



Time series

# Time series definition [1]

## Informal definition

A time-series is a set of observation  $x_t$  each one being recorder at a specific time  $t$ .

## Formal definition

A time series model for the observed data  $x_t$  is a specification of the joint distribution (or possible only the mens covariance) of a sequence of random variable  $X_t$  of which  $x_t$  is postulated to be a realization

## A binary process

Consider the sequence of iid random variables, with  $P[X_t = 1] = p$  and  $P[X_t = -1] = 1 - p$

## Random walk

The random walk is obtained by cumulatevely summing iid random variables. Thus a random walk with zero mean is obtained by defining

# Stationarity, autocovariance and autocorrelation[1]

## Mean Function

Let  $X_t$  be a time series with  $E(x_t^2) < \infty$ . The mean function of  $X_t$  is  $\mu_X(t) = E(X_t)$ . The covariance function of  $X_t$  is  $\gamma_X(r, s) = \text{Cov}(X_r, X_s) = E[(X_r - \mu_X(r))(X_s - \mu_X(s))]$   $\forall r, s$

## Weakly stationary TS

$X_t$  is weakly stationary if i)  $\mu_X(t)$  is independent from time  $t$  and ii)  $\gamma_X(t+h)$  is independent of  $t \forall h$

## Autocovariance function

At lag  $h$  the autocovariance function is defined as  $\gamma_X(h) = \text{Cov}(X_{t+h}, X_t)$

## Autocorrelation function

At lag  $h$  the autocorrelation function is defined as

$$\rho(h)_X = \frac{\gamma_X(h)}{\gamma_X(0)} = \text{Cor}(X_{t+h}, X_t)$$

# Bibliography I

- [1] Peter J Brockwell e Richard A Davis. *Introduction to time series and forecasting*. Springer, 2002.