

Geological Storage of CO₂: Sensitivity and Risk Analysis

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Dissertation for the degree of Philosophiae Doctor (PhD)

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October 2013

Scientific environment

This dissertation is submitted as a partial fulfillment of the requirements for the degree Doctor of Philosophy (PhD) at the University of Bergen. It is part of the project Mathematical Modeling and Risk Assessment of CO₂ storage, MatMoRA, which was funded by the Norwegian Research Council of Norway, Statoil and Norske Shell under grant no. 178013/I30 and lead by Professor Helge Dahle at the Department of Mathematics, University of Bergen (UiB).

The working environment have been SINTEF-ICT in Oslo, and CIPR in Bergen, and SIMTECH in Stuttgart. Professor Knut-Andreas Lie, chief scientist at SINTEF-ICT, has been the main adviser and Professor Jan M. Nordbotten at the Department of Mathematics, UiB, along with Halvor M. Nilsen, research scientist at SINTEF, have been co-advisers. With warm supports from Professor Rainer Helmig, head of Department of Hydromechanics and Modelling of Hydrosystems at Stuttgart University, last parts of the work is benefited from advices of professor Wolfgang Nowak, head of Stochastic Modelling of Hydrosystems, Dr. Sergey Oladyshkin, postdoctoral fellow at SIMTECH, and Professor Holger Class at Stuttgart University.

Acknowledgements

I dedicate this to my father.

First and foremost, Knut-Andreas Lie was more than an adviser for me, whom I had the chance and honor to work with closely and in a daily-based manner. I learned a lot from discussions with Halvor M. Nilsen, initially my only friend in Norway. Jan M. Nordbotten always inspired me in my works. Helge Dahle warmly helped me with every aspect of my studies. I have enjoyed working with Segrey Oladyshkin, Wolfgang Nowak, and Holger Class while spending a semester in Stuttgart University.

I would like also to acknowledge the sponsors of the MatMora project defined in the University of Bergen in partnership with SINTEF-ICT and the University of Stuttgart, and the German Research Foundation (DFG) for financial support of the project within the Cluster of Excellence in Simulation Technology (EXC310/1) at the University of Stuttgart.

Abstract

Geological CO₂ storage is a key technology to be utilized for preventing the industrial CO₂ emission into the atmosphere. A successful storage operation requires a safe geological structure with large storage capacity. The practicality of the technology is challenged by various operational concerns from site selection to monitoring the injected CO₂ in the long-term.

The research in this report is aimed to address the significance of a sophisticated geological modeling that can help in prediction of the storage performance. In the first part, we investigate the significance of assessing the geological uncertainty and its consequences in the site selection and early stages of the storage operations. This includes the injection period and the early migration time of the injected CO₂ plume. The key part of the research is the extensive set of realistic geological realizations used in the analysis. The heterogeneity is modeled by the most influential geological parameters in a shallow-marine system. Among those parameters are the aggradation angle, levels of barriers in the system, faults, lobosity, and progradation direction.

A typical injection scenario is simulated over 162 realizations and major flow responses are defined to measure the success of the early stages of CO₂ storage operations. These responses include the volume of trapped CO₂ by capillarity, dynamics of the plume in the medium, pressure responses, and risk of leakage through a failure in the sealing cap-rock. Impact of geological uncertainty on these responses is investigated by comparing all cases for their performance. The results show a large variations in the responses due to changing the geological parameters. Among the main influential parameters are the aggradation angle, the progradation direction, and the faults in the medium.

A sophisticated geological uncertainty study requires large number of detailed simulations that are time consuming and computationally costly. The second part of the research introduces a work-flow that employs an approximating response surface method, which is called arbitrary polynomial chaos (aPC). The aPC is fast and sophisticated enough to be used practically in the process of sensitivity analysis and uncertainty and risk assessment. We demonstrate the work-flow by combining the aPC with a global sensitivity analysis technique, namely the Sobol indices, which is a variance based method and has proven to be practical for sophisticated problems. The probabilistic uncertainty analysis is performed by applying the Monte-Carlo process using the aPC. The results show that the aPC can be used successfully in an extensive geological uncertainty study.

List of papers

1. M. Ashraf, K.A. Lie, H.M. Nilsen & A. Skorstad, *Impact of geological heterogeneity on early-stage CO₂ plume migration: CO₂ distribution sensitivity study*, submitted to the International Journal of Greenhouse Gas Control(IJGGC).
2. M. Ashraf, *Impact of geological heterogeneity on early-stage CO₂ plume migration: pressure sensitivity study*, submitted to the International Journal of Greenhouse Gas Control(IJGGC).
3. M. Ashraf, S. Oladyshkin, W. Novak, *Ashraf, Meisam, Sergey Oladyshkin, and Wolfgang Nowak. "Geological storage of CO₂: Application, feasibility and efficiency of global sensitivity analysis and risk assessment using the arbitrary polynomial chaos."*, Proceedings of the European Geosciences Union (EGU) General Assembly 2012, April, Vienna, Austria, Geophysical Research Abstracts., Vol. 14, EGU2012-9243. Published in International Journal of Greenhouse Gas Control (2013).

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Chapter 1

Introduction

1.1 Introduction

“We won’t have a society if we destroy the environment”

– Margaret Mead, American cultural anthropologist, 1901-1978

Climate changes resulting from the contribution of human CO₂ emissions as green-house gas has been shown in studies such as [41]. The underground sequestration of the CO₂ produced from localized sources like power-plants and oil and gas recovery sites is proposed as a possible solution to reduce the rate of CO₂ emission into the atmosphere [11, 40]. The technology required to inject CO₂ is similar to what is in use in the oil, gas, and mining industry. However, there are some challenges that are specific to carbon storage operations. Primary, the temporal and spatial scales in these problems are larger. Secondary, the risk of leakage of stored CO₂ up to the surface via natural features like fractures and faults or via man-made features such as leakage through ill-plugged wells and broken cap-rock due to high pressure imposed to the system during the injection operations is a major concern.

The main objectives of carbon storage operations are to maximize the storage volume and the volumetric injection rate, and to minimize the risk of leakage of the stored CO₂. The CO₂ storage operations require multidisciplinary collaborations. The work-flow from initial phases of a project until end of storage operations are divided between government and private sector, research organizations and industry. In particular, it is the task of the research community to investigate the safety of CO₂ sequestration and provide the methodology for CO₂ fate prediction [5].

Bachu [5] discusses a road-map of site selection for geological CO₂ sequestration. He defines the process in three steps: to assess the general suitability of the site, to perform an inventory study on the source point and storage location and the operational transport issues, and finally to investigate the safety and assess the capacity of the storage. Issues about safety and storage capacity are looked at differently from the perspective of immediate and ultimate results. For example, when talking about the risk of leakage, we might consider the leakage through ill-plugged wells or fractures during the injection time as the immediate risk. On the other hand, leakage caused by plume migration long time after the injection and contamination to other aquifer systems are considered as ultimate risks.

To predict the fate of the injected CO₂, it is crucial to study the dynamics of flow in the storage medium. Dynamical study of flow includes quantification of acting forces in a geological heterogeneous medium as well as involving a complicated system of mathematical equations. It seems convenient to replace the geological heterogeneous medium with an equivalent homogeneous medium to simplify the solution of the flow equations. However, proper modeling of geological heterogeneity is a major concern on reservoir assessment and carbon storage studies [6, 23, 56, 57].

In this thesis, we report a series of studies performed within a PhD program under the MatMora project. MatMora is a strategic project that is defined to address the needs of mathematical tools to develop a modeling work-flow. The work in this thesis is focusing on the fundamental uncertainty in geological description. The work is reported in a series of papers, and the objective is to perform a sensitivity analysis on a set of geological parameters used to describe the geology of shallow-marine depositional systems. Although the focus is on a particular depositional system, the procedure can be implemented for other systems of interest.

We start this introduction section by discussing the global warming and its causes, and the carbon storage as an interim proposed solution to mitigate the increasing level of industrial CO₂ emission to the atmosphere. Section 1.3 provides the work-flow of the works reported in the thesis. A brief overview of literature is given and the work is discussed in that section.

In Section 1.5, we review a systematic definition for uncertainty from the literature and after that the geological uncertainty and parameters are described. Flow equations for single-phase and two-phase

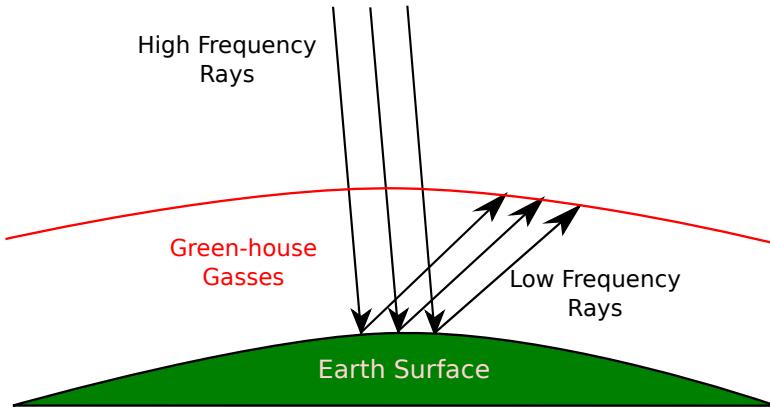


Figure 1.1: Green-house gases act like a blanket trapping the heat received from the sun.

flow problems are discussed in Section 1.6. In Section 1.7, various flow regimes occurring during geological storage of CO₂ are described [16] by discussing the force balance within the medium at different times. Next, we discuss the vertical averaging method which can be used in large aquifers to enhance the speed of simulation.

The introduction to the thesis continues in Section 1.9 by a discussion of flow simulation scenario and assumptions [17] in the work. We use a set of flow responses that monitor the performance of the operation in a typical carbon storage problem [18], with a special emphasis on the injection and early migration of CO₂ in the medium. Flow dynamics and a linear sensitivity analysis [19] formed on the simulation results are discussed in this section.

Section 1.10 provides an overview of [20] techniques that can be used for rapid flow simulation. We use a response surface method to evaluate the flow responses. This proxy model is then used in a global sensitivity analysis and Monte-Carlo risk assessment process. The introduction ends by introducing the MATLAB functions used in our work-flow.

1.2 Carbon storage

There are a number of theories that explain the causes of climate change. Milankovich theory [33] relates the energy received from the sun to the cyclical variation of earth orbit around the sun, and earth rotation around its axis. The earth orbit changes eccentricity between circular and elliptical. This influences the difference between earth and sun, and on its maximum influence can lead to about 20% difference in the energy received from the sun. The second variation occurs in the rotation of earth around its plane axis. This rotation wobbles approximately every 13600 years and the summer solstice switches from June to January. Also, a tilt variation of earth rotational axis happens approximately after every 41000 years. This can cause warmer winters and colder summers in high latitudes [33].

The solar radiation changes by a small amount of 0.1% over a 11 year cycle. Also on the scales of tens to thousands of years variations in the earth orbit result in seasonal changes and that in the past caused glacial and inter-glacial cycles.

The theory of green house effect relates the earth climatic change to the fact that the long wave radiation from earth back to atmosphere is absorbed by the green-house gases, mainly carbon dioxide, water vapor, and methane existing in the atmosphere. This results in trapping of heat energy and an increase in atmosphere temperature level (Figure 1.1) [33].

Human manipulations in the nature has led to approximately 100 ppm increase in carbon dioxide level in the atmosphere. Most scientists believe that we are already experiencing the global warming due to green house effects. The IPCC Second Assessment report states that the observed warming trend since the late 19th century is unlikely to be entirely natural in origin and is partly due to anthropogenic causes [14].

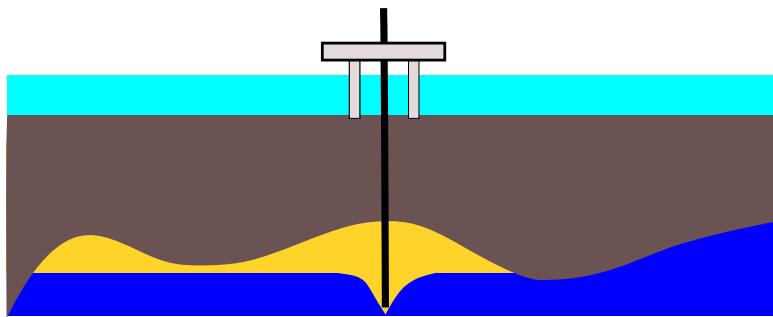


Figure 1.2: Geological sequestration is a proposed solution for mitigating the industrial CO₂ emissions.

Carbon capture and storage (CCS) has got a major attention in the industry and the scientific communities. According to the International Energy Agency (IEA), the cost of mitigating climate change by 2050 is estimated to be 70% higher without implementing CCS [44].

CCS is considered as an interim solution, because it is valid due to fossil fuel consumption, and the long term strategy of replacing fossil fuel with renewable energy will terminate the validity CCS. Therefore, initiating CCS has to be done in a reasonable fashion such that it does not slow down the research for renewable energy. Another concern regarding CCS is the acceleration of coal and fossil fuel consumption with the excuse of availability of CCS technology.

Sequestration of CO₂ at the ocean floor and also in deep underground aquifers are some of the options available for permanent storage of CO₂. Large availability of storage volumes and potential for almost permanent storage makes the geological sequestration an appropriate option (Figure 1.2). Nevertheless, this alternative is not free from economical, social and industrial concerns.

In the last decades, the scientific community has been putting efforts into convincing the public about the feasibility of these operations. A fair public acceptance must be based on social awareness. Any plan to increase the acceptance level in a society starts by measuring the current knowledge level of that society. The EU has conducted a survey to assess the public awareness in 12 European states. This survey is published in the recent Eurobarometer report in May 2011. People's awareness and acceptance of climate change and its causes, and the methods to avoid or mitigate the problems, in particular the CCS technology, was examined in the survey. The majority of European participants are either fairly or very well informed about causes and consequences of climate change. However, the awareness of CCS in between the European respondents was low. Two third of the participants in the survey have had not heard at all about CCS.

The same survey suggests that the overall trust in Europe in the sources of information regarding CCS is best in universities and other scientific institutions. Governments are investing in research, not only to move toward industrialization of CCS, but also to make it well received by public. This highlights the importance of researching the storage of CO₂ and the way it is needed both for industrial demands and social concerns.

1.3 Modeling procedure

Predicting the fate of CO₂ storage involves identification and quantification of the relevant uncertainties and risk assessment process. The procedure starts with a geological description and continues with modeling of flow in geological formations. After constructing a deterministic flow model, the stochastic nature of the problem is analyzed by studying the variation in the model outcome due to uncertainties in the system.

Figure 1.3 shows the modeling work-flow implemented in this thesis. The steps are categorized in three parts: geological uncertainty, flow dynamics, and stochastic analysis. The relations between the steps are plotted by arrows in the flow-chart. In this section, we briefly describe each step. More details

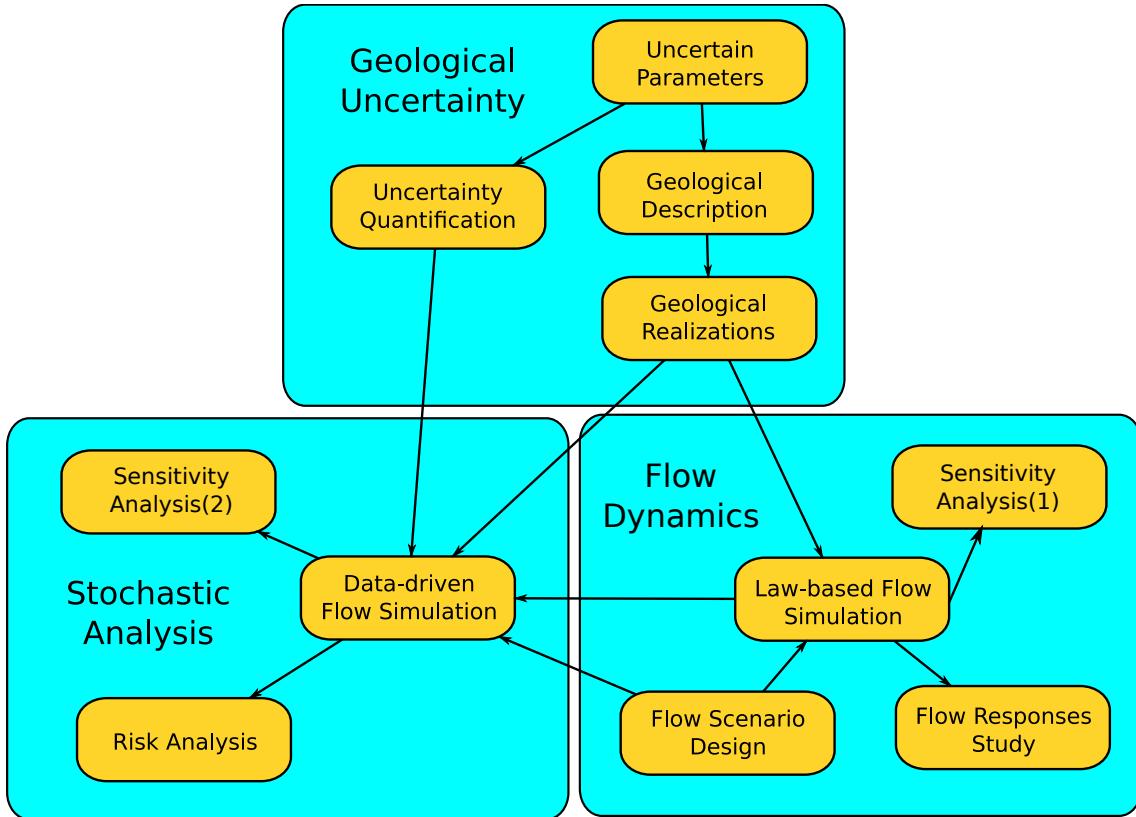


Figure 1.3: Modeling procedure diagram.

will follow in the next sections.

Uncertain parameters: In the first step, we identify the uncertain parameters of the model to study their influence in the modeling outcome. It is possible that our knowledge of model sensitivity to the parameters is limited. Then, in a conservative approach we choose a larger number of parameters and by doing a primary sensitivity analysis with a fast technique, we filter out the important parameters. Herein, the focus is on geological parameters that are determined to be the most influential sources of geological uncertainty for a shallow-marine environments [43].

Uncertainty quantification: After identification of the uncertain geological parameters, we assign a likelihood to each of the parameters. It is hardly possible to have a unique likelihood template that applies to every geological location. Thus, we note that probabilities of existence for an uncertain geological feature can change from place to place. The uncertainty enters the modeling in the form of parameter frequency histograms. The conventional practice is to consider an analytical distribution function to be assigned to the parameters. However, the sampling procedure normally ends in scarce frequency histograms that are difficult to fit into a unique analytical distribution function.

Geological description: Geological uncertainty study is normally done by series of runs to measure the sensitivity of the model to the parameter variations. Results are valid, only if the geology used in the work-flow is representative of reality. The process of geological description results in a large number of realizations to be used in the next steps of the study. Herein, we will use a set of equiprobable geological realizations of a shallow-marine reservoir.

Flow scenario design: Herein we define the initial and boundary conditions of the CO₂ injection problem. Also, we specify the injection scenarios. Possible simplifying physical assumptions will be taken here. Each scenario is implemented for all geological realizations.

Law-based flow modeling: After defining the injection problem, we simulate the flow dynamics in the chosen realizations. We use a two-phase flow model and a standard commercial simulator.

Data-driven flow modeling: Modeling the flow dynamics via formulations of physical laws nor-

mally results in complicated equations with many degrees of freedom. The computational cost of solving these equations is high, in particular for uncertainty related studies that require a large number of simulations to cover the variation in the uncertain parameters. So called data-driven methods, are mathematical functions that are specified by correlating a set of unknown flow attributes to their corresponding uncertain parameter values. These methods need to be tuned by a law-based method before employment. Because these methods are designed to be only dependent on the uncertain parameters, they are normally low in computational costs. However, they may exhibit the pitfall of not following the physical rules and in some cases produce unrealistic results.

Flow responses study: Once the simulation results are obtained from the flow modeling procedure, it is possible to calculate the important flow responses from simulation results. The fate of carbon storage and assessment of the operations can be inferred from these responses. Storage capacity, injection rate, and leakage risk are evaluated from flow responses. Responses include pressure distribution over time, CO₂ plume development, and other quantities describing the dynamics of flow.

Sensitivity and risk analysis: The sensitivity analysis is performed in two ways: firstly by using three-dimensional, two-phase flow simulations on all realizations available for demonstrating the geological variability. In the second method, we employ an approximating polynomial to perform global sensitivity analysis and stochastic uncertainty studies. Using the relatively fast data-driven method, we perform a Monte-Carlo process on 10000 simulation cases.

1.4 Uncertainty Sources

Sources of uncertainty can exist in every part of the CO₂ storage modeling process. Herein, we briefly describe each of the possible contributions to the uncertainty in modeling within various parts.

Uncertainty in physical modeling: We might ignore some phenomena during the physical modeling of CO₂ storage that can be influential in the flow behavior. This might happen due to lack of awareness of the phenomena or by underestimating the significance of it. For example, we might ignore the heat exchange within the system, assuming that heat transfer does not play an important role in the flow performance. If some parameters in the modeling are sensitive to the heat and change by temperature variations, the assumption to ignore heat transfer effect can introduce considerable bias in the outcome of the modeling.

Mathematical formulation and numerical approximation: Modeling CO₂ injection and migration in a realistic geological formation results in a complicated mathematical system that in most of the cases can not be solved analytically. The numerical approach to approximate the original mathematical system, normally introduces errors in approximation. Mathematical analysis can help in estimating the error or its order, but it might not be doable for complicated models.

A specified physical problem can be formulated mathematically in more than one way. The choice of primary unknowns to be found can change the mathematical form and nature of the equations. Degrees of non-linearity and coupling between unknowns in the equations can vary in different formulations.

Geological uncertainty: The high costs of data acquisition and technical limitations introduce a huge amount of geological uncertainties in CO₂ storage modeling. The injected CO₂ may travel in a large spatial scale and providing enough geological information and the medium attributes is a big challenge.

User introduced uncertainty: These type of uncertainties are caused by the errors introduced by a user for her/his biased choice of modeling tools and interpretations of modeling results.

1.5 Geological modeling

The central part of a successful CO₂ storage modeling is to provide aquifer models that depict the geological heterogeneity in a realistic manner. This requires having an inclusive understanding about model

sensitivity with respect to different geological parameters and quantifications of geological uncertainty and its impacts on the process.

The conventional practice of geological modeling includes using geostatistical models. It is possible that two different heterogeneity patterns produce the same geostatistical model, as discussed by Caers [12]. Therefore, a geostatistical model does not represent a unique reservoir image and if we do not include additive information in the process, we might end-up with an unrealistic heterogeneity texture[12, 23]. The primary attention in our work has been on this issue and to provide a more realistic way of geological uncertainty analysis for CO₂ sequestration by including information of geological features and textures in the process.

1.5.1 Geological description

Geological storage of CO₂ requires large accommodation of subsurface volumes. Only sedimentary basins, which hold relative large pore volumes, are generally suitable for this mean. However, not all sedimentary basins are similarly appropriate for CO₂ sequestration.

Convergent basins along active tectonic areas pose a higher risk of CO₂ leakage due to volcanism, earthquakes, and active faults. Divergent basins located on the stable lithosphere are much less prone to earthquakes or other catastrophic event that can lead to accidental releases of large CO₂ quantities. Therefore, specific considerations must be done in selecting site locations with respect to security of subsurface storage.

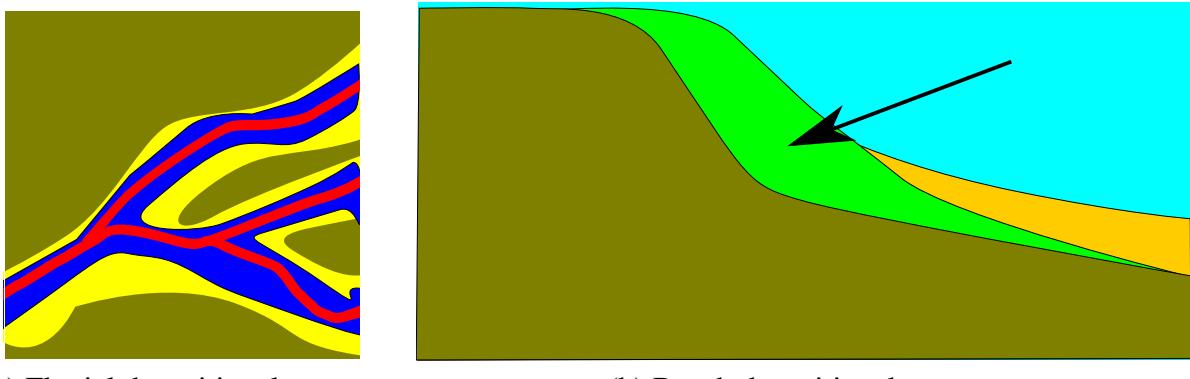
Sedimentary basins are composed of various lithological facies. Stratigraphic architecture and sand-body geometry control the capacity and effectiveness of CO₂ sequestration. As a result of various tectonic depositional and erosional process, low and high permeability rocks are accumulated on top of each other and can form stratigraphic flow-path leading to various directions and speed of subsurface flow. Three types of formations can be characterized: aquifers, aquitards, and aquiclude.

Aquifers are high permeability strata that provide major beddings for flow transport. Good rock quality in continuous sandbodies allow for efficient CO₂ storage in an acceptable capacity volumes. Aquitards are made of low permeability strata that provide beddings with orders of magnitude slower flow than aquifers. Layers of aquifers and aquitards are formed by thick accumulation of sediments that undergo burial, compaction, lithification and uplift over millions of years. They can be covered by aquiclude, which are evaporative beds that are impervious to fluid flow. Typical seal rocks include, from most ductile to most brittle: salt, anhydrite, krogen-rich shales, dense mudstone, tightly cemented sandstones, anhydrite-filled dolomite, carbonate, or silica-cemented sandstones, and cherts.

Aquifer pressure is normally close to hydrostatic, because of the conductivity within the medium allows for pressure equilibrium over long time. High pressurized compartments can exist in highly sealed structures. The pressure of the sedimentary basin has a significant impact on its suitability for CO₂ storage [5]. Trapping mechanism for CO₂ is in two main parts: stratigraphic and structural traps. Stratigraphic trapping is primarily controlled by the geometry of depositional facies and sand body continuity. These factors control the permeability distribution within the medium that controls the efficiency of injection and storage of CO₂. Structural heterogeneity factors include faults, folds, and fracture intensity. The dip angle of formation layers control the buoyancy forces that govern on CO₂ plume migrating along the conductive layers. Fractures can enhance the mobility of the plume and sealing faults can provide structural traps for long-term CO₂ storage. Anticline structures also can be permanent traps for stored CO₂.

Depositional environment varies between fluvial to marine systems. The texture and degree of sandiness of beach deposits are functions of the shore profile, typically consisting of a gently sloping formation layering in a transition from near shore to deep offshore. Deposits range from sandy, coarse grain structure near the shore, to muddy, burrowed, fine grained sand in the lower off shore. High energy near the shore that is a result of interplay between wave, fluvial, and tidal forces, filters out the larger grains in the deposition.

Therefore, formations closer to the shore contain large continuous sand bodies that have good qual-



(a) Fluvial depositional system.

(b) Beach depositional system.

Figure 1.4: Heterogeneities in fluvial and beach depositions. Arrow in Figure (b) shows that the deposition mass is less heterogeneous than the fluvial systems.

ity rock. This is the reason for shallow-marine systems to be appropriate traps for hydrocarbons and analogously, good candidates for CO₂ storage.

The beach facies normally are homogeneous rocks with internal heterogeneity due to tidal systems. In contrast, mixed-load fluvial deposits that contain both mud and sand are more heterogeneous than beach systems. The presence of numerous mud drapes as a result of periodic floods, serve as barriers to fluid flow. Heterogeneity in the fluvial systems exist on multiple scales, from small-scale variations of rock type near the river bed, to the large-scale heterogeneity in fluvial channel-fill sandstones and over-bank deposits. Heterogeneity also occurs within these systems in the form of muddy abandoned channel-fill deposits.

In theory, we prefer a medium that allows for more lateral movement to overcome the buoyancy bypassing of the flow. Heterogeneity in the vertical direction, such as shale inter-bed barriers can serve for this and disperse the flow in the lateral direction. Structural heterogeneities can have a similar impact. In addition, splitting a large plume into smaller plumes lowers the risk of leakage of huge CO₂ amounts via potential breakings in the integrity of the sealing barriers, or abandoned wells.

CO₂ injectivity is related to sequestration capacity and effectiveness, and can be defined by the conductive cross-sectional area. Stratigraphic factors that enhance injectivity are high permeability and injection interval thickness. In addition, the lateral permeability architecture can influence the injectivity quality. The lower the injectivity is, the higher will be the pressure buildup in the medium due to injection.

Over the last two decades, there have been a large number of studies concerning the subsurface storage of CO₂. Several authors investigate the efficiency of geological CO₂ storage based on regional data in a specific site location.

A case study from the Texas Gulf Coast [42] investigates the sequestration capacity and efficiency in accordance to the geological heterogeneity. The study performs a site-scale assessment of brine aquifers for geological CO₂ sequestration. Injection is considered in the Frio formation which is a sandstone-rich, high quality, overlain by a thick, regionally extensive shale in the upper Texas Gulf Coast. Migration of CO₂ during injection (20 years), and post-injection (40 years) is studied in different geological realizations. The heterogeneity represented by stochastic modeling of geological sediment. Structural heterogeneity is modeled by layers dip angle and faults at different locations. Six models are made based on regional available geological information. The study shows that in a homogeneous rock volume, CO₂ flow paths are dominated by buoyancy, bypassing much of the lateral rock volume. If the permeable rock is inter-bedded with multiple low permeability layers, the flow paths are dispersed, enhancing the lateral movements of CO₂ and allowing for larger percentage of contact with rock volume. The study suggests that dip angle enhances buoyancy forces and decreases storage capacity, while compartmentalization by faulting appears to increase sequestration capacity at the cost of increased pressure, and consequently, increased risk of CO₂ leakage.

A number of pilot sites are established worldwide to test the large-scale injection of CO₂ in the subsurface formation. The In Salah project [68] in Algeria is an industrial-scale injection project into a fracture-influenced, matrix-dominated sandstone formation. The reservoir matrix comprises tidal deltaic sandstone. The project benefits from relatively high level of data acquisition: wireline and LWD well logs, image logs and production and geophysical monitoring [68]. In addition, the most valuable monitoring method has been the use of satellite airborne radar above the injection well. Also, chemical tracers are used in the injected CO₂ to differentiate the natural CO₂ in place from the injected volumes, when the CO₂ breaks through other wells. The detailed analysis highlights the geological controls on the movement and dispersion of CO₂ plumes. The injection is performed via a horizontal well perpendicular to the geomechanical stress field and the faults present in the domain. This, along with the fracture network, enhance the plume migration- about three times faster than the flow in a homogeneous domain. Results from In Salah illustrate the value of reducing geological uncertainty by employing sufficient logging tools and monitoring techniques.

The CO₂-SINK project at Ketzin Germany [32] is another pilot site for practicing subsurface CO₂ injection. The injection is performed in the Stuttgart formation that is geologically heterogeneous within an anticline structure. The Stuttgart formation is made of sandy channel facies of good rock quality alternate with muddy flood-plain-facies. A thick cap-rock section covers the Stuttgart formation.

Practically, including all details of every scale into a flow simulation model is impossible. Various simplifications have been made to account for heterogeneities in modeling. Some earlier studies consider two dimensional modeling, with homogeneous or geostatistically populated permeabilities. The study in [52] simulates an escape rate of CO₂ in a homogeneous medium similar to Utsira formation in Norway. By changing the horizontal permeability, they demonstrate that most of the injected CO₂ volume accumulates in a fine layer beneath the cap-rock due to buoyancy forces in the long-term CO₂ migration process. However, this study assumes no vertical heterogeneities. A layered heterogeneity is examined in [78]. They used a log normal distribution of permeability in a simplified two dimensional grid to account for viscous and gravity forces. Results suggest that the sweep efficiency of CO₂ in the porous medium is low, and heterogeneity, in particular the vertical transmissibility, can have a big impact on the storage efficiency.

To examine the impact that the geological heterogeneity degree can have on the CO₂ sequestration modeling, [30] constructed a suite of three-dimensional simulation models, with varying net to gross ratios. A radial variogram, with a shale length of 300 m, was used to populate five models of varying degrees of net-sand-to-gross-shale ratios. The models were up-scaled, using flow-based methods, to make the computation feasible. The study concludes that formations containing shale barriers are effective in containing an injected CO₂ plume within the formation and that heterogeneity serves to limit the reliance of the formation seal as the only mechanism for containment.

1.5.2 Geological parameters

From the flow modeling perspective, sources of geological uncertainty can manifest themselves in the rock parameters that go in the flow equation such as permeability and porosity. However, to represent the geological uncertainty, it is not enough to randomize these parameters. This approach might work in simple geological models, but it can fail to give plausible results for the realistic heterogeneous problems with uncertain structural and depositional descriptions.

In response to the EU priorities of reducing time to first oil and of improving overall hydrocarbon recovery efficiency, the interdisciplinary SAIGUP study was initiated to increase the understanding of the influence of geological uncertainties in oil field recoveries. SAIGUP stands for 'sensitivity analysis of the impact of geological uncertainties on production forecasting in clastic hydrocarbon reservoirs'. The context in SAIGUP is defined for shallow-marine depositional systems. The main objective of the SAIGUP project has been to perform a quantitative sensitivity analysis to measure the impact of sedimentological and structural variations within geological descriptions on oilfield recovery estimates [43, 53, 54]. Herein, we will use six different rock types to investigate the impact of geological hetero-

Table 1.1: Grid specifications for fine and coarse scales in the SAIGUP modeling process.

Parameter	Fine Scale	Coarse Scale
Number of cells in the x direction	80	40
Number of cells in the y direction	240	120
Number of cells in the z direction	80	20
Number of total cells	1,500,000	96,000
Number of active cells	1,500,000	79,000
Model x dimension	<i>3 km</i>	<i>3 km</i>
Model y dimension	<i>9 km</i>	<i>9 km</i>
Model z dimension	<i>80 m</i>	<i>80 m</i>

genetics on CO₂ sequestration. The rock properties within each facies are populated based on real data. Variations are considered in a horizontal-vertical matrix in three levels of heterogeneities, low, medium, and high, as illustrated in Figure 1.5. The design focused on special considerations. For example, making complex enough heterogeneities to be a plausible representative of realistic models, and producing large enough number of realizations with sufficient overlapping to be able to perform a quantitative sensitivity analysis.

Sedimentological variability is modeled in small and large scales and combined to provide realistic variations of reservoir heterogeneities. All models are considered in a progradational sedimentary environment. A regular grid is used for all of the realizations in two gridding resolutions, fine and coarse, and the total bulk volume is the same in all cases. Each geological realization contains about 1.5 million cells in the fine model. Figure 1.6 shows the fine grid model for a selected realization with medium level of heterogeneity. A major fault in the model breaks the structure and makes large vertical depth difference in the two parts of the model (from about 1500 m to 3000 m). Thickness of the model is much smaller than these depth differences. To make it easier to see the property variations on the grid in the vertical direction, we map the properties on a flat uniform geometry (Figure 1.7).

Figure 1.8 shows the spatial distribution of the six modelled facies in the selected realization, and Figure 1.9 shows the histogram of lateral transmissibility within each facies and the total model in the logarithm scale. Each facies is modeled separately in some levels of upscaling starting from the lamina scales, before populating on the fine grid. Flow based upscaling techniques are used, and the suitability of the methods depends on the balance of forces. When the medium is conductive due to high permeability, the viscous dominated steady state method is used. In the rocks with lower transmissibility where the capillary forces are dominant, the capillary equilibrium is assumed [53].

On the last step, the fine populated grid is mapped on to the coarse grid that is to be used for the flow solver. Since the grid size in the fine model is too expensive computationally for flow simulations, the lateral dimension is doubled in each cell while every four layers are lumped into one layer in the vertical direction. Figure 1.10 shows the top view of lateral transmissibility in logarithmic scale for four consecutive layers of a selected case, and their corresponding upscaled layer in the coarse grid. Table 1.1 shows the grid specifications in the coarse and fine SAIGUP models.

A detailed discussion about the upscaling the Sedimentological and structural parameters for SAIGUP simulation models can be found at [53].

Structural aspects are modeled via fault modeling. Within the SAIGIP setup, faults are considered in different levels of intensity, orientation, and transmissibility. The orientations may vary in both lateral directions, and we consider a grid that contains faults in both directions (Figure 1.11).

Although these models were designed to study the impact of geological heterogeneity on oil recovery, they may also be used to model a scenario in which CO₂ is injected into an abandoned reservoir. Therefore, we have selected five parameters from the setup and varied these parameters by combining different levels for our CO₂ storage study. These features are lobosity, barriers, aggradation angle,

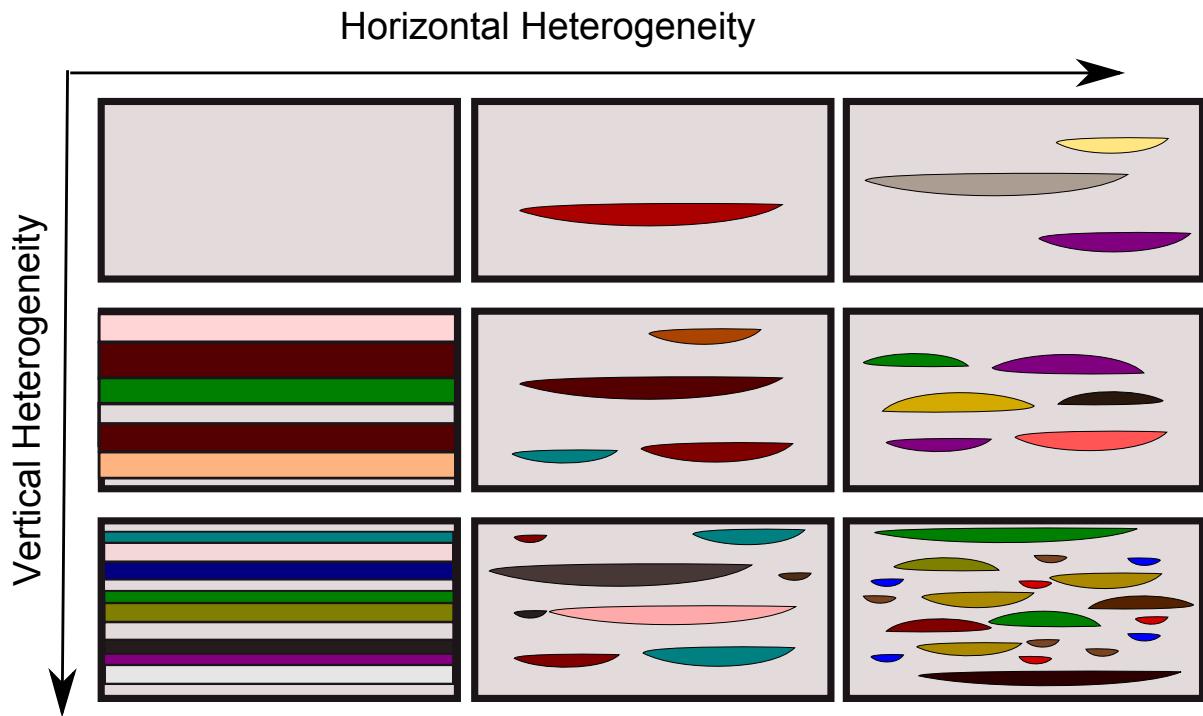


Figure 1.5: Stratigraphic heterogeneity levels in lateral and vertical directions. Modified from [53].

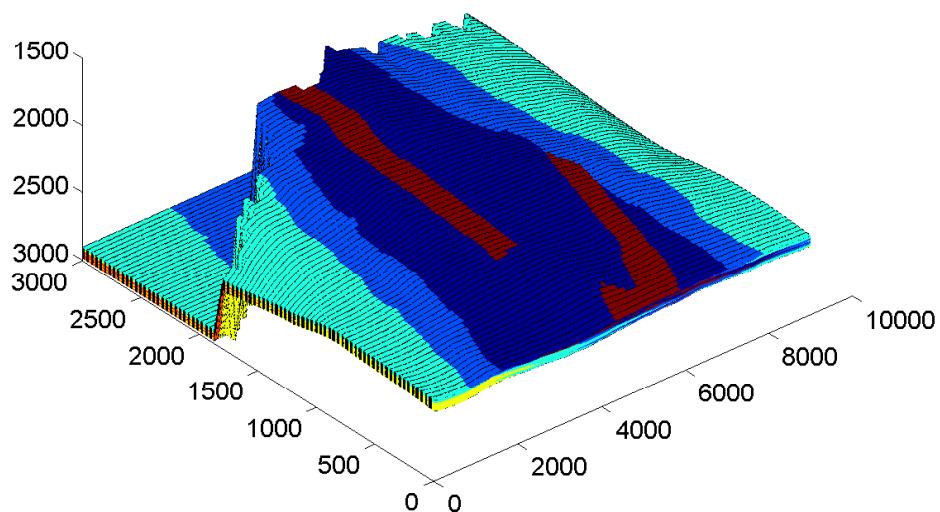


Figure 1.6: Unfaulted fine grid perspective view. Colors depict rock types, see Figures 1.7 and 1.8.

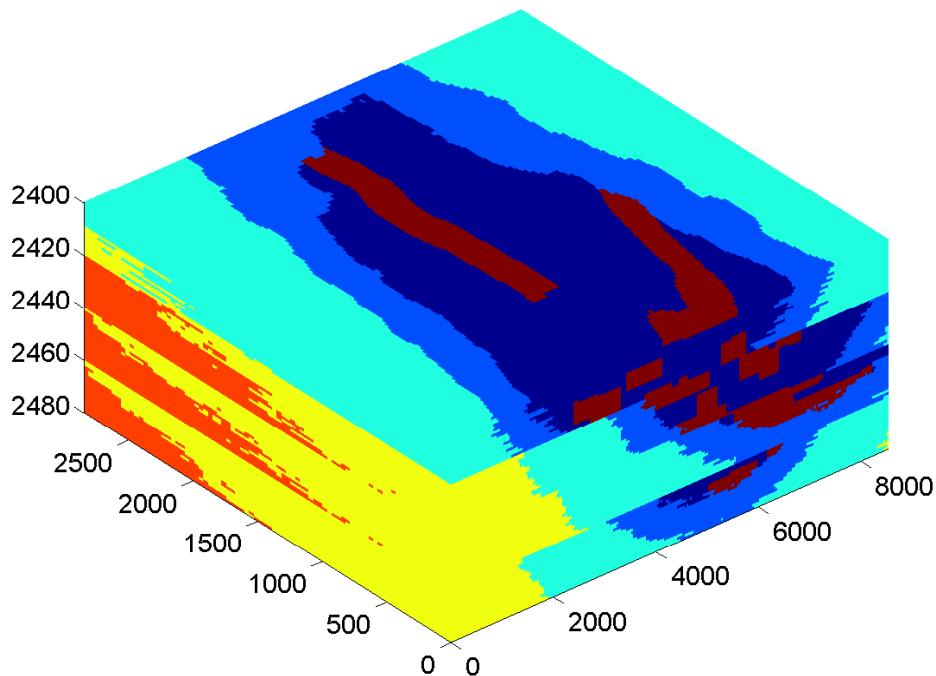


Figure 1.7: Perspective view of the rock type variations for a selected case mapped on a uniform grid.

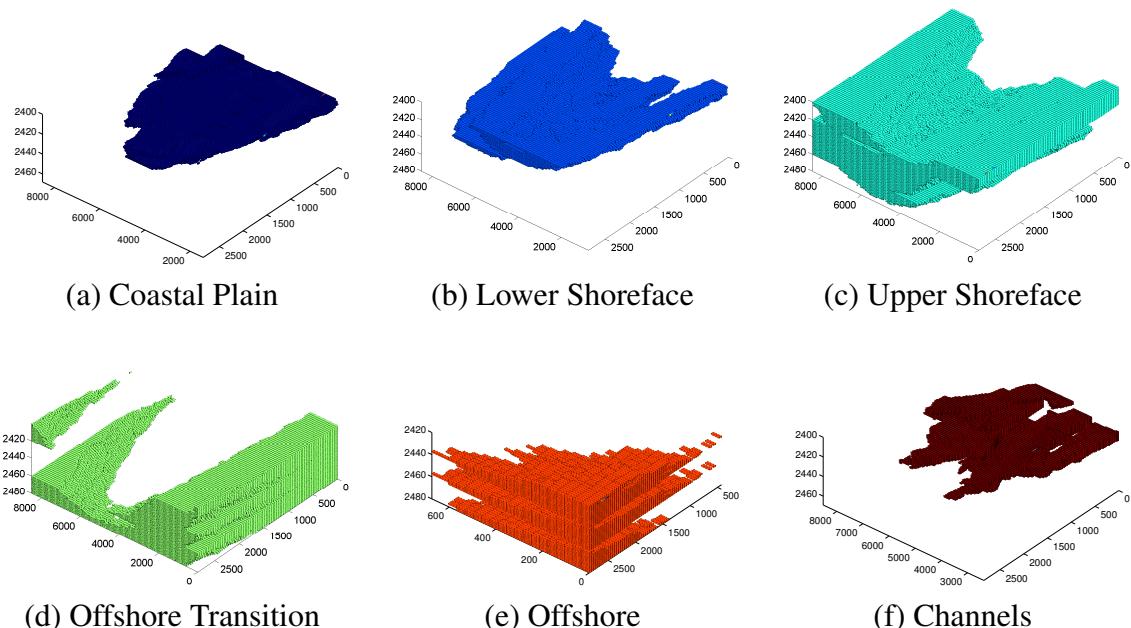


Figure 1.8: Six different rock types used in modeling the stratigraphic heterogeneities. Compare to Figure 1.7

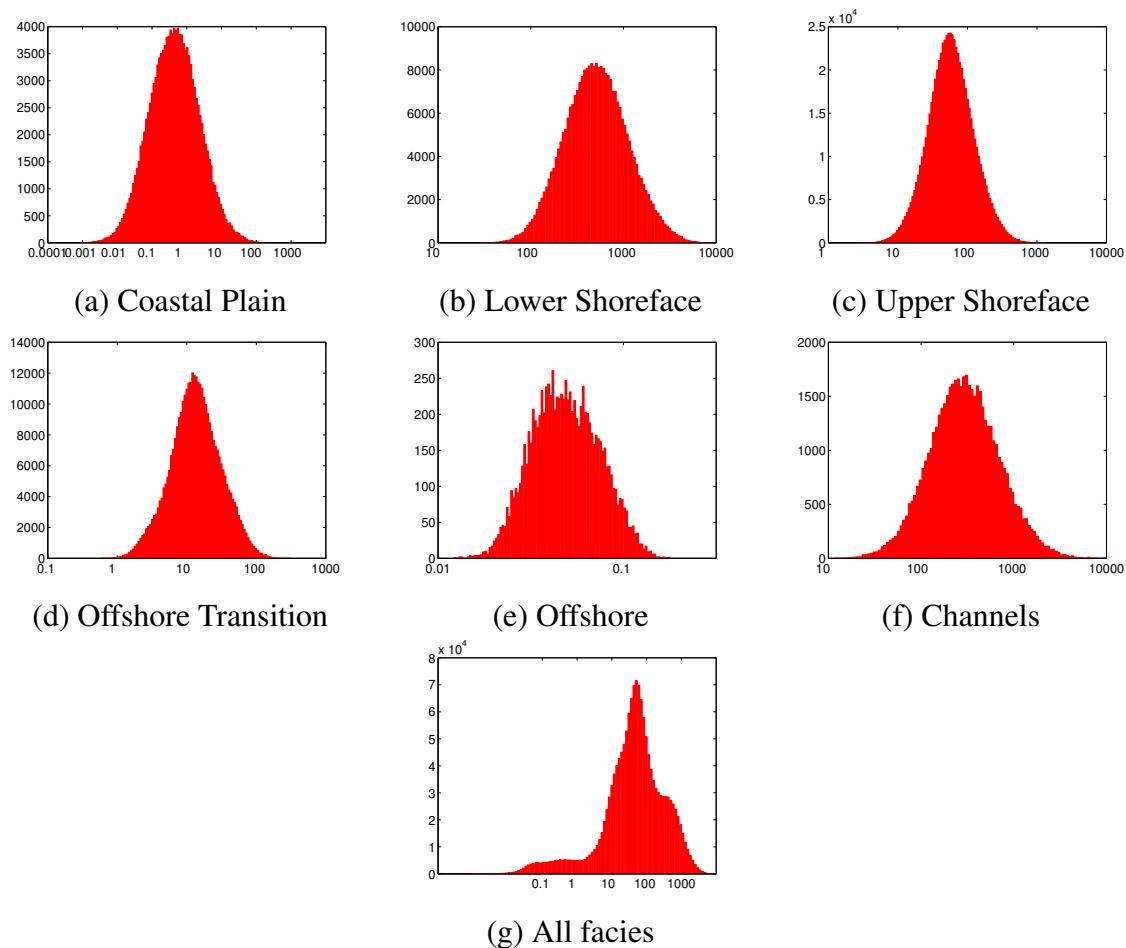


Figure 1.9: Histogram of lateral transmissibility for different facies and the entire fine model for a selected case. Scales are logarithmic in units cP.m³/day/bar.

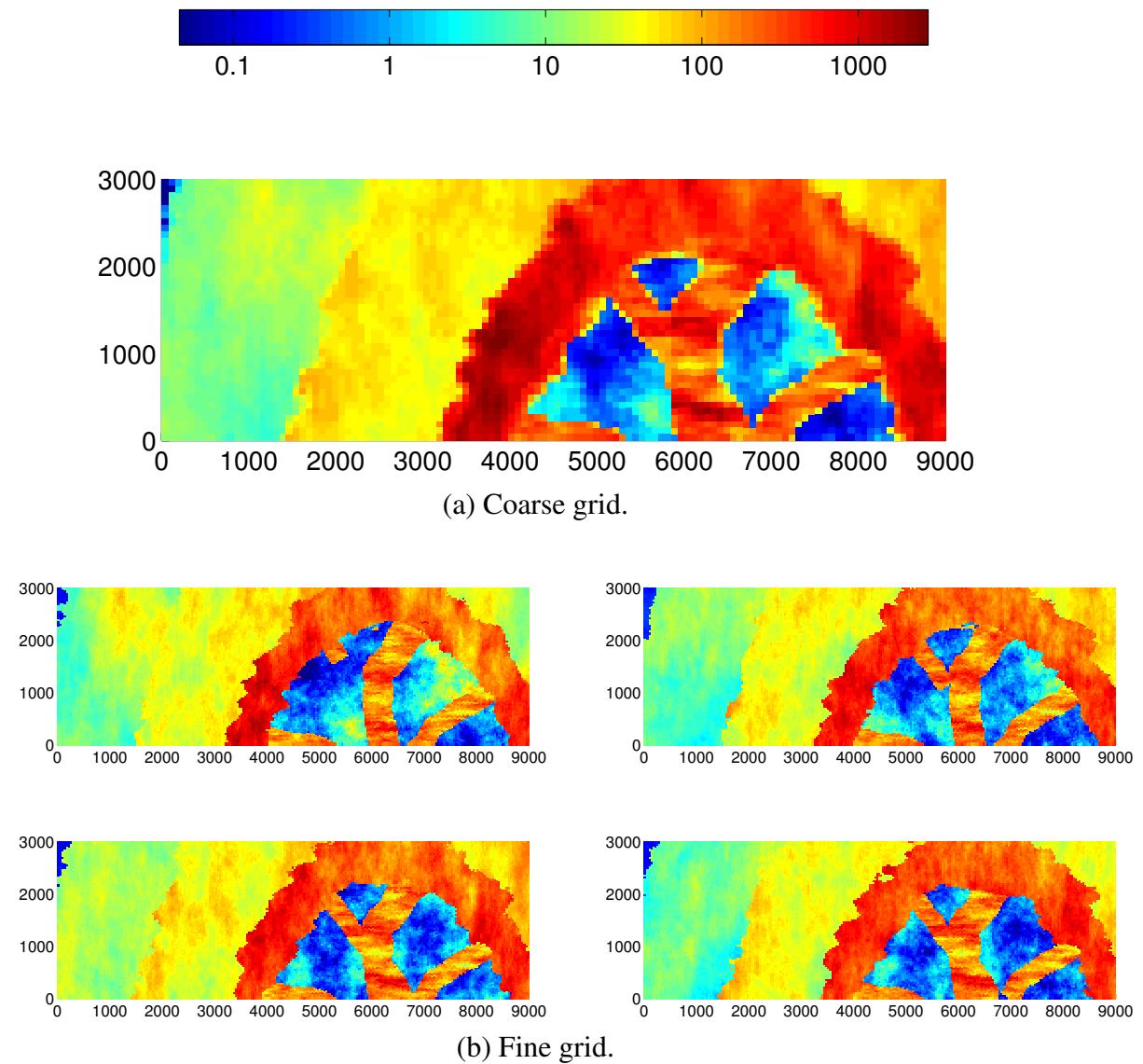


Figure 1.10: Logarithmic of lateral transmissibility plotted for four layers in fine grid versus their representative layer in the coarse grid. Plots are top view and units are $\text{cP} \cdot \text{m}^3/\text{day}/\text{bar}$.

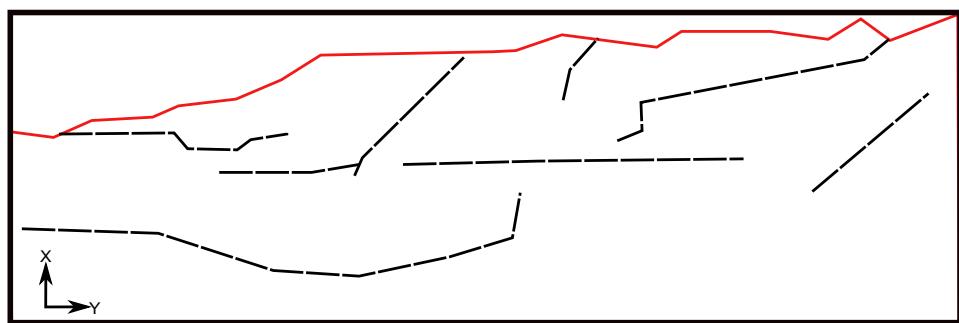


Figure 1.11: Top view illustration of faults used in the faulted grids. The fault plotted in red corresponds to the upper boundary of the model on the crest.

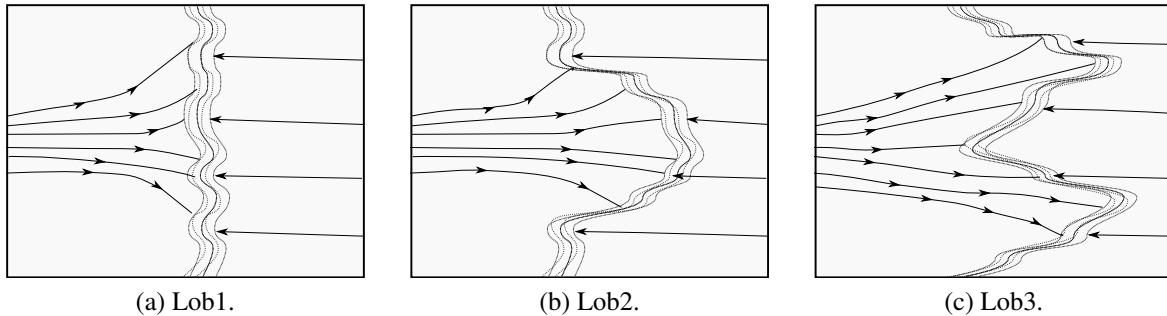


Figure 1.12: Lobosity levels are defined based on the shoreline shape, which is caused by the interplay between fluvial and wave forces.

progradation, and fault. In the following, we describe each feature briefly.

Lobosity: Lobosity is a metric for describing the interplay between fluvial and wave processes in a shallow-marine depositional system. As the river enters the mouth of the sea, the shore-line shapes where the river flux crashes with the waves from sea. The balance between the sediment supply from rivers and the available accommodation space in the shallow sea defines the shore-line shape. Sea waves smear out the shore-line, while fluvial flux from river makes branches into the sea. Less wave effect produces more pronounced lobe shapes around the river entrance into the sea.

The channels made into the sea mouth by fluvial supplies contain good quality rocks with relatively higher porosity and permeability and poor quality rock types are located between the conductive branches. Reservoir quality decreases with distance from the shore-face. Lobosity variation can influence the CO₂ injection operation and plume distribution in the aquifer. In this study, models of three levels of lobosity are used: flat shoreline, one lobe and two lobes, see Fig. 1.12.

Barriers: Barriers are mud-draped surfaces sitting between reservoir sections that are caused by periodic floods in a shallow-marine depositional system. Mud-drapes extend in both vertical and lateral directions and are potential significant barriers to flow. In the SAIGUP domain used here, these barriers were modeled by defining areas between layers with zero transmissibility multipliers. This areal coverage was designed in three levels: low (10%), medium (50%), and high (90%). We use the same variations in this study, see Fig. 1.13.

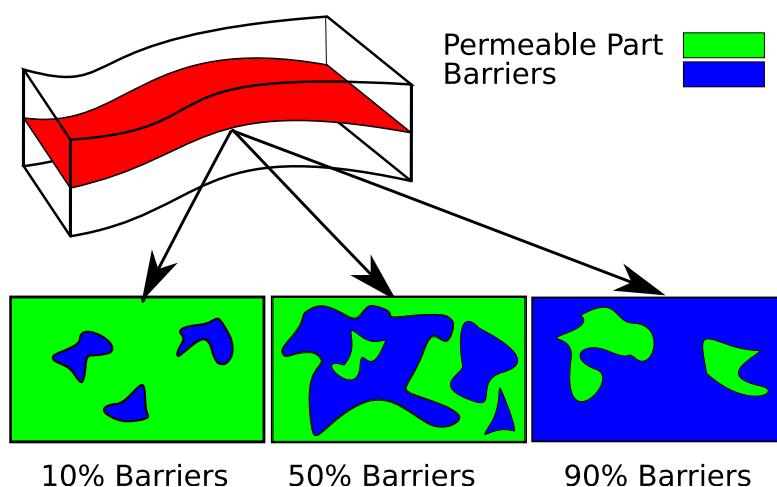


Figure 1.13: Barrier levels caused by periodic floods.

Aggradation angle: In shallow-marine systems, the sediment supply from rivers deposits in a spectrum of large size grains in the land side toward fine grains deep in the basin. Amount of deposition supplied by the river compared to the accommodation space that the sea provides defines the transition angle.

tion of different rock-types between the river and the sea. If the river flux or sea level fluctuates, the equilibrium changes into a new bedding shape based on the balance of these factors.

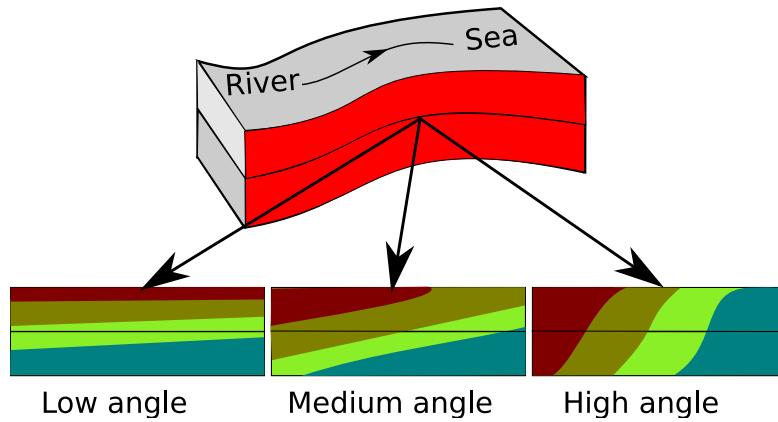


Figure 1.14: Aggradation angle levels.

When the river flux increases, it shifts the whole depositional system into the sea causing an angle between transitional deposits that are stacked on each other because of this shifting. This angle is called aggradation angle. Three levels of aggradation are modeled here: low, medium and high (Fig. 1.14). As we will see later, aggradation can have a major role in influencing the CO₂ flow direction in the medium.

Progradation: Progradation is the depositional-dip direction between the sea and the river. Two types are considered here: up and down the dominant structural dip. Progradation combined with lobosity can influence the plume development in the medium, as the injected CO₂ plume migrates upward to the crest goes through heterogeneities (Fig. 1.15).

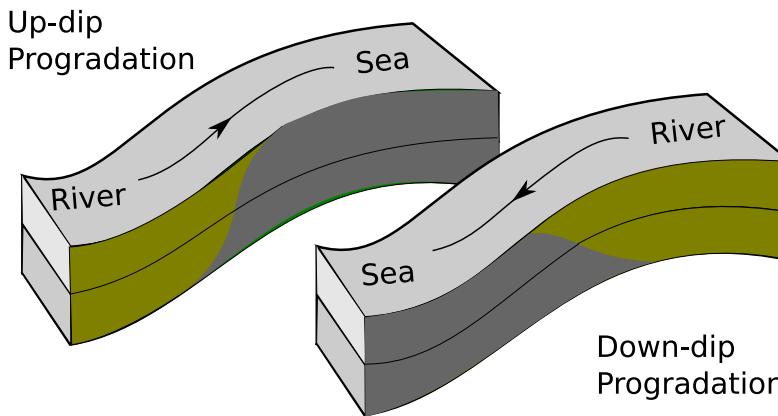


Figure 1.15: Progradation levels.

For more information about the geological modeling, see the special issue of the Petroleum Geosciences that is devoted to the SAIGUP study [54]. One selected realization of the SAIGUP models is available for download [73] and this model is used as an example in MATLAB Reservoir Simulation Toolbox (MRST) [76].

1.6 Flow equations

After introducing the parameters that make our geological model, we need to define the flow problem. In this section we discuss various formulations of the governing equations describing single and two phase flow in porous medium. Solution to these type of equations is implemented in the ECLIPSE black-oil simulator that we use to model the flow. We introduce the functionalities and axillary equations required to close the flow equation system. This section also includes a brief mathematical discussion on the flow equations. We discuss various flow regimes in the medium in the next section.

1.6.1 Single phase flow

Assume a porous domain Ω with boundary Γ as shown in Figure 1.16. We write the continuity equation in general form for a single phase flowing in the domain:

$$\text{Accumulation} + \text{In-Out Flux} = \text{Source/Sink} \quad (1.1)$$

$$\int_{\Omega} \frac{\partial}{\partial t} (\phi \rho) d\tau + \int_{\Gamma} (\rho v \cdot n) d\sigma = \int_{\Omega} q d\tau \quad (1.2)$$

In Equation 1.2, ϕ is the rock porosity, ρ is the fluid density, v is the flow velocity, and n is the normal vector to the boundary. The term q denotes the mass source or sink in the system. Integrations are taken over arbitrary domain Ω with boundary Γ (Figure 1.16). Flow velocity is considered at the representative elementary volume (REV) scale for porous media [7].

The resistance of a porous medium against flow results in a velocity that can be calculated from pressure and gravity gradient and fluid properties in the medium. This is governed by Darcy's law for single phase flow:

$$v = -\frac{K}{\mu} \cdot (\nabla P + \nabla D). \quad (1.3)$$

In Equation 1.3, K is the permeability of the medium. Permeability is a function of pore size distribution and connectivity and in the macro scale, it is a measure of medium conductivity when a fluid is flowing through the medium (Figure 1.17). D is the gravity term that is a function of fluid specific gravity and elevation in vertical direction.

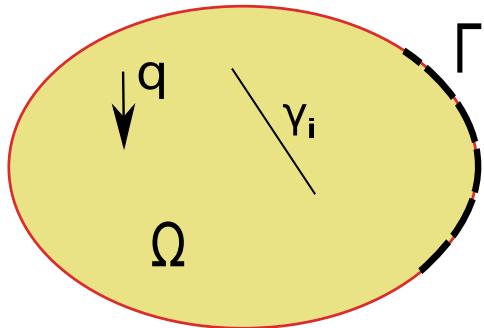


Figure 1.16: Flow domain and boundaries.

Substituting velocity term from Equation 1.3 into Equation 1.2 gives:

$$\int_{\Omega} \frac{\partial}{\partial t} (\phi \rho) d\tau - \int_{\Gamma} (\rho \left(\frac{K}{\mu} \cdot (\nabla P + \nabla D) \right) \cdot n) d\sigma = \int_{\Omega} q d\tau. \quad (1.4)$$

As a primary unknown in Equation 1.4, the pressure depends upon the boundary conditions (as the second term left hand side of Equation 1.4 is an integration over the boundaries of the domain). Also,

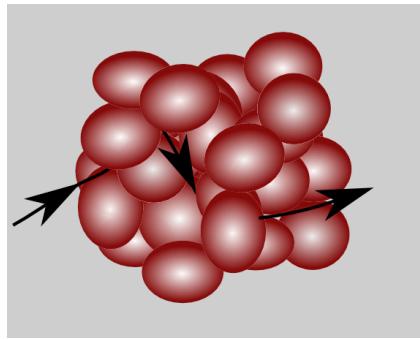


Figure 1.17: Permeability is an indication of how easy it is for the fluids to flow through the medium.

any geological discontinuities in the medium (γ_i in Figure 1.16) appears in Equation 1.4 through the K tensor and can influence pressure behavior in the domain.

The second term in Equation 1.4 can be converted into an integration over domain Ω , using divergence theorem resulting in the following:

$$\int_{\Omega} \left[\frac{\partial}{\partial t} (\phi \rho) + \nabla \cdot (\rho v) \right] d\tau = \int_{\Omega} q d\tau. \quad (1.5)$$

Equation 1.5 is valid for arbitrary domain Ω , hence the equality is valid for the integrands *almost everywhere* in domain Ω in the *general* situation:

$$\frac{\partial}{\partial t} (\phi \rho) + \nabla \cdot (\rho v) = q. \quad (1.6)$$

Fluid and rock change in volume with pressure variations. These dependencies are defined by a parameter called total compressibility, which is approximated by a combination of rock and fluid compressibilities:

$$C_T \approx C_{rock} + C_{fluid}, \quad (1.7)$$

where

$$C_{rock} = \frac{\partial \phi}{\partial P}, \quad (1.8)$$

and

$$C_{fluid} = \frac{1}{\rho} \frac{\partial \rho}{\partial P}. \quad (1.9)$$

In Equation 1.8, C_{rock} can be assumed constant in moderate pressure changes depicting a linear relation between pressure and porosity. Also, Equation 1.9 can be expanded resulting in the following [59]:

$$\rho = \rho_0 \left(\frac{P}{P_0} \right)^m \exp[C_{fluid}(P - P_0)]. \quad (1.10)$$

Assuming slight compression gives [70]:

$$\rho \simeq \rho_0 + C_{fluid} \rho_0 (P - P_0). \quad (1.11)$$

By substituting from Equations 1.7, 1.9, 1.8, and Equation 1.3 into Equation 1.6, density vanishes and by defining volumetric source/sink η , we have the single-phase diffusivity equation:

$$C_T \frac{\partial P}{\partial t} - \nabla \cdot \left[\frac{K}{\mu} (\nabla P - \nabla D) \right] = \eta. \quad (1.12)$$

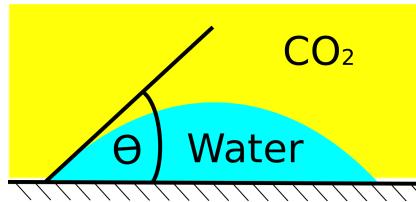


Figure 1.18: Wettability and interfacial tension in water-CO₂ system. Water is the wetting phase in this example.

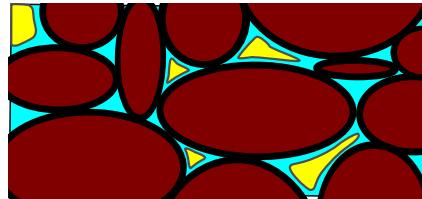


Figure 1.19: Multi-phase flow in the pore scale.

1.6.2 Two-phase flow

In a two-phase flow of CO₂ and water within porous media, interactions between phases lead to loss of energy. This introduces specific phenomena occurring in the pore scale that have impact on the macro scale flow performance. More complicated equations appear in modeling the two-phase flow compared to the single-phase problem. First, we describe some of the conceptual two-phase phenomena in the pore scale and then we will continue by deriving the flow equations for two phases in the system, i.e., CO₂ and water.

When CO₂ and water get in contact at the pore scale, an interface forms between them such that the energy in the system is minimized. Water and CO₂ are also in contact with the porous medium and the interface between them forms an angle from the solid phase in the water phase (shown by θ in Figure 1.18) that depends on their ability for wetting the rock. This is called wettability and the phase with the preference of wetting the solid phase is called the wetting phase. The other phase is called the non-wetting phase. Conventionally, θ is measured inside the denser fluid. If $\theta < \frac{\pi}{2}$ then the denser phase is the wetting phase. Wettability in a porous medium depends on the fluids and the rock. It can have a significant influence in the phase displacement within the medium. For water-CO₂ system, normally water is the wetting phase.

At very low water saturations, the water phase forms molecular films surrounding the rock grains. In this situation, the water phase is immobile and can not make a continuous phase moving through the porous medium. As water saturation in the medium increases, the layers covering the rock grains grow in size until the saturation exceeds a critical level, above which the water phase is able to flow in the medium. This saturation is called the critical or connate water saturation. In a water-wet rock, once the critical water saturation is reached (for example, during the first deposition of sediments), it can not go below that level by being displaced via a non-wetting phase. Therefore, when we inject CO₂ into an aquifer, there will always be some residual water saturation in the regions invaded by CO₂.

As a non-wetting phase, CO₂ flows in the middle part of the pore space as shown in Figure 1.19. If CO₂ saturation decreases in the medium, it reaches a critical level under which it can not make a continuous phase flowing through the pore-network. Tiny drops of CO₂ are trapped in the middle of the pore space and only very large pressure difference across the pore can move it out of the pore. This level of CO₂ saturation is called the residual saturation. Higher residual saturation is more interesting for the purpose of immobilizing more volumes of injected CO₂ in the aquifer, which reduces the risk of CO₂ leaking through any breakings in the geological formation and channeling toward surface.

Relative ease for the phase to flow within the medium is described by the relative permeability parameter. Relative permeability is a function of wettability and phase saturation. High phase saturation

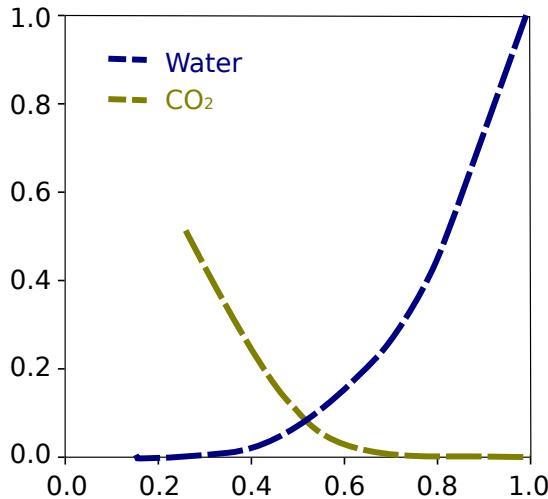


Figure 1.20: Two-phase flow and relative permeability.

indicates a higher space available for the phase to flow through that space. A sample of CO₂-water relative permeability functions are shown in Figure 1.20. A library of relative permeability curves for CO₂-water system for various rock-types is available at [8].

The difference in surface tension between water and CO₂ causes a pressure acting on the interface of the two fluids. This pressure is called capillary pressure. In addition, capillary pressure depends on the geometry of pores. Since the pore geometry is very irregular, it is more convenient to use simpler geometry to derive the concept of capillary pressure. Therefore, experimental work in the laboratory is required to specify the capillary pressure functionality in a special case.

Assuming a geometry of pipe to represent a pore structure, after balancing the forces in the pore-system capillary pressure can be written in the following form:

$$P_c = \frac{2\sigma}{r} \cos\theta, \quad (1.13)$$

where σ is the interfacial tension, θ is the angle between the interface and the solid phase, and r is the radius of the pore.

Capillary pressure is a jump in phase pressure across the interface of the two phases. Therefore, we can relate it to the phase pressures:

$$P_c = P_{nw} - P_w. \quad (1.14)$$

Here, P_{nw} is the non-wetting phase pressure and P_w is the wetting phase pressure.

Capillary pressure can be expressed in an empirical relation as a function of wetting phase saturation. Lower capillary pressure is expected for higher wetting saturations, and capillary pressure values go up for lower wetting saturations (Figure 1.21).

Assume hydrostatic equilibrium for a porous domain in which water and CO₂ are segregated due to buoyancy effect. If capillary forces are considerable in the domain, the sharp interface between water and CO₂ in the macro scale will be replaced by a transition zone with a spectrum of saturations between phases (Figure 1.23). Due to the hydrostatic equilibrium, phase pressure at each depth can be related to the hydrostatic pressure of that phase:

$$P_w = \rho_w g z, \quad (1.15)$$

$$P_{CO_2} = \rho_{CO_2} g z. \quad (1.16)$$

Having the phase pressure, capillary pressure can be calculated by Equation 1.14. As capillary pressure is a function of wetting saturation, the phase saturations can be back-calculated from this functionality and the phase saturation distribution over the medium can be found (Figure 1.22):

$$S_w = P_c^{-1}(S_w). \quad (1.17)$$

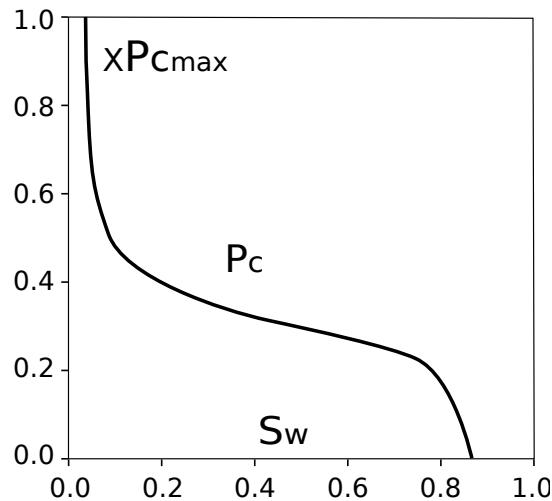


Figure 1.21: Capillary pressure can be expressed as a function of wetting saturation. The plot is a typical curve of capillary pressure function with a maximum of $P_{c\max}$

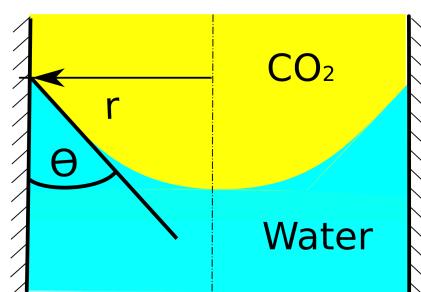


Figure 1.22: Capillary pressure distribution in the transition zone.

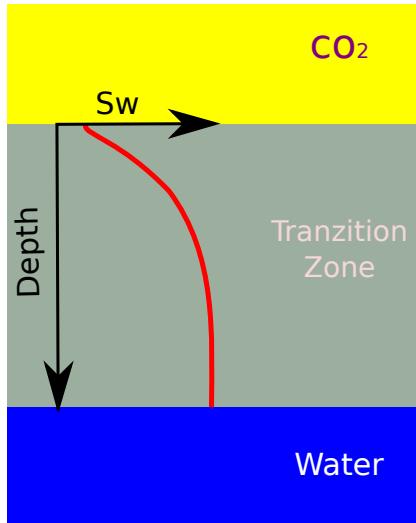


Figure 1.23: Saturation distribution in the capillary transition zone.

We can derive mass and momentum balance for two-phase flow, similar to what we have seen for single-phase flow. The equations must be written for each phase. In Equation 1.4, the accumulation term must be considered only for one phase mass calculated by multiplying the total accumulation mass by phase saturation (S_α). Also the velocity is the phase velocity v_α , and the source/sink term must be written for the phase mass rate q_α .

For phase $\alpha = \{\text{water}, \text{CO}_2\}$, we have:

$$\int_{\Omega} \frac{\partial}{\partial t} (\phi \rho_\alpha S_\alpha) d\tau + \int_{\Gamma} (\rho_\alpha v_\alpha \cdot n) d\sigma = \int_{\Omega} q d\tau. \quad (1.18)$$

Darcy equation for two phases $\alpha = \{\text{water}, \text{CO}_2\}$ can be written in the following form:

$$v_\alpha = -\frac{K_{e\alpha}}{\mu_\alpha} \cdot (\nabla P_\alpha + \nabla D_\alpha). \quad (1.19)$$

Here, $K_{e\alpha}$ is the effective permeability for phase α and can be calculated from:

$$K_{e\alpha} = K_{abs} K_{r\alpha}, \quad (1.20)$$

where K_{abs} is the absolute rock permeability and $K_{r\alpha}$ is the relative permeability of phase α . P_α is the phase pressure and D_α is the gravity term for the phase specific gravity.

Similar to Equation 1.6, differential form of mass balance equation for each of phases $\alpha = \{\text{water}, \text{CO}_2\}$ is as follows:

$$\frac{\partial}{\partial t} (\phi \rho_\alpha S_\alpha) + \nabla \cdot (\rho_\alpha v_\alpha) = q_\alpha. \quad (1.21)$$

In this equation, q_α is the source/sink mass rate for phase α .

The phase saturations are related by the following equation:

$$S_{\text{water}} + S_{\text{co}_2} = 1. \quad (1.22)$$

Fluid properties change by pressure and temperature. Density is mainly a function of pressure and viscosity depends upon temperature. These functions, called by convention equation of state (EOS), must be coupled to the system to honor fluid attribute variability [22, 36].

Mass exchange between phases may happen leading to change in composition. That also influences the fluid properties. In the immiscible fluids, the mass exchange can be in small order leading to slight changes in fluid properties. That can be modeled as a linear function with respect to pressure and temperature.

Extensive mass exchange between phases results in more nonlinear fluid property variations that require a detailed equation of state. Also for highly miscible fluids and high mass transfer between phases, it might be better to write mass and momentum balance equations for components within phases in addition to phase equations.

There are a number of approaches to formulate the primary unknowns in the system of flow equations. The direct way is to replace phase velocities from Equation 1.19 into Equation 1.21, leaving the phase pressures and water saturation as the primary unknowns. This ends in a set of strongly coupled equations.

A popular approach for formulating the set of flow equations is the fractional flow method [10]. In this method the total multiphase flow problem is treated as a single-phase flux of multi-phase mixture. Therefore, individual phases are described as a function of total flow. This leads to separate equations for pressure and saturation.

Pressure is defined for the total flow either globally or pseudo-globally and relates to the phase pressure and saturation with auxiliary equations. The fractional flow approach keeps the governing equations in the form of single flow equations, and numerical schemes for single-phase flow can be revised into efficient schemes for multi-phase problems.

Pressure and saturation equations have different mathematical nature: pressure has a diffusive character of an elliptical nature, which is numerically more stable than the saturation equation. Saturation equation is of convection-diffusion form with hyperbolic character in the convection part. The convection operator in saturation equation can be highly non-linear due to strong coupling of saturation and phase velocity. This nonlinearity can lead to shocks and discontinuities in the saturation solution.

As an example of fractional flow formulation, global pressure P_t is defined based on phase pressures:

$$P_t = \frac{1}{2}(P_w + P_{CO_2}) - \int_{S_w|_{P_c=0}}^{S_w} (f_w - \frac{1}{2}) P'_c(S_w) dS_w, \quad (1.23)$$

where water fractional flow f_w is defined as:

$$f_w(S_w) = \frac{\frac{K_{rw}}{\mu_w}}{\frac{K_{rw}}{\mu_{co_2}} + \frac{K_{rco_2}}{\mu_{co_2}}}. \quad (1.24)$$

The total velocity is defined as:

$$v_t = v_w + v_{co_2}. \quad (1.25)$$

If capillary and gravity effects are negligible, saturation equation can be solved analytically, e.g. via Buckley-Leverett technique, or method of characteristics.

1.7 Flow regimes

A major part of our studies includes modeling physical phenomena occurring within flow through porous media. Various phenomena occurs during a complete sequence of CO₂ sequestration. During injection the forces imposed by the injector dominate the flow behavior in a region around the injector. When CO₂ plumes develop in a thin layer moving along the stratigraphical structure, a large interface between water and CO₂ enhances the diffusion phenomena and lets more CO₂ be dissolved into water. Convection of water with dissolved CO₂ leads to complicated flow regimes.

The injected CO₂ undergoes various stages until it is stored underground. We consider two stages in our studies: injection, early migration, and long-term migration. Many forces act on flow within medium, each of which requires a set of modeling parameters. Simplifying assumptions for flow modeling can be justified at each stage with relevance to the dominating forces in the medium.

The following can be recognized as forces acting on the medium at the scale at which Darcy velocity is defined:

- Forces due to pressure gradients, mostly imposed by injectors (and/or producing wells).

- Buoyancy due to density contrasts between flowing phases. Gravity acts in the vertical direction.
- Capillary forces due to inter-facial tensions.
- Hysteresis due to sequencing of imbibition and drainage during flow in the porous medium.
- Convection forces due to gradients of density within one phase.
- Diffusion due to concentration gradients of one component.
- Reaction due to chemical reactions between phases and rock.

Modeling all forces acting on a porous medium is not practical, and we need to look at each flow regime separately by neglecting some forces that have a minor role. Herein, we discuss the main forces during injection and within long term migration.

1.7.1 Injection and early migration

Injection of CO₂ in the underground happens by forcing CO₂ mass through an injector into the medium. This poses a pressure gradient around the injector causing flow within the near bore region. Some authors call the force due to pressure difference ‘viscous force’, since viscosity has an important role in transferring the stress due to pressure difference in the porous medium resulting in fluid mobility. We use the same term throughout this thesis.

Viscous and gravity forces are the two major forces acting on the region around the injector during injection. Depending on fluid properties and distance from injection point, force balance changes. Gravity causes rapid phase separation resulting in upward movement of CO₂. Gravity forces dominate two-phase regions far from the injector with lower viscous flow velocity compared to near well locations, where the flow velocity is high. At each position in the medium, a force balance results in a total force vector that may cause flow in a particular direction (Figure 1.24a).

Attempts in the literature on evaluating force interplay during a multiphase flow regime incorporating injection in the porous medium, employ sensitivity analysis on flow attributes such as flow velocity and pressure. There a number of publications that discusses reducing a complicated flow problem into a simplified problem by taking plausible assumptions [9, 15, 21, 28, 29, 62, 66, 81, 82]. Utilizing analytical solutions gives the flexibility of examining a wide range of parameter variations within the model, enjoying a fast evaluation of the corresponding flow behavior. Semi-analytical and numerical sensitivity analysis are also practiced in the literature to involve more physical modeling features in the flow performance evaluations[2, 3, 69].

The flow equations can be normalized to a dimensionless version that is used in many studies discussing the capillary and gravity influence on the flow. Herein, we give the method reported in [29]. If we assume incompressible flow in a one-dimensional domain Ω without any source/sink, Equation 1.21 reduces to the following for the wetting phase:

$$\phi \frac{\partial s_w}{\partial t} + \frac{\partial v_w}{\partial x} = 0, \quad (1.26)$$

and Darcy equation for one dimension flow becomes:

$$v_w = -K \frac{k_{rw}}{\mu_w} \left(\frac{\partial P_w}{\partial t} + \rho_w g z \right). \quad (1.27)$$

Here, z is the elevation and g is the gravitational acceleration. The system is closed by Equations 1.22 and 1.14 . We can define the dimensionless variables as follows:

$$X^* = \frac{x}{L}; \quad T^* = \frac{t v_t}{L \phi}; \quad \text{and} \quad P_c^* = \frac{P_c}{\pi_c}, \quad (1.28)$$

where L is a length constant in the problem and π_c is a capillary pressure normalizing constant. After reformulating Equation 1.26, fractional flow can be written in the following form:

$$f_w = G(S_w) + C(S_w) \frac{\partial S_w}{\partial X^*}, \quad (1.29)$$

where S_w is the normalized wetting phase saturation, G is the gravity contribution, and C is the capillary contribution to the flow. The gravity and capillary contributions, G and C , are expressed by quantities relative to the viscous force [38] and we have:

$$G(S_w) = F_w(1 - N_G k_{rnw}), \quad (1.30)$$

$$C(S_w) = N_{nw} F_w k_{rnw} \frac{dP_c}{dS_w}, \quad (1.31)$$

wherein:

$$F_w = (1 + \frac{k_{rnw}}{\mu_{nw}} \frac{\mu_w}{k_{rw}})^{-1}, \quad (1.32)$$

$$N_c = \frac{k\pi_c}{\mu_{nw} L v_t}, \quad (1.33)$$

$$N_G = (k \frac{\rho_w - \rho_{nw}}{\mu_{nw} v_t} g z). \quad (1.34)$$

Having these definitions, Equation 1.26 reshapes into:

$$\frac{\partial S_w}{\partial T^*} + \frac{dG(S_w)}{dS_w} \frac{\partial S_w}{\partial X^*} + \frac{\partial}{\partial X^*} \left(C(S_w) \frac{\partial S_w}{\partial X^*} \right) = 0. \quad (1.35)$$

Applying specific type of capillary pressure and relative permeability function may lead to simplified forms of Equation 1.35 with the possibility of having an analytical solution [82].

Some important conclusions in the literature from sensitivity studies on capillary, gravity and viscous forces are summarized here and inferred for CO₂ injection application:

- Gravity and capillary pressure will only influence the flow speed significantly for slow displacement rates. Therefore, around the injection point where normally fluids are flowing with a relatively high speed, viscous forces are dominant.
- If capillary is of any significance, ignoring capillary forces in modeling the injection of CO₂ results in a pessimistic CO₂ sweep efficiency. Capillary helps in the spreading of CO₂ in the frontal CO₂-water interface.
- Less capillary forces in the porous medium allows more space for CO₂. This enhances the density segregation due to gravity forces.

The main focus in the series of work in this thesis has been to assess the flow influence by heterogeneity during injection time and early CO₂ migration. For CO₂ injection problems, one objective is to maximize the rate of injection and aligned with that we use relatively high injection rates in our studies. Therefore, we did not include capillarity forces for modeling the high displacement rates within heterogeneities, which can be justified by the results in the literature.

Table 1.2: Spatial scales for CO₂ storage. Ranges are extracted from [13].

Feature	Spatial scale
Capillary fringe	10cm->10m
Plume radius	10km->100km
Pressure perturbation	50km->500km
Migration distance	50km->500+km

Table 1.3: Temporal scales for CO₂ storage. Ranges are extracted from [13, 37].

Feature	Temporal scale
Density segregation	1 month->5+ years
Capillary segregation	1 year->50 years
Injection period	5 years->50 years
Convective mixing	20 years->1000 years
Plume migration	few hundred years->1000 years
Mineral reaction	500 years->100000 years

1.7.2 Long term migration

The injected CO₂ volume in the geological formation will travel below the sealing cap by buoyancy forces due to the density difference between water and CO₂. One concern is to have CO₂ stored safely, and the mobile CO₂ is at risk of leaking through any imperfections in the sealing layers and abandoned wells. Molecular diffusion occurs at the interface of water and CO₂ and this mass transfer from the CO₂ plume into water increases the water density. Transition of CO₂ from mobile phase into water with dissolved CO₂ is helping the safe storage of CO₂: the heavier water with dissolved CO₂ has the tendency of moving downward. Time scale for this convective mixing is of the order of several hundreds years (Table 1.3). Yet, this is not the end and the dissolved CO₂ can react with the porous medium ending up in a solid phase and it can be stored permanently in a process called mineral trapping. This is an extremely slow process and it can take thousands of years [37].

Mixing of CO₂ and water in the long-term happens through phases with various time scales and physical phenomena. Diffusion of CO₂ in water continues and layer of water with dissolved CO₂ builds up below the CO₂ plume until it forms heavy parts convecting in the form of unstable fingers, as shown in Figure 1.24b.

The onset time for the convective mixing is important in terms of storage safety. This time depends on the Rayleigh number in the medium:

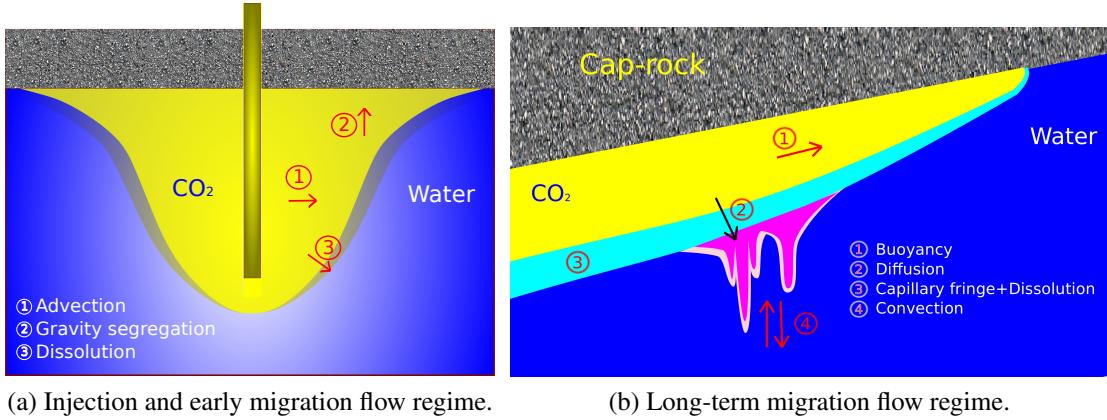
$$Ra = \frac{Kg\Delta\rho H}{D_c\phi\mu} \quad (1.36)$$

Here, $\Delta\rho$ is the density difference between the water and CO₂ phases and D_c is the diffusion coefficient of CO₂ into water phase. The higher density sitting on top of lower density makes an unstable system and the medium must have a minimum Rayleigh number to have a growing instability for a small perturbation in the medium. Heterogeneities in the medium can initiate perturbations, reducing the instability onset time [24, 39]. Therefore, heterogeneity is an important factor that must be considered when we are choosing an aquifer for CO₂ storage.

Capillary fringe in the plume can enhance the onset of the convective mixing. It can speed up the process up to five times [26].

The flow equations for convective mixing are a set of mass and momentum balance for component $c = \{\text{Water}, \text{CO}_2\}$ within phase $\alpha = \{\text{Wetting}, \text{Non-Wetting}\}$:

$$\frac{\partial}{\partial t} \sum_{\alpha} \phi S_{\alpha} \rho_{\alpha} X_{\alpha}^c + \nabla \cdot \sum_{\alpha} \rho_{\alpha} X_{\alpha}^c v_{\alpha} = 0, \quad (1.37)$$

Figure 1.24: Flow regimes in geological CO₂ storage.

and

$$\nu_\alpha = -\frac{k_{r\alpha} K}{\mu_\alpha} [\nabla P_\alpha - \rho_\alpha g z] \quad (1.38)$$

where X_α^c is the mole fraction of component c in phase α [25].

1.8 Vertical averaging

In this section we discuss the common approach used in modeling the migration of CO₂ in large scale aquifers. This is a specific type of multi-scale modeling, at which we work with two models with two and three dimensions. A 2D model is extracted from the corresponding 3D model to reduce the computation costs and in some cases the accuracy can be enhanced by reducing the numerical diffusion as we will discuss here. This section is discussing the back-ground concept used in [60] reported in Appendix A, Chapter A.2. The vertical equilibrium has been used extensively in the MatMoRA cluster of projects (e.g., MatMoRA [74], MatMoRA-II [19], VAMP [18], IGEMS [77], Numerical CO₂ lab [17]). This technique is specially useful when investigating CO₂ injection in larger models, as discussed in Section 1.9.2.

Spatial gradients of flux and pressure appear in the the flow equations can be decomposed into lateral and vertical components. For a three dimensional problem with the set of unit vectors $\{\vec{i}, \vec{j}, \vec{k}\}$, the gradient operator can be written as the following:

$$\nabla = \nabla_\ell + \nabla_v, \quad (1.39)$$

where:

$$\nabla = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}, \quad (1.40)$$

$$\nabla_\ell = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j}, \quad (1.41)$$

$$\nabla_v = \frac{\partial}{\partial z} \vec{k}. \quad (1.42)$$

x , y , and z are the location space components in the three directions.

The geological formation tops are not necessarily oriented horizontally, but they are normally close to horizontal. In CO₂ storage problems the lateral scale is orders of magnitude larger than the vertical direction. Therefore, the variations in the vertical direction are relatively negligible and for a quantity of interest Ψ we have:

$$\int_A \nabla \Psi d\tau \gg \int_{\zeta} \nabla \Psi dz, \quad (1.43)$$

where A is an arbitrary horizontal surface in the model and ζ is the vertical interval of the domain. Hence, with large lateral scales we can consider average values across the vertical direction for parameters and variables in the flow equations. This reduces the problem from 3D to 2D. In this case we need to relate the parameters in the two dimensional and three dimensional problems. If this relation is not extremely non-linear the flow solutions can be obtained considerably faster on a two dimensional problem.

In a distance from the injection point, vertical equilibrium assumption is applicable. The density difference between water and CO₂ results in a rapid segregation of the two phases. A layer of CO₂ sits on top of water table and in the presence of considerable capillary forces a capillary transition zone forms between water and CO₂ zones (Figure 1.25).

The vertical equilibrium assumption facilitates the flow equation averaging across the vertical direction ζ . The phase distribution can be calculated from the volume calculations and capillary inverse function for the transition zone as mentioned in 1.6. Phase pressure variation versus depth follows the hydrostatic gravitational gradient for each phase.

The two dimensional version of flow equation can be written in the following form, assuming incompressible fluids [13, 60]:

$$\tilde{\phi} \frac{\partial \tilde{S}_{\alpha}}{\partial t} - \nabla_{\ell} \cdot \tilde{v}_{\alpha} = \tilde{\eta}_{\alpha}, \quad (1.44)$$

which follows by two dimensional Darcy equation:

$$\tilde{v}_{\alpha} = -\tilde{K} \tilde{\lambda}_{\alpha} (\nabla_{\ell} \tilde{P}_{\alpha} - \tilde{\rho}_{\alpha} \tilde{g}). \quad (1.45)$$

Herein, tilde sign denotes the two dimensional parameter or variable that is the vertically averaged of corresponding parameters and variables in the three dimensional problem. The averaging over vertical direction care defined in the following:

$$\tilde{\phi} = \frac{1}{H} \int_0^H \phi dz, \quad (1.46)$$

$$\tilde{K} = \frac{1}{H} \int_0^H K dz, \quad (1.47)$$

$$\tilde{\lambda}_{\alpha} = \frac{1}{H} K^{-1} \int_0^H K \lambda_{\alpha} dz. \quad (1.48)$$

Note that $\tilde{\lambda}_{\alpha}$ is a tensor, because it is defined such that the 2D Darcy equation is consistent with the 3D Darcy equation. The variables in the 2D equations are defined in the following:

$$\tilde{v}_{\alpha} = \frac{1}{H} \int_0^H v_{\alpha} dz, \quad (1.49)$$

$$\tilde{s}_{\alpha} = \frac{1}{\tilde{\phi} H} \int_0^H \phi s_{\alpha} dz, \quad (1.50)$$

$$\tilde{P}_{\alpha} = p_{\alpha}(z_D), \quad (1.51)$$

$$\tilde{\eta}_{\alpha} = \frac{1}{H} \int_0^H \eta_{\alpha} dz. \quad (1.52)$$

The hydrostatic equilibrium results in a constant phase pressure gradient over the vertical direction. Therefor, the phase pressure at any datum depth z_D in the medium can be used into the 2D flow equations.

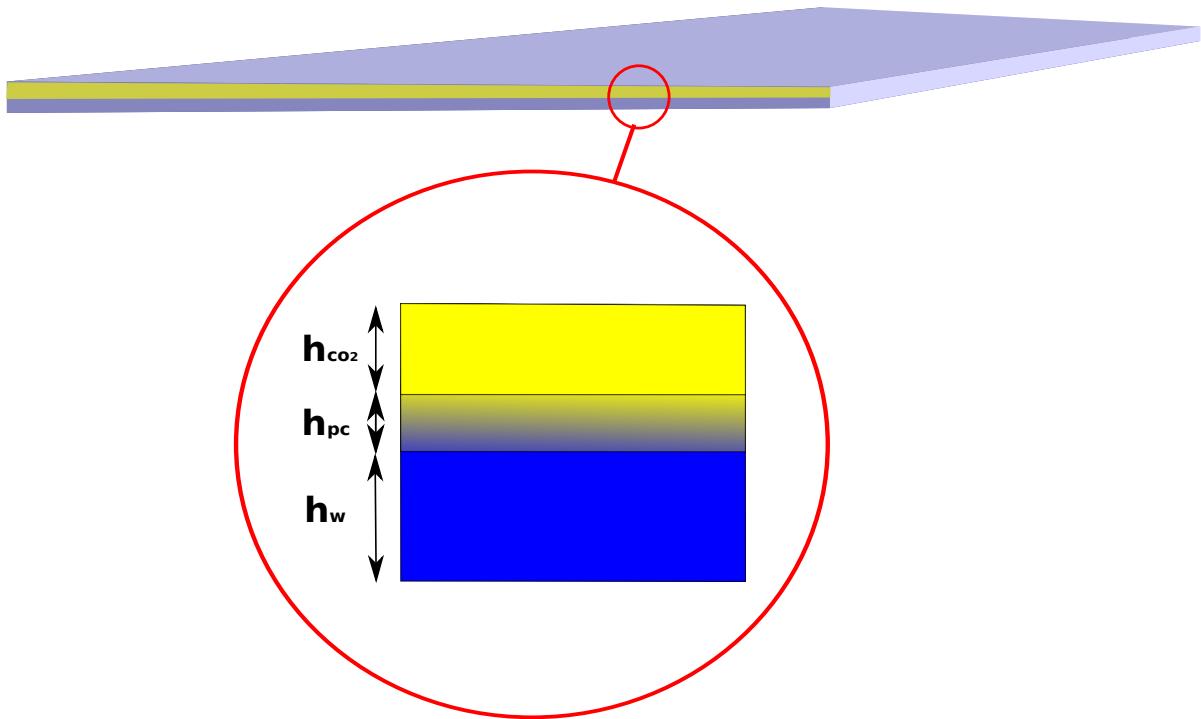


Figure 1.25: Hydrostatic equilibrium condition results in three zones: the CO_2 zone with thickness h_{co_2} , the water table with thickness h_w , and the capillary transition zone h_{pc} .

Accuracy of vertical averaging approach within CO_2 storage context has been examined in many works in the literature [16, 35, 50, 60]. It has been shown that vertical averaging is applicable to large scale CO_2 studies. The numerical aspects and speed up of the method compared to three dimensional problem are studied in [35, 51], where a second order speedup is reported. Lie et al. [50] have shown the influence of vertical discretization in the three dimensional models which can fail in low resolutions to capture the thin layer of CO_2 plume migrating beneath the top sealing surface in the formation. The vertical averaging approach can handle this issue by considering the water and CO_2 interface in calculating the vertically averaged parameters in the model. A comparison between three and two dimensional models has been done on various aspects in [60].

The influence of topography in modeling the CO_2 migration and estimating the storage fate has been shown in [35, 77] by examining different geometries and heterogeneities in the domain. These conclusions suggest that the significance of the model top surface is more than the small order flow in the model vertical thickness.

The studies reported in this thesis concern the flow near injector and early times of plume development. Therefore vertical equilibrium would benefit larger scales with big portion of model in vertical equilibrium. We do our flow modeling by including the three dimensions and for the quantitative uncertainty analysis we employ a polynomial approximation (Section 1.10).

1.9 Flow modeling

We use a standard porous media simulator [72] to solve the flow equations in the medium. The simulator is based on finite volume method and the following assumptions are taken:

- Two compressible phases are considered in the medium: water and super critical CO_2 .
- No mass exchange occurs between the two phases.
- No heat exchange is considered.

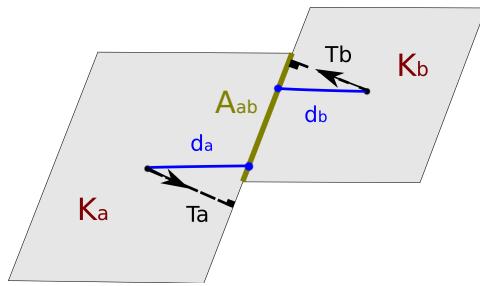


Figure 1.26: Transmissibility calculation for two cells a and b.

1.9.1 Numerical scheme

The simulator uses a standard two-point finite difference scheme to solve Equation 1.21 on a corner-point grid. The Darcy equation for two-phases can be written in a difference. This makes the flow equation into cell a from the neighboring cell b :

$$F_{ab\alpha} = T_{ab} M_{a\alpha} \Delta \psi_\alpha. \quad (1.53)$$

Here, T_{ab} is the transmissibility of the medium between the two cells. $M_{a\alpha}$ is the mobility of phase α that is taken upstream of the flow from cell a and $\Delta \psi_\alpha$ is the potential term difference between two cell centers.

Transmissibility for two neighboring cells (i.e., sharing a face area, see Figure 1.26) is calculated by harmonic average of transmissibilities from the center of each cell to the center of the common face between the two cells:

$$T_{ab} = \frac{1}{\frac{1}{T_a} + \frac{1}{T_b}}. \quad (1.54)$$

Each half transmissibility T_a or T_b is calculated by an inner product between the permeability of the cell K_a , the mutual area A_{ab} between cells, and the distance from cell center to the mutual face center d_a :

$$T_a = K_a \cdot d_a \cdot A_{ab}. \quad (1.55)$$

The mobility term in Equation 1.53 is defined as follows:

$$M_{a\alpha} = \frac{k_{r\alpha}}{B_\alpha \mu_\alpha}, \quad (1.56)$$

where $k_{r\alpha}$ is the relative permeability of phase α , μ_α is the viscosity of phase α , and B_α is the formation volume factor of phase α , which is defined as :

$$B_\alpha = \frac{\text{Volume at surface condition}}{\text{Volume at formation condition}} = \frac{V_{s\alpha}}{V_{r\alpha}}. \quad (1.57)$$

This definition is connected to compressibility of the fluid, i.e., to changes in volume at surface and at the geological formation condition, but it is defined in this way in the simulator to honor cases where a fluid like oil loses its dissolved gas while being produced at surface pressure. Since we assume no mass exchange between phases in our study, here the formation volume factor works like compressibility of the fluid. Formation volume factor is a function of pressure. Slight compressibility is considered for phases in our study, and phase density is defined as a function of pressure:

$$\rho_\alpha(P) = \frac{\rho_{0\alpha}}{B_\alpha(P)}. \quad (1.58)$$

Here, $\rho_{0\alpha}$ is the density of phase α at surface condition.

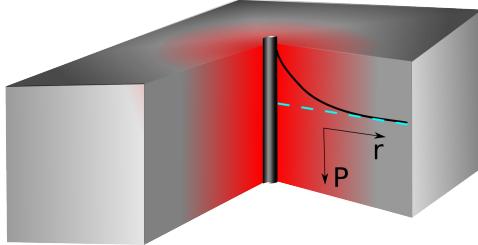


Figure 1.27: Well modeling inside a simulation cell.

Wells are defined as sources or sinks in Equation 1.21. In reality, wells are a void space drilled in the porous medium and the flow into the well-bore and up to the surface for production wells (and vice-versa for injectors) goes through a pressure change that must be modeled separately from the porous medium.

Figure 1.27 shows a schematic pressure distribution around the injector. The well radius is much smaller than the simulation cell containing the well and the pressure in the bottom-hole is different than the cell pressure. The well bottom-hole pressure can be related to the cell pressure containing the well by a separate approximation that can be coupled with the flow equations in the grid model. Flow equation for phase α between the cell center and the well for an injector is written as follows:

$$\eta_\alpha = T_w \cdot M_\alpha \cdot [P_w - P_i]. \quad (1.59)$$

Here, η_α is the volumetric injection rate of phase α , P_w is the injector bottom-hole pressure, P_i is the cell pressure, T_w is the transmissibility between the cell and the injection well-bore, and M_α is the mobility of injection flow into the cell.

A region can be assumed by a radius r_e at which the pressure is equal to the cell pressure. Approximating the flow near the well-bore by Equation 1.12, the transmissibility for this region can be found from the analytical solution to Equation 1.12:

$$T_w = \frac{K \cdot h}{\ln(\frac{r_e}{r_w})}, \quad (1.60)$$

where h is the medium thickness, K is the medium rock permeability, and r_w is the well radius. Here, we assume that the well is completed and connected in the entire thickness h of the cell and there is no skin effect in the well. The Equation 1.60 can be extended to model wells with partial completions and skins. The effective radius r_e in Equation 1.60 is estimated from Peaceman formula and can be related to the cell geometry:

$$r_e = 0.28 \frac{\left[\delta_x^2 (\frac{K_y}{K_x})^{\frac{1}{2}} + \delta_y^2 (\frac{K_x}{K_y})^{\frac{1}{2}} \right]^{\frac{1}{2}}}{\left(\frac{K_y}{K_x} \right)^{\frac{1}{4}} + \left(\frac{K_x}{K_y} \right)^{\frac{1}{4}}}. \quad (1.61)$$

Here, K_x and K_y are the permeabilities in x and y directions and δ_x and δ_y are the cell sizes in these directions. This equation assumes a vertical well and a diagonal permeability tensor. It can be modified for more general cases.

1.9.2 Flow scenarios

All of the SAIGUP realizations have dimensions of 3 km \times 9 km \times 80 m. The model spatial scales capture the typical geological features in a shallow-marine system, such as shore-line shape and aggradation angle variations. Various scales of heterogeneity can considerably impact the flow behavior. The lateral extent of the model is smaller than the scales used for CO₂ storage studies. In some storage sites, the lateral extent that the CO₂ travels can go to few hundreds of kilometers. This makes our study limited in the spatial domain around the injector. For the same reason, in the temporal scale, we are more

focused on injection and early migration time. We examine a number of injection scenarios to study the spatial distribution of CO₂ in the medium during injection and early migration periods.

Pressure study is essential for injection operations. A detailed pressure study requires larger scales than what is used here. We choose open boundaries for the model to compensate for the actual large extents of a typical storage location. The choice of open boundary is not valid in domains that are bounded by structural seals. In fact, for the close and semi-close domains the pressure is a main control on the storage capacity along with other parameters. The results of our pressure study can change significantly by choosing different boundary conditions.

We represent the boundary by large pore volumes on the outer closed cells and this makes the pressure to relax earlier than it does in a large domain. Even with such artifact, the effect of heterogeneities is clearly seen in a considerable fraction of cases with an extreme pressure build-up. We investigate the operational concerns related to pressure build-up for a typical injection scenario. Our pressure study can be used for devising mitigation plans by defining operational constraints for injectors. We perform an extensive probabilistic analysis on the CO₂ pressure behavior in the medium that can be applied in a further study with specific concerns about the pressure analysis.

We consider the injection of 20% of the total pore volume of the model(excluding the large volumes at the boundaries), which amounts to 40 MM m³. This volume is injected into all realizations in three different scenarios. In the first scenario, the injection is forced to finish in 30 years and the pressure in the system is allowed to rise unlimitedly. Linear relative permeability functions are considered in this scenario. The purpose of the first scenario is to examine the flow distribution in the medium influenced by geological heterogeneity. Linear assumption for relative permeabilities is taken to speed up the flow within the medium. We have used quadratic relative permeability function in another scenario. This scenario has shown a considerable increase in the pressure responses for many cases during CO₂ injection into the aquifer. This is mainly due to lower CO₂ mobility at low saturations compared to the linear relative permeability. Albeit, the CO₂ moving under a cap-rock will effectively have a linear relative permeability.

The third injection scenario is similar to the first studied scenario, except that the injector is controlled by pressure rather than volumetric rate. Thus, injection time is variable depending on the injectivity of the medium.

One injector is considered in the study. With one injector, it is easier to study the flow behavior and the plume development within the medium. The injector is located in the flank and to increase the sweep efficiency for the up-moving CO₂ plume, the injector is completed only in the lower part of the aquifer. The injector location and the completed layers are fixed for all of the realizations. The studies here aim to identify the influence of uncertainty on injectivity and fixing a place for injection helps in achieving this goal. As mentioned earlier, injectivity is a big player in the success of the operations. Uncertainty might be less in the near well-bore region than in the larger scale in the domain, but yet requires costly operation data aquisition. Fixing the location of the well serves specifying the probability of having a feasible injectivity in different heterogeneities.

There are few locations of distorted geometries in the faulted realizations that may be considered as structural traps for the injected CO₂. The topography in the SAIGUP realizations is simple and does not cover the variational space to be used in a sensitivity analysis. The slight inclination in the structural geometry of the medium, from the flank up to the crest, leads the injected CO₂ to accumulate in the crest and below the faulted side of the aquifer. The structural trapping due to variational morphology is studied in IGEMS, which is a sister project to MatMora (for example, see [77]).

In a homogeneous medium, we expect the CO₂ to accumulate under the cap-rock. A small fraction of the injected CO₂ will escape through the open boundary near the injection well and the rest of it will stay within the medium in two forms that we refer to as mobile and residual volumes. As the CO₂ moves through the rock, part of it stays in the smaller pores by capillary trapping process and can not be discharged by brine. The other parts move through the larger pores and can be displaced by water in an imbibition process. This volume is called mobile. As we are interested in storing the CO₂ permanently and safely, increasing the trapped volume is in line with the objective of minimizing the leakage risk

Figure 1.28: Top, bottom, and upper side boundaries are closed and the rest are open to the flow.

and maximizing the storage capacity. Likewise, the more mobile volume of CO₂ exists in the medium, the more will be the risk of leakage.

Defining the boundary condition of the aquifer of interest can influence the flow behavior in the system. Computational costs make it more feasible to model the flow locally and in the part of the aquifer that is going through more pronounced changes in flow behavior. Therefore, we can choose the boundaries of the model inside the aquifer in a volume that is containing the injection wells and the areas effected by them. Hydrostatic open boundary condition is a choice for the system boundaries to include the aquifer parts that fall outside the boundaries (Fig. 1.29).

The underground network of aquifer systems can be connected via geological channeling and conductive features. Some aquifers might be active and connected to the surface and expand in volume by variations in water influx due to seasonal rains. This can impose an external force on the system boundaries considered in a storage problem. Fig. 1.29 shows the water influx through the boundaries of the system due to external aquifer activities. We consider the external support by imposing a higher pressure than the hydrostatic pressure on the boundary of the model.

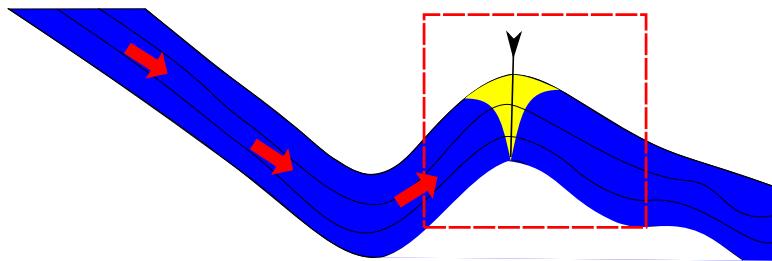


Figure 1.29: External pressure drive.

1.9.3 Flow responses

The primary unknowns in the flow model are the CO₂ pressure and the saturation distribution at different times. From the simulation output, we can derive quantities that address the feasibility of CO₂ injection, these quantities include a number of flow responses related to the CO₂ injection and migration problems. Each of these responses are directly or indirectly a measure of success for the operation within a specific realization. In the following, we give a brief description of each of them:

Boundary fluxes: The flux out of open boundaries is a measure of sweep efficiency for the CO₂ plume. Channeling can lead to early CO₂ breakthrough at boundaries and we prefer cases with less out-fluxes through open boundaries. The out-flux through the open boundary that is closer to the injector is a potential loss for the injected volume. After the injection stops, some of the CO₂ that has left the domain comes again due to gravity segregation effect.

Total mobile and residual CO₂ volume: If the CO₂ saturation is below the critical value, it will be immobile in the bulk flow, although not in the molecular sense. Less mobile CO₂ means less risk of leakage and more residual volumes (with saturations less than the critical) resulting from a more efficient volume sweep as preferable. We use critical saturation of 0.2 for both water and CO₂. During injection time the flow process is mainly drainage but after injection imbibition also happens and increases the residual trapped CO₂.

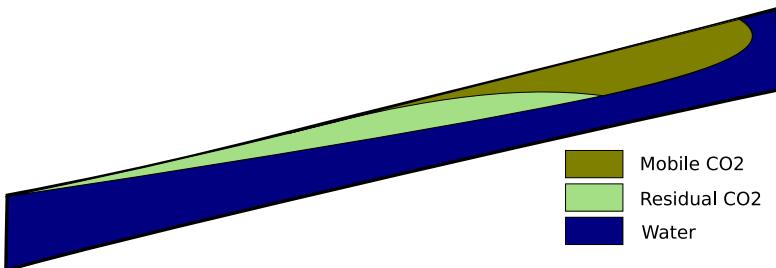


Figure 1.30: Mobile and residual CO₂ volume.

Total number of CO₂ plumes and largest plume: To estimate the risk of leakage from the cap-rock, we assume that all mobile CO₂ connected to a leakage point will escape out of the reservoir. Hence, it is preferable if the total mobile CO₂ volume is split into smaller plumes rather than forming a big mobile plume. We looked at the largest plume size, the number of plumes, and other statistical parameters.

Average aquifer pressure: Average aquifer pressure is one of the most important responses to be considered. The pressure response in general shows a sharp jump at the start of injection and a declining trend during the injection and plume migration.

As soon as the injection starts, a pulse of pressure goes through the medium, introducing a pressure buildup in the aquifer. When the pressure wave reaches the open boundary, the aquifer pressure starts declining to a level maintained by the injector. When the injector stops operation, the pressure support will be removed and the pressure drops and declines until it reaches equilibrium.

Leakage risk: During injection operation the foremost important issue is the aquifer pressure which as discussed earlier may lead to fractures in the cap-rock. On the other hand, the cap-rock break depends on lithology and sealing thickness and differs from point to point. Some weaker locations can be the most probable to break and start leaking if any mobile CO₂ exists there.

An uncertainty assessment process consisting of geo-mechanical modeling of aquifer combined with flow modeling can cost a large amount of computations. To avoid expensive computations, the idea in this thesis is to model the possible breakings on the cap-rock (considering the stress stream in the medium) by introducing a probability measure on the cap-rock. This measure can be used to evaluate different cases for their risk of leakage, considering the CO₂ distribution under the cap-rock.

Here, we define the probability of leakage as a measure on the cap-rock that assigns a value to each point of the cap-rock, modeling the relative weakness of the cap-rock and the medium at that point. If for example both the cap-rock and the aquifer are continuous homogeneous layers with constant thickness, then the point of cap-rock that sits on the highest point of the injection slice can be the most probable place for leakage in case of dramatic pressure increase in the well: the stress stream is more in the injection slice and the CO₂ accumulation occurs on the topmost part of the mentioned slice. Then one may consider a 2D-Gaussian probability distribution on the cap-rock, centered above the injection slice.

If the medium is heterogeneous or tilted, the injected CO₂ may be distributed in different number and sizes of plumes below the cap-rock. Therefore, in addition to the probability of breaking for each point of cap-rock, one must consider the CO₂ connected volume that is attached to that point.

Since we have neither the cap-rock model nor the geo-mechanical properties of the SAIGUP models, we use a simple 2D-Gaussian leakage probability distribution centered at a point on the crest which is in the same slice as the injection point (Fig. 1.31). We calculate the probability of each cell in the top layer and using the simulation results for the case, we weight it by the CO₂ saturation of that cell and the plume size that the cell is attached to. Summing up the values of the topmost cells, we assign a single number to the case which we call leakage risk of the case. One may weight the case risk value with the average pressure in the system, such that higher pressure gives a bigger weight.

Results are discussed by comparing all cases in plots. However, the conclusions are made based on detailed flow study in some picked cases. For example, Figures 1.32 to 1.36 show the rock properties

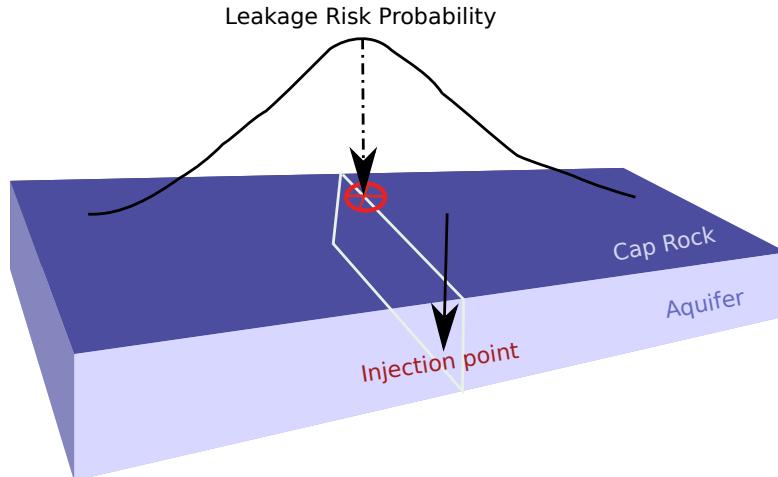


Figure 1.31: We use a 2D Gaussian distribution for leakage probability on the cap-rock.

Table 1.4: Geological heterogeneities for two selected cases.

Case	Fault	Lobosity	Barrier	Aggradation Angle	Progradational Direction
A	unfaulted	medium	medium	medium	down-dip
B	unfaulted	high	medium	low	up-dip

and CO₂ distribution in the domain at end of injection and end of simulation in two different cases. The heterogeneity description of the selected cases, called A and B, is given in Table 1.4.

CO₂ distribution in Figures 1.34 and 1.35 show that heterogeneity in case B has enhanced the lateral flow compared to case A. Direction of the flow can be seen in Figure 1.36. It is clear that heterogeneity can influence the imbibition and drainage process during and after injection. This, in turn, impacts the residual trapping process.

Figure 1.37 demonstrates the plume evolution during simulation in two different cases, i.e., case A and B. The mobile CO₂ is plotted here and it shows that the injected CO₂ moves more in the vertical direction in case A, ending up in a big mass accumulated under the cap-rock. On the contrary, the heterogeneity in case B enhances the lateral movement of the plume, resulting in a laterally spread plume within the medium.

Figure 1.38 shows the development of pressure in the same cases. We see that injection in case B causes a dramatic pressure build-up due to the poor vertical transmissibility. The pressure build-up spreads out within the medium in case A, while it is trapped in the injection layers for case B.

One way to report the described responses and their relations to the uncertain parameters in one graph is to use scatter plots. Each case will then be represented by a marker sign with attributes dedicated to the set of geological parameter levels used in that case. Figure 1.39 shows some of the codes used in the study. That will be used later in the thesis in the papers reporting from our study.

1.9.4 ECLIPSE input file

In this section, important parts of the ECLIPSE input files that are used in modeling the flow are given. We will go through different sections of the ECLIPSE input file. It is assumed that the reader is familiar with the syntax and terminology used in the ECLIPSE simulation. See [72] for more information about ECLIPSE keywords. We use the version 2009 of ECLIPSE-100 black-oil module.

Several flow scenarios were examined before concluding in a few number of scenarios to be used in the study. Two main scenarios are considered that differ mainly in defining the well operational specifications. We will explain more about these cases in the SCHEDULE section. Only the important parts of the input file are given such that it is possible to reproduce the runs.

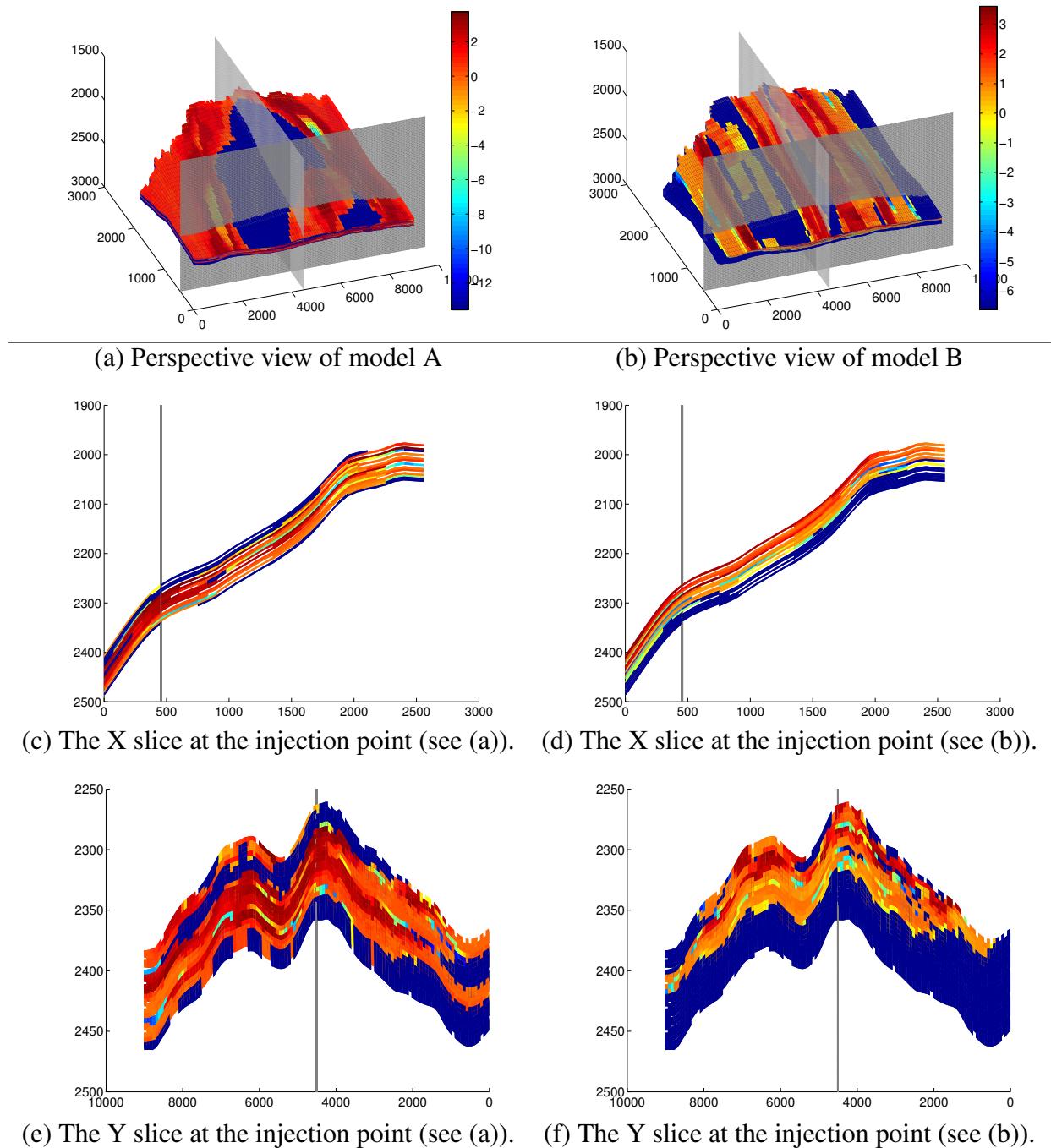


Figure 1.32: Transmissibility in the vertical direction for two selected cases. The left plots correspond to case A in Table 1.4, and the right plots belong to case B. Colors are in log scale and the scale in Figures (a) and (b) are powers of ten in $\text{cP} \cdot \text{m}^3/\text{day}/\text{bar}$ units.

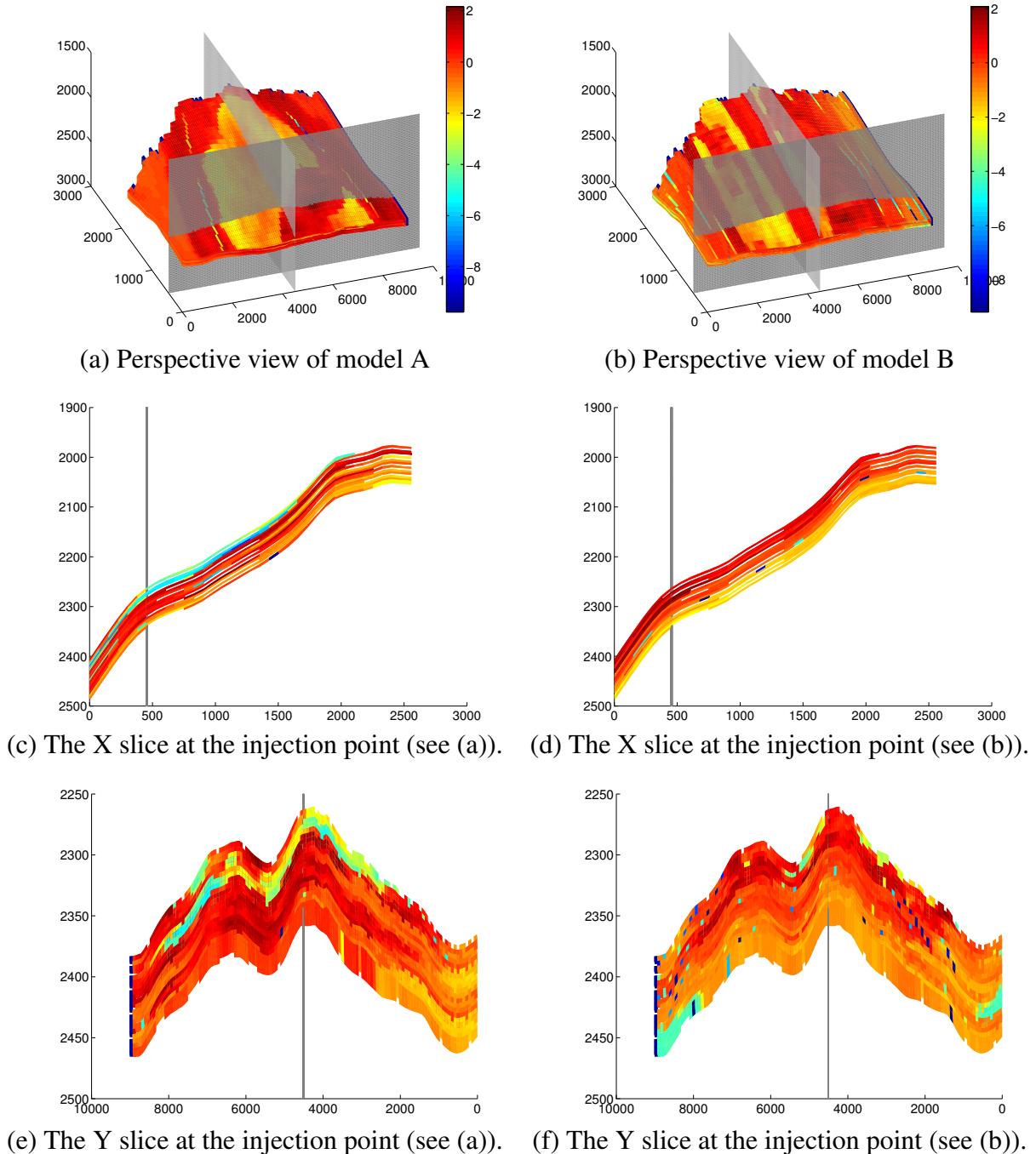


Figure 1.33: Transmissibility in the lateral direction for two selected cases. The left plots correspond to case A in Table 1.4, and the right plots belong to case B. Colors are in log scale and the scale in Figures (a) and (b) are powers of ten in $\text{cP} \cdot \text{m}^3/\text{day}/\text{bar}$ units.

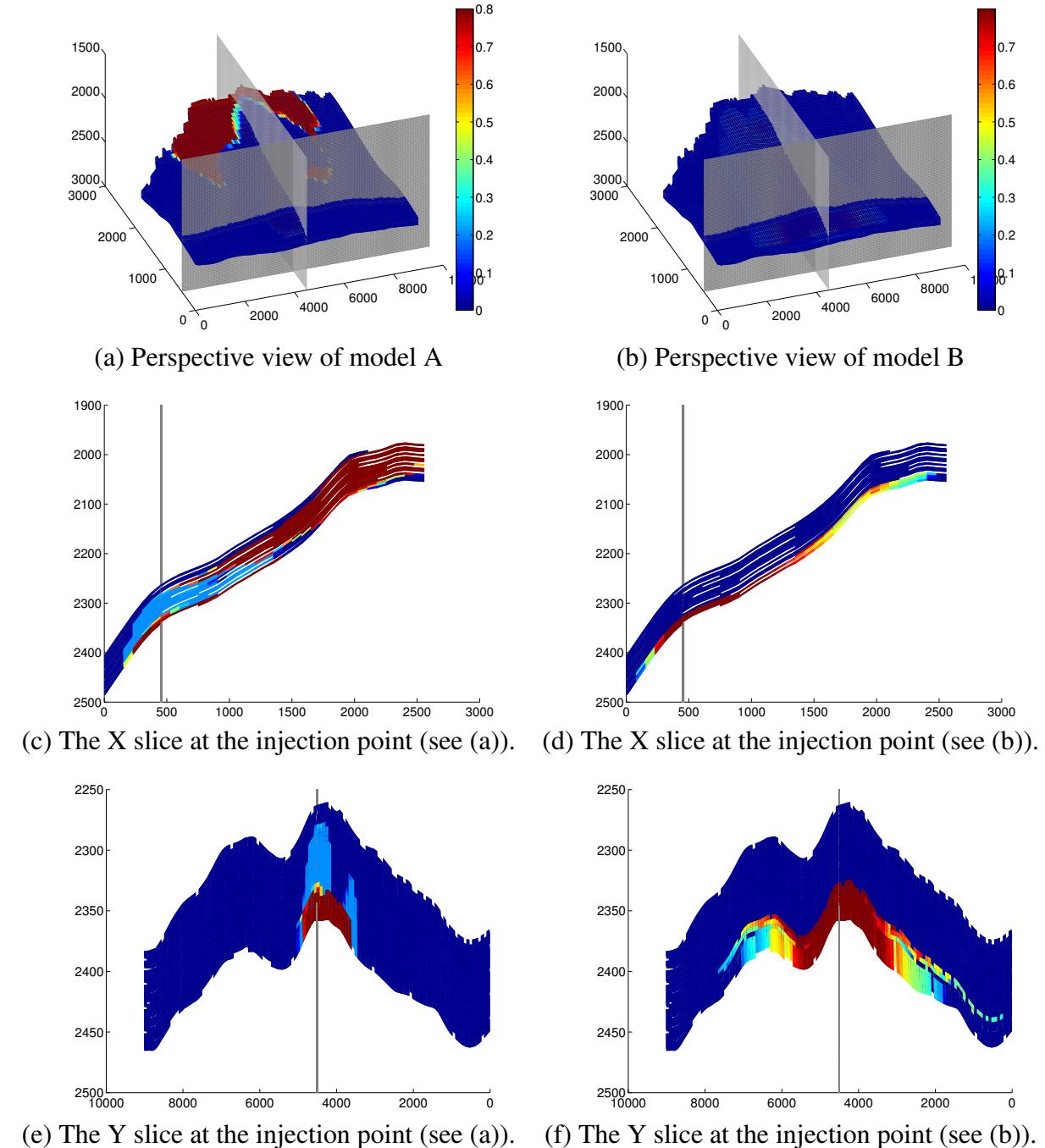


Figure 1.34: CO₂ distribution at the end of injection for two selected cases. The left plots correspond to case A in Table 1.4, and the right plots belong to case B.

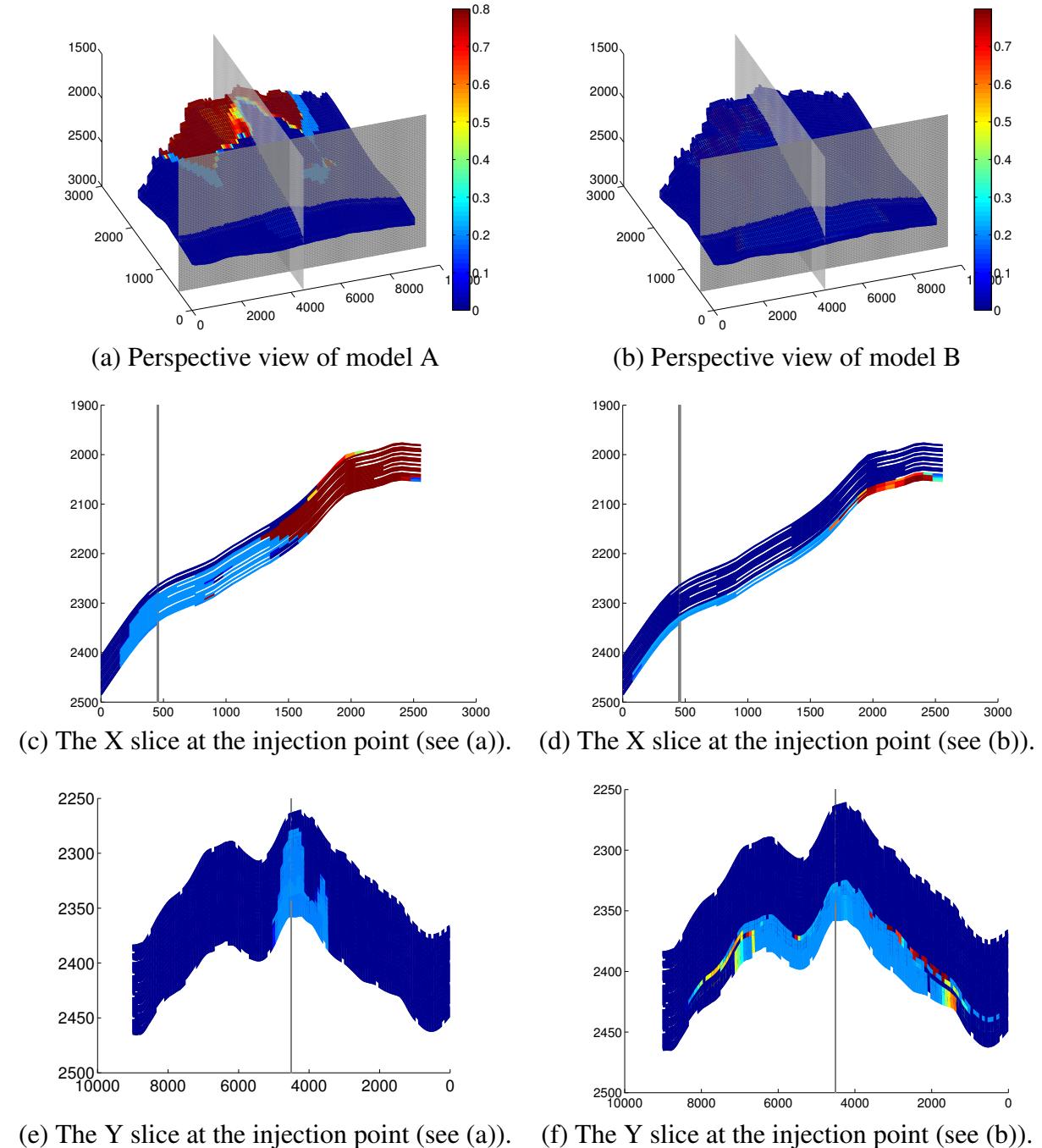


Figure 1.35: CO₂ distribution at the end of simulation for two selected cases. The left plots correspond to case A in Table 1.4, and the right plots belong to case B.

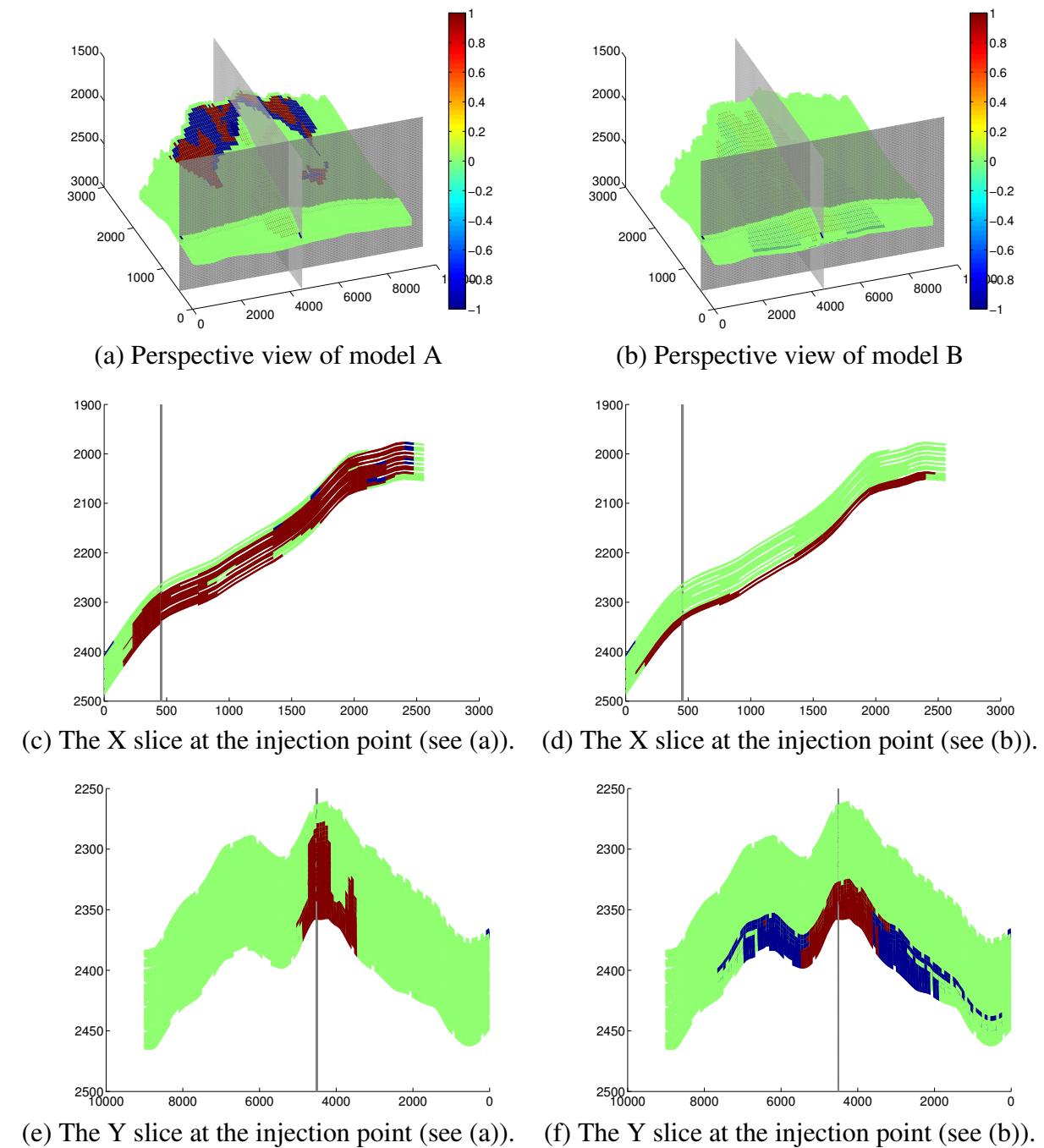


Figure 1.36: Flow sign in the Y direction at the end of injection for two selected cases. The left plots correspond to case A in Table 1.4, and the right plots belong to case B. Blue color corresponds to down-dip direction, red to up-dip direction, and green represents the stagnant fluid.

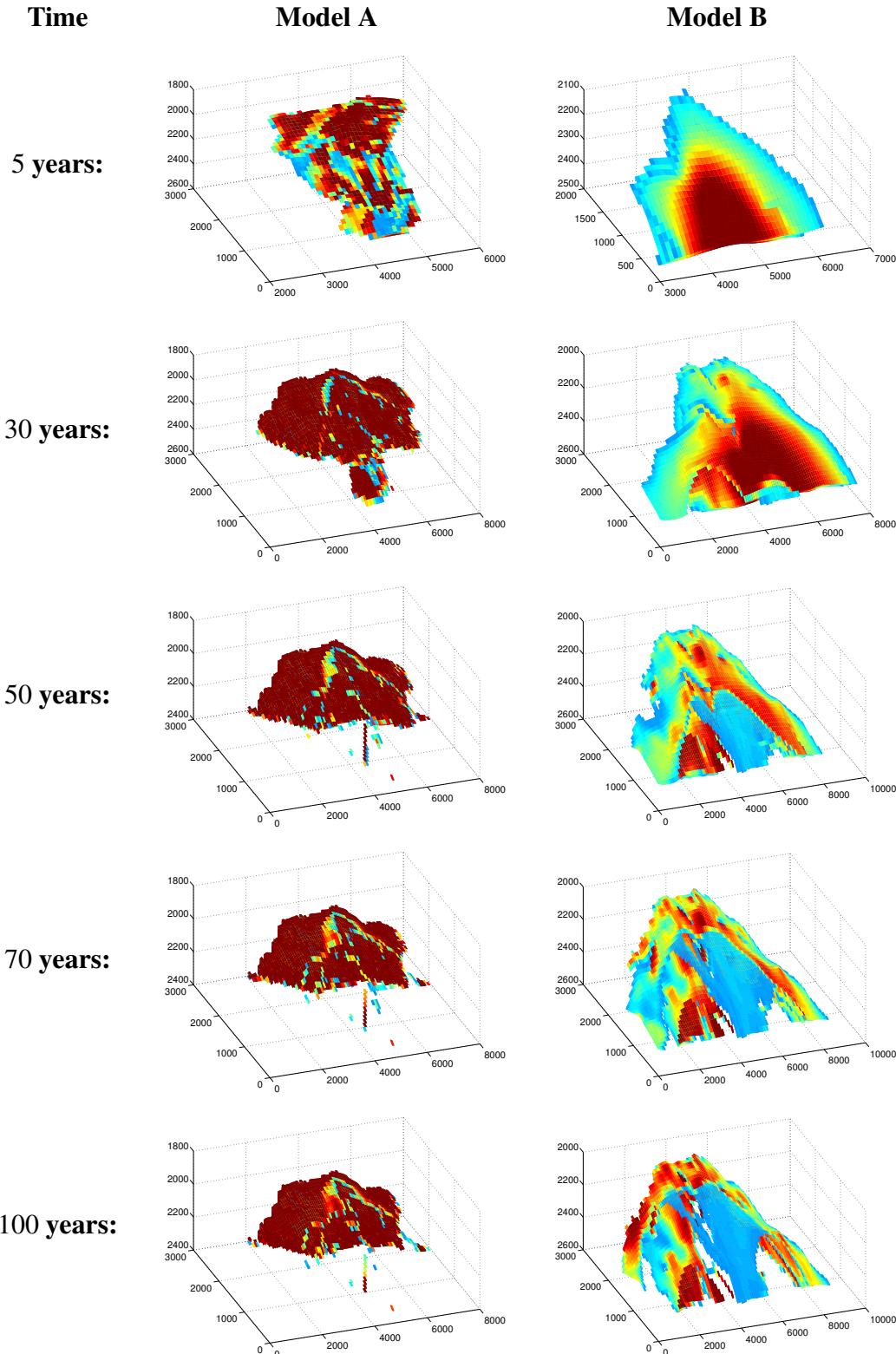


Figure 1.37: Mobile CO₂ distribution at different times for two selected cases. Cases A and B are described in Table 1.4. Compare with Figures 1.32 and 1.33 for transmissibility values in different directions.

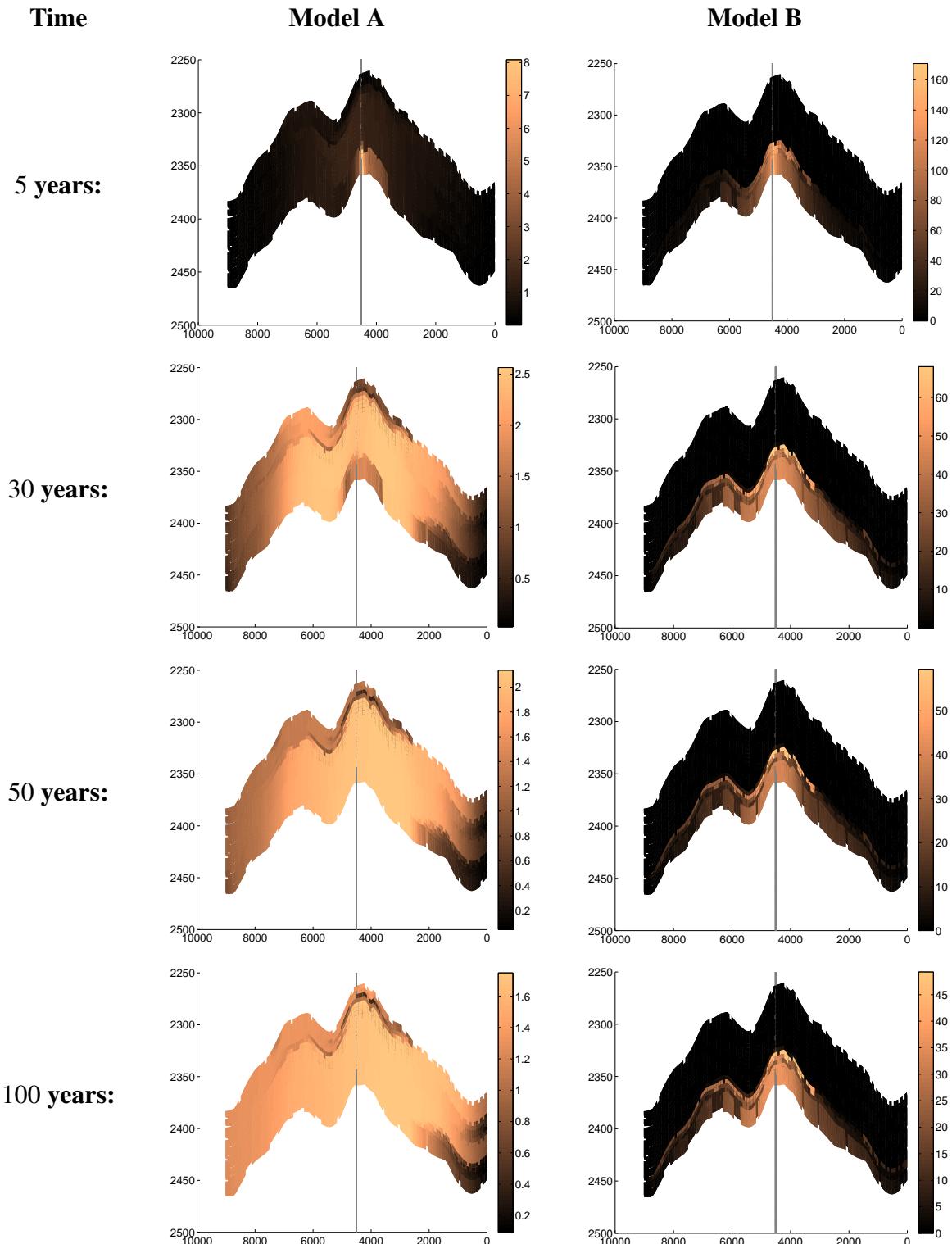


Figure 1.38: Pressure development during injection and early migration. Pressure differences from hydrostatic pressure are shown for two selected cases. Values are in bar. Cases A and B are described in Table 1.4. Compare with Figures 1.32 and 1.33 for transmissibility values in different directions.

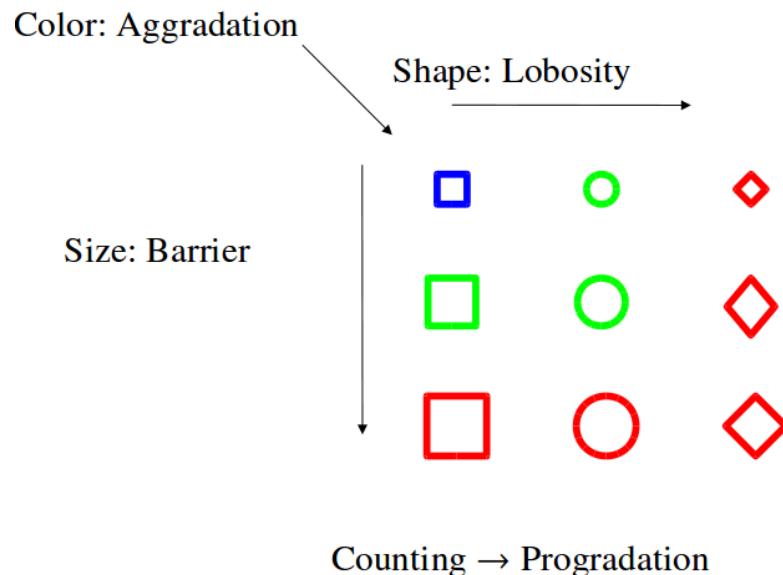


Figure 1.39: Marker codes used to plot the simulation results of all cases together.

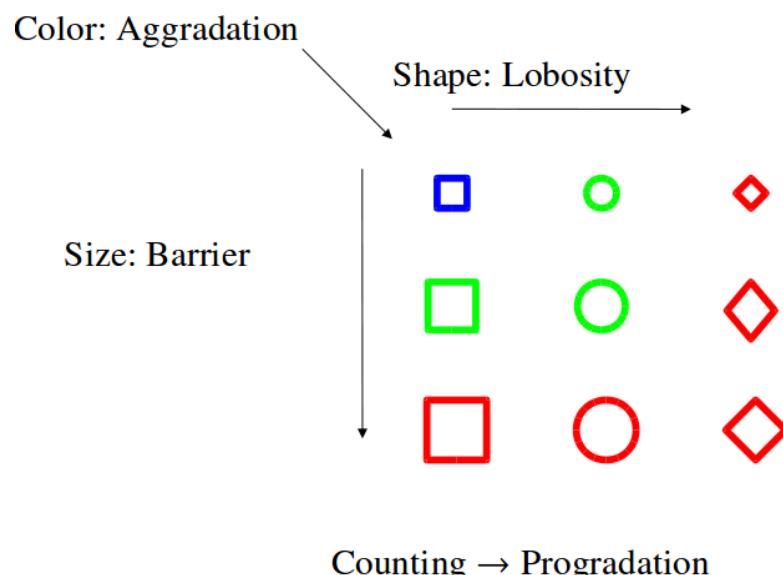


Figure 1.40: Marker codes used to plot the simulation results of all cases together.

The model starts by specifying the general simulation settings: grid dimensions, phases involved in the study, simulation start date, and so on. We consider no mass exchange between water and CO₂. Therefore, it is enough to represent the flow by oil-water system where oil represents the CO₂ phase. We use CO₂ properties for oil:

```
RUNSPEC
DIMENS —Grid dimensions
40 120 20 /
—Two-phase flow problem with no mass exchange
WATER
OIL —CO2 is treated as OIL and CO2 properties used for it.
METRIC —Metric unit system
START —Simulation start date
1 'JAN' 2000 /
```

Then, the grid information are given for each realization. The set of keywords generated in the SAIGUP study are included in the input file. Each included file contains data for a specific keyword. Each file is named after the keyword name it includes with the extension 'INC'. For example, 'PORO.INC' contains the PORO keyword, which contains the porosity value for each cell in the model. Only two INCLUDE keywords are printed here to improve the readability of the code. In the second INCLUDE we provide the pore volume multipliers for the cells on the boundary of the model. This is used to represent hydrostatic open boundaries for three sides of the model.

```
GRID
INCLUDE — Rock properties are included for each realization
'COORD. INC' / 'ZCORN. INC' / 'ACTNUM. INC' / 'NTG. INC' / 'PORO. INC' /
'PERMX. INC' / 'PERMY. INC' / 'PERMZ. INC' /
'MULTX. INC' / 'MULTY. INC' / 'MULTZ. INC' /
INCLUDE —Pore volume multipliers for the cells in the boundary
'MULTPV. INC' / —1e6 and 1e3 values are used in different parts of the boundary.
```

In the EDIT section, we provide the fault transmissibility multipliers for each faulted case.

```
EDIT
INCLUDE
'EDITNNC. INC' /
```

In the PROPS section the relative permeability data are provided in two sets of tables with two different endpoints for CO₂ to consider the hysteresis effect. In the SOLUTION section, we use the first table to initialize the model with 100% water everywhere, and in the SCHEDULE section we use the second table to consider the residual CO₂ in a drainage process followed by an imbibition. In the presented scenario, linear relative permeabilities are used. Another scenario contains quadratic relative permeabilities that are given to the model similarly. Zero capillary pressure is used here. PVT data for CO₂(modeled by OIL) and water phases, fluid viscosities, densities, and the rock-fluid compressibility models are provided here.

```
PROPS
SWFN
— Sw Krw Pcow
0.2 0.0 0
1 1.0 0
/ —First table is used for time step zero
0.2 0.0 0
0.8 1.0 0
/ —Second table is used for the simulation
SOF2
— So Kro
0.000 0.0000
0.800 1.0000
/ —First table is used for time step zero
0.200 0.0000
0.800 1.0000
```

```
/ —Second table is used for the simulation
PVTW —Water PVT model
  200.0 1.0  3.03E-06  0.4  0.0 /
PVDO —CO2 PVT model
  0.0   1.1  0.04
  400.0 0.95 0.04
/
ROCK —rock-fluid compressibility model
  400.0  0.30E-06 /
DENSITY —Phase densities
  700   1033  0.044/
```

In the REGIONS section we define different domains in the model. We specify the main domain that excludes the cells considered to represent the open boundaries. This is later used in the calculations of flow responses. Also, the saturation table is assigned here to be used in the initialization of the model as explained earlier.

```
REGIONS
INCLUDE
'LRGNS.INC' /
SATNUM
96000*1/
```

The model is initialized here for the first time step by considering the hydrostatic equilibrium in the medium prior to CO₂ injection.

```
SOLUTION
— DATUMz Pi@DATUM WOC    Pe@WOC GOC Pe@GOC
EQUIL      2000     250    100      0     0     0     /
```

In the SUMMARY section we specify the output vectors to be used in our analysis.

```
SUMMARY
— FIELD DATA
FPR
FOIP
FWIP
— REGION DATA
ROIP
/
RWIP
/
RWSAT
/
RPR
/
— WELL DATA
WBHP
/
WOIR
/
VVIR
/
```

Finally and in the main part of the model, we define the simulation scenario by providing the injector completion specifications and injection plan for the well. Here we see the SCHEDULE section that is considered for fixed injection rate over 30 years, followed by 70 years of early migration.

```
SCHEDULE
SATNUM —The second saturation table is assigned here to consider the hysteresis effects.
96000*2/
WELSPECs —Well drilling information
```

```
'I' 'G' 6 60 1* 'OIL' /
/
COMPDAT —Well completion information
'I' 6 60 17 20 'OPEN' 0 .0 0.1/
/
WCONINJE —Well injection plan
'I' 'OIL' 'OPEN' 'RESV' 1* 3650.0 /
/
RPTRST
BASIC = 3 FREQ=1 FLOWS /
TSTEP
0.1 /
TSTEP
120*90 /
WCONINJE —Well is shut-in after 30 years
'I' 'OIL' 'SHUT' 'RESV' 1* 0.0 /
/
TSTEP
280*90 /
END
```

In the other scenario we control the well by pressure constraint and we continue the injection until the aimed total CO₂ volume is injected in the aquifer, or the simulation time reaches 100 years:

```
SCHEDULE
SATNUM —The second saturation table is assigned here to consider the hysteresis effects.
96000*2/
WELSPCS —Well drilling information
'I' 'G' 6 60 1* 'OIL' /
/
COMPDAT —Well completion information
'I' 6 60 17 20 'OPEN' 3 .0 0.1/
/
WCONINJE —The injector is set to inject conditioned by a pressure lower than 400 bar
'I' 'OIL' 'OPEN' 'RESV' 1* 3650.0 400/
/
RPTRST
BASIC = 3 FREQ=8 FLOWS /
ACTION —Stop the well as soon as the total injected volume is 40000000 m3
STPINJ FOIT > 40000000 /
WCONINJE
'I' 'OIL' 'SHUT' 'RESV' 1* 0.0 /
/
ENDACTIO
TSTEP
0.1 /
TSTEP —The simulation continues for a total 100 years
120*90 /
RPTRST
BASIC=3 FREQ=8/
TSTEP
280*90 /
END
```

We used a similar to the first scenario presented here with small modifications for the stochastic analysis that we will introduce in the next section.

1.10 Sensitivity and risk analysis

Mathematical models developed to approximate the injected CO₂ in the storage sites consist of several steps, including the determination of parameters which are the most influential on the model outputs. Sensitivity analysis can serve as a guide to any further use of the model.

In the initial sensitivity analysis performed on geological uncertain parameters of our studies, we use the large number of detailed flow simulations and measure the variability of model responses with respect to each level of the uncertain parameters.

We can obtain histograms of response Γ for three different levels of parameter α (i.e., low, medium, and high) by performing simulations over all geological realizations. Measuring the mean response value on each histogram results in an average for all cases with a fixed level for parameter α . Having three average points for low, medium, and high levels of parameter α , a line can be fitted to those points that approximates the trend of variations of response Γ versus the increase in levels of para α .

Assuming an equal probability for each level, the model output variations are examined by looking at each response at two important simulation times, i.e., end of injection and end of simulation. Assessing the uncertainty with input variations over a relatively high resolution demands a fast flow modeler. We use a response surface method that is explained in the next section in details.

1.10.1 Stochastic analysis

Phenomenon for which variables are uncertain can be modeled as a stochastic process. Uncertainty reduction in different parts of the modeling requires a better understanding and description of input parameters and dependency rules within the system. Parameters can be ranked in the order of their influence on the model output. Knowing the most influential parameters helps in treating the stochastic nature of the process. Sensitivity analysis serves in identification and evaluation of important model parameters.

As discussed in the earlier sections, various sources of uncertainties are embedded within CO₂ storage modeling and operations. The focus of our research has been on geological uncertainty and its consequences. The procedure used here to identify the relative importance of uncertain geological parameters via sensitivity analysis and the corresponding risk assessment is a general work-flow that can be applied to any type of uncertainties in the model inputs.

1.10.2 Arbitrary polynomial chaos expansion

Our research continues by employing a stochastic response surface method that approximates the flow responses by projecting them on high-dimensional polynomials. In particular, we use arbitrary polynomial chaos (aPC) expansions, which consists of orthogonal polynomial bases that are constructed according to the uncertainties in the input parameters. The approach is flexible with respect to the quantification of probabilities for uncertain parameters and can be applied in studies with limited knowledge of probabilities.

The reduced model approximated by aPC is considerably faster than the original detailed one, and thus it provides a promising start point for global sensitivity analysis and probabilistic risk assessment. Variance-based global-sensitivity analysis methods have shown success in non-linear and complex problems [67]. The system can be decomposed into approximating functions of input parameters, and this makes it easy to implement methods based on variance. The bottle-neck of variance-based approaches can be their computational costs. In our case, the variance in output responses can be set equal to the variance of polynomial components calculated for each input parameters. Polynomials are inexpensive to evaluate compared with a full simulation. This makes it efficient to implement a variance based sensitivity analysis using polynomial approximation. Furthermore, the fact that the response surface has known polynomial properties simplifies the approach significantly. The speed of polynomial approximation makes it feasible to perform an intensive probabilistic risk assessment via a Monte-Carlo process over a high resolution input variation.

Statistical accuracy of a Monte-Carlo process is highly sensitive to the resolution of variational inputs. A response surface method assisting a Monte-Carlo procedure must be constructed on a dense Cartesian grid, which will be computationally demanding. We explore an alternative method, which is a polynomial chaos expansion (PCE), and only requires a minimum number of model evaluations to construct the approximating response surface. The approach we use is based on the aPC as described in [63]. The main idea is to construct the approximating response surface by projecting the response on orthogonal polynomial bases within the uncertain parameter space. Therefore, uncertainty in input pa-

rameters is involved in the process from the initial steps of the work-flow. This approach is an advanced statistical regression method that offers an efficient and accurate way of including nonlinear effects in stochastic analysis, see e.g., [27, 31, 83]. The method is examined for accuracy and one attractive feature of PCE is the high-order approximation of error propagation as well as its computational speed [64] when compared to Monte-Carlo processes.

Earlier PCE techniques put the restriction of specified types of uncertainty distribution functions to be used in the work-flow. In contrast, the arbitrary polynomial chaos (aPC) is flexible to accommodate for a wide range of data distribution [63]. The aPC can work even in cases with limited uncertainty information reduced in a few statistical moments of samples. They can be specified either analytically (as probability density, cumulative distribution functions), numerically as histograms, or as raw data sets. In terms of performance, the aPC approach shows an improved convergence applied to input distributions that fall outside the range of classical PCE.

In general, an approximation of system response Γ can be written as a function of the uncertain input parameters Θ :

$$\Gamma \approx \Upsilon(\Theta). \quad (1.62)$$

Uncertainty of input parameters Θ can be represented by a mapping h from random variable space ξ to random input space Θ

$$\Theta = h(\xi). \quad (1.63)$$

As discussed earlier, h can be an analytical or numerical representation.

The responses of the system can be expanded into the space of the approximating polynomial bases. This expression is specified by constant coefficients c_i :

$$\Gamma \approx \sum_{i=1}^{n_c} c_i \Pi_i(\Theta). \quad (1.64)$$

Here, n_c is the number of expansion terms, c_i are the expansion coefficients, and Π_i are the multi-dimensional polynomials for the variables $\Theta = [\theta_1, \dots, \theta_n]$. The number n_c of unknown coefficients c_i depends on the degree d of the approximating polynomial, and the number of considered parameters n :

$$n_c = \frac{(d+n)!}{d! \times n!}. \quad (1.65)$$

Polynomials Π_i in the Expansion 1.64 consist of polynomial bases P_l , with l ranging from zero up to the approximation degree d . These bases are orthogonal, i.e., every pair of non-identical bases fulfill the following condition:

$$\int_{I \in \Omega} \omega P_l P_m d\tau(\Theta) = \delta_{lm}, \quad (1.66)$$

where ω is a weight function, δ is the Kronecker delta function, and τ is the measure for input variable space. We choose the weight to be one, i.e., $\omega := 1$.

The orthogonal polynomial basis satisfying Equation (1.66) can be obtained from the solution of the following linear system of equations [63]:

$$\begin{bmatrix} \mu_0 & \mu_1 & \dots & \mu_k \\ \mu_1 & \mu_2 & \dots & \mu_{k+1} \\ \dots & \dots & \dots & \dots \\ \mu_{k-1} & \mu_k & \dots & \mu_{2k-1} \\ 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} P_0^{(k)} \\ P_1^{(k)} \\ \dots \\ P_{k-1}^{(k)} \\ P_k^{(k)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 1 \end{bmatrix}. \quad (1.67)$$

Here, μ_k is the k^{th} non-central (raw) statistical moment of the random input variable, which is defined as:

$$\mu_k = \int_{\Theta \in \Omega} \Theta^k d\tau(\Theta). \quad (1.68)$$

Thus, arbitrary polynomial chaos expansion base on Equation 1.67 only demands the existence of a finite number of moments, and does not require the exact knowledge or even existence of probability density functions. An interesting aspect is that only moments up to twice the order of the expansion matter. This means that there is no need for any kind of assumptions for data probability distribution leading to subjectivity artifacts as discussed earlier.

The PCE techniques are divided into intrusive [34, 55, 80] and non-intrusive [45, 47, 48, 64] approaches. Intrusive techniques require modifications in the system of governing equations (e.g., the flow model system). In some cases, this can end up in semi-analytical methods that are used for simpler stochastic analysis studies (e.g., stochastic Galerkin method). However, the intrusive approaches can be very complex and analytically cumbersome and can not be implemented for industrial applications. In contrast to intrusive techniques, the non-intrusive methods are vastly used in practical studies. These methods do not require any symbolic manipulations of the governing equations. The sparse quadrature and the probabilistic collocation method (PCM, [48, 64]) are among the non-intrusive technique. In a simple sense, PCM can be considered as a mathematically optimized interpolation of model output for various parameter sets. The polynomial interpolation is based on minimal model evaluations in an optimally chosen set of parameter locations that are called collocation points. Hence, the challenge in these techniques is to find a balance between accuracy and speed to evaluate the uncertainty in the physical processes.

The collocation formulation has the advantage of treating the model as a black-box. This formulation requires the corresponding output to be known in the collocation set of input parameters.

According to [79], the optimal choice of collocation points corresponds to the roots of the polynomial of one degree higher ($d + 1$) than the order used in the chaos expansion (d). This strategy is based on the theory of Gaussian integration (e.g., [1]).

For multi-parameter analysis, the full tensor grid of available points from the original integration rule is $(d + 1)^n$, which is larger than the necessary number M of collocation points. This might be used for low-order (1st, 2nd) analysis of limited number of parameters. However, for a large number of parameters and high order of polynomial approximations, the full grid becomes computationally cumbersome. In the collocation approach, the minimal set of points is chosen from the most probable regions based on the parameter uncertainty information(See [48, 63, 64]).

We implement the probabilistic collocation method for computing the coefficients c_i in Equation 1.64. The weighted-residual method in the random space is defined as [48]:

$$\int \left[\Gamma - \sum_{i=1}^{n_c} c_i \Pi_i(\Theta) \right] w(\Theta) p(\Theta) d\tau = 0, \quad (1.69)$$

where $w(\Theta)$ is the weighting function and $p(\Theta)$ is the joint probability density function of Θ . By substituting the weight function in Equation 1.69 with Delta function, the equation reduces to

$$\Gamma_c - \sum_{i=1}^{n_c} c_i \Pi_i(\Theta_c) = 0. \quad (1.70)$$

In this equation, Γ_c and Θ_c are the responses and input parameters in the collocation points. Having the Θ_c chosen based on the probability distributions of input parameters, and Γ_c from the minimal model evaluations on Θ_c , we can solve Equation 1.70 and find the coefficients c_i .

1.10.3 Sensitivity analysis

Sensitivity analysis helps in understanding the degree of dependency of system responses to the input parameters. When the input parameters are uncertain in value, the model predictions will consist of uncertainties that must be eliminated for a robust and precise prediction. Therefore, sensitivity analysis can be useful both in optimizing the system performance and in studying the variation in performance coming from the stochastic nature of the system.

Global sensitivity analysis covers the entire variational space for uncertain parameters, while other methods, like the gradient-based methods, are limited in the scope influence of the parameters.

Variance-based methods are very popular among different types of sensitivity analysis methods. Variance-based methods provide global sensitivity and work for general non-linear problems. When the response is decomposed into simpler components (for instance, polynomial bases), it is easier to decompose the unconditional variance in the output into terms due to individual parameters and the interaction between them. It is possible then to rank the input parameters based on their contribution to the output variance [67, 71].

Following the linear sensitivity analysis performed initially in the study on the extensive detailed simulations, we tackle the global sensitivity analysis based on the aPC technique. This approach is well described in [63, 65]. Morris method [58] considers a uniform importance of input parameters within predefined intervals. We use a weighted global sensitivity in a more flexible approach accounting for arbitrary bounds and parameters with different importance defined by weighting functions. The big advantage of aPC-based sensitivity methods is their low computational costs for obtaining global sensitivity analysis. The aPC based-method places the parameter sets for model evaluation at an optimized spacing in parameter space. This can be interpreted as fitting polynomials to the model response. These polynomials approximate the model over the entire parameter space in a weighted least-square sense. This is more beneficial to computing a tangent or local second derivatives (compare FORM, SORM methods, e.g., [46]) that approximate the model well just around one point in the parameter space.

As an advantage, in variance based methods one can work with arbitrary system as a black-box and calculations are based on inputs and outputs only. More recent works are concerned about expediting calculation pace [20, 63, 65]. The idea is to replace the system with an approximating function which gives benefits in sensitivity calculations, because it is easy to relate the output variances to the input variables.

We expand the variance of output solution into components. Assume that we break the system output into components:

$$\Gamma = \Gamma_0 + \sum_i \Gamma_i + \sum_{i > j} \Gamma_{ij} + \dots \quad (1.71)$$

A single index shows dependency to a specific input variable, whereas more than one index shows interaction of input variables. If we consider input vector Θ to be of n components θ_i for $i = 1, \dots, n$, then $\Gamma_i = f_i(\theta_i)$ and $\Gamma_{ij} = f_{ij}(\theta_i, \theta_j)$. In practice, we consider a finite number of terms in Equation (1.71). The first order sensitivity index, so called Sobol index, is defined as follows [71]:

$$S_i = \frac{V[E(\Gamma | \theta_i)]}{V(\Gamma)}, \quad (1.72)$$

where $E(\Gamma | \theta_i)$ is the conditional expectation of output Γ given θ_i and V is the variance operator. Since θ_i can be fixed at any value in its uncertainty interval, each of those values produce a distinct expectation $E(\Gamma | \theta_i)$. Equation 1.72 is a measure for variations of these expectations, which indicates the direct contribution of parameter θ_i in the output variance. For more than one index, a higher-order Sobol index can be defined as:

$$S_{ij} = \frac{V[E(\Gamma | \theta_i, \theta_j)] - V[E(\Gamma | \theta_i)] - V[E(\Gamma | \theta_j)]}{V(\Gamma)}. \quad (1.73)$$

Here, $V[E(\Gamma | \theta_i, \theta_j)]$ is the variance of output expectations after fixing θ_i and θ_j . This index represents significance of variation in output generated from uncertainty in input variables together, i.e., the interaction of uncertain parameters. If we add all indices that contain variable θ_i , the sum is called the total Sobol index:

$$S_{Ti} = S_i + \sum_{j \neq i} S_{ij} + \sum_{j \neq i} \sum_{k \neq i} S_{ijk} + \dots \quad (1.74)$$

To clarify the subject, we go through a simple analytical example given in [4]. Suppose that the exact expression for response Γ to be known and can be written as a polynomial with parameters θ_1 , θ_2 , and θ_3 :

$$\Gamma(\theta_1, \theta_2, \theta_3) = \theta_1^2 + \theta_2^4 + \theta_1 \theta_2 + \theta_2 \theta_3^4. \quad (1.75)$$

The Sobol indices can be calculated from functions F that are defined based on orthogonality condition used to decompose the solution and for input with Gaussian distribution Φ_n in uncertainty domain R^n they are as follows:

$$F_0 = \int_{R^n} \Gamma(\Theta) \Phi_n(\Theta) d\Theta, \quad (1.76)$$

$$F_i = \frac{\int_{R^{n-1}} \Gamma|_{\theta_i} \Phi_{n-1}(\theta_{\sim i}) d\theta_{\sim i}}{\Phi_1(\theta_i)} - F_0, \quad (1.77)$$

$$F_{i,j} = \frac{\int_{R^{n-2}} \Gamma|_{\theta_i, \theta_j} \Phi_{n-2}(\theta_{\sim i, \sim j}) d\theta_{\sim i} d\theta_{\sim j}}{\Phi_2(\theta_i, \theta_j)} - F_0 - F_i(\theta_i) - F_j(\theta_j). \quad (1.78)$$

$\Gamma|_{\theta_i}$ and $\Gamma|_{\theta_i, \theta_j}$ are the Γ values at fixed θ_i and $\{\theta_i, \theta_j\}$ respectively. $\theta_{\sim i}$ is the vector of dummy variables corresponding to all but the component θ_i of uncertain parameters Θ .

Let us denote the variances by D :

$$D = V[F(\Theta)] = \int F^2(\Theta) d\Theta - F_0, \quad (1.79)$$

that can be decomposed into

$$D_i = \int F_i^2(\theta_i) d\theta_i, \quad (1.80)$$

and

$$D_{i,j} = \int F_{i,j}^2(\theta_i, \theta_j) d\theta_i d\theta_j. \quad (1.81)$$

Then the Sobol indices can be found from:

$$S_i = \frac{D_i}{D}, \quad (1.82)$$

$$S_{i,j} = \frac{D_{i,j}}{D}. \quad (1.83)$$

Finally, the total Sobol index can be found from Equation 1.74. Applying the calculations of Equations 1.76 to 1.81 for our example (i.e., the expression in Equation 1.75) we can obtain the following Sobol indices, assuming Gaussian distributions for the parameters over the interval $[0, 1]$:

$$S_1 = 0.0005 \quad S_2 = 0.4281 \quad S_3 = 0.0000$$

$$S_{12} = 0.0007 \quad S_{13} = 0.0000 \quad S_{23} = 0.5708$$

$$S_{123} = 0.0000$$

and the total sobol indices are:

$$S_{T1} = 0.0012 \quad S_{T2} = 0.9996 \quad S_{T3} = 0.5708.$$

The total Sobol index can be used as a sensitivity measure to rank parameters for their influence in the results variation. In this example, we can see that the ranking that the total Sobol indices suggests is consistent with what can be inferred directly from the simple expression in Equation 1.75: θ_2 is the most influential parameters, because it appears in three terms, and in one of them with a forth degree. Interactions are represented by two indices, and S_{13} is zero, because there exist no term in Equation 1.75 that contains both θ_1 and θ_3 .

With known polynomial coefficients, Sobol indices are easy to calculate. When the number of parameters is large, it is possible to do initial sensitivity analysis with lower degree polynomials to filter out pertinent parameters. Then the analysis continues on the filtered parameters with a higher degree polynomial approximation.

1.10.4 Risk analysis

The risk is the impact of uncertainty on objectives. Quantifying the risk requires calculating this impact, which consist of two parts: quantification of the uncertainty and evaluation of the system consequences. Risk R of a process is quantitatively defined as the consequence C caused by the process multiplied by the probability P of that consequence to happen:

$$R = P \times C. \quad (1.84)$$

In the case of CO₂ injection into deep aquifers, the amount of CO₂ which stays mobile and undissolved in the medium for a time after injection can be considered as a consequence, bearing the potential of leakage up to the surface if exposed to a geological leakage point. The risk could be the expected amount of CO₂ that will leak through ill-plugged abandoned wells or cracks in the sealing rocks.

We consider looking at responses and the probability of them to happen. We initially examine this probability by drawing the histogram of response values obtained from detailed simulations on large number of realizations and corresponding to important time, like end of injection and end of simulation. Yet larger number of points in the uncertain parameter space are studied employing the data-driven aPC method, which requires a considerably shorter time for evaluating the responses than what takes for a full simulation. This way it is possible to perform an intensive Monte-Carlo process in a full tensor grid of input parameter variational space, resulting in a high resolution output probability distribution.

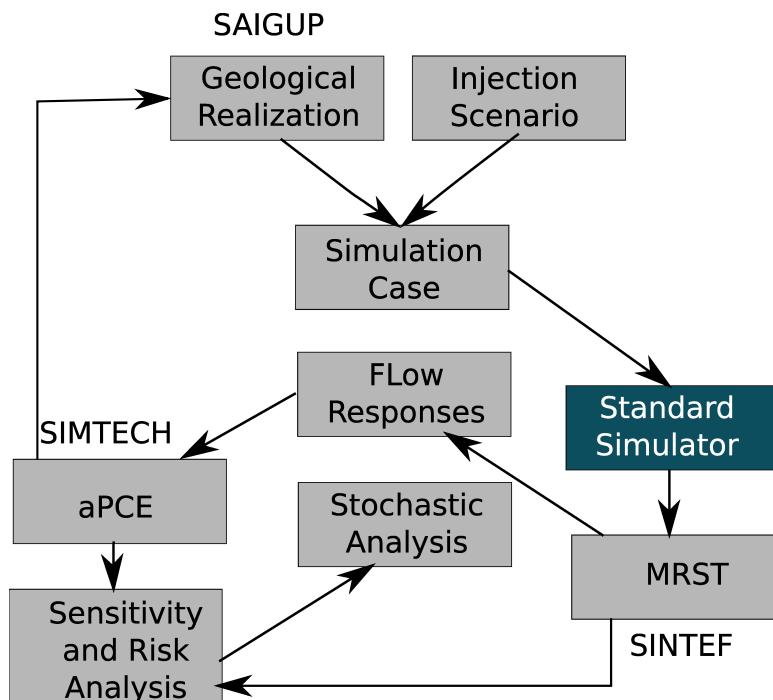


Figure 1.41: Flow-chart of work process implemented in an automated procedure.

1.11 General summary

The work objectives have been two folds:

- Assessing the significance of geological modeling in CO₂ injection and early migration studies.
- Introducing a frame-work for extensive realistic sensitivity analysis and risk assessment of geological CO₂ storage.

The significance of geological uncertainty is discussed by extensive study on CO₂ flow in different geological models. Sensitivity analysis and risk assessment have resulted in ranking of the studied geological parameters for various flow responses in the medium. The work-flow implemented in this work is a stepwise procedure that can be generalized to be used in any similar large scale analysis.

1.11.1 Implementation of the work-flow

This thesis incorporated working with large number of realizations, various flow scenarios, and different procedures and softwares. While the study progresses, new ideas and challenges rises that require manipulation of parts of the work-flow. In order to achieve the defined goals of the research, an automated work-flow is required that connects different parts of the study and converts data type in-between them. This makes the study feasible in terms of saving time for any necessary modifications.

MATLAB programming language is used for implementing the work-flow in this research. The main reason for this choice, apart from the rich facilities available within MATLAB toolboxes, is to use many functions within the MATLAB Reservoir Simulation Toolbox (MRST) [49, 75] available as free and open-source software in MATLAB language. For flow simulation a commercial software is used, which is a standard simulator for oil and gas industry and research. The simulator is using finite difference for flow modeling.

Figure 1.41 shows the elements of the work-flow implemented by a large number of MATLAB functions. Functions from MRST at SINTEF and the stochastic tools of SIMTECH group in Stuttgart university are utilized and merged into the work-flow. The design is made such that the work-flow is flexible and general. Some research has been done by replacing the commercial simulator with in-house simulators at SINTEF, but the main study was performed using a commercial standard simulator.

1.11.2 Generic application of results

The main focus of the work is two folds:

- Significance of geological uncertainty in the early stages of CO₂ storage operations.
- Applying a mathematical tool to perform sensitivity analysis and probabilistic risk study for assessing the geological uncertainty impacts on the success of CO₂ storage.

We rank the most influential geological parameters. The demonstrated work-flow can be used in any study concerning the site selection and early stages of CO₂ storage. However, there are some limitations in our presentation of the work-flow that needs to be considered when this work is to be consulted for any similar study.

First limitation is the SAIGUP model size. CO₂ storage studies requires large models that can cover the CO₂ spatial traveling extents within the aquifer. Therefore, our study is limited to the domain around the injector.

An over-pressurized injection can introduce breakings in the sealing cap-rock that is aimed to be used for structural strapping of CO₂. It is more feasible to use minimum number of wells to minimize the costs of the project and also the risk of CO₂ leakage via drilled wells after they are abandoned. Therefor, a typical injection scenario include few number of injectors with no production well to balance the injection pressure. The elliptic nature of pressure equation and the small compressibility of the medium result in a large area influenced by injection pressure. Therefor, the pressure related studies need a large size of model to study the influence of the impulse imposed by the injector on the entire region connected to that impulse.

To overcome this limitation in the SAIGUP models, we exaggerated the volume of the cells in the boundaries of the model that are supposed to be open. The large pore volumes on the boundaries avoid extreme pressure build-up due to injecting in a closed system. On the other hand, the study is limited to the region near the well. Since the high pressures are happening near the injector, this is

more interesting to study for a pressure build-up around the well rather than looking at the entire region influenced by the injection pressure.

The pressure behavior is very sensitive to the way the boundaries are defined. In reality, there are different systems of aquifers. Some of them are large with huge pore-volumes. To model these aquifers, we can use smaller size models with open boundaries. However, some aquifers are medium and small in size. To model these aquifers, we can assume semi-close and close boundaries. For any aquifer system, we can define the boundary by exaggerating the pore volume of the cells on the boundaries of the model. The transmissibilities of the boundary cells can be also modified to represent the size of the aquifer system. This controls the amount of pressure relaxation in the medium through the boundaries. If CO₂ exists in the boundaries, relative permeability functions at the boundary can be modified in addition to the transmissibilities.

The open boundaries in our study are considered to be fully open. This makes the pressure to relax through the boundaries and the results of our pressure study are influenced by this choice. While we have observed a considerable portion of cases with extreme pressures due to heterogeneity effects, the pressures reported in our study are moderate compared to not fully opened boundaries. The sensitivity analysis is based on comparing cases for their pressure values. Therefor, the outcome of sensitivity analysis should be valid regardless of the boundary choices. The size limitation in the SAIGUP models resulted in an extension to the current study in a project, which is called IGEMS [77].

The IGEMS models have larger size compared to the SAIGUP models. There is only one major structural trap in the SAIGUP models that allows for most of the injected CO₂ to accumulate under the cap-rock. This is not sufficient for studying the effect of variations in the top-surface topography on the CO₂ movement in the medium. The IGEMS study has focused on the structural trapping due to deformations in the top-surface morphology and faults. The results show that the structural trapping can be important in the amount of storage due to structural trapping and it can control the speed of the plume migrating under the top sealing cap-rock.

In the vertical scale the study can be improved by using a higher grid resolution. Variations in the vertical direction exist in considerably smaller scales than the lateral direction. In particular, this is more crucial for the long-term migration of CO₂ where a thin plume of CO₂ migrates beneath a sealing layer due to the buoyancy forces.

Another issue to be mentioned is the assumption for geological uncertainties that are used in the stochastic analysis. We consider near uniform distributions for the probabilities of uncertain parameter values. While there is no loss of generality, there are too comments that could improve our analysis:

- In general, the uncertainty probability can be different than uniform distribution. Actually, these information are very case dependent and can change from one location to another.
- One advantage of the aPC method is the flexibility to apply for arbitrary forms of uncertainty data. Choosing various distributions for the geological parameters would be more demonstrative of the aPC method strength.

Choosing uniform uncertainty distributions for our study is due to our limitation to use the SAIGUP models that are made based on equi-probable values for the geological uncertain parameters. A general stochastic process using the aPC must be considered in the following steps:

- Use the techniques from the aPC to derive appropriate sample points for geological parameters.
- Construct geological models at these sample points.
- Perform flow simulations for each sample point.
- Construct the proxy model.
- Perform global sensitivity analysis using the Sobol indexes method and the proxy model.

- Perform the Monte-Carlo using the aPC study to assess the uncertainty and risk.

One thing to notice in Figure 1.41 is the link between designing geological realizations and implementation of aPCE method. The sensitivity analysis and risk assessment procedure must start from the 'aPCE' box in Figure 1.41, by finding the collocation points from the given geological uncertainty, and then based on those collocation points we design the geological realizations. However, for the availability of a large set of SAIGUP realizations that were generated before this study, our start point was from the 'Geological Realization' box in Figure 1.41. This change in start point resulted in assuming a given geological uncertainty knowledge that suits the SAIGUP geological design. Nevertheless, we practice the procedure in a scope of geological modeling and flow analysis that is novel in its kind and can be consulted for further extensive studies.

Chapter 2

Introduction to the papers

2.1 Introduction

The main scientific part of this thesis consists of three papers. They come in a sequence to show the research progress within this PhD program. Paper I includes a detailed study on the flow responses and effect of relative permeability on the flow and it is submitted to the International Journal of Greenhouse Gas Control (IJGGC). Pressure is an important model response during injection operations. Therefore, a special study is dedicated to pressure analysis in the system. This is reported in Paper II, which is submitted to the IJGGC. Finally, Paper III reports modern stochastic techniques used to perform detailed quantitative sensitivity analysis and probabilistic risk assessments. This paper is accepted for publication in the IJGGC.

2.2 Summary of papers

Paper I: Impact of geological heterogeneity on early-stage CO₂ plume migration

Summary:

It is a conventional practice in the context of CO₂ storage study to simplify the geological modeling to achieve an easier force balance study in the medium. Assuming a homogeneous medium is the first step to quantify the temporal and spatial scales in CO₂ storage problems by a dimensionless analysis on the analytical solution of flow equation. However, in practice the real flow performance is very much influenced by geological heterogeneity.

We use a set of SAIGUP realizations selected to cover the variability of four sedimentological and structural parameters. The selected parameters are lobosity, barriers, aggradation angle, progradation direction. Each of these parameters varies over three levels, except the progradation direction, which includes up-dip and down-dip directions. Combining the available parameters makes 54 realizations.

30 years of injection and 70 years of early migration of CO₂ are simulated and flow responses related to the storage capacity and leakage risk objectives are defined and calculated from the simulation results. The responses are reported in scatter plots at the end of injection and at the end of early migration time.

This work is specific in examining how heterogeneity influences flow behavior by using a number of geological realizations. Flow responses defined in this work are specific to CO₂ studies and differ from the responses used in the original SAIGUP project to study oil recovery. We simulate the aquifer average pressure, model boundary fluxes, residual and mobile CO₂ saturation, and spatial distribution of connected CO₂ volumes. These responses can be considered to evaluate the site storage capacity and risk of CO₂ leakage to surface.

Comments:

This work initially was presented at the Edinburgh 2010 ACM conference, in a brief proceedings presentation, and more details of the work are reported in a proceedings for CMWR 2010 conference in Barcelona. The aim of this paper is to investigate the impact of geological heterogeneity on a typical CO₂ injection problem.

The main concern here is to include realistic geological heterogeneity knowledge into flow simulation work-flow, which is specific to the CO₂ storage problem. Results of this work conclude that the range of variability in flow responses indicate the significance of geological heterogeneity in modeling the CO₂ flow. Geological features are ranked by for their impact on each of the defined flow responses and in particular, aggradation angle has shown a big impact in most of the responses.

This paper lacks the quantitative sensitivity analysis, which was added in later works.

Contribution of the candidate:

Major part of the work is done by the candidate. This includes:

- Defining the problem.

- Designing the simulation scenarios.
- Defining the simulation responses.
- Performing the simulations and processing the simulation results.
- Analyzing the results.
- Writing the report.

The candidate has received helps from the co-authors mainly on analyzing the simulation results and writing the report. This has been through discussion meetings and review comments on the draft report originally written by the candidate.

Paper II: Impact of geological heterogeneity on early-stage CO₂ plume migration: sensitivity study

Summary:

In this paper, we consider faults in addition to the four geological parameters used in Paper I. In the SAIGUP study, fault modeling is performed in an intensive variability over structural parameters and transmissibility across the fault. To keep the work less succinct and conclusive in studying the dynamics of flow, we fix the structural variability in its medium level and vary the transmissibility over three main levels: unfaulted, open faults and closed faults.

The flow responses analyzed are the same as defined in Paper I. In addition, we perform a linear sensitivity analysis for the flow responses with respect to the geological variations. The outcome of the sensitivity analysis shows that the flow behavior is highly sensitive to aggradation angle. Barriers and faulting will also influence the flow responses significantly. In this work, similar to Paper I, we clearly see the range variation in flow responses which demonstrates how important it is to model the geological features accurately.

Comments:

- *The SAIGUP realizations*

This paper is presented in ECMOR conference in Oxford, 2010. This is a complementary to proceedings works presented in ACM Edinburgh, 2009 and CMWR in Barcelona, 2010.

Topography is a major player in the gravity dominated flow behavior. The SAIGUP realizations include variability in topography of the geological layering via structural changes due to faults and also barriers in the model. These are good enough for early migration when the CO₂ and water segregate and plumes accumulate below cap-rock and start the longer migration. In the long-term migration, top surface geometry is an important geological parameter and larger models than the SAIGUP models with a better resolution of the top surface are needed to get good predictions of the long-term migration phase. This was considered in the next generation of geological studies performed following this study [61, 77] under the IGEMS research project.

- *Physical assumptions*

The work concentrates on how geological heterogeneity impacts the flow performance. We need to measure the volumetric sweep efficiency of CO₂ plumes to evaluate the residual trapping. The most important parameter is the rock transmissibility, rather than fluid properties. Including more physics in the modeling will add the computational costs specially when the flow modeling is used in a sensitivity analysis or risk assessment process. Therefore, we used simple fluid models for PVT.

To accelerate the flow, we used linear relative permeability curves. This increases the flow mobility in the low saturation values. However, in CO₂-water system the relative permeability curves

are closer to a quadratic function. This is investigated in the next paper (Paper III). Hysteresis effects are modeled by changing the endpoints in the relative permeability curves and no scanning-curves is considered here. During injection, the main process is drainage. After injection, the imbibition process starts and mostly is a replacement of CO₂ by water due to gravity segregation. This justifies the usage of simple hysteresis model, and more detailed study can be done to investigate this influence in a quantitative manner.

- *Uncertainty considerations*

Our main motivation for using the SAIGUP data was the extensive work that was put into building realistic geological realizations. The geological parameters are changed in value between low and high levels. These values are assumed with the same probability, which is a reasonable start point for sensitivity analysis. In general, this probability might not be uniform, depending to the regional geology of the storage site.

Contribution of the candidate:

The study was a joint work between the candidate and the co-authors on the following steps:

- Defining the problem.
- Designing the simulation scenarios.
- Defining the simulation responses.
- Performing the simulations and processing the simulation results.
- Analyzing the results.
- Writing the report.

The candidate had a solid and major contribution in every step. Performing simulations and processing the simulation results were done solely by the candidate.

Paper III: Impact of geological heterogeneity on early-stage CO₂ plume migration: sensitivity study

Summary:

In this paper, we use the same setup that is used in Paper II. Five geological features are examined, which are lobosity, barriers, aggradational angle, progradational direction, and faults. Also, the same injection scenario is used: 30 years of CO₂ injection and 70 years of early migration. The injector is controlled by a constant rate and no pressure constraint is set to allow for all ranges of pressure, including those that are unrealistic. Moreover, we define an additional model output that is related to the risk of CO₂ leakage through any breakages in the cap-rock.

We examine the influence of simplifying assumptions considered in our works regarding linearity of relative permeability function. We perform a detailed flow analysis on various geological realizations using two different relative permeability relations: linear and quadratic functions. The non-linearity in relative permeability hinders the flow in the lower saturation values of the displaced phase, i.e., water phase. We discuss the influence of curvature on CO₂ flow dynamics within the aquifer. Finally, we perform a quantitative sensitivity analysis by using the flow simulation results. The sensitivity analysis results suggest that aggradational angle, fault criteria, and progradational direction are the most influential geological parameter in our study.

Comments:

Results show that using linear relative permeability function ends up in conservative conclusions with respect to CO₂ distributions in the domain in terms of storage safety. Since computational costs

are much lower for linear relative permeability scenario than quadratic relative permeability, it sounds a good strategy to perform sensitivity analysis by using linear relative permeability function on flow responses that are based on CO₂ distribution in the domain.

However, dramatic pressure build-up can happen in the medium during injection in the quadratic relative permeability scenario. This suggests that for pressure studies we must use more realistic relative permeability functions. Following this work, a detailed pressure study with more realistic relative permeability curves is performed, which is reported in the next paper.

Within one geological realization, injection location can dramatically impact the injectivity of the well. In fact, this is an uncertain parameter in the CO₂ storage operations. Choosing to inject in the river channels or in the permeable homogeneous parts near the shore will enhance the injectivity and the CO₂ sweep efficiency in the medium. On the other hand, injecting in the locations with low permeabilities and pore-volumes can significantly increase the injection pressure, while limiting the transport of CO₂ in the medium. Studying the impact of injection location can be performed by injecting in many different points in one realization and comparing the corresponding flow responses. However, this will considerably increase the number of detailed simulations in the study.

For the allowed time, we limited our study to a fixed point by injecting via one well in the flank part of the SAIGUP models. This location is selected after qualitative analysis of a detailed study on a homogeneous case. There, we aimed to fulfill the criterion of maximizing the CO₂ storage volume via increasing the vertical travel path towards the structural trap location under the cap-rock. One mitigating strategy for minimizing the effect of injection location can be to inject via several wells in different location in the medium.

Similar argument applies to the leakage risk study reported here. We use a leakage probability over the cap-rock that can dramatically influence the calculated leakage risk. We take this assumption to simplify the way we introduce the method.

Contribution of the candidate:

Major part of the work is done by the candidate. This includes:

- Defining the problem.
- Designing the simulation scenarios.
- Defining the simulation responses.
- Performing the simulations and processing the simulation results.
- Analyzing the results.
- Writing the report.

The candidate has received helps from the co-authors mainly on analyzing the simulation results and writing the report. This has been through discussion meetings and review comments on the draft report originally written by the candidate.

Paper IV: Geological storage of CO₂: heterogeneity impact on pressure behavior

Summary:

After observing the influence of relative permeability curvature on pressure response for CO₂ injection studies, in this paper we perform a detailed pressure study on the chosen geological realizations.

Pressure build-up is an important criterion that can determine the success and failure of CO₂ storage operations. Over-pressurized injections can induce new fractures and open the existing faults and fractures that increases the risk of leakage for the mobile CO₂ in the domain. On the other hand, the pressure disturbance imposed on the system travels within the domain beyond the scales of CO₂ distribution. If the CO₂ is injected into a saline aquifer connected to fresh water aquifers, the pressure pulse

may result in fresh water contaminations by the brine far from the injection point. We define specific pressure responses to examine the pressure disturbance in the system during injection.

Two injection scenarios are examined for the same 160 geological realizations setup. In the first scenario, the injector is set to a fixed volumetric rate to inject the CO₂ volume in 30 years into the domain, allowing for an unlimited pressure build-up. In the second scenario, a pressure constraint is set on the injector that results in various rate of injection in different geological realizations to inject the same amount of CO₂ volume considered in the first injection scenario.

Pressure response sensitivity study with respect to different geological features indicates the significance of aggradation angle, progradation direction, and faults during injection. A probabilistic pressure analysis is also performed based on the 160 simulations on the available realizations.

Contribution of the candidate:

Almost all parts of the work is done by the candidate. This includes:

- Defining the problem.
- Designing the simulation scenarios.
- Defining the simulation responses.
- Performing the simulations and processing the simulation results.
- Analyzing the results.
- Writing the report.

The candidate has received helps from the first adviser mainly on writing the report. This has been through review comments on the draft report originally written by the candidate.

Paper V: Geological storage of CO₂: global sensitivity analysis and risk assessment using the arbitrary polynomial chaos expansion

Summary:

In this paper, we perform a stochastic sensitivity and risk analysis. We obtain a high resolution global sensitivity and probabilistic study on the flow responses that are defined and discussed in the previous papers. We choose barriers, aggradation angle, and faults from the SAIGUP geological parameters. Faults are considered by changing the transmissibility value across them, which is a continuous parameter. One more parameter is added to the study which is common in the literature and models the external pressure support from other aquifers attached to the model (regional groundwater effect).

Flow simulation on high resolution variational geology demands a huge computational costs. To enhance the calculation speed, we use a data-driven method that does not need to solve the full physical flow equations. We approximate the flow solver by a response surface method that is a polynomial and relates the system output to the input with a minimal computational cost. We use the arbitrary polynomial chaos expansion (aPC) to approximate the flow responses. The aPC method considers the uncertainty in the input variables.

A global sensitivity analysis is performed by employing Sobol indices that are based on variances of responses. The method is shown to be robust in problems of high levels of complexity and non-linearity.

And finally, we perform a Monte-Carlo process using the approximating polynomial on a high resolution input variations. This makes it possible to perform a high resolution probabilistic study on the flow responses. This way, extreme cases can be identified by probability of occurrence.

Comments:

To implement our stochastic technique, we choose geological parameters in this study that can be interpolated between two levels of their values. For example, it makes sense to use barriers coverage

level of 25% between the low (10%) and medium (50%) levels used in the previous studies. Some of the geological parameters are discrete and can not be interpolated between two values. For instance, lobosity can only be varied over three points and we can not define a 1.5-lobe.

Having a large number of points in the input values interval requires intensive geological modelings to be used in the flow simulations. Using the data-driven polynomial, the approach only needs evaluating the polynomial in the defined values, and there is no need for full geological modeling except in the collocation points, i.e., point values that the polynomial coefficients must be calculated.

The work reported here is to demonstrate the work-flow of using the aPC for global sensitivity analysis and probabilistic risk assessment. A normal work-flow starts by defining the uncertainties in the input parameters and follows by building the geological models for the aPC collocation points that are based on those uncertainties. To perform this study on the SAIGUP models that are consistent with a uniform uncertainty in the geological parameters, with no loss of generality, we used uniform uncertainty distributions for our study. However, the aPC method is not limited to uniform uncertainty descriptions.

Geological features are ranked based on the sensitivity analysis results. The results are in agreement with dynamics of the flow in the aquifer. Aggradation angle is the most influential parameter, while the regional groundwater has the least influence in the model responses. The study is not limited to the assumed uncertainty of input parameters and the conclusion may change for a very different uncertainty description.

Contribution of the candidate:

Major part of the work is done by the candidate. This includes:

- Defining the problem.
- Designing the simulation scenarios.
- Designing the work-flow.
- Integrating the aPC MATLAB code into the work-flow.
- Performing the runs and processing the results.
- Performing the global sensitivity analysis.
- Performing the risk assessment.
- Analyzing the results and preparing plots.
- Writing the report.

The candidate received help from the co-authors and others involved in defining the problem and writing the report. The report has gone through extensive reviews, after it was originally written by the candidate. The core of the report, including the formulation of the method, the description of the modeling, the presentation of the results, and discussions on the results are originally written by the candidate. The review comments resulted in additional elaborations on the pressure issue and some discussions on the aPC technique when it is compared to other techniques in the literature. The candidate received support from the co-authors on the aPC directly related comments.

Chapter 3

Scientific results

Paper I

3.1 Impact of geological heterogeneity on early-stage CO₂ plume migration: pressure behavior study

M. Ashraf, K.A. Lie, H.M. Nilsen & A. Skorstad

Submitted to *International Journal of Greenhouse Gas Control (IJGGC)*

**UNDER
CONSTRUCTION**

Paper II

3.2 Geological storage of CO₂: heterogeneity impact on pressure behavior

M. Ashraf

Submitted to *International Journal of Greenhouse Gas Control (IJGGC)*

**UNDER
CONSTRUCTION**

Paper III

3.3 Geological storage of CO₂: Application, feasibility and efficiency of global sensitivity analysis and risk assessment using the arbitrary polynomial chaos.

,
M. Ashraf, S. Oladyshkin, W. Nowak

Proceedings of the European Geosciences Union (EGU) General Assembly 2012, April, Vienna, Austria, Geophysical Research Abstracts., Vol. 14, EGU2012-9243. Published in International Journal of Greenhouse Gas Control (2013).

**UNDER
CONSTRUCTION**

Appendix A

**Additional manuscripts not evaluated as part of
thesis**

Paper A

A.1 Impact of geological heterogeneity on early-stage CO₂ plume migration

Ashraf, M., Lie, K.A., Nilsen, H.M., Nordbotten, J. M., and Skorstad, A.

presented and published in the proceedings of the Computational Methods in Water Resources (CMWR) conference in Barcelona, 2010.

**UNDER
CONSTRUCTION**

Paper B

A.2 Impact of geological heterogeneity on early-stage CO₂ plume migration: sensitivity study

Ashraf, M., Lie, K.A., Nilsen, H.M., and Skorstad, A.

Presented and published in the proceedings of the ECMOR XII in Oxford, 2010.

**UNDER
CONSTRUCTION**

Paper C

A.3 Field-case simulation of CO₂ plume migration using vertical-equilibrium models.

Nilsen , H.M., Herrera , P.A., Ashraf, M., Ligaarden , I.S., Iding , M., Hermanrud , C., Lie , K.A., Nordbotten , J.M., Dahle , H.K., and Keilegavlen , E.

Presented in GHGT10, Amsterdam, 2011. Published in 2011, Energy Procedia 4 (2011): 3801-3808.

**UNDER
CONSTRUCTION**

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