Appendix C

TRANSFORMATION TO THE ξ-COORDINATE

Define new time and zonal distance variables as

$$\tau = t, \quad \xi = x - ct, \tag{C.1}$$

respectively. Note that to remain at the same point in ξ as t increases, it is necessary to move eastward at a constant rate c. To perform the transformation $(x, t) \rightarrow (\xi, \tau)$ on our set of equations, a relationship between the derivative operators of each coordinate frame is needed. The necessary relationships can be attained by expanding the x and t derivative operators in terms of ξ and τ using the chain rule

$$\frac{\partial}{\partial t} = \frac{\partial \xi}{\partial t}\frac{\partial}{\partial \xi} + \frac{\partial \tau}{\partial t}\frac{\partial}{\partial \tau}, \tag{C.2}$$

$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} + \frac{\partial \tau}{\partial x}\frac{\partial}{\partial \tau}. \tag{C.3}$$

The derivative terms evaluate to, $\partial\xi/\partial t = -c$, $\partial\tau/\partial t = 1$, $\partial\xi/\partial x = 1$, and $\partial\tau/\partial x = 0$, which reduces (C.2) and (C.3) to $\partial/\partial t = -c\,\partial/\partial\xi + \partial/\partial\tau$ and $\partial/\partial x = \partial/\partial\xi$, respectively. If we can assume that the flow does not change in time, τ, as seen from a reference frame translating at the same constant speed as the forcing (c), then mathematically we have that $\partial F/\partial\tau = 0$, for any field variable F. Applying this “steady state assumption” to the expression for the t derivative operator the final form of (C.2) and (C.3) becomes

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \tag{C.4}$$

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \xi}, \tag{C.5}$$

which are the same forms seen in section 2.4.