Econometrica, Vol. 51, No. 4 (July, 1983)

ON STATE DEPENDENT PREFERENCES AND SUBJECTIVE PROBABILITIES

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This paper presents an expected utility theory for state-dependent preferences. It proposes axioms that permit the joint derivation of subjective probabilities and utilities when the decision maker's preferences are not independent of the prevailing state of nature. In addition to the usual von Neumann–Morgenstern axioms, these axioms also include the requirement that the decision-maker's actual preferences are consistent with his preferences contingent on an hypothetical probability distribution over the states of nature. Two versions of the consistency axiom are introduced and their significance in the context of Bayesian decision theory is discussed.

1. INTRODUCTION

THE STANDARD PROBLEM of decision-making under uncertainty involves three basic elements: acts, states of nature, and consequences. Each combination of an act and a state of nature determines a unique consequence. Uncertainty, captured by the notion of states of nature, results from the lack of advance knowledge of the exact consequence that follows from a given act. The decision-maker's purpose is to choose an act which results in the most desirable consequences. This requires a criterion for judging the desirability of acts. Expected utility is one such criterion.

Expected utility theory assumes that decision-makers have preferences over acts and postulates that these preferences have a structure that permits their representation by an expected utility index. The use of the expectation operator presumes the existence of a utility index defined over the set of consequences and a subjective probability distribution over states of nature. This presumption is made plausible by Savage's theory, which builds upon the work of Ramsey, de Finetti, von Neumann–Morgenstern, and others.

There are circumstances, however, in which the evaluation of the consequences is not independent of the prevailing state of nature. These include a class of insurance problems involving irreplaceable objects such as life, health, and heirlooms. Another example is criminal activity where one possible outcome is loss of freedom. Expected utility theory can be extended to include state-dependent preferences by assigning a utility index to the set of consequences for each state of nature. The utility of two consequence state-of-nature pairs may differ, even if the consequence is the same in both.

Some implications of expected utility maximizing behavior with state-dependent preferences for optimal insurance have been studied by Arrow [**2**], Cook and Graham [**3**], and Karni [**6**]. Recognizing that the then existing expected utility theory did not apply to state dependent preferences, Arrow [**2**, p. 61] was

[1]We would like to thank the Editors and an anonymous referee for very useful comments and suggestions.

careful to assume the existence of objective probabilities over states of nature. Furthermore, quoting Rubin, he claimed that with state dependent preferences it is impossible to separate tastes (as represented by a utility function) and beliefs (as represented by probabilities) in a unique way with the use of observations. This claim depends, of course, on the extent to which behavior can be reasonablv restricted.

Our paper studies axiomatizations of expected utility theory for state-dependent preferences some of which lead to a unique subjective probability distribution on the set of states of nature. Departing from the original framework of Savage, and following, say, Anscombe–Aumann [**1**], we make the simplifying structural assumption that there are finitely many states of nature and that *a consequence is a lottery with objectively known probabilities over a finite nonempty set of prizes*. Let the set of states of nature be denoted by S. For each state $s \in S$ the set of prizes is denoted by $X(s)$. A lottery over $X(s)$ is represented by a nonnegative vector in Euclidian space of dimension $|X(s)|$, the coordinates of which sum to one. The set of all such lotteries is denoted by $Y(s)$. Hence, denoting $\sum_{s \in S} |X(s)|$ by l we have, $L = \{f \in R^l \,|\, f(s) \in Y(s) \text{ for } s \in S\}$.

A preference relation $\succsim$ over acts is a binary relation on L. Given such a binary relation $\succsim$ we define the relations $\succ$ and $\sim$ as usual: For all f, g in $L: f \succ g$ if and only if $f \succsim g$ and not $g \succsim f$, and $f \sim g$ if and only if $f \succsim g$ and $g \succsim f$.

We denote by Ω the set of all preference relations over L satisfying the following three NM (von Neumann—Morgenstern) axioms:

AXIOM A.1 (*Weak Order*): (a) *for all f and g in $L: f \succsim g$ or $g \succsim f$.* (b) *For all f, g and h in L: if $f \succsim g$ and $g \succsim h$, then $f \succsim h$.*

AXIOM A.2 (*Independence*): *For all f, g, and h in L, and for all $a \in R_+$, $0 < a < 1$: if $f \succ g$, then $af + (1-a)h \succ ag + (1-a)h$.*

AXIOM A.3 (*Continuity*): *For f, g, and h in L: if $f \succ g$ and $g \succ h$, then there exist a, b in $(0,1)$ such that $af + (1-a)h \succ g$ and $g \succ bf + (1-b)h$.*

A NM utility is a function which assigns a number, say $w(s,x)$, to each pair $(s,x) s \in S$ and $x \in X(s)$. A NM utility is a vector in R^l. We will use the notation $w = (w(s)) \in R^l$ and denote the set of NM utilities by W where

$$W = \left\{ w \in R^l \,|\, w(s) \in R^{|X(s)|} \text{ for all } s \in S \right\}.$$

Given $f \in L$, $w \in W$, and $s \in S$, $w(s)f(s)$ is the inner product

$$\sum_{x \in X(s)} w(s,x) f(s,x)$$

in $R^{|X(s)|}$.

We can now state a version of the classical von Neumann–Morgenstern Theorem.

NM THEOREM: *Given $\succsim$ in Ω, there exists $w \in W$ such that for all f and $g \in L$:*

$$(1) \qquad f \succsim g \quad \textit{iff} \quad \sum_{s \in S} w(s)f(s) \geqq \sum_{s \in S} w(s)g(s).$$

Furthermore, if for some $w' \in W$ and for all f and g in L, $[f \succsim g$ iff $\sum_{s \in S} w'(s)f(s) \geqq \sum_{s \in S} w'(s)g(s)]$, then there exist $c > 0$ and real numbers $d(s)$ one for each state such that $w'(s) = cw(s) + d(s)$ for all $s \in S$. We shall refer to such transformation as cardinal unit comparable (across states).

The representation of preferences $\succsim$ by the function w, as above, does not permit the separation of tastes from beliefs, as represented by subjective probabilities of the states of nature. Indeed let $p(s)$, $s \in S$, be an arbitrary list of positive numbers with $\sum_{s \in S} p(s) = 1$. Then $f \succsim g$ iff $\sum_{s \in S} p(s)v(s)[f(s) - g(s)] \geqq 0$ provided that $p(s)v(s) = w(s)$ for all $s \in S$. In other words, the representation of preferences by the function w, or the preferences themselves, are consistent with any positive subjective probability over S.

To obtain a unique subjective probability distribution on S additional restrictions must be imposed on the preferences relation $\succsim$. Our topic rules out the restriction that preferences are state-independent.

Ramsey's [9] method of deriving subjective probabilities does not rule out the possibility of state-dependent preferences. It does require, however, that there be "ethically neutral" propositions, or states, i.e. states such that if the prize is the same the decision-maker is indifferent as to which state prevails. Ramsey did not elaborate upon this point. The theory presented below does not require Ramsey's assumption.

Fishburn [5] proposed another axiomatization which allows for state-dependent preferences. He extended the set of acts L in the following manner: For each event (nonempty subsets of the set of states of nature) he defines the set of acts conditioned on events and assumes the existence of a preference relation over all conditional acts. This preference relation is assumed to satisfy axioms analogous to A.1–A.3 and several additional structural restrictions. Specifically his Axiom 6 requires that for every two disjoint events, not all of the consequences conditioned upon one event are preferred to all of the consequences conditioned upon the other event. This is irreconcilable with some applications (e.g. life insurance problems) that motivated our research.

2. CONSISTENT PREFERENCE RELATIONS AND SUBJECTIVE PROBABILITY

The historical interest in the separation of subjective probabilities and utilities stems from the distinction between the transitory nature of probabilistic beliefs and the unchanging nature of tastes. The use of probabilities is a convenient method of incorporating new information. Starting from prior subjective probabilities and applying the Bayesian statistical machinery, new (posterior) probabilities are obtained and used in the decision-making process. In the economic literature this point is emphasized by Arrow [**2**] and Dréze [**4**].

To capture the unchanging nature of tastes we introduce a notion of consistent preference relations. Suppose that two preference relations $\succsim$ and $\succsim' \in \Omega$ are given. Intuitively speaking, these preference relations are consistent, if they are induced by the same utilities and the difference between them is explained fully by the different underlying subjective probability distributions, say, a prior and a posterior. More specifically, suppose that the preference relation $\succsim$ is represented, according to the NM Theorem, by $w \in W$, and for all $s \in S$, $w(s) = p(s)u(s)$ where u represents the intrinsic, unchanging preferences and p is a prior probability distribution on S. Further suppose that the incorporation of new information results in a posterior probability distribution p' on S; then the preference relation $\succsim'$ can be represented by $w' \in W$ where for all $s \in S$, $w'(s) = p'(s)u(s)$. If for some $s \in S$ both $p(s)$ and $p'(s)$ are positive, then $w(s) = c(s)w'(s)$ where $c(s)$ is a positive constant. This, in turn, implies that the preference relations $\succsim$ and $\succsim'$ restricted to the state s coincide.

Rather than regarding p' as posterior probabilities, it is possible to conceive of it as hypothetical probabilities on S. To clarify this idea consider a decision-maker who must choose between watching a football game in an open stadium and watching it at home on television. As he faces uncertainty with regard to the possibility of rain, we are clearly in the state-dependent preferences framework. His decision reflects the preference relation $\succsim$. The assumption that he has preferences $\succsim'$ compatible with probability distribution p' means that statements such as "If the probability of rain during the game does not exceed 10 per cent, then I shall prefer watching the game in the stadium" are meaningful in the sense that the decision-maker is able to plan his own choices under at least one conceivable probability distribution on the states of nature.

Our main theorem, stated below, asserts that a unique subjective probability distribution on S can be derived from the following data: a hypothetical (conceivable) strictly positive probability distribution on S; the decision-maker's choices between each and every pair of acts contingent upon the hypothetical probability; the decision-maker's actual choices between each and every pair of acts. This derivation depends, of course, on the axioms imposed on the preference relations.

To facilitate the formal exposition we introduce the following notations and definitions: Let $L_{p'}$ denote the set of acts contingent upon a strictly positive probability distribution p' on S; then

$$L_{p'} = \left\{ f' \in R^l_+ \mid \text{for all } s \in S \sum_{x \in X(s)} f'(s,x) = p'(s) \right\}.$$

We can now present the preference relation on acts contingent on p' as a preference relation $\succsim'$ on $L_{p'}$. Since $L_{p'}$ is a mixture set, if $\succsim'$ satisfies the three

TABLE I

$\succ_s$ \ $\succ'_s$	$\succ'_s \neq \emptyset$	$\succ'_s = \emptyset$
$\succ_s \neq \emptyset$	Evidently nonnull	$\emptyset$
$\succ_s = \emptyset$	Evidently null	Indeterminate case

NM axioms, the NM Theorem applies. Notice also that if f' in some $L_{p'}$ is given, the probabilities p' can be recovered by defining $p'(s) = \sum_{x \in X(s)} f'(s, x)$ for all $s \in S$.

To introduce formally the notion of consistency between $\succsim$ and $\succsim'$, we define the mapping H from $L_{p'}$ to L. For all $f' \in L_{p'}$, $H(f'(s)) = f'(s)/p'(s)$. The mapping H is a one-to-one onto.

Consider next the notation of a null state. For a given $\succsim \in \Omega$ and $s \in S$ we define $\succsim_s$ as the induced relation on $Y(s)$ as follows: For y and $y' \in Y(s)$, $y \succsim_s y'$ if and only if for all f and g in L, $f \succsim g$ where $f(s) = y$, $g(s) = y'$ and $f(s') = g(s')$ for all $s' \neq s$, $s' \in S$. Using this definition we note that when preferences over L are state-independent, a state $s \in S$ is considered null if and only if $\succ_s$ is empty. However, with state dependent preferences $\succ_s$ may be empty only because all the prizes in $X(s)$ are equally preferred. Hence state s for which $\succ_s$ is empty is certainly null if we have independent evidence that not all the prizes in $X(s)$ are equally preferred. This evidence can be obtained from another preference relation in Ω say $\succsim'$.

Thus, we define a state $s \in S$ to be *evidently null* with respect to $\succsim'$ if $\succ_s = \emptyset$ and $\succ'_s \neq \emptyset$. Clearly a state $s \in S$ is *evidently nonnull* if $\succ_s \neq \emptyset$. In between we have the indeterminate case, i.e., states that are neither evidently null nor evidently nonnull. We summarize these possibilities in Table I.

If some state $s \in S$ is evidently nonnull, i.e. $\succ_s \neq \emptyset$, then $\succ \neq \emptyset$. Note, however, that the opposite direction holds as well. If the preference relation $\succsim$ is *nontrivial*, i.e. $\succ \neq \emptyset$, then the NM utility representation of $\succsim$ (say w) is not constant for at least one $s \in S$. Since $w(s)$ represents $\succsim_s$, $w(s)$ not constant implies $\succ_s \neq \emptyset$. Hence, there exists at least one nonnull state.

We are now in a position to introduce the Consistency Axiom:

CONSISTENCY AXIOM: *For all $s \in S$ and all f' and g' in $L_{p'}$, if f' agrees with g' outside s and $H(f') \succ H(g')$, then $f' \succ' g'$. Moreover, if s is evidently nonnull, then if f' agrees with g' outside s, $f' \succ' g'$ implies $H(f') \succ H(g')$.*

MAIN THEOREM: *Suppose that two binary relations are given: a relation $\succsim$ on L such that $\succ$ is nonempty and $\succsim'$ on $L_{p'}$ for some strictly positive p'. Assume also that each of the two relations satisfies the* NM *Axioms* A.1, A.2, A.3, *and jointly they satisfy the Consistency Axiom. Then*:

(a) *there exists an* NM *utility* $u \in W$ *and a probability distribution* p *on* S *such that for all* f *and* $g \in L$: $f \succsim g$ *if and only if*

$$\sum_{s \in S} p(s)u(s)\left[f(s) - g(s)\right] \geqq 0 \tag{1}$$

and for all f' *and* g' *in* $L_{p'}$: $f' \succsim' g'$ *if and only if*

$$\sum_{s \in S} u(s)\left[f'(s) - g'(s)\right] \geqq 0. \tag{2}$$

(b) *The* u *of part* (a) *is unique up to a cardinal unit comparable transformation, as in* NM *Theorem in Section* 1.

(c) *For* s *evidently null* $p(s) = 0$, *and for* s *evidently nonnull* $p(s) > 0$. *Furthermore the probability* p *restricted to the event of all evidently nonnull states is unique.*

Notice that if we denote $H(f')$ by f and $H(g')$ by g, then the second inequality of part (a) of the Main Theorem can be written analogously to the first inequality as:

$$\sum_{s \in S} p'(s)u(s)\left[f(s) - g(s)\right] \geqq 0.$$

Before proving the Main Theorem we state a related result from Karni and Schmeidler [7]. This result is obtained under more restrictive assumptions and has a stronger conclusion.

We denote by $\hat{L}$ the union of $L_{p'}$ over all strictly positive probability distributions over S. Thus

$$\hat{L} = \left\{ \hat{f} \in R_+^l \,\middle|\, \sum_{s \in S} \sum_{x \in X(s)} \hat{f}(s,x) = 1 \right.$$
$$\left. \text{and for all } s \in S, \sum_{x \in X(s)} \hat{f}(s,x) > 0 \right\}.$$

Given $\hat{f} \in \hat{L}$, the probability distribution p' on S such that $\hat{f} \in L_{p'}$ can be recovered by defining $p'(s) = \sum_{x \in X(s)} \hat{f}(s,x)$ for all $s \in S$. Also, the function H defined above can be extended to all of $\hat{L}$. Since no confusion will result we denote the extension by H. Hence for all $\hat{f} \in \hat{L}$,

$$H(\hat{f}(s,x)) = \hat{f}(s,x) \Big/ \sum_{y \in X(s)} \hat{f}(s,y).$$

Suppose that a preference relation $\hat{\succsim}$ on $\hat{L}$ is given. If this relation satisfies the three NM axioms, then the conclusion of NM Theorem applies since $\hat{L}$ is convex.

Within this new framework we shall call a state of nature s *obviously null* if for some strictly positive probability p' s is evidently null according to the previous definition (using the restriction of $\hat{\succsim}$ to $L_{p'}$ as $\succsim'$).

We now introduce a strengthening of the consistency axiom:

STRONG CONSISTENCY AXIOM: *For all* $s \in S$ *and all* $\hat{f}$ *and* $\hat{g}$ *in* $\hat{L}$ *such that* $\hat{f}$ *agrees with* $\hat{g}$ *outside* s: *if* $H(\hat{f}) \succ H(\hat{g})$, *then* $\hat{f} \mathrel{\hat{\succ}} \hat{g}$, *and if* s *is obviously null and* $\hat{f} \mathrel{\hat{\succ}} \hat{g}$, *then* $H(\hat{f}) \succ H(\hat{g})$.

With these definitions in mind, the statement of the KS Theorem is straightforward.

KS THEOREM: *The main Theorem holds if the following changes are made*: $L_{p'}$ *is replaced by* $\hat{L}$, *the relation* $\succsim'$ *is replaced by* $\hat{\succsim}$, *the Consistency Axiom is replaced by the Strong Consistency Axiom, and evidently null is replaced by obviously null.*

Clearly the assumption of existence of a preference relation on $\hat{L}$ which is strongly consistent with $\succsim$ and satisfies the NM axioms is considerably more restrictive than the analogue assumption in the Main Theorem. The critical aspect of the stronger assumption is that comparisons are required between elements of $\hat{L}$, say $\hat{f}$ and $\hat{g}$, such that $\hat{f} \in L_{p'}$, $\hat{g} \in L_{p''}$, and $p' \neq p''$. On the other hand a stronger conclusion holds in the KS Theorem, i.e. that inequality (2) in part (a) applies to all of $\hat{L}$. The significance of this result is explained in the Concluding Discussion Section below. We shall not include a proof of the KS Theorem since it can easily be reconstructed from the proof of the Main Theorem.

3. PROOF OF THE MAIN THEOREM

To begin with, we observe that the NM Theorem applies to $\succsim'$ on $L_{p'}$. (One way of showing this is by noting that the mapping H from $L_{p'}$ onto L is affine.)

Let w and u represent $\succsim$ and $\succsim'$ respectively according to the NM Theorem. Let h' be an arbitrary element in $L_{p'}$ and for each $s \in S$ denote by L'_s the subset of $L_{p'}$, whose elements agree with h' outside s. Denoting $H(h')$ by $\tilde{h}$ we define similarly the subset L_s of L, $s \in S$. Clearly for each $s \in S$, $w(s)$ and $u(s)$ represent $\succsim$ restricted to L_s and $\succsim'$ restricted to L'_s respectively. Finally, let us denote by S' the subset of evidently null states and by S'' the subset of evidently nonnull states.

For $s \in S$ the definition of H restricted to L'_s and the consistency axiom imply that $u(s)$ represent $\succsim$ restricted to L_s. By the uniqueness part of the NM Theorem there are $c(s) > 0$ and $d(s)$ such that on $X(s) w(s,x) = c(s)u(s,x) + d(s)$. For $s \in S'$ define $c(s) = 0$ and define $d(s) = w(s,x)$ for any $x \in X(s)$. For s in the complement of $S' \cup S''$ define $c(s)$ to be any nonnegative number and $d(s) = w(s,x)$ for any $x \in X(s)$. Notice that for all s not in S'', $w(s)$ is constant. Since $\succ$ is nonempty, S'' is nonempty. Hence $\sum_{s \in S} c(s) = \bar{c} > 0$ and for all $s \in S$ we define $p(s) = c(s)/\bar{c}$. Thus $[p(s)u(s)]_{s \in S} \in W$ is obtained from w by a cardinal unit comparable transformation, and by the NM Theorem it also represents $\succsim$ on L. This implies (a)(1) of the Main Theorem, (a)(2) follows trivially from the definition of u.

Part (b) of the Main Theorem follows immediately from the corresponding uniqueness part of the NM Theorem.

Proof of part (c). The definition of evidently null states and inequality (a)(1) and (a)(2) imply immediately that $p(s) = 0$ for all $s \in S'$. For evidently nonnull states, (a)(1) and (a)(2) imply $p(s) > 0$.

We now prove the uniqueness of p on S'' up to a positive multiplicative coefficient. The proof is by way of negation. Suppose that two probability distributions on S, p, and $\hat{p}$ are given and suppose that both satisfy (a)(1) on L. Since for each s in the complement of $S' \cup S''$ $u(s)$ is constant, the summation over those states in (a)(1) is zero and they do not affect the inequality. Thus it may be assumed without loss of generality that $\sum_{s \in S''} p(s) = \sum_{s \in S''} \hat{p}(s) = 1$. Thus, the negation assumption implies that for some s and t in S, $p(s) > \hat{p}(s)$ and $p(t) < \hat{p}(t)$. For all r in $[0, 1]$ let g_r and h_r in L be such that they agree outside s and t and

$$g_r(\bar{x}_s, s) = h_r(\underline{x}_t, t) = r,$$

$$g_r(\underline{x}_s, s) = h_r(\bar{x}_t, t) = 1 - r,$$

$$g_r(\underline{x}_t, t) = h_r(\underline{x}_s, s) = 1,$$

where $\bar{x}_j$, $\underline{x}_j$, $j = t, s$, are the most preferred and the least preferred prizes in state j respectively. Using condition (1) in the NM Theorem,

$$rp(s)\left[u(\bar{x}_s, s) - u(\underline{x}_s, s)\right] + (1 - r)p(t)\left[u(\underline{x}_t, t) - u(\bar{x}_t, t)\right] \geqq 0$$

if and only if

$$r\hat{p}(s)\left[u(\bar{x}_s, s)\right] - u(\underline{x}_s, s)\right] + (1 - r)\hat{p}(t)\left[u(\underline{x}_t, t) - u(\bar{x}_t, t)\right] \geqq 0.$$

By the nondegeneracy assumption, the difference between the values of u in the first squared brackets is positive and the second is negative. The inequalities between the values of p and $\hat{p}$ imply that both $p(s)$ and $\hat{p}(t)$ are positive. Thus there exists $\bar{r}$ which turns the first inequality to equality. Clearly for the same $\bar{r}$ the second inequality is false and the required contradiction has been obtained. *Q.E.D.*

4. CONCLUDING DISCUSSION

As has been mentioned previously the main motivation for deriving the prior probability of a decision-maker is to enable the processing of new information using Bayes theorem. More specifically suppose that before choosing an act $f \in L$ the decision-maker can perform an experiment the outcome of which is an element e in the finite set E. Furthermore, it is assumed that the decision-maker knows the conditional probabilities $q(e \mid s) \geqq 0$, $e \in E$ and $s \in S$ ($q(e \mid s) \geqq 0$ and

$\sum_{e \in E} q(e \mid s) = 1$ for all $s \in S$). Suppose, in addition that the decision-maker has a preference relation over L which satisfies Axioms A.1–A.3 of Section 1. Hence (by NM Theorem), his preferences can be represented by NM utility $w \in W$. It appears as if it is possible to apply Bayes theorem and calculate a posterior NM utility $w' \in W$ conditional on the outcome of the experiment without any additional restrictions on his prior preference relation. This idea is captured by:

LEMMA: *Given conditional probabilities q as above, a result of the experiment $e \in E$, and $w \in W$, the posterior preference relation represented by $w' \in W$, where w' is computed using Bayes theorem, is independent of the prior probability p as long as $p(s) > 0$ for all $s \in S$.*

PROOF: Suppose that an arbitrary p satisfying the condition of the Lemma is given. Define for each $s \in S$: $u(s,x) = w(s,x)/p(s)$. By Bayes Theorem, for each $s \in S$, $p(s \mid e) = p(s)q(e \mid s)/\sum_{t \in S} p(t)q(e \mid t)$. Since, for all $s \in S$, $w'(s,x) = p(s \mid e)u(s,x)$ by substitution we get $w'(s,x) = q(e \mid s)w(s,x)/\sum_{t \in S} p(t)q(e \mid t)$.

The dependence of $w'(s,x)$ on p is through the coefficient $[\sum_{t \in S} p(t)q(e \mid t)]^{-1}$ which is independent of $s \in S$. Since the preference relation represented by w' is not changed when w is multiplied by a positive constant we get the same posterior preference relation whatever the positive prior probability p. *Q.E.D.*

However, appearances notwithstanding, without additional assumptions on the decision-maker's preferences the posterior preference relation computed in the proof of the Lemma does not necessarily represent the decision-maker's posterior preferences. This last point merits some elaboration. A decision-maker is referred to as Bayesian if he is an expected utility maximizer and he applies Bayes theorem to update his prior in view of new information. If one rejects the extreme Bayesian doctrine that every decision maker is Bayesian, there is a problem of recognizing Bayesian decision makers. The accepted practice is to consider a decision-maker as Bayesian if his preferences over acts can be represented by an expected utility with a unique prior.

We face three possibilities: First to adopt this practice which we may call a test of Bayesianity. The second possibility is to weaken the test of Bayesianity by not insisting on the uniqueness of the prior. The third possibility is to strengthen the test by verifying the consistency of his preferences with every conceivable posterior. Next we consider each of these possibilities in turn.

As was mentioned in the paper, Axioms A.1–A.3 above do not define a unique prior. The main result of the paper is that to obtain a unique prior it suffices to have preferences contingent on some conceivable posterior which satisfy A.1–A.3 and our Consistency Axiom. In other words in order *to predict* the behavior of the decision-maker, in view of the Lemma, it is enough to know that he is Bayesian and to know his initial preferences on L. However, to make sure that he is Bayesian according to the accepted practice the additional data, namely, that his posterior preferences with respect to some given posterior probability p' satisfy our axioms, is required.

The weak test of Bayesianity may be thought of as based on the premise that a decision-maker is Bayesian unless proved otherwise. Preferences on L satisfying Axioms A.1–A.3 pass this test. Furthermore, as was mentioned already, the Lemma implies that this information suffices to predict the decision-maker's posterior preferences.

The interest in the third possibility stems from the presumption that Bayesian decision theory is not as normatively compelling as to exclude other decision theories. Therefore, there is special interest in formulating the complete set of axioms that characterizes a Bayesian decision-maker in the aforementioned sense. In particular, we are interested in the conditions that not only permit the representation of the prior preferences by expected utility with a uniquely defined prior probability as is the case in accepted practice, but also guarantee that the posterior preferences are arrived at according to Bayes' theorem. This approach requires the study of preference relations defined on the space consisting of acts, experiments, and results. (This structure is mentioned but not studied in Raiffa and Schlaifer [**8**].) Such a system of axioms will preclude the following kind of situation. The decision-maker has prior preferences satisfying Axioms A.1–A.3 and posterior preferences contingent on a conceivable posterior probability p' satisfying the same Axioms A.1–A.3, and consistency with the prior preference relation. The same decision-maker when faced with another conceivable posterior probability p'' has other posterior preferences which also satisfy Axioms A.1–A.3 and consistency, i.e. the conditions of the Theorem of Section 2. Applying the Theorem twice we get each time a unique prior but the two priors are not identical. This situation is excluded by the hypothesis of the KS Theorem. Therefore the strong test of Bayesianity consists of the hypothesis of the KS Theorem, which includes the strong consistency requirement.

Nevertheless the strong test of Bayesianity suggested above does not guarantee that the decision-maker updates his prior probability according to Bayes' Theorem. This conclusion, however, is not specific to state-dependent preferences. It applies equally to the standard theory with state-independent preferences.

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Manuscript recieved March, 1982; revision received September, 1982.

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