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MEASURING NONMONETARY UTILITIES IN UNCERTAIN CHOICES: THE ELLSBERG URN

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Several years ago Ellsberg [1] raised some very fundamental decision theoretic questions regarding the probabilistic interpretation of experimental choice behavior in contingency situations not characterized by von Mises type public probability measures. Fellner [2] has since elaborated what he calls a semiprobabilistic interpretation of the kind of behavior (revealed in various experimental settings) which violates the Savage axioms of personalistic expected utility theory.

My purpose here is to explicate the utility interpretation of these violations by means of the expected utility calculus, and to suggest a scheme for measuring the utility of money rewards in such contingency situations. Fellner speaks of the utility interpretation of such violations as "Version 2," but prefers to discuss the problem in terms of the "Version 1," probabilistic interpretation.

Let me make my own position clear at the outset: I stand with those, like Savage, Raiffa, and Schlaifer, who say they would not want to violate the axioms consciously. The axioms and their implications are a guide to patterns of behavior under uncertainty which I find desirable for me. If *I* violate the axioms I have made a mistake. Certainly, I *think* this would be my position in most ordinary decision situations under uncertainty, although I am not prepared to rule out the possibility that under some circumstances I would find the decision implications of the axiom unacceptable.

However, having stated this, I am not prepared to assert that he who seriously and consciously violates the axioms, and in my judgment "knows what he is doing," is thereby simply making a mistake," and should be given a little more conditioning and "educa-

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1. D. Ellsberg, W. Fellner, and H. Raiffa, "Symposium: Decisions under Uncertainty," this *Journal*, LXXV (Nov. 1961), 643–94. See also H. Roberts, "Risk, Ambiguity, and the Savage Axioms: Comment," this *Journal*, LXXVII (May 1963).
2. W. Fellner, *Probability and Profit* (Homewood, Ill.: Irwin, 1965).

tion." To me, his thoughtful choices are inviolate, and Fellner is right in exploring the implications of such violations. Even if the axioms are to be regarded as basically a normative theory, that theory can also do valuable service in helping us to understand actual behavior. But I do not care for the probabilistic interpretation of the violations. To me probabilities are probabilities in the sense of nonnegativity, additivity and the property of the unit measure over the whole event space. I grant the right of a man to have systematic and deliberate preferences for rewards based on dice game contingencies over the same rewards based on Dow-Jones stock price contingencies. But if he insists also that he is less than certain that the Dow-Jones average will either rise or not rise by five points or more tomorrow, then so far as I am concerned he is now making a "mistake." He does not understand what is (or should be) meant by probability. He is entitled to his tastes, but not to any new definitions of probability. Fortunately, this is not what subject violators do. They merely violate the axioms, without the necessity for the probabilistic interpretation.

As I see it, it is much more plausible to say that violators in "nonstandard process" contingencies, such as the stock price example, suffer utility losses (or gains) relative to what is experienced in less controversial "standard process" contingencies, such as dice games. In nonstandard process collectives, characterized by subjective probabilities which vary widely among individuals, there may be real or imagined elements of skill which increase or reduce the subjective value of the outcomes "lose" or "win." Alternatively, an individual may have a low psychological tolerance for the "ambiguity" associated with nonstandard process events, which we can very reasonably and naturally describe in terms of "utility losses." Thus, if a man loses a dice game bet he and his associates might consider that he was merely the victim of bad luck, and the poor showing may be socially excusable. But if he loses from incorrectly predicting a rise in the Dow-Jones, he may perceive that his colleagues feel that he should have known better, that he is not so smart after all, that they are glad to see his "ignorance" revealed, and so on. Or, if he knows nothing about the stock market, then the mysteries and ambiguities in the Dow-Jones may generate special uncomfort and anxieties when he gambles on such contingencies. In all such cases, we are simply saying that the utility of money or other rewards is not independent of the circumstances under which it is obtained. The utilities in the payoff matrix may have arguments other than what appears to be the "objective" reward.

I. The Ellsberg Problem

In the Ellsberg counterexample we imagine confronting a subject with two urns. Urn I contains 100 red or black balls. The number of red and the number of black are not specified, nor is there specified a *procedure* whereby the composition of balls in the urn is determined. Urn II contains 50 red and 50 black balls.

Define the following four "random" variables or contingencies based on a random draw from Urns I or II:

Urn I	Urn II
N red, 100–N black N unknown	50 red, 50 black

$$R_1 = \begin{cases} x, \text{ if red } (p_1 \text{ unknown}) \\ 0, \text{ if black } (q_1 \text{ unknown}) \end{cases} \qquad R_2 = \begin{cases} x, \text{ if red } (p_2 = \frac{1}{2}) \\ 0, \text{ if black } (q_2 = \frac{1}{2}) \end{cases}$$

$$B_1 = \begin{cases} 0, \text{ if red } (p_1 \text{ unknown}) \\ x, \text{ if black } (q_1 \text{ unknown}) \end{cases} \qquad B_2 = \begin{cases} 0, \text{ if red } (p_2 = \frac{1}{2}) \\ x, \text{ if black } (q_2 = \frac{1}{2}) \end{cases}$$

The results of Ellsberg's interrogations and Fellner's similar experiments with monetary rewards are that many subjects show the following pattern of tastes:

1. $R_1 \sim B_1$
2. $R_2 \sim B_2$
3. $R_2 P R_1$
4. $B_2 P B_1$

where $\sim$ means "indifferent to" and P means "preferred to." That is, the subjects are indifferent between the contingencies "win \$$x$ on red, \$0 on black," and "win \$0 on red, \$$x$ on black" in Urn I, and similarly for Urn II. But as between Urn I and Urn II, "win \$$x$ on red, \$0 on black" in Urn II is preferred over the same contingency in Urn I, and similarly "win \$0 on red, \$$x$ on black" in Urn II is preferred over the same contingency in Urn I. There is a consistent preference for Urn II contingencies over Urn I contingencies.

II. Probabilistic Interpretation

Assume now that we can represent an individual's choices in regard to R_2 and B_2, in terms of an expected utility calculation using the "objective" probabilities $p_2 = q_2 = \frac{1}{2}$. Assume further that his choices involving R_1 and B_1 can be represented by an expected utility calculation in which he behaves as if he imputes subjective probability weights, or degrees of belief, p_1 to red and q_1 to black. Then, if $u(m)$ is the utility of m units of money:

$$(2.1)\qquad R_1 \sim B_1 => E[U(R_1)] = p_1 u(x) + q_1 u(0) = E[U(B_1)] = p_1 u(0) + q_1 u(x),$$

and, therefore, $p_1 = q_1$, if $u(x) \neq u(0)$. Indifference between red and black in Urn I implies that red and black are equally probable (but not necessarily $= \frac{1}{2}$). Also,

$$(2.2)\qquad R_2 \sim B_2 => E[U(R_2)] = p_2 u(x) + q_2 u(0) = E[U(B_2)] = p_2 u(0) + q_2 u(x),$$

which is consistent with the hypothesis that the individual acts as on an expected utility calculation, with his subjective probabilities, p_2 and q_2, equal to the "objective" value $p_2 = q_2 = \frac{1}{2}$. Behaviorally we can only say that $p_2 = q_2$, but we assume for the individual that he takes $p_2 = q_2 = \frac{1}{2}$, as in the probability textbooks.

We also have

$$(2.3)\qquad R_2 P R_1 => E[U(R_2)] = p_2 u(x) + q_2 u(0) > E[U(R_1)] = p_1 u(x) + q_1 u(0)$$

and

$$(2.4)\qquad B_2 P B_1 => E[U(B_2)] = p_2 u(0) + q_2 u(x) > E[U(B_1)] = p_1 u(0) + q_1 u(x).$$

Hence, adding the inequations (2.3) and (2.4), $p_2 + q_2 > p_1 + q_1$, if $u(0) + u(x) > 0$; therefore $p_1 + q_1 < 1$, in violation of the Savage axioms and the usual probability calculus.

III. Utility Interpretation

In Urn I we write p_1 for the "unknown" subjective probability of red, on a single draw, just as we might write p for the "unknown" subjective probability that the Vietnam War will end in December. Moreover, in the Ellsberg process, we can surely write $p_1 = \frac{N}{100}$, and $q_1 = \frac{100-N}{100}$ for Urn I. (This follows if we are justified in writing $p_2 = \frac{50}{100}$, $q_2 = \frac{50}{100}$ for Urn II, and if we cannot do the latter, then we cannot even assert $p_2 = q_2 = \frac{1}{2}$ for the "standard process" Urn II.) It just happens that for Urn I we are totally ignorant of numerical values for p_1 and q_1. It seems very difficult, at this point, to avoid the conclusion that $p_1+q_1=\frac{N}{100}+\frac{100-N}{100}=1$, whatever the values of p_1 and q_1.

Now assume that utility depends upon the prize $\$x$, *and* the contingency situation under which x is won, but not upon the event (e.g., no color preference). We use the notation u_1 for utility in Urn I, u_2 for Urn II.

From the taste pattern above we can state:

(3.1) $R_1 \sim B_1 => E[U(R_1)] = p_1u_1(x)+q_1u_1(0) = E[U(B_1)] = p_1u_1(0)+q_1u_1(x)$,

and $p_1=q_1$. Since $p_1+q_1=1$, $p_1=q_1=\frac{1}{2}$.

(3.2) $R_2 \sim B_2 => E[U(R_2)] = p_2u_2(x)+q_2u_2(0) = E[U(B_2)] = p_2u_2(0)+q_2u_2(x)$.

As before, this is consistent with $p_2=q_2=\frac{1}{2}$.

(3.3) $R_2PR_1 => E[U(R_2)] = p_2u_2(x)+q_2u_2(0) > E[U(R_1)] = p_1u_1(x)+q_1u_1(0)$.

But from (3.1) and (3.2), $p_1=q_1=p_2=q_2=\frac{1}{2}$, and, the

$u_2(x)+u_2(0) > u_1(x)+u_1(0)$

(3.4) $B_2PB_1 => E[U(B_2)] = p_2u_2(0)+q_2u_2(x) > E[U(B_1)] = p_1u_1(0)+q_1u_1(x)$,

and, from (3.1) and (3.2), $u_2(x)+u_2(0) > u_1(x)+u_1(0$ preferences are consistent provided that $u_2(m) > u_1(m)$

IV. Measuring Nonmonetary Utility

From the above analysis, our subject is behaving as if he suffered utility losses at all levels of money reward m for Urn I contingencies as compared with Urn II contingencies. Suppose we assume that such losses are additive with monetary utilities. Then

(4.1) $u_2(m) = u_1(m) + \lambda(m)$

where $\lambda(m)$ is utility loss, due to ambiguity, in situation 1.

$u_1(m)$ and $\lambda(m)$ can be measured, given an individual's von Neumann-Morgenstern utility function $u_2(m)$ for standard process collectives (in this case, Urn II), by finding the premium $\pi(x)>0$, for each $x>0$, such that $R'_1 \sim R_2$, where

$$R'_1 = \begin{cases} x+\pi(x), & p_1 = \frac{N}{100} = \frac{1}{2} \text{ (since } R_1 \sim B_1\text{)}, \\ 0, & q_1 = \frac{100-N}{100} = \frac{1}{2} \text{ (since } R_1 \sim B_1\text{)}; \end{cases}$$

$$R_2 = \begin{cases} x, & p_2 = \frac{1}{2}, \\ 0, & q_2 = \frac{1}{2}. \end{cases}$$

Given x, and $\pi(x)$, such that $R'_1 \sim R_2$, we can write

$$E[U(R'_1)] = \tfrac{1}{2}u_1[x+\pi(x)] + \tfrac{1}{2}u_1(0) =$$
$$E[U(R_2)] = \tfrac{1}{2}u_2(x) + \tfrac{1}{2}u_2(0).$$

If we assume $u_1(0) = u_2(0)$, then

(4.2) $\quad u_1[x+\pi(x)] = u_2(x)$;

or, from (4.1)

(4.3) $\quad \lambda[x+\pi(x)] = u_2[x+\pi(x)] - u_1[x+\pi(x)]$
$\qquad\qquad = u_2[x+\pi(x)] - u_2(x).$

Note that $u_1(0) = u_2(0)$ implies $\pi(0) = 0$.

A graphic illustration of the measured functions $u_1(m)$, $\lambda(m)$, given $u_2(m)$, is shown below:

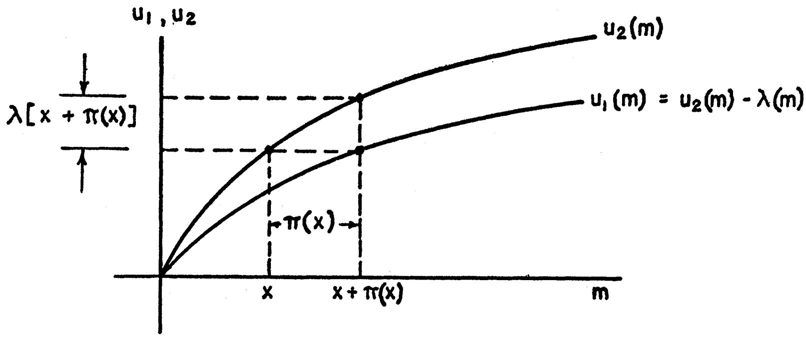

FIGURE I

In closing, I would conjecture that the kind of choice behavior revealed by the Ellsberg counterexample is not just a characteristic of "known" versus "unknown" probability situations. I suspect that the preference for Urn II gambles over Urn I gambles would be little changed for many, if not most, subjects, if they were guaranteed that the number of red balls in Urn I had been determined by a random draw from the integers 0–100. Urn I is then a 50–50 gamble, except that it is a compound gamble, rather than a simple gamble as in Urn II. The question then is not "Are there uncertainties that are not risks?", as posed by Ellsberg, but "Are there risks that are not risks?"

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