Journal of Risk and Uncertainty, 4:285-297 (1991)

Ambiguity, Probability, Preference, and Decision Analysis

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Key words: ambiguity, probability, preference, decision analysis

Abstract

The possible influence of ambiguity on decisions about probabilities has received considerable attention, with most modeling efforts focusing on the probabilities. In this article, it is argued that effects of ambiguity on decision making often operate via the decision maker's preferences rather than probabilities. Therefore, a natural way to model ambiguity is in terms of preferences, via the consequence space and utilities. Some difficulties in developing ambiguity-related models to encompass the full range of realistic decision-making problems and in implementing such models are discussed briefly. Finally, the question of whether ambiguity should matter prescriptively is addressed.

1. Introduction

According to theories of subjective probability (e.g., de Finetti, 1937; Savage, 1954), an individual's uncertainty about an event or variable can be represented probabilistically. In decision analysis, such subjective probabilities are assessed and used as inputs to decision models. Decision makers may feel quite comfortable with some probability assessments but more shaky about others, and the vagueness in the latter instances is often referred to as ambiguity about the probabilities. Wallsten (1990) argues that "vague uncertainties" would be a better label than "ambiguous probabilities" for this phenomenon, but the latter will be used here because it has become common usage.

The paradox presented and discussed by Ellsberg (1961) suggests that ambiguity can have an influence on an individual's decisions. The paradoxical nature of this suggestion lies in the fact that it is seen as contrary to the usual modeling of decision-making problems via expected utilities in decision analysis. As a result, the Ellsberg paradox has generated considerable interest and stimulated a lot of work relating to ambiguity. The work most pertinent to this article has involved descriptive and normative models for

This research was supported in part by the National Science Foundation under Grants SES-8721423 and SES-9008767. Many lengthy discussions with Rakesh K. Sarin and our collaboration on a preference-based approach for modeling decision making under ambiguity have played a key role in the evolution of my thoughts regarding ambiguity and its possible role in decision modeling.

considering ambiguity in decision making. Most of these efforts have focused on probabilities, typically modifying probabilities in such a manner that they no longer are required to obey the usual rules of probability. One objective of this article is to argue that the effects of ambiguity on decision making typically operate through the decision maker's preferences rather than probabilities. Therefore, a natural way to model ambiguity effects is through preferences, in particular by expanding the space of consequences and modifying the utilities accordingly. A model in this spirit is developed in Sarin and Winkler (1990).

Models for dealing with ambiguity have typically been limited to simple situations with a single event (and hence a single probability) of interest. A second objective of this article is to suggest what types of generalizations are needed to make such models available for possible use in realistic decision-making situations and to point out that such generalizations are by no means trivial. Of course, the nature of a model is directed in part by whether it is intended to describe actual decision-making behavior, to be used prescriptively in decision analysis, or to fulfill yet another purpose. The question of whether ambiguity should matter prescriptively is controversial, and the final objective of this article is to present my current views on this issue.

The article is organized as follows. A description of an Ellsberg choice situation involving the drawing of a ball from an urn is presented in section 2 along with some typical responses. Ambiguity and decision modeling are discussed in section 3, with emphasis on the distinction between models that focus on the probabilities and models that deal with ambiguity via preferences. Section 4 describes some characteristics of realistic decision-making problems that must be addressed by models if the models are to have serious practical value. Finally, section 5 attempts to address the question as to whether ambiguity should matter prescriptively.

2. Ambiguity and Decision Making

Discussions concerning ambiguity often relate to the Ellsberg paradox (Ellsberg, 1961), one version of which involves two urns. Urn I (the ambiguous urn) contains 100 balls, each of which is either red or black, but you do not know the proportion of each color; Urn II (the unambiguous urn) contains exactly 50 red and 50 black balls. Betting on Red_{I} means that a ball will be drawn at random from Urn I and you will win a prize if the ball is red and nothing if the ball is black; Black_{I}, Red_{II}, and Black_{II} are defined similarly.

Suppose that the prize associated with a winning bet is \$10,000. For each of the following pairs of bets, consider whether you prefer the first bet, prefer the second bet, or are indifferent between the bets:

1. Red_{I} vs. Black_{I}
2. Red_{II} vs. Black_{II}
3. Red_{I} vs. Red_{II}
4. Black_{I} vs. Black_{II}.

What if the prize were set at \$100 instead of \$10,000? Would you modify any of your choices for the pairs of bets? How would you choose with a prize of \$1?

Most people are indifferent between Red_I and $Black_I$ and likewise for Red_{II} and $Black_{II}$. In choosing between Red_I and Red_{II} or between $Black_I$ and $Black_{II}$, however, many take Red_{II} or $Black_{II}$, preferring to deal with the unambiguous urn. Under the usual analysis of this problem with the standard expected utility model, the expected utility of a bet on event E (e.g., Red_I) is simply pu (prize) $+ (1 - p)u$ (no prize), where p is the individual's subjective probability that event E will occur and u represents the individual's utility function. Indifference between Red_I and $Black_I$ implies that the individual views each color as having probability one-half of occurring in a draw from Urn I, and similarly for Urn II. But then Red_I and Red_{II} both have probability one-half, and the model prescribes indifference between urns.

The preference pattern noted above is by no means universal. Quite a few people are indifferent between Red_I and Red_{II} and between $Black_I$ and $Black_{II}$, and some even prefer to bet on the ambiguous urn. Moreover, informal experimentation suggests that the choices may change when the prize changes as, for example, in the questions posed earlier in this section. In particular, the paradoxical combinations of choices seem to have less appeal as the value of the prize diminishes. Naturally, as the prize approaches $0, we would expect individuals to approach indifference for all four pairs of bets. The point is simply that the size of the prize may have an effect on the choices. Some individuals even flip-flop, preferring Red_{II} to Red_I for large prizes and vice versa for small prizes.

Of course, the Ellsberg situation is idealized and not typical of real-world decision-making situations. Important uncertainties seldom can be represented by simple chance mechanisms with fixed, given probabilities. Much of the evidence regarding decision making under ambiguity has arisen from simple experimental situations, and the role of ambiguity in the real world is not clear, as indicated by Heath and Tversky (1991, p. 6):

> Ellsberg's example and most of the subsequent experimental research on the response to ambiguity or vagueness were confined to chance processes, such as drawing a ball from a box, or problems in which the decision maker is provided with a probability estimate. The potential significance of ambiguity, however, stems from its relevance to the evaluation of evidence in the real world. Is ambiguity aversion limited to games of chance and stated probabilities, or does it also hold for judgmental probabilities? We found no answer to this question, but there is evidence that casts some doubt on the generality of ambiguity aversion.

Some ambiguity research has involved medicine (Curley, Eraker, and Yates, 1984) and insurance (Hogarth and Kunreuther, 1989; Kunreuther and Hogarth, 1990). More realistic studies are needed to gauge the impact of ambiguity on decision making in practice and the influence of characteristics such as the size of the payoffs.

3. Ambiguity and Decision Modeling

The evidence, although limited in real situations, indicates that ambiguity about probabilities could influence decision makers' choices in some circumstances. How might this be allowed for in decision modeling? The answer depends on whether the model's orientation is descriptive or prescriptive. Section 5 will provide some discussion of the controversial question as to whether ambiguity should even be considered in prescriptive decision modeling. The concern of the present section is how it might be considered if we indeed want to do so.

Most efforts to allow for ambiguity formally in decision-making models have focused on the probabilities. After all, discussions of ambiguity have generally addressed ambiguity about probabilities, as in the Ellsberg situation described in section 2. These probability-based models have typically led to modified probabilities, sometimes called decision weights, that do not necessarily obey the rules of probability. In particular, the modified probabilities are often not additive.

Models with modified probabilities may do a good job of describing data on choices in certain situations with ambiguity about probabilities. For example, Einhorn and Hogarth (1985) develop a model involving an anchoring and adjustment process in which an individual anchors on an estimate of a probability and then adjusts by imagining other possible values of the probability. The resulting adjusted probabilities are not additive.

Describing observed behavior well, of course, is not sufficient to warrant advocating the use of a model for prescriptive or normative purposes. Models with modified probabilities have also been developed from a normative perspective, usually by changing the standard axioms of decision theory. Examples include Fishburn (1983, 1986), Gilboa (1987), Segal (1987), and Schmeidler (1989). But modified probabilities that do not obey the usual rules of probability seem more problematic in normative models than in descriptive models. Probability is the language of uncertainty, but is that language understandable or sensible when the rules of the language are violated? What does it mean to say that if a ball will be drawn from Urn I in the Ellsberg situation, the probability is 0.4 that it will be red and 0.4 that it will be black, when we know that it must be one or the other? To say that a weighted average, with 0.4 and 0.4 as weights, closely describes the value of a bet on, say, $\mathrm{Red}_{\mathrm{I}}$ implied by someone's choice of bets is one thing. To advocate the normative merits of such modified probabilities is a much greater and more controversial step.

In his discussion of the Ellsberg paradox, Smith (1969, p. 325) comes out strongly against "probabilities" that do not behave like probabilities:

> But I do not care for the probabilistic interpretation of the violations. To me probabilities are probabilities in the sense of nonnegativity, additivity and the property of the unit measure over the whole event space. I grant the right of a man to have systematic and deliberate preferences for rewards based on dice game contingencies over the same rewards based on Dow-Jones stock price contingencies. But if he insists also that he is less than certain that the Dow-Jones average will either rise or not rise by five points or more tomorrow, then so far as I am concerned he is now making a "mistake." He does not understand what is (or should be) meant by probability. He is entitled to his tastes, but not to any new definitions of probability.

I heartily endorse Smith's statement.

Arguments put forth by Nau (1990), Kadane (1990), and others view the Ellsberg paradox and ambiguity in general in terms of possible manipulation by an opponent, thus bringing in game-theoretic considerations. Frisch and Baron (1988) note possible concerns that an opponent knows more or can bias a situation to your disadvantage; and Ellsberg (1961, p. 658) himself notes this possibility: "The subject can always ask himself, 'What is the likelihood that the experimenter has rigged this urn?'"

When game-theoretic considerations are present, the appropriate strategy is not to modify probabilities, but to reframe the question and assess different probabilities. Instead of just assessing probabilities of red and black being drawn from Urn I in the Ellsberg situation, the decision maker can assess probabilities for these events conditional on betting on Red_{I} and conditional on betting on Black_{I}. For example, probabilities for red of 0.4 following a bet on Red_{I} and 0.6 following a bet on Black_{I} are legitimate assessments that would lead to indifference between Red_{I} and Black_{I} but a preference for bets on the unambiguous urn. Bordley and Hazen (1991) relate assessments of this type to "suspicion" (see also Viscusi, 1989) and to Hazen's (1987) subjectively weighted linear utility model. To the extent that "ambiguity effects" in a decision-making problem can be dealt with by reframing the analysis, the problem might not really fall into the category of decision modeling under ambiguity that is of interest here.

Modified probabilities, then, seem unappealing from a prescriptive viewpoint. But if probabilities are not to be modified in modeling responses to ambiguity, what is to be done? Although ambiguity about probabilities is the ambiguity of concern in this article, I would argue that the influence of this ambiguity on decision-making behavior generally operates through preferences. Thus, attention should be focused on the preference side of modeling rather than on probabilities. The preference side involves the consequences in the decision model and the value function or utility function over those consequences.

How does the influence of ambiguity on decision-making behavior operate through preferences? Intuitively, ambiguity about a probability of an event seems to be akin to an increase in the riskiness of a course of action with consequences related to that event. This perceived additional "riskiness" might be related to potential feelings of regret or blame if a poor outcome results and satisfaction or joy if a particularly good outcome results. Such concerns could manifest themselves through, for instance, feelings of anxiety and discomfort and periods of sleeplessness for some people and excitement and a rush of adrenaline for others. As anyone who has endured sleepless nights or enjoyed euphoric moments can testify, these are very real consequences that merit serious consideration. Reactions to ambiguity might naturally be viewed as related to preferences for such types of consequences, and such preferences are modeled in the expected utility approach through utilities, not probabilities.

The idea of looking at the preference side is not at all new. Fellner (1961) notes that ambiguity can be thought of in terms of modifications of utility as well as modifications of probability, but he does not pursue this notion. Roberts (1963, p. 332) also comments on this line of attack:

> Ellsberg's analysis of his paradox is based on the assumption that the utilities of outcomes are a function only of the monetary consequences. But this is not necessarily the assumption that the subject was making. As just one illustration (For

> another in the same vein, the subject may fear that Urn I might contribute to an ulcer, regardless of what a rational analysis of other aspects of the problem may suggest), the subject might feel that his choice of Red_I could lead to unpleasant second guesses by someone who observed the experiment; he could be criticized, however unfairly, for not taking an apparently "safe" course of action (Red_{II}) if he lost by taking an "unsafe" one (Red_I). An analysis of utility, formal or informal, might reveal the reason for the subject's willingness to pay more for Red_{II} than Red_I, and similarly for $Black_{II}$ over $Black_I$.

More recently, Lindley (1990, p. 54) comments on consequences in a slightly different context (a discussion of the Allais paradox):

> Utility is attached to a consequence . . . , and in evaluating that consequence You are free to take into account any aspect of it that You wish. In particular, You may wish to include the surprise that will delight You if (Your action) results in the unexpected (outcome). What is happening is that You are not just considering money but other aspects of the situation as well. Once relevant aspects are included the difficulty disappears.

Heath and Tversky (1991, p. 7–8) take a similar tack, expanding the consequences to include nonmonetary aspects, and they relate the degree of importance of such aspects to the decision maker's competence in the area of interest:

> We propose that the consequences of each bet include, besides the monetary payoffs, the credit or blame associated with the outcome. Psychic payoffs of satisfaction or embarrassment can result from self-evaluation or from an evaluation by others. In either case, the credit and the blame associated with an outcome depend, we suggest, on the attributions for success and failure. In the domain of chance, both success and failure are attributed primarily to luck. The situation is different when a person bets on his or her judgment. If the decision maker has limited understanding of the problem at hand, failure will be attributed to ignorance whereas success is likely to be attributed to chance. In contrast, if the decision maker is an "expert," success is attributable to knowledge whereas failure can sometimes be attributed to chance.

Also, Frisch and Baron (1988, p. 153) include among their proposed reasons for why ambiguity may affect behavior the notion that "issues of blame, responsibility and regret are more salient in situations of ambiguity." In a more general philosophical discussion relating to the Ellsberg paradox, Weirich (1986) focuses on the interpretation of consequences.

In most prescriptive modeling of decision making under ambiguity, then, it seems reasonable that modifications in the standard model should involve utilities, which reflect preferences, instead of probabilities. Smith (1969, p. 325) agrees, noting that subjects

> . . . merely violate the axioms, without the necessity for the probabilistic interpretation. . . . As I see it, it is much more plausible to say that violators in "nonstandard process" contingencies, such as the stock price example, suffer utility losses (or gains) relative to what is experienced in less controversial "standard process" contingencies, such as dice games. . . . an individual may have a low psychological tolerance for the "ambiguity" associated with nonstandard process events, which we can very reasonably and naturally describe in terms of "utility losses."

Smith goes on to develop a simple model with the utility of a consequence reduced in the presence of ambiguity by a "utility loss due to ambiguity." This is appealing intuitively but has some undesirable implications, such as violations of dominance.

A more general approach is to assess utilities for an expanded consequence space that includes attributes other than the monetary payoff, positive or negative, that is received. Despite the above quotes indicating support for this direction, little has been done in the way of actual modeling. Bell (1981, p. 37) provides an example, using the two attributes of reward and regret, in which "concave preference for regret leads people to select (the unambiguous urn)." More generally, in Sarin and Winkler (1990), a model is developed in which the utilities are modified to allow in each case for a payoff that might have been received but was not. If a difference in the utilities of monetary payoffs is thought of as a type of regret, then under certain circumstances the model reduces to a multiattribute utility model with attributes "payoff received" and "regret." Of course, other attributes, such as health effects (anxiety, stress, etc.) and effects due to perceived evaluations by others (e.g., reduced prospects for advancement) could also be considered directly. For example, in a different context not dealing with ambiguity, Harvey (1989) develops a model attempting to take into consideration preferences for what he calls psychological effects (e.g., effects of self-evaluation or evaluation by others). The point is that multiattribute models can, at least in principle, get to the preference-related heart of the ambiguity problem. Potential difficulties in implementing ambiguity-related models are considered next.

4. Beyond models of simple situations

Models attempting to deal with ambiguity in decision making, whether concentrating on modifications of probabilities or utilities, have been limited almost exclusively to very simple situations. They typically deal with single-probability situations, where the uncertainty is about an event and its complement and the ambiguity therefore concerns only a single probability. Almost all interesting real-world decision-making problems are more complex. To be helpful in practice, then, models must be generalized to handle ambiguity in more complex situations.

Perhaps the most obvious generalization is to multiple-event situations. Real-world uncertainties often have more than two possible outcomes. As a result, the uncertainty cannot be described by just a single probability. With K mutually exclusive outcomes,

$K - 1$ probabilities are needed; in the continuous case, an entire density function is required. Unfortunately, questions regarding ambiguity become more difficult as we move to multiple outcomes. The ambiguity felt by the decision maker may take on a variety of patterns. For example, she may feel a great deal of ambiguity about one particular probability and much less about the others, or she may feel equally vague about all of the probabilities. These two possibilities could have very different implications for her decision-making behavior and would have to be treated differently in descriptive models. To the extent that they lead to different sets of nonmonetary consequences (e.g., different levels of anxiety associated with a course of action), any consideration of ambiguity from a prescriptive standpoint would also have to treat them differently.

Another characteristic of many interesting decision-making problems is that they are dynamic in nature. In applications, even after related but less important actions and events are ignored in order to keep a decision tree at a manageable size, the tree typically reflects sequences of several actions and events. In such "bushy" trees, numerous probabilities must be assessed for different event branches, and the decision maker may feel ambiguity about various probabilities here and there in the tree. This raises the tricky issue of the propagation of ambiguity in a decision tree. If an action will lead to a series of events with probabilities having varying degrees of ambiguity (perhaps depending on the preceding events in the tree), how can the influence, if any, of these ambiguities be taken into consideration in the evaluation of the action? Put another way, if ambiguity is to be modeled through preferences, how can the utilities at the end of the tree be modified to allow for the ambiguities encountered en route to the end points?

Probabilities can be manipulated in decision models via the standard calculus of probabilities. Moreover, multiattribute utility theory provides a framework for combining different aspects of preference. In contrast, part of the difficulty in thinking about modeling ambiguity for multiple-event or dynamic situations is that there is no readily available "calculus of ambiguity." Little attention has been given to possible relationships among ambiguities and to improving our understanding of the basic notion of ambiguity. Fishburn (1990) treats ambiguity as a primitive concept and considers various axioms in an attempt to move toward a general theory of ambiguity. For example, one axiom that seems intuitively reasonable states that the ambiguity about an event (or, more in keeping with the phraseology used here, about the probability of the event) equals the ambiguity about the complement of the event. A condition that is less compelling but deserves further attention is submodularity, which states that the ambiguity about the union of two events is less than or equal to the sum of their individual ambiguities minus the ambiguity about their intersection.

A general theory of ambiguity could be quite useful. To apply it in decision modeling, of course, would require practical procedures for measuring the degree of ambiguity about a probability. An ambiguity of zero, which is associated with the null set, a sure event, and any completely unambiguous event (perhaps Red_{II} or Black_{II} in the Ellsberg situation in section 2 *if* there are no doubts about the contents of the urn or the randomness of the draw), has an obvious interpretation. Among events with nonzero ambiguity, it is easy to think of cases where one event is more ambiguous than another. For example, Red_{I} seems more ambiguous than Red_{III}, where Urn III contains 100 balls, each of which is either red or black, and you know that the number of red balls is between 40 and 60. But attaching a specific number to the ambiguity about a particular event such as Red_{III} is a large step beyond saying that the ambiguity of Red_{III} is less than the ambiguity of Red_{I}. This measurement problem is further complicated by the fact that it would seem to be desirable to measure an individual's utilities in order to distinguish between attitude toward ambiguity and attitude toward risk. Just as potential problems arise in the separation of probability elicitation from utilities (e.g., see Kadane and Winkler, 1988), the separation of ambiguity elicitation from utilities may be a tricky issue.

Complicating matters further is the fact that reactions to ambiguity are likely to be quite situation-dependent. Potentially relevant factors include not only the degree of ambiguity but also the size of the payoffs and the probabilities and the perceived role of individuals other than the decision maker (e.g., the decision maker's boss or spouse). To the extent that we may wish to include such factors, they certainly render the modeling process more complex.

5. Should ambiguity matter prescriptively?

Although the evidence is not unambiguous and comes largely from somewhat artificial experimental situations, it is nonetheless apparent that ambiguity about probabilities might influence decisions made by some people in some circumstances. In descriptive models of decision-making behavior, then, it is reasonable to attempt to consider ambiguity. The role of ambiguity from a prescriptive standpoint is not so clear. Saying that decision makers should take ambiguity into consideration is a much stronger statement than observing that they sometimes do so when left to their own devices. Since the spirit of applied decision analysis is prescriptive, the question of whether ambiguity should matter prescriptively is important from a practical perspective as well as from a theoretical perspective.

At a foundational level, decision makers who are concerned about ambiguity may want more than they should reasonably expect. At least in part, the desire to remove any ambiguity about a probability reflects a desire for a "true" probability. But the theory of subjective probability, as developed by de Finetti (1937) and Savage (1954), does not admit the notion of a "true" probability. We can speak of *my* probability that the Dow-Jones Industrial Average (DJIA) will increase during the next six months, and *your* probability that the DJIA will increase during the next six months, but there is no such thing as *the* probability that the DJIA will increase during the next six months. Under some circumstances, such as Urn II in the Ellsberg situation, the case for a particular value for a probability (e.g., one-half for the probability of Red_{II}) seems clear, and there is general agreement on that value. Thus, in Urn II, a decision maker might find it natural to speak of *the* probability of Red_{II}, although no such animal exists; a probability of one-half for Red_{II} for an individual is based on assumptions such as the contents of the urn being as stated and the draw being random, and such assumptions are subjective. In Urn I, too, the setup encourages thought of a "true" probability for Red_{I}, since there is a fixed number of red balls out of the 100 balls in the urn; the decision maker just doesn't happen to know that number. In most important real-world decisions, the uncertainty does not lend itself to this type of straightforward setup, but people may still like to think (erroneously) that there is a "true" probability and may feel uncomfortable not knowing this so-called "true" value. As noted in section 2, further work is needed on the impact of ambiguity on decision makers in realistic situations, as opposed to artificial Ellsberg-type situations or scenarios with realistic cover stories but externally given probabilities.

One approach, then, would be to emphasize to a decision maker that no "true" probabilities exist and that their own subjective probabilities are of interest, however ambiguous they may feel about them. These probabilities can then be assessed, and the analysis can proceed in the usual manner. To the extent that a decision maker feels ambiguous or "shaky" about some of the assessed probabilities, a sensitivity analysis can indicate how robust the model of the decision-making situation is with respect to changes in these probabilities. To help the decision maker assess probabilities and think about the degree of ambiguity, eliciting ranges of values or even second-order probabilities to represent uncertainty about probabilities of interest might prove useful. Such elicited ranges or second-order distributions can also be valuable in guiding sensitivity analysis and in suggesting where additional information might be valuable.

A careful analysis, including sensitivity analysis, can provide insight into a decision-making situation and can help the decision maker feel comfortable with whatever action is chosen. If different actions have similar expected utilities and certainty equivalents, then extraneous factors such as ambiguity can be brought into play informally in the final decision-making process. This sort of approach might operate in the Ellsberg situation, where indiffference between red and black in each urn implies that a standard analysis without ambiguity will yield equal expected utilities for $\mathrm{Red}_{\mathrm{I}}$ and $\mathrm{Red}_{\mathrm{II}}$. Ambiguity can then be used as a tiebreaker, so to speak, and might carry the day even when the expected utilities differ slightly. Remember that a prescriptive decision analysis provides guidance but does not bind the decision maker.

When the differences among expected utilities and certainty equivalents are not so small, the analysis can provide justification for the course of action that is chosen and can thereby facilitate the separation of "good decision making" from "good outcomes" in the mind of anyone evaluating the decision or the decision maker (including the decision maker herself in the case of self-evaluation). Perhaps this can reduce worries about second guessing by others noted by Roberts (1963) and concerns about credit and blame noted by Heath and Tversky (1990) and by Frisch and Baron (1988) (see section 3). Such worries might also be reduced by consulting acknowledged experts for probability assessments of events about which the decision maker feels ambiguous.

A counterargument to sweeping ambiguity under the rug is that some people experience ambiguity effects that are quite tangible. For example, a decision that leaves a person in a situation with ambiguity about probabilities representing important uncertainties may have adverse health effects associated with anxiety, stress, sleeplessness, and so on. Obviously some people can deal with such problems more easily than others, and some decision makers might even view the effects in a positive vein, enjoying a rush of adrenaline associated with an action involving vague uncertainties. To the extent that a decision maker experiences such effects, positive or negative, they can be very real, and it would be desirable for a prescriptive analysis to incorporate them insofar as feasible. As

Raiffa (1985, p. 113) notes with respect to prescriptive analysis, "The analyst can query the decision maker about what are his or her real concerns and if these cognitive concerns loom large, they can be incorporated into the analysis." Furthermore, since such effects are obviously preference-related, the natural strategy for considering them in decision-making models is through preference assessment and preference modeling. The model developed in Sarin and Winkler (1990) represents a step in this direction, but as indicated in section 4, much more work on some tricky issues is required to generalize this approach to realistic decision-making situations with characteristics such as multiple-event branches and sequences of events and actions.

The formal treatment of ambiguity in decision models will inevitably complicate the analysis and place greater assessment burdens on the decision maker. It may be difficult to incorporate attitude toward ambiguity in modeling in practice. Just as methods are available to represent various attitudes toward risk (e.g., decreasing risk aversion) by appropriate classes of utility functions, perhaps simplified procedures for dealing with preferences regarding ambiguity can be developed. Moreover, in the spirit of value-focused thinking (Keeney, 1988, 1992), a decision maker who is seriously bothered by ambiguity might include the objective of minimizing ambiguity among her objectives and might creatively seek out options that would place her in less ambiguous situations without forcing her to give up much on other dimensions.

In summary, prescriptive modeling of ambiguity in decision making is not an easy issue with which to deal. Reactions to ambiguity are, when carefully examined, typically preference-related, and any prescriptive modeling of such relations should be handled through the preference side of the modeling process. In any event, modified probabilities that no longer satisfy the usual rules of probability seem totally unappealing from a prescriptive viewpoint. Expansion of the space of consequences and modification of utilities are more appealing and are in line with the preference-related nature of attitude toward ambiguity. Such steps, however, are not without their own difficulties, both practical and philosophical, and should certainly not be taken lightly.

To the extent that ambiguity is considered formally or informally in the decision-modeling and decision-making process, decision makers should be cognizant of the distinction between risk (as normally defined in decision theory) and ambiguity. Moreover, they should be fully aware of what is being given up on other dimensions in order to accommodate attitude toward ambiguity. M.B.A. students studying decision analysis are often quite surprised at how risk averse their assessed utility functions are and at how much they must give up in expected value to accommodate their assessed risk attitudes. This realization often leads them to move toward less risk-averse positions, and the same might happen with respect to ambiguity. My present inclination is to discourage the consideration of ambiguity in decision modeling in practice, but to encourage further study aimed at improving the general understanding of ambiguity and potential ways of handling and modeling ambiguity.

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