Semiparametric (Distribution-Free) Testing of the Expectations Hypothesis in a Parimutuel Gambling Market

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The expectations hypothesis maintains that a current forward or futures price should be an unbiased forecast of the expected future price. This article tests the expectations hypothesis in the parimutuel gambling market for greyhound racing using parametric and semiparametric estimators. Parimutuel gambling markets are similar to speculative asset markets in many regards. Conventional maximum likelihood tests of asset pricing require a priori specification of the statistical distribution governing agents' expectations. Distributional misspecifications may bias conventional tests. The semiparametric estimators applied in this article overcome these problems and, in addition, maintain consistency under heteroscedasticity. The results reject the expectations hypothesis.

KEY WORDS: Maximum score estimation; Nonparametric regression.

The expectations hypothesis maintains that a current forward or futures price should be an unbiased forecast of the expected future price. The expectations hypothesis provides a useful benchmark from which to evaluate asset-price relationships. Modern asset-pricing models that relate prices or expected returns to economywide risk factors explain deviations from the expectations hypothesis. Some assets, such as parimutuel wagers, however, are likely to be independent of economywide risk factors and thus may be interesting candidates for tests of the expectations hypothesis.

This article uses semiparametric techniques to evaluate the expectations hypothesis in the greyhound parimutuel gambling market. Parimutuel gambling markets bear several similarities to speculative markets for commodities and financial instruments. As Gabriel and Marsden (1990) noted, both types of markets have many investors (gamblers) with access to widely available information. Investments in both markets are risky and uncertain, and both share the potential for profitable trading using insider information. In addition, once an investment (bet) is made, the return cannot be guaranteed and is thus uncertain.

Parimutuel gambling markets may offer advantages for investigations of asset pricing. A wide range of market information known to be used by investors (bettors) is available in the racing program. Probabilistic measurements of market expectations are also available in the parimutuel odds. Expert opinions of forecasters are available in the form of professional handicapping services. Parimutuel assets (wagers) also have common, well-defined termination points at which values become certain (Thaler and Ziemba 1988). Finally, aggregation biases that may affect expectations tests (Keane and Runkle 1990) are less likely to occur for the (timewise) disaggregated data available for a parimutuel market.

1. THE GREYHOUND PARIMUTUEL GAMBLING MARKET

This analysis evaluates the expectations hypothesis using data collected over a two-and-one-quarter-month period (March through early May of 1991) at the Woodlands greyhound track in Kansas City, Kansas. There are currently 60 greyhound racetracks in the United States. In 1990, nearly $3.5 billion was wagered on greyhound racing in the United States (American Greyhound Track Operators' Association 1991). According to the American Greyhound Track Operators' Association (1991), greyhound racing is considered by many to be the most consistent and predictable form of parimutuel wagering.

Each greyhound performance involves 13 to 15 races with eight dogs in each race. Betting windows open 20 minutes before each race. Opening odds (the morning line) are set by the track management and reflect their expectation of post-time odds. The odds are adjusted approximately every 30 seconds throughout the betting period. As bets are posted, the odds may shift significantly. The betting window closes immediately before the race and the final odds that determine actual payouts become known approximately 30 seconds later. In contrast to conventional games of chance with known probabilities, in which participants wager against the house, parimutuel gamblers wager against one another under uncertain odds. Thus, bettors with superior forecasting ability will be successful at the expense of less-informed gamblers. Although the opening odds may influence the final odds, the two can differ substantially. Thus, bettors have limited and uncertain information about final odds while the betting window is open.

A range of wagers is available to greyhound parimutuel gamblers. Win bets pay if the selected entry wins. Place wagers pay if the entry finishes first or second. Show bets pay if the entry finishes first, second, or third. Several "exotic" wagers involving combinations of two or more dogs are also available. A quinella requires selection of first and second

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place without regard to order. An exacta requires selection of first and second place in order. A trifecta requires selection of first, second, and third place in order. With eight entries, there are 28, 56, and 336 possible quinella, exacta, and trifecta wagers, respectively. Exotic wagers are not available in every race. Each wager costs $2.00, and payouts usually reflect the number of possible wagers. Exact payouts are determined by dividing the total (commission-adjusted) pool wagered on that type of bet among the winning tickets.

Under the expectations hypothesis, the final odds on a given bet can be interpreted as reflecting the market's estimate of the probability associated with winning that bet. The exact market odds will differ slightly from the posted final odds because the latter are adjusted for the commission (take) of the State and track (18% of the total pool on win, place, and show bets and 19% or 20% of the total pool on exotic bets) and are rounded down to the nearest tenth. This second adjustment, known as "breakage," is retained for purse distributions. As Asch, Malkiel, and Quandt (1984) noted, because of the take, the expectations hypothesis suggests that returns to any gambling strategy should be approximately −18% on win, place, and show bets and −19% or −20% on exotic bets. Over the 1990–1991 season at the Woodlands, breakage averaged .35%, implying that expected returns should be reduced by .35% to account for breakage. Because the odds for each entry are affected in a homogeneous manner by the take, the final odds provide a suitable metric for measuring the final market odds.

A range of information is available to bettors in parimutuel markets, including subjective information such as professional handicappers' forecasts, market odds, and factual performance and physical-characteristics data relevant to bettors' subjective probability forecasts. Under the expectations hypothesis, market odds represent the market's estimate of winning probabilities. Most studies have limited the consideration of market information to odds and professional handicappers' forecasts and have ignored other potentially relevant information such as the factual performance data that are available to bettors in the racing program.

An extensive literature evaluating investor behavior in parimutuel gambling markets exists. This literature was surveyed by Thaler and Ziemba (1988). A comprehensive collection of papers was presented by Hausch, Lo, and Ziemba (1994). Most studies reject the expectations hypothesis by finding that rates of return above the track commission can be earned through a variety of gambling strategies (e.g. see, Ali 1979; Asch et al. 1984; Bolton and Chapman 1986; Figlewski 1979; Gabriel and Marsden 1990; Hausch, Ziemba, and Rubinstein 1981). Several studies, including those of Ali (1977), Ziemba and Hausch (1986), and Snyder (1978), confirm a "longshot-favorite" bias, suggesting that expected returns to longshot wagers are lower than returns to wagers on favorites. This bias also implies a rejection of the expectations hypothesis.

2. MODEL SPECIFICATION AND ECONOMETRIC PROCEDURES

Studies of investor behavior in parimutuel gambling markets typically relate discrete finish outcomes (i.e., wins) to market odds, handicappers' forecasts, and other publicly available information as follows:

$$\delta_{ij} = f(\text{market odds}_{ij}, \text{handicapper forecasts}_{ij}, \text{other information}_{ij}), \quad (1)$$

where δ_{ij} is 1 if the ith entry wins the jth race and 0 otherwise. The expectations hypothesis implies that the conditional expectations of the binary outcome variable should equal the probabilities implied by the market odds. Thus, if the handicappers' forecasts or other data are statistically significant in the presence of final odds, the expectations hypothesis is rejected.

Empirical tests typically use a version of the following binary response model:

$$\begin{aligned} \delta_i &= 1 \text{ if } X_j\beta + e_i \geq 0 \\ &= 0 \text{ otherwise}, \end{aligned} \quad (2)$$

where δ_i is an $n \times 1$ vector of discrete indicators of the response, X_i is an $n \times k$ matrix of explanatory variables, and β is a $k \times 1$ vector of parameters. A key assumption is that all information related to discrete outcomes can be summarized in a single index (i.e., of the form $X\beta$). If one knows a priori the statistical distribution of e_i, the parameter vector β can be consistently estimated up to a scalar multiple using conventional maximum likelihood techniques. It is usually assumed that e_i has the normal or logistic distribution independent of X_i.

This parametric framework may be inappropriate for several reasons. First, such tests implicitly assume a particular distribution (e.g., logistic) for expectation errors. This distinction is important because misspecification may lead to biased forecasts and inferences. As Horowitz (1993) noted, there is seldom sufficient justification for assuming that the distribution of e_i belongs to a known parametric family. In addition, heteroscedasticity leads to inconsistency in parametric maximum likelihood estimates of models of discrete events (Yatchew and Griliches 1984). Heteroscedasticity is of particular concern because entries are grouped into five different quality grades on the basis of past performance to ensure competitive racing. Finally, as Greene (1990, p. 683) noted, parametric maximum likelihood estimators are not chosen to maximize a criterion based on correct prediction and thus may not be compatible with optimal prediction.

Conventional maximum likelihood estimation of Equation (2) requires specification of a distribution function that relates $P(\delta = 1)$ to $X\beta$. The binomial logit model has the specification

$$P(\delta = 1|X) = F(X\beta), \quad (3)$$

where $F(\cdot)$ is the logistic cumulative distribution function. As an alternative to maximum likelihood estimators, this analysis uses the nonparametric maximum score (MS) estimators developed by Manski (1975, 1985, 1988) and Horowitz (1992, 1993) to test the expectations hypothesis. MS is a nonparametric estimator based on an optimal prediction rule. Alternative nonparametric estimators were developed by Cosslett (1983), Ichimura (1988), Klein and Spady (1989), Powell, Stock, and Stoker (1989), Chamberlain (1986), and Matzkin (1992). Consistent MS estimation requires only that a single quantile of the error distribution be independent of the explanatory variables. Unlike conventional maximum likelihood procedures, consistency is maintained in the presence of heteroscedasticity (see Manski 1985, 1988).

The MS estimator estimates the parameters of a linear quantile regression model $(X\beta)$ by choosing the β that maximizes a step function representing the sample score:

$$S_{N\alpha}(\beta) \equiv (1/N) \sum_{i=1}^{N} [y_i - (1-2\alpha)] \operatorname{sgn}(X_i\beta), \quad (4)$$

where y_i is equal to 1 for a positive occurrence of the event and -1 otherwise, $\operatorname{sgn}(X_i\beta)$ equals 1 or -1 as $X_i\beta$ is positive or negative, and α is a weighting constant $(0 < \alpha < 1)$ defining the regression quantile. Because β can only be identified up to scale, a normalization rule is needed. Manski's (1975, 1985, 1988) MS estimator uses the normalization $\beta \in R^n$: $\|\beta\| = 1$.

Maximizing the sample score is analogous to finding the parameter estimate that maximizes the weighted average number of correct predictions. The weighting constant α relates the estimated β to the conditional response probabilities such that $X\beta = 0$ corresponds to $P(y = 1|X\beta) = 1 - \alpha, X\beta < 0$ corresponds to $P(y = 1|X\beta) < 1 - \alpha$, and so on. Because there are eight entries in each race, if event outcomes were completely random each entry would have a .125 probability of winning. Thus, we adopt $\alpha = .875$, corresponding to $P(y = 1|X\beta) = .125$ when $X\beta = 0$. If one assumes a monotonic relationship between $X\beta$ and the response probability, the choice of α is relevant only to the precision of the estimated β. The appropriate choice of α under alternative assumptions regarding the relationship between $X\beta$ and the response probabilities was discussed by Manski and Thompson (1989). In this application, a range of constants was considered and $\alpha = .875$ was found to give superior performance. In particular, standard errors for the MS estimator were generally smallest using this weighting constant.

Manski (1975, 1985) and Kim and Pollard (1990) developed the asymptotic theory associated with the MS estimator. Kim and Pollard (1990) showed that the MS estimator has a nonnormal distribution and converges at the relatively slow rate of $N^{-1/3}$ and thus may not be appropriate for making inferences. Manski and Thompson (1986) suggested that bootstrapping may perform well in estimating standard errors for MS parameter estimates (especially for a smooth probability response). Because of its nonstandard distribution and slow rate of convergence, however, inferences for the MS estimator are purely heuristic.

Horowitz (1992, 1993) developed a smoothed version of the MS estimator that has an asymptotically normal limiting distribution. Under Horowitz's approach, the score function is modified so that it becomes continuous and differentiable. The modified score function is

$$S^*_{N\alpha}(\beta) = (1/N) \sum_{i=1}^{N} [y_i - (1-2\alpha)] K(X\beta/h_N), \quad (5)$$

where $K(\cdot)$ is a continuous, twice-differentiable smoothing function and $\{h_N\}$ is a sequence of bandwidths that converge to 0 at the rate $N^{-1/(2k+1)}$, where k is the order of the kernel. In the applications that follow, the cumulative normal probability distribution function (a second-order kernel) is used as the smoothing function. Horowitz (1992) showed that the smoothed MS estimator converges at the rate $N^{-k/(2k+1)}$. Thus, for a second-order kernel, rescaling the centered, bias-corrected estimator by $N^{2/5}$ yields MS estimates that are asymptotically normal. Horowitz (1992) gave the asymptotic theory of the smoothed estimator and discussed methods for removing its asymptotic bias, estimating asymptotic standard errors, and estimating the optimal bandwidth.

As is the case with the unsmoothed estimator, β can be identified only up to scale and a normalization rule is thus necessary. Horowitz (1992, 1993) recommended selecting an element of X, x_i, that has an everywhere positive Lebesgue density and fixing its coefficient such that $|\beta_i| = 1$. Horowitz (1992) suggested using a global maximization method, such as simulated annealing, to obtain starting values for the nonlinear regression.

Nonparametric kernel regression techniques require selection of an appropriate bandwidth parameter. Bandwidth selection procedures were reviewed by Marron (1988). Horowitz (1992) used a plug-in approach. Under the plug-in approach, an initial estimate of the bandwidth (h) is used to obtain preliminary estimates of the MS parameters. From these preliminary estimates, an optimal (mean squared error minimizing) bandwidth can be calculated.

Nonparametric kernel regression of y on $X\beta$ can be used to obtain conditional response probabilities. Response probabilities that are not monotonically increasing in $X\beta$ lead one to question homoscedastic logit or probit specifications. Estimation of the response probabilities implicitly assumes that the conditional distribution of the errors depends on X only through the index $X\beta$ (i.e., the "index-sufficiency" restriction). Under the index-sufficiency restriction, the conditional probability $P(\cdot)$ can be expressed as $E(y|X\beta)$.

The nonparametric kernel regression estimate of the conditional response probability $\hat{P}(\cdot) = \hat{F}(X\hat{\beta})$ is given by

$$\hat{F}(X\hat{\beta}) = \frac{\sum_{i=1}^{N} \{k((X_i\hat{\beta} - X\hat{\beta})/h)\delta_i\}}{\sum_{i=1}^{N} k((X_i\hat{\beta} - X\hat{\beta})/h)}, \quad (6)$$

where δ_i is 1 if the ith dog wins and 0 otherwise, $k(\cdot)$ is a kernel function [see Silverman (1986) for a discussion of kernel function choice], and h is the kernel bandwidth (given by $h = \lambda\hat{\sigma}$, where $\hat{\sigma}$ is the standard deviation of $X\hat{\beta}$ and λ is a smoothing constant). In this application, the logistic density kernel (a second-order kernel) is used and the optimal bandwidths are chosen by minimizing the cross-validation prediction error criterion of Härdle and Marron (1985):

$$\mathrm{CV}(h) = 1/N \sum_{i=1}^{N} [\delta_i - \hat{R}_{-i}(X_i\hat{\beta})]^2, \quad (7)$$

where $\hat{R}_{-i}(X_i\hat{\beta})$ is the kernel estimate excluding the ith observation.

Horowitz (1993) showed that a parametric specification can be tested against a nonparametric alternative by considering whether the parametric specification falls within a confidence band for the nonparametric regression. Härdle (1990, pp. 98–128) discussed determination of pointwise confidence intervals and confidence bands for nonparametric regressions. Confidence intervals for a point can be constructed using the bootstrap procedures proposed by McDonald (1982). Horowitz's (1993, p. 60) Proposition 1 constructed a confidence band for the nonparametric regression function using a modification of Härdle's (1990, pp. 116–117) theorem 4.3.1. Härdle showed that a $(1-\alpha) \times 100\%$ confidence band for the nonparametric regression function $y = \hat{F}(x)$ is given by

$$\hat{F}(x) +/- [c_\alpha/(2\log(1/h)^{1/2} + d_N)] \times [\hat{\sigma}^2(x)/(\hat{f}(x)nh)]^{1/2}/c_K, \quad (8)$$

where

$$\hat{\sigma}(x) = N^{-1}\sum_{i=1}^{N} K((x-x_i)/h)y_i^2/\hat{f}(x) - \hat{F}(x)^2, \quad (9)$$

$$\hat{f}(x) = N^{-1}\sum_{i=1}^{N} K((x-x_i)/h), \quad (10)$$

$$d_N = (2\log(1/h))^{1/2} + (2\log(1/h)^{-1/2}\log(C/(2\pi^2)))^{1/2}, \quad (11)$$

$$c_K = \int_{-\infty}^{\infty} K^2(x)\,dx, \quad (12)$$

and

$$C = (2c_K)^{-1}\int_{-\infty}^{\infty} (K'(x))^2\,dx, \quad (13)$$

and c_α is chosen such that $\exp(-2\exp(-c_\alpha)) = 1-\alpha$. Horowitz's (1993) modification involves using $X\hat{\beta}$ for x, where $\hat{\beta}$ is the maximum likelihood estimate. This is justified by the fact that the maximum likelihood estimator has a faster rate of convergence than the nonparametric estimator. Horowitz (1993) also showed that the MS specification, which assumes that $P(y=1|X\beta=0) = .125$, can be tested by considering whether a pointwise confidence interval at $X\beta = 0$ contains .125.

3. EMPIRICAL APPLICATION AND IN-SAMPLE TESTS

Market odds, a professional handicapper's forecasts, relevant performance variables, and finish statistics were collected from the daily racing program (the *Official Program of the Woodlands*) for 600 races (4,690 observations) at the Woodlands greyhound track. The data were divided into two portions to pursue in-sample and out-of-sample tests of the expectations hypothesis. The first portion contained data for 20 performances (1,912 observations), and the second portion contained data for 28 performances (2,778 observations). The second portion was used for out-of-sample simulations as well as in-sample tests (on a subset of 1,993 observations).

Three models were considered. The first model contained only final odds. The second model contained data relevant to subjective ex ante bettor handicapping, including the first- and second-place forecasts of a professional handicapper, morning-line odds, the average of the entry's five best times from its previous 10 races (average best time), a dummy variable (mid-track post) equal to 1 if the entry started in positions 3–6, and an index representing the entry's past performances (performance index). The dummy variable accounts for the mid-track disadvantage caused by crowding. The performance index is the average of four individual indexes, representing past performances at the same distance, grade, starting position, and track. Each index is the weighted average of the entry's proportions of first-, second-, and third-place finishes (weighted by one-half, one-third, and one-sixth, respectively). For each entry, the two performance indicators were subtracted from the mean for all eight entries, allowing relative comparisons. The third model combined final odds and the handicapping data. Because final odds are not known until after the betting windows close, only Model II can be used for out-of-sample forecasting. Summary statistics for the explanatory variables are contained in Table 1.

Three different estimators were used for each model—a maximum likelihood logit model, the unsmoothed MS estimator, and the smoothed MS estimator. The semiparametric models are of the form $y_{ij} = \mathrm{sgn}[X_{ij}\beta + \varepsilon_{ij}]$, where $y_{ij} = 1$ if the ith entry won the jth race and -1 otherwise and ε_{ij} is an error term. The logit models are of the form $\delta_{ij} = F(X_{ij}\beta) + \varepsilon_{ij}$, where $F(\cdot)$ is the logistic cumulative distribution function (cdf) and $\delta_{ij} = 1$ if the ith entry won and 0 otherwise.

Table 1. Summary Statistics of Explanatory Variables

Variable	Mean	Std. dev.	Minimum	Maximum
Final odds	8.358	6.052	.200	69.700
Morning line odds	7.065	3.723	1.200	15.000
Average best time	.000	.207	−1.595	1.359
Mid-track post	.500	.500	.000	1.000
Handicapper first	.125	.331	.000	1.000
Handicapper second	.125	.332	.000	1.000
Performance index	.000	.059	−.196	.688

The logit, unsmoothed MS, and smoothed MS parameter estimates and relevant statistics for the first subset of data are presented in Table 2. To simplify comparison, parameter estimates and standard errors for the logit and unsmoothed MS models were rescaled by dividing by the absolute value of the estimates of the parameters used for normalization in the smoothed MS estimates (i.e., final odds in Model I and the performance index in Models II and III). This imposes an absolute value of 1 for the coefficients on these variables. Hypothesis tests, normalized scores, and conditional probabilities were calculated using unscaled parameter estimates. The interpretation of the parameters is transparent to the rescaling.

Nonparametric bootstrapping (500 replications) was used to obtain standard errors for the unsmoothed MS estimates. The bootstrap procedure involves randomly drawing with replacement many bootstrap samples of size N (where N is the number of observations) and estimating the parameters for each sample. The deviations of each replicated parameter set from the original estimates are used to construct a covariance matrix. Because the unsmoothed MS estimates have a nonstandard distribution, care must be used in interpreting hypothesis testing results. Asymptotic standard errors are presented for the logit and smoothed MS estimates.

In light of the grouped nature of the data, error terms for dogs within the same race may not be independent. Nonindependence could lead to misleading inferences (Keane and Runkle 1990). An alternative bootstrapping approach is used to recognize such correlation. Individual races are treated as independent events, and bootstrap samples are constructed by randomly drawing 246 races from the estimation data with replacement. Because sampling is from the races rather than from individual entries, within-race correlation is maintained. Standard errors calculated using this form of bootstrapping (200 replications) are also presented in Table 2.

The unsmoothed and smoothed MS parameter estimates for Models I and II are similar although estimates of the standard errors differ substantially in several cases. A similar result was found by Horowitz (1993). In most cases, the unsmoothed MS parameters have smaller standard er-

Table 2. Logit, Unsmoothed, and Smoothed Maximum Score Estimates and Summary Statistics for Models of Parimutuel Market

	ML logit estimates			Maximum score estimates			Smoothed maximum score estimates		
Variable	Model I	Model II	Joint model	Model I	Model II	Joint model	Model I	Model II	Joint model
Constant	4.1379	−.3807	−.4387	6.6985	.1548	36.0579	6.8525	.1499	35.9516
	(.7258)[a]	(.0566)	(.0894)	(.0437)	(.1928)	(1.5068)	(.3975)	(.1529)	(29.5699)
	[.8795][b]	[.0655]	[.0978]	[.0423]	[.2079]	[4.4265]	[.0928]	[.1167]	[12.0914]
Final odds	−1.0000		−.0382	−1.0000		−6.5743	−1.0000[c]		−6.6559
	(.1120)		(.0894)	(.2101)		(.7320)			(5.4194)
	[.1686]		[.0419]	[.1944]		[3.6079]			[2.1989]
Morning line odds		−.0129	.0034		−.0477	1.2537		−.0458	1.3387
		(.0067)	(.0111)		(.0373)	(.6832)		(.0231)	(1.0376)
		[.0073]	[.0692]		[.0397]	[2.6064]		[.0248]	[.4301]
Average best time		−.4418	−.4966		−1.4895	−37.3388		−1.3717	−37.4194
		(.1037)	(.1572)		(.1833)	(2.2790)		(.6213)	(30.0753)
		[.1207]	[.1751]		[.2636]	[7.6036]		[.5414]	[11.7796]
Mid-track post		−.0679	−.0722		.0715	−9.5450		.0516	−9.2581
		(.0330)	(.0496)		(.1782)	(2.3343)		(.1744)	(7.0161)
		[.0361]	[.0539]		[.3447]	[8.8343]		[.1394]	[.4301]
Handicapper first		.0871	.0787		.1701	−7.5726		.1728	−7.6667
		(.0491)	(.0734)		(.1661)	(2.9223)		(.6838)	(7.0753)
		[.0569]	[.0833]		[.2845]	[11.6895]		[.1871]	[3.1613]
Handicapper second		.0128	−.0069		.1710	−1.5581		.1579	−1.6022
		(.0498)	(.0739)		(.1561)	(1.9498)		(.1629)	(1.6022)
		[.0525]	[.0777]		[.9227]	[8.1396]		[.2252]	[4.5699]
Performance index		1.0000	1.0000		1.0000	1.0000		1.000	1.000
		(.2867)	(.4372)		(.1630)	(1.5799)			
		[.3259]	[.4626]		[1.1017]	[6.7663]			
Maximum normalized score	.4964	.5092	.5062	.6715	.6584	.7033	.6697	.6510	.7016
Proportion of correct predictions in-sample using sgn($X\beta$)	.8734	.8740	.8750	.5915	.6344	.6553	.5853	.6370	.6506
Proportion of correct predictions out-of-sample using sgn($X\beta$)	.8737	.8747	.8751	.5708	.6171	.6349	.5644	.6271	.6361
Proportion of correct wins in-sample using conditional Pr($X\beta$)	.2967	.2846	.2927	.2967	.2846	.3172	.2967	.2927	.2764
Proportion of correct wins out-of-sample using conditional Pr($X\beta$)	.3220	.2910	.3220	.3220	.3051	.2764	.3220	.3023	.3051

[a] Numbers in parentheses are approximate standard errors. For the logit and smoothed maximum score models, standard errors are given by asymptotic estimates. For the unsmoothed maximum score model, standard errors are calculated by bootstrapping.

[b] Numbers in brackets are bootstrapped standard errors allowing for within-race correlation.

[c] Parameter values fixed by normalization. Parameter estimates and standard errors for logit and MS models are rescaled by dividing by $|\beta_k|$.

Table 3. Within-Sample Wald Tests of the Expectations Hypothesis

Description of test	ML logit estimates			Maximum score estimates			Smoothed maximum score estimates		
	Test statistic	df	p value	Test statistic	df	p value	Test statistic	df	p value
Test of bettor handicapping data in joint model for first subsample	19.6771	6	.0032	236.8301	6	.0001	33.5774	5	.0001 [.0200]*
Test of bettor handicapping data in joint model for second subsample	25.1814	6	.0001	23.7681	6	.0006	40.6076	5	.0001 [.0570]
Test of handicapper forecasts in joint model for first subsample	1.4973	2	.4730	8.1591	2	.0169	2.9302	2	.2429 [.0700]
Test of handicapper forecasts in joint model for second subsample	4.7979	2	.0908	1.4012	2	.4963	15.2753	2	.0005 [.2266]
Test of bettor handicapping data in joint model for first subsample with correction for within-race correlation	14.7843	6	.0220	37.9564	6	.0001	19.7426	5	.0014 [.0600]
Test of bettor handicapping data in joint model for second subsample with correction for within-race correlation	24.2553	6	.0004	42.7779	6	.0001	94.6537	5	.0001 [.0206]
Test of handicapper forecasts in joint model for first subsample with correction for within-race correlation	1.2279	2	.5412	1.4153	2	.4928	3.6462	2	.1615 [.1200]
Test of handicapper forecasts in joint model for second subsample with correction for within-race correlation	4.0949	2	.1291	6.07815	2	.0479	4.5541	2	.1026 [.1281]

* Numbers in brackets are approximate p values obtained from Beran's (1986, 1988) bootstrap method. For purposes of comparison, chi-squared critical values at the $\alpha = .05$ level are 12.59 (df = 6), 11.07 (df = 5), and 5.99 (df = 2). Because their distribution may be nonstandard, tests for the unsmoothed estimates provide heuristic evidence only and thus should be viewed cautiously.

rors than the asymptotic standard errors of the smoothed MS parameters. This is especially true for the joint model. Because the unsmoothed MS estimates have a nonstandard distribution and slower rate of convergence, however, inferences are of a heuristic nature only.

Two local maxima, differing primarily in the performance-index parameter estimate, were revealed for the unsmoothed MS estimates of the joint model. The global maximum had a small performance-index coefficient, implying relatively large values for other parameters when the estimates were rescaled. Smoothed and unsmoothed MS estimates for the joint model are similar. The smoothed MS estimates of the joint model were sensitive to the initial bandwidth. Parameter estimates varied by as much as 10% when the bandwidth was doubled. The maximized score and hypothesis testing results were similar, however, across a range of initial bandwidths. This may reflect correlation between final odds and the handicapping data. Models I and II were not sensitive to the initial bandwidth choice.

The logit parameter estimates differ substantially from the MS estimates in many cases. Standard errors estimated by the two alternative methods are quite similar in most cases. The standard errors that allow for within-race correlation are slightly larger in most cases than those that do not allow for such correlation.

The normalized score, reflecting the weighted average of correct predictions, is given by

$$\tilde{S}_{N\alpha}(\beta) \equiv (1/N)\left[S_{N\alpha}\Big/\left((1/N)\sum_{i=1}^{N}|y_i-(1-2\alpha)|\right)+1\right]. \tag{14}$$

Maximizing the normalized score is equivalent to maximizing the sample score. If $\alpha = .5$, the normalized score represents the unweighted fraction of the observations correctly classified. In our case, where $\alpha = .875$, greater weight is placed on predicting winners than losers. For both MS estimators, the joint models yield the highest normalized scores, followed by Model I. Normalized scores for the logit estimates are considerably below those of the MS estimates. Using the sign of $X\beta$ as a scoring rule, the joint models yielded the largest number of correct in-sample and out-of-sample predictions, followed by Model II. Using the sign of $X\beta$ as a scoring rule, the logit estimates generate the greatest proportions of correct predictions, both in-sample and out-of-sample. Because the logit specification is centered on $X\beta = 0$ [such that $P(y=1|X\beta) \to .5$ as $X\beta \to 0$], however, the logit model rarely predicts entries to win. Because 87.5% of the dogs do indeed lose, this prediction strategy generates the most correct predictions.

The expectations hypothesis is tested by evaluating the significance of the handicapper's forecasts and other handicapping data in a joint model also containing final odds. Wald tests of the expectations hypothesis are presented in Table 3. For the first subsample, five of six tests indicate that the handicapper's forecasts are not significant. Similar conclusions are reached by four of the six tests for the second subsample. Two of the three tests suggesting a significant effect for the handicapper's forecasts are for the unsmoothed MS estimator and thus are suspect. Results are similar when an allowance is made for within-race correlation.

In contrast, Wald tests of the lack of significance of the bettor handicapping data in the presence of final odds are rejected in every case, suggesting that expectations of win

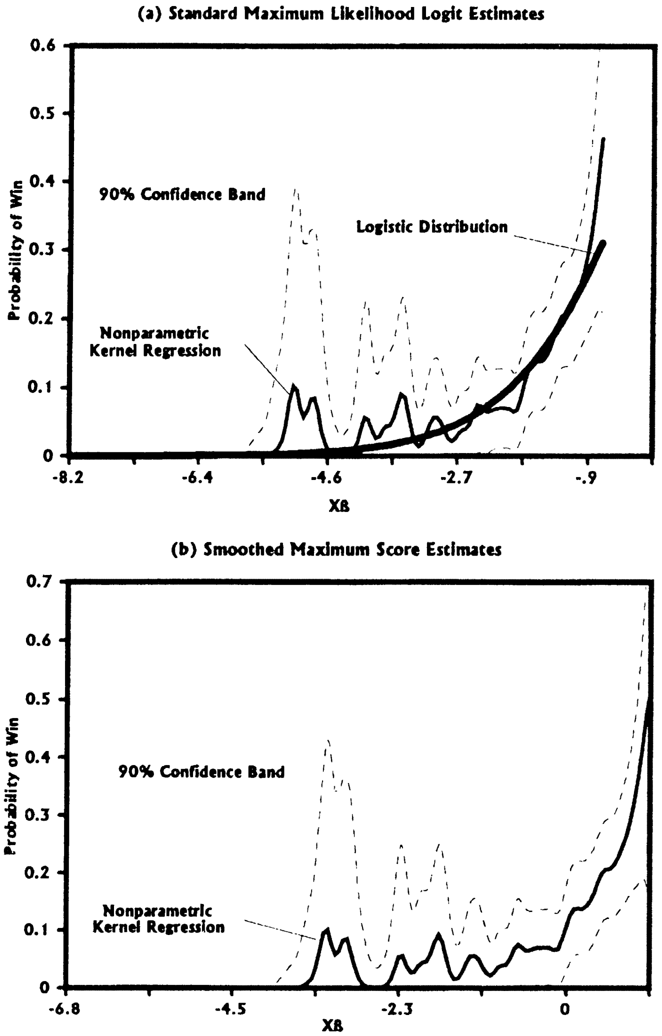

Figure 1. Nonparametric Estimates of the Probability of a Win for Model of Market Odds (Model I).

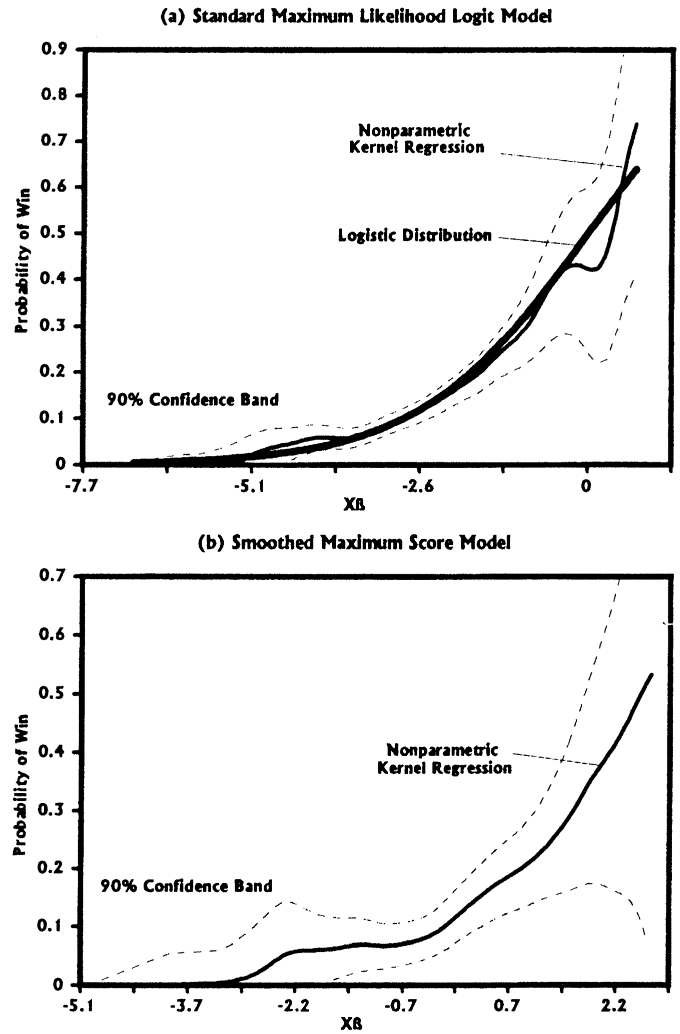

Figure 2. Nonparametric Estimates of the Probability of a Win for Model of Bettor Handicapping Data (Model II).

probabilities made conditional on handicapping data are significantly different from the market odds. This suggests that performance information could be used to improve finish forecasts and thus rejects the expectations hypothesis. Similar results are obtained when within-race correlation is recognized. Note that the tests are conditional on the model's linear specification.

Horowitz (1992) found that asymptotic standard errors for the smoothed MS estimates may be inaccurate in small samples. To verify test results based on asymptotic theory, Horowitz (1992, 1993) suggested applying the prepivoting bootstrap techniques of Beran (1986, 1988) to construct simulated null distributions for the test statistics. Under this approach, the parameters are first estimated from the sample data. Bootstrap samples of size N are then randomly drawn from the estimation data with replacement. For each sample, the parameters are estimated along with their covariance matrices and the null hypothesis that each replicated parameter set is equal to the original estimates is tested. The distribution of the replicated test statistics is used to identify appropriate critical values and probability values for the test statistics.

Beran's procedures were applied to the smoothed MS estimates. The estimated $\alpha = .05$-level critical value for the Wald test of the bettor handicapping data in the first subsample is 18.89. This is above the chi-squared critical value (12.59) but leads to the same conclusion. The simulated distribution implies a p value of .02 for the test statistic. The simulated $\alpha = .05$ critical value for the Wald test of the handicapper's forecasts was 4.89 for the first subsample, which verifies the original test. Similar results were obtained for the second subset of data. The simulated distributions implied p values of .06 and .23 for the tests of the bettor handicapping data and handicapper's forecasts, respectively. When the prepivoted bootstrapping was repeated using random draws taken from the races, similar results and conclusions were obtained.

Nonparametric kernel regression estimates of the response probabilities for the logit and smoothed MS estimators are presented in Figures 1–3. The estimates were very similar for the smoothed and unsmoothed MS parameter estimates. Thus, probability responses for the unsmoothed MS estimates are not presented. To simplify comparisons, the $X\beta$'s are rescaled to have unitary variances by dividing through by the indexes' respective standard deviations.

Nonparametric estimates of the probability responses for Model I are presented in Figure 1 along with 90% confi-

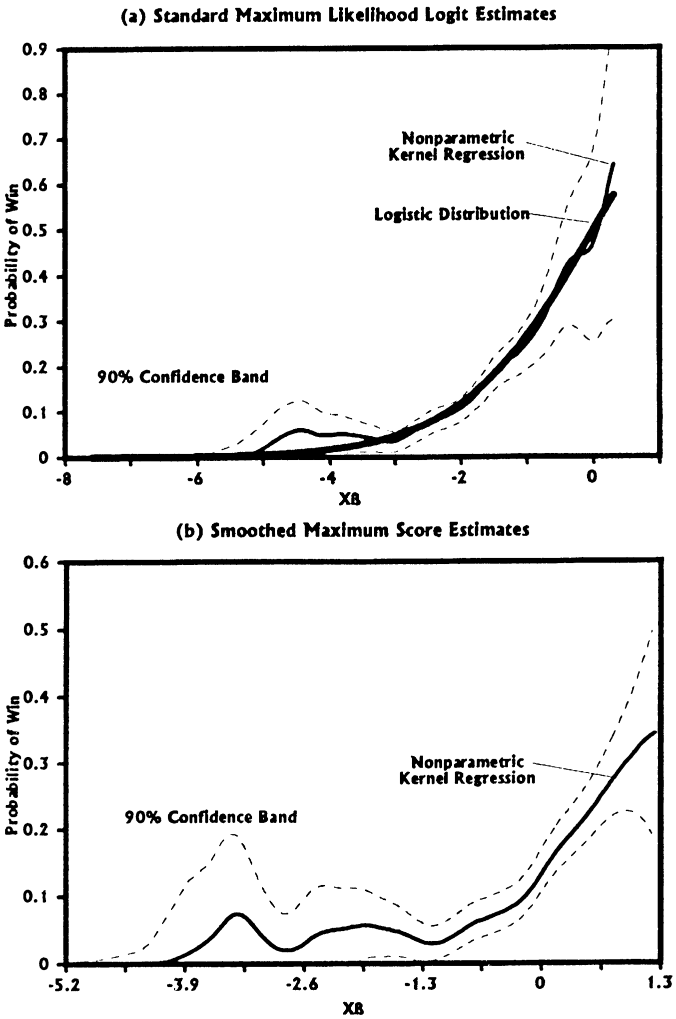

Figure 3. Nonparametric Estimates of the Probability of a Win for Joint Model.

dence bands. Panel (a) contains estimates calculated from the logit parameter estimates along with the logistic cdf. The conditional response probabilities for Model I are not monotonic and do not resemble the logistic cdf. Thus, a parametric, homoscedastic logit model seems inappropriate. The logistic cdf, however, is entirely contained within the 90% confidence band. Thus, the logit specification is not rejected at the $\alpha = .10$ level of significance.

Panel (b) of Figure 1 contains the nonparametric estimate of the response probability for the smoothed MS estimates. At $X\beta = 0$, the implied conditional probability is .1073 and the confidence band contains .125. Following Horowitz (1993), a specification test statistic was constructed by dividing the absolute value of the deviation of the conditional probability from .125 by the bootstrapped estimate of the (pointwise) standard deviation of the kernel regression at $X\beta = 0$. The statistic's value of .5668 was below the standard normal critical value of 1.65 at the $\alpha = .10$ level. Thus, the data appear to be consistent with the smoothed MS specification.

Nonparametric kernel regressions representing conditional probabilities for logit and smoothed MS estimates of Model II are presented in Figure 2. The probability responses are generally monotonic and closely resemble a logistic distribution. The 90% confidence band for the logit specification [Panel (a)] entirely contains the logistic cdf and thus supports the logit specification. The probability response for the smoothed MS estimates of Model II [Panel (b)] closely resembles a logistic cdf. At $X\beta = 0$, the conditional probability is .1154. The test statistic for $P(y = 1|X\beta = 0) = .125$ is .7102, which supports the smoothed MS specification.

Conditional probability response estimates for the joint model are presented in Figure 3. The responses are not entirely monotonic, but more closely resemble the logistic cdf than was the case for Model I. The confidence band contains the logistic cdf and thus supports the logit specification. For the smoothed MS specification, the conditional probability is .1325 at $X\beta = 0$. The specification test statistic is .3609, which supports the smoothed MS specification.

Nonparametric regression estimates may be sensitive to the choice of bandwidth and kernel function. The regressions were reestimated using the normal density kernel function, the Epanechnikov kernel function, and the triangular kernel function and were found to yield nearly indistinguishable results. The bandwidth parameter affects the shape of the regression function and thus may affect the monotonicity of the probability response. The response for Model II remained nonmonotonic for bandwidth parameters up to four times larger than the optimal value implied by cross-validation. Specification tests imply the same conclusions in every case.

Under the index-sufficiency assumption, the response probabilities represent forecasts of win probabilities. Using predicted win probabilities, Model I correctly predicts the winner in 29.7% of the races, whereas Model II is correct 28.5% to 29.3% of the time, depending on the estimator. The in-sample and out-of-sample predictive power of the alternative estimators is similar. The professional handicapper correctly predicted the winner in 25.6% of the races.

4. OUT-OF-SAMPLE GAMBLING SIMULATIONS AND TESTS

The professional handicapper's forecasts and the ex ante conditional probability forecasts generated from the MS estimates can be used to pursue an alternative test of the expectations hypothesis. Using predicted finish orders, simulated wagers can be used to determine whether rates of return significantly above those expected for the market (−18% for win, place, and show bets, −19% for exacta and quinella bets, and −20% for trifecta wagers) could have been earned. Simulated returns above expected rates reject the expectations hypothesis. Of course, the alternative wagers are not independent because all are constructed from the same set of information.

Simulated win, place, show, quinella, exacta, and trifecta wagers were conducted using the professional handicapper's forecasts and the ex ante conditional probability forecasts generated from MS estimates of Model II for 354 races from 28 consecutive performances. Simulated wagers were placed on the entry with the highest forecasted probability of winning and on the professional handicapper's

Table 4. *Out-of-Sample Gambling Simulation Results: Rates of Return to Gambling (%)*

	Rate of return using handicapper's forecasts (%)						Rate of return using econometric model forecasts (%)					
Performance	Win	Place	Show	Quinella	Exacta	Trifecta	Win	Place	Show	Quinella	Exacta	Trifecta
1	83.57	4.29	−20.00	−76.92	−100.00	−100.00	−1.43	−10.00	−9.29	50.00	−100.00	−100.00
2	76.15	56.92	20.77	45.00	750.00	−100.00	−36.92	−6.15	−17.69	−18.33	335.00	−100.00
3	−24.17	−33.33	−26.67	−14.55	163.33	−100.00	5.83	−16.67	26.67	−59.09	−100.00	−100.00
4	−55.00	−32.92	−17.92	−75.45	−100.00	−100.00	45.00	27.92	13.75	99.09	290.00	10.00
5	−5.00	−0.83	0.83	14.55	−100.00	472.00	30.00	−20.83	0.00	71.82	−100.00	347.27
6	−9.17	−35.00	−5.00	122.73	−100.00	−100.00	−9.17	16.67	10.83	−41.82	−100.00	−100.00
7	−2.50	−23.33	−22.50	−28.18	−100.00	−100.00	−31.67	23.33	−10.00	92.73	−100.00	−100.00
8	−15.83	−26.67	−24.17	−11.82	−75.00	−100.00	50.83	−10.00	−11.67	256.36	−100.00	−100.00
9	−9.29	−26.43	−29.29	−30.77	47.50	−100.00	44.29	−15.00	8.57	8.46	−100.00	−100.00
10	−65.33	−39.33	−46.00	−100.00	−100.00	−100.00	−28.67	1.33	−19.33	−57.14	−100.00	−100.00
11	46.67	20.00	−1.67	−100.00	−100.00	−100.00	146.67	66.67	25.00	70.91	−100.00	1391.00
12	−20.00	−26.67	−30.83	−100.00	−100.00	−100.00	27.50	−4.17	−22.50	−100.00	−100.00	−100.00
13	−60.00	−39.17	−50.83	71.82	−100.00	−100.00	−60.00	−10.00	−36.67	71.82	−100.00	−100.00
14	−18.33	−12.50	−18.75	−40.00	90.00	465.56	−2.50	−24.17	−7.92	−100.00	−100.00	−100.00
15	−0.77	−13.08	−28.46	195.83	−100.00	−100.00	12.31	10.77	−0.77	96.67	−100.00	−628.00
16	74.17	5.83	−10.83	−100.00	−100.00	−100.00	−6.67	−22.50	9.17	−100.00	−100.00	−100.00
17	−44.17	13.33	−23.33	−12.73	−100.00	−100.00	−40.00	60.00	45.00	−100.00	−100.00	−100.00
18	−26.43	−22.86	−42.86	36.92	−100.00	−100.00	38.57	14.29	10.00	−24.29	−100.00	−100.00
19	0.71	−9.29	11.07	−14.62	−100.00	−100.00	−0.71	−3.57	−0.36	37.86	216.67	−30.83
20	−50.83	−30.83	−35.83	−68.18	−100.00	−100.00	−0.83	−47.50	−12.50	41.82	−15.00	814.44
21	−14.62	−20.00	−12.31	−100.00	−100.00	−100.00	−6.15	10.00	16.15	1.67	−100.00	−100.00
22	11.67	−16.67	−20.00	−47.27	−100.00	−100.00	4.17	0.00	−0.83	−25.45	64.00	−100.00
23	−15.83	2.50	2.08	9.09	176.67	−100.00	−1.67	17.50	10.42	9.09	−100.00	−100.00
24	−54.17	−54.17	4.17	12.50	186.67	−100.00	−54.17	−20.00	15.83	−60.00	−100.00	−100.00
25	−78.33	−51.67	−18.33	−82.73	−100.00	−100.00	−30.83	12.50	20.00	−81.00	−100.00	−100.00
26	25.00	−5.83	−3.33	29.09	−100.00	−100.00	−41.67	−22.50	4.17	32.73	340.00	−100.00
27	0.77	−23.08	−4.62	71.67	−100.00	−100.00	−22.14	−20.00	−21.43	10.77	3.33	−100.00
28	−52.86	−26.43	−34.29	−42.31	−100.00	−100.00	0.77	13.85	28.46	260.00	−100.00	−100.00
Total net returns	−75.80	−117.70	−127.20	−102.60	−49.40	−347.80	7.40	4.50	16.40	113.60	−49.20	169.00
Average rate of return	−10.85	−16.69	−17.46	−15.58	−27.17	−59.37	1.10	0.78	2.61	15.88	−30.93	34.28
t test of expectations hypothesis*	.90	.30	.16	.25	−.24	−1.40	2.45*	4.00*	5.93*	1.97*	−.44	.82
Wilcoxon signed-rank test of expectations hypothesis*	.68	−.11	.07	−.18	−1.62	−3.37	2.25*	3.51*	4.10*	1.69*	−1.59	−2.16

* Tests of equality of mean rates of return to expected aggregate market values. Critical values at α = .05 for one-tailed tests are 1.70 for the *t* statistics and 1.65 for the Wilcoxon signed-rank statistics. An asterisk indicates rejection of the null hypothesis at the .05 level.

forecasted winner in each race. For the quinella, exacta, and trifecta wagers, the entries with the first, second, and third highest win probabilities and the handicapper's predicted first, second, and third place finishers were used to construct simulated bets.

Simulated rates of return and net returns to $2 bets are presented in Table 4. In reality, the simulated bets would have had a very small effect on the odds and payouts. This effect is negligible, however, because the average pool per race at the Woodlands in 1990—1991 was over $30,000 (Kansas Racing Commission 1991). One-tailed t tests were used to determine whether the average rates of return were significantly above those expected for the market. Vannebo (1980) and Gabriel and Marsden (1990) argued that gambling returns are often skewed and thus that nonparametric tests should be used. Thus, Wilcoxon matched-pairs signed-ranks nonparametric tests that assumed values of -18% to -20% for the comparison groups were also applied.

Average rates of return to gambling using the handicapper's forecasts are all negative and none are significantly above expected market rates. Thus, the results support the expectations hypothesis and suggest that professional handicappers' forecasts are fully incorporated in market expectations. The simulation using predictions from MS estimates of Model II, however, suggests that positive rates of return could have been earned for five of the six wagers. Four wagers have average rates of return that are significantly above expected market rates. The results are confirmed both by one-tailed t tests and by nonparametric Wilcoxon ranked-signs tests. The largest significant rate of return was earned by quinella wagers, which would have paid a bettor $113.60 (a 15.88% average rate of return). These results imply a rejection of the expectations hypothesis in that available data could have been used to improve market forecasts and generate above-normal profits. Because of an unusually large return to win wagers, performance 11 could be suspected to be an outlier. When this performance is excluded, the average rate of return to win wagers falls to -4.3% and the t statistic falls to 2.34, leading to the same conclusions.

5. SUMMARY AND CONCLUDING REMARKS

This investigation tests the expectations hypothesis in the parimutuel gambling market for greyhound racing. The article argues that conventional tests may be flawed because of difficulties in defining relevant prices, returns, and information and because of aggregation biases. In addition, misspecification of the statistical distribution governing agents' expectations may bias conventional tests. As an alternative, this analysis considers nonparametric (distribution-free) tests. Expert forecasts and econometric forecasts generated from market data are compared to the aggregate market's forecasts, which are represented by the parimutuel odds.

The results suggest that the expert opinions of professional handicappers cannot be used to improve forecasts of finish probabilities but that other relevant factual performance data are not completely incorporated into market forecasts and expected returns. Similar results are obtained by a parametric logit model and by semiparametric maximum score estimators. The results are supported both by in-sample and out-of-sample tests. The findings suggest that opportunities to earn returns above those expected for the aggregate market existed, implying a rejection of the expectations hypothesis. Several caveats apply to the results in that they were generated over a short (two-and-one-quarter month) period of study. In addition, the costs of collecting information and generating forecasts were ignored. Future research may profit from extending the out-of-sample applications of the forecasting models.

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