Review of Economic Studies (1980) XLVII, 305-320
0034-6527/80/00150305 $02.00

Adaptive Expectations and Uncertainty

T. LAWSON
University of Cambridge

1. INTRODUCTION AND SUMMARY

According to the simple adaptive expectations hypothesis, expectations adjust by a constant proportion of the previous discrepancy:

$$\pi_{t+1}^e = \pi_t^e + \lambda(\pi_t - \pi_t^e), \quad 0 \leqq \lambda \leqq 1 \qquad \text{...(1.1.1)}$$

where π_t is the series being predicted and π_t^e is the expectation of π_t formed at $t-1$.

This model is widely used by economists; partly for theoretical reasons[1] but largely for properties it possesses which are convenient for econometric work.[2] In the field of inflation-rate expectations there is also some empirical support. This has been obtained by estimating the model on expectations data collected through sample surveys.[3]

Such work has also revealed two possible limitations of the model, and it is these possible limitations which provide the major motivation of this paper. First the empirical results seem to contradict the explicit assumption that the adjustment coefficient, λ, remains constant throughout. In fact the evidence suggests that λ increases in times of accelerating inflation and general uncertainty.[4] The second problem is that when the inflation-rate is accelerating, the model is certain to produce inaccurate forecasts.[5] Empirical work has revealed that higher order error-learning mechanisms are better at explaining the data for such periods.[6]

In Sections 2–5 a modified adaptive expectations model is developed which is consistent with the empirical evidence. It is found, in Sections 2 and 3 that by introducing certain realistic elements of uncertainty, an adaptive model with a time-varying coefficient is the result. The model is subsequently extended in Section 4 to allow expectations to be formed of the incremental growth rate of inflation.

Finally in Section 6 an expectations series derived from the model, is used as an explanatory variable in an econometric analysis of personal savings behaviour.

2.1. THE MODEL

A key assumption is that each economic agent possesses a subjective probability distribution concerning the rate of change of his " own " price index which varies from period to period. So for any named future date the agent assigns to all possible outcomes a place on a scale measuring his degree of belief that the outcome will be realized. With the passage of time as new relevant information comes to light, the agent's degree of belief in all outcomes will be updated. Prices will differ from market to market due to fluctuations in relative demands and so forth; and so agents will form beliefs concerning the rate of change of the prices of commodities which they themselves buy.

These markets may be separated spatially for example, or perhaps more importantly by social economic class. We shall be considering agents in one market only.

We further take the view that observations obtained in this market are composed of two parts which cannot be separately identified. The first is a " normal " or " permanent " component which is present in all markets. The second is a " transitory " or random " noise " term, with an expected value of zero. The noise component is present in this market but may not be in any of the others.

There is some evidence to suggest that agents are aware that (or act as if) this situation prevails.[7] They thus set about disentangling, or " filtering-out ", the permanent component from the observation; and subsequently use it as the basis for the period ahead prediction. To make the model operational we shall indeed assume that agents act in this manner. In other words we assume that agents interpret the permanent component as the deterministic part of the observation; believing there to exist a functional relationship between it and the permanent component of the next period. In fact we initially make the simple though not unrealistic assumption that people believe that the inflation rate over the ensuing period will remain constant. However if *ex post* it is discovered that this model has produced a significant forecast error the agent may decide that the permanent component has undergone a " step-change ".

Denoting the permanent component at time t by π_t, the " noise " or transitory term by ε_t and the observation obtained at time t by M_t, then the agent's belief concerning the system may be described by the following equations:

$$M_t = \pi_t + \varepsilon_t \qquad ...(2.1.1)$$

$$\pi_t = \pi_{t-1} + u_t \qquad ...(2.1.2)$$

where it is assumed that ε_t and u_t are independently distributed with zero means. In the absence of contrary evidence we shall also assume throughout that all random variables are normally distributed.[8]

Finally and consistent with much of the empirical evidence referred to in Section 1 we assume that present and past realizations of the inflation-rate are the only determinants of inflationary expectations.[9]

2.2. NOTATION

It is convenient at this stage to introduce some notation which will facilitate the exposition of the model. We denote by $\hat{\pi}((t+k)/(t-j))$ the agent's conditional estimate of π_{t+k} formed at $t-j$. Similarly the density function of π_{t+k} formed at $t-j$ will be denoted $\pi((t+k)/(t-j))$. In this notation the simple adaptive schema of Section 1 can be rewritten:

$$\hat{\pi}((t+1)/t) = \hat{\pi}(t/(t-1)) + \lambda(M_t - \hat{\pi}_t(t/(t-1))); \quad 0 \leqq \lambda \leqq 1 \qquad ...(2.2.1)$$

3.1. VARIABLE ADJUSTMENT COEFFICIENT

With this view of the world it follows that the prior expectation of the π_t held at $t-1$ (denoted $\hat{\pi}(t/(t-1))$) will be revised after the observation M_t has been observed providing a new estimate of π_t (denoted $\hat{\pi}(t/t)$).

If an adaptive mechanism similar to equation (1.1.1) is appropriate for modelling this revision, then there are two types of uncertainty which might be expected to influence the size of the adjustment coefficient, λ, and so cause it to be time varying. The first relates to the agent's degree of belief that the observation M_t is free of " noise "; that is that ε_t will be zero. The second relates to the degree of belief attached to the prior estimate of π_t. If the degree of belief that M_t is *un*disturbed by noise is relatively large then *ceteris paribus* relatively little weight will be given to M_t when calculating $\hat{\pi}(t/t)$. Similarly this will also be the outcome if *ceteris paribus* a relatively high degree of belief is attached to $\hat{\pi}(t/(t-1))$. In these situations λ could be expected to be relatively small. We shall see that having introduced these elements of uncertainty, an adaptive model with an adjustment coefficient which varies through time in the suggested manner, is precisely the result.

We have supposed that the prior distribution $\pi(t/(t-1))$ is normal; therefore it can be characterized by its first and second moments. The mean is $\hat{\pi}(t/(t-1))$, and the variance, which reflects the agent's degree of belief in his estimate, is denoted Σ_t. Denoting the variance of ε_t in equation (2.1.1) by W_t and the variance of u_t in equation (2.1.2) by Y_t, and using simple conditional probability theory (see Appendix A), it is now possible to derive the agent's posterior distribution of π_t after M_t has been observed.[10]

$$\pi(t/t) \sim N(\hat{\pi}(t/t); \quad \sigma_t) \qquad \ldots(3.1.1)$$

where

$$\hat{\pi}(t/t) = \hat{\pi}(t/(t-1)) + \lambda_t(M_t - \hat{\pi}(t/(t-1))) \qquad \ldots(3.1.2)$$

and

$$\lambda_t = \frac{\Sigma_t}{W_t + \Sigma_t} \qquad \ldots(3.1.3)$$

and where

$$\sigma_t = \Sigma_t - \frac{\Sigma_t^2}{W_t + \Sigma_t}. \qquad \ldots(3.1.4)$$

Clearly[11] the smaller the agent's degree of belief that noise will be absent from the observation, the larger will be W_t, and thus the smaller *ceteris paribus* will be λ_t. On the other hand if the agent's degree of belief in his prior estimate $\hat{\pi}(t/(t-1))$ is relatively small, then it will be reflected in a relatively large Σ_t, and so λ_t.

It follows from (2.1.2) that the agent's posterior estimate of π_t will also be his prior forecast of π_{t+1}:

$$\hat{\pi}((t+1)/t) = \hat{\pi}(t/t). \qquad \ldots(3.1.5)$$

The agent's prior distribution of π_{t+1} can therefore be written:

$$\pi((t+1)/t) \sim N(\hat{\pi}((t+1)/t); \quad \Sigma_{t+1}) \qquad \ldots(3.1.6)$$

where

$$\hat{\pi}((t+1)/t) = \hat{\pi}(t/(t-1)) + \lambda_t(M_t - \hat{\pi}(t/(t-1))) \qquad \ldots(3.1.7)$$

$$\lambda_t = \frac{\Sigma_t}{W_t + \Sigma_t} \qquad \ldots(3.1.8)$$

and

$$\Sigma_{t+1} = \sigma_t + Y_t. \qquad \ldots(3.1.9)$$

Clearly $\Sigma_{t+1} \geqq \sigma_t$ reflecting the possibility that u_t in equation (2.1.2) may be different from zero thus providing an extra element of uncertainty.

The result is a modified adaptive expectations hypothesis similar to (2.2.1) but with a time-varying coefficient of adjustment. In fact we have obtained a recursive algorithm with which in principle an expectations series can be derived. First, however, it is necessary to describe the method by which the initial prior distributions are found; and also how the posterior estimate of the error variance terms, W_t and Y_t, are obtained. The discussion of these problems is postponed until Section 5.

3.2. INFLATION PROCESS STATES

Some properties of the model can be brought into focus by considering some numerical examples.

First consider the extreme case in which the agent at time $t-1$ is very confident in his prior estimate of π_t. Assume also that he attaches a relatively low degree of belief to the possibility of M_t being undisturbed by noise.

Thus Σ_t is relatively small, let us say 1·0 and W_t is large at say, 100. Let us suppose that the measurement obtained at t is 12 per cent, whilst the prior prediction formed at $t-1$ is only 8 per cent. The posterior estimate of the normal component can be found from (3.1.2):

$$\begin{aligned}\hat{\pi}(t/t) &= \hat{\pi}(t/(t-1)) + \lambda_t(M_t - \hat{\pi}(t/(t-1))) \qquad \ldots(3.2.1)\\ &= 8 + \lambda_t(12-8)\end{aligned}$$

where

$$\lambda_t = \frac{\Sigma_t}{W_t+\Sigma_t} = \frac{1}{100+1} \simeq 0{\cdot}01. \quad \text{...(3.2.2)}$$

Thus

$$\hat{\pi}(t/(t+1)) = \hat{\pi}(t/t) \simeq 8{\cdot}04 \quad \text{...(3.2.3)}$$

We shall call an observation which contains a relatively large proportion of noise, a transient. Clearly when the agent believes a transient is occurring, he as good as ignores it.

Contrast this result with the outcome when at time $t-1$ the agent is fairly confident in his posterior estimate of π_{t-1} but attaches an extremely low degree of belief to the possibility that the realization of u_t will be zero. Clearly he will not be greatly surprised if the systems equation, $\pi_t = \pi_{t-1}$, does not hold. Assume also that he assigns a relatively high degree of belief to the possibility of zero noise in the measurement M_t. This situation can be characterized by a relatively high Y_t, let us say 99, a small σ_{t-1}, say 1, and a relatively small W_t, let us say 1 again. Assume as in the last example that the measurement obtained at t is 12 per cent whilst the agent predicted 8 per cent. Proceeding as before we find that in this situation:

$$\lambda_t = \frac{99+1}{99+1+1} \simeq 0{\cdot}99 \quad \text{...(3.2.4)}$$

and

$$\hat{\pi}((t+1)/t) = \hat{\pi}(t/t) = 11{\cdot}96 \quad \text{...(3.2.5)}$$

Clearly the agent is prepared to believe that a structural change, we shall call it a " step-change ", is occurring to the system; with the result that he adjusts almost completely to the current measurement.

These examples represent extreme situations in which the agent either adjusts completely to the current observation or he does not adjust at all. In most situations the agent will neither be certain that the noise content of his observation is zero, nor will he attach full weight to his prior estimate. In particular he will probably assume the relationship $\pi_t = \pi_{t-1}$ holds, but will expect some variation in the observed series, due to a modest amount of randomly distributed noise. Consequently he will continually adapt slightly, but not completely, to the current measurement, hoping to obtain a satisfactory estimate of the " true " normal component. If the inflation-rate series does indeed exhibit these characteristics we shall refer to it as being in the " no-change " state. The situation in which the agent believes the no-change state will occur can be characterized by a relatively small W_t and a zero Y_t.

Thus if system (2.1.1) and (2.1.2) is used by the agent when forming his expectations, it is possible that he will distinguish three different inflationary process states. These are the no-change, step-change, and transient states, which can be characterized by the following error-variance descriptions:

	W_t	Y_t	
Transient	large	zero	
Step Change	small	large	...(3.2.6)
No Change	small	zero	

4.1. A TRENDED INFLATION RATE

It is of course the case that historically the inflation rate has exhibited a non-zero trend, which has also been non-constant. We therefore extend the system outlined in the previous sections and model the incremental growth rate of inflation, g_t, as a random walk process. Maintaining the assumption that disturbances are independently and normally distributed it is now possible to formulate a more realistic model of the agent's view of the inflationary process:

$$M_t = \pi_t + \varepsilon_t \qquad \varepsilon_t \sim N(0;\ W_t) \qquad \ldots(4.1.1)$$

$$\pi_t = \pi_{t-1} + g_t + u_t \quad u_t \sim N(0;\ Y_t) \qquad \ldots(4.1.2)$$

$$g_t = g_{t-1} + v_t \qquad v_t \sim N(0;\ Z_t). \qquad \ldots(4.1.3)$$

It is clearly possible to extend the system by adding rates of change of growth rates, and so on. Which order of process to " stop-at " will depend on the series being predicted and to some extent on the particular agents making the forecasts. Judging from recent UK experience, a second order process is sufficient to allow the decision maker the possibility of forecasting accurately. In any case the problem of specifying which variables expectations are formed over can be overcome at a theoretical level by transforming to vector notation.

Let $\underline{P}$ be the vector whose elements consist of those variables, such as the level of the inflation-rate, its incremental growth rate, and so on, over which expectations are assumed to be formed.

In the above example, assuming the system described by (4.1.2) and (4.1.3) we obtain:

$$\underline{P}_t = \begin{pmatrix} \pi_t \\ g_t \end{pmatrix}; \quad \text{let } \underline{\alpha}_t = \begin{pmatrix} u_t \\ v_t \end{pmatrix} \qquad \ldots(4.1.4)$$

and assume

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim N\left(\begin{matrix} 0 \\ 0 \end{matrix};\ \underline{F}_t\right) \quad \text{so that } \underline{F}_t = \begin{pmatrix} Y_t & 0 \\ 0 & Z_t \end{pmatrix}. \qquad \ldots(4.1.5)$$

Let

$$\underline{S}_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}; \quad \underline{S}_2 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \underline{X} = (1 \quad 0).$$

We can thus rewrite the system as

$$M_t = \underline{X}\underline{P}_t + \varepsilon_t \qquad \varepsilon_t \sim N(0;\ W_t) \qquad \ldots(4.1.6)$$

$$\underline{P}_t = \underline{S}_1\underline{P}_{t-1} + \underline{S}_2\underline{\alpha}_t \quad \underline{\alpha}_t \sim N(0;\ \underline{F}_t). \qquad \ldots(4.1.7)$$

At time $t-1$ the agent possesses a prior distribution of $\underline{P}_t$. We have supposed it to be normal and so it is characterized by its mean $\hat{\underline{P}}(t/(t-1))$ and variance–covariance matrix $\underline{\Sigma}_t$.

$$\underline{P}(t/(t-1)) \sim N(\hat{\underline{P}}(t/(t-1));\ \ \underline{\Sigma}_t). \qquad \ldots(4.1.8)$$

Using results from conditional probability theory (see Graybill (1961, for example Theorem 3.10, p. 63)) we can determine the posterior distribution $\underline{P}(t/t)$ after new information M_t is observed:

$$\underline{P}(t/t) \sim N(\hat{\underline{P}}(t/t);\ \ \underline{\sigma}_t) \qquad \ldots(4.1.9)$$

where

$$\hat{\underline{P}}(t/t) = \hat{\underline{P}}(t/(t-1)) + \underline{\lambda}_t(M_t - \underline{X}\hat{\underline{P}}(t/(t-1))) \qquad \ldots(4.1.10)$$

$$\underline{\lambda}_t = \underline{\Sigma}_t\underline{X}'(\underline{X}\underline{\Sigma}_t\underline{X}' + W_t)^{-1} \qquad \ldots(4.1.11)$$

and

$$\underline{\sigma}_t = \underline{\Sigma}_t\underline{X}'(\underline{X}\underline{\Sigma}_t\underline{X}' + W_t)^{-1}\underline{X}\underline{\Sigma}_t \qquad \ldots(4.1.12)$$

From (4.1.10), (4.1.12) and (4.1.7) the prior distribution of $\underline{P}_{t+1}$ is similarly derived:

$$\underline{P}(t+1)/t) \sim N(\hat{\underline{P}}((t+1)/t);\ \ \underline{\Sigma}_{t+1}) \qquad \ldots(4.1.13)$$

where

$$\hat{\underline{P}}((t+1)/t) = \underline{S}_1\hat{\underline{P}}(t/t) \qquad \ldots(4.1.14)$$

and where

$$\underline{\Sigma}_{t+1} = \underline{S}_1\sigma_t\underline{S}_1' + \underline{S}_2\underline{F}_t\underline{S}_2'. \qquad \ldots(4.1.15)$$

The result is a modified adaptive expectations mechanism, which does not suffer from the suggested limitations of the simple adaptive hypothesis. The adjustment coefficients are time varying in an a-priori sensible fashion and the model is no longer certain to produce inaccurate forecasts when the inflation-rate is trended.

In its present form the model is similar to those which have long existed in control theory and operations-research journals.[12] The interpretation here is different. However we are able to refer to this literature for efficient methods of making the model functional.

First however we refer to Section 3 where it was found that three possible states of the inflationary process could occur under system (2.1.1) and (2.1.2). These were the no-change, step-change and transient states. The system has now been extended to include an incremental growth variable and is described by equations (4.1.1), (4.1.2) and (4.1.3). As a result there is the additional " extreme " situation to consider in which the agent attaches a very low degree of belief to the possibility of the realization of v_t being zero, when he confidently expects M_t to be relatively undisturbed by noise. Such a structural change representing a (temporary) breakdown in the System (4.1.3) will be called a slope-change.

There are now four basic process states in the model which may be characterized by the following error-variance descriptions.

	W_t	Y_t	Z_t
No change	Small	Zero	Zero
Step change	Small	Large	Zero
Slope change	Small	Zero	Large
Transient	Large	Zero	Zero

5. SETTING THE ERROR-VARIANCES

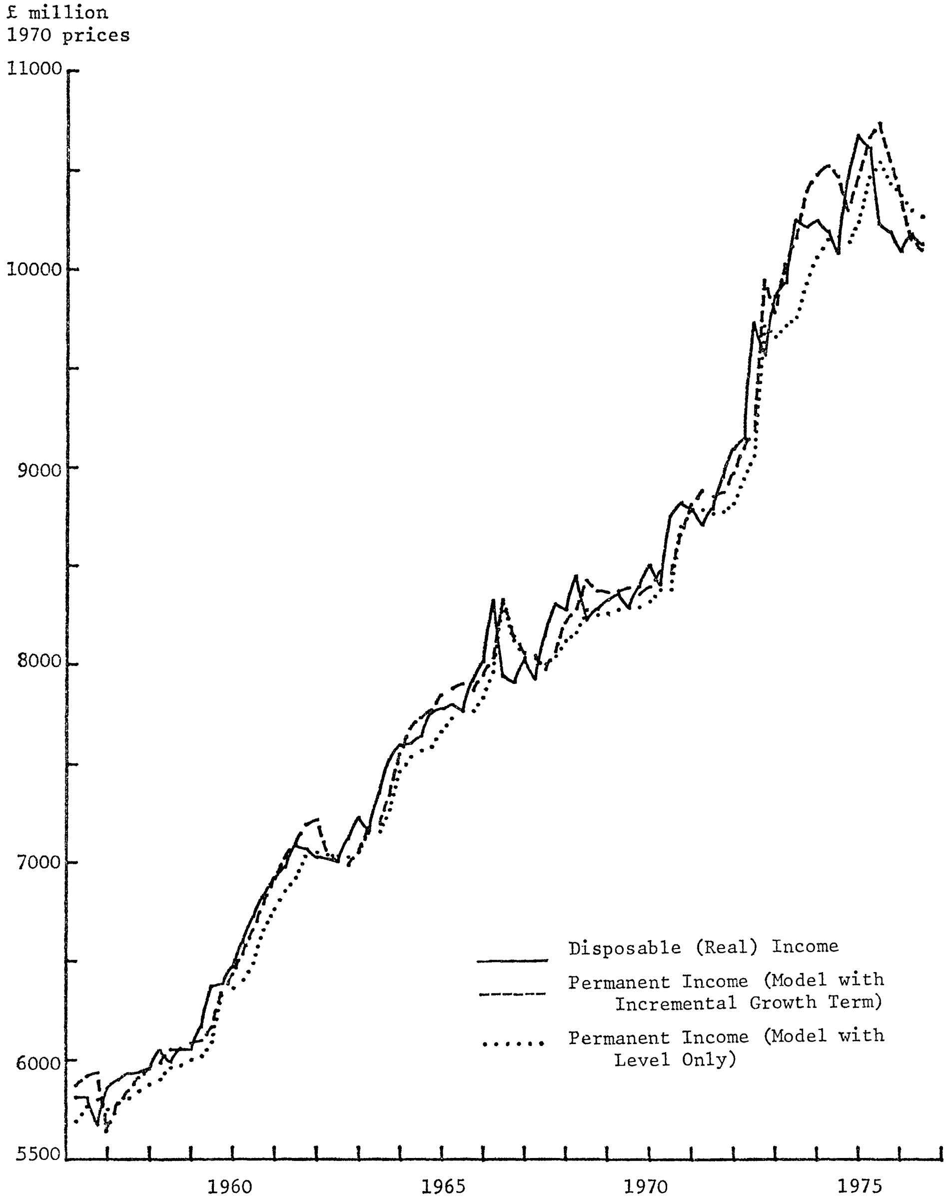
We are set to describe how the agent obtains his beliefs about the current state of the inflationary process. This is equivalent to deriving the current estimates of W_t, Y_t and Z_t.

The method adopted is an adapted version of that proposed by Harrison and Stevens (1971), in the context of demand forecasting. Briefly the idea is to assign four sets of values to the error-variances; one describing each of the four process states defined in Section 4. Having done so, and assuming that we know the prior distribution at some point, then using the formulae of Section 4, the posterior estimate and the forecast of the following periods inflation-rate can be obtained for each process state. Thus one forecast is derived using the error-variance magnitudes which characterize the no-change state; another is obtained under the step-change state description, and so on. At any time it is unlikely that the forecaster will know which state obtains, with certainty. He will therefore attach a probability to the event that each state is the correct one. Consequently the overall forecast for any period is found as the weighted average of the separate state forecasts; where the weights are the probabilities that the respective state prevails. The mathematical details concerning the calculation of these probabilities is found in Appendix C.

There is an additional problem arising from this procedure. By starting with one prior and updating in the suggested manner, four posterior estimates are obtained, and also four prior forecasts of the following periods inflation-rate. If this procedure is continued, we now start with four priors, not one as before, and consequently we derive sixteen posteriors and so sixteen prior predictions of the inflation-rate for the following period. As the process continues the number of distributions in each period increases by a factor of four. This result is not incorrect but is clearly computationally unmanageable.

The following procedure is therefore adopted to handle this problem. Starting with four prior distributions of the inflation-rate in some period, the sixteen distributions subsequently arrived at are condensed into four distributions; so that the four distributions with the same current posterior state are combined. Thus the four posterior distributions derived with the step-change state are combined, and so on. Precise, mathematical details of this procedure are found in Appendix D of this paper.

This approach is not entirely satisfactory. Essentially we are making a discrete approximation to what should be a continuous distribution of (W_t, Y_t, Z_t). A more satisfactory approximation might be to define several step-change state descriptions (by assigning a different value to Y_t in each), several slope-change state descriptions, etc. However this would prove very expensive computationally, whilst practical trials with the



Source: C.S.O. (1976)

FIGURE 1

error-variances taking different values, have revealed that this second approximation is unnecessary. As long as certain preconceived beliefs concerning the manner in which expectations are formed are adhered to, the performance of the model does not depend critically on the choice of these parameters. In particular we suggest that agents are more likely to adapt their estimates of the level of the inflation-rate than of its rate of change; and that they are less likely again to interpret forecast errors as changes in a yet higher order of process. Consequently the model is designed to be less sensitive in slope than in level.[13]

Such a model is characterized by the following error-variance magnitudes:

	W	Y	Z	
No-change State	1	0	0	
Step-change State	1	100	0	...(5.1.1)
Slope-change State	1	0	10	
Transient State	101	0	0	

This model was employed to obtain a series representing estimates of the permanent component of real disposable income.[14] This series is graphed in Figure 1 where it is contrasted with the series obtained using the following model, which does not allow expectations to be formed of a rate of growth variable:

	W	Y	
No-change State	1	0	
Step-change State	1	100	...(5.1.2)
Transient State	101	0	

These graphs reveal certain, interesting characteristics of the models. The pattern of the series derived using the model without a slope-change is similar to that which would be produced using the simple adaptive hypothesis of Section 1. It is however somewhat smoother reflecting the refusal to respond to transients. A feature of the second series derived using the extended model (5.1.1) is its close tracking performance over most of the period.[15] It is also evident that following a period of reasonably accurate forecasting, confidence in current predictions is relatively high, so that when an unforeseen structural change occurs the model is slow to respond. This is the situation in 1961 and more markedly so when the downturn in the actual series occurs in 1973. By way of contrast, by the time a similar downturn occurs somewhat later in 1974, confidence in current predictions is lower with the result that the system is quicker to adjust.

6.1. AN APPLICATION IN PERSONAL SAVINGS BEHAVIOUR

A meaningful test of the modified adaptive expectations hypothesis requires a theory in which inflationary expectations are of central importance. Deaton (1976) provides such a theory when he argues that unanticipated changes in the general price level have played a central role in determining the abnormally high savings-ratios recently experienced in the UK. As consumers only possess up-to-date information on the prices of goods they are purchasing, they may not always be able to distinguish relative price changes from absolute price changes. For example if the inflation rate is accelerating, the consumer may find that prices of commodities he is intending to buy are higher than expected. If he erroneously believes that these price changes are associated only with these particular goods he will almost definitely not buy as many as he originally intended. At a given moment different consumers will be shopping for different commodities. As each individual finds that the prices of the particular goods that he wishes to buy are higher than expected he will similarly purchase less than he originally intended. There is therefore " mass illusion that all goods are relatively more expensive so that, as each consumer attempts to adjust his purchases, real consumption falls and if real income is maintained, the savings-ratio rises". Of course consumers will reassess the situation and on discovering their mistakes they will attempt to rectify them. It may be however that expectations are slow to adjust; so that while the inflation-rate continues to accelerate the savings ratio will remain abnormally high.

Deaton formalizes these ideas and obtains the following equation:

$$\frac{\sigma}{\mu} = \frac{\sigma^*}{\mu^*} + \left\{\log\left(\frac{\mu}{P}\right) - \log\left(\frac{\mu}{P}\right)^*\right\} - \phi\{\log P - \log P^*\} \qquad \text{...(6.1.1)}$$

where μ denotes current disposable income, which is taken to be the sum of current expenditure ε and savings σ. The general price level is denoted P.

To present this result in a form capable of being contradicted by the data, Deaton assumes the following supplementary specifications, in which π denotes the price inflation-rate, ρ the rate of change of real disposable income, and the symbol * denotes expected, desired or permanent components as appropriate. Δ is the first different operator.

Aggregate Consumption Function (*Friedman* (1957))

$$\varepsilon^* = k\mu^*; \quad k \text{ constant} \qquad \text{...(6.1.2)}$$

Expectations Mechanisms:

Rate of Change of Income

$$\rho^* = \beta\rho + (1-\beta)\rho^0; \quad 0 \leqq \beta \leqq 1; \quad \rho^0 \text{ constant.} \qquad \text{...(6.1.3)}$$

Rate of Change of Prices

$$\pi^* = \pi^0, \quad \text{a constant.} \qquad \text{...(6.1.4)}$$

Desired Savings Ratio Adjustment Mechanism:

$$\left(\frac{\sigma}{\mu}\right)^*_t - \left(\frac{\sigma}{\mu}\right)^*_{t-1} = \eta\left\{\left(\frac{\sigma}{\mu}\right)^*_{t-1} - \left(\frac{\sigma}{\mu}\right)_{t-1}\right\} = \eta\left((1-k) - \left(\frac{\sigma}{\mu}\right)_{t-1}\right) \qquad \text{...(6.1.5)}$$

Substituting (6.1.2), (6.1.3), (6.1.4) and (6.1.5) into (6.1.1) gives:

$$\Delta\frac{\sigma}{\mu} = \{\eta(1-k) - (1-\beta)\rho^0 + \phi\pi^0\} + (1-\beta)\rho_t - \phi\pi_t - \eta\left(\frac{\sigma}{\mu}\right)_{t-1} \qquad \text{...(6.1.6)}$$

which is the equation Deaton estimates.

Before turning to the empirical analysis we consider some theoretical issues:

(i) The coefficient, ϕ, attached to the inflation-rate variable in equation (6.1.6) has the following interpretation:

$$\phi = \Sigma e_{kk} . w_k^* \qquad \text{...(6.1.7)}$$

where e_{kk} is the (uncompensated) elasticity of demand for good k, and w_k^* is the value share:

$$w_k^* = \frac{p_k^* . q_k^*}{\varepsilon_k^*} \qquad \text{...(6.1.8)}$$

q_k is the quantity of good k that is purchased and p_k is its price. It follows that if demand curves slope downwards then ϕ must be negative. Moreover it can be shown that under the assumption of additive preferences, ϕ is approximately equal to the inverse of the elasticity of the marginal utility of income with respect to income.[16] Several studies have obtained an estimate of this last quantity which suggest that its value is of the order of $-0{\cdot}5$.[17] Consequently we may expect to find that our estimate of ϕ is negative and in the region of $-0{\cdot}5$.

(ii) Equation (6.1.3) specifies that the expected rate of growth of real income is a weighted average of the actual rate of growth and some base or normal rate ρ^0. This is a fair assumption because there has been little apparent trend in the actual values of ρ over the sample period. Consequently it seems reasonable to suppose that the estimate of ρ_0 (if it can be unravelled from the regression parameter estimates) will be approximately equal to the average rate of growth of real disposable income over this period. This figure is calculated to be 0·00771 or 0·77 per cent.

(iii) Equation (6.1.4) is the improbable hypothesis that inflationary expectations have been constant, even during the post 1972 period. A sophisticated theory such as Deaton's in which incorrect inflation-rate expectations play an important role would seem to require a slightly more sophisticated theory of expectations formation. In fact Deaton also experiments with an alternative formulation, but with little success. In the empirical work which we shall report the modified adaptive expectations hypothesis developed in the previous sections is employed. π^* is thus no longer a constant and so the estimating equation becomes:

$$\Delta\frac{\sigma}{\mu} = \{\eta(1-k)-(1-\beta)\rho^0\}+(1-\beta)\rho_t-\phi(\pi_t-\pi_t^*)-\eta\left(\frac{\sigma}{\mu}\right)_{t-1} \qquad \text{...(6.1.9)}$$

(iv) Finally we note the savings-ratio adjustment mechanism (6.1.5) according to which the planned ratio adjusts by a constant proportion of the discrepancy between the previous outcome and a constant long-run ratio. This equilibrium ratio is in turn derived from the Friedman consumption function (6.1.2).

The notion that consumers will adjust desired expenditure patterns in order to compensate for past *flow* ratios having diverged from desired levels seems a little unrealistic. Instead we examine the hypothesis that discrepancies between past savings-ratios and their desired outcomes are treated strictly as bygones; so that the current desired savings-ratio is determined independently of such discrepancies. In fact we assume that permanent income and expected income are identical, from which it follows that the desired savings-ratio is Deaton's equilibrium value, $1-k$, in every period:[18]

$$\varepsilon^* = k\mu^p \equiv k\mu^* \qquad \text{...(6.1.10)}$$

so that

$$\left(\frac{\sigma}{\mu}\right)^* = (1-k). \qquad \text{...(6.1.11)}$$

It therefore follows that

$$\left(\frac{\sigma}{\mu}\right)_t^* - \left(\frac{\sigma}{\mu}\right)_{t-1}^* = (1-k)-(1-k) = 0. \qquad \text{...(6.1.12)}$$

Consequently the equation to be estimated must be reformulated. Using (6.1.1), (6.1.2), (6.1.3) and (6.1.12) the following equation is obtained.

$$\Delta\frac{\sigma}{\mu} = -(1-\beta)\rho^0+(1-\beta)-\phi(\pi-\pi^*). \qquad \text{...(6.1.13)}$$

The outcome is that the lagged savings-ratio term disappears from the equation and the constant term is now negative.[19]

6.2. EMPIRICAL ANALYSIS

The data used in the estimations are quarterly, seasonally adjusted observations on personal disposable income, personal expenditure and prices. The data are for the UK, taken from the Central Statistical Office (1976), for the period 1957(ii) to 1975(ii). Estimations were carried out by a Cochrane–Orcutt type iterative procedure using J. White's Econometrics Computer Program: Shazam.

In all estimations a statistically significant first-order correlation coefficient was obtained in the region of $-0{\cdot}004$. This information is not however repeatedly reported below, except where further information can be drawn from it.

Parameter estimates were first obtained for equation (6.1.9) using data for the whole period:

Sample period	Constant	$(1-\beta)$	$-\phi$	$-\eta$	R^2	DW
1957(ii)–1975(ii) (73 observations)	−0·0049 (1·780)	0·6601 (8·710)	0·4034 (2·638)	0·0081 (0·268)	0·5532	2·1701

NOTES: ( ) denotes t-ratio and DW is the Durbin–Watson statistic.

Perhaps the most interesting result is that the estimate of ϕ has the correct sign attached, and whilst being significantly different from zero is not significantly different from the predicted value of $-0{\cdot}5$. Also apparent is the statistical non-significance of the estimated coefficient of the lagged savings ratio term. This is consistent with equation (6.1.13) and thus provides some support for specification (6.1.10). Further support is suggested by the negative constant term. However this is not statistically significant at the 5 per cent level.

In the light of these results equation (6.1.13) was estimated:

Sample period	Constant	$(1-\beta)$	$-\phi$	R^2	DW	SEE
1957(ii)–1975(ii)	−0·0042 (4·359)	0·6592 (8·779)	0·4116 (2·749)	0·5529	2·1596	0·00867

NOTES: ( ) denotes t-ratio as before and SEE is the standard error of the estimate.

The estimated value of ϕ is again in the region of $-0{\cdot}5$, whilst the constant term, which is negative as predicted, is now statistically significant.

The parameter estimates obtained were found to be fairly stable when equation (6.1.13) was re-estimated over different periods. In fact when the sample period was shortened by omitting several data points from either " end " of the total sample, it was found that the coefficient estimates barely altered. For a formal test the sample was split into two periods so that the last 5 years[20]—twenty data points—were considered separately.

Sample period	Constant	$(1-\beta)$	$-\phi$	R^2	DW	SEE
1957(ii)–1970(ii) (53 observations)	−0·0040 (3·635)	0·6436 (6·973)	0·3686 (1·848)	0·5059	2·1976	0·00859
1970(iii)–1975(ii) (20 observations)	−0·0047 (2·103)	0·6904 (4·581)	0·5099 (1·959)	0·6218	2·0629	0·00942

NOTE: ( ) denotes t-ratio.

Using the standard Chow test of structural stability an $F(4, 65)$ value of 0·488 was obtained on the null hypothesis that the parameters were the same in both periods. The probability of $F(4, 65)$ taking a value greater than 0·488 is 0·744, from which it can be seen that the hypothesis of structural stability is not easily rejected.

Overall these results do provide some support for the view that equation (6.1.13) is the correct formulation of Deaton's theory. The coefficient attached to the rate of change of income variable is in the region of 0·66 so that consumers are revealed to attach weights to the current rate of change of real income and to some normal or base rate of change, in a ratio of approximately 2 to 1. The base rate itself, ρ^0, which was predicted to be of the order of 0·77 per cent can be unravelled from the estimated constant terms with the aid of the estimated first-order auto-correlation coefficient, δ say, which was estimated as $-0{\cdot}0410$.

$$-(1-\beta)\rho^0(1-\delta) = 0{\cdot}0042$$
$$(1-\beta) = 0{\cdot}6592$$
$$(1-\delta) = 1{\cdot}0410$$

$\therefore \rho^0 \simeq 0{\cdot}0062.$

Thus the estimated value of ρ^0 is 0·62 per cent.

In all the estimations the R^2-statistic was in the region of 0·6. This must be interpreted in the light of the behaviour of the $\Delta(\sigma/\mu)$ series which is highly erratic and could not easily be explained by any simple function.

The fact that a good deal of variance is explained, coupled with the close conformity of the parameter estimates with prior predictions, suggests the relevance of Deaton's theory of " Involuntary Savings ". In addition the performance of the expectations series seems to provide some modest support for the modified adaptive expectations hypothesis.

APPENDIX A. DETERMINING THE POSTERIOR DISTRIBUTION OF π_t

In the notation of Section 2.2, Bayes Theorem may be written

$$\pi(t/t) = \frac{L(M_t/\pi_t)\pi(t/(t-1))}{\int \pi L(M_t/\pi_t)\pi(t/(t-1))d\pi} \qquad \text{...(A1)}$$

where L denotes likelihood.

A.(i) $\pi(t/(t-1)) = \dfrac{1}{\Sigma\sqrt{2\pi}} \exp\left\{-\dfrac{(\pi-\hat{\pi}(t/(t-1)))^2}{2\Sigma}\right\} \equiv N(\hat{\pi}(t/(t-1)); \Sigma).$

A.(ii) $L(M/\pi) = \dfrac{1}{W\sqrt{2\pi}} \exp\left\{-\dfrac{(M-\pi)^2}{2W}\right\}.$

A.(iii) The denominator of (A1) is not a function of π and can be treated as a constant.

A.(iv) The overall constant of proportionality can be calculated to ensure that

$$\int \pi(t/t)d\pi = 1.$$

Thus:

$$\pi(t/t) \propto \exp\left\{-\frac{(\pi-\hat{\pi}(t/(t-1)))^2}{2\Sigma} - \frac{(M-\pi)^2}{2W}\right\}$$

$$= \exp\left\{-\frac{(\pi^2(\Sigma+W)-2\pi(\hat{\pi}(t/(t-1))W+M\Sigma)+\hat{\pi}(t/(t-1))^2W+M^2\Sigma)}{2\Sigma W}\right\}$$

$$\propto \exp\left\{\frac{-\left(\pi^2(\Sigma+W)-2\pi(\hat{\pi}(t/(t-1))W+M\Sigma)+\dfrac{(\pi(t/(t-1))W+M\Sigma)^2}{\Sigma+W}\right)}{2\Sigma W}\right\}$$

$$= \exp\left\{\frac{-\left(\pi-\dfrac{(\hat{\pi}(t/(t-1))W+M\Sigma)}{\Sigma+W}\right)^2}{\dfrac{2\Sigma W}{\Sigma+W}}\right\}.$$

Using (iv) the posterior distribution of π_t can now be found as:

$$\pi(t/t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{(-(\pi-\hat{\pi}(t/t))^2}{2\sigma}\right\} \equiv N(\hat{\pi}(t/t);\, \sigma)$$

where

$$\hat{\pi}(t/t) = \frac{\hat{\pi}(t/(t-1))W + M\Sigma}{\Sigma + W}$$

i.e.

$$\hat{\pi}(t/t) = \hat{\pi}(t/(t-1)) + \frac{\Sigma}{\Sigma+W}(M - \hat{\pi}(t/(t-1)))$$

and

$$\sigma = \frac{\Sigma W}{\Sigma + W} = \Sigma - \frac{\Sigma^2}{\Sigma + W}.$$

APPENDIX B. STARTING VALUES

B.(i) *Forecasts and Associated Degrees of Belief*

The purpose of this study is to produce a series representing inflationary-expectations which can be used as an independent variable in further econometric work. The preferred method of setting " starting values " requires data from the period immediately prior to the period of this econometric study. The purpose is to examine this data in the hope of detecting a pattern in the series, which may suggest the state of expectations. If for example it is found that the general level of prices has been constant over a prolonged period, then it seems reasonable to believe that, towards the end of this period at least, agents will be predicting a constant, zero rate of inflation, with a high degree of belief attached to this forecast. Similarly if the inflation-rate has revealed a constant trend for some period, then it can be assumed that it will eventually be anticipated with some confidence. Of course it will not always be the case that such clear-cut trends reveal themselves. However it was found in practical trials that by starting the process from anywhere within a fairly wide range of starting values, the process converged to more or less the same predictions each time, after about seven or eight periods.

B.(ii) *Probabilities that Particular Process States will Occur*

In Appendix C the method by which the posterior state-probabilities are calculated is described. The method requires that we present the initial probabilities that each state will occur; these probabilities being continually updated in a manner described in Appendix C(iii). The initial values chosen must clearly reflect preconceived ideas about the probability of each state occurring. In the case of the inflation-rate series for example, the large number of occurrences of the no-change state indicates that a relatively high probability should be assigned to another one occurring. In any case, it was again found that given a particular set of starting values, after about seven periods or so, the probabilities converged to more or less the same values as those obtained using reasonably different starting values.

APPENDIX C. DERIVATION OF THE STATE PROBABILITIES

The required statistic is P_t^{ij}, the probability that the inflationary process was in state i at $t-1$, and is in state j at time t. This is easily found using Bayes Theorem:

$$P_t^{ij} = \text{prob}\,(S_t = j,\, S_{t-1} = i/M_t,\, \phi_{t-1})$$

where $S_t = j$ means the state j prevailed at t and

$$\phi_{t-1} = \{M_{t-1}, M_{t-2}, \ldots\}$$

$$P_t^{ij} \alpha L(M_t/S_t = j, S_{t-1} = i, \phi_{t-1}) \times \text{prob}\,(S_t = j, S_{t-1} = i/\phi_{t-1})$$
$$= L(M_t/S_t = j, S_{t-1}, \phi_{t-1}) \times \text{prob}\,(S_t = j/S_{t-1} = i, \phi_{t-1}) \times \text{prob}\,(S_{t-1} = i/\phi_{t-1})$$
$$= L^{ij} \times X_{t-1}^j \times P_{t-1}^i \quad \text{say.}$$

C.(i) The likelihood function, L^{ij}, presents no problems as the relevant variance magnitudes are known.

C.(ii) The probability that state i occurred at $t-1$ is calculated:

$$P_{t-1}^i = \textstyle\sum_k P_{t-1}^{ki}$$

C.(iii) To determine how the prior probability of state j occurring, is formed and updated, we need some more assumptions. In particular we take the view that if agents have in the past frequently witnessed a particular process state, then they will attach a particularly high probability to its reoccurrence. We therefore need a method of inferring the number of times a particular state obtains. In particular we need a running total for each process-state, which is incremented as that particular state is believed to have occurred. This is effectively achieved by taking the pre-set probabilities that each state will occur (see Appendix A.(ii)) and incrementing them in each period by the probability that the respective state obtained.

First we notice that it is easiest to use the current observation to infer which state obtained in the previous period. For example suppose the observed inflation-rate is far higher than expected. Then after obtaining the next observation it is easier to determine whether or not the first was a lone transient. Consequently we first determine the probability at $t-1$, that the inflationary system was in state j at $t-2$:

$$Q_{t-1}^j = \textstyle\sum_k P_{t-1}^{jk}.$$

The prior probability that state j will occur at $t-1$, denoted X_{t-2}^j, can now be updated to give the prior for $t-1$ as follows:

$$X_{t-1}^j = \frac{X_{t-2}^j + Q_{t-1}^j}{\sum_j (X_{t-2}^j + Q_{t-1}^j)}.$$

Alternatively the following formula can be assumed:

$$X_{t-1}^j = \frac{\psi X_{t-2}^j + Q_{t-1}^j}{\sum_j (\psi X_{t-2}^j + Q_{t-1}^j)}$$

where ψ is a pre-set weighting factor reflecting a-priori theory concerning the importance of recent as compared with not so recent information.

APPENDIX D. COMBINING ALL DISTRIBUTIONS GOING TO PROCESS STATE J

D.(i) *First Moments.*

$$P_t^j = \textstyle\sum_i P_t^{ij}$$
$$\hat{\pi}^j(t/t) = \textstyle\sum_i P_t^{ij} \hat{\pi}^{ij}(t/t)/P_t^j$$
$$\hat{g}^j(t/t) = \textstyle\sum_i P_t^{ij} \hat{g}^{ij}(t/t)/P_t^j.$$

D.(ii) *Second Moments.*

Let

$$\underline{\sigma}_{t-1}^{ij} = \begin{bmatrix} \sigma_{\pi\pi,t}^{ij} & \sigma_{\pi g,t}^{ij} \\ \sigma_{\pi g,t}^{ij} & \sigma_{gg,t}^{ij} \end{bmatrix}$$

and

$$\underline{\sigma}_{t-1}^{j} = \begin{bmatrix} \sigma_{\pi\pi,t}^{j} & \sigma_{\pi g,t}^{j} \\ \sigma_{\pi g,t}^{j} & \sigma_{gg,t}^{j} \end{bmatrix}$$

Then $\underline{\sigma}^{ij}_{t-1}$ is condensed into $\underline{\sigma}^{j}_{t-1}$ as follows:

$$\sigma^j_{\pi\pi,t} = \sum_i P^{ij}_t[\sigma^{ij}_{\pi\pi,t}+(\hat{\pi}^{ij}(t/t)-\hat{\pi}^j(t/t))^2]/P^j_t$$

$$\sigma^j_{\pi g,t} = \sum_i P^{ij}_t[\sigma^{ij}_{\pi g,t}+(\hat{\pi}^{ij}(t/t)-\hat{\pi}^j(t/t))(\hat{g}^{ij}(t/t)-\hat{g}^j(t/t))]/P^j_t$$

$$\sigma^j_{gg,t} = \sum_i P^{ij}_t[\sigma^{ij}_{gg,t}+(\hat{g}^{ij}(t/t)-\hat{g}^j(t/t))^2]/P^j_t.$$

First version received September 1977; *final version accepted March* 1979 (*Eds.*).

I am indebted to Terry Barker and Richard Stone for encouraging me to write this paper. I am also very grateful to Angus Deaton, Frank Hahn, Oliver Hart and Gene Savin for subsequent comment and criticisms.

NOTES

1. See for example Muth (1960).

2. An expectations term usually enters the regression equation as an independent, " forcing " variable. However it is necessary for econometric work that the independent variables be observable. If we assume that expectations are formed using the simple adaptive schema, then by making a Koyck transformation we can eliminate the expectations term from the equation.

3. For example, since 1949, a Mr Joseph A. Livingstone has conducted a semi-annual survey in the US, in which respondents from a variety of occupations have given their wage and price predictions. For a detailed description see Turnovsky (1970), Turnovsky and Wachter (1972), Gibson (1972).

Attempts have been made to explain this and other data with the following model.

$$\pi^e_{t+1} = \alpha_0+\alpha_1\pi^e_t+\alpha_2\pi_t, \quad \alpha_0, \alpha_1, \alpha_2, \text{const.}$$

Typically, the tests have produced a statistically insignificant estimate of α_0, and estimates of α_1 and α_2 which sum to unity.

Using this model with the Livingstone data for example the following estimates were obtained:

Lahiri (1976):	$\alpha_1 = 0{\cdot}534$; $\alpha_2 = 0{\cdot}426$	Period
	$\alpha_1+\alpha_2 = 0{\cdot}960$	1952–1970
Turnovsky (1970):	$\alpha_1 = 0{\cdot}226$; $\alpha_2 = 0{\cdot}781$	Period
	$\alpha_1+\alpha_2 = 1{\cdot}007.$	1962–1969

4. For example compare the results (Note 3) of Turnovsky (1970) and Lahiri (1976).

5. Equation (1.1.1) can also be written as

$$(\pi^e_{t+1}-\pi^e_t) = \lambda(\pi_t-\pi^e_t)$$

If π_t is trended, then π^e_t can only be so if π^e_t is consistently less than π_t. Thus expectations are certain to be inaccurate when the inflation-rate is accelerating. Moreover π^e_{t+1} is a convex combination of π^e_t and π_t and so the maximum value it can obtain in these circumstances is that of π_t.

6. See the work of Turnovsky (1970) and Lahiri (1976) for the US, and that of Carlson and Parkin (1975) for the UK.

7. See for example the work of Kane and Malkiel (1976) who find that " return-to-normality elements " dominate the forecasts of future inflation-rates.

8. Of course with this assumption any problems of choosing a point estimate are reduced; for example the maximum likelihood, minimum variance, and minimum error estimates coincide.

9. For example Turnovsky (1970).

10. The notation $\sim N(A; B)$ is used to denote a normal distribution with mean A, and variance B.

11. If there is uncertainty in the prior estimate of π_t then $\Sigma_t > 0$. This in turn implies that $\sigma_t < \Sigma_t$. This means that new information M_t always tends to reduce uncertainty in the estimate of π_t.

12. As they stand equations (4.1.6) and (4.1.7) are expressed in what control engineers call " state space form ". The recursive algorithm displayed in equations (4.1.8)–(4.1.15) is known as the Kalman Filter (Kalman (1960)). An interesting discussion of Kalman Filtering Methods and their usefulness for Economics can be found in Athans (1974).

13. In fact the discrete approximation adopted, perhaps represents the most inuitively appealing approach. We should be updating the distribution of error-variances which represent the agent's degree of belief in all possible outcomes of the respective error terms. Instead we allow him to identify four particular process states, which are really no more than " patterns ", in the inflationary process. At any time two or more of these states might obtain simultaneously and so a weighted average forecast is appropriate. Uncertainty remains of central importance, of course, but now we must make subjective judgements concerning the influence of the different elements. By making the system less sensitive in slope than in level, we have adopted the view that the level of the inflation-rate is likely to be more variable than its rate of growth. Thus uncertainty concerning the system $\pi_t = \pi_{t-1}+g_t$ will lead the agent to make a larger revision when new information comes to light, than with similar uncertainty concerning the relationship $g_t = g_{t-1}$.

14. It may at first seem curious that in a paper on inflation-rate expectations the model's predictive or tracking ability is illustrated using data on real disposable income. However the model lends itself to forecasting any economic variable which has a permanent component interpretation; whilst real income also exhibits the pattern of continuous trend which, in diagrammatic form, best reveals the adaptive properties of the model.

15. This suggests that it is possible for the Bayesian agents to predict in a satisfactory manner using a very simple model requiring extremely limited resources. It seems reasonable to conclude therefore that if agents can forecast more or less accurately using the information available, then agents will.

16. See Deaton (1976, p. 17).

17. Brown and Deaton (1972).

18. The Friedman consumption function is clearly inadequate for dealing with theories of optimal portfolio adjustment, and so on. However for the purpose of the present exercise only, we shall stick with the " fixed-k " assumption.

19. If we combine equations (6.1.1), (6.1.2) and (6.1.11) only; we obtain the equation:

$$\Delta\frac{\sigma}{\mu} = (\rho-\rho^*)-\phi(\pi-\pi^*).$$

We have seen that the Bayesian agent can use the expectations model to closely track the series he is predicting; so let us assume that in the long run and on average expectations are correct. Thus $(\rho-\rho^*)$ and $(\pi-\pi^*)$ are zero, which we may accept corresponds to $((\mu/P)-(\mu/P)^*)$ and $(P-P^*)$ being also equal to zero. It thus follows from equation (6.1.1) that in the long run, the savings-ratio will equal the desired savings ratio, on average. The desired savings ratio is in turn derived from the specificed aggregate consumption function. If we assume the overly simple specification of Friedman's—equation (6.1.2)—then we find that

$$\frac{\sigma}{\mu} = \left(\frac{\sigma}{\mu}\right)^* = 1-k.$$

Most empirical studies have obtained an estimate of k of the order of 0·9. The theory of involuntary savings as formulated therefore suggests that unanticipated price-changes and unanticipated real-income changes cause the savings-ratio to fluctuate around a constant level of about 10 per cent.

N.B. If there are reasons for believing that an inflationary (deflationary) policy will cause agents to consistently under-predict (over-predict) for a substantial period of time—maybe following a long period of zero-inflation a permanent jump is experienced and is erroneously treated as a transient; or perhaps the incremental growth rate of the inflation-rate exhibits a trend—then it follows that such a policy will have deflationary (inflationary) consequences for the level of output and employment.

20. The path of the savings-ratio was markedly different in the second period from that in the first.

REFERENCES

ATHANS, M. (1974), " The importance of Kalman Filtering Methods for Economic Systems ", *The Annals of Economic and Social Measurement*, **3** (1), 49-64.

BROWN, J. A. C. and DEATON, A. S. (1972), " Models of Consumer Behaviour: A Survey ", *Economic Journal*, **82**, 1145-1236.

CARLSON, J. A. and PARKIN, M. (1975), " Inflationary Expectations ", *Economica*, 123-138.

DEATON, A. S. (1976), (1977), " Involuntary Savings Through Unanticipated Inflation " (Cambridge Growth Project Research Paper No. 438). Revised version: *American Economic Review* (Nov. 1977).

FRIEDMAN, M. (1957) *A Theory of the Consumption Function* (Princeton University Press).

GIBSON, W. E. (1972), " Interest Rates and Inflationary Expectations ", *American Economic Review*, **62**, 854-865.

GRAYBILL, F. A. (1961) *An Introduction to Linear Statistical Methods*, **1** (New York: McGraw-Hill).

HARRISON, P. J. and STEVENS, C. F. (1971), " Short-term Forecasting ", *Operations Research Quarterly*, **22** (4), 341-362.

KALMAN, R. E. (1960), " A New Approach to Linear Filtering and Prediction Problems ", *Journal of Basic Engineering*, 35-45.

KANE, E. J. and MALKIEL, B. G. (1976), " Autoregressive and non-autoregressive elements in cross-section forecasts of inflation ", *Econometrica*, **44**, 1-16.

LAHIRI, K. (1976), " Inflationary expectations: their formation and interest rate effects ", *American Economic Review*, **66**, 124-131.

MUTH, J. F. (1960), " Optimal properties of exponentially weighted forecasts ", *Journal of American Statistical Association*, **55**, 299-306.

TURNOVSKY, S. J. (1970), " Empirical evidence of the formation of price expectations ", *Journal of the American Statistical Association*, **65**, 1441-1454.

TURNOVSKY, S. J. and WACHTER, M. L. (1972), " A test of the ' expectations hypothesis ' using directly observed wage and price expectations ", *Review of Economic Statistics*, **54**, 47-54.

UK CENTRAL STATISTICAL OFFICE (1976) *Economic Trends* (Annual Supplement) (London: HMSO).