

KnightTour 304x304 start=(0,0) end=(2,1)
solutions=0 try=92471 solutions=1 step=92413

Challenging the record for the maximum solution to the Knight's Tour Problem

Masakatsu Okuno* Eiji Miyano**

(*ZOMA Co., Ltd. **Kyushu Institute of Technology)

Results

Search Order Sorting

Search Order Random

304x304 (Background Image)

48x48

The Knight's Tour Problem: Research Background and Objectives

Research Background

The Knight's Tour problem is a puzzle where a knight must traverse every square on a chessboard.

It is possible on boards of various sizes beyond 8x8.

A history of exploration predating computers.

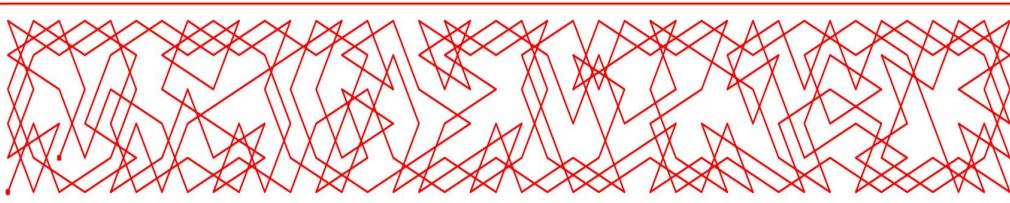
No research has attempted larger sizes within practical time limits.

Research Goal

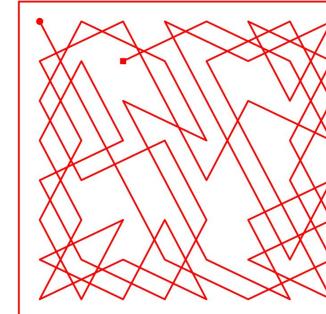
To determine the largest size solvable on a standard computer.

KnightTour 40x6 start=(0,0) end=(2,1)
seed=5 try=261 solutions=1

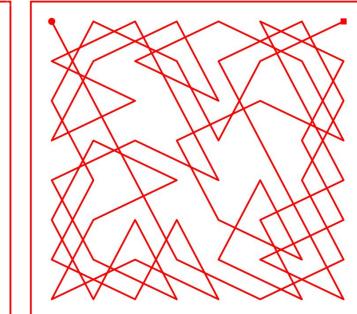
6X40 closed



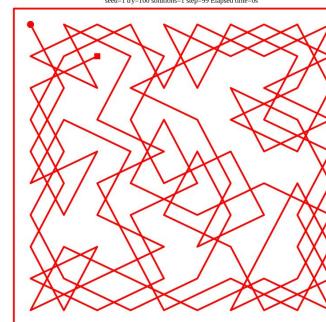
8X8 closed
KnightTour 8x8 start=(0,0) end=(2,1)
seed=0 try=64 solutions=1 step=63 Elapsed time=517.4ms



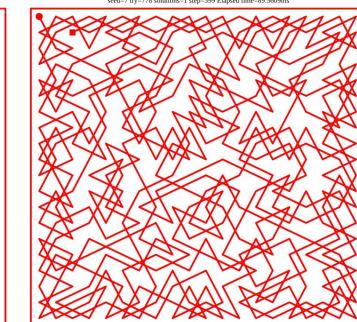
8X8 open
KnightTour 8x8 start=(0,0) end=(7,7)
seed=0 try=64 solutions=1 step=63 Elapsed time=520.1ms



10X10 closed
KnightTour 10x10 start=(0,0) end=(2,2)
seed=1 try=100 solutions=1 step=99 Elapsed time=10s



20X20 closed
KnightTour 20x20 start=(0,0) end=(19,19)
seed=7 try=778 solutions=1 step=399 Elapsed time=85.569ms



3X90 closed



Algorithm Overview

Input: An undirected graph and the Start and End nodes contained within the graph

Strategy: Provide a Seed to shuffle the search order, examine whether moving from Now (the current node) to Next (the forward candidate) is valid, and explore the path using the Backtrack method

The following explains the algorithm and strategy by item.

1. Edge pruning

Identify and prune unnecessary edges

2. Dead End Detection

Find dead ends and avoid them

3. Islands, Bridges, and Eulerian Paths

Decompose unvisited nodes into islands and bridges, then apply Eulerian path conditions

4. Parallel Processing (Threads)

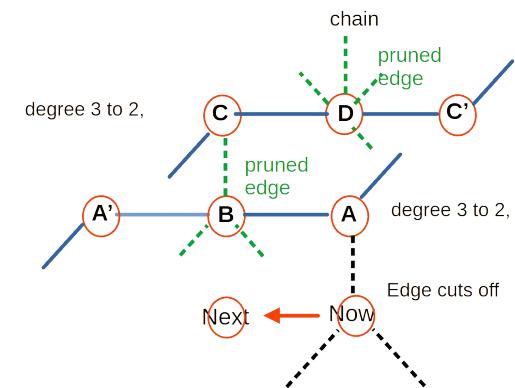
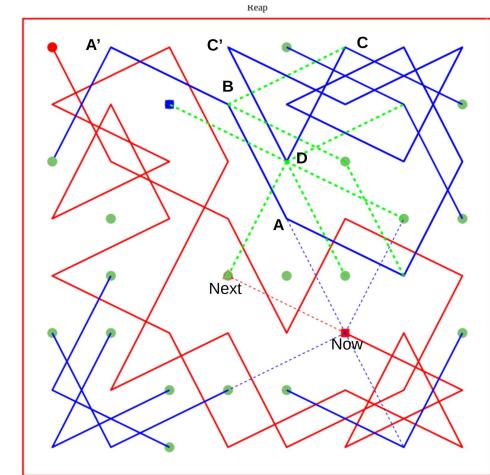
Utilize parallel processing features provided by the programming language

1. Edge pruning

Example of pruning edges

1. When attempting to move from Now to Next, the edges (black dashed lines) connecting Now to other nodes are severed because the path is lost.
2. If the degree of A is 3, then the degree becomes 2, and B is contained in the single path of A and A'.
3. Since the only entrances and exits to B are A and A', the extra edges of B (green dashed lines) can be pruned.
4. As a result, C, which had a degree of 3, now has a degree of 2, and pruning becomes possible for D as well.

Pruning is chained, increasing the number of nodes contained within a single path (blue line).
The efficiency of exploration increases dramatically as the number of single paths increases.



2 Dead End Detection

- A dead end is a node that can no longer be traversed.
- As a result of pruning, Dead End is now identifiable.
- Dead End (Aota-maru) refers to the following three types:
- Figure 1: Nodes whose degree has become 1 or less due to pruning
- Figure 2: Nodes where no further candidate can be selected regardless of the direction from which they were approached
- Figure 3: Nodes that have formed a loop
- Upon detecting these, stop further searching.
- Pruning and dead-end detection are highly efficient.
- By avoiding the loop in Figure 3, Figure 4 confirms a direct path to End by the 44th move.

Figure 1

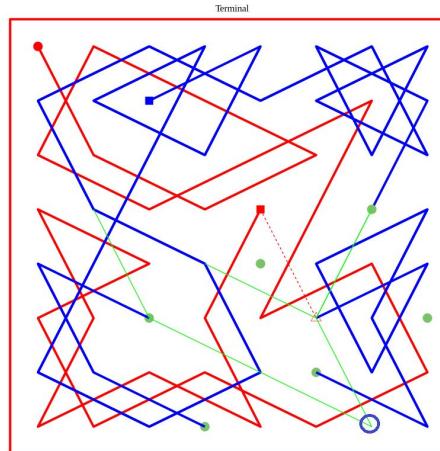


Figure 2

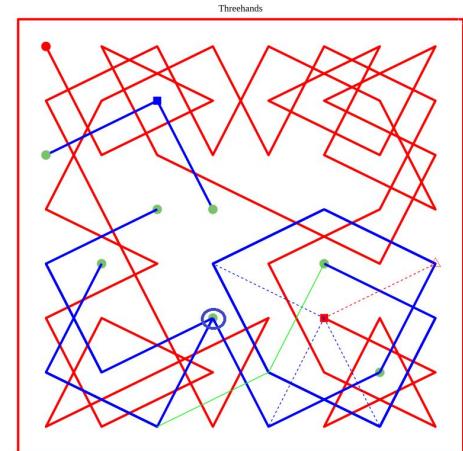


Figure 3

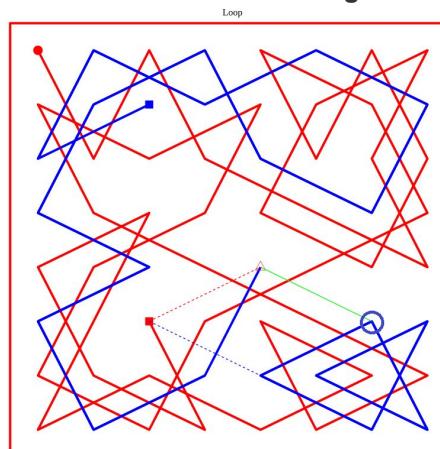
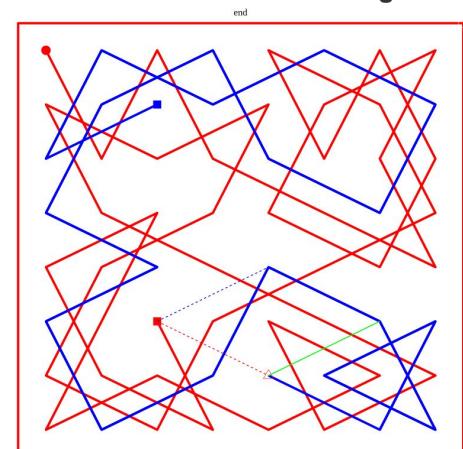


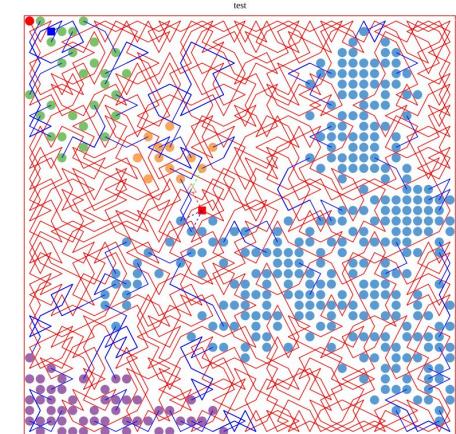
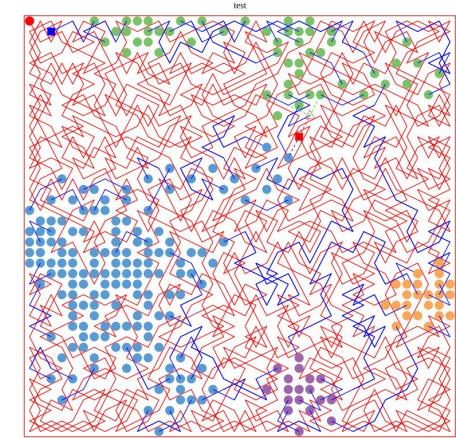
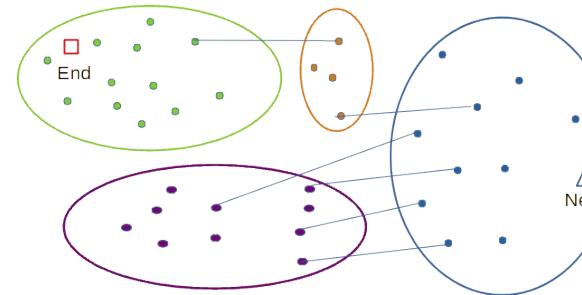
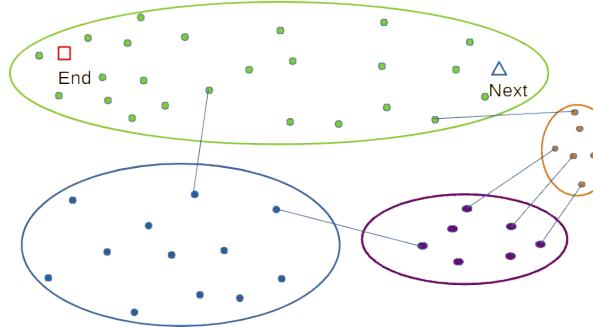
Figure 4



3 Islands and Bridges, Eulerian Paths

From Eulerian closed-loop property,
the following holds:

1. If all bridges on the islands are in even numbers, Next and End must be on the same island.
2. If there are an odd number of islands connected by bridges, there can only be two such islands, and Next and End must be contained within separate islands with an odd number of bridges.
3. There must be no island without a bridge.



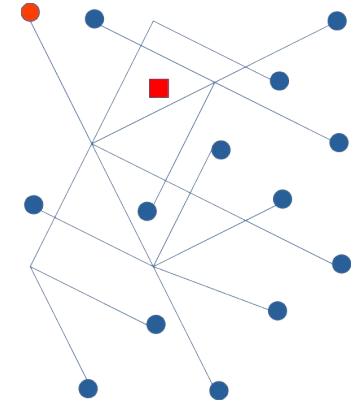
- Global structural analysis yields clear results and is highly effective when dealing with large sizes.
- On the other hand, the computational cost is high.

Colored circles indicate islands; blue lines indicate bridges. Red lines indicate areas already passed through.

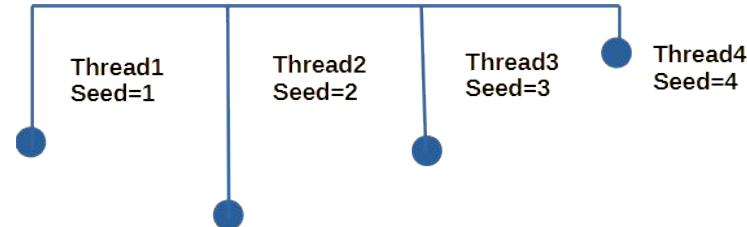
4 Parallel Processing (Threads)

- When exploring a path from Start (red circle) to End (red square), decompose it into independent paths of a certain number of steps from Start (3 steps in the diagram).
- Launching multiple threads from a new Start (blue circle) to End achieves non-overlapping parallel execution.
- The more threads there are (the larger N is), the more efficient it becomes, but this comes at the cost of thread switching overhead.

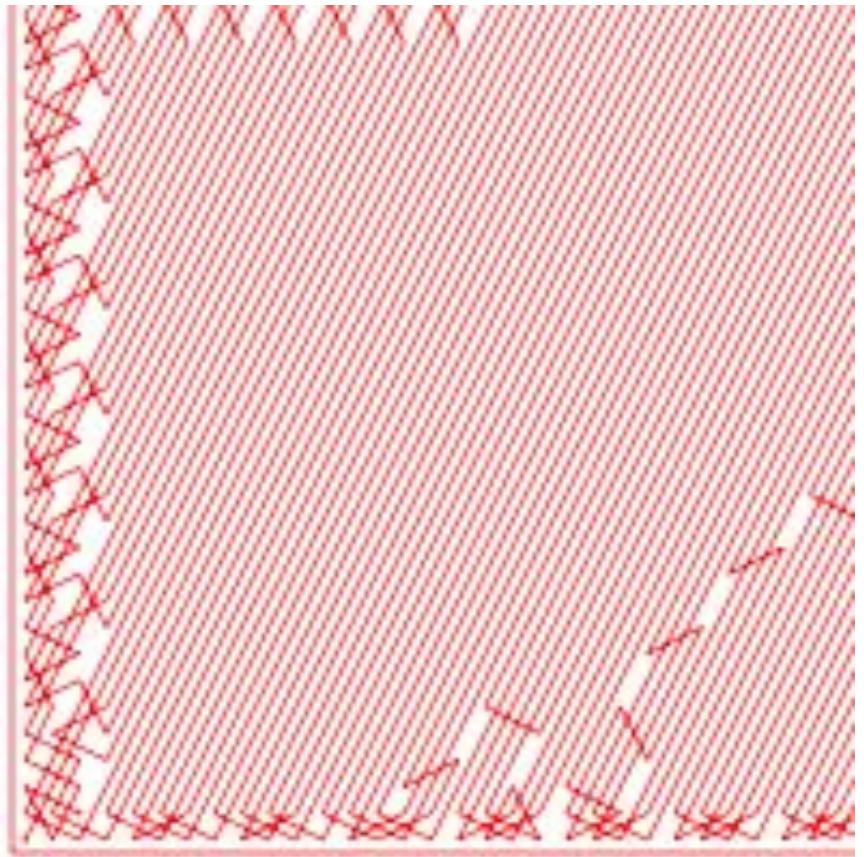
- Start and End are the same; launch multiple threads with different search orders.
- Depending on the combination of search order, it may be possible to reach the solution quickly with fewer attempts. Start and End are the same
- By pitting threads against each other in competition, solutions can be found in a shorter time. Start and End are identical; multiple threads are launched with different search orders.



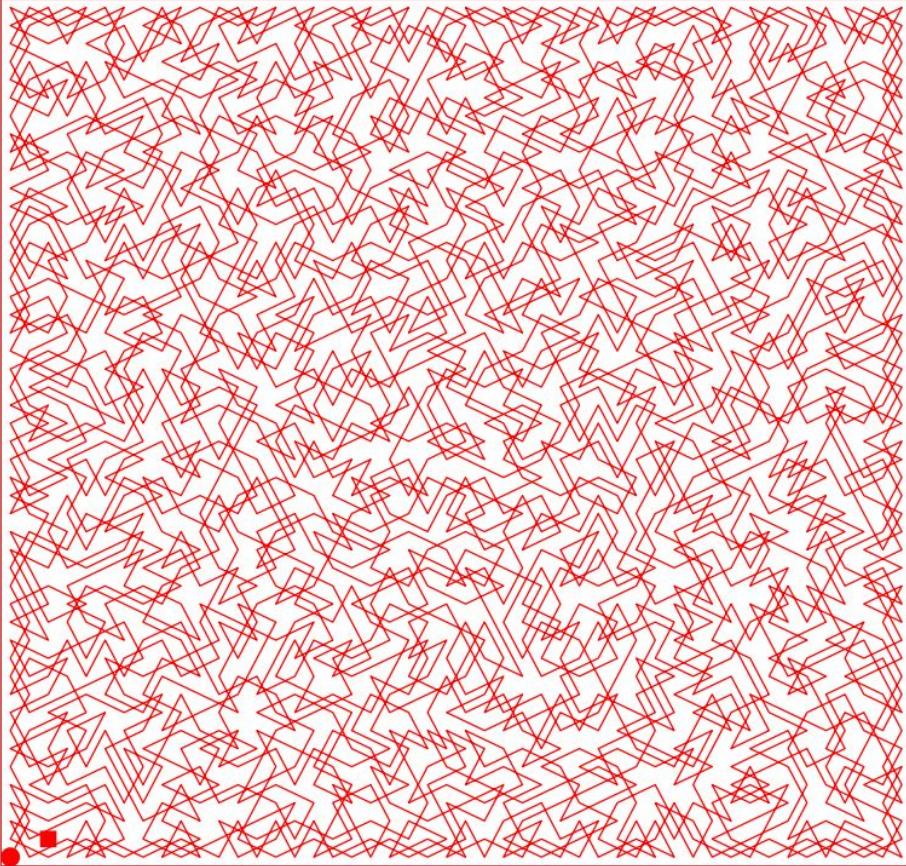
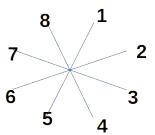
Threads enable finding large-scale solutions
Start and End are identical; multiple threa



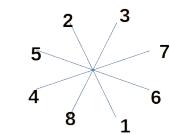
Knight Tour 48x48 start=(0,0) end=(2,1)
seed=151 try=35737 solutions=1 step=2303 Elapsed time=29m41.2948412s



Search Order Sorting 304X304 Part



Search Order Random 48x48



Conclusion

- Search Order Sorting Type 304X304 and Search Order Random Type 48X48 are, as far as we know, the largest sizes found to date, excluding those composed of smaller boards.
- Therefore, it is reasonable to assert this result as the maximum record.
- Furthermore, algorithm improvements and record updates will continue in the future.

Halfway through the journey, In the helmet, infinite.

Thank you for your kind attention.