Introduction to Modern Macroeconomics I: A New Keynesian Model with Price Stickiness Problem Set 10

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Consider an economy which is modeled with a New Keynesian model with price stickiness, and its agent's problem is described as below:

1 Household problem

There is a representative household that consumes, supplies labor, accumulates bonds, holds shares in firms, and accumulates money. It gets utility from holding real balances. Its problem is:

$$\max_{C_t,N_t,B_{t+1},M_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\eta}}{1+\eta} + \theta \ln\left(\frac{M_t}{P_t}\right)\right)$$

S.t

$$P_t C_t + B_{t+1} + M_t - M_{t-1} \leq W_t N_t + \Pi_t - P_t T_t + (1 + i_{t-1}) B_t$$

1. Form the lagrangian and write down the Focs
2. Interpret this Focs

2 Production

2.1 Final Good Producer

The final goods sector is perfectly competitive and aggregates intermediates goods into a final good for consumption. In fact, in each period final good producer chooses intermediate goods, $Y_j(t)$, according to their price, $P_j(t)$, And combine The continuum of These intermediate goods with CES aggregator production function, $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$, to maximize its profit.

1. Write down the final good producer maximization problem.

2. Solve it and derive intermediate good demand function in term of $Y_t, P_t, P_j(t), \epsilon$.

3. Derive the aggregate price level, P_t.

2.2 Intermediate Producers

A typical intermediate producer produces output according to a constant returns to scale technology in labor, with a common productivity shock, A_t :

$$Y_t(j) = A_t N_t(j)$$

Intermediate producers face a common wage. They are not freely able to adjust price so as to maximize profit each period, but will always act to minimize cost. The cost minimization problem is to minimize total cost subject to the constraint of producing enough to meet demand.

1. Write down the intermediate good producer cost minimization problem.

2. Drive Foc and interpret the meaning of the Lagrange multiplier

3. Now write down the intermediate good producer's real profit function in term of $Y_t, P_t, P_t(j)$, and real marginal cost, mc_t.

Now Consider in each period there is a fixed probability of $1-\phi$ that a firm can adjust its price. This means that the probability a firm will be stuck with a price one period is ϕ , for two periods is ϕ^2, and so on. Consider the pricing problem of a firm given the opportunity to adjust its price in a given period. Since there is a chance that the firm will get stuck with its price for multiple periods, the pricing problem becomes dynamic:

$$\max_{P_t(j)} E_t \sum_{s=0}^{\infty} (\beta\phi)^s \frac{u'(C_{t+s})}{u'(C_t)} \left(\frac{P_t(j)}{P_{t+s}} Y_{t+s}(j) - mc_{t+s} Y_{t+s}(j)\right)$$

If We impose that output will equal demand. we get:

$$\max_{P_t(j)} \quad E_t \sum_{s=0}^{\infty} (\beta\phi)^s \frac{u'(C_{t+s})}{u'(C_t)} \left(P_t(j)^{1-\epsilon} P_{t+s}^{\epsilon-1} Y_{t+s} - mc_{t+s} P_t(j)^{-\epsilon} P_{t+s}^{\epsilon} Y_{t+s}\right)$$

1. Derive Foc and simplify it to yield an equation for an intermediate good price, $P_j(t)$. Does it depend on intermediate good j?
2. Define $X_{1,t} = u'(C_t)\, mc_t P_t^{\epsilon} Y_t + \phi\beta E_t X_{1,t+1}$ and $X_{2,t} = u'(C_t)\, P_t^{\epsilon-1} Y_t + \phi\beta E_t X_{2,t+1}$ and write $P_j(t)$ in term of ϵ,$X_{1,t}$ and $X_{2,t}$.
3. Drive Optimal reset price in case when $\phi = 0$

3 Equilibrium and Aggregation

To close the model we need to specify an exogenous process for A_t, some kind of monetary policy rule to determine M_t, and a fiscal rule to determine T_t. Let the aggregate productivity term follow a mean zero AR(1) in the log:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t}$$

Let's suppose that the money supply follows an AR(1) in the growth rate, where $\Delta \ln M_t = \ln M_t - \ln M_{t-1}$:

$$\Delta \ln M_t = (1 - \rho_m)\, \pi + \rho_m \Delta \ln M_{t-1} + \varepsilon_{m,t}$$

Since the government prints money, it effectively earns some revenue. Right now we're abstracting from the government doing any spending, and for simplicity We're going to assume that the government does not operate in bond markets (this is innocuous since it raises revenue only through a lump sum tax) So we can write Fiscal rule for T_t from government budget constraint as below:

$$T_t = -\frac{M_t - M_{t-1}}{P_t}$$

1. Using the fact that in equilibrium, bond-holding is always zero in all periods: $B_t = 0$, plus the relationship between the lump sum tax and money growth derive the relation between C_t and Y_t.
2. Using production function Derive Y_t in term of A_t, B_t, v_t^p ($v_t^p = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj$).
3. Now write down the full set of equilibrium conditions. Write 14 equations in 14 aggregate variables $\left(C_t, i_t, P_t, N_t, w_t, M_t, mc_t, A_t, Y_t, v_t^p, P_t^{\#}, X_{1,t}, X_{2,t}, \Delta \ln M_t\right)$.

4 Re-writing the equilibrium conditions

We want to re-write these conditions (i) only in terms of inflation, eliminating the price level; and (ii) getting rid of the heterogeneity, which the Calvo (1983) assumption allows us to do; and (iii) in terms of real money balances, $m_t = \frac{M_t}{P_t}$, instead of nominal money balances.

The Calvo assumption[1] allows us to write the aggregate price level (raised to $1-\epsilon$) as a convex combination of the reset price and lagged price level :

$$P_t^{1-\epsilon} = (1-\phi)P_t^{\#,1-\epsilon} + \phi P_{t-1}^{1-\epsilon}$$

And it also allows us to write the price dispersion term as below:

$$v_t^p = (1-\phi)\left(1+\pi_t^{\#}\right)^{-\epsilon}(1+\pi_t)^{\epsilon} + (1+\pi_t)^{\epsilon}\,\phi v_{t-1}^p$$

For eliminating the price level define inflation as $\pi_t = \frac{P_t}{P_{t-1}} - 1$ and define $\pi_t^{\#} = \frac{P_t^{\#}}{P_{t-1}} - 1$ as reset price inflation:

1. Now rewrite the equilibrium condition in desired way which is mentioned at first.

5 The Steady State

In Steady state A = 1. And Steady state inflation is equal to the exogenous target, π.

1. Now derive the steady state value of model's variables

6 The Flexible Price Equilibrium

A useful concept that will come in handy, particularly when thinking about welfare, is a hypothetical equilibrium allocation in which prices are flexible, which corresponds to the case when $\phi = 0$. Because there is no endogenous state variable in this model when prices are flexible, we can actually solve for the flexible price equilibrium by hand.

1. Use superscript f to denote the hypothetical flexible price allocation. And derive $v_t^{f,p}$, mc_t^f , w_t^f ,N_t^f and Y_t^f

2. Consider the case in which $\sigma = 1$, and explain how Productivity fluctuation, A_t affect labor hours , N_t^f.

[1]Calvo assumption allows us to integrate out the heterogeneity and not worry about keeping track of what each firm is doing from the perspective of looking at the behavior of aggregates.

7 Log-Linearization

For simplicity, assume that $\pi = 0$ (i.e. a ”zero-inflation steady state”), and answer the following questions:

1. **New Keynesian IS Curve.**
   (a) impose the accounting identity ($C_t = Y_t$) in the Euler equation and log-linearize it .
   (b) interpret the differences between the **New Keynesian IS Curve** which you derived and the **Old Keynesian IS Curve**.

2. **New Keynesian Phillips Curve**
   (a) Log linearize the v_t^p around the zero-inflation steady state.
   (b) Log-linearize the production function.
   (c) Log-linearize the static labor demand specification, And eliminate $\tilde{N}_t$ from log linearized production function.
   (d) Now log-linearize expression for the flexible price level of output,Y_t^f. And use log linearized production function to derive $\tilde{mc}_t$ in term of output gap, $\tilde{Y}_t - \tilde{Y}_t^f$, and models parameters.
   (e) Log-linearize the $\tilde{x}_{1,t}$ and $\tilde{x}_{2,t}$. ($x_{1,t} \equiv \frac{X_{1,t}}{P_t^\epsilon}$, $x_{2,t} \equiv \frac{X_{2,t}}{P_t^\epsilon}$).
   (f) Log-linearize reset price expression and impose $\tilde{x}_{1,t}$ and $\tilde{x}_{2,t}$ to derive the New Keynesian Phillips Curve.
   (g) Use the terminal condition that inflation will return to steady state eventually, And solve the New Keynesian Phillips Curve forward to get an equation for $\tilde{\pi}_t$. And interpret it.

8 Quatitative Analysis

Now, solve the model quantitatively in Dynare using a first order approximation about the steady state. Use the following parameter values (more on this later): $\phi = 0.75, \sigma = 1, \eta = 1, \psi = 1, \epsilon = 10, \theta = 1, \rho_a = 0.95, \rho_m = 0.0, and \pi = 0$. Assume that the standard deviation of both shocks are 0.01. Draw impulse responses to the both shocks.