



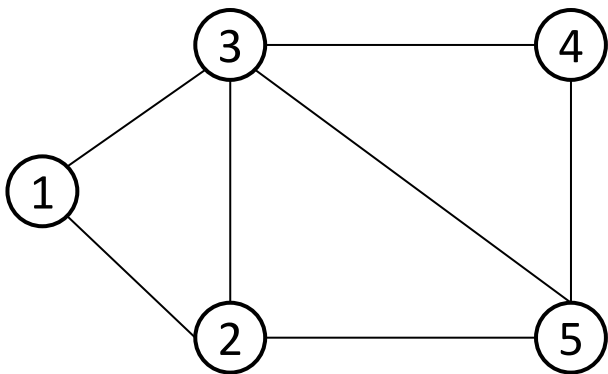
Katedra za računarstvo  
Elektronski fakultet, Univerzitet u Nišu

# Veštačka inteligencija

CSP – Constraint Satisfaction Problems

# Bojenje grafa

- ▶ Kako obojiti graf tako da ni jedna dva susedna čvora nemaju istu boju.
  - ▶  $V$  (čvorovi grafa)
  - ▶  $D$  (moguće boje za svaki čvor)
  - ▶  $C$  (ni jedna dva susedna čvora nemaju istu boju)



Promenljive:  $\{1, 2, 3, 4, 5\}$

Domeni:  $\{D_1, D_2, D_2, D_4, D_5\}$   $D_{i(1,5)} = \{R, G, B\}$

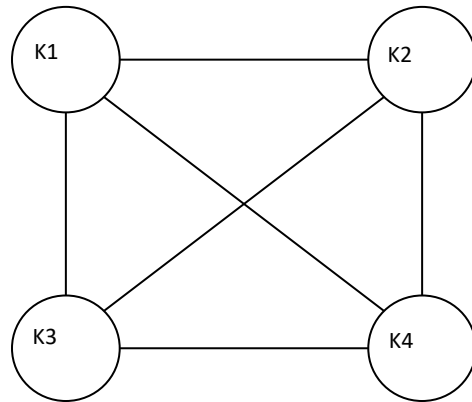
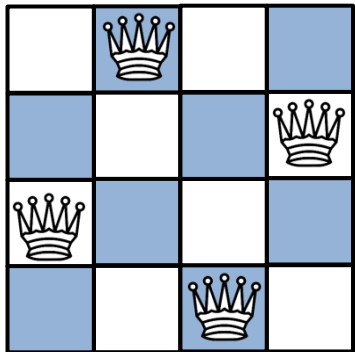
**U ovom primeru domeni su isti za sve promenljive!**

Ograničenja:  $\{1 \neq 2, 1 \neq 3, 2 \neq 1, 2 \neq 3, 2 \neq 5, 3 \neq 2, 3 \neq 4, 3 \neq 5, 4 \neq 3, 4 \neq 5\}$



# N Kraljica

- ▶ Kako rasporediti n kraljica na šahovskoj tabli dimenzija  $n \times n$  tako da se ne napadaju
- ▶  $n=4$



Promenljive:  $\{K1, K2, K3, K4\}$   $K_{i(1,4)}$  – kraljica u i-tom redu

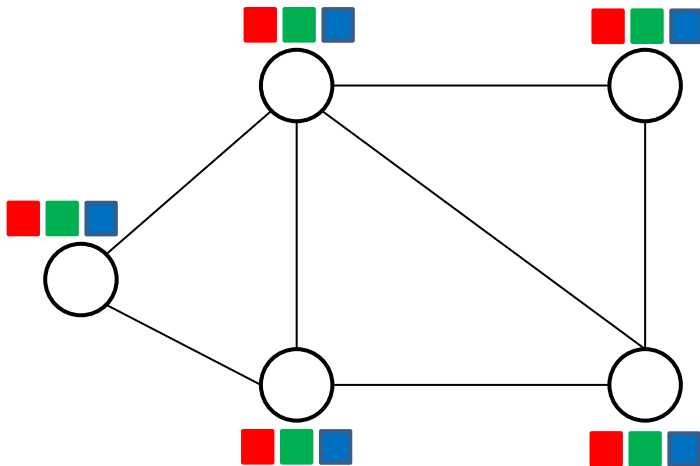
Domeni:  $\{D1, D2, D3, D4\}$   $D_{i(1,4)} = \{1, 2, 3, 4\}$  opseg mogućih kolona za  $K_i$  kraljicu

Ograničenja:  $(K_i, K_j)$  ne mogu biti u istoj koloni ili na dijagonali



# Backtracking (BT)

## Polazno stanje

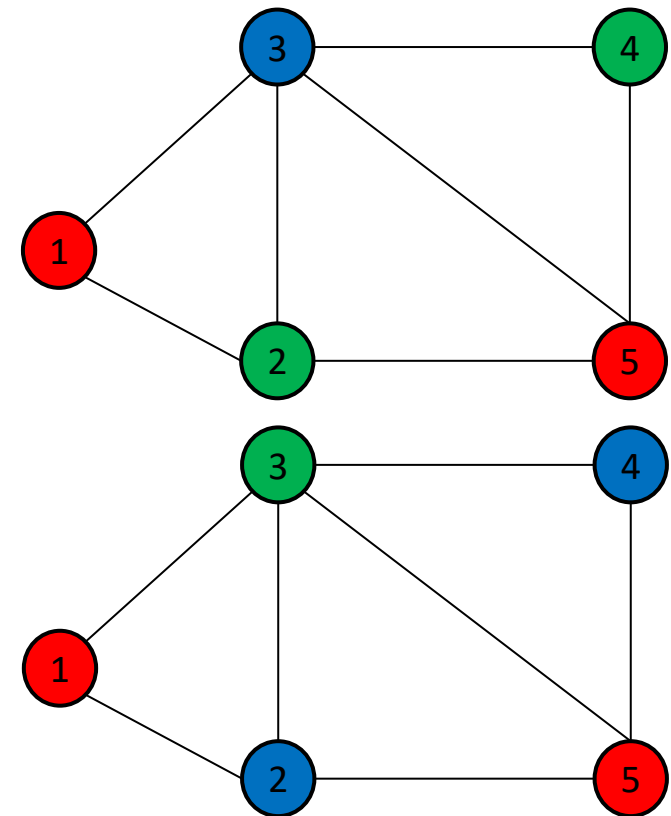


Promenljive: {1,2,3,4,5}

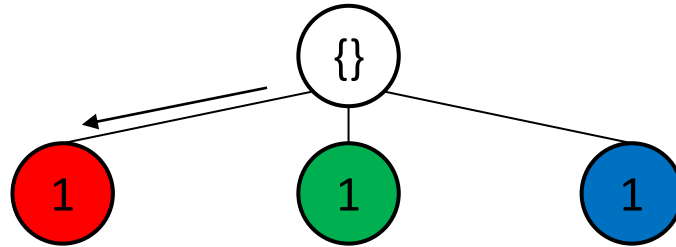
Domeni: {D1,D2,D2,D4,D5}  $D_{i(1,5)} = \{R,G,B\}$

Ograničenja: {1≠2,1≠3,2≠1,2≠3, 2≠5,3≠2,3≠4,3≠5,4≠3,4≠5}

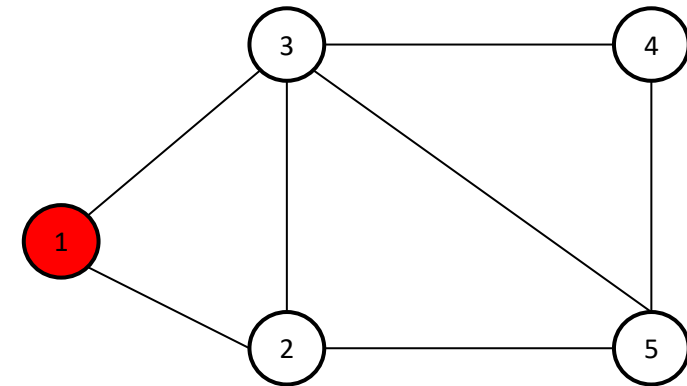
## Ciljno stanje/a



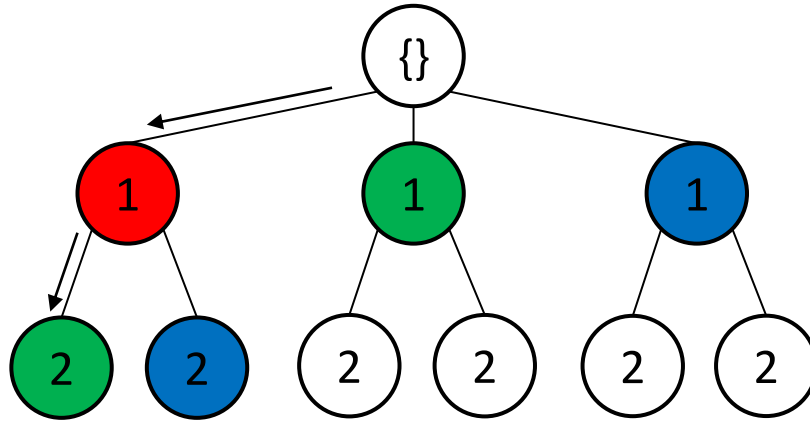
# Backtracking



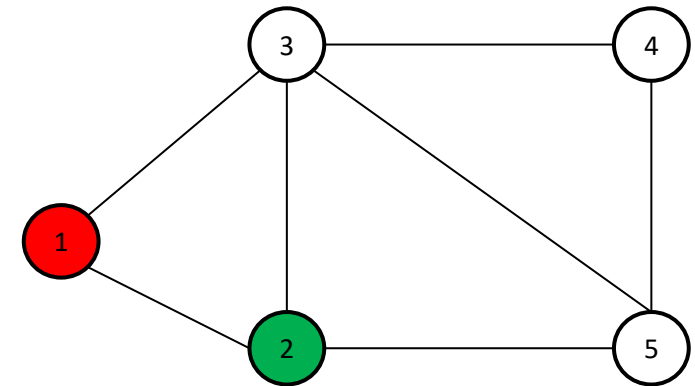
Obilazak: 1,2,5,4,3



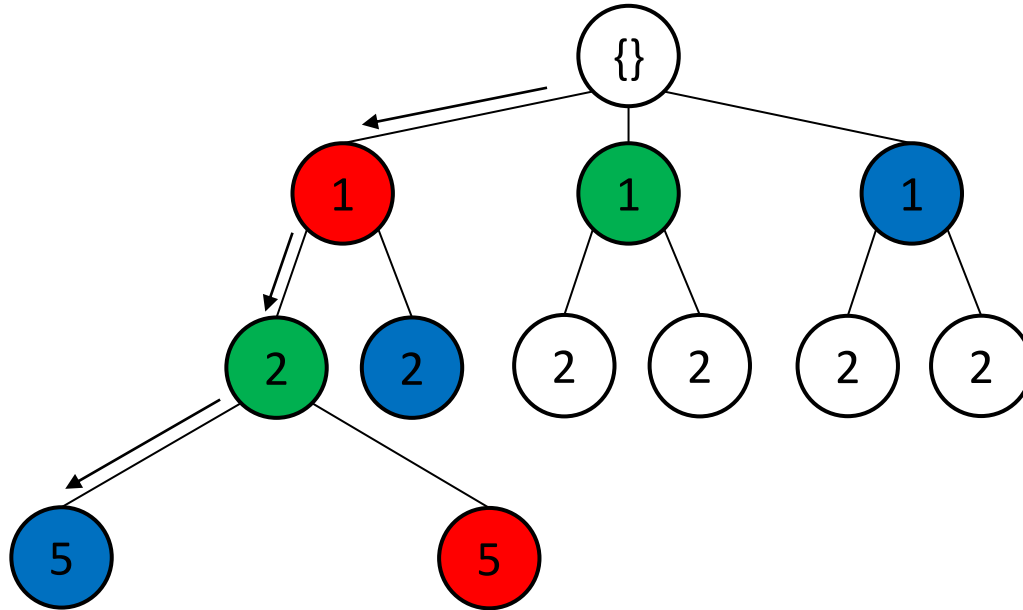
# Backtracking



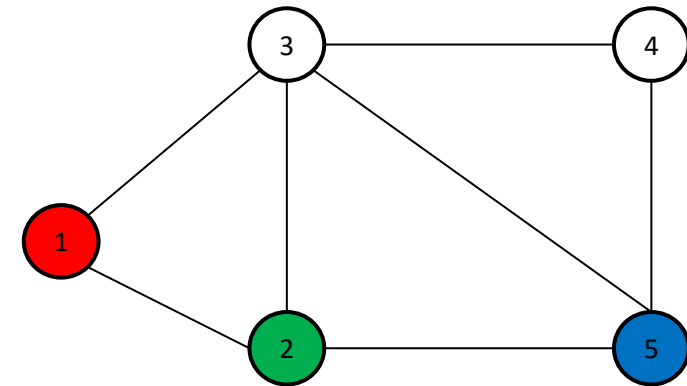
Obilazak: 1,2,5,4,3



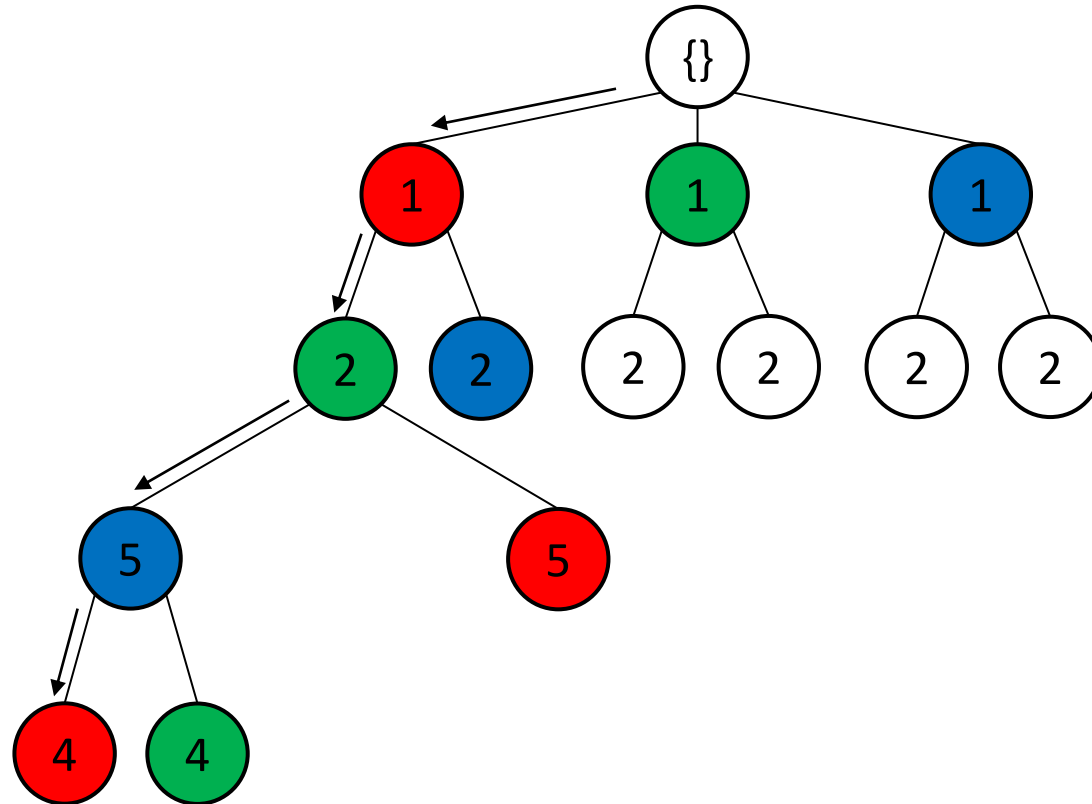
# Backtracking



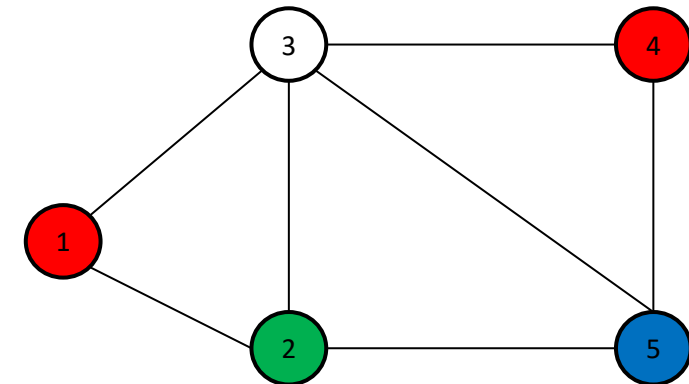
Obilazak: 1,2,5,4,3



# Backtracking

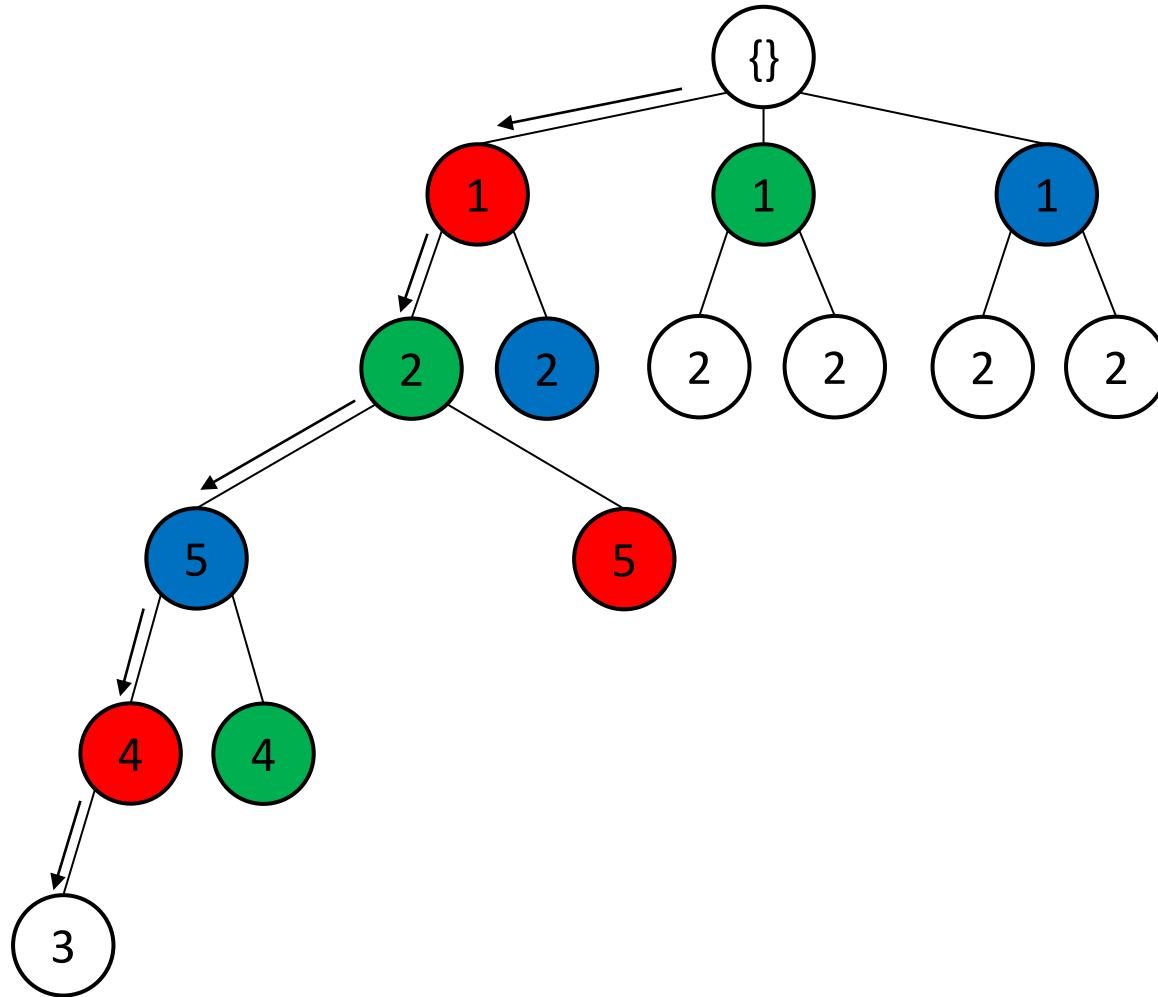


Obilazak: 1,2,5,4,3

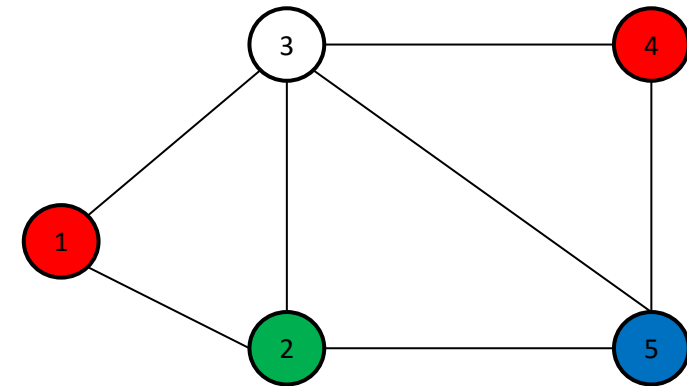




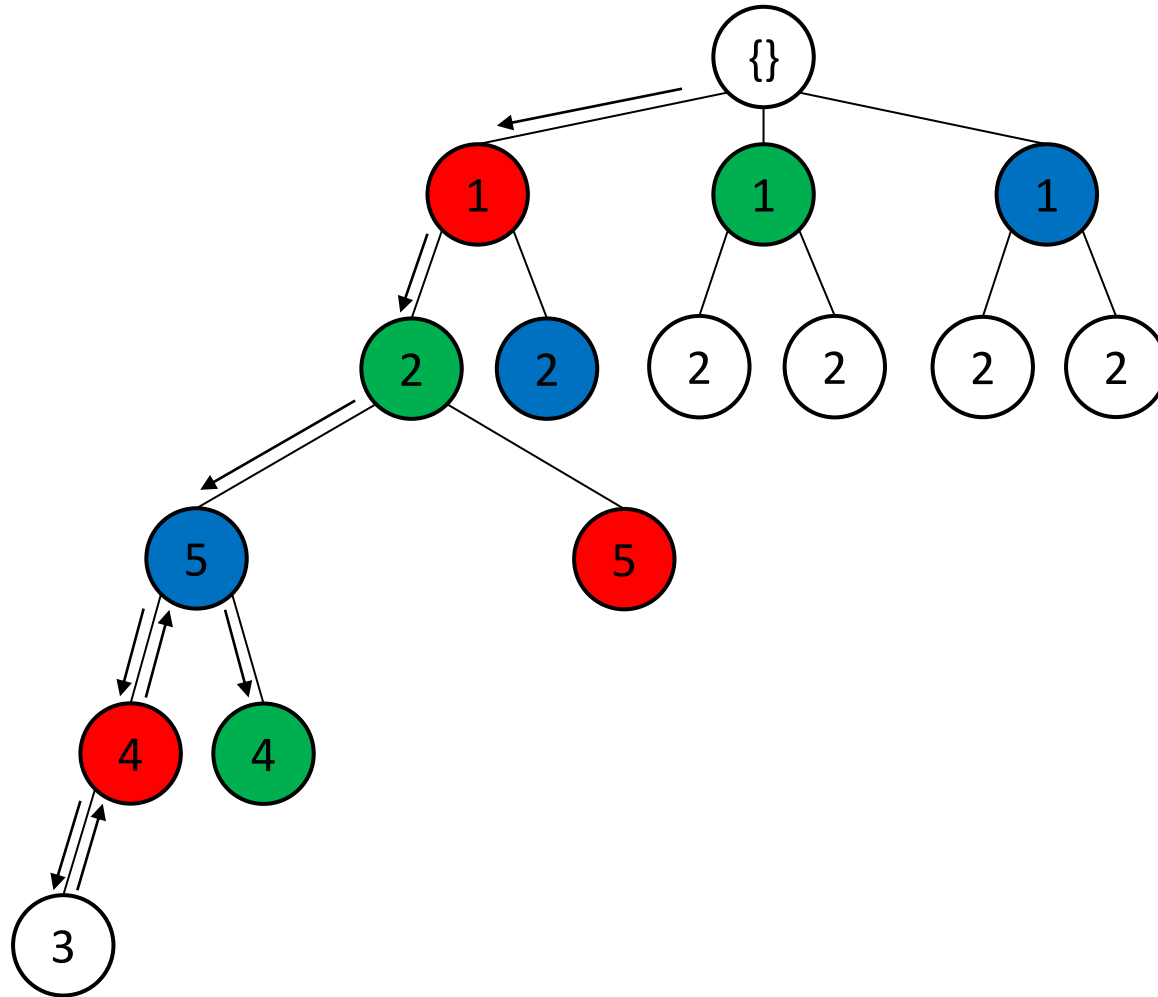
# Backtracking



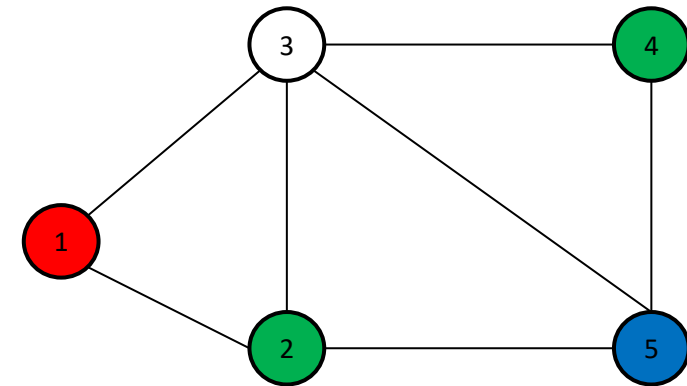
Obilazak: 1,2,5,4,3



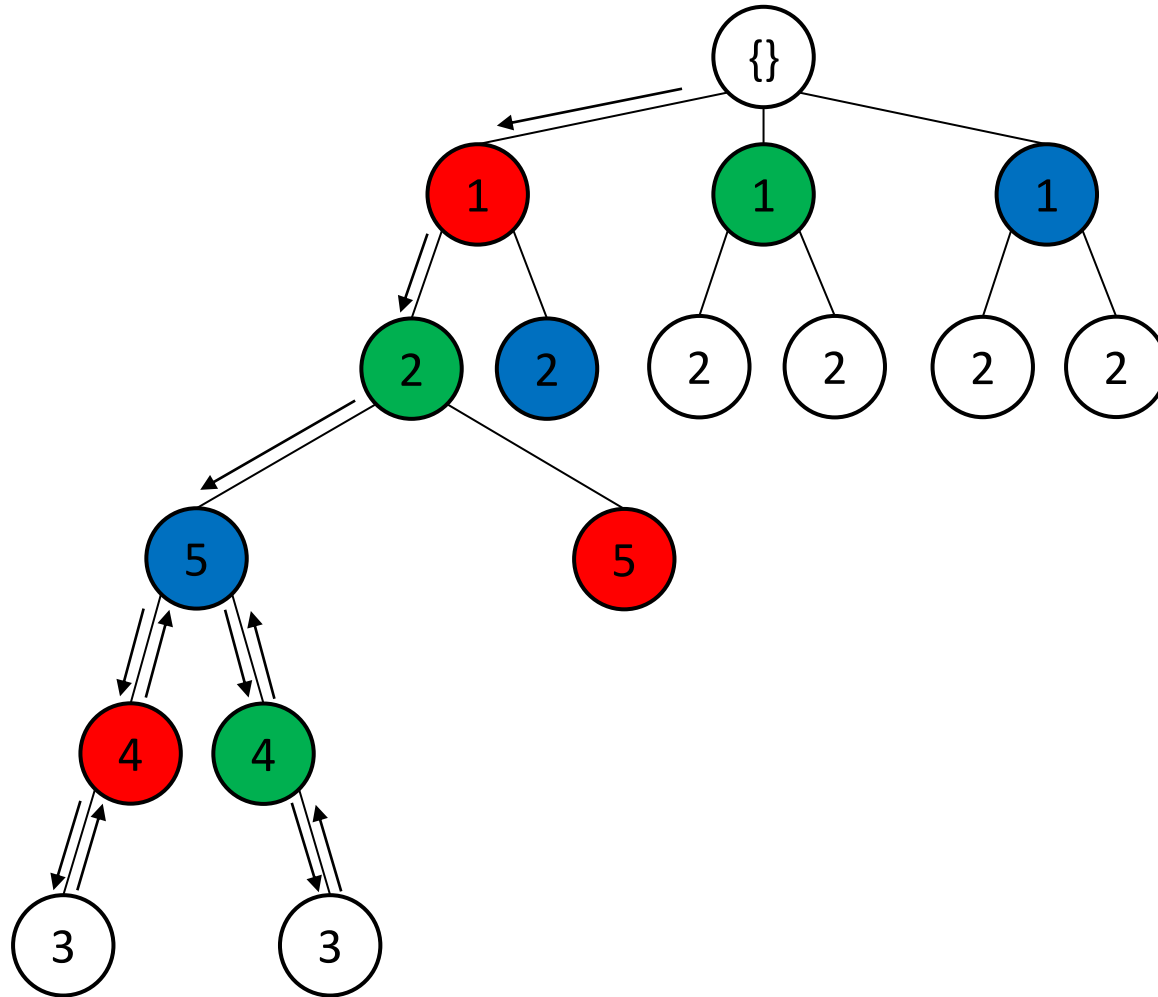
# Backtracking



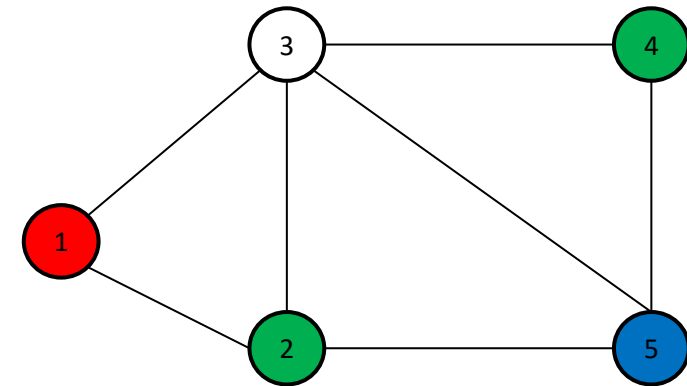
Obilazak: 1,2,5,4,3



# Backtracking



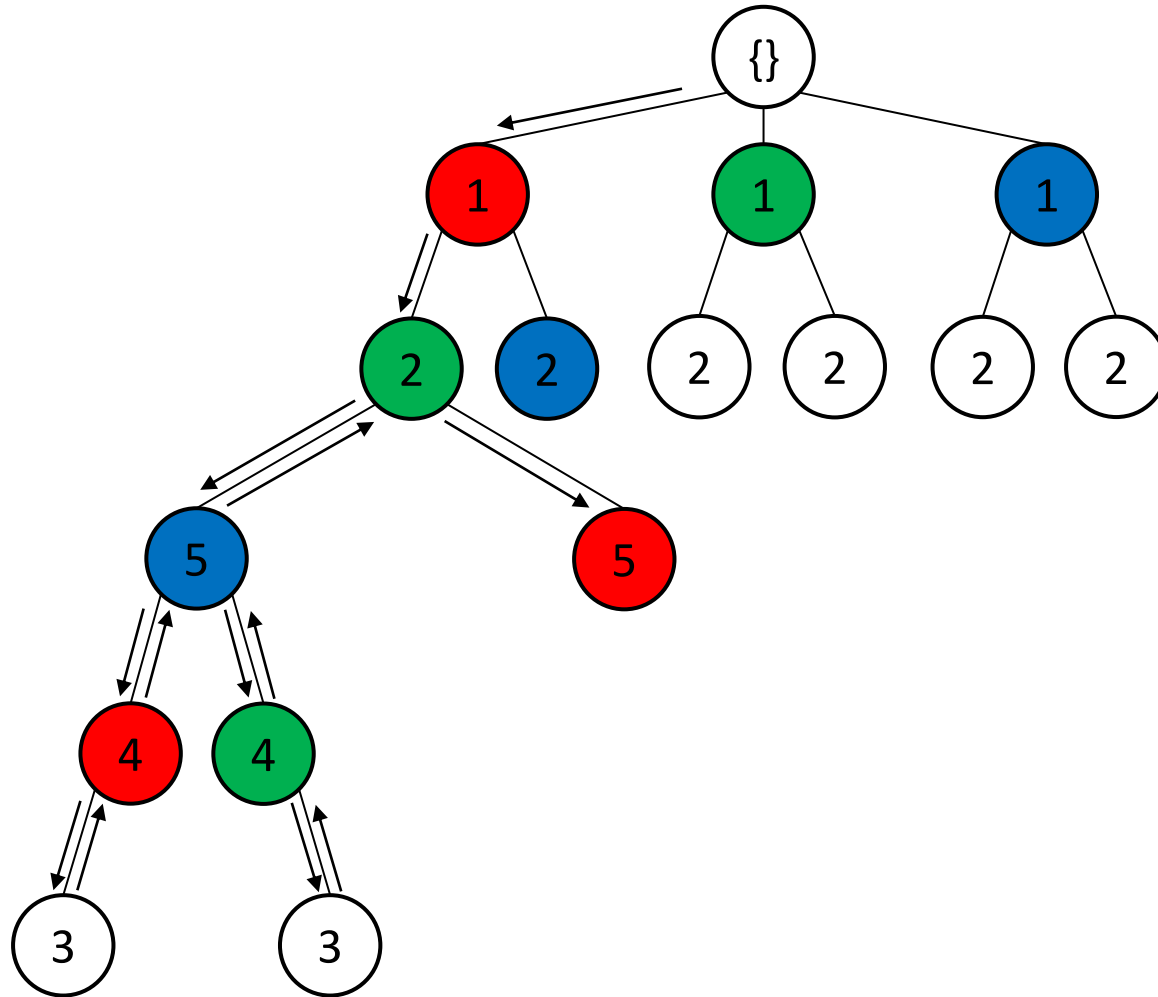
Obilazak: 1,2,5,4,3



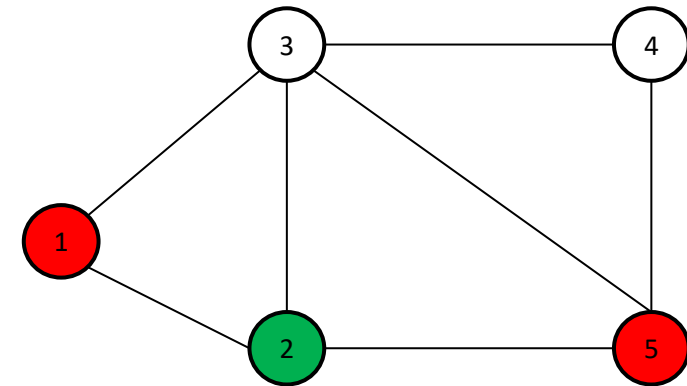
Backtrack:  
 $3 \neq 5, 3 \neq 4, 3 \neq 2, 3 \neq 1$



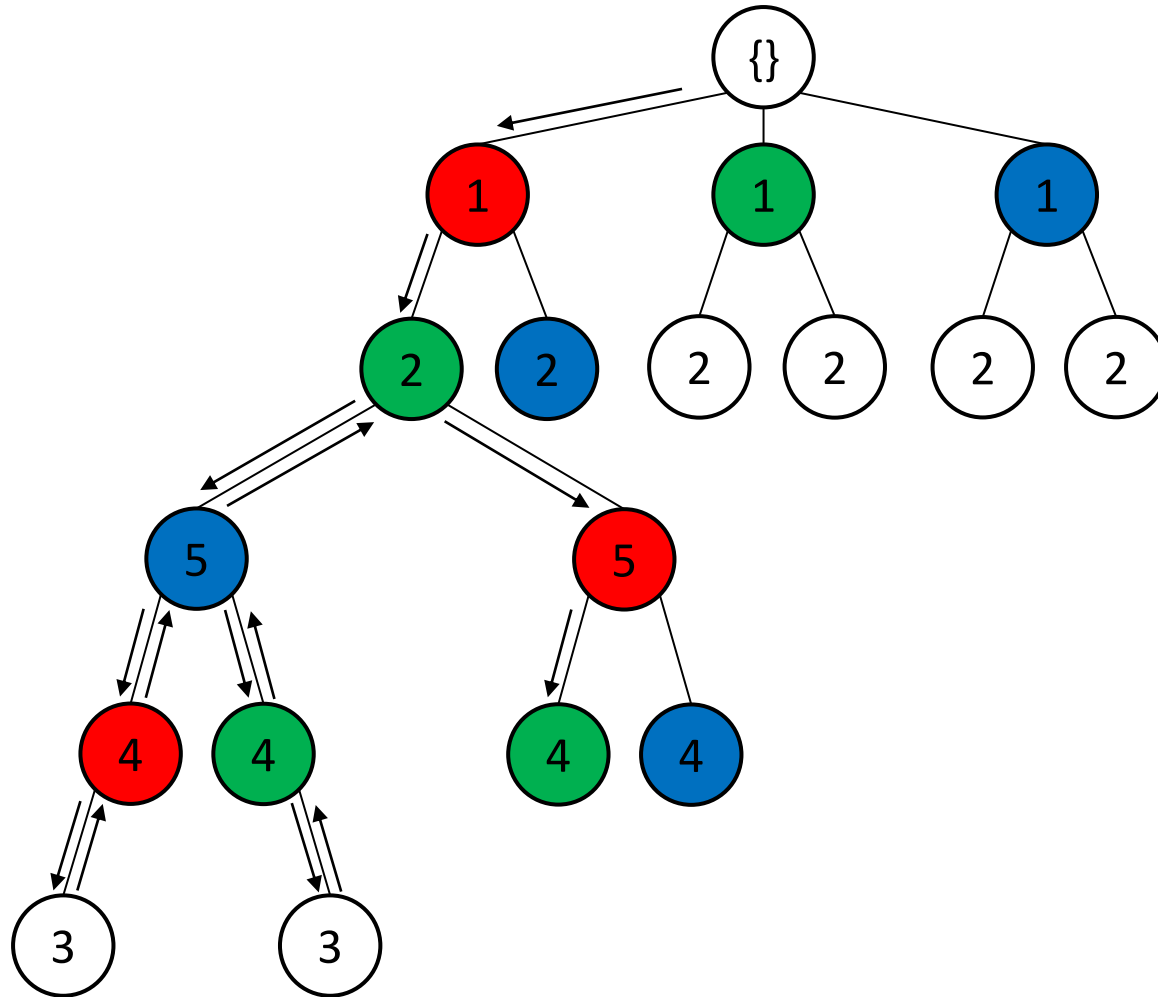
# Backtracking



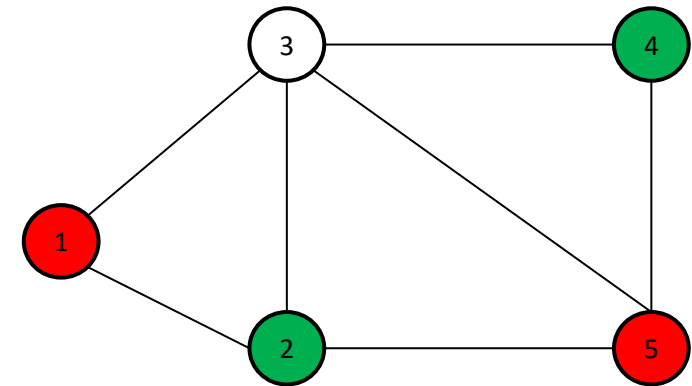
Obilazak: 1,2,5,4,3



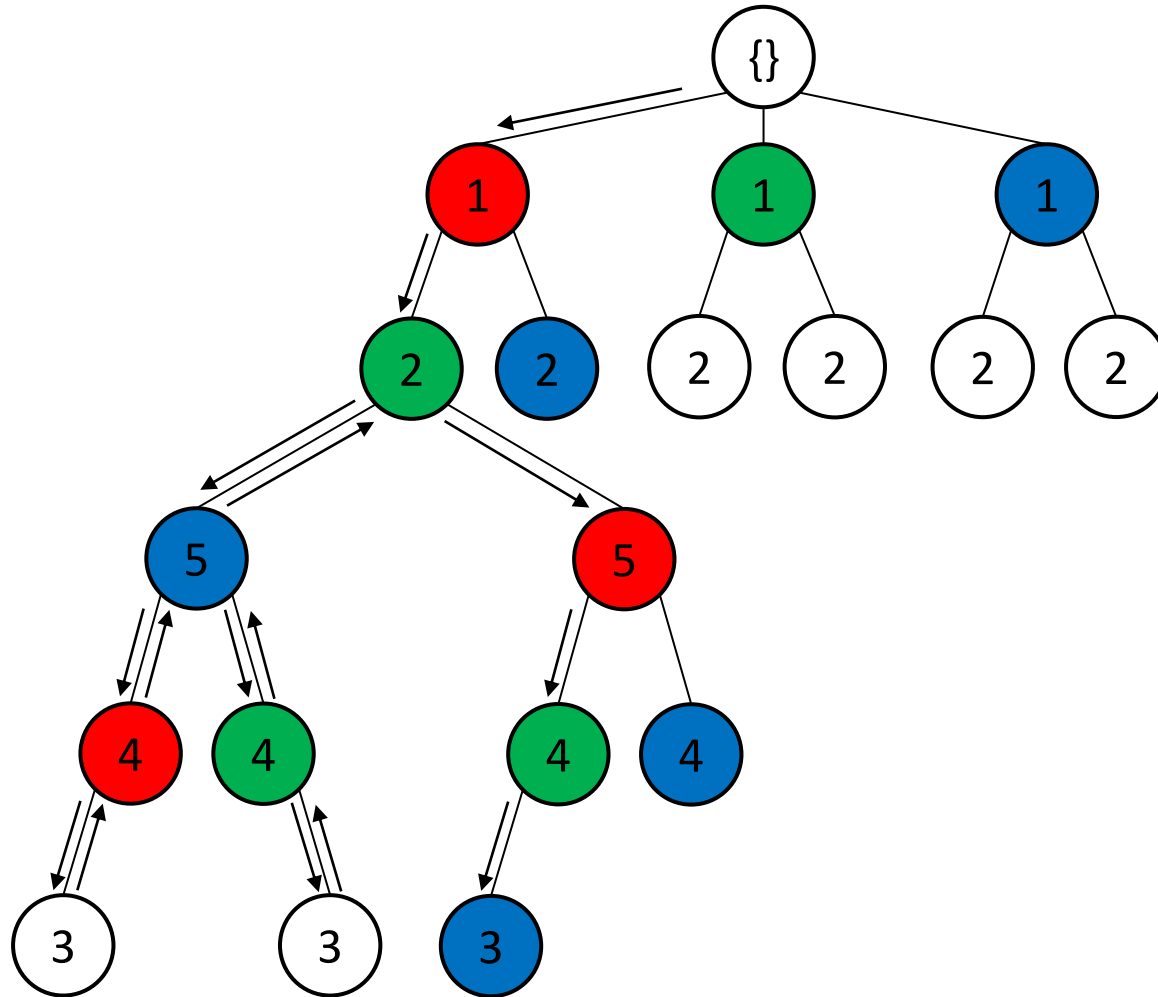
# Backtracking



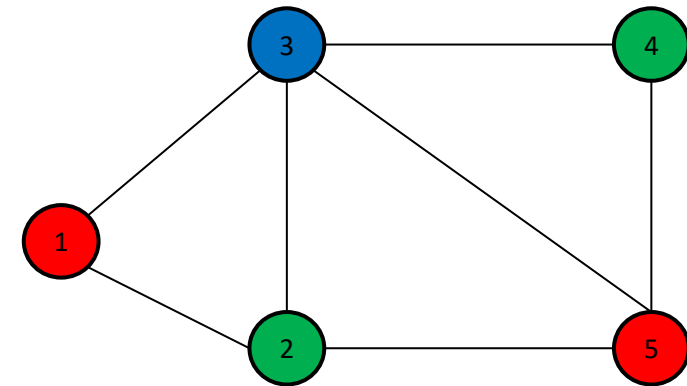
Obilazak: 1,2,5,4,3



# Backtracking














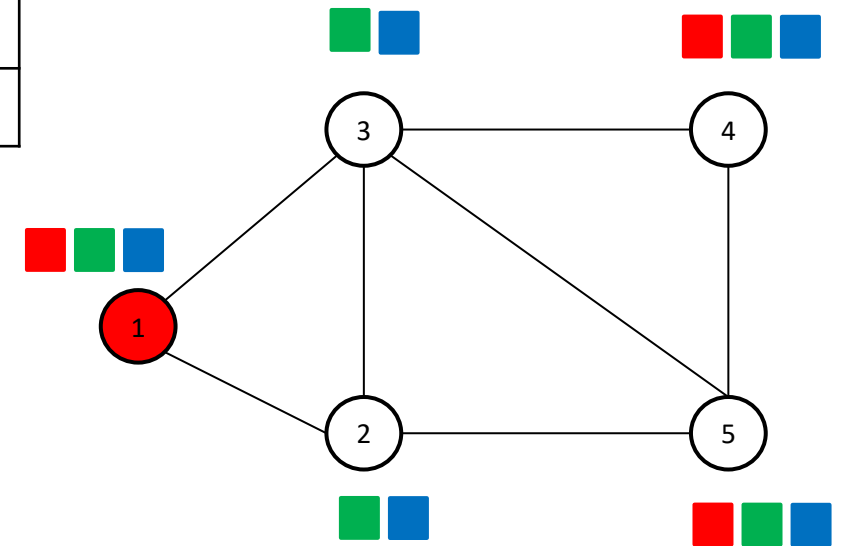
Obilazak: 1,2,5,4,3



# Forward Checking (1)




















Order: 1,2,5,4,3

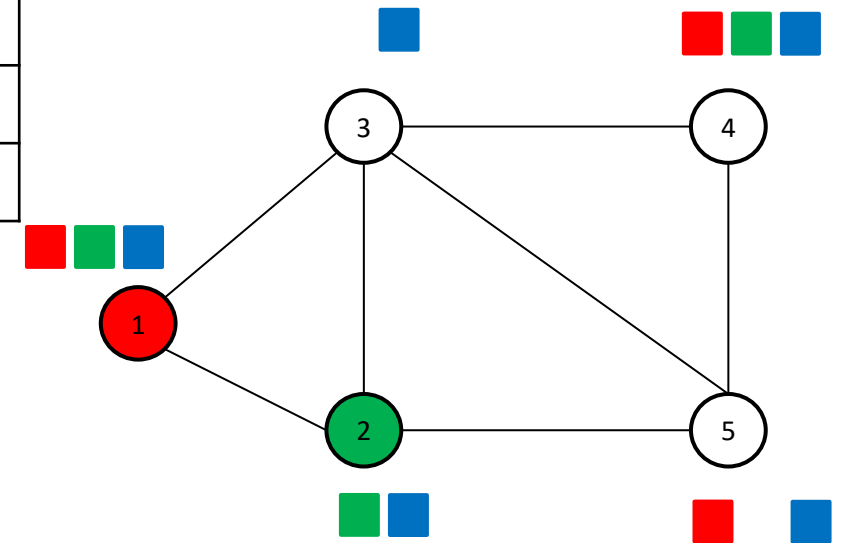
1	2	5	4	3
	 	  	  	 



# Forward Checking (2)

Order: 1,2,5,4,3










1	2	5	4	3
	 	  	  	 
		 	  	

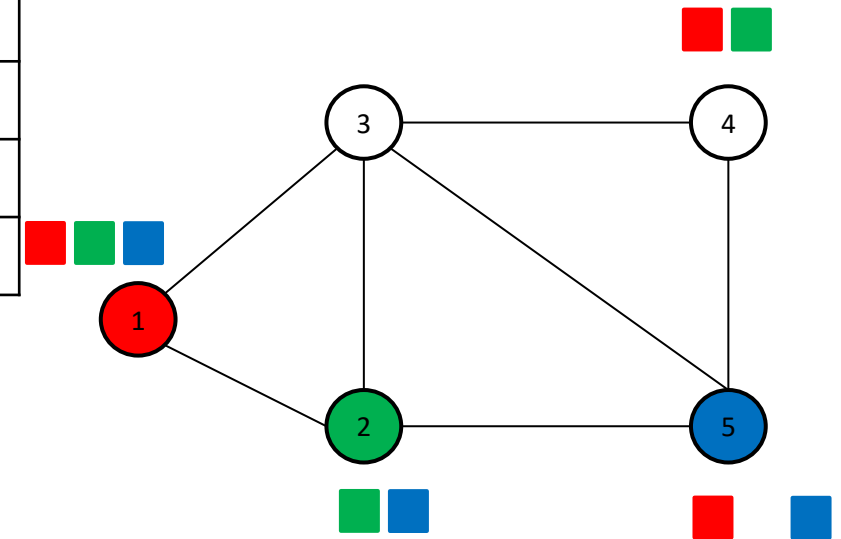




# Forward Checking (3)































Order: 1,2,5,4,3

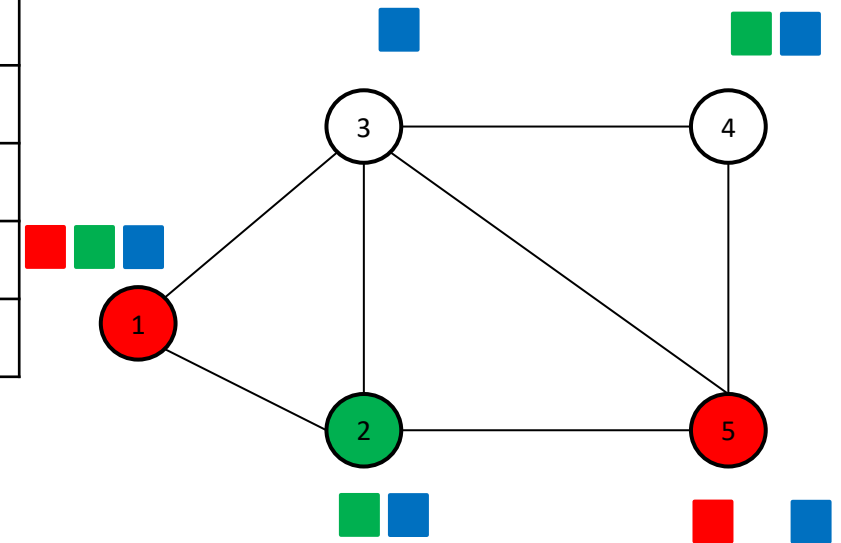
1	2	5	4	3
	 	  	  	 
		 	  	
			 	/



# Forward Checking (4)

































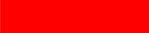


Order: 1,2,5,4,3

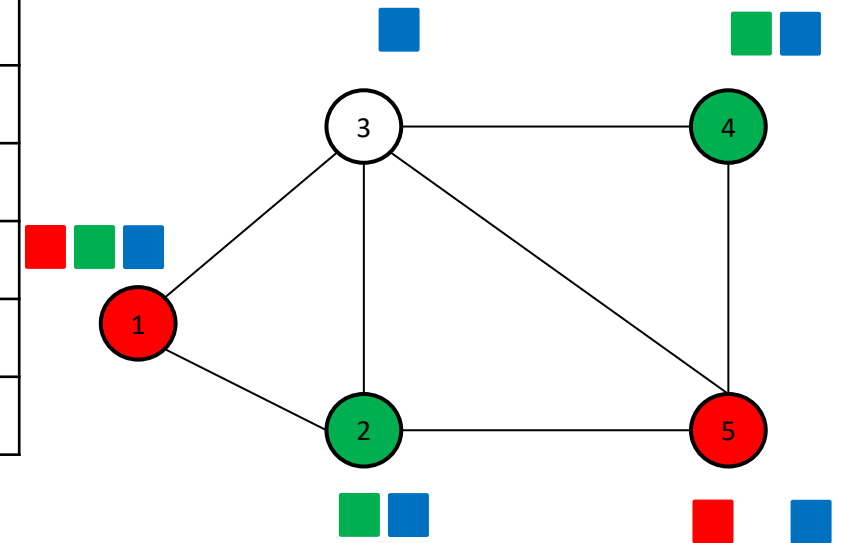
1	2	5	4	3
	 	  	  	 
		 	  	
			 	/
			 	



# Forward Checking (5)






























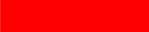


Order: 1,2,5,4,3

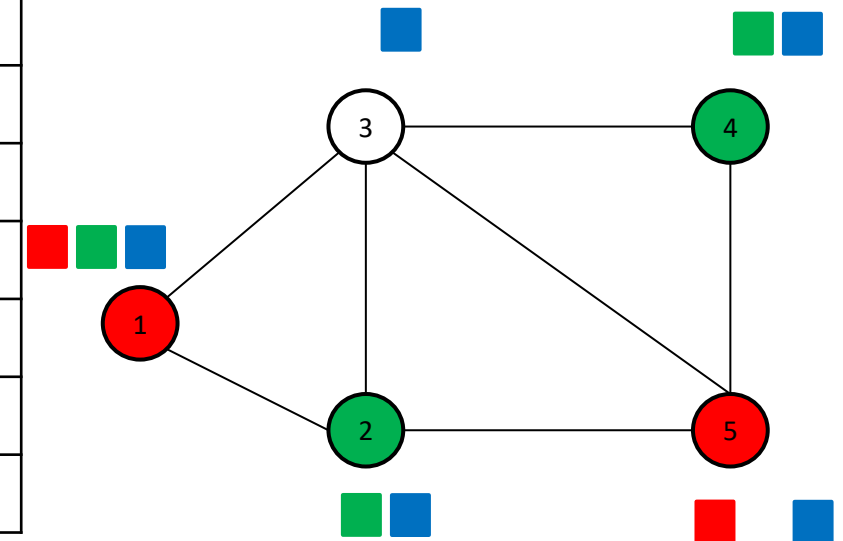
1	2	5	4	3
	 	  	  	 
		 	  	
			 	/
			 	
				



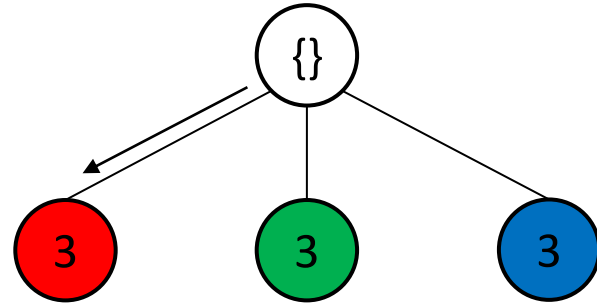
# Forward Checking (6)

Order: 1,2,5,4,3

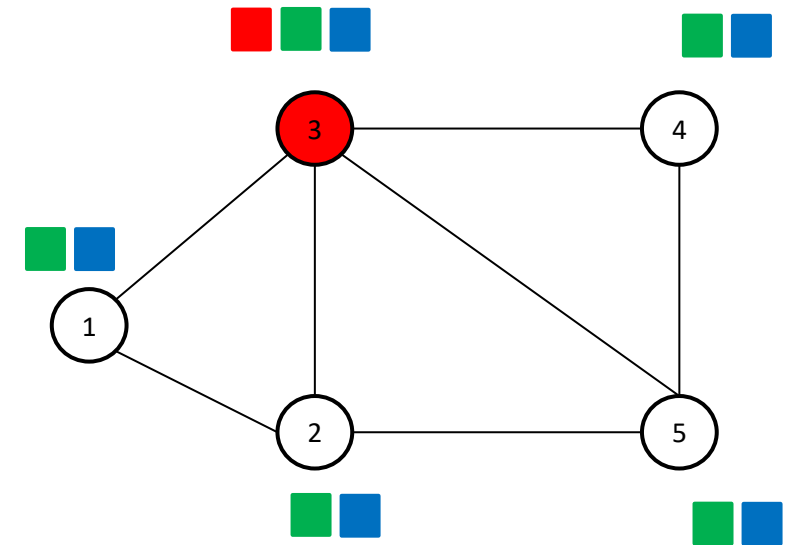
1	2	5	4	3
	 	  	  	 
		 	  	
			 	/
			 	
				
				



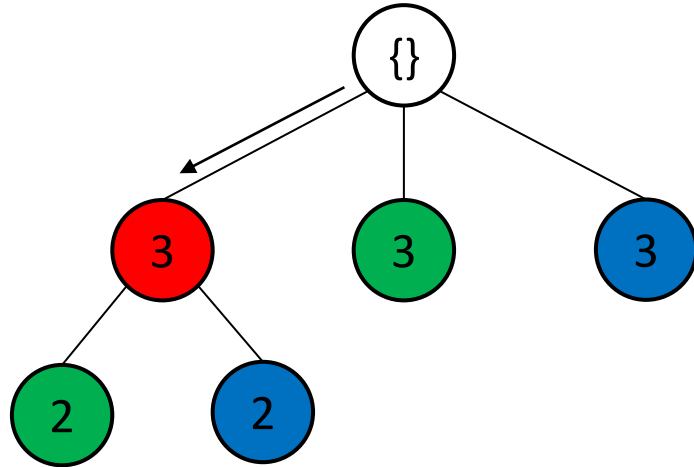
# Forward checking & Degree Heuristics (1)



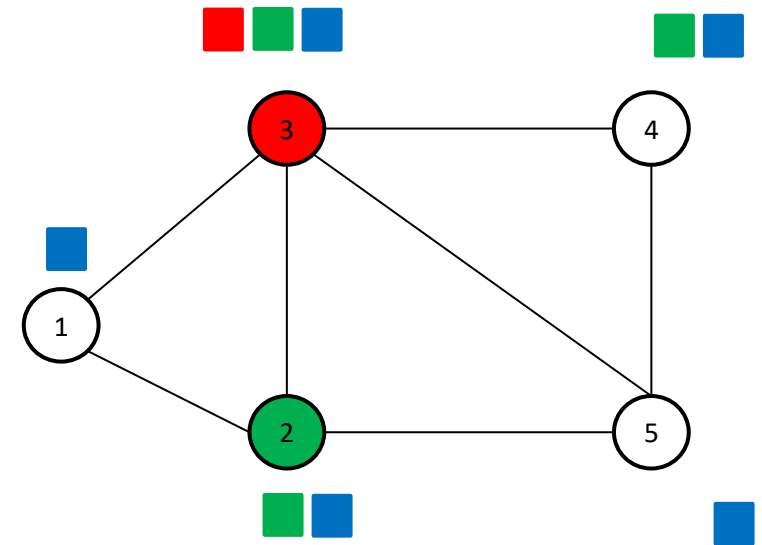
Obilazak: 3,2,5,4,1



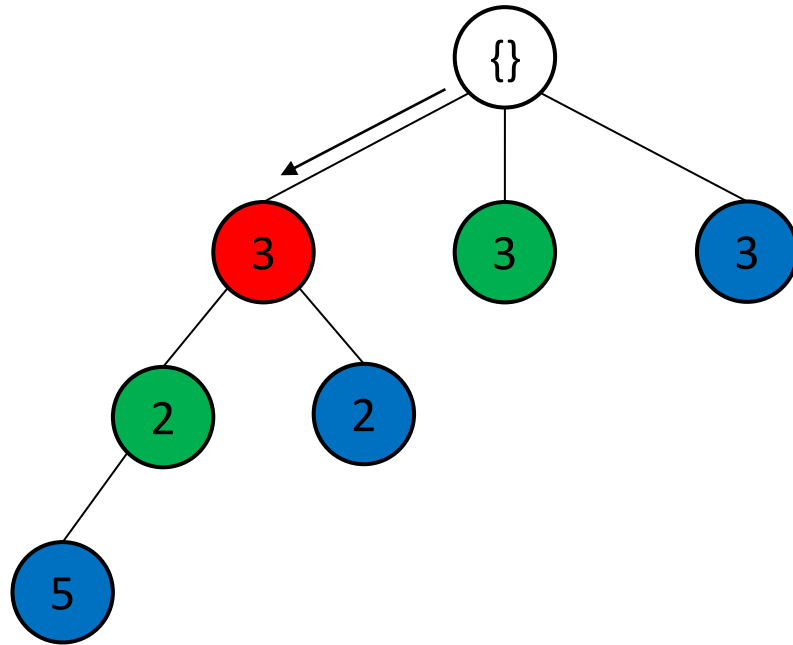
# Forward checking & Degree Heuristics (2)



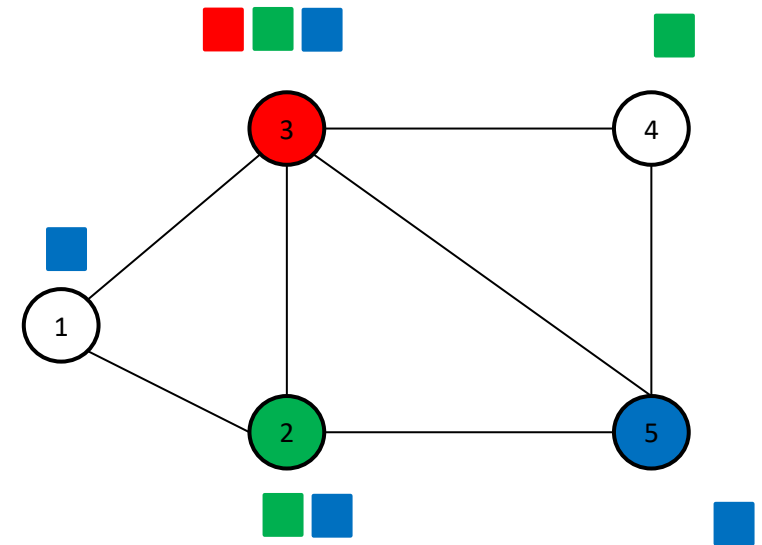
Obilazak: 3,2,5,4,1



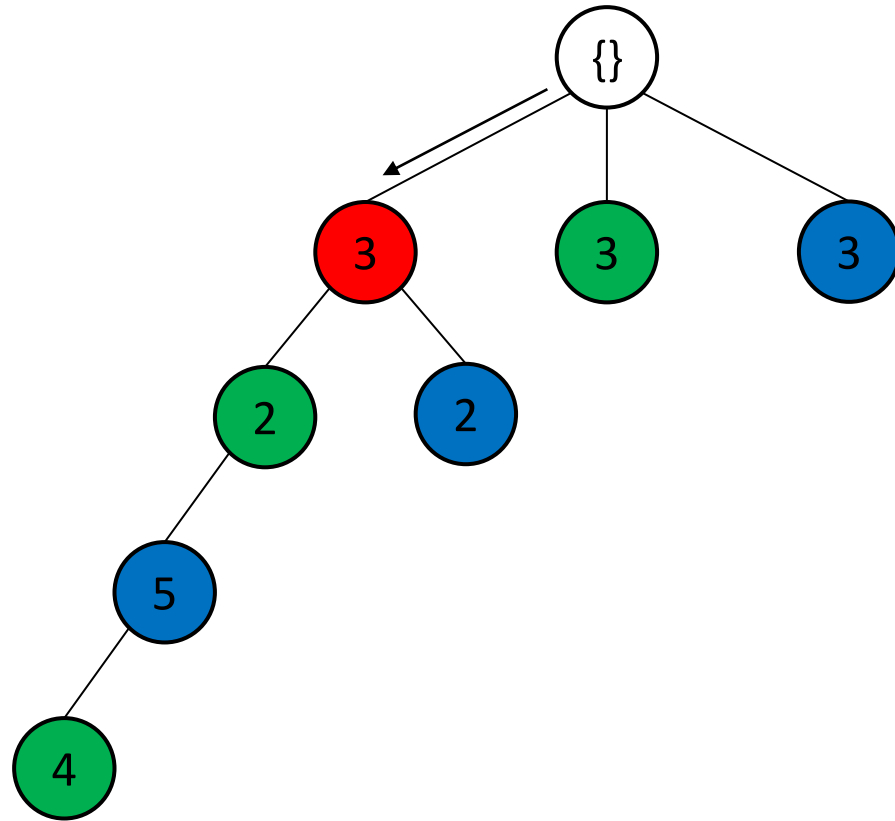
# Forward checking & Degree Heuristics (3)



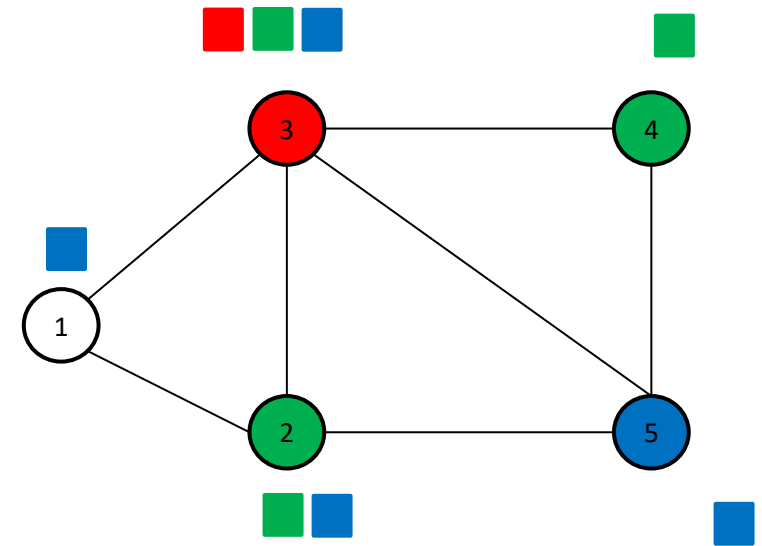
Obilazak: 3,2,5,4,1



# Forward checking & Degree Heuristics (4)

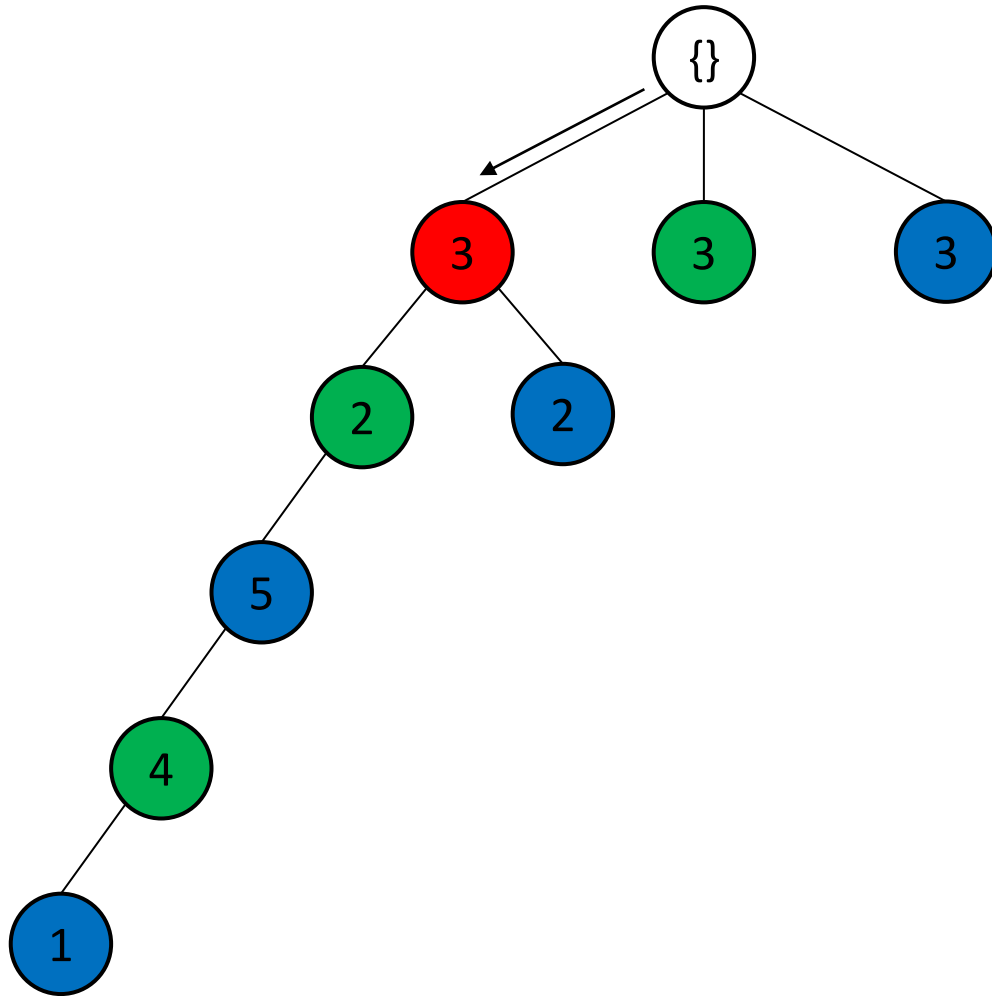


Obilazak: 3,2,5,4,1

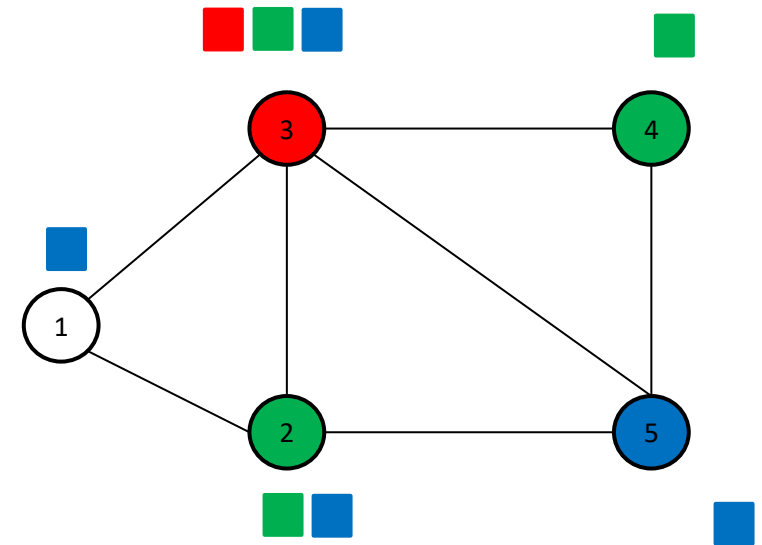




# Forward checking & Degree Heuristics (5)








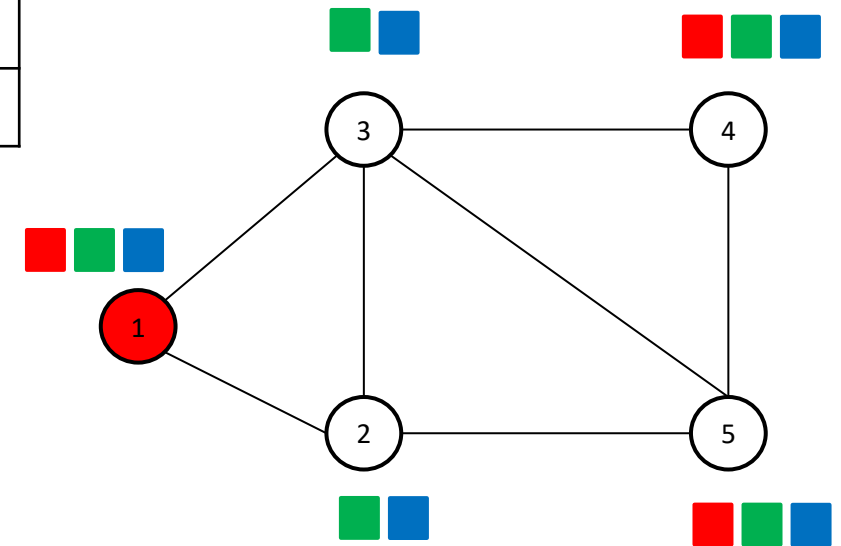
Obilazak: 3,2,5,4,1













# Forward checking & MRV (1)

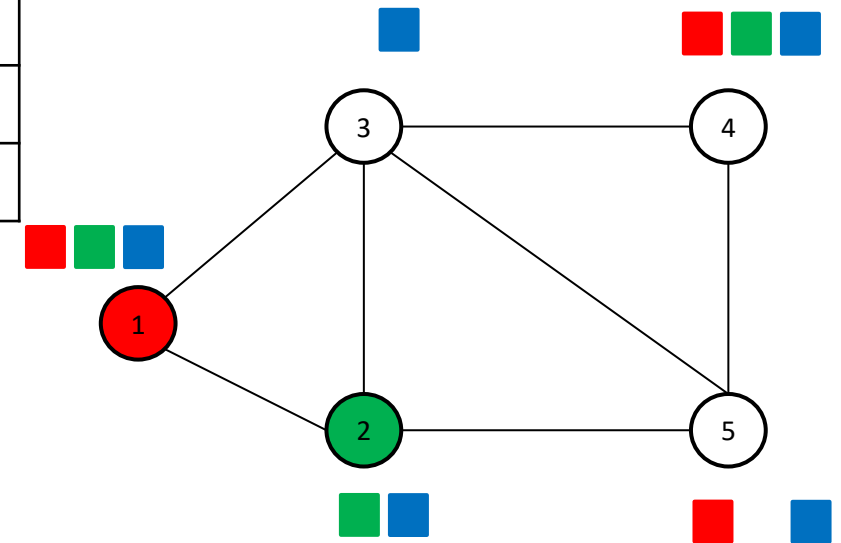
Start with order: 1,2,5,4,3

1	2	5	4	3
				




























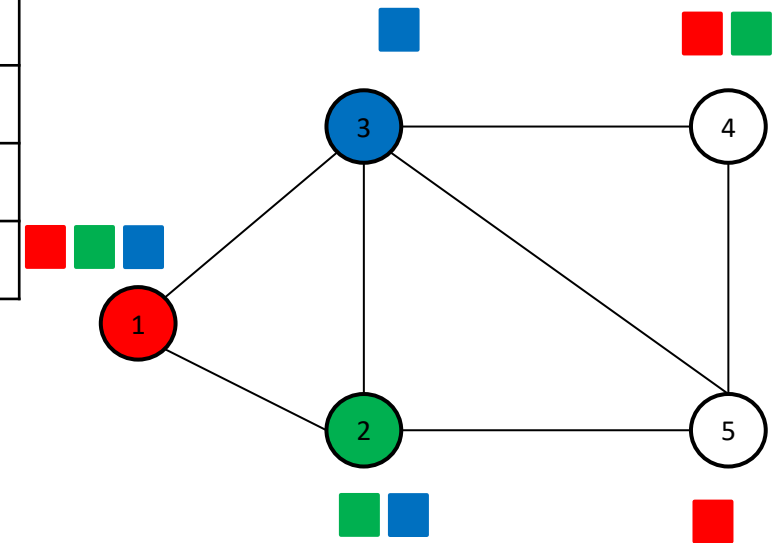
# Forward checking & MRV (2)

1	2	5	4	3
				
				

































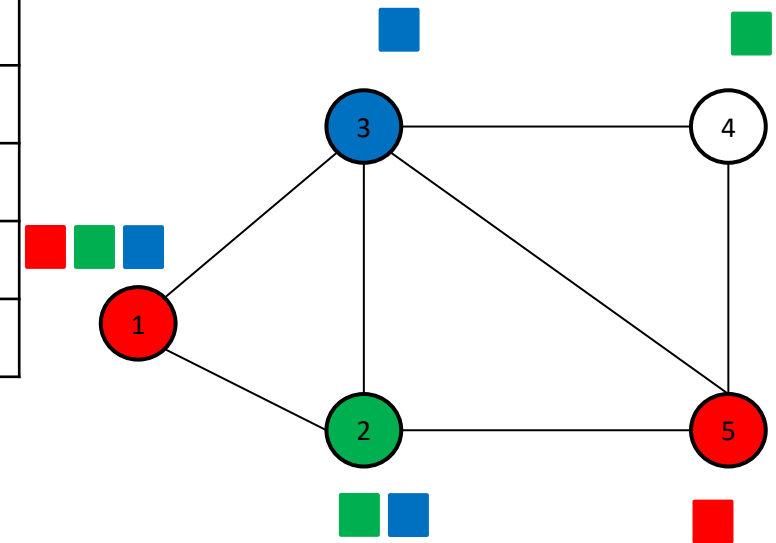
# Forward checking & MRV (3)

1	2	5	4	3
	 	  	  	 
		 	  	
			 	






































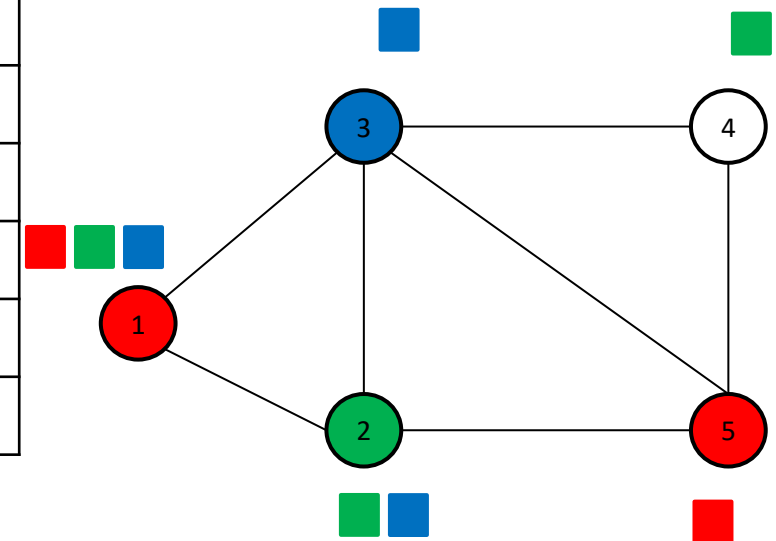
# Forward checking & MRV (4)

1	2	5	4	3
	 	  	  	 
		 	  	
			 	
				








# Forward checking & MRV (5)

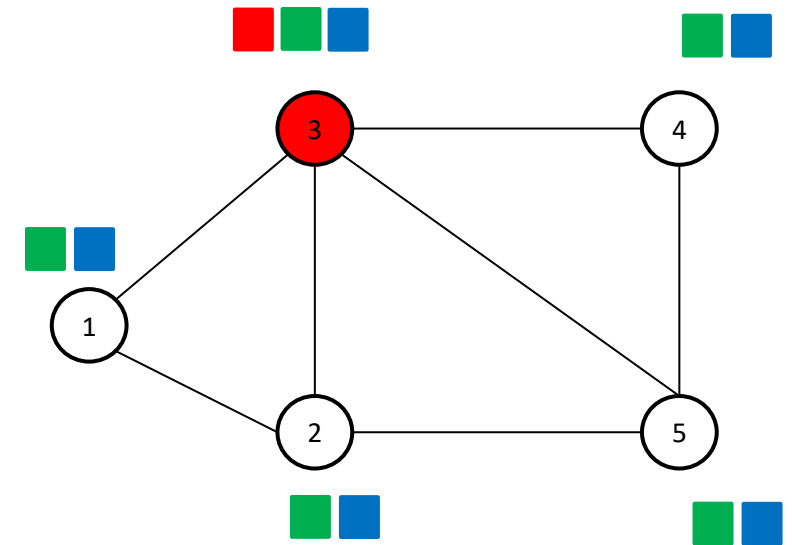
1	2	5	4	3
	 	  	  	 
		 	  	
			 	
				
				



# Forward checking & MRV & DH (1)
















Start: 3 (Degree Heuristics)

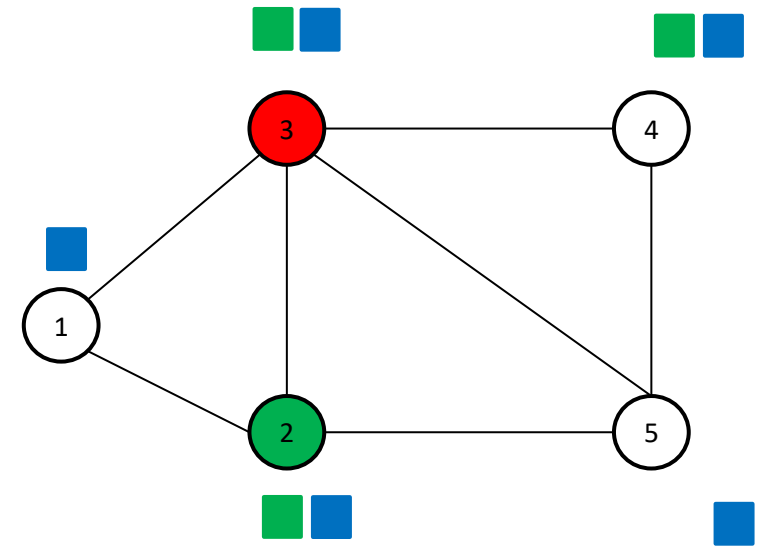
3	2	5	4	1
				



# Forward checking & MRV & DH (2)

Start: 3 (Degree Heuristics)
















3	2	5	4	1
	 	 	 	 
			 	

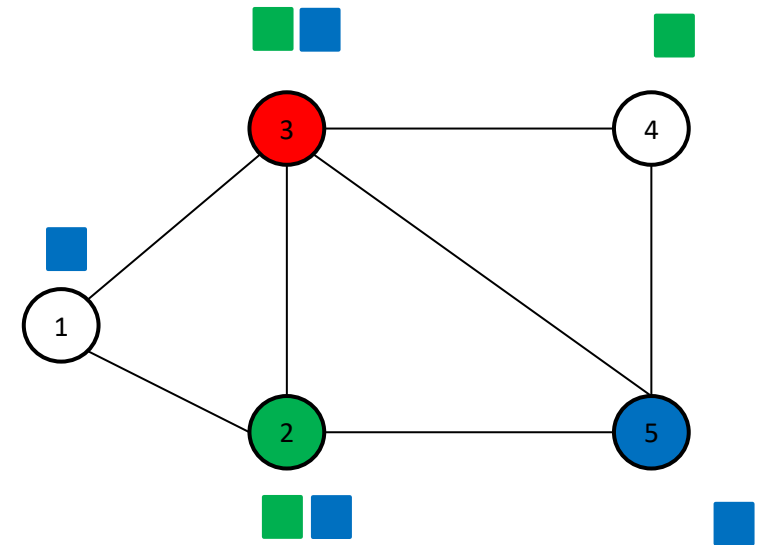




# Forward checking & MRV & DH (3)





















Start: 3 (Degree Heuristics)

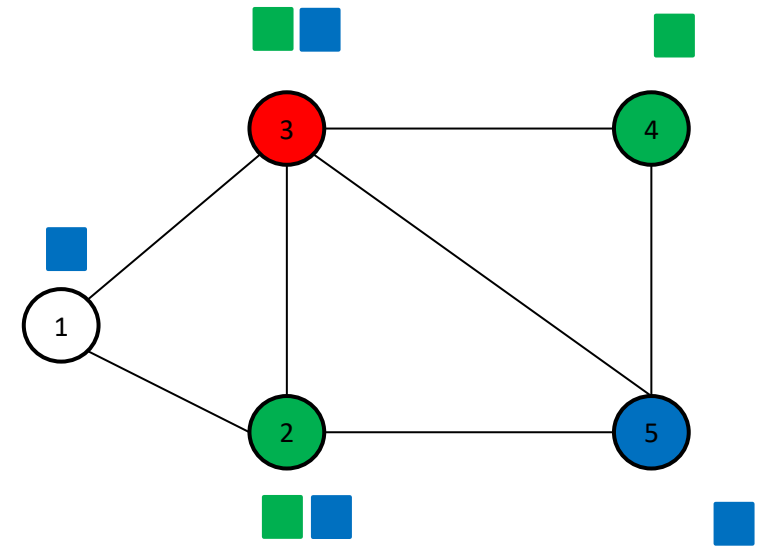
3	2	5	4	1
				
				
				



# Forward checking & MRV & DH (4)


























Start: 3 (Degree Heuristics)

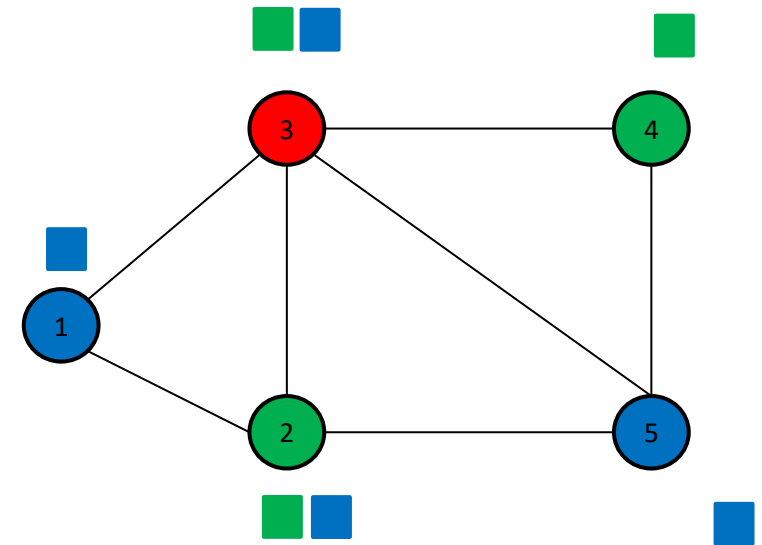
3	2	5	4	1
				
				
				
				



# Forward checking & MRV & DH (5)




Start: 3 (Degree Heuristics)

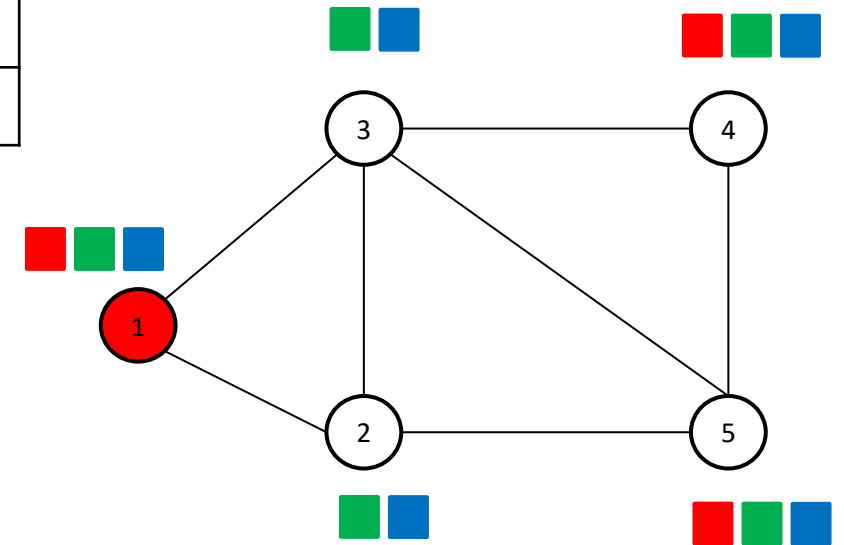
3	2	5	4	1
				
				
				
				
				



# Forward checking & LCV (1)




















Order: 1,2,5,4,3

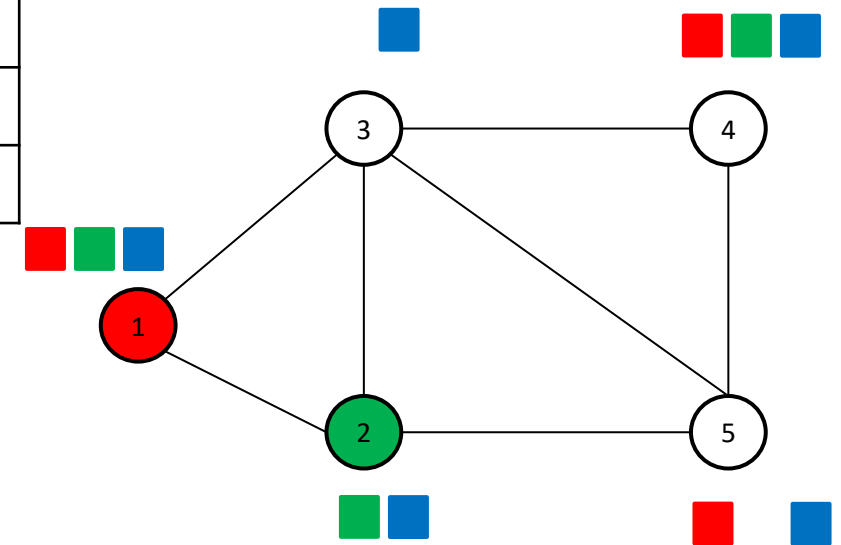
1	2	5	4	3
				



# Forward checking & LCV (2)






















Order: 1,2,5,4,3

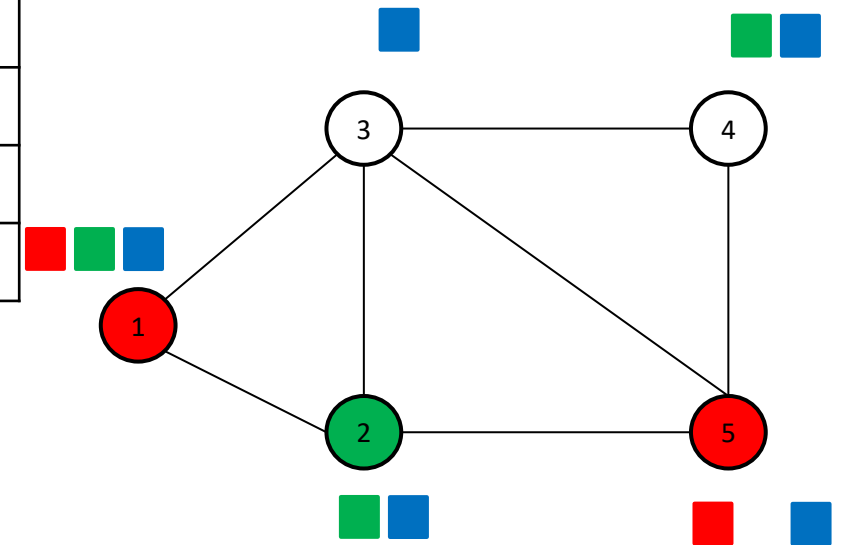
1	2	5	4	3
	 	  	  	 
		 	  	



# Forward checking & LCV (3)































Order: 1,2,5,4,3

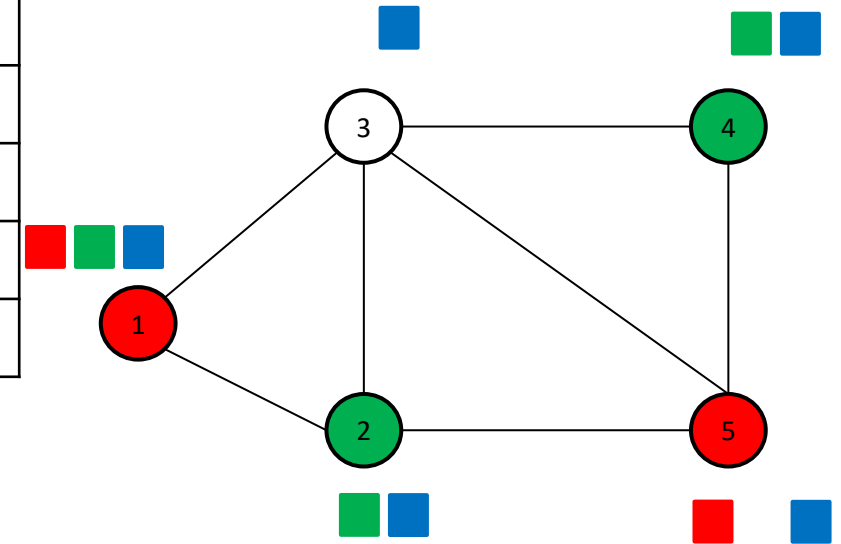
1	2	5	4	3
	 	  	  	 
		 	  	
			 	



# Forward checking & LCV (4)












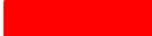























Order: 1,2,5,4,3

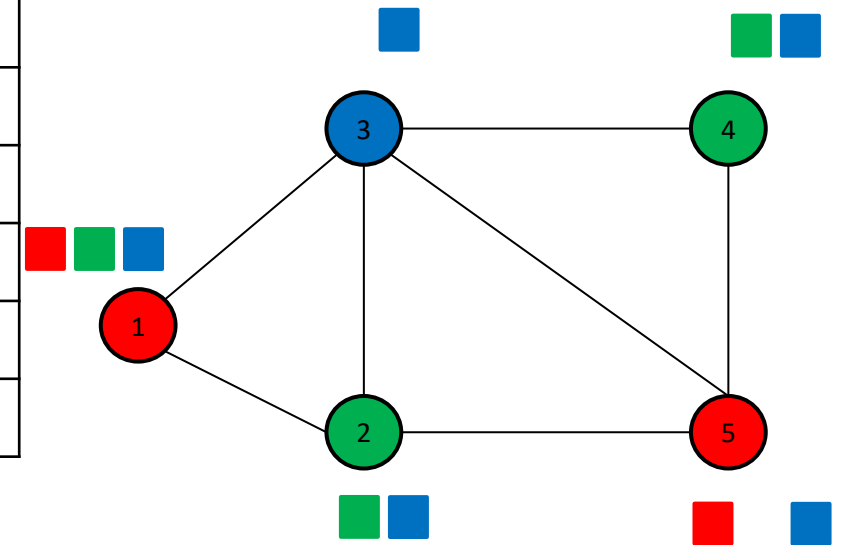
1	2	5	4	3
	 	  	  	 
		 	  	
			 	
				



# Forward checking & LCV (5)

Order: 1,2,5,4,3

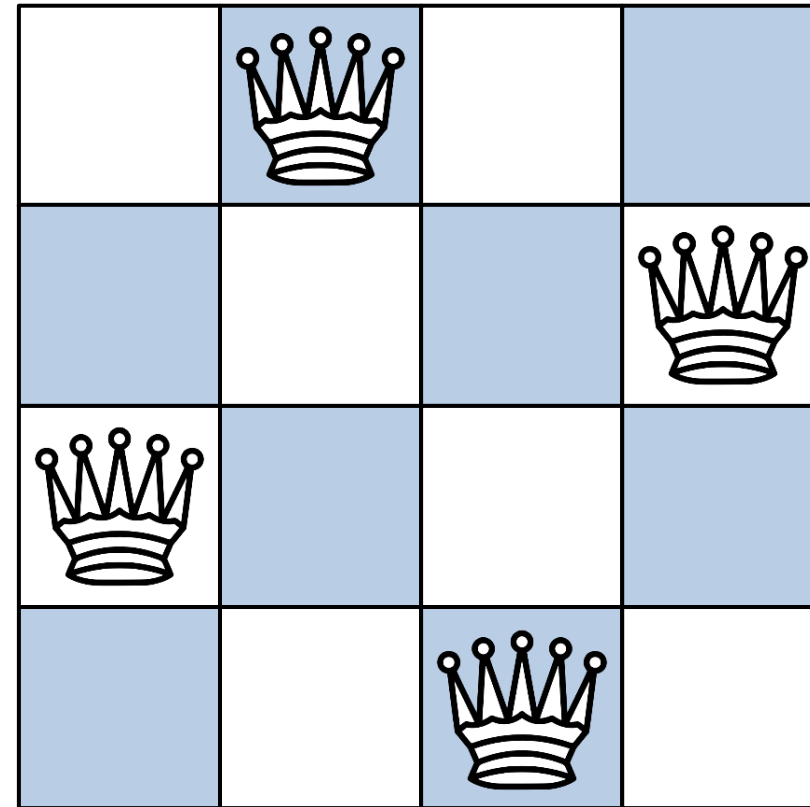
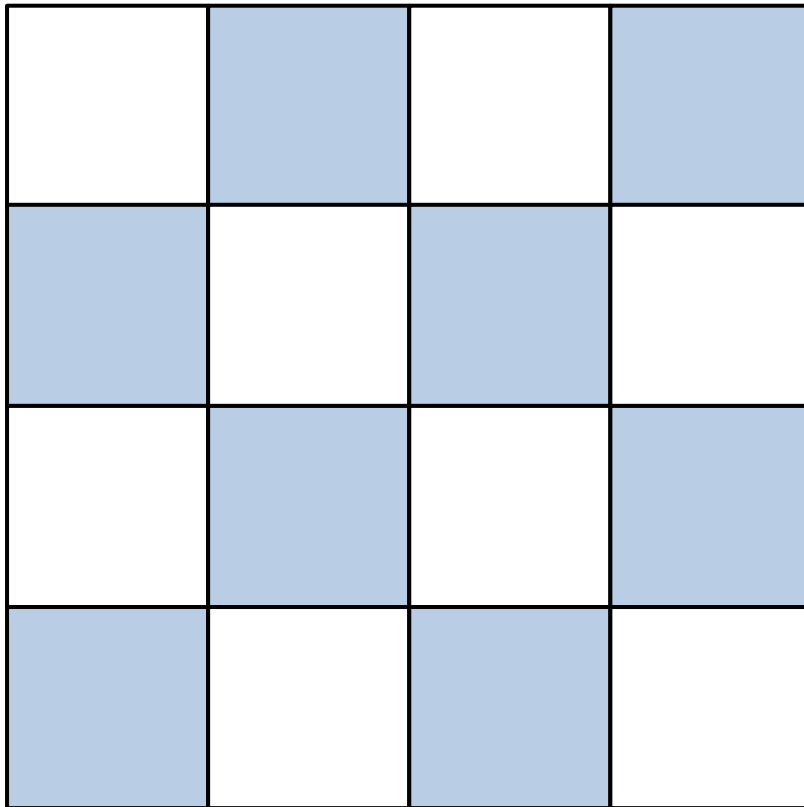
1	2	5	4	3
	 	  	  	 
		 	  	
			 	
				
				



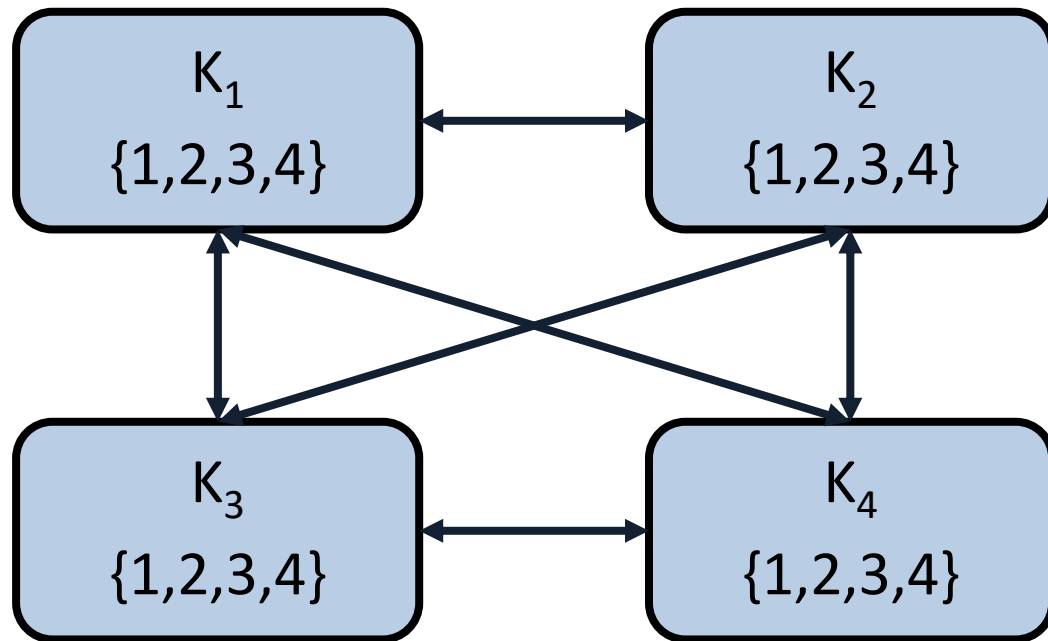


# CSP za problem 4 kraljice

## ► Početno i ciljno stanje problema

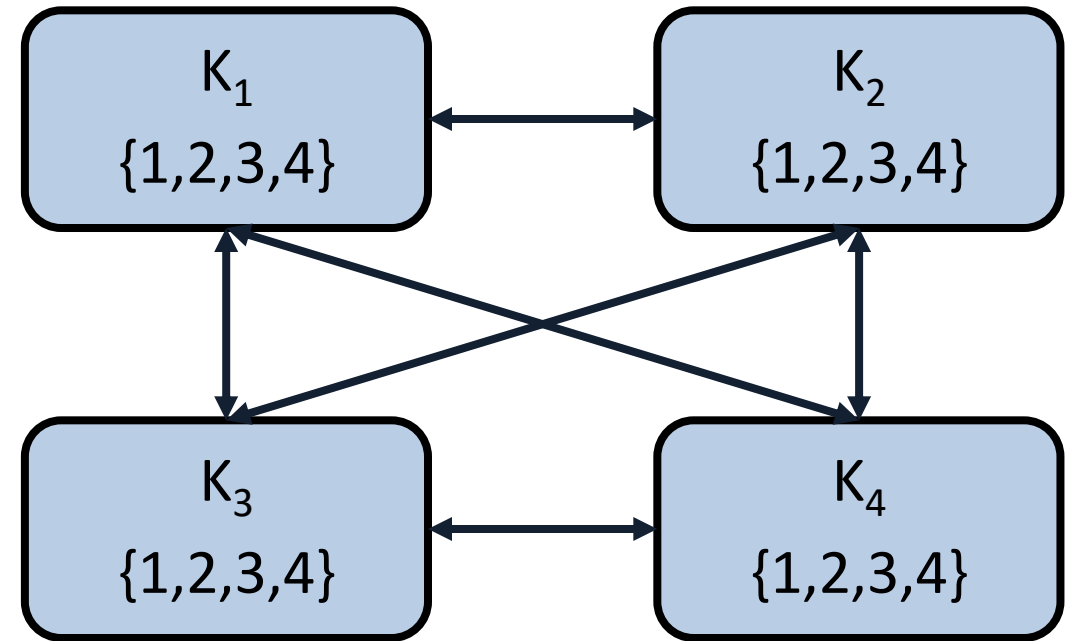


# CSP za problem 4 kraljice



- ▶ Stanje problema odgovara dodeljivanju vrednosti jednoj od promenljivih, tj. izboru kolone u kojoj će se naći jedna od kraljica
- ▶ Redosled izbora kraljice kojoj treba dodeliti kolonu je proizvoljan

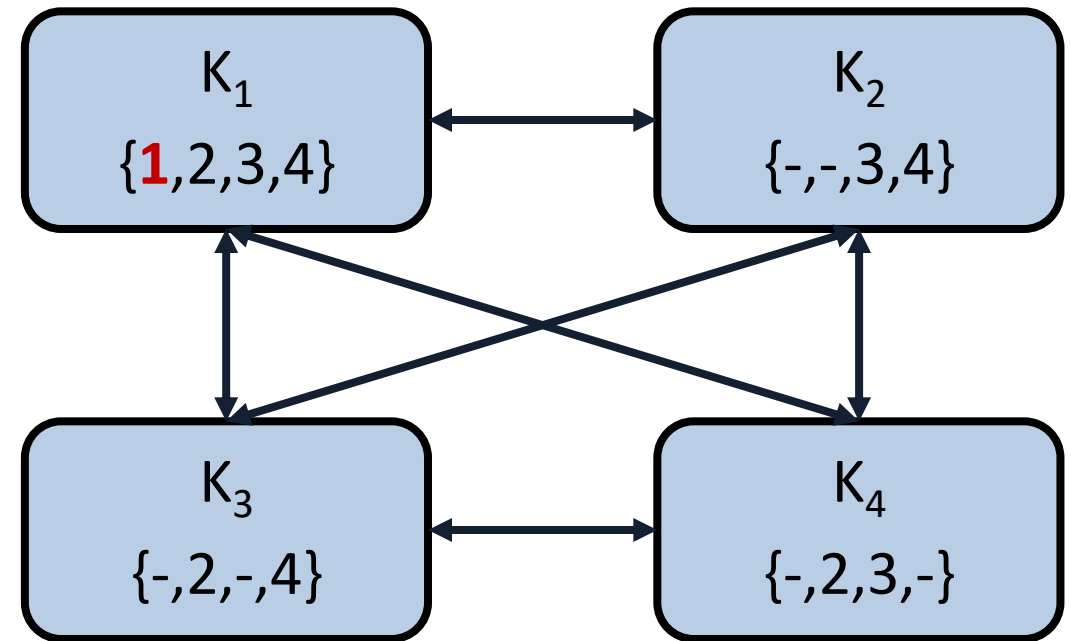
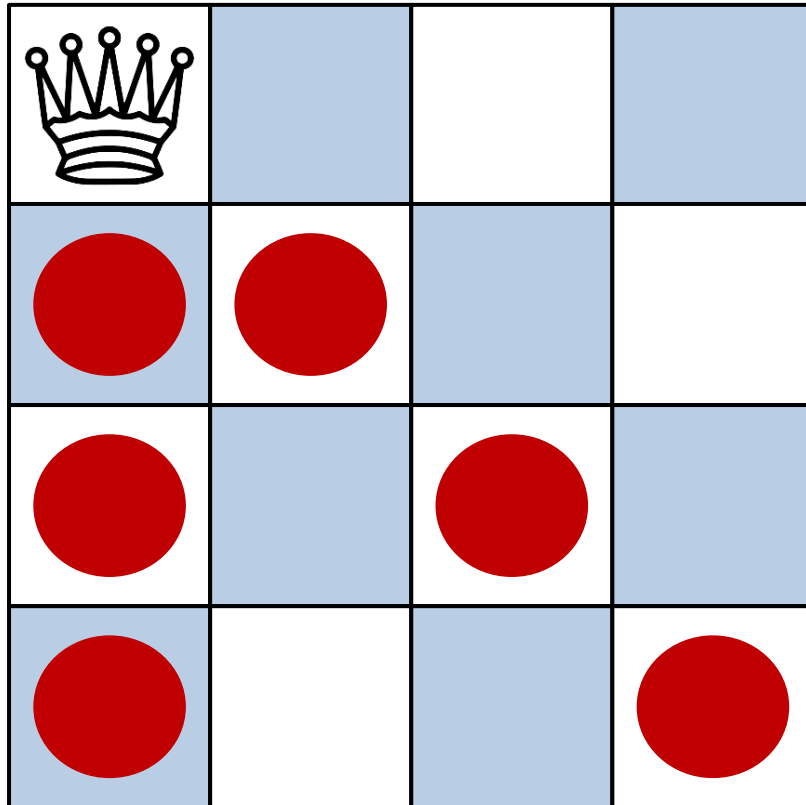
# CSP FC MRV 4 kraljice

MRV:  $K_1 K_2 K_3 K_4$



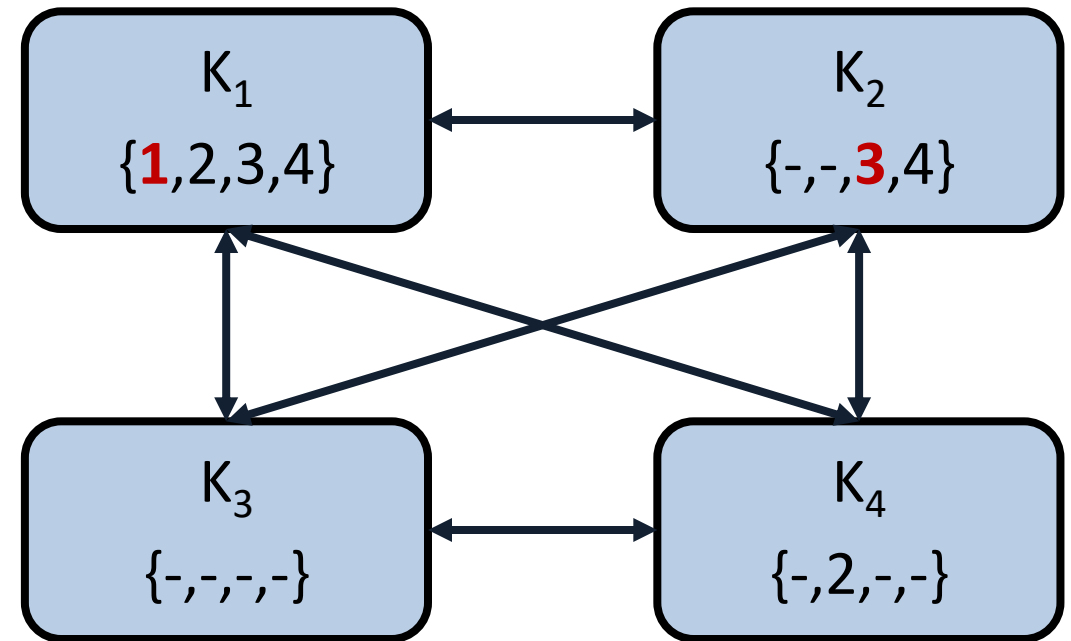
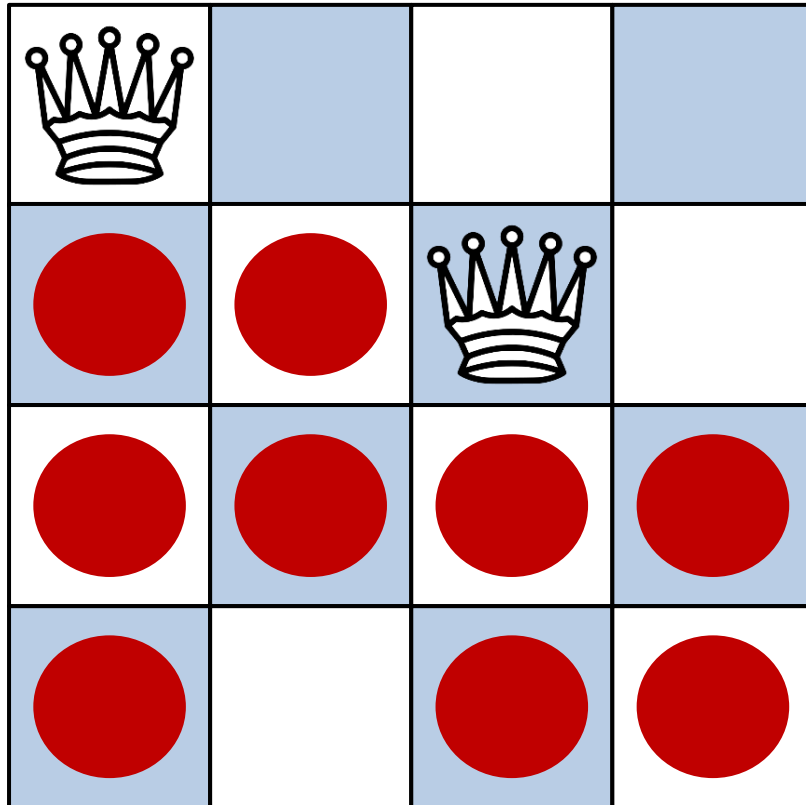
# CSP FC MRV 4 kraljice (2)



MRV:  $K_2$   $K_3$   $K_4$



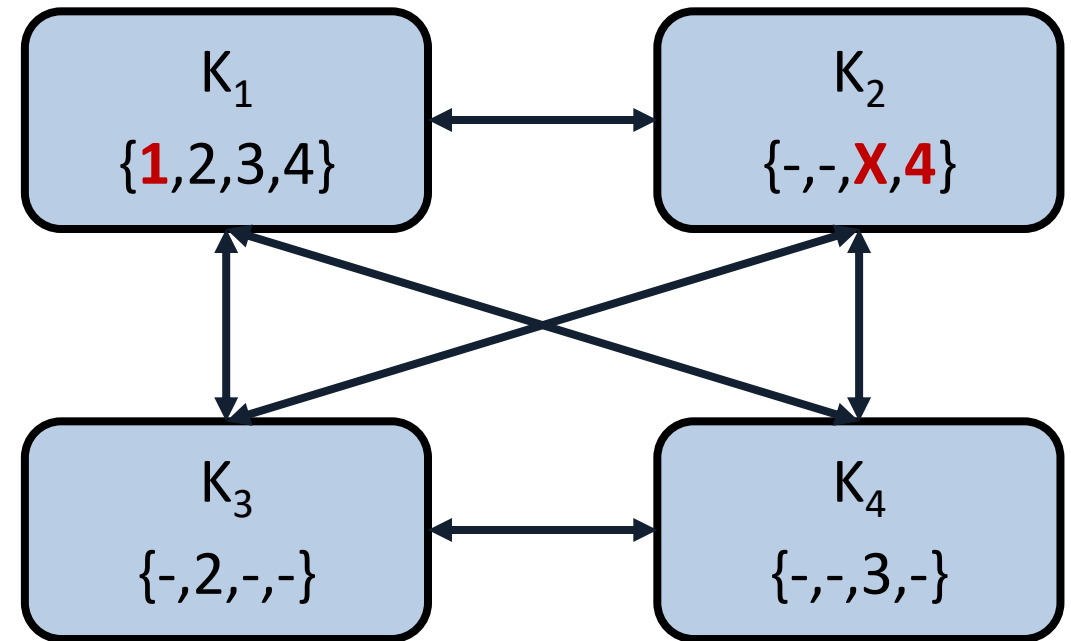
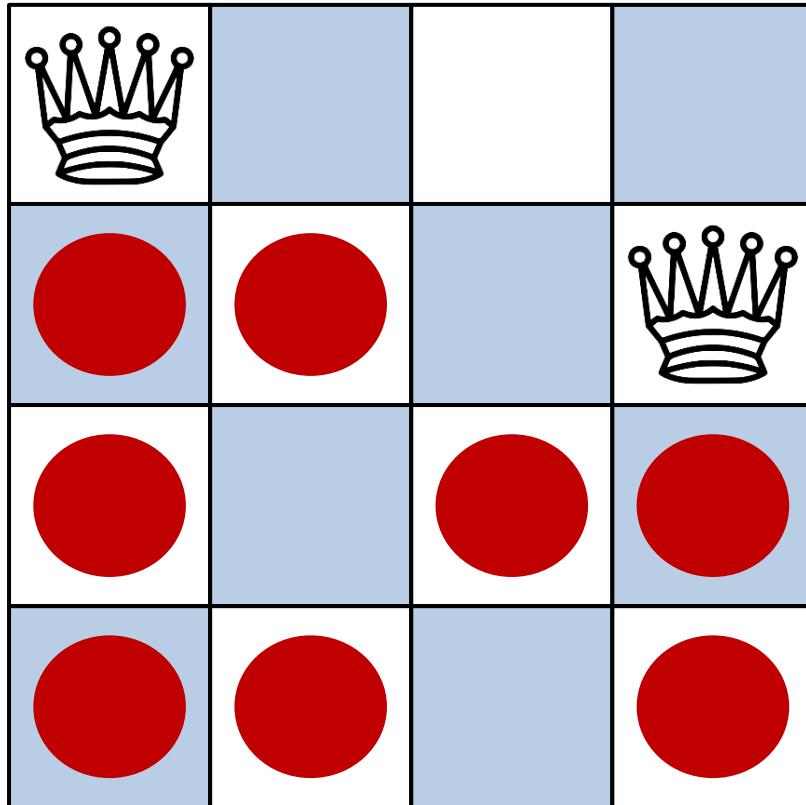
# CSP FC MRV 4 kraljice (3)



FC: failure



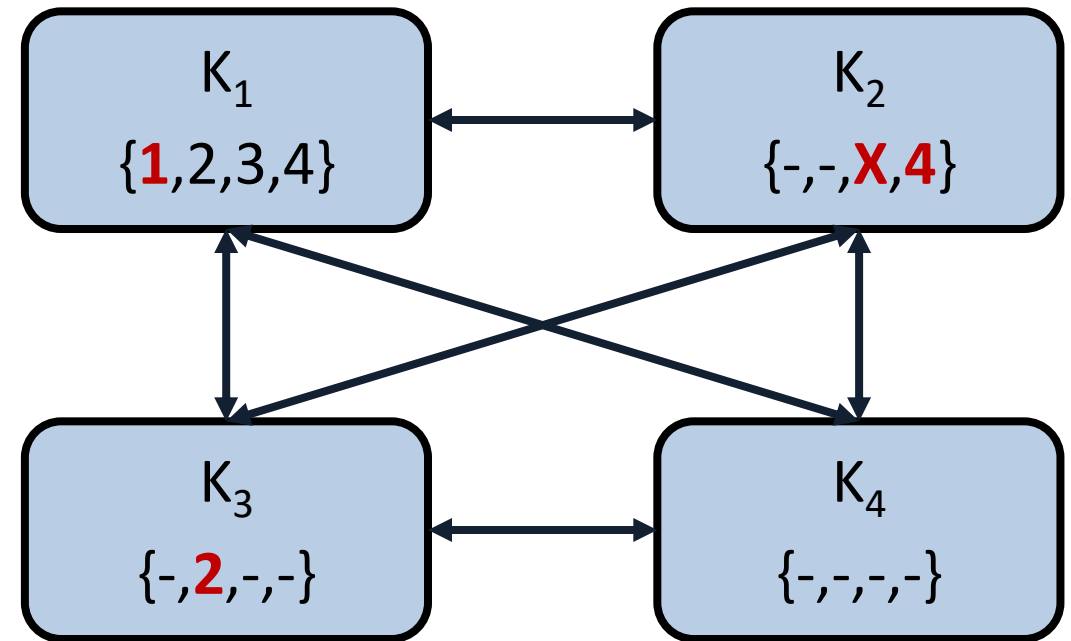
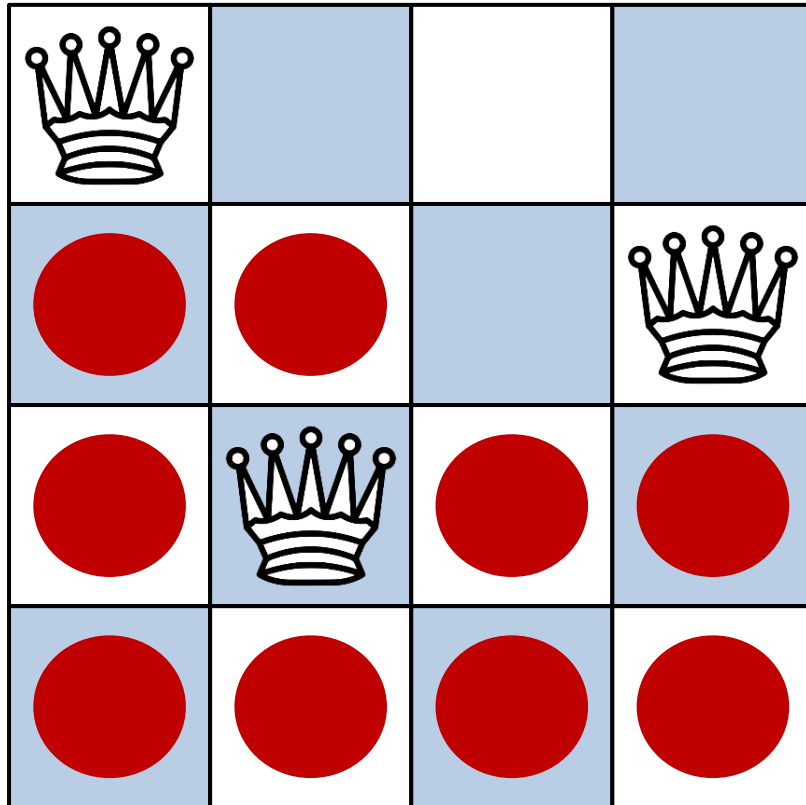
# CSP FC MRV 4 kraljice (4)



MRV:  $K_3 K_4$



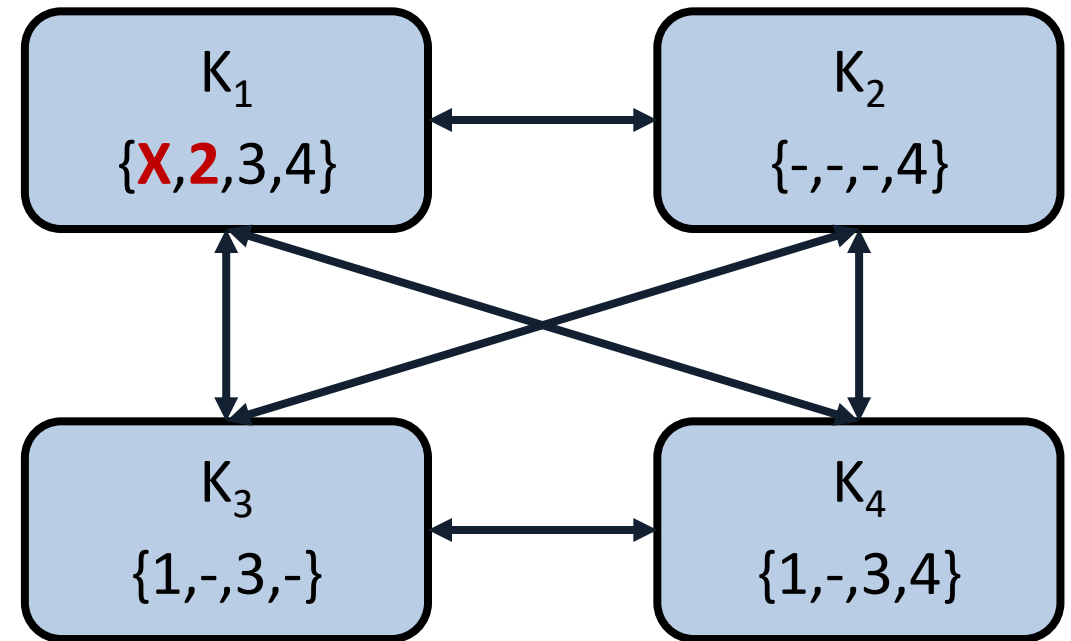
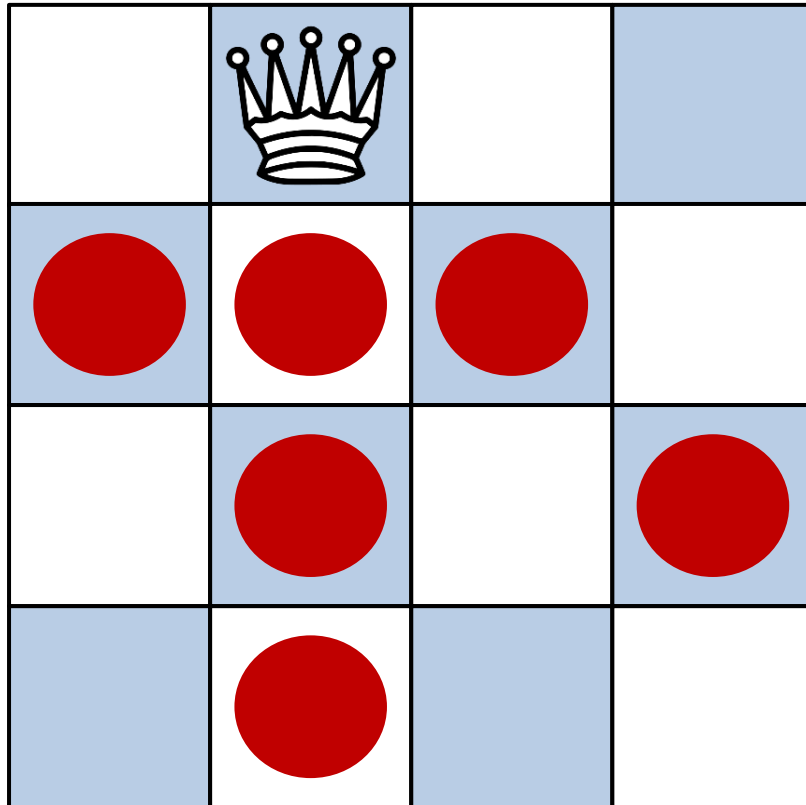
# CSP FC MRV 4 kraljice (5)



FC: failure



# CSP FC MRV 4 kraljice (6)

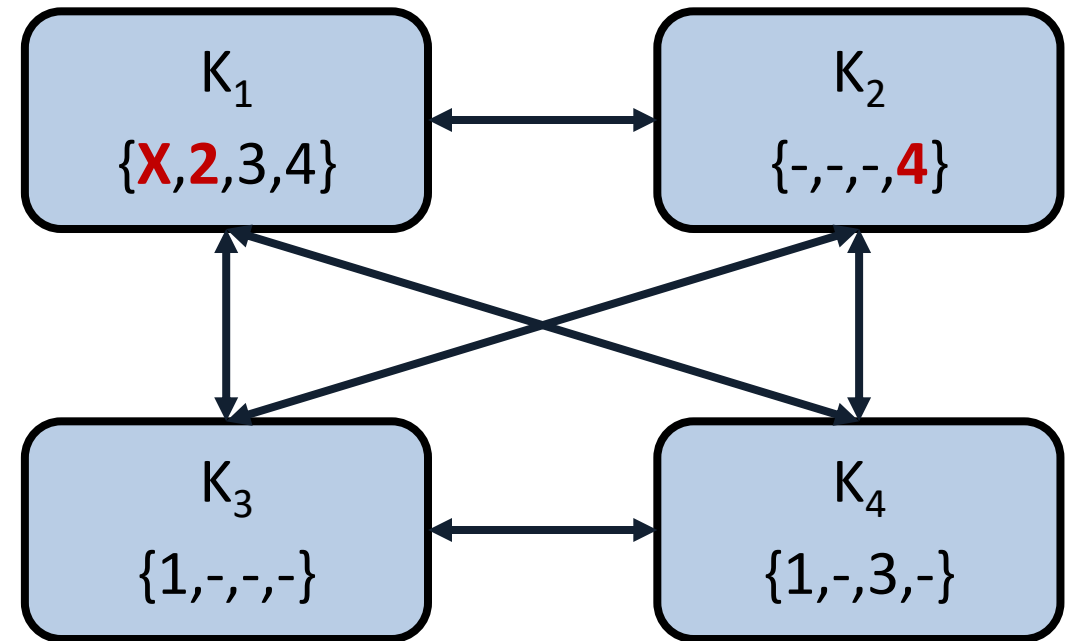
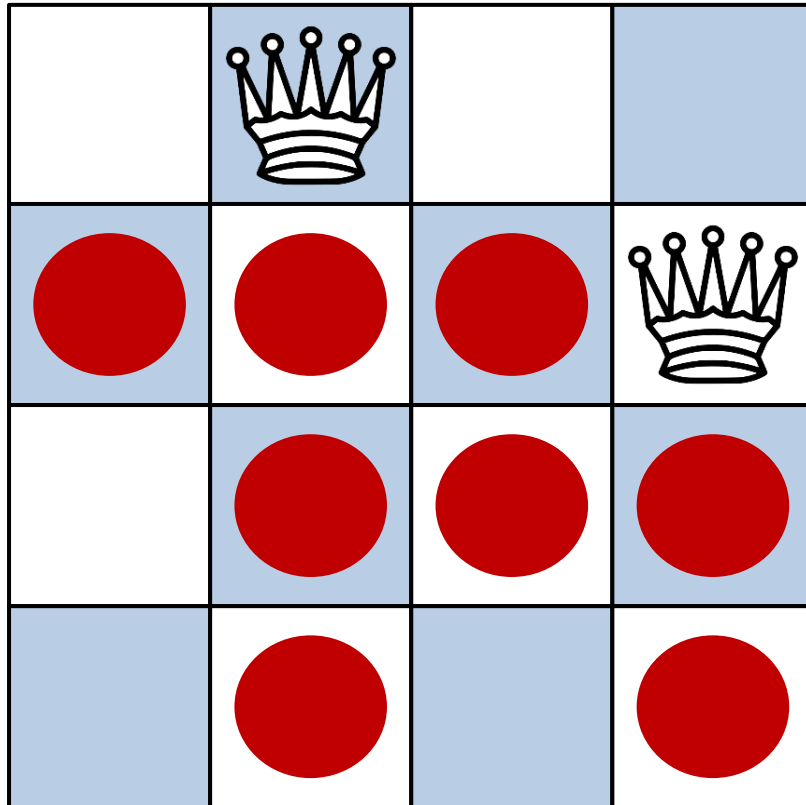


MRV:  $K_2$





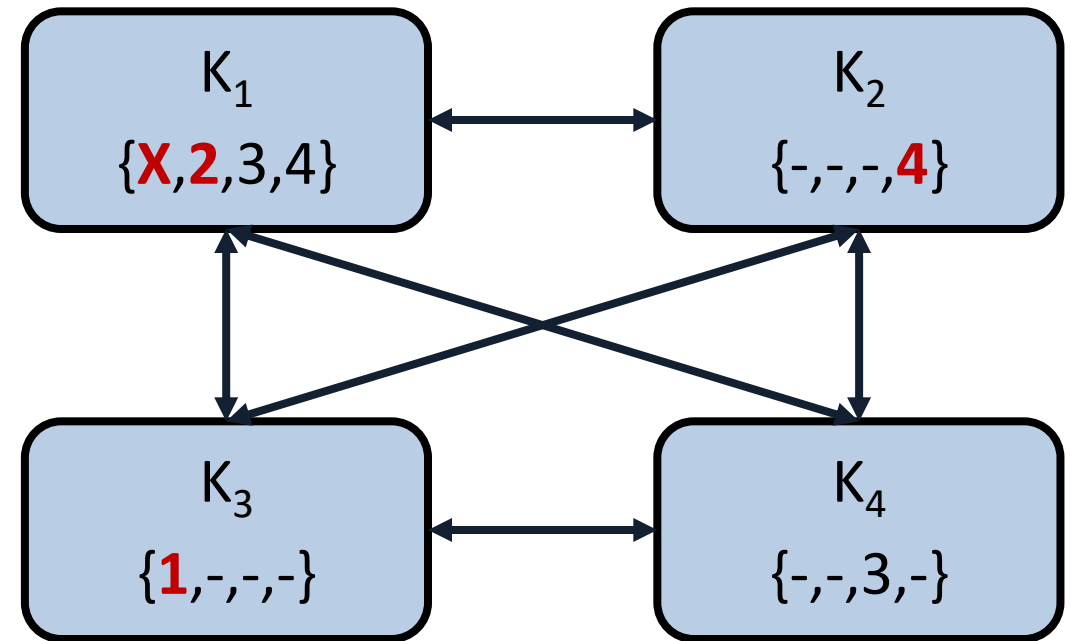
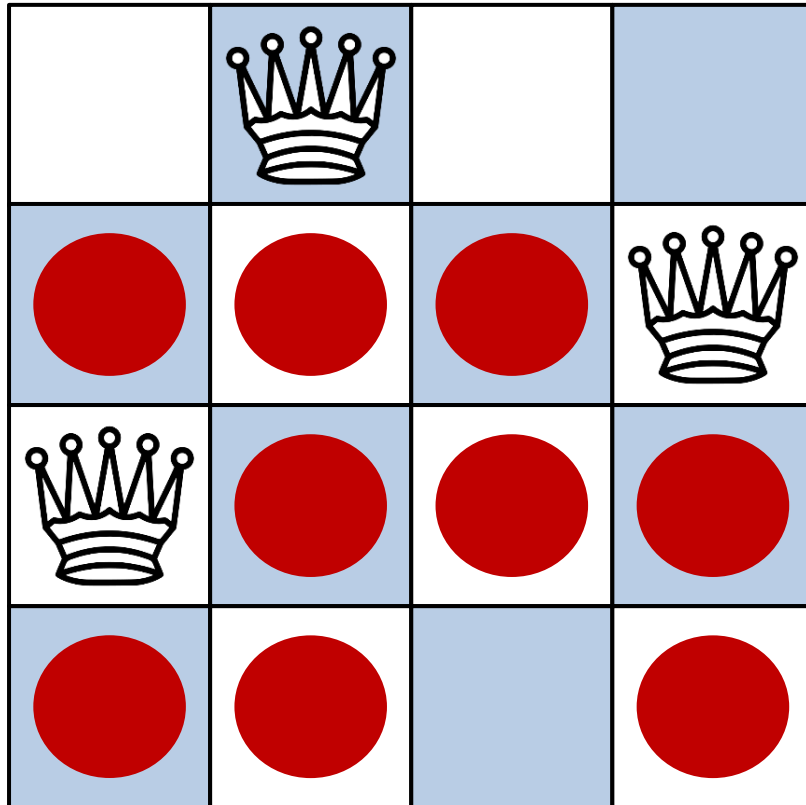
# CSP FC MRV 4 kraljice (7)



MRV:  $K_3$



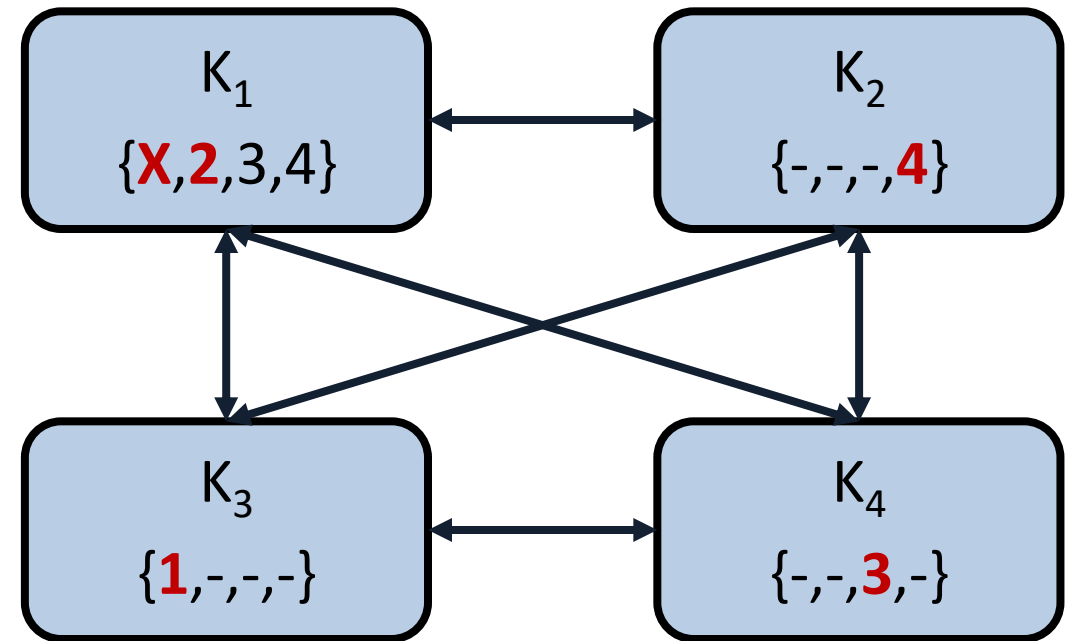
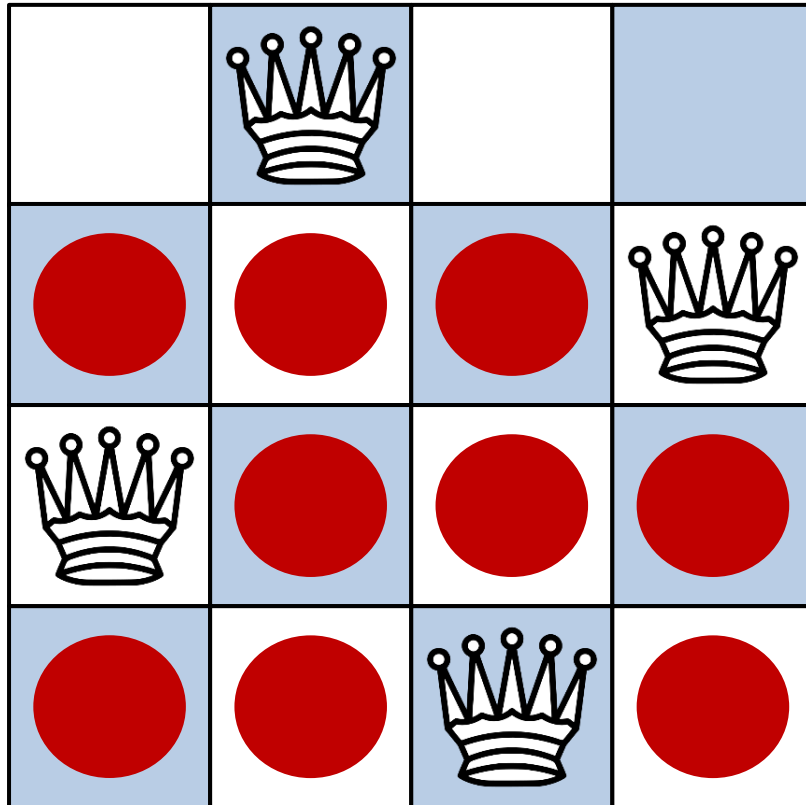
# CSP FC MRV 4 kraljice (8)



MVR:  $K_4$



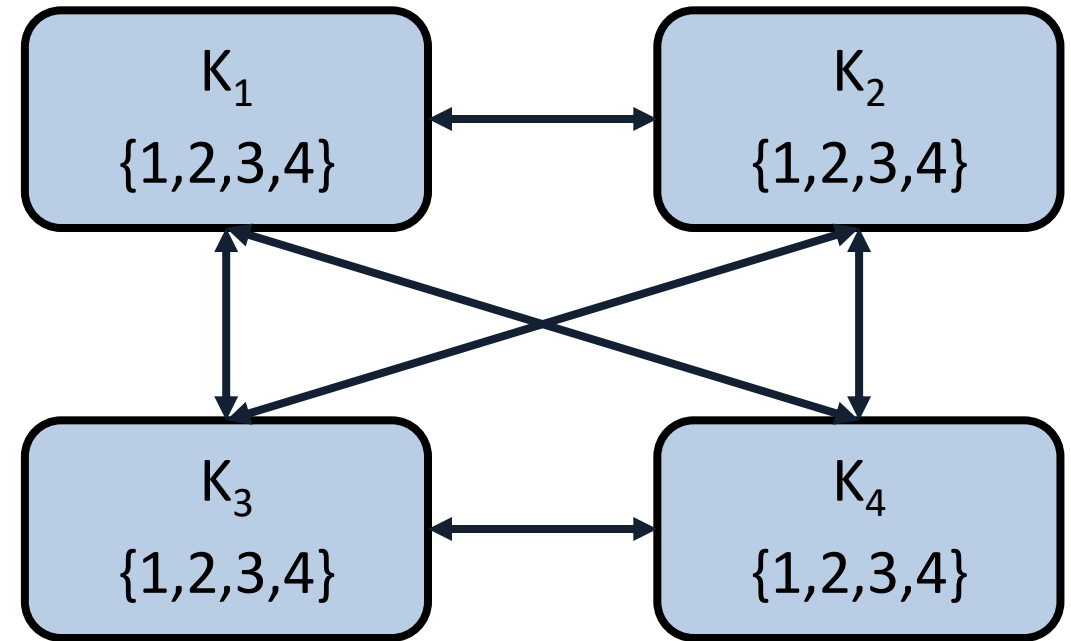
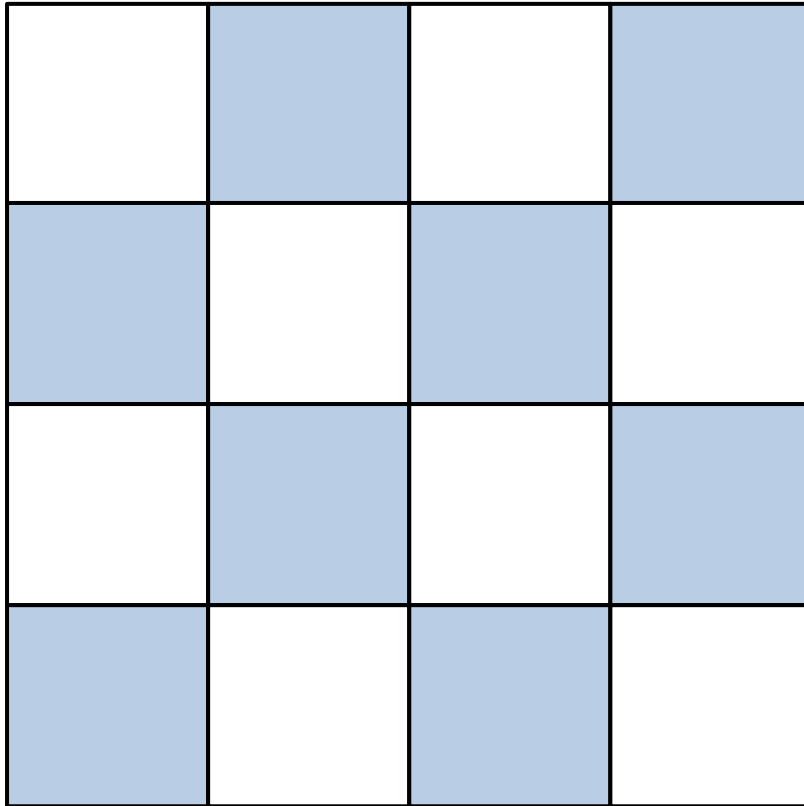
# CSP FC MRV 4 kraljice (9)



$(K_1, K_2, K_3, K_4) = (2, 4, 1, 3)$



# CSP FC LCV 4 kraljice

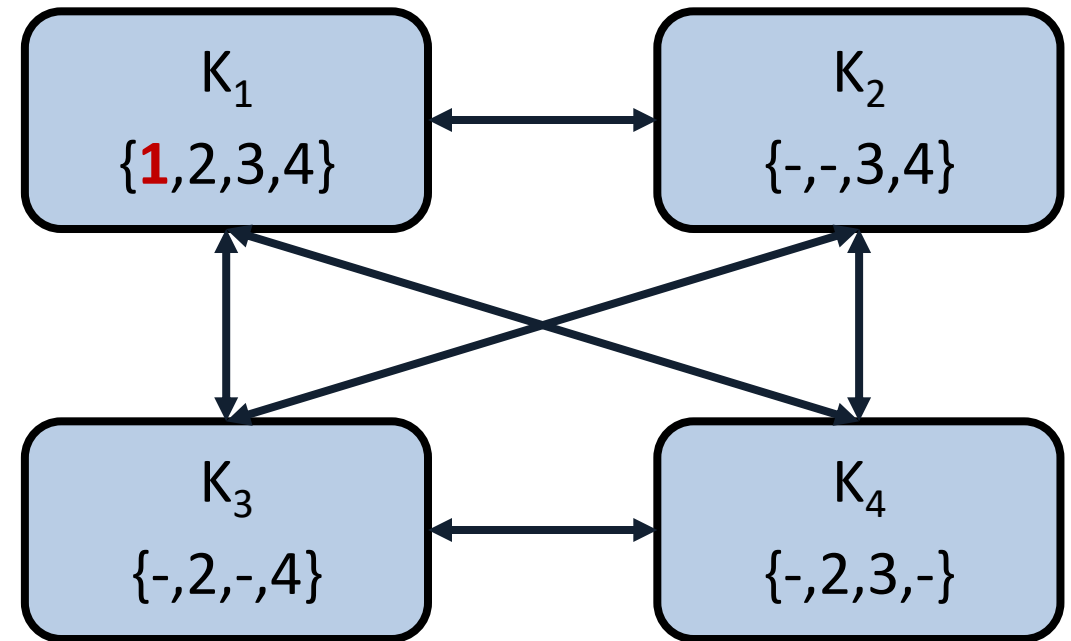
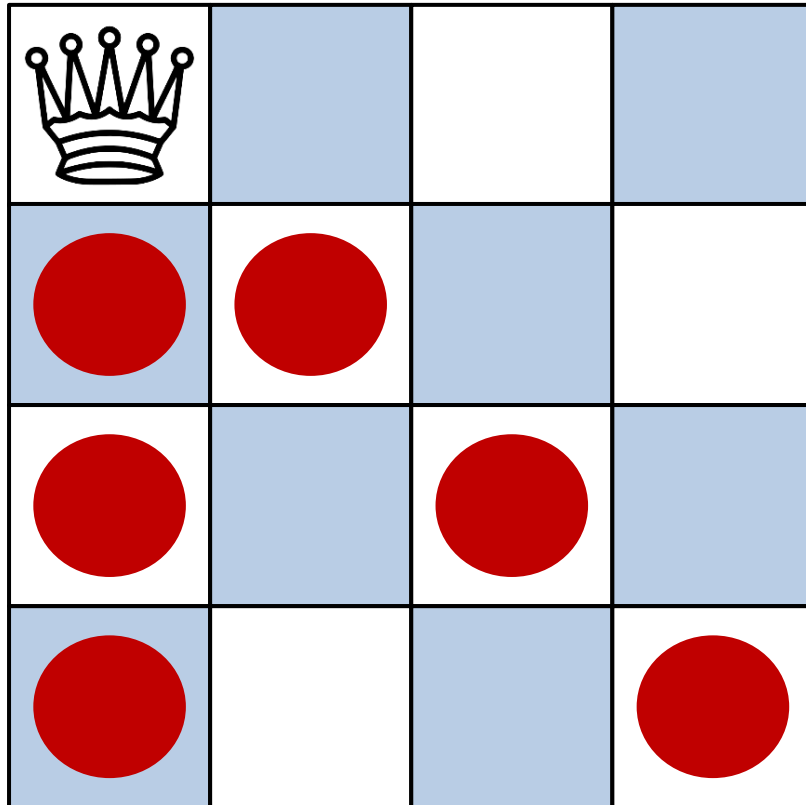


LCV:  $K_1 - 1$

Obilazak:  $K_1 K_2 K_3 K_4$



# CSP FC LCV 4 kraljice (2)

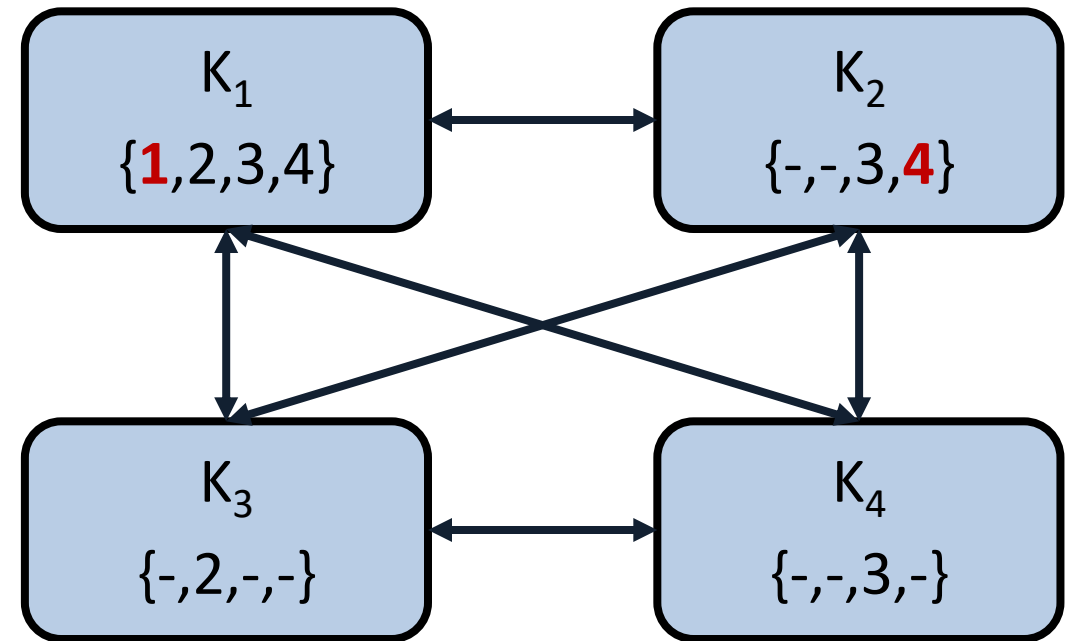
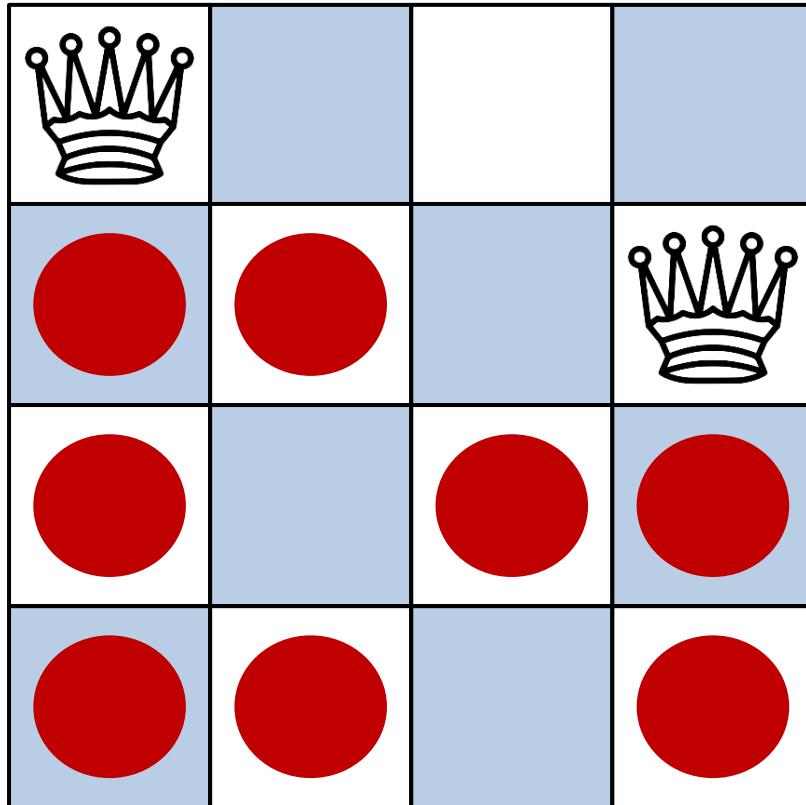


LCV:  $K_2 - 4$

Obilazak:  $K_1 K_2 K_3 K_4$



# CSP FC LCV 4 kraljice (3)

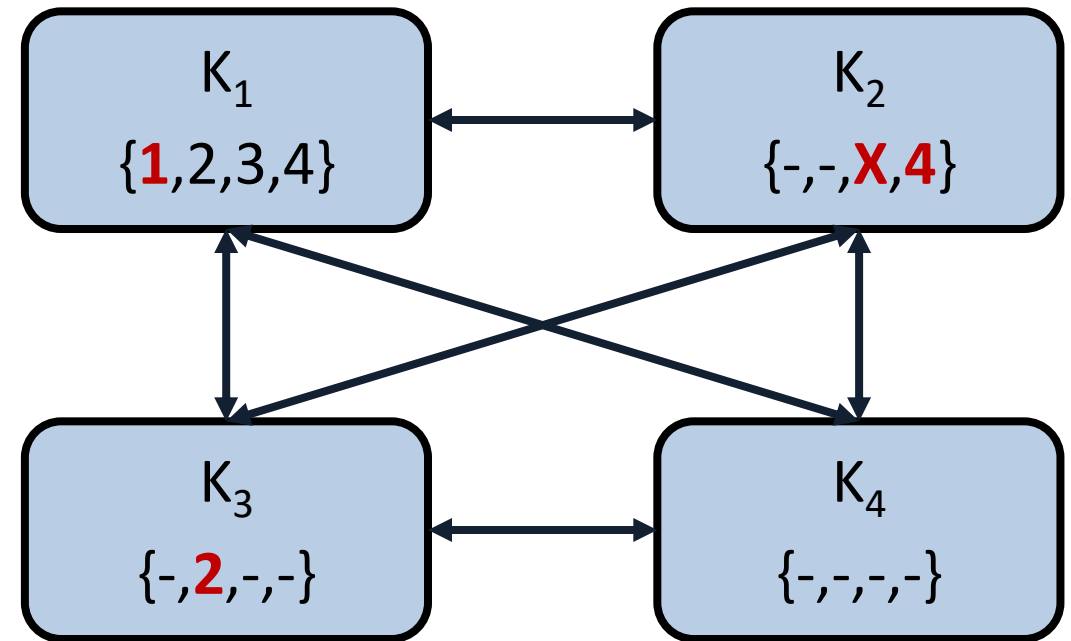
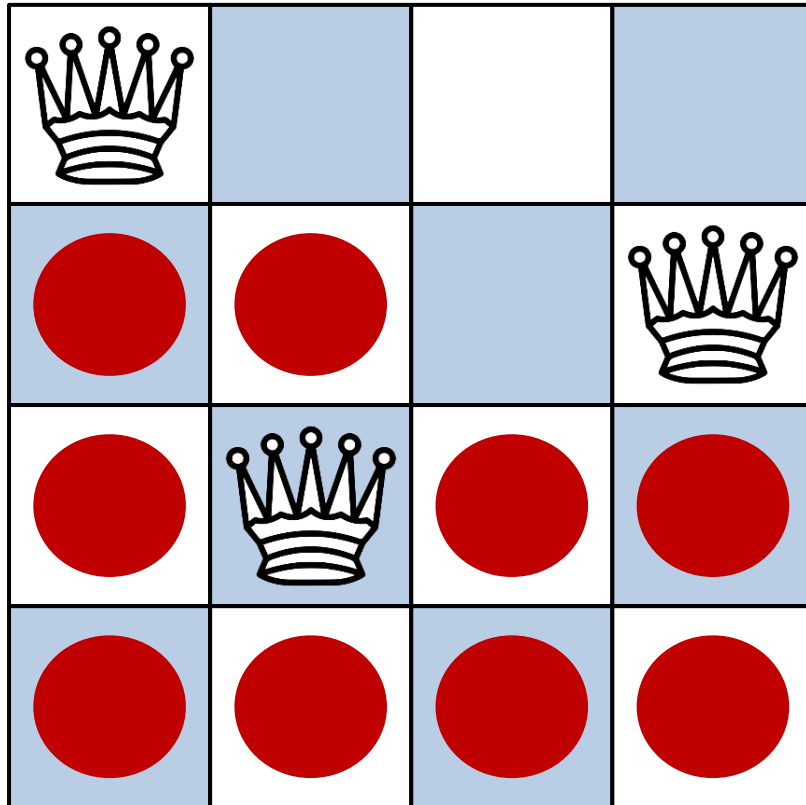


LCV:  $K_3 - 2$

Obilazak:  $K_1 K_2 K_3 K_4$



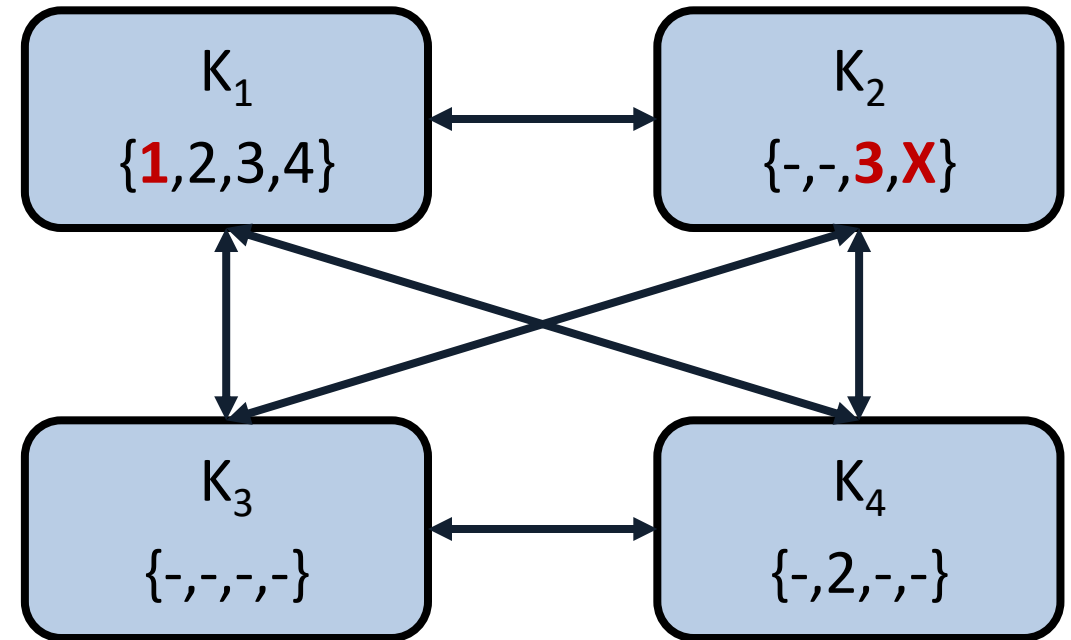
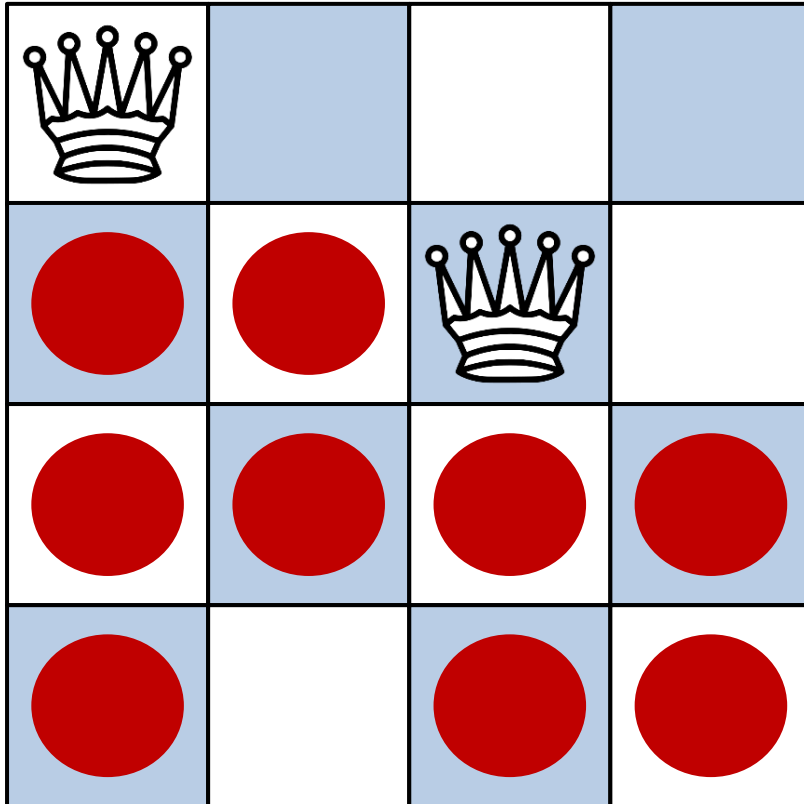
# CSP FC LCV 4 kraljice (4)



FC: failure



# CSP FC LCV 4 kraljice (5)

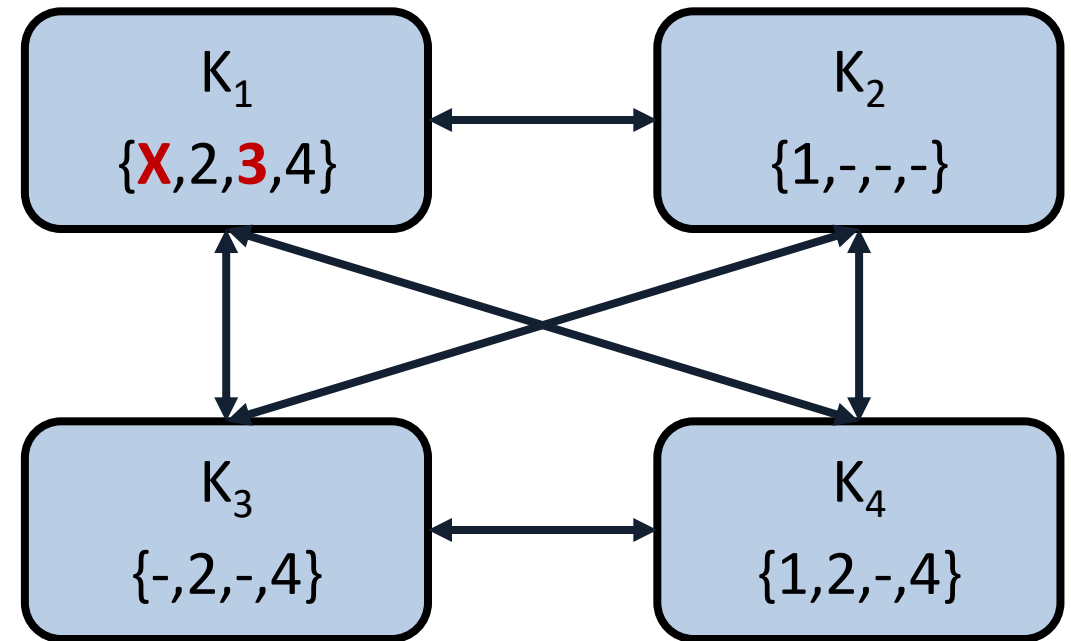
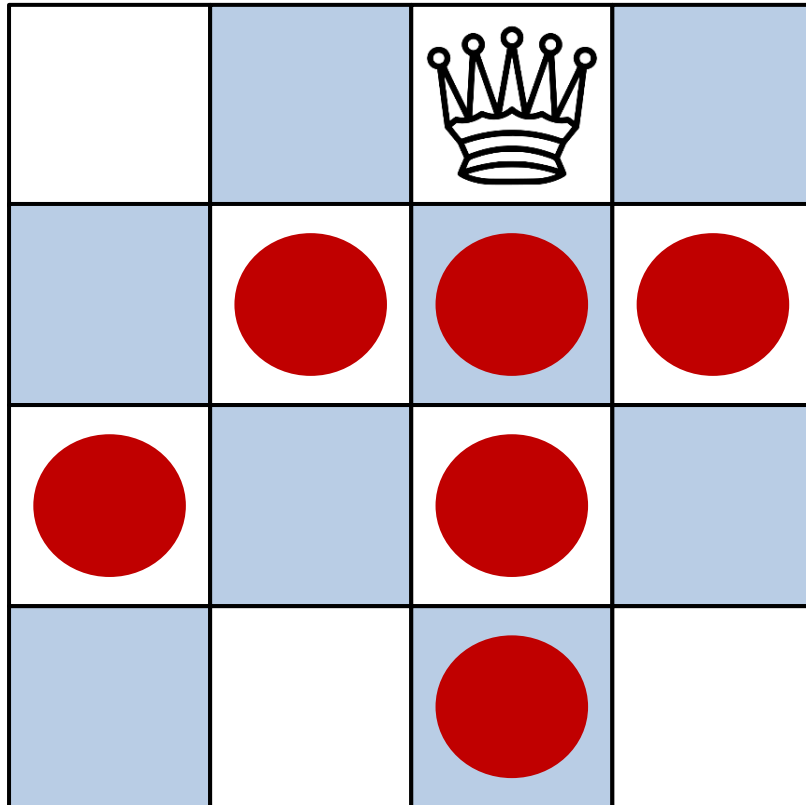


FC: failure





# CSP FC LCV 4 kraljice (6)

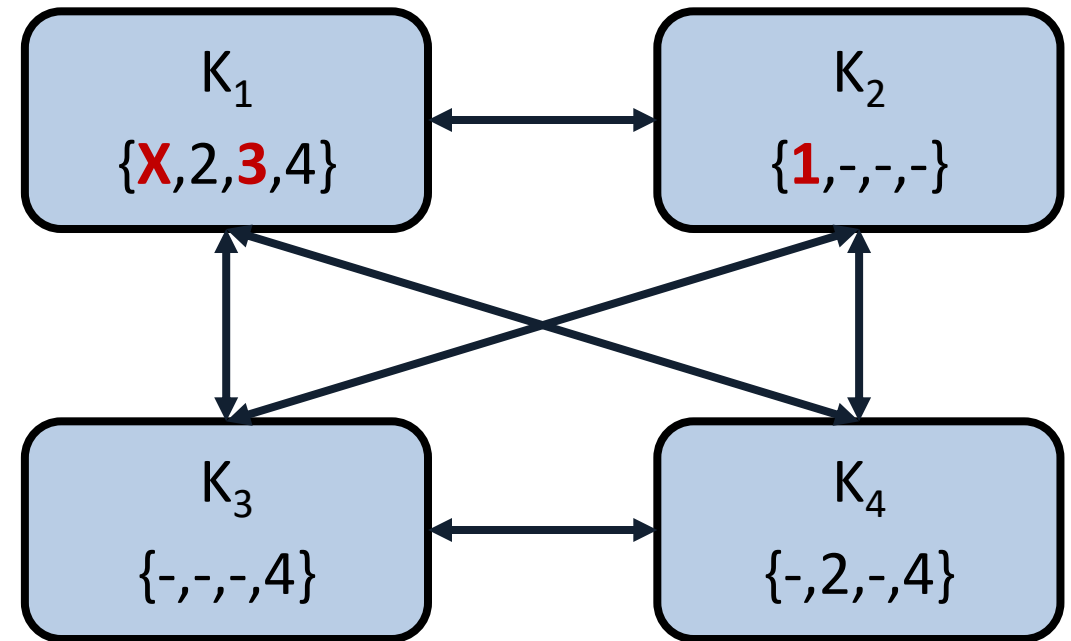
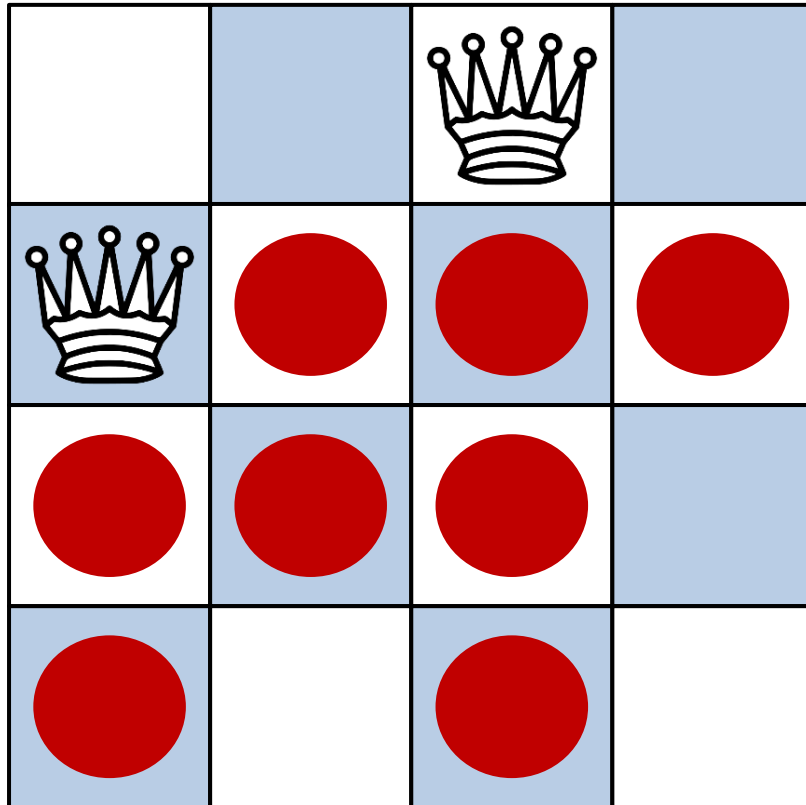


LCV:  $K_1 - 3$

Obilazak:  $K_1 K_2 K_3 K_4$



# CSP FC LCV 4 kraljice (7)

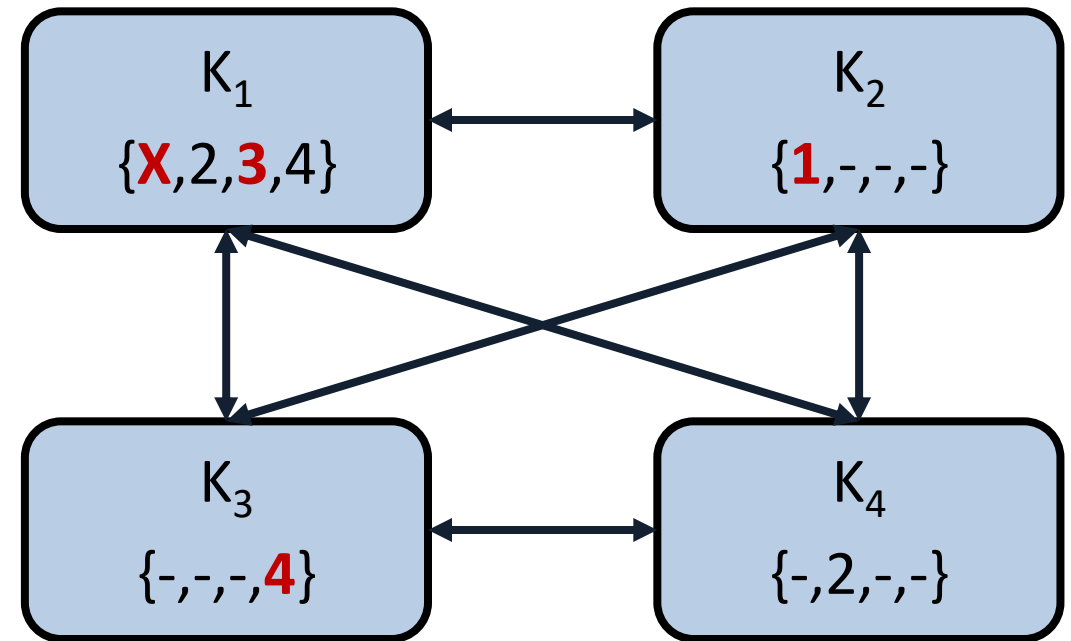
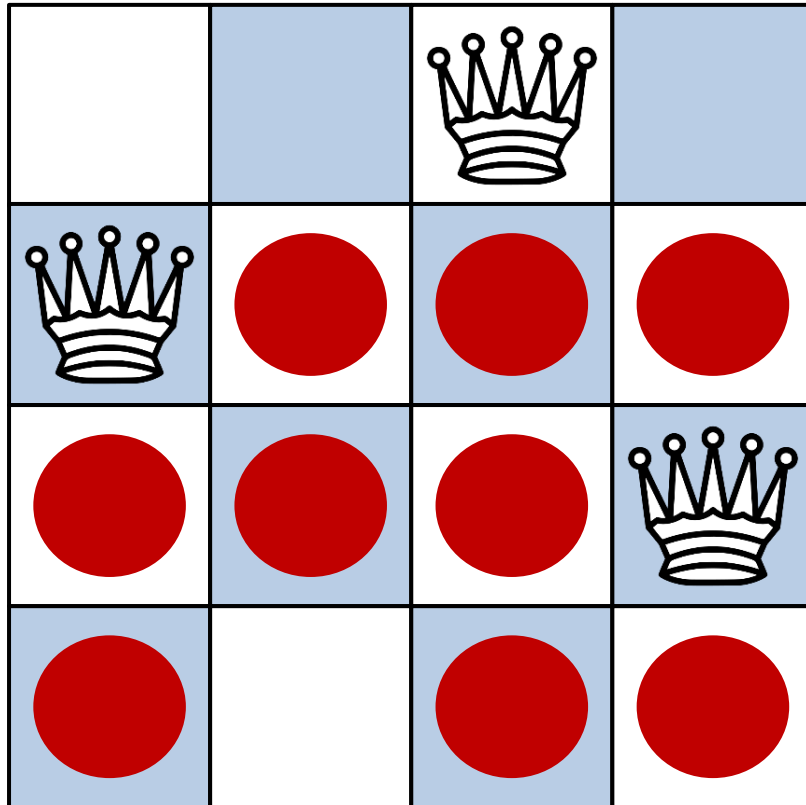


LCV:  $K_2 - 1$

Obilazak:  $K_1 K_2 K_3 K_4$



# CSP FC LCV 4 kraljice (8)

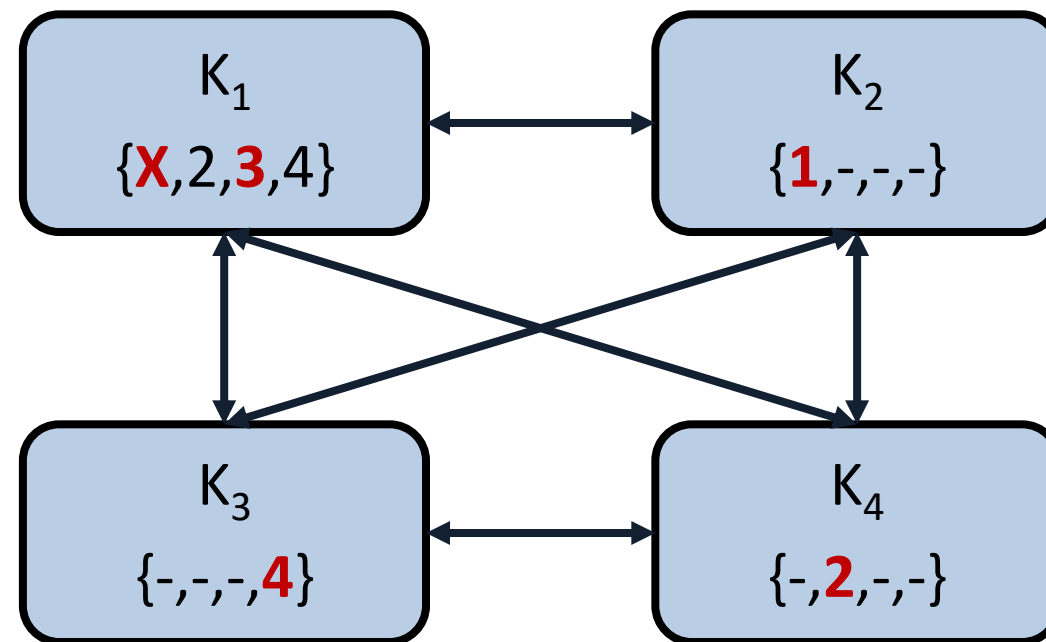
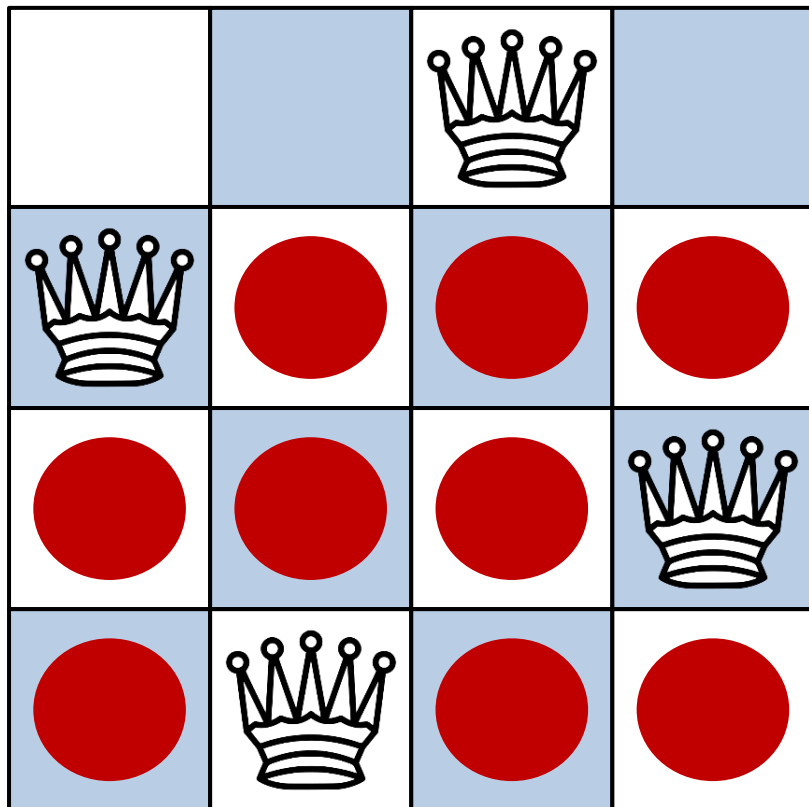


LCV:  $K_3 - 4$

Obilazak:  $K_1 K_2 K_3 K_4$



# CSP FC LCV 4 kraljice (9)



LCV:  $K_4 - 2$

$(K_1, K_2, K_3, K_4) = (3, 1, 4, 2)$

