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Choice-Based Elicitation and Decomposition of Decision Weights for Gains and Losses Under Uncertainty

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This paper reports the results of an experimental parameter-free elicitation and decomposition of decision weights under uncertainty. Assuming cumulative prospect theory, utility functions were elicited for gains and losses at an individual level using the tradeoff method. Subsequently, decision weights were elicited through certainty equivalents of uncertain two-outcome prospects. Furthermore, decision weights were decomposed using observable choice instead of invoking other empirical primitives, as in previous experimental studies. The choice-based elicitation of decision weights allows for a quantitative study of their characteristics, and also allows, among other things, for the examination of the sign-dependence hypothesis for observed choice under uncertainty. Our results confirm concavity of the utility function in the gain domain and bounded subadditivity of decision weights and choice-based subjective probabilities. We also find evidence for sign dependence of decision weights.

Key words: decision under uncertainty; Choquet expected utility; cumulative prospect theory; decision weights; choice-based probabilities; probability weighting
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1. Introduction

Subjective expected utility (SEU) theory (Savage 1954) evaluates an uncertain alternative as the sum of utilities from outcomes weighted by the corresponding subjective probabilities, and consequently establishes a simple and intuitive separation of value and belief. If this theory is abandoned in favor of more general theories that replace probabilities by nonadditive decision weights, the above-mentioned separation between value and belief becomes less clear. Among the most influential of these theories are Choquet expected utility (CEU) theory (Gilboa 1987, Schmeidler 1989) and cumulative prospect theory (CPT) (Tversky and Kahneman 1992). CPT is more general than CEU because decision weights are sign dependent (i.e., they depend on whether the associated consequence is a gain or a loss). Under these theories, the decision weight attached to an event depends on the rank ordering of outcomes in the uncertain alternative under consideration. In other words, it reflects additional considerations attached to the particular decision context, related to decision attitude, over and above pure belief (Wakker 2004).

Consistent with the common intuition that belief precedes preference (people prefer to bet on event A rather than on event B primarily because they believe that A is more likely than B), researchers suggested decomposing decision weights into a component of belief (e.g., a subjective probability), independent of the particular decision context, and a component reflecting decision attitude. Tversky and Fox (1995), Fox et al. (1996), Fox and Tversky (1998), Wu and Gonzalez (1999), and Kilka and Weber (2001) used a decomposition model in which the decision weight assigned to an uncertain event results from a two-stage process. The decision maker first judges the probability of the event, and then transforms this judged probability by a transformation function known from decision under risk assuming CPT. In this decomposition, the belief component results from direct judgment (i.e., an additional empirical primitive) and not from choice. More recently, Wakker (2004) proposed formal foundations of a decomposition model that is based solely on observable preference. While not immune from criticism either (Wakker 2004, p. 238), this approach is more consistent with behavioral concepts used in standard economics.

Decision weights have been investigated using two research strategies. The first strategy consists of testing simple preference conditions to obtain information about the shape of the weighting function (Wu and Gonzalez 1999). The second strategy consists of eliciting decision weights from individual preferences. Tversky and Fox (1995) and Fox et al. (1996) used a certainty equivalent method and two-outcome prospects to elicit individual decision weights in the gain domain, although utility functions were not elicited at the individual level. Indeed, Tversky and Fox (1995) used the same power utility function for all subjects, whereas Fox et al. (1996) used a similar methodology with a linear utility function (inferred from a preceding experimental study).

In the vein of the above-mentioned second strategy, this paper uses a two-step method to elicit CPT decision weights for gains and losses under uncertainty at an individual level. This is accomplished without any assumptions regarding the shape of the individual's utility function, its parametric form, or the necessity to use more than two-outcome prospects. The first step employs the tradeoff method (Wakker and Deneffe 1996) to elicit the utility function. Based on the comparison of two-outcome prospects, this technique allows eliciting utilities without potential distortions due to nonlinear decision weights. In the second step, decision weights are elicited through certainty equivalents of prospects involving only one nonzero outcome. At this stage, our study extends the studies by Abdellaoui (2000) and Bleichrodt and Pinto (2000) from risk to uncertainty.

Furthermore, this paper experimentally operationalizes the choice-based decomposition of decision weights proposed by Wakker (2004). For this purpose, we elicit choice-based degrees of belief (henceforth called choice-based probabilities). We then estimate the decision attitude component of the decomposition model (i.e., the transformation function that maps choice-based probabilities to decision weights) separately for gains and losses.

The contribution of our study is complementary to previous experimental investigations (e.g., Fox et al. 1996, Wu and Gonzalez 1999). For the gain domain, it allows for a verification of the robustness of some results of these experimental studies. This verification is also a means by which we can compare the results of decision weight decompositions based on standard behavioral concepts and those resulting from a decomposition based on nonbehavioral concepts studied in psychology. The confirmation of the results previously obtained through the latter approach is fruitful for all fields involved. For the loss domain, our results shed light on a less-investigated side of individual decision making under risk and under uncertainty. They particularly allow for a straightforward comparison between the properties of decision weights for gains and losses.

This paper is structured as follows. Section 2 briefly reviews CPT and introduces the two-stage approach linking decision making under uncertainty to decision making under risk. Section 3 describes the procedure to successively elicit the utility function and decision weights. Section 4 completes the description of our experimental setup. The results of our study are presented in §5. The paper concludes with a summary and discussion of the main findings in §6.

2. Theoretical Framework

In decision theory, uncertainty is modeled through a set S, called the *state space*. The decision maker knows that exactly one of these states will obtain, but she does not know which one. Subsets of S are called *events*. The objects of choice in this paper are binary *prospects* $(x_1, A; x_2)$ yielding a monetary outcome x_1 if event A obtains and a monetary outcome x_2 otherwise. Outcomes are expressed as changes with respect to the status quo, i.e., gains or losses. A prospect that involves both a gain and a loss outcome is called *mixed*. Other prospects are called *nonmixed*.

If the prospect $(x_1, A; x_2)$ is evaluated according to CEU and $x_1 \geq x_2$, its value is given by

$$\pi_1 \cdot u(x_1) + \pi_2 \cdot u(x_2), \tag{1}$$

where π_1 and π_2 are *decision weights* depending on the ranking of outcomes, and $u(\cdot)$ is a strictly increasing *utility function* from $\mathbb{R}$ to $\mathbb{R}$. The decision weights π_1 and π_2 are defined by

$$\pi_1 = W(A) \quad \text{and} \quad \pi_2 = 1 - W(A), \tag{2}$$

where $W(\cdot)$ is a *weighting function*, i.e., a function from 2^S to $[0, 1]$, satisfying monotonicity with respect to set inclusion ($A \subset B \Rightarrow W(A) \leq W(B)$), and the normalization conditions $W(\varnothing) = 0$ and $W(S) = 1$. Under SEU, due to additivity of $W(\cdot)$, decision weights coincide with subjective probabilities.

Under CPT, the utility function satisfies $u(0) = 0$, and decision weights are determined through two weighting functions: $W^+(\cdot)$ for gains, and $W^-(\cdot)$ for losses. The value of the prospect $(x_1, A; x_2)$ is still given by Equation (1). If it involves only gains (losses) with $x_1 \geq x_2 \geq 0$ ($x_1 \leq x_2 \leq 0$), $W(\cdot)$ is replaced by $W^+(\cdot)$ ($W^-(\cdot)$) in the decision weights π_1 and π_2 defined by Equation (2). For mixed prospects with $x_1 > 0 > x_2$, decision weights are defined by $\pi_1 = W^+(A)$ and $\pi_2 = W^-(S - A)$.

If the utility function satisfies $u(0) = 0$ and the *duality condition* $W^-(A) = 1 - W^+(S - A)$ holds for all events A, CPT and CEU coincide. For nonmixed prospects, the model given by Equation (1) agrees with the multiple-priors model (Gilboa and Schmeidler 1989) and Gul's (1991) disappointment theory.

Decision under risk can be seen as a special case of decision under uncertainty in which events are replaced by objective probabilities. A prospect $(x_1, p; x_2)$ yields monetary outcome x_1 (x_2) with probability p $(1-p)$. In the evaluation of risky prospects, the weighting function $W(\cdot)$ in Equation (2) is replaced by a *probability weighting function* $w(\cdot)$, i.e., a strictly increasing function from $[0,1]$ to $[0,1]$ satisfying $w(0)=0$ and $w(1)=1$. Analogously, under CPT the weighting functions $W^+(\cdot)$ and $W^-(\cdot)$ are replaced by $w^+(\cdot)$ and $w^-(\cdot)$, respectively.

Following Schmeidler (1989), convexity of the weighting function has been used to formalize ambiguity aversion (see, however, Epstein 1999). Subsequently, convexity was shown to correspond to a pessimistic preference relation under uncertainty (Wakker 2001), where pessimism means that events receive more attention as their associated outcomes are ranked lower. Empirical research has shown, however, that the weighting function does not exhibit pessimism universally. Rather, it reflects diminishing sensitivity, resulting in an overweighting of unlikely events and an underweighting of likely events (e.g., Tversky and Fox 1995, Wu and Gonzalez 1999, Kilka and Weber 2001). Prominent manifestations of this principle are the possibility effect and the certainty effect, according to which the impact of an event is particularly strong when it turns impossibility into possibility, or possibility into certainty. These findings were formalized by means of *bounded subadditivity* (Tversky and Wakker 1995), which comprises *lower subadditivity* (LSA) and *upper subadditivity* (USA). LSA and USA of $W(\cdot)$ are defined as follows:

$$\text{LSA:} \quad W(A) \geq W(A \cup B) - W(B), \tag{3}$$

$$\text{USA:} \quad 1 - W(S-A) \geq W(A \cup B) - W(B), \tag{4}$$

provided that A and B are disjoint and $W(A \cup B)$ and $W(B)$ are bounded away from 1 and 0, respectively.[1] Note that LSA of $W(\cdot)$ contradicts convexity.

Under CPT, decision weights are no longer pure measures of belief. Building on the empirical finding that subadditivity is more pronounced for uncertainty than for risk, Tversky and Fox (1995) and Fox and Tversky (1998) proposed a two-stage model in which the decision maker first assesses the probability of an uncertain event and then transforms this value by the risky probability weighting function. In their model, the belief component is captured by direct judgment quantifications of degrees of belief, i.e., judged probabilities, which are assumed to satisfy support theory (Tversky and Koehler 1994). Fox and Tversky (1998) found that certainty equivalents of uncertain prospects $(x, A; 0)$ are well approximated by certainty equivalents of corresponding risky prospects $(x, q(A); 0)$, with $q(A)$ denoting the judged probability of event A.

A different approach by Wakker (2004) assumes CPT for prospects of the form $(x, A; 0)$ and $(x, p; 0)$, $x > 0$, and formally shows that the observation that decision makers are less sensitive to uncertainty than to risk is equivalent to the existence of a subadditive and choice-based function $\hat{q}(\cdot)$ such that

$$W(A) = w(\hat{q}(A)), \tag{5}$$

where $\hat{q}(A)$ denotes the choice-based probability of event A. It is uniquely determined by $\hat{q}(A) = p$, with p such that indifference between $(x, A; 0)$ and $(x, p; 0)$ holds (see Wakker 2004, Theorem 1, p. 238).[2] From this indifference statement, it follows that the function $w(\cdot)$ in Equation (5) is the probability weighting function (for risk).

In the above choice-based decomposition, the belief component can have some source preference in it, i.e., some preference for one source of uncertainty over another.[3] The Ellsberg (1961) paradox represents the most famous illustration of source preference. In this example, one ball is drawn at random from an urn containing 50 red balls and 50 black balls, and from an urn containing 100 red and black balls in an unknown proportion. Ellsberg argued that most people would rather bet on a ball of either color drawn from the first urn than a ball of either color drawn from the second. This corresponds to source preference for the "known" urn.[4] The above betting preference implies that the choice-based probability of the event "A red (black) ball is drawn from the 'unknown' urn" is smaller than 0.5 (see also Wakker 2004, p. 238). In contrast, the judged probability of either event is commonly indicated as 0.5. The relation between choice-based probabilities and judged probabilities is further addressed in §5.4.

3. Elicitation of Utility Function and Decision Weights

Our approach for eliciting decision weights amounts to a two-step procedure. The first step, which is based

[1] Rigorous formulations of the boundary conditions are available in Tversky and Wakker (1995).

[2] The uniqueness of $\hat{q}(\cdot)$ follows from the fact that $w(\cdot)$ is strictly increasing, so that the p satisfying the indifference between $(x, A; 0)$ and $(x, p; 0)$ is unique for $u(x) > 0$; see Wakker (2004, p. 241).

[3] The issue of source preference also poses a challenge to the two-stage model of Tversky and Fox (1995); see the discussion in Fox and Tversky (1998, p. 892).

[4] For a formal definition of source preference, see Tversky and Wakker (1995).

on the tradeoff method (Wakker and Deneffe 1996), elicits the utility function by determining a standard sequence of outcomes, i.e., a sequence of outcomes equally spaced in utility units. In the second step, decision weights can be computed using the utility values obtained in the first step as inputs.

As in Fennema and van Assen (1999) and Etchart-Vincent (2003), we employ mixed prospects for the elicitation of the utility function. The use of mixed prospects allows us to obtain standard sequences of outcomes beginning with the status quo outcome, i.e., $x = 0$. Due to this choice of design, the elicitation of decision weights can in turn be based on prospects that are particularly easy for participants to compare, as they consist of only one nonnull outcome.

3.1. Elicitation of the Utility Function

In our experimental investigation, standard sequences are elicited for gains and losses using monetary outcomes. The elicitation of the standard sequence for gains is performed as follows. Let $R^- < r^- < x_0 = 0$ denote three fixed outcomes and E a specified event. As a first step, the (positive) outcome x_1^+ is determined such that the decision maker is indifferent between the prospects $(x_0, E; r^-)$ and $(x_1^+, E; R^-)$. As a second step, the decision maker is called to state the outcome x_2^+ such that indifference between the prospects $(x_1^+, E; r^-)$ and $(x_2^+, E; R^-)$ holds. Assuming that CPT is an adequate descriptive theory of choice, the combination of the equations resulting from the above two indifference statements implies the equality of $u(x_2^+) - u(x_1^+)$ and $u(x_1^+) - u(x_0)$.

The next steps follow the general principle that once outcome x_i^+ has been elicited, outcome x_{i+1}^+ leading to indifference between $(x_i^+, E; r^-)$ and $(x_{i+1}^+, E; R^-)$ has to be determined. The elicitation procedure results in an increasing sequence of outcomes $x_0, x_1^+, \ldots, x_n^+$ such that

$$u(x_{i+1}^+) - u(x_i^+) = u(x_i^+) - u(x_{i-1}^+), \quad i = 1, \ldots, n-1. \tag{6}$$

Likewise, with $R^+ > r^+ > x_0 = 0$ and the same event E, a decreasing standard sequence of losses $x_0, x_1^-, \ldots, x_n^-$ is elicited.

3.2. Elicitation of Decision Weights

Let A_j denote an element of the family of events representing the relevant source of uncertainty. The determination of the decision weights for the gain domain, which builds on the standard sequence of outcomes for gains $x_0 = 0, x_1^+, \ldots, x_n^+$, proceeds as follows. For each event A_j, an outcome CE_j^+ is assessed such that the decision maker is indifferent between the prospect $(x_n^+, A_j; 0)$ and the certain receipt of CE_j^+. Under CPT, and with the normalization conditions $u(0) = 0$ and $u(x_n^+) = 1$, this indifference statement translates into

$$W^+(A_j) = u(CE_j^+). \tag{7}$$

Using the normalization conditions $u(0) = 0$ and $u(x_n^-) = -1$, decision weights for the loss domain are obtained by means of $W^-(A_j) = -u(CE_j^-)$.[5]

Except by coincidence, $CE_j^\bullet$ itself is not an element of the standard sequence of outcomes, which means that $u(CE_j^\bullet)$ is not immediately available.[6] Because utility is approximately linear over small intervals of outcomes, we determine $u(CE_j^\bullet)$ by linear interpolation between the two adjacent elements of the standard sequence (e.g., Bleichrodt and Pinto 2000). We additionally fit a number of parametric families to our data and use the estimated functions to determine utilities to check the robustness of our results. Details will be explained in §5.2.

3.3. Elicitation of Choice-Based Probabilities

The central idea underlying the two-stage approach is that decision weights comprise a component reflecting belief and a component reflecting decision attitude. To make the belief component explicit, a choice-based probability $\hat{q}(A_j)$ is assessed such that the decision maker is indifferent between the risky prospect $(x_n^+, \hat{q}(A_j); 0)$ and the uncertain prospect $(x_n^+, A_j; 0)$ for each event A_j. Assuming CPT, this indifference statement implies

$$w^+(\hat{q}(A_j)) = W^+(A_j). \tag{8}$$

It should be noted that the latter equality is derived, without invoking the original two-stage approach, purely by matching a risky simple prospect to an uncertain simple prospect involving the same nonnull outcome. The belief component is thus elicited in a "choice-based" manner, as opposed to the approach of asking subjects to state judged probabilities utilized by, for instance, Tversky and Fox (1995) or Fox et al. (1996). Equation (8) corresponds to the relation obtained in Wakker (2004, Theorem 1). Choice-based probabilities can be applied to decompose decision weights for losses according to a similar equation with $w^-(\cdot)$ and $W^-(\cdot)$ instead of $w^+(\cdot)$ and $W^+(\cdot)$.

4. Experiment

4.1. Subjects and Procedure

Forty-one subjects (33 male, 8 female) participated in our study. All of them were graduate students of business administration at the University of Mannheim.

[5] The utility of a loss $x < 0$ is given by $\lambda u(x)$, where λ is the loss-aversion coefficient. Obviously, this parameter cancels out when calculating decision weights for losses.

[6] Here and henceforth, the superscript $^\bullet$ stands for either $^+$ (gains) or $^-$ (losses).

They were enrolled in a decision analysis course, and hence were familiar with probability and expected-utility basics. Each subject received a fixed payment of DM 50 (≈25 US$) for participation. The experiment took place in February–March 2001.

The experiment was conducted in the form of computer-based individual interview sessions. Special software had been developed for the purpose of the present experiment. The subject and the experimenter were seated in front of a personal computer. The experimenter presented the subject with various choice situations associated with the different experimental tasks and entered the subject's statements into the computer after clear confirmation. Compared to procedures entirely driven by a computer program (e.g., Abdellaoui 2000), this procedure allows the subject to focus only on the displayed choice situation, and moreover, helps to prevent her from completing the experimental task without due diligence. Subjects were encouraged to take as much time for reflection as they considered necessary.

Each subject participated in two successive and separate sessions: one for gains and the other for losses. As in Abdellaoui (2000), the gains session was carried out first. The gains session consisted, in order, of tradeoff experiments (TO experiments), decision weights experiments (DW experiments), and choice-based probabilities experiments (CBP experiments). The losses session consisted of TO experiments and DW experiments. The TO experiments preceded the DW experiments because the latter required as input the respective final element $x_n^{\cdot}$ of the standard sequence of outcomes. Subjects were given the opportunity of a break of a few minutes after the first session. The mean time needed to complete the experiment as a whole amounted to approximately two hours.

Participants were not directly asked for the specific (outcome or probability) value leading to indifference, i.e., we did not employ a matching procedure. Instead, every value was assessed by means of a series of binary choice questions resulting from a "bisection-like" process as described in the next subsection.

4.2. Method and Stimuli

The utility function was elicited from six indifferences through TO experiments. For gains (losses), this amounts to the construction of a standard sequence $x_0 = 0, x_1^+, \ldots, x_6^+$ ($x_0 = 0, x_1^-, \ldots, x_6^-$). We chose $R^- = -400$ DM ($R^+ = 400$ DM) and $r^- = -100$ DM ($r^+ = 100$ DM). The event E was defined as "CDU wins the German general election in 2002." The stimuli were chosen to guarantee that the utility curvature over the interval $[0, x_6^+]$ ($[x_6^-, 0]$) would not be negligible (see Wakker and Deneffe 1996, p. 1139).

The source of uncertainty used in DW experiments and CBP experiments (i.e., used to elicit and decompose decision weights) was the stock index DAX, which is computed as a capitalization-weighted average of the stock prices of the 30 largest companies listed in Germany. We defined the relevant source of uncertainty as "percentage change of the DAX index over the next six months," measured from the respective day on which the experiment took place.

By the construction of a stock index, the state space is bounded below by −100%, whereas there is no logical upper bound. We partitioned the space of feasible percentage changes to create five elementary events. Abbreviating "percentage change of the DAX index over the next six months" by ΔDAX, these are $A_1 = \{\Delta\text{DAX} < -13\%\}$, $A_2 = \{-13\% \leq \Delta\text{DAX} < -4\%\}$, $A_3 = \{-4\% \leq \Delta\text{DAX} < +4\%\}$, $A_4 = \{+4\% \leq \Delta\text{DAX} < +13\%\}$, and $A_5 = \{+13\% \leq \Delta\text{DAX}\}$.[7] Our event space comprises the five elementary events plus all unions formed from the elementary events that result in contiguous intervals. The event space is depicted in Figure 1.

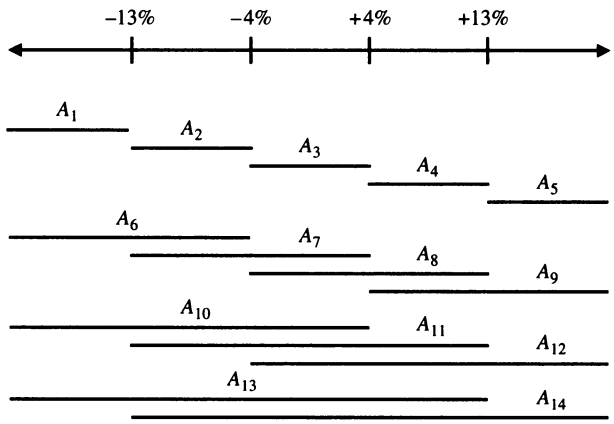
Screenshots A, B, and C in Figure 2 display the typical choice situation that participants faced during TO experiments, DW experiments, and CBP experiments, respectively. Participants could express either strict preference for one of the prospects or indifference.[8] The assessment procedure used was exclusively choice based and similar for TO, DW, and CBP experiments. In the TO experiments and the DW experiments, outcomes could be varied in steps of 50 DM (using the horizontal scroll bar located in the "choice frame"). For CBP experiments, probabilities could be varied in steps of 0.01.

To explain how our assessment procedure works, let us take the case of the TO experiments for gains and assume that outcome x_{i-1}^+ is known. To elicit outcome x_i^+, the subject was first asked to choose between the prospects $a = (x_{i-1}^+, E; r^-)$ and $b = (x_{i-1}^+ + \delta', E; R^-)$. δ' was fixed by the experimenter (using the scroll bar) approximately at the midpoint of the interval $[0, \Delta]$, with Δ set at the level of 6,000 DM that guaranteed strict preference of prospect b over prospect a for $\delta' = \Delta$. If the subject expressed strict preference for prospect a (b), the next choice situation involved a modification of prospect b to become more (less) attractive by replacing δ' with δ'' fixed near the

[7] The rationale behind the chosen partition is as follows. Assuming that the index change roughly follows a normal distribution, we estimated its standard deviation based on current financial data (adjusting for the time horizon in our study) and set the expected change equal to zero (because of the relatively short time horizon in our study). We then fixed interval boundaries such that the cumulative density of each interval amounts to 0.2.

[8] We used a "bisection-like" version of the procedure used in Cohen et al. (1987).

Figure 1 Event Space: "Percentage Change of the DAX Index Over the Next Six Months"



midpoint of the narrower interval $[\delta', \Delta]$ ($[0, \delta']$). This adjustment process continued until the subject felt indifferent between the two prospects on the screen.

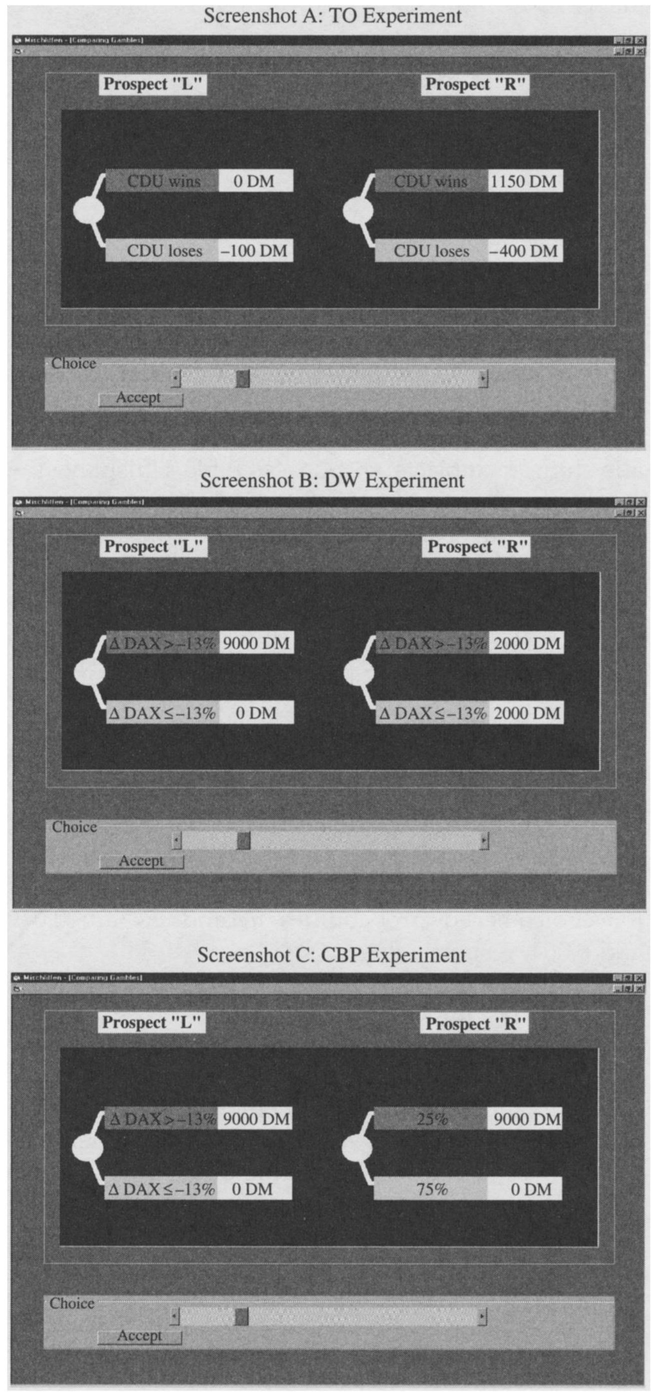
Once an indifference statement had been obtained, the experimenter explored whether small variations of δ (equal to the minimum step size of 50 DM) in either direction immediately resulted in a statement of strict preference. If so, which was the normal case, a unique feasible outcome leading to indifference had been found. If not, the range of feasible outcomes leading to indifference was explored; ultimately, the outcome value for which the subject expressed greatest hesitation between the prospects was selected.

Next, the button "Accept" had to be clicked on. Even after that, a participant noticing a decision error could re-enter the adjustment process. The next task appeared on the screen only after clicking on the button "Confirm."

For DW experiments, the procedure aiming to find the certainty equivalent of a prospect $(x_n^{\cdot}, A_j; 0)$ was fundamentally the same as in TO experiments. The subject was first asked to choose between the prospect $(x_n^{\cdot}, A_j; 0)$ and the certain receipt of δ', where the outcome δ' was fixed by the experimenter near the midpoint of the interval $[0, x_n^+]$ ($[x_n^-, 0]$) for gains (losses). For CBP experiments, the "bisection-like" approach was applied to the probability of the risky prospect. Accordingly, the subject was first asked to choose between the uncertain prospect $(x_n^{\cdot}, A_j; 0)$ and the risky prospect $(x_n^{\cdot}, p; 0)$, where the probability p was fixed by the experimenter near the midpoint of the unit interval. The process continued as described above for the TO experiments, with the difference that probability values take the place of outcome values.

The order in which the events $A_1, \ldots, A_{14}$ appeared during the experiment was determined randomly for each subject. It was fixed for all tasks that were based on this set of events (i.e., the DW experiments (gains and losses) and the CBP experiments).

Figure 2 Screenshots of Typical Choice Tasks



To be able to assess the reliability of subjects' answers, we presented them twice with the choice situations leading to the determination of x_1^+, x_1^-, CE_2^+, CE_2^-, CE_{12}^+, CE_{12}^-, $\hat{q}_2$, and $\hat{q}_{12}$. The choice tasks corresponding to CE_2^+ and CE_{12}^+ / CE_2^- and CE_{12}^- / $\hat{q}_2$ and $\hat{q}_{12}$ reappeared at the end of the DW (gains)/DW (losses)/CBP experiments, whereas the choice tasks corresponding to x_1^+ and x_1^- reappeared at the very end of the experiment as a whole.

Table 1 **Reliability (First Assessment vs. Second Assessment)**

	x_1^+	x_1^-	CE_2^+	CE_2^-	CE_{12}^+	CE_{12}^-	$\hat{q}_2$	$\hat{q}_{12}$
Paired t tests (two-tailed)	$t_{29} = 1.46^{ns}$	$t_{29} = 1.42^{ns}$	$t_{40} = 0.40^{ns}$	$t_{40} = 0.71^{ns}$	$t_{40} = -1.73^{ns}$	$t_{40} = 1.60^{ns}$	$t_{40} = -0.08^{ns}$	$t_{40} = -2.05^{*}$
Pearson correlation	0.90	0.75	0.92	0.95	0.79	0.99	0.72	0.78

Notes. The sign of t reflects the sign of the mean difference between the first and second assessments. *ns*: nonsignificant for $\alpha = 0.05$; *: $p < 0.05$.

5. Results

5.1. Reliability

For purposes of the present study, reliability refers to the stability of subjects' responses when an identical choice task is presented twice. Reliability is measured by means of the Pearson correlation coefficient. Paired t tests are conducted to check for systematic shifts in subjects' responses. Table 1 displays the results.

The overall picture confirms the consistency of subjects' responses. The observed correlation coefficients are quite high, ranging from 0.72 to 0.99. For only one of the choice-based probabilities tasks ($\hat{q}_{12}$), a significant difference is detected by the paired t test ($t_{40} = -2.05$, $p = 0.05$; two-tailed). The mean absolute differences between the first and the second assessments, measured on the normalized scale of utility for outcomes using linear interpolation, are 0.063 (0.070), 0.055 (0.065), and 0.091 (0.068) for gain (loss) outcomes x_1^+ (x_1^-), CE_2^+ (CE_2^-), and CE_{12}^+ (CE_{12}^-), respectively. The mean absolute differences for choice-based probabilities $\hat{q}_2$ and $\hat{q}_{12}$ are 0.049 and 0.064, respectively.

5.2. Utility Function

Both nonparametric and parametric classifications of the shape of the elicited utility functions reveal a predominance of concavity in the gain domain. We obtain some, albeit not very pronounced, evidence in favor of convexity in the loss domain. Our results are quite consistent with previous studies except for a relatively high number of utility functions classified as having a mixed shape through the nonparametric approach. It will be argued below that this finding might be related to the size of the outcome intervals over which the utility functions are constructed.

Nonparametric Results. To classify the curvature of participants' utility functions, we define $d_i^{\cdot} = x_i^{\cdot} - x_{i-1}^{\cdot}$, $i = 1, \ldots, 6$, and $\Delta_j^{\cdot} = \text{sign}(d_{j+1}^{\cdot} - d_j^{\cdot})$, $j = 1, \ldots, 5$, for both domains separately. A utility function is labeled concave/linear/convex in a particular domain if at least three out of five $\Delta_j^{\cdot}$s equal $1/0/-1$, and mixed otherwise. The idea behind this classification scheme is that even if the utility function of the subject's "true" preference functional (which we assume is CPT) exhibits a unique curvature, the stochastic component of her preference is likely to prevent all $\Delta_j^{\cdot}$s from being equal. An equivalent classification scheme has been applied in the studies of Abdellaoui (2000), Bleichrodt and Pinto (2000), and Etchart-Vincent (2003, 2004). In contrast to the present study, however, these studies employed risky prospects for the elicitation of the utility function.

The results of the present study are displayed in the first row of Table 2. With respect to the gain domain, 19 subjects (46.3%) are classified as having a concave utility function. Of the other subjects, 3 are classified as linear (7.3%), 8 are classified as convex (19.5%), and 11 are classified as mixed (26.8%). The modal curvature of the utility function in the gain domain is thus concavity, in accordance with both the contention in Tversky and Kahneman (1992) and the respective results of the two related studies. The relative frequency of concave shapes conditional on a classification as nonmixed amounts to 63.3%, which is quite in line with the corresponding figures of 58.3% in Abdellaoui (2000) and 67.4% in Bleichrodt and Pinto (2000). What distinguishes our findings from the two related studies is the comparatively high percentage of mixed classifications.

With respect to the loss domain, only 10 subjects (24.4%) exhibit the hypothesized convex shape of the utility function. Of the other subjects, 9 are classified

Table 2 **Classification of Utility Functions (Tradeoff Method)**

	Gains				Losses			
	Concave	Linear	Convex	Mixed	Concave	Linear	Convex	Mixed
Present study* (%)	46.3	7.3	19.5	26.8	22.0	22.0	24.4	31.7
Abdellaoui (2000)** (%)	52.5	17.5	20.0	10.0	20.0	25.0	42.5	12.5
Etchart-Vincent (2004)** (%)	no data				14.3	25.7	37.2	22.8
Bleichrodt and Pinto (2000)*** (%)	59.2	26.5	2.0	12.2	no data			

*Utility elicited with unknown probabilities; *Utility elicited with known probabilities; ***Utility elicited with known probabilities and nonmonetary outcomes.

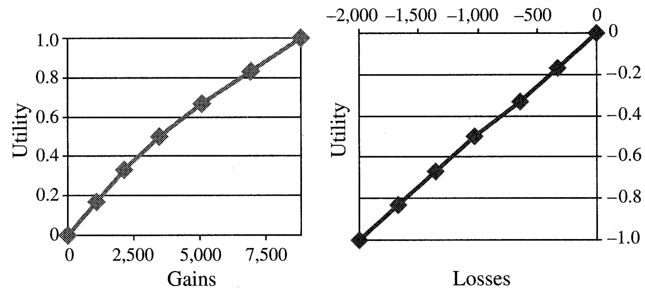
as linear (22.0%), 9 are classified as concave (22.0%), and 13 are classified as mixed (31.7%). The preponderance of mixed shapes parallels the earlier observation of the relatively high number of mixed classifications in the gain domain. The lack of a distinct pattern among the nonmixed classifications might be due to the fact that the average size of the outcome interval $[x_6^-, 0]$ is relatively small. This conjecture is consistent with a considerable body of evidence suggesting that the utility function is approximately linear over small and moderate stakes (see, e.g., Lopes and Oden 1999, Rabin 2000). Interestingly, Etchart-Vincent (2004) found that the percentage of mixed cases declines in favor of concavity when the average size of the loss interval under investigation is higher (22.8% mixed cases for small losses as given in Table 2, and 11.5% mixed cases for higher losses).[9] The percentages of linear and convex cases remained constant. Moreover, both in Abdellaoui (2000) and in Fennema and van Assen (1999) the evidence in favor of convexity in the loss domain is somewhat weaker than the evidence in favor of concavity in the gain domain.

When only concave and convex shapes are taken into account, a binomial test reveals that there are significantly more concave utility functions in the gain domain than convex ones ($p = 0.03$, one-tailed), whereas the reverse statement for the loss domain cannot be established ($p = 0.5$, one-tailed). These findings are also mirrored in the aggregate data. Figure 3 displays the utility functions, separately for each domain, that result when the elements of the respective standard sequences are averaged over participants.

Parametric Results. In addition to the nonparametric classification of utility functions presented above, parametric estimations were conducted using nonlinear least squares fits. Their purpose is twofold. First, they provide another way of looking at the utility function data and allow for comparisons with studies that adopt parametric approaches. Second, the estimated utility functions serve as input in the determination of decision weights as explained in §3.2. Two parametric forms were used: power and exponential (Table 3).

The power family is frequently employed in experimental studies on utility measurement. It exhibits constant relative risk aversion, and its widespread use possibly has roots in the power law of psychophysics (Stevens 1957); for an early application in risky decision making, see Tversky (1967). Exhibiting the property of constant absolute risk aversion, the exponential specification is also extensively used in utility assessment from laboratory experiments as well as

Figure 3 Utility Functions (for Mean Values)



decision analysis interviews. The domain $[0, x_6^+]$ for gains ($[x_6^-, 0]$ for losses) is mapped into the positive (negative) unit interval through the rescaling $z = x/x_6^+$ ($z = -x/x_6^-$).

For gains, both the power utility function and the exponential utility function exhibit a concave shape for the mean and median parameter estimates, so that the nonparametric findings are confirmed. At an individual level, both power and exponential estimates show that the predominant shape is concavity (26 out of 41 subjects satisfy $\alpha^+ < 1$ and $\lambda^+ > 0$, respectively). As shown in Table 4, the estimate for the power utility function almost coincides with the median estimate of 0.89 in Abdellaoui (2000) and the median estimate of 0.88 in Tversky and Kahneman (1992). With regard to the loss domain, the parametric fitting also supports the nonparametric results that were at variance with the hypothesized convex shape. For the power utility function and the exponential utility function, the mean and median parameter estimates are near to or even on either side of the respective borderline separating concave and convex shapes. At an individual level, convexity for losses seems less dominant than concavity for gains (23 out of 41 utility functions are classified as convex). The parameter estimate for the power utility function therefore deviates from the median estimate of 0.92 reported in Abdellaoui (2000) and the median estimate of 0.88 obtained by Tversky and Kahneman (1992).

5.3. Decision Weights

Decision weights conform to the property of bounded subadditivity almost without exception, which strongly questions the interpretation of subjects' behavior as "SEU plus noise." Whereas the degree of subadditivity does not differ systematically between the gain and loss domains, decision weights appear to exhibit higher elevation for losses.

Table 3 Parametric Specifications

	Power	Exponential
Gains	$z^{\alpha+}$	$(1-\exp(-\lambda^+ z))/(1-\exp(-\lambda^+))$
Losses	$-(-z)^{\alpha-}$	$-(1-\exp(\lambda^- z))/(1-\exp(-\lambda^-))$

[9] In Etchart-Vincent (2004), the mean interval for small losses is [FF − 8,460; FF − 1,500]; the mean interval for high losses is [FF − 100,000; FF − 20,000] (1 US$ ≈ 6 FF).

Table 4 Summary Statistics for Parameters of the Utility Function

	Gains			Losses		
	Median	Mean	Std. dev.	Median	Mean	Std. dev.
Power: α^*	0.91	0.91	0.32	0.96	1.08	0.47
Exponential: λ^*	0.28	0.57	1.21	0.09	0.03	1.19

We do not find evidence for a systematic violation of the duality condition. The aforementioned results do not critically depend on the particular interpolation method used for the utility function.

Decision Weights for Gains and Losses. As explained in §3.2, the decision weight of event A_j, $W^+(A_j)$ ($W^-(A_j)$), equals the (absolute value of the) utility of the certainty equivalent of the prospect $(x_n^+, A_j; 0)$ $((x_n^-, A_j; 0))$. The utility function is obtained by both linear interpolation between the outcomes of the standard sequence and (individual) parametric fitting of the families listed in Table 3, separately for each subject.

Figure 4 displays median decision weights for gains and losses. It can be seen that median decision weights satisfy all monotonicity conditions imposed by the structure of the event space as depicted in Figure 1. Overall, Figure 4 suggests that for each outcome domain, median decision weights are not sensitive to the parametric specification used for the utility function. One-factor ANOVA tests with repeated measures detect significant differences between the three specifications in 11 (5) out of 28 cases for α fixed at 0.05 (0.01). It seems, however, that the "traditional" power family produces decision weights that are closer to those computed by means of linear interpolation. This is confirmed by (two-tailed) paired t tests that detect significant differences between linear interpolation and the power specification in only 4 (2) out of 28 cases if α is fixed at 0.05 (0.01).

Table 5 Elevation of Decision Weights: $W^+(A)$ vs. $W^-(A)$ (Paired t Tests, One-Tailed)

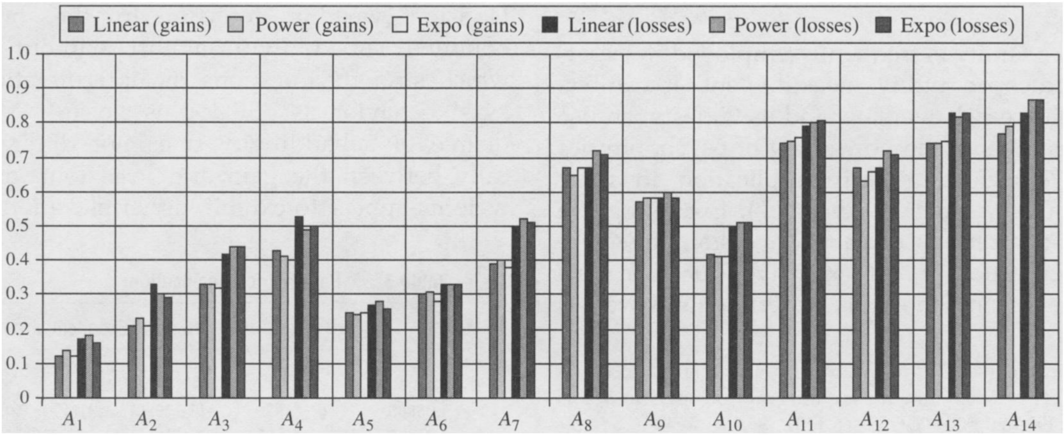
	Linear interpolation t_{40}	Power approximation t_{40}	Exponential approximation t_{40}
A_1	0.72^{ns}	0.26^{ns}	0.48^{ns}
A_2	2.26^*	1.75^*	1.80^*
A_3	2.13^*	1.39^{ns}	1.51^{ns}
A_4	1.67^{ns}	1.36^{ns}	1.39^{ns}
A_5	0.65^{ns}	0.05^{ns}	0.15^{ns}
A_6	1.02^{ns}	0.64^{ns}	0.52^{ns}
A_7	1.25^{ns}	0.96^{ns}	0.87^{ns}
A_8	1.81^*	2.25^*	1.83^*
A_9	1.00^{ns}	1.04^{ns}	0.87^{ns}
A_{10}	2.23^*	2.20^*	1.99^*
A_{11}	2.13^*	2.42^*	2.00^*
A_{12}	1.62^{ns}	2.17^*	1.83^*
A_{13}	1.83^*	2.21^*	1.74^*
A_{14}	1.29^{ns}	1.70^*	1.14^{ns}

Notes. *ns*: nonsignificant for $\alpha = 0.05$; *: $p < 0.05$.

Figure 4 also shows that median decision weights referring to the loss domain exceed their gain domain counterparts for all events. In the terminology used for the probability weighting function under risk (Gonzalez and Wu 1999), decision weights for the loss domain seem to exhibit more elevation. This statement is statistically supported more for likely events than for unlikely ones. As can be seen from Table 5, paired t tests, conducted separately for each event, lead to a rejection of the null hypothesis $W^+(A_j) = W^-(A_j)$ in favor of $W^+(A_j) < W^-(A_j)$ in about one half of the 14 cases ($\alpha = 0.05$, one-tailed).

It should be observed, however, that the eventwise comparison of decision weights is not necessarily a very effective technique to detect potential differences between the two domains. This limitation can be

Figure 4 Median Decision Weights



overcome using the two-stage approach that permits a characterization of the elevation of a participant's decision weights by means of a single overall parameter. We will therefore return to the issue of elevation in §5.5.

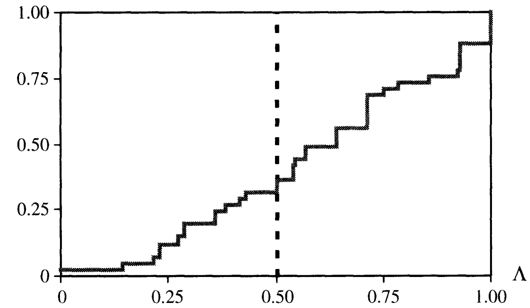
For an additional within-subject analysis, let Σ^- (Σ^+) denote the number of events for which $W^-(A_j) > W^+(A_j)$ ($W^-(A_j) < W^+(A_j)$) holds, separately for each subject. For each subject, we compute the ratio $\Lambda = \Sigma^-/(\Sigma^- + \Sigma^+)$, which should on average be close to 0.5 if no systematic differences in elevation are present. Figure 5 displays the corresponding empirical distribution function (for linear interpolation). The ratio Λ is larger (smaller) than 0.5 for 26 (13) out of 41 subjects. The null hypothesis that Λ values larger than 0.5 and smaller than 0.5 are equally likely is rejected by a sign test ($p = 0.03$, one-tailed). The within-subject analysis thus confirms the results of the eventwise comparisons presented before.

Sensitivity of Elicited Decision Weights. The notion of sensitivity is related to the bounded subadditivity property of decision weights presented in §2, which says that the impact of an event is particularly strong if it is added to the null event or subtracted from the universal event. The structure of the event space in the present study leads to 16 conditions to test for LSA[10] and to 6 conditions to test for USA,[11] separately for each domain. For each participant, the respective number of conditions satisfied is computed. To obtain conservative results, a condition counts as satisfied only if strict inequality holds. Figure 6 displays the empirical distribution functions of conditions satisfied (for linear interpolation).

If decision weights were perfectly additive, the number of conditions satisfied would equal zero in all cases. If decision weights were fundamentally additive but with a nonsystematic error component, the number of conditions satisfied would be stochastic and its distribution would be centered around one half of the number of available conditions.

Figure 6 provides strong evidence against "noisy additivity" of decision weights and, therefore, against the descriptive validity of SEU. With respect to LSA (USA), more than 8 (3) conditions are satisfied

Figure 5 **Empirical Distribution for Elevation Comparison of Decision Weights**



by 90.2% (95.1%) of the subjects in the gain domain and by 90.2% (82.9%) of the subjects in the loss domain. Aggregating over all subjects, the proportion of LSA (USA) conditions satisfied amounts to 78.7% (87.8%) in the gain domain and to 86.4% (84.1%) in the loss domain. These figures closely resemble the results reported in Tversky and Fox (1995, Table 4, p. 276). The median number of LSA (USA) conditions satisfied is 13 (6) in the gain domain and 15 (6) in the loss domain.

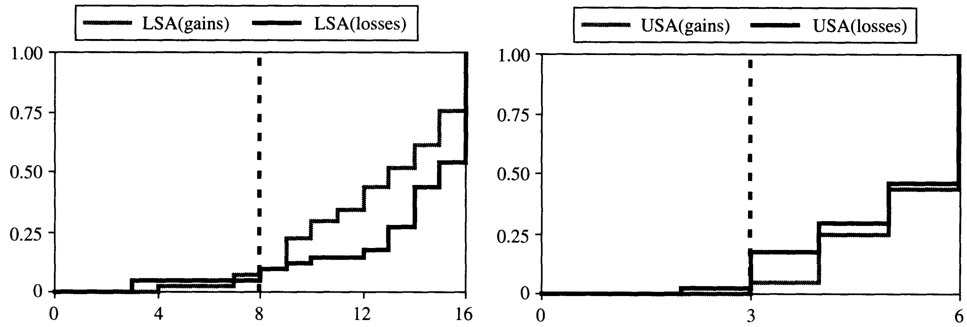
To also obtain a quantification of the degree of subadditivity of decision weights, the measure initially applied by Tversky and Fox (1995) and thereafter used, for instance, by Kilka and Weber (2001) is computed. An equivalent statement of the condition defining LSA (USA) is $W^\bullet(A_i) + W^\bullet(A_j) - W^\bullet(A_i \cup A_j) \geq 0$ ($1 - W^\bullet(S - A_i) - (W^\bullet(A_i \cup A_j) - W^\bullet(A_j)) \geq 0$). The index of LSA (USA), henceforth denoted by $\mu_W^\bullet(\text{LSA})$ ($\mu_W^\bullet(\text{USA})$), is computed as the mean value of the left-hand side of the above inequalities, taken over the 16 (6) triples of events allowing a test of the respective condition. It is computed separately for each participant and each domain. The median values (where medians are taken over participants) of the indices amount to $\mu_W^+(\text{LSA}) = 0.18$ ($\mu_W^+(\text{USA}) = 0.20$) in the gain domain and to $\mu_W^-(\text{LSA}) = 0.25$ ($\mu_W^-(\text{USA}) = 0.18$) in the loss domain. The degree of subadditivity in our data is consistent with the findings of Tversky and Fox (1995, Table 5, p. 277), and with the findings concerning the source of uncertainty with higher perceived competence in Kilka and Weber (2001, Table 1, p. 1719). The pervasiveness of subadditivity is also mirrored by the fact that the LSA (USA) index is strictly positive for 90.2% (100%) of the subjects in the gain domain and for 92.7% (92.7%) of the subjects in the loss domain. Both t tests of the hypothesis that the mean index value is zero and sign tests of the hypothesis that positive and negative index values are equally likely result in $p < 0.001$ for each of the four domain-property combinations.

We also considered whether the degree of subadditivity of decision weights differs across the two domains. The null hypothesis that the mean LSA

[10] LSA implies, e.g., that $W^\bullet(A_1) \geq W^\bullet(A_6) - W^\bullet(A_2)$, where $A_6 = A_1 \cup A_2$. The 15 other triples of events $(A_i; A_i \cup A_j, A_j)$ allowing a test of LSA are $(A_2; A_7, A_3)$, $(A_3; A_8, A_4)$, $(A_4; A_9, A_5)$, $(A_1; A_{10}, A_7)$, $(A_2; A_{11}, A_8)$, $(A_3; A_{10}, A_6)$, $(A_3; A_{12}, A_9)$, $(A_4; A_{11}, A_7)$, $(A_5; A_{12}, A_8)$, $(A_1; A_{13}, A_{11})$, $(A_2; A_{14}, A_{12})$, $(A_4; A_{13}, A_{10})$, $(A_5; A_{14}, A_{11})$, $(A_6; A_{13}, A_8)$, and $(A_7; A_{14}, A_9)$. All the triples satisfy the boundary condition that $W^\bullet(A_i \cup A_j)$ is bounded away from 1.

[11] USA implies, e.g., that $1 - W^\bullet(A_{14}) \geq W^\bullet(A_6) - W^\bullet(A_2)$, where $S - A_{14} = A_6 - A_2 = A_1$. The five other triples of events $(S - A_i; A_i \cup A_j, A_j)$ allowing a test of USA are $(A_{14}; A_{10}, A_7)$, $(A_{14}; A_{13}, A_{11})$, $(A_{13}; A_9, A_4)$, $(A_{13}; A_{12}, A_8)$, and $(A_{12}; A_{10}, A_3)$. All the triples satisfy the boundary condition that $W^\bullet(A_j)$ is bounded away from 0.

Figure 6 Empirical Distributions of Subadditivity Conditions Satisfied for $W^{\bullet}(\cdot)$



(USA) index value is equal for gains and losses cannot be rejected by either paired t tests or sign tests at conventional levels of significance (each $p > 0.05$, one-tailed). This result suggests that the sensitivity of decision weights does not exhibit domain dependence. An alternative characterization of decision weights in terms of elevation and sensitivity will be presented in §5.5.

Duality of Elicited Decision Weights. If subjects' decision weights systematically violate duality, one of the additional degrees of freedom introduced by CPT (as compared to CEU) is empirically warranted. Owing to the structure of the event space in the present study, there are eight pairs of events and complementary events that provide a test of the duality condition. Table 6 displays mean values of the respective sums of decision weights. Additionally, it shows t values for tests of the hypothesis that mean sums equal 1.

The overall picture is remarkably clear. The respective duality conditions are not significantly violated ($\alpha = 0.05$, two-tailed).

Error Propagation. It is generally agreed that individuals' decision-making behavior involves some element of randomness, usually interpreted as decision error (see, e.g., Hey 1999). In the tradeoff method, subjects' responses are "chained" (i.e., the elicitation of $x_i^{\bullet}$ requires $x_{i-1}^{\bullet}$ as input), which might give rise to propagation of such decision errors. Due to the fact that DW experiments require $x_6^{\bullet}$ (resulting from TO experiments) as input, error propagation in TO experiments might in turn impair the reliability of the measurement of decision weights.

To be able to assess the impact of error propagation, we performed a simulation based on an error theory comparable to Hey and Orme (1994). More specifically, we assumed that the utility difference between two successive elements of the standard sequence of outcomes is distorted by an additive error term. The error terms were assumed to be independently and identically distributed normal random variables with mean 0 and standard deviation 0.05. For each individual subject, hypothetical decision weights were calculated on the basis of distorted utility values (using linear interpolation) and subsequently contrasted with the empirically determined decision weights.[12]

Over 1,600 simulations (40 runs with 41 subjects each) were performed, separately for gains and losses. For each event, we computed the mean absolute value of the difference between simulated and empirical decision weights and normalized this quantity by dividing through the mean empirical decision weight. This index does not exceed 3.5% for gains as well as losses. If the mean absolute value is replaced by the standard deviation, the index does not exceed 4.5%. To sum up, the simulation shows that decision errors in the tradeoff method do not entail excessive variability of decision weights, which confirms the results presented earlier in this subsection.

5.4. Probabilities and Decision Weights

As was found in previous experimental investigations for judged probabilities, the elicited choice-based probabilities exhibit bounded subadditivity. The degree of subadditivity is lower than for decision weights, which is also in line with previous research. The comparison of decision weights and choice-based probabilities reveals an overweighting of unlikely events for both gains and losses. An underweighting of likely events occurs for gains but not for losses.

Choice-Based Probabilities. Table 7 contains summary statistics (median, mean, and standard deviation) for the choice-based probabilities. Parallel to the finding for decision weights, it can be verified that median and mean probabilities satisfy all monotonicity conditions imposed by the structure of the event space.[13]

[12] Formally, the distorted utility scale $\hat{u}(\cdot)$ is based on the following error model: $\hat{u}(x_i) = \hat{u}(x_{i-1}) + 1/6 + \varepsilon_i$ for $i = 1, \ldots, 6$ ($\hat{u}(x_0) = 0$), where ε_i denotes the ith error term. After a utility rescaling ($\hat{u}(x_i)/\hat{u}(x_6)$ for $i = 1, \ldots, 6$), the simulated decision weights were determined through linear interpolation using the simulated utility function.

[13] Assuming the two-stage decomposition $W^{\bullet}(A_i) = w^{\bullet}(\hat{q}(A_i))$, the monotonicity condition for subjective choice-based probabilities,

Table 6 Tests of Duality (t Tests, Two-Tailed)

	Linear interpolation		Power approximation		Exponential approximation	
	Mean	t_{40}	Mean	t_{40}	Mean	t_{40}
$W^-(A_1)+W^+(S-A_1)$	0.955	-1.42^{ns}	0.954	-1.41^{ns}	0.965	-1.09^{ns}
$W^-(A_5)+W^+(S-A_5)$	1.012	0.29^{ns}	0.997	-0.08^{ns}	1.012	0.30^{ns}
$W^-(A_6)+W^+(S-A_6)$	0.989	-0.28^{ns}	0.971	-0.74^{ns}	0.985	-0.39^{ns}
$W^-(A_9)+W^+(S-A_9)$	1.020	0.48^{ns}	1.013	0.33^{ns}	1.032	0.74^{ns}
$W^-(A_{10})+W^+(S-A_{10})$	1.067	1.62^{ns}	1.054	1.25^{ns}	1.073	1.63^{ns}
$W^-(A_{12})+W^+(S-A_{12})$	1.012	0.32^{ns}	1.023	0.69^{ns}	1.032	0.88^{ns}
$W^-(A_{13})+W^+(S-A_{13})$	1.044	1.30^{ns}	1.065	1.90^{ns}	1.064	1.77^{ns}
$W^-(A_{14})+W^+(S-A_{14})$	0.973	-0.80^{ns}	0.998	-0.05^{ns}	0.985	-0.42^{ns}

Note. *ns*: nonsignificant for $\alpha = 0.05$.

Subadditivity of Choice-Based Probabilities. As pointed out in detail in §§2 and 3.3, degrees of belief in the present study are determined through choices and in this respect differ from judged probabilities elicited in studies relying on the original two-stage model. This aspect naturally leads to the question of whether the properties of the elicited degrees of belief are affected by the elicitation mode. Subadditivity is one of the key properties of judged probabilities in support theory (Tversky and Koehler 1994) and also crucial in the derivation of the decomposition in Wakker (2004). Parallel to the analysis of the subadditivity property for decision weights, Figure 7 displays the empirical distribution functions of LSA/USA conditions satisfied.

The graphs in Figure 7 closely resemble their decision weights counterparts in Figure 6. With respect to LSA (USA) of choice-based probabilities, more than 8 (3) conditions are satisfied by 82.9% (87.8%) of the subjects. We also computed indices of subadditivity for choice-based probabilities, denoted by μ_q(LSA) and μ_q(USA), which are defined analogously to the respective measures for decision weights, μ_W(LSA) and μ_W(USA), presented in §5.3. The LSA (USA) index is strictly positive for 90.2% (97.6%) of the subjects. The median values (where medians are taken over participants) of these indices amount to $\mu_q(\text{LSA}) = 0.12$ and $\mu_q(\text{USA}) = 0.13$. Two conclusions can be drawn from these figures. First, they provide strong empirical support for subadditivity of choice-based probabilities, which thereby share an important property with judged probabilities (Tversky and Koehler 1994, Tversky and Fox 1995, Fox et al. 1996, Wu and Gonzalez 1999, Kilka and Weber 2001). Second, because the indices of subadditivity are smaller for probabilities than for decision weights, our data are consistent with an interpretation of decision weights as a subadditive measure of belief transformed by a subadditive function (Fox and Tversky 1998, Wakker 2004).

Decision Weights vs. Choice-Based Probabilities. It was already mentioned that CEU coincides with SEU for the special case that decision weights are additive subjective probabilities. Contrasting decision weights and choice-based probabilities, it can be asked whether they coincide for every event considered. Table 8 displays the results of paired t tests conducted to check whether mean decision weights equal mean choice-based probabilities, separately for each event and each domain. To facilitate interpretation, the events are ordered according to their median choice-based probabilities, reproduced from Table 7.

The pattern in the gain domain reflects an overweighting of unlikely events and an underweighting of likely events (relative to the respective choice-based probabilities). This finding is broadly consistent with evidence from studies of decision making under risk, particularly an inverse S-shaped transformation function (see, e.g., Tversky and Kahneman 1992, Wu and Gonzalez 1996, Bleichrodt and Pinto 2000). The statistical significance of the observed differences is not high, however. The pattern in the loss domain, which is foreshadowed by the pattern in the gain domain and the comparison of decision weights across domains presented in Figure 4, indicates a marked overweighting of unlikely events. An underweighting of likely events cannot be found in the data, yet the decrease of the difference between decision weights and choice-based probabilities as one moves towards more likely events is common to both domains. From an overall perspective, the results contained in Table 8 suggest that CPT is descriptively more adequate than SEU.

i.e., $\hat{q}(A) \le \hat{q}(B)$ for all events A, B with $A \subset B$, follows from the corresponding condition for decision weights and the fact that the transformation function $w^{\bullet}(\cdot)$ is increasing.

Table 7 Summary Statistics for Choice-Based Probabilities

	$\hat{q}_1$	$\hat{q}_2$	$\hat{q}_3$	$\hat{q}_4$	$\hat{q}_5$	$\hat{q}_6$	$\hat{q}_7$	$\hat{q}_8$	$\hat{q}_9$	$\hat{q}_{10}$	$\hat{q}_{11}$	$\hat{q}_{12}$	$\hat{q}_{13}$	$\hat{q}_{14}$
Median	0.10	0.17	0.35	0.42	0.20	0.25	0.40	0.66	0.55	0.50	0.75	0.72	0.75	0.85
Mean	0.13	0.20	0.36	0.43	0.23	0.27	0.42	0.62	0.54	0.48	0.73	0.68	0.75	0.82
Std. dev.	0.09	0.13	0.19	0.21	0.15	0.15	0.19	0.20	0.21	0.17	0.16	0.17	0.13	0.13

5.5. Fitting the Two-Stage Model

Fitting the two-stage model to our data amounts to estimating the parameter(s) of the probability weighting function via nonlinear regression with decision weights as dependent variable and choice-based probabilities as explanatory variable (Figure 8 plots median choice-based probabilities against median decision weights). We focus on parametric specifications of the probability weighting function that permit a clear separation between elevation and curvature. Whereas the curvature parameter does not differ significantly between the gain and loss domains, the elevation parameter is markedly higher for losses. The results obtained under the two-stage model therefore mirror their counterparts from the holistic analysis of decision weights in §5.3.

The estimations are conducted separately for each participant. We employ several parametric forms originally proposed for the probability weighting function under risk. One parametric specification that proves to be particularly useful is the linear-in-log-odds form applied by, for instance, Goldstein and Einhorn (1987), Lattimore et al. (1992), and Gonzalez and Wu (1999):

$$w(p) = \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1-p)^{\gamma}}.\text{[14]} \tag{9}$$

Its usefulness derives from the fact that it permits a distinction between two essential features of the probability weighting function: elevation and curvature (i.e., sensitivity in the context of probability weighting). For the linear-in-log-odds specification, the parameter δ mainly controls elevation, whereas the parameter γ mainly controls curvature. The issues of elevation and curvature of decision weights addressed in §5.3 can therefore be reanalyzed in the light of the respective parameter estimates of the probability weighting function in the two-stage model.[15]

A parsimonious parametric form for the probability weighting function that nevertheless incorporates a clear separation between elevation and curvature is the linear approximation

$$\begin{aligned} w(p) &= \alpha + \beta \cdot p \quad \text{for } p \in (0,1), \\ w(0) &= 0, \quad w(1) = 1, \end{aligned} \tag{10}$$

applied, for instance, in Wu and Gonzalez (1996) and Kilka and Weber (2001). Curvature is captured by the parameter β, whereas elevation can be characterized through $\int w(p)\,dp$.[16]

By construction, probability weighting functions with a single free parameter like the one used by Karmarkar (1978) or Tversky and Kahneman (1992), or the one-parameter variant in Prelec (1998) do not permit an independent variation of elevation and curvature. For this reason, the following presentation of results is restricted to the two-parameter specifications.

Table 9 conveys parameter estimates of the probability weighting function for median data resulting from nonlinear least squares with a normally distributed error term.[17] It can be seen that, for linear interpolation as well as power or exponential approximations, the curvature parameter for median data is quite similar across the two domains for all parametric specifications of the probability weighting function. In contrast, the estimates of δ show that the probability weighting function for median data exhibits more elevation for losses than for gains.[18] In decision

[14] The linear-in-log-odds property is demonstrated in Gonzalez and Wu (1999, p. 139).

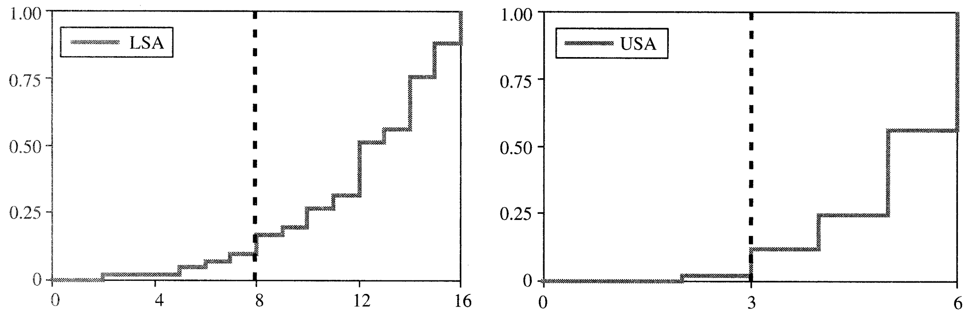
[15] The same kind of decomposition is feasible for the (two-parameter) probability weighting function derived axiomatically by Prelec (1998): $w(p) = \exp(-\beta \cdot (-\log(p))^{\alpha})$, where the parameter β mainly controls elevation and the parameter α mainly controls curvature. We also applied this probability weighting function in the analyses to be presented subsequently. For a small number of subjects, we obtained parameter estimates—especially for the parameter β—that were quite high, which would have unduly affected summary statistics and statistical inference. We therefore focus on the linear-in-log-odds function, complementing it with a parsimonious and particularly robust specification of the probability weighting function to avoid dependence on a single parametric form.

[16] Bounded subadditivity of the probability weighting function corresponds to $\alpha > 0$ and $\alpha + \beta < 1$, in which case the measure of elevation equals $\alpha + \beta/2$ (see also Kilka and Weber 2001, p. 1717).

[17] We also conducted the nonlinear regressions using individual subject data. The median (across subjects) estimated parameter values are quite in line with the results presented in Table 9.

[18] The quality of fit is systematically higher under the linear-in-log-odds specification for all domains and all kinds of utility interpolation. The mean adjusted R^2 ranges from 0.94 to 0.97 for the linear-in-log-odds specification and from 0.74 to 0.83 for the linear approximation.

Figure 7 Empirical Distributions of Subadditivity Conditions Satisfied for $\hat{q}(\cdot)$



making under risk, higher elevation of the probability weighting function for losses has been found by both Abdellaoui (2000) and Wu et al. (2004) in their reanalysis of the Tversky and Kahneman (1992) data using a two-parameter function.

At the level of individual subjects, paired t tests are conducted to investigate the hypothesis of equal parameter values in the gain and loss domains. Whereas the curvature parameter (γ) is statistically indistinguishable across domains at conventional levels of significance ($t_{40} = 0.00$, $p = 1.00$ for the linear interpolation; $t_{40} = 1.26$, $p = 0.21$ for the power approximation; $t_{40} = 1.18$, $p = 0.25$ for the exponential approximation; two-tailed), the domain dependence of the elevation parameter (δ) is confirmed ($t_{40} = 2.04$, $p = 0.02$ for the linear interpolation; $t_{40} = 2.08$, $p = 0.02$ for the power approximation; $t_{40} = 1.23$, $p = 0.11$ for the exponential approximation; one-tailed).

The conclusions derived for the linear-in-log-odds specification turn out to be robust when the simple linear probability weighting function is considered instead. Paired t tests show no significant difference between the curvature parameter (β) of the gain and loss domains ($t_{40} = 1.12$, $p = 0.27$ for the linear interpolation; $t_{40} = 1.55$, $p = 0.13$ for the power approximation; $t_{40} = 0.90$, $p = 0.37$ for the exponential approximation; two-tailed). The measure of elevation in the loss domain exceeds its gain domain counterpart significantly ($t_{40} = 2.31$, $p = 0.01$ for the linear interpolation; $t_{40} = 1.91$, $p = 0.03$ for the power approximation; $t_{40} = 1.78$, $p = 0.04$ for the exponential approximation; one-tailed).

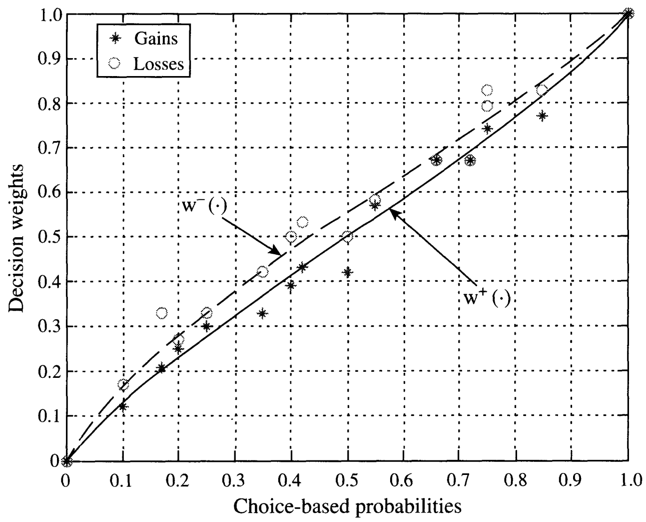
It must be mentioned that the picture is somewhat less clear at the level of individual subjects with 26 out of 41 participants satisfying $\delta^- > \delta^+$ (for linear interpolation, power approximation, and exponential approximation), $p = 0.06$ for a sign test (one-tailed). The general conclusion is strengthened again by the results of the linear probability weighting function where the measure of elevation is higher in the loss domain for 28 out of 41 subjects (for linear interpola-

Table 8 Decision Weights vs. Choice-Based Probabilities (Paired t Tests, One-Tailed)

		Linear interpolation		Power approximation		Exponential approximation	
	Median $\hat{q}(\cdot)$	Gains t_{40}	Losses t_{40}	Gains t_{40}	Losses t_{40}	Gains t_{40}	Losses t_{40}
$W^{\bullet}(A_1) - \hat{q}_1$	0.10	1.51^{ns}	3.92^{***}	2.06^{*}	3.63^{***}	1.56^{ns}	3.63^{***}
$W^{\bullet}(A_2) - \hat{q}_2$	0.17	1.56^{ns}	3.63^{***}	1.40^{ns}	4.17^{***}	1.06^{ns}	4.09^{***}
$W^{\bullet}(A_5) - \hat{q}_5$	0.20	0.05^{ns}	1.79^{*}	2.20^{*}	3.18^{**}	1.99^{*}	3.20^{**}
$W^{\bullet}(A_6) - \hat{q}_6$	0.25	1.99^{*}	3.20^{**}	2.06^{*}	2.89^{**}	2.05^{*}	2.98^{**}
$W^{\bullet}(A_3) - \hat{q}_3$	0.35	1.06^{ns}	4.09^{***}	0.87^{ns}	3.06^{**}	0.92^{ns}	3.46^{***}
$W^{\bullet}(A_7) - \hat{q}_7$	0.40	2.05^{*}	2.98^{**}	0.76^{ns}	2.08^{*}	1.00^{ns}	2.29^{*}
$W^{\bullet}(A_4) - \hat{q}_4$	0.42	0.92^{ns}	3.46^{***}	-0.06^{ns}	1.64^{ns}	0.05^{ns}	1.79^{*}
$W^{\bullet}(A_{10}) - \hat{q}_{10}$	0.50	0.25^{ns}	1.20^{ns}	-1.54^{ns}	0.97^{ns}	-1.14^{ns}	1.21^{ns}
$W^{\bullet}(A_9) - \hat{q}_9$	0.55	0.27^{ns}	2.89^{**}	-0.20^{ns}	0.97^{ns}	0.25^{ns}	1.20^{ns}
$W^{\bullet}(A_8) - \hat{q}_8$	0.66	1.00^{ns}	2.29^{*}	-0.20^{ns}	2.87^{**}	0.27^{ns}	2.89^{**}
$W^{\bullet}(A_{12}) - \hat{q}_{12}$	0.72	-1.33^{ns}	1.17^{ns}	-1.44^{ns}	1.31^{ns}	-0.98^{ns}	1.43^{ns}
$W^{\bullet}(A_{11}) - \hat{q}_{11}$	0.75	-1.14^{ns}	1.21^{ns}	-1.81^{*}	1.11^{ns}	-1.33^{ns}	1.17^{ns}
$W^{\bullet}(A_{13}) - \hat{q}_{13}$	0.75	-0.98^{ns}	1.43^{ns}	-1.82^{*}	1.28^{ns}	-1.18^{ns}	1.41^{ns}
$W^{\bullet}(A_{14}) - \hat{q}_{14}$	0.85	-1.18^{ns}	1.41^{ns}	-2.12^{*}	-0.71^{ns}	-1.57^{ns}	-0.68^{ns}

Notes. *ns*: nonsignificant for $\alpha = 0.05$; *: $p < 0.05$; **: $p < 0.01$; ***: $p < 0.001$.

Figure 8 Median Decision Weights as a Function of Choice-Based Probability and Estimated Weighting Functions for Linear-in-Log-Odds (Linear Interpolation)



tion, power approximation, and exponential approximation), $p = 0.01$ for a sign test (one-tailed).

The parametric estimates resulting from the linear-in-log-odds specification can also be used to test the duality of the probability weighting function in the framework of the two-stage model. As pointed out by Abdellaoui (2000), the duality condition for the probability weighting function (i.e., $w^-(p) = 1 - w^+(1 - p)$ for all p), applied to the linear-in-log-odds specification, implies $\gamma^+ = \gamma^-$ and $\delta^+ = 1/\delta^-$. The results of the paired t tests of the equality $\gamma^+ = \gamma^-$ have already been presented above. They are confirmed by sign tests ($p = 0.76$ for linear interpolation and power approximation; $p = 0.35$ for the exponential approximation; two-tailed). Sign tests of the restriction $\delta^+ = 1/\delta^-$ lead to qualitatively similar results ($p = 0.21$ for linear interpolation and power approximation; $p = 0.12$ for the exponential approximation; two-tailed). In summary, a violation of the duality condition for the probability weighting function cannot be established at conventional levels of significance.

Table 9 Parameter Estimates of the Probability Weighting Function (Median Data)

	Linear-in-log-odds		Linear weighting	
	Elevation: δ	Curvature: γ	Elevation: $\alpha + (\beta/2)$	Curvature: β
Linear interpolation				
Gains	0.987 (0.04)	0.860 (0.04)	0.497 (0.008)	0.889 (0.03)
Losses	1.277 (0.06)	0.786 (0.05)	0.551 (0.011)	0.838 (0.04)
Power approximation				
Gains	0.975 (0.04)	0.832 (0.04)	0.495 (0.009)	0.865 (0.03)
Losses	1.345 (0.05)	0.842 (0.03)	0.561 (0.007)	0.884 (0.03)
Exponential approximation				
Gains	0.981 (0.04)	0.907 (0.05)	0.498 (0.009)	0.922 (0.04)
Losses	1.318 (0.05)	0.866 (0.04)	0.556 (0.008)	0.905 (0.03)

Note. Values in parentheses are standard errors.

6. Discussion and Conclusion

This paper provides a parameter-free and fully choice-based elicitation and decomposition of decision weights under CPT. We found that SEU is violated in a systematic fashion in both the gain and loss domains. The elicited weighting functions and the choice-based probabilities seem to be consistent with the psychological principle of diminishing sensitivity, stipulating a decrease in marginal effect as distance from a reference point increases. The reference points are 0 and 1 for $w^+(\cdot)$ and $w^-(\cdot)$, and $\varnothing$ and S for $W^+(\cdot)$, $W^-(\cdot)$, and $\hat{q}(\cdot)$. This suggests that the subjective treatment of uncertainty, in the presence of exogenously given probabilities as well as in their absence, is mainly governed by a simple psychological principle. On this point our paper extends the previous experimental findings by Tversky and Fox (1995), Fox and Tversky (1998), Wu and Gonzalez (1999), and Kilka and Weber (2001). The similarity of the properties of judged probabilities and choice-based probabilities comes as good news for the link between the psychological concept of judged probabilities and the more standard economic concept of choice-based probabilities.

The other findings regard the shape of the utility function in both gain and loss domains and the usefulness of the introduction by CPT of a specific weighting function for losses. The paper reports the results of an experimental elicitation of utility using the tradeoff method with unknown probabilities. For gains, concavity is the predominant shape of the utility function. Our results are particularly similar to those obtained under risk and without taking into account the null monetary outcome as a reference point (Wakker and Deneffe 1996, Abdellaoui 2000). For losses, no clear evidence in favor of convexity was observed, and this result is also consistent with previous findings under risk (Etchart-Vincent 2003, 2004).

Decision weights for the loss domain exhibit more elevation, particularly for likely events. When the two-stage model is assumed, the resulting probability weighting function exhibits more elevation for losses. Furthermore, the hypothesis of equal curvature across domains is not rejected. This is consistent with similar findings under risk (Abdellaoui 2000). The duality condition is not contradicted by our data, suggesting that CEU might approximate CPT in particular choice situations.

Notwithstanding the empirical parallels between judged probabilities and choice-based probabilities, the interpretation of the latter as a pure measure of belief becomes questionable in the presence of source preference as in the Ellsberg paradox (Wakker 2004, p. 238). Moreover, choice-based probabilities might be affected by an effect found in experiments by Fox and Tversky (1995), referred to as "comparative ignorance hypothesis." Fox and Tversky (1995) argue that ambiguity aversion or, more generally, source preference is an inherently comparative phenomenon that (largely) disappears when uncertain prospects contingent on a particular source are considered in isolation. The elicitation of choice-based probabilities needs to invoke a contrast between two sources of uncertainty: chance and a specific source of uncertainty. This poses the problem of the impact of the presence of a second source (i.e., chance) on the "elicited link" between choice-based probabilities and decision weights. In contrast, when judged probabilities are elicited, the subject focuses on a single source of uncertainty.

In the absence of a set of data allowing an independent elicitation of the probability weighting function under risk and a decomposition of decision weights, it is, however, hardly possible to gain a precise idea about the "descriptive performance" of a decomposition of decision weights. This suggests a possible extension of our work in future research. An elicitation of the belief component $\hat{q}(\cdot)$ for gains and losses is also desirable.

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