SE 210 Worksheet 1 Sets and Counting

1) a) $A = \{1,2,3,4,5\}$

b) $A^3 = A \times A \times A$

c) $|A^3| = |A|^3 = 5^3 = 125$

2) Binary choices are 0 & 1

$A = \{0,1\}$

$K = 500$

$|A^K| = |A|^K = 2^{500}$

3)

a) largest $A \cup B$ = 7

b) smallest $A \cup B$ = 4

c) largest $A \cap B$ = 3

d) largest $A \cap B$ = 0

4) $4 \cdot 3 \cdot 2 \cdot 1 = 4! = 24$

5) $A = \{$ English characters $\}$

$|A| = 26$

$|A|^5 = 26^5 = 11,881,376$

6) $\binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 120$

7) $\binom{10}{5} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5!} \cdot 5! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30,240$

2E210 | Worksheet 2 | Probability Spaces

1) a) $\Omega = \{A, B\}$

b) $\Omega = \{H, T\}$

c) $\Omega = \Omega_0 \times \Omega_1$

$\Omega_0 = \{$Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec$\}$

$\Omega_1 = \{1, 2, 3, 4, 5, 6, 7\}$

d) $\Omega = \{1, 2, \ldots 10\}$

e) $\Omega = \Omega_0 \times \Omega_1$

$\Omega_0 = \{$red, black, white, green$\}$

$\Omega_1 = \{$clock, beige$\}$

2) a) $\Omega = \{H, T\}^{200}$

b) $\Omega = \{$Non-negative integers$\}$

c) $\Omega = \{$words in Hamlet$\}$

3) a) $A \cap B \cap C$

b) $A \cup B \cup C$

c) $(A \cap B) \setminus C$

4) $\Omega = \{a, b, c\}$

$Pr(a) = 1/2$

$Pr(b) = 1/3$

a) $Pr(c) = 1 - (Pr(a) + Pr(b))$

$Pr(c) = 1 - (1/2 + 1/3)$

$Pr(c) = 1/6$

b)

5 a) First toss is heads
b) All three tosses result in either all heads or all tails
c) One of the three tosses result in either all heads or all tails

$\{H, T\}^3$

$|\{H, T\}^3| = 2^3 = 8$

$Pr(E_1) = 4/8 = 1/2$

$Pr(E_2) = 2/8 = 1/4$

$Pr(E_3) = 3/8$

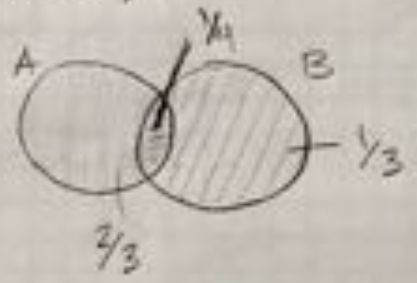
6) $Pr(A \cap B) = 1/4$

$Pr(A^c) = 1/3$, $Pr(A) = 2/3$

$Pr(B) = 1/2$

$A^c = \Omega \setminus A$ the event A does not happen

$Pr(A \cup B)$?



$Pr(A) + Pr(B) - Pr(A \cap B)$

$\frac{2}{3} + \frac{1}{2} - \frac{1}{4} = \frac{8}{12} + \frac{6}{12} - \frac{3}{12} = \frac{11}{12}$

7) $\Omega = \{1,2,3,4,5,6\}^2$

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$|\Omega| = 6^2 = 36$

$Pr(\omega) = 1/36$

$|A| = 6$

$Pr(A) = 6/36 = 1/6$

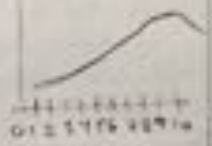
(a) Sample space $\Omega = \{\text{good, rotten}\}^{10}$

Probability of outcomes: $Pr(\omega) = Pr(1 \text{ GA's}) + Pr(9 \text{ A's}) + Pr(\ldots Pr(0 \text{ A's}) = 1$

Event of interest: the set of outcomes

$A = \{\omega : \omega \text{ is 10 good apples}\} \subset \Omega$

$$\sum_{\omega \in A} Pr(\omega) = -1$$



How many ways to pick 10 apples from the barrel? $\binom{100}{10} = |\Omega|$

$Pr(\text{good}) = 90 \text{ good from } 100$

$$|A| = \binom{90}{10}$$

$$Pr(A) = \frac{\binom{90}{10}}{\binom{100}{10}} = \frac{90 \cdot 89 \cdot 88 \cdot 87 \cdot 86 \cdot 85 \cdot 84 \cdot 83 \cdot 82 \cdot 81}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 33.0489$$

(b) Sample space $\Omega = \{\text{girl, boy}\}^6$

$$|\Omega| = 2^6 = 64$$

Event of interest $A = \{3 \text{ boys, } 3 \text{ girls}\}$

$$|A| = \binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2} = 20$$

$$Pr(A) = \frac{20}{64} = \frac{10}{32} = \frac{5}{16}$$

1) Sample space $\Omega = \{H,T\}^3$

HHH TTT
HHT THT
HTH THH
HTT TTH

$|\Omega| = 2^3$ $\quad \Pr(2H) = \frac{3}{8} = \frac{1}{4}$

Event A_1 = {first outcome is a head}

$|A_1| = 4$

a) $\Pr(2 \text{ Heads} \mid 1^{st} \text{ toss } H) = \frac{\Pr(2 \text{ Heads} \cap 1^{st} \text{ toss } H)}{\Pr(1^{st} \text{ toss } H)} = \frac{2}{4} = \frac{1}{2}$

b) $\Pr(2 \text{ Heads} \mid 1^{st} \text{ toss } T) = \frac{\Pr(2 \text{ Heads} \cap 1^{st} \text{ toss } T)}{\Pr(1^{st} \text{ toss } T)} = \frac{1}{4}$

c) $\Pr(2 \text{ Heads} \mid \text{first 2 outcomes } H) = \frac{\Pr(2H \cap \text{first 2 outcomes } H)}{\Pr(\text{first 2 outcomes } H)} = \frac{2}{4} = \frac{1}{2}$

d) $\Pr(2 \text{ Heads} \mid \text{first 2 outcomes } T) = \frac{\Pr(2H \cap \text{first 2 outcomes } T)}{\Pr(\text{first 2 outcomes } T)} = 0$

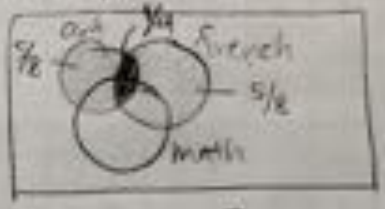
e) $\Pr\left(2 \text{ Heads} \mid \begin{matrix}\text{first outcome } H \\ \text{third outcome } T\end{matrix}\right) = \frac{\Pr(2H \mid 1^{st} H \text{ and } 3^{rd} T)}{\Pr(1^{st} H \text{ and } 3^{rd} T)} = \frac{1}{2}$

2) Sample space $\Omega = \{\text{art, French, math}\}^2$

$\Pr(\text{art}) = 5/8$

$\Pr(\text{French}) = 5/8$

$\Pr(\text{art} \cap \text{French}) = 1/4$



Sample space $\Omega = \{AF, AM, FM\}$

$\Pr(\text{Art}) = \Pr(\text{Art \& French}) + \Pr(\text{Art \& Math})$

$\Pr(\text{Art \& Math}) = \Pr(\text{Art}) - \Pr(\text{Art \& French})$
$= \frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

$\Pr(\text{French \& math}) = \Pr(\text{French}) - \Pr(\text{French \& Art})$
$= \frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

a) $\Pr(\text{math}) = \Pr(\text{French \& math}) + \Pr(\text{Art \& math})$
$= \frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$

b) Probability = 1 = 100%, he has to take Art or French

5) Have, Sarah

$Pr(H) = 60\%$

$Pr(S) = 80\%$

$Pr(H \cup S) = 90\%$

$Pr((H \cup S)^c) = 10\%$

$1 = Pr(\text{pass } H, \text{pass } S) +$
$Pr(\text{pass } H, \text{doesn't pass } S) +$
$Pr(\text{doesn't pass } H, \text{passes } S) +$
$Pr(\text{doesn't pass } H, \text{doesn't pass } S)$

$0.1 = Pr(\text{doesn't pass } H, \text{doesn't pass } S)$

$Pr(H) = 0.6 = Pr(\text{pass } H, \text{pass } S) + Pr(\text{pass } H, \text{doesn't pass } S)$
$Pr(S) = 0.8 = Pr(\text{pass } H, \text{pass } S) + Pr(\text{doesn't pass } S, \text{pass } S)$

$1 = Pr(\text{pass } H, \text{pass } S) +$
$0.6 - Pr(\text{pass } H, \text{pass } S) +$
$0.8 - Pr(\text{pass } H, \text{pass } S) +$
0.1

$1 - 1.5 = -Pr(\text{pass } H, \text{pass } S)$

$\boxed{Pr(\text{pass } H, \text{pass } S) = 0.5}$

5) Ω = {all cards}

Ω = 52

a) $Pr(\heartsuit \mid red) = \frac{Pr(\heartsuit \cap red)}{Pr(red)} = \frac{13/52}{26/52} = \frac{1}{2}$

b) $Pr(\{2,Q,K,A\spadesuit\} \mid \heartsuit) = \frac{Pr(\{2,Q,K,A\spadesuit\} \cap \heartsuit)}{Pr(\heartsuit)} = \frac{1/52}{13/52}$

$= \frac{1}{13}$

c) $Pr(J \mid \omega > 10) = \frac{Pr(J \cap \omega > 10)}{Pr(\omega > 10)} = \frac{4/52}{16/52} = \frac{1}{4}$

6) $Pr(B) = 1/4$

$Pr(A \mid B) = 1/2$

$Pr(A \cap B) = Pr(A \mid B)\, Pr(B)$

$Pr(A \cap B) = (1/2)(1-\frac{1}{4})$

$Pr(A \cap B) = 3/8$

10) Conditional Probability

$Pr(man) = Pr(woman)$

$Pr(cb \mid man) = 5\%$

$Pr(cb \mid woman) = 1\%$

looking for $Pr(male \mid cb) = ?$

$Pr(man \mid cb) = \frac{Pr(man \cap cb)}{Pr(cb)}$

$= \frac{Pr(cb \mid man)}{Pr(cb \mid man) + Pr(cb \mid woman)}$

$Pr(man \mid cb) = \frac{5\%}{5\%+1\%} = \frac{5}{6}$

15) A = Heads on 1st toss

B = Tails on 2nd toss

C = Heads on 3rd toss

D = All 3 outcomes same

E = Exactly one head

c) which pairs are dependent?

1) A, B independent

2) A, D independent

3) A, E dependent

4) D, E dependent

1 & 2

16) 1) A = {first card is a ♡}
B = {second card is a ♡}
Dependent, $Pr(B|A) \neq Pr(B)$

2) A = {first card is ♡}
B = {first card is 10}
Independent

3) A = {first card is 10}
B = {second card is 9}
Dependent, $Pr(B|A) \neq Pr(B)$

4) A = {first card is ♡}
B = {second card is 10}
Independent

(2 & 4)