幾何計画法の制御応用

小蔵　正輝

奈良先端大学
情報科学領域

TABLE 1
Logarithms of Numbers

1000–1500

No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d
100	00000	43	00043	44	00087	43	00130	43	00173	44	00217	43	00260	43	00303	43	00346	43	00389	43
101	00432	43	00475	43	00518	43	00561	43	00604	43	00647	42	00689	43	00732	43	00775	42	00817	43
102	00860	43	00903	42	00945	43	00988	42	01030	42	01072	43	01115	42	01157	42	01199	43	01242	42
103	01284	42	01326	42	01368	42	01410	42	01452	42	01494	42	01536	42	01578	42	01620	42	01662	41
104	01703	42	01745	42	01787	41	01828	42	01870	42	01912	41	01953	42	01995	41	02036	42	02078	41
105	02119	41	02160	42	02202	41	02243	41	02284	41	02325	41	02366	41	02407	42	02449	41	02490	41
106	02531	41	02572	40	02612	41	02653	41	02694	41	02735	41	02776	40	02816	41	02857	41	02898	40
107	02938	41	02979	40	03019	41	03060	40	03100	41	03141	40	03181	41	03222	40	03262	40	03302	40
108	03342	41	03383	40	03423	40	03463	40	03503	40	03543	40	03583	40	03623	40	03663	40	03703	40
109	03743	39	03782	40	03822	40	03862	40	03902	39	03941	40	03981	40	04021	39	04060	40	04100	39
110	04139	40	04179	39	04218	40	04258	39	04297	39	04336	40	04376	39	04415	39	04454	39	04493	39
111	04532	39	04571	39	04610	40	04650	39	04689	38	04727	39	04766	39	04805	39	04844	39	04883	39
112	04922	39	04961	38	04999	39	05038	39	05077	38	05115	39	05154	38	05192	39	05231	38	05269	39
113	05308	38	05346	39	05385	38	05423	38	05461	39	05500	38	05538	38	05576	38	05614	38	05652	38
114	05690	39	05729	38	05767	38	05805	38	05843	38	05881	37	05918	38	05956	38	05994	38	06032	38
115	06070	38	06108	37	06145	38	06183	38	06221	37	06258	38	06296	37	06333	38	06371	37	06408	38
116	06446	37	06483	38	06521	37	06558	37	06595	38	06633	37	06670	37	06707	37	06744	37	06781	38
117	06819	37	06856	37	06893	37	06930	37	06967	37	07004	37	07041	37	07078	37	07115	36	07151	37
118	07188	37	07225	37	07262	36	07298	37	07335	37	07372	36	07408	37	07445	37	07482	36	07518	37
119	07555	36	07591	37	07628	36	07664	36	07700	37	07737	36	07773	36	07809	37	07846	36	07882	36
120	07918	36	07954	36	07990	37	08027	36	08063	36	08099	36	08135	36	08171	36	08207	36	08243	36
121	08279	35	08314	36	08350	36	08386	36	08422	36	08458	35	08493	36	08529	36	08565	35	08600	36
122	08636	36	08672	35	08707	36	08743	35	08778	36	08814	35	08849	35	08884	36	08920	35	08955	36
123	08991	35	09026	35	09061	35	09096	36	09132	35	09167	35	09202	35	09237	35	09272	35	09307	35
124	09342	35	09377	35	09412	35	09447	35	09482	35	09517	35	09552	35	09587	34	09621	35	09656	35
125	09691	35	09726	34	09760	35	09795	35	09830	34	09864	35	09899	35	09934	34	09968	35	10003	34
126	10037	35	10072	34	10106	34	10140	35	10175	34	10209	34	10243	35	10278	34	10312	34	10346	34
127	10380	35	10415	34	10449	34	10483	34	10517	34	10551	34	10585	34	10619	34	10653	34	10687	34
128	10721	34	10755	34	10789	34	10823	34	10857	33	10890	34	10924	34	10958	34	10992	33	11025	34
129	11059	34	11093	33	11126	34	11160	33	11193	34	11227	34	11261	33	11294	33	11327	34	11361	33
130	11394	34	11428	33	11461	33	11494	34	11528	33	11561	33	11594	34	11628	33	11661	33	11694	33
131	11727	33	11760	33	11793	33	11826	34	11860	33	11893	33	11926	33	11959	33	11992	32	12024	33
132	12057	33	12090	33	12123	33	12156	33	12189	33	12222	32	12254	33	12287	33	12320	32	12352	33
133	12385	33	12418	32	12450	33	12483	33	12516	32	12548	33	12581	32	12613	33	12646	32	12678	32
134	12710	33	12743	32	12775	33	12808	32	12840	32	12872	33	12905	32	12937	32	12969	32	13001	32
135	13033	33	13066	32	13098	32	13130	32	13162	32	13194	32	13226	32	13258	32	13290	32	13322	32
136	13354	32	13386	32	13418	32	13450	31	13481	32	13513	32	13545	32	13577	32	13609	31	13640	32
137	13672	32	13704	31	13735	32	13767	32	13799	31	13830	32	13862	31	13893	32	13925	31	13956	32
138	13988	31	14019	32	14051	31	14082	32	14114	31	14145	31	14176	32	14208	31	14239	31	14270	31
139	14301	32	14333	31	14364	31	14395	31	14426	31	14457	32	14489	31	14520	31	14551	31	14582	31
140	14613	31	14644	31	14675	31	14706	31	14737	31	14768	31	14799	30	14829	31	14860	31	14891	31
141	14922	31	14953	30	14983	31	15014	31	15045	31	15076	30	15106	31	15137	31	15168	30	15198	31
142	15229	30	15259	31	15290	30	15320	31	15351	30	15381	31	15412	30	15442	31	15473	30	15503	31
143	15534	30	15564	30	15594	31	15625	30	15655	30	15685	30	15715	31	15746	30	15776	30	15806	30
144	15836	30	15866	31	15897	30	15927	30	15957	30	15987	30	16017	30	16047	30	16077	30	16107	30
145	16137	30	16167	30	16197	30	16227	29	16256	30	16286	30	16316	30	16346	30	16376	30	16406	29
146	16435	30	16465	30	16495	29	16524	30	16554	30	16584	29	16613	30	16643	30	16673	29	16702	30
147	16732	29	16761	30	16791	29	16820	30	16850	29	16879	30	16909	29	16938	29	16967	30	16997	29
148	17026	30	17056	29	17085	29	17114	29	17143	30	17173	29	17202	29	17231	29	17260	29	17289	30
149	17319	29	17348	29	17377	29	17406	29	17435	29	17464	29	17493	29	17522	29	17551	29	17580	29
150	17609	29	17638	29	17667	29	17696	29	17725	29	17754	28	17782	29	17811	29	17840	29	17869	29
No.	0	d	1	d	2	d	3	d	4	d	5	d	6	d	7	d	8	d	9	d

Prop. parts		
	44	43
1	4	4
2	9	9
3	13	13
4	18	17
5	22	22
6	26	26
7	31	30
8	35	34
9	40	39
	42	41
1	4	4
2	8	8
3	13	12
4	17	16
5	21	20
6	25	25
7	29	29
8	34	33
9	38	37
	40	39
1	4	4
2	8	8
3	12	12
4	16	16
5	20	20
6	24	23
7	28	27
8	32	31
9	36	35
	38	37
1	4	4
2	8	7
3	11	11
4	15	15
5	19	18
6	23	22
7	27	26
8	30	30
9	34	33
	36	35
1	4	4
2	7	7
3	11	10
4	14	14
5	18	18
6	22	21
7	25	24
8	29	28
9	32	32
	34	33
1	3	3
2	7	7
3	10	10
4	14	13
5	17	16
6	20	20
7	24	23
8	27	26
9	31	30

はじめに

非負システム

幾何計画

感染症の制御

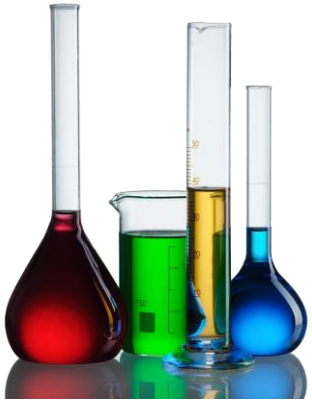
その他の例

非負システム

非負の量

確率・統計

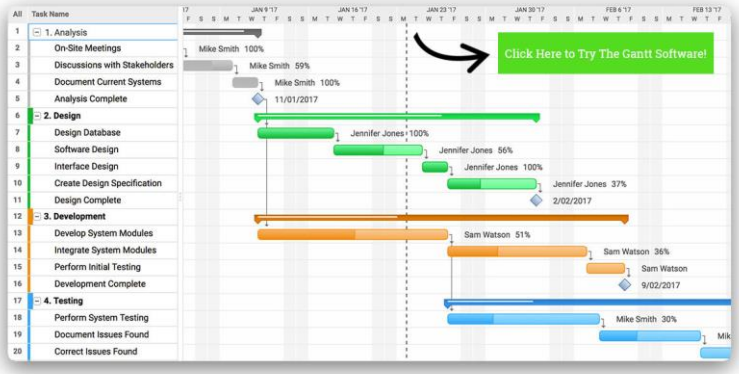
化学



経済

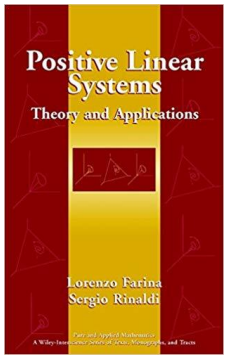
数理生物学

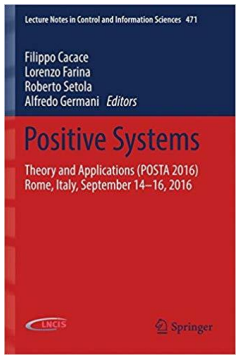
プロジェクト管理



非負システム

変数が非負である動的システム





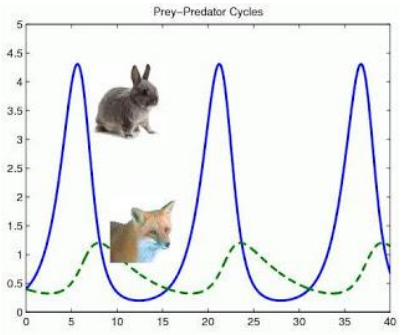
システム/制御/情報, Vol. 58, No. 12, pp. 524–525, 2014

アイ・サイ問答教室

非負システムっておもしろいの？

- Lotka-Volterra 方程式

$$\frac{dx}{dt} = \alpha x - \beta xy,$$
$$\frac{dy}{dt} = \delta xy - \gamma y,$$



- Buffer network

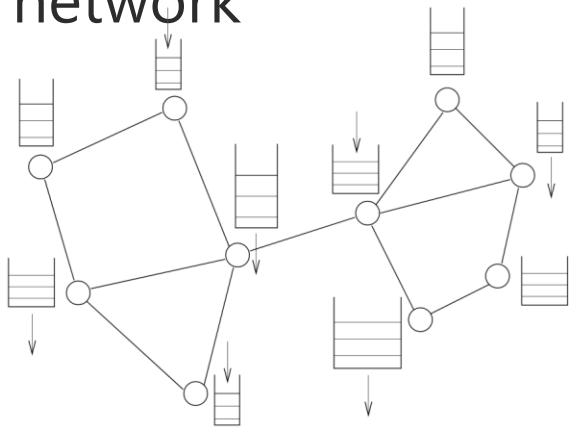
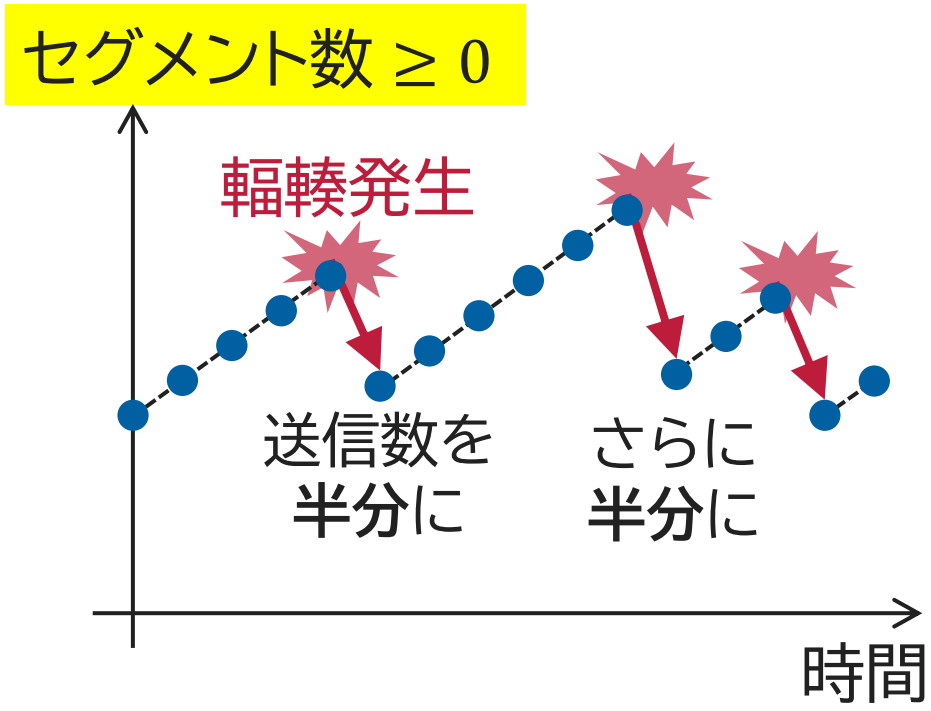


Fig. 1. Positive systems are commonly used to model dynamics of buffer networks (1). Each state represents the content of a buffer. Content can be transferred from one buffer to another via the network links. The content of a buffer can also change as a result of local production or consumption.

Rantzer, Valcher, “A tutorial on positive systems and large scale control,” *2018 IEEE Conference on Decision and Control*, 2018.

AIMDアルゴリズム

- Additive increase multiplicative decreaseの略
- Transmission Control Protocol（TCP）



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LINEAR ALGEBRA AND ITS APPLICATIONS
ELSEVIER
Linear Algebra and its Applications 393 (2004) 47–54
www.elsevier.com/locate/laa

Positive matrices associated with synchronised communication networks

Abraham Berman [a,*], Robert Shorten [b], Douglas Leith [b]

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Automatica 41 (2005) 725–730
www.elsevier.com/locate/automatica

Technical communique

Analysis and design of AIMD congestion control algorithms in communication networks☆

R.N. Shorten[1], D.J. Leith*[,1], J. Foy, R. Kilduff

616 IEEE/ACM TRANSACTIONS ON NETWORKING, VOL. 14, NO. 3, JUNE 2006

A Positive Systems Model of TCP-Like Congestion Control: Asymptotic Results

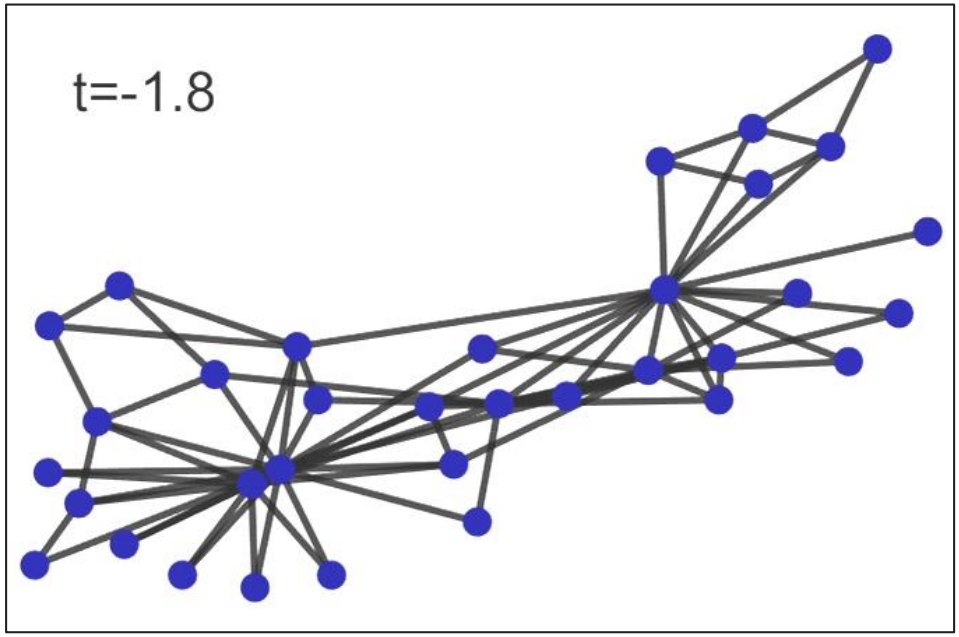
Robert Shorten, Fabian Wirth, and Douglas Leith

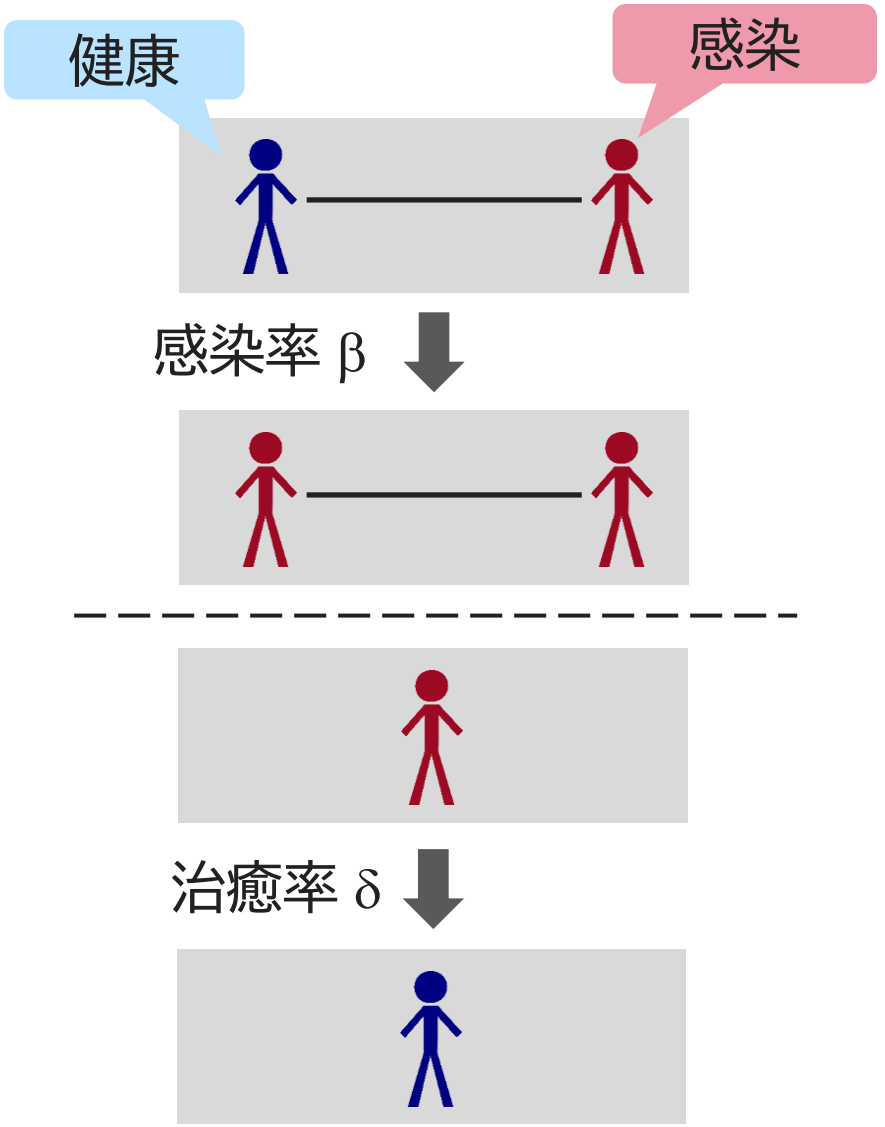
感染症の伝播

ネットワーク上のSISモデル

頂点 ＝ 個体

辺 ＝ 関係性





感染確率, 感染者数 ≥ 0

うれしいこと

$$\dot{x}(t) = Ax(t)$$

	通常のシステム
エネルギー関数	$x^\top Px$ P：正定値行列

安定性解析

$$\dot{x}(t) = Ax(t)$$

通常のシステム

- $\dfrac{dV\left(x(t)\right)}{dt}$

$$= \frac{dx^{\mathsf{T}}}{dt}Px + x^{\mathsf{T}}P\frac{dx}{dt}$$

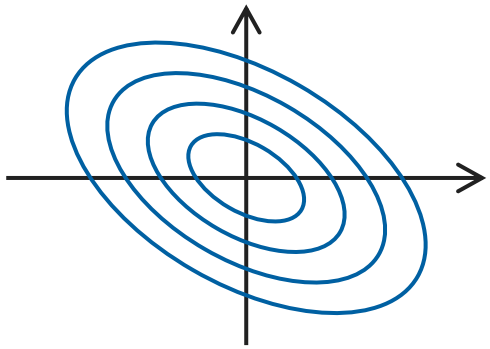
$$= (Ax)^{\mathsf{T}}Px + x^{\mathsf{T}}P(Ax)$$

$$= x^{\mathsf{T}}(A^{\mathsf{T}}P + PA)x$$

- 安定性 ⇔ $A^{\mathsf{T}}P + PA$ が負定

うれしいこと

$$\dot{x}(t) = Ax(t)$$

	通常のシステム
エネルギー関数	$x^\top Px$ P：正定値行列
等高線	
ツール	線形行列不等式

安定性解析

$$\dot{x}(t) = Ax(t)$$

通常のシステム

- $\dfrac{dV(x(t))}{dt} = \dfrac{d}{dt}(x^\top Px)$

$$= \frac{dx^\top}{dt}Px + x^\top P\frac{dx}{dt}$$

$$= (Ax)^\top Px + x^\top P(Ax)$$

$$= x^\top(A^\top P + PA)x$$

- 安定性 ⇔ $A^\top P + PA$ が負定

非負システム

- $\dfrac{dV(x(t))}{dt} = \dfrac{d}{dt}v^\top x$

$$= v^\top Ax$$

- 安定性 ⇔ $v^\top A$ が負のベクトル

線形計画によるアプローチ（for 線形システム）

ロバスト性解析

- Briat2012 (IJRNC), Chen2013 (Automatica), Shen2013 (Automatica)

切り替えシステム

- Gurvitz2007 (IEEETAC), Fornasini2010 (IEEETAC), Bolzern2010 (IEEETAC), Blanchini2012 (IEEETAC), Colaneri2014 (Automatica)

大規模結合系

- Ebihara2017 (IEEETAC)

線形計画の限界

- 非負システム

パラメータ

$$\Sigma\colon\ dx/dt = A(\theta)x$$

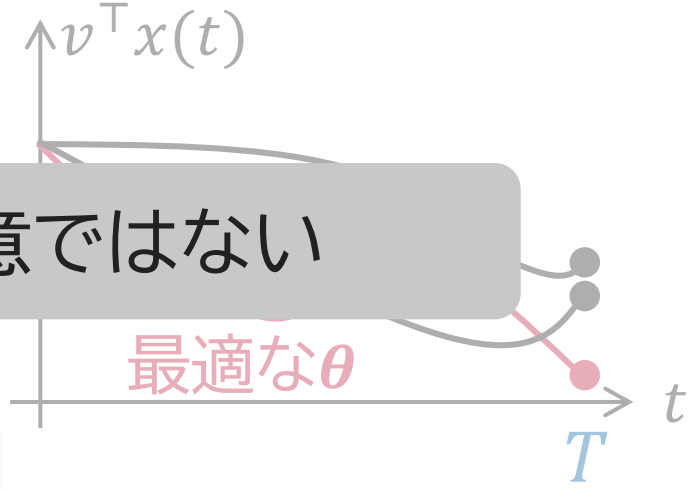
- 有限時間整定問題

整定時間

$$\text{minimize} \quad v^\top x(T)$$

$$\theta \in \Theta$$

パラメータ集合



パラメータ調整は得意ではない

- $x(T) = \exp(A(\theta)T)x$

幾何計画

幾何計画

幾何計画

Optim Eng (2007) 8: 67–127
DOI 10.1007/s11081-007-9001-7
EDUCATIONAL SECTION

A tutorial on geometric programming

Stephen Boyd · Seung-Jean Kim · Lieven Vandenberghe · Arash Hassibi

Posynomialの最適化

Monomial

$$g(x) = c\, {x_1}^{a_1} \cdots {x_n}^{a_n}$$

- 正の変数 $x_1, \ldots, x_n > 0$
- 正の係数 $c > 0$
- 任意の冪 $a_1, \ldots, a_n \in \mathbb{R}$

Posynomial

$$f(x) = (\text{monomialの有限和})$$

- 多項式の「お化け」

幾何計画問題

Posynomial

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & f_i(x) \le 1 \ \ (i = 1, \dots, p) \\ & g_j(x) = 1 \ \ (j = 1, \dots, q) \end{aligned}$$

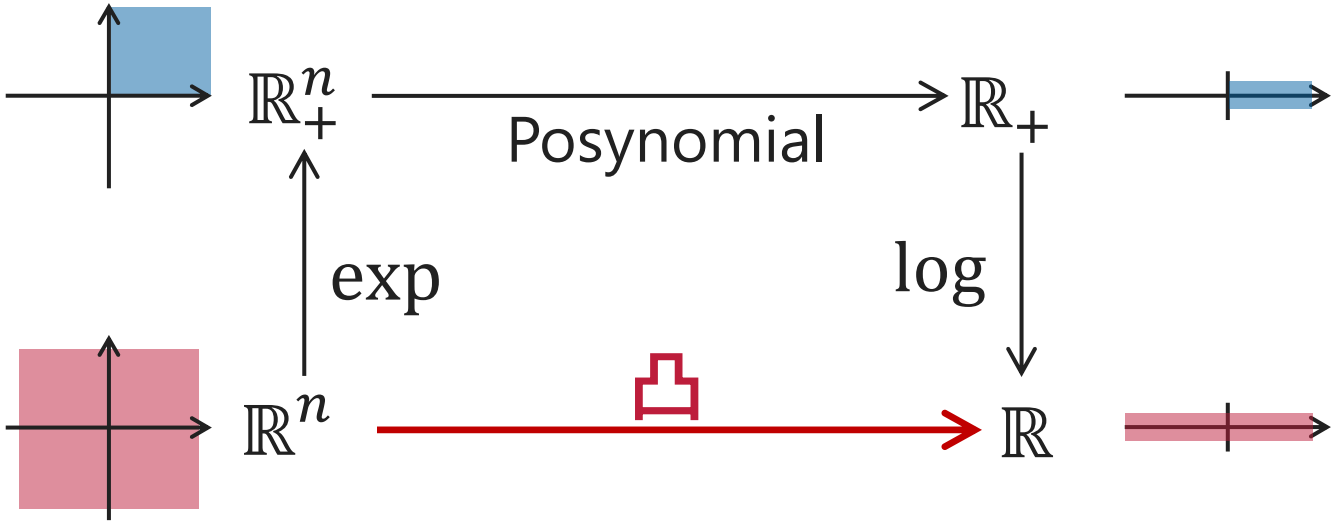
Monomial

- 例

$$\begin{aligned} \text{minimize} \quad & x^{-1}y^{-1/2}z^{-1} + 2.3xz + 4xyz \\ \text{subject to} \quad & (1/3)x^{-2}y^{-2} + (4/3)y^{1/2}z^{-1} \le 1, \\ & x + 2y + 3z \le 1, \\ & (1/2)xy = 1, \end{aligned}$$

- 多項式計画の「お化け」
- 凸最適化問題に帰着

対数領域での凸性



対数変換

幾何計画 **凸最適化**

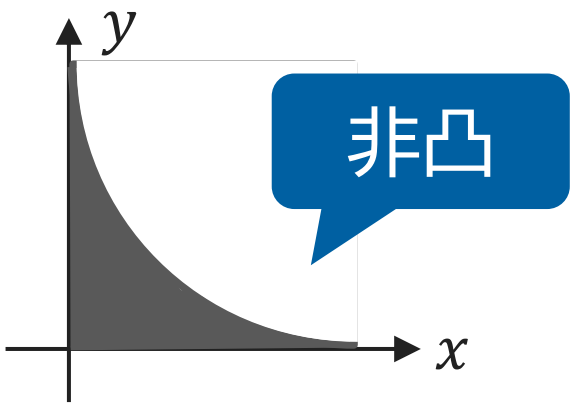
（指数変換）

例

- 幾何計画問題

minimize $x^{-1/3}y^{-1/4}$

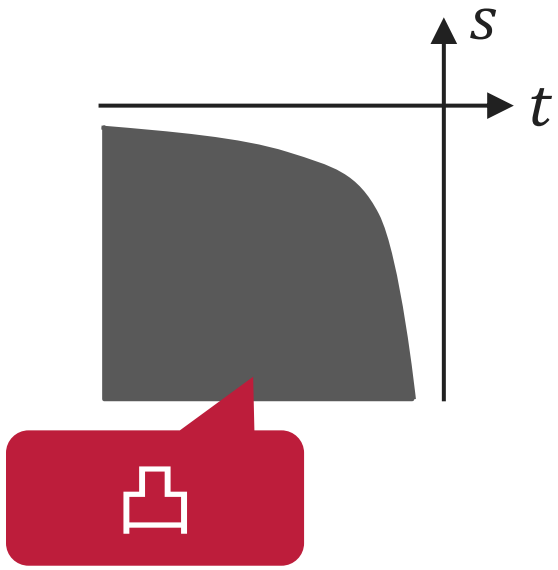
subject to $x^{1/2} + y^{1/2} \leq 1$



- 変数変換: $x = e^t, y = e^s$

minimize $e^{\left(-\frac{t}{3}-\frac{s}{4}\right)}$

subject to $e^{t/2} + e^{s/2} \leq 1$



- 等価な凸最適化問題

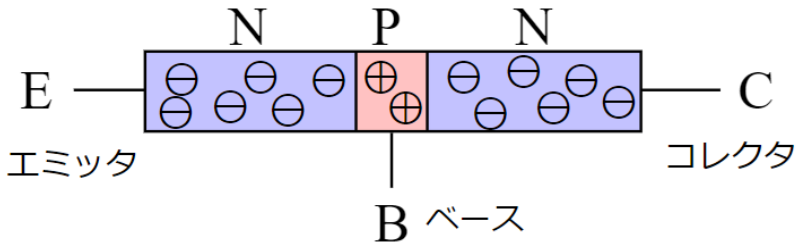
minimize $-\frac{t}{3} - \frac{s}{4}$

subject to $s \leq 2\log(1 - e^{t/2})$

応用

トランジスタ設計

- ベースへの不純物添加
- ベース走行時間の最小化



デジタル回路設計

- Gate sizing
- Wire sizing
- 歩留まり最適化

情報理論

- Channel capacity problem
- Optimal permutation codes

非負システムに使えないか？

有限時間整定問題

- 非負システム

$$dx/dt = \begin{bmatrix} a & b \\ c & d \end{bmatrix} x$$

- 有限時間整定問題

対数領域で凸

$$\begin{aligned} &\text{minimize} && v^\top x(T) \\ &\text{subject to} && a, b, c, d \geq 0 \end{aligned}$$

- $v^\top x(T) = v^\top \exp\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} T\right) x_0$

Posynomial

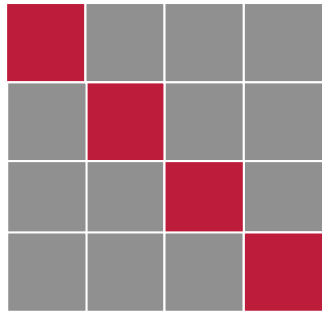
ロバスト制御問題

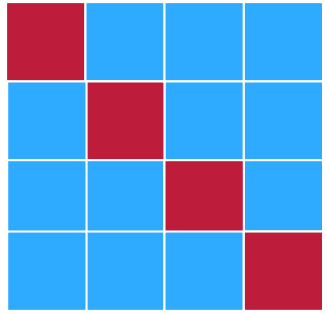
入出力のある非負システム

$$\Sigma: \begin{cases} dx/dt &= A(\theta)x + B(\theta)u \\ y &= C(\theta)x + D(\theta)u \end{cases}$$

$A(\theta)$の対角要素に関する凸性

- 構造的安定性：Colombino2016 (IEEETAC)
- H^2, H^∞ノルム：Dhingra2018 (IEEETCNS)





全ての要素に関する対数凸性 *

* O, Kishida, Lam, “Geometric programming for optimal positive linear systems”, *under preparation*, 2019

感染症の抑え込み

O, Harada, “Resource allocation for containing epidemics from temporal network data,” in *23rd International Symposium on Mathematical Theory of Networks and Systems*, 2018

感染症

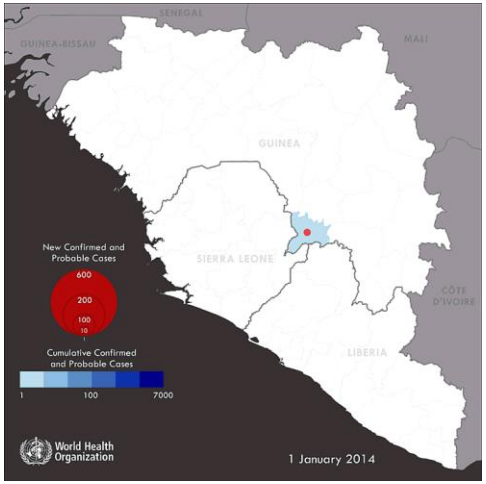
アテネの疫病 429–427 BC

Michiel Sweerts: Plague in an Ancient City

アントニヌス疫病

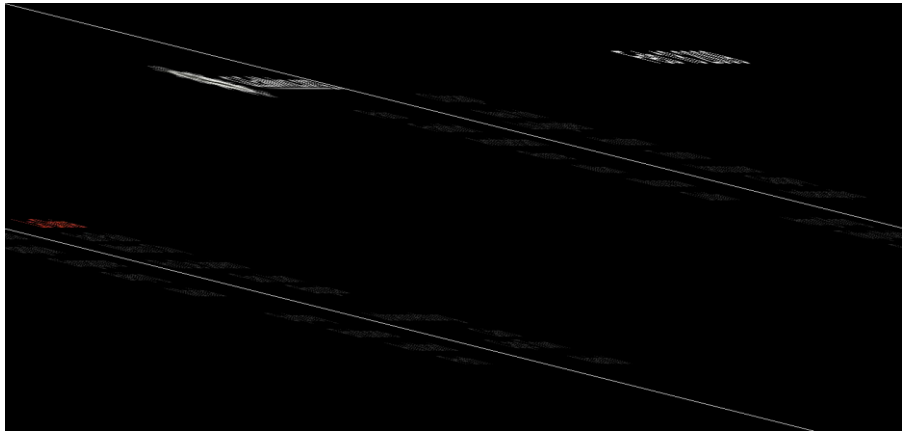
The angel of death striking a door during the plague of Rome. Engraving by Levasseur after J. Delaunay.

エボラ出血熱 2014



World Health Organization,
http://www.who.int/csr/disease/ebola/maps/en/

H1N1インフルエンザ 2009



ネットワーク科学

NetSci®

International School and Conference on Network Science

	WED, June 13		THU, June 14		
8.30am	Opening		8.15-8.45am	Chat with the experts	
9am	**Fil Menczer** - The spread of misinformation in social media [PLOS Lecture]	Chair: A. Barrat	9am	**Mason Porter** - Centrality in Time-Dependent Networks [Springer Complexity Lecture]	Chair: Y. Moreno
9.45am	**Miriah Meyer** – Designing effective visualizations		9.45am	**Sonia Kéfi** - Diversity of interaction types and the functioning of ecological communities	
			10.15am	**Sam Scarpino** - Network heterogeneity induces entropy barriers in social contagion	
10.30-11.15am	*Coffee break in the poster area*		10.45-11.30am	*Coffee break in the poster area*	
11.15am	Epidemics-1 Theory-1 Social systems-1 Brain-1 Structures-1		11.30am	Epidemics-3 Theory-3 Social systems-3 Biology Economics-1	
1-2.30pm	*Lunch break in the poster area*		1-2.30pm	*Lunch break in the poster area*	*#ShareYourCode* twitter chat with @PLOSONE
2.30pm	Epidemics-2 Theory-2 Social systems-2 Ecology Structures-2		2.30pm	Lightning talks	
4.30-5.15pm	*Coffee break in the poster area*		3.35-3.50pm	*Short break*	
5.15pm	**Brooke Foucault Welles** - Network Science, Activism, and Social Change: The Rise of Networked Counterpublics	Chair: F. Menczer	3.50pm	**Erdős-Rényi Award**	
5.45pm	**Stefano Battiston** - The price of complexity in financial networks [Chaos, Solitons & Fractals Lecture]		5.10-6.15pm	*Coffee break in the poster area*	
			5.30-6.15	*Scientific publishing and communication* – Federico Levi (Nature Physics)	*Society Board Meeting*
6.15-7pm	Poster Session 1		6.15-7pm	Poster Session 2	

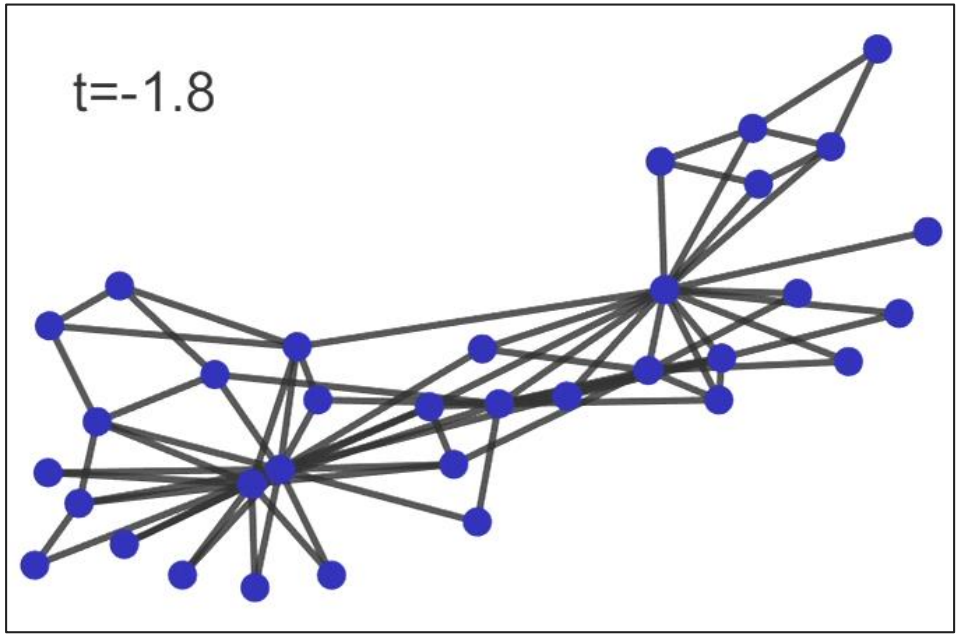
	the network of cattle movements in Great Britain to understand a complex multi-host pathogen system	Chair: V. Colizz
9.45am	**Claudia Wagner** - Minorities in social networks	
10.15am	**Amy Wesolowski** - Understanding the role of human connectivity on the spatial dynamics of malaria	
10.45-11.30am	*Coffee break in the poster area*	
11.30am	Spread-1 Theory-4 Social systems-4 Theory-5 Economics-2	
1-2.30pm	*Lunch break in the poster area*	
1.30-2.15	*Ask me anything* w/ Physical Review editors	
2.30pm	Epidemics-4 Spread-2 Social systems-5 Brain-2 Structures-3	
4.15-5pm	*Coffee break in the poster area*	
5pm	**Brenda McCowan** - Characterizing social stability and its effects on individual and societal health using network dynamics	Chair: M. Porter
5.45	**Sophie Achard** - Assessing reliability of resting-state fMRI graph analysis: challenges in measuring brain connectivity networks alterations for clinical applications	
6.15-6.30pm	Awards & Closure	

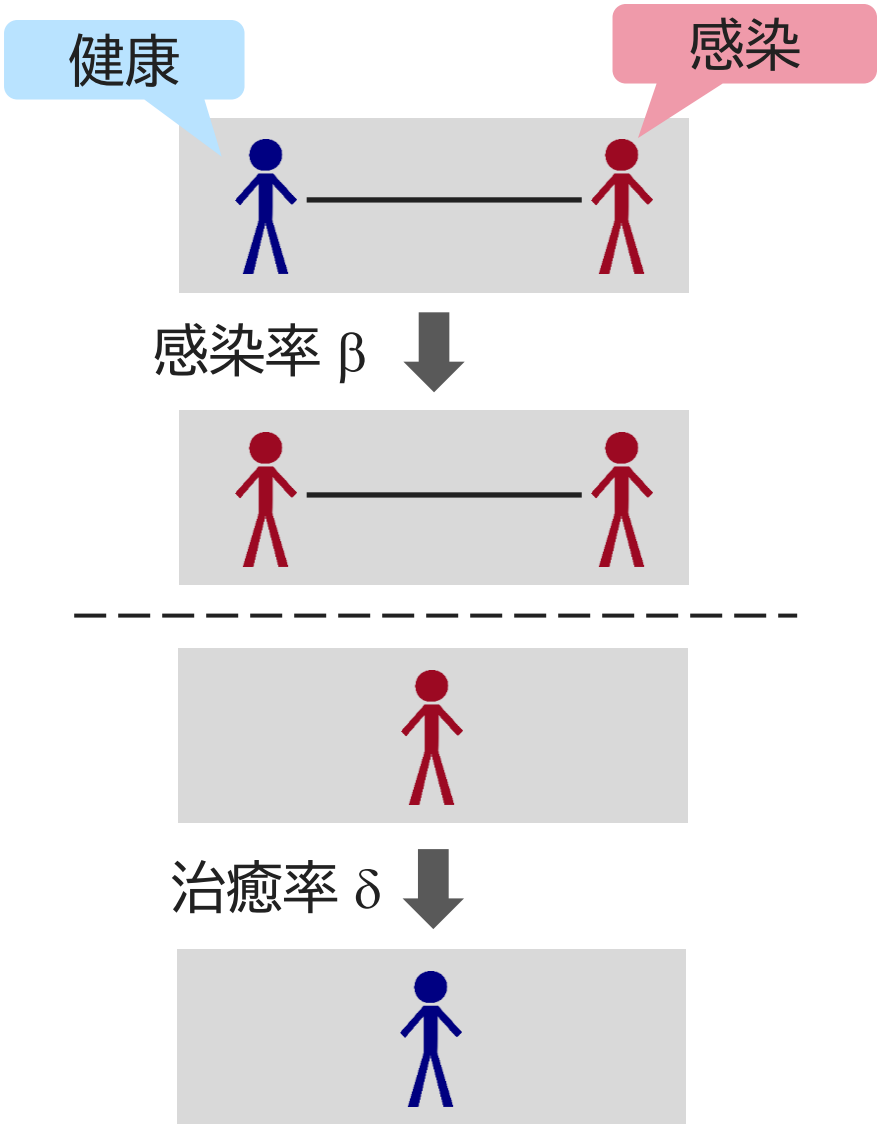
ネットワーク上のSISモデル

グラフ＋確率過程

頂点 ＝ 個体

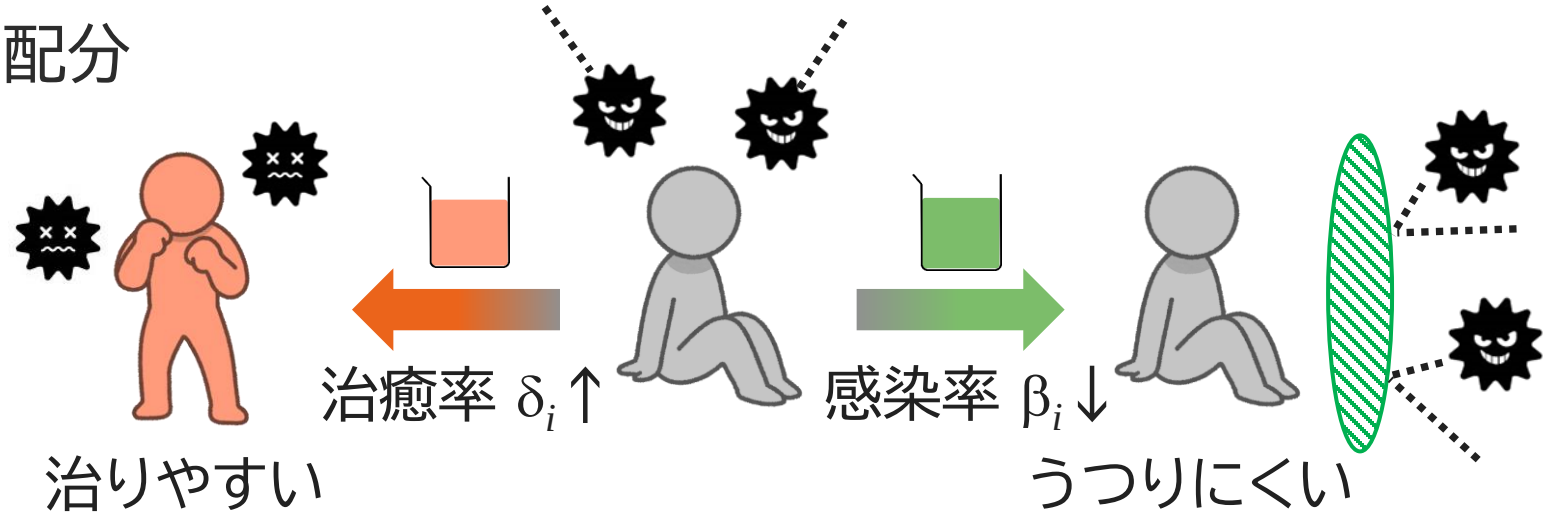
辺 ＝ 関係性





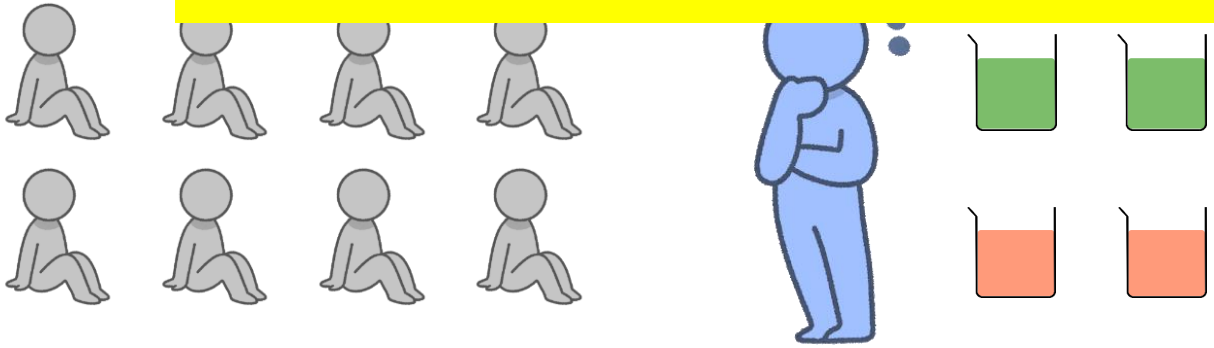
資源配分問題

薬を配分



薬は有限

仮定：ネットワーク構造が固定

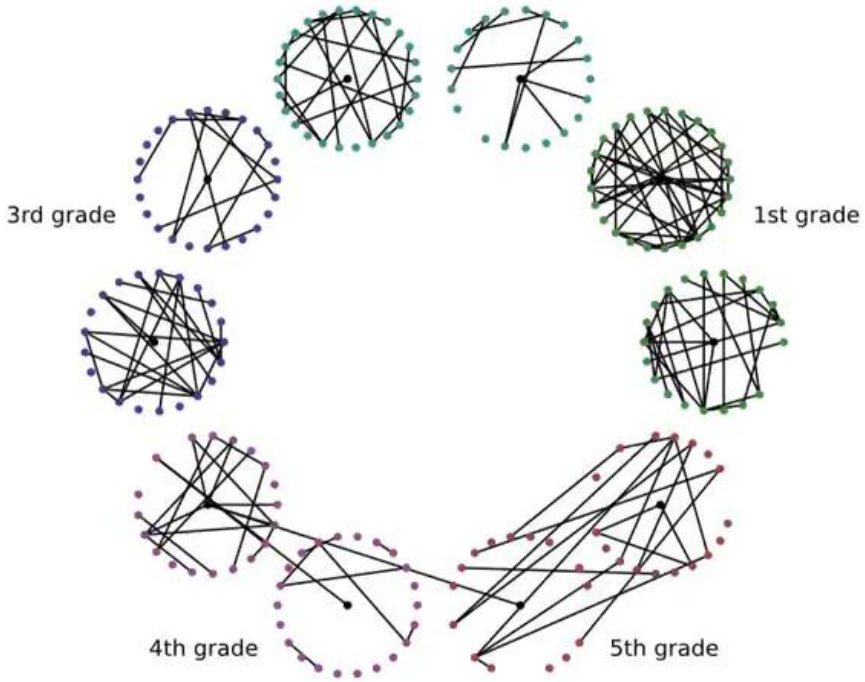


Preciado et al., “Optimal resource allocation for network protection against spreading processes,” *IEEE Transactions on Control of Network Systems*, 2014.

ノード＋枝＋時間

技術の進展→データの取得が容易に

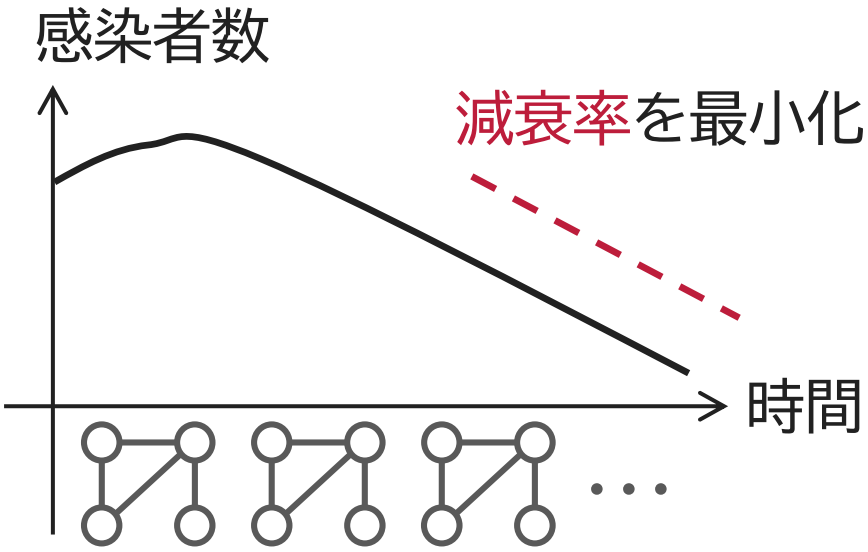
例（フランスの小学校）



J. Stehl'e *et al.*, "High-resolution measurements of face-to-face contact patterns in a primary school," *PLoS ONE*, 2011.

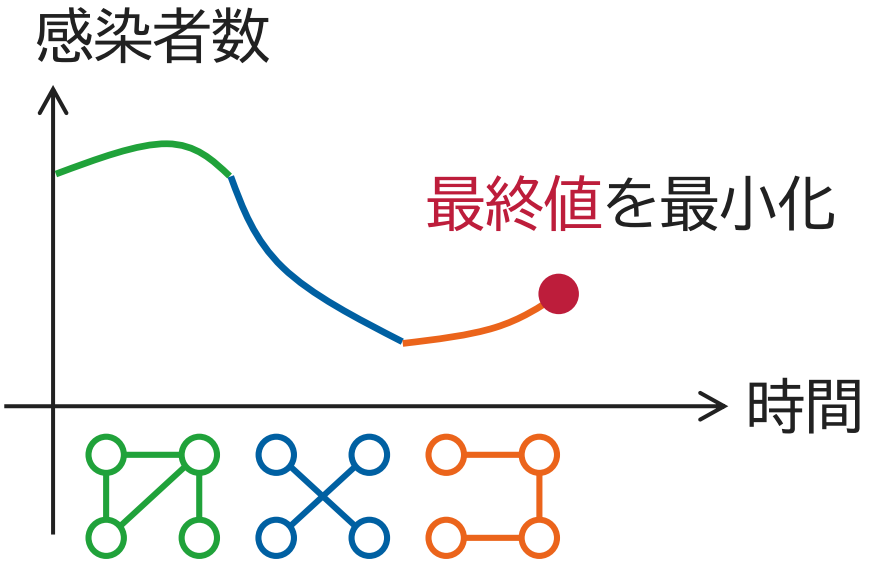
幾何計画へ

静的ネットワーク



- 漸近安定化問題
- 評価指標＝最大固有値

テンポラルネットワーク

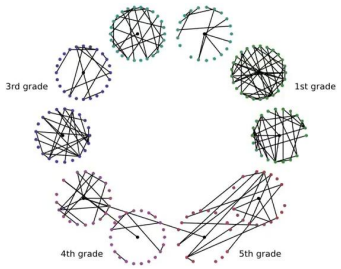


- 有限時間整定問題！
- 幾何計画に帰着

シミュレーション

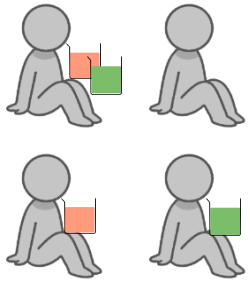
- 比較

テンポラル
ネットワーク



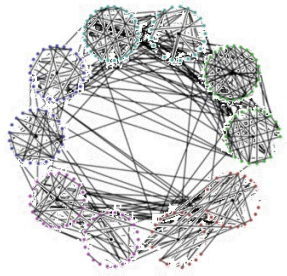
提案法

医療資源の配分



時間情報の
集約

静的ネットワーク

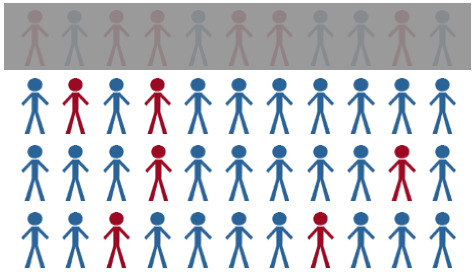


従来法

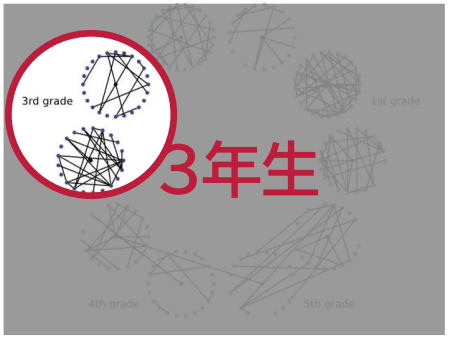
- 追加感染者数を最小化

9:00

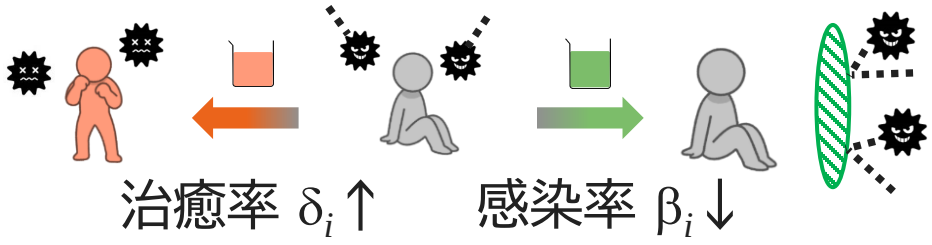
17:40



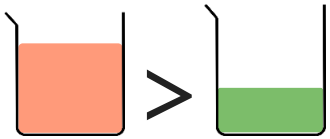
- 3年生に限定



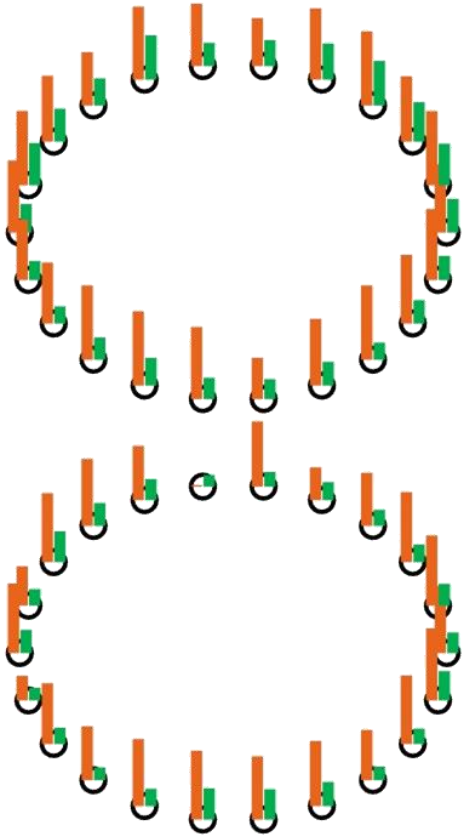
最適資源配置



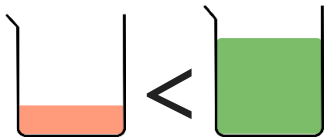
従来法



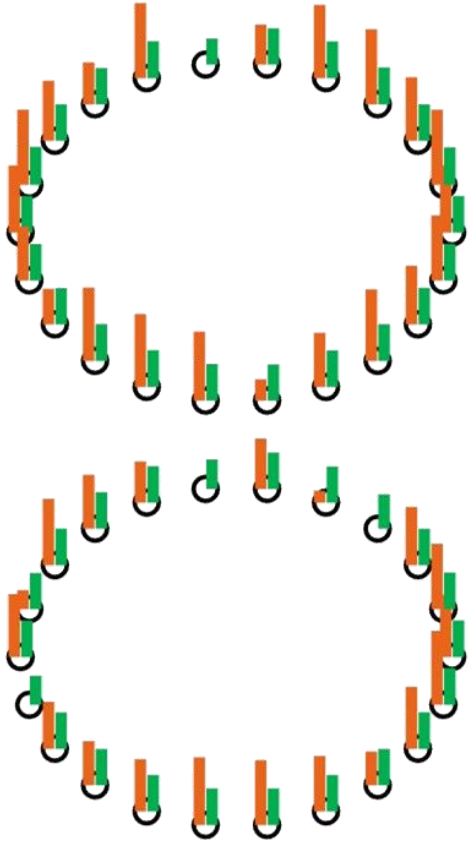
治癒を促進



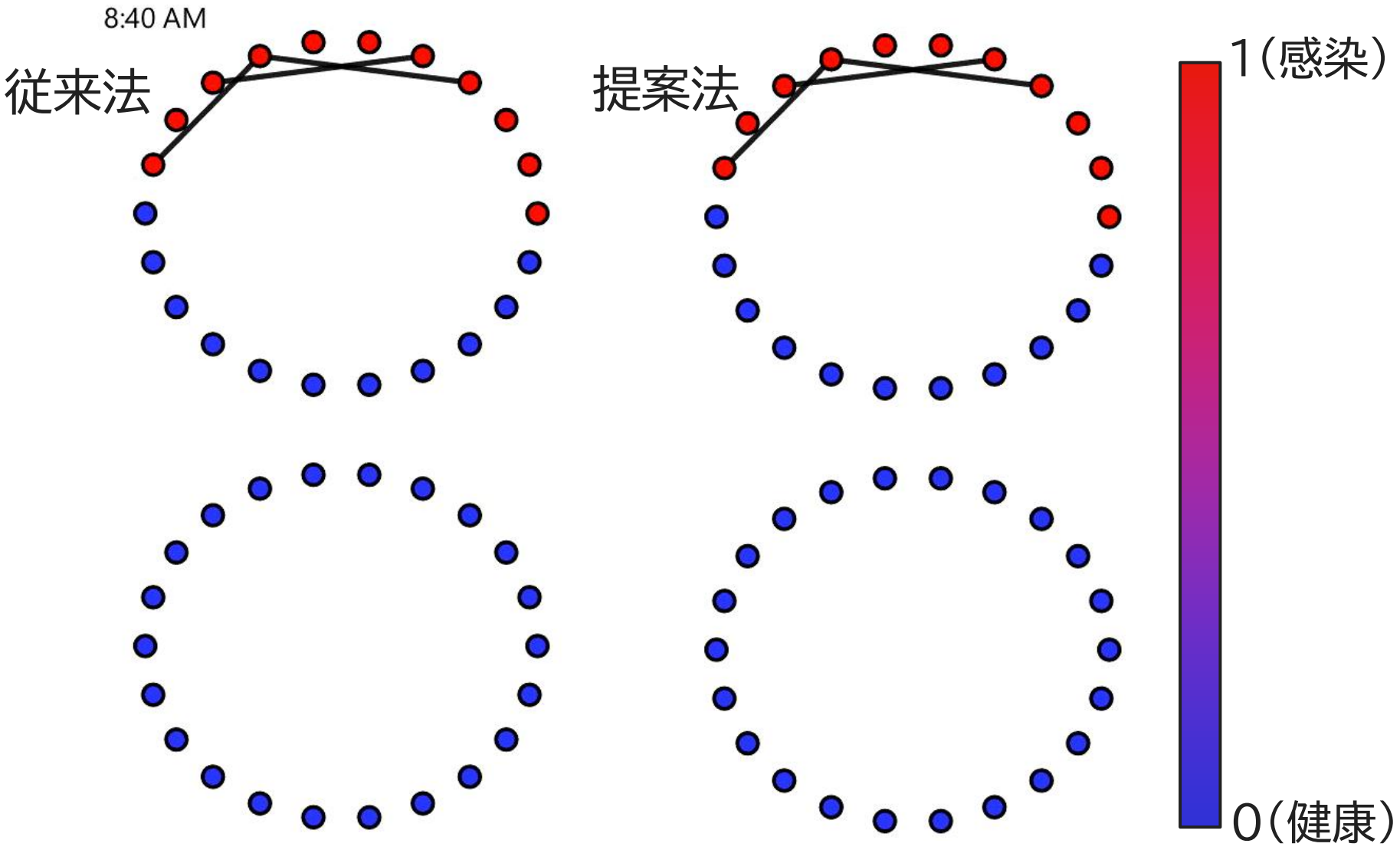
提案法



伝染を防止



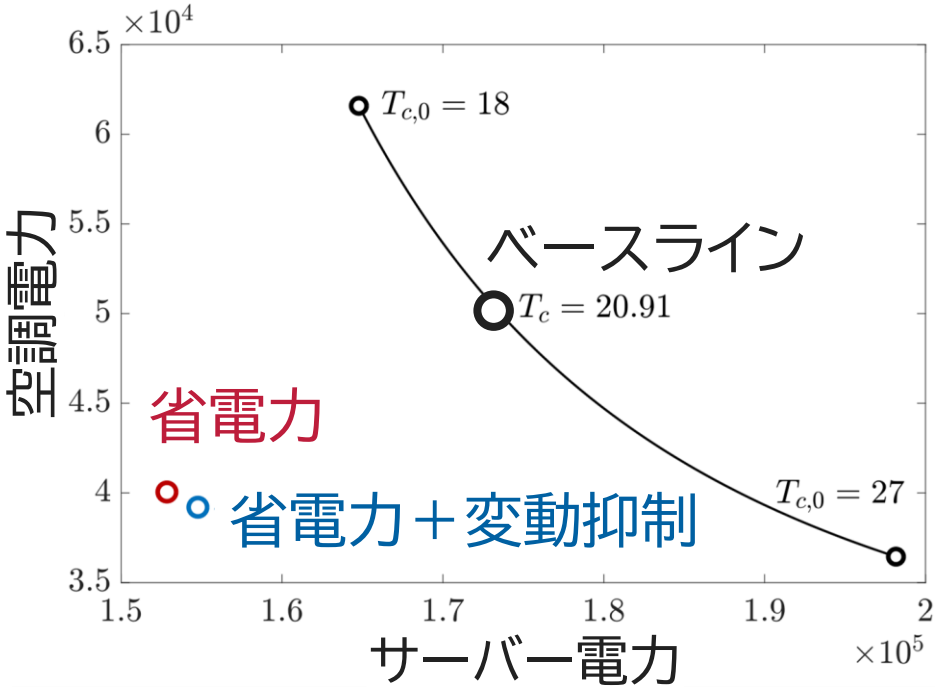
感染確率のシミュレーション

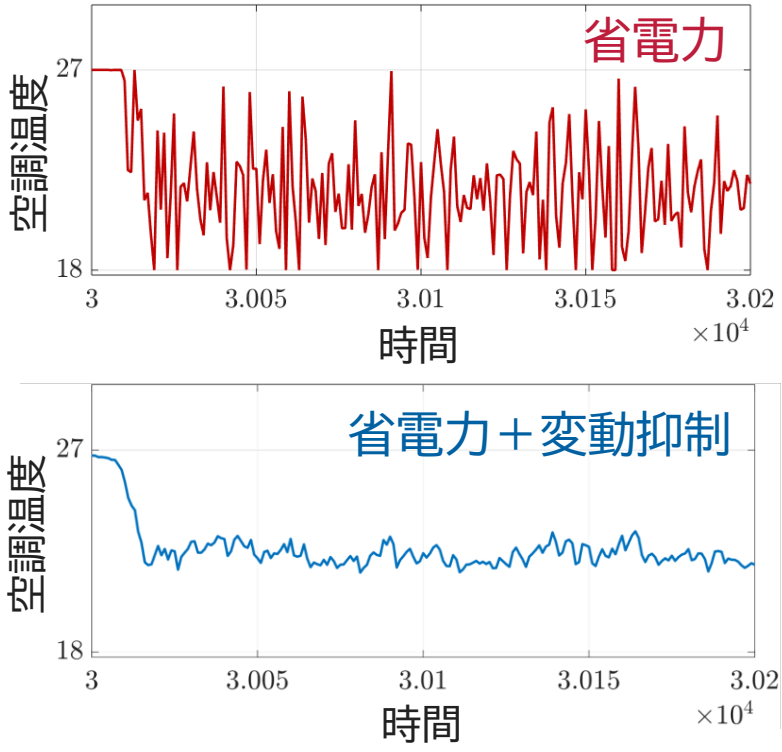


その他の例

データセンターの省電力化

- 空調とサーバーの別個制御が一般的
- 空調温度と可動サーバー数を幾何計画で同時最適化
- ベースラインから18%の消費電力削減





O, Wan, Kasahara, “Model predictive control for energy-efficient operation of data centers with cold aisle containments,” in *6th IFAC Conference on Nonlinear Model Predictive Control*, 2018.

分散送信電力制御アルゴリズムのロバスト性

- 3G通信の電力制御アルゴリズム
- 干渉ゲインの不確定性への頑健性は未解決課題

ロバスト安定性解析

$$\begin{bmatrix} 0 & g_{12} & g_{13} \\ g_{21} & 0 & g_{13} \\ g_{31} & g_{32} & 0 \end{bmatrix} + E \Delta F$$

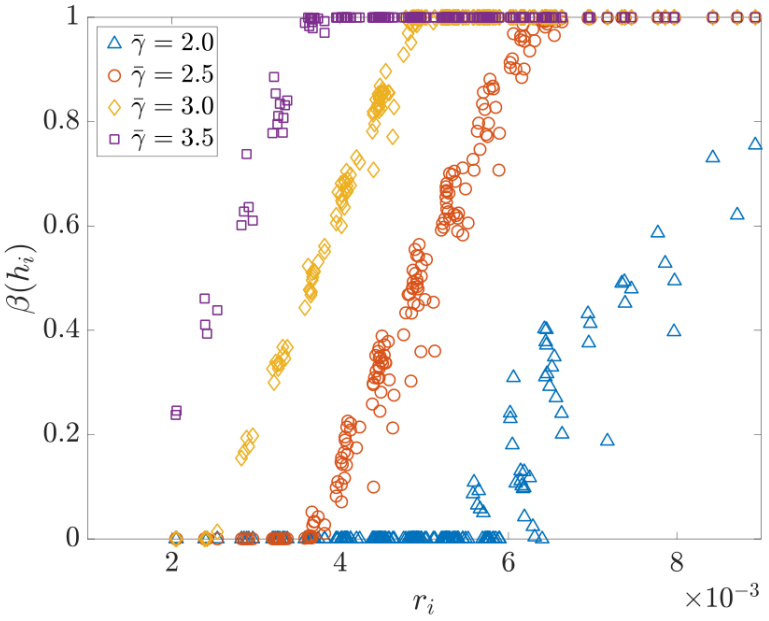
不確かさの構造

不確かさの構造

ノミナルな干渉ゲイン

不確かさの原因
ノルム ≤ 1

ロバスト安定化



資源配置とPageRankの関係

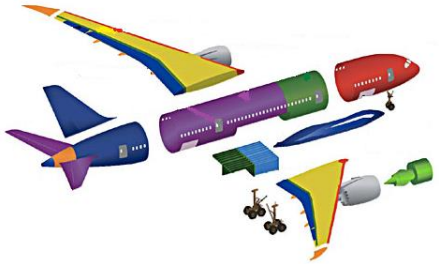
O, Kishida, Hayashi, Lam, “Resource allocation for robust stabilization of Foschini-Miljanic Algorithm,” in *2019 American Control Conference* (accepted), 2019.

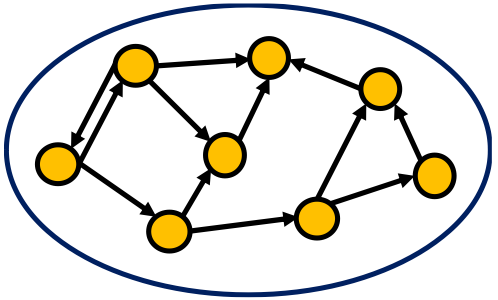
製品開発のマネジメント

- 相互依存しあう開発コンポーネント
- 投資戦略:コンポーネント vs 依存性
- 既存の手法: 経験則に依存

✔ 予算を考慮した最適資源配置

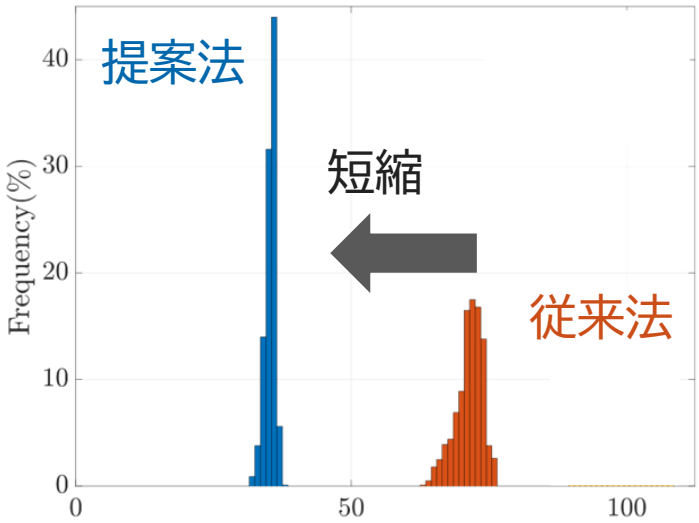
✔ 設計完了時間を最小化





依存関係のネットワーク

設計完了時刻の分布



O, Harada, Kishida, Yassine, “Resource optimization of product development projects with time-varying dependency structure,” submitted to *Research in Engineering Design*

まとめと展望

「静的」な写像
通信・デジタル回路・
情報理論...

ダイナミクスを含む
写像→制御系設計