Introduction to Modern Macroeconomics I
Problem Set 9

Hosein Joshaghani, Mahdi Mir

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1 The Budget Constraint with Money

Consider following definitions of variables:

- Let M_{t-1} denote the nominal money holdings brought into period t. So at period t this is a predetermined variable. Likewise, M_t denotes new money holdings (determined at time t) that will be brought into $t+1$.
- Let p_t denote the nominal price of goods.
- i_{t-1} denotes the nominal interest rate on nominal bonds. It observed at $t-1$ and pays off in time t.
- w_t is real wage.
- T_t is the lump sum tax, that household pays.
- Consider $R_t K_t$ as the real income from leasing capital.
- C_t and K_t are in terms of the final good.

1. Explain why the standard household budget constraint from the RBC model in the presence of money becomes:

$$\begin{aligned} &C_t + K_{t+1} - (1-\delta)K_t \\ &\quad + \left(\frac{B_{t+1} - B_t}{P_t}\right) + \left(\frac{M_t - M_{t-1}}{P_t}\right) = w_t N_t + R_t K_t - T_t + \Pi_t + i_{t-1}\frac{B_t}{P_t} \end{aligned}$$

2. Show that the firm problem from the RBC model remains unchanged.
3. Write the FOCs for the firm problem.

2 The Exogenous Money Supply

Suppose the central bank supplies money in the form of an exogenous AR(1) process in the growth rate that is:

$$\ln M_t - \ln M_{t-1} = (1 - \rho_m)\,\pi^* + \rho_m\,(\ln M_{t-1} - \ln M_{t-2}) + \varepsilon_{m,t}$$

Here π^* is the steady-state growth rate of the money supply.

Because the government produces money, it effectively earns some revenue. We call this revenue 'seignorage revenue.'

1. Explain why the seignorage revenue is like a tax and why it is equal to:

$$\frac{M_t - M_{t-1}}{P_t}$$

2. Suppose that government spending is G_t. Write down the government budget constraint in each period.

3 Money in the Utility Function

We assume households get utility from consumption, leisure, and holding real money balances – M_t/P_t.

1. Explain why M_t is the household money holdings during period t.

Now we introduce the functional forms assumptions. Suppose households utility function is:

$$\ln C_t + \theta \ln (1 - N_t) + \psi \frac{\left(\frac{M_t}{P_t}\right)^{1-\zeta} - 1}{1 - \zeta}$$

1. Write down the household problem.

2. Form the current value Lagrangian.

3. Write FOCs.

3.1 Fisher Equation

Consider i_t is nominal interest rate and r_t is the real interest rate in period t. Also assume π_t is the inflation rate in period t. Show that the following equation holds:

$$(1 + i_t) = (1 + r_t)\,(1 + \pi_{t+1})$$

The above equation is called the Fisher equation, named after Irving Fisher, an American economist.

1. Now, rearrange the Fisher equation to yield:

$$1 + r_t = (1 + i_t)\,\frac{P_t}{P_{t+1}}$$

2. Then, rearrange household FOCs and use the Fisher relationship to get ***exactly*** the same FOCs obtain in the standard RBC model.

3. Simplify the first order condition for money holdings by using the first order condition for bonds. Define $m_t = \frac{M_t}{P_t}$ as 'real money balances' to yield:

$$m_t = \psi^{\zeta} C_t^{\zeta} \left(\frac{1 + i_t}{i_t}\right)^{\zeta}$$

4. The last equation essentially is the money demand. Anlayze variations in money demand with variation in:

   - The price level
   - Consumption
   - Nominal interest rate

5. Use m_t as defind earlier and define $\pi_t = \ln P_t - \ln P_{t-1}$, rearrange the process for money supply to obtain:

$$\Delta \ln m_t + \pi_t = (1 - \rho_m)\,\pi^* + \rho_m \pi_{t-1} + \rho_m \Delta \ln m_{t-1} + \varepsilon_{m,t}$$

3.2 The Equilibrium

Assume the same process for technology and the same production function as we have in the standard RBC model. Consider the standard capital accumulation equation. Moreover, assume the government spending is zero in the equilibrium, and its budget constraint holds with equality.

1. We have 13 variables that use all of the earlier conditions and assumptions to write a full set of 13 equations determining the model's equilibrium.

2. Explain how the yielded equations imply that the response of the real variables to a technology shock will be identical in this set up to earlier, and real variables will not respond to monetary shocks. Put differently, money is completely neutral concerning real variables, and the classical dichotomy holds – real variables are determined first, and then nominal variables are determined.

3.3 Quantitative Analysis with Dynare

Use the same parameters as before in the standard RBC model, for new parameters set $\rho_m = 0.5$, and the standard deviation of the monetary policy shock to 0.01 (i.e., 1 percent). Let $\pi^* = 0.00$, so that there is no inflation in the steady-state. Let $\psi = 1$ and $\zeta = 2$.

- Use Dynare and plot the IRFs to the mentioned monetary shock. Interpret the results. What happens to the real and nominal interest rate, real and nominal balances? Interpret.
- Assume $\rho_m = 0$ now plot again the IRFs and explains the results intuitively.

4 Cash in Advance

Now we are going to get money into our basic RBC model in another way. This framework will show that money is not entirely neutral, and the classical dichotomy does not hold.

We introduce the cash in advance constraint, which says that one must have enough money on hand to finance all nominal purchases of consumption goods.

- Assume the household utility function is:

$$\ln C_t + \theta \ln (1 - N_t)$$

1. Formally, write down the cash in advance constraint in each period.
2. Write down the household problem in the presence of money and the cash in advance constraint.
3. Form the current value lagrangian. Consider λ_t as the Lagrangian multiplier for the household's budget constraint and μ_t as the Lagrangian multiplier for the cash in advance constraint.

4. Write first-order conditions. You should obtain these equations.

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= 0 \Leftrightarrow \frac{1}{C_t} = \lambda_t + \mu_t \\
\frac{\partial \mathcal{L}}{\partial N_t} &= 0 \Leftrightarrow \frac{\theta}{1 - N_t} = \lambda_t w_t \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= 0 \Leftrightarrow \lambda_t = \beta \, \mathbb{E}_t \left(\lambda_{t+1} \left(R_{t+1} + (1 - \delta) \right) \right) \\
\frac{\partial \mathcal{L}}{\partial B_{t+1}} &= 0 \Leftrightarrow \lambda_t = \beta \, \mathbb{E}_t \, \lambda_{t+1} \left((1 + i_t) \left(\frac{P_t}{P_{t+1}} \right) \right) \\
\frac{\partial \mathcal{L}}{\partial M_t} &= 0 \Leftrightarrow -\frac{\lambda_t}{P_t} + \beta \, \mathbb{E}_t \, \frac{\mu_{t+1}}{P_{t+1}} + \beta \, \mathbb{E}_t \, \frac{\lambda_{t+1}}{p_{t+1}} = 0
\end{aligned}
$$

5. As a particular case, suppose that one has enough money relative to consumption that the cash in advance constraint was never binding. What would this assumption imply for the Lagrangian multiplier of cash in advance restriction?

6. Show that with the last assumption, the nominal interest rate in each period must be zero. Interpret this result intuitively.

- For the rest of the problems, assume the cash in advance constraint always binds.

4.1 The Equilibrium

1. Write the firm problem and its FOCs exactly as before.

   Consider the same assumptions for the equilibrium as money in the utility function case.

2. Show that the assumption of zero government spending and holding its budget constraint with equality result in the accounting identity.

3. Now we have 16 variables and equations, write down the full set of equilibrium equations. You should end up with the following equations. Note that we need not eliminate Lagrangian multipliers.

$$
\begin{aligned}
Y_t &= A_t K_t^{\alpha} N_t^{1-\alpha} && (1)\\
R_t &= \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} && (2)\\
w_t &= (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} && (3)\\
K_{t+1} &= I_t + (1-\delta) K_t && (4)\\
Y_t &= C_t + I_t && (5)\\
\Delta \ln m_t &= (1-\rho_m)\, \pi^* - \pi_t + \rho_m \pi_{t-1} + \rho_m \Delta \ln m_{t-1} + \varepsilon_{m,t} && (6)\\
\ln A_t &= \rho \ln A_{t-1} + \varepsilon_t && (7)\\
\frac{1}{C_t} &= \lambda_t + \mu_t && (8)\\
\frac{\theta}{1-N_t} &= \lambda_t w_t && (9)\\
\lambda_t &= \beta\, \mathbb{E}_t\, \lambda_{t+1} \left((1+i_t) \left(\frac{P_t}{P_{t+1}} \right) \right) && (10)\\
\lambda_t &= \beta\, \mathbb{E}_t \left(\mu_{t+1} \frac{P_t}{P_{t+1}} + \lambda_{t+1} \frac{P_t}{P_{t+1}} \right) && (11)\\
\lambda_t &= \beta\, \mathbb{E}_t \left(\lambda_{t+1} \left(R_{t+1} + (1-\delta) \right) \right) && (12)\\
m_t &= C_t && (13)\\
\Delta \ln m_t &= \ln m_t - \ln m_{t-1} && (14)\\
\pi_t &= \ln P_t - \ln P_{t-1} && (15)\\
1 + r_t &= (1+i_t)\, \mathbb{E}_t \left(1 + \pi_{t+1}\right)^{-1} && (16)
\end{aligned}
$$

4.2 Quantitative Analysis with Dynare

Use the same parameters as money in the utility case earlier.

1. Using Dynare show that the IRFs for a technology shock is exactly the same as money in the utility case and the standard RBC model.
2. Show that however it is small, money does have real effects in this framework.
3. Specifically, show that a positive monetary shock contracts output. Explain economically why this happened.