State-feedback stabilization of Markov jump linear systems with randomly observed Markov states

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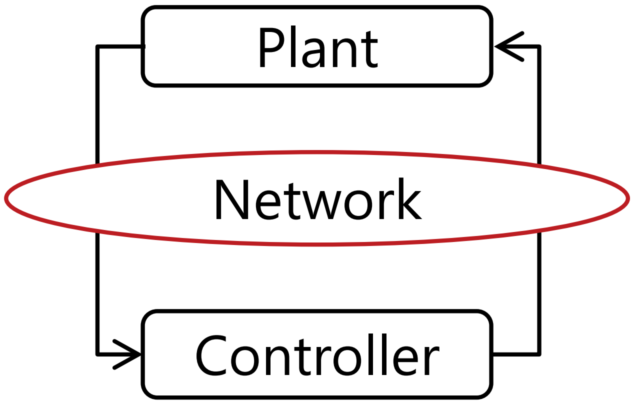
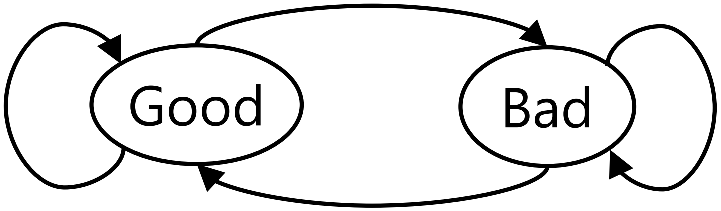
Markov jump linear systems

$$x(k+1) = A_{r(k)}x(k) + B_{r(k)}u(k)$$

- r: time-homogeneous Markov chain (Markov state)
- Example: Networked control system



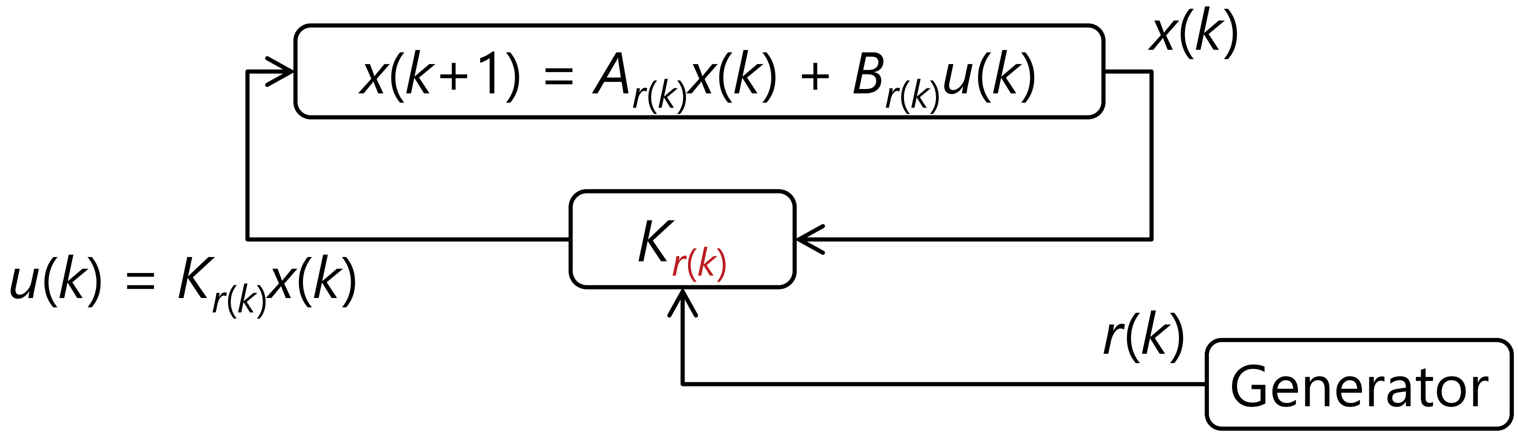
Gilbert-Elliot channel



- Feedback control
  - Need to measure both the state and the Markov state

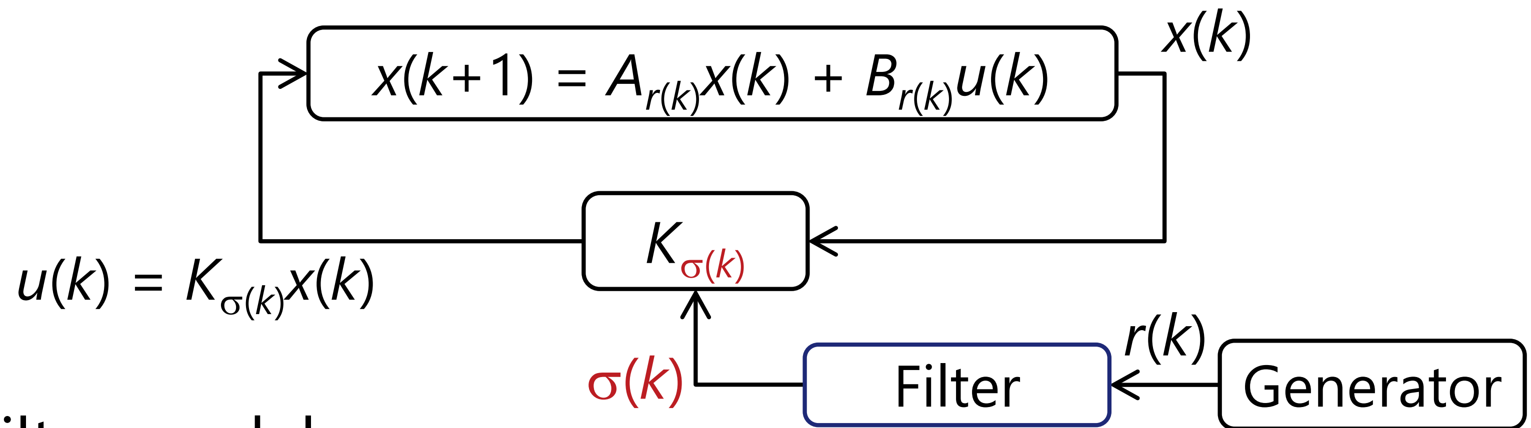
State-feedback control

- If the controller knows the Markov state:



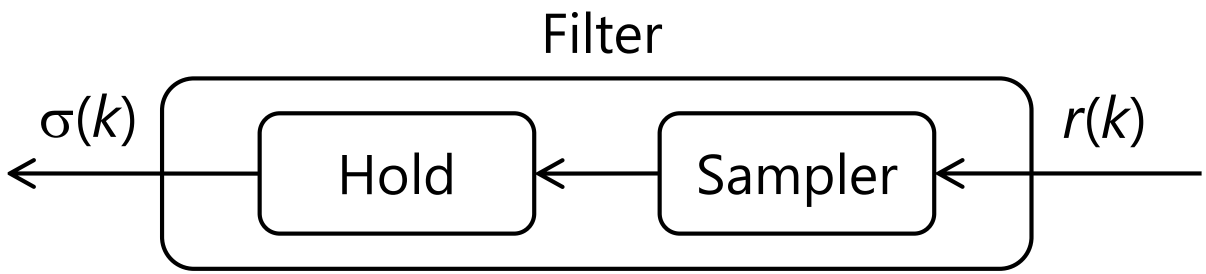
- Well-developed theory
  - Stabilization, H^2 control, H^∞ control, … [Costa et al. '05]
  - Linear matrix inequalities
- Can we always measure the Markov state?

Observation through filters

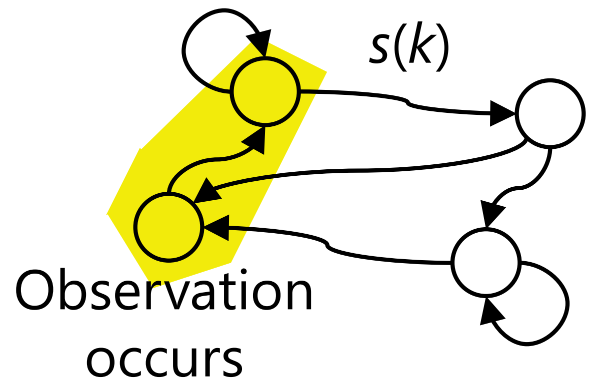


- Filter models
  - Deterministic case
    - Cluster observation [Val et al. '02], Periodic observation [Cetinkaya & Hayakawa '15]
  - Stochastic case
    - Unreliable observation [Costa et al. '15]
  - Deterministic + Stochastic?
    - Periodic observation with probabilistic failures

General filter model



- Sampler
  - has its own Markov chain
  - samples *when and only when* the chain hits a subset



- Examples

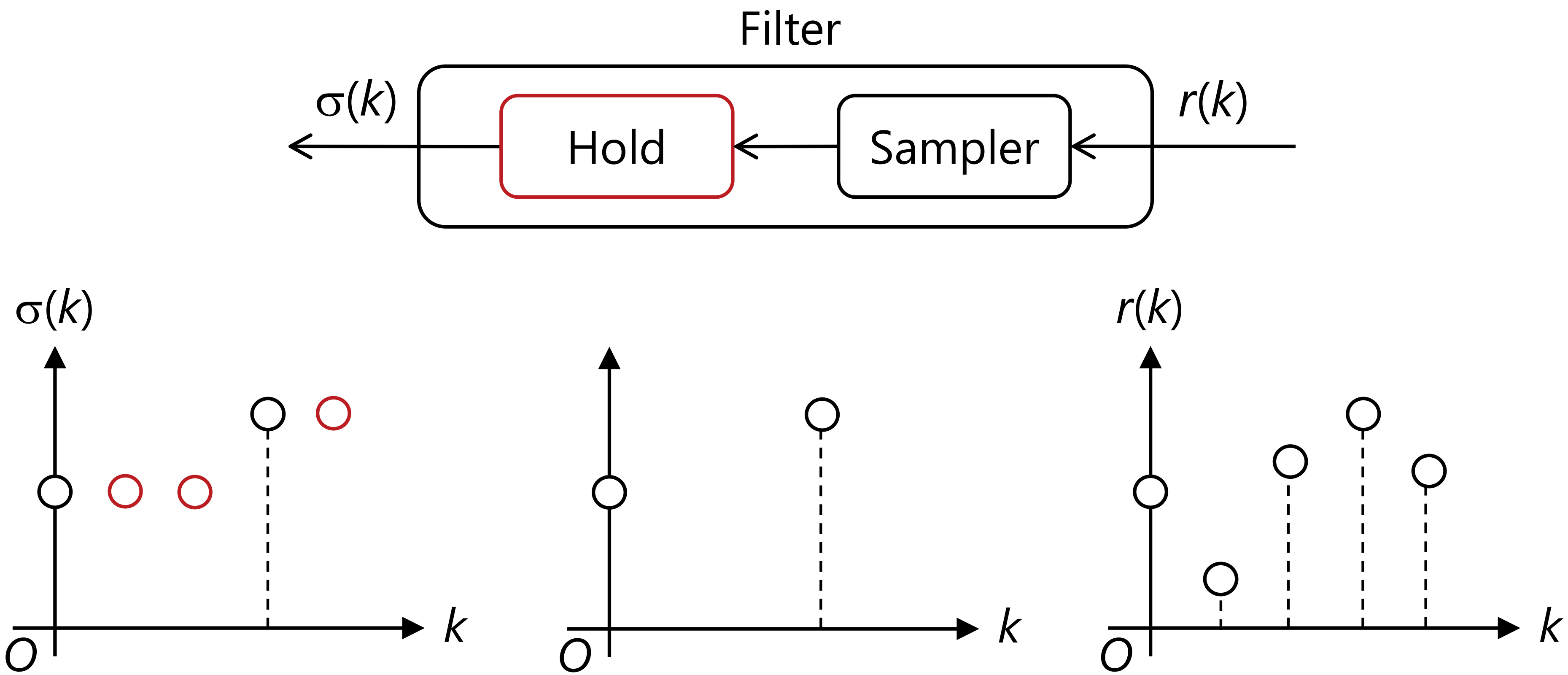
$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Sampling with period three

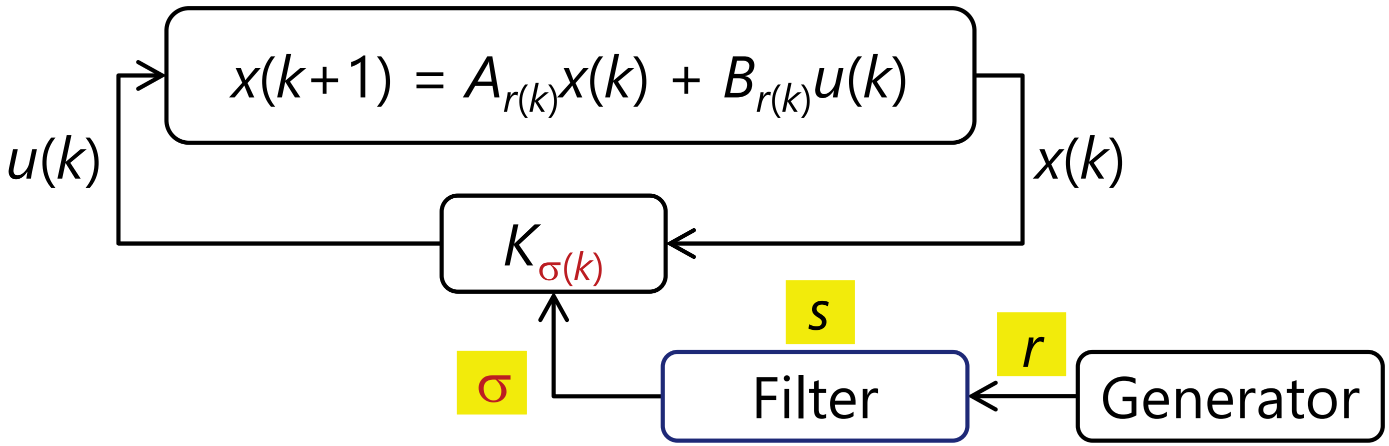
$$\begin{bmatrix} p & 1-p \\ p & 1-p \end{bmatrix}$$

Sampling with success probability p

General filter model

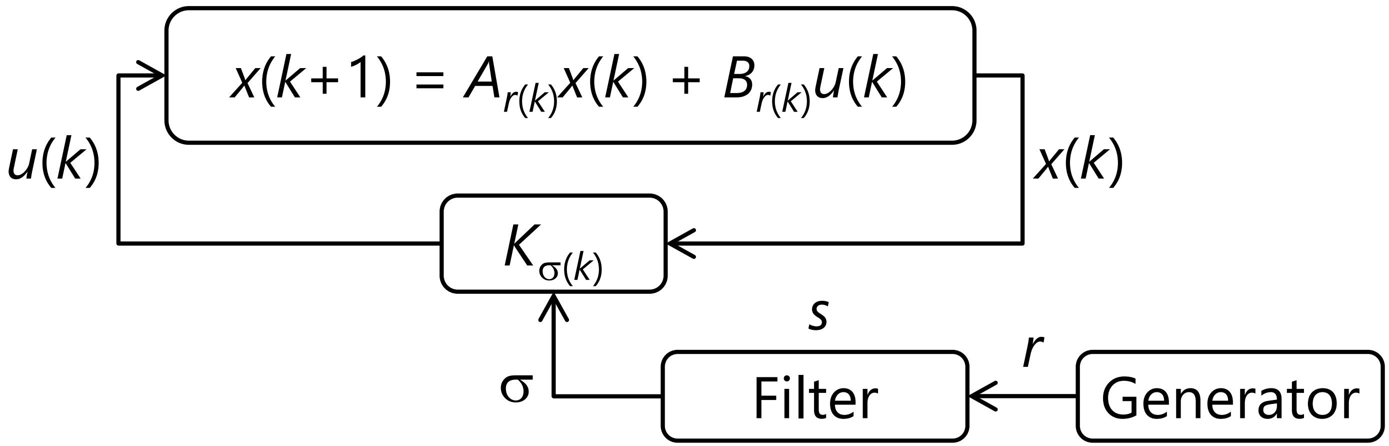


State-feedback control (cont'd)

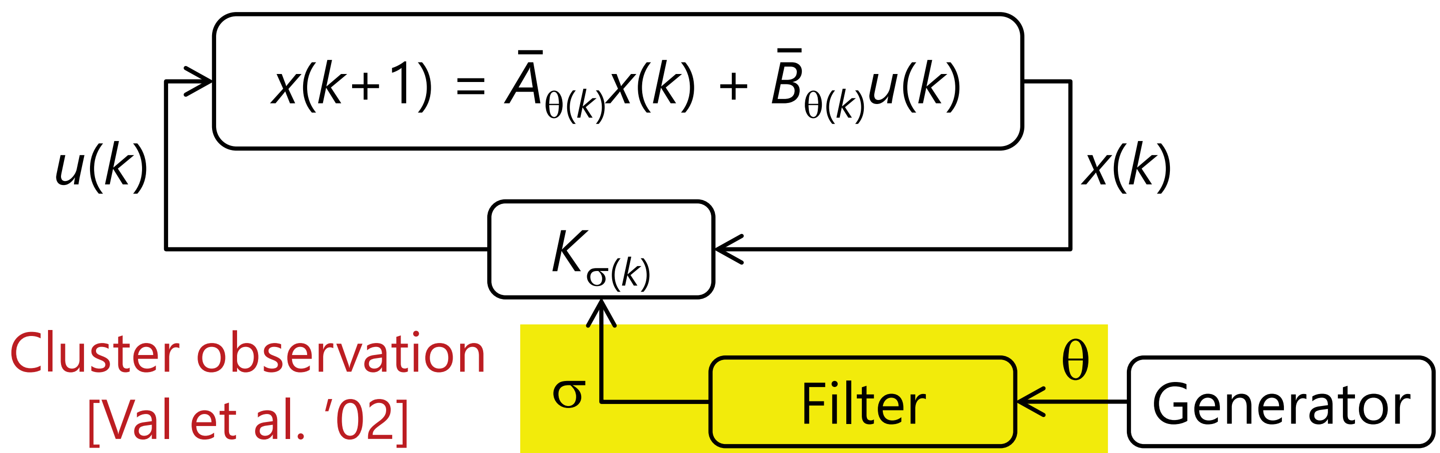


- σ is not a Markov chain
  - cannot use the theory of Markov jump linear systems
- But, the triple (r, s, σ) is a Markov chain
  - Extended Markov chain
  - Embed the closed-loop system into another Markov jump linear system with a big mode space

Equivalent system



$\theta = (r, s, \sigma)$, $\bar{A}_\theta = A_r$, $\bar{B}_\theta = B_r$



Stabilization

- **Theorem** Assume that the matrices $R_{\alpha,\beta,\gamma}$, G_γ, and F_γ satisfy the linear matrix inequalities

$$\begin{bmatrix} R_{\alpha,\beta,\gamma} & A_\alpha G_\gamma + B_\alpha F_\gamma \\ G_\gamma^\top A_\alpha^\top + F_\gamma^\top B_\alpha^\top & G_\gamma + G_\gamma^\top - \mathcal{D}_{\alpha,\beta,\gamma}(R) \end{bmatrix} > 0$$

$$\mathcal{D}_{\alpha,\beta,\gamma}(R) = \sum_{\alpha',\beta',\gamma,} \bar{p}_{(\alpha',\beta',\gamma'),(\alpha,\beta,\gamma)} R_{\alpha',\beta',\gamma'}$$

transition prob. of extended Markov chain

Define the feedback gains by

$$K_\gamma = F_\gamma G_\gamma^{-1}$$

Then, $E[||x(k)||^2]$ converges to zero exponentially fast.

  - Proof: Use the idea in cluster observation [Val et al. '02]

Example

- Markov jump linear system with 3 modes

$$A_1 = \begin{bmatrix} -0.45 & -0.3 \\ 1.2 & 0.45 \end{bmatrix}, \; A_2 = A_3 = \begin{bmatrix} -0.7 & 0.7 \\ 0.2 & 0.8 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \; B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; B_3 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

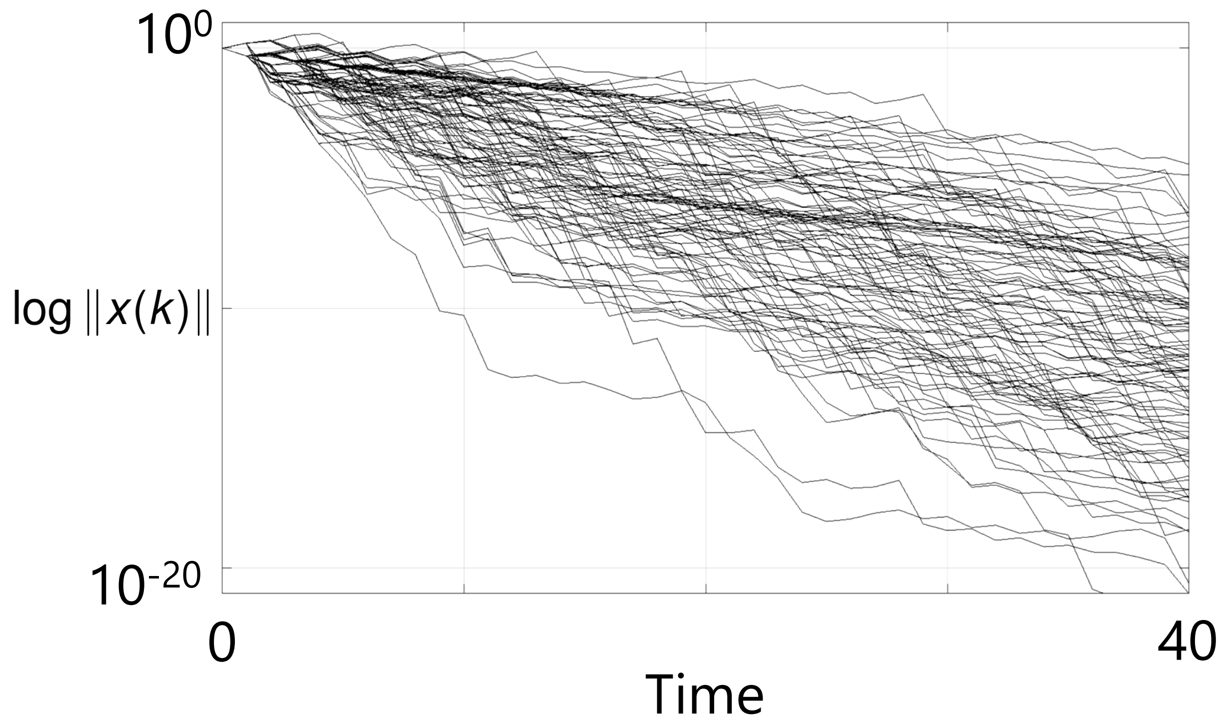
  - Transition probabilities

$$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$$

  - Observation: Periodic (T=4) observation with failure probability 0.5

Numerical simulation

- Sample paths



Conclusion

- State feedback stabilization of Markov jump linear systems
  - General model of Markov-state observation
    - Extends various frameworks in the literature
  - Feedback gains via solving linear matrix inequalities