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DECISION WEIGHTS IN ANTICIPATED UTILITY THEORY

Response to Segal

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The Anticipated Utility model permits the incorporation of decision weights into a model of choice under uncertainty which maintains the desirable properties of transitivity and preservation of dominance. In order to model choice it is necessary to impose appropriate conditions on the decision weighting function. Segal has argued that the function should be concave so that less favorable outcomes are always given more weight than the favorable ones. In this paper it is argued that outlying low probability events, whether favorable or unfavorable, are likely to be 'overweighted'.

The primary motivation behind the development of anticipated utility (AU) theory was the desire to incorporate the concept of decision weights [Edwards (1955), Fellner (1961)] in a theory of choice under uncertainty which maintained desirable properties of the expected utility (EU) approach, such as transitivity and preservation of stochastic dominance. Given the central role of decision weights, discussion and analysis of the form of the decision weighting function, such as that offered by Segal (1986), is obviously a vital part of the development of AU theory.

In his remarks, Segal makes three basic points in relation to the AU theory presented in Quiggin (1982). These are

(i) AU permits the modelling of the Allais 'paradox' and the common ratio effect in a framework of rational choice.
(ii) The axiomatic requirement, imposed in Quiggin (1982), that $f(1/2)=1/2$ is unduly restrictive.
(iii) The most plausible behavioral assumption is that f is concave.

I agree with the first two points but not with the third. While the pattern of preferences represented by an AU functional with concave f and U is a perfectly reasonable extension of the EU concept of risk aversion (though not the only possible one), the empirical evidence does not provide a strong case for suggesting that it is predominant. Rather, there is considerable empirical evidence supporting the pattern described in Quiggin (1982) as 'overweighting of extreme events'. If the requirement that $f(1/2)=1/2$ is dropped, this pattern may be expressed by the condition

(A**) There exists p^0 such that f is concave on $[0,p^0]$ and convex on $[p^0,1]$.

This condition implies that for small p $f(p)>p$ and $g(p)>p$, where $g(p)=1-f(1-p)$. A typical preference pattern of this kind is illustrated in fig. 1. For each of the problems cited by Segal, the modal choice pattern is consistent with preferences of this kind. For example, Quiggin (1985) describes a weighting function satisfying condition (A**) which resolves the Allais 'paradox'.

As Segal shows, preferences satisfying A** are inconsistent with the Generalised Allais Paradox (as well as being, like all non-trivial AU preference patterns) inconsistent with Machina's (1982) Generalised Common Ratio Effect. Although the existence of Allais-type preferences and common-ratio effects has been demonstrated in a large number of experiments, the proposed generalisations rest on the interpretation of a much smaller number of observations, primarily those of MacCrimmon and Larsson (1979).

MacCrimmon and Larsson presented a group of 19 subjects with a range of choices in which one prospect was of the form (r,p) while the other was of the form $(5r,0.8p)$. Their objective was to test the prediction of EU theory that variations in p should not affect the choice of prospect. Their results are illustrated in fig. 2.

It is apparent that these results are consistent with the GCRE. However, they are also quite consistent with postulate A**. What is required is that $f(p)/f(0.8p)$ should decrease as p takes the successive values 1.00, 0.75, 0.5, 0.25, 0.1 and 0.05. A numerical example is given in table 1. The crucial feature of this example is that the extreme tails of the distribution should receive very high weights but that these should decline rapidly so that $f'(p)$ is near 1 over most of its range. A similar weighting pattern is also necessary for the Allais 'paradox' to occur [Quiggin (1985)]. MacCrimmon and Larsson also examined the Allais paradox but observed a maximum of 6 violations from their 18 subjects. Such a tiny group can scarcely yield strong evidence on the pattern of probability weights, but the observations do seem to be in line with postulate A**.

Thus, the main data adduced in support of GCRE cannot be regarded as evidence against the overweighting postulate A**. By contrast, there is fairly stong evidence, both from experiments and from observed behavior, in support of overweighting of extreme events. The most important behavioral evidence is the widely observed coincidence of gambling and insurance, first examined by Friedman and Savage (1948). Although Friedman and Savage,

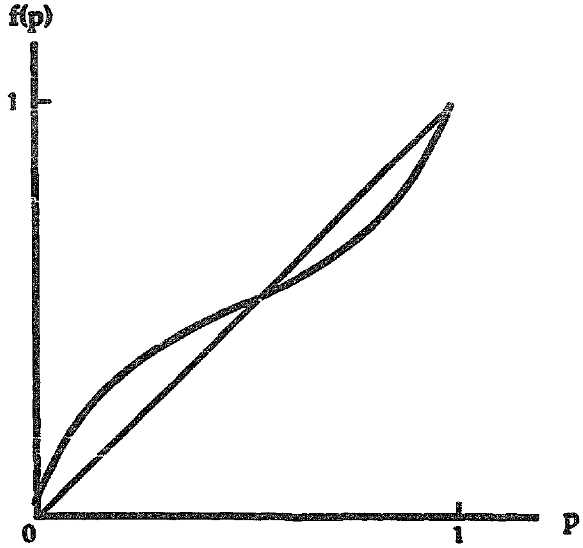
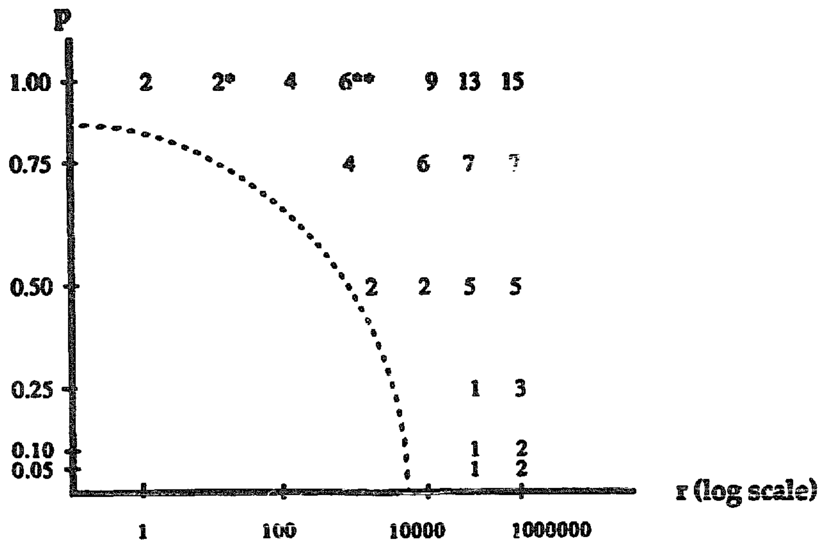


Fig. 1. Probability weighting function.



*One subject chose A on one presentation of this set, and B on the other.
**Two subjects chose A on one presentation of this set, and B on the other.

Fig. 2. The number of subjects, out of 19, selecting the alternative (r, p) over the alternative $(5r, 4/5p)$. (*Source*, MacCrimmon and Larsson (1979)).

and a number of subsequent writers, attempted to explain this behavior in terms of EU theory, all of these attempts are open to serious objections [Machina (1983), Quiggin (1987)]. By contrast, in AU theory, simultaneous gambling and insurance can be explained quite easily by postulates such as A**.

Experimental evidence also seems to support the hypothesis that low-

Table 1
Probability weighting function example.

p	$f(p)$	$f(p)/f(0.8p)$
0.04	0.09	
0.05	0.10	1.11
0.08	0.13	
0.10	0.15	1.15
0.20	0.24	
0.25	0.285	1.19
0.4	0.415	
0.5	0.5	1.20
0.6	0.585	
0.75	0.715	1.22
0.80	0.76	
1.00	1.00	1.32

probability extreme events receive a relatively high weight, although interpretation is made somewhat more difficult by the fact that most experiments have been conducted within the framework of EU theory, or without any adequate theoretical framework. Among studies suggesting an overweighting of low-probability extreme events are Preston and Baratta (1948), Griffith (1949), Edwards (1955) and Ali (1977). Not all of these experiments can be regarded as evidence for the postulate A**, as against the concave weighting function proposed by Segal. It is clear that both postulates will imply a high weight for events at the bottom tail of the distribution, and some of the studies deal primarily with this case. The studies of Griffith and Ali, however, deal with racetrack bettors and show overweighting for long-priced horses, that is, for events at the upper tail of the distribution.

One specific test of this hypothesis in an AU context was presented by Quiggin (1981). In this paper, a survey of risk attitudes amongst Australian farmers was re-analysed to fit an AU preference function, and overweighting of low probability events was found to be predominant. However, this study was based on a model incorporating the requirement $f(1/2)=1/2$, and thus the possibility of a concave weighting function was excluded from consideration.

To summarise, preferences characterised by postulate A**, although inconsistent with global risk-aversion, appear to be quite widespread. It is obviously undesirable that global risk-aversion should be excluded from the domain of permissible preference patterns, and hence Segal's criticism of the *axiomatic* requirement that $f(1/2)=1/2$ must be accepted. On the other hand, the evidence suggests that as a behavioral hypothesis, A** fits many, and perhaps most, people, who are risk averse in most cases but are willing to accept some unfair gambles at long odds.

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