# PL

August 13, 2021

We want to find the maximum solution to:

$\max_{x_1,x_2} 4x_1 + 3x_2$

In this example, the objective function is subject to the following constraints:

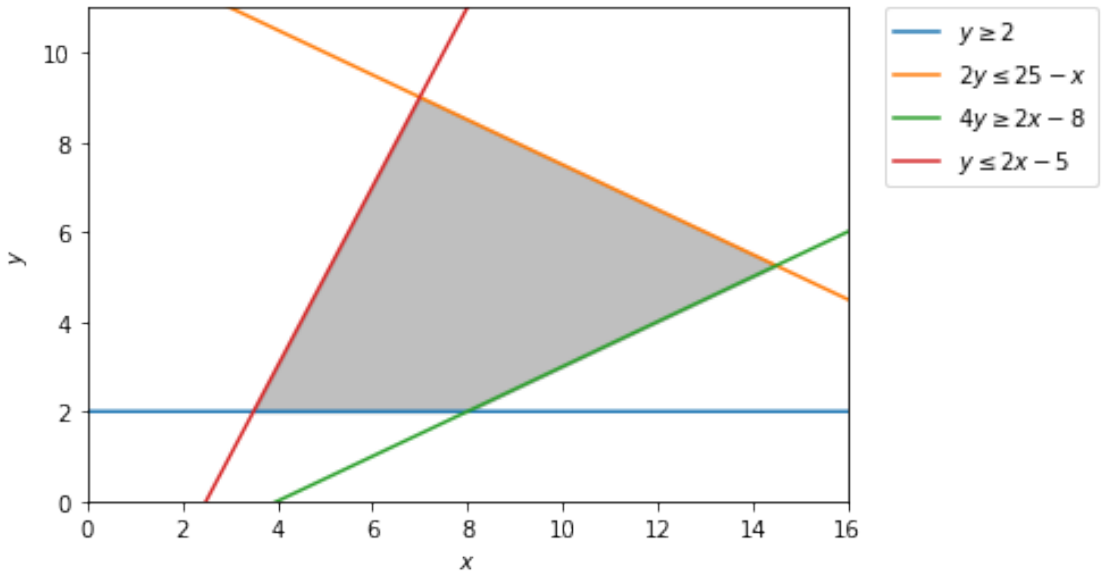
$$\begin{aligned} x_1 &\geq 0 \\ x_2 &\geq 2 \\ 2x_2 &\leq 25 - x_1 \\ 4x_2 &\geq 2x_1 - 8 \\ x_2 &\leq 2x_1 - 5 \end{aligned}$$

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import linprog
%matplotlib inline
```

[2]:
```
# Construct lines
# x > 0
x = np.linspace(0, 20, 2000)

# y >= 2
y1 = (x*0) + 2
# 2y <= 25 - x
y2 = (25-x)/2.0
# 4y >= 2x - 8
y3 = (2*x-8)/4.0
# y <= 2x - 5
y4 = 2 * x -5

# Make plot
plt.plot(x, y1, label=r'$y\geq2$')
plt.plot(x, y2, label=r'$2y\leq25-x$')
plt.plot(x, y3, label=r'$4y\geq 2x - 8$')
plt.plot(x, y4, label=r'$y\leq 2x-5$')
plt.xlim((0, 16))
plt.ylim((0, 11))
```

```
plt.xlabel(r'$x$')
plt.ylabel(r'$y$')

# Fill feasible region
y5 = np.minimum(y2, y4)
y6 = np.maximum(y1, y3)
plt.fill_between(x, y5, y6, where=y5>y6, color='grey', alpha=0.5)
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
```

[2]: <matplotlib.legend.Legend at 0x7fc1cd59c970>




We need some mathematical manipulations to convert the target problem to the form accepted by linprog

$\min_{x_1,x_2} -4x_1 - 3x_2$

vector of decision:

$$c = [-4, -3]$$

Inequality constraints:

$$x_1 + 2x_2 \leq 25$$
$$2x_1 - 4x_2 \leq 8$$
$$-2x_1 + x_2 \leq -5$$

where

$$\mathbf{A_{ub}} = \begin{bmatrix} 1 & 2 \\ 2 & -4 \\ -2 & 1 \end{bmatrix}$$

$$\mathbf{b_{ub}} = \begin{bmatrix} 25 \\ 8 \\ -5 \end{bmatrix}$$

[3]:
```
c = np.array([-4.0, -3.0])

A_ub = np.array([[1.0, 2.0],
                 [2.0, -4.0],
                 [-2.0, 1.0]])

b_ub = np.array([25.0, 8.0, -5.0])

x1_bouns = (0, None)
x2_bouns = (2, None)

bounds = [x1_bouns, x2_bouns]

result = linprog(c, A_ub=A_ub, b_ub=b_ub, bounds=bounds)
```

[4]:
```
print(result)
```

```
     con: array([], dtype=float64)
     fun: -73.74999999974425
 message: 'Optimization terminated successfully.'
     nit: 5
   slack: array([7.55449037e-11, 7.67919062e-11, 1.87500000e+01])
  status: 0
 success: True
       x: array([14.5 ,  5.25])
```