Image processing operators (2)

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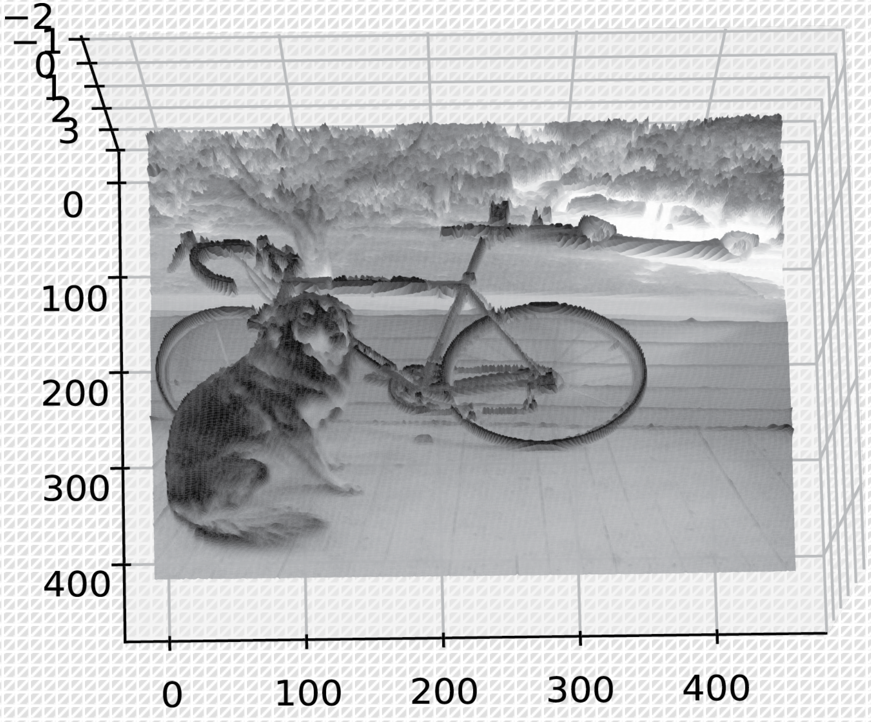
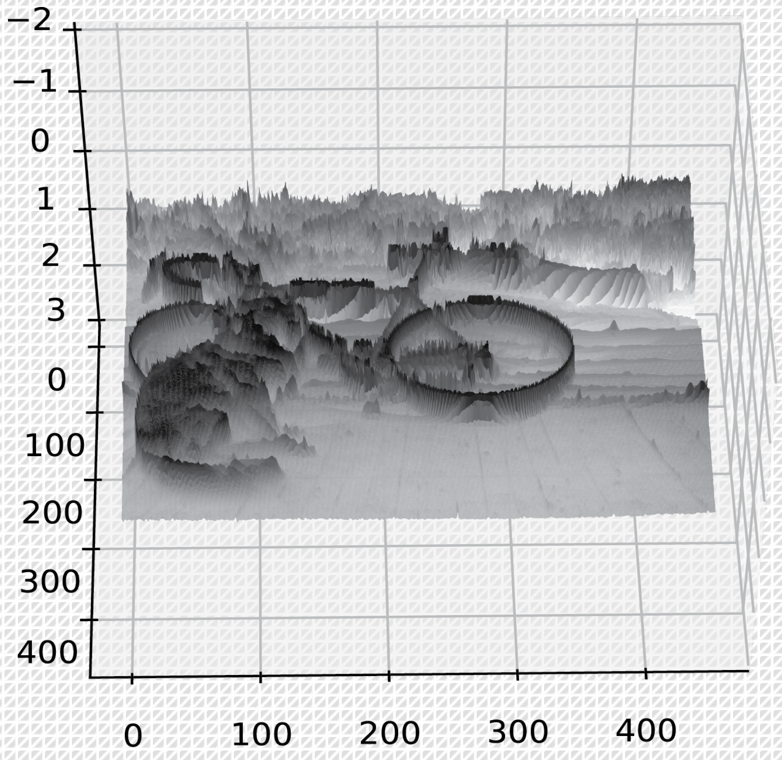
Edges

- What is an “edge”?





Edges

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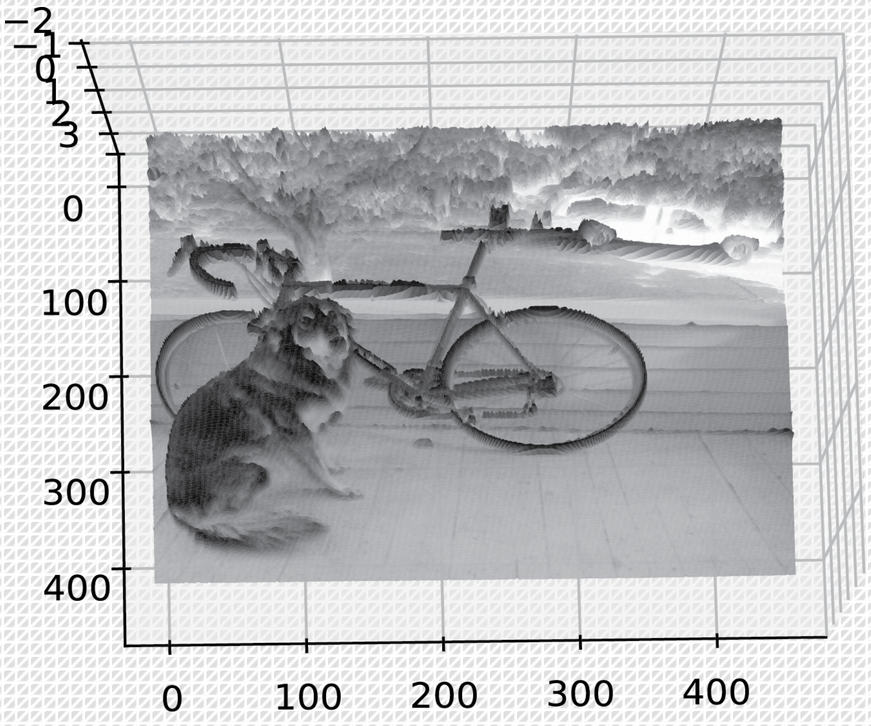
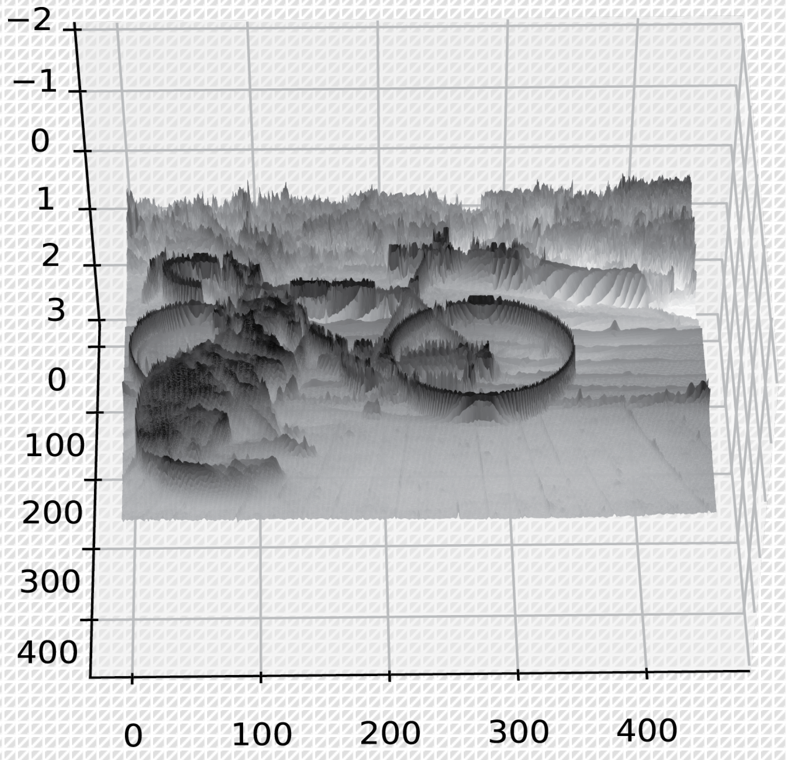
- Image is a function.
- Think of the gray tones as HEIGHTS.
- Edges are rapid changes in this function





Edges

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- Image is a function
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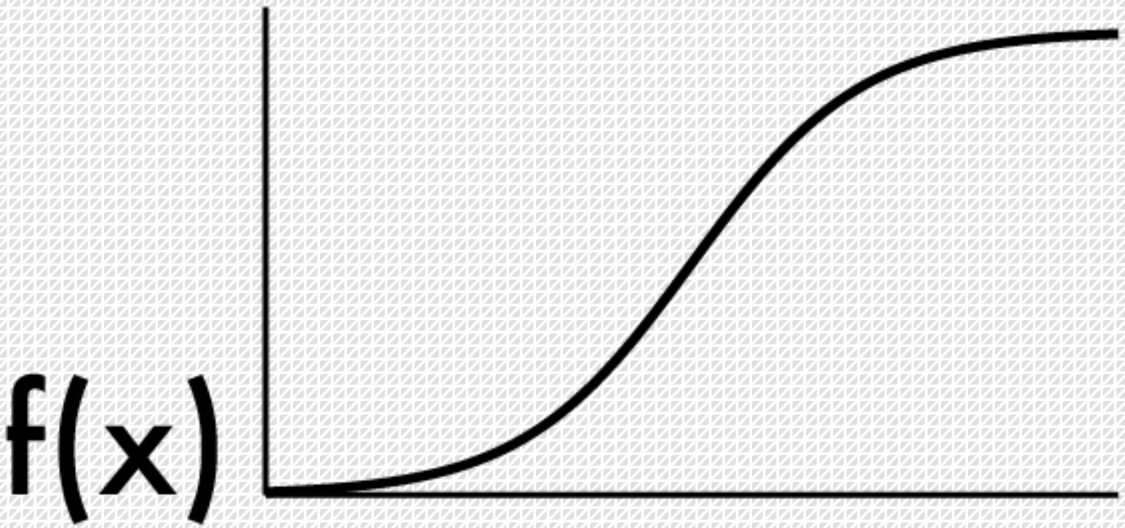

Edges

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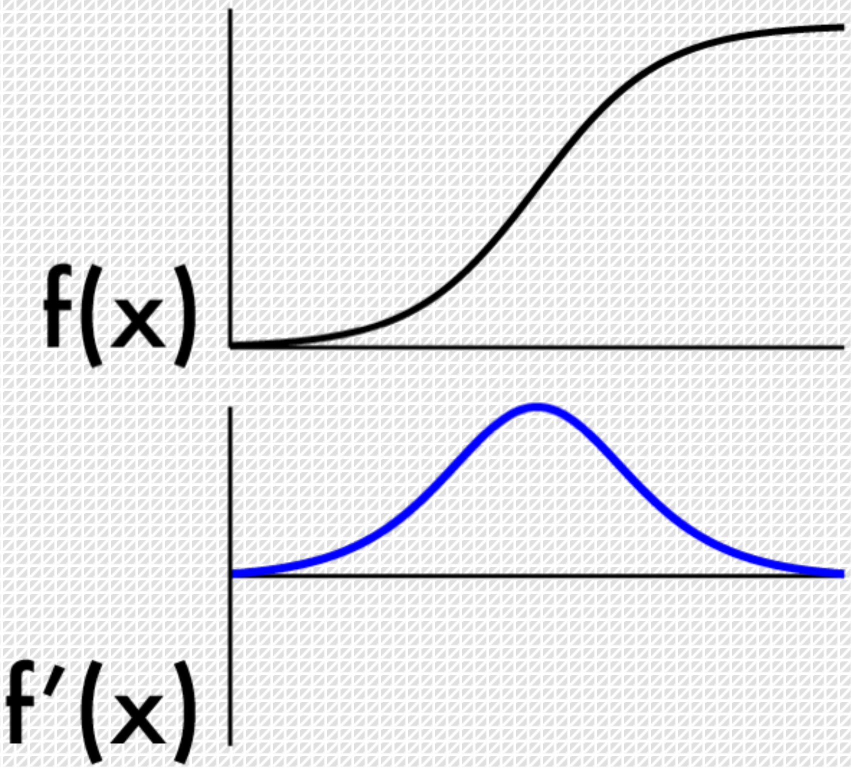

Edges

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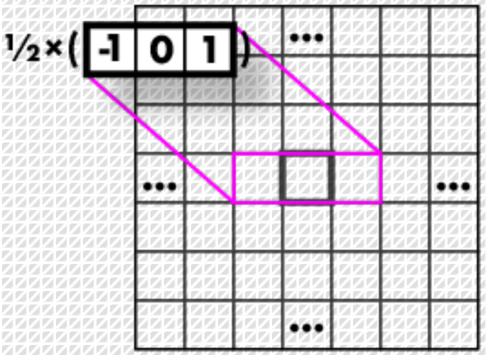
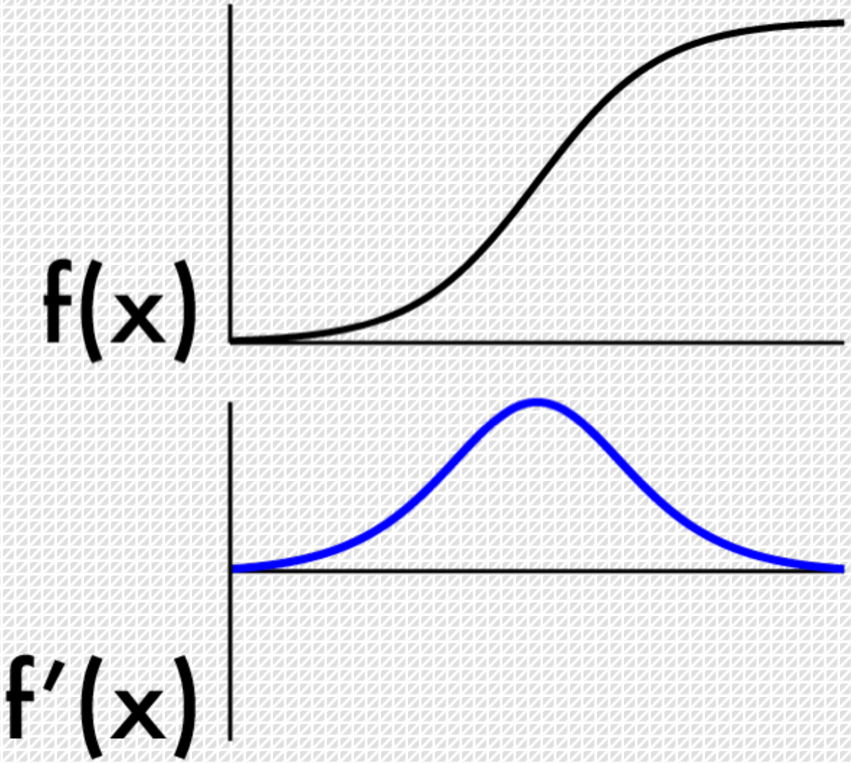
- Recall:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

- We don't have an "actual" function, must estimate
- Possibility: set h = 2
- What will that look like?





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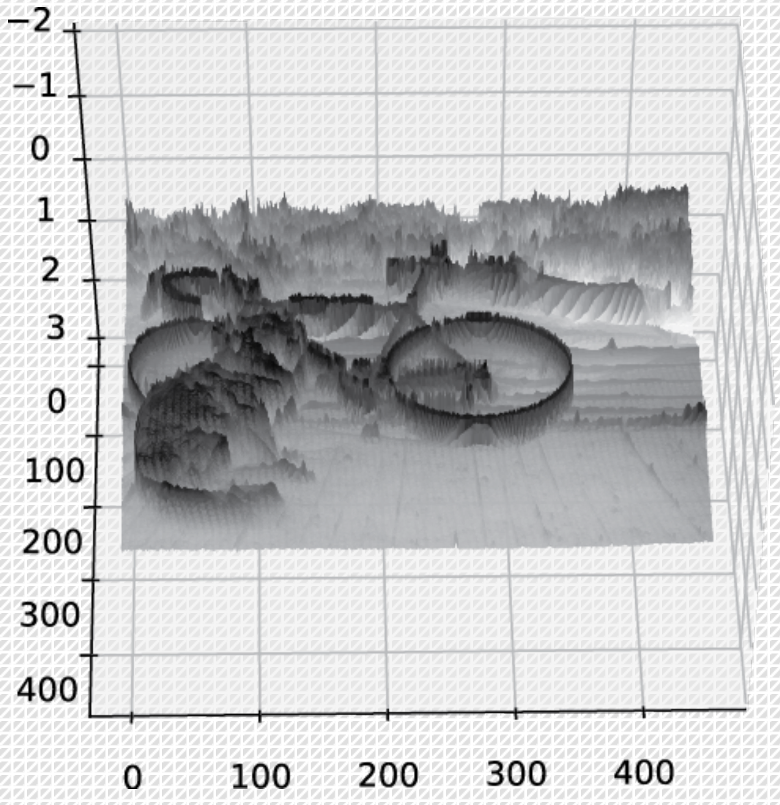
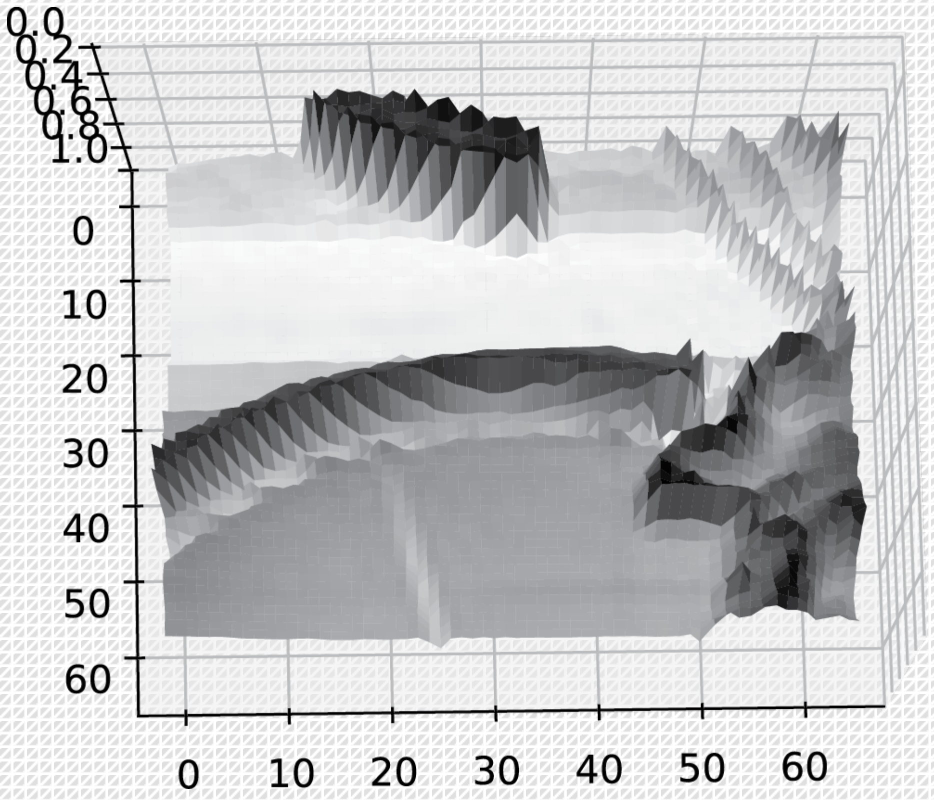
Edges

- Images are noisy!





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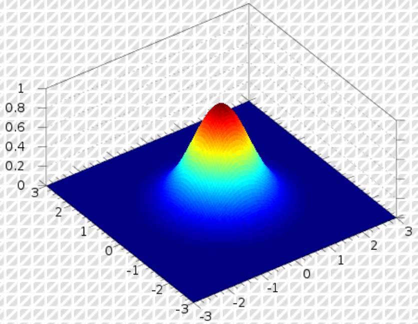

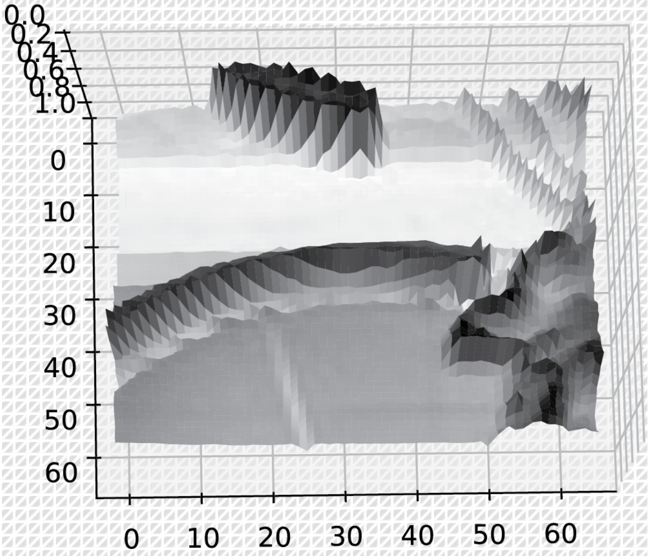

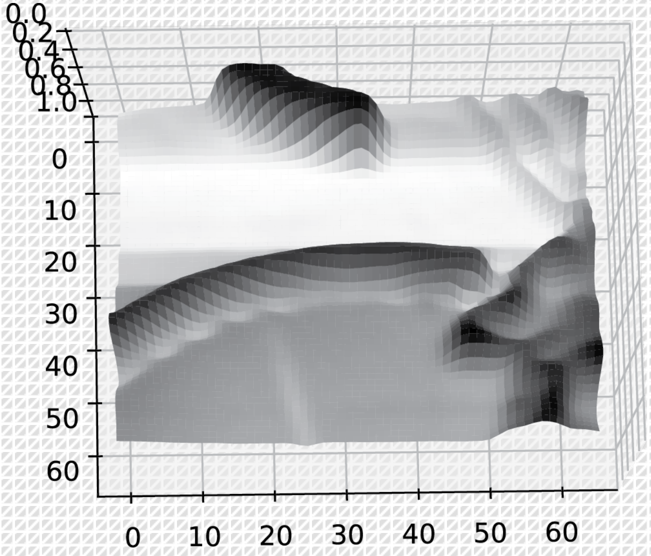

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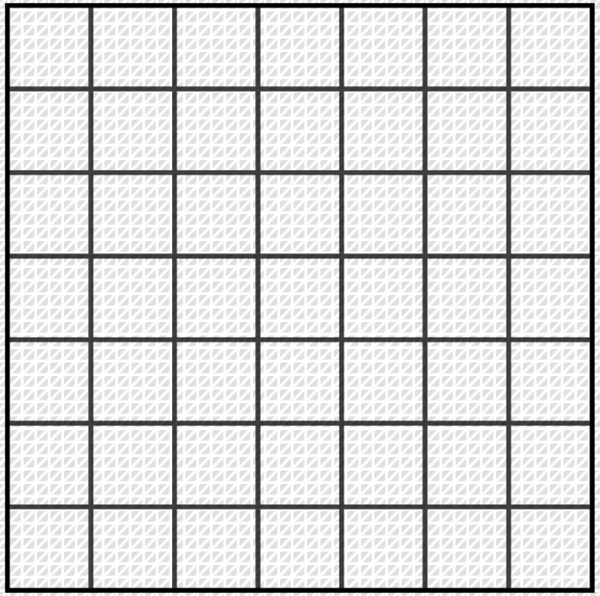
Edges

- Images are noisy!

$$\overset{E}{\tfrac{1}{2}\times\left(\begin{bmatrix}-1 & 0 & 1\end{bmatrix}\right)} * \left(\overset{G}{\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}} * \quad \right)$$

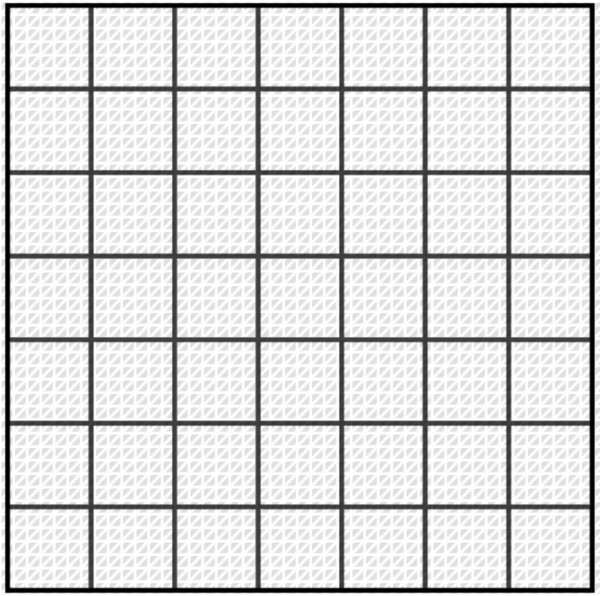


Edges

Source: https://courses.cs.washington.edu/courses/cse576/23sp/notes.html

- Smooth first, then derivative

$$\tfrac{1}{2}\times\left(\begin{bmatrix}-1 & 0 & 1\end{bmatrix} * \begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}\right) *$$



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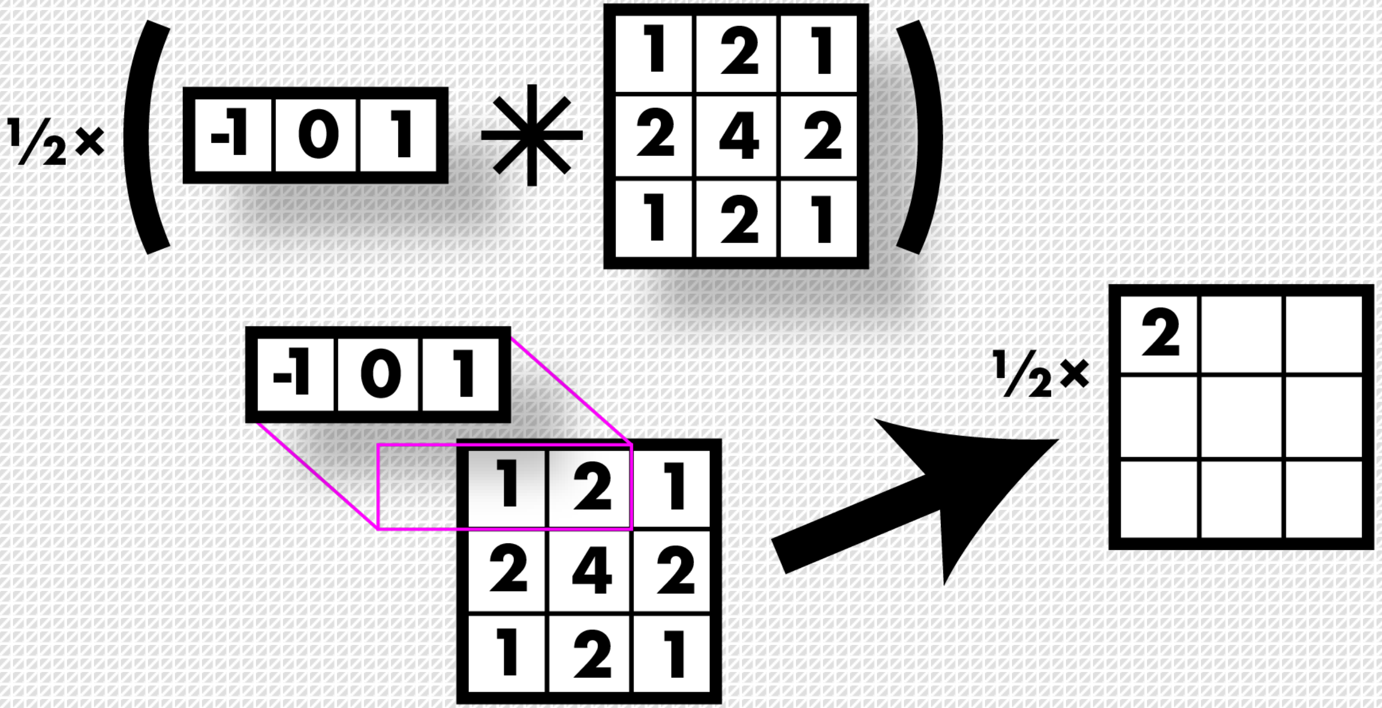

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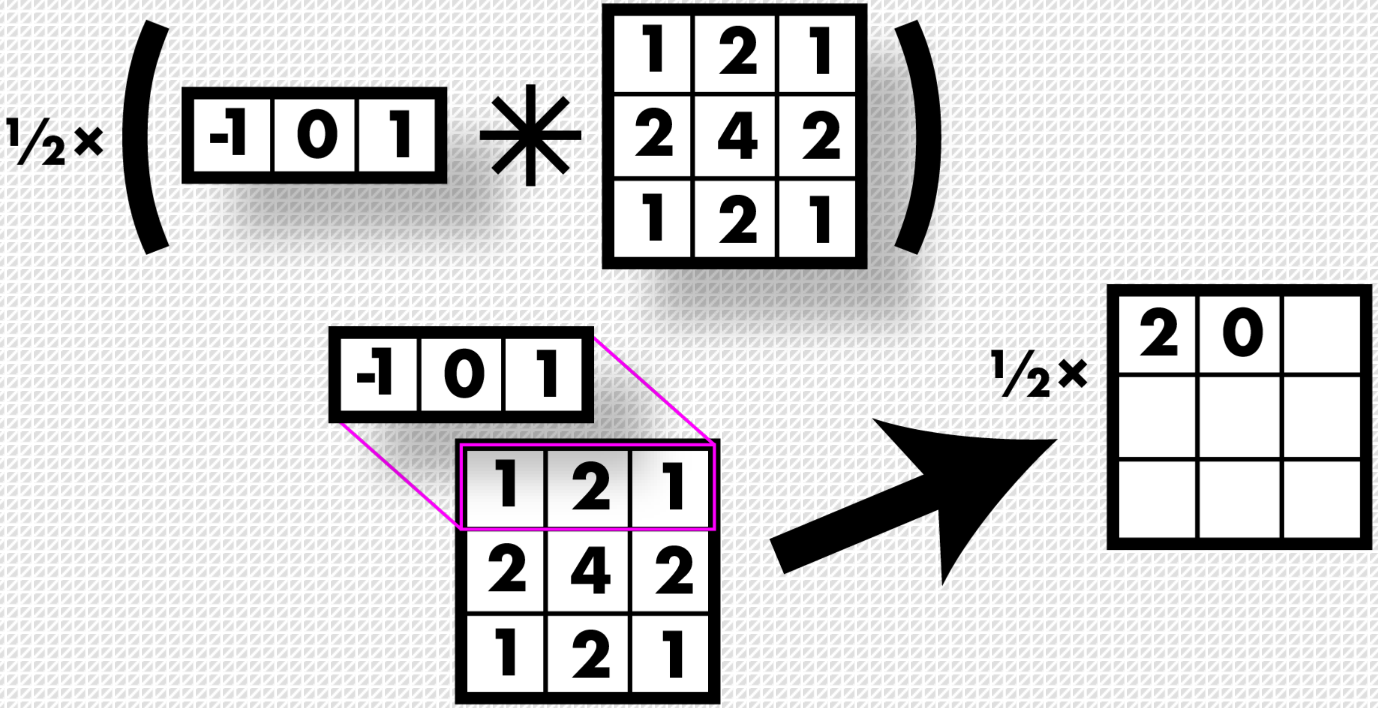

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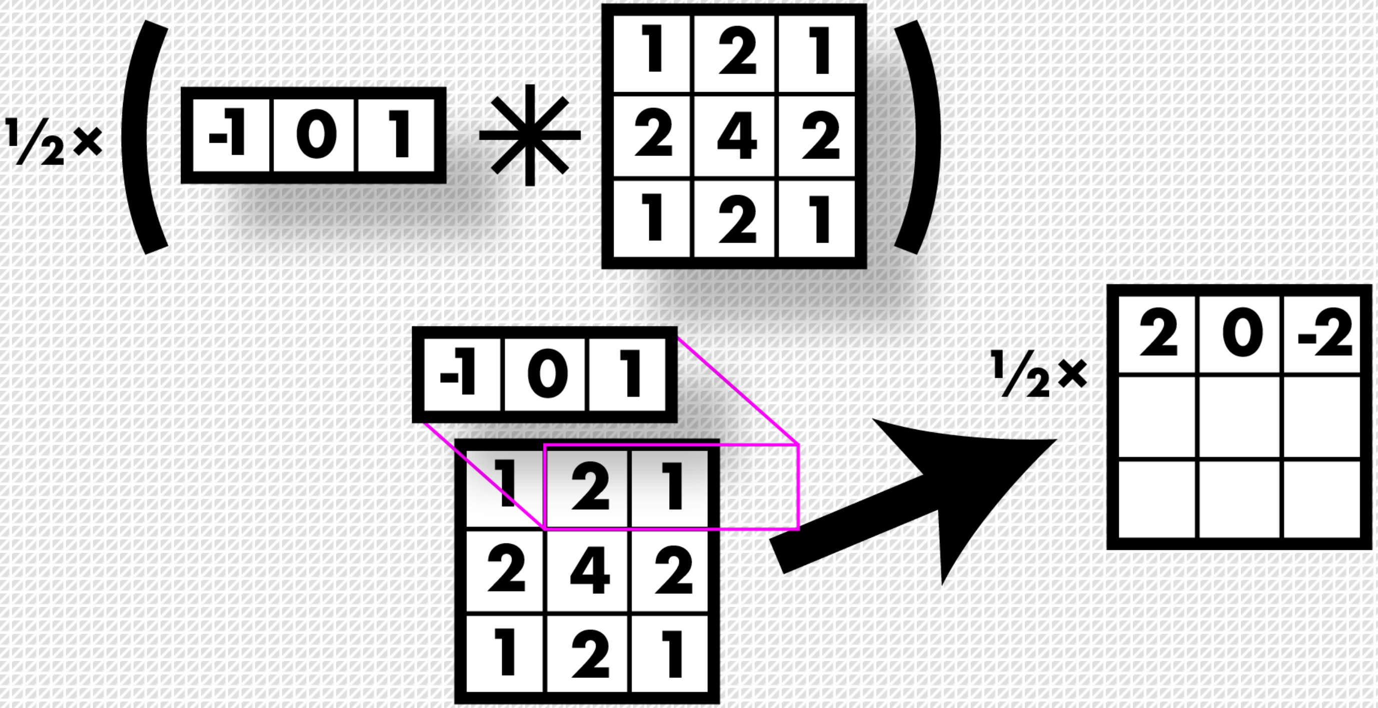
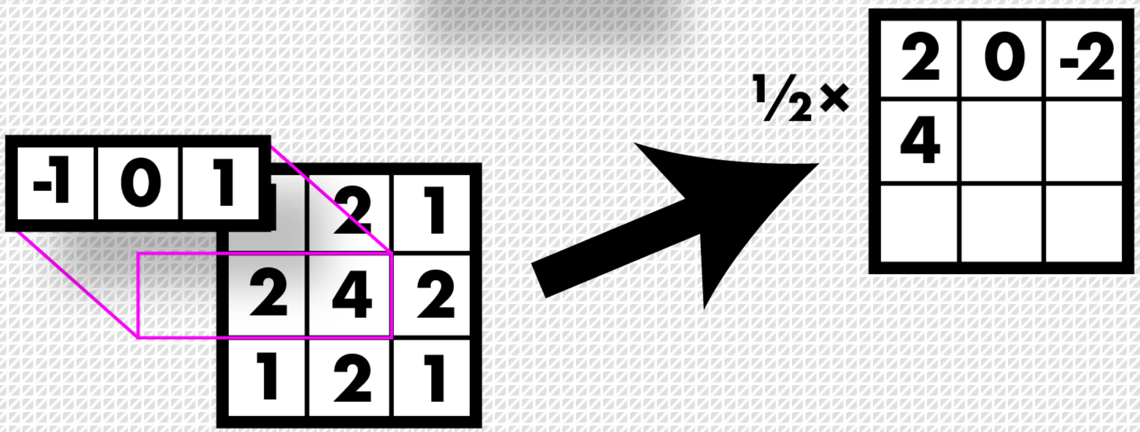


Edges

Source: https://courses.cs.washington.edu/courses/cse576/23sp/notes.html

- Smooth first, then derivative

$$\tfrac{1}{2}\times\left(\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline\end{array} * \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline\end{array}\right)$$



Edges

Source: https://courses.cs.washington.edu/courses/cse576/23sp/notes.html

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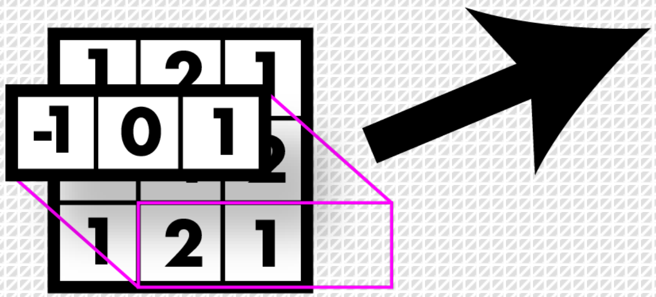
$$\tfrac{1}{2}\times\left(\begin{bmatrix}-1 & 0 & 1\end{bmatrix} * \begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}\right)$$



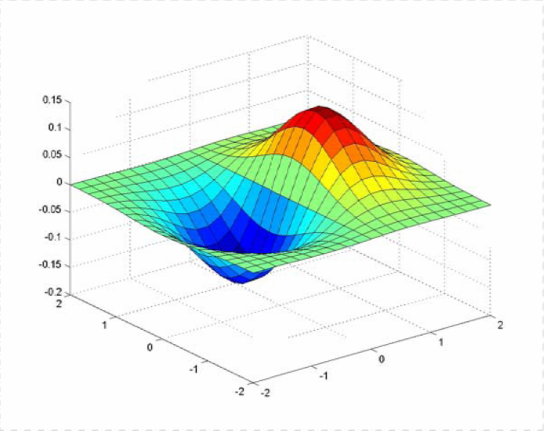
$$\tfrac{1}{2}\times\begin{bmatrix}2 & 0 & -2\\ 4 & 0 & -4\\ 2 & 0 & -2\end{bmatrix}$$

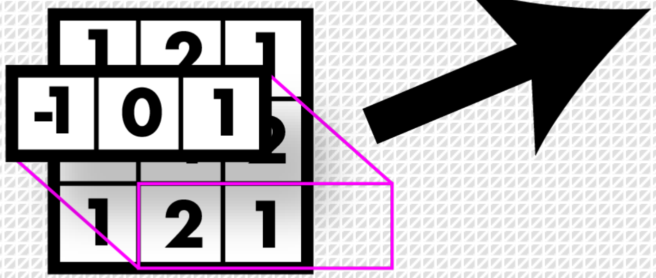
Edges

Source: https://courses.cs.washington.edu/courses/cse576/23sp/notes.html

- Smooth first, then derivative

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$





$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Edges

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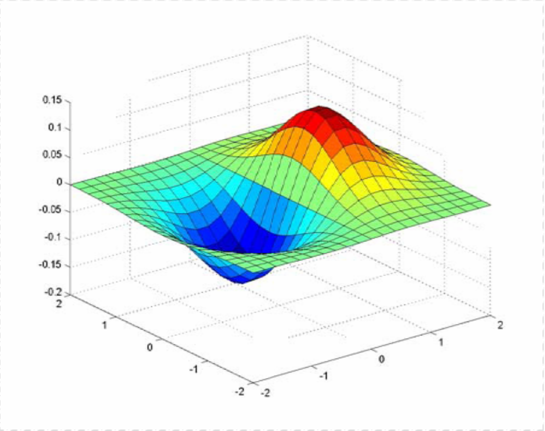
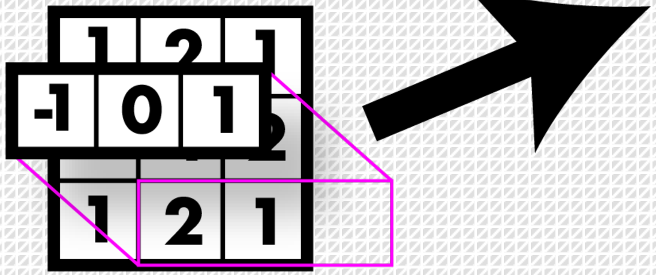
- Smooth first, then derivative

$$\frac{1}{2} \times \left(\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \right)$$





$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Edges

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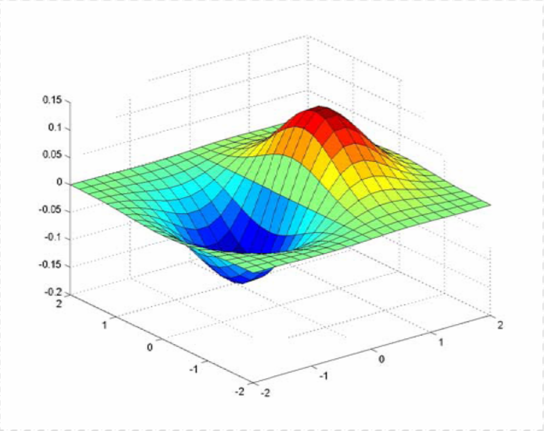
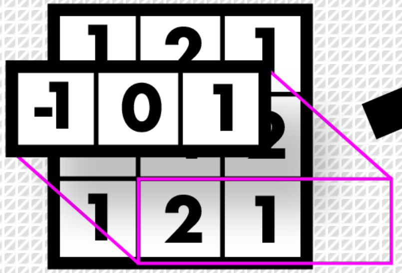
- Smooth first, then derivative → Sobel filter!

$$\tfrac{1}{2}\times\left(\begin{bmatrix}-1 & 0 & 1\end{bmatrix} * \begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix}\right)$$





$$\rightarrow \begin{bmatrix}1 & 0 & -1\\ 2 & 0 & -2\\ 1 & 0 & -1\end{bmatrix}$$

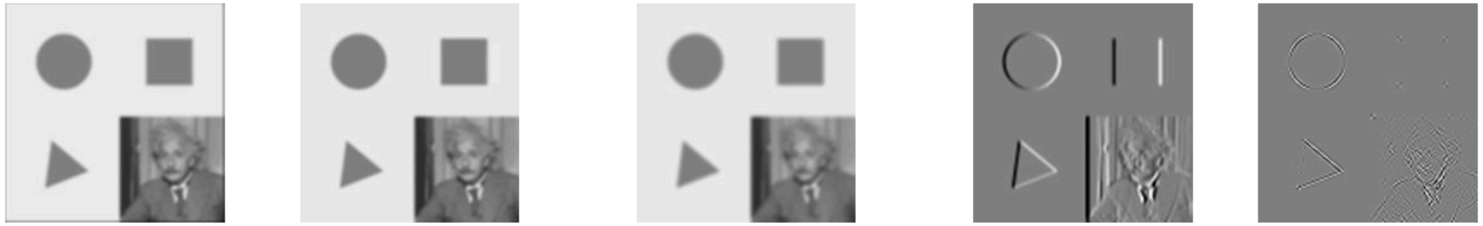
Edges

Source: https://courses.cs.washington.edu/courses/cse576/23sp/notes.html

- Smooth first, then derivative

$$\frac{1}{K^2}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline 1 & 1 & \cdots & 1 \\ \hline \vdots & \vdots & 1 & \vdots \\ \hline 1 & 1 & \cdots & 1 \\ \hline \end{array} \quad \frac{1}{16}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \quad \frac{1}{256}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 6 & 24 & 36 & 24 & 6 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array} \quad \frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \quad \frac{1}{4}\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline \end{array}$$

$$\frac{1}{K}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline \end{array} \quad \frac{1}{4}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline \end{array} \quad \frac{1}{16}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array} \quad \frac{1}{2}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline \end{array} \quad \frac{1}{2}\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline \end{array}$$



(a) box, $K = 5$ (b) bilinear (c) "Gaussian" (d) Sobel (e) corner

Figure 3.14 Separable linear filters: For each image (a)–(e), we show the 2D filter kernel (top), the corresponding horizontal 1D kernel (middle), and the filtered image (bottom). The filtered Sobel and corner images are signed, scaled up by $2\times$ and $4\times$, respectively, and added to a gray offset before display.

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Edges

- But

$$\frac{1}{K^2}\begin{array}{|c|c|c|c|}\hline 1 & 1 & \cdots & 1 \\ \hline 1 & 1 & \cdots & 1 \\ \hline \vdots & \vdots & 1 & \vdots \\ \hline 1 & 1 & \cdots & 1 \\ \hline \end{array} \qquad \frac{1}{16}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} \qquad \frac{1}{256}\begin{array}{|c|c|c|c|c|}\hline 1 & 4 & 6 & 4 & 1 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 6 & 24 & 36 & 24 & 6 \\ \hline 4 & 16 & 24 & 16 & 4 \\ \hline 1 & 4 & 6 & 4 & 1 \\ \hline \end{array} \qquad \frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad \frac{1}{4}\begin{array}{|c|c|c|}\hline 1 & -2 & 1 \\ \hline -2 & 4 & -2 \\ \hline 1 & -2 & 1 \\ \hline \end{array}$$

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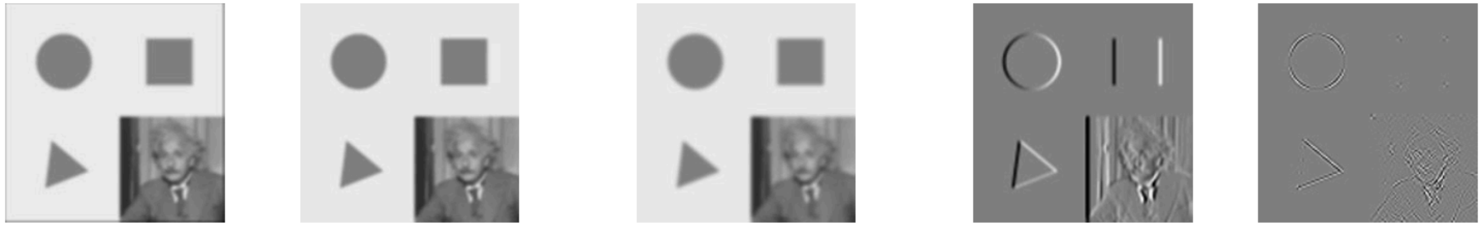

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Edges

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- Could take derivative
- Find high responses
- Sobel filters!
- But…
- Edges go both ways
- Want to find extrema

Edges

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- Could take derivative
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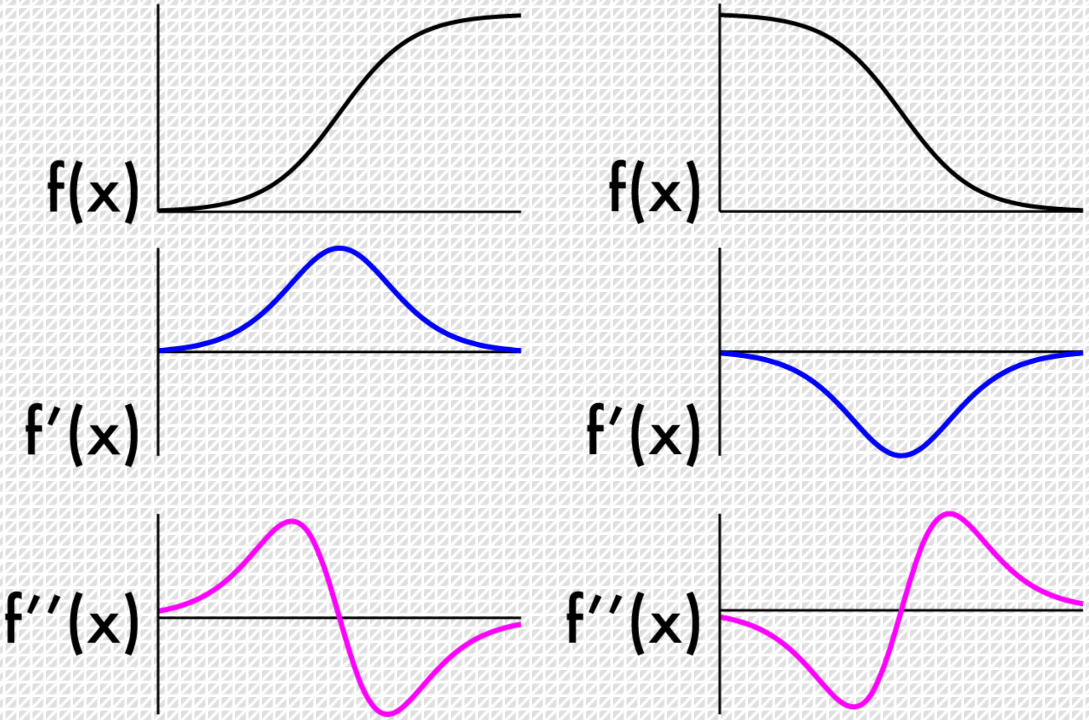

Edges

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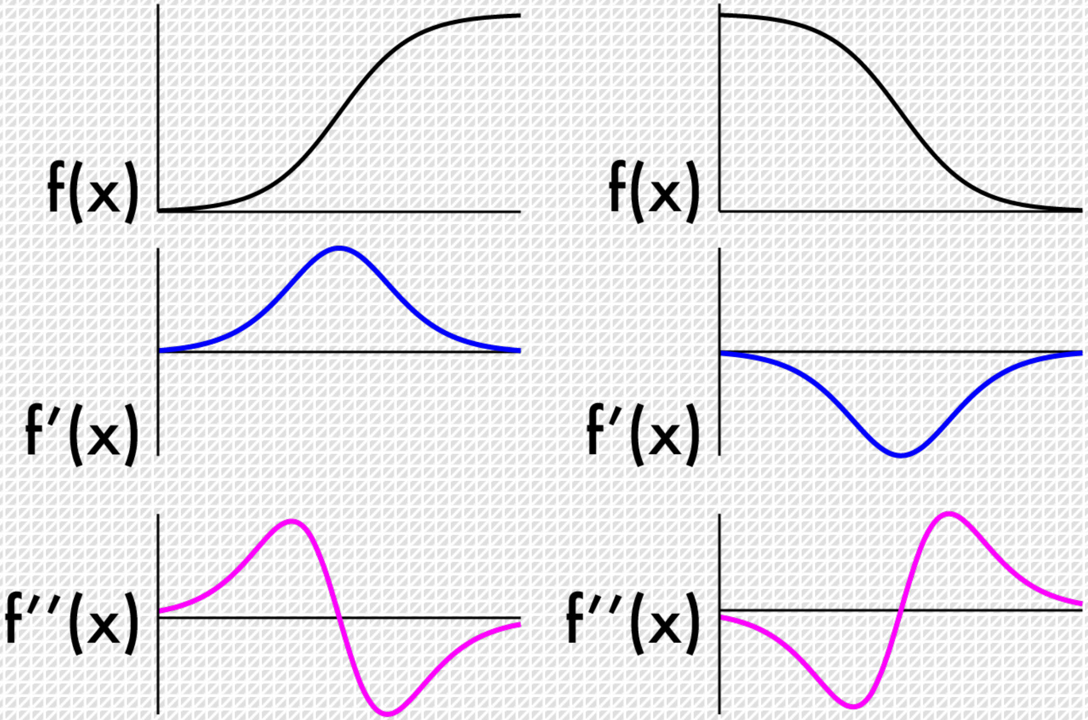
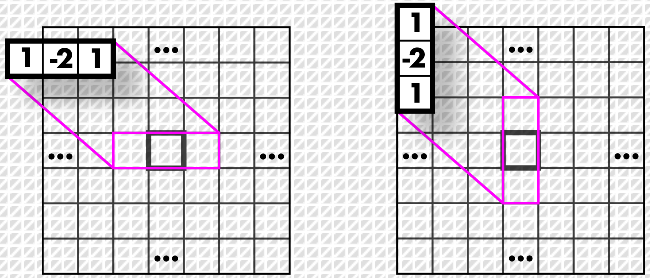
- Crosses zero at extrema
- Recall:
  - $$f''(x) = \lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$
- Laplacian:
  - $$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$
- Again, have to estimate f''(x):





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Edges

- Laplacians

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

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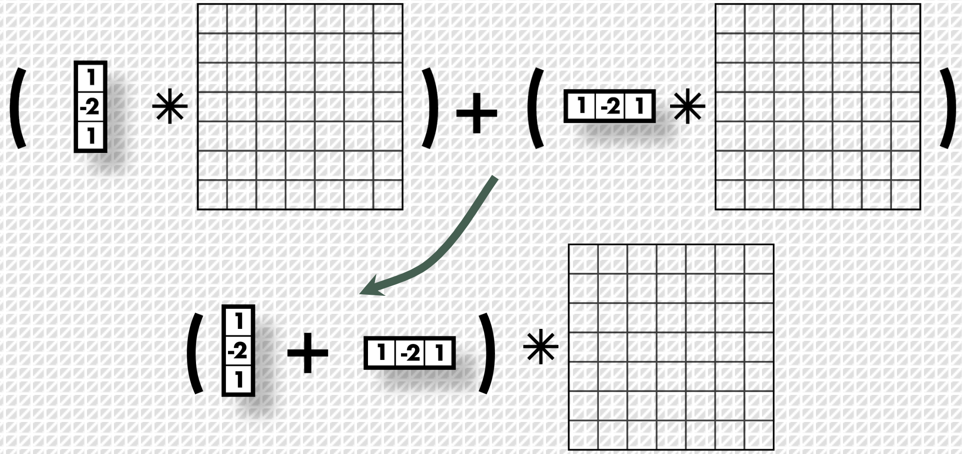

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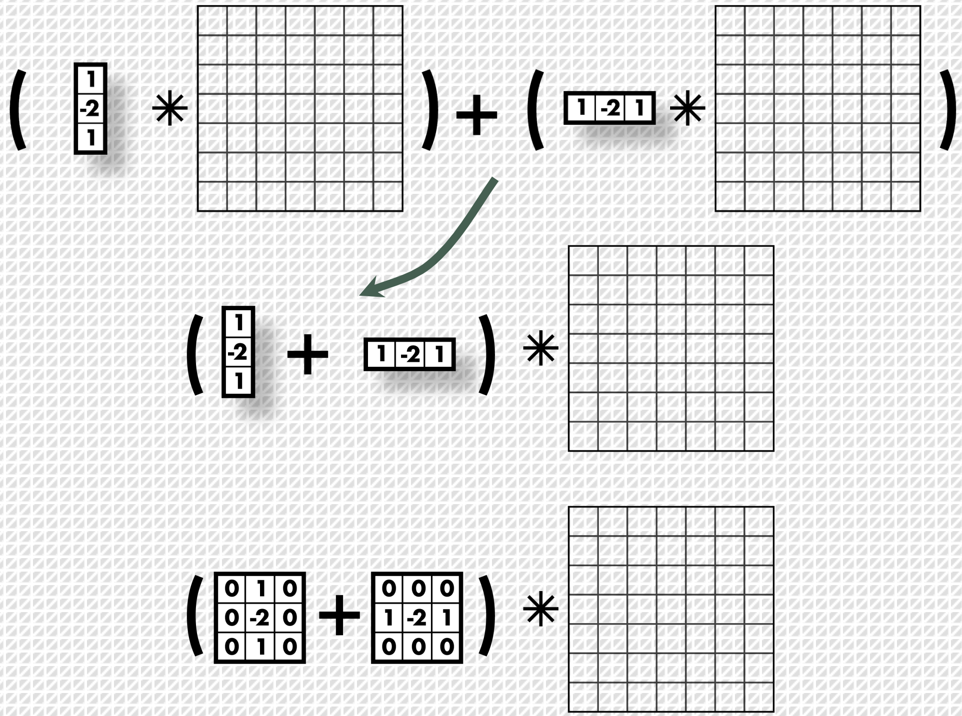

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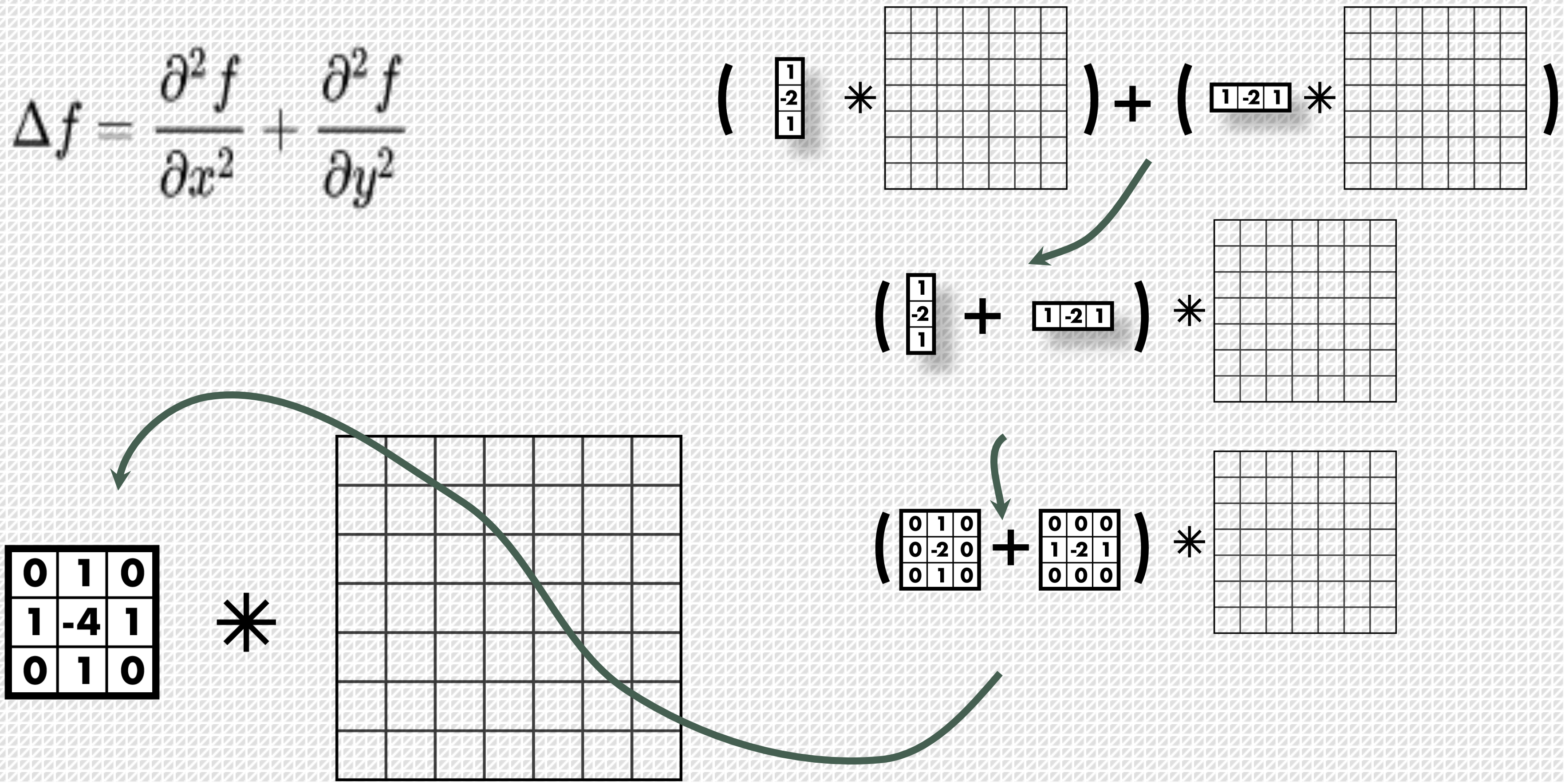

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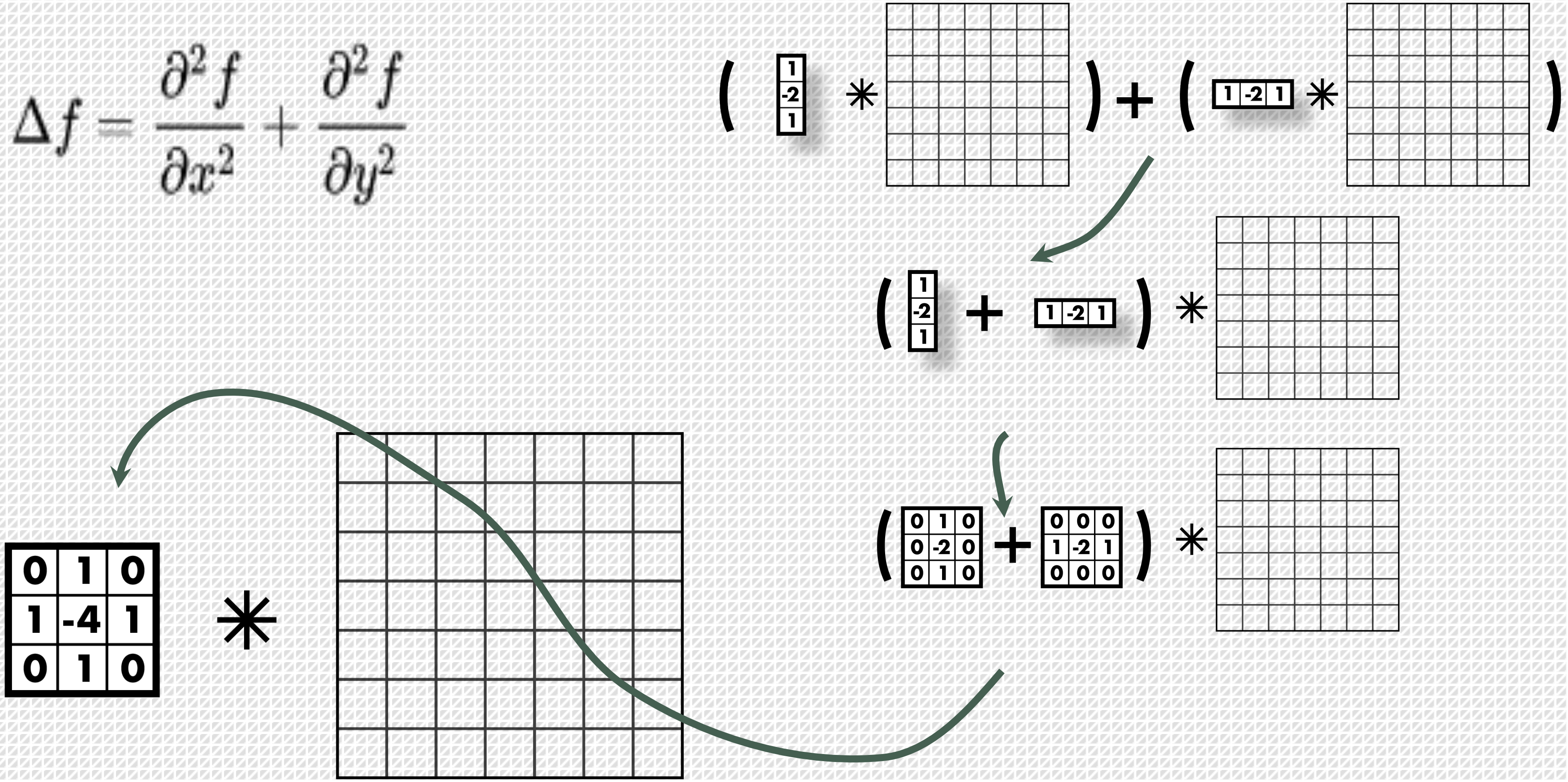
- Laplacians

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$



Other linear filters

- summed area tables (*integral image*)
- … is used for face detection to compute simple multi-scale low-level features

3	2	7	2	3
1	5	1	3	4
5	1	**3**	5	1
4	3	2	1	6
2	4	1	4	8

(a) S = 24

3	5	12	14	17
4	*11*	**19**	24	31
9	**17**	28	38	46
13	24	37	48	62
15	30	44	59	81

(b) s = 28

3	5	12	*14*	17
4	11	19	24	31
9	17	28	38	46
13	24	37	**48**	62
15	30	44	59	81

(c) S = 24

Figure 3.17 Summed area tables: (a) original image; (b) summed area table; (c) computation of area sum. Each value in the summed area table $s(i, j)$ (red) is computed recursively from its three adjacent (blue) neighbors (3.31). Area sums S (green) are computed by combining the four values at the rectangle corners (purple) (3.32). Positive values are shown in **bold** and negative values in *italics*.

Talk from the top scientist

AI 如何拯救人性

Neighborhood operators

- Gaussian filter