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VENTURE THEORY: A MODEL OF DECISION WEIGHTS*

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Several theories suggest that people replace probabilities by decision weights when evaluating risky outcomes. This paper proposes a model, called venture theory, of how people assess decision weights. It is assumed that people first anchor on a stated probability and then adjust this by mentally simulating other possible values. The amount of mental simulation is affected by the absolute size of payoffs, the extent to which the anchor deviates from the extremes of 0 and 1, and the level of perceived ambiguity concerning the relevant probability. The net effect of the adjustment (i.e., up or down vis-à-vis the anchor) reflects the relative weight given in imagination to values above as opposed to below the anchor. This, in turn, is taken to be a function of both individual and situational variables, and in particular, the sign and size of payoffs. Cognitive and motivational factors therefore both play important roles in determining decision weights. Assuming that people evaluate outcomes by a prospect theory value function (Kahneman and Tversky 1979) and are cautious in the face of risk, fourteen predictions are derived concerning attitudes toward risk and ambiguity as functions of different levels of payoffs and probabilities. The results of three experiments are reported. Whereas only a subset of the model's predictions can be tested in Experiment 1, all fourteen are tested in Experiments 2 and 3 using hypothetical and real payoffs, respectively. Several of the model's predictions are not supported in Experiment 2 but almost all are validated in Experiments 1 and 3. The failures relate to the exact nature of probability \times payoff interactions in attitudes toward risk and ambiguity for losses. The theory and results are discussed in relation to other experimental evidence, future tests of the theory, alternative models of risky choice, and implications of venture theory for explaining further phenomena.
(RISK; AMBIGUITY; PROBABILITY \times UTILITY INTERACTIONS)

Recent years have seen a surge of interest in models of risky decision making. (For overviews, see Schoemaker 1982; Tversky and Kahneman 1986; Fishburn 1988; Machina 1987; Weber and Camerer 1987.) Much of this research attempts to develop prescriptively appealing axioms for risky choice that can account for certain well-established paradoxes. In some cases, appeal is made to additional, psychological arguments such as notions of regret (see e.g., Bell 1982; Loomes and Sugden 1982). The present paper does not follow an axiomatic approach; nor is our intention prescriptive. Instead, we seek to develop a *descriptive* model that can account for choice behavior in a wide range of situations involving risk and uncertainty.

There are many ways of modeling choice under risk and uncertainty. The view adopted here is that the value of an outcome received following a choice made under certainty does not differ intrinsically from the value of the same outcome received following a choice made under risk or uncertainty. Thus, for example, \$100 has the same value to a person whether it was won in a lottery, acquired for performing a specific task, and so on. On the other hand, what does change is how, prior to making a choice, a person evaluates probabilistic information bearing on the attractiveness of taking an action that could lead to the outcome. We therefore model the subjective evaluation of decision outcomes by psychophysical functions while the weights given to probabilities are conceptualized as the end result of mental processes that reflect both cognitive and motivational factors. Specifically, venture theory uses the value function proposed in Kahneman and Tversky's (1979) prospect theory and provides an account of how decision

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weights are influenced by variables that are both cognitive and motivational in origin. In common with many theories in which probabilities are “distorted” (see, e.g., Karmarkar 1978; Quiggin 1982), venture theory can be used to explain the well-known choice paradoxes such as those proposed by both Allais (1953) and Ellsberg (1961). However, such explanations are not the focus of this paper. Instead we use venture theory to explore the nature of probability \times utility interactions.

The paper is organized as follows. We first discuss some of the issues underlying people’s use of probabilistic information in risky decision making. This leads to the development of the venture theory model of decision weights. By combining venture theory decision weights with the value function of prospect theory (Kahneman and Tversky 1979), we make a series of predictions concerning attitudes toward risk and ambiguity as a function of different levels of probabilities and payoffs. These predictions are then tested in three experiments. Finally, we discuss venture theory in relation to (a) our own and other experimental findings, (b) future tests of the theory, (c) alternative models of risky choice, and (d) implications for further work.

Probabilities \neq Decision Weights

To motivate our theory, consider first the distinction between the value assigned to receiving an outcome denoted x under conditions of both certainty and uncertainty. When certain of receiving the outcome, assume that the value assigned to it is $v(x)$ where $v(\cdot)$ represents a prospect theory value function in which (i) outcomes are evaluated as gains or losses relative to a reference point, (ii) $v(\cdot)$ is concave over gains but convex over losses, and (iii) $v(\cdot)$ is steeper for losses than for gains (Kahneman and Tversky 1979). Assume further that under certainty of *not* receiving the outcome its value is 0. Thus, it is reasonable to conclude that the value of receiving the outcome under uncertainty lies somewhere between 0 and $v(x)$. This, in turn, implies that under uncertainty $v(x)$ has been discounted by some factor or “decision weight” that is bounded between 0 and 1. To those familiar with the axioms of decision theory, it seems natural to use probability as a decision weight. However, this hides important psychological concerns. These center on the meaning of probability, the effects of uncertainty or ambiguity about probability estimates, and possible effects due to the sign and/or size of outcomes.

The meaning of probability has long been controversial. Whereas subjectivists, following de Finetti (1937) and Savage (1954), are willing to equate probabilities with the general concept of “degrees of belief,” we maintain that most people’s intuitions about probabilities are more narrowly equated with the concept of long-run relative frequencies (cf. Lopes 1981; Keren and Wagenaar 1987). In teaching probabilistic concepts, for example, most introductory statistics texts use familiar gambling devices such as dice thereby implicitly, and often explicitly, motivating the concept of probability as the limit of long-run relative frequency. In addition, when students first encounter expected utility theory (von Neumann and Morgenstern 1947), many find the notion of applying the expected utility principle to one-shot as well as multiple plays of a gamble counterintuitive. In other words, whereas the use of probability as a decision weight is intuitively appealing when considering the value of multiple plays of a gamble, this notion does not have the same appeal for single-shot gambles. To illustrate, consider the difference in observable outcomes between playing, once and many times, a gamble characterized by a p -chance of winning $\$x$ and a $(1 - p)$ chance of $\$0$. In the multiple-shot case, whereas many different outcomes are possible, it is highly probable that the *net* outcome will be close to the expected value, and more so the greater the number of plays. Thus, the use of probability as a decision weight in characterizing the net expected outcome does correspond to some physical reality. In contrast, a similar calculation in the single-shot case has little meaning in that the extreme values of $\$x$ and $\$0$ are the only two possible

outcomes. Thus, from a *psychological* viewpoint there is greater uncertainty concerning the outcome of a single gamble as opposed to the net outcome of multiple plays even when the probability determining outcomes is the same in both cases. In what follows, we refer to this additional source of uncertainty as *outcome uncertainty*.

In experimental work on risky decision making, probabilities are typically provided or assumed to be known with precision. However, this contrasts with experience in the real world which is more commonly characterized by vagueness or ambiguity concerning probabilities. Moreover, as originally demonstrated by Ellsberg (1961), people do not weight known and ambiguous probabilities equally in choice (see also Einhorn & Hogarth 1985, 1986). Thus, any theory of decision weights must also account for the effects of *ambiguity* or vagueness about probabilities.

There is a common intuition that the nature of outcomes (i.e., sign and size of payoffs) can affect the weight given to probabilities in decision making. Indeed, from a prescriptive viewpoint an advantage of expected utility theory is that it exhorts people to assess probabilities and utilities independently in order to avoid the traps of “wishful thinking” or “persecution mania.” (However, see Kadane and Winkler 1988.) Descriptively, the nature of probability \times utility interactions has been hard to specify from empirical studies. Some researchers (e.g., Edwards 1962) have argued for the existence of effects due to sign of payoff but not size; however, since theories have not explicitly predicted when and where such interactions might occur, the evidence for or against interactions is hard to evaluate. We shall argue that payoffs do exert an influence on decision weights but that, in some circumstances, these effects are offset by other variables.

The Decision Weight Model

The process and rationale underlying venture theory are similar to those of the ambiguity model proposed by Einhorn and Hogarth (1985, 1986). See also Hogarth (1989). The key notion is that decision weights used to discount values of outcomes for uncertainty are the end result of a process that involves first anchoring on an estimate of probability and then adjusting this by imagining other possible values for the weights. In the typical experimental task, the anchor is the probability supplied by the experimenter; in more realistic situations, it could be a figure suggested by experience, an estimate provided by an expert or from statistical data, and so on. The adjustment is the net effect of a mental simulation process in which the decision maker “tries out” various weights suggested by different possible scenarios. This process, we argue, is affected by the presence of outcome uncertainty, the sign and size of payoffs, and ambiguity.

To illustrate, imagine being faced with a 0.5 chance of winning $\$x$ (and a 0.5 chance of $\$0$) in a simple, one-shot gamble. First, note that this situation is characterized by outcome uncertainty in that although the probability of winning is 0.5, the outcome can only be either $\$x$ or $\$0$. Indeed, it is the presence of outcome uncertainty that encourages mental simulation of different possible decision weights. Second, although outcome uncertainty encourages mental simulation, one would not expect people to expend the same amount of energy in imagining different outcomes irrespective of the amount at stake, i.e., the size of x . Instead, it seems reasonable to assume that the greater the payoffs, the more people invest in mental simulation, i.e., they think more about large as opposed to small payoffs. Third, depending on the sign of x , equal weight would not necessarily be accorded in imagination to values above and below 0.5. Indeed, assuming caution in the face of risk, positive payoffs would lead to more weight being given to possible values below rather than above 0.5; for negative payoffs, it would be the reverse. Finally, consider the situation where 0.5 is the best estimate of an ambiguous probability. Relative to cases involving known probabilities, ambiguity would increase uncertainty thereby inducing more simulation of alternative decision weights.

This process can be represented algebraically in the form

$$w(p_A) = p_A + k \quad (1)$$

where p_A is the anchor and k the adjustment. In our theory, $k = (k_g - k_s)$ is the *net* effect of simulating values greater and smaller than the anchor where k_g represents the weight accorded to possible decision weights greater than the anchor, and k_s corresponds to the weighted values below the anchor. More specifically, k_g (k_s) is a composite of the range of values greater (smaller) than the anchor and the importance attached to those values. To make the model operational, it is necessary to specify (1) how the anchor, p_A , is established, (2) what affects the *amount* of mental simulation, and (3) what determines the *sign* or direction of the adjustment process.

(1) As implied in the example above, when the probability of obtaining an outcome is known, this forms the anchor. In ambiguous circumstances, the anchor is assumed to be some initial value of the probability that is typically available to the decision maker. This may be a figure based on historical data, provided by experts, or a judgment founded on other sources of information including memory.

(2) The amount of mental simulation is assumed to increase with outcome uncertainty, the size of payoffs, and ambiguity. Consider first the effect of outcome uncertainty for a fixed level of payoffs when there is no ambiguity. For a two-outcome gamble, there is no outcome uncertainty when the probability of obtaining one of the outcomes is known to be either 0 or 1. For a probability of 0.5, however, outcome uncertainty is maximized. Thus both outcome uncertainty and mental simulation increase from 0 to a maximum as the probability increases from 0 to 0.5 and decreases from 1 to 0.5. The effects of both payoff size and ambiguity are to increase the amount of mental simulation. For the former, the rationale is that large payoffs attract more attention than small ones thereby implying more simulation of alternative decision weights; for the latter, because ambiguity increases uncertainty about the probabilities, it is assumed to increase the amount of mental simulation over and above the nonambiguous case. However, it is important to note that, unlike outcome uncertainty, anchors of 0 or 1 do not imply lack of mental simulation in the presence of ambiguity. (We return to this point below.)

(3) The sign of the adjustment process (i.e., k), reflects two factors. These are (a) the location of p_A , and (b) the relative weight given to imagined values above and below the anchor. The location of p_A affects the net effect of the adjustment process in that, if $p_A = 0$, the adjustment must be nonnegative (because $k_s = 0$), and nonpositive if $p_A = 1$ (because $k_g = 0$). It also follows that for small values of p_A there is a greater range of values that can be imagined above the anchor than below it; for large values of p_A , it is the reverse.

Two kinds of variables can affect the weighting of values above and below the anchor. First, there are individual differences. For example, when assessing the chances of obtaining a good outcome (e.g., a large sum of money), people may differ in the extent to which they imagine values above and below the anchor. Second, differential weight reflects the context of the decision. In this paper, we assume that people are cautious rather than reckless when taking decisions under uncertainty (see, e.g., the literature on “defensive pessimism,” Norem and Cantor 1986). For decisions involving good or positive payoffs, values greater than the anchor are given less weight than those below; moreover, the degree of differential weighting increases with the size of the payoffs. Conversely, bad or negative payoffs imply that greater weight is accorded to values above rather than below the anchor, and the extent of differential weighting increases with the absolute size of the negative payoffs. In other words, cautious or “defensively pessimistic” behavior is generally characterized by “underweighting” probabilities of gains and “overweighting” probabilities of losses where the absolute size of payoffs affects the degree of “under-” and “overweighting.”

The assumptions concerning the sign and size of the adjustment in equation (1), i.e., $k = (k_g - k_s)$, can be summarized by writing

$$k_g = f[\sigma, \theta, p_A, v(x)] \quad \text{and} \quad (2a)$$

$$k_s = g[\sigma, \theta, p_A, v(x)] \quad (2b)$$

where both k_g and k_s are increasing functions of outcome uncertainty (σ) and perceived ambiguity (θ), k_g is a decreasing function of p_A but k_s is an increasing function of p_A . The absolute size of $v(x)$ increases both k_g and k_s , and together with its sign, determines the extent to which more weight is given in imagination to values above or below the anchor.

Further Specifications

Since the above functions are loosely specified, we now consider restrictions that correspond with the underlying psychological intuitions.

To motivate the exposition, imagine a simple, one-shot gamble involving a nonambiguous chance p_A of winning \$10 and a $(1 - p_A)$ chance of \$0. In the absence of ambiguity, recall that we have already specified $w(p_A) = 0$ when $p_A = 0$, and $w(p_A) = 1$ when $p_A = 1$. However, how do other values of p_A relate to $w(p_A)$?

The sign and size of the adjustment, k , is determined by the relative sizes of the possible ranges of decision weights above and below the anchor as well as the differential weight accorded to values within both ranges. Consider values of p_A less than 0.5. Note first that the range of possible decision weights above the anchor (i.e., $1 - p_A$) is greater than that below (i.e., p_A). Second, because we are dealing with a p_A chance of *winning* \$10, more weight is accorded in the simulation to values of possible decision weights below rather than above the anchor. However, this does not mean that the adjustment, k , is negative (i.e., $k_s > k_g$), because although more weight is given in imagination to possible values of decision weights in the range below the anchor, there is a larger range of possible values above the anchor (i.e., $1 - p_A > p_A$). In fact, the adjustment, k , will be positive when the *combined* effect of the range of possible weights above the anchor and the weight attached to those values exceeds the *combined* effect of their counterparts below the anchor. Thus, for very small values of p_A , one would expect $k_g > k_s$.

Next, holding all else constant, consider what happens as p_A increases. The range of possible decision weights below the anchor becomes larger whereas that above becomes smaller. This, in turn, means that k_s increases while k_g decreases, and that their difference, k , decreases with increases in p_A . The sign of k therefore changes from positive to negative, i.e., from “overweighting” where $k_g > k_s$ to “underweighting” where $k_g < k_s$. Moreover, this argument implies a unique “crossover point,” denoted p_c , between 0 and 0.5 where $k_s = k_g$.

Now examine values of p_A larger than 0.5. Because $p_A > (1 - p_A)$, it follows that $k_s > k_g$, i.e., the adjustment will always be negative.

When faced with a p_A chance of *losing* \$10 and a $(1 - p_A)$ chance of \$0, values accorded to possible decision weights above the anchor are assumed to be given more weight than those below. This therefore means that the adjustment, k , will be positive ($k_g > k_s$) over all values of $p_A \leq 0.5$. Moreover, the unique crossover point, p_c , where $k = 0$ (and $k_g = k_s$) will occur at some value above $p_A = 0.5$.

Consider now the effects of both outcome uncertainty and variations in the size and sign of payoffs. Recall that outcome uncertainty affects the amount of mental simulation; moreover, there is no outcome uncertainty at $p_A = 0$, it rises to a maximum at $p_A = 0.5$ and then declines to zero at $p_A = 1$. This specification implies that outcome uncertainty will amplify or dampen differences between k_g and k_s and thus the amount of over- or underweighting of probabilities. At $p_A = 0.5$ outcome uncertainty has its maximum effect

on accentuating the difference between k_g and k_s ; however, this effect diminishes as p_A approaches both 0 and 1.

Payoffs have two types of effect on the venture function. The absolute size of the payoffs affects the amount of mental simulation such that increases in payoffs increase k , i.e., differences between k_g and k_s . Second, the sign and size of payoffs affect the location of the crossover point, p_c , i.e., where $k_g = k_s$. More specifically, for positive payoffs where more weight is given in imagination to possible values of decision weights below rather than above the anchor, $p_c < 0.5$; for negative payoffs where more weight is given to possible values above rather than below, $p_c > 0.5$. Moreover, the larger the positive payoffs, the smaller the value of p_c ; the larger the absolute size of negative payoffs, the larger the value of p_c .

Figures 1 and 2 illustrate several venture functions involving nonambiguous probabilities for positive payoffs (gains) and negative payoffs (losses), respectively. In Figure 1, curves (b) and (c) illustrate the general behavior of venture functions for gains discussed above. Small probabilities are overweighted and large probabilities are underweighted. Similarly, curves (b) and (c) in Figure 2 show the analogous pattern for losses except that since the crossover points (or p_c values) are above $p_A = 0.5$, overweighting of probabilities occurs for all but large values of p_A . A second feature to be noticed in both figures is that deviations from the diagonal which represent the size of differences between k_g and k_s are typically large in the region of $p_A = 0.5$ but decline as p_A approaches 0 and 1 (this statement excludes, of course, the nature of the functions in the neighborhood of their p_c values).

In Figure 1, the curves have been drawn to represent venture functions with different payoff sizes which can be ordered in magnitude as $(a) > (b) > (c)$. These payoff differences have two effects on the venture functions. First, the p_c values of the different curves are smaller the larger the payoffs. Second, the absolute size of k ($= |k_g - k_s|$ or "distortions" vis-à-vis the diagonal) is greater the larger the payoffs. Figure 2 shows the analogous effects with respect to losses. In this figure, the absolute losses associated with the venture functions can be ordered $(a) > (b) > (c)$. In this case, the p_c values are greater the larger the absolute sizes of the negative payoffs as are the deviations from the diagonal. Finally, the diagonal lines in both figures represent special cases of the venture function where there is no mental simulation of alternative values.

To model the effects of ambiguity, recall that this increases the amount of mental simulation and thus the extent to which $w(p_A)$ deviates from p_A . Consider first the $w(p_A)$

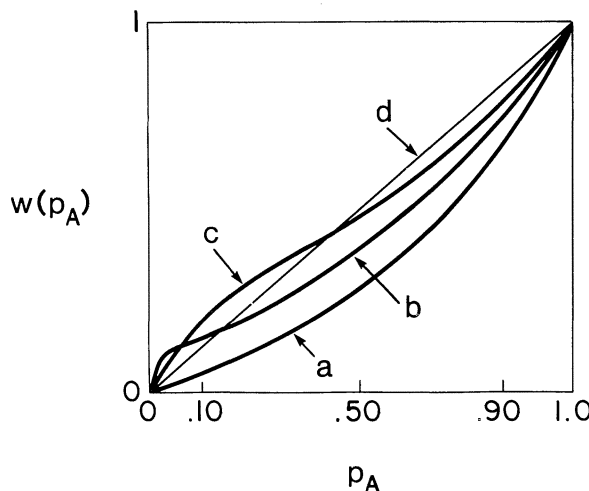


FIGURE 1. Graphs of Venture Functions for Gains Involving Payoffs of Different Sizes with $a > b > c > d$.

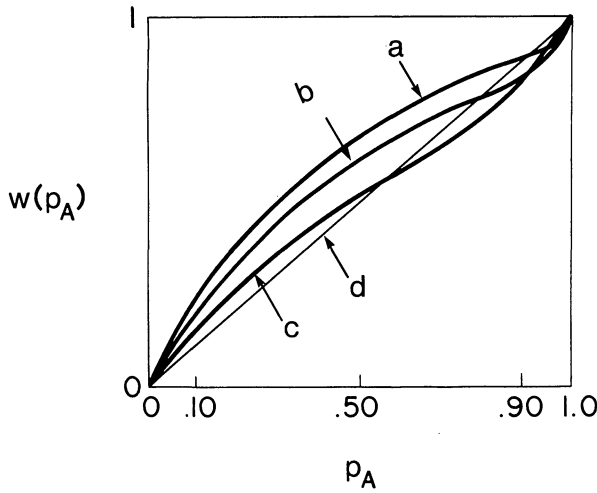


FIGURE 2. Graphs of Venture Functions for Losses Involving Payoffs of Different Sizes with $|a| > |b| > |c| > |d|$.

values associated with anchors of 0 and 1. In the presence of ambiguity, these are adjusted, up for $p_A = 0$, and down for $p_A = 1$. Moreover, the amount of the adjustment reflects the degree of perceived ambiguity. (In thinking through this, ask yourself: Would you prefer having a “known zero” chance of winning a prize as opposed to an “ambiguous zero” chance?) Assume further that the amount by which $w(p_A)$ overweights p_A when $p_A = 0$ is the same as the amount of underweighting when $p_A = 1$. Relative to venture functions with nonambiguous probabilities, the general effect of ambiguity is to overweight small probabilities (more accurately decision weights) and to underweight large ones. This is illustrated in Figures 3 and 4 which show, for gains and losses respectively, the effects of venture functions incorporating ambiguity relative to venture functions with nonambiguous probabilities. Figures 3 and 4 also show the implications of variations in the sign and size of payoffs which are analogous to the reasoning depicted in Figures 1 and 2. The crossover point is a decreasing function of payoffs, i.e., the crossover point

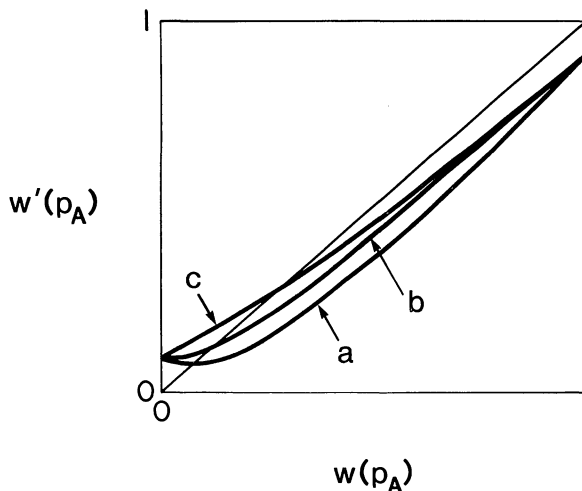


FIGURE 3. Venture Functions for Gains with Ambiguous Probabilities, $w'(p_A)$, as a Function of Venture Functions with Nonambiguous Probabilities, $w(p_A)$ for Different Levels of Payoffs Where $a > b > c$.

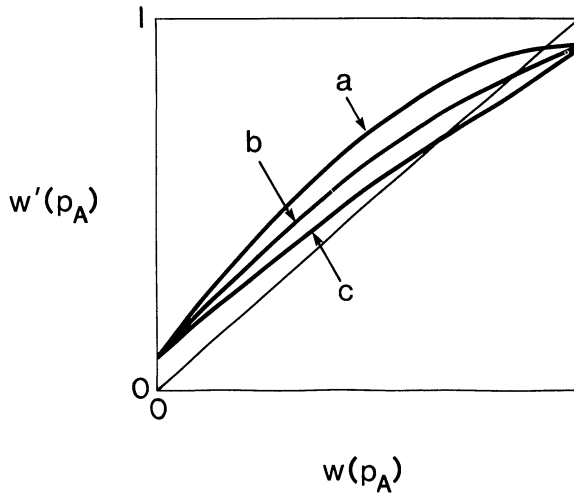


FIGURE 4. Venture Functions for Losses with Ambiguous Probabilities, $w'(p_A)$, as a Function of Venture Functions with Nonambiguous Probabilities, $w(p_A)$ for Different Levels of Payoffs Where $|a| > |b| > |c|$.

for large positive payoffs is at a small value of $w(p_A)$; the crossover point for large negative payoffs is at a large value of $w(p_A)$, and crossover points for intermediate payoffs lie between these extremes.

Summary and Implications

In general, the venture function starts by “overweighting,” has a crossover point (p_c), and then “underweights” the anchor. The location of the crossover point depends on the relative weight given in imagination to values of possible decision weights above and below the anchor. This, in turn, is assumed to depend on both the sign and size of payoffs such that, for positive payoffs, $p_c < 0.5$ and is smaller the larger the stakes; for negative payoffs, $p_c > 0.5$ and is larger the greater the absolute size of the payoff. Incidentally, it should be noted that by fitting responses to a series of questions given in the 1950’s to several illustrious subjects (including de Finetti and Malinvaud), Allais (1986) has developed a three-parameter model that yields curves similar to those of venture theory. In fact, his empirical results for positive payoffs indicate curves similar to those illustrated in Figure 1.

Second, the extent to which $w(p_A)$ deviates from p_A over the range of the latter depends on outcome uncertainty, the size of payoffs, and the amount of perceived ambiguity, i.e., the greater the mental simulation induced by these factors, the greater the deviation. Note further that, for multiple plays of a gamble, outcome uncertainty is reduced so that, in the absence of ambiguity, $w(p_A) \rightarrow p_A$. The limit of p_A would not, however, be reached in the case of ambiguity.

Third, since venture functions are not linear in p_A , decision weights associated with complementary probabilities, i.e., p_A and $(1 - p_A)$, will not necessarily sum to one.

Finally, the model can be used to predict how attitudes toward risk and ambiguity vary as a function of sign and size of payoffs as well as levels of probability.

Attitudes toward Risk and Ambiguity

The predictions made below correspond to expectations concerning the modal behavior of subjects since it is assumed that all subjects have the same attitude of caution (“defensive pessimism”) in the face of risk and ambiguity. Attitudes toward risk and ambiguity are defined with respect to specific choices. Thus, a choice is said to be risk averse if a person

prefers a sure amount equal to the expected value of a gamble to the gamble itself, and ambiguity averse if a gamble with a known probability is preferred to one involving an uncertain probability that has been equated in other respects with the known probability. By extension, the terms risk- and ambiguity seeking are the contrary of risk- and ambiguity averse. The statement that, for example, risk aversion increases as a function of a given variable, is taken to mean that, in a sample of individuals, the observed proportion of risk-averse choices increases with that variable.

In order to make our predictions, we need to specify further the value function proposed by Kahneman and Tversky (1979). Specifically, we shall assume that the subjective values of gains and losses can be modeled by power functions of the underlying physical stimuli but with different exponents for gains and losses such that the function for losses is steeper than for gains.

It is instructive to consider predicted attitudes toward risk and ambiguity separately for gains and losses. For gains, there are two forces that induce tendencies toward risk aversion. These are the concavity of the prospect theory value function, and the general underweighting of probabilities implied by the venture function. Whereas small probabilities can be overweighted (thereby implying a force toward risk seeking), underweighting predominates as probabilities increase—see Figure 1. We therefore predict that risk aversion increases with the size of probabilities. We label this Prediction 1.

In the absence of probability “distortions,” risk attitudes are invariant to the size of payoffs when values are power functions of outcomes. However, the effect of increasing payoffs is to lower p_c such that the range of probabilities over which underweighting occurs is greater for large as opposed to small payoffs. This implies greater risk aversion for large as opposed to small payoffs—Prediction 2. Predictions 1 and 2 are summarized in Table 1 along with other predictions detailed below.

The effects of payoff size on risk attitudes in the domain of gains can be further illuminated by considering the curves (a), (b), and (c) in Figure 1 which represent venture functions with payoffs of different sizes, [$(a) > (b) > (c)$]. As discussed above, the functions imply relative differences in risk attitudes between small and large payoffs. However, they also imply that differences in risk attitudes due to size of payoff vary with probability level. Specifically, Figure 1 shows that large differences in risk attitudes due to payoff size would be expected for probabilities (p_A) in the region of 0.5 (as measured by the distances between the curves representing different payoff sizes at that point); moreover, as p_A increases, these differences decrease. For small values of p_A , predictions are unclear since the sizes of differences between venture functions for large and small payoffs depend on the relative locations of the p_c values for large and small payoffs. Compare, for example, the differences in Figure 1 at $p_A = 0.10$ between curves (a) and (c), on the one hand, and curve (b) and the diagonal, on the other. In short, for gains venture theory predicts a probability \times payoff interaction with respect to medium and large probabilities—Prediction 3.

Predictions concerning the effects of payoffs and probabilities on attitudes toward ambiguity can be inferred by consulting Figure 3. In general, ambiguity aversion in the domain of gains is expected to: (i) increase with probability level—Prediction 4; (ii) increase with payoff size—Prediction 5; and (iii) in analogous fashion to Prediction 3, be more sensitive to payoff size at medium as opposed to high levels of probability—Prediction 6.

For losses, there are two forces that conflict in their impact on risk attitudes. On the one hand, the convex nature of the prospect theory value function over losses implies risk seeking. On the other, since $p_c > 0.5$ for losses, probabilities are generally overweighted thereby implying a force toward risk aversion.

Effects of probabilities and payoffs on risk attitudes can be predicted by considering Figure 2 where the curves (a), (b), and (c) represent venture functions for negative payoffs [$|(a)| > |(b)| > |(c)|$]. Examination of Figure 2 leads to the following predic-

TABLE 1

Summary of Venture Theory Predictions Concerning Attitudes toward Risk and Ambiguity

Gains

Attitudes toward risk

1. Main effect for probability: The proportion of risk-averse choices is predicted to increase as probabilities increase.
2. Main effect for payoffs: The proportion of risk-averse choices is predicted to increase as payoffs increase.
3. Probability \times payoff interaction: Effects of payoff size will be larger for medium-sized probabilities (in the region of $p_A = 0.5$) and diminish as probabilities increase.

Attitudes toward ambiguity

4. Main effect for probability: The proportion of ambiguity-averse choices is predicted to increase as probabilities increase.
5. Main effect for payoffs: The proportion of ambiguity-averse choices is predicted to increase as payoffs increase.
6. Probability \times payoff interaction: Effects of payoff on ambiguity aversion will be larger at medium (p_A about 0.5) as opposed to high levels of probability.

Losses

Attitudes toward risk

7. Main effect for probability: The proportion of risk-averse choices is predicted to decrease as probabilities increase.
8. Main effect for payoffs: The proportion of risk-averse choices is predicted to increase as the absolute size of payoffs increases.
9. Probability \times payoff interaction: Effects of payoff size will be larger for medium-sized probabilities (in the region of $p_A = 0.5$) and diminish as probabilities decrease.

Attitudes toward ambiguity

10. Main effect for probability: The proportion of ambiguity-averse choices is predicted to decrease as probabilities increase.
11. Main effect for payoffs: The proportion of ambiguity-averse choices is predicted to increase as the absolute size of payoffs increases.
12. Probability \times payoff interaction: Effects of payoff on ambiguity aversion will be larger at medium (p_A about 0.5) as opposed to low levels of probability.

Asymmetries in attitudes toward losses and gains

13. As payoffs increase, the level of risk aversion for gains will increase relative to the level of risk seeking over losses.
14. For low probabilities, ambiguity aversion for losses will be more prevalent than ambiguity seeking for gains; however, at high probabilities, there will be more ambiguity aversion for gains than ambiguity seeking for losses.

tions: (i) decreasing risk aversion as probabilities increase—Prediction 7; (ii) increases in risk aversion as payoffs increase—Prediction 8; and (iii) greater effects of payoffs on risk attitudes at medium as opposed to low probability levels—Prediction 9. In mirror-image fashion to Prediction 3 for gains, we note the inability of the model to make precise predictions concerning payoff differences in regions close to p_c , in this case for larger probabilities.

By consulting Figure 4, the effects of payoffs and probabilities on attitudes toward ambiguity in the domain of losses can be inferred. These are that ambiguity aversion is expected to: (i) decrease as probabilities increase—Prediction 10; (ii) increase as the absolute size of payoffs increases—Prediction 11; and (iii) in mirror-image fashion to Prediction 6, be more sensitive to payoff size at medium as opposed to low probability levels—Prediction 12.

Two further predictions concern asymmetries between attitudes toward risk and ambiguity in the domains of losses and gains. For risk attitudes, recall that whereas for gains both the general underweighting of probabilities implied by the venture function and the concavity of the value function complement each other in inducing risk aversion, the forces implied by the venture and value functions for losses conflict, i.e., overweighting

of probabilities and the convexity of the value function. This might seem to imply that, for medium and large probabilities, there would be greater risk aversion for gains than corresponding risk seeking for losses. However, this prediction cannot be made unambiguously since the value function is steeper for losses than for gains. On the other hand, because increasing payoffs accentuate the effect of caution on the venture function (for both losses and gains), venture theory does predict that, as payoffs increase, the level of risk aversion for gains will increase relative to the level of risk seeking over losses—Prediction 13.

Venture function also predicts asymmetries in attitudes toward ambiguity in the domains of losses and gains that can be inferred from Figures 3 and 4. In general, for low probabilities one would expect to see more ambiguity aversion for losses than ambiguity seeking for gains; however, at high probabilities, there would be more ambiguity aversion for gains than ambiguity seeking for losses—Prediction 14.

Existing Evidence

Existing literature already speaks to the validity of certain aspects of venture theory as well as some of its predictions. Several investigators, for example, have noted increasing risk aversion for gains associated with increases in probability levels—Prediction 1 (see, e.g., McCord and de Neufville 1984), and the notion that risk aversion should increase with size of payoffs is not foreign (see, e.g., MacCrimmon and Larsson 1979; Machina 1982)—Prediction 2.

Evidence that ambiguity aversion for gains should increase with probability level (Prediction 5) has been documented by Einhorn and Hogarth (1986) and Curley and Yates (1985); similarly, decreases in ambiguity aversion over losses as probabilities increase (Prediction 10) have been reported by Einhorn and Hogarth (1986) and Hogarth and Kunreuther (1989). An important implication of the effect of ambiguity in the venture function is that, relative to the nonambiguous case, there should be little or no “reflection effect” in risk attitudes for medium-sized probabilities. And indeed, this has been verified by Hogarth (1989) using a legal decision-making scenario in which hypothetical plaintiffs and defendants had to decide whether to go to or settle out of court.

There is evidence that probability weighting functions differ in respect to losses and gains for both nonambiguous and ambiguous probabilities. For the former, see Marks (1951), Irwin (1953), Slovic and Lichtenstein (1968), Nygren and Isen (1985), and Cohen, Jaffray, and Said (1985). For the latter, see Einhorn and Hogarth (1986).

Differences in risk attitudes due to the sign of payoffs have been noted in many studies (see Edwards 1962). Moreover, in a recent study of professional actuaries, Hogarth and Kunreuther (1988) varied probability levels, payoff size, ambiguity, and type of risk (independent versus correlated) in a factorial design concerning the pricing of a warranty. In addition to significant main effects for probability level and ambiguity, their data revealed a significant main effect for independent versus correlated risks (which proxies for payoff size by varying the potential loss at stake) and an interaction between payoff size and ambiguity.

Finally, in an attempt to calibrate prospect theory decision-weight or π -functions separately for gains and losses, Currim and Sarin (1989) reported data contrary to venture theory in that π -functions for losses gave smaller $\pi(p)$ values at the same levels of p as π -functions for gains. We discuss this result below after considering our own experimental evidence.

Experimental Evidence

To test predictions concerning the effects of payoffs and probabilities on risk attitudes, it is important to vary independently both payoffs and probabilities. To achieve this,

three experiments were performed. In the first, payoffs were varied at different probability levels; in the second and third, probabilities were varied across two different payoff levels (small and large) and ambiguity was also manipulated. The third experiment differed from the others in that choices led to real as opposed to hypothetical monetary consequences.

Experiment 1

Task. The experimental stimuli were hypothetical gambles involving choices between, on the one hand, a p chance at winning (losing) $\$x$ and a $(1 - p)$ chance of $\$0$, and, on the other hand, a riskless amount equal to the expected value of the gamble, i.e., $\$px$. Three probability levels were investigated, 0.10, 0.50, and 0.80 with three payoff levels corresponding to expected values of $\$2$, $\$200$, and $\$20,000$ (i.e., low, medium, and high payoffs). There were thus nine choices to be made in respect of gains (3 probability levels \times 3 payoff levels) and nine choices in respect of losses.

Subjects and method. Subjects were 96 graduate and undergraduate students recruited through advertisements on the university campus. They were paid at the rate of $\$5$ per hour to participate in this and other experiments on decision making in a laboratory setting. Each subject responded to all 18 choices which were presented in random order in an experimental booklet. There were three possible responses for each choice: prefer the gamble (risky choice); prefer the riskless amount (i.e., sure thing); or indifference.

Results. Results are presented in Table 2. Table 3 provides a tally of the results of this and the other experiments relative to the venture theory predictions specified in Table 1. It is important to realize that in this experiment probabilities and payoffs were *not* varied factorially and that the design implies a negative correlation between payoff size and probability. For example, for the $\$20,000$ sure win subjects were faced with a 0.10 chance of winning $\$200,000$, a 0.50 chance of winning $\$40,000$, and a 0.80 chance of winning $\$25,000$. Thus, since probabilities have been held constant and outcomes varied, the relevant comparisons are *down the columns* within each probability level. This means that the data can only be used to make formal tests of Predictions 2, 8, and 13 (see Table 1) concerning the effects of payoffs on attitudes toward risk. However, the results can also suggest whether the predicted probability \times payoff interactions occur (Predictions 3 and 9).

The results for gains demonstrate the predicted effect of size of outcomes on attitudes toward risk (Prediction 2). At each probability level, risk-averse behavior increases as payoffs increase ($p < 0.0001$ by Cochran's test at all 3 probability levels).¹ For example, note from Table 2 that at the 0.10 probability level, 26% of subjects prefer the sure thing at $\$2$ and this number increases to 92% at $\$20,000$. The same pattern occurs at the 0.50 probability level (25% to 80%), and again at 0.80 (35% to 69%). However, at the 0.80 probability level there is little difference in risk attitudes between the sure things involving $\$200$ and $\$20,000$ (63% and 69%). Note too, that although probabilities and payoffs were not varied factorially, the results do suggest the presence of the predicted probability \times payoff level interaction on risk attitudes, i.e., Prediction 3. Specifically, at the 0.50 probability level, whereas 80% of subjects prefer the sure thing for the largest payoff, only 25% prefer the sure thing for the smallest payoff, i.e., a difference of (80% - 25%) or 55%. At the 0.80 probability level, the comparable figures are 69% and 35% thereby implying a smaller difference of 34%.

¹ It is unclear how one should perform an analysis of variance with repeated measures on a 0-1 dependent variable. Thus, although we have performed several such analyses, we have adopted the conservative strategy of testing our hypotheses by reporting the results of specific contrasts based on Cochran's test. Substantive conclusions have not proved sensitive to the statistical procedures adopted.

TABLE 2
*Experiment 1: Percentages of Subjects' Choices Indicating Risk Aversion
for Gains and Risk Seeking over Losses*

Probability Levels:	.10	.50	.80
% Risk averse for gains ¹			
Payoff levels			
\$2	26	25	35
\$200	78	62	63
\$20,000	92	80	69
% Risk seeking over losses ²			
Payoff levels			
-\$2	40	61	61
-\$200	50	43	54
-\$20,000	49	41	51
% Risk averse for gains/% Risk seeking over losses			
Payoff levels			
\$2	0.66	0.41	0.58
\$200	1.56	1.44	1.15
\$20,000	1.88	1.96	1.35

¹ Based on number of subjects choosing the "sure thing" ($n = 96$).
² Based on number of subjects choosing the "risky" option ($n = 96$).

For losses, the pattern of results is less apparent than for gains. However, risk averse behavior is observed to increase as payoffs increase (Prediction 8) and there is suggestive evidence of the predicted interaction between payoff levels and probability (Prediction 9). These effects are demonstrated by the lack of an effect of payoff size on risk attitudes

TABLE 3
Summary of Venture Theory Predictions and Experimental Results

Predictions	Experiment 1	Experiment 2	Experiment 3
Gains			
Attitudes toward risk			
1. Main effect for probability	NA ¹	Yes	Yes
2. Main effect for payoffs	Yes	Yes	Yes
3. Probability \times payoff interaction	Suggested	Yes	Yes
Attitudes toward ambiguity			
4. Main effect for probability	NA	Yes	Yes
5. Main effect for payoffs	NA	Yes	Yes
6. Probability \times payoff interaction	NA	No	Yes
Losses			
Attitudes toward risk			
7. Main effect for probability	NA	Yes	Yes
8. Main effect for payoffs	Yes	No	Yes
9. Probability \times payoff interaction	Suggested	No	No
Attitudes toward ambiguity			
10. Main effect for probability	NA	Yes	Yes
11. Main effect for payoffs	NA	No	Yes
12. Probability \times payoff interaction	NA	No	No
Asymmetries in attitudes towards losses and gains			
13. Risk aversion (gains)-Risk seeking (losses)	Yes	Yes	Yes
14. Ambiguity aversion/seeking	NA	Yes	Yes

¹ NA means "not applicable".

at the 0.10 probability level (Cochran's test, $df = 2$, $Q = 2.48$, $p = 0.289$), but increasing risk-averse behavior (i.e., proportion of subjects choosing the sure thing) at 0.50 and 0.80 ($p < 0.0005$ by Cochran's test at both levels).

Prediction 13 is a refinement of the prospect theory reflection effect and states that, at a given probability level, the ratio of choices that are risk averse for gains relative to those that are risk seeking over losses will increase as the level of payoffs increases. These ratios are presented at the foot of Table 2. For all three probability levels, the ratio increases with payoffs.

Experiments 2 and 3

Because these two experiments were identical in design except for one important feature, they will be presented and discussed together.

Task. Subjects were required to rank three possible options. These were: (1) choosing a certain sum, i.e., sure thing; (2) selecting a ball at random from an urn (designated #1) where the prize was contingent on drawing a ball of a specified color (with zero otherwise) and where the composition of the urn (numbers of balls and their colors) was known, i.e., choice with known probability; and (3) selecting a ball at random from an urn (designated #2) where the prize was contingent on drawing a ball of a specified color (with zero otherwise) but where the composition of the urn was not specified, i.e., choice with ambiguous probability. To operationalize ambiguity in the third option, subjects were told "Imagine that you have been allowed to view the contents of Urn #2 for a few seconds. You estimate that it contains . . . balls but are not too sure of your estimate." Moreover, note that to equate the same beliefs about probability levels in the corresponding ambiguous and nonambiguous conditions, we explicitly endowed subjects with specific probabilistic beliefs, e.g., "*you* estimate that it contains . . ." (emphasis added here). The task was constructed so that, for each choice, the sure thing was equal to the expected value of drawing a ball at random from the urn with known composition. Moreover, the estimate of the number of winning balls in the ambiguous urn was equal to the number of winning balls in the urn of known composition.

Whereas Experiment 2 was conducted using hypothetical payoffs, choices in Experiment 3 had real monetary consequences for the subjects. Both studies involved a factorial arrangement of three within-subject variables; size of payoffs, probabilities, and sign of payoffs. There were two levels of payoff, small and large; three probability levels, 0.10, 0.50, and 0.90; and versions involving both gains and losses. Thus each subject faced twelve choice situations (i.e., $2 \times 3 \times 2$). In Experiment 2 (hypothetical choices) the small and large payoffs were \$1 and \$10,000, respectively; the corresponding amounts in Experiment 3 (real choices) were 10 cents and \$10. The design of both experiments therefore permits testing Predictions 1 through 14. In addition, comparison of the results of these two experiments can shed light on the issue of whether the nature of payoffs (real or hypothetical) affects the behavior of experimental subjects.

Subjects and method. There were 146 subjects in Experiment 2 from the same population as Experiment 1, recruited and remunerated in similar manner. Subjects responded to the stimuli presented on the screen of a microcomputer. However, these were not all presented in sequence; instead, blocks of stimuli were interspersed between other decision-making tasks. For 82 of the subjects, the stimuli were presented in the same random order; the remaining 64 subjects saw the stimuli in individually randomized orders. Since this difference in method made no difference to results, it is ignored in the subsequent analyses.

Although recruited from the same population as the other experiments, the remuneration of the 49 subjects in Experiment 3 depended on their actual choices. To overcome problems associated with subjects losing money in experimental tasks, the following procedure was adopted. When subjects first came to the laboratory they were told they

were going to participate in two tasks and were being paid \$10 up front. Subjects were then given \$10 and signed a receipt for this sum but on the understanding that if they failed to complete the two tasks they would forfeit the \$10. Next, they were informed about the tasks. In the first, they would be required to solve a puzzle and answer some questions. For the second, they would be required to make a series of decisions with important consequences. Specifically, the second experiment could result in their “winning or losing a sum between \$0 and \$10.”

By giving the subjects \$10 before the first experimental task, we deliberately attempted to induce a sense of ownership or “endowment” before the subjects were asked to risk this money in the second task (cf. Cohen, Jaffray and Said 1985). We also “framed” the instructions to emphasize “winning” or “losing” \$10. The first task was unrelated to the object of this study and took about 15 minutes to complete.

At the outset of the second task, subjects were informed that they would make choices in twelve situations that could each involve *winning* or *losing* amounts between \$0 and \$10. After this, one of the twelve situations would be chosen at random, and they would then obtain the outcome associated with their choice in that situation. Before proceeding, subjects were also explicitly given the option of withdrawing from the experiment; none did so.

Stimuli were administered to subjects in individually randomized orders using the same procedures as in Experiment 2 except that subjects completed all twelve choices at one sitting and there were no intermediate tasks. After completing the task, each subject was faced with the consequences of the choices made in one of the situations chosen at random. This was done in a separate room which contained random devices to simulate the twelve different situations. Overall, subjects earned between \$0 and \$20 for participating in the experimental session and the mean was close to \$10. Subjects who lost \$10 in the second task (i.e., net pay of \$0) were actually given \$2 as compensation before leaving the laboratory.

Results. The results of Experiments 2 and 3 are presented in Table 4. These are also summarized in Figures 5 and 6 which show the percentages of subjects whose choices were risk- and ambiguity averse as a function of probability levels, sign of payoff (gain or loss), and size of payoff (large or small) in Experiments 2 and 3, respectively.

Consider the results in respect of risk attitudes toward gains. As shown in Figure 5a for Experiment 2, these conform with venture theory in that risk-averse behavior increases with both probabilities (Prediction 1) and payoffs (Prediction 2), and differences in risk attitudes between the two payoff conditions are greater at the medium as opposed to the high probability level (Prediction 3). For example, with small payoffs (\$1) 39% of subjects exhibit risk-averse behavior at the 0.50 probability level and this rises to 53% at the 0.90 level—see Table 4. However, the corresponding figures are 82% and 84% for large gains (\$10,000). (Cochran’s test shows significant effects across probability levels for small gains, $Q = 40.29$, $df = 2$, $p < 0.0001$, but not large gains, $Q = 0.73$, $df = 2$, $p = 0.694$.)

Experiment 3 shows similar results—see Table 4 and Figure 6a. The proportion of risk averse choices increases with both probabilities (Prediction 1) and payoffs (Prediction 2). Moreover, the effect of payoff is more pronounced at 0.50 than at 0.90 (Prediction 3). To see this, note that the difference between the proportions of risk averse choices at $p_A = 0.50$ is 45% (i.e., 78% – 33%) but that this drops to 23% at $p_A = 0.90$ (i.e., 90% – 67%). (Cochran’s test shows significant effects for probabilities for both small and large gains, $p < 0.0001$. In addition, differences between small and large payoffs are significant at each probability level, $p < 0.001$.)

In Experiment 2, increases in ambiguity-averse behavior are observed across probability levels—see Figure 5b ($p < 0.0001$ by Cochran’s test for both large and small gains). There are also small but statistically significant effects due to payoff size (Cochran’s test of differences between payoff sizes at the 0.10, 0.50, and 0.90 probability levels gave

TABLE 4
Experiments 2 and 3: Risk- and Ambiguity Aversion

Payoff:	Gains				Losses			
	Experiment 2		Experiment 3		Experiment 2		Experiment 3	
	Small	Large	Small	Large	Small	Large	Small	Large
(a) % Risk averse choices								
<u>Probability</u>								
0.10	20	80	20	43	33	36	39	61
0.50	39	82	33	78	21	18	18	33
0.90	53	84	67	90	17	23	14	14
(b) % Ambiguity averse choices								
<u>Probability</u>								
0.10	49	56	51	55	67	68	63	80
0.50	71	82	69	82	54	57	49	65
0.90	79	86	80	78	47	44	51	41
(c) % Risk averse for gains/ % Risk seeking over losses								
Payoff:	Experiment 2		Experiment 3					
	Small	Large	Small	Large				
<u>Probability</u>								
0.10	0.28	1.25	0.33	1.10				
0.50	0.49	1.00	0.40	1.16				
0.90	0.63	1.09	0.80	1.05				

values, $df = 1$, of $Q = 3.46$, $p = 0.063$; $Q = 6.82$, $p = 0.009$; and $Q = 3.52$, $p = 0.061$, respectively). However, there is no interaction between payoff size and probabilities. Thus, whereas the data support Predictions 4 and 5, Prediction 6 is not validated.

In Experiment 3, on the other hand, the data do support Predictions 4, 5, and 6—see Figure 6b and Table 4. Ambiguity aversion increases with probability level, is generally more pronounced for large as opposed to small payoffs, and the effect of payoff is greater at $p_A = 0.50$ than at 0.90. (Cochran's test shows significant effects across probability levels for both small and large payoffs, $p < 0.01$. Significant effects of payoffs are found for the 0.50 probability level, $Q = 4.5$, $df = 1$, $p < 0.05$, but not at the other probability levels.)

For losses, the data summarized in Figure 6 for Experiment 2 show both decreasing risk- and ambiguity aversion as probabilities increase ($p < 0.001$ by Cochran's test for both large and small losses), but no effects for differences in payoff size, and no probability \times payoff interactions.² Thus, whereas Predictions 7 and 10 were validated, Predictions 8, 9, 11, and 12 were not.

The data from Experiment 3 reveal a different picture—see Figure 6 and Table 4. Risk- and ambiguity aversion decrease as probabilities increase (Predictions 7 and 10); there is also increasing risk- and ambiguity aversion for large as opposed to small payoffs (Predictions 8 and 11). On the other hand, the predicted interactions (Predictions 9 and 12) do not obtain. The differences between the sizes of the payoff effects for both risk- and ambiguity aversion do not differ significantly between $p_A = 0.50$ and $p_A = 0.10$.

² Due to a programming error, responses concerning losses provided by 17 subjects were lost. Data for losses therefore involve 129 as opposed to 146 subjects.

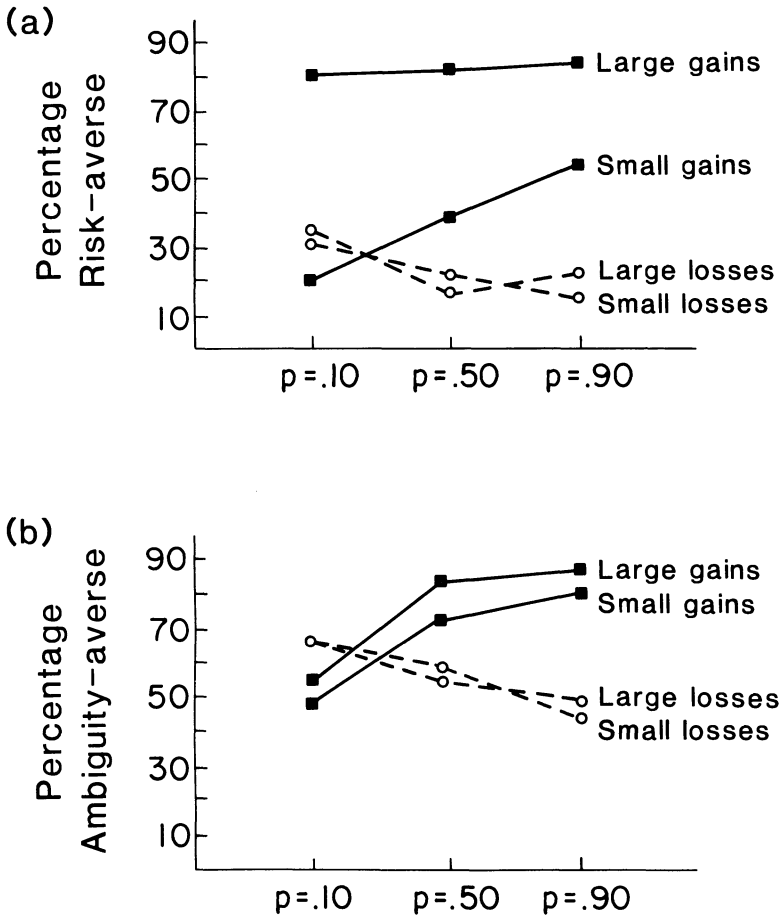


FIGURE 5. (a) Experiment 2: Percentages of Risk-Averse Subjects in Different Conditions. (b) Experiment 2: Percentages of Ambiguity-Averse Subjects in Different Conditions.

(Cochran's test reveals significant effects due to probabilities on risk attitudes for both small and large payoffs, $p < 0.01$. A similar main effect is also found for large payoffs on attitudes toward ambiguity, $p < 0.0001$. For risk attitudes, effects of payoffs at the 0.10, 0.50, and 0.90 probability levels gave values, $df = 1$, of $Q = 6.4$, $p = 0.012$; $Q = 2.9$, $p = 0.089$; and $Q = 0$, $p = 1.0$, respectively. The analogous figures in respect of attitudes toward ambiguity were $Q = 4.6$, $p = 0.033$; $Q = 4.6$, $p = 0.033$; and $Q = 1.9$, $p = 0.166$.)

Prediction 13 refers to the effect of payoff size on the ratio of choices that are risk averse for gains relative to those that are risk seeking over losses. These ratios are shown at the foot of Table 4 for both Experiments 2 and 3. As can be observed, the predicted increases in the ratios (from small to large payoffs) are observed in all cases.

Prediction 14 refers to asymmetries in attitudes toward ambiguity. Specifically, it holds that at low probabilities ambiguity aversion over losses will be greater than ambiguity seeking for gains, and that at high probabilities, ambiguity aversion for gains will be greater than ambiguity seeking over losses. This prediction can be verified by consulting Table 4. Consider, for example, the 67% ambiguity aversion for losses at the 0.10 probability level in the small payoff condition of Experiment 2. This is greater than the corresponding ambiguity seeking for gains which is 51% and can be deduced from the figure of 49% for ambiguity aversion provided in Table 4 (i.e., $100\% - 49\% = 51\%$). Similarly, the other comparisons at the 0.10 probability level are: $68\% > 44\%$ (or 100%

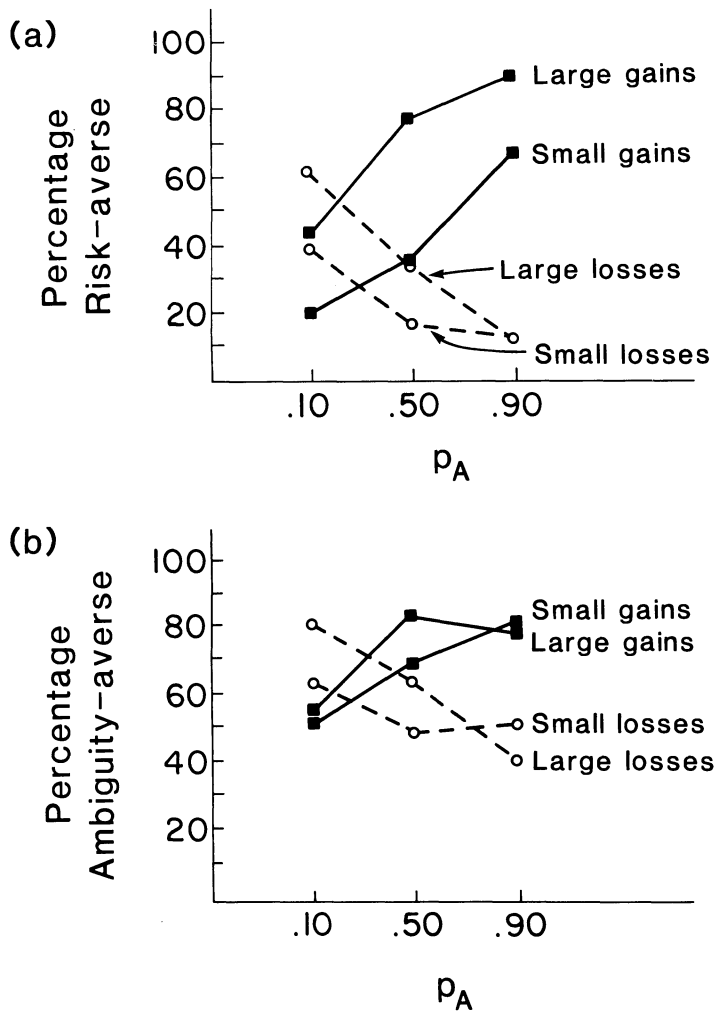


FIGURE 6. (a) Experiment 3: Percentages of Risk-Averse Subjects in Different Conditions. (b) Experiment 3: Percentages of Ambiguity-Averse Subjects in Different Conditions.

– 56%); 63% > 49%; and 80% > 45%. At the 0.90 probability level, ambiguity aversion for gains always exceeds the corresponding ambiguity seeking over losses, the figures being 79% > 53% (or 100% – 47%), 86% > 56%, 80% > 49%, and 78% > 59%.

Finally, since in these experiments attitudes toward risk and ambiguity were simultaneously measured within subjects, it is of interest to ask whether these attitudes were correlated. In general, the data revealed no consistent pattern in that, on average, across all 12 conditions of the experiments, subjects' attitudes toward risk and ambiguity only coincide (i.e., are both risk- and ambiguity averse or risk- and ambiguity seeking) 55% of times in Experiment 2 and 57% in Experiment 3. These data are consistent with the findings of Curley, Yates, and Abrams (1987).

Discussion

Taken as a whole, the experimental evidence supports a model that combines the value function of prospect theory with venture theory decision weights. Our discussion is organized around four topics: (a) the experimental evidence; (b) future tests of venture

theory; (c) the relation of venture theory to other models of risky choice; and (d) implications of venture theory for explaining other phenomena.

Experimental Evidence

For each of the three experiments, Table 3 provides a count of venture theory's success in validating the fourteen predictions outlined in Table 1. In Experiment 1, ambiguity was not manipulated and venture theory was successful in meeting the three predictions that could be clearly made (2, 8, and 13); this experiment also provided suggestive evidence in favor of Predictions 3 and 9.

From the viewpoint of venture theory, Experiment 2 provided the weakest evidence. This was especially true of losses where, for attitudes toward both risk and ambiguity, neither main effects nor interactions were noted involving effects of payoffs (Predictions 8, 9, 11, and 12). In addition, there was no probability \times payoff interaction for ambiguity in respect of gains (Prediction 6). In direct contrast, however, are the results of Experiment 3 which involved the same design as Experiment 2 but used real as opposed to hypothetical payoffs. For gains, the predicted probability \times payoff interaction with respect to ambiguity did obtain (Prediction 6). For losses, main effects were noted for attitudes toward both risk and ambiguity (Predictions 8 and 11); and although the precise probability \times payoff interactions predicted by venture theory did not occur, probability and payoff did interact for attitudes toward both risk and ambiguity (see Figure 6).

We note, incidentally, that contrary to previous experimental work aimed at detecting utility \times probability interactions, the research strategy adopted here was to use a theoretical model that explicitly suggested where such effects would be most likely to occur. Moreover, this model predicted a highly interactive pattern of outcomes. More work is needed to address the exact nature of these interactions.

In comparing the results of Experiments 2 and 3, there are few qualitative differences concerning gains but large differences for losses. Why? We suggest two hypotheses. The first arises from the observation that, because of the costs of facing pure loss gambles, we typically structure our lives to avoid such situations. Thus, gambles involving gains are easier to imagine than those involving losses and differences between real and hypothetical behavior are smaller for gains. (Although we also note that in an experiment involving hypothetical payoffs, Goldstein, Levi and Coombs 1987, found effects on decision weights for gains due to size of payoff but not for losses.) Second, whereas the shapes of both the venture and value functions generally reinforce risk aversion for gains, the same functions imply conflicting forces on risk attitudes for losses. The resulting conflict implies greater inherent instability in attitudes toward losses.

Existing evidence concerning the effects of real as opposed to hypothetical payoffs in gambling experiments does not reveal a simple set of findings (see, e.g., Edwards 1953; Feather 1959; Slovic 1969), and more work is needed to address this issue. It is significant, however, that in a recent experiment comparing risk attitudes toward hypothetical and real payoffs (involving \$100 at the 0.50 probability level), Schoemaker (1988) also reported no differences in risk attitudes toward gains but greater risk aversion for losses in respect of real payoffs.

This discussion may also relate to Currim and Sarin's (1989) attempt to calibrate prospect theory models, where, contrary to venture theory, π -functions for losses were found to imply more underweighting of probabilities than π -functions for gains. We note, however, that Currim and Sarin used hypothetical gambles and scenarios mainly involving consumer decisions. Either or both of these factors could have contributed to the differences between their results and ours.

The venture theory predictions assumed that all subjects had the same attitude of caution in the face of risk. In contrast, Schneider and Lopes (1986) preselected risk-averse and risk-seeking subjects from the extremes of a large subject pool (greater than

1,000) and found systematic differences in subsequent choices. Their data were consistent with Lopes' (1987) two-factor theory whereby, for risk-averse subjects, risk aversion in the domain of gains results from positive correlation between the two forces of "security-potential" and "aspiration level"; however, these same two forces conflict in determining attitudes toward losses. For risk-seeking subjects, on the other hand, it is the reverse: conflict on the gain side, but no conflict concerning losses.

It is straightforward within venture theory to model optimism (as opposed to caution) such that the conflict between forces that produce risk attitudes reflect in the same way as in Lopes' (1987) model. Within venture theory, one simply reverses the assumptions that determine the location of p_c . This means that when these functions are combined with the prospect theory value function, the shapes of the venture and value functions would both favor risk seeking over losses but conflict with respect to risk attitudes toward gains.

Future Tests of Venture Theory

The notion that "distortions of probability" can be exploited to explain deviations from expected utility theory is not new (see, e.g., Bernard 1974). However, little attention has been paid to date as to *why* decision weights differ from probabilities although Rachlin et al. (1986) have analyzed the prospect theory π -function from a behaviorist viewpoint. Kahneman and Tversky (1979, 1984) have provided some psychophysical arguments (see also Grossberg and Gutowski 1987), but Lopes (1987) points out that these arguments reduce decisions concerning risk to a series of psychophysical reactions and leave no room for explanations of risky behavior that incorporate either emotive aspects such as fear or cognitive processes such as imagination.

In constructing venture theory, we assumed that payoffs were evaluated by psychophysical functions but conceptualized decision weights as resulting from the outcome of a mental simulation process affected by both cognitive and motivational influences. To make these assumptions operational, it is important to recall that we modeled the prospect theory value function by power functions even though Kahneman and Tversky (1979) defined this more generally as S-shaped. Indeed, it could be argued that some of the effects of payoffs on risk attitudes observed in our experiments could be attributed to an S-shaped value function which is relatively flat around the reference point (cf. Markowitz 1952) as opposed to changes in the venture function. Our evidence, and that of others, leads us to believe that there are different decision weight functions for gains and losses and that payoffs do exert effects on decision weights. However, it is important that future experimental work address the exact shape of the value function so that, without having to make *a priori* assumptions about either the value or venture functions, it will be possible to attribute changes in risk attitudes to the value and venture functions as appropriate. Concerning attitudes toward ambiguity, on the other hand, our predictions hold irrespective of the specific forms assumed for the value function. This is because attitudes toward ambiguity are defined relative to choices involving known probabilities.

Our emphasis on modeling the venture function is supported by a belief that the roles of cognition and motivation become particularly important as one moves from studying gambles in stylized laboratory settings to decisions taken in more realistic settings where, although people may have knowledge about payoffs, information about uncertainties or probabilities is typically incomplete (March and Shapira 1987). Because the notion of mental simulation is central to our model, a legitimate question centers on what evidence exists for this process over and above the fact that our outcome data are consistent with it. Whereas we have collected no systematic process data for gambles involving non-ambiguous probabilities, process data reported in both Einhorn and Hogarth (1985) and Hogarth and Kunreuther (1988) explicitly support the notion of a mental simulation process for choices involving ambiguous probabilities. To provide tests of the simulation

hypothesis in the presence of nonambiguous probabilities, it would be particularly revealing to collect and compare process data for situations where mental simulation is and is not predicted by the theory. For the latter, consider cases where there is little or no outcome uncertainty as when the amount to be won in a gambling situation depends on the *net* outcome of many trials involving the same random device, e.g., 30 trials of coin flipping.

Relations to Other Models of Risky Choice

Other models with decision weighting functions have been proposed in the literature (see, e.g., Karmarkar 1978; Quiggin 1982; Yaari 1987). Moreover, most of these models explain the “standard” paradoxes such as that of Allais (1953) as can models using standard probabilities (e.g., Chew and MacCrimmon 1979; Machina 1982). However, the decision weight functions adopted by these researchers are restricted in ways that would not predict all the findings described in this paper. In the models of both Karmarkar (1978) and Quiggin (1982), for instance, the decision weight at $p = 0.5$ is constrained to equal 0.5. Like Tversky and Kahneman (1986), we also doubt whether one can capture the richness of choice behavior by way of axiomatically consistent theories. (See also their comments, p. S259, concerning the “regret” models of Bell 1982, and Loomes and Sugden 1982.)

As a guide to decision making, venture theory has several unattractive features. Recall, however, that our intent is not to prescribe, but to describe. One “disturbing” feature of the model is that because the venture function is not necessarily a monotonic function of p_A , it could predict violations of dominance in choice. In choice problems where dominance relations are transparent, we do not believe that people will violate this normative principle (cf. Kahneman and Tversky 1979). However, when the relation is not transparent or people make independent judgments of the value of uncertain outcomes, violations of dominance can still occur (see, e.g., Goldstein and Einhorn 1987).

Further Implications

Whereas both attitudes toward risk and ambiguity and the nature of probability \times utility interactions were the focus of this paper, it is important to emphasize that venture theory can “explain” the well-known choice paradoxes of Allais (1953) and Ellsberg (1961). However, because venture theory explicitly allows for the effects of size of payoff, it can also explain the fact that the rate of violation of the substitution axiom in the Allais paradox is greater for large as opposed to small payoffs (see, e.g., MacCrimmon and Larsson 1979; Wothke 1985). This is equivalent to the “fanning out” hypothesis of Machina (1982) and would not be predicted by, for example, prospect theory. (See the recent, extensive, theoretical, and experimental comparisons of generalized utility theories by Camerer 1989.)

The structure of venture theory allows explanations of contextual effects on decision making. To do this, however, it is first necessary to specify how, in given circumstances, context affects differential weighting in imagination of possible values of decision weights above and below the anchor. For example, the simultaneous existence of gambling and the purchase of insurance has challenged many theorists working within the expected utility framework. This phenomenon can be explained within both venture theory and prospect theory by noting that small probabilities tend to be overweighted. However, consider the findings of Hershey and Schoemaker (1980) who have shown systematic differences when the same economic choices are presented as gambles, on the one hand, or insurance contracts, on the other. Specifically, the gambling context induces less risk-averse behavior than insurance. To explain this phenomenon within prospect theory requires a framing assumption whereby “losses” are assumed to be more aversive than “costs” (Kahneman and Tversky 1984). Within venture theory, however, these findings

are accommodated by noting that people are liable to have different attitudes toward uncertainty when gambling as opposed to buying insurance. Because the social context of insurance suggests caution (cf. Hershey and Schoemaker 1980), it is reasonable to assume that more weight is given in imagination to values of possible decision weights above the anchor in the insurance as opposed to gambling scenarios such that $p_{c(\text{insurance})} > p_{c(\text{gambling})}$ (recall one is dealing with probabilities of losses).

Venture theory also promises to provide an attractive means of exploring other phenomena. For example, when playing sequences of gambles people have been observed to change risk attitudes as a function of experiencing wins or losses (Thaler and Johnson 1986; McGlothlin 1956). Given the relatively small stakes involved, it seems implausible that this behavior can be explained by shifts in utility or value functions. In addition, explanations that posit changing reference points become somewhat involved. Instead, it seems more likely that past successes or failures lead to shifts in optimism or caution that affect the differential accessibility of scenarios people can imagine for future gambles. In venture theory, differential weighting of such imaginary scenarios imply values of p_c that can change from trial to trial. Similarly, the puzzling effects of affect on risk attitudes might be amenable to analysis by considering the likely impact of affect on p_c (see, e.g., Isen and Geva 1987, and references). We stress, however, that in suggesting these topics, it is important to specify how differential weighting in imagination can lead to experimental predictions. Given the flexibility inherent in venture theory, “explanations” of existing phenomena are relatively easy to construct.

One intriguing avenue of research is the role of individual differences. These were marked in the studies reported here by considerable variation around the modal class of behavior predicted by venture theory. Indeed, this is not unusual in work on risky decision making and several researchers go so far as to report separate analysis for subjects who are predominantly risk-averse or risk-seeking (e.g., Hershey and Schoemaker 1985; Schneider and Lopes 1986). To the extent that revealed p_c values capture systematic individual tendencies, venture theory is capable of accounting for individual differences. However, the problem, as in much research on this topic, is to find variables that can be reliably related to individual differences as opposed to classifying subjects on the basis of the very behavior being studied.

Finally, we note that the extensive literature on risky decision making relies heavily on stylized experimental gambles and, in particular, the explanation of a limited number of “paradoxical” findings. Indeed, an important cost of proposing a model of risky decision making is that one must, at least, be able to explain this literature. However, there is now growing awareness that the gambling metaphor of risky choice is limited in its application to many real world situations (cf. March and Shapira 1987). Since venture theory models how decision weights are affected by psychological constructs of emotion (e.g., caution) and cognition (e.g., imagination), we believe it provides a useful structure for studying the impact of uncertainty in a wide range of tasks both in and outside the psychological laboratory.³

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References

- ALLAIS, M., “Le comportement de l’homme rationnel devant le risque: Critique des postulats et axiomes de l’Ecole Américaine,” *Econometrica*, 21 (1953), 503–546.
- , “The General Theory of Random Choices in Relation to the Invariant Cardinal Utility Function and

- the Specific Probability Function," Paper presented at the 3rd Internat. Conf. Foundations and Applications of Utility, Risk and Decision Theories, Aix-en-Provence, France, 1986.
- BELL, D. E., "Regret in Decision Making Under Uncertainty," *Oper. Res.*, 30 (1982), 961-981.
- BERNARD G., "On Utility Functions," *Theory and Decision*, 5 (1974), 205-242.
- CAMERER, C. F., "An Experimental Test of Several Generalized Utility Theories," *J. Risk and Uncertainty*, 2 (1989), 61-104.
- CHEW, S. H. AND K. R. MACCRIMMON, "Alpha-nu Choice Theory: A Generalization of Expected Utility Theory," Working paper No. 669. Vancouver: University of British Columbia, Faculty of Commerce and Business Administration, 1979.
- COHEN, M., J. Y. JAFFRAY AND T. SAID, "Individual Behavior under Risk and under Uncertainty: An Experimental Study," *Theory and Decision*, 18 (1985), 203-228.
- CURLEY, S. P. AND J. F. YATES, "The Center and Range of the Probability Interval as Factors Affecting Ambiguity Preferences," *Organizational Behavior and Human Decision Processes*, 36 (1985), 273-287.
- , ——— AND R. A. ABRAMS, "Psychological Sources of Ambiguity Avoidance," *Organizational Behavior and Human Decision Processes*, 38 (1987), 230-256.
- CURRIM, I. S. AND R. K. SARIN, "Prospect versus Utility," *Management Sci.*, 35, 1 (1989), 22-41.
- DE FINETTI, B., "Foresight: Its Logical Laws, Its Subjective Sources," *Anna. Inst. Henri Poincaré*, 7 (1937). Translation in H. E. Kyburg Jr. & H. E. Smokler (Eds.), *Studies in Subjective Probability*, Wiley, New York, 1964.
- EDWARDS, W., "Probability Preferences in Gambling," *Amer. J. Psychology*, 66 (1953), 349-364.
- , "Utility, Subjective Probability, Their Interaction, and Variance Preferences," *J. Conflict Resolution*, 6 (1962), 42-51.
- EINHORN, H. J. AND R. M. HOGARTH, "Ambiguity and Uncertainty in Probabilistic Inference," *Psychological Rev.*, 93 (1985), 433-461.
- AND ———, "Decision Making under Ambiguity," *J. Business*, 59, 4, 2 (1986), S225-S250.
- ELLSBERG, D., "Risk, Ambiguity, and the Savage Axioms," *Quart. J. Economics*, 75 (1961), 643-669.
- FEATHER, N. T., "Subjective Probability and Decision under Uncertainty," *Psychological Rev.*, 66 (1959), 150-164.
- FISHBURN, P. C., *Nonlinear Preference and Utility Theory*, The Johns Hopkins University Press, Baltimore, 1988.
- GOLDSTEIN, W. M. AND H. J. EINHORN, "Expression Theory and the Preference Reversal Phenomena," *Psychological Rev.*, 94 (1987), 236-254.
- , K. R. LEVI AND C. H. COOMBS, "Optimistic and Pessimistic Decisions: Effect of Outcome Desirability on the Impact of Uncertainty," Unpublished manuscript, University of Chicago, Center for Decision Research, 1987.
- GROSSBERG, S. AND W. E. GUTOWSKI, "Neural Dynamics of Decision Making under Risk: Affective Balance and Cognitive-Emotional Interactions," *Psychological Rev.*, 94 (1987), 300-318.
- HERSHEY, J. C. AND P. J. H. SCHOEMAKER, "Risk Taking and Problem Context in the Domain of Losses: An Expected Utility Analysis," *J. Risk and Insurance*, 47 (1980), 111-132.
- AND ———, "Probability versus Certainty Equivalence Methods in Utility Measurement: Are They Equivalent?," *Management Sci.*, 31 (1985), 1213-1231.
- HOGARTH, R. M., "Ambiguity and Competitive Decision Making: Some Implications and Tests," In P. C. Fishburn and I. H. LaValle (Eds.), *Choice under Uncertainty. Ann. Oper. Res.*, 19 (1989), 31-50.
- AND H. KUNREUTHER, "Pricing Insurance and Warranties: Ambiguity and Correlated Risks," Unpublished manuscript, University of Chicago, Center for Decision Research, 1988.
- AND ———, "Risk, Ambiguity, and Insurance," *J. Risk and Uncertainty*, 2 (1989), 5-35.
- IRWIN, F. W., "Stated Expectations as Functions of Probability and Desirability of Outcomes," *J. Personality*, 21 (1953), 329-335.
- ISEN, A. M. AND N. GEVA, "The Influence of Positive Affect on Acceptable Level of Risk: The Person with a Large Canoe Has a Large Worry," *Organizational Behavior and Human Decision Processes*, 39 (1987), 145-154.
- KADANE, J. B. AND R. L. WINKLER, "Separating Probability Elicitation from Utilities," *J. Amer. Statist. Assoc.*, 83, 401 (1988), 357-363.
- KAHNEMAN, D. AND A. TVERSKY, "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica*, 47 (1979), 263-291.
- AND ———, "Choices, Values, and Frames," *Amer. Psychologist*, 39 (1984), 341-350.
- KARMAKAR, U. S., "Subjectively Weighted Utility: A Descriptive Extension of the Expected Utility Model," *Organizational Behavior and Human Performance*, 21 (1978), 61-72.
- KEREN, G. AND W. A. WAGENAAR, "Violation of Utility Theory in Unique and Repeated Gambles," *J. Experimental Psychology: Learning, Memory, and Cognition*, 13 (1987), 387-391.
- LOOMES, G. AND R. SUGDEN, "Regret Theory: An Alternative Theory of Rational Choice under Uncertainty," *Economic J.*, 92 (1982), 805-824.

- LOPES, L. L., "Decision Making in the Short Run," *J. Experimental Psychology: Human Learning and Memory*, 7 (1981), 377-385.
- , "Between Hope and Fear: The Psychology of Risk," *Adv. in Experimental Social Psychology*, 20 (1987), 255-295.
- MACHINA, M. J., "'Expected Utility' Analysis without the Independence Axiom," *Econometrica*, 50 (1982), 277-323.
- , "Decision-Making in the Presence of Risk," *Science*, 236 (1987), 537-543.
- MACCRIMMON, K. R. AND S. LARSSON, "Utility Theory: Axioms versus 'Paradoxes,'" In M. Allais & O. Hagen (Eds.), *Expected Utility Theory and the Allais Paradox*. Dordrecht, Holland, 1979, pp. 333-409.
- MARCH, J. G. AND Z. SHAPIRA, "Managerial Perspectives on Risk and Risk Taking," *Management Sci.*, 33 (1987), 1404-1418.
- MARKOWITZ, H., "The Utility of Wealth," *J. Political Economy*, 60 (1952), 151-158.
- MARKS, R. W., "The Effect of Probability, Desirability, and 'Privilege' on the Stated Expectations of Children," *J. Personality*, 19 (1951), 332-351.
- MCCORD, M. R. AND R. DE NEUFVILLE, "Utility Dependence upon Probability: An Empirical Demonstration," *J. Large Scale Systems*, 6 (1984), 91-103.
- MCGLOTHLIN, W. H., "Stability of Choices among Uncertain Alternatives," *Amer. J. Psychology*, 69 (1956), 604-615.
- NOREM, J. K. AND N. CANTOR, "Anticipatory and Post Hoc Cushioning Strategies: Optimism and Defensive Pessimism in 'Risky' Situations," *Cognitive Theory and Research*, 10 (1986), 347-362.
- NYGREN, T. E. AND A. M. ISEN, "Examining Probability Estimation: Evidence for Dual Subjective Probability Functions," Paper presented at the meeting of the Psychonomics Society, Boston, 1985.
- QUIGGIN, J., "A Theory of Anticipated Utility," *J. Economic Behavior and Organization*, 3 (1982), 323-343.
- RACHLIN, H., A. W. LOGUE, J. GIBBON AND M. FRANKEL, "Cognition and Behavior in Studies of Choice," *Psychological Rev.*, 93 (1986), 33-45.
- SAVAGE, L. J., *The Foundations of Statistics*, Wiley, New York; 1954.
- SCHNEIDER, S. L. AND L. L. LOPES, "Reflection in Preferences under Risk: Who and When May Suggest Why," *J. Experimental Psychology: Human Perception and Performance*, 12 (1986), 535-548.
- SCHOEMAKER, P. J. H., "The Expected Utility Model: Its Variants, Purposes, Evidence and Limitations," *J. Economic Literature*, 20 (1982), 529-563.
- , "Are Risk-Attitudes Related across Domains and Response Modes?," Unpublished manuscript, University of Chicago, Center for Decision Research, 1988.
- SLOVIC, P., "Differential Effects of Real versus Hypothetical Payoffs on Choices among Gambles," *J. Experimental Psychology*, 80, 3 (1969), 434-437.
- AND S. LICHTENSTEIN, "Relative Importance of Probabilities and Payoffs in Risk Taking," *J. Experimental Psychology Monograph*, 78, 3, 2 (1968), 1-18.
- THALER, R. AND E. JOHNSON, "Hedonic Framing and the Break-Even Effect," Unpublished manuscript, Cornell University, Johnson Graduate School of Management, 1986.
- TVERSKY, A. AND D. KAHNEMAN, "Rational Choice and the Framing of Decisions," *Journal of Business*, 59, 4, 2 (1986), S251-S278.
- VON NEUMANN, J. AND O. MORGENSTERN, *Theory of Games and Economic Behavior*. Second ed., Princeton University Press, Princeton, NJ; 1947.
- WEBER, M. AND C. CAMERER, "Recent Developments in Modelling Preferences under Risk," *OR Spektrum*, 9 (1987), 129-151.
- WOTHKE, W., "Allais' Paradox Revisited: The Implications of the Ambiguity Adjustment Model," Unpublished manuscript, Northwestern University, Department of Psychology, 1985.
- YAARI, M. E., "The Dual Theory of Choice under Risk," *Econometrica*, 55 (1987), 95-115.