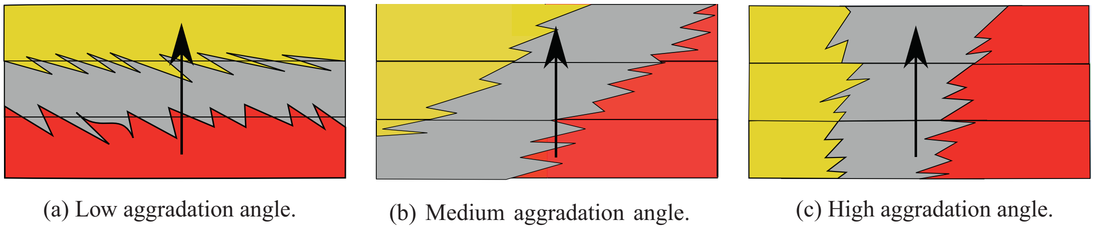

Fig. 9. Illustration of how the aggradation angle affects the effective vertical conductivity.

Using polynomials for this approximation is convenient, because it is easy to analytically obtain the output variances from the statistics of the input variables of the polynomials. In our case, the solution is approximated by orthogonal polynomials with ascending polynomial degree. We expand the variance of model output into individual components originating from all possible combinations of input parameters. Assume that we break the system output into components as follows:

$$\Gamma = \Gamma_0 + \sum_i \Gamma_i + \sum_i \sum_{j>i} \Gamma_{ij} + \cdots \quad (8)$$

A single index (here: i) shows dependency to a specific input variable. More than one index (e.g.: i and j) shows interaction of two or more input variables. If we consider the input vector Θ to have n components θ_i for $i = 1, \ldots, n$, then $\Gamma_i = f_i(\theta_i)$ and $\Gamma_{ij} = f_{ij}(\theta_i, \theta_j)$. In practice, we stop at a finite number of terms in Eq. (8). The first order sensitivity index, the so called Sobol index, is defined statistically as follows (Saltelli, 2008):

$$S_i = \frac{V[E(\Gamma \mid \theta_i)]}{V(\Gamma)}, \quad (9)$$

where $E(\Gamma \mid \theta_i)$ is the conditional expectation of output Γ for a given value of θ_i and V is the variance operator. In plain words, S_i is the fraction of total variance $V(\Gamma)$ that can be explained by the parameter θ_i. Since θ_i can be fixed at any value in its uncertainty interval, each of those values produces a distinct expectation. In Eq. (9), the variance of those expectations is divided by the unconditional variance of output (i.e., with no input variable fixed). For more than one index, a higher-order Sobol index can be defined as:

$$S_{ij} = \frac{V[E(\Gamma \mid \theta_i, \theta_j)] - V[E(\Gamma \mid \theta_i)] - V[E(\Gamma \mid \theta_j)]}{V(\Gamma)}. \quad (10)$$

Here, $V[E(\Gamma \mid \theta_i, \theta_j)]$ is the variance of output expectations after fixing θ_i and θ_j. This index represents the significance of variation in output generated from the joint uncertainty in several input

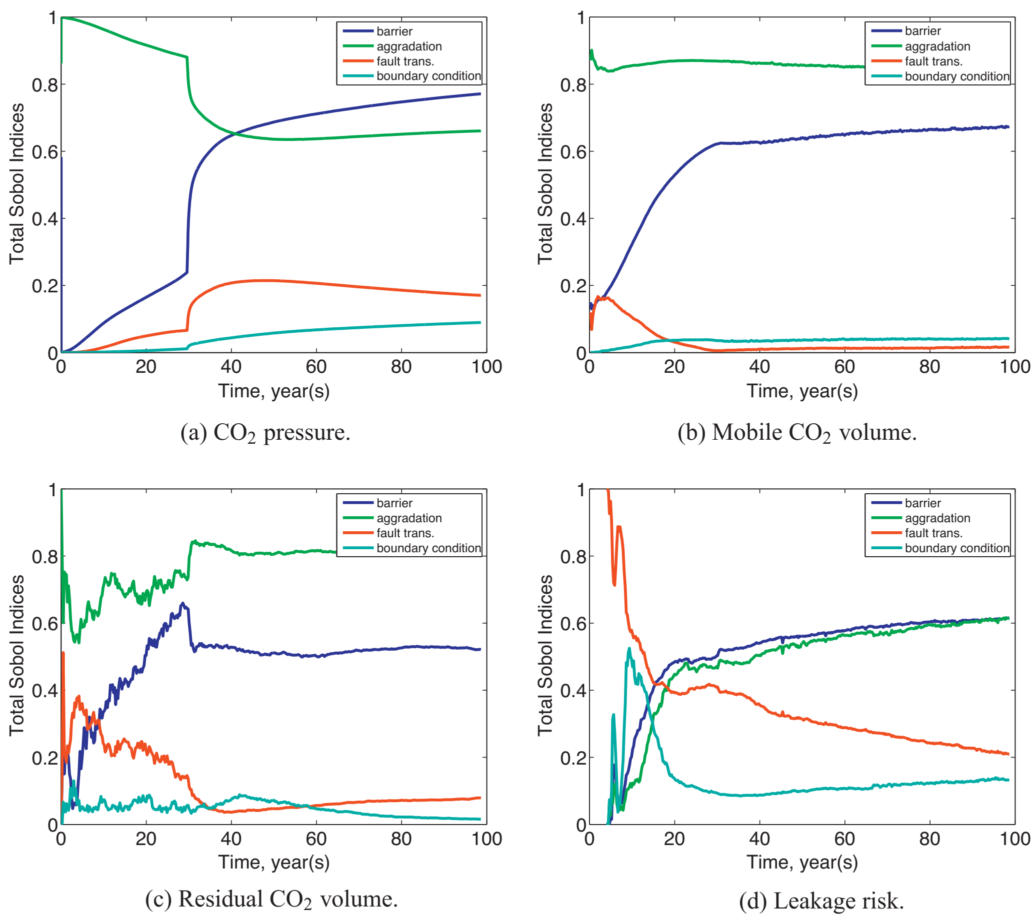

Fig. 10. Sensitivities (expressed by total Sobol indices) plotted versus time for different responses.

Please cite this article in press as: Ashraf, M., et al., Geological storage of CO_2: Application, feasibility and efficiency of global sensitivity analysis and risk assessment using the arbitrary polynomial chaos. Int. J. Greenhouse Gas Control (2013), http://dx.doi.org/10.1016/j.ijggc.2013.03.023