Firm Dynamics with Downward Nominal Wage Rigidity

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Motivation

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  3. long-term contracting subject to downward nominal wage rigidity
- The goal is to use the framework to study
  1. the long-run implications of changes in inflation target
  2. the short-run implications for moneatry policy by embedding into NK (not today)

Model

Environment

- Continuous time, $t \in [0,\infty)$
- Focus on the steady state for today
- A continuum of risk-neutral workers with discount rate r
  - Unemployed: flow value of leisure b, off-the-job search λ^U
  - Employed: endogenous wage w, on-the-job search $\lambda^E \equiv \zeta\lambda^U$
- A continuum of heterogenous firms hire workers
- Random search with CRS matching function $M(\tilde{u}, V)$, where $\tilde{u} \equiv u + \zeta(1-u)$

Technology

- Firm's production function

$$y = z^{1-\alpha} n^{\alpha}$$

- Idiosyncratic productivity z follows geometric Brownian motion:

$$dz = \mu z dt + \sigma z dW$$

- The evolution of employment n consists of:
  - hiring with cost $c(h)n$, where h is the hiring rate and $c(h) \equiv \frac{\kappa}{1+\nu} h^{1+\nu}$
    - no vacancy cost
  - poaching from other firms
  - exogenous separation at rate s
  - firing
- Firms exit at rate $\varkappa$ and replaced by the new entrants

Wage Setting

- Assume equal treatment within a firm (same wages & randomized firing)
- Firms offer recursive contracts with full commitment. Workers can't commit.
- Given $(z_{t-dt}, n_{t-dt}, W_{t-dt}, w_{t-dt})$, firms offer $\{w_t, W_t\}$ subject to

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  1. Promise keeping ($dt \to 0$):

$$rWdt \leq wdt + (s + \varkappa)(U - W)dt + \lambda^E \int \max\{\tilde{W} - W, 0\} dF(\tilde{W})dt + dW$$

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  2. Downward nominal wage rigidity

$$dw \geq -\pi w dt$$

     - w_t: real wage, π: inflation rate (set by the central bank)

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- Then $\tilde{J}(n, z, W, w) = J(\hat{n}, W, w)z$, where $\hat{n} \equiv n/z$ and $J \equiv J(\hat{n}, W, w)$ solves HJB-QVI:

$$\min \left\{ \begin{array}{c} \rho J - \max\limits_{d\hat{n},h} \left\{ \hat{n}^\alpha - w\hat{n} - c(h)\hat{n} + d\hat{n}\partial_{\hat{n}} J + \frac{\sigma^2}{2}\hat{n}^2\partial^2_{\hat{n}\hat{n}} J + dW\partial_W J - \pi w \partial_w J \right\}, \\ J - J^*(\hat{n}, W, w) \end{array} \right\} = 0$$

where $\rho \equiv r + \varkappa - \mu$,

$$d\hat{n} = \hat{n}\left(h - \mu - s - \lambda^E(1 - F(W))\right)$$

$$dW = rW - \left[w + (s + \varkappa)(U - W) + \lambda^E \int \max\{W' - W, 0\} dF(W')\right]$$

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$$J^*(\hat{n}, W, w) \equiv \max_{\hat{n}^* \leq \hat{n}, W^* \geq W, w^* \geq w} J(\hat{n}^*, W^*, w^*)$$

s.t. $\quad W^* \frac{n^*}{n} + U(1 - \frac{n^*}{n}) \geq W$

Closing the Model

- The entrants draw $(\hat{n}^0, z^0)$ from cdf $\Psi(\hat{n}^0, z^0)$ and solve

$$(w^0(\hat{n}^0), W^0(\hat{n}^0)) \in \arg\max_{W,w} J(\hat{n}^0, W, w)$$

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- The value of unemployment is

$$rU = b + \lambda^U \int \max\{W - U, 0\} dF(W) + \frac{\chi \int \hat{n} z d\Psi(\hat{n}, z)}{u} \int \frac{\hat{n} z}{\int \hat{n} z d\Psi(\hat{n}, z)} W^0(\hat{n}) d\Psi(\hat{n}, z)$$

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- Consistency

$$F(W) = \int_{\tilde{W} \le W} \frac{v(\hat{n}, \tilde{W}, w)\hat{n} z}{V} dG(z, \hat{n}, \tilde{W}, w)$$

$$v(\hat{n}, W, w) = \frac{h(\hat{n}, W, w)\hat{n} z}{\lambda^F(p^u + p^e H(\tilde{W}))}, \quad H(W) = \int_{\tilde{W} \le W} \frac{\hat{n} z}{N} dG(z, \hat{n}, \tilde{W}, w)$$

and $\lambda^U = \dfrac{M(V, \tilde{u})}{\tilde{u}}, \lambda^F = \dfrac{M(V, \tilde{u})}{V}$

Equilibrium

Flexible Wage

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$$\rho S(\hat{n}) = \max_{h \geq 0, W' \geq U} \hat{n}^{\alpha} - c(h)\hat{n} - W'h\hat{n} + \lambda^E \int_{\tilde{W} \geq W'} \tilde{W} dF(\tilde{W})\hat{n} + (\varkappa + s)U\hat{n}$$

$$+ S_n(\hat{n})\hat{n}(h - \mu - s - \lambda^E(1 - F(W'))) + S_{nn}(\hat{n})\frac{1}{2}(\sigma\hat{n})^2$$

with $S_n(\hat{n}^*) = U$. Note W' is now a jump variable achieved through $w = \pm\infty$.

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- FOCs:

$$\left(S_n(\hat{n}) - W'\right)\lambda^E f(W') = h, \quad \left(S_n(\hat{n}) - W'\right) = \underbrace{\kappa h^{\nu}}_{c'(h)}$$

  - $\nu > 1$: W' is increasing in $S_n(\hat{n})$
  - $\nu = 1$: W' is independent of $S_n(\hat{n})$
  - $\nu < 1$: W' is decreasing in $S_n(\hat{n})$

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- Now bring back DNWR constraint
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- Computational algorithm:
  - Guess $F(W)$
  - Solve firm's HJB-QVI to obtain policy functions
  - Obtain steady state distribution using KFE
  - Compute implied $F^{new}(W)$ and check $F^{new}(W) \approx F(W)$
  - If not, update $F(W)$ using $F^{new}(W)$

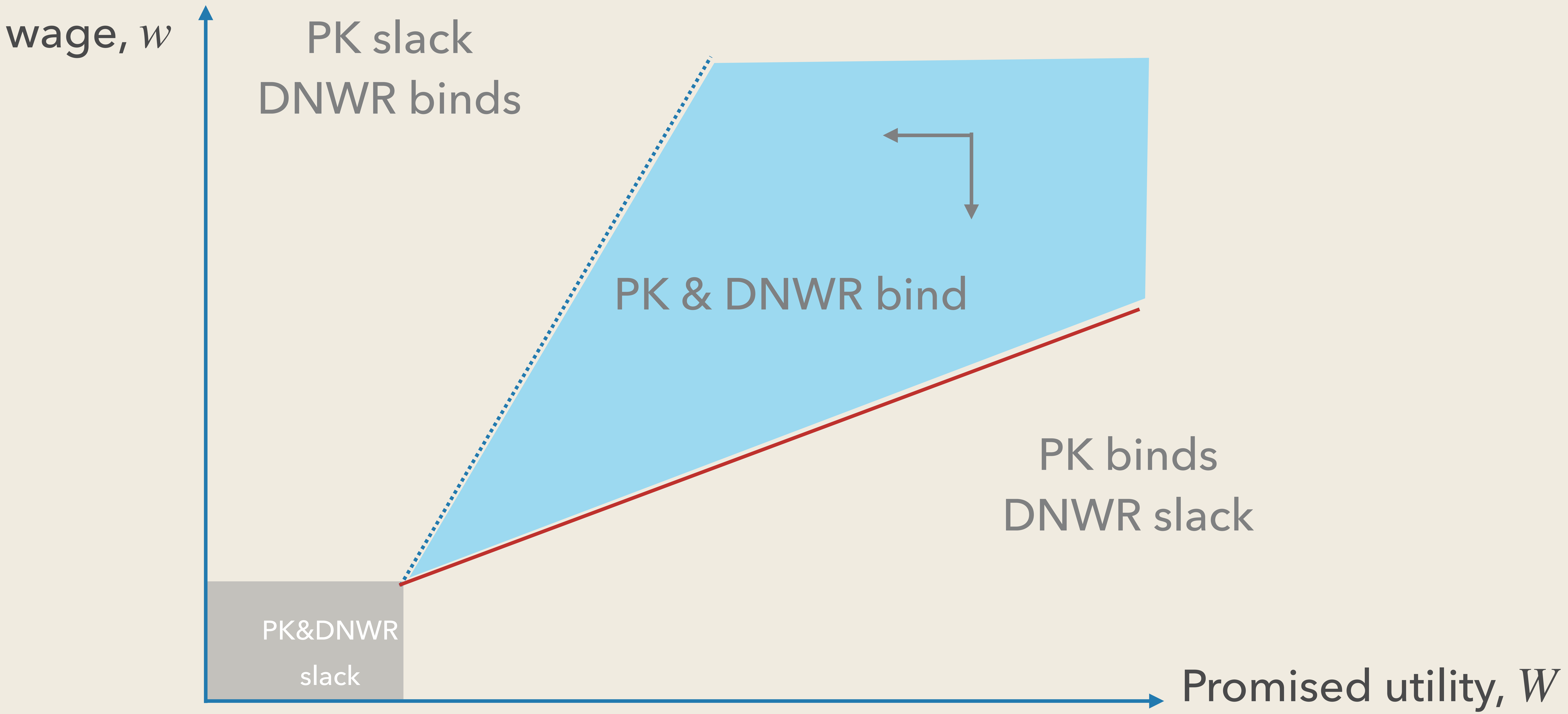
Parameter Values

- A period is a quarter
- Assume Cobb-Douglas matching function $M(\tilde{u}, V) = \tilde{u}^{\eta} V^{1-\eta}$
- Parameter values:

$$
\begin{aligned}
\alpha &= 2/3 \\
s &= 0.068 \\
\varkappa &= 0.021 \\
\mu &= -0.01 \\
\sigma &= 0.15 \\
\eta &= 0.5 \\
\pi &= 0.5\,\% \\
\nu &= 4 \\
\zeta &= 0.2
\end{aligned}
$$

- Choose (b, κ) to target unemployment rate 5% & aggregate wage markdown 20%

Wage and Promised Utility Dynamics



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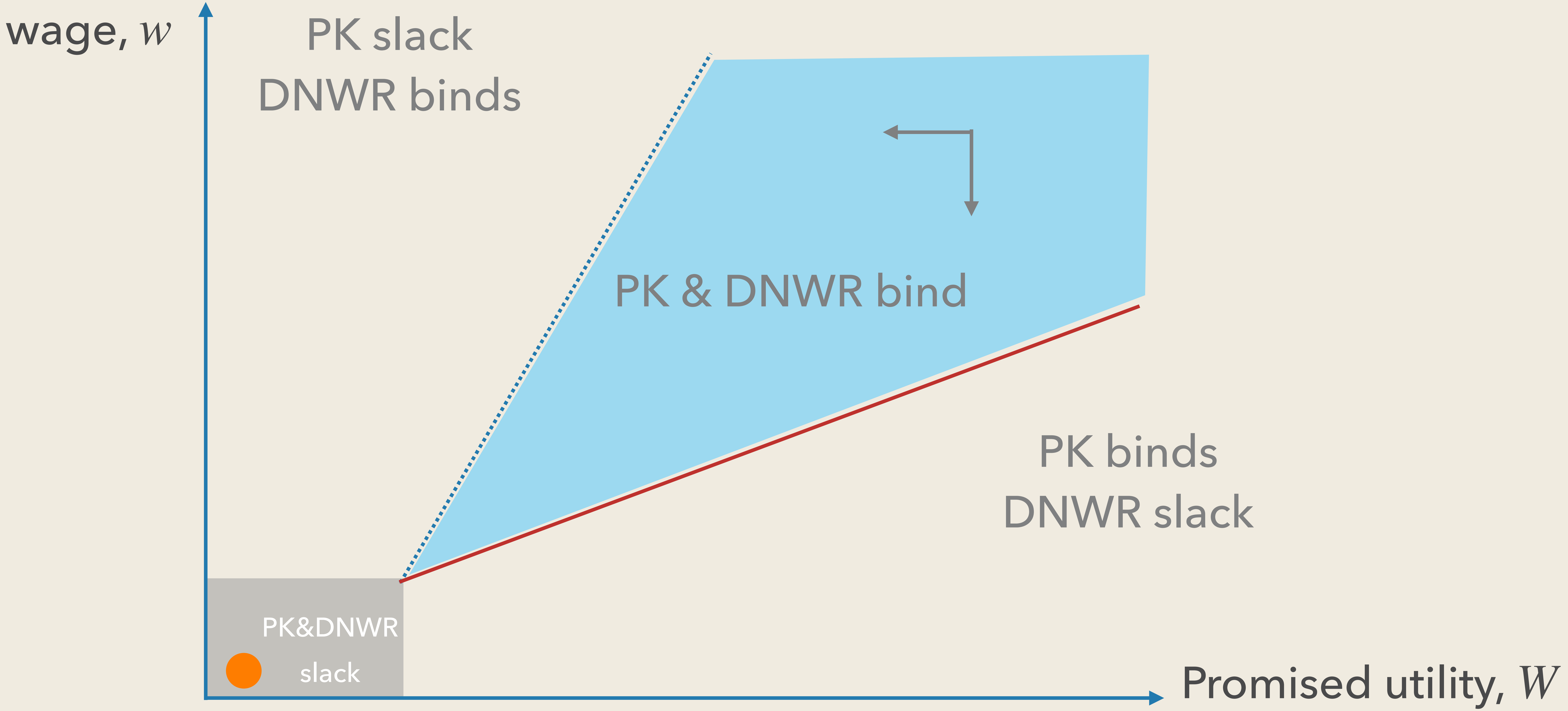

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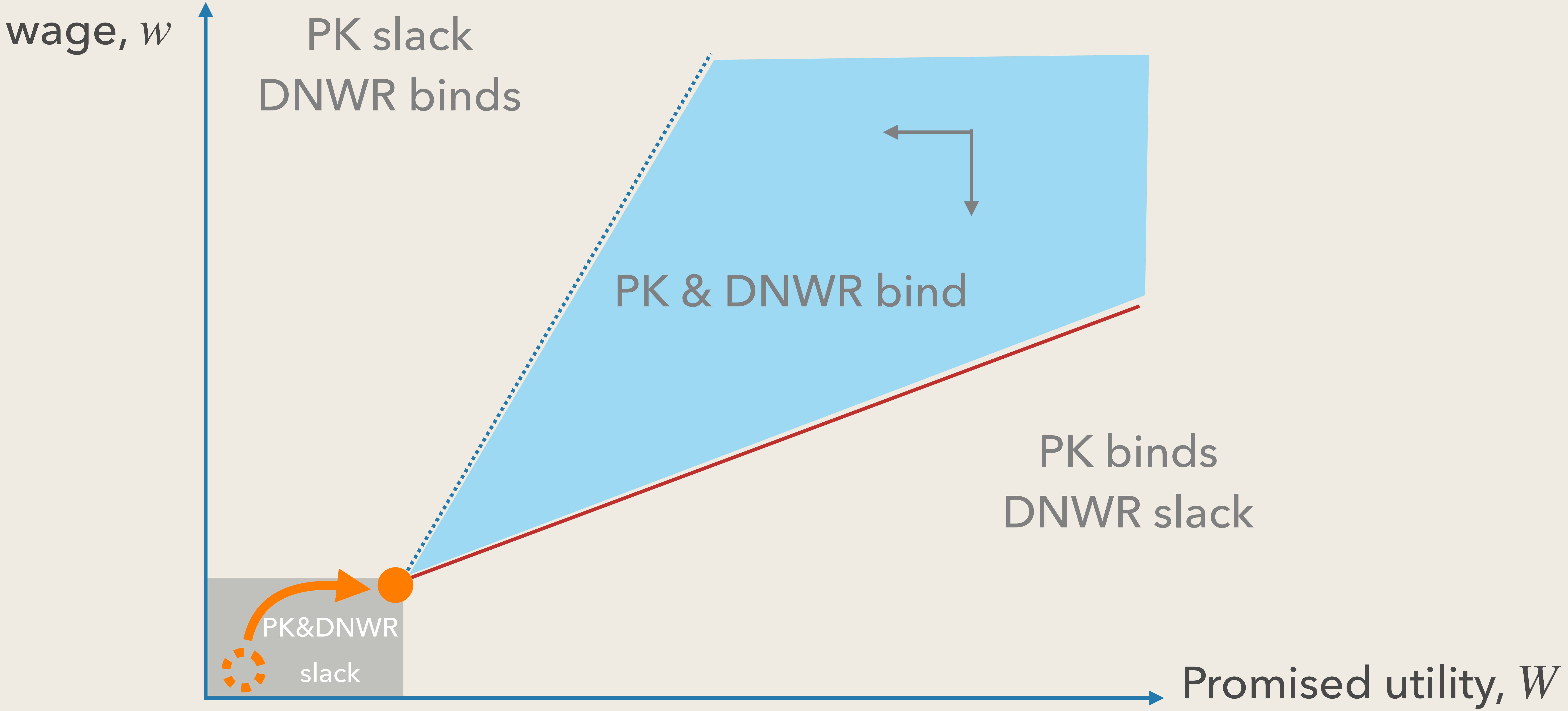

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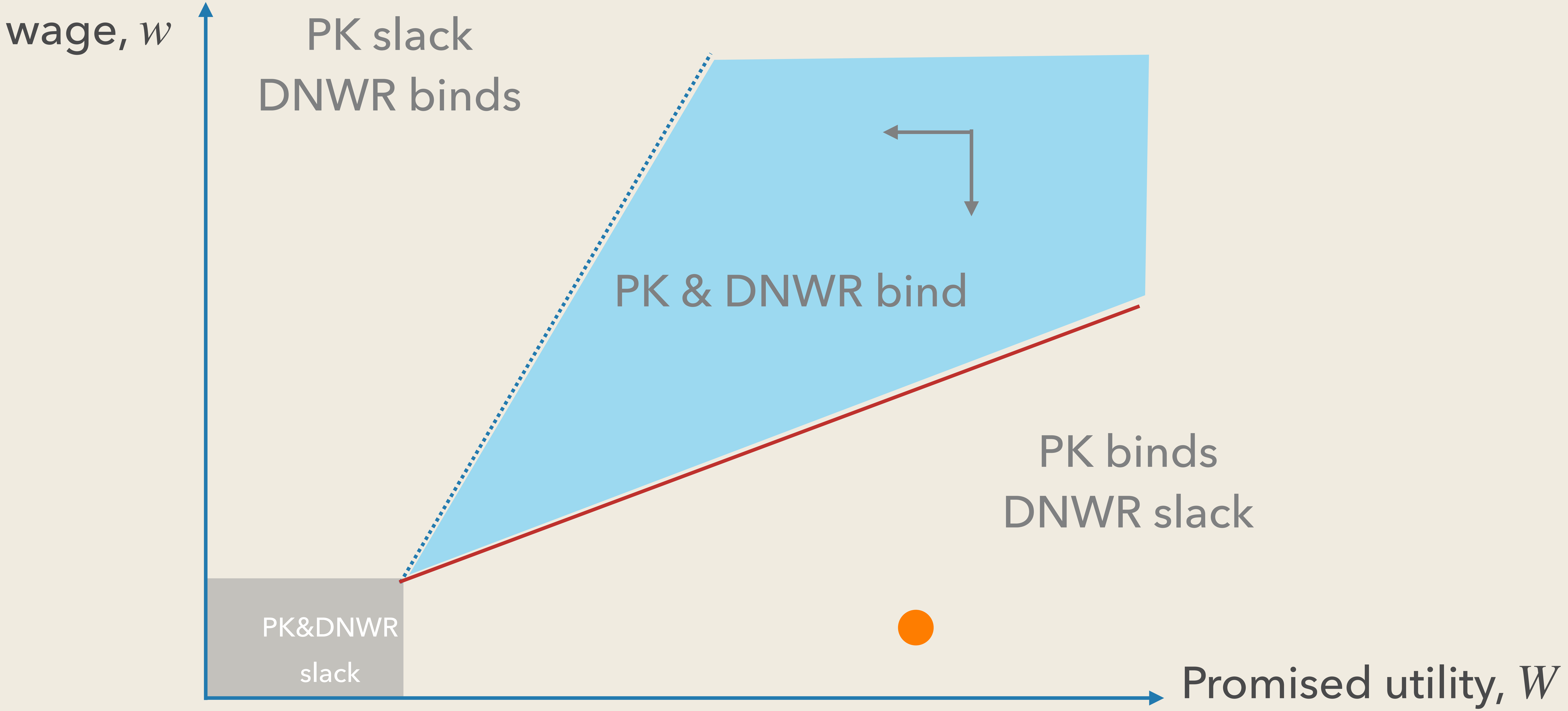

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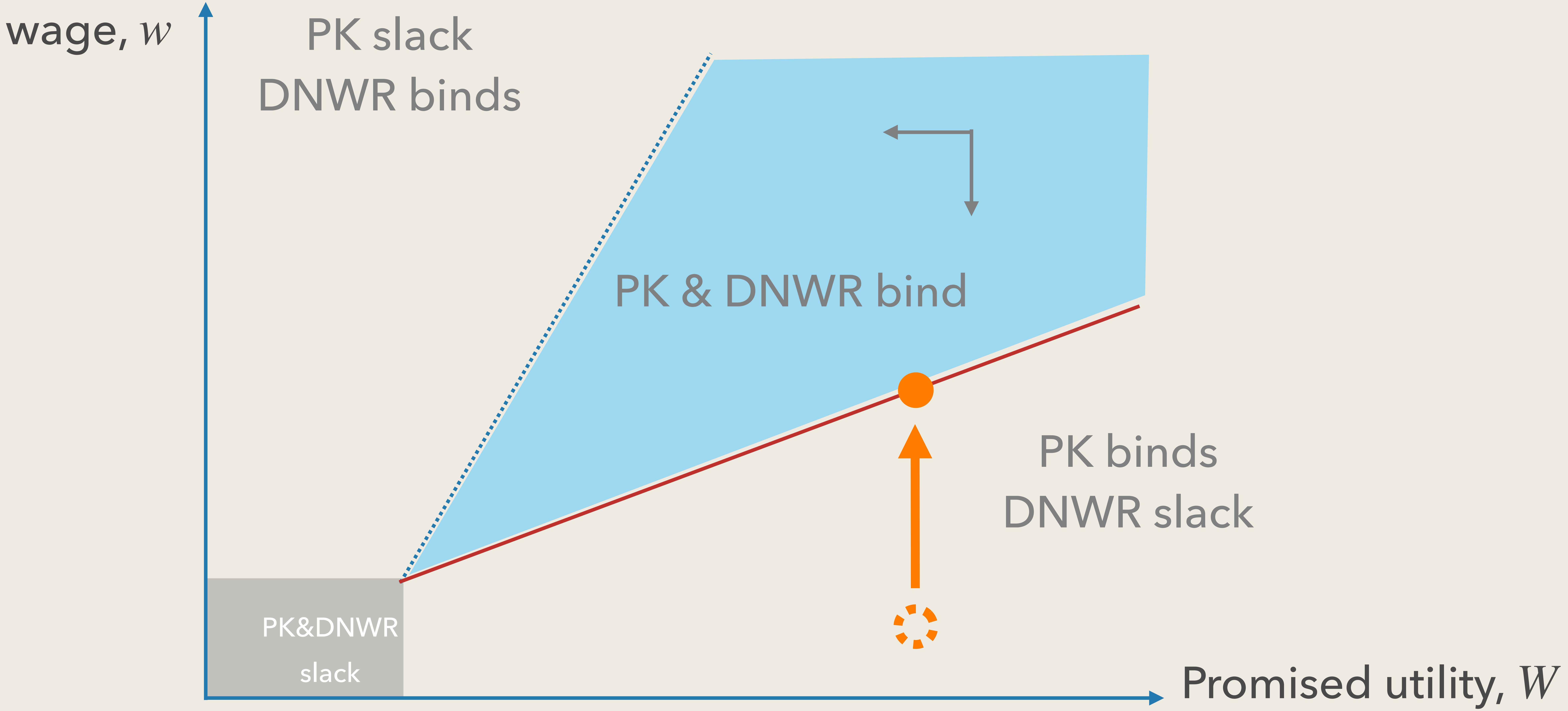

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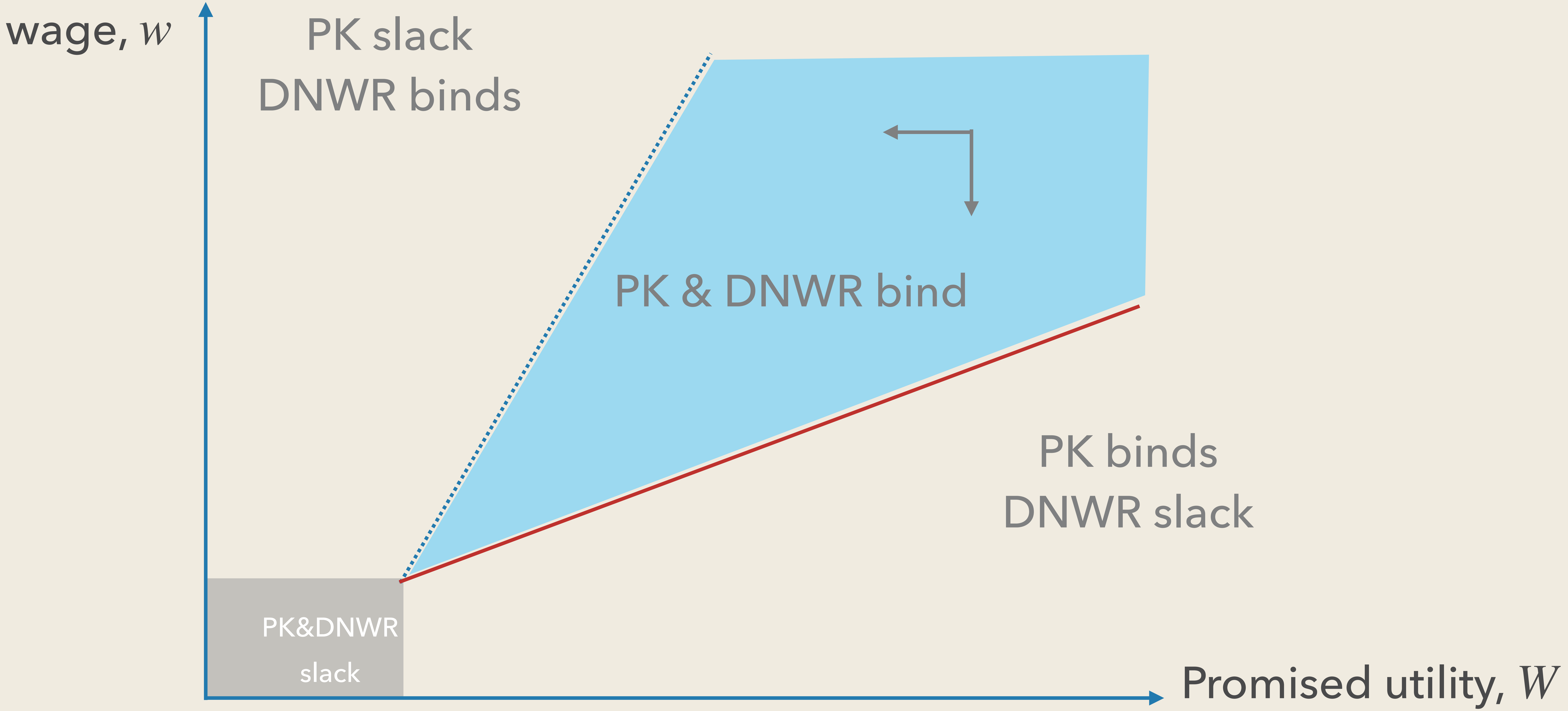

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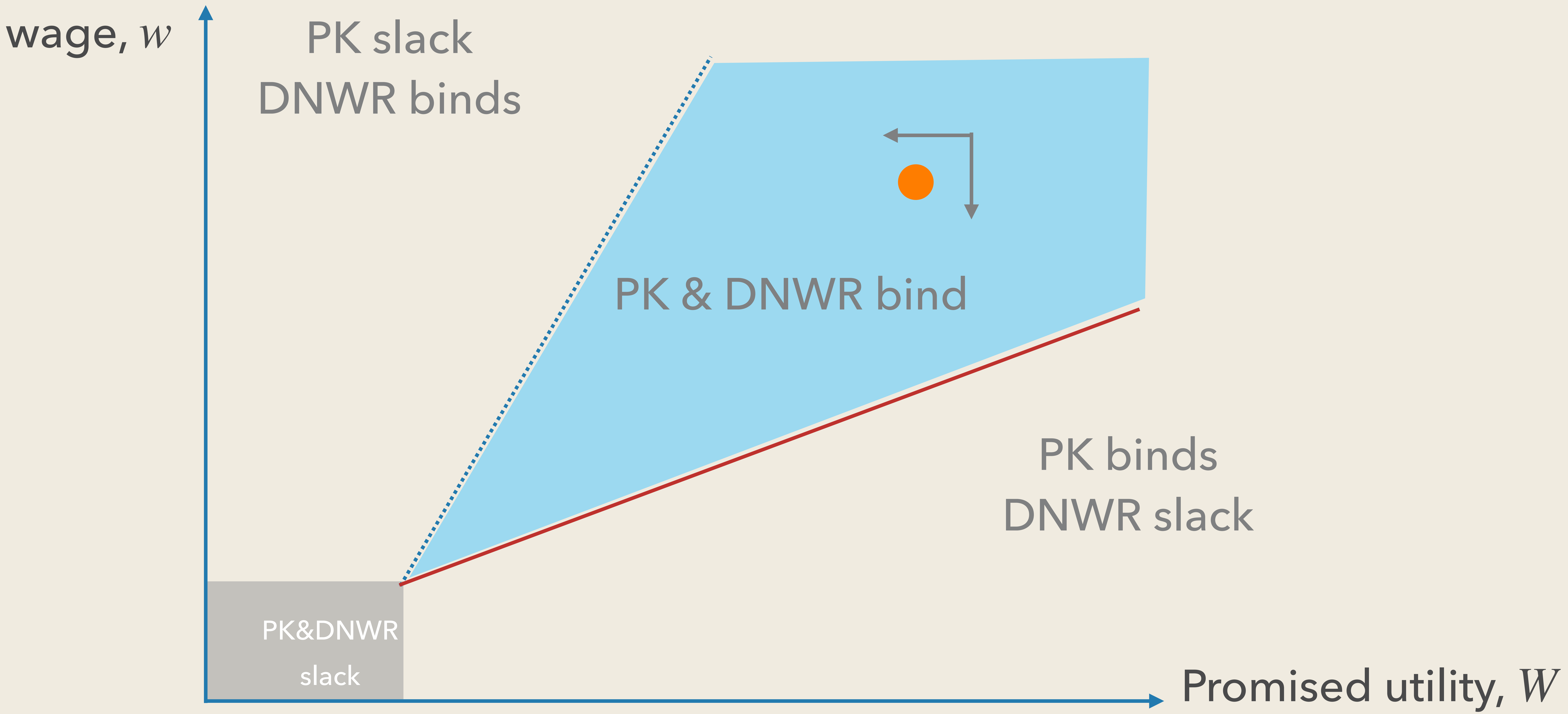

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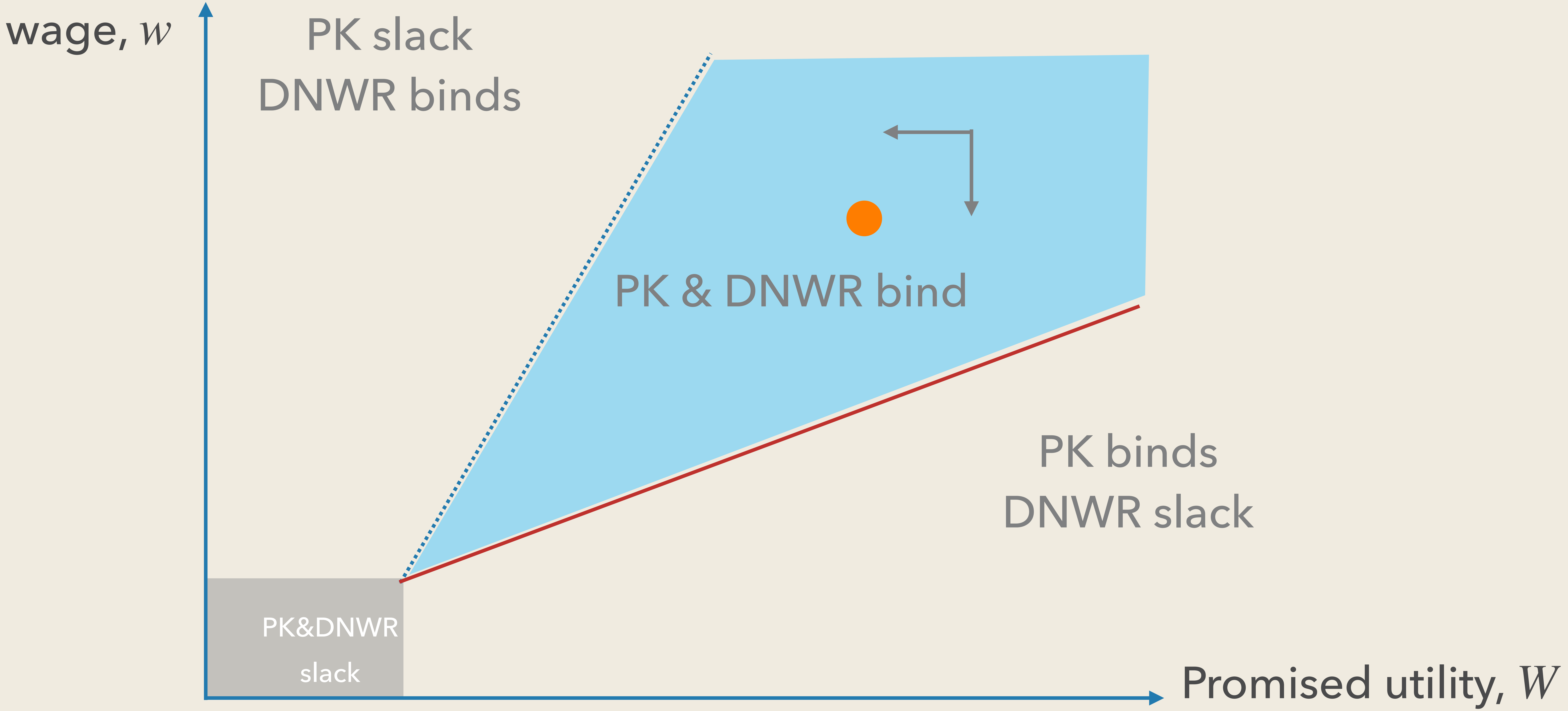

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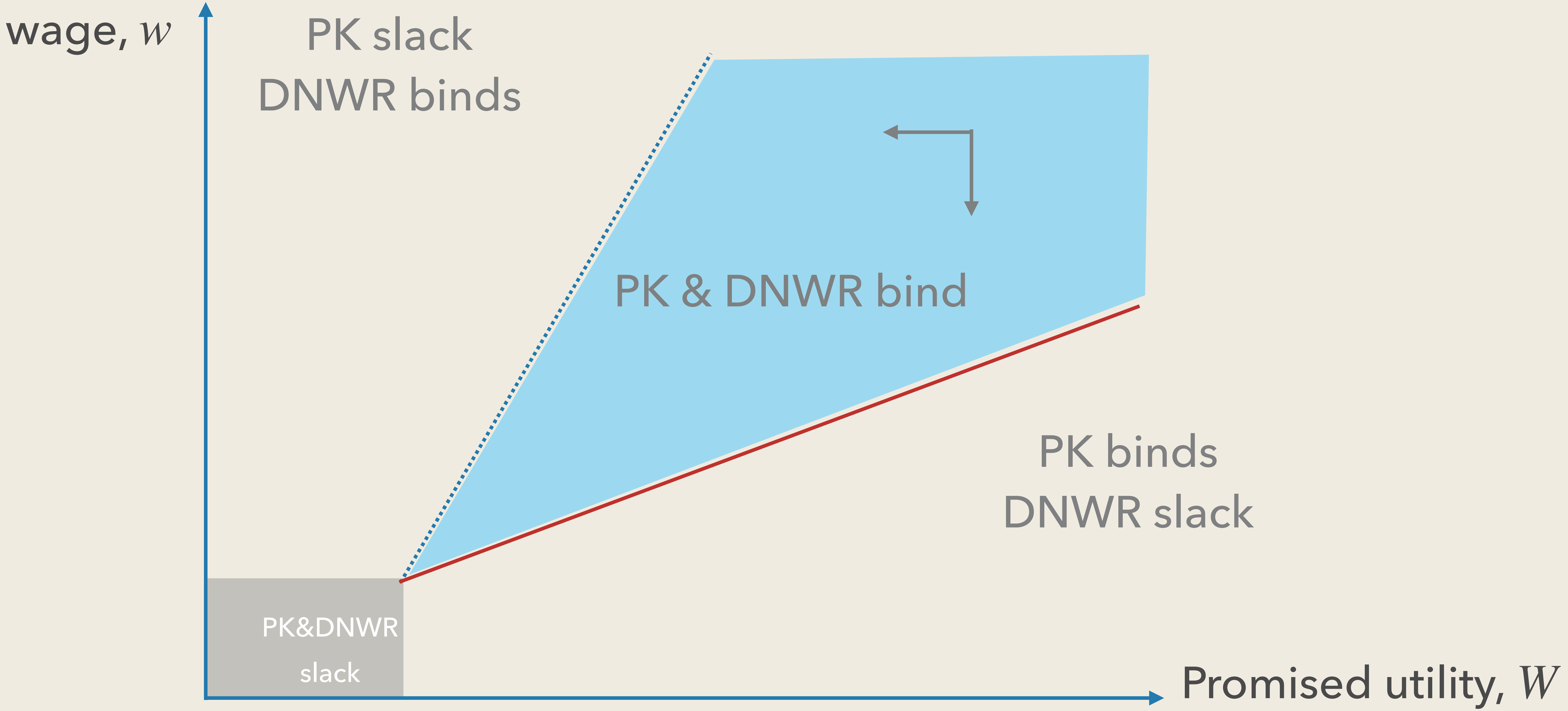

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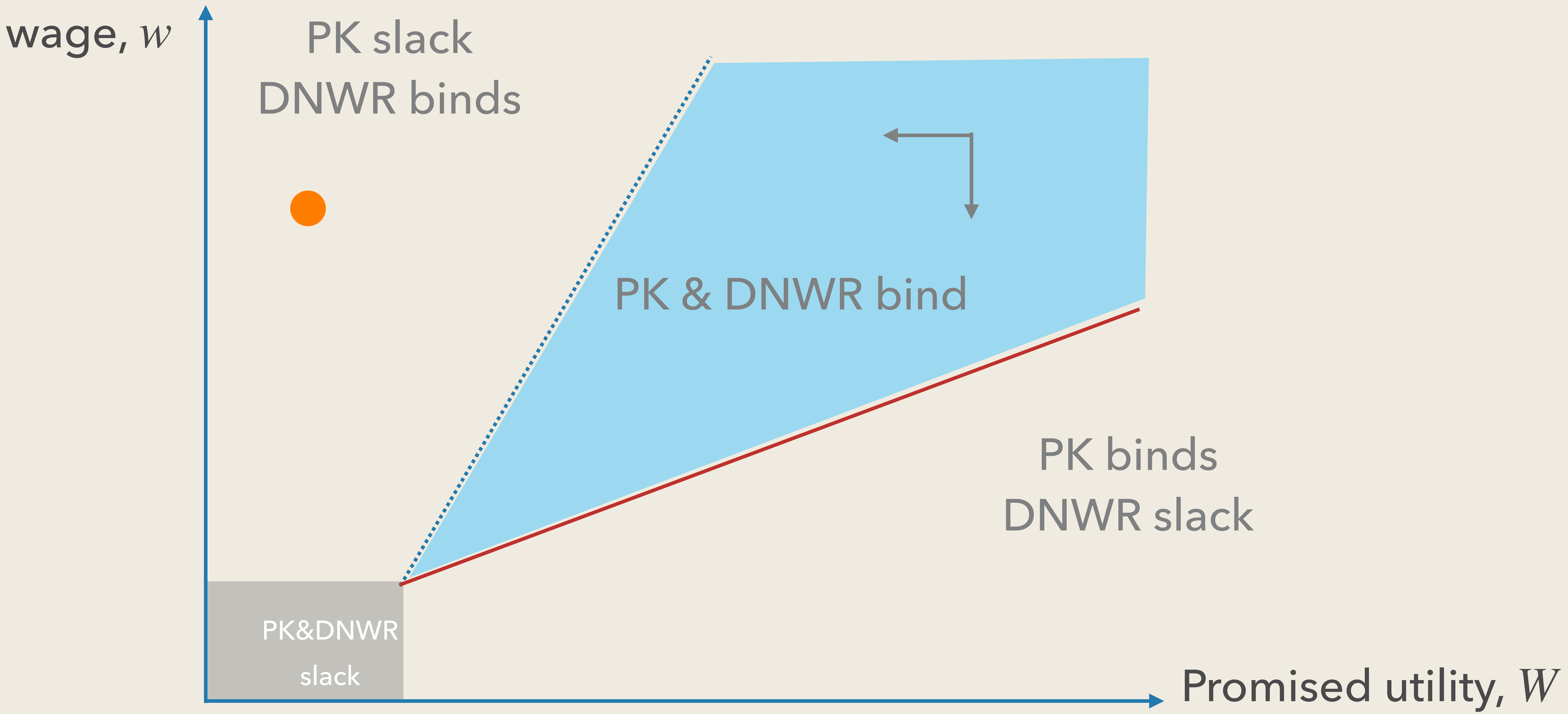

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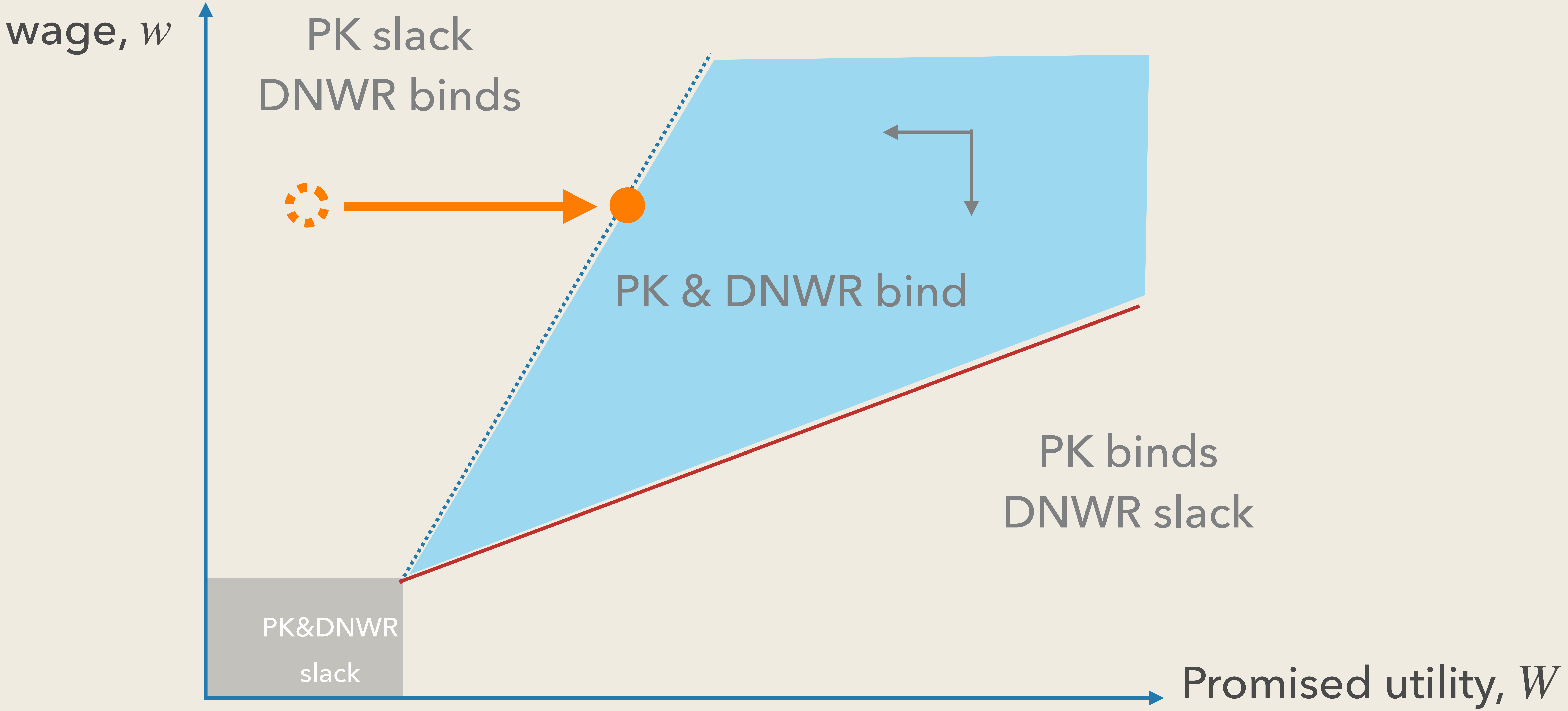
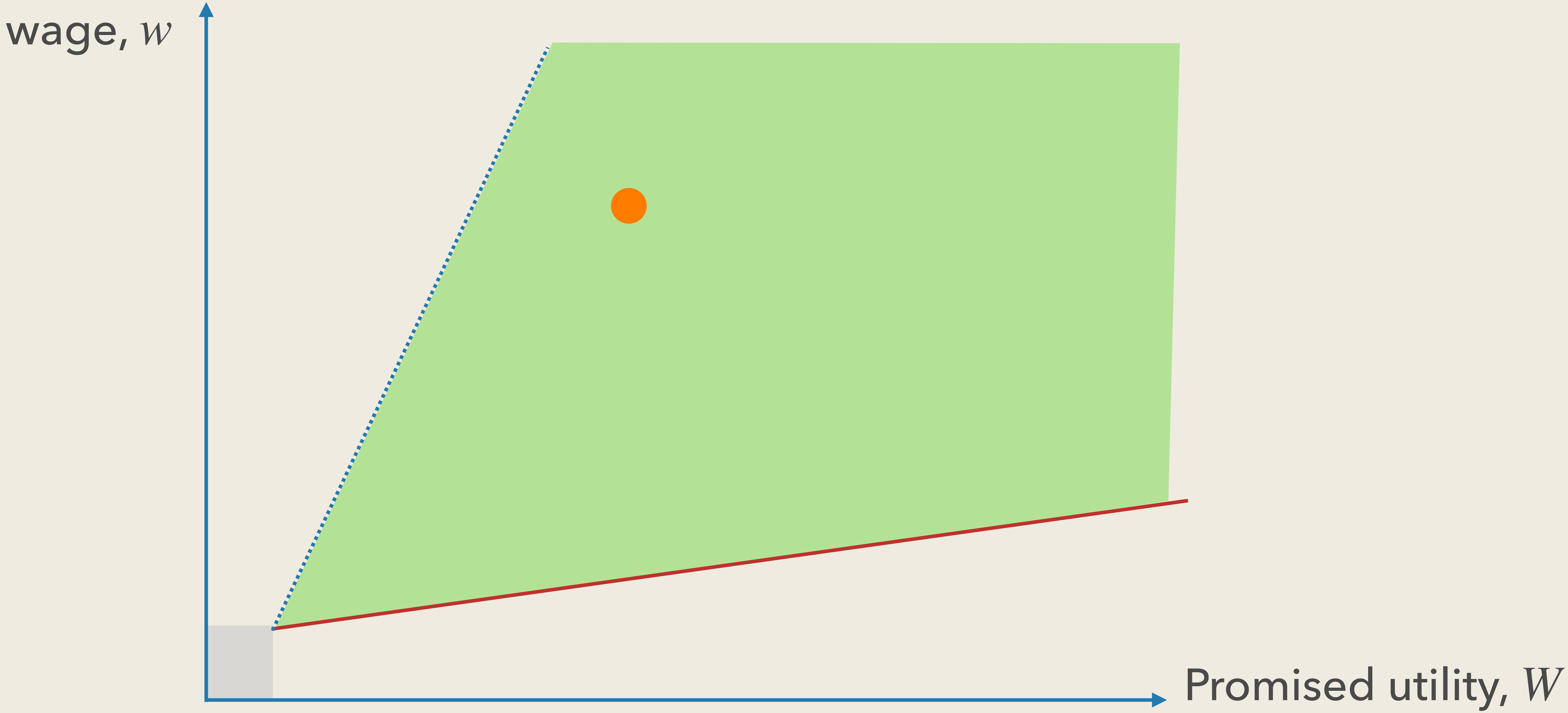


Negative Productivity Shock



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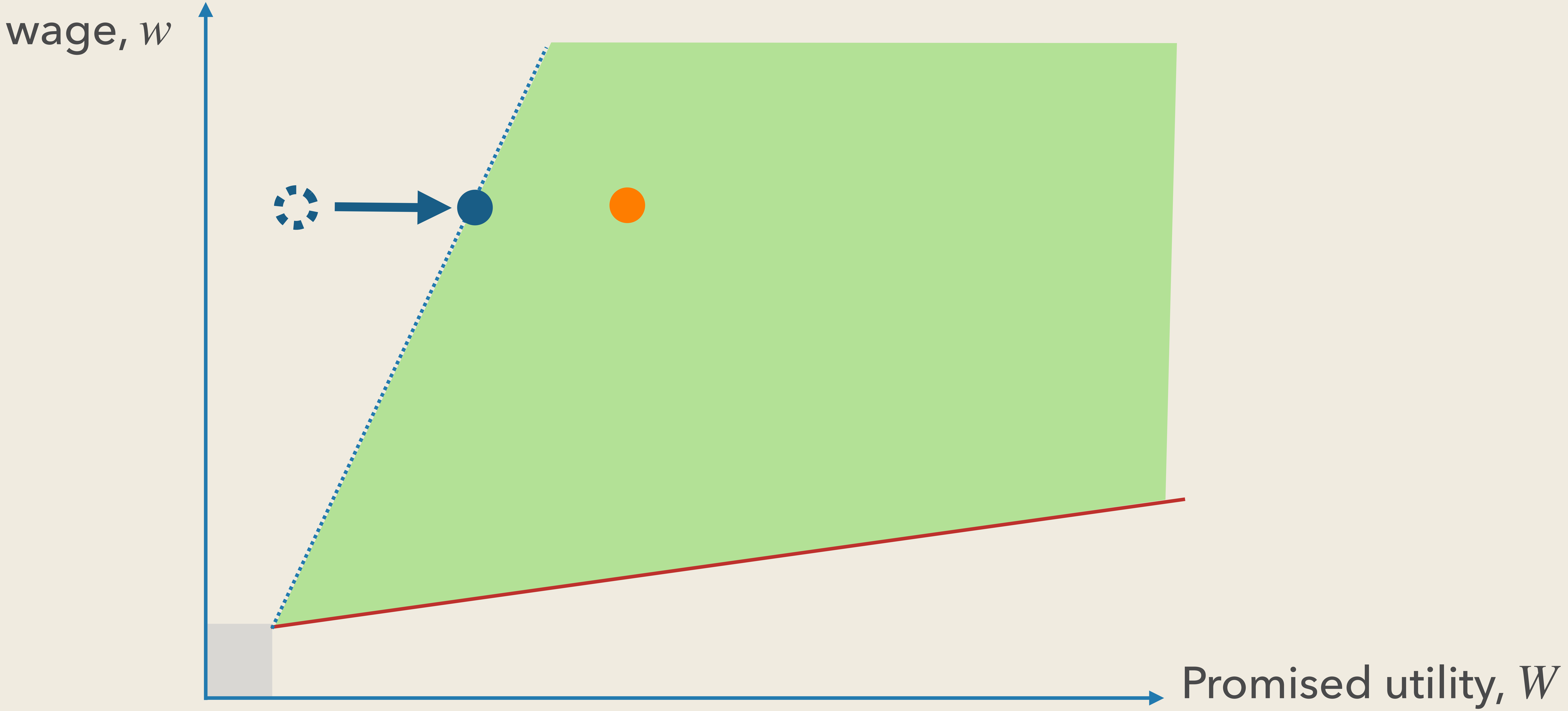

Increasing Long-run Inflation Target

	Annual 2% Inflation	Annual 4% Inflation
EE rate	5.81%	5.94%
Output	0.440	0.448
Unemployment Rate	5%	2.1%
Labor Share	52.1%	52.7%
Top 1% Firm Emp Share	8.0%	8.1%

Conclusion

- A framework that integrates
  1. firm dynamics
  2. frictional labor market
  3. frictional wage settings
- Many more things to do…