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Probability and Utility Estimates for Racetrack Bettors

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Subjective and estimated objective winning probabilities are obtained from 20,247 harness horse races. It is shown that subjectively a horse with a low winning probability is exaggerated and one with a high probability of winning is depressed. Various hypotheses characterizing the bettors' behavior to explain the observed subjective-objective probability relation are explored. Under some simplified assumptions, a utility of wealth function of a decision maker is derived, and a quantitative summary measure of his risk attitude is defined. Attitude toward risk of a representative bettor is examined. It is found that he is a risk lover and tends to take more risk as his capital dwindles.

Several laboratory experiments (Preston and Baratta 1948; Yaari 1965; Rosett 1971) suggest that in making a decision under uncertainty low probability events are overbet and high probability events are underbet. None of these experiments was conducted under a natural environment of the decision makers. In two separate studies of actual bettings in horse races, Grifith (1949, 1961) arrived at the conclusion similar to those of Preston and Baratta and Yaari. McGlothlin (1956) repeated the Grifith (1949) study with a different set of data on horse races, and among other things he confirmed Preston and Baratta's (1948) findings. None of the Grifith and McGlothlin studies considered more than 1,500 races to estimate several probabilities. Martin Weitzman (1965) analyzed over 12,000 races to obtain a relationship between the objective probability of

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TABLE 1

DATA DESCRIPTION

Race Track and Year	Racing Dates (N)	Races (N)	Average Daily Attendance	Average Bet per Race ($)*	Average Bet per Person in a Race ($)
Saratoga:					
1970	193	1,909	3,784	23,701	6.26
1971	182	1,779	3,873	25,500	6.58
1972	187	1,829	3,393	23,706	6.98
1973	189	1,834	3,486	25,350	7.27
1974	172	1,721	3,541	25,694	7.26
Roosevelt:					
1970–71† ..	192	1,698	20,426	228,884	11.21
1972	154	1,355	17,014	213,082	12.52
1973	153	1,347	17,148	214,110	12.49
1974	159	1,406	15,789	216,435	13.71
Yonkers:					
1971	155	1,381	18,025	224,289	12.44
1972	145	1,268	17,258	234,837	13.61
1973	160	1,407	15,871	225,639	14.22
1974	148	1,313	15,988	227,853	14.25

NOTE.—All tracks are in New York.
* Includes all possible betting opportunities.
† Oct.–Dec. 1970, and March–Oct. 1971. In the analysis the time period is treated as the year 1971.

a horse to win and the return for a dollar bet if the horse wins. The subjective probability defined in the Grifith or McGlothlin studies is proportional to the reciprocal of this return. The estimated relationship confirms the Grifith and McGlothlin findings.

The studies of Grifith, McGlothlin, and Weitzman were limited to the thoroughbred races. The probability estimates of these studies have serious estimation errors. The estimates obtained by Weitzman are the most reliable in which a coefficient of variation of 16 percent or higher is common.[1] This can be considered large to detect even a difference of say, 20 percent, between objective and subjective probabilities. Moreover, serious technical difficulties, though ignored, arise in their grouping of observations (see Sec. I) to estimate the objective probabilities.

The first theoretical attempt to explain the observed discrepancies between objective and subjective probabilities is by Rosett (1965). Weitzman (1965) constructed a representative utility of wealth function as an implication from the relationship between objective and subjective probabilities.

In this paper, public betting behavior in 20,247 harness horse races is analyzed. The data are described in table 1. Subjective and estimated

[1] Improved estimates can be obtained from his estimated objective probability-return relationship, but the significance of this improvement cannot be ascertained from the available data.

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objective winning probabilities are obtained in Section I. It is found that subjectively a horse with a low probability of winning is overstated and that a high probability of winning is understated. Section II explores various hypotheses characterizing the bettors' behavior to explain the observed subjective-objective probability relation. Specifically, a theorem is proved showing a possible alternative explanation of these biases. Section III is devoted to studying the betting public's risk attitudes, especially as they are affected by the amount of money available to bet. A numerical measure of risk attitude is defined. It is found that the betting public behaves as risk lovers. The risk attitude seems to change with the amount of money (capital) available to bet. The representative bettor becomes more risk loving as his capital declines. Concluding remarks are in Section IV.

I. Probability Estimates

Besides the popular betting of the "place," "show," "daily double," etc., there is one regular betting opportunity in a race known as "win" bet. Betting on a horse to win pays only when the horse finishes first in the race.[2] Payoffs are according to the odds. Odds are profits per dollar bet to a successful bettor. Odds are determined from the bets made by the public, the track take, and breakage.[3] Odds for different types of bets—win, place, show, etc.—are computed separately. Suppose there are H horses numbered[4] $1, 2, \ldots, h, \ldots, H$, and let X_h be the amount of money that has been bet for horse h to win. Then,

$$W = \sum_{h=1}^{H} X_h$$

is the total win bet in the race and is called the "win pool." Let the track take and breakage be α. Then the odds on horse h are

$$a_h = [(1 - \alpha)W - X_h]/X_h = (1 - \alpha)W/X_h - 1, \; h = 1, 2, \ldots, H. \tag{1}$$

[2] Races with "dead heat" at any finishing position are not considered.

[3] A fixed proportion of the amount bet in a race is taken out by the track before it distributes the rest to the successful bettors. This proportion is known as the track take. The breakage arises because of the following two restrictions: (*a*) odds cannot be below a certain minimum, and (*b*) odds have to be rounded downward except when the former restriction is in effect, in which case, it is rounded upward. For the races that are analyzed, all odds are rounded to 10 cents and the minimum odds are also 10 cents.

[4] Sometimes several horses are grouped, known as "entry" or "field," for a single betting interest, and the group is assigned a single number. A bet on this number is successful if one of the horses in the group is successful. Thus, if it is a win bet, the bet is successful if one of the horses in the group finishes first. Without loss of generality, this group will be taken as a single horse.

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The estimates of a_h's which are reported to the public once in every minute till the race starts fluctuate over the entire betting period. However, as fluctuation is minimum in the last few minutes of the betting period when the major amount of bets are placed, and as our data are limited to only final odds, we will proceed as if the final a_h's prevailed throughout the period (for an alternative justification of this assumption see Rosett [1965]). Our data are also limited to the odds on win bets, and therefore we will confine ourselves to the win bet only and refer to it as a bet.

Following Grifith and McGlothlin, the subjective probability of horse h to win is defined as the proportion of the win pool that is bet on it,[5] (i.e., X_h/W). This can be shown to be equal to $(1 - \alpha)/(1 + a_h)$, and from (1) it can be verified that

$$\sum_{h=1}^{H} \left(\frac{1 - \alpha}{1 + a_h}\right) = 1 \tag{2}$$

where H is the number of horses in the race.

The objective winning probability of a horse is defined to be the proportion of times the horse wins when the race is repeated an infinitely large number of times. Thus, a race can be taken as a binomial trial with this objective probability for the winning outcome. In a race the total of the objective probabilities is one.

Both subjective and objective probabilities are different for horses in a race, and also they differ in different races. Let the subjective probability for a horse to win the ith race be ρ_i, the objective probability be π_i, and the odds be a_i. Then $\rho_i = (1 - \alpha)/(1 + a_i)$, where α is the track take and breakage. From our data on the odds, α can be computed using (2), and hence ρ_i can be obtained. However, we have only one observation to estimate π_i. Thus, no reasonably reliable estimate of π_i can be obtained. This estimation difficulty is inherent, though ignored in the studies of Grifith, McGlothlin, and Weitzman.

In our study the horses are grouped, and their average subjective probability is compared with an unbiased estimate of the corresponding average objective probability. Horses in the same group are identified with a single horse. Let there be N races competed in by a horse with ρ_i and π_i as its subjective and objective probabilities to win the ith race. Then its average subjective and objective probabilities are defined as

[5] Considering each dollar bet as a vote to win, the subjective probability is the total public support for the horse to win the race. It should be noted that this definition of subjective probability is not to imply expected net gain from betting a dollar on a horse is $-\alpha$, though the subjective expected net gain is $-\alpha$ and is the same for every horse. This subjective probability is also the reciprocal of the return to a dollar bet if the horse wins when there is no track take or breakage. Thus, when there is no possibility of confusion, these two terms, subjective probability and return, will be used interchangeably.

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$\bar{\rho} = \sum \rho_i/N$, and $\bar{\pi} = \sum \pi_i/N$, respectively. When there is no possibility of confusion, these averages will be referred to as the respective winning probabilities of the horse. Once ρ_i's are computed $\bar{\rho}$ can be obtained. An unbiased estimate of $\bar{\pi}$ is $\hat{\pi} = \sum Y_i/N$, where $Y_i = 1$ if the horse wins the ith race and equals zero otherwise. As the races are considered independent binomial trials, it can be shown that $E(\hat{\pi}) = \bar{\pi}$, and $\text{var}(\hat{\pi}) = \bar{\pi}(1 - \bar{\pi})/N - \sigma_\pi^2/N$, where $\sigma_\pi^2 = \sum (\pi_i - \bar{\pi})^2/N$. In the absence of a reasonable estimate of σ_π^2, we neglect it in estimating var $(\hat{\pi})$ which is estimated as $\hat{\pi}(1 - \hat{\pi})/N$. Thus, the reported standard error of $\hat{\pi}$ which is the square root of this variance-estimate is exaggerated.

In grouping the horses we use the following criteria: (*a*) N should be as large as possible; and (*b*) σ_π^2 should be as small as possible. By choosing a large N, var $(\hat{\pi})$ is reduced so that $\hat{\pi}$ is a meaningful estimate of $\bar{\pi}$. Criterion (*b*) is necessary for a valid test of the hypothesis of subjective over- or underestimation of objective probabilities by comparing $\bar{\pi}$ with $\bar{\rho}$. Previous studies have grouped the horses according to their odds falling in various selected ranges. For this grouping, it is very likely to have more than one horse in a group competing in the same race. Thus, their identity with a single horse is destroyed. To avoid this possibility, such competing horses can be treated as an "entry" or "field" (see n. 4). However, the odds on this field which are to be estimated should be lower than the individual odds on these horses, and thus the field may not belong to the group from which the individual horses are drawn. Hence, not only need we estimate the odds on the field but also the groups need to be rearranged. Thus, we abandon this method of grouping.

It can be argued that the variation in the winning probabilities of the horses most likely to win the races may not be substantial, that is, σ_π^2 would be small for this group of horses. Also, for this group N would be as large as the total number of races. Hence this grouping closely satisfies criteria *a* and *b*. Similar arguments can be put forward for the horses which are second most likely to win the races. Thus, without any further information, the horses are grouped according to "favorites." The horse with the lowest odds in a race is called the first favorite; the horse with the next lowest odds is known as the second favorite, and so on. The hth favorite in a race will be called horse h,[6] and its subjective and objective winning probabilities will be denoted by $\bar{\rho}_h$ and $\bar{\pi}_h$, respectively.

In the following discussion, no reference is made to horses 9 and 10 because the number of races in which they competed is small. The estimates $\hat{\pi}_h$'s and their standard errors are in columns 3 and 4 of table 2, respectively. The standard errors expressed as percentages of the estimates

[6] A race is limited to the maximum of 10 horses, and often eight horses compete in a race. In our data, eight horses competed in 15,749 out of 20,247 races, whereas nine horses competed in only 299 races and 10 horses competed in only 71 races.

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TABLE 2

PROBABILITY ESTIMATES

Horse (h) (1)	Races Competed (N) (2)	$\hat{\pi}_h$ (3)	SE ($\hat{\pi}_h$) (4)	$100\left(\frac{\text{SE}}{\hat{\pi}_h}\right)$ (5)	$\bar{\rho}_h$ (6)	$\frac{(\bar{\rho}_h - \hat{\pi}_h)}{\text{SE}(\hat{\pi}_h)}$ (7)
1	20,247	.3583	.0034	0.95	.3237	−10.29
2	20,247	.2049	.0028	1.37	.2077	0.99
3	20,247	.1526	.0025	1.64	.1513	−0.52
4	20,247	.1047	.0022	2.10	.1121	3.45
5	20,231	.0762	.0019	2.49	.0827	3.49
6	20,088	.0552	.0016	2.90	.0601	3.01
7	19,281	.0341	.0013	3.81	.0417	5.80
8	15,749	.0206	.0011	5.34	.0276	6.20
9	299	.0033	.0033	. . .	. . .	. . .
10	71	.0141	.0140	. . .	. . .	. . .

NOTE.—Subjective probabilities for horses 9 and 10 are not shown because they are based on a very small number of observations. Corresponding objective probability estimates with their standard errors are shown to give an idea of the unreliability of these estimates.

(col. 5) vary from 0.95 percent to 5.34 percent and they can be considered small. The subjective probabilities are in column 6. Differences between subjective and estimated objective probabilities are expressed as ratios to the respective standard errors and are shown in column 7. Except for horses 2 and 3, these differences can be considered statistically significant. Every subjective probability with the exception of that of the first favorite exceeds the corresponding objective probability estimate. For the first favorite the subjective probability is significantly less than the corresponding objective probability. Thus, subjectively, a horse with a high winning probability is understated and one with a low winning probability is overstated. Qualitatively, this finding is in accord with that observed by Preston and Baratta (1948), Grifith (1949, 1961), McGlothlin (1956), and Weitzman (1965).

The present data extend over a number of years (1970–74), various tracks, and numerous race conditions. To examine the influence of some of these factors,[7] all the data were divided into 15 data sets. The previous analysis applied to each of these data sets pointed to the same conclusion as has already been reached.

II. Theoretical Explanation of the Differences between Subjective and Objective Probabilities

Despite the sampling errors and the grouping problem, the Grifith, McGlothlin, and Weitzman findings about the relationship between subjective and objective probabilities are qualitatively in accord with

[7] Some characteristics such as average daily attendance, average bet per race, average bet per race per person, the years of racing, and the racetracks are in table 1.

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the present study. Grifith and McGlothlin as well as Preston and Baratta seemed to imply that the observed differences between subjective and objective probabilities are psychological. If the bettors are sophisticated the observed subjective-objective probability relation can be explained by adopting the Friedman-Savage (1948) expected utility hypothesis (EUH) if the utility functions are restricted to a properly chosen class.[8]

Rosett (1965) claims that the observed relationship is consistent with the hypothesis that bettors are rational, sophisticated, and have strong preference for low-probability–high-return bets.[9] This hypothesis is one manifestation of EUH with the properly restricted class of utility functions. To see this let us define the equilibrium betting opportunity set as the one constituting only the nondominant opportunities; that is, for any two opportunities in the set no one is preferred to the other by every bettor. Henceforth, we also maintain that an opportunity A is preferred to another opportunity B by *every* rational bettor if and only if the winning probability of A is at least as large as that of B and the return of A is larger than that of B or the winning probability of A is larger than that of B and the return of A is at least as large as that of B. Then one can prove (left to the reader):

Theorem 1.—For any two betting opportunities, A and B, A is not preferred to B by every rational bettor if and only if A is not preferred to B by every bettor with an increasing utility function who follows EUH. As can be shown from the above theorem, the equilibrium betting opportunity set as characterized by the rationality hypothesis is identical with the one obtained by adopting the EUH and restricting the utility functions to a class of increasing functions. In this case, EUH with a class of increasing utility functions is equivalent to the rationality hypothesis. The hypothesis that bettors have strong preference for low-probability–high-return bets restricts further the class of utility functions, and following Rosett (1965) it reduces the equilibrium set to the set where objective probability, π, is related to return, R, by $R = \pi^{\beta}$ where $\beta < 0$.

Rosett's postulated objective probability-return relationship was consistent with only a part of his data, and it systematically overstated the returns when the winning probabilities were smaller. In what follows we show that if the assumption of sophisticated bettors is replaced by the assumption that each bettor knows something about the objective winning probabilities but no one knows them exactly, then for a wide range of

[8] The EUH asserts that each bettor has a unique utility of wealth function and each maximizes his expected utility to choose his preferred bet. Utility functions are unique only up to positive linear transformations.

[9] Bettors are rational in the sense that no one prefers a bet with a smaller winning probability and the same or lower return, or with a lower return and the same or lower winning probability to that available to him. Bettors are sophisticated in the sense that the objective winning probabilities are known to them.

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assumptions about these estimation errors the data can be consistent with the hypothesis that bettors are rational and risk neutral.

For simplicity of exposition, let us assume that there are only two horses, H_1 and H_2, in a race. Let π_h be the objective (true) probability for H_h to win the race, ρ_h be an individual's subjective estimate of π_h, and a_h be the market-established final odds on H_h. For every individual, ρ_h's are nonnegative and $\rho_1 + \rho_2 = 1$; ρ_h's vary from person to person. Let the distribution of ρ_h be $F_h(\cdot)$ so that $F_h(\rho)$ is the proportion of individual estimates of π_h not exceeding ρ. To avoid mathematical complexity, we assume that the probability of an individual estimating π_h exactly is zero. Bettors are assumed rational, and the amount of a bet for an individual is decided upon in advance and it is the same for everyone. With the above assumptions we prove:

Theorem 2.—If π_h is the median of the distribution, $F_h(\cdot)$, and the bettors are risk neutral, the subjective probability for H_h to win, $\bar{\rho}_h = (1 - \alpha)/(1 + a_h)$ where $0 \leq \alpha < 1$ is the track take and breakage, exceeds the corresponding objective probability, π_h, if and only if $\pi_h < \pi_{h'} (h \neq h')$.

Proof.[10]—As bettors are risk neutral, the utility functions are linear; and as they are rational, horse H_h will be bet on if and only if

$$[(1 + a_h)\rho_h - 1] > \max \{0, [(1 + a_{h'})\rho_{h'} - 1]\}. \qquad (3)$$

Without loss of generality, we assume $\alpha = 0$. As $\bar{\rho}_1 + \bar{\rho}_2 = 1$, $1/(1 + a_1) + 1/(1 + a_2) = 1$, so that from (3), the individual with subjective probability estimate ρ_h and $\rho_{h'}$ will bet on H_h if and only if $\rho_h > 1/(1 + a_h)$. Thus, the proportion of all the bettors wagering on H_h is $1 - F_h[1/(1 + a_h)] = 1 - F_h(\bar{\rho}_h)$, and this must equal $\bar{\rho}_h$ because bettors are assumed to bet the same amount. Hence, $\bar{\rho}_h$ can be obtained as a solution to the equation,

$$F_h(\bar{\rho}_h) = 1 - \bar{\rho}_h, \qquad h = 1, 2. \qquad (4)$$

As $F_h(\cdot)$ is a monotonic function, the solution is unique.

If $\pi_1 = \pi_2$, then $\pi_1 = \pi_2 = \frac{1}{2}$, and as π_h is the median of the distribution, $F_h(\pi_h) = 1 - \pi_h$, so that $\bar{\rho}_h = \pi_h = \frac{1}{2}$. If $\pi_1 > \pi_2$, then $\pi_1 > \frac{1}{2}$ and $\pi_2 < \frac{1}{2}$. Consequently, $F_1(\pi_1) > 1 - \pi_1$ and $F_2(\pi_2) < 1 - \pi_2$; and as $F_h(\cdot)$s are nondecreasing functions, it follows from (4) that $\bar{\rho}_1 < \pi_1$ and $\bar{\rho}_2 > \pi_2$. Similarly, it can be shown that if $\pi_1 < \pi_2$, then $\bar{\rho}_1 > \pi_1$ and $\bar{\rho}_2 < \pi_2$. Hence the theorem.

Following the theorem, if $\pi_1 \neq \pi_2$, the larger objective probability is understated whereas the smaller objective probability is overstated. This is in accord with our empirical findings.

[10] Some of the assumptions and conditions of the theorem can be relaxed. For example, the condition that π_h is the median of the distribution, $F_h(.)$, can be weakened and the theorem holds if the larger of π_1 and π_2 does not fall below the median of the distribution of its subjective estimates.

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If we want to maintain the hypothesis that bettors are sophisticated, the observed objective probability-return relation can be explained, as noted earlier, by EUH with a suitable restriction on the class of utility functions. One such attempt was made by Weitzman (1965) where it is assumed that bettors are homogeneous, each having the same utility function. The utility function is chosen so that the observed probability-return relation coincides with the expected utility indifference curve. It can be shown that bettors with such a utility function are not rational.[11] The source of this counterintuitive behavioral implication is the expansion possibility of the betting opportunity set through various combinations (parlay, martingale, etc.) of the simple bets available in individual races. Thus, this undesired implication can be avoided by restricting the betting opportunities to a single race. It is shown in the next section that the implied utility function is increasing with increasing rate; that is, the bettors are risk lovers. Hence, the observed discrepancies between subjective and objective probabilities may be due to estimation errors by the bettors who are risk neutral or because the bettors are sophisticated but they are risk lovers and behave as if the betting opportunities are limited to a single race.

III. Risk Attitudes

In what follows we assume that bettors are expected utility maximizers, sophisticated, and behave as if the betting opportunities are limited to a single race. Let Mr. B, like Mr. Avmart in Weitzman (1965), be a representative of all the bettors who establish the final odds. He may not be one of the bettors. Let $u(\cdot)$ be his utility of wealth function. In general, utility is also a function of various circumstantial factors. In what follows we abstract from these nuances and assume utility is exclusively a function of wealth. Let us now construct a representative race in which horses 1, 2, . . . , 8 of table 2 are competing. Let the true objective probability of horse h to win a race be $\hat{\pi}_h$. By definition, horse h did not have unique odds in all the races in which it competed. The representative (estimated) odds are taken to be a weighted average of all its realized odds. The weights are chosen so that the actual average net gain per dollar bet for horse h to win over all the races it competed in is the amount that would have been obtained if this weighted average of the odds had prevailed for

[11] The expected utility indifference curve is estimated as

$$\pi = [1.011 - 0.087 \log_{10}(1 + R)]/R,$$

where R is the return and π is the winning probability of a bet. A parlay (for the definition, see Rosett 1965) constructed from 10 races and using a betting opportunity with $R = 10$ and π as obtained from the indifference curve would have a probability better than three times the probability of a bet from the indifference curve with the same return as this parlay. Thus, if this is an indifference curve, bettors cannot be rational.

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TABLE 3

UTILITY FUNCTION AND AVERAGE AND STANDARD DEVIATION OF GAINS FROM A DOLLAR BET

Horse (h) (1)	Estimated Odds ($\bar{a}_h$) (2)	$x = 1 + \bar{a}_h$ (3)	Utility Function $u(x)$ (4)	Average Gain ($) (5)	SD ($) (6)
1	1.5	2.5	.0574	−0.10	1.26
2	3.0	4.0	.1004	−0.19	1.63
3	4.4	5.4	.1348	−0.17	2.00
4	6.2	7.2	.1966	−0.24	2.30
5	8.7	9.7	.2699	−0.26	2.68
6	12.3	13.3	.3726	−0.27	3.22
7	17.7	18.7	.6028	−0.36	3.64
8	26.4	27.4	1.00	−0.44	4.27

every race. Let $\bar{a}_h$ be the estimated odds for horse h. These are reported in column 2 of table 3. For any race the objective as well as the subjective probabilities must add up to 1. In our case, these totals are, respectively, 1.01 and 0.99. Thus, the conditions are reasonably met. As these $\hat{\pi}_h$'s and $\bar{a}_h$'s are constructed from a number of races, the present race can be considered a representative of them.

With the choice of $\hat{\pi}_h$'s and $\bar{a}_h$'s, Mr. B's expected utility of betting M dollars on horse h is

$$E_h(u) = (1 - \hat{\pi}_h)u(X_0 - M) + \hat{\pi}_h u[(X_0 - M) + M(1 + \bar{a}_h)] \tag{5}$$

where X_0 is the initial capital of Mr. B. As the $\bar{a}_h$'s are the final odds, Mr. B must be indifferent between betting on any two of these eight horses. Thus, Mr. B being an expected utility maximizer,

$$E_1(u) = E_2(u) = \cdots = E_8(u). \tag{6}$$

Without loss of generality, we assume that $X_0 - M = 0$, and the odds are in the units of M dollars where M is the common denominator of all the bets; and as the utility function is unique only up to positive linear transformations, we assume that $u(0) = 0$ and $u(1 + \bar{a}_8) = 1$, where $\bar{a}_8$ are the highest odds. Then from (5) and (6),

$$u(1 + \bar{a}_h) = \hat{\pi}_8/\hat{\pi}_h, \qquad h = 1, 2, \ldots, 7. \tag{7}$$

Using (7), the specific values of the function, $u(\cdot)$, are computed and reported in column 4 of table 3; $u(\cdot)$ is plotted in figure 1. All the points lie below the line joining the two chosen points $[0, u(0) = 0]$ and $[1 + \bar{a}_8, u(1 + \bar{a}_8) = 1]$. Hence, the utility function of Mr. B must be

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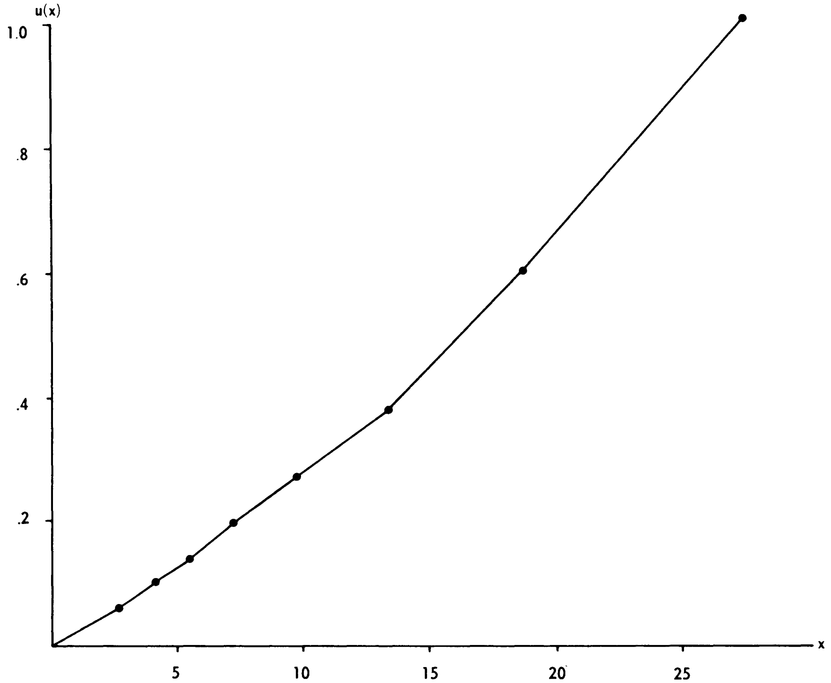

FIG. 1.—Utility function

convex and he must be a risk lover.[12] These points are joined by a broken line which is the estimated utility curve.

To obtain a mathematical function representing the estimated utility curve in figure 1, $\log_{10} u(x_h)$ is regressed on $\log_{10} x_h$ where $x_h = 1 + \bar{a}_h$. The estimated relationship is given by

$$\log_{10} u(x) = \underset{(.0206)}{0.2794} + \underset{(.0210)}{1.1784} \log_{10} x, \qquad R^2 = .9981.$$

The numbers in the parentheses are the estimated standard errors. Thus, the estimated utility function is $u(x) = 1.91x^{1.1784}$. This shows $u'(x) > 0$ and $u''(x) > 0$. Though the relative risk-aversion measure of Arrow (1971), $-xu''(x)/u'(x)$, is constant, his absolute risk-aversion measure, $-u''(x)/u'(x)$, increases with wealth, implying the representative bettor takes more risk as his capital declines. This finding supports the conjecture of Markowitz (1952) for the range of the utility function under consideration.

[12] For each of the eight horses, the comparison of the average and standard deviation of the gains from a dollar bet computed over the 20,247 races as reported in cols. 5 and 6 of table 3 points to the same conclusion.

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For a further analysis, we define a measure (new) of overall risk attitude, δ, as follows:

$$\delta = 1 - \frac{\int_a^b u(x)\, dx}{(1/b - a) \int_a^b x dx}$$

where $a = 0$, $b = 1 + \bar{a}_8$, $u(a) = 0$, $u(b) = 1$. If $\delta > 0$, the bettor is said to be a risk lover; if $\delta < 0$, he is a risk averter, and if $\delta = 0$, he is a risk neutral. The absolute magnitude of δ does not exceed 1.

For our data, $\delta = .1239$. In many respects (see table 1) the races at Roosevelt are alike to those at Yonkers. But Saratoga differs considerably from either Roosevelt or Yonkers. One striking difference is that the average bet per person per race at Saratoga is about half as much as that at either Roosevelt or Yonkers. This can be interpreted as the representative bettor at Saratoga having smaller betting capital than the one at Roosevelt or Yonkers. To investigate its effects on the risk attitude, δ values are computed from the data corresponding to each of these tracks. These are respectively, .1742, .1009, and .0652. It seems the representative at Saratoga tends to take more risk than the one at Roosevelt who in turn takes more risk than the one at Yonkers. However, the difference in attitude toward risk is less pronounced between Roosevelt and Yonkers. This can be taken as extra evidence supporting the observation that the representative bettor with less capital tends to take more risk.

To investigate further the effects of capital on risk attitude, we note that out of every dollar bet, the track take and breakage are 18¢. Thus, the total capital available to bet declines through the races during a racing night. There are usually nine races in a racing night; sometimes there are 10 or even 11, but let us proceed as if there are 9 in a racing night. If we assume that 10 percent of the money available to bet at the beginning of a race is actually put down as a bet on the race, and the track take and breakage are 18 percent (average α over all the races is .180002), then the total money available to bet at the last race will be 14 percent less than it is at the beginning of the first race. We are assuming that all the bettors arrive at the first race and stay until the last. As there are a considerable number of late arrivals, we compute δ for the first two and the last. These are, respectively, .2228, .2038, and .3040; δ values are practically the same for the first two races but are considerably larger for the last. This is in accord with our previous finding about the effect of a change in capital on the representative's risk attitude.

IV. Conclusion

The subjective winning probabilities are obtained as an average of the proportion of the total money bet on a horse. Horses 1, 2, . . . , 8 are defined as first, second, . . . , eighth favorites in any race. The objective

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winning probability is computed as the proportion of times a horse finishes first out of all the races in which it competes. The data consist of 20,247 races from three tracks and over a period of 5 years. It is shown that subjectively a horse with a high objective probability of winning is understated and one with a low objective probability of winning is overstated. The pattern seems to be consistent over the years of the sample period, among the chosen tracks, and under various race conditions. Theories explaining these discrepancies between objective and subjective winning probabilities are explored. It is shown that if bettors are risk neutral but not sophisticated, the market mechanism may generate the observed objective-subjective probability relationship.

Maintaining the assumption that bettors are sophisticated, have the same utility function, and behave as if the betting opportunities are limited to a single race, the representative utility of wealth function is constructed. Identifying this utility function with a representative bettor, it is found that he is a risk lover. A measure of risk attitude, δ, is defined which varies from -1 to 1. If $\delta > 0$, the person is a risk lover; if $\delta < 0$, he is a risk averter, and $\delta = 0$ means he is a risk neutral. The representative is found to have $\delta = .1239$. The effects of capital on risk attitude are investigated, and it seems that the more capital the representative has, the less he tends to be a risk lover.

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