



Image processing operators (2)

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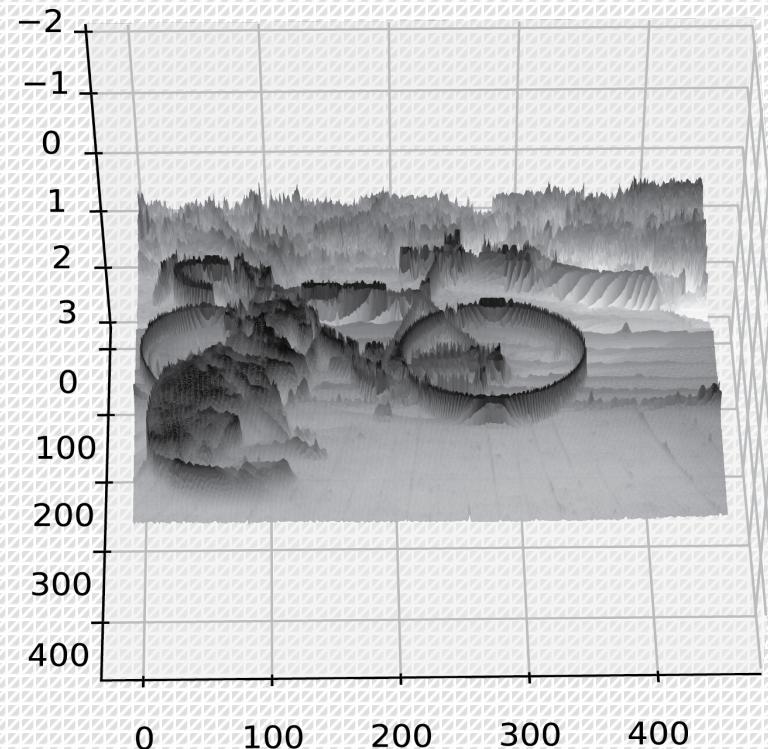
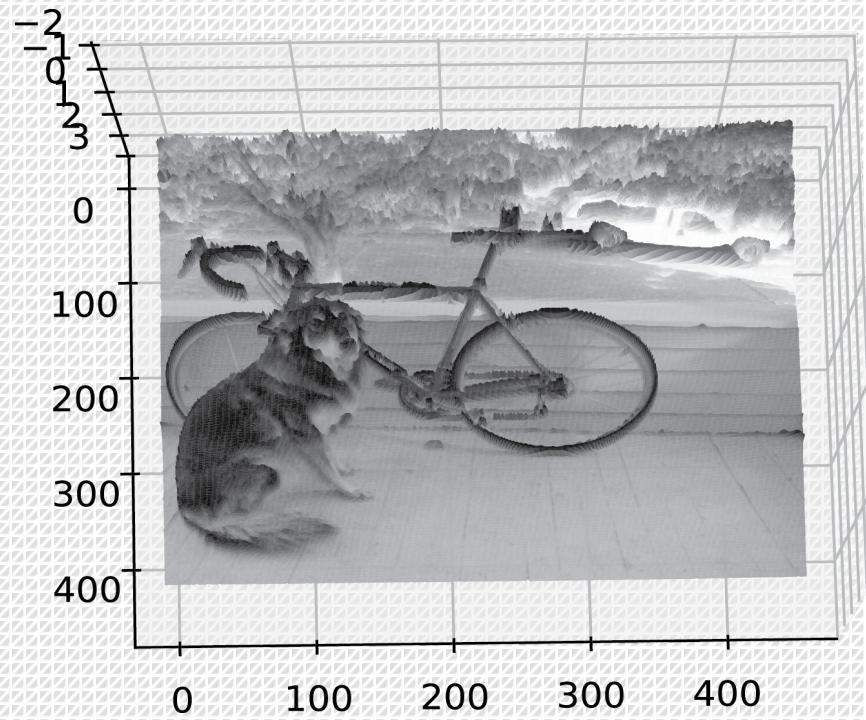


Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- What is an “edge”?

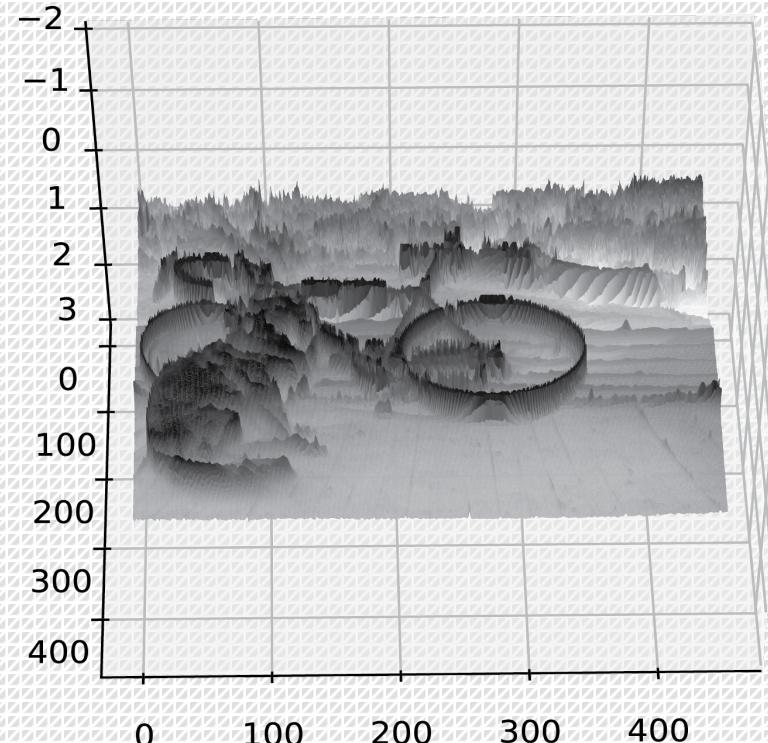
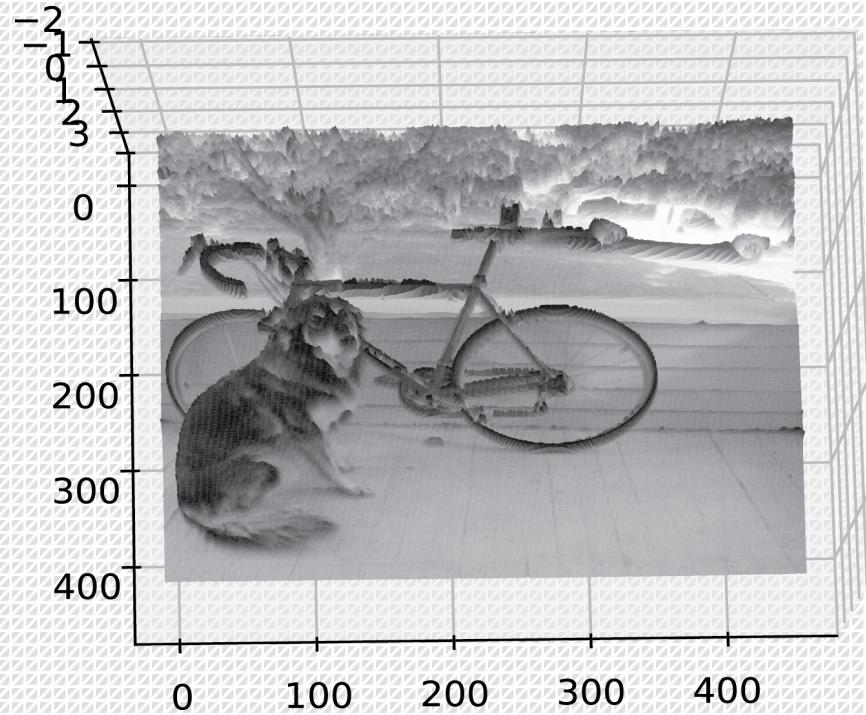


Edges

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- Image is a function.
- Think of the gray tones as HEIGHTS.
- Edges are rapid changes in this function

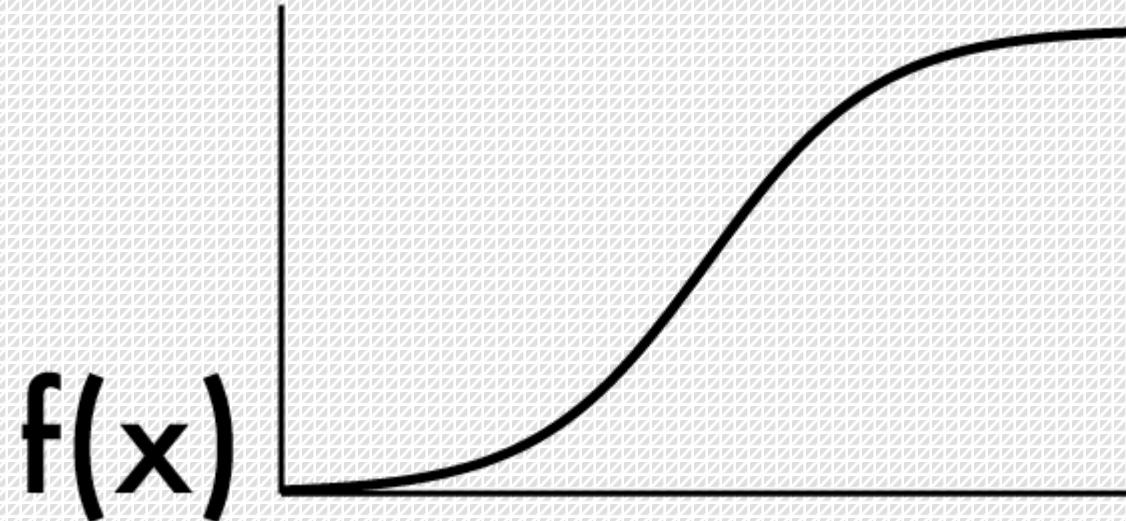


Edges

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- Image is a function
- Edges are rapid changes in this function



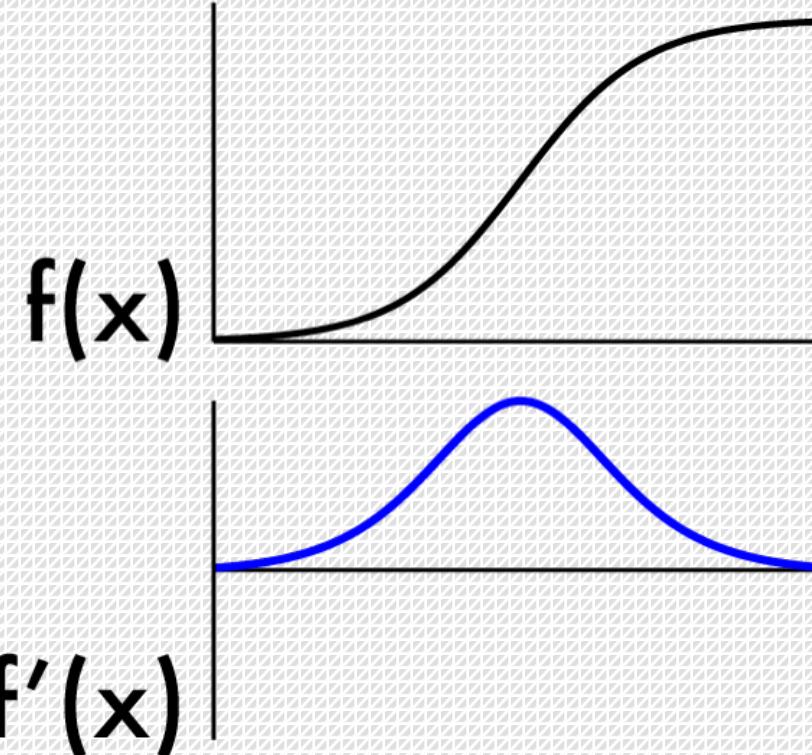


Edges

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- Image is a function
- Edges are rapid changes in this function



Edges

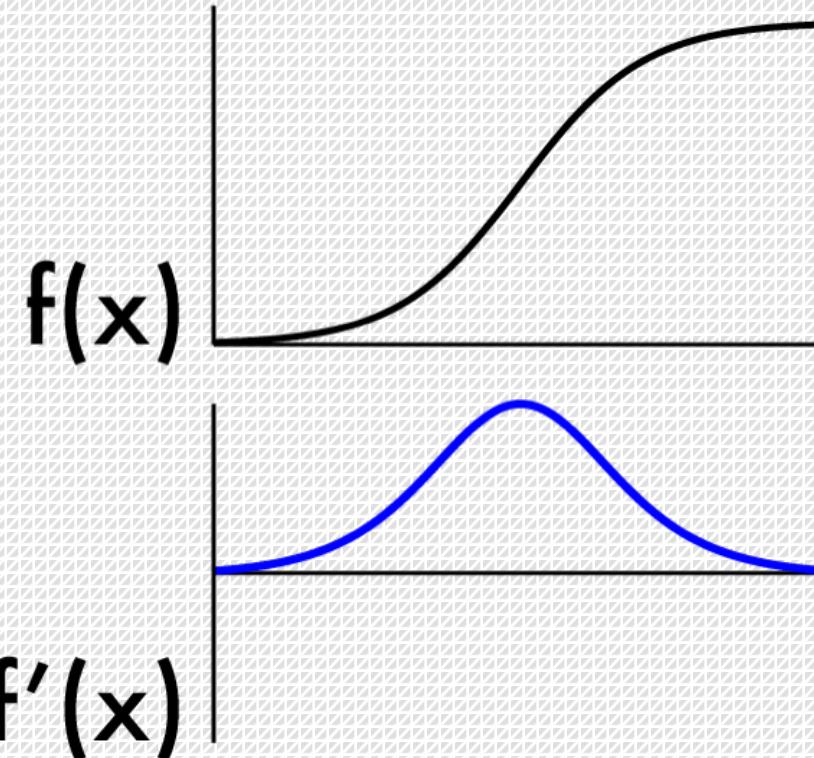
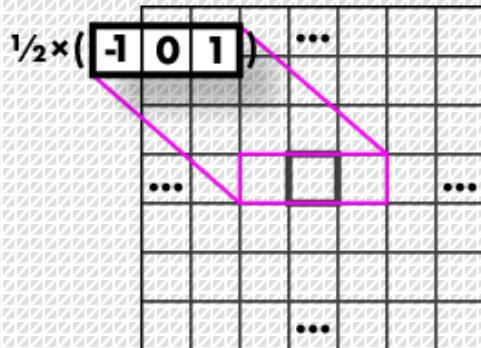
Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Recall:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$

- We don't have an "actual" function, must estimate
- Possibility: set $h = 2$
- What will that look like?

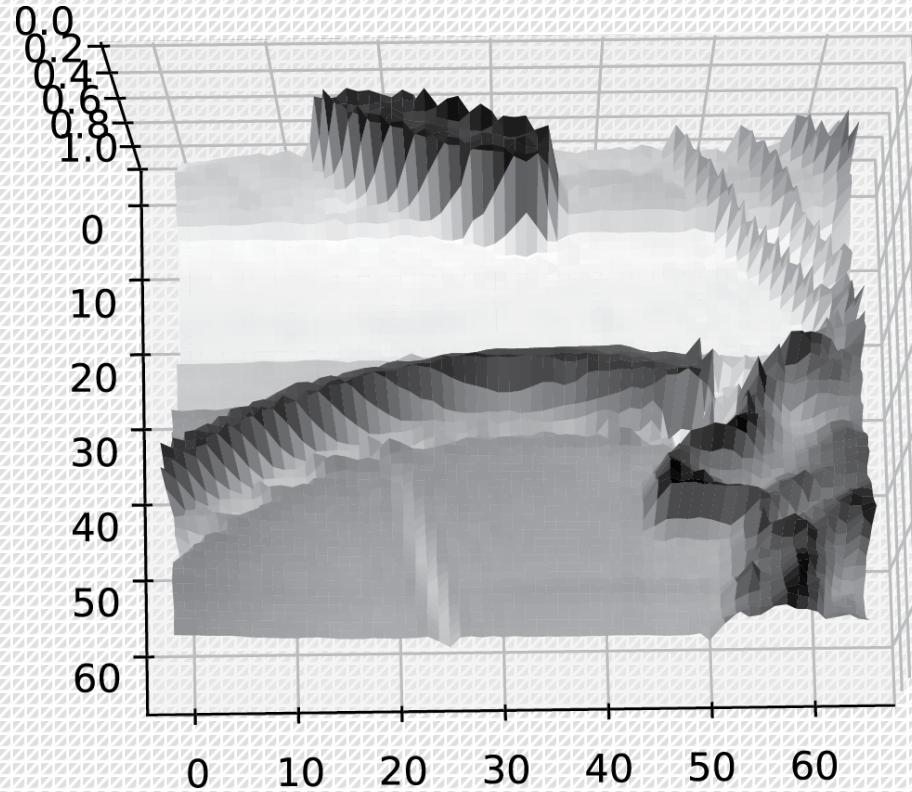
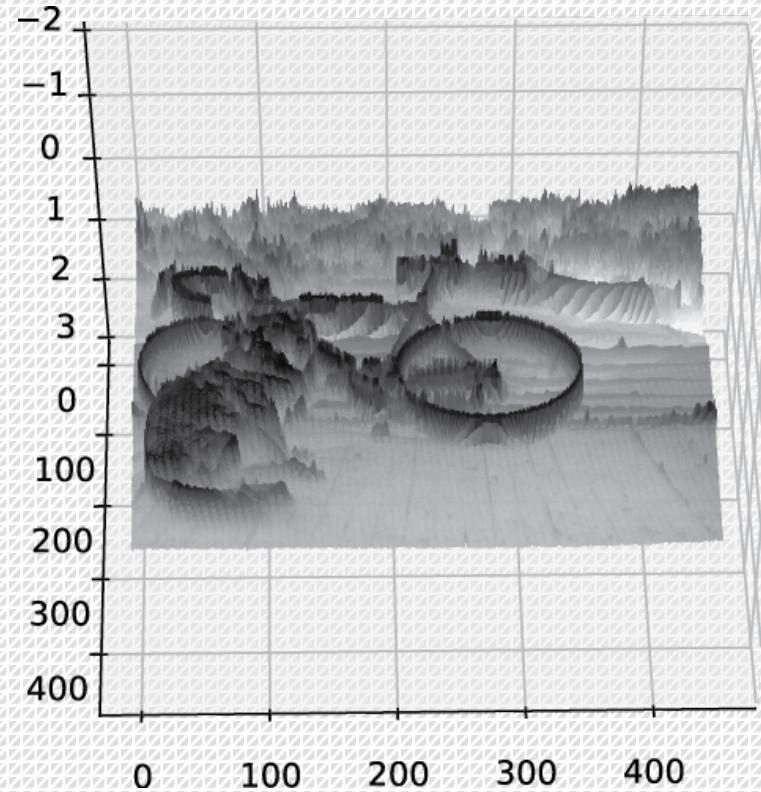


Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Images are noisy!

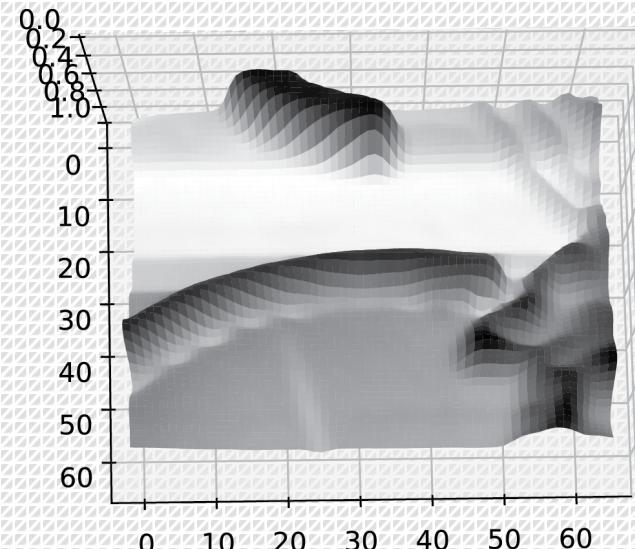
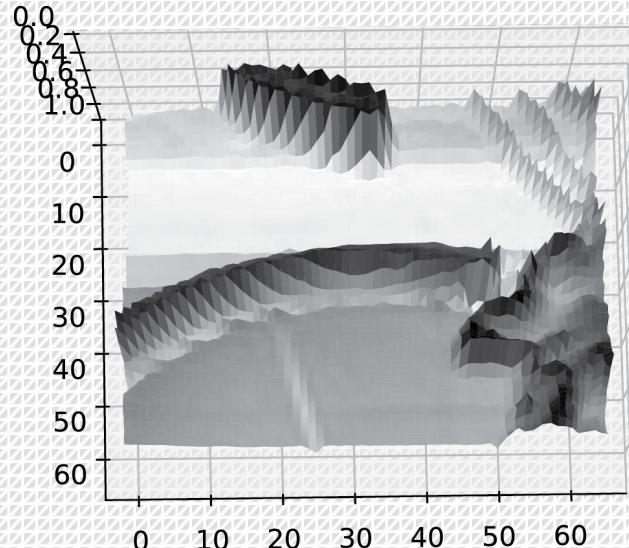
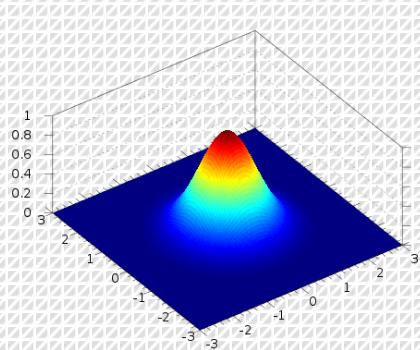


Edges

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- Images are noisy!



Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Images are noisy!

Diagram illustrating the convolution operation for edge detection:

Input image (G) is a 3x3 grid:

1	2	1
2	4	2
1	2	1

Kernel (K) is a 1x3 filter:

-1	0	1
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The result of the convolution is a 3x3 output grid:

Handwritten annotations include a red 'G' above the input grid and a red bracket above the kernel.

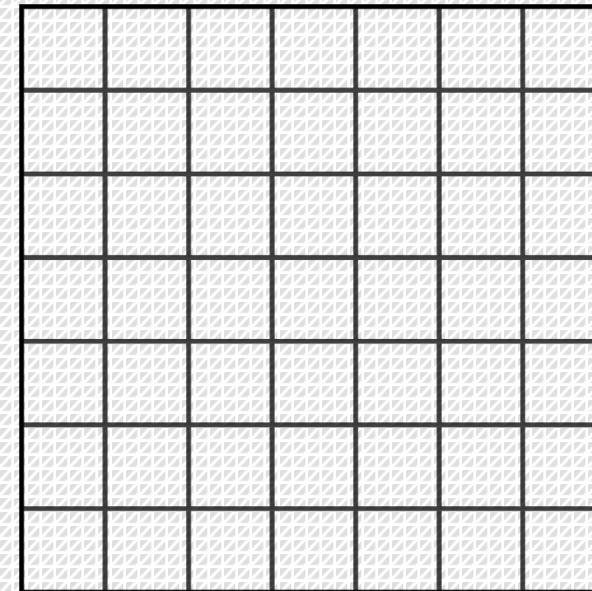
Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Smooth first, then derivative

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right) *$$



Edges

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- Smooth first, then derivative

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$

The diagram illustrates the convolution operation. A 1x3 kernel $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ is applied to a 3x3 input matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$. The result is a 1x1 output $\begin{bmatrix} 2 \end{bmatrix}$, indicated by a large arrow. A pink line connects the top-left element of the kernel to the top-left element of the input matrix, highlighting the receptive field of that output unit.

Edges

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- Smooth first, then derivative

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$

$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

The diagram illustrates a convolution operation. A 3x3 input matrix is multiplied by a 3x3 kernel. The result is then scaled by $\frac{1}{2}$. The input matrix has its top-right 2x2 submatrix highlighted in pink. The output is a 1x2 matrix with values 2 and 0.

$$\frac{1}{2} \times \begin{bmatrix} 2 & 0 \\ \vdots & \vdots \end{bmatrix}$$

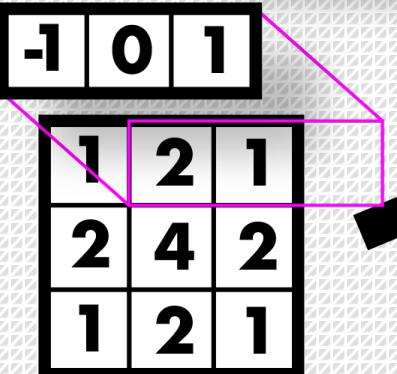
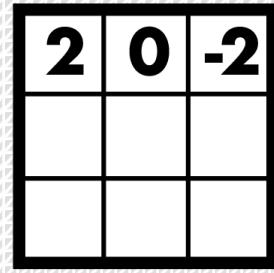
Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Smooth first, then derivative

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$

$\frac{1}{2} \times$  \rightarrow $\frac{1}{2} \times$ 

A diagram illustrating a 3x3 convolution operation. A 3x3 input matrix is multiplied by a 3x3 kernel. The result is then scaled by $\frac{1}{2}$. The input matrix has values 1, 2, 1 in its first row, and 2, 4, 2, 1, 2, 1 in its second and third rows respectively. The kernel has values -1, 0, 1 in its first row. The result matrix has values 2, 0, -2 in its first row, and empty cells in its second and third rows. A pink arrow points from the input matrix to the result matrix, indicating the flow of data through the convolution process.

Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Smooth first, then derivative

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$

The diagram illustrates a convolution operation. A 3x3 input matrix (bottom) is multiplied by a 3x3 kernel (top). The result is a 2x2 output matrix (right). A pink arrow points from the top-left element of the kernel to the top-left element of the input, indicating the receptive field of that output unit. The output matrix is scaled by $\frac{1}{2}$.

-1	0	1						
2	4	2						
1	2	1						

2	0	-2						
4								

Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Smooth first, then derivative

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$

The diagram illustrates a convolution operation. On the left, a 3x3 input matrix with values 1, 2, 1; 2, 4, 2; and 1, 2, 1 is multiplied by a 3x3 kernel with values -1, 0, 1; 0, 2, 1; and 1, 2, 1. The result is scaled by $\frac{1}{2}$. On the right, a 2x2 output matrix with values 2, 0, -2; 4, 0, -4; and 2, 0, -2 is shown, resulting from a stride of 2. A pink arrow points from the input to the output, indicating the receptive field of the output unit.

$$\frac{1}{2} \times \begin{bmatrix} 2 & 0 & -2 \\ 4 & 0 & -4 \\ 2 & 0 & -2 \end{bmatrix}$$

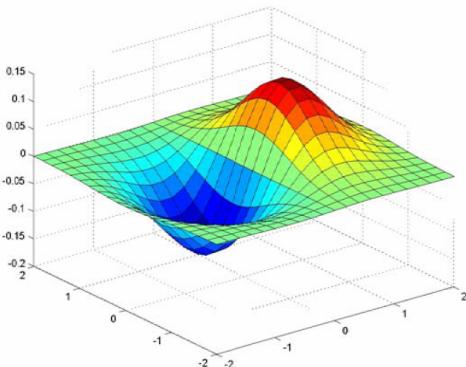
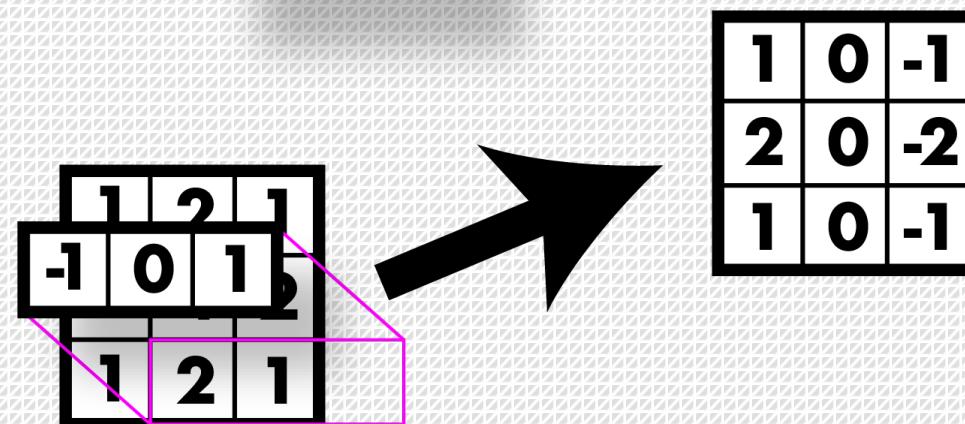
Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Smooth first, then derivative \rightarrow Sobel filter!

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$



Edges

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- Smooth first, then derivative

$$\frac{1}{K^2} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

$$\frac{1}{8} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{4} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

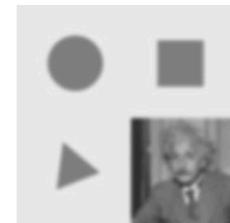
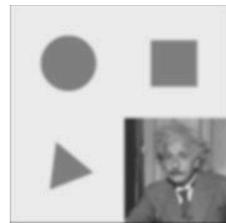
$$\frac{1}{K} \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$\frac{1}{4} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$\frac{1}{16} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$



(a) box, $K = 5$

(b) bilinear

(c) “Gaussian”

(d) Sobel

(e) corner

Figure 3.14 Separable linear filters: For each image (a)–(e), we show the 2D filter kernel (top), the corresponding horizontal 1D kernel (middle), and the filtered image (bottom). The filtered Sobel and corner images are signed, scaled up by $2\times$ and $4\times$, respectively, and added to a gray offset before display.

Edges

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- But

$$\frac{1}{K^2} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

$$\frac{1}{8} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{4} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

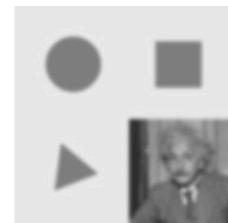
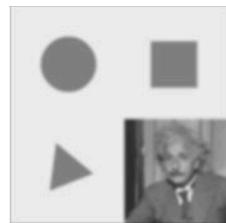
$$\frac{1}{K} \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$\frac{1}{4} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

$$\frac{1}{16} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$



(a) box, $K = 5$

(b) bilinear

(c) “Gaussian”

(d) Sobel

(e) corner

Figure 3.14 Separable linear filters: For each image (a)–(e), we show the 2D filter kernel (top), the corresponding horizontal 1D kernel (middle), and the filtered image (bottom). The filtered Sobel and corner images are signed, scaled up by $2\times$ and $4\times$, respectively, and added to a gray offset before display.

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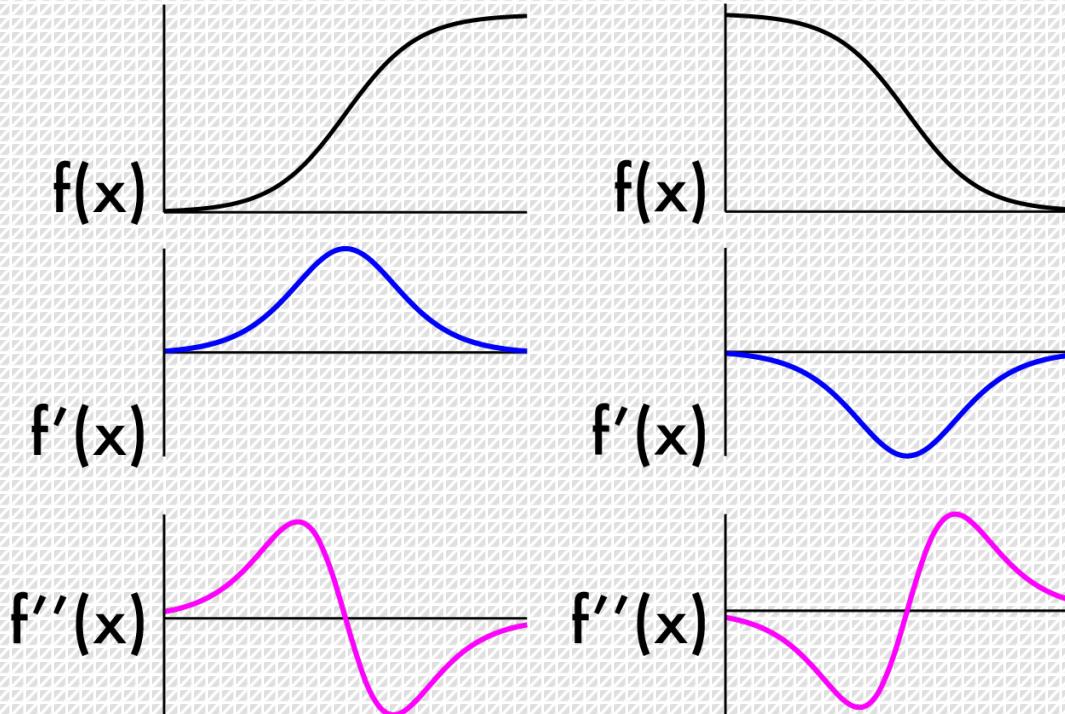
- Could take derivative
- Find high responses
- Sobel filters!
- But...
- Edges go both ways
- Want to find extrema

Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Could take derivative
- Find high responses
- Sobel filters!
- But...
- Edges go both ways
- Want to find extrema



Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>



- Crosses zero at extrema

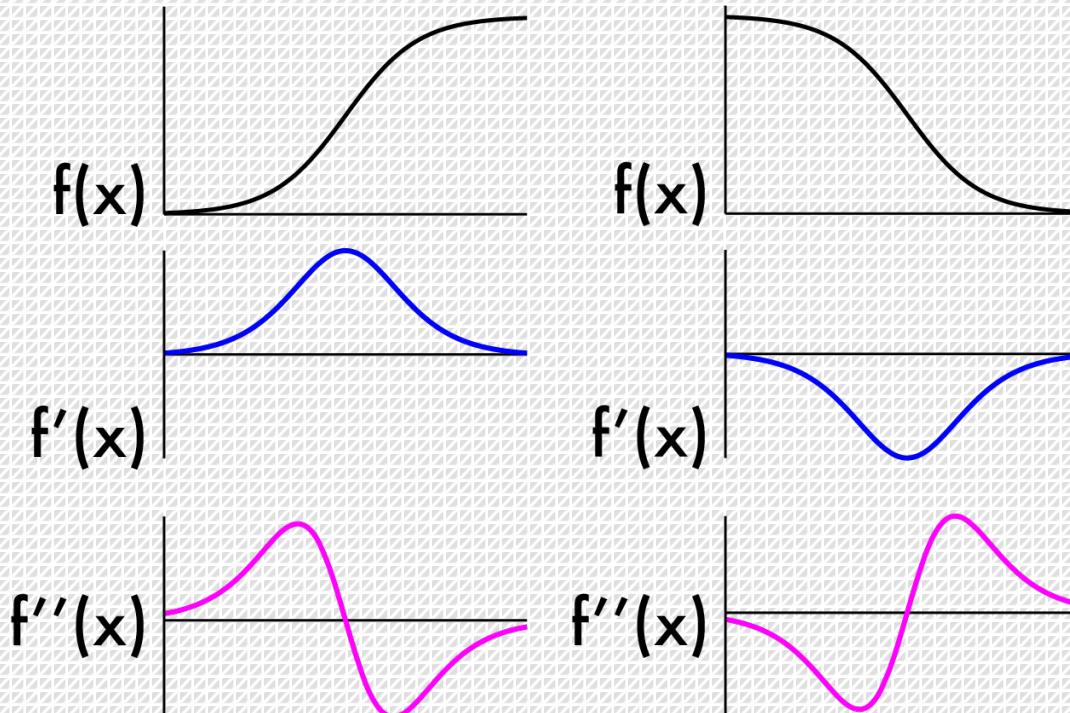
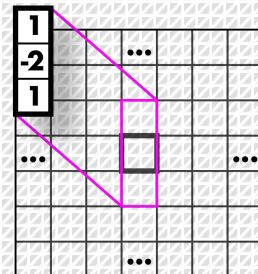
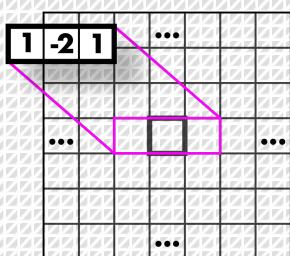
- Recall:

- $$f''(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

- Laplacian:

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Again, have to estimate $f''(x)$:



Edges

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- Laplacians

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

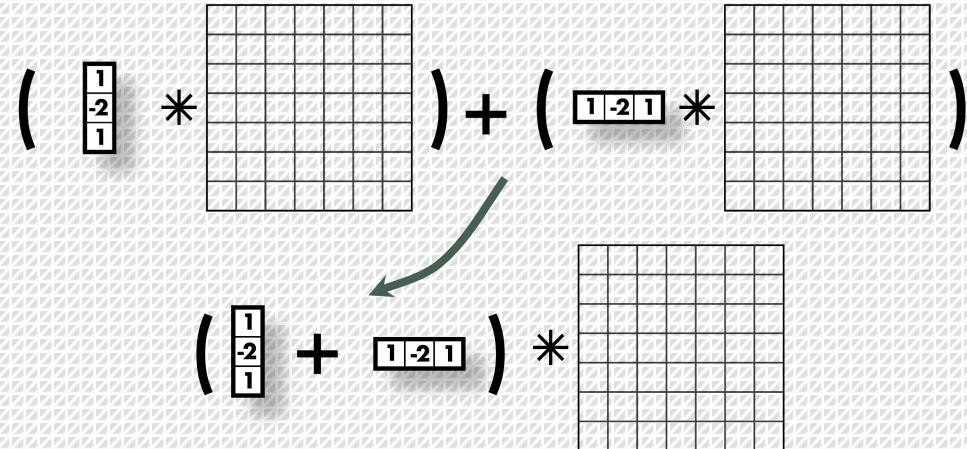
Edges

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- Laplacians

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



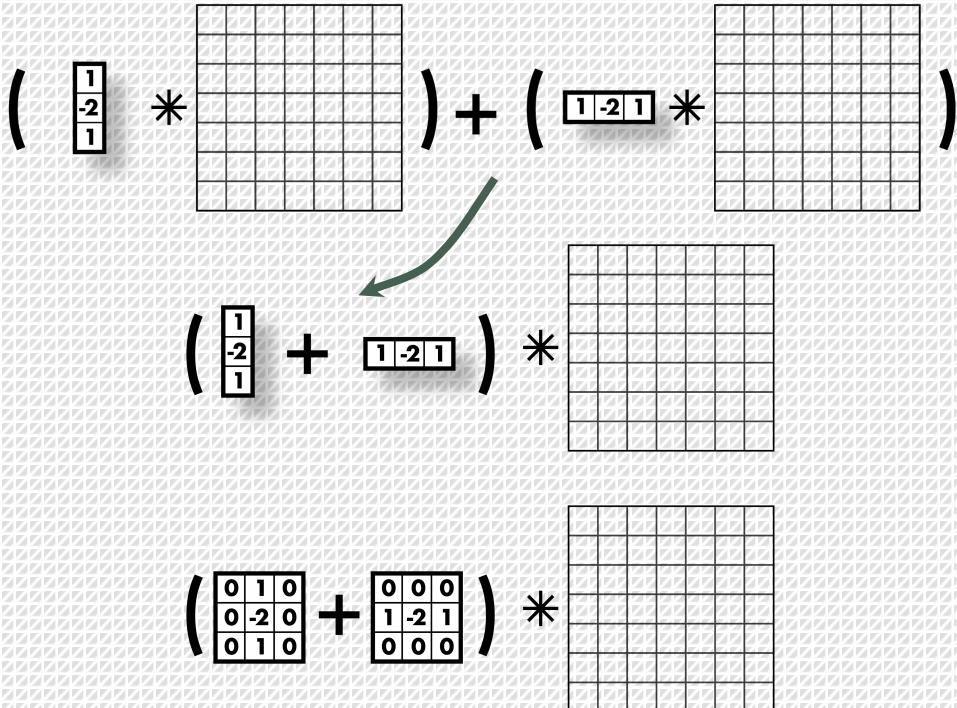
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- Laplacians

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



Edges

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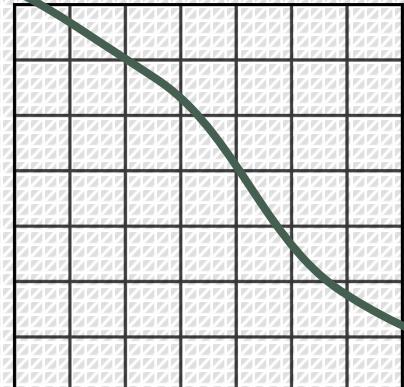


- Laplacians

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\begin{array}{|c|c|c|} \hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

*



$$\left(\begin{array}{c|c} 1 & \\ -2 & \\ 1 & \end{array} + \begin{array}{c|c|c} 1 & -2 & 1 \end{array} \right) *$$

Edges

Source: <https://courses.cs.washington.edu/courses/cse576/23sp/notes.html>

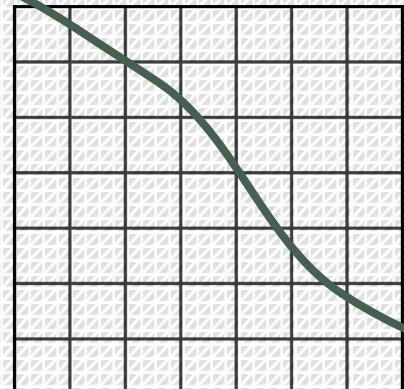


- Laplacians

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

*



$$\left(\begin{array}{|c|c|c|} \hline 1 & & \\ \hline -2 & & \\ \hline 1 & & \\ \hline \end{array} \right) * \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline \end{array} \right) + \left(\begin{array}{|c|c|c|} \hline 1 & -2 & 1 \\ \hline \end{array} \right) * \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|} \hline \end{array} \right)$$

$$\left(\begin{array}{c|c} 1 & \\ -2 & \\ 1 & \end{array} + \boxed{1 \cdot 2 \cdot 1} \right) *$$

$$\left(\begin{array}{ccc} 0 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 1 & 0 \end{array} \right) + \left(\begin{array}{ccc} 0 & 0 & 0 \\ 1 & -2 & 1 \\ 0 & 0 & 0 \end{array} \right) \quad *$$

Other linear filters

- Summed area tables (*integral image*)
- ... is used for face detection to compute simple multi-scale low-level features

3	2	7	2	3
1	5	1	3	4
5	1	3	5	1
4	3	2	1	6
2	4	1	4	8

(a) $S = 24$

3	5	12	14	17
4	11	19	24	31
9	17	28	38	46
13	24	37	48	62
15	30	44	59	81

(b) $s = 28$

3	5	12	14	17
4	11	19	24	31
9	17	28	38	46
13	24	37	48	62
15	30	44	59	81

(c) $S = 24$

Figure 3.17 Summed area tables: (a) original image; (b) summed area table; (c) computation of area sum. Each value in the summed area table $s(i, j)$ (red) is computed recursively from its three adjacent (blue) neighbors (3.31). Area sums S (green) are computed by combining the four values at the rectangle corners (purple) (3.32). Positive values are shown in **bold** and negative values in *italics*.

Non-linear filter

- Why we need non-linear filter?
- Regular blurring with a Gaussian filter fails to remove the noisy pixels and instead turns them into softer (but still visible) spots

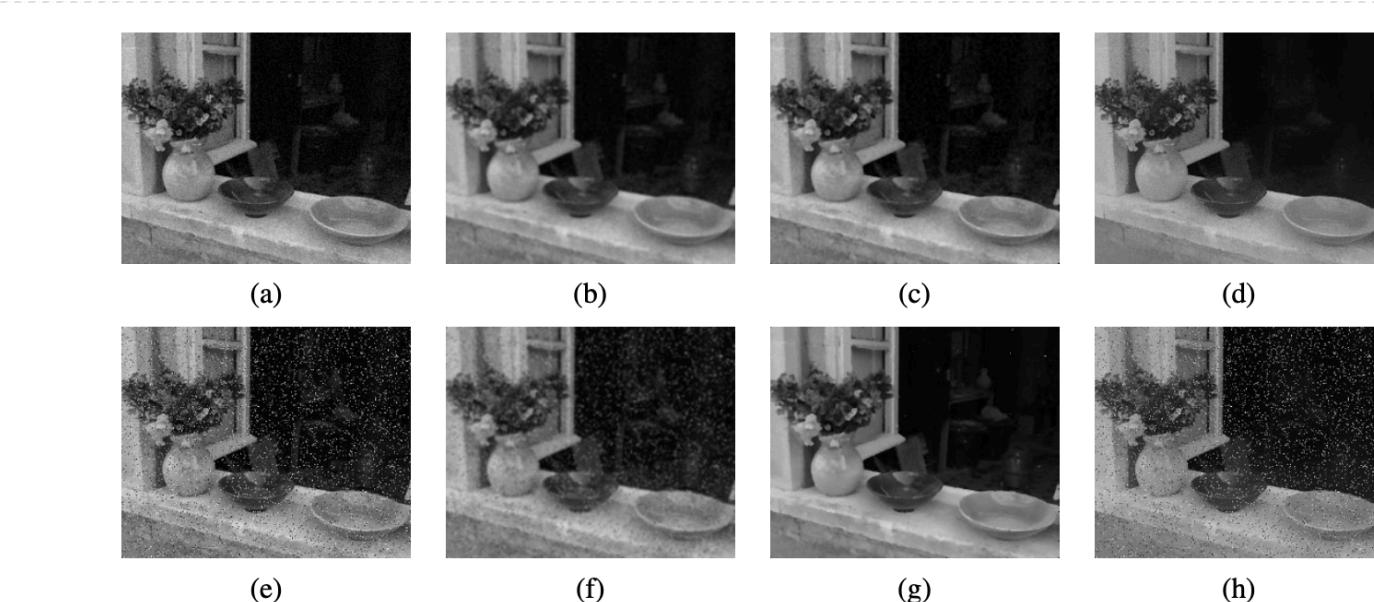


Figure 3.18 Median and bilateral filtering: (a) original image with Gaussian noise; (b) Gaussian filtered; (c) median filtered; (d) bilaterally filtered; (e) original image with shot noise; (f) Gaussian filtered; (g) median filtered; (h) bilaterally filtered. Note that the bilateral filter fails to remove the shot noise because the noisy pixels are too different from their neighbors.

Non-linear filter

- Median Filtering:

...selects the median value from each pixel's neighborhood.

...since the shot noise value usually lies well outside the true values in the neighborhood, the median filter is able to filter away such bad pixels.

1	2	1	2	4
2	1	3	5	8
1	3	7	6	9
3	4	8	6	7
4	5	7	8	9

(a) median = 4

Non-linear filter



- Median Filtering (α -trimmed mean):

....averages together all of the pixels except for the α fraction that are the smallest and the largest

1	2	1	2	4
2	1	3	5	8
1	3	7	6	9
3	4	8	6	7
4	5	7	8	9

(b) α -mean= 4.6

Non-linear filter



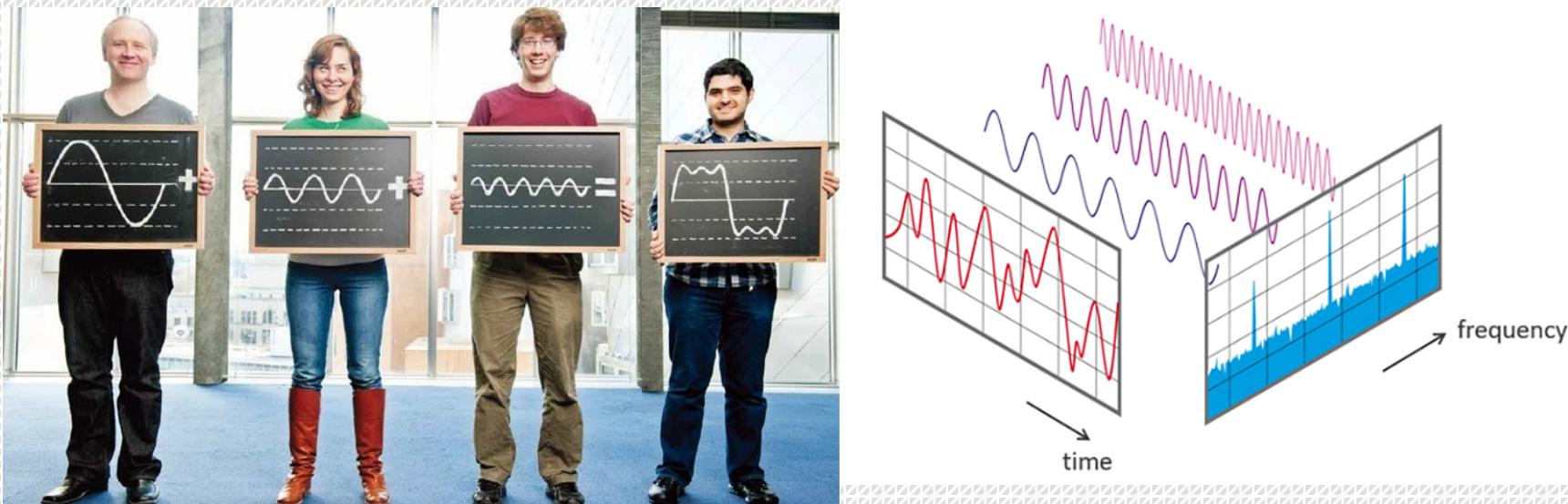
- Median Filtering (weighted median):
- each pixel is used a number of times depending on its distance from the center

1	2	1	2	4
2	1	3	5	8
1	3	7	6	9
3	4	8	6	7
4	5	7	8	9

(b) α -mean= 4.6

Fourier transforms

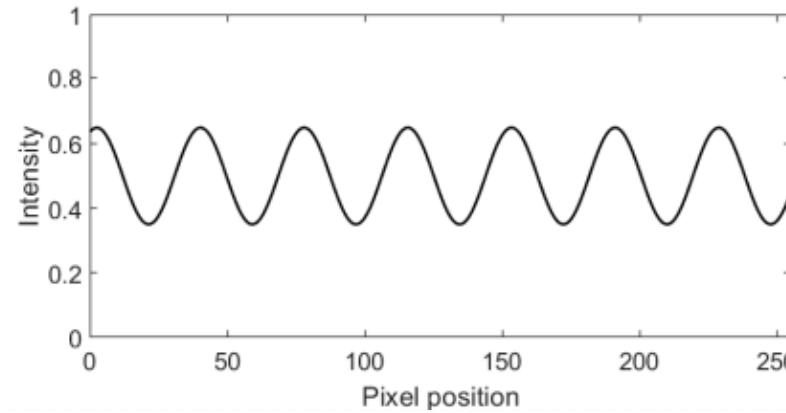
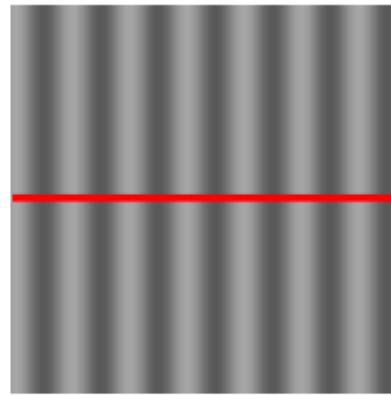
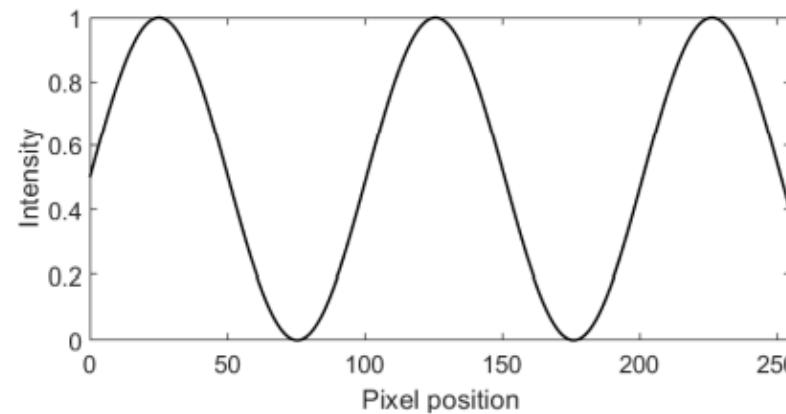
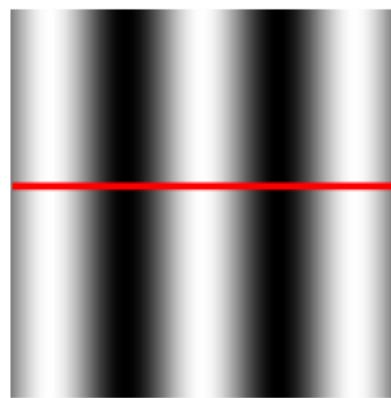
- Spatial domain → Frequency domain



https://www.bilibili.com/video/BV1uY411z7uk/?spm_id_from=333.788.recommend_more_video.0&vd_source=1d98d45f728c64dff48447f704f66bae

Fourier transforms

- Spatial domain → Frequency domain

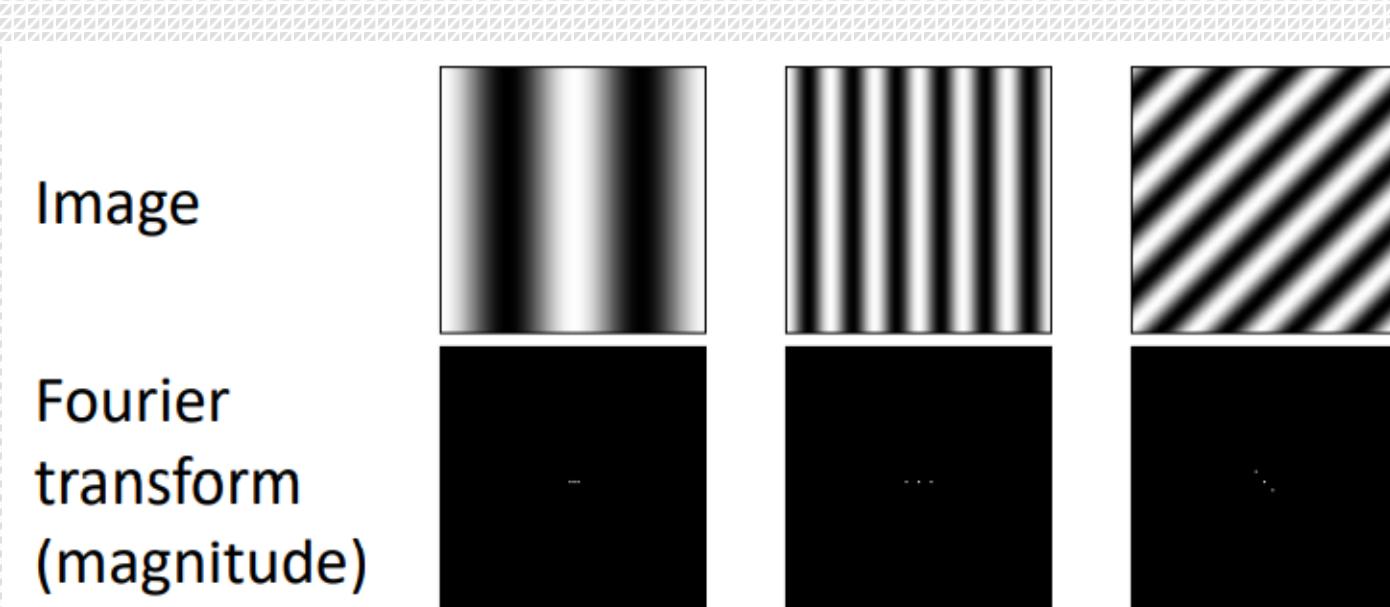


Fourier transforms



- Spatial domain → Frequency domain

- 逆傅里叶变换将频域转化为空间域
- 频率域处理是先将图像变换到频率域，然后在频率域对图像进行处理，最后通过反变换将图像变为空间域。





Talk from the top scientist



AI 如何拯救人性