) = c'(q^*)$.
- Then, $p^m > p^*$ and $q^m < q^*$.

Profit Maximization



Profit Maximization

- Marginal revenue in monopoly

$$MR = p(q^m) + p'(q^m)q^m$$

MR describes two effects:

  – **A *direct* (positive) effect**: an additional unit can be sold at $p(q^m)$, thus increasing revenue by $p(q^m)$.

  – **An *indirect* (negative) effect**: selling an additional unit can only be done by reducing the market price of all units (the new and all previous units), ultimately reducing revenue by $p'(q^m)q^m$.

    - *Inframarginal units* – initial units before the marginal increase in output.

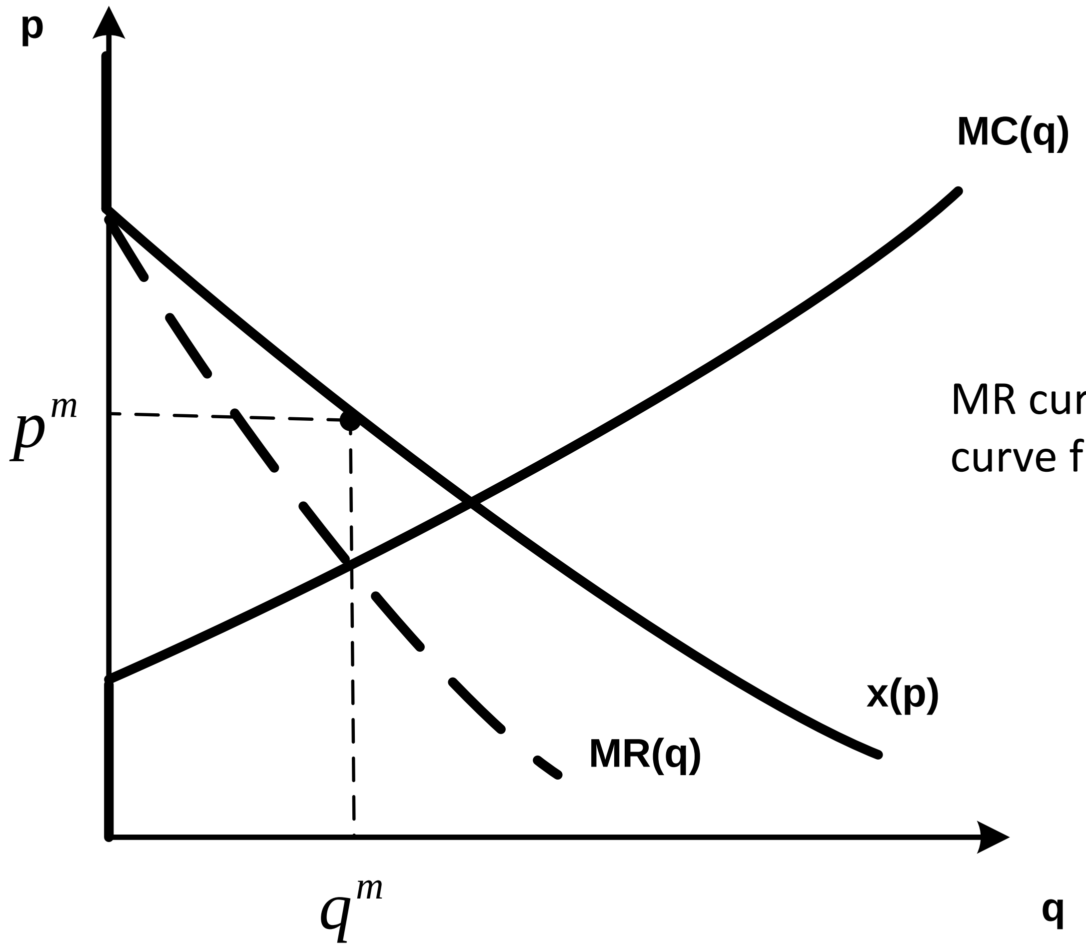
Profit Maximization

- Is the above FOC also sufficient?
  - Let's take the FOC $p(q^m) + p'(q^m)q^m - c'(q^m)$, and differentiate it wrt q,

$$\underbrace{p'(q) + p'(q) + p''(q)q}_{\frac{dMR}{dq}} - \underbrace{c''(q)}_{\frac{dMC}{dq}} \leq 0$$

  - That is, $\frac{dMR}{dq} \leq \frac{dMC}{dq}$.
  - Since MR curve is decreasing and MC curve is weakly increasing, the second-order condition is satisfied for all q.

Profit Maximization

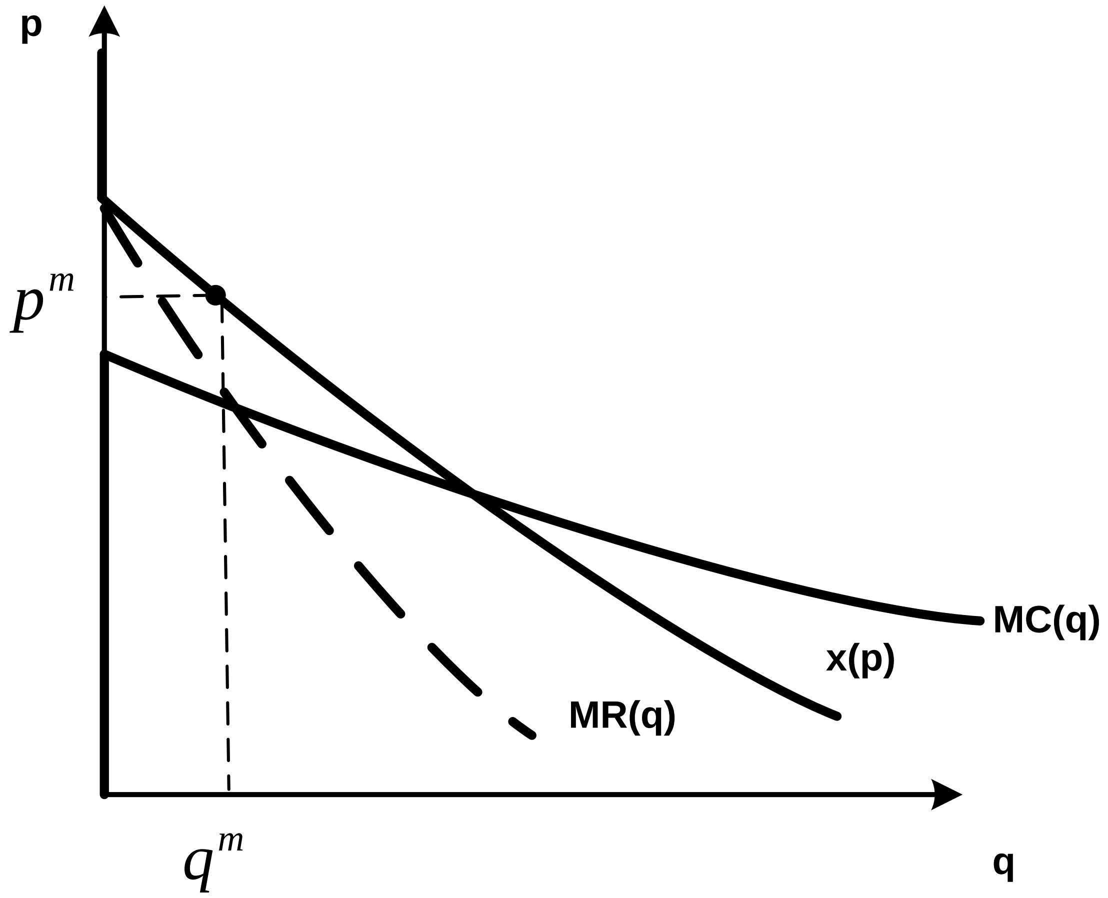


MR curve crosses MC curve from above

Profit Maximization

- What would happen if MC curve was decreasing in q (e.g., concave technology given the presence of increasing returns to scale)?
  – Then, the slopes of MR and MC curves are both decreasing.
  – At the optimum, MR curve must be steeper MC curve.

Profit Maximization



Profit Maximization: Lerner Index

- Can we re-write the FOC in a more intuitive way? Yes.
  - Just take $MR = p(q) + p'(q)q = p + \frac{\partial p}{\partial q} q$ and multiply by $\frac{p}{p}$,

$$MR = p\frac{p}{p} + \underbrace{\frac{\partial p}{\partial q}\frac{q}{p}}_{1/\varepsilon_d} p = p + \frac{1}{\varepsilon_d} p$$

  - In equilibrium, $MR(q) = MC(q)$. Hence, we can replace MR with MC in the above expression.

Profit Maximization: Lerner Index

- Rearranging yields

$$\frac{p - MC(q)}{p} = -\frac{1}{\varepsilon_d}$$

- This is the ***Lerner index*** of market power
  - The price mark-up over marginal cost that a monopolist can charge is a function of the elasticity of demand.
- Note:
  - If $\varepsilon_d \to \infty$, then $\frac{p-MC(q)}{p} \to 0 \implies p = MC(q)$
  - If $\varepsilon_d \to 0$, then $\frac{p-MC(q)}{p} \to \infty \implies$ substantial mark-up

Profit Maximization: Lerner Index

- The Lerner index can also be written as

$$p = \frac{MC(q)}{1+\frac{1}{\varepsilon_d}}$$

  which is referred to as the ***Inverse Elasticity Pricing Rule*** (IEPR).

- *Example* (Perloff, 2012):
  - Prilosec OTC: $\varepsilon_d = -1.2$. Then price should be $p = \frac{MC(q)}{1+\frac{1}{-1.2}} = 5.88MC$
  - Designed jeans: $\varepsilon_d = -2$. Then price should be $p = \frac{MC(q)}{1+\frac{1}{-2}} = 2MC$

Profit Maximization: Lerner Index

- ***Example 1*** (linear demand):
  - Market inverse demand function is
  $$p(q) = a - bq$$
  where $b > 0$
  - Monopolist's cost function is $c(q) = cq$
  - We usually assume that $a > c \geq 0$
    - To guarantee $p(0) > c'(0)$
    - That is, $p(0) = a - b0 = a$ and $c'(q) = c$, thus implying $c'(0) = c$

Profit Maximization: Lerner Index

- ***Example 1*** (continued):
  - Monopolist's objective function

$$\pi(q) = (a - bq)q - cq$$

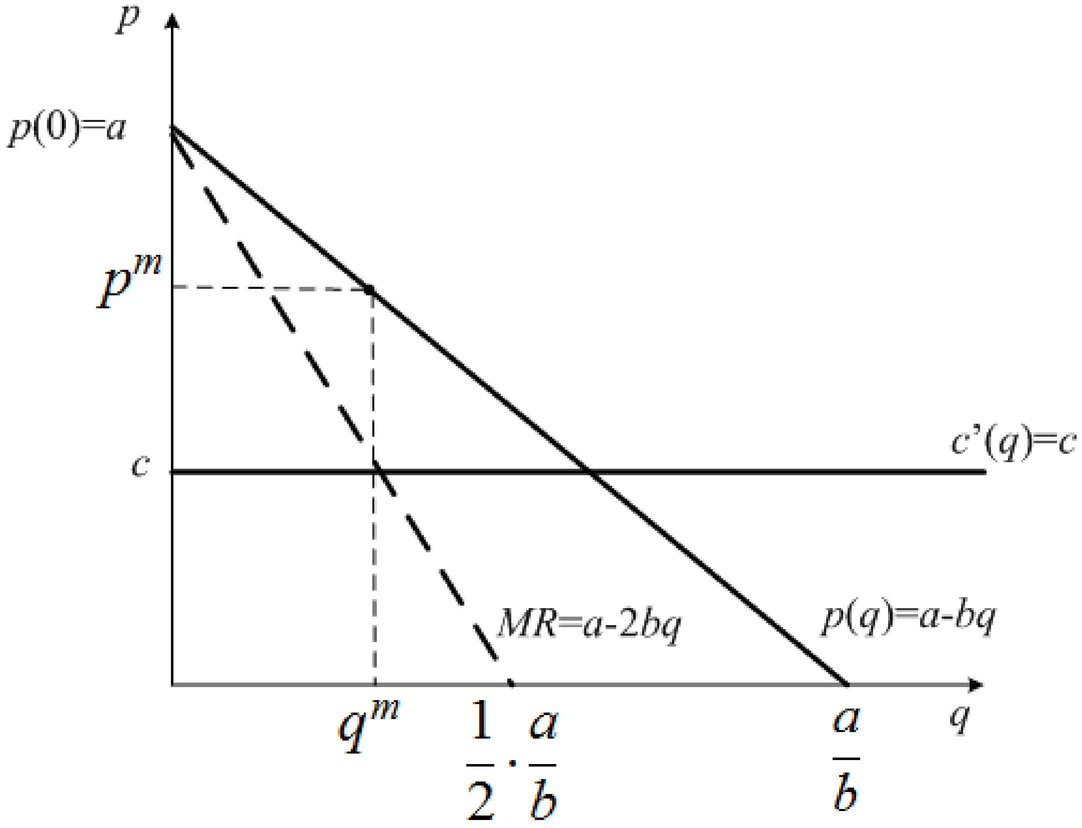
  - FOC: $a - 2bq - c = 0$
  - SOC: $-2b < 0$ (concave)
    - Note that as long as $b > 0$, i.e., negatively sloped demand function, profits will be concave in output.
    - Otherwise (i.e., Giffen good, with positively sloped demand function) profits will be convex in output.

Profit Maximization: Lerner Index

- ***Example 1*** (continued):
  - Solving for the optimal q^m in the FOC, we find monopoly output
$$q^m = \frac{a-c}{2b}$$
  - Inserting $q^m = \frac{a-c}{2b}$ in the demand function, we obtain monopoly price
$$p^m = a - b\left(\frac{a-c}{2b}\right) = \frac{a+c}{2}$$
  - Hence, monopoly profits are)
$$\pi^m = p^m q^m - cq^m = \frac{(a-c)^2}{4b}$$

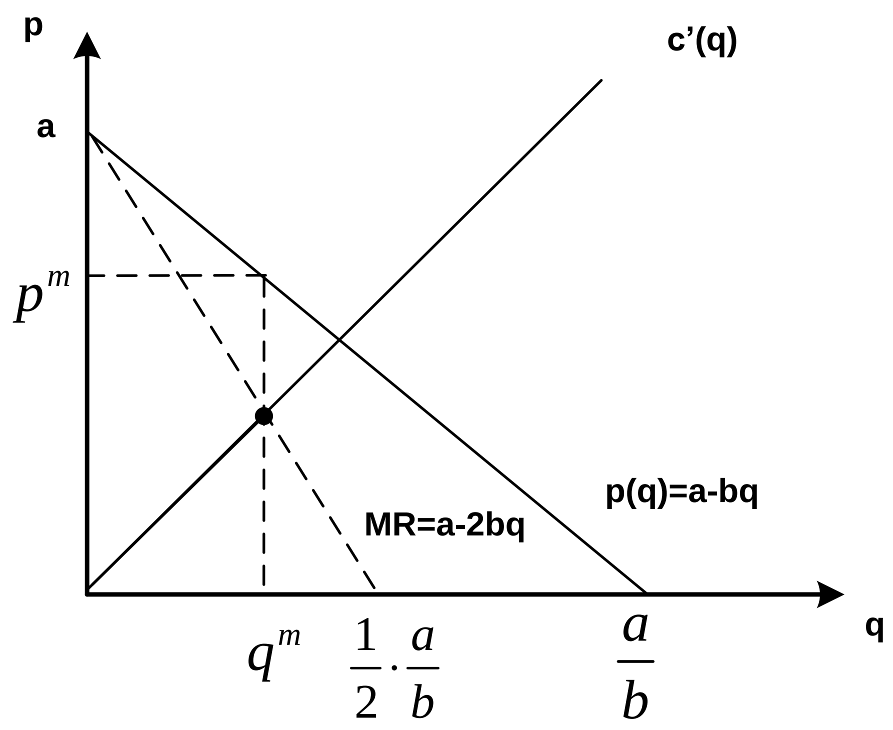
Profit Maximization: Lerner Index

- ***Example 1*** (continued):



Profit Maximization: Lerner Index

- ***Example 1*** (continued):
  - Non-constant marginal cost
  - The cost function is convex in output
    $$c(q) = cq^2$$
  - Marginal cost is
    $$c'(q) = 2cq$$



Profit Maximization: Lerner Index

- ***Example 2*** (Constant elasticity demand):
  - The demand function is
  $$q(q) = Ap^{-b}$$
  - We can show that $\varepsilon(q) = -b$ for all q, i.e.,
  $$\varepsilon(q) = \frac{\partial q(p)}{\partial p}\frac{p}{q} = \underbrace{(-b)Ap^{-b-1}}_{\frac{\partial q(p)}{\partial p}}\underbrace{\frac{p}{Ap^{-b}}}_{\frac{p}{q}}$$
  $$= -b\frac{p^{-b}}{p}\frac{p}{p^{-b}} = -b$$

Profit Maximization: Lerner Index

- ***Example 2*** (continued):
  – We can now plug $\varepsilon(q) = -b$ into the Lerner index,

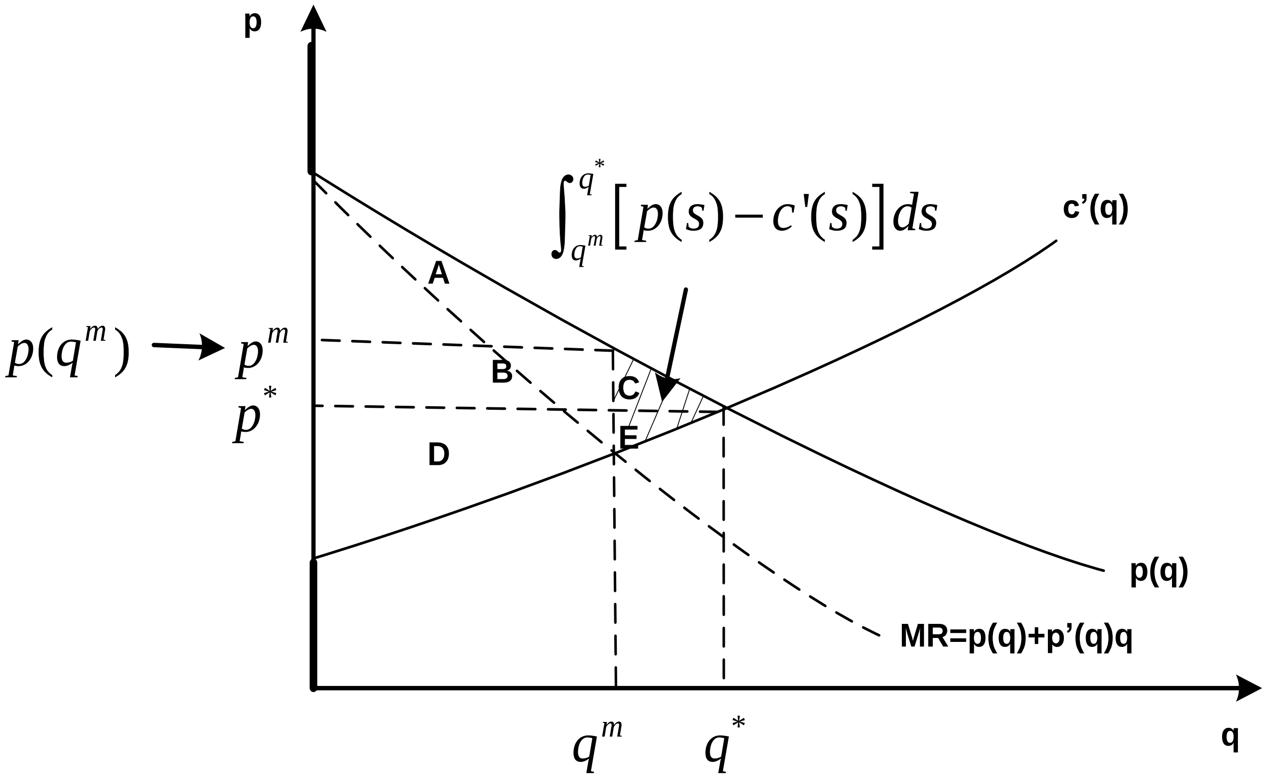
$$p^m = \frac{c}{1 + \frac{1}{\varepsilon(q)}} = \frac{c}{1 - \frac{1}{b}}$$

  – That is, price is a **constant mark-up over marginal cost.**

Welfare Loss of Monopoly

Welfare Loss of Monopoly

- Welfare comparison for perfect competition and monopoly.



Welfare Loss of Monopoly

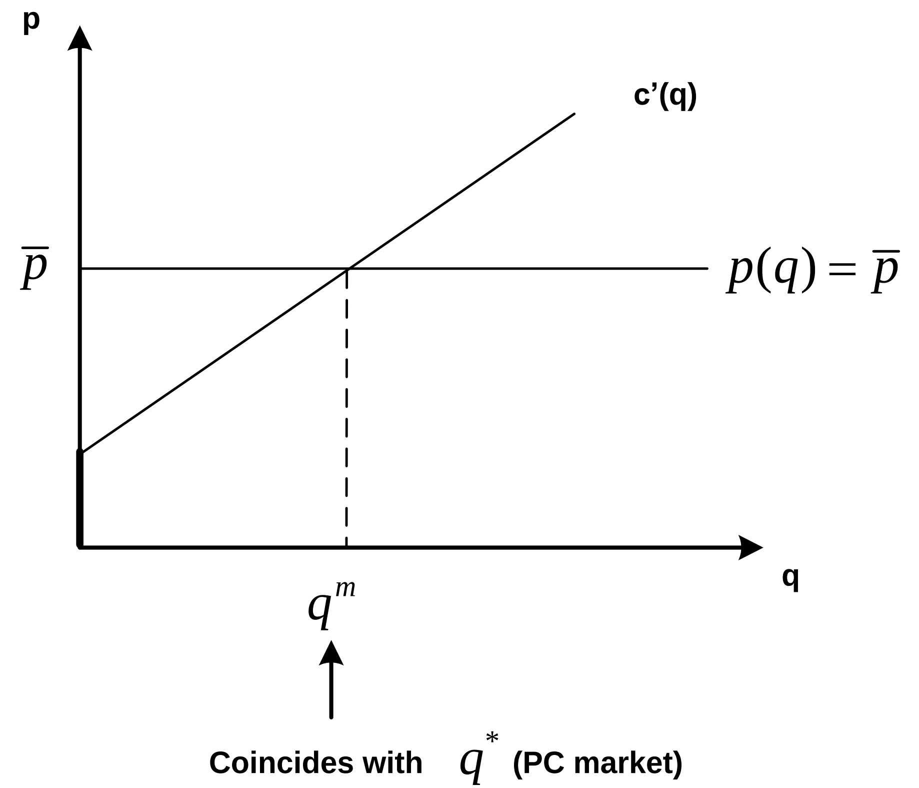
- Consumer surplus
  - Perfect competition: *A*+*B*+*C*
  - Monopoly: *A*
- Producer surplus:
  - Perfect competition: *D*+*E*
  - Monopoly: *D*+*B*
- **Deadweight loss of monopoly (DWL)**: *C*+*E*

$$DWL = \int_{q^m}^{q^*} [p(s) - c'(s)]ds$$

- **DWL decreases as demand and/or supply become more elastic.**

Welfare Loss of Monopoly

- Infinitely elastic demand
$$p'(q) = 0$$
- The inverse demand curve becomes totally flat.
- Marginal revenue coincides with inverse demand:
$$MR(q) = p(q) + 0 \cdot q$$
$$= p(q)$$
- Profit-maximizing q
$$MR(q) = MC(q) \Longrightarrow$$
$$p(q) = MC(q)$$
- Hence, $q^m = q^*$ and $DWL = 0$.



Welfare Loss of Monopoly

- ***Example*** (Welfare losses and elasticity):
  - Consider a monopolist with constant marginal and average costs, $c'(q) = c$, who faces a market demand with constant elasticity

  $q(p) = p^e$ with $e < -1$ (clear later....)

  where e is the price elasticity of demand ($e < -1$)
  - Perfect competition: $p_c = c$
  - Monopoly: using the IEPR

$$p^m = \frac{c}{1 + \frac{1}{e}}$$

Welfare Loss of Monopoly

- ***Example*** (continued):
  - The consumer surplus associated with any price (p_0) can be computed as

$$CS = \int_{p_0}^{\infty} q(p)dp = \int_{p_0}^{\infty} p^e dp = \frac{p^{e+1}}{e+1}\bigg|_{p_0}^{\infty} - \frac{p_0^{e+1}}{e+1}$$

  - Under perfect competition, $p_c = c$,

$$CS = -\frac{c^{e+1}}{e+1}$$

  - Under monopoly, $p^m = \frac{c}{1+1/e}$,

$$CS_m = -\frac{\left(\frac{c}{1+1/e}\right)^{e+1}}{e+1}$$

Welfare Loss of Monopoly

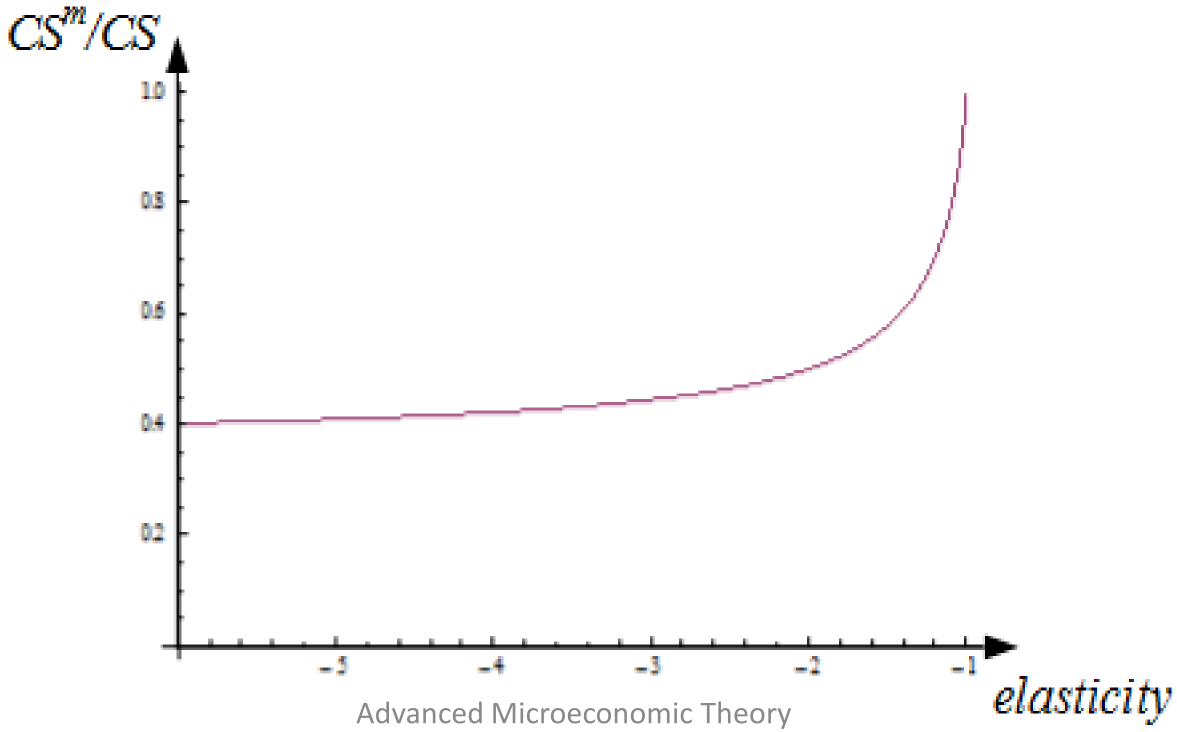
- ***Example*** (continued):
  - Taking the ratio of these two surpluses

$$\frac{CS_m}{CS} = \left(\frac{1}{1 + 1/e}\right)^{e+1}$$

  - If $e = -2$, this ratio is ½
    - CS under monopoly is half of that under perfectly competitive markets

Welfare Loss of Monopoly

- ***Example*** (continued):
  - The ratio $\frac{CS_m}{CS} = \left(\frac{1}{1+1/e}\right)^{e+1}$ decreases as demand becomes more elastic.



Welfare Loss of Monopoly

- ***Example*** (continued):
  - Monopoly profits are given by

$$\pi^m = p^m q^m - c q^m = \left(\frac{c}{1 + 1/e} - c\right) q^m$$

where $q^m(p) = p^e = \left(\frac{c}{1+1/e}\right)^e$.

  - Re-arranging,

$$\pi^m = \left(\frac{-c/e}{1 + 1/e}\right)\left(\frac{c}{1 + 1/e}\right)^e$$

$$= -\left(\frac{c}{1 + 1/e}\right)^{e+1} \cdot \frac{1}{e}$$

Welfare Loss of Monopoly

- ***Example*** (continued):
  - To find the transfer from CS into monopoly profits that consumers experience when moving from a perfectly competition to a monopoly, divide monopoly profits ($\pi^m = -\left(\frac{c}{1+\frac{1}{e}}\right)^{e+1} \cdot \frac{1}{e}$) by the competitive CS ($CS = -\frac{c^{e+1}}{e+1}$)

$$\frac{\pi^m}{CS} = \left(\frac{e+1}{e}\right)\left(\frac{1}{1+1/e}\right)^{e+1} = \left(\frac{e}{1+e}\right)^e$$

  - If $e = -2$, this ratio is ¼
    - One fourth of the consumer surplus under perfectly competitive markets is transferred to monopoly profits

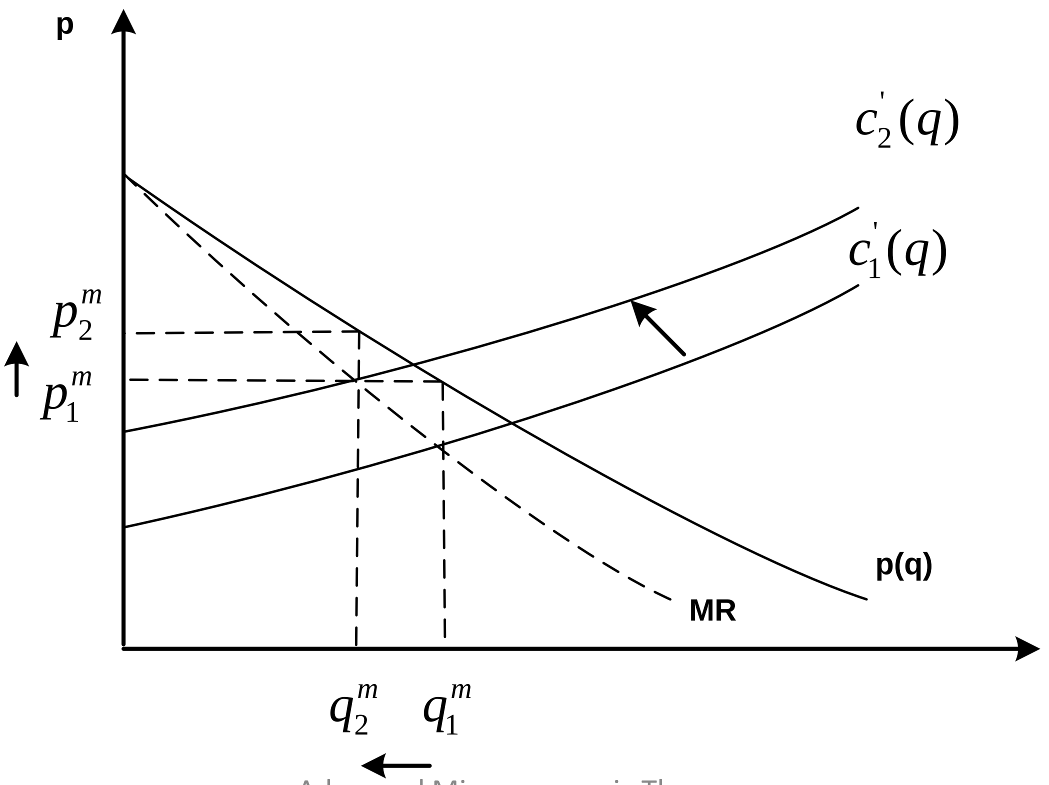
Welfare Loss of Monopoly

- More social costs of monopoly:
  - Excessive R&D expenditure (patent race)
  - Persuasive (not informative) advertising
  - Lobbying costs (different from bribes)
  - Resources to avoid entry of potential firms in the industry

Comparative Statics

Comparative Statics

- We want to understand how q^m varies as a function of monopolist's marginal cost



Comparative Statics

- Formally, we know that at the optimum, $q^m(c)$, the monopolist maximizes its profits

$$\frac{\partial \pi(q^m(c), c)}{\partial q^m} = 0$$

- Differentiating wrt c, and using the chain rule,
- $\frac{\partial^2 \pi(q^m(c),c)}{\partial q^m \partial q^m} \frac{dq^m(c)}{dc} + \frac{\partial^2 \pi(q^m(c),c)}{\partial q^m \partial c} = 0$

- Solving for $\frac{dq^m(c)}{dc}$, we have

$$\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{(\partial q)^2}}$$

Comparative Statics

- ***Example***:
  - Assume linear demand curve $p(q) = a - bq$
  - Then, the cross-derivative is

$$\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c} = \frac{\partial \left(\frac{\partial [(a - bq)q - cq]}{\partial q} \right)}{\partial c}$$

$$= \frac{\partial [a - 2bq - c]}{\partial c} = -1$$

and

$$\frac{dq^m(c)}{dc} = -\frac{\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}}{\frac{\partial^2 \pi(q^m(c), c)}{\partial q^2}} = -\frac{-1}{-2b} < 0$$

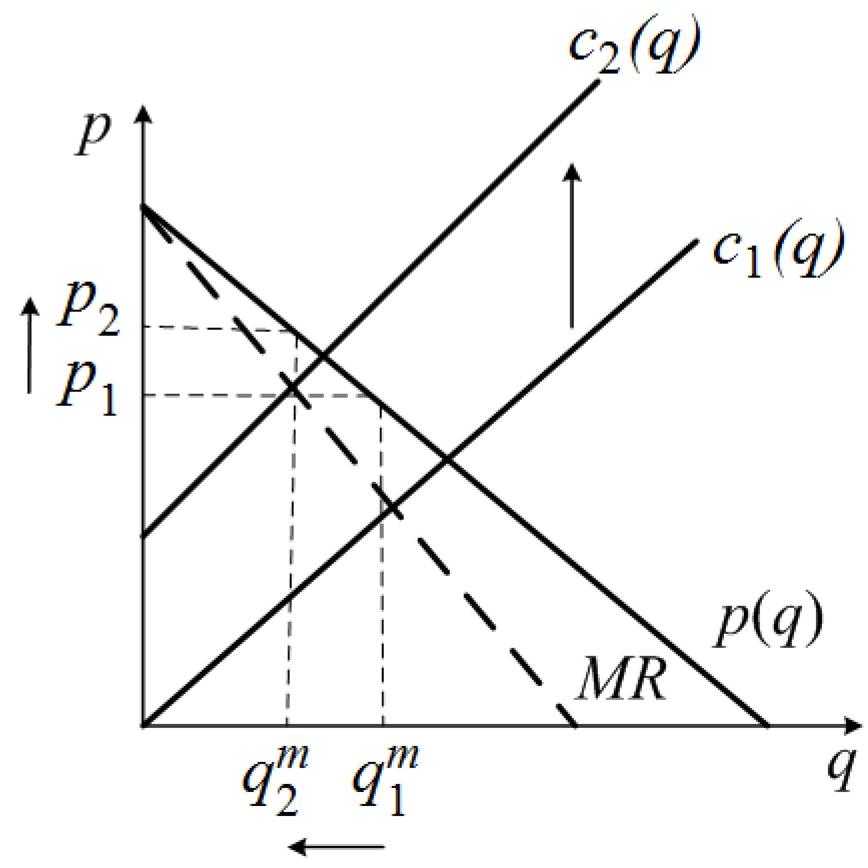
Comparative Statics

- ***Example*** (continued):
  - That is, an increase in marginal cost, c, decreases monopoly output, q^m.
  - See examples for other demand functions in the book
  - Even if we don't know the precise demand function, but know the sign of

$$\frac{\partial^2 \pi(q^m(c), c)}{\partial q \partial c}$$

Comparative Statics (DbY)

- ***Example*** (continued):
  - Marginal costs are increasing in q
  - For convex cost curve $c(q) = cq^2$, monopoly output is
    $$q^m(c) = \frac{a}{2(b+c)}$$
  - Here,
    $$\frac{dq^m(c)}{dc} = -\frac{a}{2(b+c)^2} < 0$$

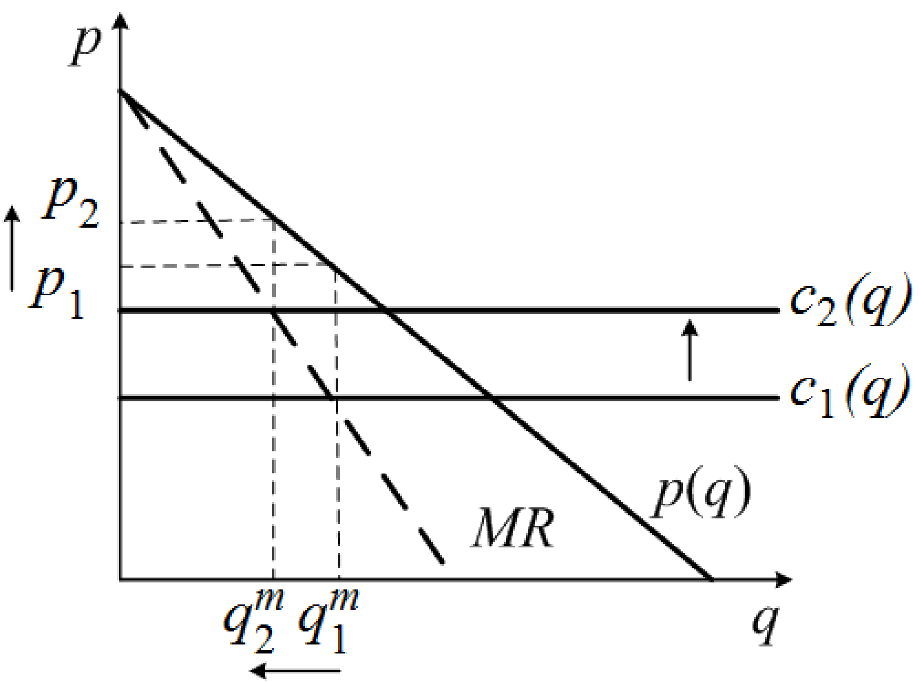


Comparative Statics (DbY)

- ***Example*** (continued):
  - Constant marginal cost
  - For the constant-elasticity demand curve $q(p) = p^e$, we have $p^m = \frac{c}{1+1/e}$ and $q^m(c) = \left(\frac{ec}{1+e}\right)^e$
  - Here,

$$\frac{dq^m(c)}{dc} = \frac{e}{c}\left(\frac{ec}{1+e}\right)^e$$

$$= \frac{e}{c} q^m < 0$$



Multiplant Monopolist

Multiplant Monopolist

- Monopolist produces output $q_1, q_2, \ldots, q_N$ across N plants it operates, with total costs $TC_i(q_i)$ at each plant $i = \{1,2,\ldots,N\}$.
- Profits-maximization problem

$$\max_{q_1,\ldots,q_N} \left[a - b\sum_{i=1}^{N} q_i\right] \sum_{i=1}^{N} q_i - \sum_{i=1}^{N} TC_i(q_i)$$

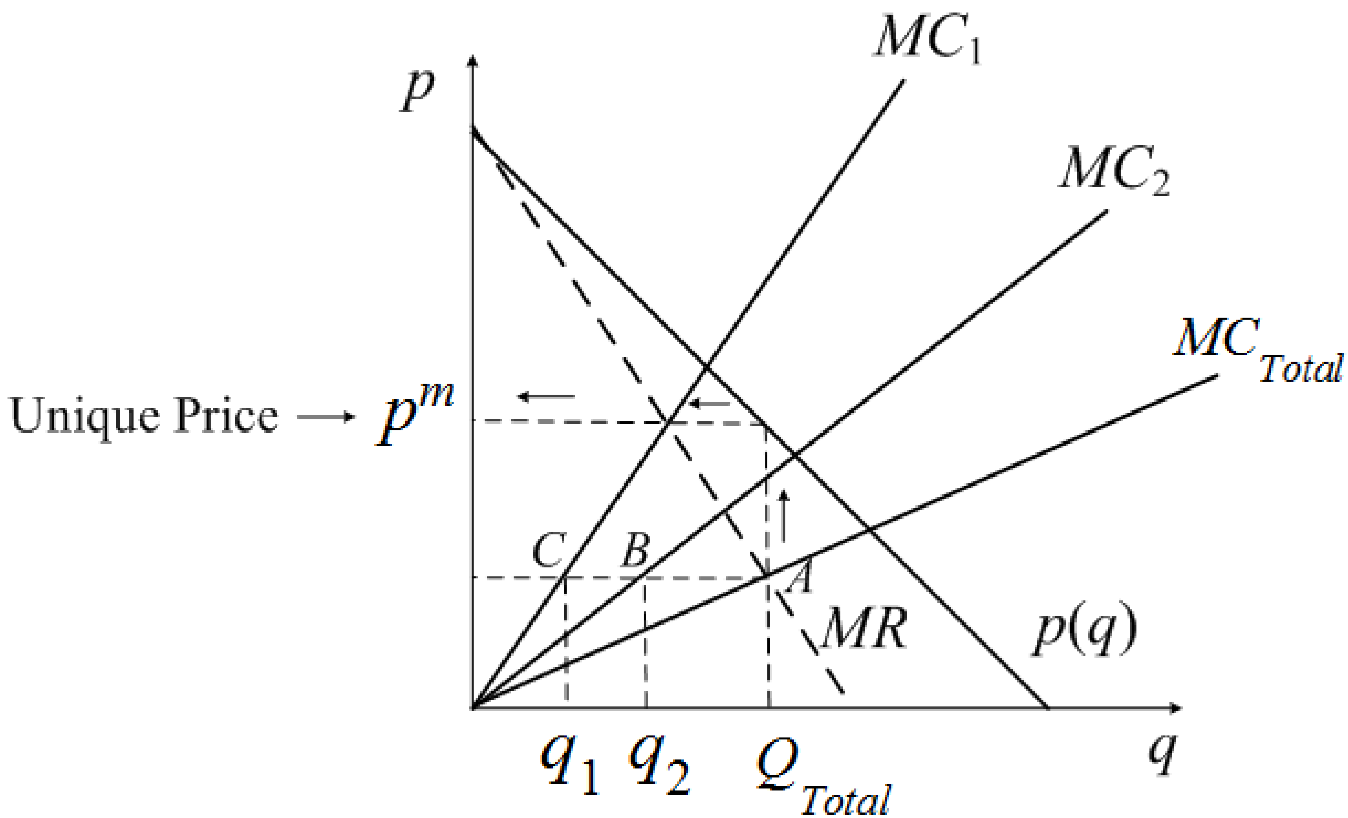
- FOCs wrt production level at every plant j

$$a - 2b\sum_{i=1}^{N} q_i - MC_j(q_j) = 0$$
$$\Leftrightarrow MR(Q) = MC_j(q_j)$$

for all j.

Multiplant Monopolist

- Multiplant monopolist operating two plants with marginal costs MC_1 and MC_2.



Multiplant Monopolist

- Total marginal cost is $MC_{total} = MC_1 + MC_2$ (i.e., horizontal sum)
- Q_{total} is determined by $MR = MC_{total}$ (i.e., point *A*)
- Mapping Q_{total} in the demand curve, we obtain price p^m (both plants sell at the same price)
- At the MC level for which $MR = MC_{total}$ (i.e., point *A*), extend a line to the left crossing MC_1 and MC_2.
- This will give us output levels q_1 and q_2 that plants 1 and 2 produce, respectively.

Multiplant Monopolist

- ***Example 1*** (symmetric plants):
  - Consider a monopolist operating N plants, where all plants have the *same* cost function $TC_i(q_i) = F + cq_i^2$. Hence, all plants produce the same output level $q_1 = q_2 = \cdots = q_N = q$ and $Q = Nq$ The linear demand function is given by $p = a - bQ$.
  - FOCs:

$$a - 2b\sum_{j=1}^{N} q = 2cq \;\; \text{or} \;\; a - 2bNq = 2cq$$

$$q = \frac{a}{2(bN + c)}$$

Multiplant Monopolist

- ***Example 1*** (continued):
  - Total output produced by the monopolist is
$$Q = Nq = \frac{Na}{2(bN + c)}$$
and market price is
$$p = a - bQ = a - b\frac{Na}{2(bN + c)} = \frac{a(bN + 2c)}{2(bN + c)}$$
  - Hence, the profits of every plant j are $\pi_j = \frac{a^2}{4(bN+c)} - F$, with total profits of
$$\pi_{total} = \frac{Na^2}{4(bN + c)} - NF$$

Multiplant Monopolist

- ***Example 1*** (continued):
  – The optimal number of plants N^* is determined by
  $$\frac{d\pi_{total}}{dN} = \frac{a^2}{4}\frac{c}{(bN+c)^2} - F = 0$$
  and solving for N
  $$N^* = \frac{1}{b}\left(\frac{a}{2}\sqrt{\frac{c}{F}} - c\right)$$
  – N^* is decreasing in the fixed costs F, and also decreasing in c, as long as $a < 4\sqrt{cF}$.

Multiplant Monopolist

- ***Example 1*** (continued):
  - Note that when $N = 1$, $Q = q^m$ and $p = p^m$.
  - Note that an increase in N decreases q_j(=q) and π_j.

Multiplant Monopolist (DbY)

- ***Example 2*** (asymmetric plants):
  - Consider a monopolist operating two plants with marginal costs $MC_1(q_1) = 10 + 20q_1$ and $MC_2(q_2) = 60 + 5q_2$, respectively. A linear demand function is give by $p(Q) = 120 - 3Q$.
  - Note that $MC_{total} \neq MC_1(q_1) + MC_2(q_2)$
    - This is a vertical (not a horizontal) sum.
  - Instead, first invert the marginal cost functions

$$MC_1(q_1) = 10 + 20q_1 \Leftrightarrow q_1 = \frac{MC_1}{20} - \frac{1}{2}$$

$$MC_2(q_2) = 60 + 5q_2 \Leftrightarrow q_2 = \frac{MC_2}{5} - 12$$

Multiplant Monopolist (DbY)

- ***Example 2*** (continued):
  - Second,

$$Q_{total} = q_1 + q_2 = \frac{MC_{total}}{20} - \frac{1}{2} + \frac{MC_{total}}{5} - 12$$
$$= \frac{1}{4} MC_{total} - 12.5$$

  - Hence, $MC_{total} = 50 + 4Q_{total}$
  - Setting $MR(Q) = MC_{total}$, we obtain $Q_{total} = 7$ and $p = 120 - 3 \cdot 7 = 99$.
  - Since $MR(Q_{total}) = 120 - 6 \cdot 7 = 78$, then

$$MR(Q_{total}) = MC_1(q_1) \Longrightarrow 78 = 10 + 20q_1 \Longrightarrow q_1 = 3.4$$
$$MR(Q_{total}) = MC_2(q_2) \Longrightarrow 78 = 60 + 5q_2 \Longrightarrow q_2 = 3.6$$