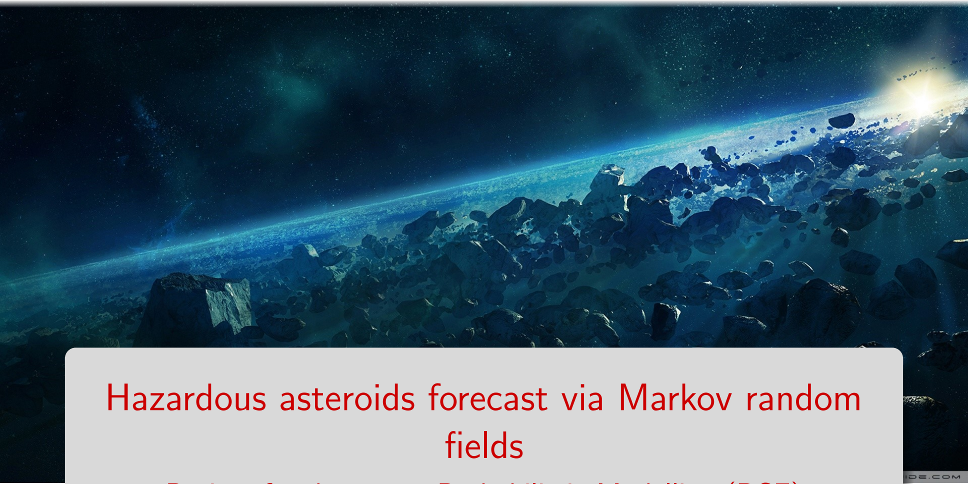

Hazardous asteroids forecast via Markov random fields

Project for the exam: Probabilistic Modelling (DSE)

Marzio De Corato

Introduction

- **Final Goal** Assessment of forecasts and interpretability for different machine learning algorithms, including the probabilistic models
- **Method** Use a dataset for which the laws that interconnects the different features are known from general principles
- **Dataset** CNEOS asteroids dataset for more than 3500 asteroids
- **Theoretical laws** Celestial mechanics
- **Algorithms involved - probabilistic models** GLASSO, mgm, minforest, mmod
- **Algorithms involved - others** Random forest, Support Vector Machines, Quadratic Discriminant Analysis, Logistic Regression

Celestial mechanics

$$\mathbf{F}_1 = \mathcal{G} \cdot \frac{m_1 m_2}{r^3}\mathbf{r} = m_1 \ddot{\mathbf{r}}_1 \tag{1}$$

$$\mathbf{F}_2 = -\mathcal{G} \cdot \frac{m_1 m_2}{r^3}\mathbf{r} = m_1 \ddot{\mathbf{r}}_2 \tag{2}$$

If we consider the motion of the second item with respect to the first one

$$\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_2 - \ddot{\mathbf{r}}_1 \quad \mu = \mathcal{G}(m_1 + m_2) \tag{3}$$

$$\frac{d^2\mathbf{r}}{dt^2} + \mu\frac{\mathbf{r}}{r^3} = 0 \tag{4}$$

$\mathbf{r} \times \ddot{\mathbf{r}} = 0 \implies \mathbf{r}$ and $\dot{\mathbf{r}}$ lies in the same plane

Celestial mechanics

With polar coordinates $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$

$$\mathbf{r} = r\hat{\mathbf{r}} \tag{5}$$

$$\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} \tag{6}$$

$$\ddot{\mathbf{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left[\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)\right]\hat{\boldsymbol{\theta}} \tag{7}$$

$$\mathbf{h} = r^2\dot{\theta}\hat{\mathbf{z}} \tag{8}$$

$$h = r^2\dot{\theta} \tag{9}$$

Celestial mechanics [1]: 2^{th} Kepler law

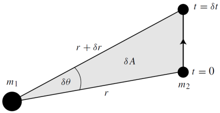

Figure 1: [1]

$$\delta A \approx \frac{1}{2} r(r + dr)\sin(\delta\theta) \approx \frac{1}{2} r^2 \delta\theta \tag{10}$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} h \tag{11}$$

h is constant $\implies$ 2^{th} Kepler law

Celestial mechanics [1]: 1^{th} Kepler law

Using the substitution $u = \frac{1}{r}$ $h = r^2\dot{\theta}$

$$\dot{r} = -\frac{1}{u}\frac{du}{d\theta}\dot{\theta} = -h\frac{du}{d\theta} \tag{12}$$

$$\ddot{r} = -h\frac{d^2u}{d\theta^2}\dot{\theta} = -h^2u^2\frac{d^2u}{d\theta^2} \tag{13}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2} \tag{14}$$

$$u = \frac{\mu}{h^2}\left[1 + e\cos(\theta - \phi)\right] \tag{15}$$

Celestial mechanics [1]: 1^{th} Kepler law

$$r = \frac{p}{1 + e\cos(\theta - \phi)} \qquad (16)$$

e is eccentricity

- circle: $e = 0 \qquad p = a$
- ellipse: $0 < e < 1 \qquad p = b$
- parabola: $e = 1 \qquad p = 2q$
- hyperbola: $e > 1$
  $p = a(e^2 - 1)$

b is the semi-major axis of the conic

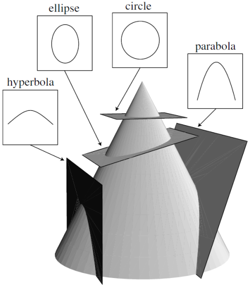

Figure 2: [1]

Celestial mechanics [1]: 3^{th} Kepler law

$$b^2 = a^2(1 - e^2) \tag{17}$$

$$r = \frac{a(1 - e^2)}{1 + e \cdot cos(\theta - \phi)} \tag{18}$$

Area swept in one orbital period T $\implies A = \pi ab$ We know that: $hT/2 \quad h^2 = \mu a(1 - e^2)$

Therefore

$$T^2 = \frac{4\pi^2}{\mu} a^3 \tag{19}$$

3^{th} Kepler law

$$\frac{m_c + m}{m_c + m'} = \left(\frac{a}{a'}\right)\left(\frac{T'}{T}\right)^2 \tag{20}$$

But since $m, m' << m_c$

$$(\acute{a})^3 \approx (T')^2 \tag{21}$$

And therefore

$$T' \approx a'^{3/2} \tag{22}$$

Orbital parameters

Mean motion

$$n = \frac{2\pi}{T} \tag{23}$$

$$v_{perihelion} = na\sqrt{\frac{1+e}{1-e}} \tag{24}$$

$$v_{aphelion} = na\sqrt{\frac{1-e}{1+e}} \tag{25}$$

Orbital parameters

Mean anomaly

$$M = n(t - \tau) \tag{26}$$

- $M = f = 0 \quad t = \tau \quad$ Perihelion
- $M = f = \pi \quad t = \tau + T/2 \quad$ Aphelion

$$M = E - e \sin E \tag{27}$$

Jupiter Tisserard invariant

$$T_P = \frac{a_p}{a} + 2 \cos I \sqrt{\frac{a}{a_p}(1 - e^2)} \tag{28}$$

References I

[1] Carl D Murray and Stanley F Dermott. *Solar system dynamics*. Cambridge university press, 1999.