Homework #1

Masafumi Endo, M.S. Student in Robotics ROB534 - Sequential Decision Making in Robotics OREGON STATE UNIVERSITY

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Questions

 \mathbf{a}

i

The cost-to-come function is shown in Table 1. Note that ∞ expresses the cost when the vertices cannot be visited.

Table 1: Cost-to-come function										
	a	b	\mathbf{c}	d	е					
$\overline{G*_5}$	∞	∞	∞	∞	0					
$G*_4$	∞	∞	7	1	∞					
$G*_3$	∞	11	4	2	∞					
$G*_2$	13	8	5	3	∞					
$G*_1$	10	9	6	6	∞					

ii

The cost-to-go function is shown in Table 2. Note that ∞ expresses the cost when the vertices cannot be visited.

Table 2: Cost-to-go function									
	a	b	\mathbf{c}	d	е				
$C*_1$	0	∞	∞	∞	∞				
$C*_2$	∞	2	∞	∞	∞				
$C*_3$	3	∞	6	∞	∞				
$C*_4$	∞	5	∞	9	13				
$C*_5$	6	∞	9	10	10				

b

i Sum of Manhattan distance of all tiles to their goal state

Admissible: It doesn't overestimate the optimal cost to reach the goal state.

ii Sum of Manhattan distance of all tiles to their goal state times 2

Inadmissible: It overestimates the optimal cost to reach the goal state.

iii Sum of Euclidean distance of all tiles to their goal state

Admissible: It doesn't overestimate the optimal cost to reach the goal state.

iv Number of tiles not in their goal state

Admissible: It doesn't overestimate the optimal cost to reach the goal state, while it is least informed.

v Number of moves remaining in optimal solution to reach goal state

Admissible: It is equal to the optimal cost to reach the goal state.

 \mathbf{c}

Based on the above answers, the heuristics are ordered as iv < iii < i < v.

Programming Assignment

Step 1

Fig. 1 shows flow charts of (a) A^* and (b) RRT algorithm. Note that O and C in Fig. 1 (a) express the open list and closed list.

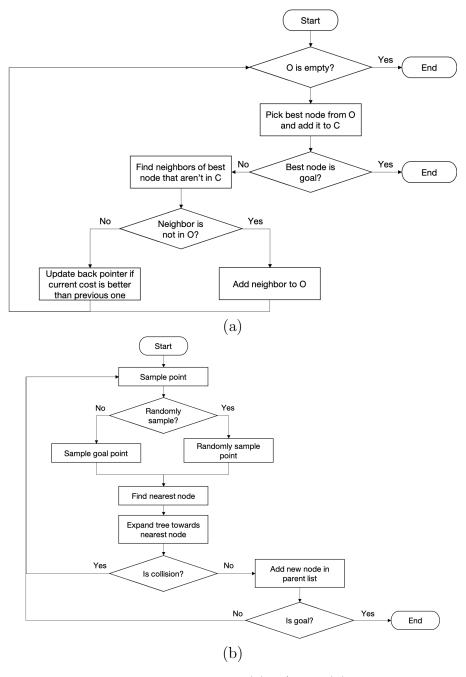


Figure 1: Flowchart of (a) A^* and (b) RRT.

Step 2

i

The paths found by A^* implementation for 2D space are shown in Fig. 2. I used euclidean distance as admissible heuristic.

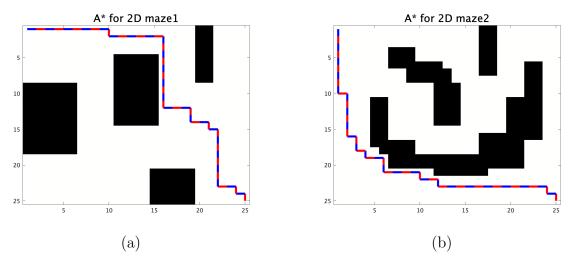


Figure 2: Planned path by A^* for 2D maze environments. Note that euclidean distance is used as admissible heuristic.

ii

Table 3 and 4 summarize the number of nodes expanded and path length on maze 1 and maze 2 environments for each ϵ value, respectively. In addition, the paths found by greedy-A* implementation for 2D space are shown in Fig. 3. I used euclidean distance as admissible heuristic.

Table 3: Record of ϵ , path length, and number of nodes expanded for maze 1 in 2D space.

time limit: 0.05 sec		time limit: 0.25 sec			time limit: 1.0 sec			
ϵ	length	nodes	ϵ	length	nodes	ϵ	length	nodes
10	48	49	10	48	49	10	48	49
5.5	48	49	5.5	48	49	5.5	48	49
3.25	48	49	3.25	48	49	3.25	48	49
-	-	-	2.12	48	49	2.12	48	49
-	-	-	1.56	48	50	1.56	48	50
-	-	-	1.28	48	289	1.28	48	289
-	-	-	1.14	48	350	1.14	48	350
-	-	-	1.07	48	369	1.07	48	369
-	-	-	_	-	-	1.03	48	378
-	-	-	_	-	-	1.01	48	379
-	-	-	_	-	-	1.008	48	379
-	-	-	_	-	-	1.004	48	379
-	-	-	-	-	-	1.002	48	379
-	-	-	_	-	-	1.001	48	379
	-	-	_	-	-	1	48	385

Table 4: Record of ϵ , path length, and number of nodes expanded for maze 2 in 2D space.

time limit: 0.05 sec			time limit: 0.25 sec			time limit: 1.0 sec		
ϵ	length	nodes	ϵ	length	nodes	ϵ	length	nodes
10	68	155	10	68	155	10	68	155
5.5	68	168	5.5	68	168	5.5	68	168
3.25	52	169	3.25	52	169	3.25	52	169
-	-	-	2.12	48	154	2.12	48	154
-	-	-	1.56	48	284	1.56	48	284
-	-	-	1.28	48	297	1.28	48	297
-	-	-	1.14	48	324	1.14	48	324
-	-	-	1.07	48	351	1.07	48	351
-	-	-	_	-	-	1.03	48	362
-	-	-	_	-	-	1.01	48	363
-	-	-	_	-	-	1.008	48	363
-	-	-	_	-	-	1.004	48	363
-	-	-	_	-	-	1.002	48	363
-	-	-	_	-	-	1.001	48	363
	-	-	_	-	-	1	48	369

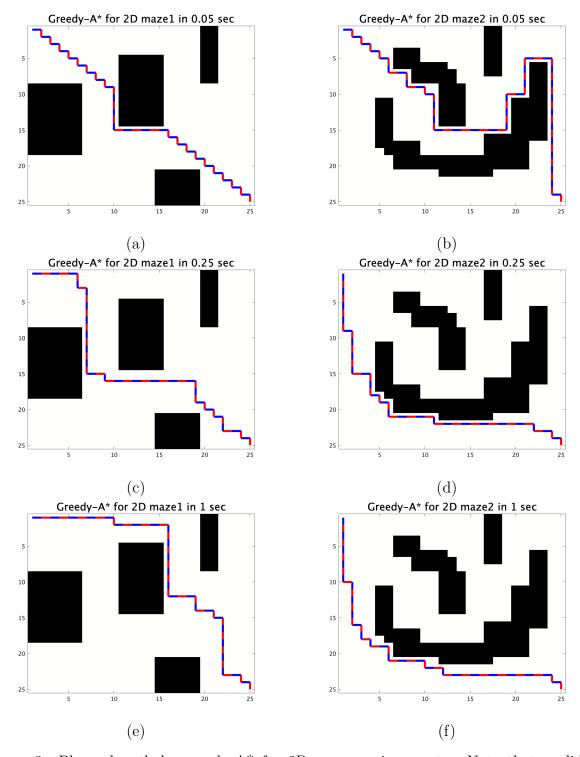


Figure 3: Planned path by greedy-A* for 2D maze environments. Note that euclidean distance is used as admissible heuristic. (a), (c), and (e) show planned paths for maze1 with time limits $0.05,\,0.25,\,$ and 1 second. (b), (d), and (f) show planned paths for maze2 with time limits $0.05,\,0.25,\,$ and 1 second.

The paths found by RRT implementation are shown in Fig. 4. For maze 1 environment, the length of found path is 42 and running time is 0.24 second. For maze 2 environment, the length of found path is 45 and running time is 0.29 second. Note that the length of path and running time may change due to its randomness. Also, the performance will be affected by several parameters: sampling rate, fixed distance to steer towards sampled points, and range to sample random points. I set these parameters as shown in Table 5.

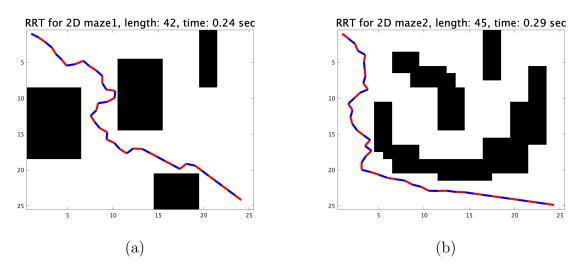


Figure 4: Planned path by RRT for 2D maze environments.

Table 5: User-specified parameters for RRT sampling rate 0.2 fixed distance 1 sampling range [0, 25]

Step 3

i

The paths found by A* implementation for 4D space are shown in Fig. 5. I used euclidean distance as admissible heuristic.

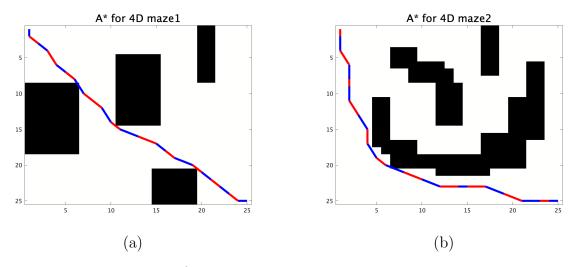


Figure 5: Planned path by A^* for 4D maze environments. Note that euclidean distance is used as admissible heuristic.

ii

Table 6 and 7 summarize the number of nodes expanded and path length on maze 1 and maze 2 environments for each ϵ value, respectively. In addition, the paths found by greedy-A* implementation for 4D space are shown in Fig. 6. I used euclidean distance as admissible heuristic.

Table 6: Record of ϵ , path length, and number of nodes expanded for maze 1 in 4D space.

time limit: 0.05 sec		time limit: 0.25 sec			time limit: 1.0 sec			
ϵ	length	nodes	ϵ	length	nodes	ϵ	length	nodes
10	36.6	30	10	36.6	30	10	36.6	30
5.5	36.6	30	5.5	36.6	30	5.5	36.6	30
3.25	36.6	29	3.25	36.6	29	3.25	36.6	29
2.12	36.6	29	2.12	36.6	29	2.12	36.6	29
1.56	36.6	29	1.56	36.6	29	1.56	36.6	29
1.28	36.6	35	1.28	36.6	35	1.28	36.6	35
1.14	36.6	92	1.14	36.6	92	1.14	36.6	92
-	-	-	1.07	35.4	122	1.07	35.4	122
-	-	-	1.03	35.2	176	1.03	35.2	176
-	-	-	1.01	35.2	364	1.01	35.2	364
-	-	-	1.008	35.2	475	1.008	35.2	475
-	-	-	-	-	-	1.004	35.2	528
-	-	-	_	-	-	1.002	35.2	560
-	-	-	_	-	-	1.001	35.2	565
	_	_	_	_	_	1	35.2	581

Table 7: Record of ϵ , path length, and number of nodes expanded for maze 2 in 4D space.

time limit: 0.05 sec		time limit: 0.25 sec			time limit: 1.0 sec			
ϵ	length	nodes	ϵ	length	nodes	ϵ	length	nodes
10	42.8	523	10	42.8	523	10	42.8	523
-	-	-	5.5	42.4	523	5.5	42.4	523
-	-	-	3.25	42.4	523	3.25	42.4	523
-	-	-	2.12	41.8	537	2.12	41.8	537
-	-	-	_	-	-	1.56	41.8	1193
-	-	-	_	-	-	1.28	41.1	1452
-	-	-	_	-	-	1.14	40.7	1529
-	-	-	-	-	-	1.07	40.5	1557

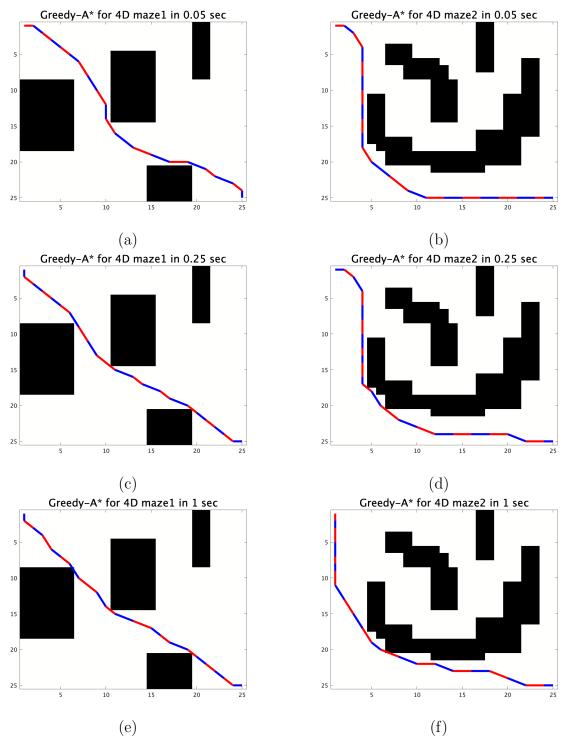


Figure 6: Planned path by greedy-A* for 4D maze environments. Note that euclidean distance is used as admissible heuristic. (a), (c), and (e) show planned paths for maze1 with time limits 0.05, 0.25, and 1 second. (b), (d), and (f) show planned paths for maze2 with time limits 0.05, 0.25, and 1 second.

Discussion Questions

1

The observation is that the larger ϵ causes the less node expansion and longer length of path found by greedy A*. This implies the solution is not optimal when ϵ value is increased. The complexity of maze environment and dimensionality of state space affect the heuristic inflation factor as shown in Table 3, 4, 6, and 7. ϵ depends on heuristic so it is important to first choose the heuristic function then pick ϵ value and deflation rate.

$\mathbf{2}$

The main issue of using RRT to solve the 4D problem is the difficulty of sampling due to the increase of the possible state spaces. To deal with this issue, the more directed sampling strategy would be necessary. In addition, instead of searching state space, RRT can search the path in the configuration space. Searching in C-space would make its random sampling more directed one since it can avoid the increase of the possible state space.

3

The advantage of using A* is that A* can find optimal and complete path if it exist while RRT doesn't guarantee optimality of the path and only guarantee probabilistic completeness. On the other hand, if the size of state space will be increase, A* requires significant computational efforts to find the path. To avoid this issue, we can decrease the amount of discretizing state space. Note that it also sacrifice the path resolution so it is necessary to check whether the path resolution is well enough or not for the robot.

4

 A^* needs to be modified to re-plan its path online any changes occur in the environment. To do so, it is necessary to use two estimates for its state evaluation: the estimate of cost based on the movement so far and the estimate of cost based on the next motion. To incorporate them for evaluating the priority queue, A^* can dynamically modify its path.

5

RRT needs to evaluate its paths with a probabilistic distribution since the situation is partially observable. To evaluate the path towards partially unknown environment, we must estimate the uncertainty and continue to find the path.

Appendix

A* Code for 2D Space

```
classdef Astar
      % set properties
      properties
           start
           goal
           map
           h_type
      end
10
11
      methods
12
           % constructor
13
           function obj = Astar(start, goal, map, h_type)
               obj.start = start;
14
               obj.goal = goal;
15
               obj.map = map;
16
17
               obj.h_type = h_type;
           end
18
19
           function [path] = search(obj)
20
21
22
               % initialization
23
               [~, num_nodes] = get_start(obj.map);
               priority_list = inf * ones(num_nodes, 1); % list containing priorities
24
               open_list = pq_init(1e+4); % open list
25
               closed_list = []; % closed list
26
27
               back_pointer = nan * ones(num_nodes, 1); % back-pointer attribute
28
               % starting position
               priority_list(obj.start) = h(obj, obj.start);
29
               open_list = pq_set(open_list, obj.start, priority_list(obj.start));
30
               back_pointer(obj.start) = -1;
31
32
               while true
                   % pick best node and remove it from open_list
33
34
                    [open_list, n_best] = pq_pop(open_list);
                   % add best node to closed_list
35
                   closed_list = [closed_list; n_best];
36
37
38
                   % check n_best is goal or not
39
                   if n_best == obj.goal
40
                        break
41
42
43
                   \% expand all nodes that are neibhbors of n_best
                   [neighbors, ~] = get_neighbors(obj.map, n_best);
44
                   neighbors = neighbors((neighbors == n_best) ~= 1);
45
                    for index_neighbor=1:length(neighbors)
46
                        neighbor = neighbors(index_neighbor);
47
                        % calculate each cost
48
49
                        cost_g = g(obj, n_best, priority_list);
                        cost_n = cost(obj, n_best, neighbor);
50
51
                        cost_h = h(obj, neighbor);
                        priority = cost_g + cost_n + cost_h;
52
                        % neighbor is in open_list
53
                        if pq_test(open_list, neighbor)
54
55
                            if priority < priority_list(neighbor)</pre>
56
                                % update open_list
57
                                priority_list(neighbor) = priority;
                                open_list = pq_set(open_list, neighbor, priority);
58
59
                                back_pointer(neighbor) = n_best;
60
61
                        % neighbor is not in open_list
62
                        else
```

```
63
                             if ismember(neighbor, closed_list)
64
                                 continue
65
                             % add neighbor to open_list
66
67
                                 priority_list(neighbor) = priority;
                                 open_list = pq_set(open_list, neighbor, priority);
                                 back_pointer(neighbor) = n_best;
69
70
                             end
                         end
71
                    end
72
73
                end
74
75
                % after finishing search process, get an optimal path from solution
76
                path = get_path(obj, back_pointer);
77
            end
78
            function [path] = get_path(obj, back_pointer)
79
80
                % initialization
                [x_g, y_g] = state_from_index(obj.map, obj.goal);
81
                path = [x_g, y_g];
82
83
                n_best = back_pointer(obj.goal);
                while true
84
85
                     [x_best, y_best] = state_from_index(obj.map, n_best);
                    path = [path; x_best, y_best];
86
                    n_best = back_pointer(n_best);
87
88
                    if n_best == -1
89
                         break
90
                    end
91
                end
92
            end
93
94
            % function to calculate actual cost of n_best
95
            function [cost_g] = g(obj, n_best, priority_list)
                cost_g = priority_list(n_best) - h(obj, n_best);
96
97
            end
98
            % function to calculate actual cost from n_best to neighbor
99
            function [cost_n] = cost(obj, n_best, neighbor)
100
                [x_best, y_best] = state_from_index(obj.map, n_best);
101
102
                [x_neig, y_neig] = state_from_index(obj.map, neighbor);
103
                cost_n = sqrt((x_neig - x_best)^2 + (y_neig - y_best)^2);
104
            end
105
            % function to calculate heuristic cost of the neighbor
106
107
            function [cost_h] = h(obj, neighbor)
                [x_neig, y_neig] = state_from_index(obj.map, neighbor);
108
                [x_goal, y_goal] = state_from_index(obj.map, get_goal(obj.map));
109
                if obj.h_type == 'm'
110
                    % manhattan distance
111
112
                    cost_h = abs(x_neig - x_goal) + abs(y_neig - y_goal);
                elseif obj.h_type == 'e
113
114
                    % euclidian distance
                    cost_h = sqrt((x_neig - x_goal)^2 + (y_neig - y_goal)^2);
115
116
                    disp('select a type of heuristic function!')
117
118
                end
119
            end
       end
120
```

Greedy-A* Code for 2D Space

```
classdef greedy_Astar

% set properties
```

```
properties
5
          start
6
          goal
          map
          h_type
          epsilon
          t_limit
10
11
      end
12
      methods
13
14
          % constructor
          function obj = greedy_Astar(start, goal, map, h_type, epsilon, t_limit)
15
              obj.start = start;
16
              obj.goal = goal;
17
              obj.map = map;
18
19
              obj.h_type = h_type;
              obj.epsilon = epsilon;
20
21
              obj.t_limit = t_limit;
22
          end
23
          function [path] = greedy_search(obj)
24
25
              tic
26
              while toc < obj.t_limit</pre>
27
                  [path, nodes_expanded] = search(obj);
                  path_length = get_path_length(path);
28
                  disp(' ')
29
                  disp('----')
30
                   fprintf('epsilon: %f \n', obj.epsilon)
31
                  32
                      path_length)
                  disp(',-----')
33
                  disp(' ')
34
35
36
                  obj.epsilon = obj.epsilon - 0.5 * (obj.epsilon - 1);
37
                  if obj.epsilon == 1
                      break
38
39
                  if obj.epsilon < 1.001
40
41
                      obj.epsilon = 1;
                  end
42
43
              end
          end
44
45
          function [path, nodes_expanded] = search(obj)
46
47
              % initialization
48
49
              [~, num_nodes] = get_start(obj.map);
              priority_list = inf * ones(num_nodes, 1); % list containing priorities
50
              open_list = pq_init(1e+4); % open list
51
52
              closed_list = []; % closed list
53
              back_pointer = nan * ones(num_nodes, 1); % back-pointer attribute
54
              % starting position
              priority_list(obj.start) = obj.epsilon * h(obj, obj.start);
55
              open_list = pq_set(open_list, obj.start, priority_list(obj.start));
56
57
              back_pointer(obj.start) = -1;
58
              while true
59
                  % pick best node and remove it from open_list
                  [open_list, n_best] = pq_pop(open_list);
60
61
                  % add best node to closed_list
                  closed_list = [closed_list; n_best];
62
63
64
                  % check n_best is goal or not
                  if n_best == obj.goal
65
66
                      break
67
                  end
68
                  % expand all nodes that are neibhbors of n_best
69
70
                  [neighbors, ~] = get_neighbors(obj.map, n_best);
```

```
71
                     neighbors = neighbors((neighbors == n_best) ~= 1);
                     for index_neighbor=1:length(neighbors)
72
73
                         neighbor = neighbors(index_neighbor);
74
                         % calculate each cost
75
                         cost_g = g(obj, n_best, priority_list);
76
                         cost_n = cost(obj, n_best, neighbor);
77
                         cost_h = obj.epsilon * h(obj, neighbor);
78
                         priority = cost_g + cost_n + cost_h;
79
                          % neighbor is in open_list
80
                         if pq_test(open_list, neighbor)
81
                              if priority < priority_list(neighbor)</pre>
82
                                  % update open_list
                                  priority_list(neighbor) = priority;
83
84
                                  open_list = pq_set(open_list, neighbor, priority);
85
                                  back_pointer(neighbor) = n_best;
86
                             end
                         % neighbor is not in open_list
87
88
                         else
                             if ismember(neighbor, closed_list)
80
90
                                 continue
                             % add neighbor to open_list
91
92
                             else
93
                                  priority_list(neighbor) = priority;
                                  open_list = pq_set(open_list, neighbor, priority);
94
                                  back_pointer(neighbor) = n_best;
95
96
                             end
97
                         end
98
                     end
99
                end
100
                \mbox{\ensuremath{\mbox{\%}}} after finishing search process, get an optimal path from solution
101
102
                path = get_path(obj, back_pointer);
103
                nodes_expanded = length(closed_list);
            end
104
105
            function [path] = get_path(obj, back_pointer)
106
                % initialization
107
                [x_g, y_g] = state_from_index(obj.map, obj.goal);
108
                path = [x_g, y_g];
109
110
                n_best = back_pointer(obj.goal);
111
                while true
                     [x_best, y_best] = state_from_index(obj.map, n_best);
112
                     path = [path; x_best, y_best];
113
                     n_best = back_pointer(n_best);
114
115
                     if n best == -1
                         break
116
                     end
117
                end
118
119
120
121
            % function to calculate actual cost of n best
122
            function [cost_g] = g(obj, n_best, priority_list)
                cost_g = priority_list(n_best) - obj.epsilon * h(obj, n_best);
123
124
125
126
            % function to calculate actual cost from n_best to neighbor
127
            function [cost_n] = cost(obj, n_best, neighbor)
                [x_best, y_best] = state_from_index(obj.map, n_best);
128
129
                [x_neig, y_neig] = state_from_index(obj.map, neighbor);
                cost_n = sqrt((x_neig - x_best)^2 + (y_neig - y_best)^2);
130
            end
131
132
            % function to calculate heuristic cost of the neighbor
133
            function [cost_h] = h(obj, neighbor)
134
                [x_neig, y_neig] = state_from_index(obj.map, neighbor);
135
136
                 [x_goal, y_goal] = state_from_index(obj.map, get_goal(obj.map));
                if obj.h_type == 'm'
137
                     % manhattan distance
138
```

```
139
                     cost_h = abs(x_neig - x_goal) + abs(y_neig - y_goal);
140
                elseif obj.h_type == 'e'
141
                     % euclidian distance
                     cost_h = sqrt((x_neig - x_goal)^2 + (y_neig - y_goal)^2);
142
143
                     disp('select a type of heuristic function!')
144
                end
145
            end
146
       end
147
   end
148
```

RRT Code for 2D Space

```
classdef RRT
       % set properties
       properties
           start
           goal
           map
           dist
           sampling_rate
10
           min rand
           max_rand
11
       end
12
13
14
       methods
15
           % constructor
16
           function obj = RRT(start, goal, map, dist, sampling_rate, rand_range)
               obj.start = start;
17
               obj.goal = goal;
18
19
               obj.map = map;
               obj.dist = dist; % dist to steer sampled point
20
21
               obj.sampling_rate = sampling_rate; % rate to sample goal point
22
               obj.min_rand = rand_range(1);
23
               obj.max_rand = rand_range(2);
           end
24
25
26
           function [path, runtime] = search(obj)
27
28
               % initialization
               [x_start, y_start] = state_from_index(obj.map, obj.start);
29
30
               [x_goal, y_goal] = state_from_index(obj.map, obj.goal);
31
               node_list = [x_start, y_start];
32
               parent_list = [nan];
33
               node_cnt = 1;
               tic
34
               while true
35
                    % random sampling
36
37
                    [x_rand, y_rand] = get_node(obj);
38
                   % find nearest node
39
40
                   index_nearest = knnsearch(node_list, [x_rand, y_rand]);
                   x_near = node_list(index_nearest, 1);
41
42
                   y_near = node_list(index_nearest, 2);
43
                   % expand tree towards [x_near, y_near]
44
45
                   [x_new, y_new] = steer(obj, [x_near, y_near], [x_rand, y_rand]);
46
47
                   % check collision
                   dx = x_new - x_near;
48
49
                   dy = y_new - y_near;
                    if ~check_hit(obj.map, x_near, y_near, dx, dy)
50
51
                        % add new node
52
                        node_list = [node_list; x_new, y_new];
```

```
parent_list = [parent_list; index_nearest];
                         node_cnt = node_cnt + 1;
54
55
56
57
                     % check goal
                     if abs(x_goal - x_new) < 1 && abs(y_goal - y_new) < 1
59
                         break
                     elseif node_cnt >= 1e+4 % max iteration is 10000
60
                         disp('error: RRT could not reach goal within 10000 nodes')
61
62
                         break
63
                     end
64
                end
65
                runtime = toc;
                \mbox{\ensuremath{\%}} after finishing search process, get a path from solution
66
67
                path = get_path(obj, node_list, parent_list);
68
            end
69
70
            % function to get node
            function [x_rand, y_rand] = get_node(obj)
71
72
                if rand(1) > obj.sampling_rate
73
                     x_rand = (obj.max_rand - obj.min_rand) * rand(1, 1) + obj.min_rand;
74
                     y_rand = (obj.max_rand - obj.min_rand) * rand(1, 1) + obj.min_rand;
75
                     [x_goal, y_goal] = state_from_index(obj.map, obj.goal);
76
77
                     x_{rand} = x_{goal};
78
                     y_{rand} = y_{goal};
                end
79
80
            end
81
            % function to steer towards new node
82
            function [x_new, y_new] = steer(obj, node_near, node_rand)
83
                grad = atan2(node_rand(2) - node_near(2), node_rand(1) - node_near(1));
84
85
                dx = cos(grad) * obj.dist;
                dy = sin(grad) * obj.dist;
86
87
                x_new = node_near(1) + dx;
                y_new = node_near(2) + dy;
88
89
90
91
            % function to get a final path from goal to start
92
            function [path] = get_path(obj, node_list, parent_list)
93
                % initialization
94
                path = [];
95
96
                index_node = length(node_list);
97
                while true
                     path = [path; node_list(index_node, :)];
98
                     index_node = parent_list(index_node, :);
99
                     if isnan(index_node)
100
101
                     end
102
                end
103
104
            end
       end
105
106
   end
```

A* Code for 4D Space

```
classdef Astar_dynamic

% set properties
properties
start
goal
map
h_type
```

```
end
10
11
      methods
12
           % constructor
           function obj = Astar_dynamic(start, goal, map, h_type)
13
               obj.start = start;
15
               obj.goal = goal;
16
               obj.map = map;
17
               obj.h_type = h_type;
18
           end
19
           function [path] = search(obj)
20
21
22
               % initialization
23
               [~, num_nodes] = get_start_dynamic(obj.map);
24
               priority_list = inf * ones(num_nodes, 1); % list containing priorities
               open_list = pq_init(1e+4); % open list
25
26
               closed_list = []; % closed list
               back_pointer = nan * ones(num_nodes, 1); % back-pointer attribute
27
28
               % starting position
29
               priority_list(obj.start) = h(obj, obj.start);
30
               open_list = pq_set(open_list, obj.start, priority_list(obj.start));
31
               back_pointer(obj.start) = -1;
32
               while true
33
                   % pick best node and remove it from open_list
34
                   [open_list, n_best] = pq_pop(open_list);
                   % add best node to closed_list
35
36
                   closed_list = [closed_list; n_best];
37
                   % check n_best is goal or not
38
39
                   if n_best == obj.goal
40
                       break
41
                   end
42
43
                   % expand all nodes that are neibhbors of n_best
                   [neighbors, ~] = get_neighbors_dynamic(obj.map, n_best);
44
                   neighbors = neighbors((neighbors == n_best) ~= 1);
45
                   for index_neighbor=1:length(neighbors)
46
47
                       neighbor = neighbors(index_neighbor);
48
                       % calculate each cost
49
                       cost_g = g(obj, n_best, priority_list);
50
                       cost_n = cost(obj, n_best, neighbor);
                       cost_h = h(obj, neighbor);
51
52
                       priority = cost_g + cost_n + cost_h;
                        % neighbor is in open_list
53
54
                       if pq_test(open_list, neighbor)
55
                            if priority < priority_list(neighbor)</pre>
56
                                % update open_list
                                priority_list(neighbor) = priority;
57
58
                                open_list = pq_set(open_list, neighbor, priority);
                                back_pointer(neighbor) = n_best;
59
60
                            end
                       % neighbor is not in open_list
61
62
63
                            if ismember(neighbor, closed_list)
64
                                continue
                            65
66
                            else
67
                                priority_list(neighbor) = priority;
                                open_list = pq_set(open_list, neighbor, priority);
68
69
                                back_pointer(neighbor) = n_best;
70
                            end
71
                       end
72
                   \verb"end"
73
               end
74
75
               % after finishing search process, get an optimal path from solution
76
               path = get_path(obj, back_pointer);
```

```
77
             end
 78
 79
             function [path] = get_path(obj, back_pointer)
 80
                  % initialization
                  [x_g, y_g, \tilde{}, \tilde{}] = dynamic_state_from_index(obj.map, obj.goal);
 81
                  path = [x_g, y_g];
                  n_best = back_pointer(obj.goal);
 83
                  while true
 84
                       [x_best, y_best, ~, ~] = dynamic_state_from_index(obj.map, n_best);
 85
                       path = [path; x_best, y_best];
 86
 87
                       n_best = back_pointer(n_best);
                       if n_best == -1
 88
 89
                            break
 90
                       end
 91
                  end
             end
 92
 93
 94
             \% function to calculate actual cost of n_best
             function [cost_g] = g(obj, n_best, priority_list)
 95
 96
                  cost_g = priority_list(n_best) - h(obj, n_best);
 97
 98
 99
             \% function to calculate actual cost from n_best to neighbor
             function [cost_n] = cost(obj, n_best, neighbor)
100
                  [x_best, y_best, ~, ~] = dynamic_state_from_index(obj.map, n_best);
[x_neig, y_neig, ~, ~] = dynamic_state_from_index(obj.map, neighbor);
101
102
103
                  cost_n = sqrt((x_neig - x_best)^2 + (y_neig - y_best)^2);
104
             end
105
             % function to calculate heuristic cost of the neighbor
106
             function [cost_h] = h(obj, neighbor)
107
                  [x_neig, y_neig, ~, ~] = dynamic_state_from_index(obj.map, neighbor);
[x_goal, y_goal, ~, ~] = dynamic_state_from_index(obj.map, get_goal(obj.map));
108
109
                  if obj.h_type == 'm'
110
                       % manhattan distance
                       cost_h = abs(x_neig - x_goal) + abs(y_neig - y_goal);
112
                  elseif obj.h_type == 'e'
113
                       % euclidian distance
114
                       cost_h = sqrt((x_neig - x_goal)^2 + (y_neig - y_goal)^2);
115
116
117
                       disp('select a type of heuristic function!')
118
             end
119
120
        end
121
   end
```

Greedy-A* Code for 4D Space

```
classdef greedy_Astar_dynamic
      % set properties
      properties
           start
           goal
           map
           h_type
           epsilon
10
           t_limit
      end
11
12
13
      methods
14
15
           function obj = greedy_Astar_dynamic(start, goal, map, h_type, epsilon, t_limit)
               obj.start = start;
16
               obj.goal = goal;
17
```

```
18
               obj.map = map;
19
               obj.h_type = h_type;
20
               obj.epsilon = epsilon;
21
               obj.t_limit = t_limit;
           end
22
23
           function [path] = greedy_search(obj)
24
25
26
               while toc < obj.t_limit
                   [path, nodes_expanded] = search(obj);
27
28
                   path_length = get_path_length(path);
                   disp(' ')
29
                   disp('----')
30
                   fprintf('epsilon: %f \n', obj.epsilon)
31
                   fprintf('number of nodes expanded: %d, path length: %f \n', nodes_expanded,
32
                       path_length)
                   disp('----')
33
                   disp(' ')
34
35
36
                   obj.epsilon = obj.epsilon - 0.5 * (obj.epsilon - 1);
37
                   if obj.epsilon == 1
38
                       break
39
                   if obj.epsilon < 1.001
40
41
                       obj.epsilon = 1;
                   end
42
43
               end
44
           end
45
           function [path, nodes_expanded] = search(obj)
46
47
48
               % initialization
49
               [~, num_nodes] = get_start_dynamic(obj.map);
               priority_list = inf * ones(num_nodes, 1); % list containing priorities
50
51
               open_list = pq_init(1e+4); % open list
               closed_list = []; % closed list
52
               back_pointer = nan * ones(num_nodes, 1); % back-pointer attribute
53
               % starting position
54
55
               priority_list(obj.start) = obj.epsilon * h(obj, obj.start);
56
               open_list = pq_set(open_list, obj.start, priority_list(obj.start));
57
               back_pointer(obj.start) = -1;
               while true
58
                   % pick best node and remove it from open_list
59
60
                   [open_list, n_best] = pq_pop(open_list);
                   % add best node to closed_list
61
                   closed_list = [closed_list; n_best];
62
63
                   \mbox{\ensuremath{\mbox{\%}}} check n_best is goal or not
64
                   if n_best == obj.goal
65
66
                       break
                   end
67
68
                   % expand all nodes that are neibhbors of n_best
69
                   [neighbors, ~] = get_neighbors_dynamic(obj.map, n_best);
70
                   neighbors = neighbors((neighbors == n_best) ~= 1);
71
72
                   for index_neighbor=1:length(neighbors)
73
                       neighbor = neighbors(index_neighbor);
74
                        % calculate each cost
75
                       cost_g = g(obj, n_best, priority_list);
76
                       cost_n = cost(obj, n_best, neighbor);
77
                       cost_h = obj.epsilon * h(obj, neighbor);
78
                        priority = cost_g + cost_n + cost_h;
                        % neighbor is in open_list
79
80
                        if pq_test(open_list, neighbor)
81
                            if priority < priority_list(neighbor)</pre>
82
                                % update open_list
                                priority_list(neighbor) = priority;
83
                                open_list = pq_set(open_list, neighbor, priority);
84
```

```
85
                                    back_pointer(neighbor) = n_best;
 86
                                end
 87
                           % neighbor is not in open_list
                           else
 88
                                if ismember(neighbor, closed_list)
 89
 90
                                    continue
                                \% add neighbor to open_list
 91
 92
                                else
 93
                                    priority_list(neighbor) = priority;
                                    open_list = pq_set(open_list, neighbor, priority);
 94
 95
                                    back_pointer(neighbor) = n_best;
 96
                                end
                           end
 97
 98
                      end
 99
                 end
100
                  % after finishing search process, get an optimal path from solution
101
102
                  path = get_path(obj, back_pointer);
                  nodes_expanded = length(closed_list);
103
104
105
             function [path] = get_path(obj, back_pointer)
106
107
                  % initialization
                  [x_g, y_g, \tilde{}, \tilde{}] = dynamic_state_from_index(obj.map, obj.goal);
108
                  path = [x_g, y_g];
109
110
                  n_best = back_pointer(obj.goal);
111
                  while true
                      [x_best, y_best, ~, ~] = dynamic_state_from_index(obj.map, n_best);
112
                      path = [path; x_best, y_best];
113
                      n_best = back_pointer(n_best);
114
                      if n_best == -1
115
116
                           break
117
                      end
                  end
118
119
             end
120
             % function to calculate actual cost of n_best
121
             function [cost_g] = g(obj, n_best, priority_list)
122
123
                 cost_g = priority_list(n_best) - obj.epsilon * h(obj, n_best);
124
             end
125
             % function to calculate actual cost from n_best to neighbor
126
             function [cost_n] = cost(obj, n_best, neighbor)
127
                  [x_best, y_best, ~, ~] = dynamic_state_from_index(obj.map, n_best);
[x_neig, y_neig, ~, ~] = dynamic_state_from_index(obj.map, neighbor);
128
129
                  cost_n = sqrt((x_neig - x_best)^2 + (y_neig - y_best)^2);
130
131
132
             % function to calculate heuristic cost of the neighbor
133
             function [cost_h] = h(obj, neighbor)
134
                  [x_neig, y_neig, ~, ~] = dynamic_state_from_index(obj.map, neighbor);
[x_goal, y_goal, ~, ~] = dynamic_state_from_index(obj.map, get_goal(obj.map));
135
136
                  if obj.h_type == 'm'
137
                      % manhattan distance
138
139
                      cost_h = abs(x_neig - x_goal) + abs(y_neig - y_goal);
140
                  elseif obj.h_type == 'e'
141
                      % euclidian distance
                      cost_h = sqrt((x_neig - x_goal)^2 + (y_neig - y_goal)^2);
142
143
                      disp('select a type of heuristic function!')
144
                  end
145
146
             end
147
        end
   end
```