Chapter 2

PRIMITIVE EQUATION MODEL

2.1 Governing Equations

The model used in this study is admittedly simple, though we believe for the expressed goals it is quite appropriate. Before proceeding, some brief physical considerations regarding the MJO are given to argue the models suitability.

The MJO is normally found in a narrow band about the equator ($\sim$ 10°S,10°N) so using an equatorial β-plane to approximate this region is very reasonable. The observed horizontal circulation patterns are $\mathrm{O}(10,000$ km), making the quasi-static approximation well-founded. Latent heating effects due to water vapor in the atmosphere are especially important in the tropics. While they most certainly play an important role in the evolution of this tropical weather system, the essential large scale features, which are of concern here, should be satisfactorily resolved without including a detailed moisture budget, but rather by simply including a specified diabatic heating. Based on these judgements, the equatorial β-plane quasi-static primitive equations are a legitimate theoretical framework. These are listed as the set (2.1)–(2.5),

$$\frac{Du}{Dt} - \beta y v + \frac{1}{\rho}\frac{\partial p}{\partial x} = F_x, \tag{2.1}$$

$$\frac{Dv}{Dt} + \beta y u + \frac{1}{\rho}\frac{\partial p}{\partial y} = F_y, \tag{2.2}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g = 0, \tag{2.3}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \tag{2.4}$$

$$c_p \frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = \mathcal{D}. \tag{2.5}$$

For a stratified, compressible atmosphere the standard forms of the hydrostatic and continuity equations are given by (2.3) and (2.4) respectively. The horizontal momentum equations (2.1), (2.2) have frictional forcing terms F_x and F_y. The last equation, the first law of thermodynamics, has a forcing term ($\mathcal{D}$) that accounts for all diabatic effects.

For convenvience we choose to use the vertical log-pressure coordinate, $z = \ln(p_0/p)$, where $p_0 = 1010$ hPa is a constant "surface" pressure. Conversion of equations (2.1)–(2.4) to this vertical coordinate is the topic of Appendix A. Restricting our attention to small amplitude disturbances that arise from perturbing a resting basic state, we reach the following form of the governing equations,

$$\begin{aligned}
\frac{\partial u}{\partial t} - \beta y v + \frac{\partial \phi}{\partial x} &= -\alpha u, \\
\frac{\partial v}{\partial t} + \beta y u + \frac{\partial \phi}{\partial y} &= -\alpha v, \\
\frac{\partial \phi}{\partial z} &= RT, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} - w &= 0, \\
\frac{\partial T}{\partial t} + \Gamma w &= -\alpha T + \frac{Q}{c_p},
\end{aligned} \tag{2.6}$$

where u is the eastward component of velocity, v the northward component, $w = Dz/Dt$ the "vertical log-pressure velocity," ϕ the perturbation geopotential, T the perturbation temperature, $\beta = 2\Omega/a$ the equatorial value of the northward gradient of the Coriolis parameter, Ω the Earth's rotation rate, a the Earth's radius, α the constant coefficient for Rayleigh friction and Newtonian cooling. We have assumed the frictional dissipation, F_x, F_y, takes the form, $-\alpha u$, $-\alpha v$, respectively. The total diabatic effect term, $\mathcal{D}$, has been partitioned into dissipation due to radiative cooling, $-\alpha T$, and generation due to convective heating, Q/c_p. The basic state static stability, $\Gamma = d\bar{T}/dz + \kappa\bar{T}$ ($\kappa = R/c_p$), is computed from the basic state temperature profile, $\bar{T}(z)$. For simplicity we assume that Γ is a constant, and choose the tropical tropospheric mean value $\Gamma = 23.79$ K. We seek solutions of (2.6) on a domain that is infinite in y, periodic over $-\pi a \leq x \leq \pi a$, and confined between $z = 0$ and $z = z_T = \ln(1010/200) \approx 1.619$, with the boundary conditions $w = 0$ at $z = 0, z_T$.