Unpacking Aggregate Welfare in a Spatial Economy *

Eric Donald[†] Masao Fukui[‡] Yuhei Miyauchi[§]

University of Pittsburgh *Boston University* *Boston University*

October 31, 2024

Abstract

How do regional productivity shocks or transportation infrastructure improvement affect aggregate welfare? In a general class of spatial equilibrium models, we provide a nonparametric formula to express aggregate welfare changes as changes in technology (Fogel 1964, Hulten 1978), spatial dispersion of marginal utility, fiscal externalities, technological externalities, and redistribution. We also use this characterization to derive a general optimal spatial transfer formula and show that the technology term alone summarizes aggregate welfare changes from technological shocks whenever optimal spatial transfers are in place. We apply our framework to study welfare gains from improving the US highway network. We find that changes in spatial dispersion of marginal utility, and to a lesser extent fiscal externalities, are as important as technological externalities in explaining the average and heterogeneity of welfare gains.

*We thank Rodrigo Adão, Treb Allen, Costas Arkolakis, Yan Bai, Anmol Bhandari, John Sturm Becko, Ariel Burstein, Levi Crews, Maya Eden, Pablo Fajgelbaum, Tishara Garg, Cecile Gaubert, Benny Kleinman, David Lagakos, Ernest Liu, Ezra Oberfield, Natalia Ramondo, Andrés Rodríguez-Clare, Edouard Schaal, Conor Walsh, and Atsushi Yamagishi as well as seminar participants at Barcelona Summer Forum, Chinese University of Hong Kong, Columbia/NYU Spatial-Trade Conference, CRED Workshop on Regional and Urban Economics, Midwest Macro Meeting, Minnesota Macro, NBER SI ITI, Princeton, SED, SMU, UC Berkeley, and UEA North American Meeting for their helpful comments. We thank Hannah Rhodenhiser and Maria Mittelbach for excellent research assistance.

†University of Pittsburgh (e-mail: eric.donald@pitt.edu).

‡Boston University (e-mail: mfukui@bu.edu).

§Boston University (e-mail: miyauchi@bu.edu).

1 Introduction

How do regional productivity shocks or transportation infrastructure improvement affect aggregate welfare? To answer these questions, there has been significant progress in the development of quantitative spatial general equilibrium models. These frameworks allow researchers to fit the model to geographically disaggregated data and compute the aggregate welfare implications of a particular shock or policy. While useful, these frameworks are highly complex and parameterized, obscuring which forces, parameters, and data moments govern the aggregate welfare effects.

An alternative approach is to apply the macro envelope theorem. Hulten (1978) showed that, in a frictionless economy, the impact on aggregate GDP of microeconomic TFP shocks is equal to the shocked producer's sales as a share of GDP (i.e. Domar weights). In an evaluation of transportation infrastructure, Fogel (1964) calculated the benefit of shipment cost savings relative to the next best alternative, assuming that reallocation is second order. While these approaches are powerful in their minimal data requirements (Donaldson 2022), they are criticized for ignoring market failures such as agglomeration and congestion externalities.[1] However, how such externalities affect aggregate implications and what other considerations may arise in spatial equilibrium remains an open question.

This paper bridges the gap between these two approaches by providing a theory to unpack aggregate welfare in spatial equilibrium models. We provide a nonparametric formula for aggregate welfare changes and describe each term in terms of measurable objects. Applying our framework to potential improvements of the US highway network, we find that the welfare implications are influenced by forces that are often overlooked in the spatial equilibrium literature.

We start by setting up a general class of spatial equilibrium models. The economy is populated by a unit mass of households with two layers of heterogeneity: (i) household types representing their identities, such as race, gender, or birthplaces, and (ii) idiosyncratic additive preferences over locations, which rationalize the wide heterogeneity of living locations within a well-defined identity group. We do not impose any parametric assumptions on idiosyncratic location preferences. Our framework further accommodates flexible location-specific utility functions as well as local amenities, production functions, input-output linkages, trade frictions, agglomeration and congestion externalities, and government transfers across locations and household types. We define aggregate welfare through a social welfare function (SWF) that is utilitarian with respect to

[1] See Lebergott (1966) and David (1969) for early criticisms of Fogel (1964) due to the omission of agglomeration externalities in his calculation.

idiosyncratic location preferences, but allows arbitrary welfare weights on different household identity types. While this is the standard notion of welfare in the spatial equilibrium literature (e.g. Allen and Arkolakis 2014, Redding and Rossi-Hansberg 2017), we further discuss axiomatizations of this SWF.[2]

We start our theoretical analysis by developing a nonparametric formula for the first-order aggregate welfare impact of technology shocks or transfer policies. We show that it is summarized by five additively separable terms. The first term, (i) Δ technology, is the percentage change in productivity multiplied by the revenue of the region or sector receiving a shock, consistent with the characterization of Fogel (1964) and Hulten (1978). The remaining four terms, jointly constituting the reallocation effects, are (ii) Δ marginal utility (MU) dispersion, (iii) Δ fiscal externalities, (iv) Δ technological externalities, and (v) Δ redistribution.

The second term, (ii) Δ MU dispersion, reflects the reallocation of resources across locations that differ in their marginal utility of income. In spatial equilibrium, agents make location decisions based on utility *levels* (inclusive of idiosyncratic location preferences). This implies that the *marginal* utility of income is not necessarily equalized across locations, as first pointed out by Mirrlees (1972). As a consequence, a reallocation of consumption across space in response to a shock has a first-order effect on aggregate (utilitarian) welfare. One interpretation of this reallocation is the lack of insurance for the ex-ante uncertainty of idiosyncratic location preferences. Another interpretation is the lack of redistribution across households of the same type but with different idiosyncratic location preferences.

The rest of the reallocation terms (iii)-(v) likewise have clear economic interpretations. The third term, (iii) Δ fiscal externality, reflects changes in the government budget induced by population movements in response to shocks. This term is positive if a shock induces reallocation towards locations that are net taxpayers. The fourth term, (iv) Δ technological externality, reflects changes in the agglomeration/congestion externalities induced by population movements. This term is positive if a shock induces reallocation towards locations that generate higher technological externalities. The fifth term, (v) Δ redistribution, reflects a reallocation of resources across heterogeneous household types. This term is positive if a shock induces the reallocation of consumption toward household types with higher welfare weights.

Beyond theoretically clarifying the sources of welfare gains and losses, our formula reveals

[2] One axiomatization by Harsanyi (1955) views idiosyncratic preferences as an ex-post realizations of birth lottery. Another axiomatization by Eden (2020a) views idiosyncratic preferences as ex-ante heterogeneity among individuals and imposes a set of plausible axioms about the social ranking of allocations across these individuals (independence of irrelevant variations, Pareto, weak anonymity).

the data moments necessary for measuring aggregate welfare changes. A celebrated feature of Fogel/Hulten's analysis is that it only requires data on changes in productivity and pre-shock equilibrium sales. Our formula sheds light on when and why we require additional information once we depart from Fogel/Hulten's analysis. Beyond changes in equilibrium consumption and population, three sets of structural parameters and equilibrium objects are required for drawing welfare conclusions: the marginal utility of consumption — which governs changes in MU dispersion; agglomeration elasticities — which govern changes in technological externalities; and spatial transfers — which govern changes in fiscal externalities. We argue that all the structural parameters in our welfare formula can be nonparametrically identifiable with suitable exogenous variations, thereby providing a clear mapping from data moments to aggregate welfare changes.

We then use our characterization to derive a nonparametric formula for optimal spatial transfers, which generalizes existing results. Optimality of spatial transfers requires a marginal change in transfers not to improve welfare. Imposing this requirement in our characterization reveals the trade-offs that optimal transfers must navigate. Specifically, our formula trades off the value of closing the spatial dispersion in marginal utility against the fiscal and technological externalities induced by a transfer, echoing the Baily-Chetty formula (Baily 1978, Chetty 2006) in the context of the optimal unemployment insurance literature.

We further show that, if optimal spatial transfers are implemented in the pre-shock equilibrium, the welfare effects of technological shocks are summarized by the (i) Δ technology term alone. This is because optimal spatial transfers are set so that the reallocation terms (ii)-(v) jointly have no first-order effect on aggregate welfare. This result is also useful in designing policies which affect productivity, such as the optimal design of transportation infrastructure (e.g. Fajgelbaum and Schaal 2020, Bordeu 2023). Our results indicate that, if the government can simultaneously implement optimal spatial transfers, the marginal returns of transportation infrastructure are simple to evaluate.

Although our main focus is on aggregate welfare, a related but distinct question is how spatially disaggregated shocks affect aggregate output. We answer this question by providing an analogous formula for changes in aggregate output. The output formula is similar to the welfare formula in that three terms – technology, fiscal externality, and technological externality – enter identically, but differs in that two terms – MU dispersion and redistribution – are now absent. Instead, a new term appears which reflects the presence of compensating differentials. This term captures that, in spatial equilibrium models, location choices are based on utility and, therefore, do not necessarily maximize output. The compensating differential term is positive if the population reallocates toward locations with high nominal income, where compensating differential is low.

In the final part of our paper, we use our framework to study the welfare gains from improvements in US highway network. To do so, we first extend the existing spatial equilibrium framework with route choice and traffic congestion (Allen and Arkolakis 2022) by introducing flexibility in utility functions and spatial transfers, two additional margins highlighted in our welfare formula. We then use this expanded model to compute the welfare gains from a marginal improvement of each link of the highway network.

We first document that the (i) Δ technology term alone provides a poor approximation for welfare gains from these link improvements, both in terms of the mean and heterogeneity of gains. On average, technology alone overpredicts welfare gains by approximately 100%, while explaining only 70% of the variation in welfare gains of link improvements.

We then apply our formula to unpack the sources of these deviations. While we confirm an important role for technological externalities from traffic congestion, as originally argued by Allen and Arkolakis (2022), we find quantitatively comparable roles for (ii) Δ MU dispersion in explaining how welfare gains differ from the prediction based on the technology term alone. We also find significant roles for (iii) Δ fiscal externalities, albeit the quantitative magnitudes are more modest than the other terms. Overall, our application sheds light on important sources of welfare gains/losses that are often overlooked in the spatial equilibrium literature.

Related Literature

Our paper contributes to a large and growing literature on spatial equilibrium models (see Redding and Rossi-Hansberg (2017) for a survey) by providing a nonparametric formula for first-order changes in aggregate welfare. A growing body of research uses quantitative spatial equilibrium models to study the aggregate welfare effects of regional productivity shocks (e.g. Caliendo, Parro, Rossi-Hansberg, and Sarte (2018)) or transportation infrastructure (e.g. Allen and Arkolakis (2014, 2022), Donaldson and Hornbeck (2016)). In particular, our paper shares the focus of Tsivanidis (2023) and Zárate (2022) in studying how welfare gains deviate from the prediction based on Fogel/Hulten or the value of travel time saved approach (Fogel 1964, Small and Verhoef 2007). We provide a general formula that explicitly characterizes the sources of deviation and sheds light on the components that are not typically the focus of the literature. Our paper is also related to Kleinman, Liu, and Redding (2024) in highlighting the usefulness of first-order approximations in spatial equilibrium models.

In pointing out the spatial dispersion in marginal utility as one margin of welfare changes, our work builds on Mirrlees (1972), who discusses this issue in a model with stylized geography. In contemporaneous work, Mongey and Waugh (2024) discuss this issue in abstract discrete choice models. They interpret dispersion in marginal utility as a market failure stemming from lack of insurance over ex-ante uncertain idiosyncratic location preferences. They then characterize the allocation with and without complete insurance markets. The focus of our paper is different. Without taking a strong normative stance on the dispersion of marginal utility as a market failure, we characterize how this feature and other typical considerations in spatial equilibrium models shape aggregate welfare changes beyond Fogel/Hulten's characterization and investigate its connection to optimal spatial transfers.

Our focus on aggregate welfare also relates to Bhandari, Evans, Golosov, and Sargent (2021) and Dávila and Schaab (2022, 2023), who provide welfare decompositions in general equilibrium models with heterogeneous agents. A key distinction of our formula is that we explicitly characterize the deviations from Hulten's characterization, and relate each term to measurable objects, such as quantities, prices, and spatial transfers. This feature of our formula allows us to provide a tight connection between welfare responses to a shock and the design of optimal spatial transfers.

We also contribute to the existing literature on the optimal design of place-based policies. Our optimal spatial transfer formula generalizes existing work focusing on agglomeration externalities (Fajgelbaum and Gaubert 2020, Rossi-Hansberg, Sarte, and Schwartzman 2021) by dispensing with any parametric assumptions for location choice, as well as existing work focusing on redistribution (Gaubert, Kline, and Yagan 2021, Davis and Gregory 2021, Ales and Sleet 2022) by introducing a rich production economy, cross-regional trade, and agglomeration externalities. We also contribute to the literature on designing place-based policies that affect productivity, such as transportation infrastructure, by characterizing the impacts of technology shocks in the presence of (optimal) spatial transfers.

2 Spatial Equilibrium Framework

This section sets up the general spatial equilibrium framework for our baseline analysis. For expositional clarity, we delegate some additional features (such as shocks to non-market amenities and amenity externalities) to Section 4.

2.1 Setup

There are N locations indexed by $i, j \in \mathcal{N} \equiv \{1, \ldots, N\}$. There are S types of households indexed by $\theta \in \Theta \equiv \{\theta_1, \ldots, \theta_S\}$, which we refer to as the identity of households, such as race and gender. The mass of each type is ℓ^θ, and we normalize the total measure to one: $\sum_\theta \ell^\theta = 1$. Each household decides its residential location at the beginning of the period. Households who decide to live in location j consume the location-specific final good aggregator specific to household type θ produced using intermediate goods. There are K intermediate goods, some of which can be potentially traded across locations subject to a cost (e.g. food or manufacturing) and some of which are not traded across locations (e.g. housing or nontradable services). Intermediate goods are produced using local labor, intermediate goods, and local fixed factors (e.g. land). Households have ownership of these local fixed factors and earn factor income depending on their type θ, irrespective of their location.

Households of type θ in location j with idiosyncratic location preferences $\boldsymbol{\epsilon}^\theta = (\epsilon_1^\theta, \ldots, \epsilon_N^\theta)$ choose their location, inelastically supply one unit of labor, and consume final goods. Their utility from living in location j is given by

$$u_j(C_j^\theta) + \epsilon_j^\theta, \tag{1}$$

where the utility function can depend on location j, embracing amenity differences.[3] The idiosyncratic location preferences ϵ_j^θ captures additive heterogeneity in preferences associated with location j. In our baseline analysis, we restrict ϵ_j^θ to be additively separable for reasons we describe in Section 2.2. The idiosyncratic preferences $\boldsymbol{\epsilon}^\theta$ capture the observed heterogeneity in households' living locations within well-defined identity types.

The households' budget constraint is

$$P_j^\theta C_j^\theta = w_j^\theta + T_j^\theta + \Pi^\theta, \tag{2}$$

where P_j^θ is the price of final goods, w_j^θ is the wage, and T_j^θ is the net government transfer, all for type θ households in location j. In reality, T_j^θ includes both taxes and transfers explicitly tagged to each location (such as state taxes and transfers in the US) and those set at the national level (such as federal taxes and transfers in the US). We do not impose any additional assumptions about T_j^θ (such as the linearity with respect to nominal wages or income) beyond the condition that the

[3] Note that the independence of the utility function $u_j(\cdot)$ on types is without loss of generality, as the final consumption basket C_j^θ can flexibly depend on location j and type θ (Equation 5).

net supply of these transfers is zero. Π^θ is the income from fixed factors for type θ households.

Households choose a location that maximizes their utility. The households' optimal location choice conditional on their type θ and location preferences $\boldsymbol{\epsilon}^\theta$ solves

$$m^\theta(\boldsymbol{\epsilon}^\theta) \in \arg\max_{m \in \mathcal{N}} u_m(C_m^\theta) + \epsilon_m^\theta. \tag{3}$$

Importantly, we do not make any parametric assumptions for the distribution of $\boldsymbol{\epsilon}^\theta$ beyond the regularity condition that they have a strictly positive density everywhere on $\mathbb{R}^N$ or are degenerate. This specification nests commonly used assumptions about location decisions in the literature. For example, Rosen (1979), Roback (1982), and Allen and Arkolakis (2014) consider a case without idiosyncratic location preference, i.e. where ϵ_m^θ is degenerate for all m; Diamond (2016) considers a case where ϵ_m^θ is distributed according to an i.i.d. type-I extreme value distribution across locations; and McFadden (1978) considers a case where $\boldsymbol{\epsilon}^\theta$ is distributed according to a generalized extreme value distribution with arbitrary correlation across alternatives. By aggregating across idiosyncratic location preferences, the population size in location j of type θ is given by

$$l_j^\theta = \ell^\theta \mu_j^\theta, \qquad \mu_j^\theta = \int_{\boldsymbol{\epsilon}} \mathbb{I}\left[j = m^\theta(\boldsymbol{\epsilon}^\theta)\right] dG^\theta(\boldsymbol{\epsilon}^\theta), \tag{4}$$

where μ_j^θ is the probability that type θ households choose location j, $\mathbb{I}\left[j = m^\theta(\boldsymbol{\epsilon}^\theta)\right]$ is an indicator function signifying if the households with idiosyncratic location preferences $\boldsymbol{\epsilon}^\theta$ choose location j, and $G^\theta(\boldsymbol{\epsilon}^\theta)$ is the distribution function of idiosyncratic location preferences $\boldsymbol{\epsilon}^\theta$.

Final goods for type θ households in location j are produced using a constant returns to scale technology over intermediate goods

$$C_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta), \tag{5}$$

where $\mathbf{c}_j^\theta \equiv \{c_{ij,k}^\theta\}_{i,k}$ denotes a vector of intermediate goods used for final goods production, where k indexes intermediate goods and i indexes the origin location of these intermediate goods.

Intermediate good k produced in location i and sold in location j is produced using the following technology

$$y_{ij,k} = \mathcal{A}_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}), \tag{6}$$

where $\mathbf{l}_{ij,k} \equiv \{l_{ij,k}^\theta\}_\theta$ denotes labor inputs, $h_{ij,k}$ denotes the local fixed factor input, $\mathcal{A}_{ij,k}$ is

Hicks-neutral productivity (including iceberg trade costs), $f_{ij,k}$ is a production function (which we assume to be strictly increasing, concave, differentiable, and constant returns), and $\mathbf{x}_{ij,k} \equiv \{x_{ij,k}^{l,m}\}_{l,m}$ denotes a vector of intermediate inputs, where m indexes the intermediate goods for inputs and l indexes the origin location.[4]

We assume that the supply of the local fixed factor at location j is given exogenously by $\bar{h}_j$. We assume that each type θ household owns α^θ share of fixed factors, where $\sum_\theta \ell^\theta \alpha^\theta = 1$. We also denote the price of the local fixed factor by r_j. Then, the aggregate per-capita return from the fixed factor for a type θ household is given by

$$\Pi^\theta = \alpha^\theta \sum_j r_j \bar{h}_j. \tag{7}$$

The government budget constraint is

$$\sum_\theta \sum_j T_j^\theta l_j^\theta = 0. \tag{8}$$

Finally, we assume that productivity $\{\mathcal{A}_{ij,k}\}$ is subject to agglomeration spillovers depending on the local population of various household types:[5]

$$\mathcal{A}_{ij,k} = A_{ij,k} g_{ij,k}(\{l_i^\theta\}_\theta), \tag{9}$$

where $g_{ij,k}(\cdot)$ are the spillover functions, and $A_{ij,k}$ is the fundamental component of productivity. Note that we allow for a flexible functional form for spillovers arising from the population size of different household types θ for different locations and goods i, j, k. We denote the elasticity of agglomeration spillovers as

$$\gamma_{ij,k}^\theta \equiv \frac{\partial \ln g_{ij,k}}{\partial \ln l_i^\theta}. \tag{10}$$

We define the decentralized equilibrium of this economy as follows.

Definition 1 (Decentralized Equilibrium)**.** A decentralized equilibrium consists of prices $\{\{P_j^\theta, w_j^\theta\}, \{p_{ij,k}\}, r_j\}$, quantities $\{\{C_j^\theta, \mathbf{c}_j^\theta, \mu_j^\theta, l_j^\theta\}, \{\mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}\}\}$, transfers $\{T_j^\theta\}$, and productivities

[4]Our framework can encompass the case with decreasing returns to scale production function by interpreting the fixed factor $h_{ij,k}$ as a fictitious factor receiving profit.

[5]By interpreting some intermediate goods k as type θ's labor services, this specification nests general agglomeration spillovers from type θ to another type $\tilde{\theta}$'s labor productivity, nesting the framework of Fajgelbaum and Gaubert (2020). In Section 4.4, we consider the agglomeration externalities that depend beyond local population size, e.g. introducing cross-region productivity spillovers (e.g. Ahlfeldt, Redding, Sturm, and Wolf 2015) or agglomeration/congestion externality specific to a sector's inputs and outputs (e.g. Allen and Arkolakis 2022).

$\{\mathcal{A}_{ij,k}\}$ such that:

(i) $\{C_j^\theta\}$ satisfies households' budget constraint (2), and $\{\mu_j^\theta, l_j^\theta\}$ solves households' optimal location choice problem (3) and (4);

(ii) Firms maximize profits

$$\mathbf{c}_j^\theta \in \arg\max_{\tilde{\mathbf{c}}_j^\theta} P_j^\theta \mathcal{C}_j^\theta(\tilde{\mathbf{c}}_j^\theta) - \sum_{i,k} p_{ij,k}\tilde{c}_{ij,k}^\theta \tag{11}$$

and

$$\begin{aligned}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \in \arg\max_{\tilde{\mathbf{l}}_{ij,k},\tilde{h}_{ij,k},\tilde{\mathbf{x}}_{ij,k}} \; & p_{ij,k}\mathcal{A}_{ij,k}f_{ij,k}(\tilde{\mathbf{l}}_{ij,k}, \tilde{h}_{ij,k}, \tilde{\mathbf{x}}_{ij,k}) \\ & - \sum_\theta w_i^\theta \tilde{l}_{ij,k}^\theta - r_i\tilde{h}_{ij,k} - \sum_{l,m} p_{li,m}\tilde{x}_{ij,k}^{l,m};\end{aligned} \tag{12}$$

(iii) Goods markets clear

$$\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{jl,m}^{i,k} = \mathcal{A}_{ij,k}f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \tag{13}$$

$$C_j^\theta \ell^\theta \mu_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta); \tag{14}$$

(iv) Labor markets clear

$$\sum_{i,k} l_{ji,k}^\theta = \ell^\theta \mu_j^\theta; \tag{15}$$

(iv) Fixed factor markets clear

$$\sum_{i,k} h_{ji,k} = \bar{h}_j; \tag{16}$$

(v) Aggregate factor payments Π^θ satisfy (7);

(vi) The government budget constraint (8) holds;

(vii) Productivity $\{\mathcal{A}_{ij,k}\}$ is subject to agglomeration spillovers given by (9).

Throughout the paper, we focus on the case where the decentralized equilibrium is unique and interior ($l_j^\theta > 0$ for all j and θ). Since our approach relies on a first-order approximation, this assumption avoids dealing with the case where equilibrium outcomes are non-differentiable with respect to the shock.[6]

2.2 Aggregate Welfare

To define aggregate welfare, we consider a social welfare criterion that is represented by the following social welfare function (SWF):

$$W = \mathcal{W}(\{W^\theta\}), \qquad W^\theta \equiv \mathbb{E}[\max_j \{u_j(C_j^\theta) + \epsilon_j^\theta\}]. \tag{17}$$

We refer to the welfare weight of type θ households Λ^θ as the marginal value of their expected utility to aggregate welfare, relative to their population size:

$$\Lambda^\theta \equiv \frac{\partial \mathcal{W}(\{W^\theta\})}{\partial W^\theta} \frac{1}{\ell^\theta}. \tag{18}$$

Our social welfare function (17) assumes utilitarianism with respect to additive idiosyncratic location preferences ϵ_j^θ but allows for arbitrary comparisons across the expected utility of different household types θ.

While this is the standard notion of aggregate welfare adopted in almost all applied work (e.g. Allen and Arkolakis 2014, Redding and Rossi-Hansberg 2017), imposing utilitarianism with respect to idiosyncratic location preferences is a non-trivial assumption. Harsanyi (1955) motivates this SWF by viewing idiosyncratic preferences as the ex-post realizations of "birth lottery". However, this view has been criticized on the ground that it is unclear whose preferences one should use to evaluate this "birth lottery" if preferences are different ex-post.[7]

Alternatively, Eden (2020a) provide axiomitization of this SWF under the interpretation that ϵ_j^θ reflects ex-ante preference heterogeneity among individuals. Instead of considering a "birth lottery", she provides a set of axioms that lead to a utilitarian SWF, which we extend to multiple household types θ in Appendix B. The three axioms are (1) independence of irrelevant variations (IIV; i.e. social welfare criteria only depend on the distribution of individual welfare gains but not directly on idiosyncratic preferences ϵ_j^θ); (2) weak anonymity within θ (i.e. social welfare

[6] See Allen and Arkolakis (2014) and Allen, Arkolakis, and Li (2020) for sufficient conditions for equilibrium uniqueness in spatial equilibrium models.

[7] See Grant, Kajii, Polak, and Safra (2010) and Eden (2020b) for recent discussions on this topic.

criteria should not depend on individual identities within type θ); and (3) Pareto condition (i.e. social welfare criteria are weakly higher for allocations where all individual weakly prefer those allocations).

Interestingly, Eden (2020a) also shows that under these axioms there *has to be* a utility function representation that takes the *additive* form of idiosyncratic location preferences (1).[8] For this reason, we mainly focus on the additive specification and discuss the implications of relaxing this assumption in Section 4.1.

While we view our SWF and its underlying axioms as plausible, one may wish to consider alternative aggregate welfare criteria. We revisit this point in Section 4.3.

2.3 Useful Representation Lemmas

We present two lemmas that will be useful later. First, we introduce a convenient alternative representation of location choice decisions. Following Hofbauer and Sandholm (2002), the discrete location choice decision under additive idiosyncratic preferences (1) can be isomorphically represented by households jointly choosing the population size subject to a cost function, as summarized in the following lemma:

Lemma 1 (Hofbauer and Sandholm 2002)**.** *The share of type θ households living in each location $\{\mu_j^\theta\}_j$ can be represented as the solution to the following problem given a vector of equilibrium consumption $\{C_j^\theta\}_j$:*

$$\begin{aligned} W^\theta = & \max_{\{\mu_j^\theta\}_j} \sum_j \mu_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}) \\ \textit{s.t.} \quad & \sum_j \mu_j^\theta = 1 \end{aligned} \tag{19}$$

for some function $\psi^\theta(\{\mu_j^\theta\})$, which we provide an explicit expression for in Appendix A.1. Moreover, W^θ coincides with the expected utility with respect to $\{\epsilon_j^\theta\}_j$ in (17), i.e. $W^\theta = \mathbb{E}[\max_j\{u_j(C_j^\theta)+\epsilon_j^\theta\}]$.

The detailed proofs of this lemma and the subsequent propositions of this paper are found in Appendix A. Importantly, $\psi^\theta(\{\mu_j^\theta\})$ summarizes the influence of idiosyncratic location preferences on households' location decisions. If there are no idiosyncratic location preferences, we have $\psi^\theta(\{\mu_j^\theta\}) = 0$. If idiosyncratic location preference follow an i.i.d. type-I extreme value

[8] Intuitively, it is difficult to make interpersonal welfare comparisons if households with different ϵ_j^θ disagree on the welfare gains depending on which location's final goods the gains are evaluated by. See Gaubert et al. (2021) and Davis and Gregory (2021) for a related discussion about the challenges in deriving welfare implications with non-additive specification.

distribution with shape parameter ν, then $\psi^\theta(\{\mu_j^\theta\}) = \frac{1}{\nu}\sum_j \mu_j^\theta \ln \mu_j^\theta$ (Anderson, De Palma, and Thisse 1988). When $\{\epsilon_j^\theta\}_j$ follow a type-I generalized extreme value (GEV) with arbitrary correlations (i.e. McFadden 1978), we show in Appendix D that $\psi^\theta(\{\mu_j^\theta\}) = \frac{1}{\nu}\sum_j \mu_j^\theta \ln S_j^\theta(\{\mu_i^\theta\})$, where function $S_j^\theta(\cdot)$ depends on the correlation function of $\{\epsilon_j^\theta\}_j$ across alternatives j.[9]

Second, the following lemma shows that the decentralized equilibrium allocation can be represented as the solution to a "pseudo-planning" problem.

Lemma 2. *Any decentralized equilibrium allocation* $\{\{\check{C}_j^\theta, \check{\mathbf{c}}_j^\theta, \check{\mu}_j^\theta, \check{l}_j^\theta\}, \{\check{\mathbf{x}}_{ij,k}, \check{\mathbf{l}}_{ij,k}, \check{h}_{ij,k}, \check{\mathcal{A}}_{ij,k}\}\}$ *solves the following pseudo-planning problem*

$$W = \max_{\{W^\theta, \{C_j^\theta, \mathbf{c}_j^\theta, \mu_j^\theta\}\}, \{\mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mathcal{A}_{ij,k}\}} \mathcal{W}(\{W^\theta\}) \tag{20}$$

subject to (9), (13)-(16),

$$W^\theta = \sum_j \mu_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}) \tag{21}$$

$$\{\mu_j^\theta\}_j \in \arg\max_{\{\tilde{\mu}_j^\theta\}_j : \sum_j \tilde{\mu}_j^\theta = 1} \sum_j \tilde{\mu}_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}) \tag{22}$$

$$C_j^\theta = \check{C}_j^\theta. \tag{23}$$

The objective function is aggregate welfare. Constraint (9) stipulates the spillover functions, while constraints (13)-(16) correspond to resource constraints. Constraint (22) imposes that location choices are incentive compatible, and (23) restricts consumption to be equal to its equilibrium value. When constraints (22) and (23) are slack, this problem coincides with the first-best planning problem (see Appendix C for details), where the Planner maximizes aggregate welfare subject to resource constraints. In this case, the envelope theorem implies that the welfare effects of technology shocks are summarized by their sales, as in Hulten (1978). However, as we show below, constraints (22) and (23) are not slack in general, which implies Hulten's theorem cannot be applied to spatial equilibrium models.

3 Theoretical Analysis

This section provides our theoretical results on the first-order effects of spatially disaggregated technological shocks on aggregate welfare. Section 3.1 provides a nonparametric formula for

[9] An alternative interpretation of $\psi^\theta(\cdot)$ is that it captures congestion externalities. For example, the model with idiosyncratic location preferences following an i.i.d. type-I extreme value distribution with shape parameter ν is isomorphic to a model without idiosyncratic location preferences where utility is given by $u_j(C_j^\theta) - \frac{1}{\nu}\ln \mu_j^\theta$. See Appendix E.3 for further discussion about this isomorphism.

aggregate welfare changes. Section 3.2 discusses the data moments that are required to assess welfare changes. Section 3.3 develops an optimal spatial transfer formula and shows how such transfers influence the impact of technology shocks on aggregate welfare. Section 3.4 provides a nonparametric formula for aggregate GDP changes and discusses its difference from aggregate welfare.

3.1 Unpacking Welfare Effects of Disaggregated Shocks

For expositional purposes, we introduce the following expectation and covariance operators. The first set of operators takes the expectation and covariance of statistics associated with location j for a given type θ household, weighted by location share μ_j^θ:

$$\mathbb{E}_{j|\theta}[X_j^\theta] \equiv \sum_j \mu_j^\theta X_j^\theta, \qquad \mathrm{Cov}_{j|\theta}(X_j^\theta, Y_j^\theta) \equiv \mathbb{E}_{j|\theta}[X_j^\theta Y_j^\theta] - \mathbb{E}_{j|\theta}[X_j^\theta]\mathbb{E}_{j|\theta}[Y_j^\theta]. \tag{24}$$

The second set of operators takes the expectation and covariance of statistics associated with type θ households, weighted by population share ℓ^θ:

$$\mathbb{E}_\theta[X^\theta] \equiv \sum_\theta \ell^\theta X^\theta, \qquad \mathrm{Cov}_\theta(X^\theta, Y^\theta) \equiv \mathbb{E}_\theta[X^\theta Y^\theta] - \mathbb{E}_\theta[X^\theta]\mathbb{E}_\theta[Y^\theta]. \tag{25}$$

We also scale the SWF so that the population-weighted average of Pareto weights coincides with the weighted average of the inverse of the marginal utility of income $u_j'(C_j^\theta)/P_j^\theta$:

$$\mathbb{E}_\theta\left[\Lambda^\theta\right] = \mathbb{E}_\theta\left[\mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right]\right]. \tag{26}$$

Now consider small changes in the exogenous components of productivity specific to origin location, destination location, and sector: $\{d\ln A_{ij,k}\}$. These shocks can represent region-sector TFP shocks (e.g. Caliendo et al. 2018) or transportation infrastructure changes (e.g. Allen and Arkolakis 2014, Donaldson and Hornbeck 2016).[10] We also allow for the possibility that the structure of transfers may change simultaneously, denoted by $\{dT_j^\theta\}$, either because of exogenous policy changes or as an endogenous response to the productivity shocks.

[10]In some context, researchers are interested in the shocks to amenities instead of productivity. Our analysis includes those cases by interpreting some intermediate goods as local amenities. From a measurement perspective, applying Proposition 1 requires knowledge of prices of the amenities, which is often unobserved and needs to be calibrated or estimated. For example, if transportation infrastructure also brings amenity benefits by shortening commuting time, one can use the value of time for $p_{ij,k}$ and commuting time for $y_{ij,k}$ (i.e. Small and Verhoef 2007). In Section 4.2, we provide an alternative expression for Proposition 1 without using amenity prices.

By applying the envelope theorem to the pseudo-planning problem of Lemma 2, we obtain the following expression for welfare changes:

Proposition 1. *Consider an arbitrary set of small shocks to the exogenous components of productivity* $\{d\ln A_{ij,k}\}$*, as well as changes in transfers* $\{dT_j^\theta\}$*, in a decentralized equilibrium. The first-order impact on welfare can be expressed as*

$$\begin{aligned} dW = & \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k}}_{\text{(i) } \Delta \text{ Technology } (\Omega_T)} + \underbrace{\mathbb{E}_\theta\left[\mathit{Cov}_{j|\theta}\left(-\frac{P_j^\theta}{u_j'(C_j^\theta)}, u_j'(C_j^\theta) dC_j^\theta\right)\right]}_{\text{(ii) } \Delta \text{ MU Dispersion } (\Omega_{MU})} \\ & + \underbrace{\mathbb{E}_\theta\left[\mathit{Cov}_{j|\theta}\left(-T_j^\theta, d\ln l_j^\theta\right)\right]}_{\text{(iii) } \Delta \text{ Fiscal Externality } (\Omega_{FE})} + \underbrace{\mathbb{E}_\theta\left[\mathit{Cov}_{j|\theta}\left(\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta, d\ln l_j^\theta\right)\right]}_{\text{(iv) } \Delta \text{ Technological Externality } (\Omega_{TE})} \\ & + \underbrace{\mathit{Cov}_\theta\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right], \mathbb{E}_{j|\theta}\left[u_j'(C_j^\theta) dC_j^\theta\right]\right)}_{\text{(v) } \Delta \text{ Redistribution } (\Omega_R)}. \end{aligned} \tag{27}$$

Below, we explain each term of Proposition 1 and illustrate them through special cases. Table 1 summarizes our formula in those special cases.

Technology, Ω_T**.** The first term of Proposition 1, which we refer to as (i) Δ technology (Ω_T), captures the effects of productivity changes absent the reallocation of resources. Note that $p_{ij,k} y_{ij,k}$ corresponds to the total sales of intermediate inputs k produced in i and sold in j. The observation that total sales summarize the aggregate effects of a shock reflects the celebrated result of Hulten (1978). If the equilibrium maximizes aggregate welfare W, the first term is sufficient for the welfare consequence of disaggregated shocks, to a first-order. However, this is not true in general, and the remaining reallocation terms may become first order.

MU Dispersion, Ω_{MU}**.** The second term, which we refer to as (ii) Δ MU (marginal utility) dispersion (Ω_{MU}), captures the fact that shocks reallocate resources across locations that potentially differ in their marginal utility of income. A shock leads to an increase in utility of $u_j'(C_j^\theta) dC_j^\theta$ in each location j for type θ households. The covariance is positive if utility changes are higher in locations with a lower inverse marginal utility of income $P_j^\theta / u_j'(C_j^\theta)$, which can be interpreted as the resource cost required to increase utility of type θ household in location j by one unit. The expectation $\mathbb{E}_\theta[\cdot]$ takes the weighted average of this covariance across household types θ.

This term is generally non-zero if the marginal utility of income is not equalized across locations, and this is precisely what happens in spatial equilibrium. In equilibrium, agents make location decisions based on utility *levels* (inclusive of idiosyncratic location preference). This implies that the *marginal* utility of income is not necessarily equalized across these locations for household type θ. In fact, marginal utility $u_j'(C_j^\theta)$ never shows up in any of the equilibrium conditions.

There are two ways to interpret the dispersion in marginal utility of income. The first interpretation is the lack of insurance for the uncertainty associated with location choice. Depending on their preference draws, or depending on the random location assignment in the absence of idiosyncratic location preference, individual households may end up in a variety of locations that differ in the marginal utility of income. The second interpretation is the lack of redistribution across households with differing residential locations but with the same household type θ. The SWF (17) puts equal weight on individual utility within type θ. This implies that aggregate welfare increases by reallocating consumption toward individuals with higher marginal utility (net of resource cost).

In certain special cases, MU dispersion is absent in the spatial equilibrium. For example, this case arises under linear utility (i.e. $u_j(C_j^\theta) = C_j^\theta$)[11] *and* no trade frictions such that final good prices P_j^θ are equalized across locations j, as considered by Kline and Moretti (2014). Alternatively, if utility functions are logarithmic $u_j(C_j^\theta) = \ln C_j^\theta$, spatial transfers vary only by household type $T_j^\theta = T^\theta$, *and* nominal wages are equalized across locations $w_j^\theta = w^\theta$, spatial dispersion in the marginal utility of income is absent: $u_j'(\frac{w^\theta + T^\theta + \Pi^\theta}{P_j^\theta})/P_j^\theta = \frac{1}{w^\theta + T^\theta + \Pi^\theta}$.

An interesting case is an environment without idiosyncratic location preference as in the tradition of Rosen (1979) and Roback (1982). As originally observed by Mirrlees (1972), even without idiosyncratic location preference, equilibrium still involves spatial dispersion in the marginal utility of income. However, the *changes* in MU dispersion are always zero in this case. To see why, notice that utility levels are always equalized across locations, and therefore shocks shift the utility levels in all locations (with positive population) by the same amount, i.e. $u_j'(C_j^\theta)dC_j^\theta = dW^\theta$. Therefore, the covariance inside Ω_{MU} is always zero. In other words, shocks fail to close any gap in marginal utility across locations in the absence of idiosyncratic location preferences.

Fiscal Externality, Ω_{FE}. The third term, which we refer to as (iii) Δ fiscal externality (Ω_{FE}), comes from the fact that shocks affect the government's budget. If a shock induces population movement toward a location that pays taxes on net (higher $-T_j^\theta$), this term has a positive effect

[11] Note that additive location-and-type-specific amenity shifter can be incorporated through ϵ_j^θ.

Table 1: Formula for Aggregate Welfare Changes in Special Cases

	Ω_T	Ω_{MU}	Ω_{FE}	Ω_{TE}	Ω_R
1. Linear utility and no trade frictions	✓		✓	✓	✓
2. No idiosyncratic location preference	✓		✓	✓	✓
3. No location-specific transfers	✓	✓		✓	✓
4. No technological externalities	✓	✓	✓		✓
5. Single type	✓	✓	✓	✓	
6. No population mobility	✓				✓
7. If optimal spatial transfers are in place	✓				

on welfare.[12] This term is absent whenever there are no location-specific transfers within types θ ($T_j^\theta = T^\theta$ for all j and θ) or the shock does not induce any labor reallocation ($d \ln l_j^\theta = 0$ for all j and θ).

Technological Externality, Ω_{TE}. The fourth term, which we refer to as (iv) Δ technological externality (Ω_{TE}), captures agglomeration externalities in productivity. If a shock induces the population to move toward a location with a higher agglomeration externality $\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta$, this term has a positive effect on welfare. Clearly, this term becomes zero if there are no technological externalities in the pre-shock equilibrium, i.e. $\gamma_{ij,k}^\theta = 0$ for all i, j, k, and θ.

Importantly, assuming constant elasticity agglomeration externalities ($\gamma_{ij,k}^\theta = \gamma$) alone does not ensure that the (iv) technological externality term is zero. To see why, observe that in a special case with a single sector, single type, no fixed factor, and no intermediate inputs in production, $\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{ij,k}^\theta$ simplifies to $w_j \gamma$. Therefore, reallocating the population toward a location with a higher nominal wage generates positive effects on aggregate welfare, as originally observed by Fajgelbaum and Gaubert (2020).

Redistribution, Ω_R. The fifth term, which we refer to as (v) Δ redistribution (Ω_R), is the covariance between the marginal increase in expected utility of type θ households $\mathbb{E}_{j|\theta}[u_j'(C_j^\theta) dC_j^\theta]$ and the utility weight net of expected resource cost on those household types, $\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right]$. Obviously, with a single household type, the (v) Δ redistribution term becomes zero.

[12] In some existing models, researchers assume that some fraction of fixed factor income is rebated to local households directly (such as through local governments' ownership of local fixed factors), which implies that Π_i^θ depends on i (e.g. Caliendo et al. 2018). In such cases, the fiscal externality term is simply modified to capture these local rebates, i.e. replacing the first term in the covariance of Ω_{FE} by $-(T_j^\theta + \Pi_j^\theta)$.

No Population Mobility. Our formula nests the case without population mobility by setting $S = N$ and having type θ_i households always locate themselves in location i: $\mu_i^{\theta_i} = 1$. If the population is immobile ($d \ln l_j^\theta = 0$ for all j and θ), then the (ii) Δ MU dispersion, (iii) Δ fiscal externality, and (iv) Δ technological externality terms all become zero.

3.2 Identifying Welfare Changes from Data

Proposition 1 also clarifies the data moments necessary for measuring aggregate welfare changes. A celebrated feature of Fogel/Hulten's analysis is that it only requires the changes in productivity $\{d \ln A_{ij,k}\}$ and pre-shock sales of goods $\{p_{ij,k} y_{ij,k}\}$, evident from our (i) Δ technology term. While it has been acknowledged that, outside of Fogel/Hulten's case, the data requirement for welfare analysis is in general more demanding, it has not been clear what equilibrium objects and structural parameters we need to draw welfare conclusions in spatial equilibrium models. Our formula makes them precise.

In particular, the reallocation terms (ii)-(v) clarifies that welfare changes additionally depend on data for pre-shock final goods prices $\{P_j^\theta\}$, spatial transfers $\{T_j^\theta\}$, and the changes in consumption and population $\{dC_j^\theta, d \ln l_j^\theta\}$.[13] In terms of the structural objects, we additionally need welfare weights $\{\Lambda^\theta\}$, agglomeration externality elasticities $\{\gamma_{ij,k}^\theta\}$, and the spatial dispersion of marginal utility $\{u_j'(C_j^\theta)\}$, evaluated around the baseline equilibrium. Choice of $\{\Lambda^\theta\}$ is a normative consideration that cannot be inferred from agents' equilibrium behavior, but we argue below that the two other objects can be nonparametrically identified with suitable exogenous variations.[14]

For the agglomeration externality elasticities $\{\gamma_{ij,k}^\theta\}$, identification requires the causal effect of exogenous population changes on productivity. The long-standing literature on agglomeration economies has extensively sought to identify these parameters.[15]

The spatial dispersion of marginal utility can be nonparametrically identified from location choice decisions. Denote the location choice probability by $\{\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)\}$ as a solution to Equation (19), where $\boldsymbol{C}^\theta$ is a vector of consumption across locations faced by household type θ. Suppose that we have exogenous variation in the consumption of each location so that we can credibly

[13] Existing research indicates that $\{dC_j^\theta\}$ and $\{d \ln l_j^\theta\}$ in response to counterfactual shocks $\{d \ln A_{ij,k}\}$ can be nonparametrically identified. In fact, they are uniquely determined by the factor supply (location choice) system, whose identification is established by Berry and Haile (2014) in general discrete choice models, and the factor demand system, whose identification is established by Adão, Costinot, and Donaldson (2017) in general multi-region trade models.

[14] One can potentially reveal $\{\Lambda^\theta\}$ by assuming that observed policies, e.g. $\{T_j^\theta\}$ in our context, are set optimally. See Adão, Costinot, Donaldson, and Sturm (2023) for such an exercise in the context of import tariffs.

[15] See Melo, Graham, and Noland (2009) for a meta-analysis of the estimates of agglomeration externalities.

identify this location choice system.[16] Then, we can identify the relative marginal utility using Lemma 1, as originally suggested by Allen and Rehbeck (2019):

$$\frac{u_j'(C_j^\theta)}{u_i'(C_i^\theta)} = \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} / \frac{\partial \hat{\mu}_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta}. \tag{28}$$

Crucially, marginal utility is nonparametrically identified separately from the idiosyncratic location preferences ϵ_j^θ.[17]

While it is reassuring that these structural parameters are in principle nonparametrically identified, implementing nonparametric estimation is unrealistic due the limited availability of exogenous variation in most applications.[18] Still, this discussion helps to clarify that the dispersion of marginal utility can be identified from location choice decisions. In our application to the US highway network in Section 5, we follow this intuition to estimate the curvature of the utility function.

3.3 Design of Place-Based Policies

In this section, we show that Proposition 1 has broader implications for the design of optimal spatial transfer policy and how such policies interact with the welfare effects of technology shocks.

3.3.1 General Optimal Spatial Transfer Formula

So far, we have focused on how technological shocks or transfer policies affect aggregate welfare. This analysis can also elucidate the optimal design of spatial transfers. The optimality of spatial transfers requires that any marginal change in consumption dC_i^θ induced by a policy reform cannot improve welfare. Imposing this requirement in Proposition 1 for the case where all technology shocks are zero implies that $dW = \Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R = 0$, where $\Omega_T = 0$. Reformulating this, we have the following formula that optimal spatial transfer must satisfy:

Proposition 2. *Assume that idiosyncratic location preferences are non-degenerate. The optimal*

[16] Berry and Haile (2014) establish nonparametric identification of such discrete choice systems.

[17] Bhattacharya (2015) provides a related but distinct nonparametric identification result of the welfare changes in discrete choice models. In defining equivalent variation, they formulate that compensation can flexibly depend on idiosyncratic preferences, and hence, marginal utility does not play a role. The above result indicates that nonparametric identification can be achieved even if we relax this stringent compensation scheme.

[18] For example, identifying the factor supply system $\{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)/\partial C_j^\theta\}$ for all i and j requires a long period of data and exogenous variation in consumption at every location.

spatial transfers must satisfy

$$\mu_i^\theta \left[\Lambda^\theta u_i'(C_i^\theta) - P_i^\theta\right] = Cov_{j|\theta}\left(T_j^\theta - \frac{1}{l_j^\theta}\sum_{l,k} p_{jl,k} y_{jl,k}\gamma_{jl,k}^\theta, \frac{\partial \ln \mu_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta}\right) \quad \textit{for all } i, \theta. \tag{29}$$

This proposition reveals the key trade-offs associated with optimal spatial transfer policy.[19] The left-hand side of this expression summarizes the marginal benefit from transferring one unit of consumption to location i for type θ. In particular, if weighted marginal utility $\Lambda^\theta u_i'(C_i^\theta)$ is high and the associated price P_i^θ is low in location i relative to other locations, the net benefit of transfers to location i tends to be high. On the right-hand side of this equation, we summarize the marginal cost of this transfer through fiscal and technological externalities. In particular, a unit increase of consumption in location i increases population by $\frac{\partial \ln \hat{\mu}_j^\theta}{\partial C_i^\theta}$ in location j. Notice that this relocation happens in all locations, not only in location i. This population relocation is associated with fiscal externality T_j^θ and technological externality $\sum_{l,k} p_{jl,k} y_{jl,k}\frac{1}{l_j^\theta}\gamma_{jl,k}^\theta$.

The above formula has a strong connection to the optimal unemployment insurance literature (Baily 1978, Chetty 2006). In fact, our formula (29) resembles what is often called the Baily-Chetty formula, which balances the trade-off between closing the gap in marginal utility across employed and unemployed workers against generating fiscal externalities by discouraging job search. Relative to the optimal unemployment insurance formula, aside from the obvious differences in context, our formula differs in that it incorporates many possible location choices and the cost includes technological externalities in addition to fiscal externalities.

Proposition 2 is a strict generalization of Fajgelbaum and Gaubert (2020), who study the same problem in a special case where there are no idiosyncratic location preferences.[20] In particular, if we take the limit of the variance of idiosyncratic location preferences to zero, hence $|\frac{\partial \ln \mu_i^\theta}{\partial C_j^\theta}| \to \infty$, the only way to satisfy Equation (29) is to set $T_j^\theta - \frac{1}{l_j^\theta}\sum_{l,k} p_{jl,k} y_{jl,k}\gamma_{jl,k}^\theta = E^\theta$ for some constant E^θ, recovering their formula. Proposition 2 also provides a strict generalization of Ales and Sleet (2022) by introducing a rich production economy, cross-regional trade, and agglomeration externalities. In particular, in the absence of trade costs and agglomeration externalities, P_i^θ drops

[19]To see the intuition, note that a perturbation of dC_i^θ leads to $\Omega_{MU} = -\ell^\theta\mu_i^\theta P_i^\theta dC_i^\theta + \mathbb{E}_{j|\theta}\left[P_j^\theta/u_j'(C_j^\theta)\right]\ell^\theta\mu_i^\theta u_i'(C_i^\theta)dC_i^\theta$, and $\Omega_R = \ell^\theta\mu_i^\theta\Lambda^\theta u_i'(C_i^\theta)dC_i^\theta - \mathbb{E}_{j|\theta}\left[P_j^\theta/u_j'(C_j^\theta)\right]\ell^\theta\mu_i^\theta u_i'(C_i^\theta)dC_i^\theta$. By noting that $d\ln l_j^\theta = \frac{\partial \ln \mu_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta}dC_i^\theta$ and $dW = \Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R = 0$, we obtain Equation (29). See Appendix A.4 for the formal planning problem and the proof. Appendix F discusses how Proposition 2 can be used to test for Pareto efficiency within a current transfer scheme, following the same approach as Werning (2007).

[20]They also consider a case where preferences take the form of $U_j(C_j, \epsilon_j) = \tilde{\epsilon}_j C_j$ and $\tilde{\epsilon}_j$ follows an independent Frechét distribution. As we show in Section 4.1, this specification is isomorphic to its log-transformation $\ln C_j + \epsilon_j$, where ϵ_j follows an i.i.d. type-I extreme value distribution. Consequently, our formula nests this case as well.

out from the left-hand side as a contributor to marginal utility dispersion, and agglomeration externalities are zero $\gamma^{\theta}_{jl,k} = 0$ on the right-hand side, recovering their formula.

3.3.2 Impacts of Technology Shocks when Optimal Spatial Transfers are in Place

We now revisit the welfare effects of technology shocks when the government implements optimal spatial transfers in the pre-shock equilibrium. For example, when the government designs transportation infrastructure (e.g. Fajgelbaum and Schaal 2020, Bordeu 2023), it might simultaneously optimize spatial transfer schemes. The following proposition provides a sharp prediction in such circumstances.

Proposition 3. *Suppose that transfers $\{T^{\theta}_j\}$ are set so that Proposition 2 holds in the pre-shock equilibrium. Then, Proposition 1 comes down to*

$$dW = \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d \ln A_{ij,k}}_{\text{(i) } \Delta \text{ Technology } (\Omega_T)} . \tag{30}$$

This result follows precisely because the prevailing optimal transfers balance the reallocation terms, $\Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R = 0$. As a result, reallocation in response to the shocks jointly has no first-order impact on aggregate welfare. Despite reaching the same conclusion as Hulten (1978), the underlying mechanisms are quite different. In a frictionless representative agent economy, any reallocation of resources has no first-order impact on aggregate welfare. In Proposition 3, the reallocation terms sum to zero, although each term is not necessarily zero. This observation resonates with Costinot and Werning (2018), who study the impact of new technologies with distributional consequences and also find that Hulten's characterization holds under second-best policy.

A flip side of Proposition 3 is that the deviation from the Δ technology term found in applied work (e.g. Caliendo et al. 2018) is attributable to the suboptimality of prevailing spatial transfers. In particular, if shocks reallocate consumption toward a location that receives less (more) transfers than what is prescribed by (29), the Δ technology term over-predicts (under-predicts) aggregate welfare changes. We also show that this deviation is relevant in our application to the US highway network in Section 5.

3.4 Aggregate Output

So far, we have focused on changes in aggregate welfare. A related but distinct question is how spatially disaggregated shocks affect aggregate output. In spatial equilibrium contexts, it is well known that aggregate welfare and output depart due to compensating differentials. While we are, of course, not the first to observe this distinction, our formula clarifies the precise sources of departure.

Let us define the change in real GDP of location i by

$$d\ln Y_i = d\ln GDP_i - d\ln P_i, \tag{31}$$

where GDP_i is nominal GDP in location i. Following Baqaee and Farhi (2024), we define aggregate real GDP using an aggregate price index weighted by each location's nominal GDP, i.e. Divisia indices:

$$d\ln Y = d\ln GDP - d\ln P, \qquad d\ln P = \sum_i \frac{GDP_i}{GDP} d\ln P_i, \tag{32}$$

where $GDP = \sum_i GDP_i$ corresponds to aggregate nominal GDP. For notational convenience, we take aggregate nominal GDP as the numeraire, so $GDP = 1$. The following proposition establishes the first-order effects of shocks or transfers on aggregate real GDP.

Proposition 4. *Consider an arbitrary set of small shocks to the exogenous components of productivity $\{d\ln A_{ij,k}\}$, as well as changes in transfers $\{dT_j^\theta\}$, in a decentralized equilibrium. Taking aggregate nominal GDP as the numeraire, the first-order impact on aggregate real GDP can be expressed as*

$$\begin{aligned} dY = & \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k}}_{\text{(i) } \Delta \text{ Technology } (\Omega_T)} + \underbrace{\mathbb{E}_\theta \left[Cov_{j|\theta} \left(-T_j^\theta, d\ln l_j^\theta \right) \right]}_{\text{(iii) } \Delta \text{ Fiscal Externality } (\Omega_{FE})} \\ & + \underbrace{\mathbb{E}_\theta \left[Cov_{j|\theta} \left(\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta, d\ln l_j^\theta \right) \right]}_{\text{(iv) } \Delta \text{ Technological Externality } (\Omega_{TE})} + \underbrace{\mathbb{E}_\theta \left[Cov_{j|\theta} \left(w_j^\theta + T_j^\theta, d\ln l_j^\theta \right) \right]}_{\text{(vi) } \Delta \text{ Compensating Differential } (\Omega_{CD})} . \end{aligned} \tag{33}$$

Relative to Proposition 1, there are both similarities and differences. Similar to the case of aggregate welfare, the (i) Δ technology, (iii) Δ fiscal externality, and (iv) Δ technological externality terms influence aggregate output changes. The presence of technological externalities and spatial transfers, both of which are forms of wedges, implies that population reallocation induces first-order effects on aggregate output in addition to changes in technology, as in Baqaee and Farhi (2020).

Different from the case of aggregate welfare, the two terms, (ii) Δ MU dispersion and (v) Δ redistribution, are absent, and there is a new term, (vi) Δ compensating differential (Ω_{CD}). The absence of MU dispersion and redistribution is intuitive because aggregate output does not take into account households' utility. The new term, (vi) Δ compensating differential, reflects the reallocation of population across locations that differ in terms of their compensating differential. Specifically, when households choose their location, they consider not only nominal income $w_j^\theta + T_j^\theta$, but also utility differences $u_j(C_j^\theta)$ (including amenity), idiosyncratic preferences ϵ_j^θ, and price indices P_j^θ. Unless all of these elements are absent, in which case nominal income is equalized across space, the (vi) Δ compensating differential term is non-zero. For example, this term is positive if a shock induces population movements toward a location with low compensating differentials (high nominal income). Such reallocation is not relevant for welfare but matters for output.

4 Extensions

We will now discuss the scope of our results and argue that our framework can accommodate further extensions and generalizations of our baseline environment.

4.1 Multiplicative Idiosyncratic Location Preference

In our baseline analysis, we have focused on specifications where idiosyncratic location preferences are additively separable. A common alternative specification in the existing literature is the case where idiosyncratic location preferences are multiplicatively separable and follow a max-stable multivariate Fréchet distribution. For concreteness, we assume that there is a single household type and drop θ superscript.[21] Formally, preferences for households living in location j are given by

$$\begin{aligned} &\tilde{U}_j(C_j, \tilde{\epsilon}_j) = \tilde{u}_j(C_j)\tilde{\epsilon}_j, \\ \text{with} \qquad &\mathbb{P}(\tilde{\epsilon}_1 \leq \bar{\epsilon}_1, \dots, \tilde{\epsilon}_N \leq \bar{\epsilon}_N) = \exp(-G(K_1(\bar{\epsilon}_1)^{-\nu}, \dots, K_N(\bar{\epsilon}_N)^{-\nu})), \end{aligned} \tag{34}$$

[21] In Appendix A.4, we prove a generalized version of Proposition 5 to multiple household types, which requires redefining Pareto weights between the two models.

where $G(.)$ is a function that is homogeneous of degree one, which we call the "correlation function". The key feature of this specification is the max-stability property, where the distribution of the maximum is Fréchet with shape parameter ν.[22]

To examine the properties of this specification, consider the log transformation of utility: $u_j(C_j) = \ln(\tilde{u}_j(C_j))$ and $\epsilon_j = \ln(\tilde{\epsilon}_j)$. It is straightforward to show that ϵ_j follows a multivariate Gumbel distribution with the same correlation function $G(\cdot)$ such that

$$U_j(C_j, \epsilon_j) = u_j(C_j) + \epsilon_j, \tag{35}$$
$$\text{with} \quad \mathbb{P}(\epsilon_1 \le \bar{\epsilon}_1, \ldots, \epsilon_N \le \bar{\epsilon}_N) = \exp(-G(K_1(\exp(-\nu\bar{\epsilon}_1)), \ldots, K_N(\exp(-\nu\bar{\epsilon}_N)))).$$

Since $\ln(\cdot)$ is a monotone transformation, the systems (34) and (35) have isomorphic *positive* predictions. The following proposition shows that these two models also deliver isomorphic *normative* predictions.

Proposition 5. *Consider the spatial equilibrium with multiplicative Fréchet idiosyncratic location preferences with arbitrary correlation* (34)*. Let* $\tilde{W} = \mathbb{E}[\max_j\{\tilde{u}_j(C_j)\tilde{\epsilon}_j\}]$ *be aggregate welfare in this economy. Consider another economy where preferences are a log transformation of the first specification, i.e. the additively separable counterpart* (35)*, and the remaining equilibrium conditions of Definition 1 are unchanged. Then, aggregate welfare, as well as the formula of Proposition 1, are identical in both economies up to a multiplicative constant.*

Proposition 5 establishes that an economy with multiplicative Fréchet shocks is isomorphic to its additively separable counterpart in both its positive *and* normative implications. This equivalence arises because of a special property of multivariate Fréchet distributions. To see this, note first that the transformation of the utility function only matters through the differences in marginal utility (see Appendix E.1). The marginal utility of consumption of households of type θ in location j in the system (34) is given by

$$\mathcal{MU}_j = u_j'(C_j)\mathbb{E}\big[\tilde{u}_j(C_j)\tilde{\epsilon}_j \big| j = \arg\max_i \tilde{u}_i(C_i)\tilde{\epsilon}_i\big] = u_j'(C_j)\tilde{W}, \tag{36}$$

where we define $u_j(C_j) = \ln(\tilde{u}_j(C_j))$ and $\epsilon_j = \ln(\tilde{\epsilon}_j)$. The second transformation of Equation (36) follows from the max-stable property of $\tilde{\epsilon}_j$: that the distribution of the maximum follows the

[22]For example, Redding (2016) is a special case with an i.i.d. Fréchet distribution, which corresponds to the case with $G(x_1, \ldots, x_N) = \sum_{j=1}^N x_j$. More generally, this preference specification delivers a generalized extreme value (GEV) demand system with flexible substitution patterns as introduced by McFadden (1978). See also Lind and Ramondo (2023) for the application of this demand system to Ricardian trade models.

same distribution irrespective of the chosen option (McFadden 1978, Lind and Ramondo 2023). Therefore, aggregate welfare implications coincide with the corresponding additive specification.[23]

4.2 Shocks to Amenities and Amenity Externalities

In Section 3, we analyzed the effects of productivity shocks on aggregate welfare. In some contexts, researchers are interested in shocks to amenities, rather than productivity. The analysis in Section 3 allows for this possibility by interpreting some intermediate goods as local amenities. From a measurement perspective, applying Proposition 1 requires knowledge of prices associated with amenities, which are often unobserved. Appendix E.2 shows an alternative expression for Proposition 1 without using prices for amenities. There, prices on amenities are simply replaced with the marginal utility of amenities.

In the quantitative spatial equilibrium literature, researchers often argue that amenity congestion externalities are isomorphic to idiosyncratic location preference and use these specifications interchangeably (e.g. Allen and Arkolakis (2014) for the isomorphism between i.i.d. Fréchet preferences and isoelastic congestion externalities). In Appendix E.3, we establish the precise conditions under which this isomorphism arises. We show that a model with idiosyncratic location preferences that follow a GEV distribution is isomorphic to a model with amenity congestion externalities, by appropriately constructing the externality function using the GEV correlation function.

4.3 Alternative Social Welfare Function

Our baseline analysis focuses on a weighted utilitarian SWF as discussed in Section 2.2. In some contexts, researchers may want to consider an alternative welfare criterion. In Appendix E.4, we consider a general SWF of the following form:

$$W = \mathcal{W}\left(\{\mathcal{U}^{SP,\theta}(\{C_j^\theta, \mu_j^\theta\})\}\right), \tag{37}$$

[23] This isomorphism may look contradictory to Eden (2020a), who claim that idiosyncratic preferences *have to be* additively separable given the form of SWF in Proposition 5 (see Section 2.2). This seeming contradiction can be resolved by observing that the multiplicative specification indeed violates their axioms applied to off-equilibrium allocations (in particular, consumption and location choice pairs that are incompatible with households optimality). If we restrict the allocation space to the equilibrium ones, the preference specifications (36) is consistent with their axioms.

where $\mathcal{U}^{SP,\theta}$ is defined arbitrarily on the distribution of consumption and population of type θ households. Appendix E.4 shows that our formula in Proposition 1 includes one additional term. The term captures the potential misalignment between the social Planner's welfare assessment of the marginal value of consumption and that of private agents.

Such an approach is also useful in considering welfare criteria and optimal policies that are exclusively based on subcomponents of our formula, as in Dávila and Schaab (2022). In Appendix E.4, we explicitly construct social welfare functions that exclusively target subcomponents of our formula and derive optimal spatial transfer formulae in each case.

4.4 General Spillovers

In our baseline model, we assumed that agglomeration externalities are purely a function of local population size (9). In some contexts, researchers specify that a higher population size in the surrounding regions also generates agglomeration spillovers (e.g. Ahlfeldt et al. 2015). In other contexts, researchers also specify that externalities arise from specific producers' input use (e.g. free entry models with labor fixed cost such as Krugman (1991)) or producers' output (e.g. congestion costs from shipment as in Allen and Arkolakis (2022)). In Appendix E.5, we generalize our results by allowing the spillover function (9) to depend on the population in all locations as well as the distribution of production in each location.

4.5 Idiosyncratic Productivity Shocks

In the baseline analysis, we have considered idiosyncratic shocks to preferences only. Some existing work (e.g. Bryan and Morten 2019) considers instead idiosyncratic shocks to household productivity. Under some additional assumptions, we can tractably incorporate idiosyncratic productivity shock in addition to idiosyncratic location preference.

We consider an environment where households of type θ have idiosyncratic productivities in each location $\boldsymbol{z}^\theta \equiv (z_1^\theta, \ldots, z_N^\theta)$ in addition to idiosyncratic location preference $\boldsymbol{\epsilon}^\theta$. The idiosyncratic productivity shocks determine households' endowment of efficiency units of labor in each location. We impose three additional assumptions relative to the baseline model. First, we restrict attention to the case of log utility: $u_j(c) = B_j \ln c + D_j$. Second, we assume away the presence of fixed factors. Third, we assume that location-specific transfers are linear in labor income.[24] With these assumptions, the budget constraint of households living in location j is $P_j^\theta c_j^\theta = (1 + \tau_j^\theta) w_j^\theta z_j^\theta$, where τ_j^θ denotes the transfer rate. We let $C_j^\theta \equiv w_j^\theta (1 + \tau_j^\theta)/P_j^\theta$ denote the

[24]This modification is inconsequential in the baseline model as we can always rewrite $T_j^\theta = \tau_j^\theta w_j^\theta$.

consumption per efficiency unit of labor.

Appendix E.6 shows that, in the above environment, Proposition 1 continues to hold with two modifications. First, C_j^θ now denotes consumption per efficiency unit of labor. Second, the (iii) fiscal externality term (Ω_{FE}) contains an additional term that takes into account the changes in the composition of households with different productivities across locations induced by the shock.

4.6 Commuting

Our baseline model assumes that households supply labor at the same location as their residential location. In the urban economics literature, it is often assumed that households make separate decisions about their residential and employment location decisions (e.g. Ahlfeldt et al. 2015, Tsivanidis 2023, Zárate 2022). Our framework can be extended to such a framework by reinterpreting household's location decisions j as a combination of residential locations j_1 and work locations j_2, (j_1, j_2). For example, the utility of agents choosing home location j_1 and work location j_2 is given by $u_{j_1 j_2}(C^\theta_{j_1 j_2}) + \epsilon^\theta_{j_1 j_2}$, where $\epsilon^\theta_{j_1 j_2}$ is home-and-work-specific idiosyncratic location preference.[25] Consequently, Proposition 1 remains unchanged by simply replacing j with (j_1, j_2) combinations.

5 Application: Welfare Gains from US Highway Network

We apply our framework to welfare gains from the US highway network. Every year, more than 150 billion dollars are spent on building, maintaining, or improving highways. Evaluating the impact of infrastructure improvement is critical to determine whether the gains justify these costs. Furthermore, understanding the heterogeneity in welfare gains from different segments of the network is critical to appropriately targeting future infrastructure investment. Allen and Arkolakis (2022), henceforth AA, study this question using a general equilibrium spatial model with traffic congestion and found large and highly variable welfare gains from improving transportation infrastructure across different links in the transportation network.

We proceed in the following steps. First, guided by our framework, we extend AA's framework in two key dimensions: by introducing flexibility in utility function curvature as well as spatial transfers. Our framework suggests that each of these two features crucially governs the welfare

[25] This extension accommodates the specification where households consume different consumption bundles depending on the home-work combination, as studied by Miyauchi, Nakajima, and Redding (2022).

gains through changes in MU dispersion and changes in fiscal externality, respectively. Second, we calibrate our model and simulate the welfare gain from improving each link of the highway network. Finally, we apply Proposition 1 to unpack the sources of both the average level of and heterogeneity in welfare gains.

5.1 Model Specification

Our specification closely follows AA, except for the utility function specification and the incorporation of spatial transfers. We specify our utility function as follows:

$$u_j(C_j) + \varepsilon_j = \frac{C_j^{1-\rho}}{1-\rho} + B_j + \varepsilon_j, \tag{38}$$

where ε_j follows an i.i.d. type-I extreme value distribution with shape parameter ν, and B_j is an exogenous amenity. The parameter ρ governs the curvature of the utility function and hence the dispersion of marginal utility. While assuming isoelastic utility with respect to consumption is restrictive, it offers a strict generalization of most of the existing literature including AA and Redding and Rossi-Hansberg (2017), who assume log utility ($\rho \to 1$) (or equivalently, a multiplicatively separable specification with an i.i.d. Fréchet distribution, as we study in Section 4.1).

Furthermore, we allow for transfers across locations T_j, so consumption is given by $C_j = (w_j + T_j)/P_j$. We assume that transfers take the following two-part structure:

$$T_j = \varkappa_j w_j + T^*, \tag{39}$$

where $\varkappa_j$ captures the rate of transfer with respect to nominal labor income, and $T^* = -\sum_j \varkappa_j w_j l_j$ is a lump-sum tax/transfer set to satisfy the government budget constraint ($\sum_j T_j l_j = 0$).

The remaining model follows AA. The nontradable final goods production technology is constant elasticity of substitution (CES), given by

$$C_j = \left(\int_k c_{j,k}^{\frac{\sigma-1}{\sigma}} dk\right)^{\frac{\sigma}{\sigma-1}}, \tag{40}$$

where $k \in K = [0,1]$ indexes the intermediate goods, and σ is the elasticity of substitution. The intermediate goods production technology is linear in labor, given by

$$y_{ij,k} = \mathcal{A}_i \tau_{ij,k} l_{ij,k}, \tag{41}$$

where $\tau_{ij,k}$ is an iceberg shipment cost, and $\mathcal{A}_i$ is the productivity of region i. Regional productivity is subject to an iso-elastic agglomeration externality in local population size given by

$$\mathcal{A}_i = A_i \left(l_i\right)^{\gamma}, \tag{42}$$

where A_i is the fundamental component of productivity.

A key feature of AA is modeling of the shipment cost $\tau_{ij,k}$ through a route choice problem. Denote by $\mathcal{R}_{ij}$ all possible routes connecting i to j. Formally, $r \in \mathcal{R}_{ij}$ is a sequence of legs (a pair of adjacent locations). Passing through each leg (k, l) incurs iceberg shipment cost t_{kl}. Conditional on region j sourcing goods k from region i, the optimal route choice implies that the shipment cost $\tau_{ij,k}$ is given by

$$\tau_{ij,k} = \min_{r \in \mathcal{R}_{ij}} \prod_{l=1}^{|r|} t_{r_{l-1} r_l} \epsilon_{ij,k}, \tag{43}$$

where $\epsilon_{ij,k}$ is an idiosyncratic cost for each (i, j, k), which follows an i.i.d. Fréchet distribution with dispersion parameter θ. Noting that $\tau_{ij,k}$ also follows a Fréchet distribution, the optimal consumption sourcing decision of region j gives rise to a gravity equation in trade flows, as in Eaton and Kortum (2002).

Finally, we assume that the leg-specific shipment cost may be subject to congestion externalities depending on the traffic passing through that leg, so

$$t_{mn} = \tilde{t}_{mn} \left(\Xi_{mn}\right)^{\lambda}, \tag{44}$$

where $\tilde{t}_{mn}$ is the exogenous component of the leg-specific shipment cost (which is in part affected by transportation infrastructure), Ξ_{mn} is the value of flows passing through leg (m, n), and λ is the parameter that captures the strength of the congestion externality in shipment costs.

We use this model to study aggregate welfare changes from a marginal decrease in $\tilde{t}_{mn}$, i.e. a link-specific improvement in transportation infrastructure. Applying our formula in Proposition 1, the first four terms – (i) technology, (ii) MU dispersion, (iii) fiscal externality, and (iv) technological externality – come down to

$$\Omega_T = -\sum_{k,l} \Xi_{kl} d \ln \tilde{t}_{kl}, \tag{45}$$

$$\Omega_{MU} = \mathrm{Cov}_j \left(-P_j / u_j'(C_j), u_j'(C_j) dC_j\right), \quad \Omega_{FE} = \mathrm{Cov}_j \left(T_j, d \ln l_j\right), \tag{46}$$

$$\Omega_{TE} = \Omega_{TE,S} + \Omega_{TE,A}, \qquad \Omega_{TE,S} = -\lambda \sum_{k,l} \Xi_{kl} d \ln \Xi_{kl}, \quad \Omega_{TE,A} = \gamma \sum_{i} w_i l_i d \ln l_i, \tag{47}$$

where $\Omega_{TE,S}$ and $\Omega_{TE,A}$ correspond to the technological externalities arising from shipment congestion and productivity agglomeration, respectively. Since we abstract from ex-ante heterogeneous household types, the redistribution term (Ω_R) is absent.

5.2 Calibration

We choose the same geographical units as AA. Using the 2012 Highway Performance Monitoring System (HPMS) dataset from the Federal Highway Administration, they create the infrastructure network across core-based statistical areas (CBSAs). The resulting network consists of 228 locations and 704 links between adjacent nodes.

To execute our counterfactual simulation, we first need to calibrate parameters $\{\gamma, \theta, \lambda, \rho, \nu, \varkappa_j\}$. We briefly describe the calibration procedure below and relegate the details to Appendix G.2. For the structural parameters that also appear in AA, we follow their baseline values. We set the shipment congestion elasticity at $\lambda = 0.092$, the elasticity of localized agglomeration externality at $\gamma = 0.1$, and the dispersion of idiosyncratic shocks for shipment route choice (also corresponding to the trade elasticity) at $\theta = 8$.

Utility function parameters $\{\rho, \nu\}$ are key parameters that govern the dispersion of marginal utility and location choice elasticities. We estimate $\{\rho, \nu\}$ using generalized method of moments (GMM) and MSA-level variation in consumption and population changes from 1980 to 2000. To build moment conditions, we construct a shift-share instrumental variable (IV) that interacts local industry composition with the national industry employment growth, similar to that of Diamond (2016). In addition, we also use the interaction of this IV with the baseline consumption level as another IV, which facilitates the identification of marginal utility at different consumption levels. To obtain meaningful variation in the baseline level of consumption, we split our samples into high-skill (4 years of college education or greater) and low-skill (less than 4 years of college education), and treat these two groups' location choice as independent samples. See Appendix G.2.1 for the detailed estimation procedure.

Table 2 shows the estimation results. We find a point estimate of ρ at 1.33 with a standard error of 0.76. Therefore, while the point estimate suggests a slightly more concave utility function, we cannot reject the null hypothesis of the commonly-used log-utility specification. We also find a point estimate of ν at 3.61 with a standard error of 1.47. Notice that, from Equation (38), the

Parameter	Estimates
ρ	1.33
	(0.76)
ν	3.61
	(1.47)

Table 2: GMM Estimates of Utility Function Parameters

Note: The table reports estimates of (ρ, ν). Standard errors are in parentheses. Estimates of ν are based on the normalization of average consumption in our sample in 2000 to one.

elasticity of location choice with respect to consumption is given by $\partial \ln l_j / \partial \ln C_j = \nu C_j^{1-\rho}(1 - l_j)$. Given our normalization of average consumption in our sample to one in 2000, this implies that the average migration elasticity with respect to consumption during the sample period is approximately 3.61. This estimate is within the range of existing estimates (e.g. Diamond 2016), as well as the baseline value used by AA. In what follows, we take our point estimates as a baseline.

The other key parameters are the rates of spatial transfers $\{\varkappa_j\}$. We calibrate these parameters using the observed pre- and post-tax-and-transfer income. Specifically, we obtain these values at the level of counties in 2012 from the Bureau of Economic Analysis (BEA). We then aggregate these values at the CBSA level and compute the ratio between tax and transfer to pre-tax income. Given that this value may not exactly satisfy the government budget constraint $\sum_j T_j l_j = 0$, we adjust $\varkappa_j$ by adding a nationwide constant term (see Appendix G.2.2 for details).

Given the above parameter choices, we solve the counterfactual equilibrium following "exact-hat algebra" approach pioneered by Dekle, Eaton, and Kortum (2007) (see Appendix G.1 for the counterfactual equilibrium system). This requires the baseline values of the equilibrium variables $\{l_i, w_i, \Xi_{ij}, C_i\}$. For $\{l_i, w_i\}$, we use the same values as used in AA. For $\{C_i\}$, we set them as post-tax-and-transfer income divided by the consumer price index (CPI) in 2012 reported by the BEA.

Finally, we infer the value of traffic over each link $\{\Xi_{ij}\}$ using the average annual daily traffic (AADT) in 2012. Given that these values are traffic counts, not the values they carry over, we infer Ξ_{ij} under the following two assumptions, which slightly differ from AA. First, while AA assume traffic value is symmetric in both directions for each leg, we allow for asymmetry to accommodate trade imbalance. Second, while AA assume that the aggregate traffic value is exactly equal to national GDP, we instead calibrate it to the observed counterpart. Specifically, we assume that average model-implied expenditure shares for goods produced in other locations coincide with the observed share of tradables in consumption basket in the US (27.6 percent; Johnson 2017). See Appendix G.2.3 for further details on this procedure. Through this procedure, we find that the sum of traffic value relative to GDP is $\sum_{i,j:i\neq j} \Xi_{ij}/(\sum_i w_i l_i) = 0.726$, somewhat less than the value of one assumed in AA.

5.3 Results

We undertake a counterfactual simulation of decreasing the exogenous component of shipment cost $\tilde{t}_{ij} = \tilde{t}_{ji}$ of each of the 704 links by 1 percent. To facilitate the interpretation of the welfare changes, we convert these values to an equivalent uniform labor productivity increase ($d \ln A_i = d \ln A$ for all i). This tells us how much of a uniform increase in labor productivity we would need to achieve the same welfare gains as from the transportation infrastructure improvement.

In Figure 1, we plot the welfare gains of each link improvement against the Δ technology term (Ω_T) in Proposition 1, which also corresponds to the calculation proposed by Fogel (1964) and Hulten (1978) within our context. Each data point corresponds to the counterfactual simulation of improving one of the 704 links. On average, we find that a 1% link improvement leads to welfare gains equivalent to 0.00042% increase in uniform productivity.[26]

There are two main takeaways from Figure 1. First, most of the data points are below the 45 degree line, implying that the technology term overpredicts welfare gains. In fact, the slope of the regression coefficient of ΔW on Ω_T is 0.49, which is substantially below one. Second, there is a great deal of heterogeneity across links in terms the deviation from the technology term. In fact, the R^2 of the regression of ΔW on Ω_T is 0.70, indicating that approximately 30 percent of the variation in welfare gains cannot be explained by the technology term Ω_T. This unexplained variation in welfare gains is important as it affects policy makers' decisions about which road link improvement to prioritize and target. Below, we unpack the sources of these large deviations from the technology term in explaining both the average of and heterogeneity in welfare gains.

5.3.1 Unpacking Deviations from Technology Term

Figure 2 shows the distribution of the percentage deviations from the prediction solely based on the technology term and its components. In the top panel, we see that the deviations ($(\Delta W - \Omega_T)/\Omega_T$) are on average negative and widely dispersed, echoing earlier findings. In the bottom

[26]This average welfare gain across link improvements is similar to the results of AA (their Figure 5), yet some departures arise from differences in model specifications and calibration of traffic values, as described earlier.

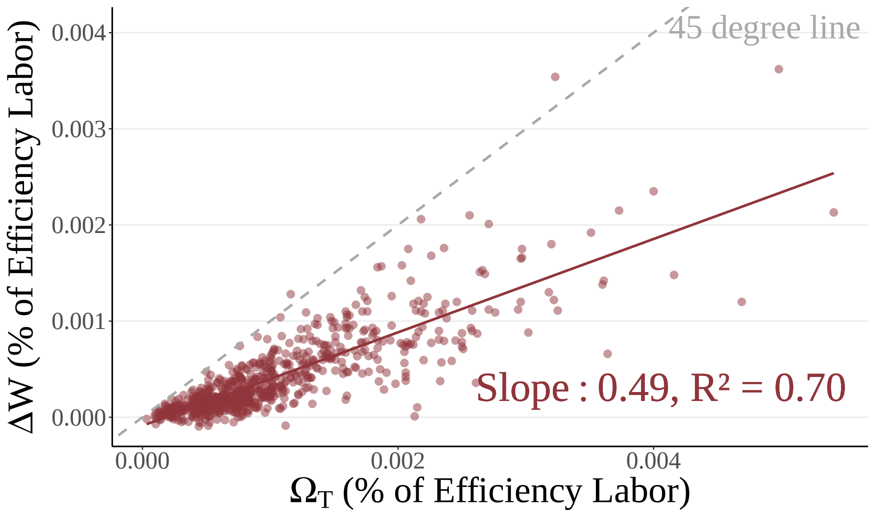

Figure 1: Overall Welfare Gains vs. Δ Technology Term

Note: This figure plots the welfare gains from 1% reductions in exogenous transportation cost for each of the 704 links in the US highway network on the y-axis, and the technology term Ω_T in Proposition 1 on the x-axis, both converted to an equivalent uniform labor productivity increase.

panel, we show the distributions of three components: Δ MU dispersion (Ω_{MU}/Ω_T), Δ fiscal externality (Ω_{FE}/Ω_T), and Δ technological externality (Ω_{TE}/Ω_T). On average, the changes in MU dispersion contribute positively to aggregate welfare and are also widely distributed. In contrast, changes in fiscal externality plays a more moderate role both on average and in explaining heterogeneity. Finally, the changes in technological externality are on average negative and highly dispersed, reiterating points emphasized in AA through the lens of our formula.

Table 3 summarizes the mean and variance of each component of deviations from the technology term. On average, the welfare gains from infrastructure improvements are 64% lower than the prediction based on the technology term, and there is a large heterogeneity with a standard deviation of 21% (a variance of 0.045). Changes in MU dispersion are quantitatively important contributors to both the average and heterogeneity of gains from infrastructure improvements. On average, changes in MU dispersion raise welfare by 11% and account for 36% ($\approx 0.016/0.045$) of the variance in deviations from the technology term across links. Positive average gains reflect that infrastructure improvements tend to raise consumption of high marginal utility locations. The large dispersion across links reflects the large spatial dispersion of marginal utility across locations. We later show that changes in MU dispersion are largely associated with differences in the nominal income of locations neighboring the improved infrastructure and, to a lesser extent, in transfers or prices.

In contrast, the contribution of changes in fiscal externalities is generally more modest. They

Variable	Mean	Variance
$(\Delta W - \Omega_T)/\Omega_T$	-0.64	0.045
Ω_{MU}/Ω_T	0.11	0.016
Ω_{FE}/Ω_T	-0.04	0.006
Ω_{TE}/Ω_T	-0.72	0.039
$\Omega_{TE,S}/\Omega_T$	-0.69	0.037
$\Omega_{TE,A}/\Omega_T$	-0.03	0.001
Residual	0.01	0.000

Table 3: Unpacking Deviations from the Δ Technology Term: Mean and Variance

Note: This table presents the mean and variance of each component of welfare gains in Proposition 1 as a ratio of the Δ technology term (Ω_T) across 704 link improvement simulations. We further decompose Ω_{TE} into the shipment congestion externality $\Omega_{TE,S}$ and agglomeration externality $\Omega_{TE,A}$ using the formula (47). Residual is defined as $\Delta W - (\Omega_T + \Omega_{MU} + \Omega_{FE} + \Omega_{TE})$ divided by Ω_T.

lower welfare by 4% on average because infrastructure improvements tend to move population toward locations that are net transfer recipients. The heterogeneity in transfers across locations naturally translates into heterogeneity in fiscal externalities. Note also that this term does not offset the other terms ($\Omega_{MU}, \Omega_{TE,S}, \Omega_{TE,A}$), reflecting the suboptimality of prevailing spatial transfers (i.e. Proposition 3).

Changes in technological externalities play an important role in explaining both the average and heterogeneity of welfare gains from infrastructure improvements. This predominantly comes from shipment congestion ($\Omega_{TE,S}/\Omega_T$). Interestingly, although the omission of agglomeration externalities is at the center of the criticisms of Fogel's approach in the transportation infrastructure literature, agglomeration externality ($\Omega_{TE,A}/\Omega_T$) is quantitatively negligible. The large attenuation of welfare gains from shipment congestion externalities is consistent with the findings in AA, which compare the counterfactual simulation results with and without the congestion externalities. Our results underscore this finding, while simultaneously highlighting that other terms are similarly important for the heterogeneity in welfare gains.

Finally, the residuals are quantitatively negligible, both for the average and heterogeneity of welfare gains, despite the non-infinitesimal shocks that we consider (1% shipment cost reduction). Therefore, in this context, the first-order approximations in Proposition 1 are virtually exact, and one should focus on the first-order terms as a source of deviation from Fogel/Hulten.

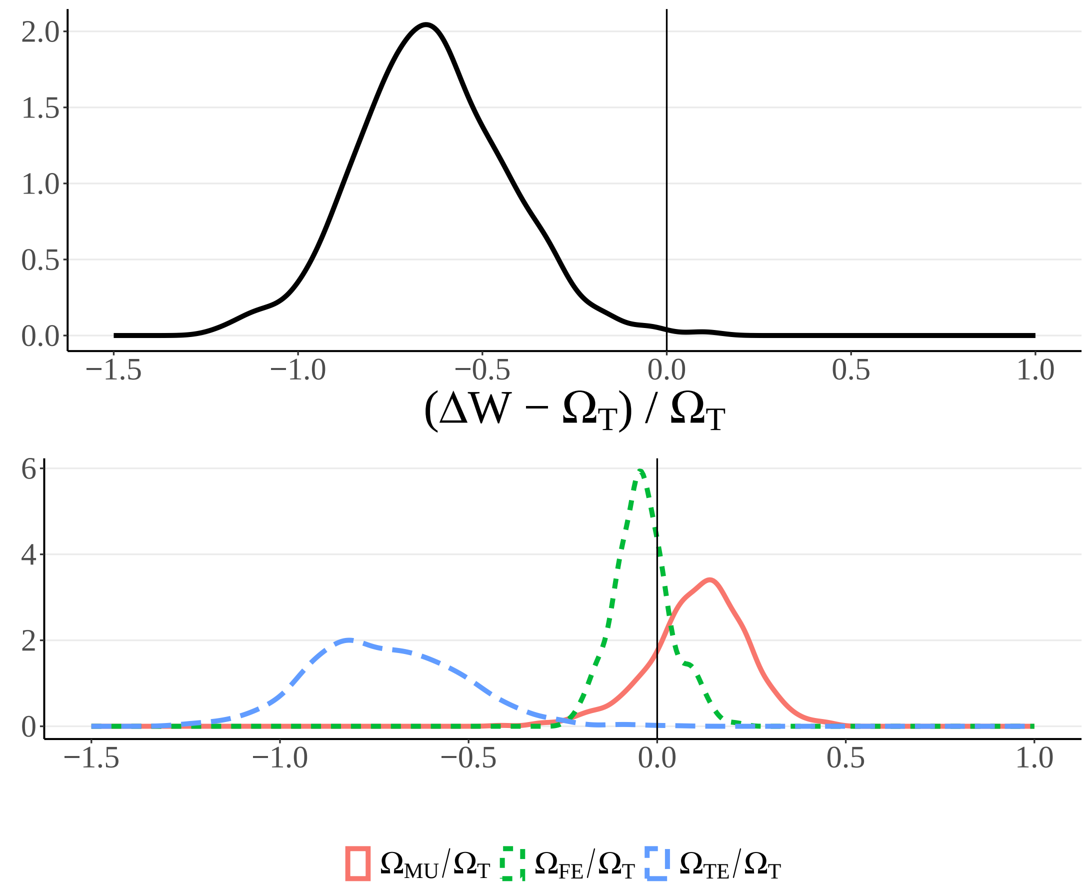

Figure 2: Unpacking Deviations from the Δ Technology Term: Density Plot

Note: The figure presents the density plots of each component of welfare gains in Proposition 1 as a ratio of the Δ technology term (Ω_T) across 704 transportation link improvement simulations.

5.3.2 Spatial Patterns of Deviations from Technology Term

To understand the spatial heterogeneity of the deviation from the technology term, Table 4 explores how they are related to a set of selected characteristics of the link and the associated nodes. To assist the comparison of the magnitudes across independent variables, all independent variables are normalized to have a standard deviation of one.

In the first column, we find that the additional welfare effects beyond the technology term are systematically related to the economic characteristics of the connected nodes (see Appendix Figure G.1 for a map). In the first row, we find that the deviation is 3 percentage points lower for links that connect nodes with income one-standard-deviation higher. Column 2 shows that this pattern is largely driven by Ω_{MU} (Column 2), consistent with the interpretation that these locations tend to have lower marginal utility, while somewhat offset by $\Omega_{TE,S}$ (Column 4) because these locations tend to have denser networks to divert traffic and hence alleviate shipment congestion. In the second row, we find that the deviation is 10 percent lower for links connecting nodes with a net transfer one-standard-deviation higher (Column 1), driven primarily by Ω_{MU} and Ω_{FE} (Columns 2 and 3). Therefore, the welfare gains of improving links around places with net transfer receipts are systematically lower, everything else equal. In the third row, we find that the differences in the CPI seem to be weakly related to the deviations from technology term, perhaps due to a smaller spatial variation compared to the nominal income and transfer rates.

We also find that the deviation is 3 percentage points higher for links with traffic one-standard-deviation higher. This pattern is primarily driven by $\Omega_{TE,S}$ (Column 4), consistent with the interpretation that links with higher traffic tend to have more substitutable routes for shipment and therefore tend to generate less congestion. At the same time, in the final row, we do not find a systematically strong relationship of $\Omega_{TE,S}$ with the number of contiguous links, perhaps due to the complex interaction of contiguous links as substitutes (diverting traffic) and complements (attracting additional shipment).

Overall, these results suggest that welfare gains from transportation infrastructure arise at various margins beyond the technology term. While the spatial economics literature typically emphasizes technological externalities as a driver of the deviation, we find that changes in MU dispersion, and to a lesser extent fiscal externalities, play quantitatively important roles in understanding both the average and heterogeneity of welfare gains from transportation infrastructure improvements.

6 Concluding Remarks

In a general class of spatial equilibrium models, we have developed a theory to unpack the sources of welfare gains from changes in technology. We provided a nonparametric formula that characterizes the deviation from Fogel/Hulten's characterization. The formula shows that first-order changes in aggregate welfare can be exactly decomposed into five terms: (i) Δ technology, (ii) Δ spatial dispersion in marginal utility, (iii) Δ fiscal externalities, (iv) Δ technological externalities, and (v) Δ redistribution. We also used this formula to study the design of place-based policy. In particular, we derived a nonparametric optimal spatial transfer formula, which generalizes those in the existing literature, and showed that welfare effects can be summarized by the (i) Δ technology term whenever optimal spatial transfers are in place. By applying our framework to the US highway network, we find that the changes in the dispersion of marginal utility, and to a lesser extent those of fiscal externalities, drive the deviation from Fogel/Hulten's characterization, in

	$(\Delta W - \Omega_T)$	Ω_{MU}	Ω_{FE}	$\Omega_{TE,S}$	$\Omega_{TE,A}$
	(1)	(2)	(3)	(4)	(5)
log Pre-Tax Income	-0.03	-0.08	-0.01	0.05	0.02
	(0.01)	(0.01)	(0.003)	(0.01)	(0.001)
Net Transfer Rates	-0.10	-0.04	-0.04	-0.02	0.01
	(0.01)	(0.005)	(0.003)	(0.01)	(0.001)
log CPI	-0.003	-0.01	0.01	-0.01	0.003
	(0.01)	(0.01)	(0.003)	(0.01)	(0.001)
log Traffic	0.03	-0.01	0.004	0.03	0.002
	(0.01)	(0.004)	(0.003)	(0.01)	(0.001)
# of Contiguous Links	-0.004	0.004	0.003	-0.01	-0.001
	(0.01)	(0.004)	(0.003)	(0.01)	(0.001)
Observations	704	704	704	704	704
Adjusted R^2	0.20	0.35	0.28	0.14	0.32

Table 4: Regressions of Welfare Components on Spatial Economic Characteristics

Note: Panel (a) presents the spatial distribution of $(\Delta W - \Omega_T)/\Omega_T$ for each link improvement. The table shows the results from a regression of deviation from technology term $(\Delta W - \Omega_T)$ as well as its decomposition on a set of characteristics of each link that we shock. All the dependent variables are normalized by Ω_T, and all the independent variables are normalized by their standard deviation. All the independent variables except for log traffic take the average value between the two nodes that are connected by the link.

addition to congestion externalities, which have been highlighted in the existing literature.

We suggest two directions for future work. First, our framework can be used for many other applications that involve a spatial dimension. In Appendix H, we provide one such application, by analyzing how observed regional growth contributes to US aggregate welfare gains. Second, while our framework is static, many interesting questions in spatial economics relate to the dynamics of economic activity and population mobility. In separate ongoing work (Donald, Fukui, and Miyauchi 2023), we tackle this question by studying optimal transfer policy in a dynamic environment.

References

Adão, R., A. Costinot, and D. Donaldson (2017). Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade. *American Economic Review 107*(3), 633–689.

Adão, R., A. Costinot, D. Donaldson, and J. Sturm (2023). Why is Trade Not Free? A Revealed Preference Approach. Working Paper 31798, National Bureau of Economic Research.

Ahlfeldt, G., S. Redding, D. Sturm, and N. Wolf (2015). The Economics of Density: Evidence from the Berlin Wall. *Econometrica 83*(6), 2127–2189.

Ales, L. and C. Sleet (2022). Optimal Taxation of Income-Generating Choice. *Econometrica 90*(5), 2397–2436.

Allen, R. and J. Rehbeck (2019). Identification with Additively Separable Heterogeneity. *Econometrica 87*(3), 1021–1054.

Allen, T. and C. Arkolakis (2014). Trade and the Topography of the Spatial Economy. *The Quarterly Journal of Economics 129*(3), 1085–1140.

Allen, T. and C. Arkolakis (2022). The Welfare Effects of Transportation Infrastructure Improvements. *The Review of Economic Studies 89*(6), 2911–2957.

Allen, T., C. Arkolakis, and X. Li (2020). On the Equilibrium Properties of Network Models with Heterogeneous Agents. Working Paper 27837, National Bureau of Economic Research.

Anderson, S., A. De Palma, and J.-F. Thisse (1988). A Representative Consumer Theory of the Logit Model. *International Economic Review*, 461–466.

Baily, M. (1978). Some Aspects of Optimal Unemployment Insurance. *Journal of Public Economics 10*(3), 379–402.

Baqaee, D. and E. Farhi (2020). Productivity and Misallocation in General Equilibrium. *The Quarterly Journal of Economics 135*(1), 105–163.

Baqaee, D. and E. Farhi (2024). Networks, Barriers, and Trade. *Econometrica 92*(2), 505–541.

Berry, S., A. Gandhi, and P. Haile (2013). Connected Substitutes and Invertibility of Demand. *Econometrica 81*(5), 2087–2111.

Berry, S. and P. Haile (2014). Identification in Differentiated Products Markets Using Market Level Data. *Econometrica 82*(5), 1749–1797.

Bhandari, A., D. Evans, M. Golosov, and T. Sargent (2021). Efficiency, Insurance, and Redistribution Effects of Government Policies. *Working Paper*.

Bhattacharya, D. (2015). Nonparametric Welfare Analysis for Discrete Choice. *Econometrica 83*(2), 617–649.

Bordeu, O. (2023). Commuting Infrastructure in Fragmented Cities. *Working Paper*.

Bryan, G. and M. Morten (2019). The Aggregate Productivity Effects of Internal Migration: Evidence from Indonesia. *Journal of Political Economy 127*(5), 2229–2268.

Caliendo, L., F. Parro, E. Rossi-Hansberg, and P.-D. Sarte (2018). The Impact of Regional and Sectoral Productivity Changes on the U.S. Economy. *The Review of Economic Studies 85*(4), 2042–2096.

Chetty, R. (2006). A General Formula for the Optimal Level of Social Insurance. *Journal of Public Economics 90*(10-11), 1879–1901.

Costinot, A. and I. Werning (2018). Robots, Trade, and Luddism: A Sufficient Statistic Approach to Optimal Technology Regulation. Working Paper 25103, National Bureau of Economic Research.

David, P. (1969). Transport Innovation and Economic Growth: Professor Fogel on and off the Rails. *The Economic History Review 22*(3), 506–525.

Dávila, E. and A. Schaab (2022). Welfare Assessments with Heterogeneous Individuals. Working Paper 30571, National Bureau of Economic Research.

Dávila, E. and A. Schaab (2023). Welfare Accounting. *Working Paper*.

Davis, M. and J. Gregory (2021). Place-Based Redistribution in Location Choice Models. Working Paper 29045, National Bureau of Economic Research.

Dekle, R., J. Eaton, and S. Kortum (2007). Unbalanced Trade. *American Economic Review 97*(2), 351–355.

Diamond, R. (2016). The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980–2000. *American Economic Review 106*(3), 479–524.

Donald, E., M. Fukui, and Y. Miyauchi (2023). Optimal Dynamic Spatial Policy. *Working Paper*.

Donaldson, D. (2022). Blending Theory and Data: A Space Odyssey. *Journal of Economic Perspectives 36*(3), 185–210.

Donaldson, D. and R. Hornbeck (2016). Railroads and American Economic Growth: A "Market Access" Approach. *The Quarterly Journal of Economics 131*(2), 799–858.

Eaton, J. and S. Kortum (2002). Technology, Geography, and Trade. *Econometrica 70*(5), 1741–1779.

Eden, M. (2020a). Aggregating Welfare Gains. Discussion Paper DP14783, Centre for Economic Policy Research.

Eden, M. (2020b). Welfare Analysis with Heterogeneous Risk Preferences. *Journal of Political Economy 128*(12), 4574–4613.

Fajgelbaum, P. and C. Gaubert (2020). Optimal Spatial Policies, Geography, and Sorting. *The Quarterly Journal of Economics 135*(2), 959–1036.

Fajgelbaum, P. and E. Schaal (2020). Optimal Transport Networks in Spatial Equilibrium. *Econometrica 88*(4), 1411–1452.

Flood, S., M. King, R. Rodgers, S. Ruggles, J. R. Warren, and M. Westberry (2023). IPUMS CPS: Version 11.0 [dataset]. *Minneapolis, MN: IPUMS 10*, D030.

Fogel, R. (1964). *Railroads and American Economic Growth*. Johns Hopkins Press Baltimore.

Gaubert, C., P. Kline, and D. Yagan (2021). Place-Based Redistribution. Working Paper 28337, National Bureau of Economic Research.

Grant, S., A. Kajii, B. Polak, and Z. Safra (2010). Generalized Utilitarianism and Harsanyi's Impartial Observer Theorem. *Econometrica 78*(6), 1939–1971.

Harsanyi, J. (1955). Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility. *Journal of Political Economy 63*(4), 309–321.

Hofbauer, J. and W. Sandholm (2002). On the Global Convergence of Stochastic Fictitious Play. *Econometrica 70*(6), 2265–2294.

Hulten, C. (1978). Growth Accounting with Intermediate Inputs. *The Review of Economic Studies 45*(3), 511–518.

Johnson, N. (2017). Tradable and Nontradable Inflation Indexes: Replicating New Zealand's Tradable Indexes with BLS CPI Data. *Monthly Labor Review*, 1–25.

Kleinman, B., E. Liu, and S. Redding (2024). The Linear Algebra of Economic Geography Models. *AEA Papers and Proceedings 114*, 328–33.

Kline, P. and E. Moretti (2014). People, Places, and Public Policy: Some Simple Welfare Economics of Local Economic Development Programs. *Annual Review of Economics 6*(1), 629–662.

Krugman, P. (1991). Increasing Returns and Economic Geography. *Journal of Political Economy 99*(3), 483–499.

Lebergott, S. (1966). United States Transport Advance and Externalities. *The Journal of Economic History 26*(4), 437–461.

Lind, N. and N. Ramondo (2023). Trade with Correlation. *American Economic Review 113*(2), 317–353.

McFadden, D. (1978). Modeling the Choice of Residential Location. *Transportation Research Record* (673).

Melo, P., D. Graham, and R. Noland (2009). A Meta-Analysis of Estimates of Urban Agglomeration Economies. *Regional Science and Urban Economics 39*(3), 332–342.

Mirrlees, J. (1972). The Optimum Town. *The Swedish Journal of Economics*, 114–135.

Miyauchi, Y., K. Nakajima, and S. Redding (2022). The Economics of Spatial Mobility: Theory and Evidence Using Smartphone Data. Working Paper 28497, National Bureau of Economic Research.

Mongey, S. and M. Waugh (2024). Discrete Choice, Complete Markets, and Equilibrium. Working Paper 32135, National Bureau of Economic Research.

Redding, S. (2016). Goods Trade, Factor Mobility, and Welfare. *Journal of International Economics 101*, 148–167.

Redding, S. and E. Rossi-Hansberg (2017). Quantitative Spatial Economics. *Annual Review of Economics 9*, 21–58.

Roback, J. (1982). Wages, Rents, and the Quality of Life. *Journal of Political Economy 90*(6), 1257–1278.

Rosen, S. (1979). Wage-Based Indexes of Urban Quality of Life. *Current Issues in Urban Economics*, 74–104.

Rossi-Hansberg, E., P.-D. Sarte, and F. Schwartzman (2021). Cognitive Hubs and Spatial Redistribution. Working Paper 26267, National Bureau of Economic Research.

Ruggles, S., S. Flood, R. Goeken, J. Grover, E. Meyer, J. Pacas, and M. Sobek (2023). IPUMS USA: Version 14.0 [dataset]. *Minneapolis, MN: IPUMS 10*, D010.

Small, K. and E. Verhoef (2007). *The Economics of Urban Transportation.* Routledge.

Tsivanidis, N. (2023). Evaluating the Impact of Urban Transit Infrastructure: Evidence from Bogota's Transmilenio. *Working Paper*.

Werning, I. (2007). Pareto Efficient Income Taxation. *Working Paper*.

Zárate, R. (2022). Spatial Misallocation, Informality, and Transit Improvements. *Working Paper*.

Online Appendix for
"Unpacking Aggregate Welfare in a Spatial Economy"

Eric Donald, Masao Fukui, Yuhei Miyauchi

A Details on Proofs

A.1 Proof of Lemma 1

Economy with Heterogeneous Preferences. Consider the problem of households of type θ deciding where to live. We index each individual by $\omega \in [0, \ell^\theta]$, and $\{\epsilon_k^\theta(\omega)\}_k$ denotes the preference draw of individual ω. Each individual solves the following problem:

$$\begin{aligned} v^\theta(\omega) = \max_{\{\mathbb{I}_j^\theta(\omega)\}_j} & \sum_j \mathbb{I}_j^\theta(\omega) \left[u_j(C_j^\theta) + \epsilon_j^\theta(\omega)\right] \\ \text{s.t.} \quad & \sum_j \mathbb{I}_j^\theta(\omega) = 1, \end{aligned} \tag{A.1}$$

where $\mathbb{I}_j^\theta(\omega) \in \{0, 1\}$ is an indicator function for the location choice of individual ω. The fraction of individuals living in location j is given by

$$\mu_j^\theta = \frac{1}{\ell^\theta} \int_0^{\ell^\theta} \mathbb{I}_j^\theta(\omega) d\omega. \tag{A.2}$$

Economy with Representative Agent. Define the following function:

$$\begin{aligned} \psi^\theta(\{\mu_j^\theta\}_j) = - \max_{\{\mathbb{I}_j^\theta(\omega)\}_{\omega,j}} & \frac{1}{\ell^\theta} \int_0^{\ell^\theta} \sum_j \epsilon_j^\theta(\omega) \mathbb{I}_j^\theta(\omega) d\omega \\ \text{s.t.} \quad & \frac{1}{\ell^\theta} \int_0^{\ell^\theta} \mathbb{I}_j^\theta(\omega) d\omega = \mu_j^\theta \\ & \sum_j \mathbb{I}_j^\theta(\omega) = 1. \end{aligned} \tag{A.3}$$

The representative agent solves

$$W^\theta = \max_{\{\mu_j^\theta\}_j : \sum_j \mu_j^\theta = 1} \sum_j \mu_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}) \tag{A.4}$$

The representative agent's first-order condition for μ_j^θ follows

$$u_j(C_j^\theta) - \frac{\partial \psi^\theta}{\partial \mu_j^\theta} = \delta^\theta, \tag{A.5}$$

where δ^θ is the Lagrange multiplier on the adding up constraint for $\{\mu_j^\theta\}_j$.

Equivalence Result. We formally restate the equivalence result of Lemma 1 as follows.

Lemma. *Suppose $\{\mathbb{I}_j^\theta(\omega)\}_j$ solves (A.1) for all ω. Then, $\{\mu_j^\theta\}_j$, given by (A.2), solves (A.4). Conversely, suppose $\{\mu_j^\theta\}_j$ solves (A.4). Then $\{\mathbb{I}_j^\theta(\omega)\}_{\omega,j}$, given by the solution to (A.3) associated with $\{\mu_j^\theta\}_j$, solves (A.1) for almost all ω. Moreover, expected utility in the economy with heterogeneous preferences equals the utility of the representative agent:*

$$\frac{1}{\ell^\theta}\int_0^{\ell^\theta} v^\theta(\omega)d\omega = W^\theta$$

Proof. We prove the first part. Suppose to the contrary, there exists $\{\tilde{\mu}_j^\theta\}_j$ such that

$$\sum_j \tilde{\mu}_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}) > \sum_j \mu_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}). \tag{A.6}$$

Let $\{\tilde{\mathbb{I}}_j^\theta(\omega)\}_{\omega,j}$ denote the solution to (A.3) associated with $\{\tilde{\mu}_j^\theta\}_j$. Plugging into (A.6),

$$\frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j(C_j^\theta)+\epsilon_j^\theta(\omega)\right]d\omega > \frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_j \mathbb{I}_j^\theta(\omega)\left[u_j(C_j^\theta)+\epsilon_j^\theta(\omega)\right]d\omega, \tag{A.7}$$

where $\sum_j \tilde{\mathbb{I}}_j^\theta(\omega)=1$ and $\sum_j \mathbb{I}_j^\theta(\omega)=1$ for all ω. However, this is a contradiction because by our presumption, for any ω,

$$\sum_j \mathbb{I}_j^\theta(\omega)\left[u_j(C_j^\theta)+\epsilon_j^\theta(\omega)\right] \geq \sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j(C_j^\theta)+\epsilon_j^\theta(\omega)\right]$$

for all $\tilde{\mathbb{I}}_j^\theta(\omega)$, which would imply

$$\frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j(C_j^\theta)+\epsilon_j^\theta(\omega)\right]d\omega \leq \frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_k \mathbb{I}_k^\theta(\omega)\left[u_j(C_j^\theta)+\epsilon_j^\theta(\omega)\right]d\omega. \tag{A.8}$$

Now we prove the converse. Suppose to the contrary, there exists $\{\tilde{\mathbb{I}}_j^\theta(\omega)\}_j$ such that

$$\sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j(C_j^\theta)+\epsilon_j^\theta(\omega)\right] > \sum_j \mathbb{I}_j^\theta(\omega)\left[u_j(C_j^\theta)+\epsilon_j^\theta(\omega)\right] \tag{A.9}$$

and $\sum_j \tilde{\mathbb{I}}_j^\theta(\omega)=1$ hold for all $\omega\in\Omega$, where $\Omega\subset[0,\ell^\theta]$ and $|\Omega|>0$. Define

$$\tilde{\mu}_j^\theta = \frac{1}{\ell^\theta}\int_0^{\ell^\theta}\tilde{\mathbb{I}}_j^\theta(\omega)d\omega. \tag{A.10}$$

Then

$$\sum_j \mu_j^\theta u_j(C_j^\theta) - \psi(\{\mu_j^\theta\}) = \frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_j \mathbb{I}_j(\omega)\left[u_j(C_j^\theta)+\epsilon_j(\omega)\right]d\omega$$

$$< \frac{1}{\ell^\theta} \int_0^{\ell^\theta} \sum_j \tilde{\mathbb{I}}_j^\theta(\omega) \left[u_j(C_j^\theta) + \epsilon_j(\omega) \right] d\omega$$
$$\leq \sum_j \tilde{\mu}_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}).$$

This is a contradiction that $\{\mu_j^\theta\}_j$ is a solution to (A.4).

We need to show that expected utility in the two economic coincides. This immediately follows given the above result. Let $\{\mathbb{I}_j^\theta(\omega)\}_{\omega,j}$ be the solution to (A.1) for all ω, and let $\{\mu_j^\theta\}_j$ denote the solution to (A.4). Then

$$\frac{1}{\ell^\theta} \int_0^{\ell^\theta} \sum_j \mathbb{I}_j^\theta(\omega) \left[u_j(C_j^\theta) + \epsilon_j^\theta(\omega) \right] d\omega = \sum_j \mu_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}). \tag{A.11}$$

□

A.2 Proof of Lemma 2

The constraints (22) and (23) immediately imply that at the solution of the pseudo-planning problem, we have $C_j^\theta = \check{C}_j^\theta$ and $\mu_j^\theta = \check{\mu}_j^\theta$, and therefore $l_j^\theta = \check{l}_j^\theta$ and $\mathcal{A}_{ij,k} = \check{\mathcal{A}}_{ij,k}$ as well.

We now show the remaining allocation of the pseudo-planning problem coincides with the decentralized equilibrium. In the decentralized equilibrium, quantities $\{\{\check{\mathbf{c}}_j^\theta\}, \{\check{\mathbf{x}}_{ij,k}, \check{\mathbf{l}}_{ij,k}, \check{h}_{ij,k}\}\}$ and prices $\{\{P_j^\theta, w_j^\theta\}, \{p_{ij,k}\}, r_j\}$ solve the resource constraints (13)-(16) as well as the following firms' optimality conditions:

$$P_j^\theta \frac{\partial \mathcal{C}_j^\theta}{\partial c_{ij,k}^\theta} = p_{ij,k}, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^\theta, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}. \tag{A.12}$$

The first-order conditions of the pseudo-planning problem with respect to $c_{ij,k}$, $l_{ij,k}^\theta$, $h_{ij,k}$, and $x_{ij,k}^{l,m}$ are

$$P_j^{L,\theta} \frac{\partial \mathcal{C}_j^\theta}{\partial c_{ij,k}^\theta} = p_{ij,k}^L, \quad p_{ij,k}^L A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^{L,\theta}, \quad p_{ij,k}^L A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i^L, \quad p_{ij,k}^L A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}^L, \tag{A.13}$$

where $\{\{P_j^{L,\theta}, w_j^{L,\theta}\}, \{p_{ij,k}^L\}, r_j^L\}$ are Lagrange multipliers on constraints (13)-(16). Therefore, the decentralized equilibrium allocation satisfies the optimality conditions of the pseudo-planning problem. Moreover, equilibrium prices and Lagrange multipliers in the pseudo-planning problem coincide up to a multiplicative constant.

□

A.3 Proof of Proposition 1

The Lagrangian for the pseudo-planning problem is

$$\begin{aligned}
\mathcal{L} = & \mathcal{W}\left(\left\{\sum_j \hat{\mu}_j^\theta(\boldsymbol{C}^\theta) u_j(C_j^\theta) - \psi^\theta(\{\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)\})\right\}\right) \\
& + \sum_{i,j,k} p_{ij,k}^L \left[A_{ij,k} g_{ij,k}(\{\ell^\theta \hat{\mu}_j^\theta(\boldsymbol{C}^\theta)\}) f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) - \left(\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{jl,m}^{i,k}\right)\right] \\
& + \sum_{j,\theta} P_j^{L,\theta} \left[\mathcal{C}_j^\theta(\mathbf{c}_j^\theta) - C_j^\theta \ell^\theta \hat{\mu}_j^\theta(\boldsymbol{C}^\theta)\right] \\
& + \sum_{j,\theta} w_j^{L,\theta} \left[\ell^\theta \hat{\mu}_j^\theta(\boldsymbol{C}^\theta) - \sum_{i,k} l_{ji,k}^\theta\right] \\
& + \sum_j r_j^L \left[\bar{h}_j - \sum_{i,k} h_{ji,k}\right] \\
& + \sum_j \eta_j^\theta \left[C_j^\theta - \check{C}_j^\theta\right],
\end{aligned}$$

where we have substituted constraints (9), (21), and (22). Since one of the constraints $C_j^\theta = \check{C}_j^\theta$ is implied by the resource constraints, we normalize $\{\eta_j^\theta\}$ such that $\sum_{j,\theta} \frac{1}{u_j'(\check{C}_j^\theta)} \eta_j^\theta = 0$. The first-order condition of the pseudo-planning problem with respect to C_j^θ is given by

$$\begin{aligned}
& \Lambda^\theta \ell^\theta \mu_j^\theta u_j'(C_j^\theta) - P_j^{L,\theta} \ell^\theta \mu_j^\theta \\
& + \sum_l \frac{\partial \hat{\mu}_l^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[w_l^{L,\theta} \ell^\theta - P_l^{L,\theta} \ell^\theta + \sum_{i,j,k} p_{ij,k}^L \mathcal{A}_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \gamma_{ij,k}^\theta \ell^\theta \frac{1}{l_l^\theta}\right] + \eta_j^\theta = 0.
\end{aligned}$$

Dividing both sides by $u_j'(C_j^\theta)$ and adding up across j and θ, we have

$$\begin{aligned}
& \sum_\theta \Lambda^\theta \ell^\theta - \sum_\theta \sum_j \frac{P_j^{L,\theta}}{u_j'(C_j^\theta)} l_j^\theta \\
& + \sum_\theta \sum_l \underbrace{\sum_j \frac{\partial \hat{\mu}_l^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \frac{1}{u_j'(C_j^\theta)}}_{=0} \left[w_l^{L,\theta} \ell^\theta - P_l^{L,\theta} \ell^\theta + \sum_{i,j,k} p_{ij,k}^L \mathcal{A}_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \gamma_{ij,k}^\theta \ell^\theta \frac{1}{l_l^\theta}\right] \\
& + \underbrace{\sum_\theta \sum_j \frac{1}{u_j'(C_j^\theta)} \eta_j^\theta}_{=0} = 0,
\end{aligned}$$

which implies

$$\sum_{\theta}\sum_{j}\frac{P_j^{L,\theta}}{u_j'(C_j^\theta)}l_j^\theta=\sum_{\theta}\Lambda^\theta\ell^\theta. \tag{A.14}$$

Since this corresponds to our normalization of SWF (26) and Lemma 2 implies that the Lagrange multipliers coincide with equilibrium prices up to scale, we have $P_j^{L,\theta}=P_j^\theta$, $p_{ij,k}^L=p_{ij,k}$, $w_i^{L,\theta}=w_i^\theta$, and $r_i^L=r_i$.

By applying the envelope theorem, we have

$$\begin{aligned}
\frac{dW}{d\ln A_{il,k}}=&\frac{d\mathcal{L}}{d\ln A_{il,k}}\\
=&p_{il,k}y_{il,k}+\sum_{\theta}\sum_{j}\eta_j^\theta\frac{d\check{C}_j^\theta}{d\ln A_{il,k}}\\
=&p_{il,k}y_{il,k}+\sum_{\theta}\sum_{j}l_j^\theta[\Lambda^\theta u_j'(C_j^\theta)-P_j^\theta]\frac{d\check{C}_j^\theta}{d\ln A_{il,k}}\\
&+\sum_{\theta}\sum_{j}\left[w_j^\theta-P_j^\theta C_j^\theta+\sum_{l,k}p_{jl,k}y_{jl,k}\frac{1}{l_j^\theta}\gamma_{jl,k}^\theta\right]\ell^\theta\frac{\partial\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta}\frac{d\check{C}_j^\theta}{d\ln A_{il,k}}.
\end{aligned}$$

Therefore, by noting that $dl_j^\theta=\ell^\theta d\mu_j^\theta$, we have

$$\begin{aligned}
dW=&\sum_{i,j,k}p_{ij,k}y_{ij,k}d\ln A_{ij,k}+\sum_{\theta}\sum_{j}l_j^\theta[\Lambda^\theta u_j'(C_j^\theta)-P_j^\theta]dC_j^\theta\\
&+\sum_{\theta}\sum_{j}[w_j^\theta-P_j^\theta C_j^\theta]dl_j^\theta+\sum_{\theta}\sum_{j}\sum_{l,k}p_{jl,k}y_{jl,k}\frac{1}{l_j^\theta}\gamma_{jl,k}^\theta dl_j^\theta.
\end{aligned} \tag{A.15}$$

Now,

$$\begin{aligned}
&\sum_{\theta}\sum_{j}l_j^\theta[\Lambda^\theta u_j'(C_j^\theta)-P_j^\theta]dC_j^\theta\\
=&\sum_{\theta}\ell^\theta\sum_{j}\mu_j^\theta[\Lambda^\theta-\frac{P_j^\theta}{u_j'(C_j^\theta)}]u_j'(C_j^\theta)dC_j^\theta\\
=&\sum_{\theta}\ell^\theta\left[\mathrm{Cov}_{j|\theta}(\Lambda^\theta-\frac{P_j^\theta}{u_j'(C_j^\theta)},u_j'(C_j^\theta)dC_j^\theta)+\mathbb{E}_{j|\theta}[\Lambda^\theta-\frac{P_j^\theta}{u_j'(C_j^\theta)}]\mathbb{E}_{j|\theta}[u_j'(C_j^\theta)dC_j^\theta]\right]\\
=&\sum_{\theta}\ell^\theta\left[\mathrm{Cov}_{j|\theta}(-\frac{P_j^\theta}{u_j'(C_j^\theta)},u_j'(C_j^\theta)dC_j^\theta)+\left(\Lambda^\theta-\mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j'(C_j^\theta)}]\right)\mathbb{E}_{j|\theta}[u_j'(C_j^\theta)dC_j^\theta]\right]\\
=&\mathbb{E}_\theta[\mathrm{Cov}_{j|\theta}(-\frac{P_j^\theta}{u_j'(C_j^\theta)},u_j'(C_j^\theta)dC_j^\theta)]+\mathrm{Cov}_\theta\left(\Lambda^\theta-\mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j'(C_j^\theta)}],\mathbb{E}_{j|\theta}[u_j'(C_j^\theta)dC_j^\theta]\right)
\end{aligned}$$

$$
\begin{aligned}
&+\mathbb{E}_\theta\left[\Lambda^\theta-\mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j'(C_j^\theta)}]\right]\mathbb{E}_\theta\left[\mathbb{E}_{j|\theta}[u_j'(C_j^\theta)dC_j^\theta]\right]\\
=&\mathbb{E}_\theta[\mathrm{Cov}_{j|\theta}(-\frac{P_j^\theta}{u_j'(C_j^\theta)},u_j'(C_j^\theta)dC_j^\theta)]+\mathrm{Cov}_\theta\left(\Lambda^\theta-\mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j'(C_j^\theta)}],\mathbb{E}_{j|\theta}[u_j'(C_j^\theta)dC_j^\theta]\right),
\end{aligned}
$$

where the last equation used the fact that $\mathbb{E}_\theta\left[\Lambda^\theta\right]=\mathbb{E}_\theta\left[\mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j'(C_j^\theta)}]\right]=\mathbb{E}_\theta\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right]$ under our price normalization (26). The two terms correspond to the (ii) MU dispersion and (v) redistribution terms in Proposition 1.

Similarly,

$$
\begin{aligned}
&\sum_\theta\sum_j[w_j^\theta-P_j^\theta C_j^\theta]dl_j^\theta\\
=&\sum_\theta\ell^\theta\sum_j\mu_j^\theta[w_j^\theta-P_j^\theta C_j^\theta]d\ln l_j^\theta\\
=&\sum_\theta\ell^\theta\left[\mathrm{Cov}_{j|\theta}(w_j^\theta-P_j^\theta C_j^\theta,d\ln l_j^\theta)+\mathbb{E}_{j|\theta}[w_j^\theta-P_j^\theta C_j^\theta]\underbrace{\mathbb{E}_{j|\theta}[d\ln l_j^\theta]}_{=0}\right]\\
=&\mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}(w_j^\theta-P_j^\theta C_j^\theta,d\ln l_j^\theta)\right]\\
=&\mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}(-\Pi^\theta-T_j^\theta,d\ln l_j^\theta)\right]\\
=&\mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}(-T_j^\theta,d\ln l_j^\theta)\right],
\end{aligned}
$$

which corresponds to the (iii) fiscal externality term. Finally,

$$
\begin{aligned}
&\sum_\theta\sum_j\sum_{l,k}p_{jl,k}y_{jl,k}\frac{1}{l_j^\theta}\gamma_{jl,k}^\theta dl_j^\theta\\
=&\sum_\theta\ell^\theta\sum_j\mu_j^\theta\sum_{l,k}p_{jl,k}y_{jl,k}\frac{1}{l_j^\theta}\gamma_{jl,k}^\theta d\ln l_j^\theta\\
=&\mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}(\sum_{l,k}p_{jl,k}y_{jl,k}\frac{1}{l_j^\theta}\gamma_{jl,k}^\theta,d\ln l_j^\theta)\right],
\end{aligned}
$$

which corresponds to the (iv) technological externality term.

□

A.4 Proof of Proposition 2

Consider the government's problem to set spatial transfers T_j^θ to maximize the SWF (17). The (constrained) Pareto efficient transfer policy solves

$$
\max_{\{\{C_j^\theta,\mathbf{c}_j^\theta,P_j^\theta,T_j^\theta,l_j^\theta,\mu_j^\theta,w_j^\theta\},\{\mathbf{x}_{ij,k},\mathbf{l}_{ij,k},h_{ij,k},\mathcal{A}_{ij,k},p_{ij,k}\},r_j,W^\theta\}}\sum_j\mathcal{W}(\{W^\theta\}),\tag{A.16}
$$

subject to (2)-(16).

To solve this problem, we follow the primal approach in the public finance literature. That is, we focus on a relaxed planning problem where the Planner picks an incentive-compatible allocation and later confirms that their chosen allocation, alongside supporting prices, is also a solution to the original problem. The relaxed planning problem is defined as follows.

Definition A.1 (Relaxed Planning Problem)**.** The Planner solves

$$\max_{\{\{C_j^\theta,\mathbf{c}_j^\theta,l_j^\theta,\mu_j^\theta\},\{\mathbf{x}_{ij,k},\mathbf{l}_{ij,k},h_{ij,k},\mathcal{A}_{ij,k}\}\}} \mathcal{W}(\{W^\theta\}) \tag{A.17}$$

subject to (9), (13)-(16)

$$\{\mu_j^\theta\}_j \in \underset{\{\tilde{\mu}_j^\theta\}_j:\sum_j \tilde{\mu}_j^\theta=1}{\arg\max} \sum_j \tilde{\mu}_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}) \tag{A.18}$$

Compared to the pseudo-planning problem in Lemma 2, the relaxed planning problem chooses consumption instead of taking the equilibrium allocation as given (Equation (23)). Furthermore, this problem is different from the first-best planning problem as considered in Appendix C because the Planner must choose an incentive-compatible population allocation (A.18). For this reason, we refer to these policies as second-best.

We first characterize the first-order conditions of the relaxed planning problem of Definition A.1. The first-order conditions with respect to $c_{ij,k}^\theta$, $l_{ij,k}^\theta$, $h_{ij,k}$, and $x_{ij,k}^{l,m}$ are given by

$$P_j^{SB,\theta}\frac{\partial C_j^\theta}{\partial c_{ij,k}^\theta} = p_{ij,k}^{SB}, \quad p_{ij,k}^{SB}A_{ij,k}\frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^{SB,\theta}, \quad p_{ij,k}^{SB}A_{ij,k}\frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i^{SB}, \quad p_{ij,k}^{SB}A_{ij,k}\frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}^{SB}, \tag{A.19}$$

where $P_j^{SB,\theta}$, $p_{ij,k}^{SB}$, $w_j^{SB,\theta}$, and r_j^{SB} are Lagrange multipliers on constraints (13)-(16). These conditions are identical to the equilibrium conditions (A.12), with $P_j^{SB,\theta}$, $p_{ij,k}^{SB}$, $w_j^{SB,\theta}$, and r_j^{SB} coinciding with P_j^θ, $p_{ij,k}$, w_j^θ, and r_j up to a multiplicative constant. The first-order condition with respect to C_j^θ is given by

$$\ell^\theta\mu_j^\theta\left[\Lambda^\theta u_j'(C_j^\theta) - P_j^{SB,\theta}\right] = \ell^\theta\sum_i \frac{\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta}\left[P_i^{SB,\theta}C_i^\theta - w_i^{SB,\theta} - \sum_{l,k} p_{il,k}^{SB}y_{il,k}\frac{1}{l_i^\theta}\gamma_{il,k}^\theta\right]. \tag{A.20}$$

Dividing both sides by $u_j^{\theta\prime}(C_j^\theta)$ and summing across j and θ, we have

$$\begin{aligned}&\sum_\theta \ell^\theta\sum_j \mu_j^\theta\left[\tilde{\Lambda}^\theta - \frac{P_j^{SB,\theta}}{u_j'(C_j^\theta)}\right]\\ &= -\sum_\theta \ell^\theta\sum_j\sum_i \frac{1}{u_j'(C_j^\theta)}\frac{\partial\mu_i^\theta}{\partial C_j^\theta}\left[w_i^{SB,\theta} - P_i^{SB,\theta}C_i^\theta + \sum_{l,k} p_{il,k}^{SB}y_{il,k}\frac{1}{l_i^\theta}\gamma_{il,k}^\theta\right]\end{aligned}$$

$$
\begin{aligned}
&= -\sum_\theta \ell^\theta \sum_i \underbrace{\sum_j \frac{1}{u'_j(C_j^\theta)} \frac{\partial \mu_i^\theta}{\partial C_j^\theta}}_{=0} \left[w_i^{SB,\theta} - P_i^{SB,\theta} C_i^\theta + \sum_{l,k} p_{il,k}^{SB} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right] \\
&= 0,
\end{aligned}
$$

where the third line comes from the fact that a uniform increase in utility in all locations will not affect location choices. This implies

$$
\sum_\theta \sum_j l_j^\theta \frac{P_j^{SB,\theta}}{u'_j(C_j^\theta)} = \sum_\theta \ell^\theta \Lambda^\theta. \tag{A.21}
$$

Comparing (A.21) and (26), and noting that $P_j^{SB,\theta}$, $p_{ij,k}^{SB}$, $w_j^{SB,\theta}$, and r_j^{SB} coincide with P_j^θ, $p_{ij,k}$, w_j^θ, and r_j up to a multiplicative constant, we have

$$
P_j^{SB,\theta} = P_j^\theta \sum_\theta \ell^\theta \Lambda^\theta, \quad p_{ij,k}^{SB} = p_{ij,k} \sum_\theta \ell^\theta \Lambda^\theta, \quad w_j^{SB,\theta} = w_j^\theta \sum_\theta \ell^\theta \Lambda^\theta, \quad r_j^{SB} = r_j \sum_\theta \ell^\theta \Lambda^\theta.
$$

In turn, we can rewrite (A.20) using equilibrium prices as

$$
\mu_j^\theta \left[\Lambda^\theta u'_j(C_j^\theta) - P_j^\theta \right] = \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[P_i^\theta C_i^\theta - w_i^\theta - \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right]. \tag{A.22}
$$

By noting $T_i^\theta = P_i^\theta C_i^\theta - w_i^\theta - \Pi^\theta$ and $\sum_i \frac{\partial \hat{\mu}_i^\theta}{\partial C_j^\theta} = 0$, we obtain (29).

Next, we need to show that all the equilibrium conditions are satisfied under $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$ where C_j^θ satisfies (29) with supporting prices $\{\{P_j^\theta, w_j^\theta\}, \{p_{ij,k}\}, r_j\}$ that satisfy (A.22). First, it is immediate to see that market clearing conditions are satisfied because (13)-(16) enter as constraints. The constraint (A.18) implies that the population distribution solves (19). Given prices $\{\{P_j^\theta, w_j^\theta\}, \{p_{ij,k}\}, r_j\}$, the firm's optimality conditions (A.13) are satisfied because they are identical to (A.19).

Finally, it remains to be shown that the government budget (8) is satisfied. Multiplying $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$ by l_j^θ and summing across j and θ, we have

$$
\begin{aligned}
&\sum_\theta \sum_j T_j^\theta l_j^\theta \\
&= \sum_\theta \sum_j P_j^\theta C_j^\theta l_j^\theta - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_\theta \sum_{i,j,k} p_{ij,k} c_{ij,k}^\theta l_j^\theta - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_{i,j,k} p_{ij,k} \left[A_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k} \mathbf{x}_{ij,k}) - \sum_{l,m} p_{ij,k} x_{jl,m}^{i,k} \right]
\end{aligned}
$$

$$
\begin{aligned}
&\quad - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_{i,j,k} p_{ij,k} \left[\sum_\theta A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}} l_{ij,k}^\theta + A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} h_{ij,k} + \sum_{l,m} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} x_{ij,k}^{l,m} - \sum_{l,m} p_{ij,k} x_{jl,m}^{i,k} \right] \\
&\quad - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_{i,j,k} \left[\sum_\theta w_i^\theta l_{ij,k}^\theta + r_i h_{ij,k} + \sum_{l,m} p_{li,m} x_{ij,k}^{l,m} - \sum_{l,m} p_{ij,k} x_{jl,m}^{i,k} \right] \\
&\quad - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_\theta \sum_j w_j^\theta l_j + \sum_j r_j \bar{h}_j - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= 0.
\end{aligned}
$$

□

Without Preference Shocks. We also discuss the case without preference shocks as considered by Fajgelbaum and Gaubert (2020). To do so, we rewrite the second-best problem as follows:

$$
\begin{aligned}
\max_{\{W^\theta, \{C_j^\theta, \mathbf{c}_j^\theta, \mu_j^\theta\}\}, \{\mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mathcal{A}_{ij,k}\}} & \sum_j \mu_j^{\bar{\theta}} u_j(C_j^{\bar{\theta}}) && \text{(A.23)} \\
\text{s.t.} \qquad & (9), (13)\text{-}(16) && \text{(A.24)} \\
u_j(C_j^\theta) &= W^\theta \text{ for all } j, \theta && \text{(A.25)} \\
\sum_j \mu_j^\theta &= 1 \text{ for all } \theta && \text{(A.26)} \\
\sum_j \mu_j^{\tilde{\theta}} u_j(C_j^{\tilde{\theta}}) &\geq \underline{W}^{\tilde{\theta}} \text{ for all } \tilde{\theta} \neq \bar{\theta} && \text{(A.27)}
\end{aligned}
$$

Note that we rewrote households' incentive compatibility constraints for location choice (22) with utility equalization (A.25) and adding up constraint (A.26). Note also that $\psi^\theta(\cdot) = 0$ without preference shocks.

The first-order condition for μ_j^θ is given by

$$
\begin{aligned}
& \ell^\theta \left[w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta \right] + \hat{\Lambda}^\theta W^\theta = \delta^{SB,\theta} \\
\Leftrightarrow \quad & w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta = \left(\delta^{SB,\theta} - \hat{\Lambda}^\theta W^\theta \right) / \ell^\theta, \qquad \text{(A.28)}
\end{aligned}
$$

where $\delta^{SB,\theta}$ denotes the Lagrange multiplier on constraint (A.26). By noting that $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$, the cross-location component of transfers only addresses technological externalities, and the cross-type component of transfers addresses redistribution concerns, as highlighted by Fajgelbaum and Gaubert (2020).

A.5 Proof of Proposition 3

By multiplying Equation (29) by $\ell^\theta dC_j^\theta$ and summing up across j and θ, we have

$$\sum_j \sum_\theta l_j^\theta \left[\Lambda^\theta u_j'(C_j^\theta) - P_j^\theta\right] dC_j^\theta$$
$$= \sum_i \sum_\theta \underbrace{\sum_j \ell^\theta dC_j^\theta \frac{\partial \mu_i^\theta}{\partial C_j^\theta}}_{=dl_i^\theta} \left[P_i^\theta C_i^\theta - w_i^\theta - \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right]. \tag{A.29}$$

Plugging this into Equation (A.15), we only have (i) technology term.

One can also prove Proposition 3 by directly applying the envelope theorem to the relaxed planning problem of Definition A.1. Despite the presence of incentive compatibility constraints for households' location decisions, there are no reallocation effects because technological effects do not directly affect these constraints.

□

A.6 Proof of Proposition 4

From Baqaee and Farhi (2024),

$$d\ln Y_i = \sum_{j,k} \frac{p_{ij,k} y_{ij,k}}{GDP_i} d\ln \mathcal{A}_{ij,k} + \sum_\theta \frac{w_i^\theta l_i^\theta}{GDP_i} d\ln l_i^\theta. \tag{A.30}$$

Using the definition of aggregate real GDP such that $d\ln Y = \sum_i GDP_i d\ln Y_i$, and using the fact that $d\ln \mathcal{A}_{ij,k} = d\ln A_{ij,k} + \sum_\theta \gamma_{ij,k}^\theta d\ln l_i^\theta$,

$$d\ln Y = \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k} + \sum_{i,j,k} \sum_\theta \gamma_{ij,k}^\theta p_{ij,k} y_{ij,k} d\ln l_i^\theta + \sum_i \sum_\theta w_i^\theta l_i^\theta d\ln l_i^\theta, \tag{A.31}$$

which gives the desired expression.

□

A.7 Proof of Proposition 5

Here, we prove a general version of Proposition 5 nesting multiple household types.

Proposition A.1. *Consider the spatial equilibrium with multiplicative Fréchet preference shocks with arbitrary correlation (34) with multiple types. Let $\tilde{W} \equiv \tilde{\mathcal{W}}(\{\tilde{W}^\theta\})$ be welfare in this economy. Consider another economy where preferences are a log transformation of the first specification, i.e. the additively separable counterpart (35) with multiple types, and the remaining equilibrium conditions* *of Definition 1 are unchanged. Let $W \equiv \mathcal{W}(\{W^\theta\})$ be welfare in this economy, where $\mathcal{W}(\{W^\theta\}) \equiv \ln \tilde{\mathcal{W}}(\{\exp(W^\theta)\})$ is the social welfare function. Then, the welfare decomposition of Proposition 1 is identical in both economies up to a multiplicative constant.*

Before starting our proof, notice that Proposition A.1 comes down to Proposition 5 with single household type.

We first note that, given the assumption that $\mathcal{W}(\{W^\theta\}) \equiv \ln \tilde{\mathcal{W}}(\{\exp(W^\theta)\})$, we have

$$dW = \sum_\theta \frac{\partial \mathcal{W}}{\partial W^\theta} dW^\theta, \qquad d\ln \tilde{W} = \sum_\theta \frac{\partial \mathcal{W}}{\partial W^\theta} d\ln \tilde{W}^\theta. \tag{A.32}$$

Therefore, to show $dW = d\ln\tilde{W}$, it is sufficient to show that the same isomorphism holds for the expected utility for each type θ, i.e. $dW^\theta = d\ln\tilde{W}^\theta$. The expected utility of households in (34) is given by

$$\tilde{W}^\theta = G^\theta(\tilde{u}_1(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N(C_N^\theta)^{\nu^\theta})^{1/\nu^\theta}, \tag{A.33}$$

and that in (35) is given by

$$W^\theta = \frac{1}{\nu^\theta} \ln G^\theta(\exp(\nu^\theta u_1(C_1^\theta)), \ldots, \exp(\nu^\theta u_N(C_N^\theta))). \tag{A.34}$$

See Appendix D for a detailed mathematical derivation. Therefore, under $u_j(C_j^\theta) = \ln(\tilde{u}_j(C_j^\theta))$ and $\epsilon_j^\theta = \ln(\tilde{\epsilon}_j^\theta)$, we have $W^\theta = \ln\tilde{W}^\theta$.

Finally, we prove that the decomposition is also identical. The Lagrangian for the pseudo-planning problem in an economy with multiplicative preference shocks (34) is

$$\begin{aligned}
\mathcal{L} = {} & \tilde{\mathcal{W}}\left(\left\{G^\theta(\tilde{u}_1(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N(C_N^\theta)^{\nu^\theta})^{1/\nu^\theta}\right\}\right) \\
& + \sum_{i,j,k} p_{ij,k}^L \left[A_{ij,k} g_{ij,k}(\{\ell^\theta \mu_j^\theta(\boldsymbol{C}^\theta)\}) f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) - \left(\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{jl,m}^{i,k}\right)\right] \\
& + \sum_{j,\theta} P_j^{L,\theta}\left[\mathcal{C}_j^\theta(\mathbf{c}_j^\theta) - C_j^\theta \ell^\theta \mu_j^\theta(\boldsymbol{C}^\theta)\right] \\
& + \sum_{j,\theta} w_j^{L,\theta}\left[\ell^\theta \hat{\mu}_j^\theta(\boldsymbol{C}^\theta) - \sum_{i,k} l_{ji,k}^\theta\right] \\
& + \sum_j r_j^L \left[\bar{h}_j - \sum_{i,k} h_{ji,k}\right] \\
& + \sum_j \eta_j^\theta \left[C_j^\theta - \check{C}_j^\theta\right],
\end{aligned}$$

where we normalize $\{\eta_j^\theta\}$ such that $\sum_\theta \sum_j \eta_j^\theta / u'(\check{C}_j^\theta) = 0$ as in the proof of Proposition 1. As in Proposition 1, the Lagrange multipliers $\{\{P_j^{L,\theta}, w_j^{L,\theta}\}, \{p_{ij,k}^L\}, r_j^L\}$ coincide with equilibrium prices up to a multiplicative constant. The first-order condition with respect to C_j^θ is

$$\ell^\theta \mu_j^\theta \left[\frac{\partial \tilde{\mathcal{W}}}{\partial \tilde{W}^\theta} \tilde{W}^\theta u_j'(C_j^\theta) - P_j^{L,\theta}\right] + \eta_j^\theta = \ell^\theta \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[P_i^{L,\theta} C_i^\theta - w_i^{L,\theta} - \sum_{l,k} p_{il,k}^L y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right],$$

where we used the fact that (see also Appendix D)

$$\frac{\partial \tilde{W}^\theta}{\partial C_j^\theta} = \tilde{W}^\theta \frac{G_j^\theta(\tilde{u}_1(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N(C_N^\theta)^{\nu^\theta})}{G^\theta(\tilde{u}_1(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N(C_N^\theta)^{\nu^\theta})} \tilde{u}_j'(C_j^\theta) = \tilde{W}^\theta \mu_j^\theta \frac{\tilde{u}_j'(C_j^\theta)}{\tilde{u}_j(C_j^\theta)} = \tilde{W}^\theta \mu_j^\theta u_j'(C_j^\theta). \tag{A.35}$$

Since $\Lambda^\theta = \frac{1}{\tilde{\mathcal{W}}} \frac{\partial \tilde{\mathcal{W}}}{\partial \tilde{W}^\theta} \tilde{W}^\theta$ under our assumption, we can rewrite the above expression as

$$\ell^\theta \mu_j^\theta \left[\Lambda^\theta \tilde{\mathcal{W}} u_j'(C_j^\theta) - P_j^{L,\theta}\right] + \eta_j^\theta = \ell^\theta \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[P_i^{L,\theta} C_i^\theta - w_i^{L,\theta} - \sum_{l,k} p_{il,k}^L y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right].$$

Dividing both sides by $u_j^{\theta\prime}(C_j^\theta)$ and summing up across j and θ,

$$\sum_\theta \sum_j \frac{P_j^{L,\theta}}{u_j'(C_j^\theta)} = \tilde{\mathcal{W}}. \tag{A.36}$$

Comparing (26) and (A.36), the Lagrange multipliers coincide with equilibrium prices up to the multiplicative constant $\tilde{\mathcal{W}}$.

Applying the envelope theorem as in the proof of Proposition 1,

$$\begin{aligned}
d\tilde{W} =& \sum_{i,j,k} p_{ij,k}^L y_{ij,k} d\ln A_{ij,k} + \sum_\theta \sum_j l_j^\theta [\tilde{\mathcal{W}} \Lambda^\theta u_j'(C_j^\theta) - P_j^{L,\theta}] dC_j^\theta \\
&+ \sum_\theta \sum_j [w_j^{L,\theta} - P_j^{L,\theta} C_j^\theta] dl_j^\theta + \sum_\theta \sum_j \sum_{l,k} p_{jl,k}^L y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta dl_j^\theta \\
=& \tilde{\mathcal{W}} \Bigg[\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k} + \sum_\theta \sum_j l_j^\theta [\Lambda^\theta u_j'(C_j^\theta) - P_j^\theta] dC_j^\theta \\
&+ \sum_\theta \sum_j [w_j^\theta - P_j^\theta C_j^\theta] dl_j^\theta + \sum_\theta \sum_j \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta dl_j^\theta \Bigg].
\end{aligned} \tag{A.37}$$

Since the term inside the square bracket is identical to what we obtain in Equation (A.15), the decomposition remains identical up to the multiplicative constant $\tilde{\mathcal{W}}$.

□

B An Axiomatization of Social Welfare Function (SWF)

In this appendix, we discuss one axiomatization of our SWF in Section 2.2. Specifically, we extend the axiomatization of Eden (2020a) with heterogeneous types θ.

We mostly follow the notation by Eden (2020a) and consider a general environment (beyond spatial equilibrium in our main paper). There are I individuals indexed by $i = 1, \ldots, I$. The set of individual allocations is $X = \mathbb{R}^J_+$, where J is the number of goods (e.g. a vector of final goods across all locations). Elements of x is denoted by $\vec{x} = (x_1, \ldots, x_J) \in X$. There are S identity groups, indexed by $s = 1, \ldots, S$. Each individual belongs to one of the identity groups, θ_s, where $\Theta \equiv \{\theta_1, \ldots, \theta_S\}$, is a partition of a set of individuals. Individual preferences $P_i \in D^*$ are defined over x, and profiles of the preferences across agents are given by $\mathbf{P} \in D^I$. We follow the same definition of welfare-equivalence relation $e(\vec{x}_i, \vec{x}'_i, P_i)$ and the index of consumption $\iota(\vec{x})$ as Eden (2020a). In the spatial context, it is natural to consider $\iota(\vec{x})$ as the set of unit vectors for each location's final goods. Similarly, we define $\lambda(\vec{x}_i, \vec{x}'_i, P_i)$ as the measure of welfare gains as in Eden (2020a), i.e. an equivalent unit increase in consumption in the index of $\vec{x}_i$ that an individual with preference P_i is indifferent with the allocation $\vec{x}'_i$. The ranking of social allocation given the preference profile $\mathbf{P}$ is given by the ordering $\preceq_{\mathbf{P}}$.

Definition B.1. Given a measure of welfare gains, λ, a constitution is said to satisfy the *Independence of Irrelevant Vaiations (IIV)* axiom if, for every $\mathbf{x}, \mathbf{x}' \in \mathbf{X^I}$ and $\mathbf{P}, \mathbf{P}' \in D^I$ such that $\lambda(\vec{x}_i, \vec{x}'_i, P_i) = \lambda(\vec{x}_i, \vec{x}'_i, P'_i)$ for every i, it holds that $\mathbf{x} \preceq_{\mathbf{P}} \mathbf{x}'$ if and only if $\mathbf{x} \preceq_{\mathbf{P}'} \mathbf{x}'$.

Definition B.2. A social preference relation $\preceq_{\mathbf{P}}$ is said to be consistent with the *Pareto Principle* if, for every two social states $\mathbf{x}, \mathbf{x}' \in \mathbf{X^I}$, it holds (a) if $\vec{x}_i \preceq_{P_i} \vec{x}'_i$ for every i, then $\mathbf{x} \preceq_{\mathbf{P}} \mathbf{x}'$, and (b) if, in addition, for some i, $\vec{x}_i \preceq_{P_i} \vec{x}'_i$, then $\mathbf{x} \preceq_{\mathbf{P}} \mathbf{x}'$.

Definition B.3. A constitution is said to satisfy *Weak Anonymity within an Identity Group* if, for every $i < i'$ and s such that $i, i' \in \theta_s$ and $P_i = P_{i'}$,

$$(\vec{x}_1, \ldots, \vec{x}_i, \ldots, \vec{x}_{i'}, \ldots \vec{x}_I) \sim (\vec{x}_1, \ldots, \vec{x}_{i'}, \ldots, \vec{x}_i, \ldots \vec{x}_I) \tag{B.1}$$

Proposition B.1. *Let $\{\iota(\vec{x})\}_{\vec{x} \in X}$ be a set of indexes. Assume that $D \subset D^*$ is a preference domain for which a welfare-equivalence relation exists, and let λ be a measure of welfare gains. Assume that there exist continuous functions $\{\mu_{\iota(\vec{x})} : \iota(\vec{x}) \to \mathbb{R}\}_{\vec{x} \in X}$ such that, for each $P \in D$, the function $u(\cdot|P) = \mu_{\iota(\cdot)}(\cdot) + \gamma(\iota(\cdot)|P)$ is a representation of P. Then there exists a constitution that jointly satisfies the IIV axiom, the Weak Anonymity within an Identity Group condition and the Pareto condition. The constitution can be represented by the social welfare function*

$$W(\mathbf{x}, \mathbf{P}) = \sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} u(\vec{x}_i | P_i) \tag{B.2}$$

Proof. The constitution represented by (B.2) trivially satisfies weak anonymity within an identity group and Pareto condition. To establish consistency with the IIV axiom, let there be $\mathbf{x}, \mathbf{x}' \in X^I$ and $\mathbf{P}, \mathbf{P}' \in D^I$ such that, for each i, $\lambda\left(\vec{x_i}, \vec{x'_i}, P_i\right) = \lambda\left(\vec{x_i}, \vec{x'_i}, P'_i\right)$. As, by Claim 1 in Eden (2020a), $\lambda\left(\vec{x}_i, \cdot, P_i\right)$ is a representation of the preferences P_i, and as $\vec{x'_i} \sim_{P_i} e\left(\vec{x_i}, \vec{x'_i}, P_i\right)$, it follows that $\lambda\left(\vec{x_i}, \vec{x'_i}, P_i\right) = \lambda\left(\vec{x}_i, e\left(\vec{x_i}, \vec{x'_i}, P_i\right), P_i\right)$ and, similarly, $\lambda\left(\vec{x_i}, \vec{x'_i}, P'_i\right) = \lambda\left(\vec{x_i}, e\left(\vec{x_i}, \vec{x'_i}, P'_i\right), P'_i\right)$. The assumption that $\lambda\left(\vec{x_i}, \vec{x'_i}, P_i\right) = \lambda\left(\vec{x_i}, \vec{x'_i}, P'_i\right)$ thus implies that

$$\lambda\left(\vec{x}_i, e\left(\vec{x}_i, \vec{x}'_i, P_i\right), P_i\right) = \lambda\left(\vec{x_i}, e\left(\vec{x}_i, \vec{x}'_i, P'_i\right), P'_i\right)$$

As $e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i'\right) \in \iota\left(\overrightarrow{x_i}\right)$, by Claim 2 it follows that the welfare gains from switching from $e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i'\right)$ to $\overrightarrow{x_i}$ are the same for all $P_i \in D$, and hence

$$\lambda\left(\overrightarrow{x_i}, e\left(\vec{x}_i, \overrightarrow{x_i'}, P_i'\right), P_i'\right) = \lambda\left(\overrightarrow{x_i}, e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i'\right), P_i\right)$$

Combining with the previous equation yields

$$\lambda\left(\overrightarrow{x_i}, e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i\right), P_i\right) = \lambda\left(\overrightarrow{x_i}, e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i'\right), P_i\right)$$

As $\lambda\left(\vec{x}_i, \cdot, P_i\right)$ is a representation of the preferences P_i and $e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i\right), e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i'\right) \in \iota\left(\overrightarrow{x_i}\right)$, given that $\iota\left(\overrightarrow{x_i}\right)$ is a weakly increasing curve, it follows that

$$e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i\right) = e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i'\right)$$

By Equation (B.2),

$$\begin{aligned} & \mathbf{x} \preceq_{\mathbf{P}} \mathbf{x}' \\ \Leftrightarrow \quad & \sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} u\left(\overrightarrow{x_i} \mid P_i\right) \leq \sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} u\left(\overrightarrow{x_i'} \mid P_i\right) = \sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} u\left(e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i\right) \mid P_i\right) \end{aligned} \tag{B.3}$$

Because $e\left(\vec{x}, \vec{x}', P_i\right) \in \iota\left(\overrightarrow{x_i}\right)$, it holds that

$$u\left(e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i\right) \mid P_i\right) = \mu_{\iota\left(\overrightarrow{x_i}\right)}\left(e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i\right)\right) + \gamma\left(\iota\left(\overrightarrow{x_i}\right) \mid P_i\right)$$

and, by definition,

$$u\left(\overrightarrow{x_i} \mid P_i\right) = \mu_{\iota\left(\overrightarrow{x_i}\right)}\left(\overrightarrow{x_i}\right) + \gamma\left(\iota\left(\overrightarrow{x_i}\right) \mid P_i\right). \tag{B.4}$$

Substituting into expression (B.3) yields the condition

$$\sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} \left(\mu_{\iota\left(\overrightarrow{x_i}\right)}\left(\overrightarrow{x_i}\right) + \gamma\left(\iota\left(\overrightarrow{x_i}\right) \mid P_i\right)\right) \leq \sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} \left(\mu_{\iota\left(\overrightarrow{x_i}\right)}\left(e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i\right)\right) + \gamma\left(\iota\left(\overrightarrow{x_i}\right) \mid P_i\right)\right) \tag{B.5}$$

$$\Leftrightarrow \quad \sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} \mu_{\iota\left(\overrightarrow{x_i}\right)}\left(\overrightarrow{x_i}\right) \leq \sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} \mu_{\iota\left(\overrightarrow{x_i}\right)}\left(e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i\right)\right) \tag{B.6}$$

Similarly, $\mathbf{x} \preceq_{\mathbf{P}'} \mathbf{x}'$ if and only if

$$\sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} \mu_{\iota\left(\overrightarrow{x_i}\right)}\left(\overrightarrow{x_i}\right) \leq \sum_{s=1}^{S} \Lambda_{\theta_s} \sum_{i \in \theta_s} \mu_{\iota\left(\overrightarrow{x_i}\right)}\left(e\left(\overrightarrow{x_i}, \overrightarrow{x_i'}, P_i'\right)\right).$$

By Equation (B.4), this is the same condition and hence $\mathbf{x} \preceq_{\mathbf{P}} \mathbf{x}'$ if and only if $\mathbf{x} \preceq_{\mathbf{P}'} \mathbf{x}'$, concluding the proof that the IIV axiom is satisfied. As the constitution represented by equation (B.2) satisfies the theorem's axioms, a constitution exists. □

Notice that the SWF (B.2) corresponds to the local approximation of the SWF used in our paper in Section 2.2.

C First-Best Allocation

In this section, we discuss the first-best planning problem, where the Planner can directly specify the allocation for both location choice and consumption. The problem is given by

$$W = \max_{\{W^\theta, \{C_j^\theta, \mathbf{c}_j^\theta, \mu_j^\theta\}\}, \{\mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mathcal{A}_{ij,k}\}} \mathcal{W}(\{W^\theta\}) \tag{C.1}$$

$$\text{s.t.} \qquad (9), (13)\text{-}(16), \tag{C.2}$$

$$W^\theta = \sum_j \mu_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}) \tag{C.3}$$

$$\sum_j \mu_j^\theta = 1. \tag{C.4}$$

In the first-best planning problem, the only constraints the Planner faces are resource constraints. Notice that this problem is different from the ones considered in Allen and Arkolakis (2014) and Fajgelbaum and Gaubert (2020) which take the incentive compatibility constraints for location choice as given, which we separately study in Appendix A.4.

The first-order conditions with respect to $c_{ij,k}$, $l_{ij,k}^\theta$, $h_{ij,k}$, and $x_{ij,k}^{l,m}$ are

$$P_j^{FB,\theta} \frac{\partial C_j^\theta}{\partial c_{ij,k}} = p_{ij,k}^{FB}, \quad p_{ij,k}^{FB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^{FB,\theta}, \quad p_{ij,k}^{FB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i^{FB}, \quad p_{ij,k}^{FB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}^{FB}, \tag{C.5}$$

where $P_j^{FB,\theta}$, $p_{ij,k}^{FB}$, $w_j^{FB,\theta}$, and r_j^{FB} are Lagrange multipliers on constraints (13)-(16). The superscript FB denotes variables from the Planner's first-best allocation. These conditions are identical to the equilibrium conditions (A.12), with the Planner's shadow prices coinciding with equilibrium prices up to a multiplicative constant. Therefore, relative quantities of inputs are not distorted in equilibrium.

The Planner's first-best allocation deviates from the equilibrium when we consider the first-order conditions for C_j^θ and μ_j^θ. The first-order condition with respect to C_j^θ gives us

$$\Lambda^{\theta,FB} u_j'(C_j^{FB,\theta}) / P_j^{FB,\theta} = 1, \tag{C.6}$$

where $\Lambda^{\theta,FB} \equiv \frac{\partial \mathcal{W}}{\partial W^\theta}$. That is, the weighted marginal utility of income is equalized across locations for type θ households.

The first-order condition for μ_j^θ is

$$\Lambda^\theta\Big(u_j(C_j^{FB,\theta}) - \frac{\partial\psi^\theta}{\partial\mu_j^\theta}\Big) + w_j^{FB,\theta} + \sum_{i,k} p_{ji,k}^{FB} y_{ji,k}^{FB} \frac{\gamma_{ji,k}^\theta}{\ell_j^\theta} - P_j^{FB,\theta} C_j^{FB,\theta} = \delta^{FB,\theta}/\ell^\theta, \tag{C.7}$$

where $\delta^{FB,\theta}$ is the Lagrange multiplier on constraint (C.4).

Now we ask whether the above two conditions can be satisfied in the decentralized equilibrium. Note that $\{\{P_j^{FB,\theta}, w_j^{FB,\theta}\}, \{p_{ij,k}^{FB}\}, r_j^{FB}\}$ coinciding with $\{\{P_j^\theta, w_j^\theta\}, \{p_{ij,k}\}, r_j\}$ up to scale. Therefore, in order for the decentralized equilibrium to satisfy (C.6), it must be that for some $J^\theta > 0$,

$$u_j'(C_j^\theta)/P_j^\theta = J^\theta \quad \text{for all } j, \theta. \tag{C.8}$$

For households' location choice in the decentralized equilibrium (A.5) to satisfy (C.7), agglomeration externalities net of transfers are invariant across locations for a given household type to be equalized:

$$\sum_{i,k} p_{ji,k} y_{ji,k} \frac{\gamma_{ji,k}^\theta}{\ell_j^\theta} - T_j^\theta = H^\theta \quad \text{for all } j, \theta \tag{C.9}$$

for some H^θ.

We summarize the results as follows.

Proposition C.1. *A decentralized equilibrium maximizes aggregate welfare for some welfare weights $\{\Lambda^\theta\}$ only if both the marginal utility of income and agglomeration externalities net of transfers are equalized across locations for a given household type (i.e.* (C.8) *and* (C.9) *hold).*

The conditions for optimality are stringent and will not generally be satisfied in a spatial economy. Indeed, there is no reason why the consumption implied by equilibrium prices and transfers set according to (C.9) will equalize marginal utility, since marginal utility never shows up in the equilibrium conditions. For example, consider the case where there are no agglomeration externalities $\gamma_{ij,k}^\theta = 0$ and no spatial transfers $T_j^\theta = 0$. The optimality of the decentralized equilibrium requires $u_j'((w_j^\theta + \Pi^\theta)/P_j^\theta)$ to be equalized across locations. This is generically not satisfied. To see why, suppose that $u_j'((w_j^\theta + \Pi^\theta)/P_j^\theta)$ is equalized across locations in the decentralized equilibrium. Then, we can always perturb the parameters governing marginal utility u_j' and utility levels u_j at the same time so that location choices are the same but u_j' will be different. Such a perturbation leads to dispersion in marginal utility across locations without changing prices or the allocation. This highlights that optimality of the decentralized equilibrium is attained only under a knife-edge set of parameters.

More generally, achieving the first-best requires agents to internalize their technological and fiscal externalities without generating dispersion in marginal utility, but the same argument as before suggests such a condition is knife-edge. To achieve the first-best, the Planner must be able to separately control consumption and the location choice decision, which would require some mechanism to directly influence location choice independent of consumption (i.e. break the incentive compatibility constraint (22)).

D Representative Agent Formulation under Generalized Extreme Value (GEV) Preference Shocks

In this section, we describe the isomorphic representative agent formulation of location choice under GEV preference shocks.

D.1 Additively Separable Case

Consider an additively separable utility function of the form

$$U_j(C_j^\theta, \epsilon_j^\theta) = u_j(C_j^\theta) + \epsilon_j^\theta, \tag{D.1}$$

where ϵ_j^θ follows a type-I generalized extreme value distribution

$$\mathbb{P}[\epsilon_1^\theta \le \bar{\epsilon}_1, \ldots, \epsilon_N^\theta \le \bar{\epsilon}_N] = \exp(-G^\theta(\exp(-\nu^\theta \bar{\epsilon}_1), \ldots, \exp(-\nu^\theta \bar{\epsilon}_N))), \tag{D.2}$$

where $G^\theta(.)$ is a correlation function that is homogeneous of degree one. As is well known since McFadden (1978), this yields the following location choice probability:

$$\mu_j^\theta = \frac{G_j^\theta(V_1^\theta, \ldots, V_N^\theta)V_j^\theta}{\sum_i G_i^\theta(V_1^\theta, \ldots, V_N^\theta)V_i^\theta}, \tag{D.3}$$

where

$$V_j^\theta \equiv \exp(\nu^\theta u_j(C_j^\theta)), \quad G_j^\theta \equiv \frac{\partial G^\theta(V_1^\theta, \ldots, V_N^\theta)}{\partial V_j^\theta}. \tag{D.4}$$

Now we construct a representative agent formulation that is isomorphic to the above model. Define a mapping $S_j^\theta(\{\mu_i^\theta\})$ that satisfies the following condition for all j:

$$G_j^\theta(S_1^\theta, \ldots, S_N^\theta)S_j^\theta = \mu_j^\theta. \tag{D.5}$$

The representative agent solves

$$W^\theta = \max_{\{\mu_j^\theta\}_j : \sum_j \mu_j^\theta = 1} \sum_j \mu_j^\theta u_j(C_j^\theta) - \frac{1}{\nu^\theta} \sum_j \mu_j^\theta \ln S_j^\theta(\{\mu_i^\theta\}). \tag{D.6}$$

The first-order condition with respect to μ_j^θ is given by

$$u_j(C_j^\theta) - \frac{1}{\nu^\theta} \ln S_j^\theta - \frac{1}{\nu^\theta} \sum_i \mu_i^\theta \frac{\partial \ln S_i^\theta}{\partial \mu_j^\theta} - \delta^\theta = 0, \tag{D.7}$$

where δ^θ is the Lagrange multiplier on the adding up constraint $\sum_j \mu_j^\theta = 1$. Note that

$$\sum_i \mu_i^\theta \frac{\partial \ln S_i^\theta}{\partial \mu_j^\theta} = 1, \tag{D.8}$$

for all j. To see this, we use the fact that $G^\theta(.)$ is homogeneous of degree one and add up (D.24) across j to have $G(S_1^\theta, \ldots, S_N^\theta) = \sum_j \mu_j^\theta$. Taking the derivative with respect to μ_j^θ gives us

$$\sum_i G_i(S_1^\theta, \ldots, S_N^\theta) S_i^\theta \frac{\partial \ln S_i^\theta}{\partial \mu_j^\theta} = 1 \tag{D.9}$$

$$\Leftrightarrow \quad \sum_i \mu_i^\theta \frac{\partial \ln S_i^\theta}{\partial \mu_j^\theta} = 1, \tag{D.10}$$

where we used (D.24) in the second line. Therefore, the first-order condition implies

$$S_j^\theta = \exp(-\nu^\theta \delta^\theta - 1 + \nu^\theta u_j(C_j^\theta)). \tag{D.11}$$

Thus, $S_j^\theta = \exp(-\nu^\theta \delta^\theta - 1) V_j^\theta$. Combining this with the fact that $G_j^\theta(.)$ is homogeneous of degree zero, we have that (D.24) implies

$$\mu_j^\theta = \exp(-\nu^\theta \delta^\theta - 1) G_j^\theta(V_1^\theta, \ldots, V_N^\theta) V_j^\theta. \tag{D.12}$$

The adding up constraint $\sum_j \mu_j^\theta = 1$ implies that

$$\exp(\nu^\theta \delta^\theta + 1) = \sum_j G_j^\theta(V_1^\theta, \ldots, V_N^\theta) V_j^\theta. \tag{D.13}$$

Therefore we obtain

$$\mu_j = \frac{G_j^\theta(V_1^\theta, \ldots, V_N^\theta) V_j^\theta}{\sum_i G_i^\theta(V_1^\theta, \ldots, V_N^\theta) V_i^\theta}, \tag{D.14}$$

coinciding with the solution to the discrete choice problem (D.20).

Finally, we confirm that indirect utility in the two representations coincides with each other. In the discrete choice problem, indirect utility is given by (see McFadden (1978))

$$W^\theta \equiv \mathbb{E}\left[\max_j \left\{u_j(C_j^\theta) + \epsilon_j^\theta\right\}\right] \tag{D.15}$$

$$= \frac{1}{\nu^\theta} \ln G^\theta(\exp(\nu^\theta u_1(C_1^\theta)), \ldots, \exp(\nu^\theta u_N(C_N^\theta))). \tag{D.16}$$

In the representative agent model, substituting (D.7) and (D.13) into (D.6), we obtain

$$W^\theta = \frac{1}{\nu^\theta} \ln G^\theta(\exp(\nu^\theta u_1(C_1^\theta)), \ldots, \exp(\nu^\theta u_N(C_N^\theta))), \tag{D.17}$$

verifying that indirect utility coincides with the original discrete choice formulation.

D.2 Multiplicatively Separable Case

Consider a multiplicatively separable utility function of the form

$$\tilde{u}_j(C_j^\theta, \epsilon_j^\theta) = \tilde{\epsilon}_j^\theta \tilde{u}_j(C_j^\theta), \tag{D.18}$$

where $\tilde{\epsilon}_j$ a follows type-II generalized extreme value distribution (multi-variate Fréchet)

$$\mathbb{P}[\tilde{\epsilon}_1^\theta \leq \bar{\epsilon}_1, \ldots, \tilde{\epsilon}_N^\theta \leq \bar{\epsilon}_N] = \exp(-G^\theta((\bar{\epsilon}_1)^{-\nu^\theta}, \ldots, (\bar{\epsilon}_N)^{-\nu^\theta})), \tag{D.19}$$

where $G^\theta(.)$ is a correlation function that is homogeneous of degree one. This yields the following location choice probability:

$$\mu_j^\theta = \frac{G_j^\theta(V_1^\theta, \ldots, V_N^\theta)V_j^\theta}{\sum_i G_i^\theta(V_1^\theta, \ldots, V_N^\theta)V_i^\theta}, \tag{D.20}$$

where

$$V_j^\theta \equiv \tilde{u}_j(C_j^\theta)^{\nu^\theta}, \quad G_j^\theta(V_1^\theta, \ldots, V_N^\theta) \equiv \frac{\partial G^\theta(V_1^\theta, \ldots, V_N^\theta)}{\partial V_j^\theta}. \tag{D.21}$$

Indirect utility follows

$$\tilde{W}^\theta = G^\theta(V_1^\theta, \ldots, V_N^\theta)^{1/\nu^\theta}. \tag{D.22}$$

Now we construct a representative agent formulation that is isomorphic to the above model. Define the utility function of the representative agent as

$$\mathcal{U}(\{\mu_j^\theta\}) = \sum_j \mu_j^\theta S_j^\theta(\{\mu_i^\theta\})^{-\frac{1}{\nu^\theta}} \tilde{u}_j(C_j^\theta), \tag{D.23}$$

where $S_j^\theta(\{\mu_i^\theta\})$ is defined, as before, to satisfy the following condition for all j:

$$G_j^\theta(S_1^\theta, \ldots, S_N^\theta)S_j^\theta = \mu_j^\theta. \tag{D.24}$$

The representative agent solves

$$\tilde{W}^\theta = \max_{\{\mu_j^\theta\}_j : \sum_j \mu_j^\theta = 1} \mathcal{U}(\{\mu_j^\theta\}). \tag{D.25}$$

The first-order condition is

$$(S_j^\theta)^{-\frac{1}{\nu^\theta}} \tilde{u}_j(C_j^\theta) - \frac{1}{\nu^\theta} \sum_i \mu_i^\theta (S_i^\theta)^{-\frac{1}{\nu^\theta}} \tilde{u}_i(C_i^\theta) \frac{\partial \ln S_i^\theta}{\partial \mu_j^\theta} = \delta^\theta \tag{D.26}$$

where δ^θ is the Lagrange multiplier on the adding up constraint $\sum_j \mu_j^\theta = 1$. Let $x_j^\theta \equiv (S_j^\theta)^{-\frac{1}{\nu^\theta}} \tilde{u}_j(C_j^\theta)$

and $\boldsymbol{x}^\theta \equiv [x_j^\theta]$. In matrix form, the set of first-order conditions can be expressed as

$$(\mathbf{I} - \boldsymbol{D}^\theta)\boldsymbol{x}^\theta = \delta^\theta \vec{\mathbf{1}}, \tag{D.27}$$

where $\boldsymbol{D}^\theta \equiv [d_{ij}^\theta]$ is an $N \times N$ matrix with $d_{ij}^\theta = \frac{1}{\nu^\theta}\mu_j^\theta \frac{\partial \ln S_j^\theta}{\partial \mu_i^\theta}$, and $\mathbf{I}$ is an $N \times N$ identity matrix. Note that (D.8) implies $\sum_j d_{ij}^\theta = 1/\nu^\theta$. Assuming that $(\mathbf{I} - \boldsymbol{D}^\theta)$ is invertible, the unique solution to (D.27) features

$$x_i^\theta = x^\theta \quad \text{for all } i, \tag{D.28}$$

which in turn implies $(S_j^\theta)^{-\frac{1}{\nu^\theta}} \tilde{u}_j(C_j^\theta) = K^\theta$ for some constant K^θ. Substituting this expression back into (D.24), we have

$$\mu_j^\theta = (K^\theta)^{-\nu^\theta} G_j^\theta(V_1^\theta, \ldots, V_N^\theta)V_j^\theta. \tag{D.29}$$

Using the adding up constraint $\sum_j \mu_j^\theta = 1$, we can solve for $(K^\theta)^{-\nu^\theta}$ to obtain

$$\mu_j^\theta = \frac{G_j^\theta(V_1^\theta, \ldots, V_N^\theta)V_j^\theta}{\sum_i G_i^\theta(V_1^\theta, \ldots, V_N^\theta)V_i^\theta}, \tag{D.30}$$

as desired. We can plug the above expression into the objective to confirm that the indirect utility also coincides with the original discrete choice formulation:

$$\tilde{W}^\theta = G^\theta(V_1^\theta, \ldots, V_N^\theta)^{1/\nu^\theta}. \tag{D.31}$$

E Details on Extensions

E.1 Non-Additive Idiosyncratic Location Preferences

In the baseline model, we have focused on a specification where idiosyncratic preferences are additively separable. This section relaxes this assumption.

We now assume that utility in location i is given by $U_i(C_i^\theta, \epsilon_i^\theta)$. Compared to the additively separable specification, marginal utility in each location now depends on preference shock draws. To see this, the average marginal utility for households deciding to live in location j is given by

$$\mathbb{E}\left[\frac{\partial}{\partial C_j^\theta} U_j(C_j^\theta, \epsilon_j^\theta) | j = \arg\max_i U_i(C_i^\theta, \epsilon_i^\theta)\right]. \tag{E.1}$$

Unlike the additively separable specification, where $\frac{\partial}{\partial C_j^\theta} U_j(C_j^\theta, \epsilon_j^\theta) = u_j'(C_j^\theta)$, the selection of preference shocks influences the marginal utility of consumption in each location.

Under this general preference specification, the isomorphic representation of households' location decisions in Lemma 1 is modified as

$$\max_{\{\mu_j^\theta\}_j : \sum_j \mu_j^\theta = 1} \mathcal{U}^\theta(\{\mu_j^\theta\}), \tag{E.2}$$

where

$$\begin{aligned}\mathcal{U}^\theta(\{\mu_j^\theta\}) = &\max_{\{\mathbb{I}_j^\theta(\omega)\}_{\omega,j}} \int_0^1 \sum_j U_j(C_j^\theta, \epsilon_j^\theta(\omega))\mathbb{I}_j^\theta(\omega)d\omega \\ \text{s.t.} \quad &\int_0^1 \mathbb{I}_j^\theta(\omega)d\omega = \mu_j^\theta \\ &\sum_j \mathbb{I}_j^\theta(\omega) = 1.\end{aligned} \tag{E.3}$$

We followed the same notation and setup as in Appendix A.1.

Under the additively separable specification, $\mathcal{U}^\theta(\{\mu_j^\theta\}) = \sum_j \mu_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\})$ and $\partial\mathcal{U}^\theta/\partial C_j^\theta = \mu_j^\theta u_j'(C_j^\theta)$, so expected marginal utility only depends on j's population and consumption. In the general case, it is affected by the entire population distribution $\{\mu_j^\theta\}_j$ and consumption vector $\{C_j^\theta\}_j$ beyond location j through the selection of preference draws.

In this generalized environment, Proposition 1 is simply modified by replacing the prior marginal utility $u_j^{\theta\prime}(C_j^\theta)$ with the one under this general specification. In particular, (ii) MU dispersion term becomes

$$\Omega_{MU} = \mathbb{E}_\theta\left[\text{Cov}_{j|\theta}\left(-\frac{P_j^\theta}{\mathcal{MU}_j^\theta}, \mathcal{MU}_j^\theta dC_j^\theta\right)\right], \tag{E.4}$$

where

$$\mathcal{MU}_j^\theta = \frac{1}{\mu_j^\theta}\frac{\partial\mathcal{U}^\theta}{\partial C_j^\theta}. \tag{E.5}$$

Conditional on the price normalization (26) using this marginal utility, all other terms are unaffected.

While this extension is straightforward in theory, it poses a challenge for the identification of aggregate welfare. To understand this challenge, consider a monotone transformation of a utility function from the additively separable class: $U_j(C_j^\theta, \epsilon_j^\theta) = m(u_j(C_j^\theta) + \epsilon_j^\theta)$ for some strictly increasing function $m(\cdot)$. This transformation does not affect the model's *positive* prediction because of the ordinal nature of the utility function over location choice decisions. However, the expected marginal utility in each location becomes

$$\mathcal{MU}_j^\theta = u_j'(C_j^\theta)\mathbb{E}\left[m'(u_j(C_j^\theta) + \epsilon_j^\theta)\middle| j = \arg\max_i m(u_i(C_i^\theta) + \epsilon_i^\theta)\right]. \tag{E.6}$$

Therefore, the function $m(\cdot)$ generally affects the marginal utility in each location. This implies that the *normative* prediction, i.e., aggregate welfare, generically depends on the choice of $m(\cdot)$. Since $m(\cdot)$ cannot be identified from the data, aggregate welfare also cannot be identified from location choice data alone.

This lack of identification is worrisome as it indicates that welfare predictions are not uniquely pinned down by observables. Even when the two models match the same data moments, the welfare conclusions drawn from these two models can be arbitrarily different. However, as we show in Section 4.1, this issue can be avoided in a special case where idiosyncratic location preferences are multiplicatively separable and follow a max-stable multivariate Fréchet distribution. In this special case, the second term with expectation operator in Equation (E.6) is constant, and hence drops out from aggregate welfare.

E.2 Shocks to Amenity and Amenity Externalities

In Section 3, we analyzed the effects of productivity shocks on aggregate welfare. In some contexts, researchers are interested in shocks to amenities instead of productivity. The analysis in Section 3 allows for this possibility by interpreting some intermediate goods as local amenities. From a measurement perspective, applying Proposition 1 requires knowledge of prices associated with amenities, which are often unobserved. Below, we provide an alternative expression for Proposition 1 without using prices for amenities.

To consider this extension, we explicitly introduce amenities as an argument in the utility function as follows:

$$U_j(C_j^\theta, \mathcal{B}_j^\theta, \epsilon_j^\theta) = u_j(C_j^\theta, \mathcal{B}_j^\theta) + \epsilon_j^\theta, \tag{E.7}$$

where $\mathcal{B}_j^\theta$ is the amenity in region j. Furthermore, we assume that amenities take the following form:

$$\mathcal{B}_i^\theta = B_i^\theta g_i^{B,\theta}(\{l_i^{\tilde{\theta}}\}), \qquad \gamma_i^{B,\tilde{\theta}\theta} = \frac{\partial \ln g_i^{B,\theta}}{\partial \ln l_i^{\tilde{\theta}}}, \tag{E.8}$$

where B_i^θ is the fundamental component of amenities, $g_i^{B,\theta}(\{l_i^{\tilde{\theta}}\})$ is the spillover function, and $\gamma_i^{B,\tilde{\theta}\theta}$ is the amenity spillover elasticity from type $\tilde{\theta}$ to type θ in location i.

Under this extension, Proposition 1 is modified as follows. Consider an arbitrary set of small shocks to the exogenous components of productivity $\{d \ln A_{ij,k}\}$ and amenities $\{d \ln B_i^\theta\}$. The first-order impact on aggregate welfare can be expressed as

$$\begin{aligned} dW = & \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d \ln A_{ij,k} + \sum_{i,\theta} l_i^\theta \partial_B u_i^\theta \mathcal{B}_i^\theta d \ln B_i^\theta}_{\text{(i) Technology } (\Omega_T)} + \underbrace{\mathbb{E}_\theta \left[\text{Cov}_{j|\theta} \left(-\frac{P_j^\theta}{\partial_C u_j^\theta}, \partial_C u_j^\theta dC_j^\theta \right) \right]}_{\text{(ii) MU Dispersion } (\Omega_{MU})} \\ & + \underbrace{\mathbb{E}_\theta \left[\text{Cov}_{j|\theta}(-T_j^\theta, d \ln l_j^\theta) \right]}_{\text{(iii) Fiscal Externality } (\Omega_{FE})} + \underbrace{\mathbb{E}_\theta \left[\text{Cov}_{j|\theta}\Big(\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta + \sum_{\tilde{\theta}} \partial_B u_j^{\tilde{\theta}} \mathcal{B}_j^{\tilde{\theta}} \gamma_j^{B,\theta\tilde{\theta}}, d \ln l_j^\theta\Big) \right]}_{\text{(iv) Technological Externality } (\Omega_{TE})} \\ & + \underbrace{\text{Cov}_\theta \left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P_j^\theta}{\partial_C u_j^\theta} \right], \mathbb{E}_{j|\theta} \left[\partial_C u_j^\theta dC_j^\theta \right] \right)}_{\text{(v) Redistribution } (\Omega_R)}. \end{aligned} \tag{E.9}$$

where $\partial_B u_j^\theta \equiv \frac{\partial u_j(C_j^\theta, \mathcal{B}_j^\theta)}{\partial B_j^\theta}$ and $\partial_C u_j^\theta \equiv \frac{\partial u_j(C_j^\theta, \mathcal{B}_j^\theta)}{\partial C_j^\theta}$. The main difference from Proposition 1 is the additional components in the (i) technology and (iv) technological externality terms. The second component inside the (i) technology term captures the effects of exogenous amenity shocks, absent reallocation effects. The coefficient in front of $d\ln B_i^\theta$, $l_i^\theta \partial_B u_i^\theta \mathcal{B}_i^\theta$, is the population-weighted sum of the marginal utility of amenities. This term strongly resembles the technology effect from productivity (the first term). In particular, if amenities are traded and priced in the market, $\partial_B u_i^\theta$ corresponds to the competitive price of the amenity, and hence $l_i^\theta \partial_B u_i^\theta \mathcal{B}_i^\theta$ is the total sales of type θ amenities in location i, corresponding to $p_{ij,k} y_{ij,k}$. The second component inside the (iv) technological externality term has the same feature: if the amenity is traded, the term reflecting changes in amenities from externalities collapses to the same form as the productivity externality term.

E.3 Isomorphism between Amenity Externalities and Preference Shocks

In the quantitative spatial equilibrium literature, researchers often argue that amenity congestion externalities are isomorphic to preference shocks and use these specifications interchangeably (e.g. Allen and Arkolakis (2014) for the isomorphism between i.i.d. Fréchet preferences and isoelastic congestion externalities). This section discusses this isomorphism through the lens of our framework.

For expositional convenience, we assume a single type and drop superscript θ. This implies $l_j = \mu_j$. Consider the following utility specification with amenity externalities, rather than preference shocks:

$$U_j(C_j, \mathcal{B}_j) = u_j(C_j) + \mathcal{B}_j, \qquad \mathcal{B}_j = -\frac{1}{\nu}\ln S_j(\{l_i\}), \tag{E.10}$$

where $S_j(\{l_i\})$ satisfies the following property

$$\frac{1}{\nu}\sum_j l_j \frac{\partial \ln S_j}{\partial l_i} = 1. \tag{E.11}$$

Note that this specification accommodates the possibility that the population in location i generates externalities in other regions. A special case of this specification is when $\ln S_j = l_j^\nu$, i.e. congestion is iso-elastic to local population size with elasticity ν.

In an interior equilibrium, utility levels are equalized across all locations:

$$u_j(C_j) - \frac{1}{\nu}\ln S_j = \bar{u}, \tag{E.12}$$

for some $\bar{u}$.

Now we show that this is isomorphic to the case where there are no amenity externalities but preference shocks follow a max-stable multivariate Gumbel distribution with shape parameter ν. That is,

$$U_j(C_j, \epsilon_j) = u_j(C_j) + \epsilon_j, \tag{E.13}$$

where $\{\epsilon_j\}$ follows specification (35). As we show in Appendix D, with a multivariate Gumbel distribution, $\psi(\{l_j\})$ in Lemma 1 takes the form $\psi(\{l_j\}) = \frac{1}{\nu}\sum_j l_j \ln S_j(\{l_i\})$, where $S_j(\{l_i\})$

satisfies (E.11). The first-order condition for the representative household problem is

$$u_j(C_j) - \frac{1}{\nu}\ln S_j - \underbrace{\frac{1}{\nu}\sum_i l_i \frac{\partial \ln S_i}{\partial l_j}}_{=1} = \delta, \tag{E.14}$$

which is the same as (E.12). Therefore, the equilibrium allocations will be identical. Moreover, it is also straightforward to see that both specifications deliver the same expected utility, thereby delivering identical normative predictions as well.

This isomorphism arises because this particular form of congestion externality does not induce misallocation. In particular, the amenity component of the (iv) technological externality term in Equation (E.9) comes down to

$$\mathrm{Cov}_j\left(-\sum_i l_i \frac{\partial \ln S_i}{\partial \ln l_j}, d\ln l_j\right) = \mathrm{Cov}_j(-\nu, d\ln l_j) = 0, \tag{E.15}$$

where we used (E.11) and $\partial_b u_i = 1$. Given that all other terms in Equation (E.9) are identical between the two specifications, the aggregate welfare predictions are also isomorphic.

This discussion also clarifies that this isomorphism only holds when preference shocks follow a max-stable multivariate Gumbel distribution, or equivalently, when the congestion externality takes the specific functional form given by (E.10) and (E.11). Outside of these special cases, congestion externalities generate misallocation and the isomorphism does not hold in general.[1]

E.4 Alternative Social Welfare Criteria

Consider a general non-welfarist welfare objective

$$W = \mathcal{W}(\{\mathcal{U}^{SP,\theta}(\{C_j^\theta, \mu_j^\theta\})\}),$$

where $\mathcal{U}^{SP,\theta}$ is defined arbitrarily on the population distribution and consumption of type θ households. Then, by applying the envelope theorem to the pseudo-planning problem as in the proof of Proposition 1, we have

$$dW = \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k} + \sum_\theta \sum_j \left[\ell^\theta \Lambda^\theta \frac{\partial \mathcal{U}^{\mathcal{SP},\theta}}{\partial C_j^\theta} - l_j^\theta P_j^\theta\right] dC_j^\theta$$

[1] Fajgelbaum and Gaubert (2020) show that under a multiplicative utility specification, spatial equilibria involve misallocation even with iso-elastic amenity externalities. Through the lens of Equation (E.9), this source of misallocation appears in the (ii) MU dispersion term. Multiplicative amenities without preference shocks imply that the marginal utility of income is not equalized across locations. Furthermore, unlike in our baseline model which abstracts from direct effects of shocks on utility, the utility changes from consumption changes dC_j are not equalized because of the changes in utility from amenities. Therefore, the (ii) MU dispersion term is not zero. Note that this specification is isomorphic to the specification with multiplicative max-stable Fréchet shocks (without amenity externality) as discussed in Section 4.1. In that case, the dispersion of marginal utility instead arises from preference shock draws.

$$
+\sum_{\theta}\sum_{j}\ell^{\theta}\Lambda^{\theta}\frac{\partial\mathcal{U}^{\mathcal{SP},\theta}}{\partial\mu_j^{\theta}}d\mu_j^{\theta}+\sum_{\theta}\sum_{j}\left[w_j^{\theta}-P_j^{\theta}C_j^{\theta}\right]dl_j^{\theta}
+\sum_{\theta}\sum_{j}\sum_{l,k}p_{jl,k}y_{jl,k}\frac{1}{l_j^{\theta}}\gamma_{jl,k}^{\theta}dl_j^{\theta} \tag{E.16}
$$

The differences from our main proposition are in the second and the third terms, which we can rewrite as

$$
\begin{aligned}
&\sum_{\theta}\ell^{\theta}\sum_{j}\left[\Lambda^{\theta}\frac{\partial\mathcal{U}^{\mathcal{SP},\theta}}{\partial C_j^{\theta}}-P_j^{\theta}\mu_j^{\theta}\right]dC_j^{\theta}\\
=&\sum_{\theta}\ell^{\theta}\sum_{j}\mu_j^{\theta}\left[\Lambda^{\theta}\left(\frac{\frac{1}{\mu_j^{\theta}}\frac{\partial\mathcal{U}^{\mathcal{SP},\theta}}{\partial C_j^{\theta}}}{u_j'(C_j^{\theta})}-1\right)+\Lambda^{\theta}-\frac{P_j^{\theta}}{u_j'(C_j^{\theta})}\right]u_j'(C_j^{\theta})dC_j^{\theta}\\
=&\mathbb{E}_{\theta}\left[\mathrm{Cov}_{j|\theta}\left(-\frac{P_j^{\theta}}{u_j'(C_j^{\theta})},u_j'(C_j^{\theta})dC_j^{\theta}\right)\right]+\mathrm{Cov}_{\theta}\left(\Lambda^{\theta}-\mathbb{E}_{j|\theta}\left[\frac{P_j^{\theta}}{u_j'(C_j^{\theta})}\right],\mathbb{E}_{j|\theta}\left[u_j'(C_j^{\theta})dC_j^{\theta}\right]\right)\\
&+\mathbb{E}_{\theta}\left[\Lambda^{\theta}\mathbb{E}_{j|\theta}\left[\left(\frac{1}{\mu_j^{\theta}}\frac{\partial\mathcal{U}^{\mathcal{SP},\theta}}{\partial C_j^{\theta}}-u_j'(C_j^{\theta})\right)dC_j^{\theta}\right]\right].
\end{aligned} \tag{E.17}
$$

Consequently, Proposition 1 comes down to

$$
dW=\Omega_T+\Omega_{MU}+\Omega_{FE}+\Omega_{TE}+\Omega_R+\Omega_{PM}, \tag{E.18}
$$

where

$$
\Omega_{PM}=\mathbb{E}_{\theta}\left[\Lambda^{\theta}\mathbb{E}_{j|\theta}\left[\left(\frac{1}{\mu_j^{\theta}}\frac{\partial\mathcal{U}^{\mathcal{SP},\theta}}{\partial C_j^{\theta}}-u_j'(C_j^{\theta})\right)dC_j^{\theta}\right]\right]+\mathbb{E}_{\theta}\left[\Lambda^{\theta}\mathbb{E}_{j|\theta}\left[\frac{\partial\mathcal{U}^{\mathcal{SP},\theta}}{\partial\mu_j^{\theta}}d\ln l_j^{\theta}\right]\right], \tag{E.19}
$$

which captures the potential misalignment between the social Planner's welfare assessment of the marginal value of consumption with that of private agents (marginal utility).

Such an approach is useful also in considering welfare criteria and optimal policies that are exclusively based on subcomponents of our decompositions, as in Dávila and Schaab (2022). Consider the following class of welfare criteria:

$$
\mathcal{U}^{SP,\theta}(\{C_j^{\theta},\mu_j^{\theta}\})=\sum_{j}(\mu_j^{\theta}+\omega_j^{\theta})u_j(C_j^{\theta})-\psi^{\theta}(\{\mu_j^{\theta}\}). \tag{E.20}
$$

Appropriate choice of the type-location specific weights ω_j^{θ} leads to the following result.

Proposition E.1. *Consider welfare criteria based on (37) and (E.20).*

1. *If $\omega_j^{\theta}=\frac{\mu_j^{\theta}}{\Lambda^{\theta}}\left[\left(\frac{P_j^{\theta}}{u_j'(C_j^{\theta})}-1\right)-\left(\Lambda^{\theta}-\mathbb{E}_{j|\theta}\left[\frac{P_j^{\theta}}{u_j'(C_j^{\theta})}\right]\right)\right]$, then the decomposition of aggregate welfare in Proposition 1 only consists of Ω_T, Ω_{FE}, and Ω_{TE}.*

2. *If* $\omega_j^\theta = -\frac{\mu_j^\theta}{\Lambda^\theta}\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right]\right)$*, then the decomposition of aggregate welfare in Proposition 1 only consists of* Ω_T*,* Ω_{MU} Ω_{FE}*, and* Ω_{TE}*.*

3. *If* $\omega_j^\theta = \frac{\mu_j^\theta}{\Lambda^\theta}\left(\frac{P_j^\theta}{u_j'(C_j^\theta)} - 1\right)$*, then the decomposition of aggregate welfare in Proposition 1 only consists of* Ω_T*,* Ω_{FE}*,* Ω_{TE}*, and* Ω_R*.*

Proof. As shown earlier, the welfare decomposition with non-welfarist welfare criteria is given by

$$dW = \Omega_T + \Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R + \Omega_{PM}, \tag{E.21}$$

where

$$\Omega_{PM} = \mathbb{E}_\theta\left[\Lambda^\theta \mathbb{E}_{j|\theta}\left[\frac{\omega_j^\theta}{\mu_j^\theta} u_j'(C_j^\theta) dC_j^\theta\right]\right] \tag{E.22}$$

with our assumption (E.20). Note that the second term in Equation (E.19) is absent owing to an envelope condition. Suppose that $\omega_j^\theta = \frac{\mu_j^\theta}{\Lambda^\theta}\left[\left(\frac{P_j^\theta}{u_j'(C_j^\theta)} - 1\right) - \left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right]\right)\right]$. Then

$$\begin{aligned}
\Omega_{MU} + \Omega_R + \Omega_{PM} = {} & \mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}\left(-\frac{P_j^\theta}{u_j'(C_j^\theta)}, u_j'(C_j^\theta) dC_j^\theta\right)\right] \\
& + \mathrm{Cov}_\theta\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right], \mathbb{E}_{j|\theta}\left[u_j'(C_j^\theta) dC_j^\theta\right]\right) \\
& + \mathbb{E}_\theta\left[\Lambda^\theta \mathbb{E}_{j|\theta}\left[\frac{\omega_j^\theta}{\mu_j^\theta} u'(C_j^\theta) dC_j^\theta\right]\right] \\
= {} & \mathbb{E}_\theta\left[\mathbb{E}_{j|\theta}\left[\left(1 - \frac{P_j^\theta}{u_j'(C_j^\theta)}\right) u_j'(C_j^\theta) dC_j^\theta\right]\right] \\
& + \mathbb{E}_\theta\left(\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right]\right) \mathbb{E}_{j|\theta}\left[u'(C_j^\theta) dC_j^\theta\right]\right) \\
& - \mathbb{E}_\theta\left[\mathbb{E}_{j|\theta}\left[\left(1 - \frac{P_j^\theta}{u_j'(C_j^\theta)}\right) u_j'(C_j^\theta) dC_j^\theta\right]\right] \\
& - \mathbb{E}_\theta\left(\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right]\right) \mathbb{E}_{j|\theta}\left[u'(C_j^\theta) dC_j^\theta\right]\right) \\
= {} & 0.
\end{aligned} \tag{E.23}$$

This proves the first claim. Likewise, if $\omega_j^\theta = -\frac{\mu_j^\theta}{\Lambda^\theta}\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j'(C_j^\theta)}\right]\right)$, then $\Omega_R + \Omega_{PM} = 0$, and if $\omega_j^\theta = \frac{\mu_j^\theta}{\Lambda^\theta}\left(\frac{P_j^\theta}{u_j'(C_j^\theta)} - 1\right)$, then $\Omega_{MU} + \Omega_{PM} = 0$. □

The first case considers a welfare criterion based entirely on aggregate efficiency considerations. The second and third cases incorporate spatial MU dispersion and redistribution considerations, respectively, as well as aggregate efficiency considerations.

Now we derive optimal policies with welfare criteria in each of the three cases discussed in Proposition E.1. In the first case, since Ω_{MU} and Ω_R cancel with Ω_{PM}, optimal transfer policy must satisfy

$$0 = -\sum_i \mu_i^\theta T_i^\theta \frac{\partial \ln \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} + \sum_i \mu_i^\theta \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \frac{\partial \ln \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta}. \tag{E.24}$$

We can rewrite the above expression to obtain the optimal spatial policy formula that exclusively targets aggregate efficiency considerations:

$$0 = -\sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right]. \tag{E.25}$$

That is, the left-hand side of our baseline formula in Proposition 2 is modified to be zero.

In the second case, the optimal policy formula is

$$\mu_j^\theta \left[u_j'(C_j^\theta) - P_j^\theta \right] = -\sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right], \tag{E.26}$$

which incorporates spatial MU dispersion as well as aggregate efficiency considerations. In the third case, the optimal policy formula is

$$\left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P_j^\theta}{u_j'(C_j^\theta)} \right] \right) \mu_j^\theta u'(C_j^\theta) dC_j^\theta = -\sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right], \tag{E.27}$$

which incorporates redistribution considerations as well as aggregate efficiency considerations.

E.5 General Spillovers

In our main model, we assumed that agglomeration externalities are purely a function of local population size (9). In some contexts, researchers specify that a higher population size in the surrounding regions also generates agglomeration spillovers (e.g. Ahlfeldt et al. 2015). In other contexts, researchers also specify that externalities arise from specific producers' input use (e.g. free entry models with labor fixed cost such as Krugman (1991)) or producers' output (e.g. congestion costs from shipment, as in Allen and Arkolakis (2022)). To capture these general externalities, we extend the spillover function (9) such that

$$\mathcal{A}_{ij,k} = A_{ij,k} g_{ij,k}(\{l_\ell^\theta\}, \{l_{ij,k}^\theta\}, y_{ij,k}), \tag{E.28}$$

where the first argument of $g_{ij,k}(\cdot)$ corresponds to the population size across types and locations, the second argument corresponds to labor inputs in production, and the third argument corresponds to output. We also denote the spillover elasticities such that

$$\gamma_{ij,k}^{P,\ell\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln l_\ell^\theta}, \qquad \gamma_{ij,k}^{L,\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln l_{ij,k}^\theta}, \qquad \gamma_{ij,k}^{Y,\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln y_{ij,k}}. \tag{E.29}$$

Under this extension, the only modification in Proposition 1 is in the (iv) technological externality term, which is modified as:

$$\Omega_{TE} = \sum_{j,l,k} p_{jl,k} y_{jl,k} \left(\sum_{\ell,\theta} \gamma_{\ell l,k}^{P,\ell\theta} d\ln l_j^\theta + \sum_\theta \gamma_{jl,k}^{L,\theta} d\ln l_{ij,k}^\theta + \gamma_{jl,k}^{Y} d\ln y_{ij,k} \right). \tag{E.30}$$

This expression recovers the (iv) technological externality term in Proposition 1 if the spillover function only depends on local population size. The only difference here is that the reallocation of population in surrounding regions and of output may have first-order effects on aggregate welfare through additional technological externalities.

E.6 Idiosyncratic Productivity Shocks

We generalize our baseline model by allowing households to draw idiosyncratic productivity $\boldsymbol{z}^\theta = (z_1^\theta, z_2^\theta, \ldots, z_N^\theta)$, in addition to preference shocks $\boldsymbol{\epsilon}^\theta$. When a household decides to live in location j, the efficiency units of labor that the household supplies is z_j^θ.

We make several modifications to our baseline model to make our analysis tractable and transparent. First, we restrict our attention to the case of log utility,

$$u_j(c_j^\theta) = B_j \ln c_j^\theta + D_j, \tag{E.31}$$

where B_j and D_j are slope and intercept parameters specific to location. Second, we assume that location-specific transfers are linear in household labor income, which we denote as τ_j^θ. Third, we assume away the presence of fixed factors.

The household's location choice problem with productivity draw $\boldsymbol{z}^\theta$ and preference draw $\boldsymbol{\epsilon}^\theta$ is

$$\max_j u_j(c_j^\theta) + \epsilon_j^\theta \tag{E.32}$$

$$\text{s.t.} \quad P_j^\theta c_j^\theta = z_j^\theta w_j^\theta (1 + \tau_j^\theta). \tag{E.33}$$

Let

$$C_j^\theta \equiv \frac{w_j^\theta (1 + \tau_j^\theta)}{P_j^\theta} \tag{E.34}$$

denote the consumption of household type θ in location j per efficiency units of labor. With our assumption on the utility function (E.31), we can write the location choice problem as

$$\max_j u_j(C_j^\theta) + \underbrace{B_j \ln z_j^\theta + \epsilon_j^\theta}_{\equiv \varepsilon_j^\theta}. \tag{E.35}$$

Viewing ε_j^θ as the convoluted idiosyncratic productivity and amenity shocks, we can apply Lemma 1 to obtain the same location choice characterization as in the baseline model:

$$\max_{\{\mu_j^\theta\}_j:\sum_j \mu_j^\theta=1} \sum_j \mu_j^\theta u_j(C_j^\theta) + \psi^\theta(\{\mu_j^\theta\}). \tag{E.36}$$

Let $\hat{\mu}_j^\theta(\{C_i^\theta\})$ be the location choice function associated with the solution to the above problem.

It will be convenient to define the average efficiency units of labor in each location-type pair as a function of a vector of $\{C_i^\theta\}$:

$$Z_j^\theta(\{C_i^\theta\}) \equiv \mathbb{E}\left[z_j^\theta \middle| j = \arg\max_i u_i(C_i^\theta) + B_i^\theta \ln z_i^\theta + \epsilon_i^\theta\right]. \tag{E.37}$$

To the extent that the location choice function is invertible,[2] i.e. the inverse function of $\hat{\mu}_j^\theta(\{C_i^\theta\})$, $C_j^\theta = \hat{C}_j^\theta(\{\mu_i^\theta\})$ exists, we can alternatively define the average efficiency units of labor as a function of location choice probabilities:

$$\mathcal{Z}_j^\theta(\{\mu_i^\theta\}) = Z_j^\theta(\{\hat{C}_l^\theta(\{\mu_i^\theta\})\}). \tag{E.38}$$

The goods market clearing conditions are modified as follows

$$\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{jl,m}^{i,k} = \mathcal{A}_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, \mathbf{x}_{ij,k}) \tag{E.39}$$

$$\mathcal{Z}_j^\theta C_j^\theta \ell^\theta \mu_j^\theta = \mathcal{C}_j^\theta(\boldsymbol{c}_j^\theta), \tag{E.40}$$

where the first equation is modified due to the absence of a fixed factor, and the second equation takes into account heterogeneity in consumption within a location-type pair. The labor market clearing condition is

$$\sum_{i,k} l_{ji,k}^\theta = \mathcal{Z}_j^\theta \ell^\theta \mu_j^\theta, \tag{E.41}$$

which takes into account heterogeneity in efficiency units of labor within a location-type pair. The rest of the equilibrium conditions remain unchanged.

It is straightforward to extend Lemma 2 to this environment. Any decentralized equilibrium solves the following pseudo-planning problem:

$$W = \max_{\{W^\theta,\{C_j^\theta,\mathbf{c}_j^\theta,\mu_j^\theta\}\},\{\mathbf{x}_{ij,k},\mathbf{l}_{ij,k},h_{ij,k},\mathcal{A}_{ij,k}\}} \mathcal{W}(\{W^\theta\}) \tag{E.42}$$

subject to (9), (E.39)-(E.41),

$$W^\theta = \sum_j \mu_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}) \tag{E.43}$$

$$\{\mu_j^\theta\}_j \in \arg\max_{\{\tilde{\mu}_j\}_j:\sum_j \tilde{\mu}_j=1} \sum_j \tilde{\mu}_j^\theta u_j(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}) \tag{E.44}$$

[2]See Berry, Gandhi, and Haile (2013) for a sufficient condition for invertibility.

$$C_j^\theta = \check{C}_j^\theta \tag{E.45}$$

Applying the envelope theorem, we obtain

$$\begin{aligned} dW &= \Omega_T + \Omega_{MU} + \Omega_{TE} + \Omega_R \\ &+ \sum_\theta \sum_j \left[w_j^\theta - P_j^\theta C_j^\theta\right] \mathcal{Z}_j^\theta \ell^\theta d\mu_j^\theta + \sum_\theta \sum_j \sum_l \left[w_l^\theta - P_l^\theta C_l^\theta\right] \ell^\theta \mu_l^\theta \frac{\partial \mathcal{Z}_l^\theta}{\partial \mu_j^\theta} d\mu_j^\theta \end{aligned}$$

Denoting $T_j^\theta = \tau_j^\theta w_j^\theta \mathcal{Z}_j^\theta$ as the average transfers that households of type θ in location j receive, we can rewrite the above expression as follows:

$$\begin{aligned} dW &= \Omega_T + \Omega_{MU} + \Omega_{TE} + \Omega_R \\ &+ \underbrace{\mathbb{E}_\theta\Big[\text{Cov}_{j|\theta}(-T_j^\theta, d\ln l_j^\theta)\Big] + \mathbb{E}_\theta\left[\text{Cov}_{j|\theta}\left(-\sum_l T_l^\theta l_l^\theta \frac{\partial \ln \mathcal{Z}_l^\theta}{\partial \ln \mu_j^\theta}, d\ln l_j^\theta\right)\right]}_{\text{(iii) Fiscal Externality } (\Omega_{FE})}. \end{aligned}$$

Therefore the only difference from Proposition 1 is the second term inside the (iii) fiscal externality term. This term arises because migration changes the composition of workers in all locations, which in turn affects the government's budget. For example, suppose that migration into location j is associated with an increase in the average productivity of workers living in location j but a decrease in other locations. If location j is a net taxpayer ($\tau_j^\theta < 0$ and thereby $T_j^\theta < 0$), then this will tend to slacken the government's budget.

F Test of Pareto Efficient Spatial Transfers

Our formula in Proposition 2 can also be used to assess whether the current transfer scheme admits of a Pareto improvement. Since our formula requires the existence of positive Pareto weights $\tilde{\Lambda}^\theta > 0$, equilibrium allocations that lead to negative inferred Pareto weights are Pareto inefficient.

Corollary 1. *If there exists j and θ such that*

$$\mu_j^\theta P_j^\theta < \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[-T_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right], \tag{F.1}$$

then there exists an alternative transfer scheme that Pareto improves the original one.

The underlying idea is the same as Werning (2007) in the context of optimal non-linear income taxation. Importantly, this test of Pareto inefficiency does not require researchers to take a stance on Pareto weights or the marginal utility of consumption in each location. The test only requires a handful of sufficient statistics such as price indices, migration elasticities, and agglomeration elasticities.

G Appendix for Application to US Highway Network

G.1 Counterfactual Equilibrium System

To align the notations of Allen and Arkolakis (2022), we define the aggregate pre-tax labor income in location j by $Y_j = w_j l_j$ and aggregate post-tax-and-transfer income by $E_j = Y_j + T_j l_j$. Using the results from Online Appendix C.2.1 of Allen and Arkolakis (2022), but by leaving the population changes $\hat{l}_j$ (by denoting the proportional change in variable x by $\hat{x} = x'/x$, where x' is the value in the counterfactual equilibrium), we have

$$\hat{A}_i^{-\theta}\hat{l}_i^{-\theta(\gamma+1)}\hat{Y}_i^{(\theta+1)}\hat{P}_i^{-\theta} = \left(\frac{E_i}{E_i + \sum_j \Xi_{ij}}\right)\hat{E}_i + \sum_j \left(\frac{\Xi_{ij}}{E_i + \sum_j \Xi_{ij}}\right)\hat{t}_{ij}^{-\frac{\theta}{1+\theta\lambda}}\hat{P}_i^{-\frac{\theta}{1+\theta\lambda}}\hat{l}_j^{-\frac{\theta(\gamma+1)}{1+\theta\lambda}}\hat{Y}_j^{\frac{\theta+1}{1+\theta\lambda}}\hat{A}_j^{-\frac{\theta}{1+\theta\lambda}} \tag{G.1}$$

$$\hat{A}_i^{-\theta}\hat{l}_i^{-\theta(\gamma+1)}\hat{Y}_i^{(\theta+1)}\hat{P}_i^{-\theta} = \left(\frac{Y_i}{Y_i + \sum_j \Xi_{ji}}\right)\hat{Y}_i + \sum_j \left(\frac{\Xi_{ji}}{Y_i + \sum_j \Xi_{ji}}\right)\hat{t}_{ji}^{-\frac{\theta}{1+\theta\lambda}}\hat{P}_j^{-\frac{\theta}{1+\theta\lambda}}\hat{l}_i^{-\frac{\theta(\gamma+1)}{1+\theta\lambda}}\hat{Y}_i^{\frac{\theta+1}{1+\theta\lambda}}\hat{A}_i^{-\frac{\theta}{1+\theta\lambda}} \tag{G.2}$$

In addition, from our location choice system (Equation 38),

$$\hat{l}_j = \frac{\exp\left(\nu C_j^{1-\rho}\left(\left(\frac{\hat{E}_j}{\hat{P}_j\hat{l}_j}\right)^{1-\rho} - 1\right)\right)}{\sum_j l_j \exp\left(\nu C_j^{1-\rho}\left(\left(\frac{\hat{E}_j}{\hat{P}_j\hat{l}_j}\right)^{1-\rho} - 1\right)\right)} \tag{G.3}$$

Given baseline values of $\{l_i, E_i, Y_i, \Xi_{ij}, C_i\}$ and parameters $\{\gamma, \theta, \lambda, \rho, \nu, \varkappa_j\}$, our counterfactuals $\{\hat{l}_i, \hat{P}_i, \hat{Y}_i, \hat{E}_i\}$ are given as the solutions to (G.1), (G.2), (G.3), and the aggregate expenditure and transfers given by

$$E_j' = Y_j' + T_j' l_j', \qquad T_j' = \varkappa_j \frac{Y_j'}{L_j'} + T^{*\prime}, \qquad T^{*\prime} = -\frac{\sum_j \varkappa_j Y_j'}{\sum_j l_j'} \tag{G.4}$$

G.2 Calibration Details

G.2.1 Estimation of Utility Function Parameters

We will now describe the details of estimating the utility function parameters, using the changes in consumption and population from 1980 to 2000. For estimation purposes, we split our samples into high-skill (4 years of college education or greater) and low-skill (less than 4 years of college education). Denoting t as year for 1980 and 2000,

$$U_{j,t}(C_{j,t}^\theta, \epsilon_{j,t}^\theta) = \frac{(C_{j,t}^\theta)^{1-\rho}}{1-\rho} + B_{j,t}^\theta + \epsilon_{j,t}^\theta, \tag{G.5}$$

where $\epsilon_{j,t}^{\theta}$ follows an independent type-I extreme value distribution with shape parameter ν. This results in the following logit location choice system

$$\mu_{j,t}^{\theta} = \frac{\exp\left(\nu\left[\frac{(C_{j,t}^{\theta})^{1-\rho}}{1-\rho} + B_{j,t}^{\theta}\right]\right)}{\sum_i \exp\left(\nu\left[\frac{(C_{i,t}^{\theta})^{1-\rho}}{1-\rho} + B_{i,t}^{\theta}\right]\right)}. \tag{G.6}$$

Taking log, time-differencing, and differencing out with location 1, we obtain

$$\Delta \ln(\mu_{j,t}^{\theta}/\mu_{1,t}^{\theta}) = \nu\left[\Delta\frac{(C_{j,t}^{\theta})^{1-\rho}}{1-\rho} - \Delta\frac{(C_{1,t}^{\theta})^{1-\rho}}{1-\rho}\right] + \nu\left[\Delta B_{j,t}^{\theta} - \Delta B_{1,t}^{\theta}\right], \tag{G.7}$$

where $\Delta x_t \equiv x_t - x_{t-1}$ denotes the time-difference for any variable x_t from 1980 to 2000. The identification threat in estimating Equation (G.7) is that unobserved location-specific amenity shocks $\Delta B_{j,t}^{\theta}$ are correlated with changes in consumption. We therefore need instrumental variables $\boldsymbol{Z}_{j,t}$ that are uncorrelated with the location-specific amenity shocks. Define the structural residual as follows:

$$e_{j,t}^{\theta}(\boldsymbol{\beta}) = \Delta \ln(\mu_{j,t}^{\theta}/\mu_{1,t}^{\theta}) - \nu\left[\Delta\frac{(C_{j,t}^{\theta})^{1-\rho}}{1-\rho} - \Delta\frac{(C_{1,t}^{\theta})^{1-\rho}}{1-\rho}\right], \tag{G.8}$$

where $\boldsymbol{\beta} = (\rho, \nu)$. Given a vector of $\boldsymbol{Z}_{j,t}$ that satisfies the following moment conditions:

$$\mathbb{E}\left[\left(\Delta B_{j,t}^{\theta} - \Delta B_{1,t}^{\theta}\right)\boldsymbol{Z}_{j,t}\right] = 0, \tag{G.9}$$

we construct a consistent GMM-estimator of (ρ, ν) that solves

$$\hat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}} \boldsymbol{e}(\boldsymbol{\beta})'\boldsymbol{Z}\Phi\boldsymbol{Z}'\boldsymbol{e}(\boldsymbol{\beta}), \tag{G.10}$$

where Φ is a weighting matrix.

To build instrument variables, we construct a shift-share instrument that interacts local industry composition with the national industry employment growth for each skill type θ, similarly to Diamond (2016). Specifically, we construct the following shift-share instrument:

$$z_{j,t}^{\theta} = \sum_k \frac{l_{j,k,t-1}^{\theta}}{\sum_k l_{j,k,t-1}^{\theta}} \Delta \ln l_{-j,k,t}^{\theta}, \tag{G.11}$$

where $l_{j,k,t-1}^{\theta}$ denotes the industry k employment of type θ in location j at time $t-1$, and $\Delta \ln l_{-j,k,t}^{\theta}$ is the national industry employment growth of skill θ excluding location j. We construct an additional instrumental variable that interacts $z_{j,t}^{\theta}$ with the baseline consumption: $z_{j,t}^{\theta} \times \ln C_{j,t}^{\theta}$. We set the weighting matrix as the identity matrix.

We report standard errors based on the consistent estimator of the asymptotic covariance matrix of the GMM estimator $\hat{V}$:

$$\hat{V} = (\hat{G}'\hat{\Omega}^{-1}\hat{G})', \tag{G.12}$$

where

$$\hat{G} \equiv \frac{\partial}{\partial \boldsymbol{\beta}} \left(\boldsymbol{e}(\hat{\boldsymbol{\beta}})' \boldsymbol{Z} \right), \quad \hat{\Omega} = \frac{1}{N} \sum_{i=1}^{N} \left(\boldsymbol{e}_i(\hat{\boldsymbol{\beta}})' \boldsymbol{Z}_i \right) \left(\boldsymbol{e}_i(\hat{\boldsymbol{\beta}})' \boldsymbol{Z}_i \right)'. \tag{G.13}$$

We run this estimation at the MSA level. For consumption, we use the pre-tax-and-transfer income from ACS data, combined with taxes, transfers, and prices at the state level, as taxes and prices are not available at the MSA level in 1980. To construct our IVs, we use 5-year samples from the ACS for the years 1980 and 2000.

G.2.2 Calibration of Transfer Rates

To estimate the rates of spatial transfers $\{\varkappa_j\}$, we use observed pre- and post-tax-and-transfer income. Specifically, we obtain these values at the level of counties in 2012 from Bureau of Economic Analysis (BEA). We then aggregate these values at the level of CBSA. Finally, we compute the ratio between the tax-and-transfer to pre-tax income and denote by $\varkappa_j^*$. Note that these transfer rates may not necessarily satisfy the government budget constraint $\sum_j T_j l_j = 0$. Therefore, we adjust these rates by a nation-wide constant $\overline{\varkappa}$ to exactly satisfy this constraint. Specifically, we set $\varkappa_j = \varkappa_j^* - \overline{\varkappa}$, where $\overline{\varkappa}$ is given by

$$\overline{\varkappa} = \frac{\sum_j \varkappa_j^* w_j l_j}{\sum_j w_j l_j}, \tag{G.14}$$

which satisfies $\sum_j T_j l_j = 0$ with the baseline value of $T^* = 0$ in Equation (39).

G.2.3 Calibration of Traffic Values

Denote Ξ_{ij}^* as the average annual daily traffic (AADT) over (i, j) in both directions. We assume that the traffic value is given by

$$\Xi_{ij} = \frac{\gamma_i}{\gamma_i + \gamma_j} \xi \left(\Xi_{ij}^* + \Xi_{ji}^* \right), \tag{G.15}$$

where $\{\xi, \{\gamma_i\}\}$ are parameters, and we normalize $\gamma_1 = 1$. Note that this assumption is consistent with the structural assumption that the link-specific transportation costs are symmetric ($\tilde{t}_{ij} = \tilde{t}_{ji}$ for all i, j).[3] We calibrate $\{\xi, \{\gamma_i\}\}$ by targeting exactly two data moments. First, transfers (trade surplus) are consistent with the net surplus in traffic inflows to outflows, such that

$$\sum_{i'} \Xi_{i'j} - \sum_{k} \Xi_{jk} = E_j - Y_j, \tag{G.16}$$

where Y_j is aggregate pre-tax labor income in location j, and E_j is aggregate post-tax-and-transfer income in location j. Second, we set the average model-implied share of consumption

[3] To see this, start with an assumption that $\Xi_{ij} = \vartheta_{ij} \xi \left(\Xi_{ij}^* + \Xi_{ji}^* \right)$. By assuming that the value of traffic in both ways is proportional to AADT, we have $\Xi_{ji} = (1 - \vartheta_{ij}) \xi \left(\Xi_{ij}^* + \Xi_{ji}^* \right)$. Furthermore, from the gravity equation of traffic (Equation 27) of Allen and Arkolakis (2022) together with the symmetric transportation costs assumption, Ξ_{ij}/Ξ_{ji} can be written as γ_i/γ_j. Combining these two expressions, we obtain the expression (Equation G.15).

expenditure over goods produced in other locations to coincide with the share of tradables in the consumption basket of the US (27.6 percent; Johnson 2017). That is,

$$\frac{\sum_{i\neq j} X_{ij}}{\sum_i Y_i} = 0.276,$$

where X_{ij} are values of goods that are produced in i and consumed in j, whose expression is given by Equation (34) of Allen and Arkolakis (2022). Given that we have N number of parameters with N number of equations (note that one equation of Equation (G.16) is redundant), we can exactly match these two sets of moments.

Through this procedure, we find that $\sum_{i,j:i\neq j} \Xi_{ij}/(\sum_i Y_i) = 0.726$. This value is somewhat smaller than the value one, assumed by Allen and Arkolakis (2022).

G.3 Additional Results

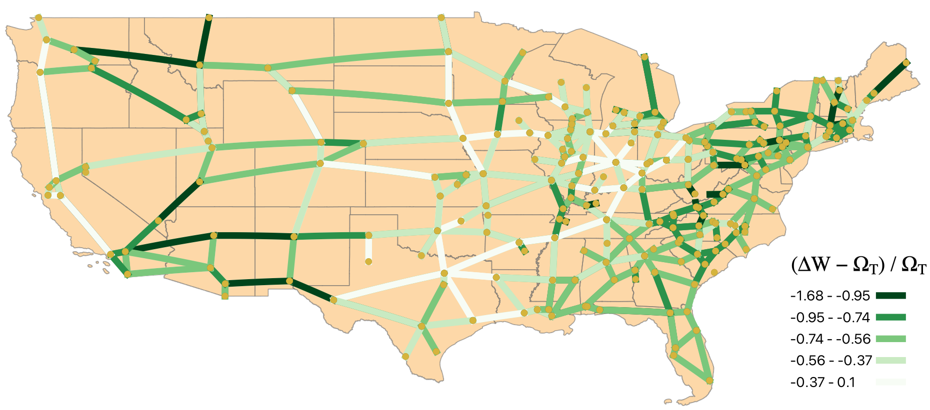

Figure G.1: Spatial Patterns of Deviations from Δ Technology Term

Note: This figure presents the spatial distribution of $(\Delta W - \Omega_T)/\Omega_T$ for each link improvement.

H Application to Ex-post Welfare Changes with Space in the US

We further demonstrate the usefulness of our formula through another application with an "ex-post" approach. We study the welfare changes in the US economy implied by the observed changes in spatial allocations during 2010-2019.

H.1 Bringing the Formula to Data

Consider a dataset generated by the model of Section 2 at different dates indexed by t. We assume that preferences satisfy $U_{j,t}(C^{\theta}_{j,t}, \epsilon^{\theta}_{j,t}) = \frac{(C^{\theta}_{j,t})^{1-\rho}}{1-\rho} + B_{j,t} + \epsilon^{\theta}_{j,t}$ where $\epsilon^{\theta}_{j,t}$ follows a type-I extreme value distribution with shape parameter ν.

For any type θ at any location i at time t, suppose we observe population $l^{\theta}_{j,t}$, pre-tax income $w^{\theta}_{j,t} + \Pi^{\theta}_{t}$, net transfers T^{θ}_{j}, and price indices $P^{\theta}_{j,t}$ as well as total sales in each location $\sum_{l,k} p_{jl,k} y_{jl,k}$. Furthermore, suppose we know the utility function parameters $\{\rho\}$ and agglomeration functions $\{g_{ij,k}(\cdot)\}$.

Under these assumptions, Proposition 1 can be used to back out the first-order welfare changes between any dates that are attributable to the (ii) Δ MU dispersion (Ω_{MU}), (iii) Δ fiscal externality (Ω_{FE}), and (iv) Δ technological externality (Ω_{TE}) terms.

Now we show that the (i) Δ technology term (Ω_T) can be read directly from the data. Let GDP_i be nominal GDP of MSA i and Y_i be real GDP of MSA i deflated using the GDP deflator of i: P^Y_i. We apply Collorary 1 in Baqaee and Farhi (2024) to express the first-order changes in real GDP of MSA i:

$$d \ln Y_i = \sum_{j,k} \frac{p_{ij,k} y_{ij,k}}{GDP_i} d \ln \mathcal{A}_{ij,k} + \sum_{\theta} \frac{w^{\theta}_i l^{\theta}_i}{GDP_i} d \ln l^{\theta}_i. \tag{H.1}$$

Furthermore, note that

$$d \ln \mathcal{A}_{ij,k} = d \ln A_{ij,k} + \sum_{\theta} \gamma^{\theta}_{ij,k} d \ln l^{\theta}_i. \tag{H.2}$$

Using these expressions and given knowledge of $\{\gamma^{\theta}_{ij,k}\}$, we construct a measure of technological changes at MSA i as follows

$$\sum_{j,k} p_{ij,k} y_{ij,t} d \ln A_{ij,k} = GDP_i \left[d \ln Y_i - \sum_{\theta} \frac{w^{\theta}_i l^{\theta}_i}{GDP_i} d \ln l^{\theta}_i - \sum_{j,k} \frac{p_{ij,k} y_{ij,k}}{GDP_i} \sum_{\theta} \gamma^{\theta}_{ij,k} d \ln l^{\theta}_i \right] \tag{H.3}$$

Summing across all MSAs, we obtain the (i) technology term:

$$\begin{aligned} \Omega_T &= \sum_i \sum_{j,k} p_{ij,k} y_{ij,k} d \ln A_{ij,k} \\ &= \sum_i GDP_i \left[d \ln Y_i - \sum_{\theta} \frac{w^{\theta}_i l^{\theta}_i}{GDP_i} d \ln l^{\theta}_i - \sum_{j,k} \frac{p_{ij,k} y_{ij,k}}{GDP_i} \sum_{\theta} \gamma^{\theta}_{ij,k} d \ln l^{\theta}_i \right]. \end{aligned} \tag{H.4}$$

By the definition of real GDP,

$$d \ln Y_i = d \ln GDP_i - d \ln P_i^Y. \tag{H.5}$$

Plugging (H.5) back into (H.4), we have

$$\begin{aligned}\Omega_T = &\sum_i \left[dGDP_i - \sum_\theta w_i^\theta l_i^\theta d \ln l_i^\theta - \sum_{j,k} p_{ij,k} y_{ij,k} \sum_\theta \gamma_{ij,k}^\theta d \ln l_i^\theta \right] \\ &- \sum_l GDP_l \sum_i \frac{GDP_i}{\sum_j GDP_j} d \ln P_i^Y. \end{aligned} \tag{H.6}$$

Given the lack of producer prices at the MSA level, we measure the last term $\sum_i \frac{GDP_i}{\sum_j GDP_j} d \ln P_i^Y$ using the GDP deflator at the national level. We assume the only factor of production is labor and assume away the presence of input-output linkages. These assumptions imply that the GDP of MSA i equals total pre-tax personal income and that sales of skill group θ equal their pre-tax personal income.

Finally, if one is willing to take a stance on welfare weights across different households $\{\Lambda^\theta\}$, then the first-order welfare changes that are attributable to the (v) redistribution term (Ω_R) are recovered as well.

Data. We implement the above approach in the context of Metropolitan Statistical Areas (MSAs) in the United States during the period 2010-2019. Our sample consists of 214 MSAs. Following Diamond (2016) and Fajgelbaum and Gaubert (2020), we consider two ex-ante types based on educational attainments: high-skill (4 years of college education or greater) and low-skill (less than 4 years of college education). We also restrict our analysis to the working-age (age 18-64) population. We construct our dataset using a combination of the BEA regional economic accounts, American Community Survey (ACS) (IPUMS-USA, Ruggles, Flood, Goeken, Grover, Meyer, Pacas, and Sobek 2023), March supplement of the Current Population Survey (March CPS) (IPUMS-CPS, Flood, King, Rodgers, Ruggles, Warren, and Westberry 2023), Consumer Expenditure Survey (CEX), and IRS Statistics of Income (SOI) data.

We first obtain population, pre-tax income, and transfer receipts by MSAs from the BEA. We allocate them to each skill group based on their shares in each MSA using 5-years samples of the ACS. We obtain tax payments by county from the IRS SOI and then aggregate them to the MSA level using the crosswalk provided by the NBER. We further allocate them to each skill group based on the aggregate shares in tax payments by each skill group using the March CPS. Net transfers $T_{j,t}^\theta$ are constructed as the difference between transfer receipts and tax payments, but we adjust them by adding a common constant so as to ensure government budget balance. Finally, we construct price indexes for each MSA and skill group as follows. The BEA provides price indexes for four broad categories at the MSA level: goods, housing, utilities, and other services. We compute the expenditure share on these four categories for each skill group using the CEX. We then construct price indexes for each MSA and skill group by weighting the price level of the four categories using the expenditure weight for each skill group. With pre-tax income, $w_{j,t}^\theta + \Pi_t^\theta$, net transfers $T_{j,t}^\theta$, and price indexes $P_{j,t}^\theta$, we compute consumption for each location-type as $C_{j,t}^\theta = \frac{w_{j,t}^\theta + \Pi_t^\theta + T_{j,t}^\theta}{P_{j,t}^\theta}$.

	dW	Ω_T	Ω_{MU}	Ω_{FE}	Ω_{TE}	Ω_R
2010-2015	2.085%	2.035%	0.128%	0.022%	0.015%	-0.116%
2015-2019	1.624%	1.367%	-0.009%	0.009%	-0.081%	0.337%

Table H.1: Welfare Changes in the US 2010-2019

Note: This table reports welfare decompositions based on Proposition 1 for observed changes in spatial allocations across US MSAs during the periods of each row. Welfare is expressed in units of GDP equivalence in the initial period. All reported numbers are annualized changes.

Parameterization. We use the same estimates of (ρ, ν) as in Table 2. We assume away input-output linkages and set the total sales of each skill type in each location as their pre-tax income. We assume constant elasticity of productivity spillovers and follow Fajgelbaum and Gaubert (2020) by setting them to $(\gamma^l_{ij,l}, \gamma^h_{ij,l}, \gamma^l_{ij,h}, \gamma^h_{ij,h}) = (0.003, 0.044, 0.02, 0.053)$, where $\gamma^\theta_{ij,\theta'}$ corresponds to the productivity spillover from type θ to θ' for the goods shipped from i to j. (We do not consider shipment congestion externality as in Section 5.) The skill type h denotes high-skill and l denotes low-skill. For welfare weights, we assume utilitarian welfare: $\Lambda^\theta = 1$ for all θ.

H.2 Results

Table H.1 shows welfare decompositions based on observed changes in spatial allocations during the periods 2010-2015 and 2015-2019. All of these numbers are annualized changes and expressed in units of GDP equivalence in the initial period. Combined with our choice of numeraire (26), these numbers answer the following question: "if we were to achieve the same welfare change by uniformly increasing utility in all locations for all household types, what percent increase in GDP would we need?"

In 2010-2015, aggregate welfare increased by a GDP equivalence of approximately 2.1%. The largest contributor is the (i) Δ technology term, but Hulten's characterization understates the welfare gain of this period. The (ii) Δ MU Dispersion term plays a non-trivial secondary role in raising welfare. This is driven by the reduction in spatial consumption inequality within skill groups, as shown by the left panel of Figure H.1. The (iii) Δ fiscal externality, (iv) Δ technological externality, and (v) Δ redistribution terms also contribute to the welfare gain, but their magnitudes are small.

In 2015-2019, aggregate welfare increased by a GDP equivalence of approximately 1.6%. The (i) Δ technology term again understates the welfare gain, but now, the (ii) Δ MU Dispersion term is close to zero. Instead, the (v) Δ redistribution term plays a substantial role in the welfare gain. The right panel of Figure H.1 explains why. During this period, the *within* group spatial distribution of consumption stopped converging, but there was convergence of consumption *across* skill groups.

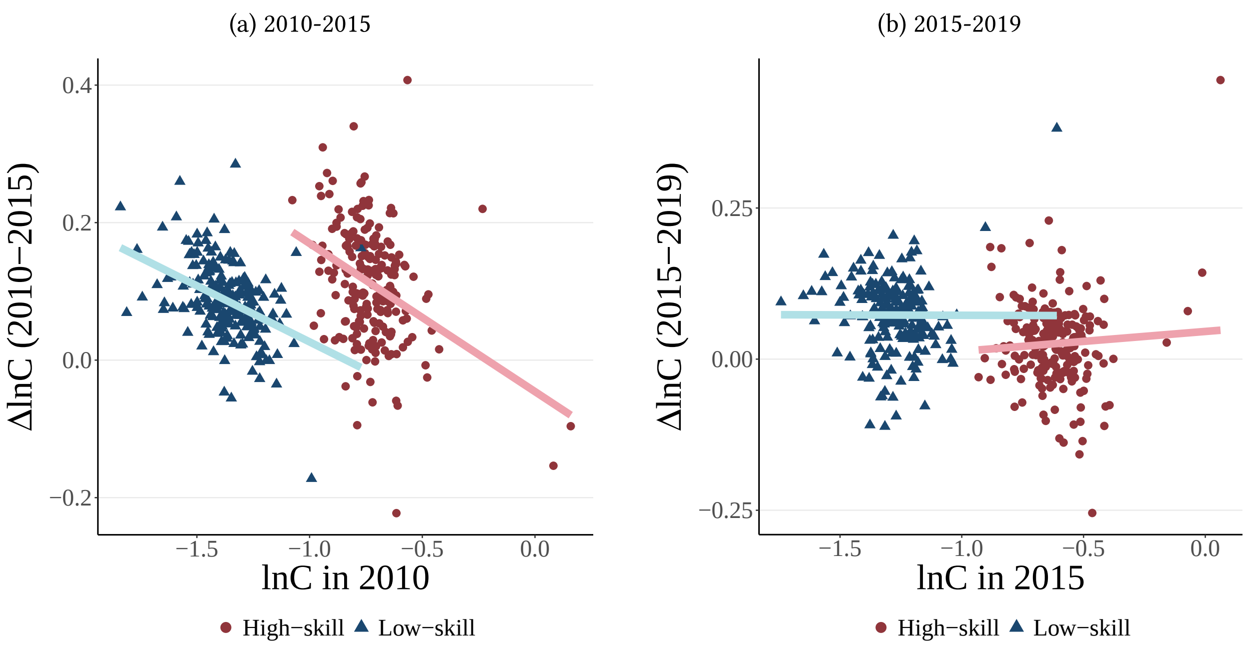

Figure H.1: Convergence in Consumption Across MSAs and Skill Groups

Note: The figure plots consumption changes for each MSA and skill group against their initial consumption level. The solid line is the best linear fit within each skill group.