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What Determines the Shape of the Probability Weighting Function Under Uncertainty?

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Decision weights are an important component in recent theories of decision making under uncertainty. To better explain these decision weights, a two-stage approach has been proposed: First, the probability of an event is judged and then this probability is transformed by the probability weighting function known from decision making under risk. We extend the two-stage approach by allowing the probability weighting function to depend on the type of uncertainty. Using this more general approach, properties of decision weights can be attributed to properties of probability judgments and/or to properties of probability weighting. We present an empirical study that shows that it is indeed necessary to allow the probability weighting function to be source dependent. The analysis includes an examination of properties of the probability weighting function under uncertainty that have not been considered yet.

(Ambiguity; Decision Weights; Prospect Theory)

1. Introduction

Expected utility theory (von Neumann and Morgenstern 1947) offers a normative basis for decision making under conditions of risk where probabilities are given. As demonstrated by Allais (1953) and many subsequent empirical studies (for an overview see Camerer 1995), expected utility fails as a descriptive theory of choice. New theories have been developed to descriptively model decision making under risk (for an overview, see Camerer 1995, Starmer 2000), with rank-dependent models being prominent among them (Quiggin 1982, Segal 1987, Wakker 1994, Yaari 1987). The defining property of rank-dependent models is that cumulative probabilities are transformed by a weighting function, usually so as to place more weight on the lowest-ranked outcomes. Rank-dependent theories have been developed that also allow for sign dependence, so as to treat gains and losses differently (Starmer and

Sugden 1989, Tversky and Kahneman 1992). Recently, the probability weighting function under risk has been studied intensively (Abdellaoui 2000, Bleichrodt and Pinto 2000, Camerer and Ho 1994, Gonzalez and Wu 1999, Prelec 1998, Tversky and Kahneman 1992, Wu and Gonzalez 1996).

In most economic applications, however, probabilities are not given, and the framework of expected utility does not apply. For such applications, Savage (1954) developed subjective expected utility theory. As demonstrated by Ellsberg (1961), this theory also fails descriptively (for an overview, see Camerer and Weber 1992). For example, there are certain sources of uncertainty a decision maker likes and certain others he or she does not like. Most persons prefer to bet on the weather of their home town rather than the weather of some unknown town; i.e., they are typically averse to the ambiguity of betting on an unknown event. Schmeidler (1989), Gilboa

(1987), and Sarin and Wakker (1992) have developed Choquet expected utility theory (CEU) and Tversky and Kahneman (1992), Luce and Fishburn (1991), and Wakker and Tversky (1993) have developed cumulative prospect theory (CPT), both of which allow for nonneutral attitudes towards ambiguity.

Various factors have been shown to explain the attitude towards ambiguity (see Camerer and Weber 1992). In this paper, we will focus on the effect of a decision maker's perceived competence in evaluating the source of uncertainty. Research initiated by Heath and Tversky (1991) showed that perceived competence is an important factor in discriminating between different sources of uncertainty (see also Keppe and Weber 1995, Fox and Tversky 1995). The higher the perceived competence, the stronger the preference for a source of uncertainty will be, i.e., the weaker the ambiguity effect will be. Strong perceived competence can even lead to ambiguity-seeking behavior.

A key element in models of decision making under uncertainty are decision weights that capture both the perceived likelihood of the event and the preference for betting on that event. Suppose you have to make a bet on whether it rains tomorrow in your home town or whether it rains tomorrow on some tropical island in the South Pacific. Your decision weight might be influenced by your probability judgment of rain in each location and by your attitude towards ambiguity. To disentangle probability judgment and attitude towards ambiguity, Tversky and Fox (1995) and Fox and Tversky (1998) have proposed a two-stage model to explain decision weights: First, the decision maker judges the probability of the event under consideration, after which this probability is transformed by the probability weighting function under risk (see also Wu and Gonzalez 1999).

Our paper extends this two-stage approach and empirically tests our extension. More specifically, we will allow the probability judgments *and* the probability weighting function both to depend on the source of uncertainty. Earlier studies have defined and empirically validated properties of decision weights. Using the extended two-stage approach, we are able to ask if probability judgments *and/or* probability weighting functions explain the properties of decision

weights. This decomposition of decision weights will allow us to better understand decision weights and to attribute important properties of decision weights to properties of probability judgments and/or properties of probability weighting. After mathematically deriving the relations between decision weights, probability judgments, and probability weighting under uncertainty, we present an empirical study that shows that it is indeed necessary to allow the probability weighting function to be source dependent. The analysis includes an examination of properties of the probability weighting function under uncertainty that have not been considered yet.

The paper is structured as follows: In §2, we set up the theoretical background of the experimental study and formulate three research hypotheses to be tested subsequently. In §3, we describe the experimental design of our study in more detail. In §4, the key results are presented. The paper concludes in §5 with a brief summary and discussion of the main findings.

2. Theoretical Background

2.1. Choquet Expected Utility Theory and Cumulative Prospect Theory

In the following, we will consider uncertain prospects $P = (x_1, A_1; \dots; x_n, A_n)$, where A_i are subsets of a set S , the set of states of nature, which are called events, and (A_1, \dots, A_n) forms a partition of S . Prospect P yields the outcome x_i if event A_i occurs, where $x_1 \leq \dots \leq x_i \leq \dots \leq x_n$. As our empirical part only involves prospects with nonnegative outcomes, we do not need to distinguish between CEU and the more general CPT which allows for a different treatment of gains and losses. Formally, the value of a prospect P under CEU can be stated as follows:

$$\text{CEU}(P) = \nu(x_n) \cdot W(A_n) + \sum_{i=1}^{n-1} \nu(x_i) \cdot [W(A_i, \dots, n) - W(A_{i+1}, \dots, n)].$$

Here, ν is a value function, W is a weighting function or capacity, and $A_{i \dots j} = A_i \cup \dots \cup A_j$.

The weighting function W is a central element in Choquet expected utility theory. It is defined on the set of all subsets of S , with $W(\emptyset) = 0$, $W(S) =$

1, and $W(A_i) \leq W(A_{ij})$ for all i, j . The decision weight associated with outcome x_i is given by $W(A_{i, \dots, n}) - W(A_{i+1, \dots, n})$, which is the difference between the capacity of the event of receiving outcome x_i or better and the capacity of the event of receiving outcome x_{i+1} or better.

DEFINITION 1. W satisfies *bounded subadditivity* (SA_W), if the following conditions hold:

(i) *Lower subadditivity* of the weighting function (LSA_W): $LSA_W(A_i, A_j) = W(A_i) + W(A_j) - W(A_{ij}) \geq 0$, whenever $W(A_{ij})$ is bounded away from one.

(ii) *Upper subadditivity* of the weighting function (USA_W): $USA_W(A_i, A_j) = 1 + W(S - A_{ij}) - W(S - A_i) - W(S - A_j) \geq 0$, whenever $W(S - A_{ij})$ is bounded away from zero.

Lower subadditivity states that the impact of an event is smaller when it is added to another event than when it is added to the null event. Upper subadditivity states that the impact of an event is larger when it is subtracted from the certain event than when it is subtracted from some intermediate event. Bounded subadditivity has been observed for weighting functions on sporting events, events defined on temperature ranges in college towns or events defined on price ranges of a specific stock; see Tversky and Fox (1995) and Fox et al. (1996). Both subadditivity conditions are more general conditions than the concavity and convexity conditions presented in Wu and Gonzalez (1999).

What is needed in addition, are criteria to compare the weighting functions for different sources of uncertainty. Tversky and Wakker (1995) formalize two source-dependent effects of weighting functions: Source preference and source sensitivity. To define these effects, let Δ and Γ be two distinct families of events, i.e., two sources of uncertainty. Following Tversky and Wakker (1995), we assume that the families are closed under union and complementation.

DEFINITION 2. The decision maker exhibits a general *source preference* (SP_W) for source Δ over source Γ if, for any event A_i in Δ and B_i in Γ , $W(A_i) = W(B_i)$ implies $W(S - A_i) \geq W(S - B_i)$.

The definition implies a relation that can be easily tested empirically (for $W(A_i) = W(B_i)$):

$$\begin{aligned} SUM_W(A_i) &= W(S - A_i) + W(A_i) \\ &\geq W(S - B_i) + W(B_i) = SUM_W(B_i). \end{aligned}$$

We will test if the average of SUM_W , i.e., $\overline{SUM_W(A)}$ or $\overline{SUM_W(B)}$, satisfies this relation.

As a second source-dependent effect, which is logically independent of source preference, Tversky and Wakker (1995) define source sensitivity.

DEFINITION 3. The decision maker exhibits less *source sensitivity* (SS_W) to source Γ than to source Δ if the two following conditions hold:

(i) If $W(A_i) = W(B_i)$ and $W(A_j) = W(B_j)$, then $W(A_{ij}) \geq W(B_{ij})$.

(ii) If $W(S - A_i) = W(S - B_i)$ and $W(S - A_j) = W(S - B_j)$, then $W(S - A_{ij}) \leq W(S - B_{ij})$, for all disjoint events A_i, A_j in Δ and disjoint events B_i, B_j in Γ , with $W(A_{ij})$ bounded away from one and $W(S - A_{ij})$ bounded away from zero.

The decision maker exhibits less source sensitivity to source Γ than source Δ if the union of disjoint events from Γ loses more relative to the corresponding single events than the union of disjoint events from Δ . Two testable conditions can be inferred (for $W(A_i) = W(B_i)$ and $W(A_j) = W(B_j)$):

$$LSA_W(A_i, A_j) \leq LSA_W(B_i, B_j)$$

and

$$USA_W(A_i, A_j) \leq USA_W(B_i, B_j).$$

Subsequently, it will be tested if the averages of LSA_W and USA_W , i.e., $\overline{LSA_W(A)}$, $\overline{LSA_W(B)}$, $\overline{USA_W(A)}$, and $\overline{USA_W(B)}$ match these conditions.

To derive testable hypotheses, we need to ex ante differentiate between sources of uncertainty. As explained in the introduction, we will hypothesize that perceived competence is an important factor in discriminating between different sources of uncertainty. For simplicity, we will speak of the perceived competence toward one source of uncertainty and not distinguish between the perceived competence toward different events of the same source.

The empirically testable properties of weighting functions are summarized as follows:

HYPOTHESIS 1. PROPERTIES OF THE WEIGHTING FUNCTION W .

(a) *Subadditivity*: W satisfies lower and upper subadditivity.

(b) *Source Preference*: W satisfies the relation $\overline{SUM}_W(A) \geq \overline{SUM}_W(B)$ when perceived competence is higher toward source Δ than source Γ , $A \in \Delta$, and $B \in \Gamma$.

(c) *Source Sensitivity*: W satisfies the relation $\overline{LSA}_W(A) \leq \overline{LSA}_W(B)$ and $\overline{USA}_W(A) \leq \overline{USA}_W(B)$ when perceived competence is higher toward source Δ than source Γ , $A \in \Delta$, and $B \in \Gamma$.

2.2. The Two-Stage Approach

Tversky and Fox (1995) and, in more detail, Fox and Tversky (1998) suggest specifying the weighting function by a two-stage approach,

$$W(A_i) = w_R(q(A_i)),$$

with W being the weighting function, q probability judgments following support theory (Tversky and Koehler 1994), A_i the event considered, and w_R the probability weighting function under risk. Fox and Tversky (1998) present evidence for such a two-stage specification. Wakker (2001) gives a formal justification for this decomposition of the weighting function.

As anticipated by Fox and Tversky (1998), this two-stage approach may be generalized by introducing a more general probability weighting function under uncertainty w of which w_R may be seen as a special case. The function w may be different for different sources of uncertainty. This leads to $W(A_i) = w_\Delta(q(A_i))$ for a source of uncertainty Δ , or for short,

$$W(A_i) = w(q(A_i)).$$

In the following, we will assume that the weighting function can be decomposed using the generalized two-stage approach. As will be described in more detail below, we will infer capacities from choices (assuming CEU), and we will ask subjects for probability judgments. Probability weighting functions are derived as residuals using the two-stage decomposition.

In §2.1, we have presented properties of weighting functions. We will now derive similar properties for judged probabilities $q(A)$ and for probability weighting functions w , assuming the generalized two-stage approach. The empirical tests of these properties will allow us to understand to what extent judged probabilities and/or probability weighting are responsible

for the observed properties of the weighting function. If w proves to be different for different sources of uncertainty, it will provide evidence in favor of the generalized two-stage approach.¹

Judged Probabilities: $q(A)$. The derivation of properties and hypotheses for judged probabilities is straightforward. Judged probabilities are assumed to follow support theory (Tversky and Koehler 1994), as probability judgments are found to be generally subadditive, i.e., $q(A_i) + q(A_j) \geq q(A_{ij})$, for disjoint A_i, A_j , with q , a capacity (Wu and Gonzalez 1999).

DEFINITION 4. Judged probabilities satisfy *binary complementarity* (BC) for a source if $q(A_i) + q(S - A_i) = 1$ holds for all A_i .

Tversky and Koehler (1994) provide empirical support for binary complementarity to hold. Following support theory, judged probabilities should satisfy *bounded subadditivity of judged probabilities* (SA_q), i.e., *lower subadditivity of judged probabilities* (LSA_q) and *upper subadditivity of judged probabilities* (USA_q). Additionally, *source sensitivity of judged probabilities* (SS_q) has yet to be investigated. We will test if the average lower (upper) subadditivity of one source Δ , i.e., $\overline{LSA}_q(A)$, $\overline{USA}_q(A)$ for $A \in \Delta$, is equal to the average lower (upper) subadditivity of another source Γ , i.e., $\overline{LSA}_q(B)$ and $\overline{USA}_q(B)$ for $B \in \Gamma$.

The empirically testable properties of judged probabilities can be summarized by the following hypothesis. Hypotheses 2a and 2b reflect the current status of the literature.

HYPOTHESIS 2. PROPERTIES OF JUDGED PROBABILITIES.

(a) *Subadditivity*: Judged probabilities satisfy upper and lower subadditivity.

(b) *Binary Complementarity*: Judged probabilities satisfy binary complementarity.

(c) *Source (In-) Sensitivity*: Judged probabilities satisfy $\overline{LSA}_q(A) = \overline{LSA}_q(B)$ and $\overline{USA}_q(A) = \overline{USA}_q(B)$ for sources Δ and Γ ; $A \in \Delta$ and $B \in \Gamma$.

¹ It is well known that the simple two-color Ellsberg paradox (Ellsberg 1961) cannot be explained by the original two-stage approach. Both for the known and unknown urn, judged probabilities will be 0.5, so they will be transformed to the same decision weights, contrary to empirical evidence. As a referee pointed out, the results of Heath and Tversky (1991) cannot be explained by the simple two-stage model either.

Probability Weighting Function: w . Up to this point, we have considered two variables: capacities and judged probabilities. Now we are going to analyze the probability weighting function, which, in light of the two-stage approach, is defined as the residual of capacities and judged probability. A probability weighting function w is a nondecreasing function from $[0, 1]$ to $[0, 1]$, with $w(0) = 0$ and $w(1) = 1$.

First we will consider lower subadditivity. The deviation from additivity of capacities can be separated into two components: $LSA_W = LSA_w + LSA_{wq}$, with

$$LSA_w(A_i, A_j) = w(q(A_i)) + w(q(A_j)) - w(q(A_i) + q(A_j))$$

and

$$LSA_{wq}(A_i, A_j) = w(q(A_i) + q(A_j)) - w(q(A_{ij})).$$

The term LSA_w captures the subadditivity of the probability weighting function given additivity of judged probabilities. LSA_{wq} is the weighted probability loss due to violation of additivity of judged probabilities.² LSA_w eliminates the impact of subadditivity of judged probabilities and captures subadditivity of probability weighting exclusively. If a person applies expected utility maximization to support theory probabilities, then $LSA_w = 0$, while, if q is an additive probability measure, then $LSA_{wq} = 0$. A similar separation exists for upper subadditivity of capacities. Here, $w(q(A_i))$ has to be replaced by the dual function $\hat{w}(q(A_i)) = 1 - w(q(S - A_i))$. Again the equation $USA_W = USA_w + USA_{wq}$ holds with the same meaning of the components.

We expect empirical results for the weighted probability judgments to be similar to the nonweighted probability judgments. SA_w corresponds to the definition of bounded subadditivity of the probability weighting function under risk in Tversky and Wakker (1995). If the weighting function satisfies source sensitivity, it can be determined to what extent this result is driven by subadditivity of probability weighting or subadditivity of judged probability.

² The index W subsequently refers to the weighting function, the index w to the probability weighting function, and the index wq to the weighted probability judgments.

Finally, we look at source preference. If W reveals source preference, again this result may be driven by judged probabilities and/or by the shape of the probability weighting function. If binary complementarity holds, different SUM_W will be exclusively determined by the probability weighting function. However, if binary complementarity does not hold, SUM_W captures the impact of differential probability weighting as well as the impact of failure of binary complementarity. To separate the pure impact of probability weighting, judged probabilities have to be normalized such that they satisfy binary complementarity, i.e.,

$$SUM_w(A_i) = w\left(\frac{q(A_i)}{q(A_i) + q(S - A_i)}\right) + w\left(\frac{q(S - A_i)}{q(A_i) + q(S - A_i)}\right).$$

SUM_w exclusively captures the effect of probability weighting. The weighted impact of differential binary complementarity is captured by $\Delta SUM_{wq} = SUM_W - SUM_w$. For binary complementarity, we have $\Delta SUM_{wq} = 0$. With $\Delta SUM_W = SUM_W - 1$ and $\Delta SUM_w = SUM_w - 1$, the equation $\Delta SUM_w + \Delta SUM_{wq} = \Delta SUM_W$ holds. Comparing SUM_w for different sources of uncertainty, we are able to check for source preference of the probability weighting function.

The testing of probability weighting functions can be summarized in the following.

HYPOTHESIS 3. PROPERTIES OF THE PROBABILITY WEIGHTING FUNCTION w UNDER UNCERTAINTY.

(a) *Subadditivity:* w satisfies lower and upper subadditivity.

(b) *Source Preference:* w satisfies $\overline{SUM_w(A)} \geq \overline{SUM_w(B)}$ when perceived competence is higher toward source Δ than source Γ , $A \in \Delta$ and $B \in \Gamma$.

(c) *Source Sensitivity:* w satisfies $\overline{LSA_w(A)} \leq \overline{LSA_w(B)}$ and $\overline{USA_w(A)} \leq \overline{USA_w(B)}$ when perceived competence is higher toward source Δ than source Γ , $A \in \Delta$ and $B \in \Gamma$.

Estimating the Probability Weighting Function. The two-stage approach allows the determination of the functional relation $W = w(q(A_i))$. The probability weighting functions for different sources can be estimated from certainty equivalents of uncertain prospects and corresponding judged probabilities q ,

as will be explained in detail in §4.4. To estimate the probability weighting function, various functional specifications originally suggested to describe the probability weighting function under risk may be used. As we consider two independent concepts to characterize sources of uncertainty, source preference, and source sensitivity, we will use the following functional forms, which both have two free parameters.³

(1) Linear-in-log-odds form: $w(q) = \lambda \cdot q^\kappa / [\lambda \cdot q^\kappa + (1 - q)^\kappa]$ with λ and κ allowing for different sources of uncertainty.⁴ The parameter λ primarily controls elevation, i.e., source preference, and the parameter κ primarily controls curvature, i.e., source sensitivity (Gonzalez and Wu 1999).

(2) Linear approximation: $w(q) = \gamma + \delta \cdot q$, $q \in (0, 1)$, $w(0) = 0$, $w(1) = 1$ with γ and δ allowing for different sources of uncertainty. The slope parameter δ controls curvature, i.e., source sensitivity. A suitable measure of elevation, i.e., source preference, is given by $\gamma + \delta/2$.

3. Experimental Design

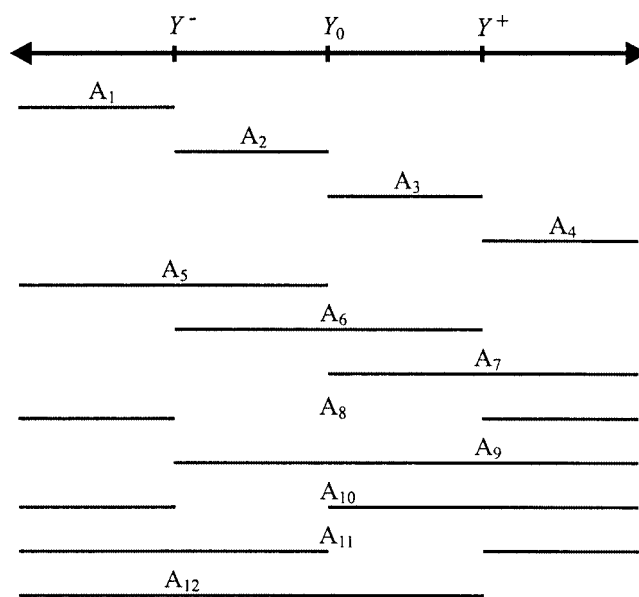
To test the hypotheses, we conducted an experiment at the University of Mannheim, Germany, in May 1997. Fifty-five students of graduate finance classes participated in the study. The study took them about one hour and was run voluntarily after class using a multipage questionnaire, which were filled out completely.

Subjects were asked to judge the stock price changes of two stocks with potentially different familiarity levels. As in Mangelsdorff and Weber (1994), we chose the stocks to be from Deutsche Bank, which is Germany's largest and probably most well-known banking group, and from Dai-Ichi Kangyo Bank, which was one of the largest Japanese banks. We expected people to consider themselves to be more competent in estimating future prices of domestic

³ We also did the analysis presented in §4 with the one-parameter function of Tversky and Kahneman (1992) and with the two-parameter function of Prelec (1998). Results are very much in line with what we present here.

⁴ The linear-in-log-odds property of $w(q)$ is demonstrated in Gonzalez and Wu (1999). This specification was used by Goldstein and Einhorn (1987) and Lattimore et al. (1992), among others.

Figure 1 Events in the Stock Price Continuum



stocks, i.e., the Deutsche Bank, than estimating future prices of foreign stocks. Both corporations were comparable in size and importance. Each preference and probability judgment refers to the closing price of the stock at July 8, 1997, in local currency, i.e., the German stock in Deutsche mark and the Japanese stock in yen. Hence, the forecasting period was about two months.

To test the hypotheses, we partitioned the stock price space in four intervals similar to the approach taken by Tversky and Fox (1995) and Fox et al. (1996). We defined 12 relevant events over these intervals in Figure 1. The 12 events consist of six partitions (event and complementary event) of the event space. The reference point Y_0 refers to the actual stock price of each stock at the time of the experiment in the stock's local currency. The interval boundaries Y^+ and Y^- are chosen such that they yield a return of +5% or -5% relative to Y_0 . A test of Hypotheses 1–3 requires the elicitation of three groups of data. Correspondingly, the questionnaire consists of three sections that were presented in the following order:

Section 1: Competence ratings

Section 2: Certainty equivalents for uncertain prospects

Section 3: Direct judgment of probabilities

Capacities can be calculated from the corresponding certainty equivalents and a particular value function. The experimental design is similar to the design applied in Fox et al. (1996) and Fox and Tversky (1998).

Section 1. Competence Ratings. First, the subjects were asked to rate their competence in estimating the stock price of the Deutsche Bank stock on July 8, 1997, on a scale from 0 (not competent at all) to 6 (very competent); the same was done for the Dai-Ichi Kangyo Bank. Such competence judgments express the individually perceived level of competence.

Section 2. Certainty Equivalents. We considered assets $P_A = (120 \text{ DM}, A; 0 \text{ DM}, \text{otherwise})$, $\$1 \approx 1.80 \text{ DM}$. The event A was defined by a range of the actual stock price of one of the two stocks on July 8, 1997, e.g., A was true if the closing price of the underlying stock on July 8, 1997, was between Y_1 and Y_2 . Alternatively, the participants were offered a sure amount of money X . Each participant had to decide whether he or she preferred the claim P_A or the sure payment X . The sure payment X was varied in steps of 10 DM from 10 DM to 110 DM, such that, for each asset P_A , a sequence of 11 preference choices had to be made. This procedure ensures that the certainty equivalent of claim P_A is elicited in a sequence of choice decisions. Finally, subjects were asked to indicate the exact certainty equivalent, which, according to the instructions, should be between the last preference for asset P_A and the first preference for the sure payment X .

Altogether, subjects evaluated 24 claims P_A . For both stocks, subjects had to evaluate one prospect $P_{A,i} = (120 \text{ DM}, A_i; 0 \text{ DM}, \text{otherwise})$ for each of the 12 events specified in Figure 1. The prospects referring to the Deutsche Bank stock are called $P_{D,1}, \dots, P_{D,12}$; the prospects referring to the Dai-Ichi Kangyo Bank stock are called $P_{J,1}, \dots, P_{J,12}$.⁵ The prospects were given in a fixed order that was randomly determined.

⁵ The prospect $P_{D,10}$ was described as follows (translation from German): The asset $P_{D,10}$ pays 120 DM, if the stock price of the Deutsche Bank on July 8, 1997, will be below 94 DM or above 99 DM, i.e., if relative to the price of 99 DM on May 16, 1997, the stock has risen or it has fallen for more than 5%, otherwise $P_{D,10}$ pays nothing. Some events overlap (depending on whether absolute prices or percentages are considered).

Section 3. Direct Judgment of Probabilities. For each of the 12 events specified in Figure 1, subjects were asked to directly assess the probability that the event occurs. Hence, each subject gave 24 probability judgments. Again, the events were given in a fixed order that was randomly determined.

Summarizing, a participant's data set for one type of stock consists of competence judgment, certainty equivalents over 12 prospects per stock $CE(P_{A,1}), \dots, CE(P_{A,12})$, and probability judgments over 12 events per stock $q(A_1), \dots, q(A_{12})$.

Besides a fixed fee of 10 DM for participation, subjects were paid a variable fee. The payment scheme, which was explained to the participants in advance, ensures that it is optimal for participants to reveal their true preference. After finishing the study, one-fifth of the participants were randomly selected to play one randomly selected preference choice for real money. For each of the selected players, one $P_{A,i}$ was randomly drawn out of the $P_{D,1}, \dots, P_{D,12}, P_{J,1}, \dots, P_{J,12}$. Since for each $P_{A,i}$ subjects had to make a sequence of choices between the claim $P_{A,i}$ and various sure payments X , it also had to be randomly determined which of the preference choices of $P_{A,i}$ was played. In case the participant had chosen the asset $P_{A,i}$, he or she received 120 DM or 0 DM on July 8, 1997, depending on the closing price of the underlying stock on this day. If he or she had chosen the indicated sure payment X , he or she got this amount instead. The respective amounts were paid in class on July 9, 1997. All random draws were taken by drawing numbered balls from an urn.⁶

4. Results⁷

4.1. Competence Judgments

Our study is based on two real stocks: Deutsche Bank and Dai-Ichi Kangyo Bank. How competent

⁶ The payment mechanism is incentive compatible for the choice questions. The elicitation of the actual certainty equivalent was not enforced by the mechanism.

⁷ As explained in §3, the uncertain prospects in our study had a single nonzero outcome. If A_i denotes the event in which the nonzero outcome is received, its decision weight, which is $W(A_i) - W(\emptyset)$, equals $W(A_i)$, the capacity of event A_i , as $W(\emptyset) = 0$. In what follows, we will use the more intuitive term "decision weight" instead of "capacity."

Table 1 Subadditivity of Decision Weights for $v(x) = x^\omega$ with Different Values of ω

	$\omega = 0.76$			$\omega = 0.88$			$\omega = 1.00$		
	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value
SUM _W	1.097	1.001	**	1.016	0.899	**	0.944	0.811	**
LSA _W	0.255	0.325	**	0.205	0.273	**	0.154	0.231	**
USA _W	0.267	0.359	*	0.302	0.399	*	0.342	0.435	*
SS _W	0.456	0.344	**	0.483	0.333	**	0.496	0.333	**

Note. Entries are medians over all participants for $v(x) = x^\omega$ with different values of ω . *p*-values refer to one-sided Wilcoxon rank sum tests Deutsche Bank vs. Dai-Ichi Kangyo Bank over all participants (*: $p < 0.05$, **: $p < 0.01$).

do the (German) participants feel about these two stocks? Subjects feel much more competent judging future stock price changes of the domestic Deutsche Bank stock than judging future stock price changes of the foreign Dai-Ichi Kangyo Bank stock. On a scale from 0–6, subjects give mean ratings of 2.87 for the Deutsche Bank and of 1.00 for the Dai-Ichi Kangyo Bank. Of 55 participants, 51 judged their competence concerning the Deutsche Bank stock to be higher than concerning the Dai-Ichi Kangyo Bank stock; 4 judged their competence as equally high.

4.2. Decision Weights

First, we calculate the decision weights from the corresponding certainty equivalents for each event for each person. Remember that a prospect $P_{A,i}$ pays 120 DM if the event A_i occurs, and 0 DM otherwise. Consequently, given a person's value function v , the decision weight $W(A_i)$ of the event A_i can be calculated from $v(CE(P_{A,i})) = v(120) \cdot W(A_i)$. This calculation requires the specification of the value function $v(\cdot)$. Following the literature, e.g., Tversky and Kahneman (1992), we use the form $v(x) = x^\omega$. For convenience, we did not estimate individual parameters ω , but used a common parameter ω for all subjects and did sensitivity analysis on that parameter. As Tversky and Kahneman (1992) found $\omega = 0.88$, we did all the analysis for $0.76 \leq \omega \leq 1$, varying ω in steps of 0.06. The decision weights of the events D_1 to D_{12} for the Deutsche Bank and the events J_1 to J_{12} for the Dai-Ichi Kangyo Bank can be calculated from the certainty equivalents assigned to the corresponding prospects $P_{D,1}$ to $P_{D,12}$ and $P_{J,1}$ to $P_{J,12}$, respectively, and the value function.

First, Hypothesis 1a claims that the weighting function satisfies lower and upper subadditivity. To test for lower subadditivity, we consider the decision weights of the event pairs (D_1, D_2) , (D_2, D_3) , (D_3, D_4) , (D_1, D_4) , and the corresponding joint events D_5, D_6, D_7, D_8 . For example, $LSA_W(D_1, D_2) = W(D_1) + W(D_2) - W(D_5)$ can be calculated for each participant from D_1 and D_2 , and the corresponding joint event, D_5 . Similar calculations are made for (D_2, D_3) , (D_3, D_4) , (D_1, D_4) , and the corresponding joint events, D_6, D_7, D_8 . The average over these four values, $\overline{LSA_W(D)}$, describes the degree of lower subadditivity of a specific participant. The $\overline{LSA_W(J)}$ numbers are calculated in a similar way.

To test for upper subadditivity, we consider the decision weights of the event pairs (D_{11}, D_{12}) , (D_9, D_{12}) , (D_9, D_{10}) , (D_{10}, D_{11}) , and the corresponding events, D_5, D_6, D_7, D_8 . For example, $USA_W(D_{11}, D_{12}) = 1 + W(D_5) - W(D_{11}) - W(D_{12})$ can be calculated for each participant from $D_{11}(= S - D_3)$, $D_{12}(= S - D_4)$ and the corresponding $D_5(= S - D_3 - D_4)$. Similar calculations are made for (D_9, D_{12}) , (D_9, D_{10}) , (D_{10}, D_{11}) , and the corresponding events, D_6, D_7, D_8 . The average over these four values $\overline{USA_W(D)}$ describes the degree of upper subadditivity of a specific participant. The $\overline{USA_W(J)}$ numbers are calculated in a similar way.⁸

Table 1 shows that, for both stocks, independently of ω , the weighting function reveals lower subadditivity as well as upper subadditivity. In each case, i.e., for each of $\overline{LSA_W(D)}$, $\overline{LSA_W(J)}$, $\overline{USA_W(D)}$, and $\overline{USA_W(J)}$, more than 90% of the subjects reveal this pattern

⁸ Comparing LSA_W for different sources, decision weights $W(A)$ and $W(B)$ have to be equal. Even if $W(J)$ in general is a bit smaller than $W(D)$, this effect is negligible.

(sign test: $p < 0.01$ in each case). This result supports Hypothesis 1a. It also replicates earlier results by Tversky and Fox (1995) and Fox et al. (1996).

Hypothesis 1b states that the weighting function satisfies source preference. As perceived competence for the Deutsche Bank is greater than for the Dai-Ichi Kangyo Bank, we check whether $\overline{SUM}_W(D) \geq \overline{SUM}_W(J)$ holds. For each subject $\overline{SUM}_W(D)$ equals the average $\overline{SUM}_W(D_i) = W(S - D_i) + W(D_i)$ over all six partitions (event and complementary event) according to Figure 1. Similar calculations apply to the Japanese stock. Table 1 shows that \overline{SUM}_W are (significantly) higher for the Deutsche Bank than for the Dai-Ichi Kangyo Bank. Subjects reveal source preference for the domestic stock over the foreign stock independently of ω . 70% (sign test $p < 0.01$) of the subjects show this pattern. This result supports Hypothesis 1b. It shows that the competence effect found by Heath and Tversky (1991) is also valid in the domain of subjective stock evaluations.

Finally, Hypothesis 1c states that the weighting function reveals higher source sensitivity for sources that subjects feel more competent about. It has to be tested whether $\overline{LSA}_W(D) \leq \overline{LSA}_W(J)$ and $\overline{USA}_W(D) \leq \overline{USA}_W(J)$ hold. The variable $SS_W(\cdot) = 1 - \overline{LSA}_W(\cdot) - \overline{USA}_W(\cdot)$ can be interpreted as a measure of sensitivity. The larger SS_W , the larger the sensitivity of the weighting function with respect to the underlying events. Results show that \overline{LSA}_W , as well as \overline{USA}_W , are significantly smaller for the Deutsche Bank than for the Dai-Ichi Kangyo Bank (\overline{LSA}_W true for at least 65%, $p < 0.02$; \overline{USA}_W true for at least 60%, $p < 0.10$). Accordingly, SS_W is larger for the Deutsche Bank than for the Dai-Ichi Kangyo Bank (at least 69%, $p < 0.01$). People reveal lower source sensitivity for less familiar stocks, i.e., weighting functions are more "subadditive" for less familiar sources. Tversky and Fox (1995) show that source sensitivity is smaller for uncertain events than for risky events. Our results show that there are also differences in sensitivity between various sources of uncertainty. Fox et al. (1996) conducted a study similar to ours in a stock market framework. They did not find systematic source dependence in their data, however, which might be because the differences in perceived competence between the underlying stocks were too small.

4.3. Judged Probabilities

Hypothesis 2a states that probability judgments satisfy lower and upper subadditivity. We consider the same pair of events and corresponding joint events as in the analysis of the weighting function. For each of the four pairs, we calculate $LSA_q(D_i, D_j) = q(D_i) + q(D_j) - q(D_{ij})$ with $D_{ij} = D_i \cup D_j$. The average over these four values, $\overline{LSA}_q(D)$, describes the degree of lower subadditivity of a specific participant. The $\overline{LSA}_q(J)$ numbers are calculated in a similar way. For example, to test for upper subadditivity as in the last section, we calculate $USA_q(D_{11}, D_{12}) = 1 + q(D_5) - q(D_{11}) - q(D_{12})$ from $D_{11} (= S - D_3)$, $D_{12} (= S - D_4)$, and $D_5 (= S - D_3 - D_4)$. The average over the four values, $\overline{USA}_q(D)$, describes the degree of upper subadditivity of a specific participant (respectively $\overline{USA}_q(J)$ for the Japanese stock). Table 2 shows that for both stocks, judged probabilities reveal lower and upper subadditivity. For both stocks, 83% of all subjects satisfy lower subadditivity (sign test: $p < 0.01$), and at least 91% of all subjects satisfy upper subadditivity (sign test: $p < 0.01$). This result supports Hypothesis 2a. Moreover, it corresponds to the findings by Tversky and Fox (1995), Fox et al. (1996), Fox and Tversky (1998), and particularly Tversky and Koehler (1994) in support theory.

Moreover, Tversky and Koehler (1994) suggest that probability judgments of complementary events add to one (Hypothesis 2b). To test Hypothesis 2b, we consider the average over the six partitions of the event space, which is termed $\overline{SUM}_q(D)$. $\overline{SUM}_q(D)$ expresses a subject's degree of binary complementarity. The same applies to the Dai-Ichi Kangyo Bank. Table 2 shows that probability judgments for the Deutsche Bank approximately satisfy

Table 2 Subadditivity of Judged Probabilities

	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value
\overline{SUM}_q	0.992	0.933	**
\overline{LSA}_q	0.038	0.050	n.s.
\overline{USA}_q	0.145	0.238	**
SS_q	0.800	0.675	**

Note. Entries are medians over all participants; *p*-values refer to one-sided Wilcoxon rank sum tests Deutsche Bank vs. Dai-Ichi Kangyo Bank over all participants (n.s.: not significant, **: $p < 0.01$).

binary complementarity. For 54.5% of the subjects, $\overline{SUM}_q(D) < 1$ and for 45.5%, $\overline{SUM}_q(D) > 1$ (sign test: $p = \text{n.s.}$), 69% are within $[0.95, 1.05]$. However, for the Dai-Ichi Kangyo Bank, this pattern does not hold. For 80% of the subjects, $\overline{SUM}_q(J) < 1$ (sign test: $p < 0.01$), and only 40% are within $[0.95, 1.05]$. Additionally, \overline{SUM}_q is significantly smaller for the Dai-Ichi Kangyo Bank than for the Deutsche Bank ($p < 0.01$); 85% of the subjects show this pattern. This result might be rather surprising. It reminds us of the source preference property of the weighting function, which might be caused by the lack of binary complementarity in the underlying probability judgments. This pattern might be due to the very low perceived competence for Dai-Ichi Kangyo Bank; see also Macchi et al. (1999). Other studies do not report a similar discrepancy (see the summary in Fox and Tversky 1998, who in Study 2, find a failure of binary complementarity).

Hypothesis 2c claims that probability judgments do not reveal source-dependent differences in subadditivity, i.e., $\overline{LSA}_q(D) = \overline{LSA}_q(J)$ and $\overline{USA}_q(D) = \overline{USA}_q(J)$. The variable $\overline{SS}_q(\cdot) = 1 - \overline{LSA}_q(\cdot) - \overline{USA}_q(\cdot)$ can be interpreted as a measure of sensitivity. The larger $\overline{SS}_q(\cdot)$, the larger the sensitivity of judged probabilities with respect to additional events. It can be seen that \overline{LSA}_q ($p = \text{n.s.}$) and \overline{USA}_q ($p < 0.01$) are smaller for the Deutsche Bank than for the Dai-Ichi Kangyo Bank. Accordingly, \overline{SS}_q is larger for the Deutsche Bank than for the Dai-Ichi Kangyo Bank ($p < 0.01$).

4.4. Probability Weighting Function

Using certainty equivalents and corresponding probability judgments for the uncertain prospects, we can estimate the parameters of the probability weighting function. If prospect $P_{A,i} = (120 \text{ DM}, A_i; 0 \text{ DM}, \text{otherwise})$ is evaluated according to CEU and the two-stage decomposition of decision weights applies, the following equation can be derived:

$$CE(P_{A,i}) = \nu^{-1}(w(q(A_i))) \cdot \nu(120 \text{ DM}).$$

To estimate the parameters of w , we assumed $v(x) = x^\omega$ for different values of ω and employed a nonlinear regression procedure minimizing squared deviations in terms of CE.

The weighting function can be estimated individually for each participant for each underlying

source of uncertainty. For comparison, we estimate a pooled probability weighting function for each source of uncertainty from the pooled data of all participants' judgments about that source of uncertainty. The probability weighting function will be formalized by the two functional specifications mentioned before, i.e., the linear-in-log-odds-form and a simple linear approximation. Table 3 shows the medians of the individually estimated parameters over all subjects and the parameter values of the pooled estimations.

The parameter values given in Table 3A show systematic differences between the two probability weighting functions for both parametric specifications. The linear-in-log-odds-function shows larger λ values and larger κ values for the Deutsche Bank than for the Dai-Ichi Kangyo Bank (λ larger for at least 54%, $p = \text{n.s.}$; κ larger for at least 59%, $p < 0.11$). λ and κ thus increase with increasing competence judgments. Lower κ values correspond to less pronounced source sensitivity, whereas lower λ values correspond to a generally lower curve indicating lower source preference. The parameter values of the linear approximation can be interpreted similarly. These findings are affirmed by the estimated parameter values of the pooled regressions (Table 3B).

Summarizing, the probability weighting function for less familiar sources of uncertainty is characterized by more subadditivity. Some of our results suggest a lower curve relative to the probability weighting function for more familiar sources of uncertainty. These results support Hypothesis 3 in tendency. These results also stress the necessity of modeling the source dependence of probability weighting, e.g., by the extended two-stage approach, $w(q(\cdot))$, introduced above.

Subsequently, the three parts of Hypothesis 3 will be tested explicitly. Hypothesis 3 is based on the separation of LSA_W , USA_W , $\Delta SUM_W = SUM_W - 1$ into LSA_w , USA_w , ΔSUM_w , and LSA_{wq} , USA_{wq} , ΔSUM_{wq} , described in §2. Subadditivity of the weighting function (index W) can be induced by weighted subadditivity of probability judgments (index wq) and/or by subadditivity of the probability weighting function (index w), with $LSA_W = LSA_w + LSA_{wq}$, and corresponding equations for the other terms.

Table 3A Estimation of the Probability Weighting Function for $v(x) = x^\omega$ with Different Values of ω : Individual Regressions

	$\omega = 0.76$			$\omega = 0.88$			$\omega = 1.00$		
	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value
Linear-in log-odds									
λ	1.315 (0.221)	1.167 (0.191)	n.s.	1.096 (0.190)	0.953 (0.163)	m.s.	0.891 (0.163)	0.790 (0.136)	m.s.
κ	0.467 (0.148)	0.389 (0.145)	*	0.489 (0.153)	0.415 (0.153)	*	0.506 (0.161)	0.442 (0.162)	*
linear approx.									
$\gamma + \delta/2$	0.560 (0.041)	0.535 (0.037)	m.s.	0.517 (0.041)	0.485 (0.039)	*	0.479 (0.043)	0.442 (0.041)	*
δ	0.567 (0.136)	0.439 (0.149)	**	0.601 (0.141)	0.466 (0.154)	**	0.607 (0.144)	0.481 (0.157)	**

Note. Entries are medians of the estimated parameters of individual regressions for each participant and median standard errors of these estimates (in parentheses). The assumed value function is $v(x) = x^\omega$ with different values of ω as indicated. *p*-values refer to one-sided Wilcoxon rank sum tests Deutsche Bank vs. Dai-Ichi Kangyo Bank for the estimated parameters (n.s.: not significant, m.s.: marginally significant, $p < 0.1$, * : $p < 0.05$, ** : $p < 0.01$).

Table 3B Estimation of the Probability Weighting Function for $v(x) = x^\omega$ with Different Values of ω : Pooled Regressions

	$\omega = 0.76$		$\omega = 0.88$		$\omega = 1.00$	
	Deutsche Bank	Dai-Ichi Kangyo	Deutsche Bank	Dai-Ichi Kangyo	Deutsche Bank	Dai-Ichi Kangyo
Linear-in log-odds						
λ	1.268 (0.041)	1.138 (0.036)	1.036 (0.035)	0.928 (0.031)	0.861 (0.030)	0.770 (0.027)
κ	0.425 (0.023)	0.298 (0.025)	0.443 (0.024)	0.312 (0.027)	0.461 (0.025)	0.326 (0.028)
linear approx.						
$\gamma + \delta/2$	0.555 (0.007)	0.532 (0.007)	0.510 (0.007)	0.483 (0.008)	0.469 (0.008)	0.439 (0.008)
δ	0.540 (0.024)	0.389 (0.029)	0.569 (0.025)	0.407 (0.030)	0.587 (0.026)	0.417 (0.031)

Note. Entries are the estimated parameters of pooled regressions over all participants' data and corresponding standard errors (in parentheses). The assumed value function is $v(x) = x^\omega$ with different values of ω as indicated.

Table 4 Subadditivity of the Probability Weighting Function for $v(x) = x^\omega$ with Different Values of ω

	$\omega = 0.76$			$\omega = 0.88$			$\omega = 1.00$		
	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value	Deutsche Bank	Dai-Ichi Kangyo	<i>p</i> -value
Linear-in log-odds									
LSA_w	0.220	0.265	*	0.172	0.211	*	0.129	0.160	*
LSA_{wq}	0.011	0.010	n.s.	0.008	0.011	n.s.	0.008	0.007	n.s.
USA_w	0.143	0.192	*	0.159	0.249	*	0.197	0.289	**
USA_{wq}	0.082	0.086	n.s.	0.070	0.087	n.s.	0.073	0.076	n.s.
SS_w	0.624	0.503	**	0.655	0.521	*	0.662	0.530	**
ΔSUM_w	0.131	0.071	*	0.041	-0.022	*	-0.049	-0.107	*
ΔSUM_{wq}	-0.003	-0.029	**	-0.002	-0.029	**	-0.002	-0.025	**
linear approx.									
LSA_w	0.246	0.275	*	0.192	0.216	m.s.	0.137	0.169	m.s.
LSA_{wq}	0.008	0.008	n.s.	0.008	0.008	n.s.	0.008	0.008	n.s.
USA_w	0.155	0.205	*	0.187	0.243	*	0.218	0.283	*
USA_{wq}	0.057	0.061	n.s.	0.062	0.067	n.s.	0.061	0.072	n.s.
SS_w	0.584	0.505	*	0.601	0.512	*	0.607	0.522	*
ΔSUM_w	0.120	0.070	m.s.	0.034	-0.029	*	-0.043	-0.116	*
ΔSUM_{wq}	-0.004	-0.023	**	-0.004	-0.024	**	-0.004	-0.024	**

Note. Entries are medians of the individual values assuming the specified probability weighting function with individually estimated parameters and value function $v(x) = x^\omega$ with different values of ω as indicated. *p*-values refer to one-sided Wilcoxon rank sum tests Deutsche Bank vs. Dai-Ichi Kangyo Bank parameters (n.s.: not significant, m.s.: marginally significant, $p < 0.1$, *: $p < 0.05$, **: $p < 0.01$).

Hypothesis 3a states that the probability weighting function satisfies lower and upper subadditivity, i.e., $\overline{LSA_w(\cdot)} \geq 0$ and $\overline{USA_w(\cdot)} \geq 0$, for both sources of uncertainty. The tests of subadditivity are based on the same set of events as in the case of decision weights and probability judgments. $LSA_{wq}(D_i, D_j)$ can be calculated using the individually estimated parametric probability weighting functions by plugging in the corresponding probability judgments. $LSA_w(D)$ and $\overline{LSA_{wq}(D)}$ are calculated for each participant as the average of the four event pairs. The calculation of $\overline{LSA_w(J)}$ and $\overline{LSA_{wq}(J)}$ is done similarly. A test of upper subadditivity is performed similarly based on the dual function.

Table 4 shows that for both probability weighting functions, $LSA_w \geq 0$ and $USA_w \geq 0$ (sign test: LSA_w true for at least 84%, $p < 0.01$; USA_w true for at least 82%, $p < 0.01$). This result supports Hypothesis 3a, the probability weighting functions under uncertainty satisfy lower and upper subadditivity. Additionally, it should be noted that on average, $LSA_{wq} \geq 0$ and

$USA_{wq} \geq 0$, although the absolute values are much smaller than the corresponding LSA_w - and USA_w - values. $LSA_{wq} \geq 0$ and $USA_{wq} \geq 0$ reflect the subadditivity of weighted probability judgments shown in Table 2. So it should be kept in mind that the subadditivity of the weighting function is driven by the curvature of the probability weighting function as well as, to a much smaller degree, by the subadditivity of judged probabilities. Such a decomposition of decision weights is directly connected to the two-stage specification proposed by Fox and Tversky (1998) and Wu and Gonzalez (1999). In accordance with the results presented above, both studies find first evidence for subadditivity of the probability weighting function in the two-stage framework. However, the empirical parts of these studies do not explicitly distinguish between the probability weighting function under risk and under uncertainty.

Hypothesis 3b states that the probability weighting function satisfies source preference. To test this condition, we take the average $SUM_w(\cdot)$ over the

considered six partitions of the event space for each participant for each stock. The resulting terms, $\overline{SUM_w(D)}$ and $\overline{SUM_w(J)}$, should satisfy $\overline{SUM_w(D)} \geq \overline{SUM_w(J)}$ according to Hypothesis 3b. $SUM_w(\cdot)$ is calculated using the individually estimated specifications of the probability weighting function and q , the corresponding judged probabilities. $SUM_w(\cdot)$ captures only the impact of the source-dependent shape of the probability weighting function, with $\Delta SUM_w(\cdot) = SUM_w(\cdot) - 1$. Departures of the probability judgments from binary complementarity are controlled for by the normalization of probabilities. The term $\Delta SUM_{wq}(\cdot)$ captures the part of the source preference effect caused by the departure of probability judgments from binary complementarity.

Table 4 shows that the median of ΔSUM_w is larger for the Deutsche Bank than for the Dai-Ichi Kangyo Bank (at least 51% of the subjects show that pattern, $p = \text{n.s.}$). This suggests that the curve of the probability weighting function for the Dai-Ichi Kangyo Bank is "below" the curve for the Deutsche Bank (see also the results in Table 3). Moreover, ΔSUM_{wq} is significantly smaller for the Dai-Ichi Kangyo Bank than for the Deutsche Bank. This can be explained by a significantly lower sum of judged probabilities for binary complementary events for the Dai-Ichi Kangyo Bank than for the Deutsche Bank (see Table 2). However, in absolute magnitude ΔSUM_{wq} values are smaller than corresponding ΔSUM_w values. Source preference of the weighting function, as reported in Table 1, is driven by both the lack of binary complementarity of judged probabilities and the elevation of the probability weighting function.

Hypothesis 3c states that the probability weighting function reveals higher source sensitivity for more familiar sources of uncertainty than for less familiar sources of uncertainty, i.e., $\overline{LSA_w(D)} \leq \overline{LSA_w(J)}$ and $\overline{USA_w(D)} \leq \overline{USA_w(J)}$. This leads to $\overline{SS_w(D)} \geq \overline{SS_w(J)}$ with $\overline{SS_w(\cdot)} = 1 - \overline{LSA_w(\cdot)} - \overline{USA_w(\cdot)}$. Table 4 shows that the median of LSA_w is smaller for the Deutsche Bank than for the Dai-Ichi Kangyo Bank (true for at least 56%, $p = \text{n.s.}$). Similarly, USA_w is smaller for the Deutsche Bank than for the Dai-Ichi Kangyo Bank (true for at least 56%, $p = \text{n.s.}$). Accordingly, SS_w of the Deutsche Bank is significantly larger than of the

Dai-Ichi Kangyo Bank. These results support Hypothesis 3c. For the linear-in-log-odds-form at least 63% of the subjects show that pattern ($p < 0.05$), while for the linear approximation at least 61% show it ($p < 0.07$). The probability weighting function of the more familiar source reveals less subadditivity and higher source sensitivity than the probability weighting function of the less familiar source of uncertainty. In contrast, the differences between the two stocks are not significant for LSA_{wq} and USA_{wq} . This result reflects the nonsignificant differences in subadditivity of probability judgments between the two stocks reported in Table 2. Summarizing, the difference in source sensitivity of the weighting function reported in Table 1 is primarily driven by the shape of the probability weighting function.

5. Summary

In this paper, we have presented a two-stage approach to explain decision weights in the context of decision making under uncertainty. We assumed and discussed a model for decision weights, where decision makers first judge the probability of an event and then transform this probability using a probability transformation function under uncertainty. This transformation function was allowed to vary with the degree of ambiguity of the event. The decomposition of decision weights into probability judgments and probability transformation made it possible to derive hypotheses about the relative influence of both components assumed to be contributing to decision weights.

In the empirical part of our study, we tested for the relative effects of and interactions among all three types of variables: decision weights, probability judgments, and probability transformation. Decision weights (using sequences of choices) and probabilities (asking subjects to judge uncertain events) formed the primitives of our analysis, which considered two types of uncertain events: bets on the stock price of Deutsche Bank and bets on the stock price of Dai-Ichi Kangyo Bank. We confirmed the usual findings for decision weights. In addition, our data indicate that properties of probability judgments were partially influenced by the source of uncertainty. The properties of the probability weighting function were also

significantly influenced by the source of uncertainty. We therefore concluded that source dependence influences probability judgments as well as probability transformation (as assumed in our model).

To understand more about the two-stage model, in future research, it would be interesting to manipulate the source preference within subjects, to see how the different components of the model react. It would also be worthwhile to extend the formal treatment in the light of Wakker (2001), who presented a theoretical foundation for the simple two-stage approach. In addition, a joint estimation of a subject's value function and decision weights should be performed.

The decomposition of decision weights advocated here might lead to a better understanding of some economic phenomena. In a cross-cultural study with subjects from the United States and Germany, Kilka and Weber (2000) asked subjects to give probability estimates on future stock prices in the United States and Germany. They found that subjects were more optimistic about those stocks they felt more competent about than about those they felt less competent about. This dependency of properties of probability judgment on sources of uncertainty can be understood in the light of a two-stage model. The model should also be applicable to economic situations where probability judgments and choices are made by different agents.

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