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THE EXCHANGE AND INTEREST RATE TERM STRUCTURE UNDER RISK AVERSION AND RATIONAL EXPECTATIONS

Charles A. WYPLOSZ*

INSEAD, Fontainebleau, France

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Recent porfolio balance models of exchange rate determination under rational expectations are extended to determine jointly the exchange rates, spot and forward, the interest rate, and the current account. With risk aversion, domestic and foreign bonds are not perfect substitutes, despite capital mobility, so that the difference between domestic and foreign interest rates is not equal to the expected rate of depreciation or, equivalently, the forward rate is a biased predictor of the spot rate. The forecast bias is studied following current account and money supply disturbances, distinguishing among permanent or temporary, and anticipated or unanticipated changes.

1. Introduction

The asset market approach has made it clear that the forward exchange rate ought to be considered jointly determined with the spot rate, the interest rate, and the market's expectations of the future spot rate.[1] More recently, borrowing from the theory of finance, several authors[2] have established the conditions under which the forward rate might systematically differ by a risk premium from the corresponding expected spot rate. A number of formal tests tend to indicate that, if one allows for a time-varying premium, it is usually impossible to reject the hypothesis of its existence.[3] The premium is also apparent, at a more casual level, in fig. 1, which presents the three-

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[1]Useful references emphasizing the joint endogeneity of the spot exchange rate and the interest rate are Black (1973), Branson et al. (1977), and Dornbusch (1975). For the role of expectations see, for example, Dornbusch (1976), Kouri (1976), and Mussa (1979).

[2]See, for example, Solnik (1974), Grauer et al. (1976), Adler and Dumas (1976), Fama and Farber (1979), and Frankel (1979a).

[3]There is a considerable amount of empirical work devoted to this issue, with a growing number of authors rejecting the no-risk premium assumption. For a survey, see Levich (1979). For a sample of more recent results, see Hakkio (1980), Bilson (1981), and Meese and Singleton (1980).

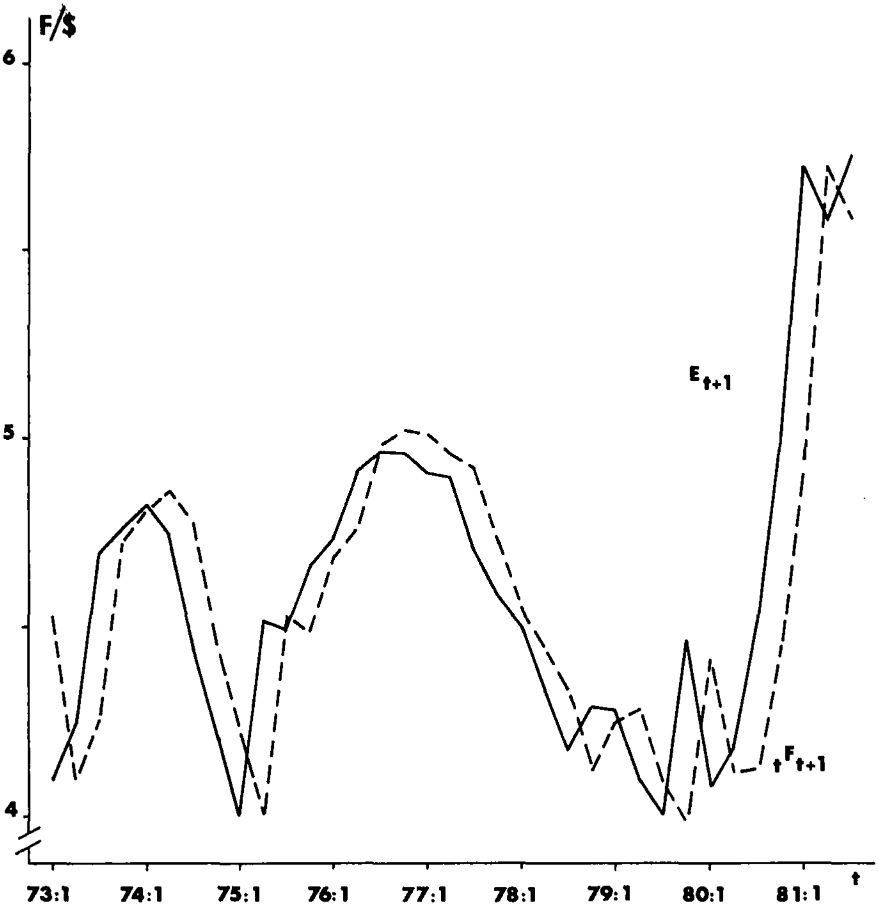

Fig. 1. The French franc: one-quarter-ahead spot (E_{t+1}) and three-month forward rate (${}_tF_{t+1}$); quarterly data. *Source:* IFS (tape of April 1982).

month forward and one-quarter-ahead spot French franc/dollar exchange rates.[4]

Yet, few papers have attempted to present models which would generate results similar to those reported in fig. 1. It seems that a central difficulty consists in giving a macroeconomic content to the usual covariance expressions which explain the forward risk premium. Inevitably, this requires

[4]This pattern can be observed for most currencies and maturities and has been reported by several authors, for example Mussa (1979). According to Mussa, such a behavior simply follows from the fact that, apart from trend movements, both rates react instantaneously to the acquisition of new information. This explains indeed the missing of turning points, but leaves us with the need to explain the systematic, yet varying, forward premia.

making some simplifying assumptions. This paper presents such an attempt. It will be asserted that net holdings of foreign assets together with trade-related heding activities determine the risk premium. As a consequence, the current account plays a crucial role, both through its impact on hedging needs and through the accumulation, or decumulation, of traded assets. The attractiveness of this feature is enhanced by recent contributions to the literature on exchange rate determination [Dornbusch and Fischer (1980), Rodriguez (1978)] which have established a direct link between exchange rate expectations and anticipations of future current account imbalances.

In the next section we present a modified version of the model built by Rodriguez (1978). Because of risk aversion, domestic and foreign assets are imperfect substitutes, allowing for their nominal returns to differ not only by the expected rate of depreciation, but also by a risk premium.[5] The stationary state properties of the resulting portfolio balance model, with three assets and a forward market, are briefly described in section 3. The model is then analytically solved, assuming rational expectations, in section 4, using a technique developed by Blanchard and Kahn (1980). In the next two sections, the model is used to describe the dynamic responses of the exchange and interest rates following first a current account disturbance, then an increase in the money stock. Distinction is made between disturbances, whether they are permanent or not, and whether they are anticipated or come as a surprise. The final section discusses the results, draws some conclusions of interest to interpret recent empirical works, and sketches possible extensions.

2. The model

To the discrete time model of Rodriguez (1978) we only add a domestic non-traded asset and we allow for the existence of a forward exchange market.[6] The model describes a country small enough not to affect the interest rate in the rest of the world. Domestic residents hold domestic money and domestic bonds, both assets being non-traded, and foreign bonds. In log-linear form, the model is as follows:

$$r_t = r_t^* + {}_t f_{t+1} - e_t, \tag{1}$$

$$m_t - e_t - H_t = h_0 - h_1(r^* + {}_t e_{t+1} - e_t) - h_2 r_t + u_t, \tag{2}$$

[5]For similar approaches, see Stein (1980), Dooley and Isard (1979), or Hooper and Morton (1980).

[6]As in the Rodriguez model, there is no attempt to describe the real side of the economy, and therefore the price and output levels are left out. Dornbusch and Fischer have an IS–LM link between the real and financial sectors of their model and describe the behavior of prices under the assumption of fixed output. Giavazzi (1980) provides an extension of their work by introducing a supply equation which enables him to describe jointly the output, the price level and the exchange rate.

$$H_{t+1}=k_{0,t}+(1-k_1)H_t-k_2(m_t-e_t)+v_{t+1}, \tag{3}$$

$$_te_{t+1}-{}_tf_{t+1}=\alpha H_t+\beta({}_tH_{t+1}-H_t), \tag{4}$$

where e_t, $_te_{t+1}$, *and* $_tf_{t+1}$ are, respectively, the logarithms of the spot exchange rate, the present expectation of next period's spot rate, and the prevailing one-period-ahead forward exchange rate; m_t is the logarithm of the present domestic money stock; r_t and r_t^* are the nominal interest rates on domestic and foreign assets, respectively; u_t and v_{t+1} are random white noise independent disturbances; $H_t=a_t-\bar{a}$, where a_t is the log of domestic holdings of foreign assets, denominated in foreign currency and where $\bar{a}$ will be explained below; and $_tH_{t+1}$ is the current expectation of next period's value for H.

Eq. (1) is the covered interest rate parity condition and translates the assumption of perfect capital mobility, so that we obtain perfect substitutability between covered foreign bonds and domestic bonds.[7] Eq. (2) is basically Rodriguez's portfolio balance condition relating the demand for money to the demand for foreign assets, with a random disturbance term u_t reflecting relative assets demand shifts.[8] The next equation, (3), again follows Rodriguez in describing the rate of accumulation of foreign assets $(H_{t+1}-H_t)$ as a Metzler-type saving function, inversely proportional to total domestic wealth, with a random disturbance v_{t+1} representing aggregate demand shifts.[9] The term $k_{0,t}$ is a 'structural' current account parameter which can change over time.

The final equation, (4), incorporates some special assumptions about the forward risk premium. This particular specification is derived from Wyplosz (1980), a simplified version of which is presented in the appendix. It acknowledges two sources of risk premia. The first term represents the role of foreign assets in the domestic portfolio, as studied in the finance theory literature.[10] It leads to a complicated hedging expression, involving the value

[7]Admittedly, then, the domestic bonds are redundant assets. For the reader uncomfortable with redundant assets, it is possible to consider (1) as an equation defining the implicit return, evaluated in domestic currency, of (the only existing) foreign assets, when they are fully covered forward.

[8]This equation is the ratio of the two equations describing the demands for money and for foreign assets as a function of existing yields, evaluated in domestic currency, and of total wealth. Consequently, h_1 is unambiguously positive while h_2 is only *assumed* to be positive. Alternatively, if domestic bonds do not exist (see footnote 7), the returns are those on the two existing interest-yielding assets: uncovered, and covered, foreign bonds. Implicit in the role of the exchange rate is the assumption of purchasing power parity.

[9]The right-hand side of (3) incorporates savings as a function of total domestic wealth which is expressed in units of foreign currency as: antilog(a_t)+antilog(m_t-e_t). When log-linearizing, we do not necessarily obtain $k_1=k_2$. If bonds are net wealth, we should introduce another term, which would eliminate the homogeneity property in (6) below. Alternatively, we can assume that there are no bonds evaluated in domestic currency (see footnote 7).

[10]For references, see footnote 2.

of domestically held foreign assets, a_t, and the covariance term obtained from mean–variance utility maximization. Since these expressions have little macroeconomic content, it is proposed here simply to represent this hedging expression as proportional to a_t, i.e. as αH_t.[11] The second term relates the risk premium to the expected current account surplus $({}_tH_{t+1}-H_t)$, acknowledging the fact that traders normally cover on the forward exchange market part of next period's payments or receipts (assuming that trade shipments occurring in period t are paid for in period $t+1$). Consequently, traders will have to offer a risk premium (resp. discount) in order to induce risk averse investors to bear part of the exchange risk which they cannot fully diversify among themselves as long as the expected trade surplus (resp. deficit) implies a net excess supply of (resp. demand for) foreign currency forward. The coefficients α, β, and $\bar{a}$, derived from optimizing behavior, are presumed to be positive. They are not time invariant since they depend upon the level of exchange rate uncertainty and total risk aversion among exchange market participants.[12] In what follows, though, these coefficients will be taken as constant.

3. The stationary state

We briefly consider the stationary state implications of the model. This is obtained when all exogenous variables and disturbances settle, i.e. when $r_t^* = \bar{r}^*$, $k_{0,t}=\bar{k}_0$, $m_t=\bar{m}$, and $v_t=u_t=0$) and when ${}_te_{t+1}=e_t=\bar{e}$, ${}_tH_{t+1}=H_{t+1} = H_t=\bar{H}$. Then (1)–(4) imply:

$$\bar{H}=\frac{\bar{k}_0-h_0k_2+k_2h\bar{r}^*}{k_1+k_2+\alpha h_2k_2}, \tag{5}$$

$$\bar{e}=\bar{m}-\frac{(1+\alpha h_2)\bar{k}_0+h_0k_1-k_1h\bar{r}^*}{k_1+k_2+\alpha h_2k_2}, \tag{6}$$

where $h=h_1+h_2$.

The stationary state characteristics are as in Rodriguez: the homogeneity property between $\bar{e}$ and $\bar{m}$ is obtained, and an increase in the foreign interest rate leads to higher holdings of foreign assets, both in volume and in value. Finally, a structural improvement in the current account (an increase in $\bar{k}_0$) leads to an increased stock of foreign assets and an exchange rate appreciation implied by the requirement of portfolio equilibrium [eq. (2)].

[11]For a similar development, but with no specific role for trade, see Dornbusch (1980), where $\bar{a}$ is shown to be the minimum variance portfolio and α is proportional to total risk aversion and to the variability of the exchange rate.

[12]As shown in the appendix, if there exists only *one* risk neutral investor with unlimited access to funds, then there is no risk premium ($\alpha=\beta=0$). If *all* traders are risk neutral, then there is no current account effect ($\beta=0$).

More novel are the stationary state characteristics of the interest and forward exchange rates:

$$\bar{f}=\bar{e}-\alpha\bar{H}, \tag{7}$$

$$\bar{r}=\bar{r}^{*}-\alpha\bar{H}. \tag{8}$$

In the presence of uncertainty, there will be a need for forward cover on foreign assets. If $\bar{H}>0$, with the stationarity state current account in balance, this implies a net supply of foreign currency forward and $\bar{f}<\bar{e}$. Consequently, foreign covered bonds being a perfect substitute for domestic ones, $\bar{r}$ will be lower than $\bar{r}^{*}$, as dictated by the interest parity condition.

These results should not be taken too literally. It has been noted that $\bar{a}$ is not really a constant and it is likely that, in the long run, $\bar{a}$ would eventually assume whatever value is needed to have $\bar{H}=0$. This does not preclude the possibility of observing, for quite a long time, divergences between r and r^{*}, as well as a risk premium on the forward exchange rate. In what follows it will be assumed that $\bar{H}=0$ is zero in the original stationary state, but may differ from zero in the new one that follows the disturbance.

4. General solution

To complete the model we need to make a further assumption about how expectations are formed. We will assume a rational expectations process whereby, for any variable x_t:

$${}_tx_{t+1}=\mathrm{E}(x_{t+1}/I_t), \tag{7}$$

where $\mathrm{E}(\cdot)$ is the expectations operator, and I_t is information available at time t.

In order to solve the model, we must distinguish the endogenous variables which are predetermined from those which are not. Non-predetermined variables in a rational expectations model are those variables which are free to react instantaneously to new information: interest and exchange rates play such a role. Predetermined variables, on the other hand, although responding to changes in expectations, are not free to vary immediately: this is the case of the stock of domestically held foreign assets, since this stock can only vary through cumulated current account imbalances. In solving the model, then, we need to obtain for the non-predetermined variables some forward-looking solutions which are independent of past expectations, while the predetermined variables should depend upon both past and present expectations of the future values of the exogenous variables. Blanchard and Kahn (1980) have shown that such a system possesses a unique saddlepointstable solution when the number of eigenvalues of the transition matrix outside the unit circle is equal to the number of non-predetermined variables.

It turns out that the present model is block-recursive since it can be solved, first for e_t and H_t, and then, through (1) and (4), for r_t and ${}_tf_{t+1}$. The reduced system is:

$$H_{t+1}=(1-k_1)H_t+k_2e_t+k_{0,t}-k_2m_t+v_{t+1}, \tag{8}$$

$${}_te_{t+1}=A_1H_t+A_2e_t+A_3+A_4k_{0,t}-r_t^*-(A_2-1)m_t+A_5u_t+A_4\cdot{}_tv_{t+1}, \tag{9}$$

with

$$A_1=(1+\alpha h_2-\beta h_2k_1)/h, \qquad A_2=(1+h+\beta h_2k_2)/h,$$

$$A_3=h_0/h, \qquad A_4=\beta h_2/h, \qquad A_5=1/h, \qquad h=h_1+h_2,$$

which can be rewritten in matrix form, with $K=1-k_1$, as:

$$\begin{bmatrix} H_{t+1} \\ {}_te_{t+1} \end{bmatrix}=A\cdot\begin{bmatrix} H_t \\ e_t \end{bmatrix}+\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix}Z_t \quad \text{and} \quad A=\begin{bmatrix} K & k_1 \\ A_1 & A_2 \end{bmatrix}, \tag{10}$$

where $Z_t'=[1,k_{0,t},r_t^*,m_t,u_t,v_{t+1},{}_tv_{t+1}]$ collects the exogenous variables. The condition for stability and unicity is that we have one eigenvalue outside the unit circle, which happens when $A_2K>A_1k_2$. We assume this to be the case and, in order to avoid oscillatory solutions, that both eigenvalues are positive:

$$0<\lambda_1<1 \quad \text{and} \quad \lambda_2>1. \tag{11}$$

We can now write, following Blanchard and Kahn, the rational expectations solution to (10):

$$H_t=\lambda_1H_{t-1}+\gamma_1Z_{t-1}-\sum_{i=0}^{\infty}\lambda_2^{-i-1}[(K-\lambda)\gamma_1+k_2\gamma_2]\cdot{}_{t-1}Z_{t+i-1}, \tag{12}$$

$$e_t=-(K-\lambda_1)k_2^{-1}H_t-\sum_{i=0}^{\infty}\lambda_2^{-i-1}[(K-\lambda)k_2^{-1}\gamma_1+\gamma_2]\cdot{}_tZ_{t+i}. \tag{13}$$

These equations, together with (1) and (4), can be used to study the system for any expected behavior of the variables collected in Z_t. Among the simplest paths, we can think of one exogenous variable undergoing a single change, and distinguish whether such a disturbance is permanent or temporary, and whether it has or has not been anticipated. The four cases which follow are meant to provide an illustration of the type of behavior generated through such simple disturbances.

5. A current account disturbance

We will first consider the case where the economy enjoys a shift in its current account, as represented by a change in $k_{0,t}$ in (3). In period zero, all other exogenous variables are expected to remain constant. Then, the expectation, as of period zero, of vector Z_t is:

$$_0Z'_t = [1, k_{0,t}, \bar{r}^*, \bar{m}, 0, 0, 0].$$

It is assumed that up until period $t=-1$, the system was resting in its stationary state, described by (5)–(8) with $k_{0,t}=\bar{k}_0$ and $\bar{H}=0$, so that $\bar{e}=\bar{f}=\bar{m}-k_0/k_2$ and $r=\bar{r}^*$. The disturbance is discovered at the beginning of period zero and we will describe the expected paths $_0H_t$, $_0e_t$, and $_0r_t$, as well as the term structure of $_0f_t$ for all $t \geqq 0$. The zero subscript is unambiguous and will be dropped, but it is important to keep in mind that we are describing period zero expectations of all variables. In this section we will allow for the change in $k_{0,t}$ to be first permanent and then temporary. In both cases it will be unanticipated. Anticipated changes will be taken up in section 6.

5.1. The final stationary state

If $k_{0,t}$ is permanently increased from $\bar{k}_0$ to $\bar{k}'_0$, the final stationary state is described as follows:

$$\bar{e}' = \bar{e} - (1+\alpha h_2)(k_1+k_2+\alpha h_2 k_2)^{-1}(\bar{k}'_0 - \bar{k}_0),$$

$$\bar{H}' = (k_1+k_2+\alpha h_2 k_2)^{-1}(\bar{k}'_0 - \bar{k}_0).$$

The change in k_0 implies an accumulation of foreign assets which, together with an exchange rate appreciation, works toward a return of the current account to equilibrium through a wealth effect on aggregate spending. The breakdown between these two adjustments is dictated by the portfolio balance condition which imposes:

$$(1+\alpha h_2)(\bar{H}' - \bar{H}) + (\bar{e}' - \bar{e}) = 0.$$

Finally, note that with $\bar{H}'>0$, we have $\bar{f}' < \bar{e}'$ and $\bar{r} < \bar{r}^*$.

5.2. The disturbance is permanent and unanticipated

We consider the case where it is discovered at time $t=0$ that $k_{0,t}$ has permanently increased to $\bar{k}'_0$. Accordingly, $Z'_t = [1, \bar{k}'_0, \bar{r}^*, \bar{m}, 0, 0, 0]$ for all $t \geqq 0$.

Then, with (12) and (13), we obtain the following paths:

$$H_t = \bar{H}'(1-\lambda_1^t), \tag{14}$$

$$\bar{e}_t = \bar{e}' + (K-\lambda_1)k_2^{-1} \cdot \bar{H}' \cdot \lambda_1^t. \tag{15}$$

The behavior of the endogenous variables is depicted in fig. 2. The current account becomes positive as the economy starts accumulating foreign assets. Portfolio equilibrium requires a corresponding exchange rate appreciation maintaining the domestic value of foreign assets compatible with a fixed money supply. Furthermore, the expectation of an appreciation implies an immediate increase in the demand for money, which has to be offset by a discrete initial drop in e_t.[13]

The forward rate term structure is given by (3′). With a current account surplus, and H_t becoming positive, there is a net supply of foreign currency forward which pushes f_{t+1} below e_{t+1} and, consequently, the domestic interest rate decreases below the world level.

5.3. The disturbance is temporary and unanticipated

We now consider the case where $k_{0,t}$ increases to $\bar{k}_0'$ from period zero until period $T-1$ and then resumes its initial value $\bar{k}_0$ from period T onwards, so that $Z_t' = [1, \bar{k}_0', \bar{r}^*, \bar{m}, 0, 0, 0]$ for $0 \leqq t < T$, and $Z_t' = [1, \bar{k}_0, \bar{r}^*, \bar{m}, 0, 0, 0]$ for $t \geqq T$. The laws of motion, derived from (12) and (13), are given below and are depicted in fig. 3:[14]

$$\begin{aligned} t \leqq T:\quad & H_t = \bar{H}'(1-\lambda_1^t) + \Delta^k \cdot \lambda_2^{-T}(\lambda_2^t + \lambda_1^t)(\lambda_2 - \lambda_1)^{-1}, \\ t > T:\quad & H_t = H_T \cdot \lambda_1^{t-T}; \end{aligned} \tag{16}$$

$$\begin{aligned} t < T:\quad & k_2(e_t - \bar{e}') = (K-\lambda_1)\bar{H}'\lambda_1^t + \Delta^k \cdot \lambda_2^{-T} \cdot \Phi(t;0), \\ t \geqq T:\quad & k_2(e_t - \bar{e}) = -(K-\lambda_1)H_T\lambda_1^{t-T}; \end{aligned} \tag{17}$$

where

$$\Delta^k = (K - \lambda_1 + A_4 k_2)(\lambda_2 - \lambda_1)^{-1}(\bar{k}_0' - \bar{k}_0) > 0,$$

[13] We note that $K - \lambda_1 > 0$ in (15) since: $\lambda_1 = 1/2[A_2 + K - ((A_2 - K)^2 + 4A_1k_2)^{1/2}]$. Also, it can be shown that $e_0 = \bar{e} - (\lambda_2 - 1)^{-1}k_2^{-1}(A_4k_2 + K - \lambda_1) < \bar{e}$.

[14] Such a temporary unanticipated increase in k_0 can be seen as the sum of an unanticipated increase and an anticipated decrease in k_0, both changes being permanent and of the same magnitude. This appears upon inspection of (16) and (17), above, and (18) and (19), below.

and

$$\Phi(t;T)=\lambda_2^{t-T}-(\lambda_2^{t-T}-\lambda_1^{t-T})(\lambda_2-\lambda_1)^{-1}(K-\lambda_1).$$

Because k_0 increases, the economy immediately starts accumulating foreign assets. The attendant increase in wealth leads to a higher demand for money which is offset through an exchange rate appreciation (to reduce e_t+H_t), an expected depreciation, or both. As a result, the exchange rate appreciates in period zero,[15] then depreciates gradually toward its (unchanged) steady state level.[16] If the disturbance is known to be short-lived (T is small), the jump is small, there will be a modest increase in foreign asset holdings until T, and, after period zero, the exchange depreciates back to its long-run value. If T is large, more foreign bonds will be accumulated, the initial appreciation in e_t is larger and followed, for a while, by a further gradual depreciation. Before period T, however, the exchange rate turns around, and embarks on a depreciation path toward its equilibrium value. Only the latter case is reported in fig. 3.

After period T, as k_0 returns to its initial lower value, the accumulated stock of foreign assets implies a current account deficit. The forward rate is unambiguously below the corresponding spot rate until period T. Thereafter, however, the sign of (e_t-f_t) in (4) is ambiguous. The hedging pressure term calls for a continuing premium, while the current account deficit implies a discount. In fig. 3 it is assumed that coefficient β is sufficiently larger than α for the latter effect to prevail.

The relationship (1) between the domestic and foreign interest rates can be rewritten as:

$$r_t-r_t^*=(f_{t+1}-e_{t+1})+(e_{t+1}-e_t). \tag{1'}$$

There is a period before T during which the exchange rate depreciation would push the domestic interest rate above the world level, while the forward premium would have the opposite effect. If both α and β are small, the depreciation effect dominates, a case described in Dornbusch and Fischer. In order to emphasize how the present paper differs from their case of perfect substitutability, it is assumed here, as well as in the other experiments, that β is larger than α, and large enough for the risk premium, related to the current account surplus, to always dominate.

[15]An alternative expression for (17) when $t<T$ is $k_2(e_t-\bar{e})=-(K-\lambda_1)\bar{H}^t(1-\lambda_1^t)-\Delta^k(1-\lambda_2^{-T}\Phi(t;0))$, which gives $k_2(e_0-\bar{e})=-\Delta^k(1-\lambda_2^{-T})<0$, and $|e_0-\bar{e}|$ increasing with T.

[16]The function $\Phi(t;T)$ can be rewritten as $\Phi(t;T)=[(\lambda_2-K)\lambda_2^{t-T}+(K-\lambda_1)\lambda_1^{t-T}](\lambda_2-\lambda_1)^{-1}$. We have $0<\lambda_1<1<\lambda_2$, $K-\lambda_1>0$ (see footnote 13), and $\lambda_2>K$ (since $K<1$). $\Phi(t;T)$ is the sum of two positive terms: the first one is increasing, the second one decreasing. Eventually the first one must dominate. The first term in (17) is decreasing and may lead to an initial decrease in e_t, especially if T is large.

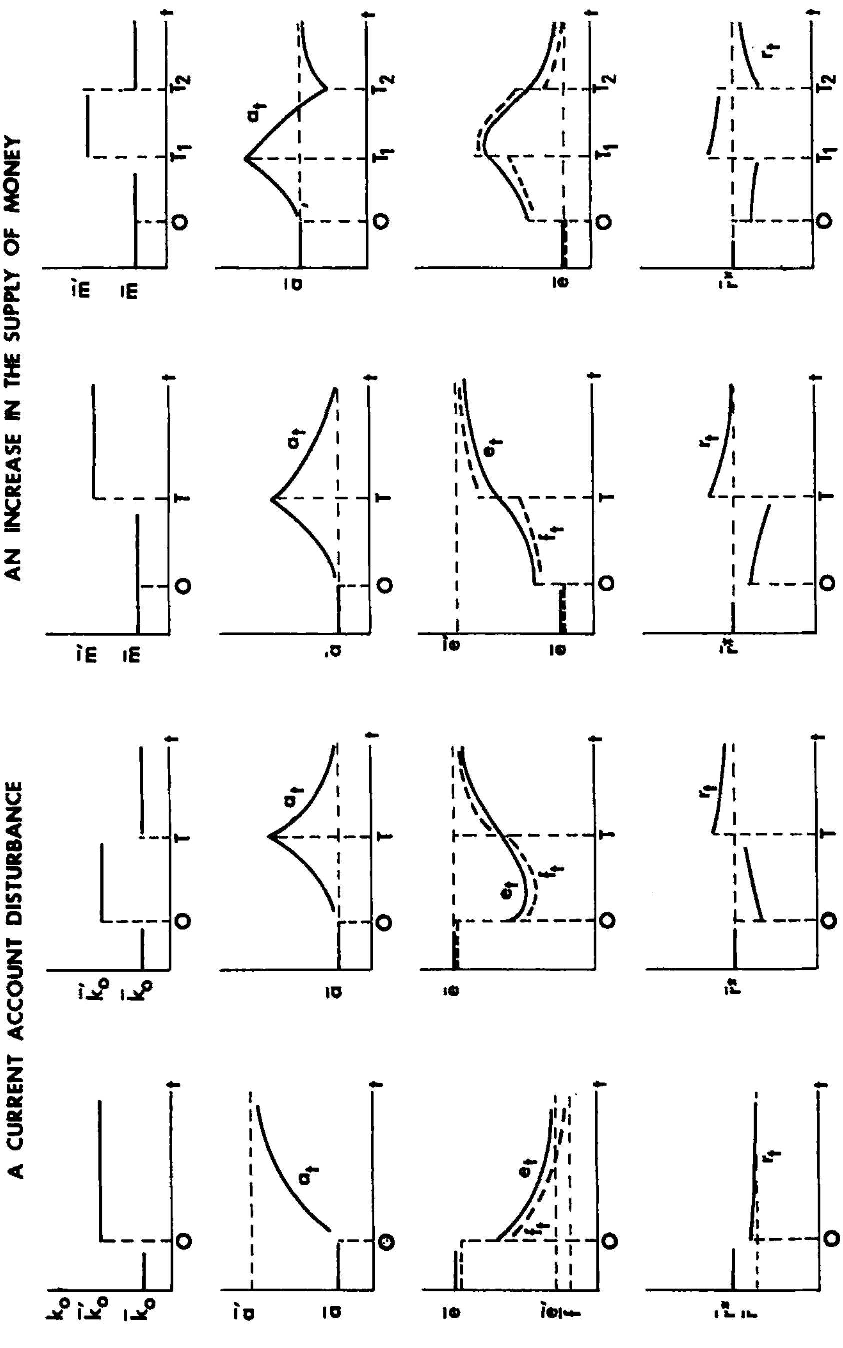

Fig. 2. Permanent unanticipated.

Fig. 3. Temporary unanticipated.

Fig. 4. Permanent anticipated.

Fig. 5. Temporary anticipated.

6. A money supply disturbance

In this section two experiments are presented involving an increase in the money stock. We will consider first a permanent and then a temporary change. Also, by way of contrasting with the previous section, both disturbances will now be anticipated. As is clear from (5) and (6), even a permanent increase in m is going to leave the steady state level of H_t unchanged: all of the effect will eventually be absorbed through an equiproportional depreciation.

6.1. The disturbance is permanent and anticipated

The increase in the money stock is announced in period zero and will be implemented in period T. Thus, the vector of exogenous variables is $Z_t' = [1, \bar{k}_0, \bar{r}^*, \bar{m}, 0, 0, 0]$ for $0 \leqq t < T$, and $Z_t' = [1, \bar{k}_0, \bar{r}^*, \bar{m}', 0, 0, 0]$ for $t \geqq T$, with $\bar{m}' > \bar{m}$. Then (12) and (13) imply:

$$\begin{aligned} t \leqq T&: \quad H_t = \Delta^m \cdot \lambda_2^{-T} \cdot (\lambda_2^t - \lambda_1^t)(\lambda_2 - \lambda_1)^{-1}, \\ t > T&: \quad H_t = H_T \cdot \lambda_1^{t-T}; \end{aligned} \tag{18}$$

$$\begin{aligned} t < T&: \quad k_2(e_t - \bar{e}) = \Delta^m \cdot \lambda_2^{-T} \cdot \Phi(t; 0), \\ t \geqq T&: \quad k_2(e_t - \bar{e}') = -(K - \lambda_1) H_T \lambda_1^{t-T}, \end{aligned} \tag{19}$$

where $\Delta^m = (K - \lambda_1 + A_2 - 1)(\lambda_2 - 1)^{-1} \cdot k_2 \cdot (\bar{m}' - \bar{m}) > 0$, and $\Phi(t; 0)$ has been defined in the previous section.[17]

In figure 4 we present the corresponding paths for the endogenous variables. Since there is an immediate expectation of a long-run depreciation, the demand for money falls, which requires a discrete jump in e_t upon announcement of the disturbance[18] so as to increase $(e_t + H_t)$ and satisfy the money market equilibrium condition (2). This depreciation leads to a current account surplus. By period T, when m is raised, e_t and H_t have both increased enough to make room for the new money stock in domestic portfolios. However, the resulting increase in private wealth turns the current account into a deficit, allowing for a decumulation of foreign assets until H_t goes back to its initial zero level.

Between period zero and T, with $H_t > 0$ and the current account in surplus, we have a positive risk premium ($f_t < e_t$). The sign of $r_t - \bar{r}^*$ is ambiguous and fig. 4 assumes that the risk premium prevails over the expected depreciation

[17] See footnote 16 above.

[18] From (19), $k_2(e_0 - \bar{e}) = \Delta^m \lambda_2^{-T}$. The depreciation will be larger the shorter the advance notice, i.e. the smaller T.

in driving r_t below $\bar{r}^*$. After period T, the hedging pressure term αH_t would still make for a positive risk premium were it not for the current account deficit term $\beta(H_{t+1}-H_t)$, presumed in fig. 4 to dominate, thus bringing r_t above $\bar{r}^*$.

6.2. *The disturbance is temporary and anticipated*

At period zero, it is announced that between T_1 and T_2 the money supply will be temporarily increased from $\bar{m}$ to $\bar{m}'$. Consequently, $Z_t' = [1, k_0, r^*, \bar{m}, 0, 0, 0]$ for both $0 \leqq t < T_1$ and $t \geqq T_2$, and $Z_t' = [1, \bar{k}_0, \bar{r}^*, \bar{m}', 0, 0, 0]$ for $T_1 \leqq t < T_2$. The results presented on fig. 5 correspond to the following laws of motion:

$$
\begin{aligned}
0 \leqq t \leqq T_1: &\quad H_t = \Delta^m(\lambda_2^{-T_1} - \lambda_2^{-T_2})(\lambda_2^t - \lambda_1^t)(\lambda_2 - \lambda_1)^{-1},\\
T_1 \leqq t \leqq T_2: &\quad H_t = H_{T_1}\lambda_1^{t-T_1} - \Delta^m \lambda_2^{-(T_2-T_1)}(\lambda_2^{t-T_1} - \lambda_1^{t-T_1})(\lambda_2 - \lambda_1)^{-1}, \qquad (20)\\
t \geqq T_2: &\quad H_t = H_{T_2}\lambda_1^{t-T_2};
\end{aligned}
$$

$$
\begin{aligned}
0 \leqq t < T_1: &\quad k_2(e_t - \bar{e}) = \Delta^m(\lambda_2^{-T_1} - \lambda_2^{-T_2})\Phi(t;0),\\
T_1 < t \leqq T_2: &\quad k_2(e_t - \bar{e}') = -(K-\lambda_1)H_{T_1}\lambda_1^{t-T_1} - \Delta^m\lambda_2^{-(T_2-T_1)}\Phi(t;T_1), \qquad (21)\\
t > T_2: &\quad k_2(e_t - \bar{e}) = -(K-\lambda_1)H_{T_2}\lambda_1^{t-T_2}.
\end{aligned}
$$

The logic of the observed behavior can be explained as follows.[19] It is foreseen that by the time of the money supply increase in period T_1, both e_t and H_t will have had to increase in order to satisfy the portfolio balance condition. The expectation of a depreciation immediately reduces the demand for money and produces, at time zero, an instantaneous depreciation, which allows for an accumulation of foreign assets. This jump will be larger the shorter the advance notice of the money supply increase, and the longer this increase is maintained.[20] When m_t actually increases in period T_1, the current account turns into a deficit. Since domestic wealth is gradually being reduced, the demand for the higher stock of money can be maintained only through an expected, and therefore actual, appreciation. When the increase in the money stock is cancelled in period T_2, with the exchange rate still above its long-run value, and appreciating, foreign assets holdings must be below their steady-state level ($H_t < 0$) to reduce the

[19] A temporary anticipated change can be seen as the sum of two anticipated permanent changes of opposite signs and identical magnitude.

[20] This follows immediately from (21) as $k_2(e_0 - \bar{e}) = \Delta^m(\lambda_2^{-T_1} - \lambda_2^{-T_2})$.

otherwise excessive demand for money.[21] The drop in wealth after period T_2 leads to lower spending and a current account surplus.

The sign of $(e_t - f_t)$ is ambiguous, except between periods zero and T_1 when both the hedging term and the current account surplus call for a positive risk premium. In fig. 5 it is assumed that the current account effect prevails in pushing the forward rate above the future spot rate between T_1 and T_2, with the opposite situation after T_2. The same ambiguity applies to the behavior of the interest rate.

7. Conclusions

The results reported here are meant to be indicative of the pattern of responses that can be generated by this model. The chosen specification of the risk premium relies on particular assumptions, yet such assumptions are indispensable if we want to incorporate it in a macroeconomic model. Other assumptions can be made, and may well lead to different results, but some general conclusions should remain.

First, since perfect capital mobility and rational expectations do not rule out the possibility of a systematic risk premium or discount,[22] the forward rate should normally be a biased forecast of the corresponding future spot rate. The forecast bias is systematic in the sense that it represents some well-defined forces at work, yet it need not be stable over time, neither in size, nor in sign, as these underlying forces vary. Thus, failure to identify a *constant* risk premium is no proof of the absence of any systematic premium. On the other hand, the existence of a shifting bias is consistent with the existence of filter rules which result in occasional profits over some short periods, but not over more extended ones, as discussed in Levich (1979).

Secondly, the joint endogeneity of the spot and forward exchange rates, together with the interest rate and the current account, results in the possible breakdown of some popular relationships based on a more partial equilibrium view. This is the case of the acceleration hypothesis [Kouri (1976)], whereby current account surpluses (resp. deficits) should be observed simultaneously with appreciating (resp. depreciating) exchange rates.[23] It is also true for the relationship between exchange rate changes and the interest rate, predicted to be positively correlated in some models, and negatively correlated in others:[24] no such regular pattern emerges from the experiments reported here. The uncovered interest rate parity does not hold here either,

[21]From (11) and (20) we have

$$H_{T_2} = -\Delta^m \lambda_1^{-T_2}[(\lambda_1^{-T_2} - \lambda_1^{-T_1}) + (\lambda_2^{-T_1} - \lambda_2^{-T_2})](\lambda_2 - \lambda_1)^{-1} < 0.$$

[22]The conditions for a non-zero risk premium are discussed in the references quoted in footnote 2.

[23]This point has already been made by several authors, for example Rodriguez or Dornbusch and Fischer. The acceleration hypothesis is usually verified along the stable path, not necessarily during transitions to this path which occur in anticipation of disturbances.

since we have several instances where the exchange rate appreciates while the interest rate first increases above the world level, then falls below it. Finally, we obtain in fig. 5 a higher interest rate precisely when the money stock is exogenously increased. More generally, the behavior of the interest rate is often found to be ambiguously explained by the model. While this ambiguity may appear to be deceptive, it is a useful reminder that this variable is subject to many more forces than simple theories allow it to be.

On a methodological level, this paper presents complete analytical solutions to a linear rational expectations model, while most authors choose graphical solutions. This is certainly a more cumbersome technique. Its main advantage is to direct attention to less obvious paths and to the role of certain parameters.[25]

Among the possible extensions, the most interesting would consist of allowing the coefficients α and β in (4) to depend on the variance of the underlying stochastic terms, as suggested in the appendix. For example, one might study the impact of randomness in monetary policy on these coefficients and therefore on the risk premium and the interest rate.

Appendix: A simplified derivation of the risk premium

This appendix presents a simplified version of the arguments developed in Wyplosz (1980).

We consider a small economy consisting of international traders who export a volume X_t and import a volume Z_t. The price of these goods is assumed to be P_t^*, fixed in foreign currency. The deals arranged during period t will be paid for in period $t+1$. A proportion α_t (resp. β_t) of exports (resp. imports) are not covered on the forward exchange market, and the amount paid will depend on the unknown value of the exchange rate $\tilde{E}_{t+1}$.

We also have investors with an initial wealth W_t. They hold domestic money M_t, domestic bonds (value$=B_t$), foreign bonds without cover (value $=B_{u,t}$), and with cover (value$=B_{c,t}$). The interest factor (interest rate $+1$) on domestic bonds is $\tilde{R}_t$, and on foreign bonds it is $\tilde{R}_t^*$. Randomness arises for $\tilde{E}_{t+1}$, $\tilde{R}_t$, and $\tilde{R}_t^*$.

The economy chooses α_t, β_t, M_t, B_t, $B_{c,t}$, and $B_{u,t}$ in order to maximize the sum Ω_t of returns from trade and wealth. By Walras law: $M_t = W_t - B_t - B_{c,t} - B_{u,t}$. Dropping the subscript t we have:

$$\begin{aligned}\Omega = {} & \alpha\tilde{E}_{+1}P^*X + (1-\alpha)FP^*X - \beta\tilde{E}_{+1}P^*Z - (1-\beta)FP^*Z + W - B - B_u - B_c \\ & + B\tilde{R} + B_u\tilde{R}^*(\tilde{E}_{+1}/E_0) + B_c\bar{R}^*(F/E_0) + B_c^*(\tilde{R}^* - \bar{R}^*)\tilde{E}_{+1}/E_0, \qquad \text{(A1)}\end{aligned}$$

[24]See the discussion and the tests reported in Frankel (1979b) and the follow up in the *American Economic Review*, December 1981.

[25]For example, the possibility of non-monotonicities in the path of e_t in figs. 3 and 5, or the role of the lag between the announcement and the occurrence of disturbances, as discussed in the text.

where $\bar{R}^* = \mathrm{E}(\tilde{R}^*)$. We can rearrange (A1) to obtain:

$$\Omega = A + B\tilde{R} + C\tilde{\lambda} + D\tilde{\psi}, \tag{A2}$$

where

$$\tilde{\lambda} = (\tilde{E}_{+1} - F)/E_0, \qquad \tilde{\psi} = \tilde{R}^* \tilde{E}_{+1},$$

and

$$A = FP^*(X - M) + W - B - B_\mathrm{u} - B_\mathrm{c},$$

$$C = FP^*(\alpha X - \beta Z) - B_\mathrm{c} \bar{R}^* F/E_0,$$

$$D = (B_\mathrm{u} + B_\mathrm{c})/E_0.$$

The economy is assumed to have a mean–variance utility function:

$$\mathrm{E}[u(\Omega)] = U[\mathrm{E}(\Omega), \mathrm{V}(\Omega)], \quad \text{with } U_1 > 0, U_2 < 0. \tag{A3}$$

The net supply of foreign currency forward is:

$$S = (1-\alpha)FP^*X - (1-\beta)FP^*Z + B_\mathrm{c}\bar{R}^*F/E_0 = FP^*(X - M) - C. \tag{A4}$$

Maximizing $\mathrm{E}[u(\Omega)]$ with respect to α, β, B, B_u, and B_c we obtain the corresponding first-order conditions. We are interested in establishing the forward market equilibrium condition $S=0$, so we need only to know the optimal value $\hat{C}$ of C. This value is given as a function of optimal values $\hat{B}$ and $\hat{D}$ of B and D in (A2):

$$\hat{C} = \frac{1}{\rho} \cdot \frac{\mathrm{E}(\tilde{\lambda})}{\mathrm{V}(\tilde{\lambda})} + \frac{\hat{B}\operatorname{cov}(\tilde{R}, \tilde{\lambda}) + \hat{D}\operatorname{cov}(\tilde{\lambda}, \tilde{\psi})}{\mathrm{V}(\tilde{\lambda})}, \tag{A5}$$

where $\rho = -U_1/2U_2$ is the degree of risk aversion. With (A4) and (A5), the forward market equilibrium condition gives:

$$\mathrm{E}(\tilde{\lambda}) = \rho \cdot \mathrm{V}(\tilde{\lambda}) \cdot FP^* \cdot (X - M) + \rho[\hat{B}\operatorname{cov}(\tilde{R}, \tilde{\lambda}) + \hat{D}\operatorname{cov}(\tilde{\lambda}, \tilde{\psi})],$$

which is used as (4) in the text. Also note that this set-up could be used to derive the portfolio balance condition (2) by solving for B, B_c, and B_u.

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