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# Monetary Policy

EC502 Macroeconomics  
Topic 10

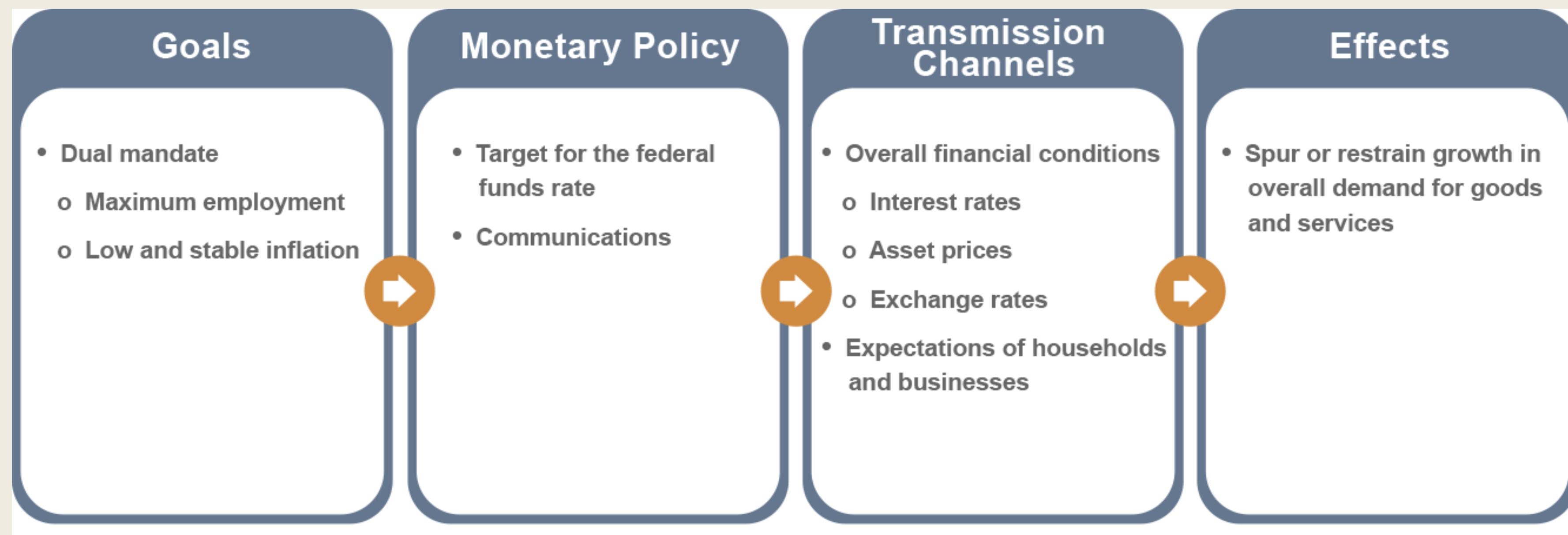
Masao Fukui

2024 Spring

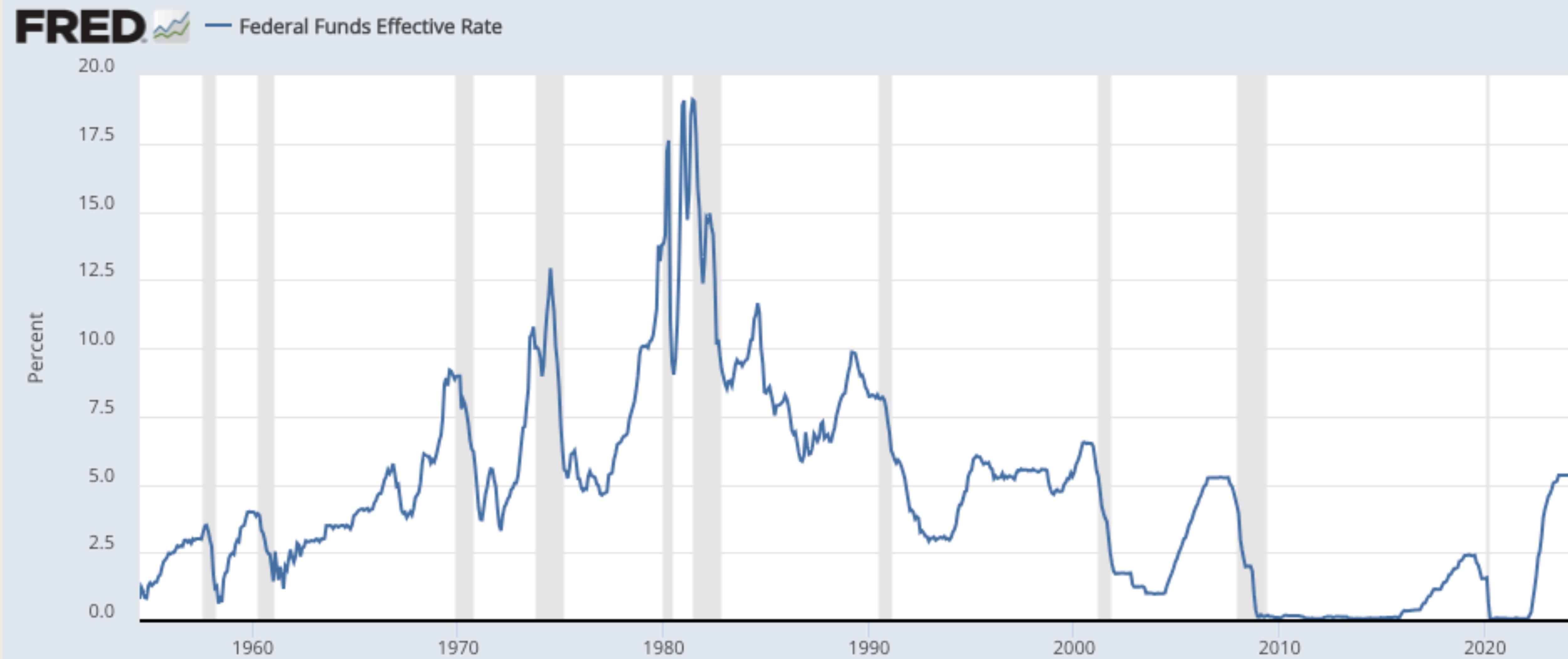
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# Monetary Policy

- Monetary policy is a central macroeconomic policy tool
- What are the goals of monetary policy? The Federal Reserve Act states:
  1. maximum employment
  2. stable prices
- How does monetary policy work? FRB website writes:



# Federal Funds Rate



# Does Monetary Policy Work in Our Model?

- FRB and many people believe monetary policy affects employment and prices
- We have already built a macroeconomic model (RBC model)
- What does our model say?
- But our model was already expressed everything in “real” term
  - in the units of consumption goods
- Let us rewrite RBC model in “nominal” term
  - in the units of dollar

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# Monetary Neutrality

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# RBC without Investment

- For the most part, we will abstract from investment
- We will add back investment at the end

# Households

- Households have the following preferences

$$u(C_0) - v(l_0) + \beta u(C_1)$$

- Now the budget constraints are

$$P_0 C_0 + A_0 = W_0 l_0 + D_0$$

$$P_1 C_1 = (1 + i) A_0 + W_1 l_1 + D_1$$

- $P_0, P_1$ : nominal price level (CPI) at  $t = 0, 1$
  - $W_0, W_1$ : nominal wage at  $t = 0, 1$
  - $1 + i$ : nominal interest rate
- Define the inflation in this economy as

$$1 + \pi_1 = \frac{P_1}{P_0}$$

# Firms

- The firms solve

$$\max_{L_0, L_1} D_0 + \frac{1}{1+i} D_1$$

subject to

$$D_0 = P_0 F_0(K_0, L_0) - W_0 L_0$$

$$D_1 = P_1 F_1(K_1, L_1) - W_1 L_1$$

$$K_1 = (1 - \delta) K_0$$

# Market Clearing Conditions

- Market clearing conditions:

$$C_0 = F_0(K_0, L_0)$$

$$C_1 = F_1(K_1, L_1)$$

$$l_0 = L_0$$

$$l_1 = L_1$$

- Monetary policy sets  $i$
- Suppose now monetary policy changes  $i$ 
  1. Can it affect prices?
  2. Can it affect employment?

# Converting into Real Model

- We can rewrite the household's budget constraint as

$$C_0 + a_0 = w_0 l_0 + d_0$$

$$C_1 = (1 + r)a_0 + w_1 l_1 + d_1$$

- $a_0 \equiv A_0/P_0$ : real saving,  $w_t \equiv W_t/P_t$ : real wage,  $d_t \equiv D_t/P_t$ : real profit
  - $1 + r \equiv (1 + i)\frac{P_0}{P_1}$ : real interest rate
- Similary, firms' profits are ( $d_t = D_t/P_t$ )

$$\max_{L_0, I_1, K_1, L_1} d_0 + \frac{1}{1+r} d_1$$

$$d_0 = F_0(K_0, L_0) - w_0 L_0$$

$$d_1 = F_1(K_1, L_1) - w_1 L_1$$

# Solutions

- $\{C_0, C_1, L_0, r\}$  solve

$$v'(L_0) = \frac{\partial F_0(K_0, L_0)}{\partial L_0} u'(C_0)$$

$$u'(C_0) = \beta(1 + r)u'(C_1)$$

$$C_0 = A_0 K_0^\alpha L_0^{1-\alpha}$$

$$C_1 = A_1 K_1^\alpha L_1^{1-\alpha}$$

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# Monetary Policy and Employment

- So, do changes  $i$  affect employment,  $L_0$ ?
- No, because  $i$  never showed up in the previous conditions.

# Monetary Policy and Prices

- Do changes  $i$  affect price levels,  $P_0$ ?
- With  $\{C_0, C_1\}$  pinned down,  $r$  is also pinned down via Euler

$$C_0^{-\sigma} = \beta(1 + r)C_1^{-\sigma}$$

- Recall

$$1 + r \equiv (1 + i)\frac{P_0}{P_1}$$

Given  $r$  and  $i$ ,  $P_0/P_1$  is pinned down from this equation

- From now on, we will fix  $P_1 = \bar{P}_1$  ( $P_1$  is generally indeterminate). Then

$$P_0 = \frac{1 + r}{1 + i}\bar{P}_1$$

A higher  $i$  lowers price level today,  $P_0$

# Monetary Neutrality

- If monetary policy raises the **nominal** interest rate  $i$ ,
  1. No effect on employment or any quantities
  2. Price level today goes down (inflation from  $t = 0$  to  $t = 1$ ,  $P_1/P_0$ , goes up)
- Monetary policy is neutral with respect to macro quantities
- Why? – Price level  $P_0$  immediately drops to keep the **real** interest rate  $r$  unchanged
- Real interest rate is what matters for the households and firms decisions
  - No one cares about nominal interest rate per se (in theory)
- Nominal wage also drops so that real wage  $w_0 = W_0/P_0$  is unchanged as well

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# Empirical Evidence on Monetary Non-Neutrality

# Naive Argument



- “Tighter monetary policy (higher  $i$ ) lowers unemployment!”
- What's wrong with such an argument?

# Monetary Policy is Endogenous

“Unfortunately for us as empirical scientists, the Federal Reserve does not randomize when setting interest rates.

Quite to the contrary, the Federal Reserve employs hundreds of PhD economists to pore over every bit of data about the economy so as to make monetary policy as endogenous as it possibly can be.”

— Nakamura and Steinsson (2018)

# Monetary Policy is Endogenous

- Fed changes interest rate for a reason
- When a recession happens, Fed lowers the interest rate
- We cannot conclude from this that a lower interest rate caused the recession
- If Fed didn't lower the rate, maybe the recession could have been worse
- Is it possible to figure out the ***causal*** effect of monetary policy?

# In Search of Exogenous Monetary Policy

- Suppose Fed ever changes interest rate for a reason unrelated to the economy
  - Not because the economy is in recession
  - Not because the economy is having unusually high inflation
- Looking at the response of the economy following such change gives us the answer
- We will cover three approaches
  1. Narrative approach (Romer-Romer, 1989)
  2. Quantitative version of narrative approach (Romer-Romer, 2004)
  3. High-frequency identification

# 1. Narrative Approach

- Romer and Romer (1989, 2023):
  - Read transcripts and records of FOMC meetings
    - 50-100 pages of detailed summaries of discussions for each meeting
  - Judge whether monetary policymakers changed interest rates for reasons unrelated to current or prospective real economic activity
  - These are their monetary policy “shocks”
    - Monetary policy changes that are not responses to economic activity

# Monetary Policy “Shocks” Dates

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## New dates

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October	1947	(–)
August	1955	(–)
September	1958	(–)
December	1968	(–)
January	1972	(+)
April	1974	(–)
August	1978	(–)
October	1979	(–)
May	1981	(–)
December	1988	(–)

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# December 1988

- 1987-1988:
  - Continuous actions toward **stabilizing** inflation
  - Not “shock”
- December 1988:
  - A desire to **reduce** inflation and a willingness to accept output consequences became widespread
  - “I think the job before us is to contain the inflation and to slow this economy down”
  - “if it is the aim of the Committee... to restore a downward trend by 1990, then it may be necessary to run the risk of some financial stress and economic weakness”
  - This counts as a shock because the shift is due to changes in policymakers’ views
  - Not because something happened in the economy in December 1988

# Impact on Unemployment

$$y_{t+h} = \beta_h S_t + \mathbf{X}'_t \boldsymbol{\gamma}_h + \epsilon_{t+h}$$

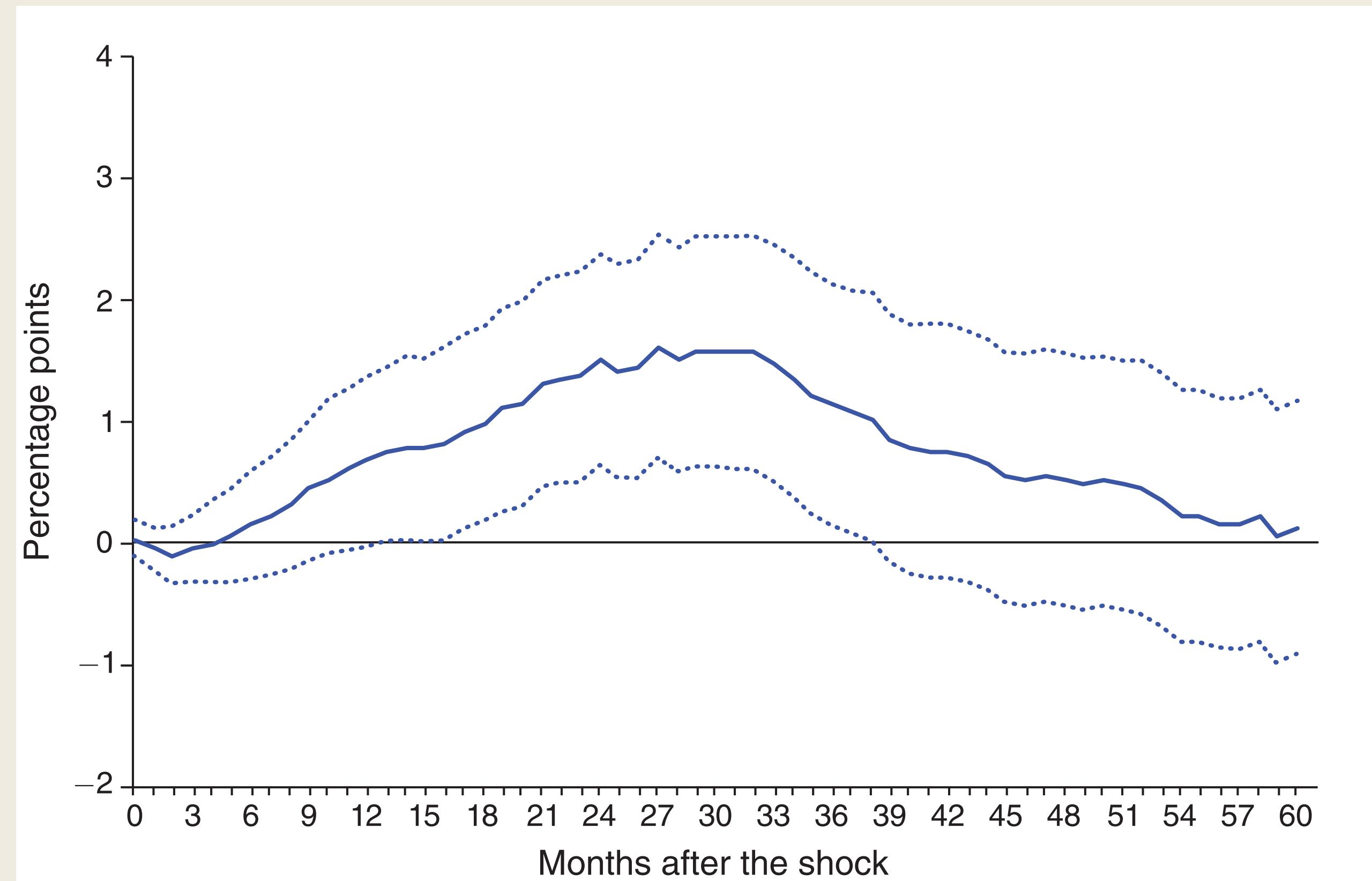
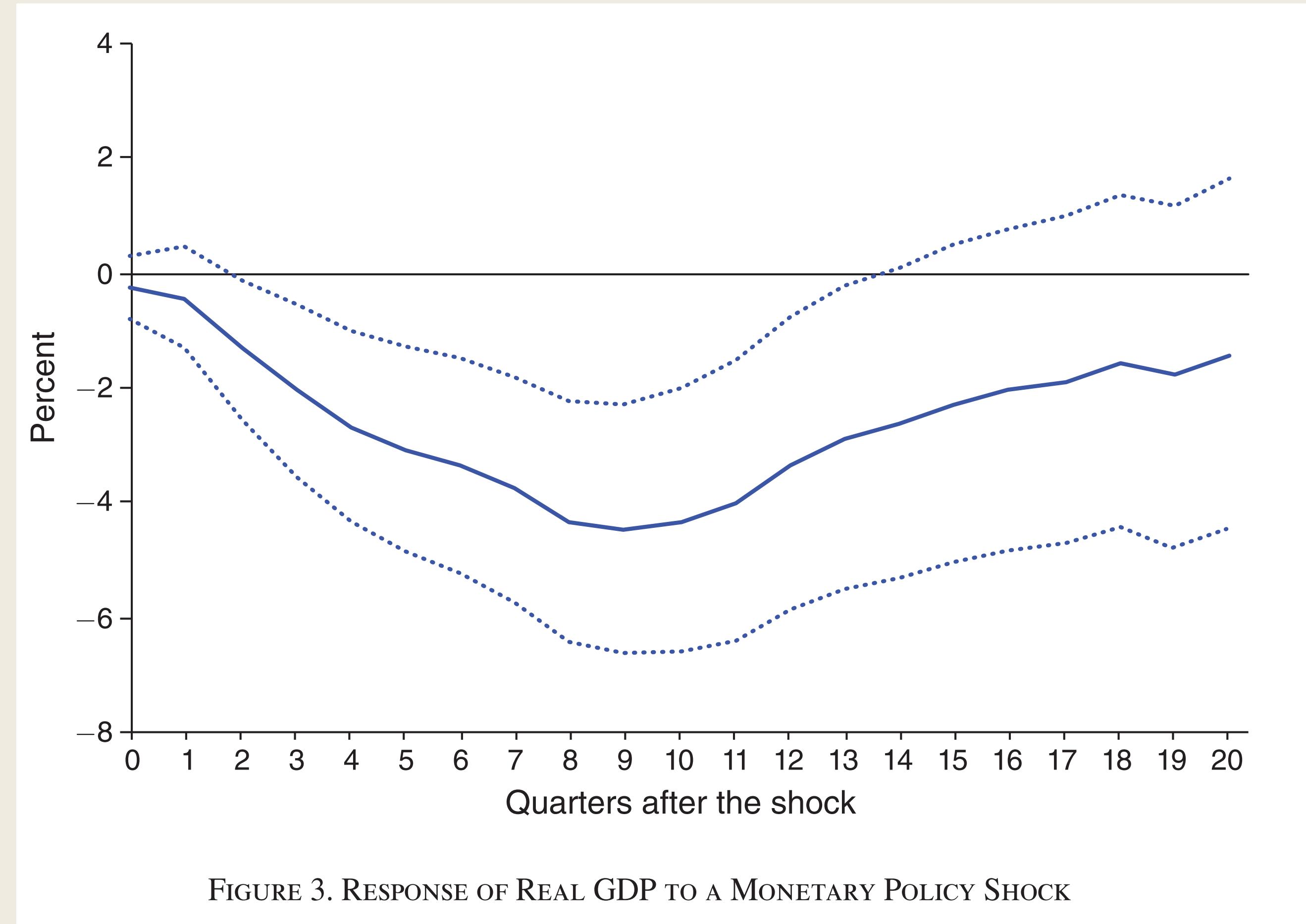
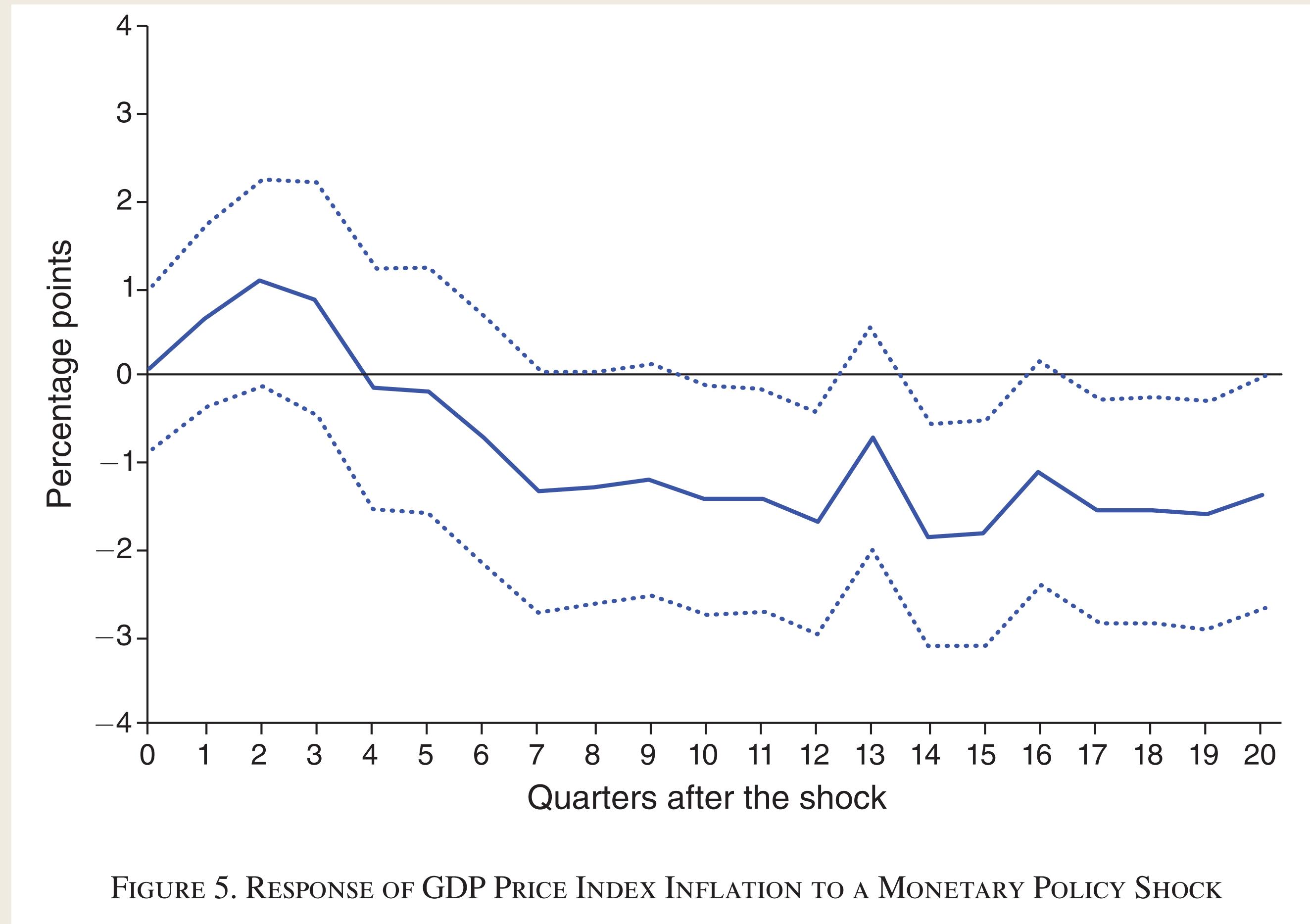


FIGURE 1. RESPONSE OF THE UNEMPLOYMENT RATE TO A MONETARY POLICY SHOCK

# Real GDP Response



# Response of Prices



## 2. Quantitative Version of Narrative Approach

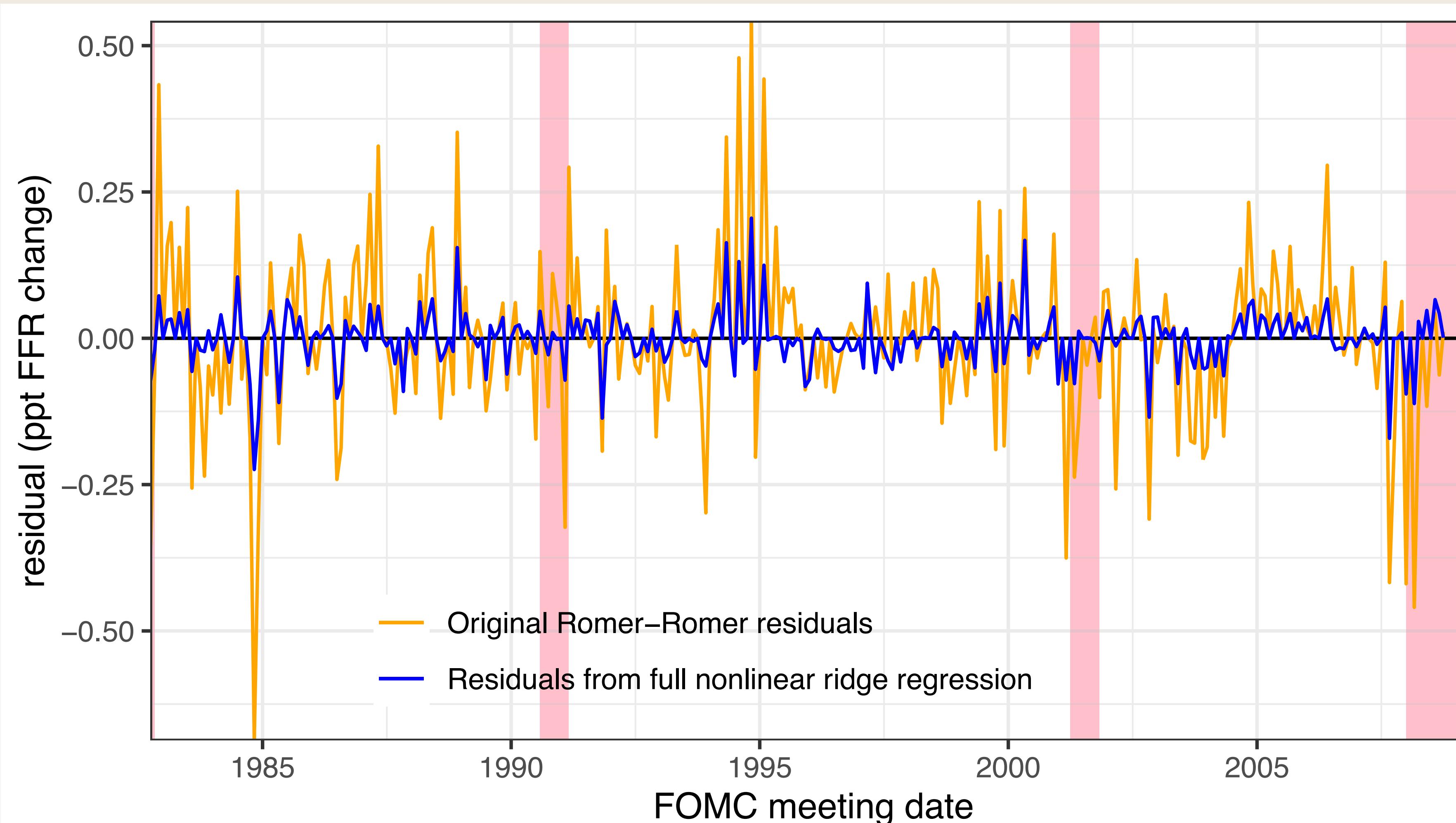
- Goal:  
Isolate policy changes for reasons unrelated to current/prospective economic activity
- Consider the following regression:

$$\Delta i_t = \mathbf{X}'_t \boldsymbol{\gamma} + \epsilon_t$$

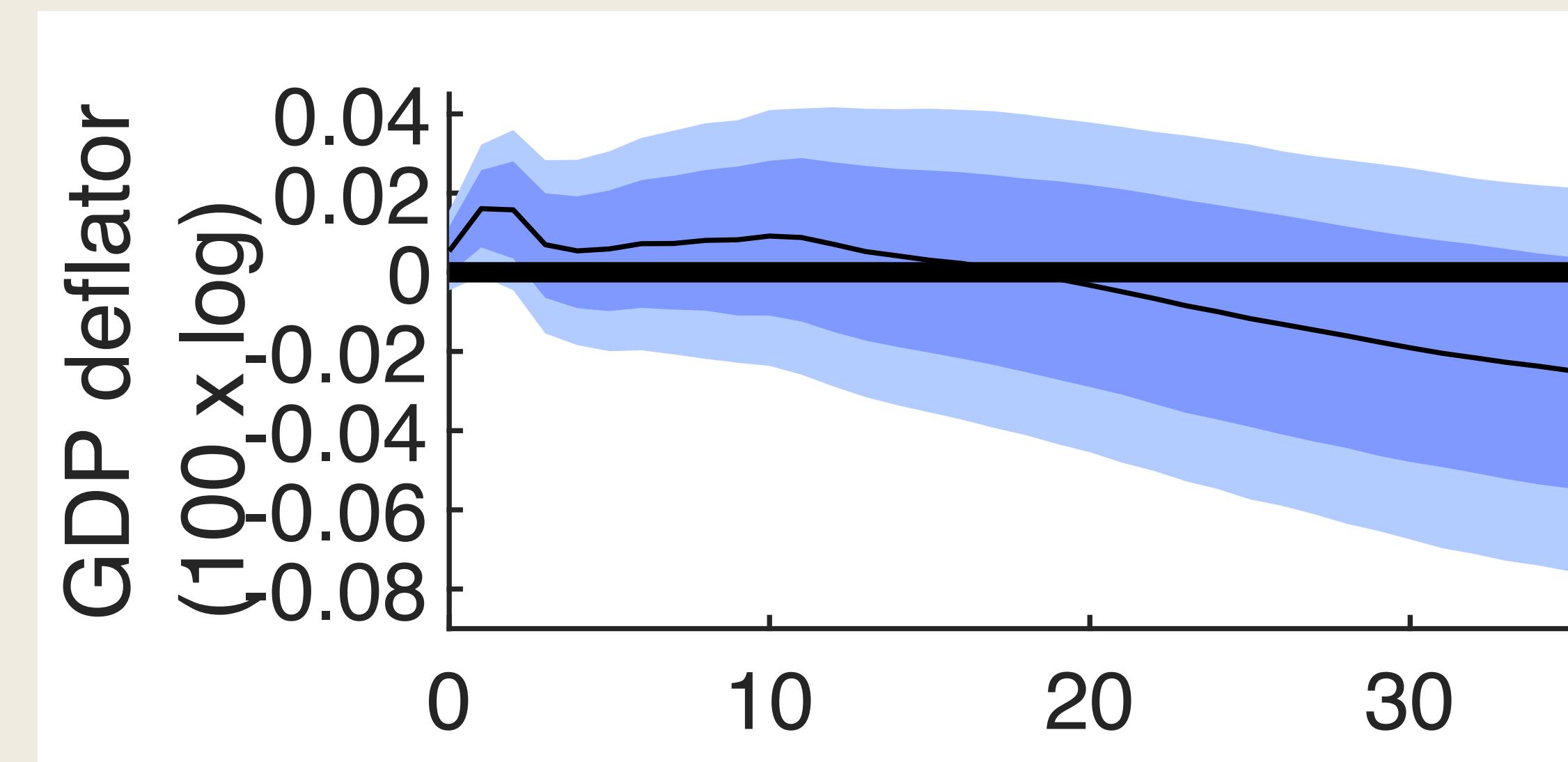
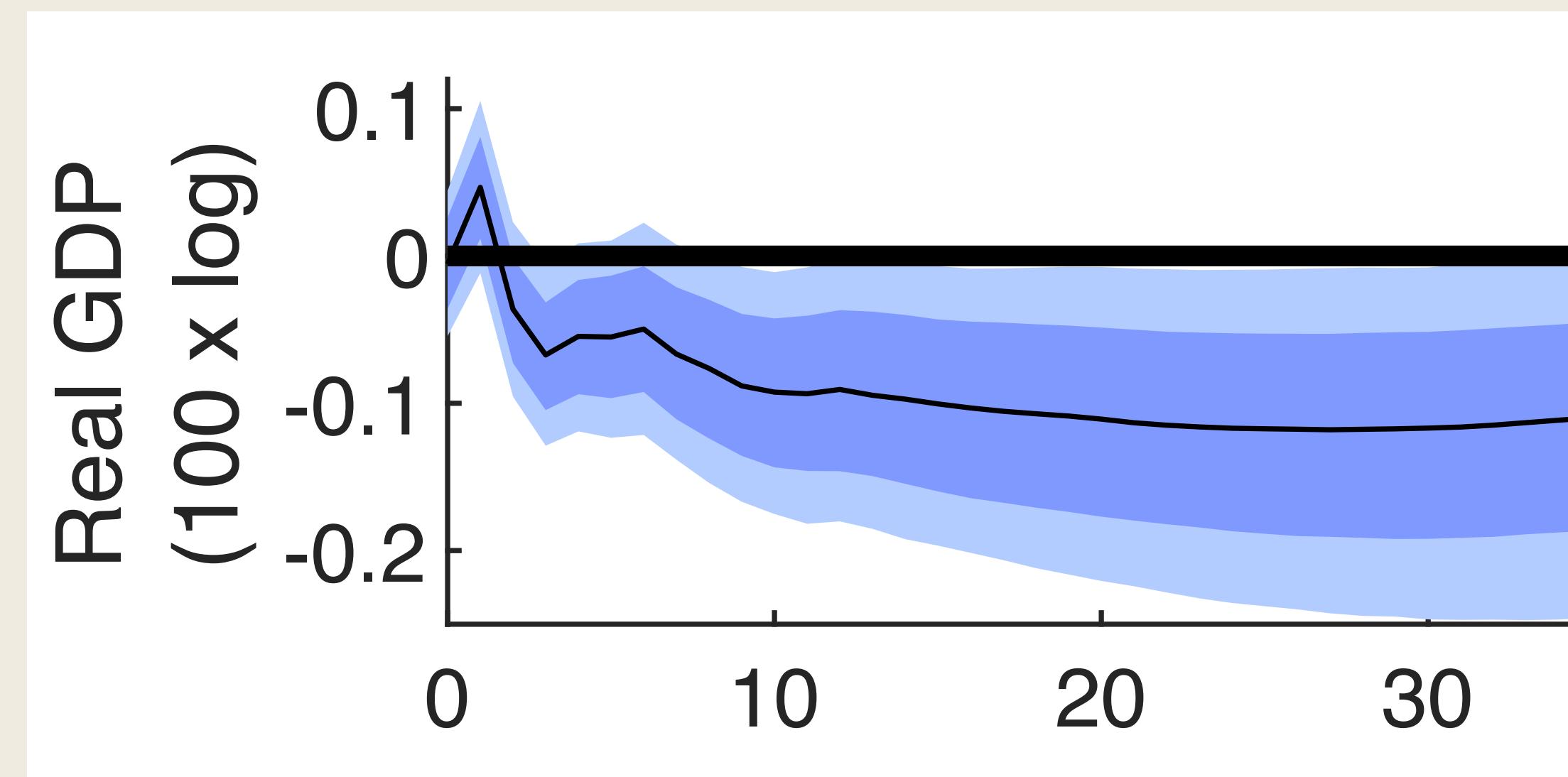
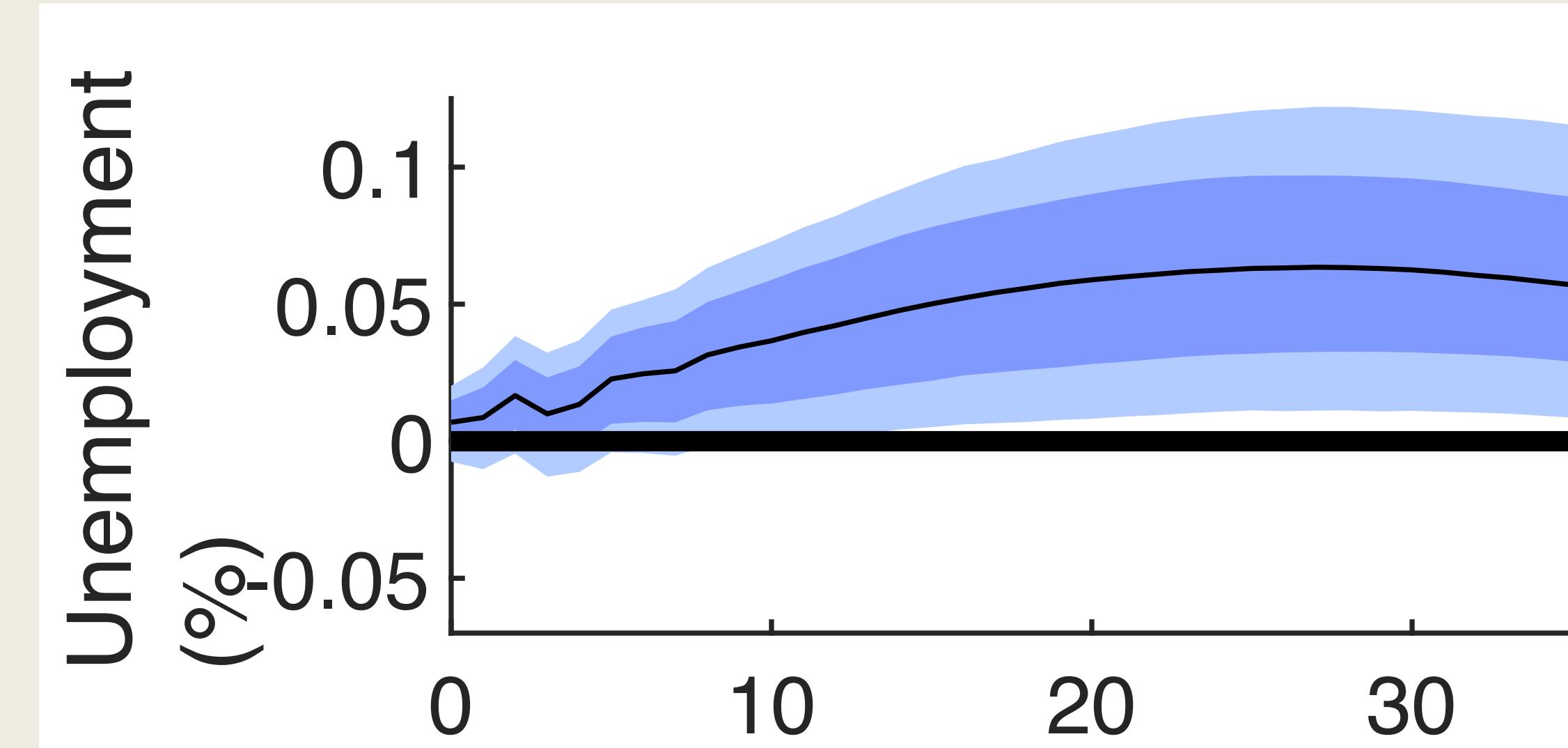
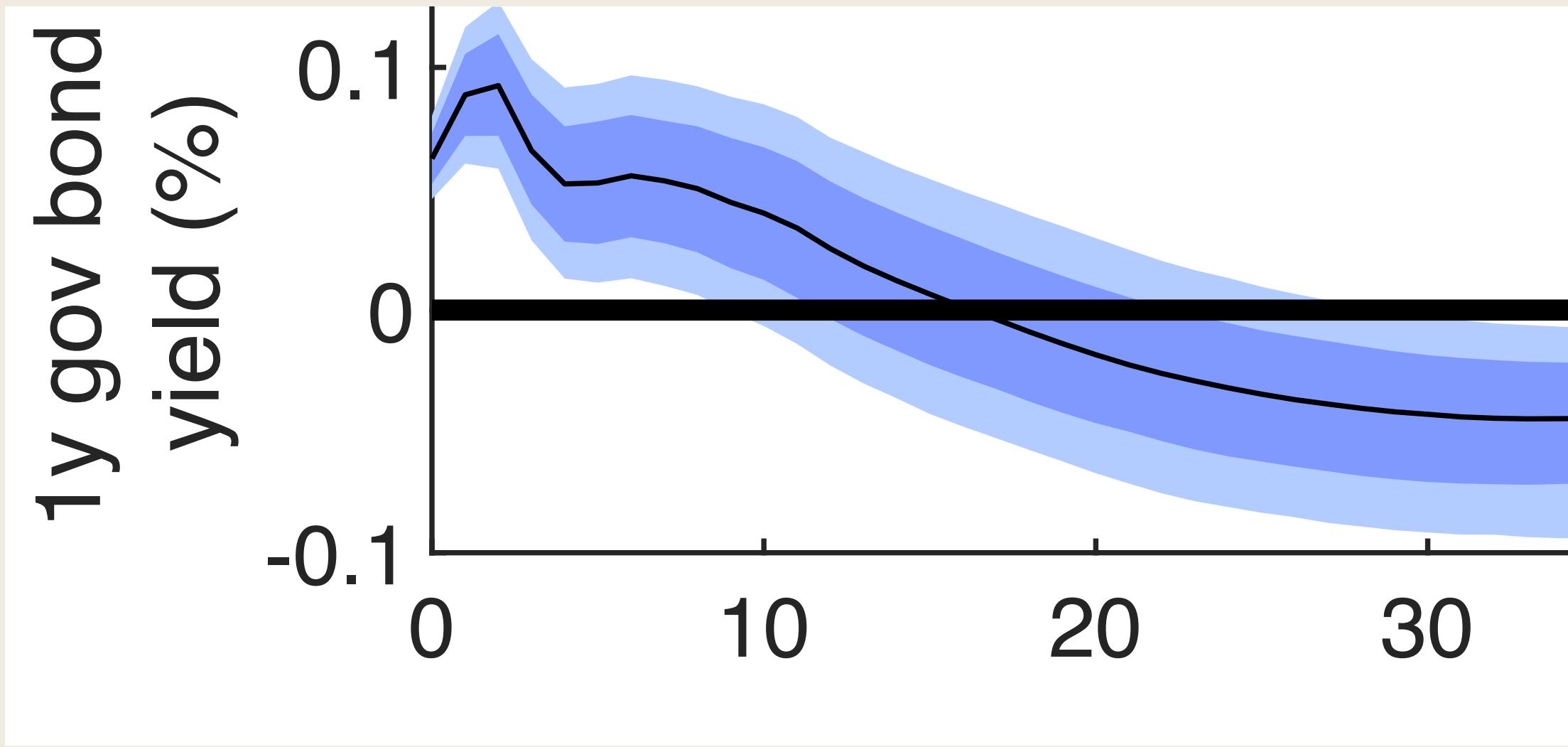
- $\Delta i_t$ : changes in Federal Funds rate (FFR)
  - $\mathbf{X}_t$ : FOMC members' forecasts or sentiments about economic activity (from FOMC meeting documents)
  - $\epsilon_t$ : changes in FFR for reasons unrelated to FOMC members' forecasts/sentiments
- We now treat the OLS residual  $\epsilon_t$  as monetary policy "shocks"
    - What are they?
      - Changes in FOMC members' tastes/goals/beliefs/moods/politics/objectives

# Monetary Policy Shock

Figure 4: ESTIMATED MONETARY POLICY SHOCKS

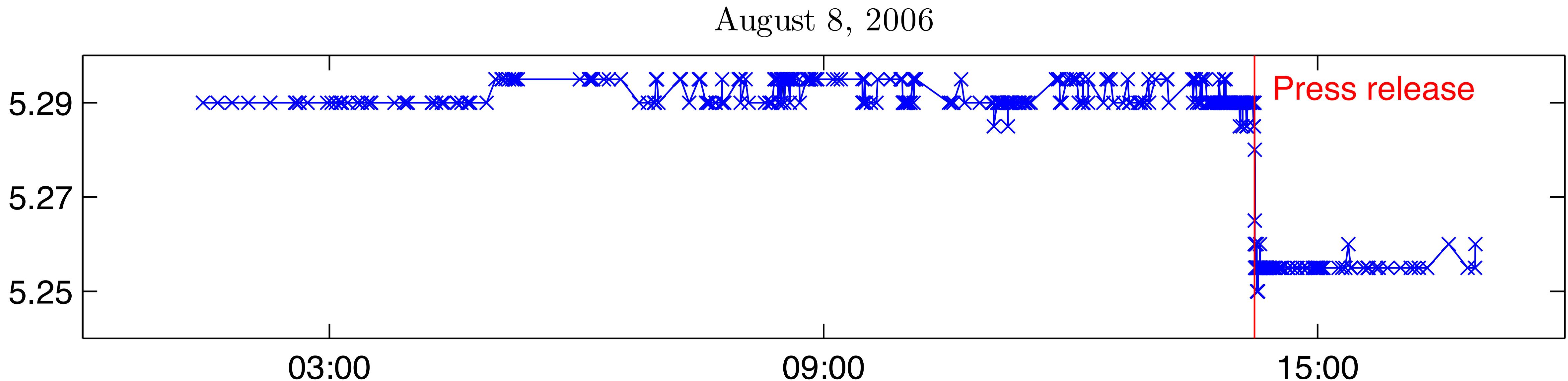


# Response of Macro Variables



### 3. High-Frequency Identification

Figure 2: Intraday Trading in Globex Federal Funds Futures



Source: Gorodnichenko and Weber (2015)

# 3. High-Frequency Identification

- Focus on 30-minutes window surrounding the FOMC announcements
- Extract changes in FFR during the 30-minutes time interval,  $\Delta i_t$ 
  - Changes in FFR unexpected by market participants
- Why is this monetary policy “shock”?
  - Nothing else other than FOMC announcements happen during the time interval
  - Not a response to changes in the economic activity
- Nakamura-Steinsson (2018) ask: Does  $\Delta i_t$  impact the real interest rate,  $r_t$ ?
  - In RBC, the answer is profound no

# Impact on Real Rate

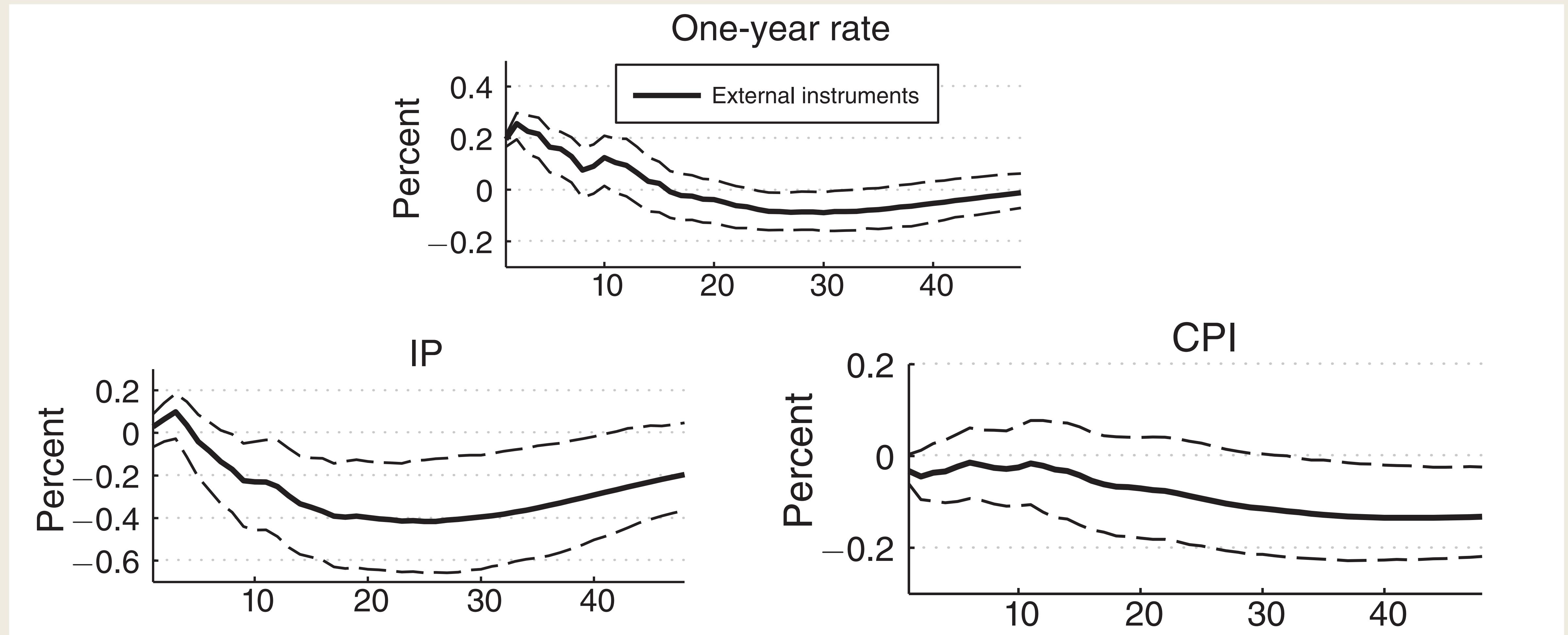
$$\Delta y_t = \beta \Delta i_t + \epsilon_t$$

TABLE I  
RESPONSE OF INTEREST RATES AND INFLATION TO THE POLICY NEWS SHOCK

	Nominal	Real	Inflation
3M Treasury yield	0.67 (0.14)		
6M Treasury yield	0.85 (0.11)		
1Y Treasury yield	1.00 (0.14)		
2Y Treasury yield	1.10 (0.33)	1.06 (0.24)	0.04 (0.18)
3Y Treasury yield	1.06 (0.36)	1.02 (0.25)	0.04 (0.17)
5Y Treasury yield	0.73 (0.20)	0.64 (0.15)	0.09 (0.11)
10Y Treasury yield	0.38 (0.17)	0.44 (0.13)	-0.06 (0.08)
2Y Treasury inst. forward rate	1.14 (0.46)	0.99 (0.29)	0.15 (0.23)
3Y Treasury inst. forward rate	0.82 (0.43)	0.88 (0.32)	-0.06 (0.15)
5Y Treasury inst. forward rate	0.26 (0.19)	0.47 (0.17)	-0.21 (0.08)
10Y Treasury inst. forward rate	-0.08 (0.18)	0.12 (0.12)	-0.20 (0.09)

# Impact of High-Frequency Shocks on Macro

- Gertler-Karadi (2015) use similar shock to investigate the impact on macro

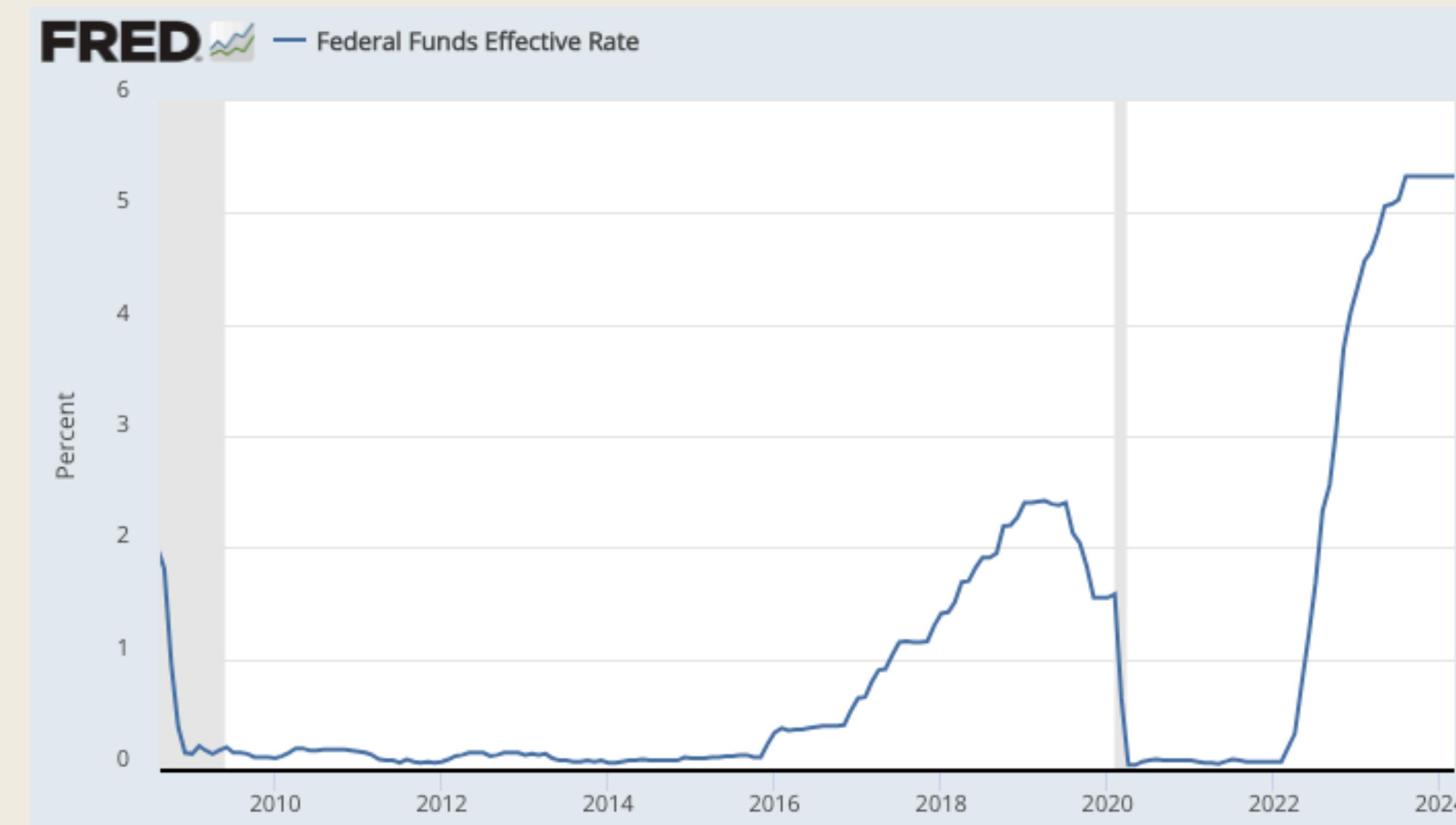


# Takeaway

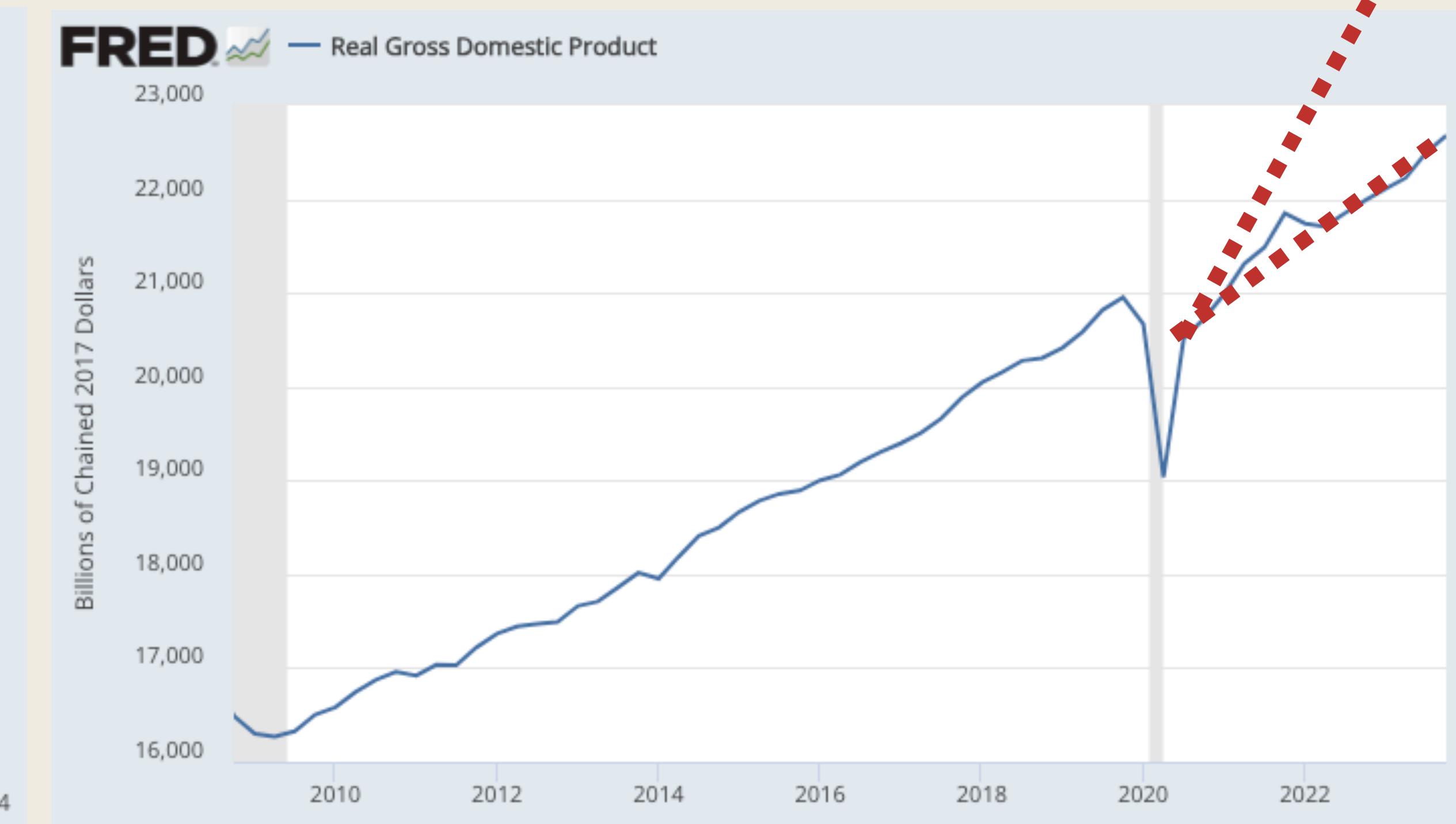
- Monetary policy is highly endogenous to economic activity
  - If it weren't, our society would be in deep trouble
- Various attempts to isolate monetary policy "shocks"
- Although none of them is a true "shock", we reach robust conclusions
- If monetary policy tightens:
  - unemployment rises
  - output falls
  - price level tends to fall
  - real interest rate rises
- Monetary policy is not neutral – a rejection of RBC model

# Looking at Recent Periods

Federal Funds Rate



Real GDP



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# Source of Monetary Non-Neutrality

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# Sticky Prices

- We have seen, empirically, that monetary policy is not neutral
- Why?
- Many believe the core underlying reason is price/wage stickiness
- Unlike RBC model, prices do not immediately adjust to keep the real rate constant

# Prices Do Not Adjust Everyday

**Maruchan - Seimen Japanese Instant Ramen Noodles Soy Sauce Taste 18.5oz (For 5 Bowls)**

by Maruchan

4.5 ★★★★★ 255 ratings

"fresh noodles -- the soy-sauce-flavor soup suitable for vegetables whose flavor of the sweet herb was effective against inside thick noodle of the smooth texture by a process while it had been nice."

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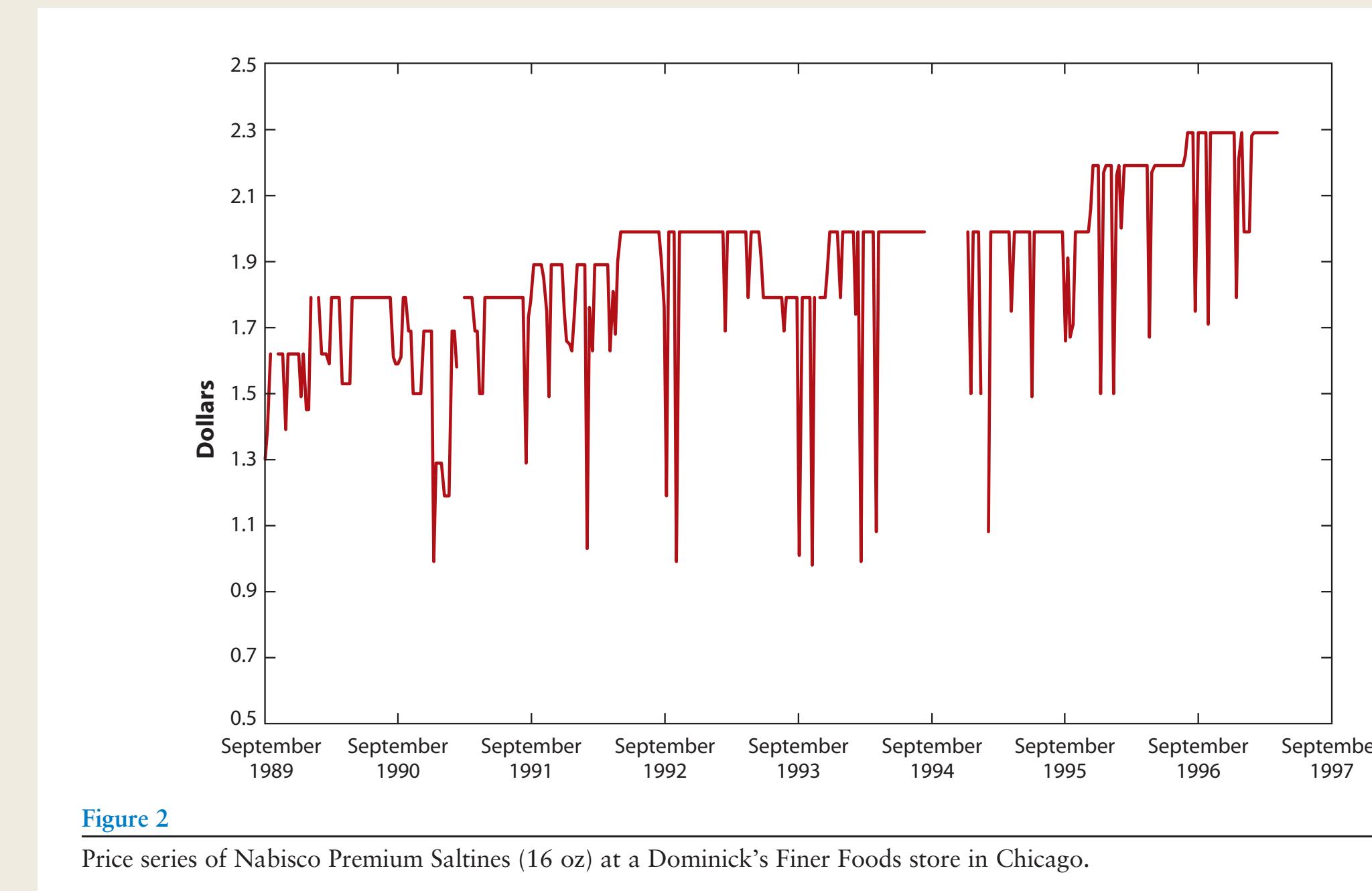
May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar

\$0 \$10 \$20 \$30

Select area to zoom in. Double-click to reset. 🚚 = shipping included

A line graph showing price history for Maruchan Seimen Japanese Instant Ramen Noodles from May to March. The Y-axis represents price in dollars, ranging from \$0 to \$30. The X-axis represents time, with months labeled from May to March. The graph shows several price fluctuations. A pink line represents the price over time, starting at approximately \$18, dropping to \$15 in June, rising to \$22 in August, then remaining flat until December. In December, there is a sharp increase to \$28, followed by another rise to \$30 in January. The price then drops to \$25 in February and remains stable through March. A blue line represents the sales rank, which shows a similar pattern of stability with occasional spikes, notably in December and January.

# Price Stickiness

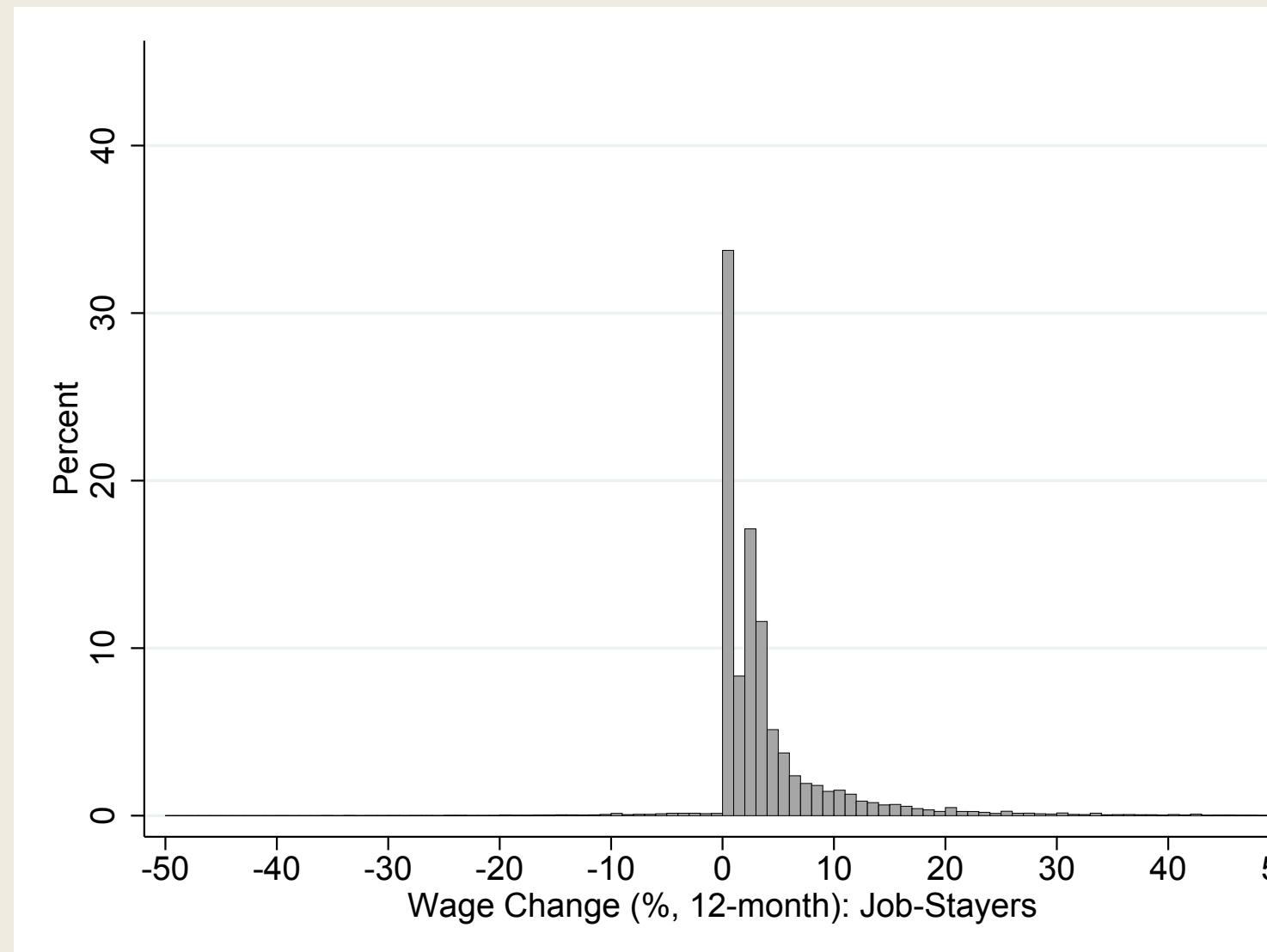


- Nakamura & Steinsson (2008) analyze microdata underlying CPI
- The median frequency of price changes is
  - 9-12% per month excluding sales
  - 19-20% per month including sales

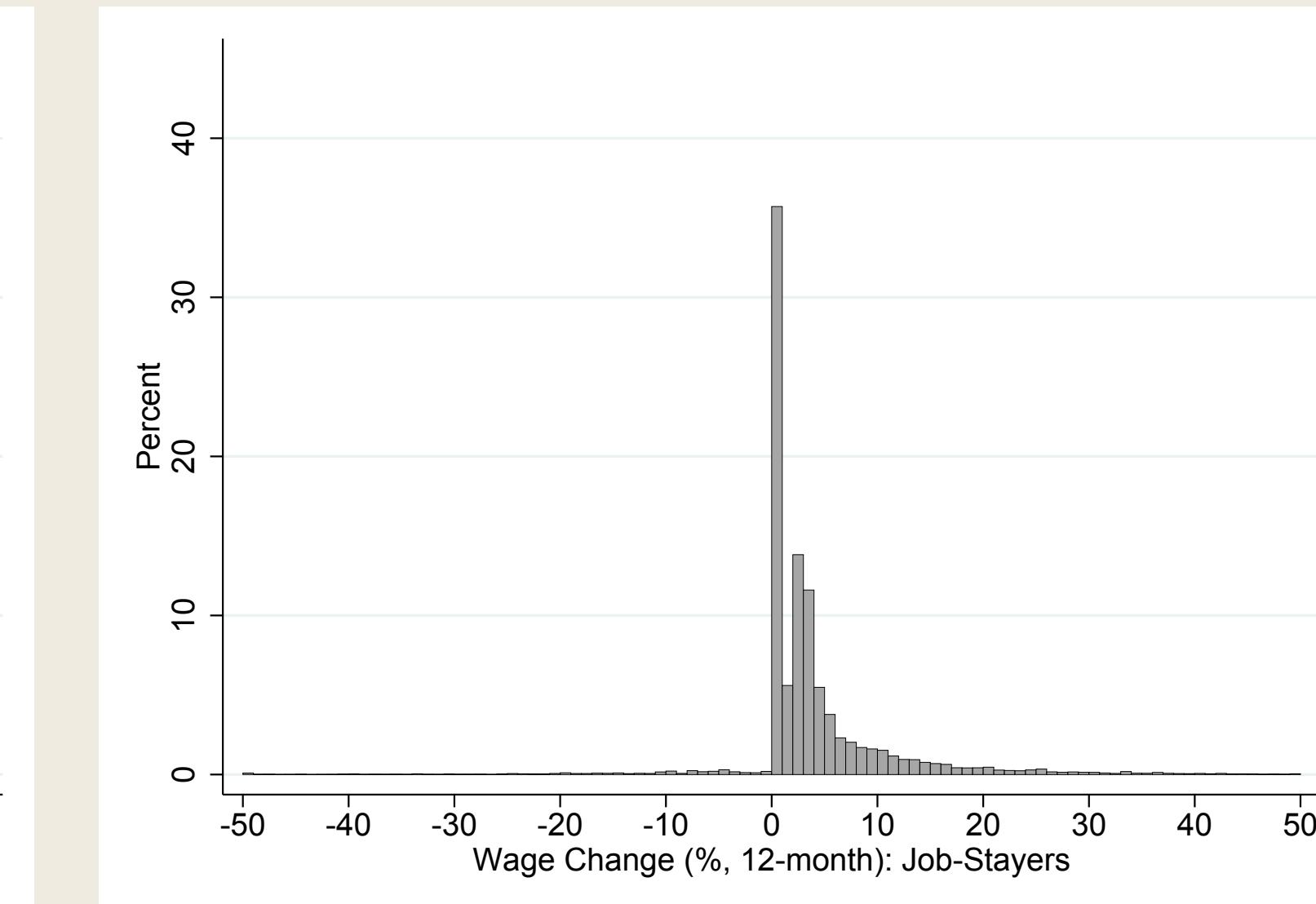
# Wage Stickiness

- Grigsby, Hurst, Yildirmaz (2021):  
Analyze payroll data of the largest U.S. payroll processing company
- Base nominal wages are sticky:
  - 35% of workers do not experience base wage changes year over year
  - Almost no worker receives nominal wage cut

Figure 2: 12-Month Nominal Base Wage Change Distribution, Job-Stayers



PANEL A: HOURLY WORKERS



PANEL B: SALARIED WORKERS

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# Monopolistic Retailer

# Goal

- Want a model that is jointly consistent with
  1. Monetary non-neutrality
  2. Sticky prices
- We will extend the RBC model by incorporating 2 and show that it implies 1

# Moving Away from Perfect Competition

- Just introducing price stickiness into the RBC model will not behave well
  1. If two firms charge different prices, no one will buy a more expensive product
  2. No firm can set prices. Not able to think about the price-setting of firms
- We therefore need to depart from a perfectly competitive product market

# Monopoly Power

- Consider continuum of identical retailers,  $j \in [0,1]$
- Assume each retailer  $i$  faces the following demand curve

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t$$

- $P_t(j)$ : the price of retailer  $j$ 's product
  - $P_t$ : average of all retailers' prices
  - $\eta$ : how much demand goes down if I over-price relative to the average (demand elasticity)
  - $Y_t$ : aggregate demand
- The perfectly competitive environment can be thought of as  $\eta \rightarrow \infty$

# Monopolist Retailer's Problem

- Retailers buy wholesale products at price  $p_t$  and sell them to customers
- Taking  $P_t$  and  $p_t$  as given, each retailer solves

$$\max_{p_t(j), y_t(j)} P_t(j)y_t(j) - p_t y_t(j) \quad \text{subject to} \quad y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} Y_t$$

- The first-order condition is

$$P_t(j) = p_t + P_t(j) \frac{1}{\eta}$$

- LHS: benefit of producing one more unit
- RHS: cost of producing one more unit
  - The marginal cost is  $p_t$
  - Producing more lowers the price by  $1/\eta$  percent

# Optimal Pricing

## ■ Rearranging

$$P_t(j) = \underbrace{\frac{\eta}{\eta - 1}}_{\text{Markup}} \times \underbrace{p_t}_{\text{Marginal Cost}}$$

- If  $\eta = \infty$ , prices are equal to the marginal cost (as in competitive models)
- Lower  $\eta$  implies firms charge higher markup and earn higher profits

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# RBC + Monopolist Retailers

# Putting into General

- We embed the above mechanism into the RBC model
- The economy now consists of three types of agents
  1. Households (nearly identical to RBC)
  2. Wholesale firms
  3. Retailers: buy wholesale goods and sell them to households and firms
- We still have flexible price

# Households

- Households purchase consumption goods from all retailers
- The price they pay per unit consumption is  $P_t$  (the average price retailers charge)
- Households solve

$$\max_{C_0, C_1, A_0, l_0} u(C_0) - v(l_0) + \beta u(C_1)$$

subject to

$$P_0 C_0 + A_0 = W_0 l_0 + D_0$$

$$P_1 C_1 = (1 + i) A_0 + W_1 l_1 + D_1$$

# Firms

- Firms sell their own product at the wholesale price  $p_t$

$$\max_{L_0, L_1} D_0 + \frac{1}{1+i} D_1$$

subject to

$$D_0 = p_0 F_0(K_0, L_0) - W_0 L_0$$

$$D_1 = p_1 F_1(K_1, L_1) - W_1 L_1$$

$$K_1 = (1 - \delta) K_0$$

# Retailers

- Continuum of retailers  $j \in [0,1]$
- They buy wholesale goods from firms and sell it to households

$$\max_{p_t(j), y_t(j)} P_t(j)y_t(j) - p_t y_t(j) \quad \text{subject to} \quad y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\eta} Y_t$$

- The market clearings are

$$C_0 = F_0(K_0, L_0)$$

$$C_1 = F_1(K_1, L_1)$$

$$l_0 = L_0$$

$$l_1 = L_1$$

# Optimal Pricing

- As before, the price of retailer  $j$  is

$$P_t(j) = \underbrace{\frac{\eta}{\eta - 1}}_{\text{Markup}} \times \underbrace{p_t}_{\text{Marginal Cost}}$$

- Since all retailers are symmetric and prices are flexible,

$$P_t = \frac{\eta}{\eta - 1} p_t$$

# Optimality Conditions

- Household labor supply is

$$u'(C_0) \frac{W_0}{P_0} = v'(L_0)$$

- Euler equation is

$$u'(C_0) = \beta(1 + i) \frac{P_0}{P_1} u'(C_1)$$

- Firm's labor demand

$$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = \frac{W_t}{p_t}$$

- Retailer's price setting

$$P_t = \frac{\eta}{\eta - 1} p_t$$

# Optimality Conditions

- Using  $1 + r = (1 + i)\frac{P_0}{P_1}$ , we can rewrite the previous conditions as follows

$$u'(C_0) \frac{\partial F_t(K_0, L_0)}{\partial L_0} \left(1 - \frac{1}{\eta}\right) = v'(L_0)$$

$$u'(C_t) = \beta(1 + r)u'(C_{t+1})$$

- The only modification from RBC model is the red parts (inverse of markup)
  - Monopoly power implies that extra production is costly. It lowers the price by  $-1/\eta$
  - This lowers both MPL

# Equilibrium Conditions

- $\{C_0, C_1, r, L_0\}$  solve

$$u'(C_0) \frac{\partial F_t(K_0, L_0)}{\partial L_0} \left( 1 - \frac{1}{\eta} \right) = v'(L_0)$$

$$u'(C_0) = \beta(1 + r)u'(C_1)$$

$$C_0 = F_0(K_0, L_0)$$

$$C_1 = F_1(K_1, L_1)$$

# Same as RBC

- At this point, nothing is really different from RBC
- It's just  $MPL$  is multiplied by a constant (inverse markup)
- As before,  $i$  never shows up in the eqm conditions, so monetary neutrality holds
- If  $i$  increases,  $P_0$  falls and  $1 + r = (1 + i) \frac{P_0}{\bar{P}_1}$  remains unchanged

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# RBC + Monopolist Retailers + Rigid Prices

# Rigid Prices

- Suppose that retailers' prices at  $t = 0$  are completely rigid

$$P_0 = \bar{P}_0$$

- Prices at  $t = 1$ :

$$P_1 = \frac{\eta}{\eta - 1} p_1 = \bar{P}_1$$

- This implies that changes in  $i$  do affect  $r$ :

$$1 + r = (1 + i) \frac{\bar{P}_0}{\bar{P}_1}$$

# Optimality Conditions

- Household labor supply is

$$u'(C_0) \frac{W_0}{P_0} = v'(L_0)$$

- Euler equation is

$$u'(C_0) = \beta(1 + i) \frac{P_0}{P_1} u'(C_1)$$

- Firm's labor demand

$$\frac{\partial F_t(K_t, L_t)}{\partial L_t} = \frac{W_t}{p_t}$$

- Retailer's price setting

$$P_t = \frac{\eta}{\eta - 1} p_t \quad P_0 = \bar{P}_0, \quad P_1 = \frac{\eta}{\eta - 1} p_1 = \bar{P}_1$$

# Consumption

- After substituting  $C_1 = A_1 K_1^\alpha L_1^{1-\alpha}$  and  $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$   
$$C_0^{-\sigma} = \beta(1+i) \frac{\bar{P}_0}{\bar{P}_1} (A_1 K_1^\alpha L_1^{1-\alpha})^{-\sigma}$$
- This equation alone pins down  $C_0$
- If  $i$  goes up,  $C_0$  goes down
- Write this relationship as  $C_0(i)$

# Rest of the Equilibrium Conditions

- The goods market clearing condition is

$$C_0(i) = F_0(K_0, L_0)$$

- This equation alone pins down  $L_0$
  - Since  $C_0(i)$  is decreasing,  $L_0$  also decreasing in  $i$
  - The economy has less aggregate demand, so we need less labor
- Combining labor supply and labor demand,

$$C_0^{-\sigma}(1 - \alpha)A_0K_0^\alpha L_0^{-\alpha} = \frac{\bar{P}_0}{p_0}\bar{v}L_0^\nu$$

- Given  $C_0$  and  $L_0$  pinned down, the above eq. residually pins down  $p_0$
- Higher  $i$  lowers  $C_0$  and  $L_0$ , and the wholesale price  $p_0$  goes down
- Fluctuations in  $\bar{P}_0/p_0$  resembles fluctuations in  $\bar{v}$  (labor disutility shock)

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# Summary

- When prices are rigid, monetary policy is no longer neutral
- Higher interest rate  $i$  lowers  $C_0, L_0, Y_0$ , consistent with the evidence

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# RBC + Monopolist Retailers + Sticky Prices

## – New Keynesian Model

# Sticky Prices

- Suppose that some firms cannot adjust prices in response to monetary policy
- A fraction  $\lambda \in [0,1]$  of retailers' prices at  $t = 0$  are
  - $\bar{p}_0$ : Wholesale price at  $t = 0$  in the absence of monetary policy changes
- The remaining fraction  $1 - \lambda$  of retailers set prices freely
- Prices at  $t = 1$  are fully flexible

$$P_0 = \bar{P}_0 = \frac{\eta}{\eta - 1} \bar{p}_0$$

# Sticky Prices

- The firms that adjust prices solve

$$\max_{p_t(j), y_t(j)} P_t(j)y_t(j) - p_t y_t(j) \quad \text{subject to} \quad y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\eta} Y_t$$

resulting in

$$P_t(j) = \frac{\eta}{\eta - 1} p_t$$

- The average price in the economy is

$$\begin{aligned} P_0 &= (1 - \lambda)P_0(j) + \lambda \bar{P}_0 \\ &= (1 - \lambda)\frac{\eta - 1}{\eta} p_0 + \lambda \bar{P}_0 \end{aligned}$$

- Nests both flexible price ( $\lambda = 0$ ) and rigid price ( $\lambda = 1$ )

# Equilibrium Conditions

- Household labor supply is

$$C_0^{-\sigma} \frac{W_0}{P_0} = \bar{v} L_0^\nu$$

- Euler equation is

$$C_0^{-\sigma} = \beta(1 + i) \frac{P_0}{P_1} C_1^{-\sigma}$$

- Firm's labor demand

$$(1 - \alpha) A_t K_t^\alpha L_t^{-\alpha} = \frac{W_t}{p_t}$$

- Retailer's price setting

$$P_0 = (1 - \lambda) \frac{\eta - 1}{\eta} p_0 + \lambda \bar{P}_0, \quad P_1 = \frac{\eta}{\eta - 1} p_1 = \bar{P}_1$$

- Goods market clearing

$$C_t = A_t K_t^\alpha L_t^{1-\alpha}$$

# Prices

- Combining labor supply, demand, and market clearing

$$\frac{p_0}{P_0} = \frac{1}{(A_0 K_0^\alpha)^{1-\sigma}} \frac{\bar{v}}{1-\alpha} L_0^{\nu+\alpha+(1-\alpha)\sigma}$$

- Solving for  $P_0$  and substituting into the retailers' pricing equation

$$P_0 = (1 - \lambda) \frac{\eta - 1}{\eta} \frac{1}{(A_0 K_0^\alpha)^{1-\sigma}} \frac{\bar{v}}{1-\alpha} L_0^{\nu+\alpha+(1-\alpha)\sigma} P_0 + \lambda \bar{P}_0$$

- Solving for  $P_0$ ,

$$P_0 = \frac{1}{1 - (1 - \lambda) \frac{\eta - 1}{\eta} \frac{1}{(A_0 K_0^\alpha)^{1-\sigma}} \frac{\bar{v}}{1-\alpha} L_0^{\nu+\alpha+(1-\alpha)\sigma}} \lambda \bar{P}_0 \quad (1)$$

# Phillips Curve

$$P_0 = \frac{1}{1 - (1 - \lambda) \frac{\eta - 1}{\eta} \frac{1}{(A_0 K_0^\alpha)^{1-\sigma}} \frac{\bar{v}}{1 - \alpha} L_0^{\nu + \alpha + (1 - \alpha)\sigma}} \lambda \bar{P}_0$$

- Assume the denominator is always positive (always true if shocks are not too big)
- Prices are higher if  $L_0$  is higher:  
households are working more
  - ⇒ wages and the wholesale price goes up
  - ⇒ retailer's marginal cost goes up
- Such a relationship is called as (New Keynesian) Phillips Curve

# Aggregate Demand

- The consumption Euler equation is

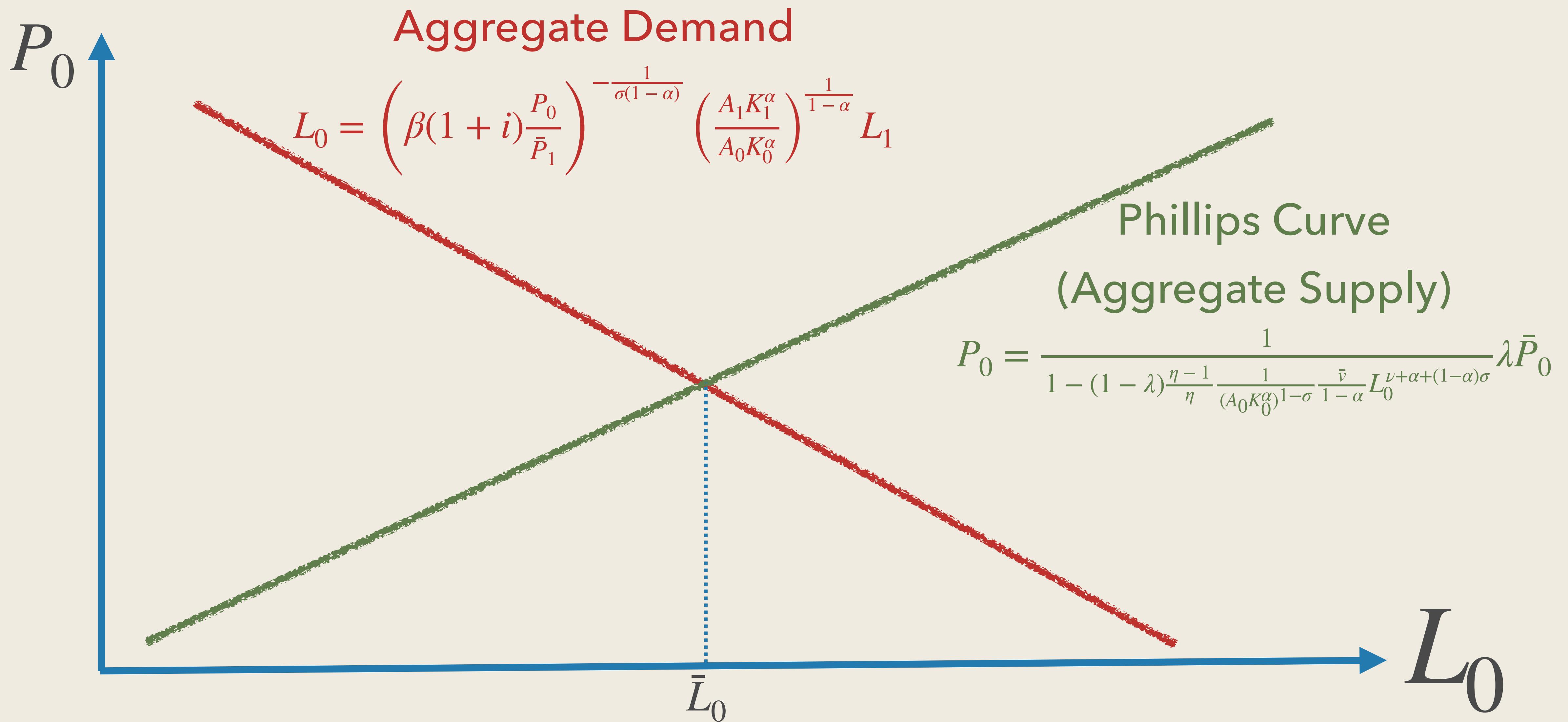
$$C_0^{-\sigma} = \beta(1 + i) \frac{P_0}{\bar{P}_1} (A_1 K_1^\alpha L_1^{1-\alpha})^{-\sigma}$$

- Given  $P_0$  and  $i$ , the above equation determines  $C_0$
- $C_0$  is decreasing in both  $P_0$  and  $i$
- Solving for  $C_0$  and plug into the goods market clearing ( $C_0 = A_0 K_0^\alpha L_0^{1-\alpha}$ ):

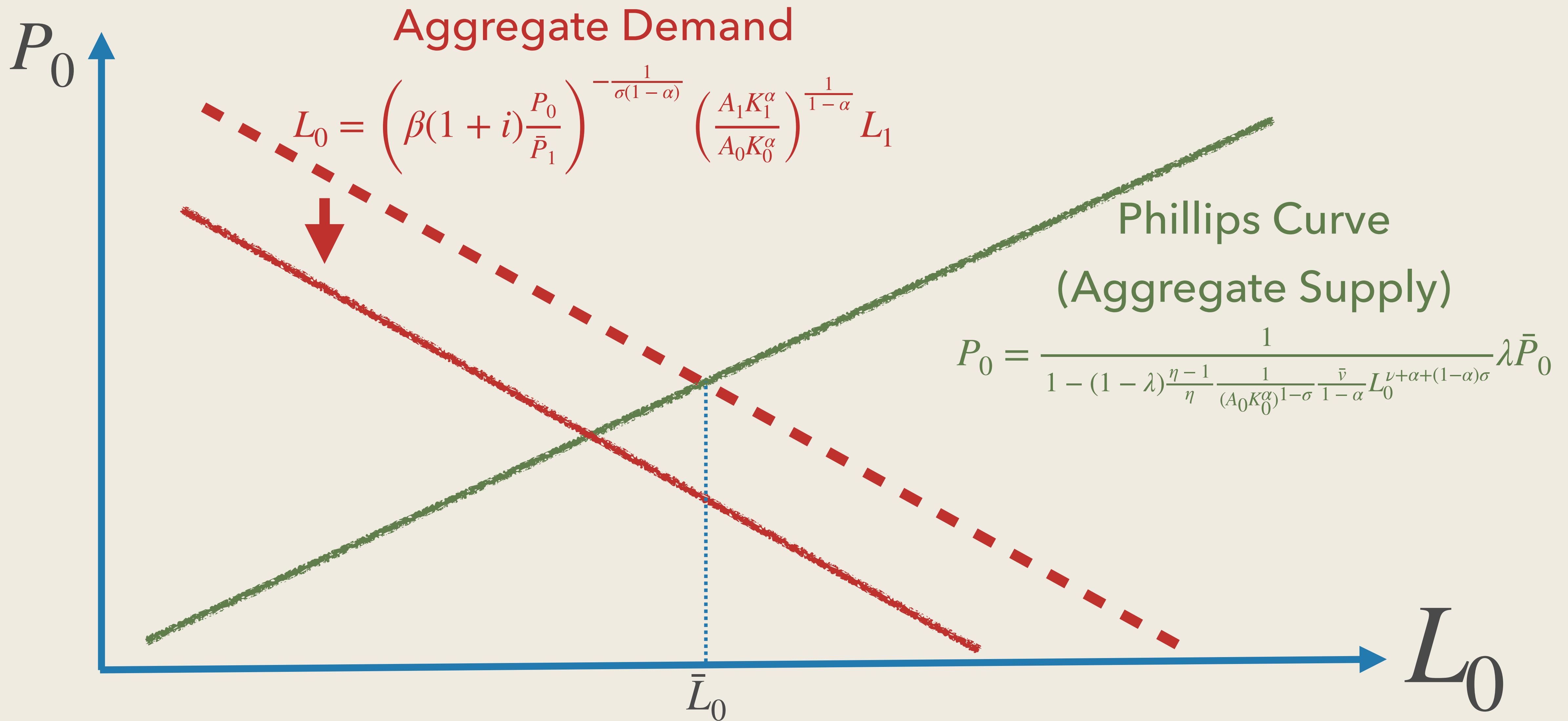
$$L_0 = \left( \beta(1 + i) \frac{P_0}{\bar{P}_1} \right)^{-\frac{1}{\sigma(1 - \alpha)}} \left( \frac{A_1 K_1^\alpha}{A_0 K_0^\alpha} \right)^{\frac{1}{1 - \alpha}} L_1$$

- $L_0$  is decreasing in both  $P_0$  and  $i$

# AS-AD Diagram



# Monetary Policy Tightening



# Monetary Policy Transmission

- When monetary policy is tightened, both  $L_0$  and  $P_0$  go down
- Higher interest rates discourage people from consuming today
- Aggregate demand drops
- Labor demand drops
- Wages and therefore wholesale price goes down
- This lowers the marginal cost of retailers and prices tend to go down
- How does this mechanism depend on price stickiness  $\lambda$ ?

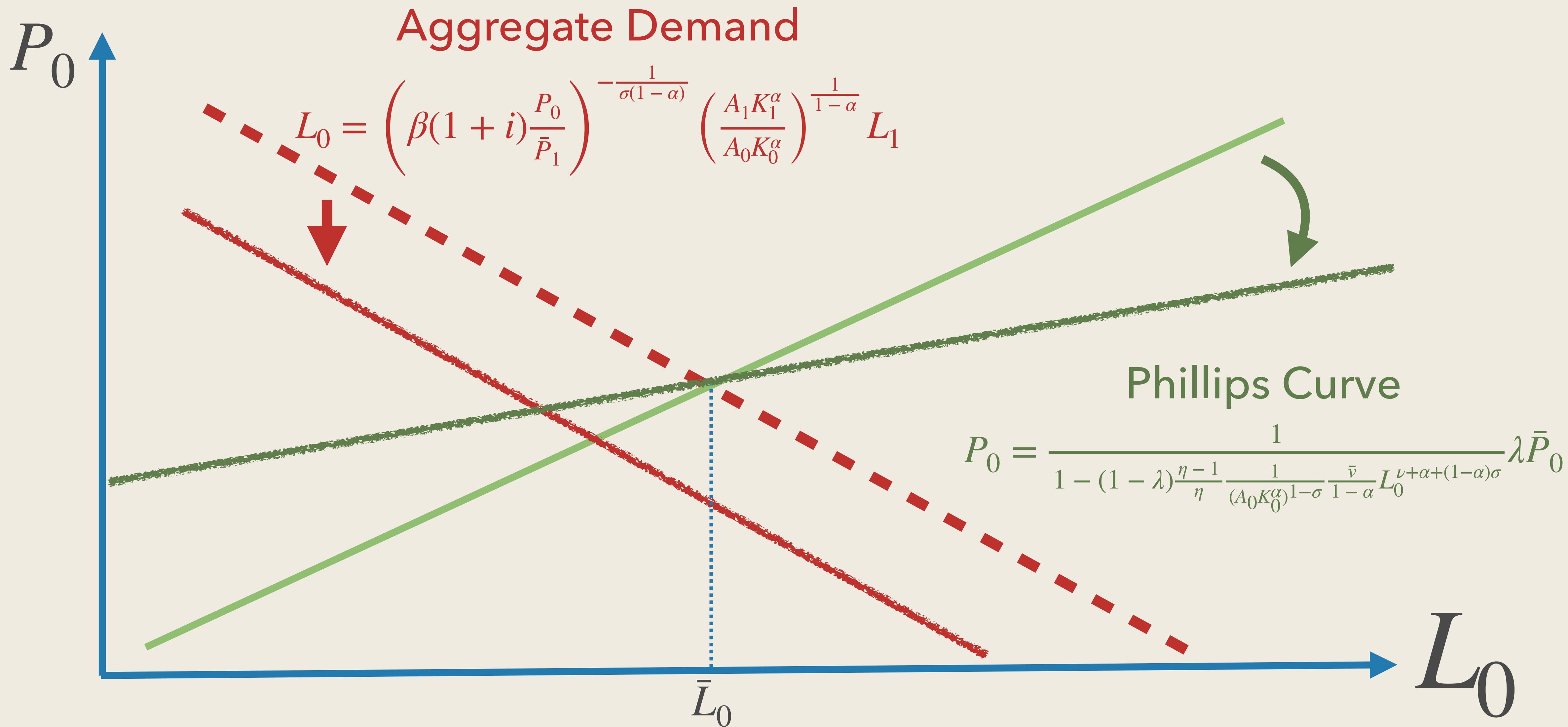
# The Slope of Phillips Curve

- The slope of Phillips curve in the neighborhood of  $L_0 = \bar{L}_0$  is

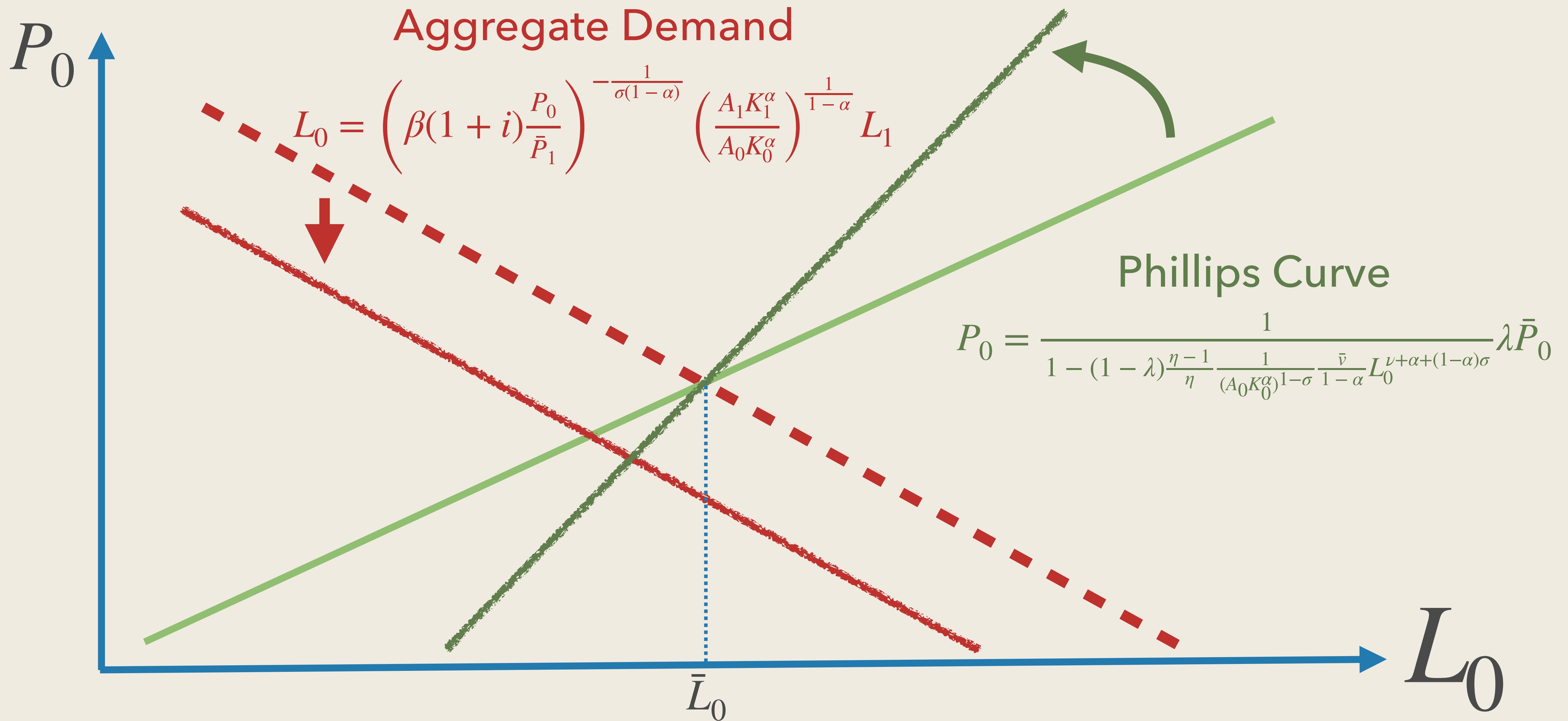
$$\frac{dP_0}{dL_0} \Big|_{L_0=\bar{L}_0} = \frac{(1-\lambda)}{\lambda} \frac{(\nu + \alpha + (1-\alpha)\sigma)}{\bar{L}_0} \bar{P}_0$$

- The slope of Phillips curve is flatter when price stickiness  $\lambda$  is higher
- Conversely, the Phillips curve is steeper when  $\lambda$  is lower

# Higher Price Stickiness $\lambda$



# Lower Price Stickiness $\lambda$



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# Takeaway

- Introducing price stickiness into the RBC model leads to monetary non-neutrality
- This is called “New Keynesian Model”
- In response to monetary policy tightening,
  1. Consumption, labor, and output all fall
  2. Prices fall
- When prices are stickier, we have more of 1 and less of 2
- When prices are more flexible, we have more of 2 and less of 1

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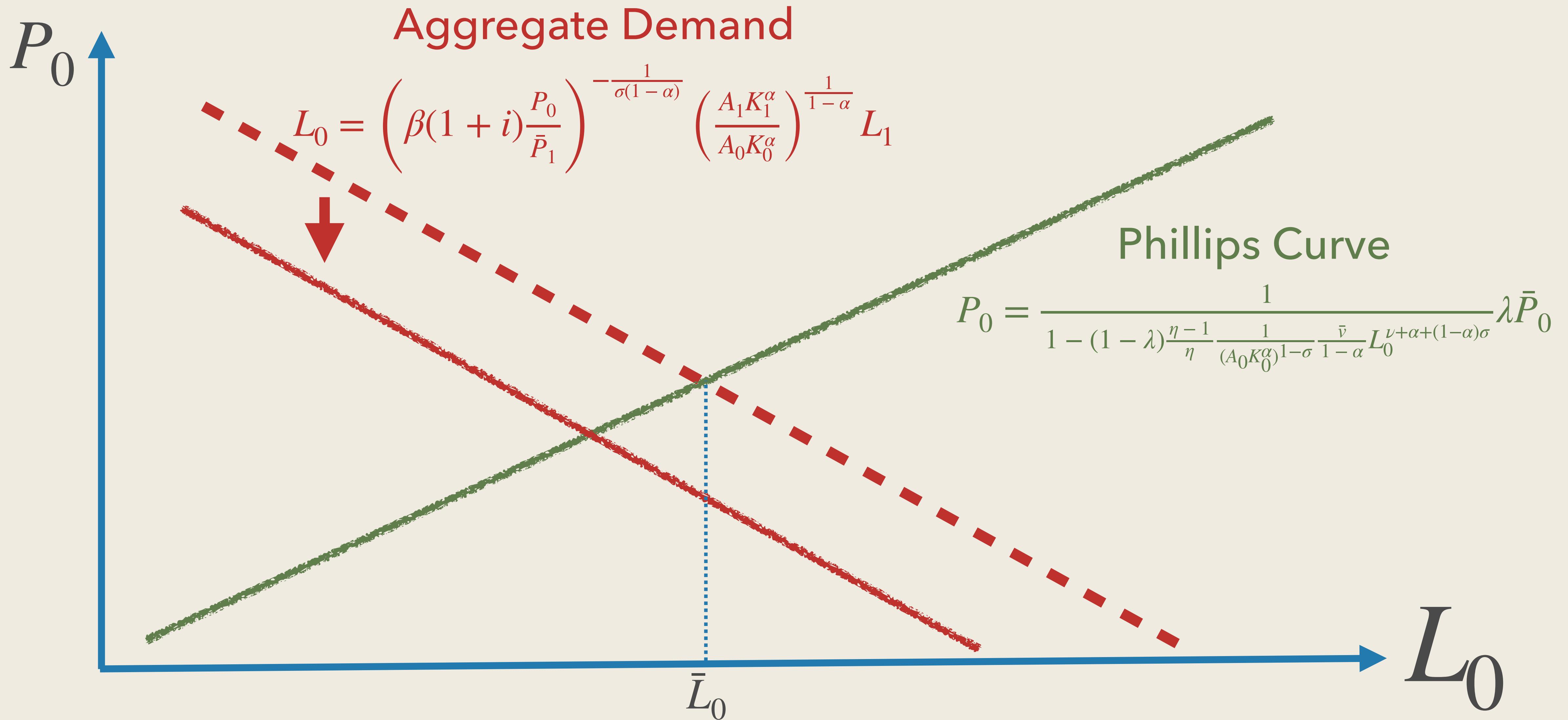
# Sources of Business Cycle Revisited

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# Business Cycles Revisited

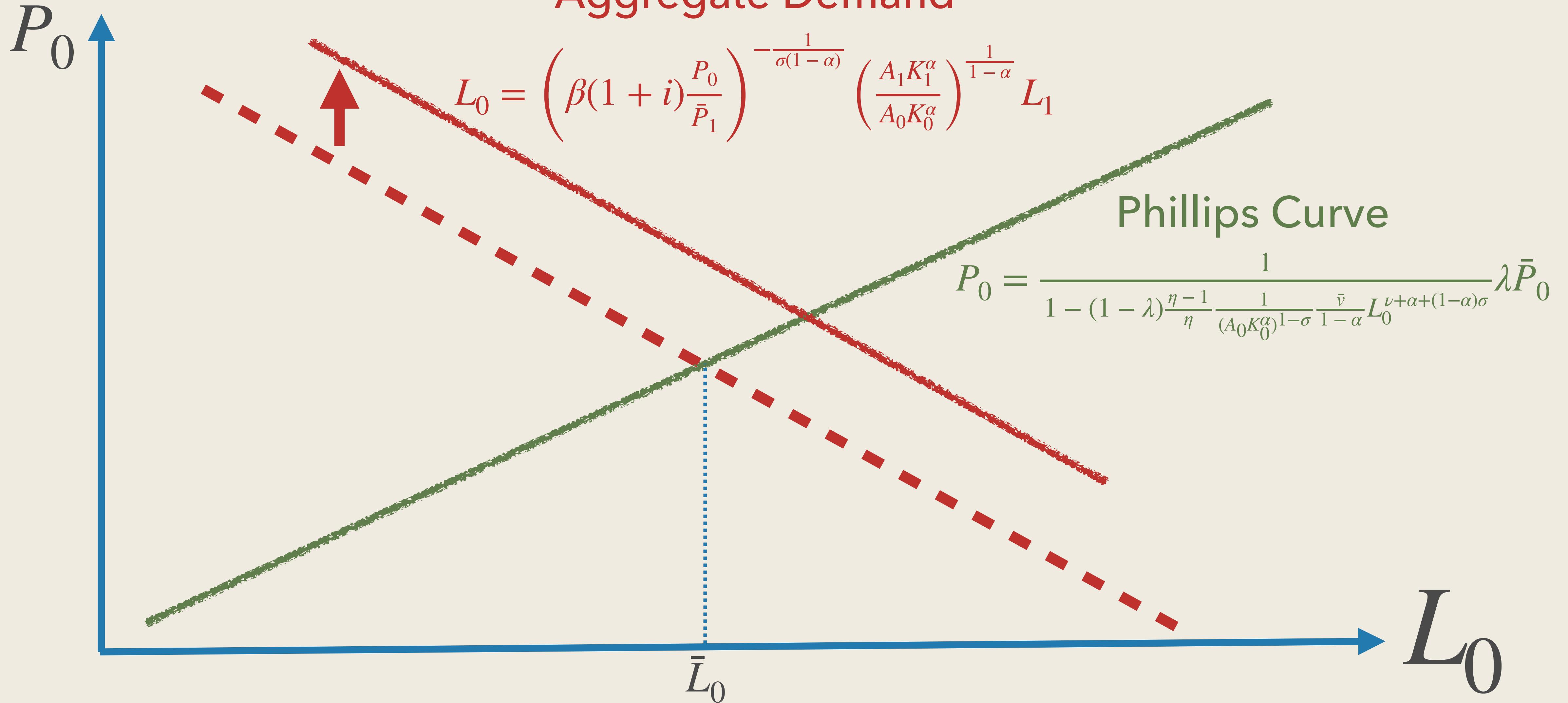
- In the RBC model, we have seen that shocks to  $\beta$  or  $A_1$  cannot generate comovement
  - $C_0$  and  $L_0$  were moving in the opposite direction
- Let us revisit it with the New Keynesian model

# Increase in $\beta$

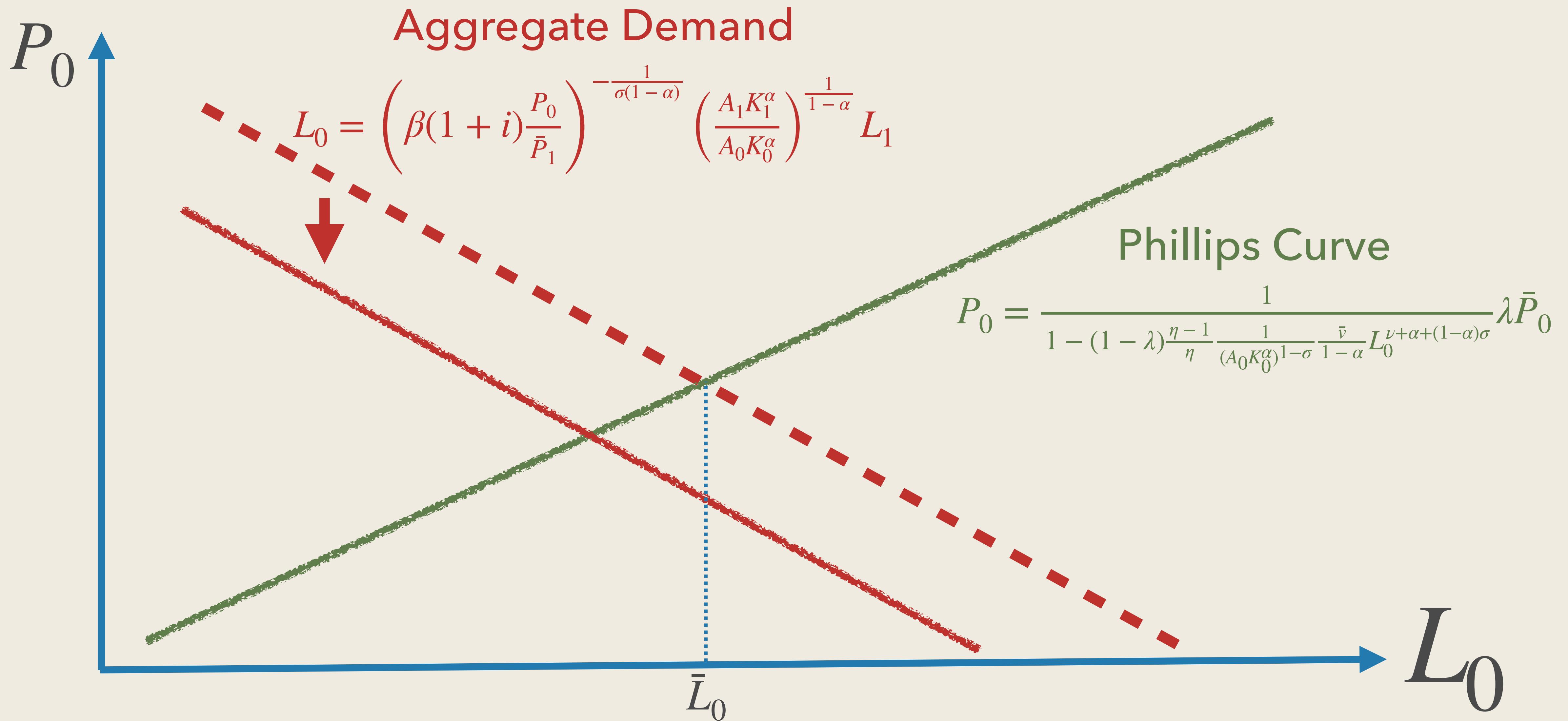


# Increase in $A_1$

Aggregate Demand



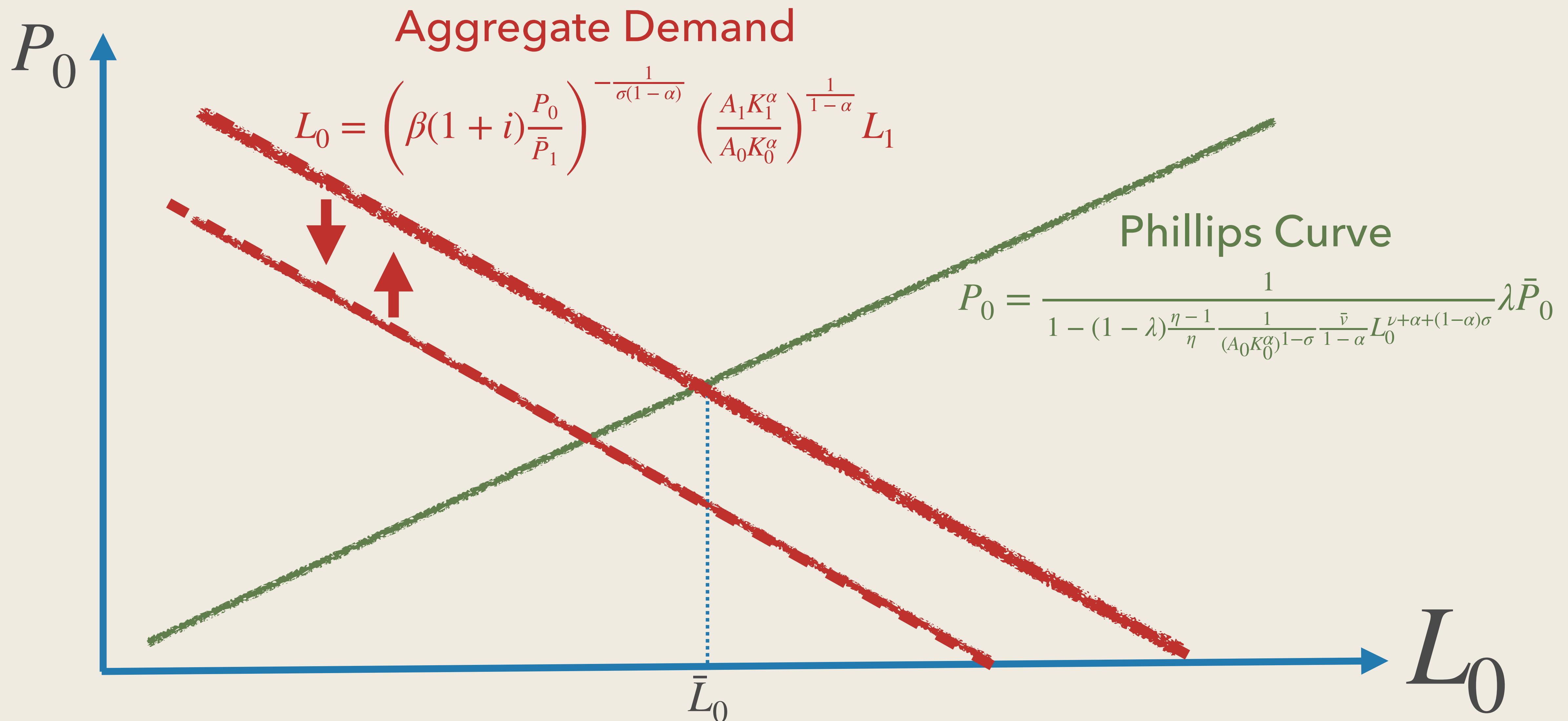
# Increase in $A_0$ when $\sigma = 1$



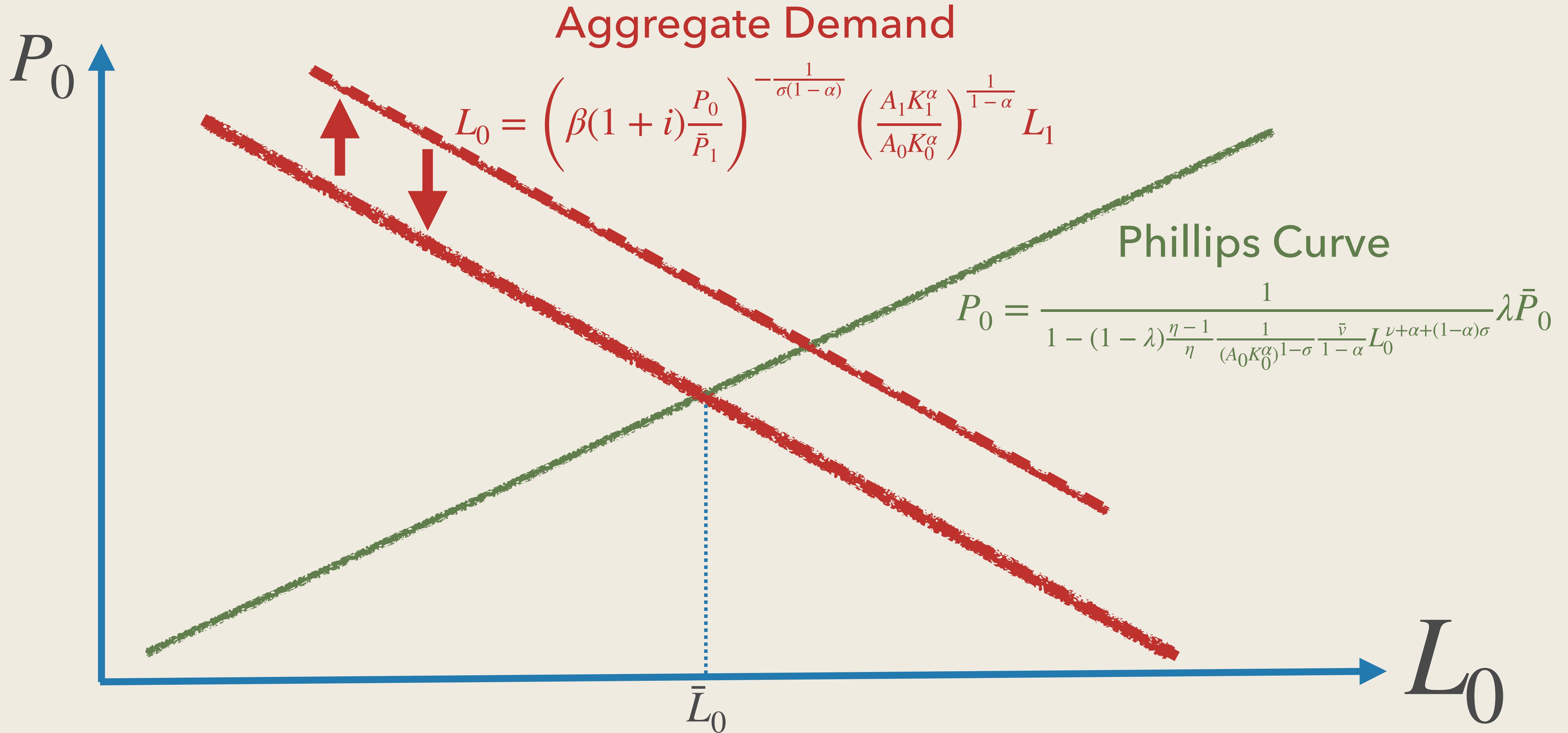
# Right Comovement

- Unlike the RBC model, patience and optimism can generate business cycles
- Why?
- When patience ( $\beta$ ) goes up, households cut spending today
- This lowers aggregate demand
- Under flexible prices, prices drop today so as to sustain aggregate demand
- When prices are sticky, prices cannot drop much, and we have lower employment
- The same mechanism operates for optimism ( $A_1$ )
- Can the Fed fight against such fluctuations?

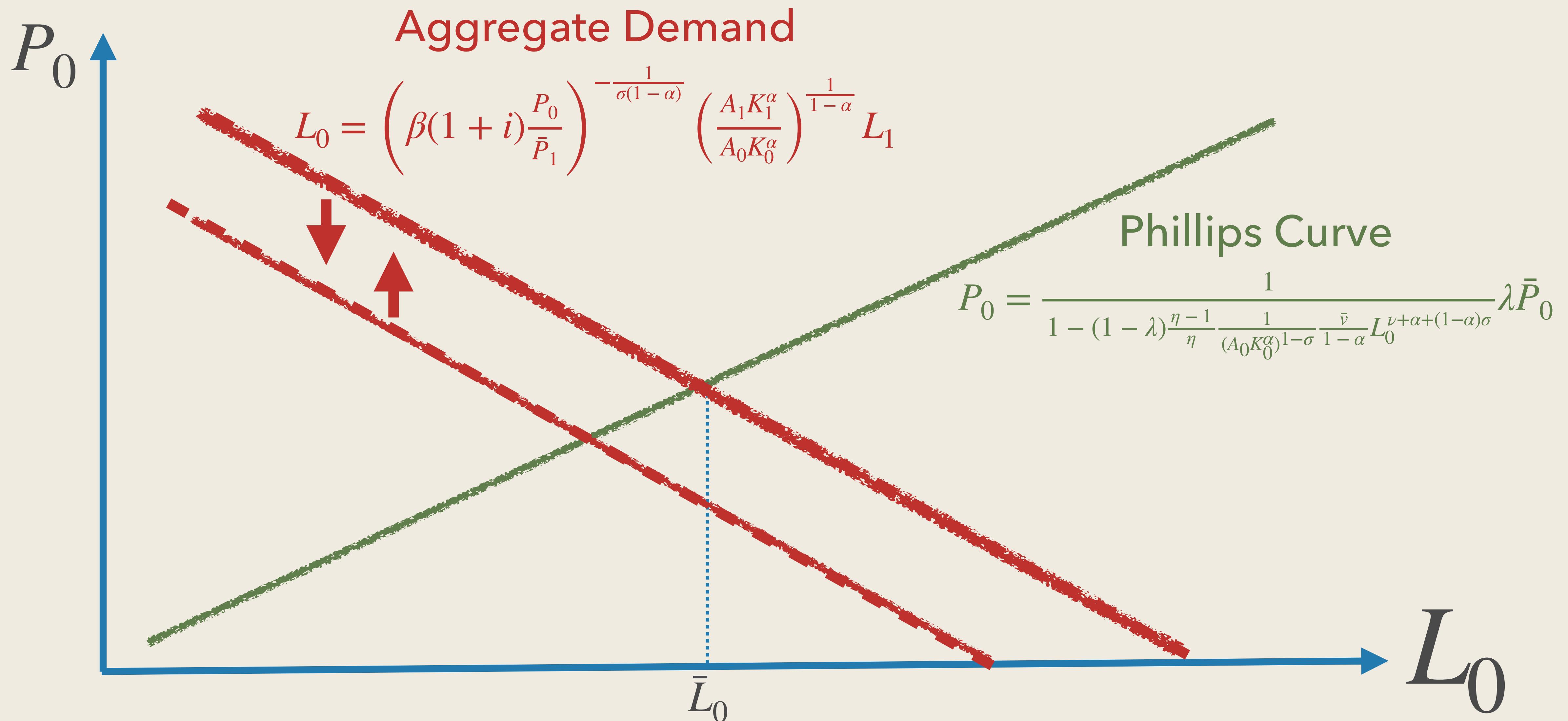
# Monetary Policy Response to Increase in $\beta$



# Monetary Policy Response to Increase in $A_1$



# Monetary Policy Response to Increase in $A_0$



# Monetary Policy Responses

- If the Fed lowers the rate appropriately, we can avoid recession in response to  $\beta \uparrow$
- If the Fed raises the rate appropriately, we can avoid boom in response to  $A_1 \uparrow$
- In both cases, monetary policy can stabilize **both** prices and employment
  - With a single instrument. This is an astonishing result.
- If the Fed cannot lower the rate, then the recession is worse
  - For example, due to the zero lower bound, as in the Great Recession

---

# Infinite Horizon New Keynesian Model

# Environment

- The economy consists of
  1. Households
  2. Firms
  3. Retailers
  4. Central bank
- Retailers purchase wholesale goods from firms
- Retailers sell the final goods to households (for  $C$ ) and firms (for  $I$ )
  - We now add back investment

# Households and Firms

- Households solve

$$\max_{\{C_t, l_t, a_t\}} \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \bar{\nu} \frac{l_t^{1+\nu}}{1+\nu} \right]$$

subject to

$$P_t C_t + A_t = (1 + i_{t-1}) a_{t-1} + W_t l_t + D_t$$

- Firms solve

$$\max_{\{I_t, K_{t+1}, D_t, L_t\}} \sum_{t=0}^{\infty} \frac{1}{\prod_{s=0}^{t-1} (1 + i_s)} D_t$$

subject to

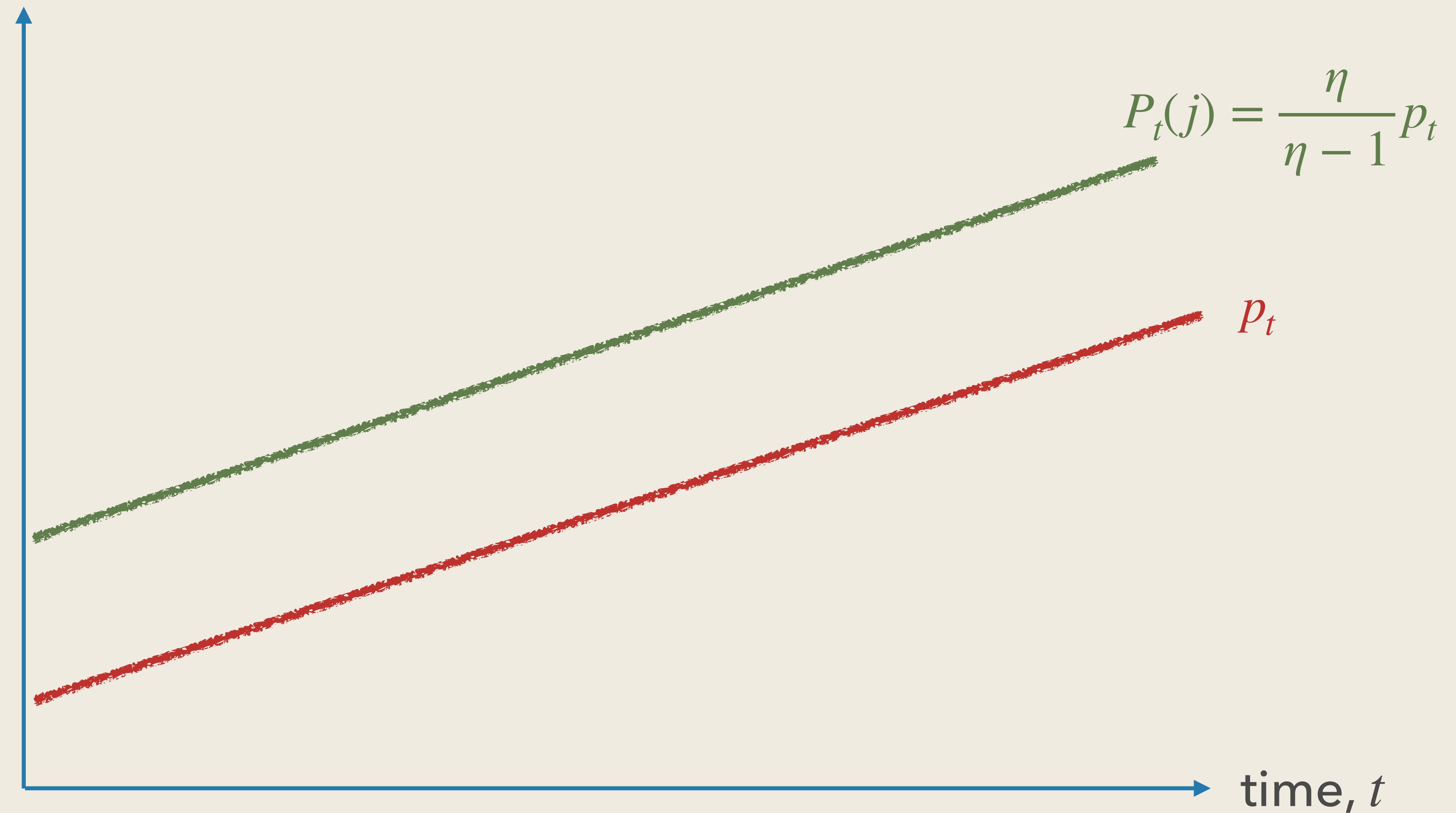
$$D_t = p_t A K_t^\alpha L_t^{1-\alpha} - W_t L_t - P_t I_t - P_t \frac{\phi}{2} \left( \frac{I_t}{K_t} \right)^2 K_t$$

$$K_{t+1} = (1 - \delta) K_t + I_t$$

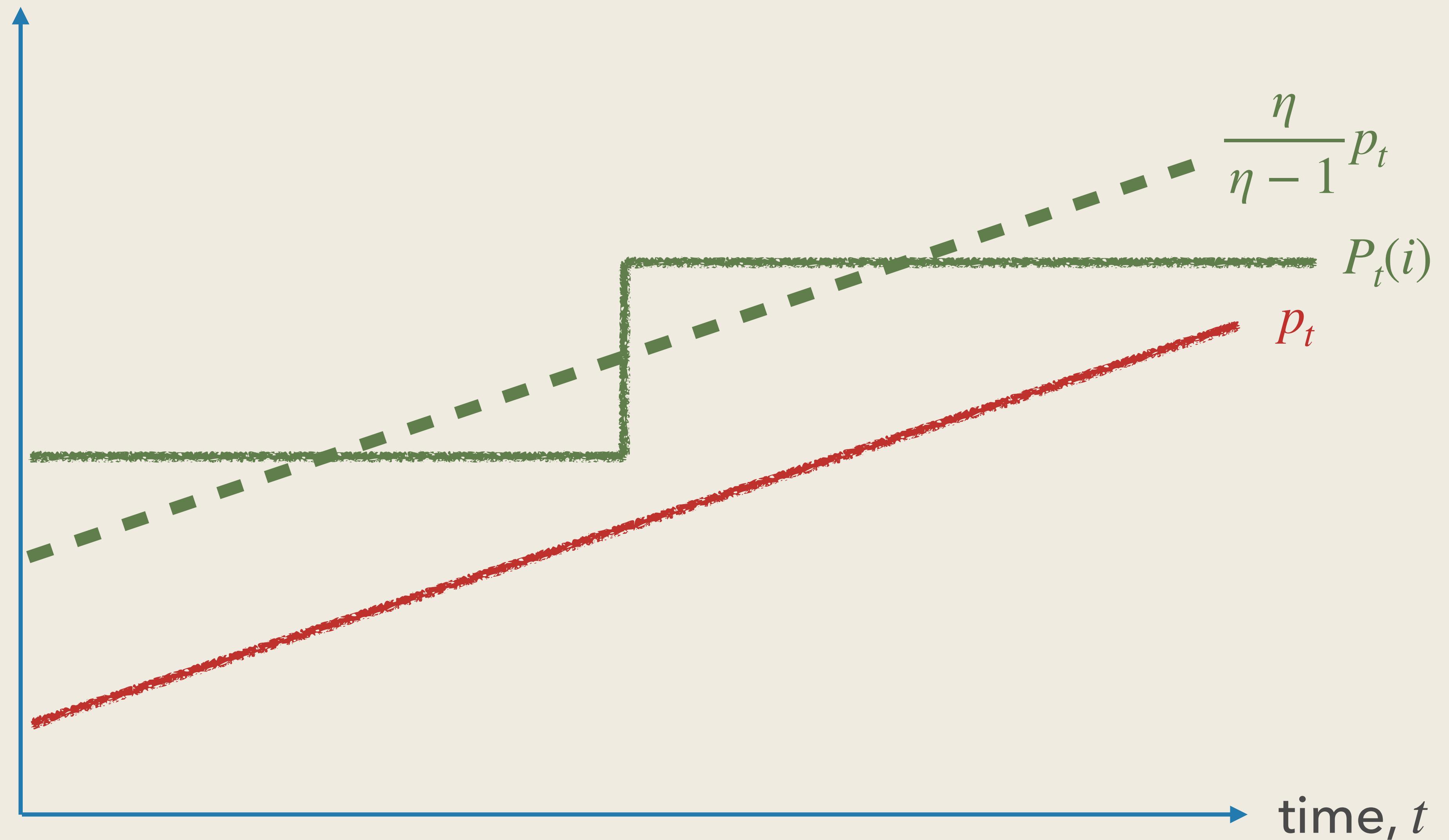
# Sticky Prices

- Retailers purchase wholesale goods at price  $p_t$  and sell it to households and firms
- Retailers can adjust their prices only with probability  $1 - \lambda$
- How should retailers set prices?

# When prices are flexible, $\lambda = 0$



# When prices are Sticky, $\lambda > 0$



# New Keynesian Phillips Curve

$$\pi_t = \kappa \left[ \frac{\eta - 1}{\eta} \frac{p_t}{P_t} - 1 \right] + \beta \pi_{t+1}$$

with  $\kappa = \frac{(1 - \beta\lambda)(1 - \lambda)}{\lambda}$  and  $\pi_t = \frac{P_t}{P_{t-1}} - 1$

- Suppose prices are flexible,  $\lambda = 0$ , then

$$P_t = \frac{\eta}{\eta - 1} p_t$$

- Suppose prices are completely rigid,  $\lambda = 1$

$$\pi_t = 0$$

# Intuition

$$\pi_t = \kappa \left[ \frac{\eta - 1}{\eta} \frac{p_t}{P_t} - 1 \right] + \beta \pi_{t+1}$$

- Inflation today depends on today's wholesale cost  $p_t$ 
  - If wholesale cost goes up, firms who can adjust prices want to raise prices
  - Inflationary.
  - The strength of the inflationary pressure is governed by  $\kappa = \frac{(1 - \beta\lambda)(1 - \lambda)}{\lambda}$
- Inflation today depends on future inflation  $\pi_{t+1}$ 
  - Suppose firms expect inflation to be high in the future
  - If firms have opportunity to adjust, they start raising today
  - Because firms may not have opportunity to raise prices when inflation happens

# Central Bank

- The central bank sets the nominal interest rate in the economy

- We assume

$$i_t = \bar{i} + \phi_\pi \pi_t + \epsilon_t$$

- $\phi_\pi$ : how much the central bank is willing to fight against inflation
  - $\epsilon_t$ : monetary policy “shock” (e.g., changes in moods of FOMC members)
- Taylor (1993) argued this is a good description of the US monetary policy

# Fisher Equation

- The relationship between nominal and real rate is

$$r_t = i_t - \pi_{t+1}$$

- This called Fisher equation

---

## Equilibrium Conditions: $\{C_t, L_t, I_t, K_{t+1}, q_t, p_t/P_t, r_t, i_t, \pi_t\}$

1. Euler equation:

$$u'(C_t) = \beta(1 + r_t)u'(C_{t+1})$$

2. Labor demand/supply:

$$\frac{p_t}{P_t} \frac{\partial F_t(K_t, L_t)}{\partial L_t} u'(C_t) = v'(L_t)$$

3. Investment:

$$\frac{I_t}{K_t} = \frac{1}{\phi} [q_t - 1], \quad q_t = \frac{1}{1 + r_t} \left[ \frac{p_t}{P_t} \frac{\partial F_{t+1}(L_{t+1}, K_{t+1})}{\partial K_{t+1}} - \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left( \frac{I_{t+1}}{K_{t+1}} \right)^2 + \left( \frac{I_{t+1}}{K_{t+1}} + (1 - \delta) \right) q_{t+1} \right]$$

4. Capital stock evolution:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

5. Goods market clearing:

$$C_t + I_t + \Phi(I_t, K_t) = F_t(K_t, L_t)$$

6. New Keynesian Phillips curve:

$$\pi_t = \kappa \left[ \frac{\eta - 1}{\eta} \frac{p_t}{P_t} - 1 \right] + \beta \pi_{t+1}$$

7. Monetary policy:

$$i_t = \bar{i} + \phi_\pi \pi_t + \epsilon_t$$

8. Fisher equation:

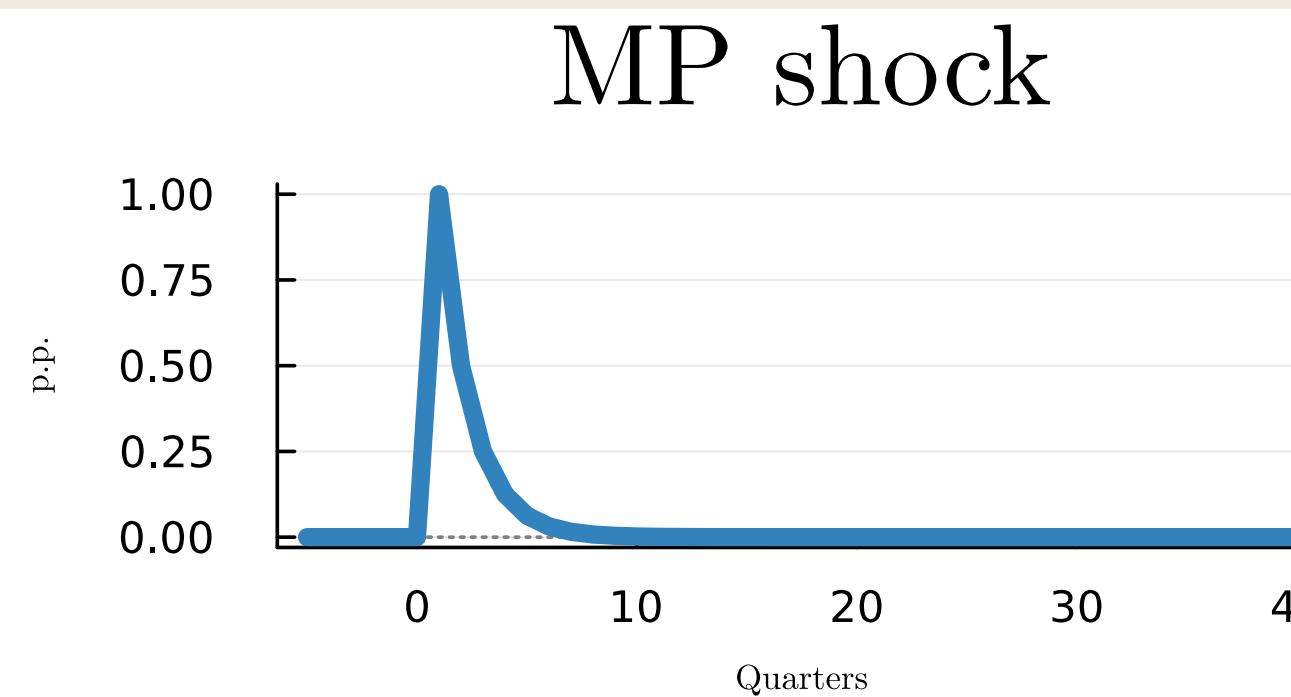
$$r_t = i_t - \pi_{t+1}$$

# Parametrization (Calibration)

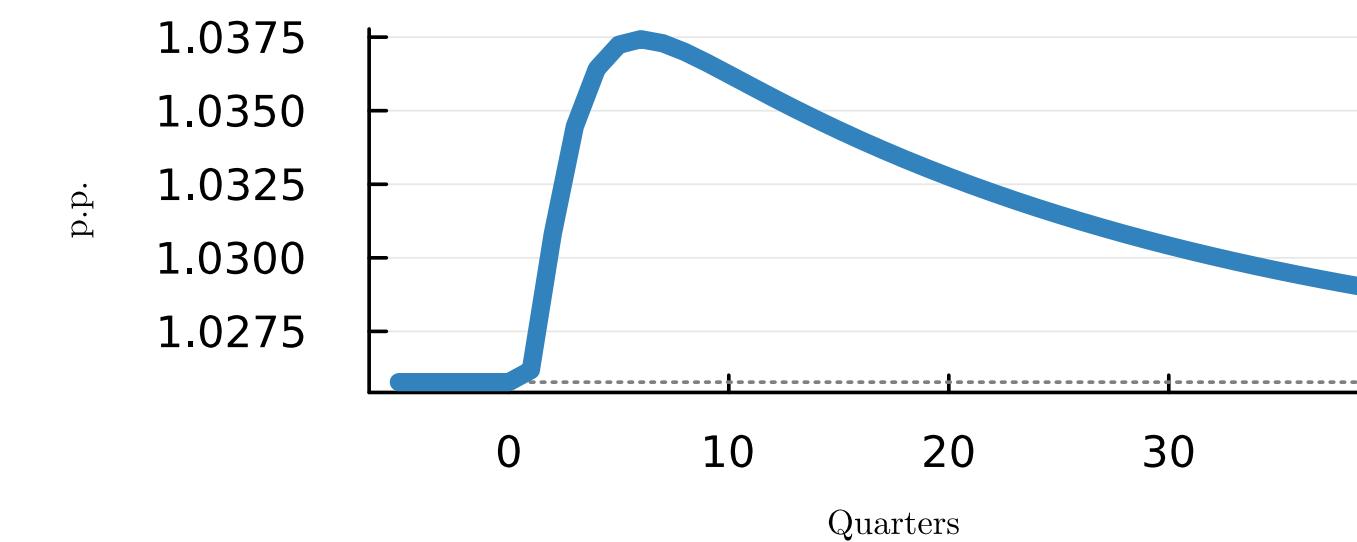
- The same parameters as in the RBC model for those in common
- We set the price stickiness to  $\lambda = 0.75$
- We set  $\phi_\pi = 1.5$ , as suggested by Taylor (1993)
- We simulate the response of the economy to monetary policy shock  $\epsilon_t$ 
  - Set the autocorrelation of the shock to 0.5

# Monetary Policy Shock

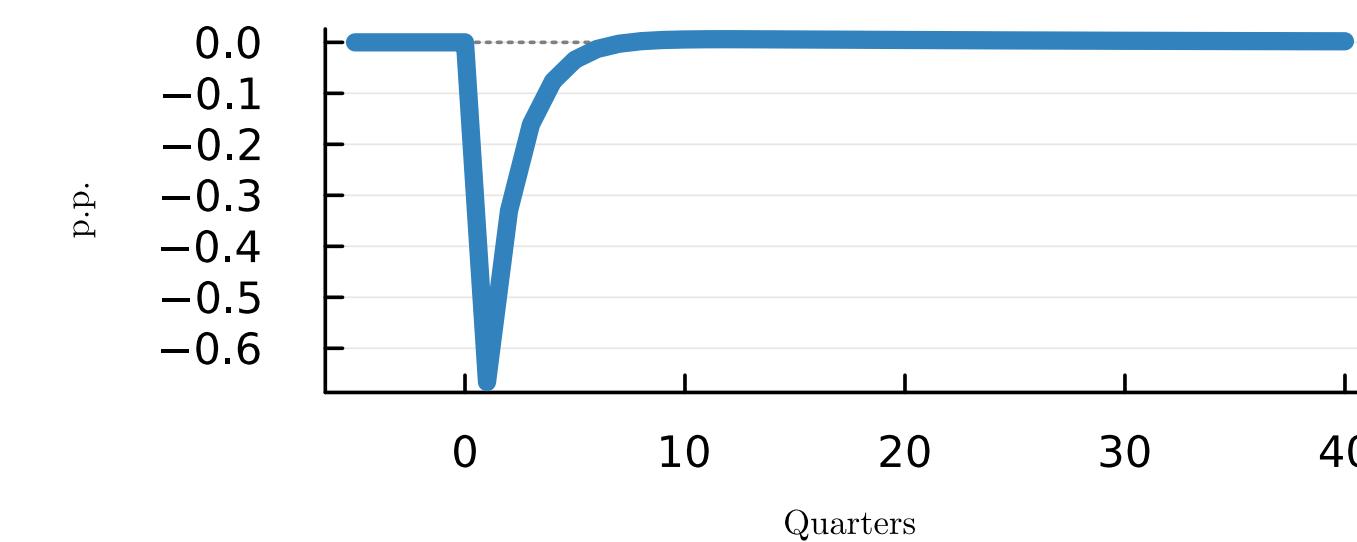
MP shock



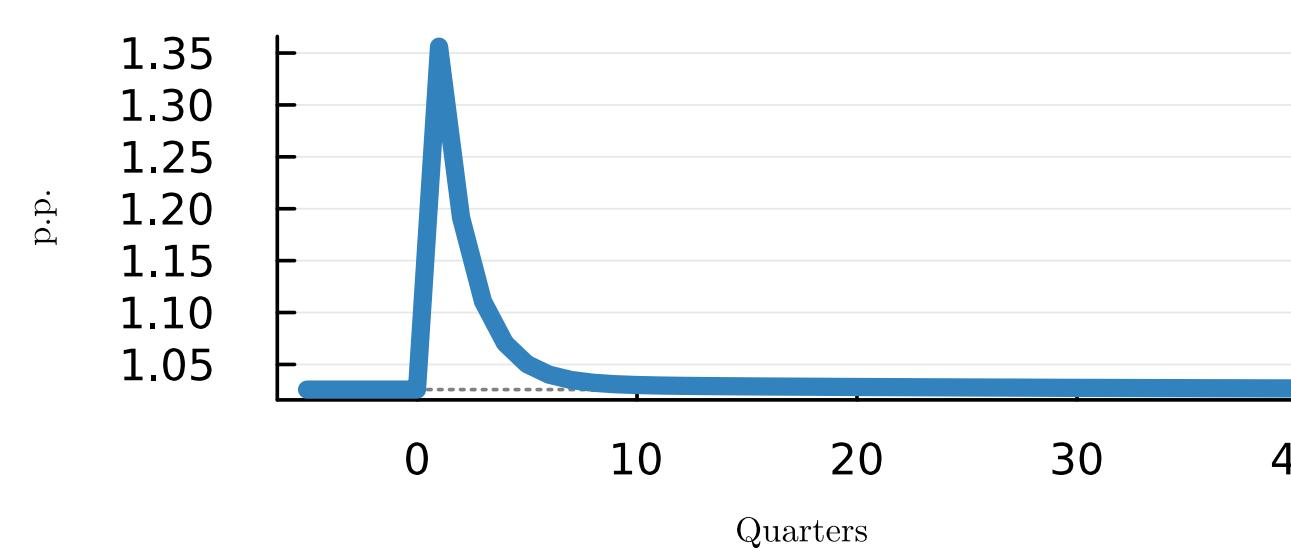
$i$



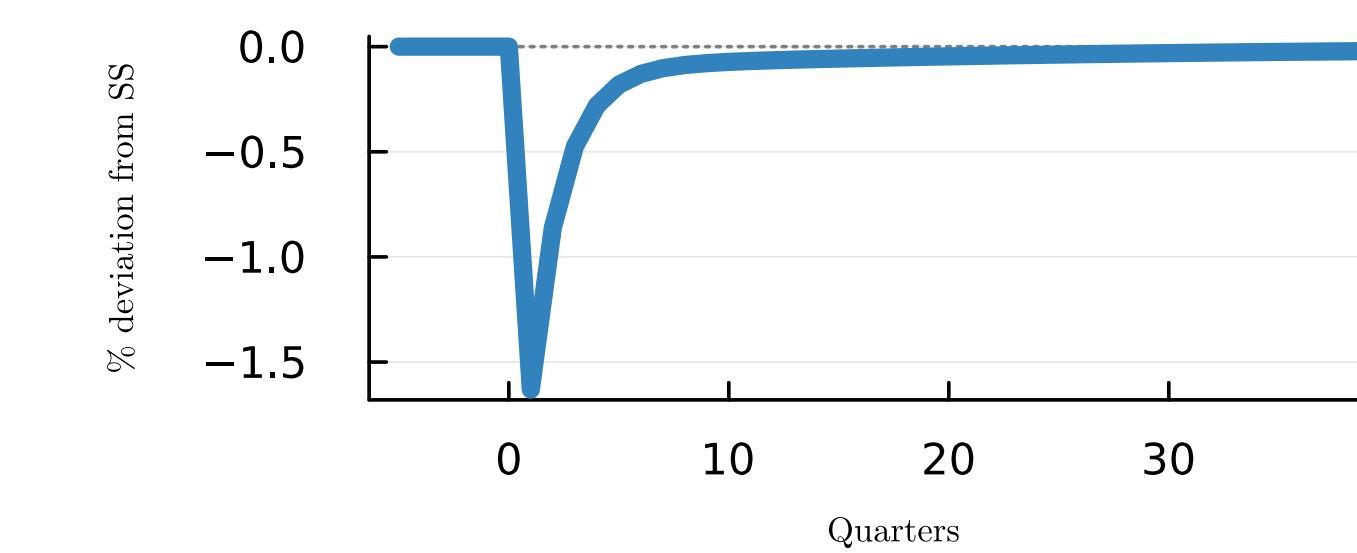
$\pi$



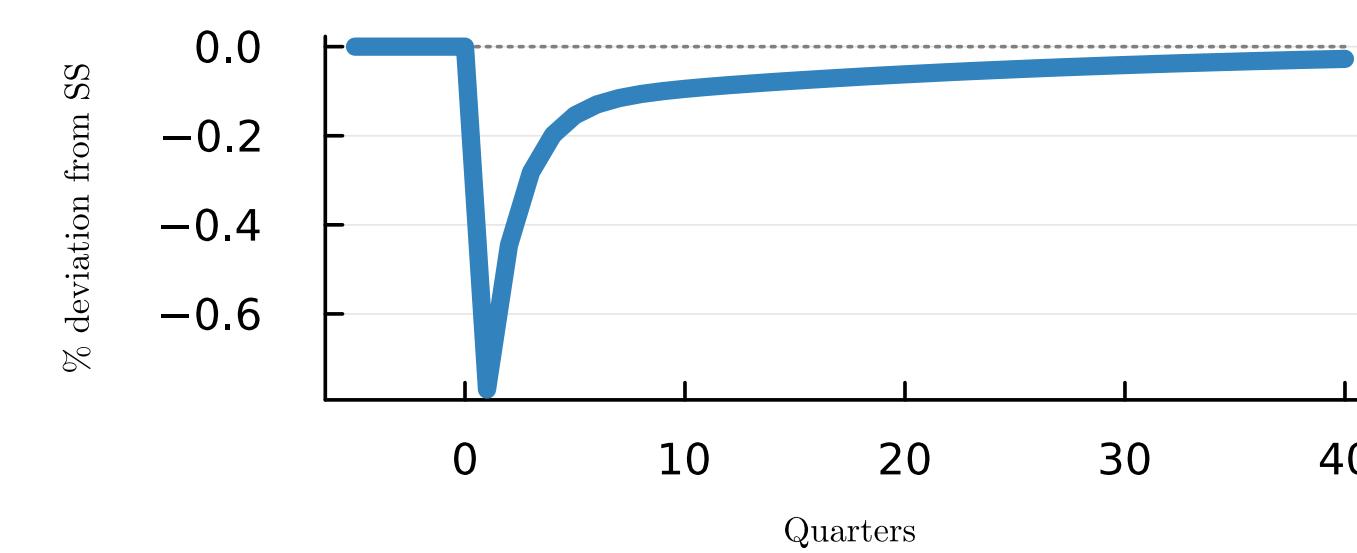
$r$



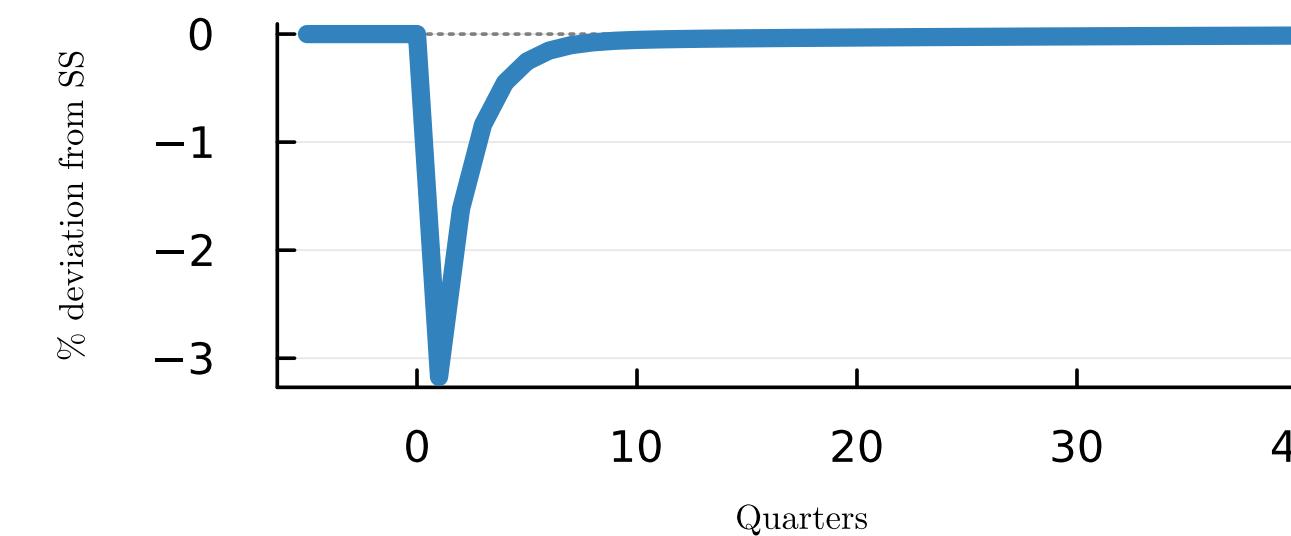
$Y$



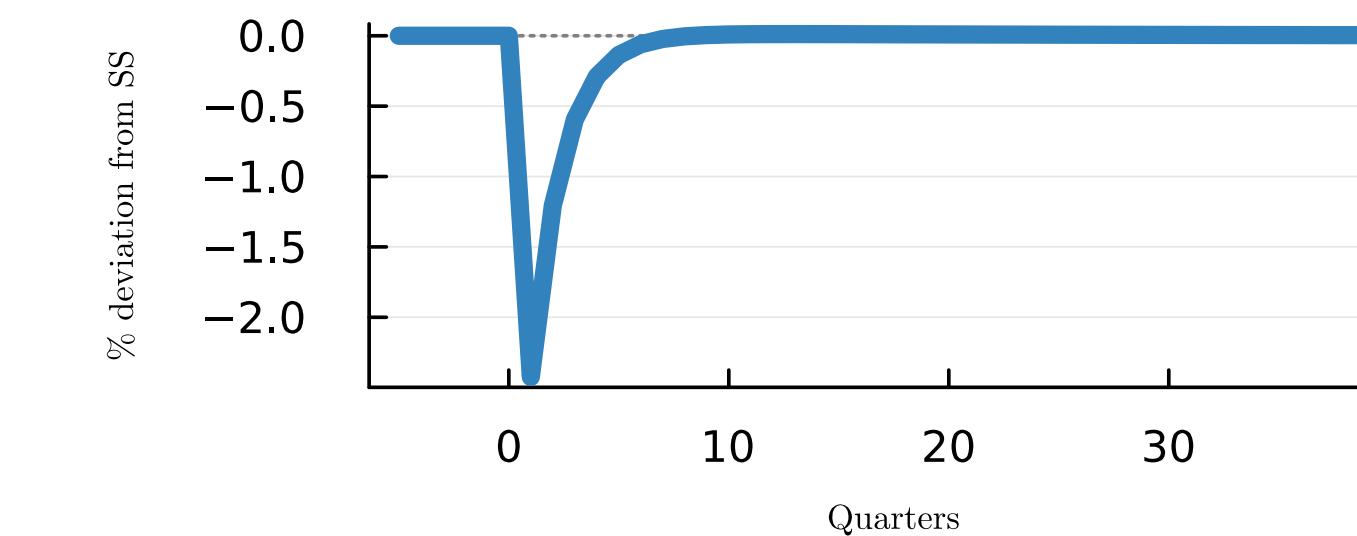
$C$



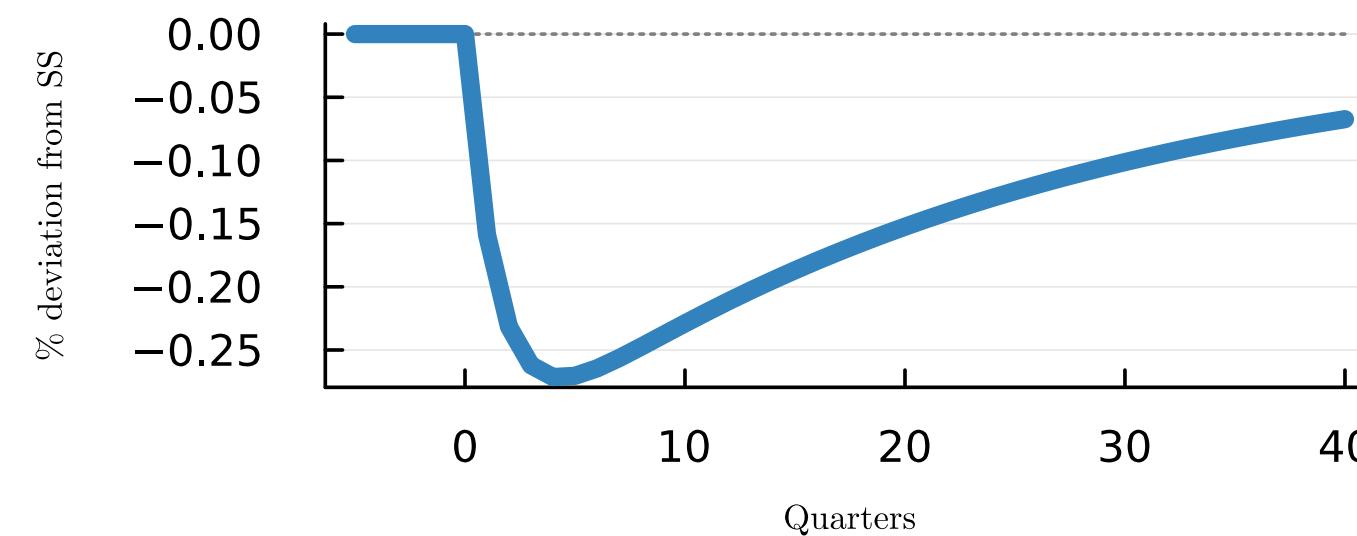
$I$



$L$



$K$



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# Zero Lower Bound