Learning under ambiguity:
An experiment using initial public offerings on a stock market

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Abstract

This paper studies the effect of learning new information on decision under uncertainty. Using new ambiguity theories, we show how learning affects both beliefs and ambiguity attitudes. We develop a new method to correct beliefs for ambiguity attitudes and decompose ambiguity attitudes into two components, pessimism (capturing ambiguity aversion) and likelihood insensitivity. We study the effect of learning in an experiment using initial public offerings (IPOs) on the New York Stock Exchange. IPOs provide a natural decision context in which no information on prior returns is available. We found that likelihood insensitivity decreased with information but pessimism was largely unaffected. Information made subjects move in the direction of expected utility, but significant deviations persisted. Subjective probabilities were well calibrated and close to market data, once they were corrected for ambiguity attitudes.

Keywords: ambiguity, learning, updating, neo-additive weighting.

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1. Introduction

This paper studies experimentally how decision makers change their behavior in the face of new information about uncertain events. If objective probabilities are unknown, the traditional approach in economics is to assume that the decision maker can assign subjective probabilities to events and behaves according to expected utility. In expected utility, subjective probabilities are used as decision weights and are updated using Bayes' rule.

While there is evidence that subjects do indeed update their beliefs upon the arrival of new information and that these updated beliefs do have predictive value (Hamermesh 1985, Smith et al. 2001), there is also substantial evidence showing that people deviate from Bayesianism when they receive new information.[1] This has important economic consequences. For example, a recent study by Ju and Miao (2012) showed that Bayesian learning cannot explain a variety of dynamic asset-pricing phenomena.

In a fundamental contribution, Ellsberg (1961) challenged the very existence of subjective probabilities. Ellsberg's paradox undermined not only the validity of subjective expected utility (which had already been done by Allais's paradox), but also the more general notion of probabilistic sophistication (Machina and Schmeidler 1992). In reaction to Ellsberg's paradox, new models of decision under uncertainty, called ambiguity models, have been developed (for overviews see Wakker 2010 ch.11, Gilboa and Marinacci forthcoming). Most of these ambiguity models allow for the possibility that decision weights differ from subjective probabilities. The decision weights reflect not only people's beliefs but also the confidence they have in these beliefs and their aversion towards ambiguity. The ambiguity models capture an intuition eloquently expressed by Keynes (1921):

[1] See Grether (1980), El-Gamal and Grether (1995), Charness and Levin (2005), Hoffman et al. (2011), Poinas et al. (2012). Psychologists and behavioral economists have identified a variety of updating biases, including under- and overconfidence (Griffin and Tversky 1992), conservatism (Phillips and Edwards 1966), representativeness (Kahneman and Tversky 1972), availability (Tversky and Kahneman 1973), and confirmatory bias (Rabin and Schrag 1999) and suggested heuristic decision models to account for these biases.

"The magnitude of the probability of an argument…depends upon a balance between what may be termed the favourable and the unfavourable evidence; a new piece of evidence which leaves this balance unchanged also leaves the probability of the argument unchanged. But it seems that there may be another respect in which some kind of quantitative comparison between arguments is possible. This comparison turns upon a balance, not between the favourable and the unfavourable evidence, but between the *absolute* amounts of relevant knowledge and relevant ignorance respectively" [p.71].

The ambiguity models make it possible to better understand the effects of learning on behavior. In Keynes`words, learning changes both the balance of evidence, and hence people`s beliefs, and the total amount of evidence, therefore the degree of ambiguity. Under expected utility, the latter plays no role and learning only affects beliefs. In the ambiguity models new information can change both beliefs and ambiguity attitudes. This possibility raises the question how decision weights are updated. [2] While several papers have approached this question from a theoretical angle and different updating rules have been proposed,[3] there is a dearth of empirical evidence on how decision weights are actually updated.[4] This motivated the current paper.

We study the updating of decision weights in the context of a general biseparable preference model (Miyamoto 1988, Ghirardato and Marinacci 2001) that includes most of the ambiguity models as special cases. We then analyze decision weights using Chateauneuf et al.'s (2007) neo-additive weighting function. This function makes it possible to disentangle beliefs from ambiguity attitudes, and to concisely describe a decision maker's ambiguity

[2] In the literature the expression "updating of non-Bayesian beliefs" is sometimes used. To emphasize that beliefs may differ from subjective probabilities under non-expected utility we use the term updating of decision weights.
[3] See Gilboa and Schmeidler (1993), Epstein 2006, Eichberger et al. 2007, Epstein and Schneider 2007, Hanany and Klibanoff 2007, Eichberger et al. 2010, Eichberger et al. 2012).
[4] Cohen et al. (2000) and Dominiak et al. (2012) experimentally studied updating under ambiguity but consider situations in which decision makers receive information that an event cannot occur. We focus on the case where decision makers accumulate evidence in favor or against events.

attitude in terms of two indices, one reflecting his pessimism (capturing ambiguity aversion) and the other reflecting his sensitivity to changes in likelihood. The latter term has been interpreted as a cognitive bias, and is arguably most affected by the arrival of new information .

Empirical evidence suggests that people are both pessimistic (ambiguity averse) and insensitive to likelihood (Wakker 2010). If new information makes the decision maker less pessimistic and allows him to be more sensitive to likelihood then he will move towards expected utility maximization. This is compatible with a common view in economics that learning and more information decrease irrationalities caused by deviations from expected utility (Myagkov and Plott 1997, List 2004, van de Kuilen and Wakker 2006).

We present a simple method to measure neo-additive decision weights and apply it in an experiment. Our method allows measuring beliefs without the distorting impact of ambiguity attitudes. Expected utility is still widely seen as the normative standard for decision under uncertainty. However, it is also well known that people`s decisions deviate from expected utility. There exists by now a substantial literature on correcting utility measurements for deviations from expected utility (McCord and de Neufville 1986, Wakker and Deneffe 1996, Delquié 1997, Bleichrodt et al. 2001). Our paper complements this literature by showing how the measurement of beliefs can be corrected for deviations from expected utility.

We applied our method in an experiment, where subjects traded options on the performance of (anonymous) initial public offerings (IPOs) of new stocks one month after their introduction. IPOs offer the possibility to study the effect of new information in a natural decision context (rather than in a somewhat contrived context using urns) for which no prior information is available. We distinguished three informational conditions and found that the arrival of new information reduced subjects' likelihood insensitivity, but not their pessimism. Beliefs were well-calibrated once they were corrected for ambiguity attitude. Subjects` behavior moved in the direction of expected utility when they obtained more information even though significant deviations remained.

2. Theoretical framework

Decision model

A decision maker faces uncertainty about the outcome he will receive at time T. The decision maker's uncertainty is modeled through a finite *state space* S_T where the subscript T denotes that the uncertainty will be resolved at time point T. The state space contains all possible *states of the world* s, only one of them finally occurring. The decision maker does not know which state will occur. *Events* are subsets of S_T. The decision maker chooses between *binary acts*, denoted by $x_E y$, giving money amount x if event E occurs at time T and money amount $y \leq x$ otherwise.

The decision maker's information about previous resolutions of uncertainty up to time $t < T$ is formalized by his *history set* $s^t = (s_1, \ldots, s_t)$, where $s_j \in S_j$ for all $1 \leq j \leq t$, and S_t denotes the state space representing the uncertainty at time t. Absence of information is denoted s^0.We assume that $S_t = S_T = S$ *for all* $t = 1, \ldots, T$. In other words, the same states are available at different points in time. Obviously, the decision maker`s beliefs may differ over time as a result of the arrival of new information. Consequently, the decision maker's preferences depend on the history s^t he has collected and are represented through a *history-dependent preference relation*. We will denote this preference relation by $\succcurlyeq_t$ where the subscript t indicates that preferences depend on the history s^t. The relations $\succ_t$ and $\sim_t$ are defined as usual. A real-valued function V_t *represents* $\succcurlyeq_t$ if for all binary acts $x_E y, v_F w$, $x_E y \succcurlyeq_t v_F w$ iff $V_t(x_E y) \geq V_t(v_F w)$.

The traditional Bayesian approach assumes that preferences $\succcurlyeq_t$ are represented by *expected utility*, i.e., $x_E y \mapsto P_t(E)U(x) + \big(1 - P_t(E)\big)U(y)$, with U a utility function defined over outcomes and P_t the subjective probability measure given s^t. In this model, the arrival of new information, which expands the history set from s^t to s^v, with $v > t$, is supposed to influence beliefs (subjective probabilities) but to leave utility unchanged. This assumption is common in the literature using expected utility and we will not depart from it even though we will consider generalizations of expected utility. It seems natural that updating takes place in the belief part of the representation and that "tastes" (utility) are not influenced by new information regarding past events. The assumption of a constant utility function is also common in the theoretical literature on the updating of decision weights under non-expected utility (e.g. Epstein 2006, Eichberger et al. 2007, Epstein and Schneider 2007).

To account for deviations from expected utility, we will consider *biseparable preferences* (Ghirardato and Marinacci 2001). Such preferences are very general and include many nonexpected utility models as special cases. Examples include contraction expected utility (Gajdos et al. 2008), maxmin expected utility (Gilboa and Schmeidler 1989), alpha-maxmin expected utility (Ghirardato et al. 2004), Choquet expected utility (Schmeidler 1989), and prospect theory (Tversky and Kahneman 1992). The biseparable preference model holds if $\succcurlyeq_t$ can be represented by

$$x_E y \mapsto W_t(E)U(x) + \big(1 - W_t(E)\big)U(y), \qquad (1)$$

with *U a real-valued function* unique up to level and unit and W_t a unique *weighing function,*[5] which need not be additive but satisfies $W_t(\varnothing) = 0$, $W_t(S_T) = 1$, and $W_t(A) \leq W_t(B)$ if $A \subseteq B$.

Chateauneuf et al. (2007) suggested a tractable way to analyze decision weights W_t. They introduced *neo-additive weighting*, in which decision weights are a linear function on

[5] Sometimes the term *capacity* is used instead of weighting function.

(0,1). For parameters a_t and b_t such that $a_t \leq 0$ and $a_t - 2 \leq b_t \leq 2 - a_t$, and for a probability measure P_t, neo-additive decision weights are defined as

$$
\begin{aligned}
&W_t(E) = \tfrac{a_t - b_t}{2} + (1-a_t)P_t(E) && \text{if } 0 < \tfrac{a_t - b_t}{2} + (1-a_t)P_t(E) < 1, \\
&W_t(E) = 0 && \text{if } \tfrac{a_t - b_t}{2} + (1-a_t)P_t(E) \leq 0, \text{ and} \\
&W_t(E) = 1 && \text{if } \tfrac{a_t - b_t}{2} + (1-a_t)P_t(E) \geq 1.
\end{aligned}
\tag{2}
$$

Chateauneuf et al. (2007) only considered the case $0 \leq a_t \leq 1$ and $b_t \leq a_t$. To allow for a broader range of attitudes, we do not impose this restriction. Equation (2) shows that neo-additive decision weighting assumes that the decision maker is *probabilistically sophisticated for a given history*, meaning that his decisions are consistent with a probability measure P_t. However, the possibility that a_t and b_t differ across histories implies that the decision maker does not satisfy probabilistic sophistication and Bayesianism in general and can deviate from it when comparing acts involving different histories.

Equation (1) with neo-additive weighting can be written as

$$x_E y \mapsto (1-a_t)\left[P_t(E)U(x) + \left(1 - P_t(E)\right)U(y)\right] + \tfrac{a_t - b_t}{2}U(x) + \tfrac{a_t + b_t}{2}U(y). \tag{3}$$

Equation (3) shows that the decision model we use in this paper is a linear combination of expected utility and the maximum, $U(x)$, and the minimum utility, $U(y)$. This will help us to explain the intuition underlying the parameters a_t and b_t as we will show next. We will refer to Eq. (3) as the *neo-additive model*.

Likelihood insensitivity

Equation (3) shows that the parameter a_t reflects the weight that the decision maker gives to expected utility in his evaluation of acts. If a_t is equal to 0 then the decision maker behaves according to expected utility. The larger is a_t, the less weight the decision maker gives to expected utility and the more he focuses on the maximum and minimum utility. In other words, the larger is a_t, the more the decision maker ignores the relative likelihoods of x and y. This can also be seen from Eq. (2). The larger is a_t, the less weight is given to $P_t(E)$.

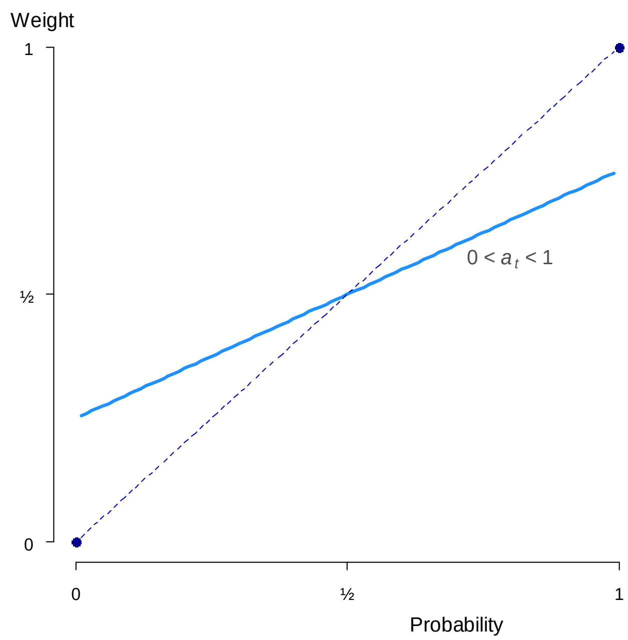


Figure 1. Likelihood insensitivity.
The figure shows the neo-additive weighting function with $a_t > 0$ and $b_t = 0$. The decision maker is insufficiently sensitive to changes in likelihood. The diagonal shows the weighting function when expected utility holds.

Figure 1 illustrates. It shows the effect of an increase in a_t when b_t is held equal to 0. When $a_t = 0$, the decision maker behaves according to expected utility (dashed line). When a_t becomes more positive the slope of the decision weighting function becomes flatter and the decision maker is less sensitive to intermediate changes in likelihood. He does not perfectly discriminate between likelihood levels and differences between (non-extreme) decision weights are less than the differences between the probabilities that they transform. The decision maker acts as if all non-extreme probabilities were close to 50%. This is called *likelihood insensitivity*. We take a_t as a *likelihood insensitivity index* with higher values of a_t corresponding with more likelihood insensitivity.

Empirical studies usually found more likelihood insensitivity for uncertainty than for risk (e.g. Kahneman and Tversky 1979, Kahn and Sarin 1988, Kilka and Weber 2001, Abdellaoui et al. 2005, Wakker 2010). There is also evidence that likelihood sensitivity is more pronounced for less familiar sources of uncertainty (Kilka and Weber 2001, Abdellaoui et al. 2011). We therefore expected that likelihood insensitivity is negatively related to the size of the history set.

Pessimism

For any a_t, Equation (3) shows that higher values of b_t imply that less weight is given to the best outcome x and, consequently, more to the worse outcome y. Figure 2 illustrates. It shows that for a given value of a_t, increases in b_t shift the weighting functions downwards and, because the decision weights reflect the weight given to the best outcome, increase the focus on the worse outcome. We will interpret b_t as an index of *pessimism* with higher values indicating more pessimism. An expected utility maximizer satisfies $b_t = 0$ and an extremely pessimistic decision maker who only considers the worst outcome regardless of its likelihood has $b_t = 1$. Negative values of b_t correspond with optimism and $b_t = -1$ for an extremely optimistic decision maker, who only considers the best outcome.

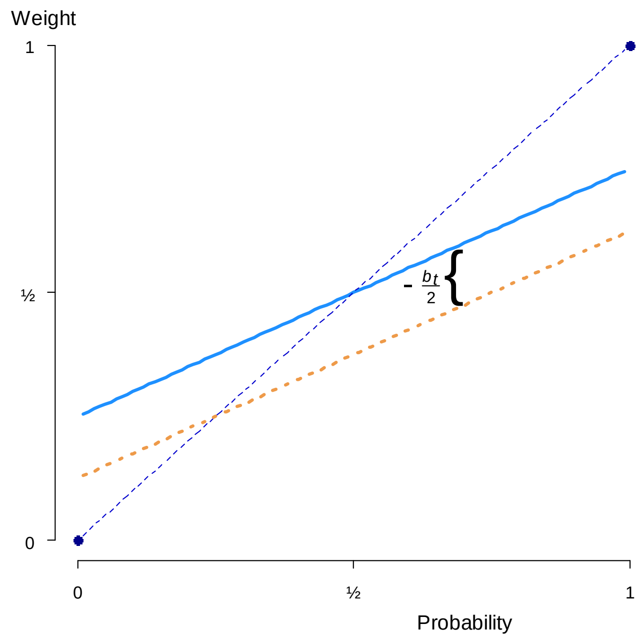


Figure 2. Pessimism.
The blue line corresponds to $a_t > 0$ and $b_t = 0$. The parallel dashed line keep a_t constant and increases b_t. The figure shows that increases in b_t shift the neo-additive weighting function downwards leading to an increase in pessimism.

Several studies found that pessimism decreased for more familiar sources of uncertainty, i.e. for sources about which the decision maker was more knowledgeable (Heath and Tversky 1991, Kilka and Weber 2001, Fox and Weber 2002, Di Mauro 2008, and Abdellaoui et al. 2011). Hence, these results suggest a decrease in pessimism when the history set becomes richer.

The impact of new information on beliefs on the one hand, and on likelihood sensitivity and pessimism on the other hand, illustrates how modern ambiguity theories can capture Keynes' (1921) intuition about the weight and the balance of evidence. If new information changes the balance of evidence in favor of an event, the decision maker will update his beliefs accordingly. But this new information also changes the balance between the "absolute amounts of relevant evidence and relevant ignorance." Our approach captures this by allowing the decision maker to also update his weighting of subjective probabilities.

The new information might induce the decision maker to rely more on his beliefs and become more sensitive to likelihood, with a_t tending to 0. In the next Section we will present a method to disentangle beliefs, pessimism, and likelihood insensitivity and to obtain beliefs that are corrected for the distorting impact of ambiguity attitudes.

3. Measuring the neo-additive model

We now explain how we identified a_t and b_t for different histories s^t. For each history s^t, we considered a three event partition of the state space *S*. In the experiment reported in the next Section, the events referred the change in the price of stocks on a specific trading day and the events were *Up*, the price goes up by at least 0.5%, *Middle,* the price varies by less than 0.5%, and *Down*, the price decreases by at least 0.5%. We also considered the event *MiddleUp* = *Middle* ∪ *Up*. For given $x > y$, we then elicited four certainty equivalents, $CE_{Up} \sim x_{Up}y$, $CE_{Middle} \sim x_{Middle}y$, $CE_{Down} \sim x_{Down}y$, and $CE_{MiddleUp} \sim x_{MiddleUp}y$. With the normalization $U(x) = 1$ and $U(y) = 0$, Eq. 1 implies $U(CE_{Up}) = W_t(Up)$, $U(CE_{Middle}) = W_t(Middle)$, $U(CE_{Down}) = W_t(Down)$, and $U(CE_{MiddleUp}) = W_t(MiddleUp)$. If the decision maker maximizes expected utility then his decision weights are equal to his subjective probabilities and, consequently, his subjective probabilities are equal to the utilities of his certainty equivalents.

Under expected utility, $U(CE_{MiddleUp}) + U(CE_{Down}) = P_t(MiddleUp) + P_t(Down) = 1$. The utilities of the complementary events *MiddleUp* and *Down* should sum to 1. We will refer to this as *complementarity*. The neo-additive model allows for violations of complementarity:

$$U(CE_{MiddleUp}) + U(CE_{Down})$$
$$= \frac{a_t - b_t}{2} + (1 - a_t)P_t(MiddleUp) + \frac{a_t - b_t}{2} + (1 - a_t)P_t(Down) = 1 - b_t. \quad (4)$$

Equation (4) shows that the more pessimistic the decision maker is, the lower will be the sum of $U(CE_{MiddleUp})$ and $U(CE_{Down})$. Hence, studying deviations of $U(CE_{MiddleUp}) + U(CE_{Down})$ from 1 allow us to identify the decision maker's degree of pessimism.

Under expected utility, the decision maker should also satisfy binary additivity: $U(CE_{Up}) + U(CE_{Middle}) - U(CE_{MiddleUp}) = P_t(Up) + P_t(Middle) - P_t(MiddleUp) = 0$. Under the neo-additive model, we obtain,

$$U(CE_{Up}) + U(CE_{Middle}) - U(CE_{MiddleUp}) = \frac{a_t - b_t}{2}. \tag{5}$$

As we know b_t from the test of complementarity, a_t can be uniquely identified.

Two points are worth making. First, the neo-additive model allows us to directly measure likelihood insensitivity and pessimism for any events, provided we can partition the state space *S* into three events and if we can measure utility. To measure utility we used the method of Abdellaoui et al. (2008), which we will explain below. Second, once we know a_t, b_t, and utility, we can also determine P_t. Assuming expected utility would give violations of additivity in the measurement of P_t. Our method filters out the non-additive part and hence, corrects P_t for ambiguity attitude.

4. Experiment

Subjects

The experiment was run at Erasmus University with 64 subjects (22 female) with a background in finance. Subjects were either third year undergraduate students majoring in finance or graduate students in finance. Their average age was 24, ranging from 21 to 33. We deliberately selected students from finance because the experimental questions involved options and we hoped that finance students would find the experimental tasks easier to understand and would be well motivated to answer the questions. Each subject received a show-up fee of €5 and in addition each subject played out one of his choices for real using a procedure described below.

Procedure

The experiment was computer-run in small group sessions involving at most 3 subjects. Subjects first received instructions and were asked to answer several questions to check their understanding of the experimental tasks. Subjects could only proceed to the actual experiment after they had answered these questions correctly. The experimental instructions including the questions to check for subjects' understanding are in Appendix B.

The source of uncertainty that we used concerned the variation in the stock returns of IPOs (Initial Public Offerings) traded at the New York Stock Exchange (NYSE). IPOs are stocks that have just entered the market. We chose IPOs for two reasons. First, stock returns are a natural source of uncertainty unlike, for example, Ellsberg urns. Second, because IPOs are new on the market, there is no previous history of prices available and learning can occur.

We used data on 328 IPOs in total. All stocks were listed on the NYSE between 1 September 2009 and 25 February 2011. At the start of the experiment, each subject drew 4 numbers, which determined the stocks he would trade in. Subjects did not know which these stocks were. This was only revealed after subjects had completed the experiment. Then we also explained subjects how they could verify the stock data on the internet.

Payoffs were determined by the performance of the stocks on the 21st trading day after their introduction on the NYSE. We defined four events: Up = (0.5, →), i.e. the stock goes up by more than 0.5% on the 21st trading day, Middle = [–0.5, 0.5], the stock varies by at most 0.5% on the 21st trading day, Down = (←, –0.5), the stock goes down by more than 0.5% on the 21st trading day, and MiddleUp = (–0.5, →), the stock goes up by more than –0.5% on the 21st trading day. In what follows, we will refer to an option that pays x if event Up obtains as an *Up -option*. *Middle-, Down-*, and *MiddleUp options* are defined in a similar way. We used the variation in the stock returns rather than the absolute prices of the stocks to make sure subjects had no information about the stocks and to avoid biases. Stocks with higher prices might attract more attention leading to biases in the elicited ambiguity attitudes.

There were three *informational conditions*, each involving a different history set. In the *no information* condition (history set s^0), subjects had no information about the underlying stock. In the *one week* condition (history set s^5), subjects were informed about the daily returns of the stock in the first 5 trading days following its introduction. Finally, in the *one month* condition (history set s^{20}), subjects were informed about the performance of the stock in the first 20 trading days following its introduction.

Table 1: The 20 choice questions

Stock	***Condition***	***y***	***x***	***Option type***		***Stock***	***Condition***	***y***	***x***	***Option type***
1	No info	0	10	Up		3	1 week	0	20	Up
1	No info	10	20	Up		3	1 week	0	20	Middle
1	No info	5	20	Up		3	1 week	0	20	Down
1	No info	10	15	Up		3	1 week	0	20	MiddleUp
1	No info	0	5	Up		3	1 week	0	20	Middle
1	No info	0	20	Up		4	1 month	0	20	Up
2	No info	0	20	Up		4	1 month	0	20	Middle
2	No info	0	20	Middle		4	1 month	0	20	Down
2	No info	0	20	Down		4	1 month	0	20	MiddleUp
2	No info	0	20	MiddleUp		4	1 month	0	20	Down

The columns labeled “Stock” refer to the four different stocks. The questions for stock 1 served to measure utility. The columns labeled “Condition” refer to the amount of information subjects received about the performance of the stock. Options were of the type $x_E y$ where the subject received €x if event E occurred and €y otherwise The columns “Option types” indicate event E.

We elicited the ask prices of 20 options, summarized in Table 1. The ask prices were determined through choice lists. Figure 3 gives an example of a choice list for a Middle-Up option. Subjects were told that they owned the option $x_E y$ and they were asked for each price on the choice list whether they wanted to sell the option. The choice lists consisted of 20 prices ranging from € (y + z) to €x in increments of z = € $\frac{x-y}{20}$. The options corresponded with the stocks subjects had drawn at the start of the experiment. We used choice lists to determine certainty equivalents because previous research suggests that choice-based procedures lead to fewer inconsistencies than directly asking subjects for their certainty equivalents (Bostic et al. 1990, Noussair et al. 2004).

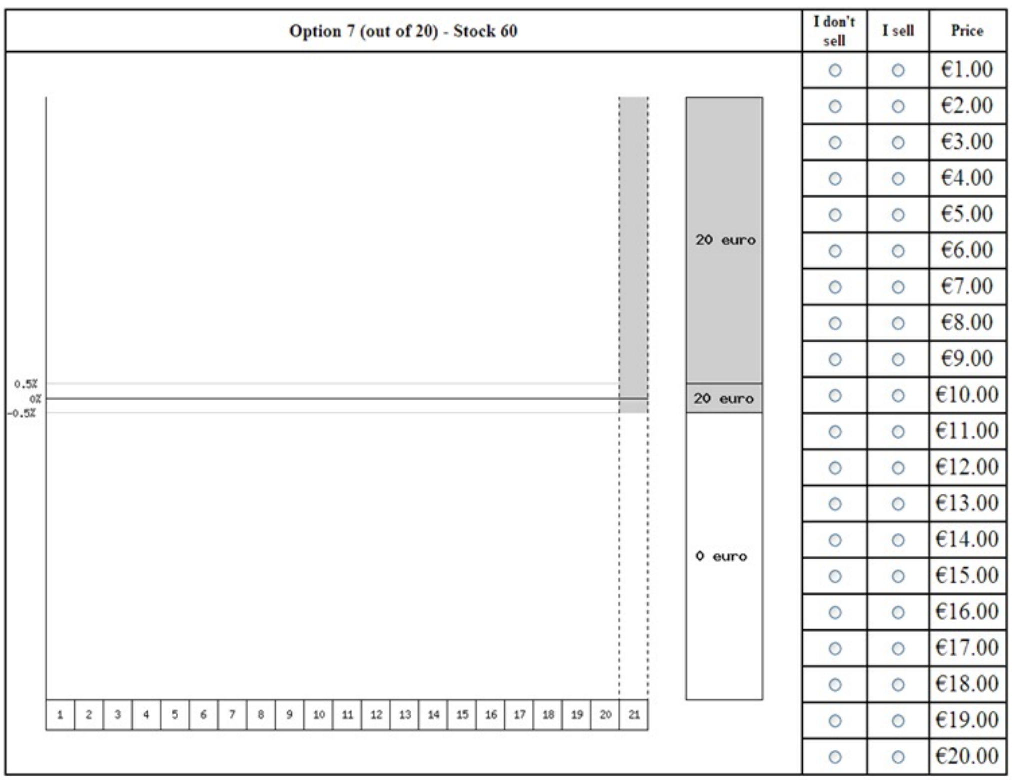


Figure 3. The choice lists subjects faced during the experiment. In this example the option pays €20 if event Middle-Up occurs on the 21st trading day following the introduction of the stock and €0 otherwise.

The 20 choices were divided into four groups (see Table 1). Group 1 consisted of 6 choices to determine utility. The questions in groups 2, 3, and 4 measured the effect of new information on ambiguity attitudes. For groups 3 and 4, we repeated one measurement to gain insight into the reliability of our measurements.

The utility questions (group 1) always came first. The order of the other groups was randomized to avoid that the effect of new information was confounded by a better understanding of the task. Accordingly, we had to use different stocks in each group. If we had used options on the same underlying stock, then subjects who had, for instance, received information on the performance of the stock in the first month would have used this information in determining their ask prices in the no information and in the one week conditions. We also randomized the order in which subjects faced the different options within each group.

Incentives

We used a random incentive system to incentivize the experiment. At the end of the experiment, subjects twice threw a twenty-sided die. The first throw selected the choice list and the second throw selected the line of that list to be played out for real. In the selected line, we implemented the choice that the subject had made during the experiment. So if the subject had chosen to sell, we paid him the price. If he had chosen not to sell, we played out the option $x_E y$ and he received €x if event E had occurred on the 21^{st} trading day and €y otherwise.

Analysis

All analyses were based on the original measurements. The repeated measurements were only used to test for the consistency of responses. For a given history, we deleted those subjects for whom the certainty equivalent of the act $20_{MiddleUp}0$ was less than the maximum of the certainty equivalents of the events $20_{Up}0$ and $20_{Middle}0$ minus €1. These subjects violated monotonicity. The €1 margin was applied to account for the fact that preferences are typically imprecise, taking into account that €1 was approximately the median absolute deviation in the consistency tests. This led to the exclusion of 3 subjects in the no information condition, 8 in the one week condition, and 9 in the one month condition. In the paired comparisons between any two conditions, subjects who violated monotonicity in at least one of the two conditions were excluded. To test for robustness we also analyzed the data excluding all subjects who violated monotonicity at least once. The results were similar.

We first analyzed the data under subjective expected utility and under the neo-additive model both assuming deterministic and assuming stochastic preferences. Utility was measured using the method of Abdellaoui et al. (2008). We fixed event Up and history s^0 and elicited certainty equivalents CE_j for the six binary acts $x_{j_{Up}}y_j, j = 1, \ldots, 6$, stated as the first entries of Table 1. By the neo-additive model:

$$U(CE_j) = W_0(Up)U(x_j) + (1 - W_0(Up))U(y_j).^6 \tag{6}$$

We assumed a power utility function, i.e., $U(x) = x^\beta$ if $\beta > 0$, $U(x) = \ln(x)$ if $\beta = 0$, and $U(x) = -x^\beta$ if $\beta < 0$. The power family is widely-used in economics and decision theory and generally gives a good fit (Stott 2006).

Nonlinear least squares was used to estimate $W_0(Up)$ and β in (6). Once we knew β, we could substitute it in Equations (4) and (5) to derive the neo-additive parameters a_t and b_t and the subjective probabilities. Dividing all money amounts by the maximum payoff €20 scales the power utility function such that $U(20) = 1$ and $U(0) = 0$.

To account for the stochastic nature of subjects' preferences, we also estimated the various parameters using structural maximum likelihood estimation. Let

[6] Under subjective expected utility $W_0(Up) = P_0(Up)$.

$\theta = (a_0, b_0, \Delta a_{1week}, \Delta b_{1week}, \Delta a_{1month}, \Delta b_{1month}, P_t(Up), P_t(Middle), \sigma, \varepsilon)$ denote the vector of estimated model parameters for t = 0, 1 week, and 1 month. For *t=1 week* and *t=1 month*, we estimated the differences in likelihood insensitivity and pessimism compared with the no-information condition: $\Delta a_t = a_t - a_0$ and $\Delta b_t = b_t - b_0$. The parameter σ denotes error and ε denotes a tremble. Following Hey and Orme (1994), we assumed a Fechner error specification, which is widely used in stochastic choice under risk (e.g. Bruhin et al. 2010, Conte et al. 2011). Let $y_{ijtr} = 1(-1)$ denotes the binary indicator that subject *i* chose to keep (sell) the option *j* with history *t* for binary choice *r*. The likelihood contribution for subject *i* facing option *j* with history *t* in choice *r* is:

$$l(y_{ijtr}|\theta,\sigma,\varepsilon) = (1-\ \varepsilon)\Phi\left(\ y_{ijtr}\frac{W_t(E_{jt})U(x_{jt}) + \left(1-W_t(E_{jt})\right)U(y_{jt}) - U(y_{jt}+(r-1)z_{jt})}{\sigma}\right) + \varepsilon/2$$

where Φ denotes the density of the standard normal distribution and *E=Up, Middle*. The overall likelihood is the product of l over all subjects, information conditions and choice lists. We performed two maximum likelihood estimations, one with the same subjects as the ma

5. Results

5.1. Consistency

Consistency was good. We observed no significant differences in ask prices in the two questions that were repeated and the correlations between the ask prices were substantial (0.86 and 0.81, both $p < 0.01$). The mean absolute differences between the ask prices were 1.09 and 1.00 in the two questions.

A comparison between the ask prices of the option 20_U0 for stocks 1 and 2 (see Table 1) yields another consistency test. In both cases the subjects had no information about the stocks and we might expect that they treated them similarly, even though they involved different underlying stocks. We indeed observed no differences between the elicited ask prices and the correlation, although lower than in the other consistency tests, was still high and clearly different from 0 ($\rho = 0.52$, $p < 0.01$). The mean absolute error was equal to 1.53.

5.2. Subjective expected utility

Appendix A shows the median ask prices under the three informational conditions. If we assume expected utility, the subjective probabilities for the events were equal to $(\frac{CE_j}{20})^{\beta}$ where β is the power coefficient obtained in the estimation of utility. Overall, there was little utility curvature, which is consistent with the hypothesis that utility is approximately linear for small stakes (Wakker 2010). The median power coefficient was equal to 1 (mean 1.41, interquartile range = [0.82, 1.25]) and the proportion of subjects with concave utility did not differ from the proportion of subjects with convex utility.

Under expected utility, the subjective probabilities should satisfy complementarity and binary additivity, as discussed in Section 3. Panel A of Figure 4 shows that complementarity held in general. We could not reject the hypothesis that *P(MiddleUp)* + *P(Down)* = 100% for all three information conditions. Moreover, we could not reject the hypothesis that the proportion of subjects for whom the sum of *P(MiddleUp)* and *P(Down)* exceeded 100% and the proportion for whom this sum was less than 100% were the same.

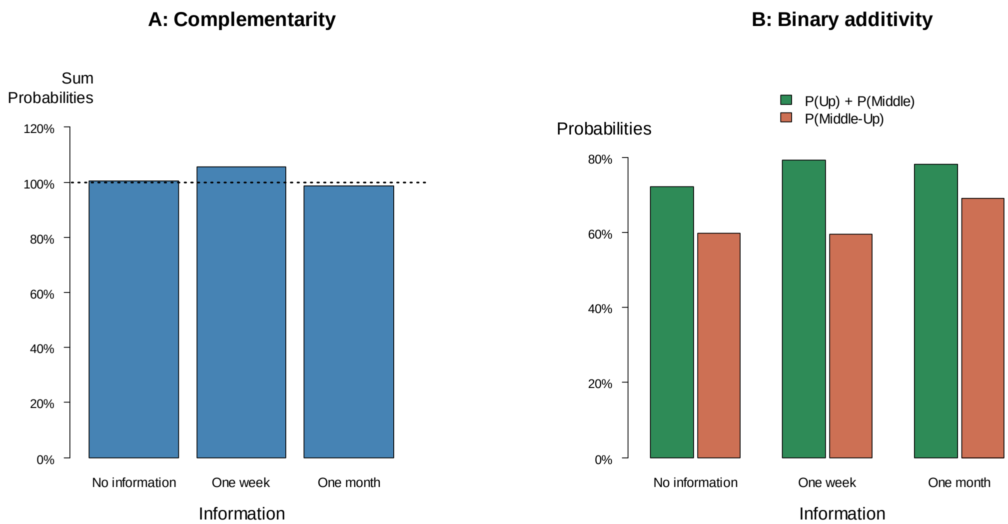


Figure 4. Tests of complementarity and binary additivity under subjective expected utility. Panel A shows that complementarity (*P(Down)* + *P(MiddleUp)* = 100%) held approximately. Panel B shows that binary additivity (*P(Up)* + *P(Middle)* = *P(MiddleUp)* was violated.

However, Panel B shows that binary additivity was clearly violated. The sum of *P(Up)* and *P(Middle)* exceeded *P(MiddleUp)* in all three conditions suggesting binary subaditivity instead of binary additivity (all $p < 0.01$). The individual analyses also showed binary subadditivity: the proportion of subjects who behaved according to binary subadditivity was significantly higher than the proportion of subjects displaying binary superadditivity in all three conditions (all $p < 0.01$).

Figure 4B also shows that the violations of binary additivity were less pronounced in the one month condition than in the other two conditions. Indeed, $P_t(Up) + P_t(Middle) - P_t(MiddleUp)$ was lower for the one month condition than the no information condition (p = 0.05) and than the one week condition ($p < 0.01$). This suggests that deviations from expected utility decreased with the provision of more information.

The dual findings of complementarity and binary subadditivity is in line with previous evidence, both based on introspective probability judgments (Tversky and Koehler 1994, Fox and Tversky 1998, Kilka and Weber 2001) and based on choice data (Baillon and Bleichrodt

2012). They are consistent with the predictions of support theory, a psychological theory of the formation of subjective probabilities (Tversky and Koehler 1994).

In summary, our data indicated violations of subjective expected utility. There was some evidence that these violations became less pronounced with the level of information, in particular when we compared the one month condition with the other two conditions.

5.3. Neo-additive model

Subjects whose derived subjective probabilities fell outside the unit interval did not behave in line with the neo-additive model and were therefore removed from the analyses. We only removed these subjects from the informational condition for which they violated the neo-additive model, but not for the other conditions. This left 54 subjects in the no information condition, 50 subjects in the one week condition, and 46 subjects in the one month condition.

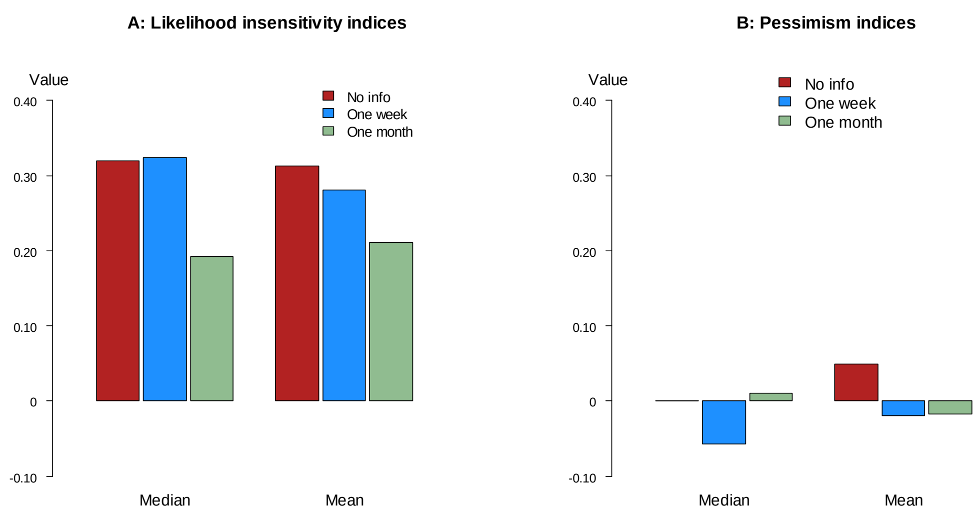


Figure 5. The likelihood insensitivity and pessimism indices. Panels A and B show the medians and means of the likelihood insensitivity and pessimism indices for the three informational conditions. Likelihood insensitivity falls with more information, but there is no clear effect on the pessimism indices.

Likelihood insensitivity

Figure 5A shows the likelihood insensitivity indices for the three conditions. All indices differed from zero ($p < 0.01$ in all three tests). They declined with the level of information, with the median decreasing from 0.32 in the no information case to 0.19 in the one month case and the mean decreasing from 0.31 to 0.21. The likelihood insensitivity index for one month was smaller than the index for no information ($p = 0.04$) and marginally smaller than the index for one week ($p = 0.08$). The indices for no information and for one week did not differ.

Figure 6 displays the relationships between the individual values of the likelihood insensitivity (LIS) indices for the three informational conditions. The horizontal axis always shows the condition in which less information was available and the diagonal corresponds with the case where the likelihood insensitivity indices were equal across informational conditions. If likelihood insensitivity decreased with the amount of information, then subjects should be located under the diagonal. This is not the case in Panel A, which depicts the relationship between the likelihood insensitivity indices for no information and the likelihood insensitivity indices for one week. However, it is true in Panels B and C, confirming that likelihood insensitivity was lower in the one month condition than in the other two informational conditions.

Figure 6 also shows that a few subjects had negative likelihood insensitivity indices, suggesting that they were overly sensitive to likelihood information. For nearly all these subjects, their oversensitivity decreased upon the arrival of new information.

The overall picture that emerges from Figure 6 is that subjects moved in the direction of "correct" sensitivity to likelihood when they received more information. The shaded areas illustrate this. Subjects located in the shaded areas moved in the direction of correct sensitivity: their likelihood insenstitivity or likelihood oversensitity decreased but they did not overshoot in the sense that they went from insensitivity to even larger oversensitivity or from oversensitivity to even larger insensitivity. While a sizeable fraction of subjects is located outside the shaded area in Panel A, in Panels B and C most points are indeed in the shaded area suggesting convergence towards expected utility.

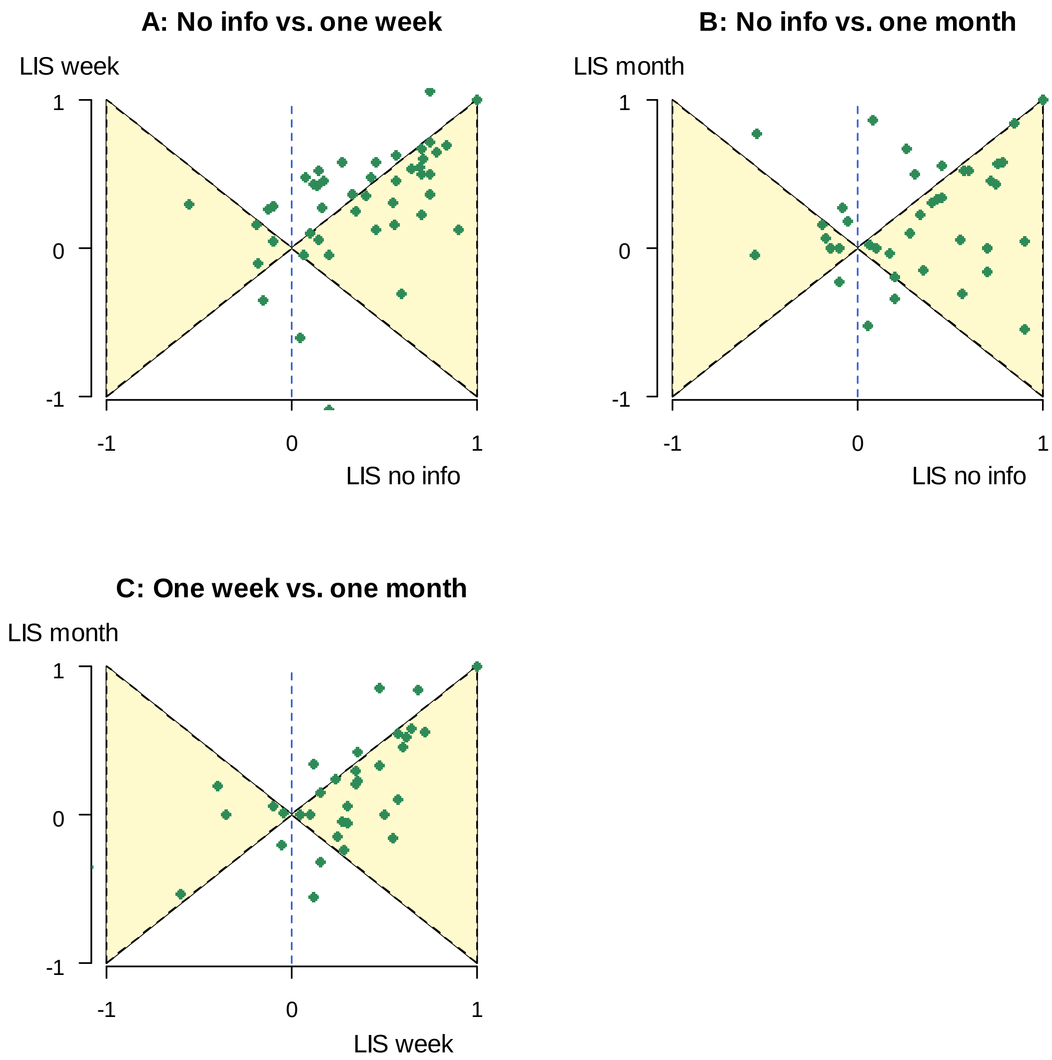


Figure 6. The relationships between the individual likelihood insensitivity (a_t) indices. If subjects converge to expected utility then the points should lie in the shaded areas. This is the case in Panels B and C.

The correlations between the likelihood insensitivity indices were fair to moderate: Spearman correlation was 0.69 between no information and one week, 0.28 between no information and one month, and 0.58 between one week and one month.

Pessimism

Figure 5B shows the median and mean values of the pessimism indices for the three informational conditions. The median values were slightly negative, indicating optimism. This is perhaps not surprising given that our subjects were students from finance and hence knowledgeable about options. Previous studies found that subjects tended to be more ambiguity seeking in domains where they feel competent (Heath and Tversky 1991, Kilka and Weber 2001). For the no information condition, the mean was positive indicating that some subjects were strongly pessimistic under that condition. No pessimism index differed from zero. Figure 5B suggests a tendency towards more optimism for the one week condition, but only the difference between the no information and the one week conditions was significant ($p < 0.01$).

Figure 7 shows the individual data. The figure plots the pessimism (b_t) indices for different informational conditions against each other. The condition with less information is always plotted on the horizontal axis. The diagonal corresponds with the case where pessimism indices are equal across informational conditions.

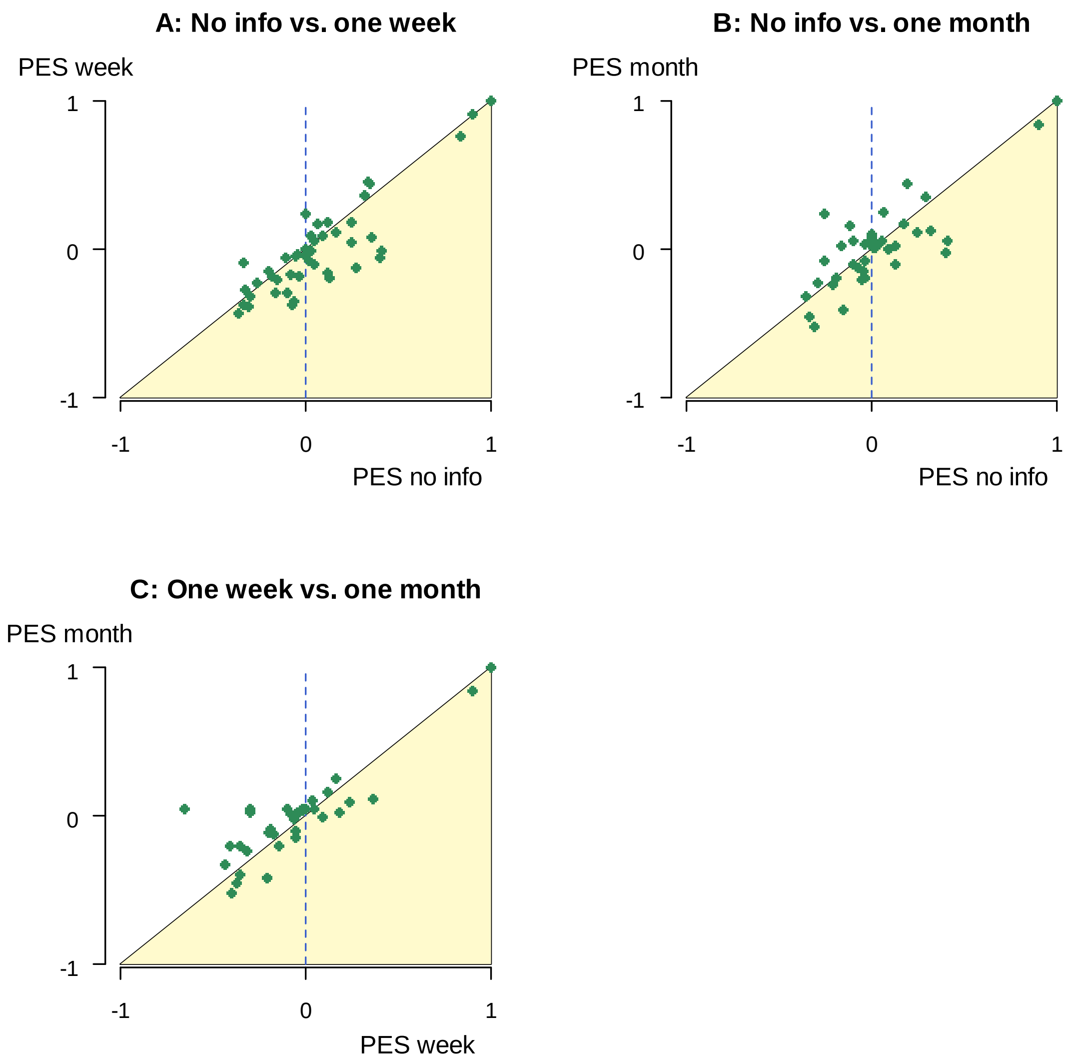


Figure 7. The relationships between the individual pessimism (b_t) indices. If pessimism falls with more information then the points should be located in the shaded areas.

If pessimism decreases across conditions then individual points should be concentrated in the shaded lower halves of the figures. There was no evidence for this. If differences existed then they were small. The figure conveys the impression that pessimism was a stable trait of individuals as the points were clustered around the diagonal. The correlations between the pessimism indices were substantial Spearman correlation was 0.79 between the no information and the one week conditions, 0.68 between the no information and one month conditions, and 0.77 between the one week and the one month conditions. They were also higher than the correlations between the likelihood insensitivity indices, suggesting that likelihood insensitivity was less stable than pessimism and that it was more strongly affected by the arrival of new information.

5.4. Neo-additive decision weights with stochastic preferences

Table 2 summarizes the results of the maximum likelihood estimation. We estimated two models. The first model was based on the same dataset we used in the previous analysis and excluded individual choices that violated monotonicity. The second model used all individual choices, including those that violated monotonicity.

The results of the maximum likelihood estimation were largely similar to those of the individual analyses. Likelihood insensitivity was similar for the no information and the one week conditions were similar, but it decreased in the one month condition. This effect was present in both models, but it was stronger in model 1. Our subjects were slightly optimistic (ambiguity seeking), particularly so in the one week condition.

As in the individual analysis, subjects moved in the direction of expected utility upon the arrival of information. On the other hand, the null that the parameters for the one month condition were equal to the expected utility parameters could clearly be rejected. Both likelihood insensitivity and pessimism differed from zero at the 1% significance level.

Table 2: Maximum likelihood estimations

		Model 1	Model 2
Violations of monotonicity included		No	Yes
a (likelihood insensititivity)		0.353***	0.384***
		(0.032)	(0.033)
	(one week effect)	−0.058	−0.032
		(0.046)	(0.047)
	(one month effect)	−0.172***	−0.089*
		(0.047)	(0.048)
b (pessimism)		−0.078***	−0.079***
		(0.018)	(0.019)
	(one week effect)	−0.064***	−0.052***
		(0.015)	(0.016)
	(one month effect)	−0.018	−0.004
		(0.016)	(0.016)
P(Up)	No info	0.394***	0.378***
		(0.01)	(0.011)
	1 week	0.334***	0.355***
		(0.009)	(0.011)
	1 month	0.333***	0.333***
		(0.008)	(0.01)
P(Middle)	No info	0.269***	0.283***
		(0.009)	(0.01)
	1 week	0.289***	0.278***
		(0.009)	(0.01)
	1 month	0.357***	0.365***
		(0.009)	(0.011)
Utility		0.899***	0.888***
		(0.022)	(0.022)
Noise		0.097***	0.098***
		(0.003)	(0.003)
Tremble		0.047***	0.073***
		(0.004)	(0.005)
Log-likelihood		−6550.2	−7661
N		128640	138240

Standard errors in parentheses. ***: significant At 1%, **, significant at 5%, * significant at 10%.

In the theoretical literature, at least three distinct methods for updating decision weights have been suggested (Eichberger et al. 2010, Gilboa and Marinacci forthcoming): optimistic updating (Gilboa and Schmeidler 1993), the Dempster-Shafer rule which corresponds with pessimistic updating (Dempster 1968, Shafer 1976), and full or generalized Bayesian updating (Walley 1991, Jaffray 1992, Chateauneuf et al. 2011). Eichberger et al. (2010) derived how the parameters of the neo additive model should be updated under each of these three approaches. Optimistic updating predicts that the ratio $\frac{a_t - b_t}{2a_t}$ will go to 1, whereas pessimistic updating predicts that $\frac{a_t + b_t}{2a_t}$ will go to 1. We found no support for these predictions. On the other hand, full Bayesian updating predicts that $\frac{a_t - b_t}{2a_t}$ will remain constant. This ratio was equal to 0.46, 0.54, and 0.45 for the o information, one week, and one month conditions, respectively. Hence, our results seem to offer some support for full Bayesian updating over the other two updating rules.

Subjective probabilities

Table 2 also shows subjects' beliefs about the different events. There is a tendency for the P(Up) and P(Down) to decrease with more information and P(Middle) to increase.[7] This latter finding is consistent with our prior expectation. Subjects might underestimate the probability of Middle as the event Middle covered the smallest portion of the bar depicting the events (see Figure 3), perhaps suggesting that this event was relatively unlikely. The arrival of new information might have led to the realization that the event Middle was more likely than initially thought and, consequently, to the updating of its subjective probability.

The elicited probabilities were well-calibrated. For each day from their introduction to the 21st trading day we computed the proportion of the 328 IPOs that were included in our study that went up by more than 0.5% (corresponding with the event Up), the proportion that varied by at most 0.5% (corresponding with the event Middle), and the proportion that went down by more than 0.5% (corresponding with the event Down). A frequentist can interpret these proportions as the actual probabilities of the events Up, Middle, and Down at each date t in the history.

[7] Most differences are significant (all $p < 0.01$) except for the difference between P(Up) in the one week and the one month condition, between P(Middle) in the one week and the no information condition, and between P(Down) in the no information and the one month condition.

Figure 8 shows the results of this analysis. Panel A shows the proportions for the event Up, Panel B for the event Middle, and Panel C for the event Down. The figures also shows the estimated median probabilities of *P(Up)*, *P(Middle)*, and *P(Down)* for our subjects and for the three informational conditions. These are displayed as dots at the end of the line.

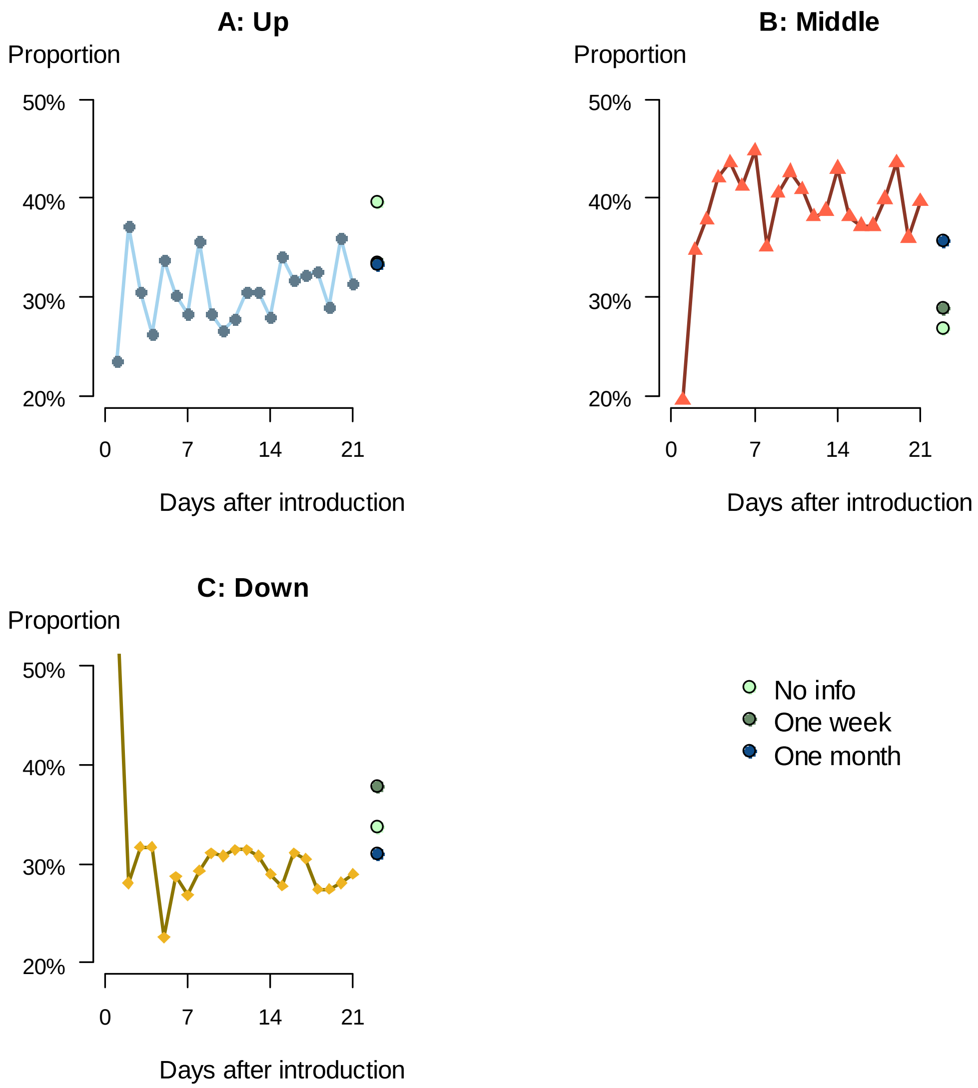


Figure 8. Stock history and beliefs. Panel A shows the proportion of the 328 IPOs in our database that went up by more than 0.5% on each trading day from their introduction to the 21st trading day. Panel B and C show the proportions that varied by at most 0.5% and went down by more than 0.5%, respectively. The dots at the end show the estimated probabilities of *P(Up)* (Panel A), *P(Middle)* (Panel B), and *P(Down)* (Panel C) under the various informational conditions (in Panel A the points for one week and one month overlap).

All subjective probabilities converged to the actual probabilities. Panels A and C show that subjects overestimated the probabilities of the events Up and Down. Upon the arrival of information, they adjusted their estimates downwards although a slight overestimation persisted. On the other hand, subjects clearly underestimated the probability that the event Middle would occur. The underestimation was reduced with information, particularly in the one month condition, but some underestimation remained.

In summary, Figure 8 shows that the representative subject tended to overestimate the frequency of the more extreme events Up and Down and to underestimate the frequency of the intermediate event Middle. The over- and underestimation was least for the one month condition, suggesting that subjects adjusted their probabilities with the arrival of new information and did so in the correct direction. The reason that the adjustment was not very good for the one week condition might be due to the fact that the returns of IPOs are highly volatile in the first few trading days and the information provided for the first week was therefore not really informative. The final probabilities were well-calibrated and close to the aggregate frequencies in the market.

6. Discussion

In this paper, we have explored the effect of learning new information on ambiguity attitudes using an easily implementable procedure that allows correcting beliefs for likelihood insensitivity and pessimism. We observed that likelihood insensitivity decreased with more information. In the no information and the one week conditions, we found substantial likelihood insensitivity, even though we used experienced subjects. In the one month condition, likelihood insensitivity was approximately halved. What is more, we noticed that subjects did not overshoot (i.e. going from likelihood insensitivity to likelihood oversensitivity), but went in the direction of correct sensitivity to likelihood information and converged to expected utility. In the literature likelihood insensitivity is often regarded as a cognitive bias. Our findings suggest that this cognitive bias is reduced with more information.

We found little effect of new information on pessimism and the correlations between the pessimism indices were high for the three informational conditions. Our data suggest that pessimism is a stable trait of decision makers, not affected much by information received. This finding is consistent with the suggestion that pessimism reflects the motivational part of ambiguity attitude (Wakker 2010). If people are naturally inclined to be pessimistic or optimistic, then new information is unlikely to change this inclination.

We observed little pessimism and in the one week condition even some optimism. This could be caused by the fact that the outcomes in our experiment were options on the performance of stocks and our subjects were students in finance who were supposedly knowledgeable about stocks. It is well known that ambiguity aversion tends to decrease and even turns into ambiguity seeking when subjects feel competent about the outcomes of the acts.

Another reason for the low amount of pessimism that we found could be that we used ask prices to determine the certainty equivalents of options, Ask prices, which reflect willingness to accept, can lead to endowment effects (see e.g. Kahneman, Knetsch, and Thaler 1991) and, consequently, to an overestimation of the certainty equivalents of the options. This would be consistent with the tendency towards optimism that we observed (see also Trautmann et al. 2011). Note that endowment effects have a similar impact on the different history sets and, hence, they do not impact our main conclusions about the effect of learning on ambiguity attitudes and beliefs.

The joint finding that pessimism was close to zero and that the likelihood insensitivity index moved towards zero upon the arrival of new information suggests that our subjects converged towards expected utility upon the receipt of information. The finding that people behaved more rationally, i.e. consistently with expected utility, when they received new information is in agreement with findings in the literature that experience and learning reduce biases. On the other hand, the weighting functions differed significantly from expected utility, even in the one month condition. Moreover, we found that the subjective probabilities violated complementarity and binary additivity when we assumed expected utility. Even though our subjects moved in the direction of expected utility, important deviations remained.

Utility curvature had little impact in our analysis; we conducted further robustness checks (not reported) assuming linear probability and obtained similar results. If utility is linear, subjective probabilities and the neo-additive decision weights can be computed directly from the raw data without the need for any econometric estimation. The similarity in results suggests that this method is reliable, at least for the stakes that we used. It can easily be implemented and any inconsistencies can immediately be traced back to the original data. This is an important advantage for practical applications that seek to correct beliefs from ambiguity attitudes and cognitive biases. The similarity in results also suggests that utility curvature played only a minor role in explaining attitudes towards uncertainty.

Our results were robust to the method of analysis used. In the paper we reported two ways of analyzing the data, one deterministic but allowing for some preference imprecision and the other stochastic, assuming a Fechner error structure. In addition to these two methods we tried several other approaches, including interval arithmetics computation to account for imprecision in preferences. The results of these additional analyses were similar providing confidence in the robustness of the conclusions.

Let us end by pointing out some caveats. We made several assumptions throughout the paper. First, we assumed that utility was constant for different levels of information. As explained, this assumption is common in the literature on updating and we believe it is reasonable. New information provides the decision maker with information about the environment, the subjective perception of which is reflected by his subjective probabilities or decision weights, and not about his preferences over outcomes, which are reflected by his utility function. Behavior vis-à-vis uncertain acts was primarily driven by the weighting function. Abdellaoui et al. (2011) measured utility for different sources of uncertainty and could not reject the null hypothesis that utility was the same across sources.

A second and more controversial assumption is that probabilistic sophistication holds within histories and, hence, that subjective probabilities can be defined. Different histories can be interpreted as different sources of uncertainty. The notion of sources of uncertainty was first proposed by Amos Tversky in the 1990s (Tversky and Kahneman 1992, Tversky and Fox 1995, Tversky and Wakker 1995). Chew and Sagi (2006, 2008) showed that subjective probabilities can be defined within sources even though probabilistic sophistication does not hold between sources, provided that preferences satisfy an exchangeability condition. Our analysis implicitly assumed this condition. Abdellaoui et al. (2011) obtained support for it in all but one of their tests. The only exception was a test involving an unfamiliar source and hypothetical choice. For real incentives, exchangeability always held. The real incentive system they used was similar to the one we used. Moreover, because our subjects were finance students, all sources that they encountered were familiar. As further evidence that the assumption appears reasonable, note that the estimated probabilities of the different events were well-calibrated. They were sensitive to the provision of new information and their updating reflected aggregate behavior of the stocks in the market.

We finally assumed that the weighting function could be described by the neo-additive form. This assumption is not too restrictive as the neo-additive weighting function provides a good approximation to more general weighting functions (Diecidue et al. 2009, Abdellaoui et al. 2010). For most subjects the estimated model parameters were plausible and within the range allowed by the model.

7. Conclusion

Ambiguity theories are useful to study the effects of learning in decision under uncertainty. Learning affects both beliefs and ambiguity attitudes, We developed a new method that allows correcting beliefs from ambiguity attitudes. The method further decomposed ambiguity attitudes into a component reflecting likelihood insensitivity and a component reflecting pessimism. Likelihood insensitivity dropped considerably upon the receipt of information, but pessimism was relatively unaffected. This is consistent with the suggestion that likelihood insensitivity reflects a cognitive bias and pessimism the motivational part of ambiguity attitude. We found that subjects behaved more in line with expected utility when they received extra information, but significant deviations from expected utility persisted. The estimated beliefs, when corrected for ambiguity attitudes, changed in the direction of aggregate market behavior indicating that subjects took the information into account and reacted to it in a reasonable manner.

Appendix A: Median ask prices

Option	Up	Middle	Down	Middle-Up
No information	8.50	7.50	7.50	12.50
1 week	8.50	8.00	9.00	12.50
1 month	7.50	7.50	7.50	13.50

Appendix B (Experimental instructions)

Instructions

Thank you for participating in our experiment. For your participation, you will receive a show up fee of €5 and an extra payment depending on your choices during the experiment. Please read the instructions carefully. Before starting the experiment, we will ask you several questions to test your understanding of the instructions. If you answer every question correctly, you will proceed to the experiment; otherwise, we will ask you to read the instructions once more and re-answer the questions until all your answers are correct. We want to be sure that you have understood the instructions so that your answers in the experiment reflect your preferences and are not caused by any misunderstandings. If you have any questions, please feel free to ask the experimenter.

During the experiment, you have to answer a series of choice questions. There are **no right or wrong answers** to these questions. We are interested in ***your*** preferences. Your final payment will be determined by the choices you make during the experiment. Hence it is in your own interest to reveal your true preferences in the choices you will face.

***

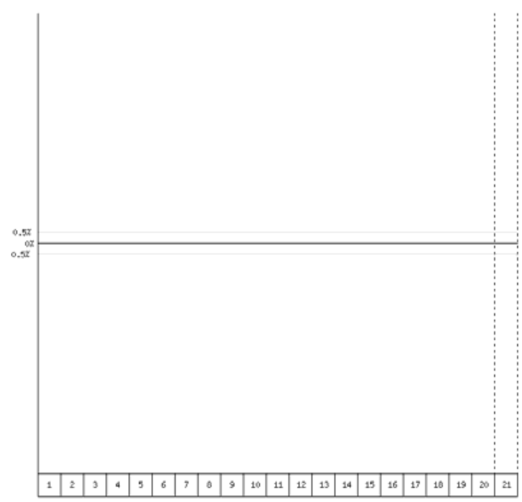
During the experiment, you will be asked to choose between a ***digital option*** for an underlying stock and a sure money amount.

A digital option for an underlying stock pays a pre-specified money amount **H** if a given event occurs and **L** otherwise.

The **underlying stock** is randomly chosen from a database of stocks that were newly-listed on the NYSE between 1 **January** 2009 and 25 February 2011. The stocks in the database are randomly numbered from 1 to 328. At the beginning of the experiment, you will draw 4 numbers from a box, and the 4 corresponding stocks will be used as the underlying stocks of your digital options. At the end of the experiment, the names of the stocks will be revealed, and you can check the historical quotes of the stock prices on Yahoo Finance afterwards. Note that we cannot manipulate the price distribution of the stocks as these are historically given.

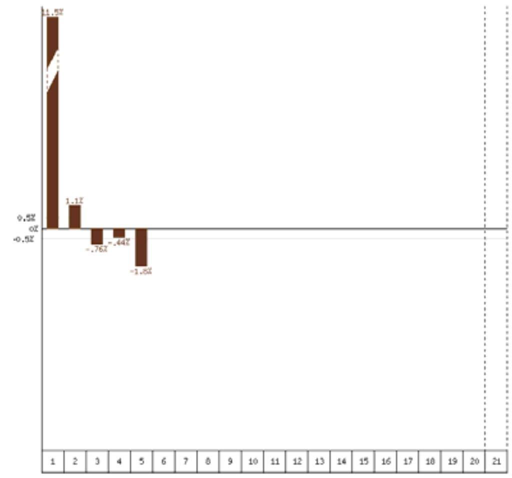
You will face 3 different situations.

Situation 1



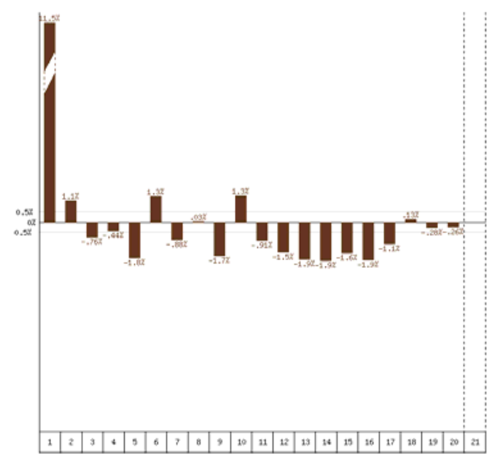
- **Situation 1:** You have an option for an underlying stock, which has just been listed on the Stock Exchange. Consequently, you have no quotes of the historical stock price. You know that the expiration date of the option is the 21st trading day of the stock, and the payoff of the option depends on the daily return of the stock on the 21st trading day. (More explanation about the option payoff will be presented later.)

Situation 2



Situation 2: You have an option for an underlying stock, which has been listed on the Stock Exchange for one week. You have 5 quotes of the historical daily return of the stock, which have been depicted by the brown bars. You know that the expiration date of the option is the (same) 21st trading day of the stock, and the payoff of the option depends on the daily return of the stock on the 21st trading day.

Situation 3

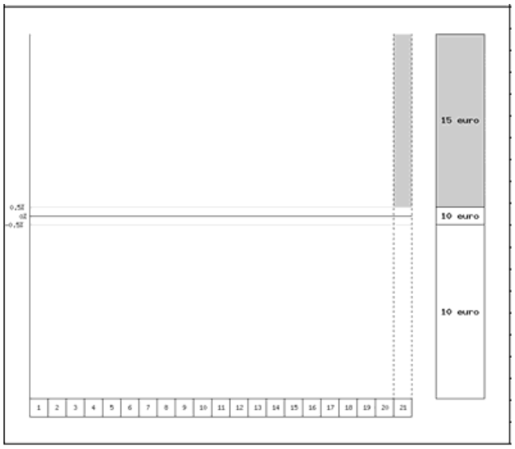


- **Situation 3:** You have an option of an underlying stock, which has been listed on the Stock Exchange for 20 days. You have 20 quotes of the historical daily return of the stock, which have been depicted by the brown bars. You know that the expiration date of the option is the (same) 21st trading day of the stock, and the payoff of the option depends on the daily return of the stock on the 21st trading day.

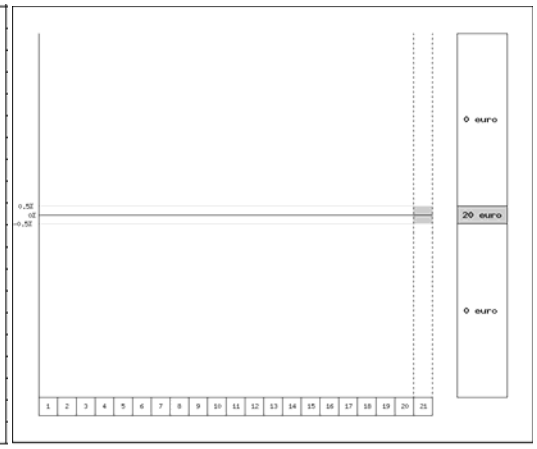
You will face 4 types of *digital option*

For each situation described above, you may face 4 types of digital options. Here, we use the first situation as an example to illustrate the 4 types of digital options.

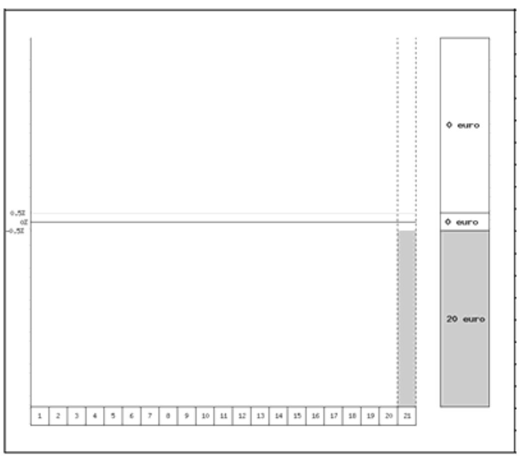
Up-Option



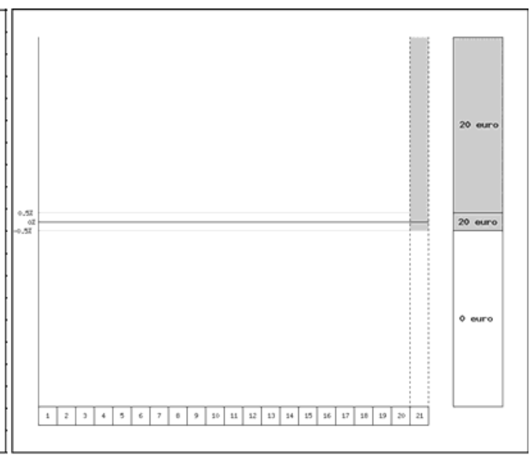
Middle-Option



Down-Option

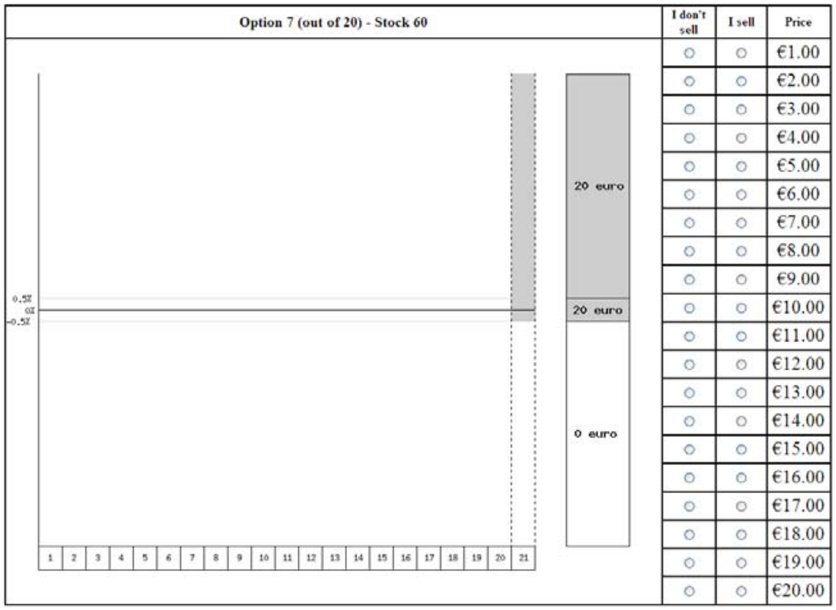


MiddleUp-Option



- An **Up-option** pays €H if the daily return (r) of the underlying stock on its expiration day exceeds +0.5% (r > +0.5%) and €L otherwise;
- **A Middle-option** pays €H if the daily return (r) of the underlying stock on its expiration day varied between –0.5% and +0.5% (–0.5% ≤ r ≤ +0.5%) and €L otherwise;
- A **Down-option** pays €H if the daily return (r) of the underlying stock on its expiration day is less than –0.5% (r < –0.5%) and €L otherwise.
- A **MiddleUp-option** pays €H if the daily return (r) of the underlying stock on its expiration day exceeds –0.5% (r ≥ –0.5%) and €L otherwise;

€H and €L are pre-specified money amounts. For instance, the figure above displays an Up-option with H=15 and L=10, and the other three types with H=20 and L=0. You may encounter different H and L in the experiment.



We will determine your selling price of 20 different options through a series of choices between **the option** and **a certain money amount**. An example is given in the above figure. For each of the 20 prices, you are asked to indicate whether you would like to sell the option or not. The money amount where you switch your choice from 'I don't sell' to 'I sell' is taken as your selling price. All sales will be realized on the 21st day.

If you sell at €x, do you agree that you also want to sell at prices higher than €x? Y/N

If you don't sell at €y, do you agree that you don't want to sell at prices lower than €y? Y/N

Payment

After making all the 20 choices, please call the experimenter. The experimenter will let you throw a 20-sided dice twice. The first throw will determine which of your choices will be played for real. The second throw determines the price you are offered.

As an example, imagine that you throw 7 on your first throw and 6 on your second. Hence the 7th choice will be selected and the price you are offered for the option in the 7th choice is €6. Suppose that option in the 7th choice is a MiddleUp-option with H=20 and L=0, as in the figure above. Suppose further that your selling price for the 7th option was found to be €9.

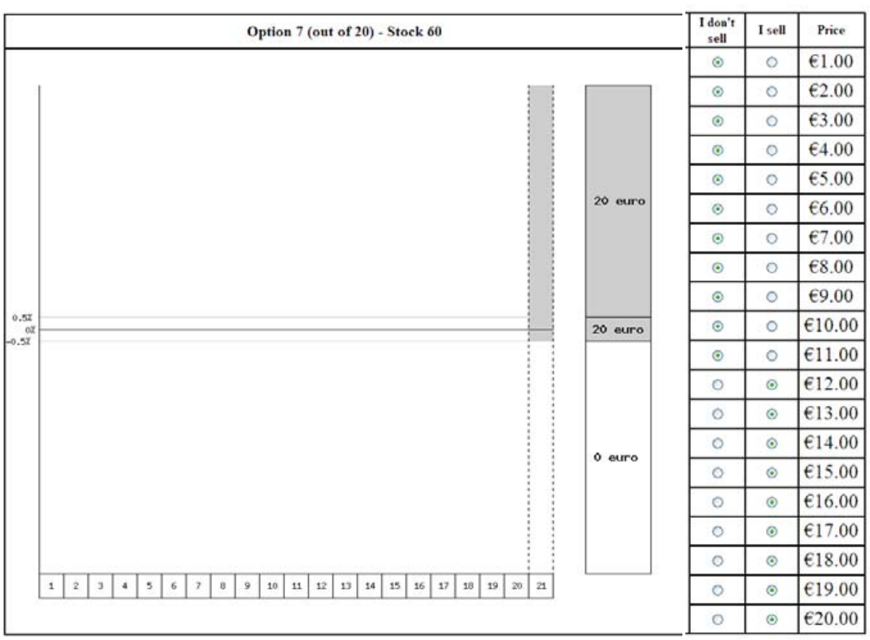
This means that you are not willing to sell the option for a price less than €9 and, hence, you don't accept the offered price of €6 and **thus you keep the option**;

- If the daily return on the 21st trading day of the underlying stock is at least –0.5% (e.g. 0.15%), then we pay you €20 plus the €5 show-up fee. In total you get €25.
- If the daily return on the 21st trading day of the underlying stock is smaller than –0.5% (e.g. –1.49%), then we pay you €0 plus the €5 show-up fee. In total you get €5.

Now imagine that you throw 7 and 10. Then the price offer you are offered is €10. Because you are willing to sell the option if the price is at least €9, you accept the offered price of €10 and thus we pay you €10 plus the €5 show-up fee. In total you get €15.

Note that it is in your best interests to state your selling price truthfully. To see that, suppose your true selling price is €9, but you state a selling price of €11. Then if the price we offer for the option is €10, you keep the option even though it is worth less to you than €10.

Questions:



Suppose you are going to play the choice in the picture above for real.

1. What is the minimum selling price?
2. What is the payoff of the plotted option, if the daily return on the 21st trading day is:
    - 1.4%?
    - –0.45%?

- -1.4%?

3. Suppose that the daily return on the 21st trading day is 1.4%, what is the total payment you get if the second number you throw is 1?
4. Suppose the daily return on the 21st trading day is 1.4%, what is the total payment you get if the second number you throw is 15?

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