SE 210 | Worksheet 1 | Sets and Counting

1) a) $A = \{1, 2, 3, 4, 5\}$

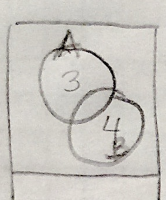
b) $A^3 = A \times A \times A$

c) $|A^3| = |A|^3 = 5^3 = 125$

2) Binary choices are 0 & 1
$A = \{0, 1\}$
$K = 500$

$|A^K| = |A|^K = 2^{500}$

3)

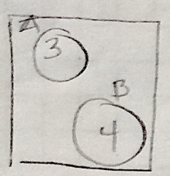


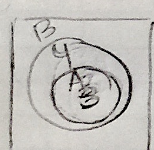
a) largest $A \cup B = 7$

b) smallest $A \cup B = 4$

c) largest $A \cap B = 3$

d) largest $A \cap B = 0$





4) $\underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 4! = 24$

5) $A = \{$English Characters$\}$
$|A| = 26$

$|A|^5 = 26^5 = 11{,}881{,}376$

6) $\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 120$

7) $\binom{10}{5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5!} \cdot 5! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30{,}240$

1) a) $\Omega = \{A, B\}$

b) $\Omega = \{H, T\}$

c) $\Omega = \Omega_0 \times \Omega_1$

$\Omega_0 = \{$Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec$\}$

$\Omega_1 = \{1, 2, 3, 4, 5, 6, 7\}$

d) $\Omega = \{1, 2, \dots 10\}$

e) $\Omega = \Omega_0 \times \Omega_1$

$\Omega_0 = \{$red, black, silver, green$\}$

$\Omega_1 = \{$black, beige$\}$

2) a) $\Omega = \{H, T\}^{200}$

b) $\Omega = \{$Non-negative integers$\}$

c) $\Omega = \{$words in Hamlet$\}$

3) a) $A \cap B \cap C$

b) $A \cup B \cup C$

c) $(A \cap B) \setminus C$

4) $\Omega = \{a, b, c\}$

$Pr(a) = 1/2$

$Pr(b) = 1/3$

a) $Pr(c) = 1 - (Pr(a) + Pr(b))$

$Pr(c) = 1 - (\frac{1}{2} + \frac{1}{3})$

$Pr(c) = 1/6$

b)

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5 a) First toss is heads
b) All three tosses result in either all heads or all tails
c) One of the three tosses result in either all heads or all tails

$\{H, T\}^3$

$|\{H, T\}|^3 = 2^3 = 8$

$Pr(E_1) = 4/8 = 1/2$

$Pr(E_2) = 2/8 = 1/4$

$Pr(E_3) = 3/8$

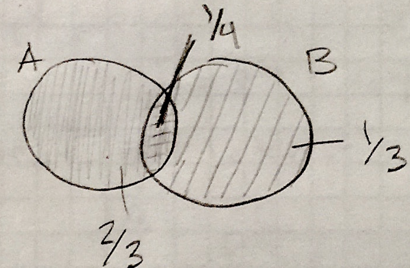
6) $Pr(A \cap B) = 1/4$

$Pr(A^c) = 1/3$, $Pr(A) = 2/3$

$Pr(B) = 1/2$

$A^c = \Omega \setminus A$ in the event A does not happen

$Pr(A \cup B)$?



$Pr(A) + Pr(B) - Pr(A \cap B)$

$$\frac{2}{3} + \frac{1}{2} - \frac{1}{4} = \frac{8}{12} + \frac{6}{12} - \frac{3}{12} = \frac{11}{12}$$

7) $\Omega = \{1,2,3,4,5,6\}^2$

$A = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$|\Omega| = 6^2 = 36$

$Pr(\omega) = 1/36$

$|A| = 6$

$Pr(A) = 6/36 = 1/6$

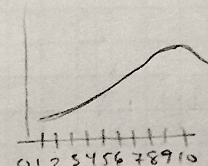
14) Sample space $\Omega = \{\text{good}, \text{rotten}\}^{10}$

Probability of outcomes: $Pr(\omega) = Pr(10A's) + Pr(9A's) + Pr \ldots Pr(0A's) = 1$

Event of interest: the set of outcomes

$A = \{\omega : \omega \text{ is 10 good apples}\} \subset \Omega$

$$\sum_{\omega \in A} Pr(\omega) = = 1$$



How many ways to pick 10 apples from the barrel? $\binom{100}{10} = |\Omega|$

Pr(good) = 90 good from 100

$|A| = \binom{90}{10}$

$$Pr(A) = \frac{\binom{90}{10}}{\binom{100}{10}} = \frac{\frac{90 \cdot 89 \cdot 88 \cdot 87 \cdot 86 \cdot 85 \cdot 84 \cdot 83 \cdot 82 \cdot 81}{\cancel{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}}{\frac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96 \cdot 95 \cdot 94 \cdot 93 \cdot 92 \cdot 91}{\cancel{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}} = 33.0489$$

16) Sample space $\Omega = \{\text{girl}, \text{boy}\}^6$

$|\Omega| = 2^6 = 64$

Event of interest $A = \{3 \text{ boys}, 3 \text{ girls}\}$

$$|A| = \binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2} = 20$$

$$Pr(A) = \frac{20}{64} = \frac{10}{32} = \frac{5}{16}$$

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1) Sample space $\Omega = \{H, T\}^3$

HHH TTT
HHT THT
HTH THH
HTT TTH

$|\Omega| = 2^3$

$|\Omega| = 8$

$Pr(2H) = \frac{2}{8} = \frac{1}{4}$

Event $A_1 = \{$ first outcome is a head $\}$

$|A_1| = 4$

a) $Pr(2 \text{ Heads} \mid 1^{st} \text{ toss } H) = \frac{Pr(2 \text{ Heads} \cap 1^{st} \text{ toss } H)}{Pr(1^{st} \text{ toss } H)} = \frac{2}{4} = \frac{1}{2}$

b) $Pr(2 \text{ Heads} \mid 1^{st} \text{ toss } T) = \frac{Pr(2 \text{ Heads} \cap 1^{st} \text{ toss } T)}{Pr(1^{st} \text{ toss } T)} = \frac{1}{4}$

c) $Pr(2 \text{ Heads} \mid \text{first 2 outcomes } H) = \frac{Pr(2 \text{Hds} \cap 1^{st} \text{ 2 outcomes } H)}{Pr(1^{st} \text{ 2 outcomes } H)} = \frac{2}{4} = \frac{1}{2}$

d) $Pr(2 \text{ Heads} \mid \text{first 2 outcomes } T) = \frac{Pr(2 \text{Hds} \cap 1^{st} \text{ 2 outcomes } T)}{Pr(1^{st} \text{ 2 outcomes } T)} = 0$

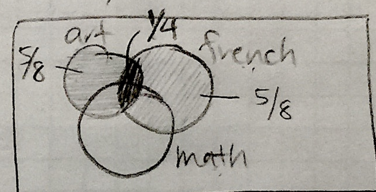
e) $Pr\left(2 \text{ Heads} \middle| \begin{array}{l} \text{first outcome } H \\ \text{third outcome } T \end{array}\right) = \frac{Pr\left(2 \text{Hds} \middle| \begin{array}{l} 1^{st} H \\ 3^{rd} T \end{array}\right)}{Pr\left(\begin{array}{l} 1^{st} H \\ 3^{rd} T \end{array}\right)} = \frac{1}{2}$

2) Sample space $\Omega = \{\text{art, french, math}\}^2$

$Pr(\text{art}) = 5/8$

$Pr(\text{french}) = 5/8$

$Pr(\text{art} \cap \text{french}) = 1/4$



Sample space $\Omega = \{AF, AM, FM\}$

$Pr(\text{Art}) = Pr(\text{Art \& French}) + Pr(\text{Art \& math})$

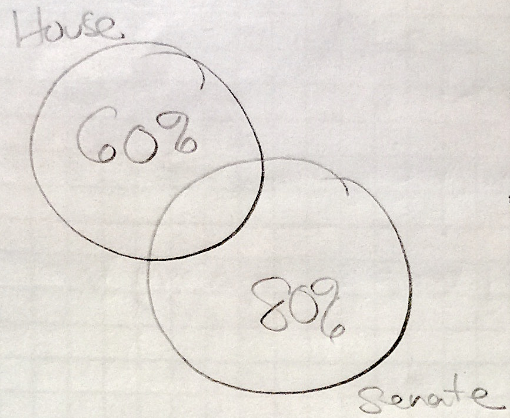
$Pr(\text{Art \& math}) = Pr(\text{Art}) - Pr(\text{Art \& French})$
$= \frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

$Pr(\text{French \& math}) = Pr(\text{French}) - Pr(\text{French \& Art})$
$= \frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

a) $Pr(\text{math}) = Pr(\text{French \& math}) + Pr(\text{Art \& math})$
$= \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$

b) Probability = 1 = 100%, he has to take Art or French

3)



$Pr(H) = 60\%$

$Pr(S) = 80\%$

$Pr(H \cup S) = 90\%$

$Pr((H \cup S)^c) = 10\%$

1 = Pr(pass H, pass S) +
Pr(pass H, doesn't pass S) +
Pr(doesn't pass H, passes S) +
Pr(doesn't pass H, doesn't pass S)

0.1 = Pr(doesn't pass H, doesent pass S)

Pr(H) = 0.6 = Pr(pass H, pass S) + Pr(pass H, doesn't pass S)
Pr(S) = 0.8 = Pr(pass H, pass S) + Pr(doesn't pass S, pass S)

1 = Pr(pass H, pass S) +
0.6 − Pr(pass H, pass S) +
0.8 − Pr(pass H, pass S) +
0.1

1 − 1.5 = − Pr(pass H, pass S)

Pr(pass H, pass S) = 0.5

5) $\Omega = \{\text{all cards}\}$ =

$\omega = 1/52$

a) $Pr(\heartsuit \mid red) = \frac{Pr(\heartsuit \cup red)}{Pr(red)} = \frac{13/52}{26/52} = \frac{1}{2}$

b) $Pr(\{J, Q, K, A\} \mid \heartsuit) = \frac{Pr(\{J,Q,K,A\} \cup \heartsuit)}{Pr(\heartsuit)} = \frac{4/52}{13/52}$

$= 4/13$

c) $Pr(J \mid \omega > 10) = \frac{Pr(J \cup \omega > 10)}{Pr(\omega > 10)} = \frac{4/52}{16/52} = \frac{1}{4}$

6) $Pr(B^c) = 1/4$

$Pr(A \mid B) = 1/2$

$Pr(A \cap B) = Pr(A \mid B)\, Pr(B)$

$Pr(A \cap B) = (1/2)(1 - \frac{1}{4})$

$Pr(A \cap B) = 3/8$

10) Conditional Probability

$Pr(men) = Pr(women)$

$Pr(cb \mid man) = 5\%$

$Pr(cb \mid woman) = 1\%$

looking for $Pr(male \mid cb) = ?$

$Pr(man \mid cb) = \frac{Pr(man \cap cb)}{Pr(cb)}$

$= \frac{Pr(cb \mid man)}{Pr(cb \mid man) + Pr(cb \mid woman)}$

$Pr(man \mid cb) = \frac{5\%}{5\% + 1\%} = \frac{5}{6}$

13) A : Heads on 1st toss

B : Tails on 2nd toss

C : Heads on 3rd toss

D : All 3 outcomes same

E : Exactly one head

a) which pairs are dependent?

1) A, B independent

2) A, D independent

3) A, E dependent

4) D, E dependent

1 & 2

16) 1) A= {first card is a ♡}

B= {second card is a ♡}

Dependent, $Pr(B|A) \neq Pr(B)$

2) A= {first card is ♡}

B= {first card is 10}

Independent

3) A= {first card is 10}

B= {second card is 9}

Dependent, $Pr(B|A) \neq Pr(B)$

4) A= {first card is ♡}

B= {second card is 10}

Independent

2 & 4