M. Ogura, "Modelling, analysis, and control of networked epidemic processes," *CiNet Friday Lunch Seminar*, 2020.

Modelling, analysis, and control of networked epidemic processes

Masaki Ogura

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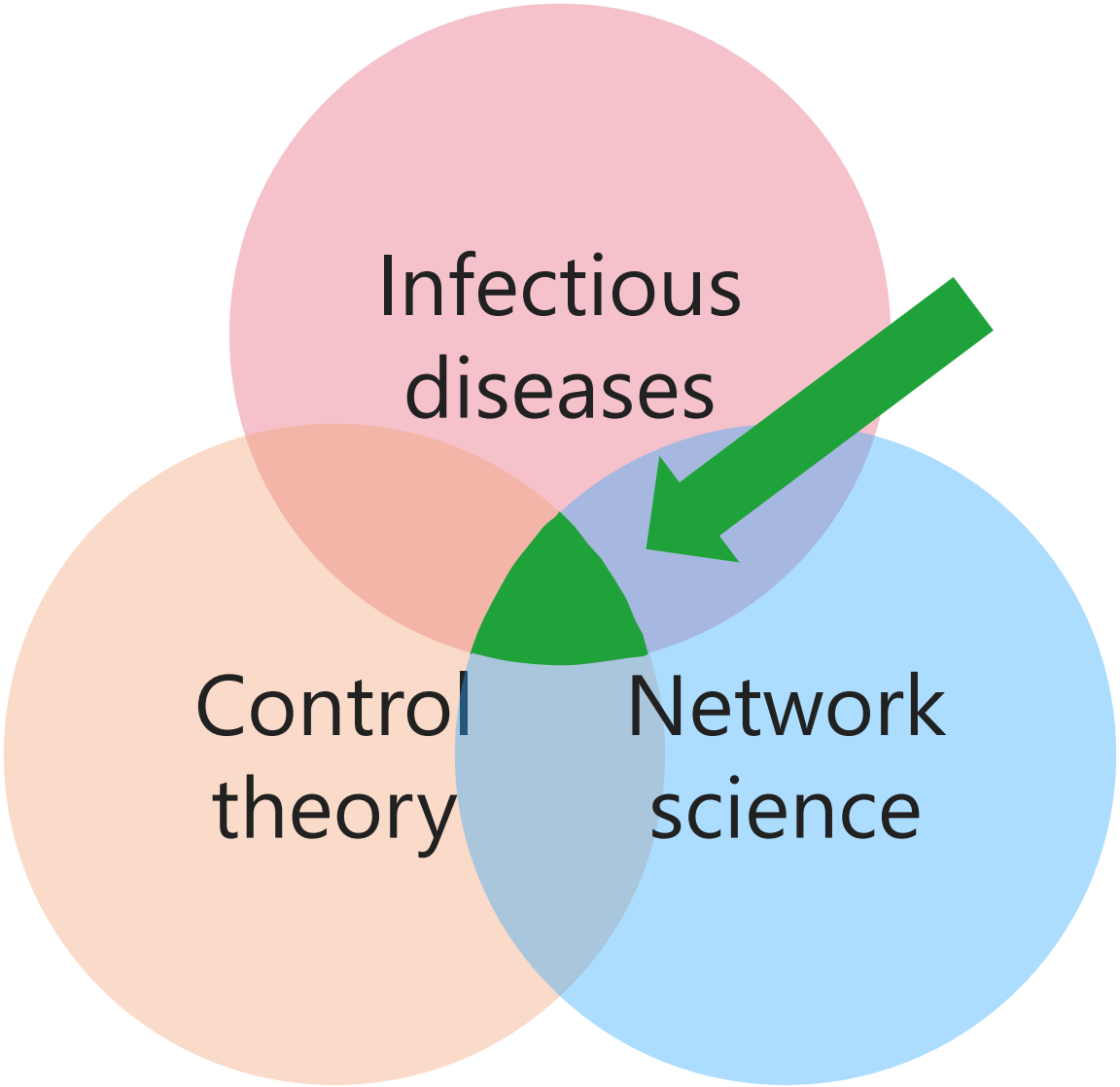
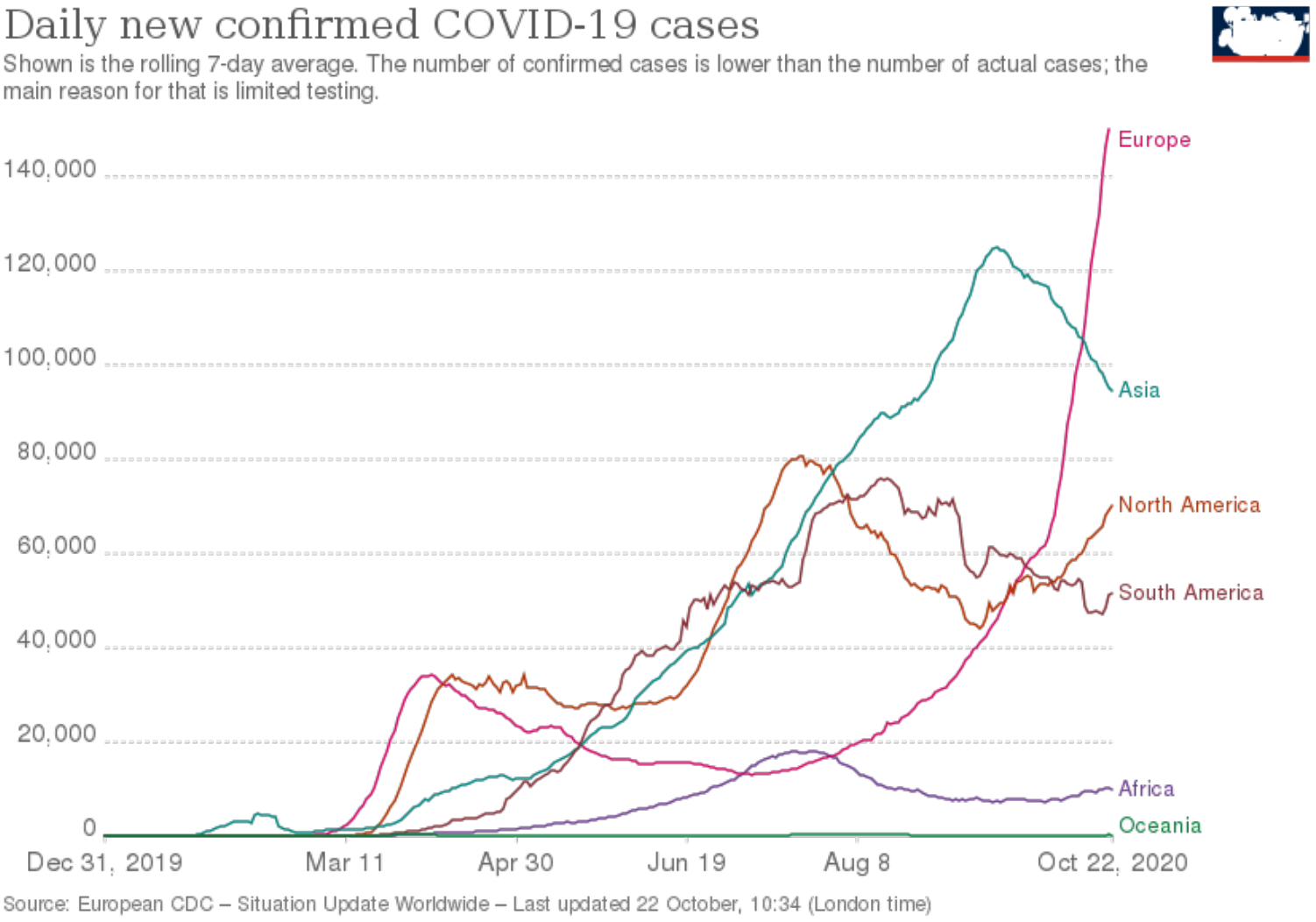
In today's seminar...



Infectious Diseases



Epidemic curve showing the 7 day rolling average of daily new cases of COVID-19 by continent [Our World in Data]

We want vaccine

天然痘

1958年～世界天然痘根絶計画を実施。
世界を挙げて徹底的なワクチン接種を実施。

400万人
1958年 推定死亡者数(世界)

根絶
1980年5月 根絶

［参考］国立感染症研究所HP

ポリオ

1988年～世界ポリオ根絶計画を実施。
世界を挙げて予防接種を実施する取り組みを続け、症例数は28年間で99%減少。世界からポリオを根絶するまであと一歩。

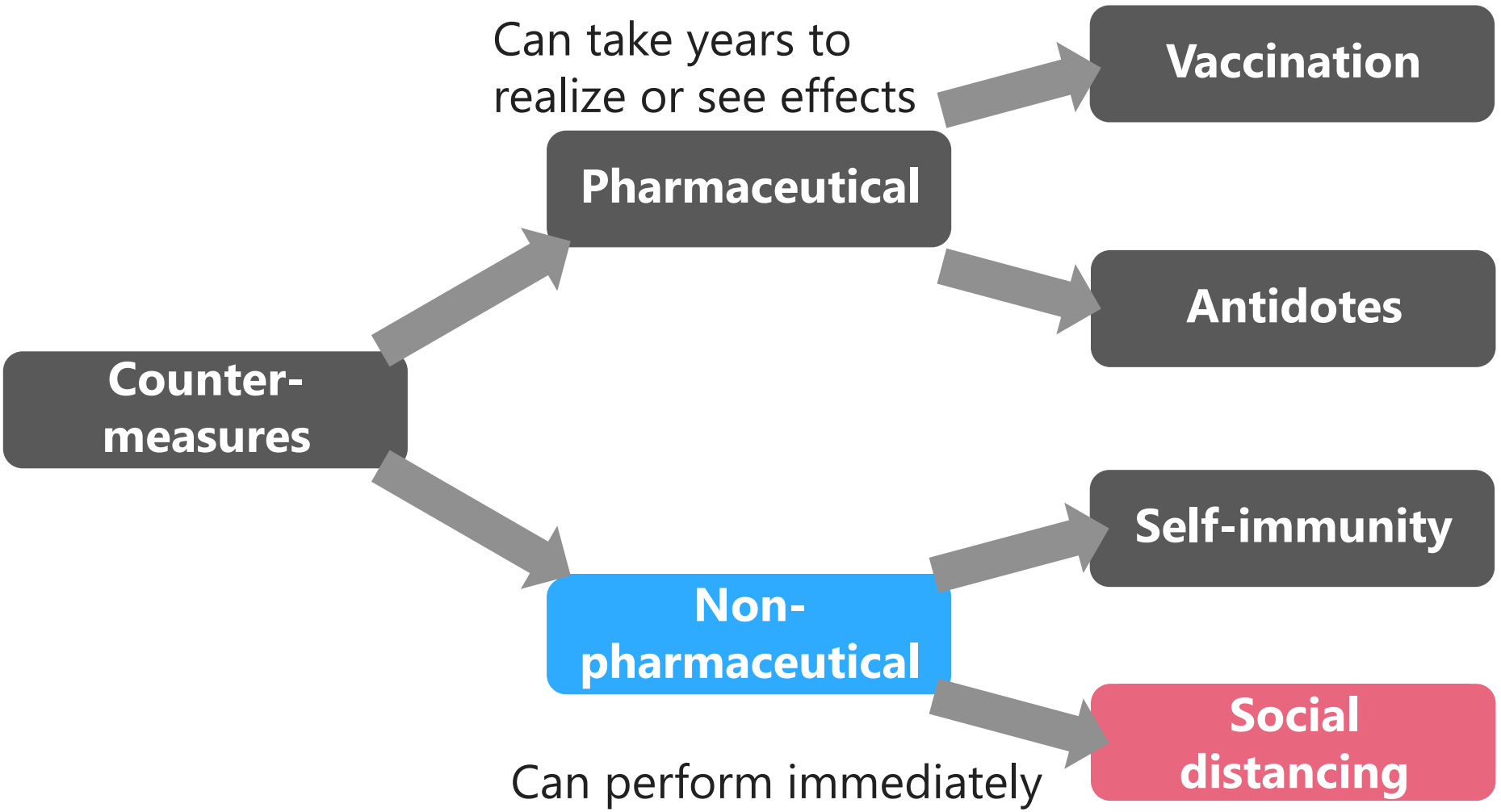
35万件
1988年 症例数(世界)

27件
2016年 症例数(世界)

［参考］日本ユニセフ協会HP

https://www.biken.or.jp/about_vaccine/support-society

How to contain infectious diseases



Social distancing

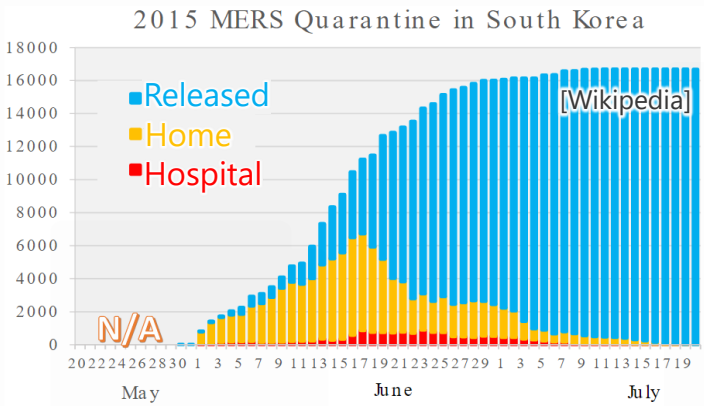
Keep a safe distance between you and others

Wearing masks

School closures

Quarantine

- MERS outbreak, South Korea
- COVID-19

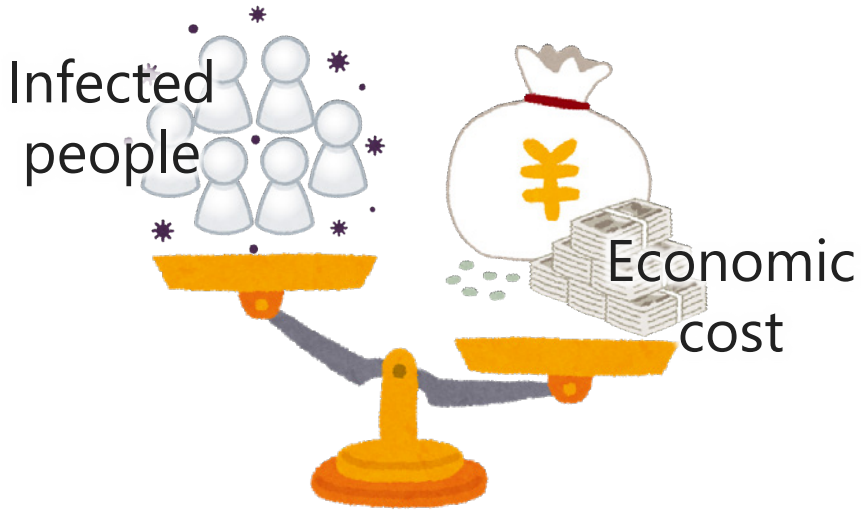


Social distancing

- Saying is one thing and doing another

Who?	**How strict?**	**How long?**
Changing situation	**Information uncertainty**	**Big data**

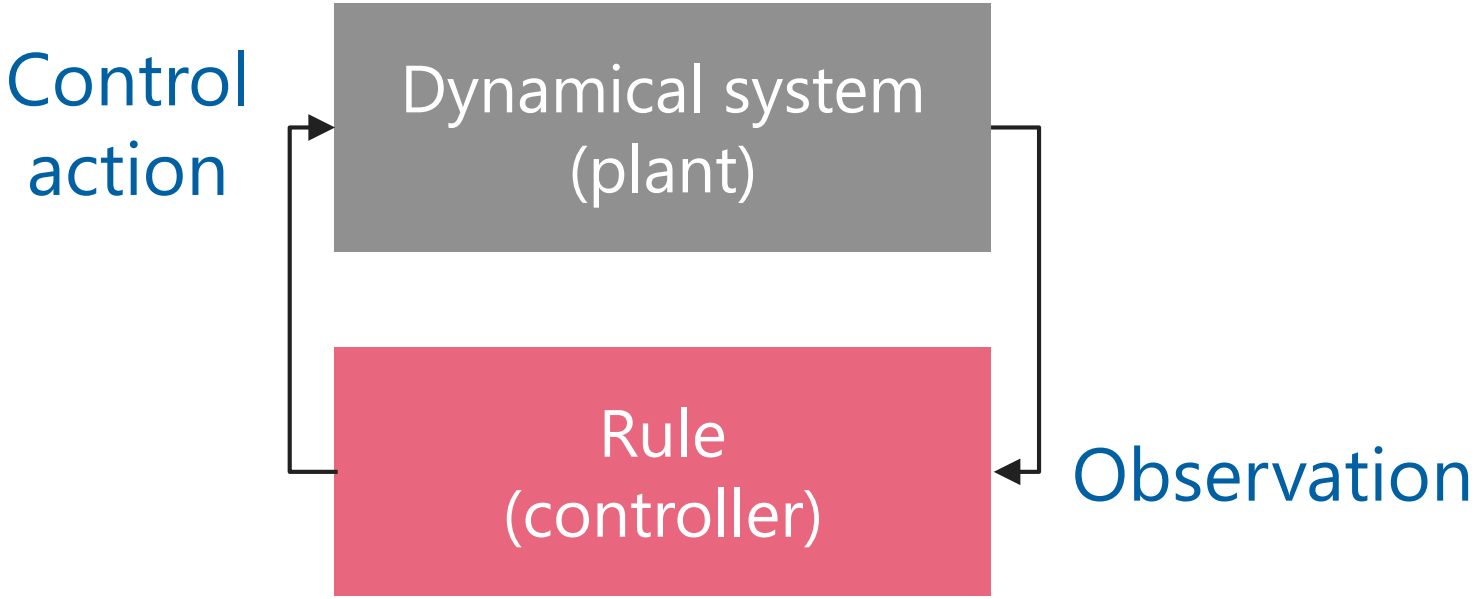
- Trade off: cost and benefit



Control theory

Control theory

- Deals with dynamical systems
  - Differential or difference equations
- Control rules to be designed (controller)
  - Make the plant behave in a desired manner
  - Costs and constraints
- Feedback control



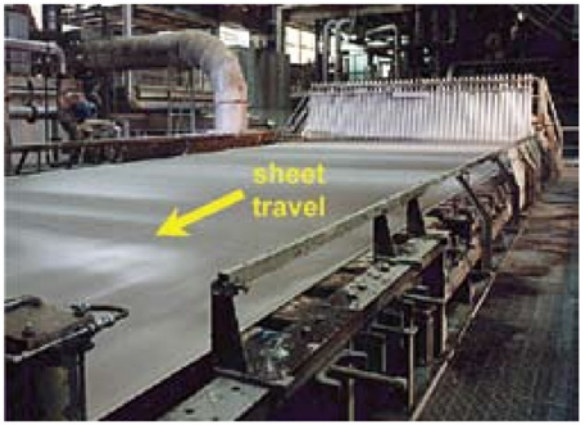
Some applications

Auto-tuning controller

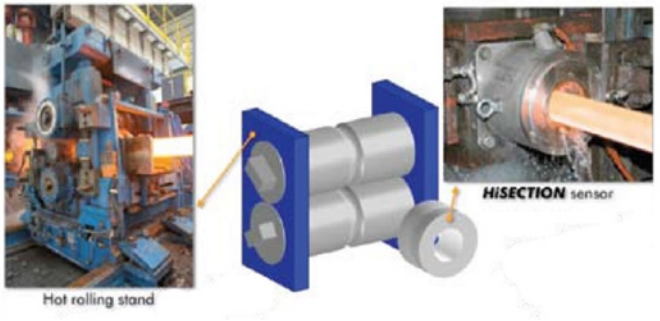
Figure 2: PID auto-tuners

Relay Auto-tuning

Paper machines



Steel rolling mills



Satellites

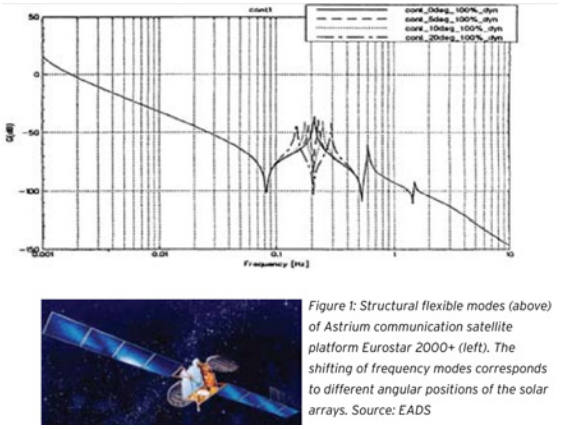


Figure 1: Structural flexible modes (above) of Astrium communication satellite platform Eurostar 2000+ (left). The shifting of frequency modes corresponds to different angular positions of the solar arrays. Source: EADS

F-35

F-35

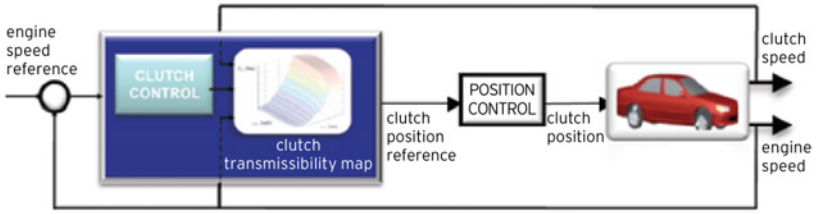
Joint direct attack munition

Laser guided MK - 82 scores direct hit against a moving target during tests at Eglin AFB

Affordable hit-to-kill accuracy minimizes collateral damage

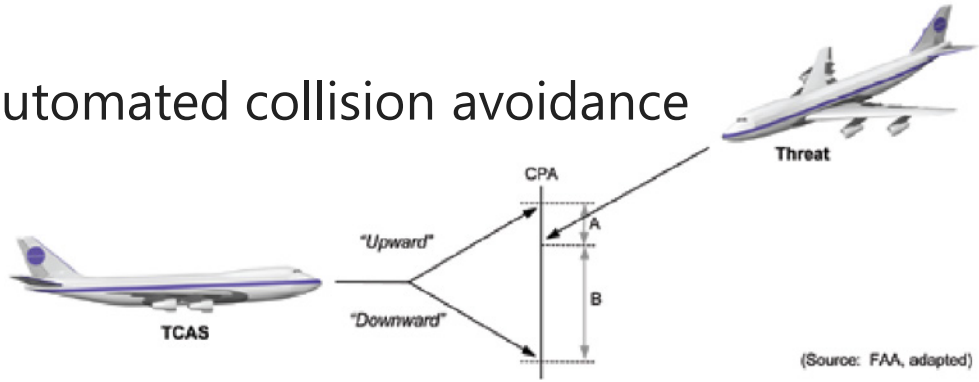
Some applications

Automated manual transmissions



F1

Automated collision avoidance



Digital signal processing

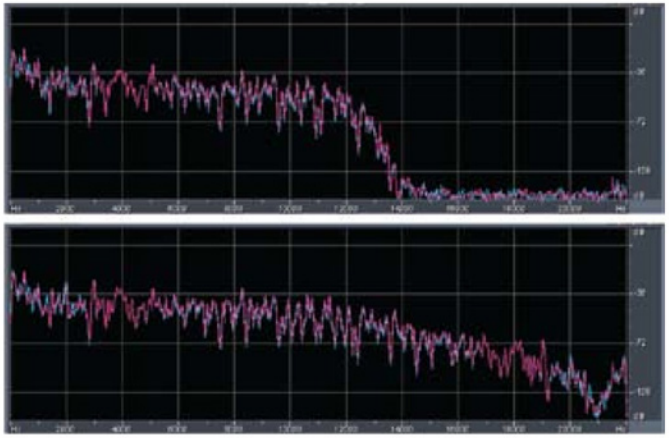
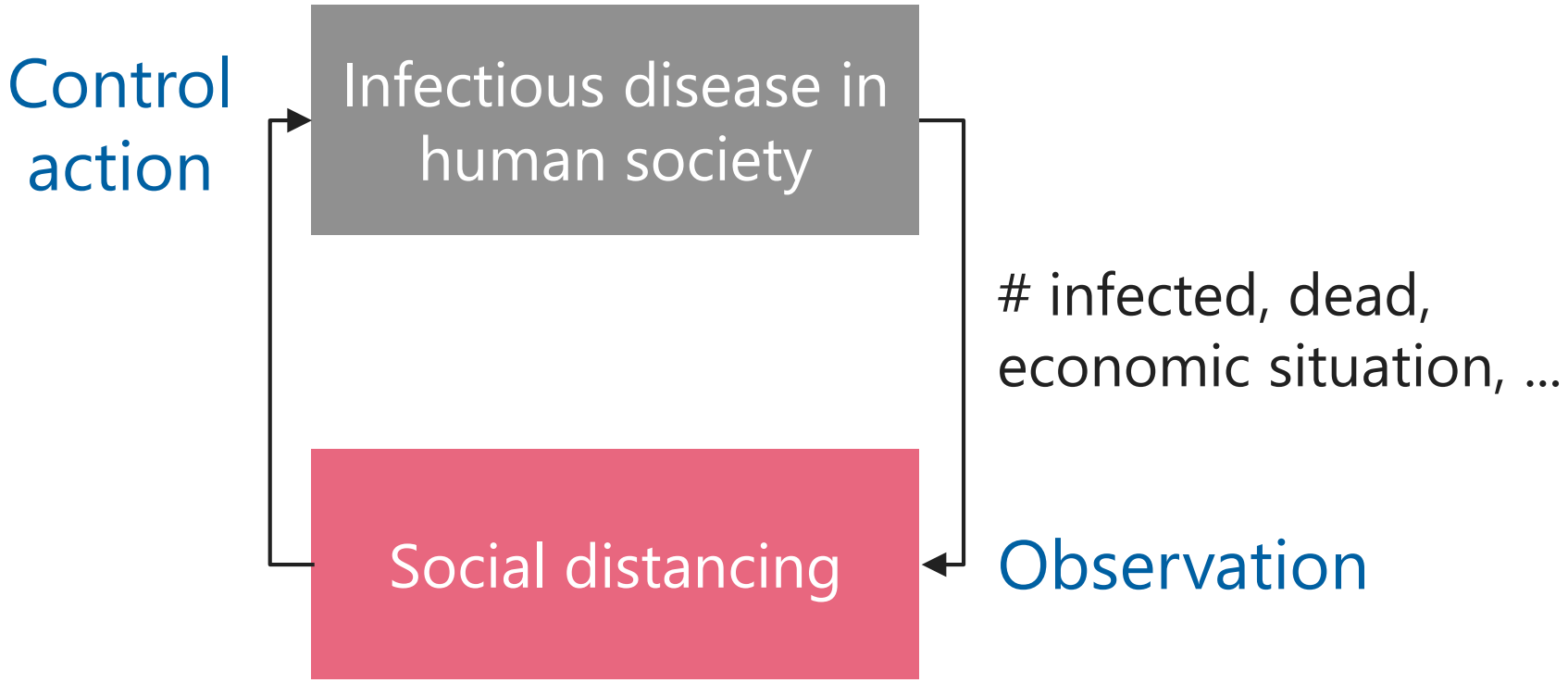


Figure courtesy of Sanyo Corporation.

Trip optimizer for railroads

Social distancing as a controller?



Mathematical model

Human society as a complex network

SOCIETY | **Factoid:**

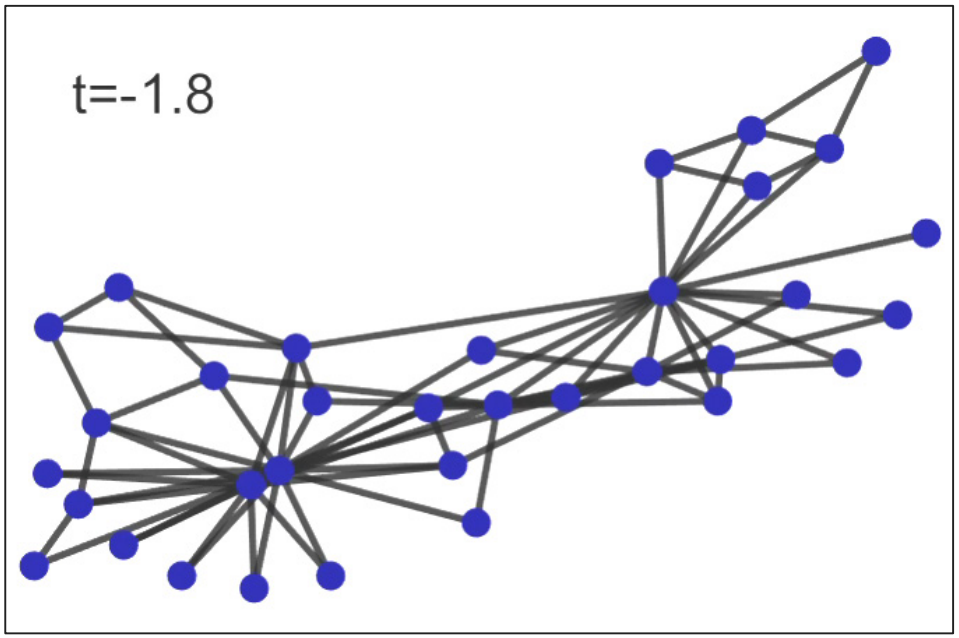
The “Social Graph” behind Facebook

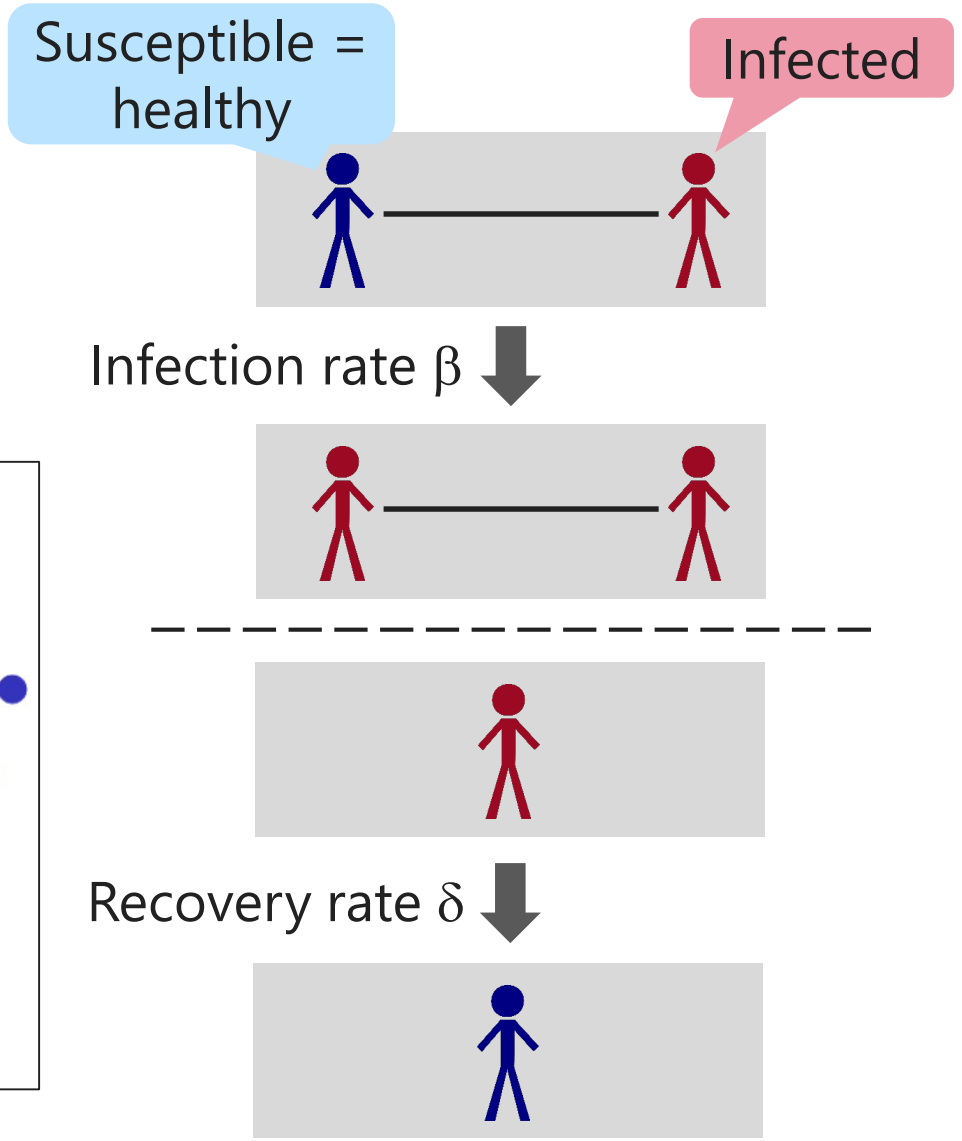
Keith Shepherd's "Sunday Best". http://baseballart.com/2010/07/shades-of-greatness-a-story-that-needed-to-be-told/

Networked susceptible-infected-susceptible model

Graph + node dynamics

- A standard model
  - Nodes = individuals
  - Edges = relationships

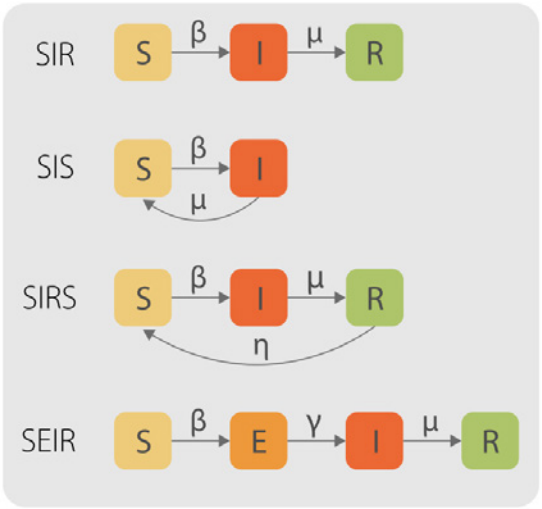




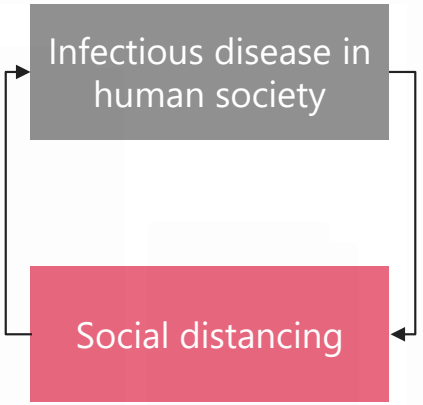
Remarks

Why SIS model?

- Analytically tractable
  - Only two states: susceptible and infected
- Often used in Systems and Control Theory
  - Not realistic but would capture something
  - SIR also used in Network Science
- No social distancing dynamics
  - Let us "attach" a controller on top of SIS model

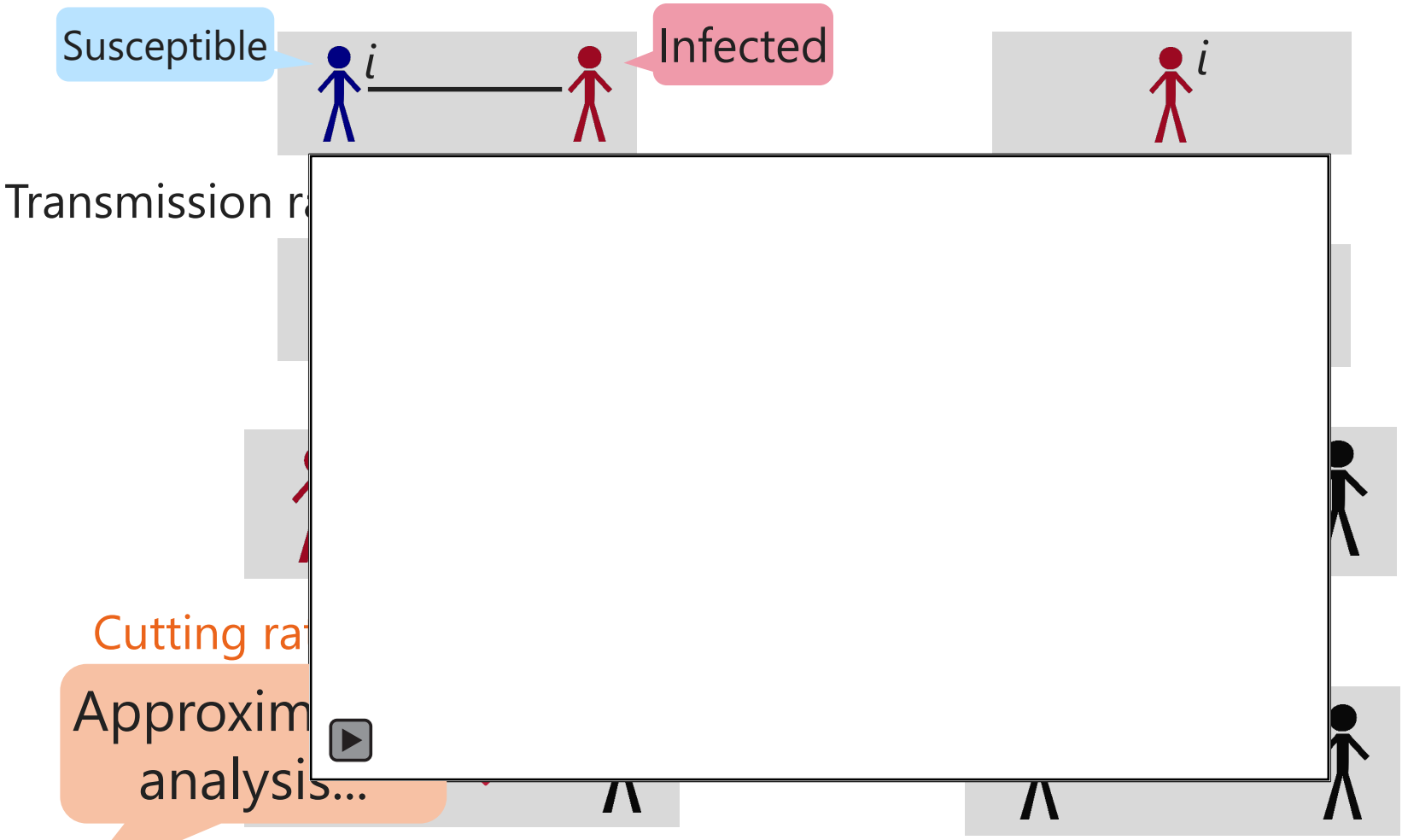


Pastor-Satorras et al., "Epidemic processes in complex networks," Reviews of Modern Physics, 2015.



SIS model w/ social distancing

Adaptive SIS model

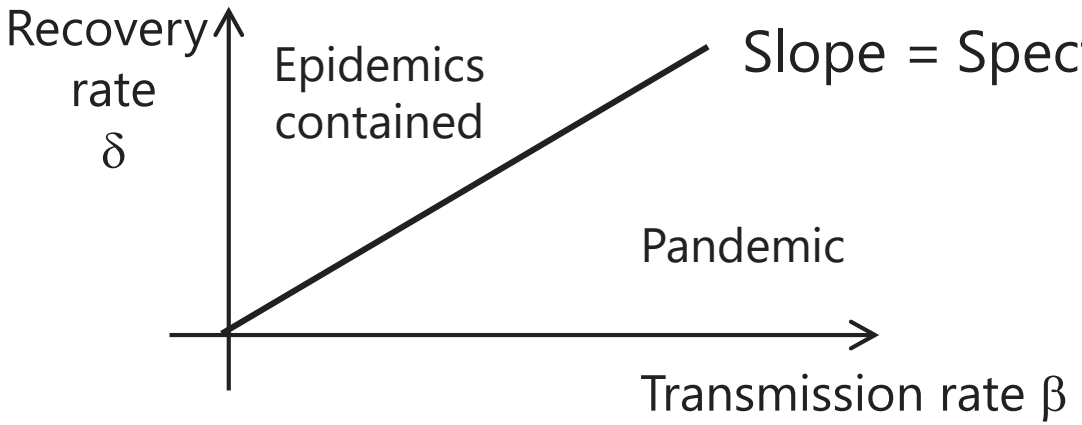


Guo et al., "Epidemic threshold and topological structure of susceptible-infectious-susceptible epidemics in adaptive networks," *Physical Review E*, 2013.

Ogura, Preciado, "Epidemic processes over adaptive state-dependent networks," *Physical Review E*, 2016.

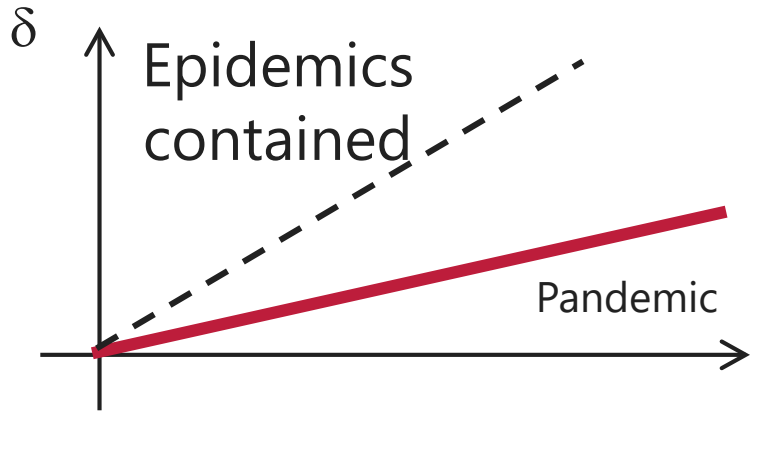
Asymptotic behavior, homogeneous case

- SIS model



Slope = Spectral radius of network

- Adaptive SIS model



$$\text{Slope} = \frac{\text{Spectral radius}}{1 + \frac{\phi}{\delta + \psi}}$$

Effective cutting rate

Ogura, Preciado, "Epidemic processes over adaptive state-dependent networks," *Physical Review E*, 2016.

Achterberg, Dubbeldam, Stam, and Van Mieghem, "Classification of link-breaking and link-creation updating rules in susceptible-infected-susceptible epidemics on adaptive networks," *Physical Review E*, 2020

Poisson-type SDE + control theoretical argument

ASIS model = Markov process w/ exponentially many states

- Computationally intractable

SDE representation

- Nodal states $x_i \in \{0, 1\}$

$$dx_i(t) = -x_i(t)dN_{\delta_i}(t) + (1 - x_i(t)) \sum_{k \in \mathcal{N}_i(0)} a_{ik}(t)x_k(t)dN_{\beta_i}(t)$$

- Edge states $a_{ij} \in \{0, 1\}$

$$da_{ij}(t) = (1 - a_{ij}(t))dN_{\psi_{ij}}(t) - a_{ij}(t)\left(x_i(t)dN_{\phi_{ij}}(t) + x_j(t)dN_{\phi_{ji}}(t)\right)$$

Upper-bound for expectations $p_i(t) = E[x_i(t)]$ $q_{ij}(t) = E[a_{ij}(t)x_i(t)]$

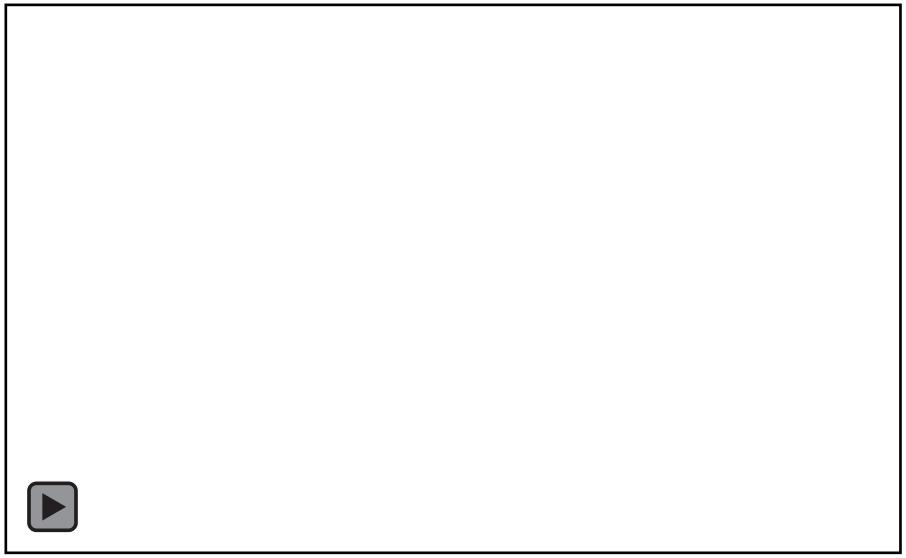
$$\frac{dp_i}{dt} = -\delta_i p_i + \beta_i \sum_{k \in \mathcal{N}_i(0)} q_{ki}$$

$$\frac{dq_{ij}}{dt} \leq -\phi_{ij}q_{ij} + \psi_{ij}(p_i - q_{ij}) - \delta_i q_{ij} + \beta_i \sum_{k \in \mathcal{N}_i(0)} q_{ki}$$

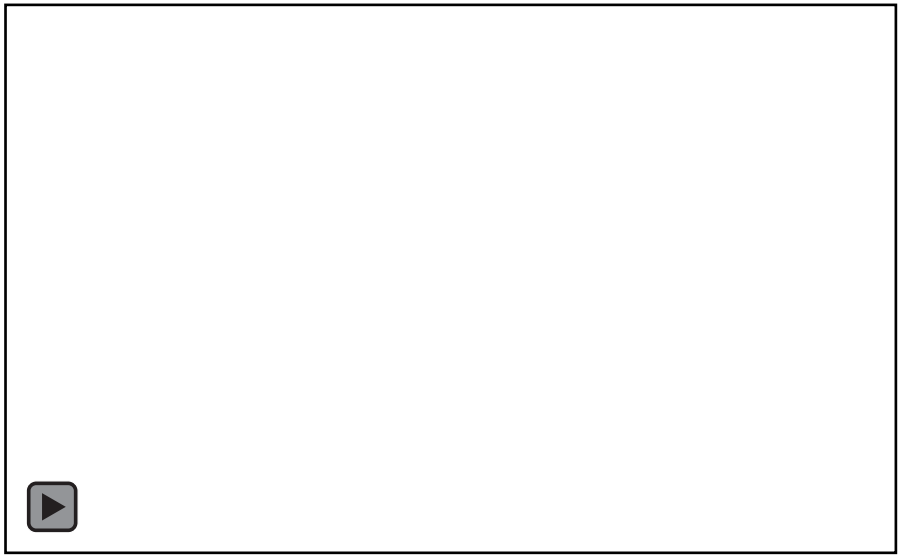
Simulation

- Karate network
  - homogeneous infection, recovery, reconnection rates
- Critical cutting rate ϕ^* computed
  - Higher cutting rate leads to containment, while lower rate to pandemic

Cutting insufficient ($\phi = \phi^*/10$)



Sufficiently fast cutting ($\phi = 2\phi^*$)



Optimal distancing strategy

Disease eradication ⟷ Trade-off ⟷ Keeping societal functions

- Question:

  How can we tune adaptation rates to eradicate spreading processes while minimizing societal loss?

- Optimization problem:

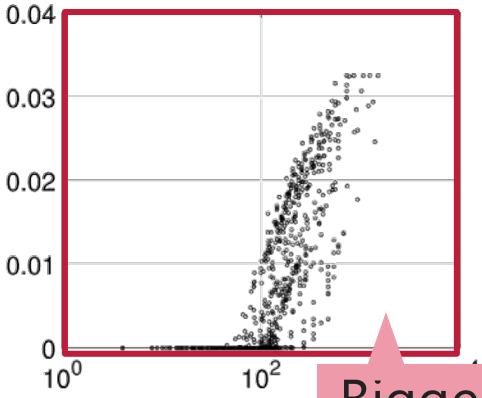
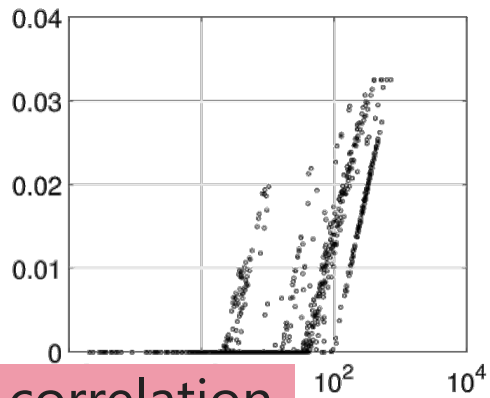
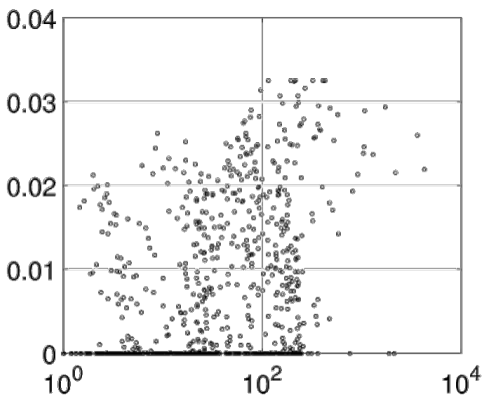
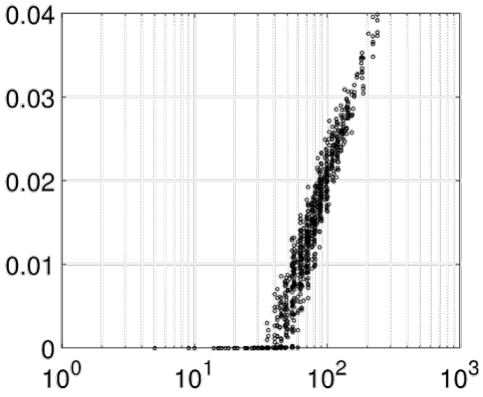
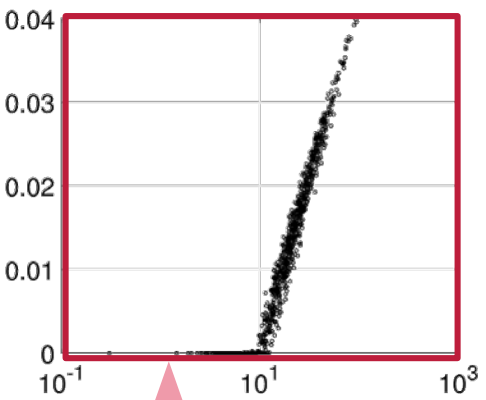
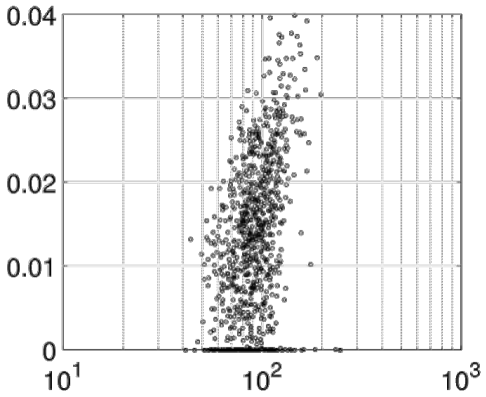
$$\text{minimize} \quad \sum_{i,j} \text{cost}(\phi_{ij})$$

$$\text{subject to} \quad \lambda_{\max}\left(\begin{bmatrix} -D_1 & B_1 \\ \Psi_1 & B_2 - D_2 - \Phi - \Psi_2 \end{bmatrix}\right) < 0$$

Can be equivalently converted to a **geometric program**

Optimal social-distancing rates ϕ_{ij}

- Optimal cost (vertical) and centralities (horizontal)

	Prod. of degrees of i and j	Prod. of eig. centralities of i and j	Edge-betweeness centrality of (i, j)
A Facebook network			
Erdös-Rényi random graph			

Summary

- Attempt for studying social distancing from systems and control theory
  - Society = system to be controlled
  - Social distancing = controller to be designed
- Several open research questions and technical challenges