Introduction to Modern Macroeconomics I
Final Exam
Quantitative Analysis of NK Model
Using Dynare

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100 points - 24 hour

1 New Keynesian Model

Consider a New Keynesian model with price stickiness similar to the one we presented in the first part of the final exam.

1.1 Households' Problem

There is a representative household that consumes the final good, supplies labor, holds bonds and shares in firms, and accumulates money. It gets utility from holding real balances. Its problem is:

$$\max_{C_t,N_t,B_{t+1},M_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t + \theta \ln (1 - N_t) + \frac{(M_t/P_t)^{1-\chi}}{1-\chi} \right) \tag{1}$$

such that:

$$P_t C_t + B_{t+1} + M_t - M_{t-1} \leq W_t N_t + \Pi_t - P_t T_t + (1 + i_{t-1}) B_t \tag{2}$$

1.2 Production

1.2.1 Final Good Producer

There is a competitive, representative final good firm which produces final output, Y_t, according to a CES aggregator function of a continuum of intermediate goods, $Y_t(j)$, $j \in (0, 1)$.

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{3}$$

1.2.2 Intermediate Producers

Intermediate goods producers produce output using a linear production technology in labor input:

$$Y_t(j) = A_t N_t(j) \tag{4}$$

Intermediate goods producers are price-takers in input markets, taking $W_t$as given.

Intermediate goods producers cannot freely adjust their prices period-by-period. Each period $1 - \phi$ portion of them randomely selected, can adjust their prices($\phi \in (0, 1)$). The probability of being able to adjust is independent of when they last updated their price. Firms discount future profit flows by the stochastic discount factor and the probability of being stuck with their currently chosen price, $\widetilde{M}_{t+s}\phi^s$.

$$\widetilde{M}_{t+s} = \beta^s \frac{u'(C_{t+s})}{u'(C_t)}$$

1.3 Equilibrium and Aggregation

1.3.1 The Flexible Price Equilibrium and Gap

A useful concept that will come in handy, particularly when thinking about welfare, is a hypothetical equilibrium allocation in which prices are flexible, which corresponds to the case when $\phi = 0$. Because there is no endogenous state variable in this model when prices are flexible, we can actually solve for the flexible price equilibrium by hand. It can be shown that flexible price output is:

$$Y_t^f = A_t N_t^f = A_t \left(\frac{\frac{\epsilon-1}{\epsilon}}{\frac{\epsilon-1}{\epsilon} + \theta} \right) \tag{5}$$

From now on, we refer to output gap as the gap between output of the model and output of an economy with flexible prices. Output gap defined as:

$$\ln X_t = \ln Y_t - \ln Y_t^f \tag{6}$$

1.3.2 Advanced Taylor Rule

Assume that the central bank sets the nominal interest rate according to a simple Taylor rule of the form:

$$i_t = (1 - \rho_i)\, i^* + \rho_i i_{t-1} + (1 - \rho_i)\left(\phi_\pi \left(\pi_t - \pi^*\right) + \phi_x(\ln(X_t) - \ln(X))\right) + \varepsilon_{i,t} \tag{7}$$

where $\phi_\pi > 1$ and $0 \leq \rho_i < 1$ are policy parameters. Here i^* is the steady-state nominal interest rate and π^* is the target inflation rate, which will also be equal to the inflation rate in the non-stochastic steady state,$ln(X_t)$ is the output gap, lnX is the steady state output gap, and $\varepsilon_{i,t}$ is a monetary policy shock, analogous to the $\varepsilon_{m,t}$ in the money growth specification. ϕ_π and ϕ_X are non-negative coefficients. And ρ_i is a smoothing parameter between zero and one. Assume also that A_t follows a mean-zero stationary AR(1) in the log:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t} \tag{8}$$

Since the government prints money, it effectively earns some revenue. Right now we're abstracting from the government doing any spending, and assume that the government does not operate in bond markets So we can write Fiscal rule for T_t from government budget constraint as below:

$$T_t = -\frac{M_t - M_{t-1}}{P_t}$$

1.3.3 Equilibrium Equations

$$\frac{\theta}{1-N_t} = \frac{w_t}{C_t} \tag{9}$$

$$(m_t)^{-\chi} = \frac{1}{C_t}\left(\frac{i_t}{1+i_t}\right) \tag{10}$$

$$\frac{1}{C_t} = \beta\,\mathbb{E}_t\,(1+i_t)\,\frac{1}{C_{t+1}}\,(1+\pi_{t+1})^{-1} \tag{11}$$

$$mc_t = \frac{w_t}{A_t} \tag{12}$$

$$C_t = Y_t \tag{13}$$

$$Y_t = \frac{A_t N_t}{v_t^P} \tag{14}$$

$$v_t^p = (1-\phi)(1+\pi_t^{\#})^{-\epsilon}\,(1+\pi_t)^{\epsilon} + (1+\pi_t)^{\epsilon}\,\phi v_{t-1}^p \tag{15}$$

$$(1+\pi_t)^{1-\epsilon} = (1-\phi)(1+\pi_t^{\#})^{1-\epsilon} + \phi \tag{16}$$

$$x_{1,t} = C_t^{-1} mc_t Y_t + \phi\beta\,\mathbb{E}_t\,(1+\pi_{t+1})^{\epsilon}\,x_{1,t+1} \tag{17}$$

$$x_{2,t} = C_t^{-1} Y_t + \phi\beta E_t\,(1+\pi_{t+1})^{\epsilon-1}\,x_{2,t+1} \tag{18}$$

$$1+\pi_t^{\#} = \frac{\epsilon}{\epsilon-1}\,(1+\pi_t)\,\frac{x_{1,t}}{x_{2,t}} \tag{19}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t} \tag{20}$$

$$i_t = (1-\rho_i)\,i^* + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi\left(\pi_t - \pi^*\right) + \phi_x(\ln(X_t) - \ln(X))\right) + \varepsilon_{i,t} \tag{21}$$

1.4 Equilibrium and Solving for Steady-State

The steady-state value of the model's variable is as follows. (Note : The steady state is a situation in which $A = 1$ and inflation is equal to the exogenous target, π.

$$(1+\pi) = \beta(1+i) \tag{22}$$

$$1+\pi^{\#} = \left(\frac{(1+\pi)^{1-\epsilon} - \phi}{1-\phi}\right)^{\frac{1}{1-\epsilon}} \tag{23}$$

$$\left(1-(1+\pi)^{\epsilon}\phi\right) v^p = (1-\phi)\left(\frac{1+\pi}{1+\pi^{\#}}\right)^{\epsilon} \tag{24}$$

$$x_1 = \frac{mc}{1-\phi\beta(1+\pi)^{\epsilon}} \tag{25}$$

$$x_2 = \frac{1}{1-\phi\beta(1+\pi)^{\epsilon-1}} \tag{26}$$

$$mc = \frac{1-\phi\beta(1+\pi)^{\epsilon}}{1-\phi\beta(1+\pi)^{\epsilon-1}} \frac{1+\pi^{\#}}{1+\pi} \frac{\epsilon-1}{\epsilon} \tag{27}$$

$$w = mc \tag{28}$$

$$N = \frac{mc \times v^p}{mc \times v^p + \theta} \tag{29}$$

$$Y = \frac{N}{v^p} \tag{30}$$

$$(m)^{-\chi} = \frac{1}{Y}\left(\frac{i}{1+i}\right) \tag{31}$$

$$Y = C \tag{32}$$

1.5 Parametrization

Parameter	Value	Description
β	0.99	Discount rate
σ_{ε}	0.01	Standard deviation for stochastic process
ϵ	10	Elasticity of substitution
π^*	0	steady state level of inflation
i^*	$\frac{1}{\beta} - 1$	steady state level of interest rate
ϕ	0.75	probability a firm will be stuck with a price for one period
ϕ_{π}	1.5	Taylor rule parameter
ϕ_x	0	Taylor rule parameter
θ	1	Utility function
ρ_a	0.95	Shock duration parameter
ρ_i	0.8	Shock duration parameter
χ	0.5	Utility function

2 Questions

2.1 Comparative Statics (30 points)

1. Report steady-state values for endogenous variables and interpret them. (10 points)
2. Sweep ϕ (the probability a firm will be stuck with a price for one period) in order to calculate the steady-state for each endogenous variables and draw a graph for each variable in which steady-state values of endogenous variables are on y-axis and different values of ϕ on x-axis. Interpret your result. (20 points)

2.2 IRF (30 points)

1. **Productivity and Monetary Shock:** In the baseline model, apply uncorrelated 1% shocks to both effectiveness of labor and monetary policy one at a time. Plot the impulse response functions (IRFs) for 100 periods (use `order=1` in `stoch simul` options). Explain the behavior of the model intuitively. (10 points)
2. **Elasticity of Substitution:** Redo the same simulation using first $\epsilon = 2$ and next using $\epsilon = 15$. Draw IRFs for the three cases on the same plot. Baseline IRF with solid black line, IRFs of $\epsilon = 15$ with dashed red line and IRFs of $\epsilon = 2$ with dashed blue line.

   Explain the dependence of model variables to this parameter (elasticity of substitution) by comparing these three IRF plots. Describe the observed differences in terms of wealth effect and intertemporal substitution effect. (10 points)
3. **Business Cycles:** Simulate the model economy for 200 periods. In the simulations, as the seed number, use two last numbers of your student ID (e.g., if your ID is ######45, use the number 45 as the seed). Namely, before the `stoch_simul` command add a another line with the command: `set_dynare_seed(45);`

   Now, interpret your graph in *business cycles*. Measure length of BC in terms of periods of the model. (10 points)

2.3 Taylor Rule (40 points)

1. Redo the same simulation using first $\phi_\pi = 1.2$ and next using $\phi_\pi = 1.02$. Draw IRFs for the three cases on the same plot. Baseline IRF with solid black line, IRFs of $\phi_\pi = 1.2$ with dashed red line and IRFs of $\phi_\pi = 1.02$ with dashed blue line. Explain the dependence of model variables to this parameter by comparing these three IRF plots. Describe the observed differences in terms of wealth effect and intertemporal substitution effect. (15 points)

2. Redo the same simulation using first $\phi_x = 2$ and next using $\phi_x = 1$. Draw IRFs for the three cases on the same plot. Baseline IRF with solid black line, IRFs of $\phi_x = 1$ with dashed red line and IRFs of $\phi_x = 2$ with dashed blue line. Explain the dependence of model variables to this parameter by comparing these three IRF plots. Describe the observed differences in terms of wealth effect and intertemporal substitution effect. (15 points)

3. Read the Wikipedia page for Taylor rule and use the intuition from the above simulations to answer the following question: While inflation is above the target in Iran, in order to stabilize the economy, should the central bank of Iran increase or decrease the interest rate? (10 points)