Labor Reallocation and Misallocation

741 Macroeconomics
Topic 7

Masao Fukui

Fall 2025

Firm Employment is Log-Linear in TFP

- In Hopenhayn-Rogerson, firm-level employment is given by

$$n = (\underbrace{z^{1-\alpha}}_{\equiv Z} \alpha/w)^{\frac{1}{1-\alpha}}$$

$$\Leftrightarrow \quad \log n = \frac{1}{1-\alpha} \log Z + const$$

- Taking the first difference,

$$\Delta \log n = \frac{1}{1-\alpha} \Delta \log Z$$

$\Rightarrow$ Firms react symmetrically to positive and negative TFP shocks

- Is this true in the data?

Ilut, Kehrig & Schneider (2018)

- Focus on US manufacturing establishments (Census data)
- Construct firm-level TFP using Solow residual:

$$\log sr_{it} = \log y_{it} - \left(\beta_n \log n_{it} + \beta_k \log k_{it} + \beta_m \log m_{it}\right)$$

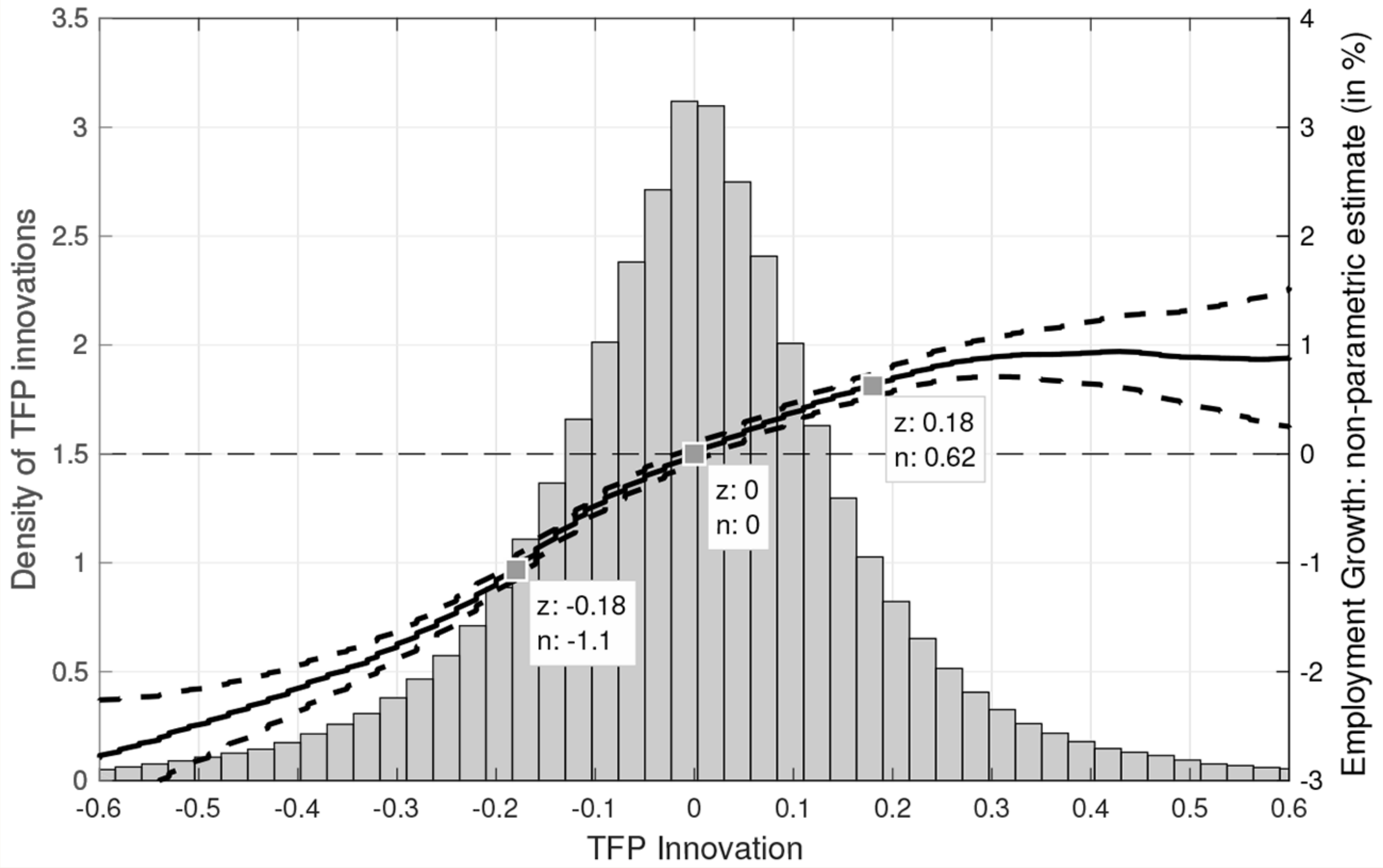
- Construct firm-level TFP shocks, Z_{it}, assuming

$$\log sr_{it} = g \times t + \alpha^i + \log Z_{it}$$

- Q: How does firm-level employment respond to TFP shocks?

$$\Delta \log n_{it} = h(\Delta \log Z_{it}) + \gamma' \boldsymbol{X}_{it} + \epsilon_{it}$$

Concave Hiring Rule



Firm Dynamics with Labor Adjustment Costs – Hopenhayn & Rogerson (1993)

Slow to Hire, Quick to Fire

- The simplest explanation:
  - it is costly to hire workers
  - less so to fire workers
- We incorporate employment adjustment costs into Hopenhayn-Rogerson

Labor Adjustment Cost

- Suppose that employment stock is costly to adjust
- Every period, $\delta \in [0,1]$ fraction of workers exogenously separate
- Firms can hire $h \times n$ workers with hiring cost $\Phi(h, n)$
  - $h < 0$ corresponds to firing
- The stock-flow equation of employment:

$$n_t = n_{t-1}(1 - \delta + h_t)$$

Bellman Equation

- Bellman equation:

$$v(n_{-1}, z) = \max \left\{ v^*(n_{-1}, z),\ -\Phi(-(1-\delta), n_{-1}) \right\}$$

where v^* is the continuation value

$$v^*(n_{-1}, z) = \max_{h, n}\ f(n, z) - wn - c_f - \Phi(h, n_{-1}) + \beta \mathbb{E} v(n, z')$$

$$\text{s.t.} \quad n = n_{-1}(1 - \delta + h)$$

- Policy functions:
  - $\chi(n_{-1}, z) \in \{0,1\}$: whether to exit or not
  - $h(n_{-1}, z)$: hiring rate
  - $n(n_{-1}, z)$: employment

Rest of the Equilibrium Conditions

- Assume that the initial firm size is given by n_0
- The free-entry condition is

$$\int v(n_0, z)\psi_0(z)dz = c_e$$

- Let $g(n_{-1}, z)$ denote the steady-state distribution, which satisfies

$$g(n, z') = \int\int \Pi(z'|z)(1 - \chi(n_{-1}, z))\mathbb{I}[n(n_{-1}, z) = n]g(n_{-1}, z)dzdn_{-1} + m\psi_0(z)\mathbb{I}[n = n_0]$$

- The labor market clearing condition is

$$\int\int n(n_{-1}, z)g(n_{-1}, z)dn_{-1}dz = L$$

Equilibrium Definition

- Recursive equilibrium: $\{v(n,z), \chi(n,z), n'(n,z), w\}$ and $\{g(n_{-1},z), m\}$ such that:
  1. Given w, $\{v(n,z), \chi(n,z), n'(n,z)\}$ solve the Bellman equation
  2. Free entry holds, $\int v(n_0,z)\psi_0(z)dz = c_e$
  3. $\{g(n_{-1},z), m\}$ satisfies the steady state law of motion
  4. Labor market clears

- The equilibrium retains the same structure as before:
  1. Block recursive property: value and policy functions independent of distribution
  2. $g(n_{-1},z)$ homogenous in m:
     - can solve for $\hat{g} \equiv g/m$ first $\Rightarrow$ solve for m using labor market clearing

Equilibrium Definition

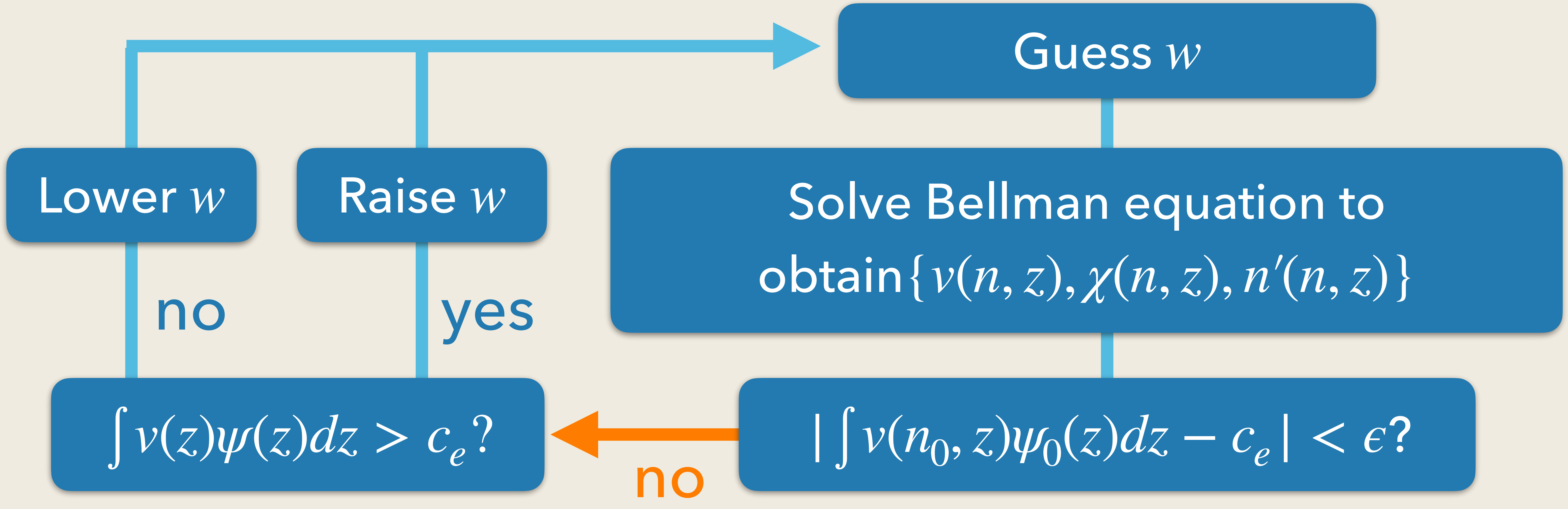
- Recursive equilibrium: $\{v(n,z), \chi(n,z), n'(n,z), w\}$ and $\{g(n_{-1},z), m\}$ such that:
  1. Given w, $\{v(n,z), \chi(n,z), n'(n,z)\}$ solve the Bellman equation
  2. Free entry holds, $\int v(n_0,z)\psi_0(z)dz = c_e$
  3. $\{g(n_{-1},z), m\}$ satisfies the steady state law of motion
  4. Labor market clears

1 & 2 alone

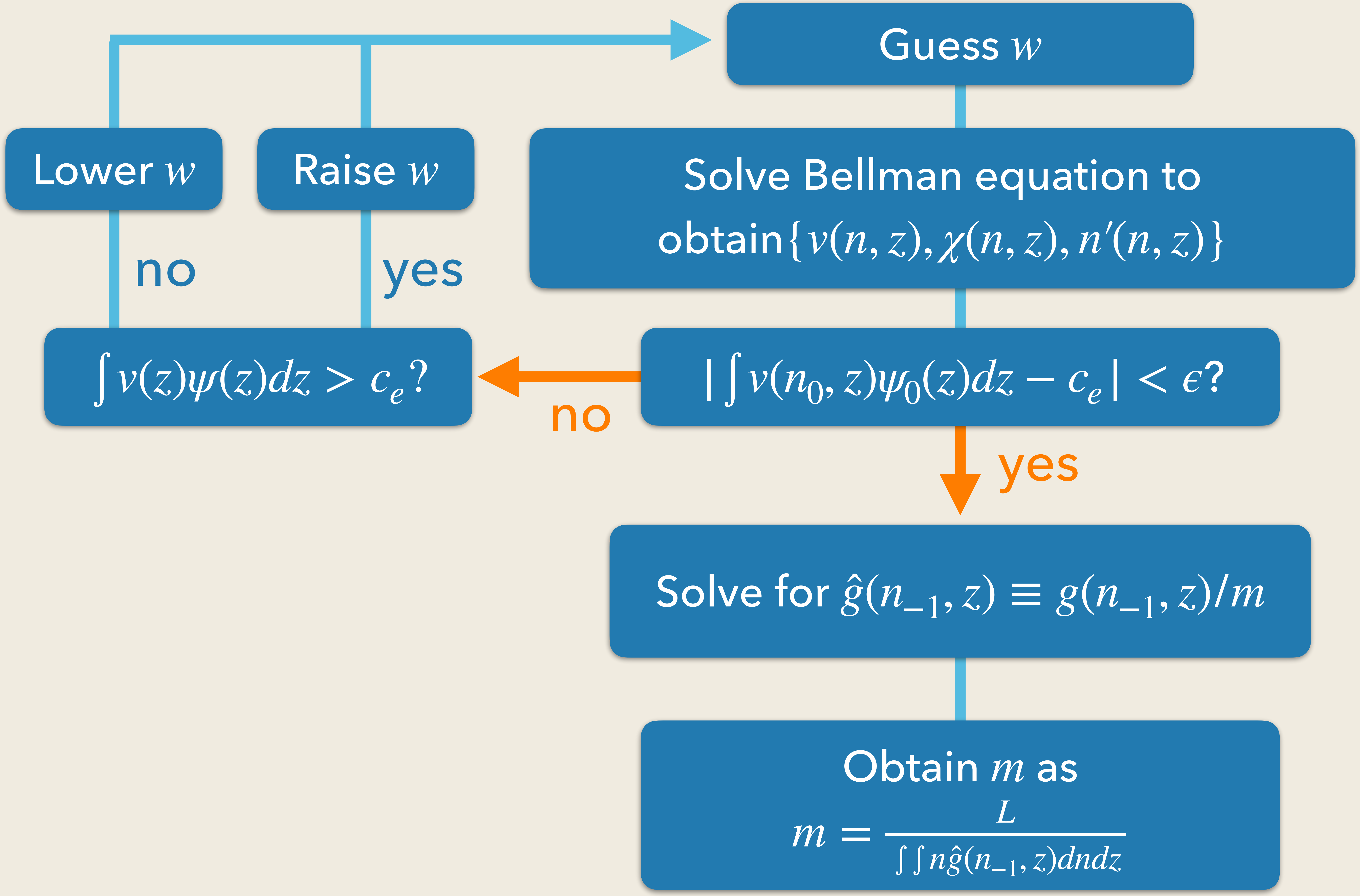
$\Rightarrow \{v(n,z), \chi(n,z), n'(n,z), w\}$

- The equilibrium retains the same structure as before:
  1. Block recursive property: value and policy functions independent of distribution
  2. $g(n_{-1},z)$ homogenous in m:
     - can solve for $\hat{g} \equiv g/m$ first $\Rightarrow$ solve for m using labor market clearing

Computational Algorithm



Computational Algorithm



Calibration

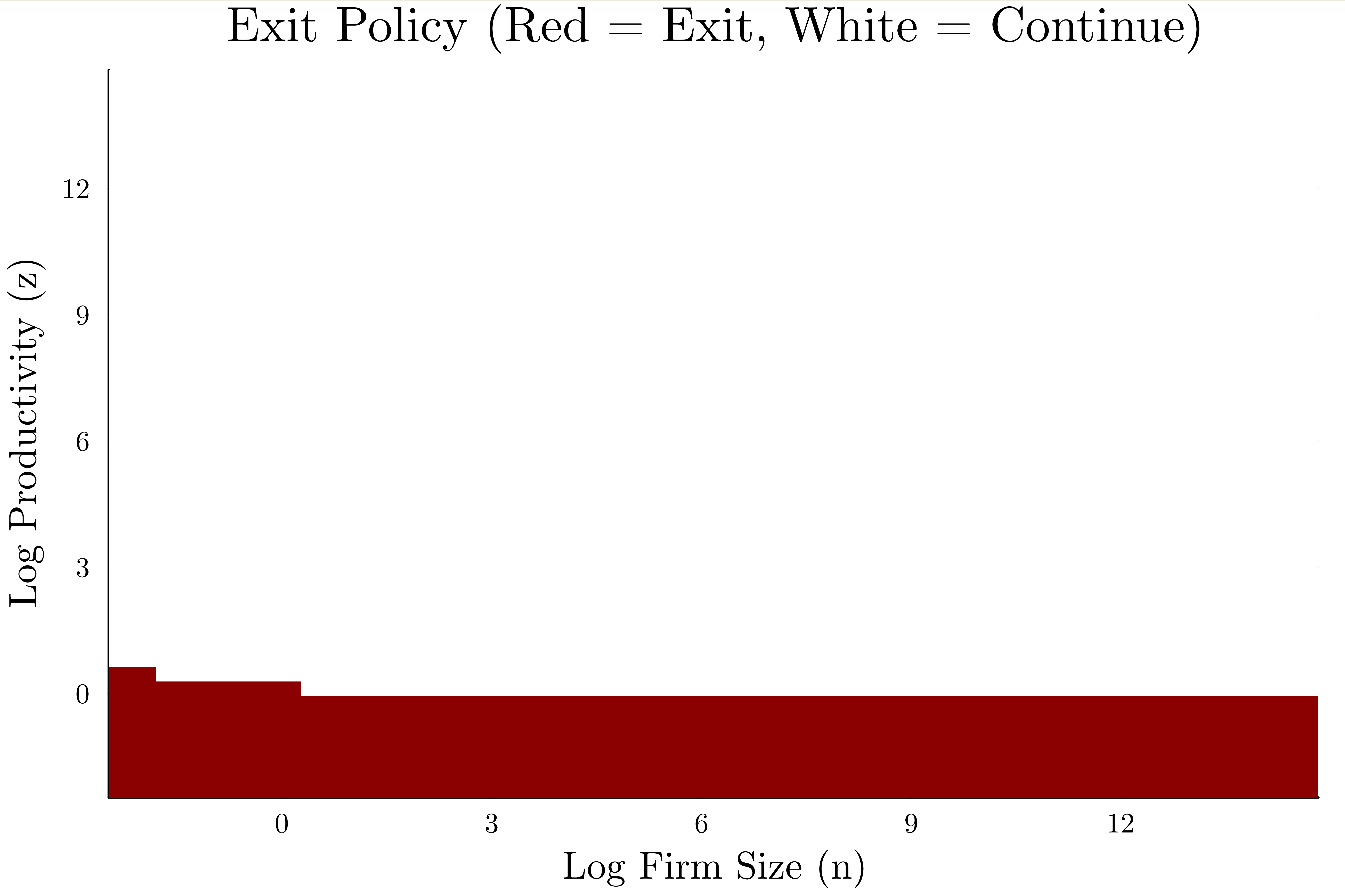
- Same parameter values for those that appear in the previous lecture note
- Set

$$\Phi(h, n_{-1}) = \begin{cases} \frac{\phi_+}{2} h^2 n_{-1} & \text{if } h > 0 \\ \frac{\phi_-}{2} h^2 n_{-1} & \text{if } h < 0 \end{cases}$$

  and assume $\phi_- = 0$ and $\phi_+ = 5$

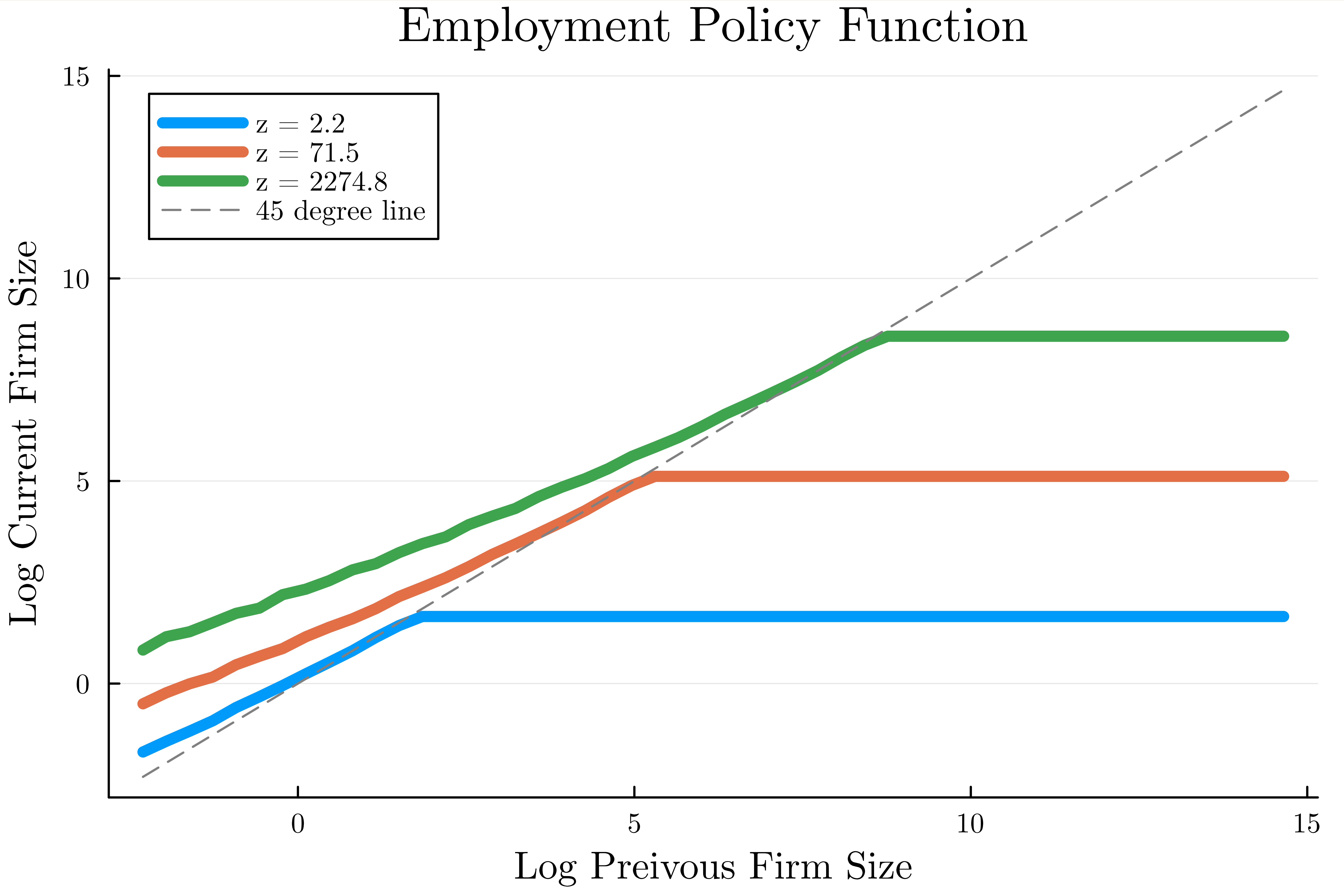
- Set $\delta = 0.1$
- Assume $n_0 = 5$ to roughly match the initial firm size

Exit Policy

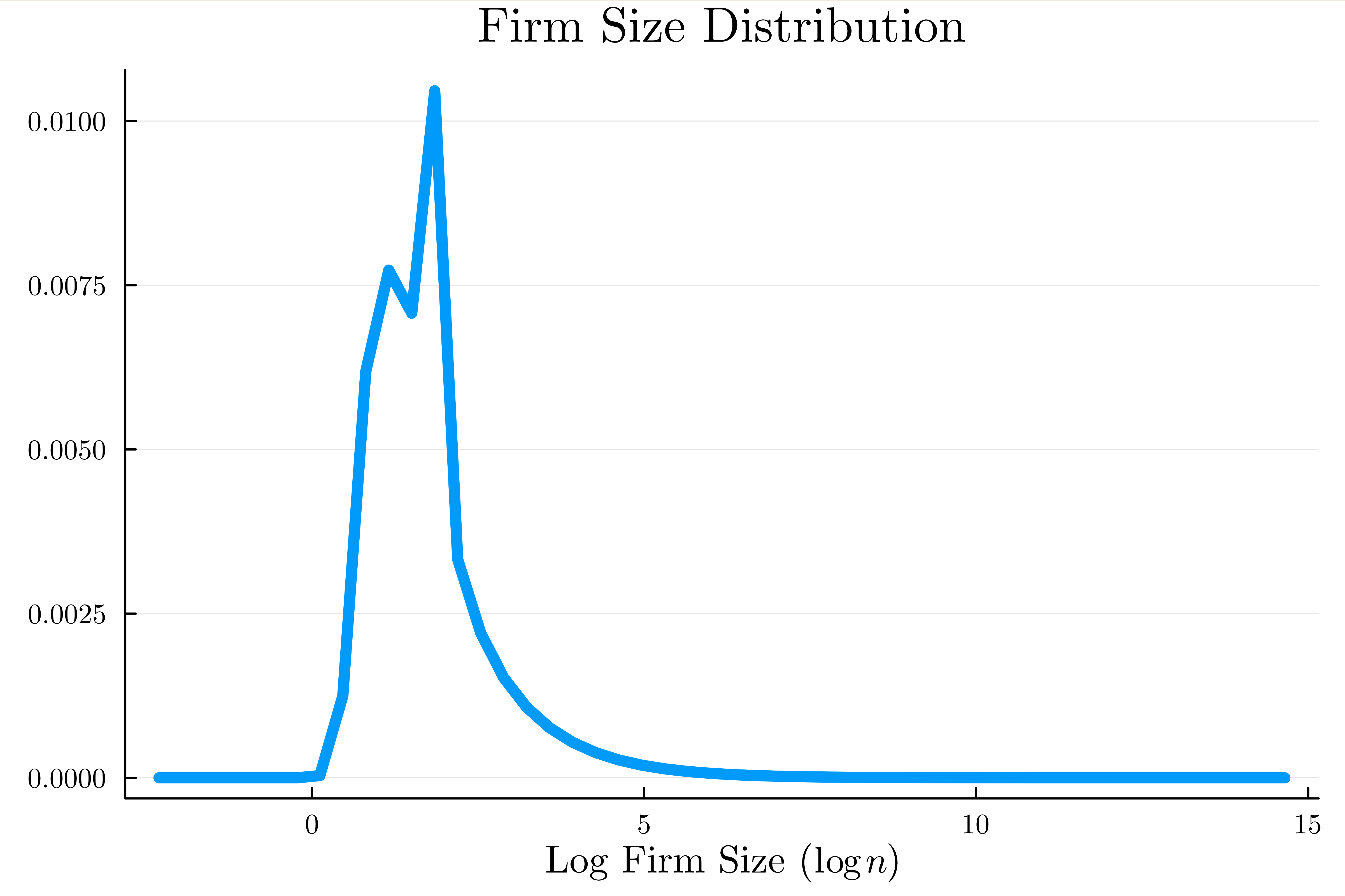


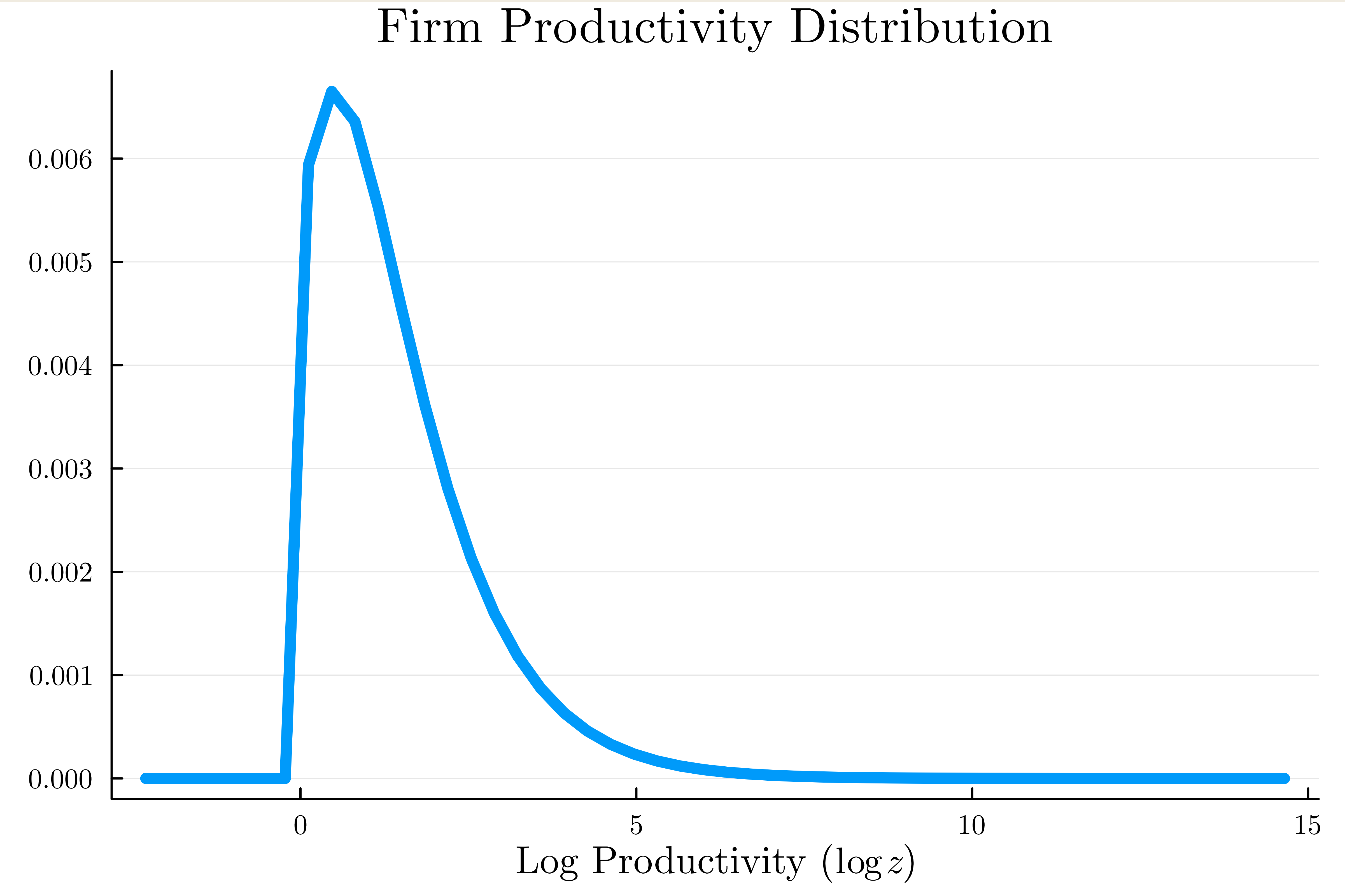
Exit Policy (Red = Exit, White = Continue)
Log Productivity (z)
12
9
6
3
0
0
3
6
9
12
Log Firm Size (n)

Employment Policy

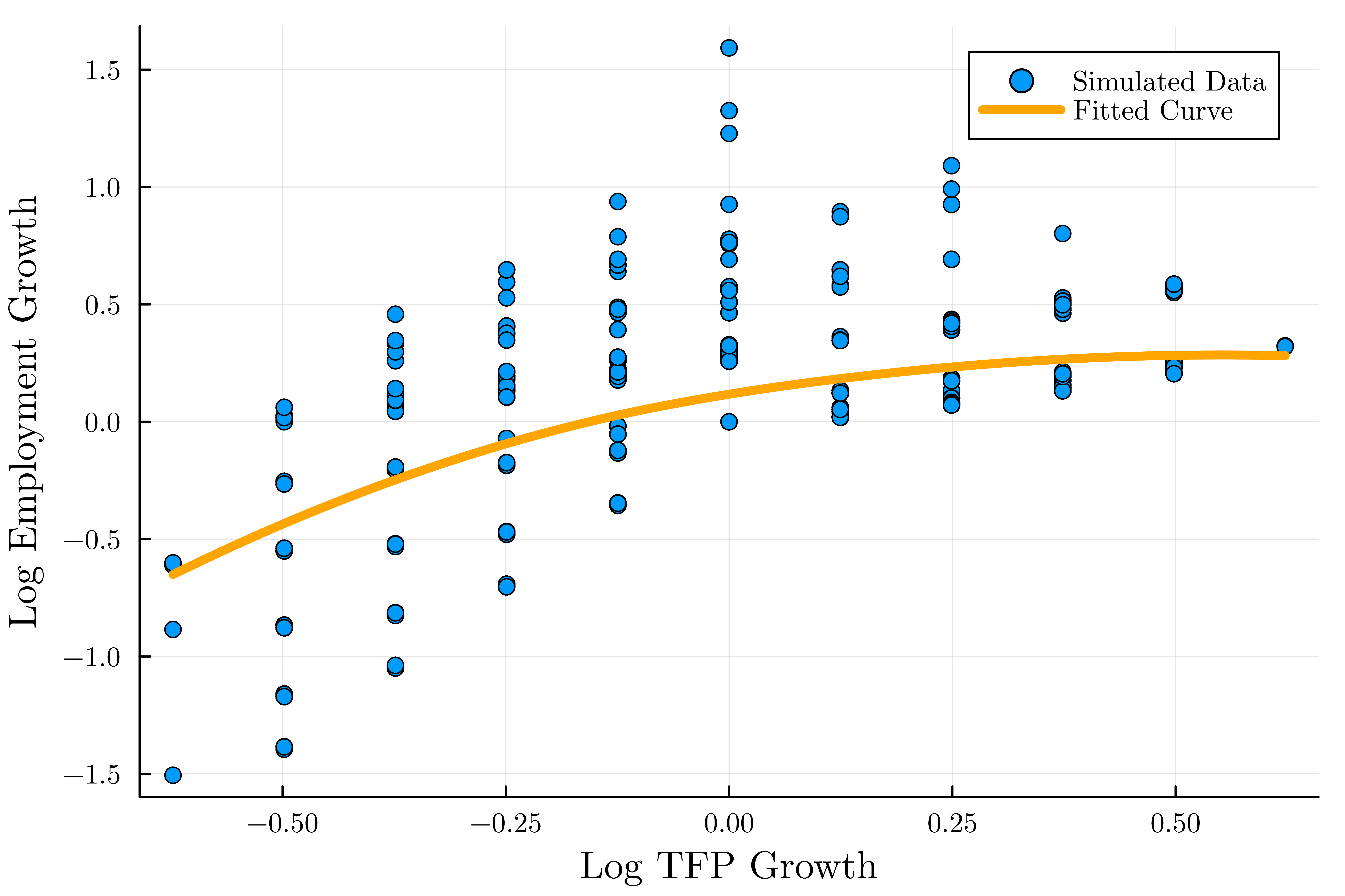


Distribution

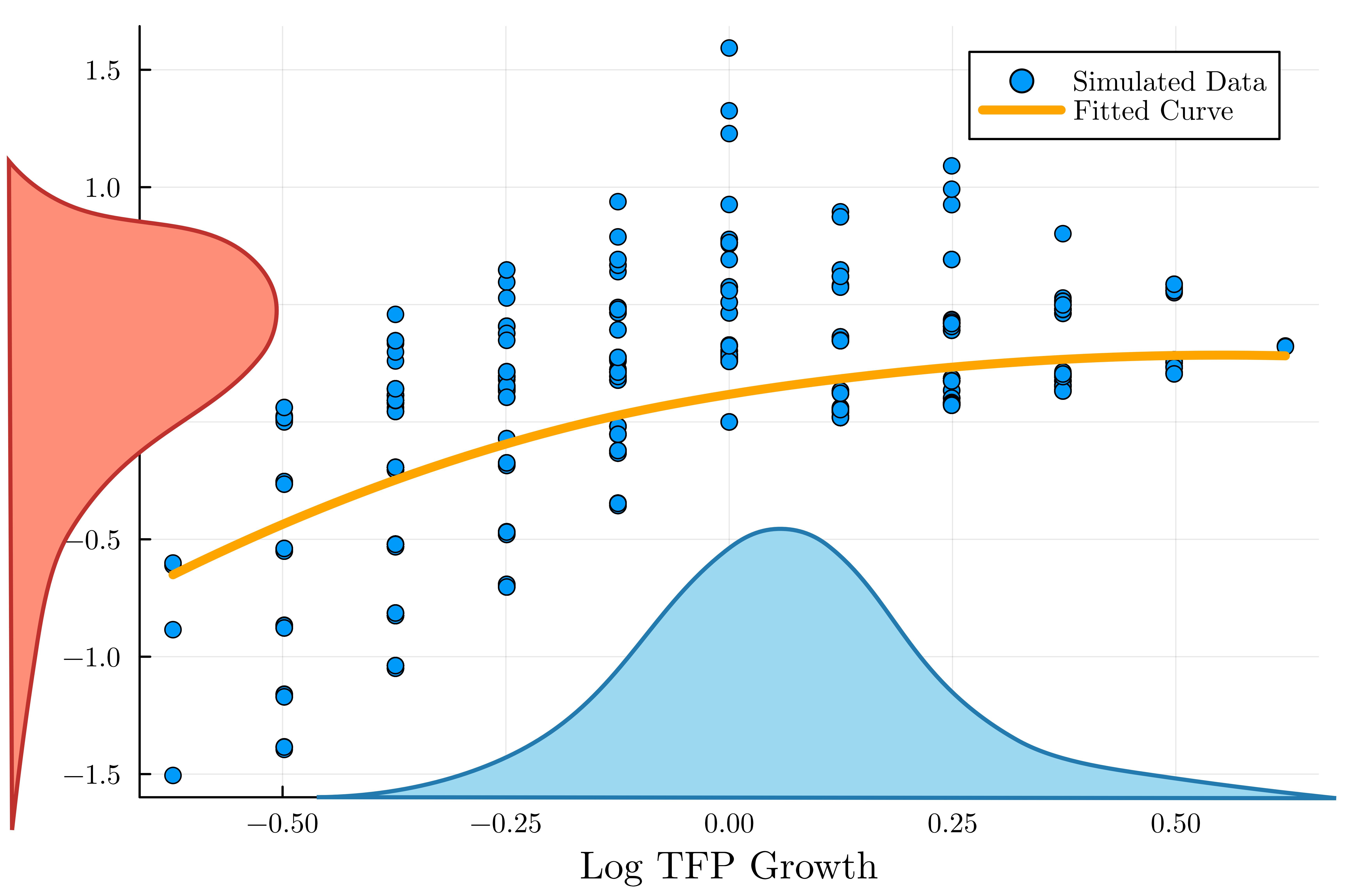




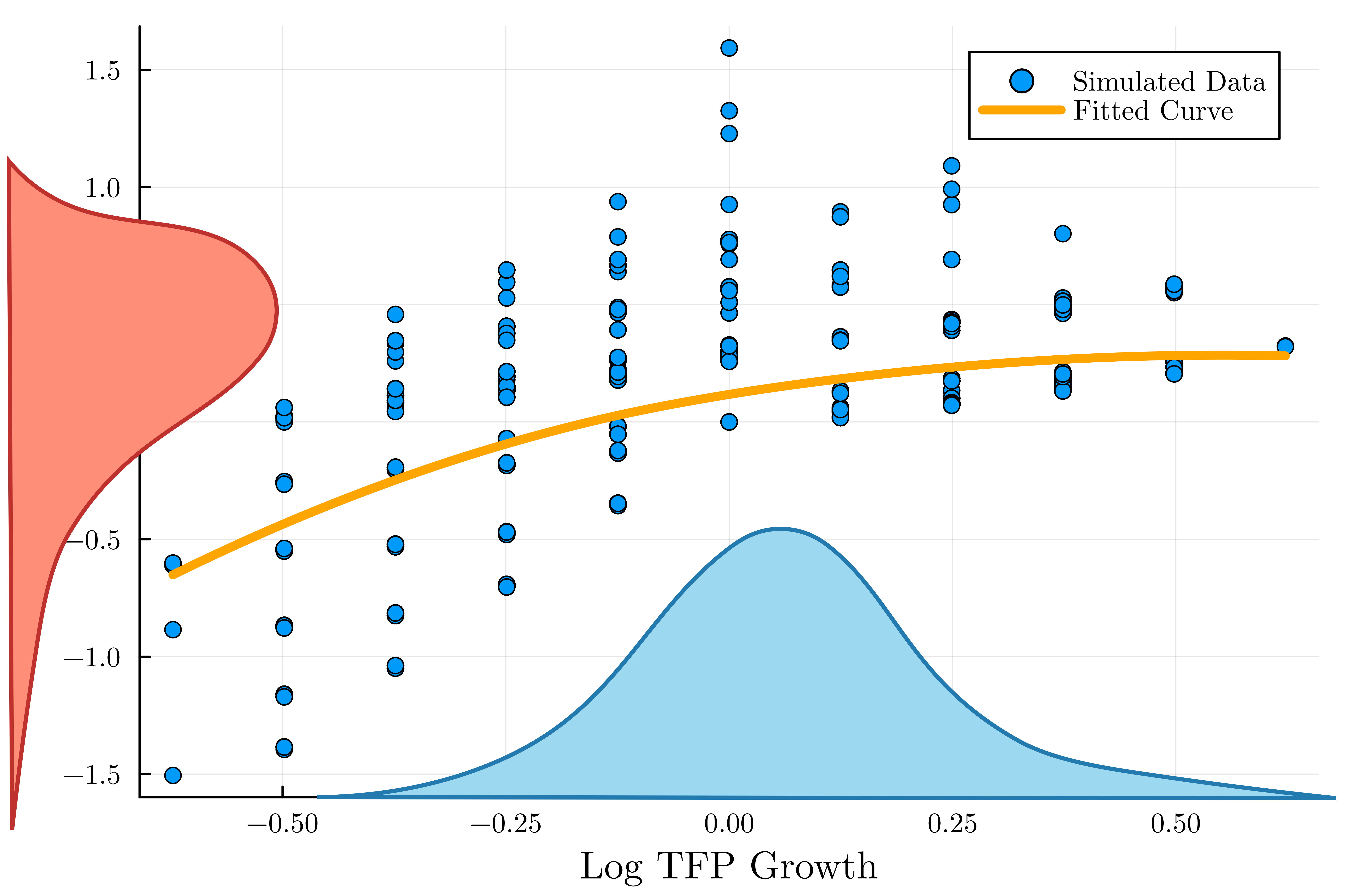
Concave Hiring Rule



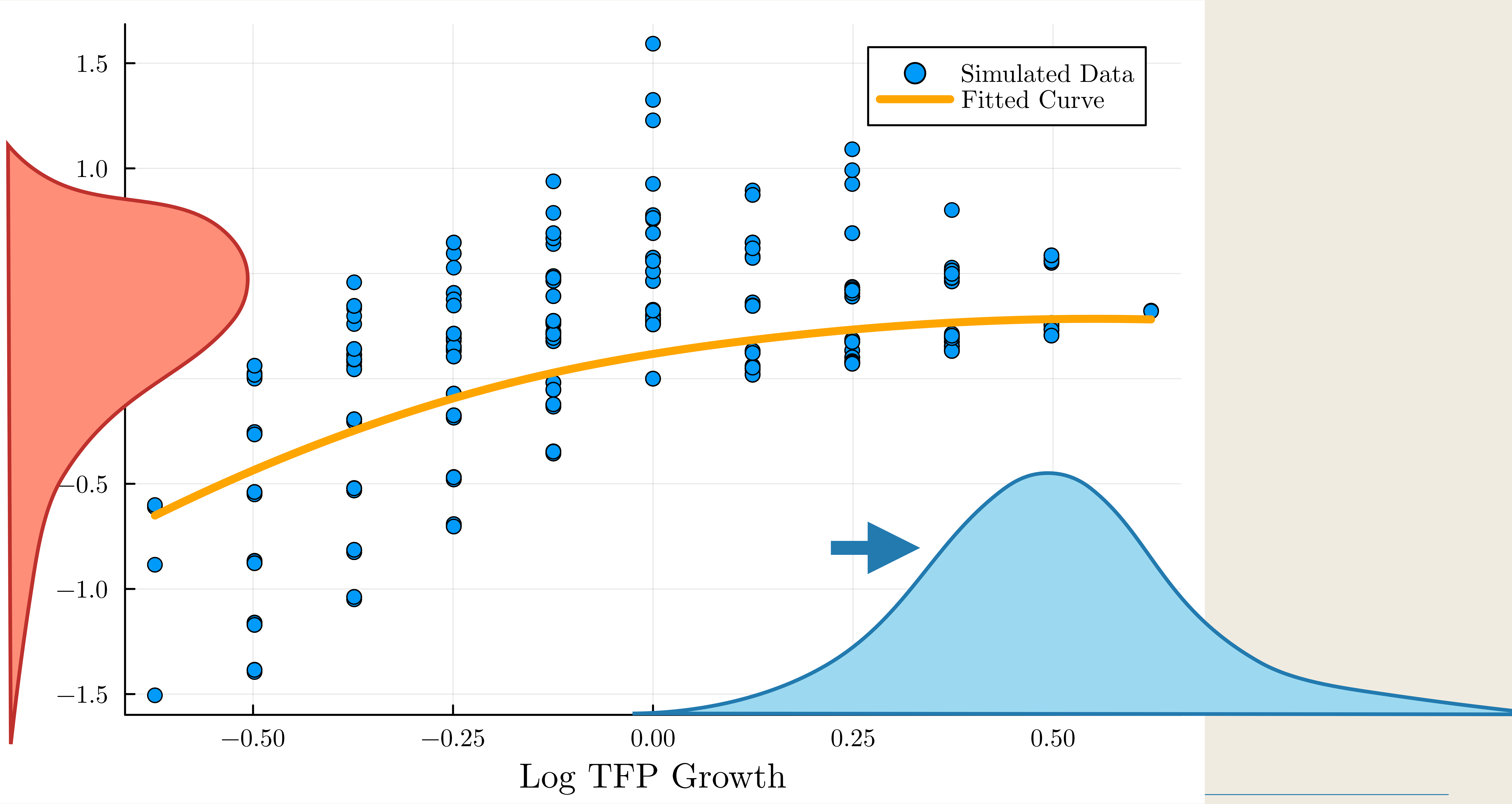
Distribution



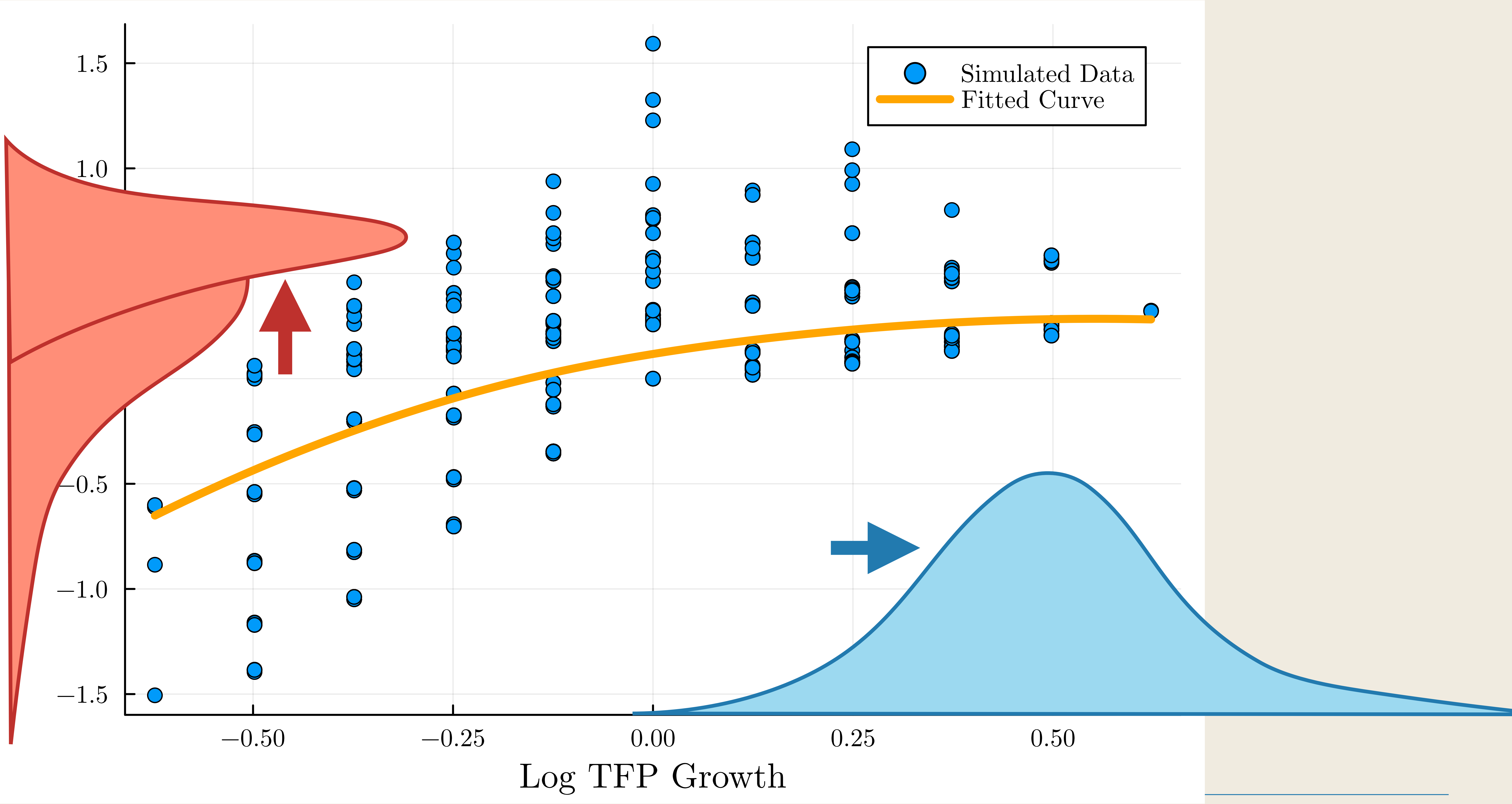
Positive Aggregate Shock



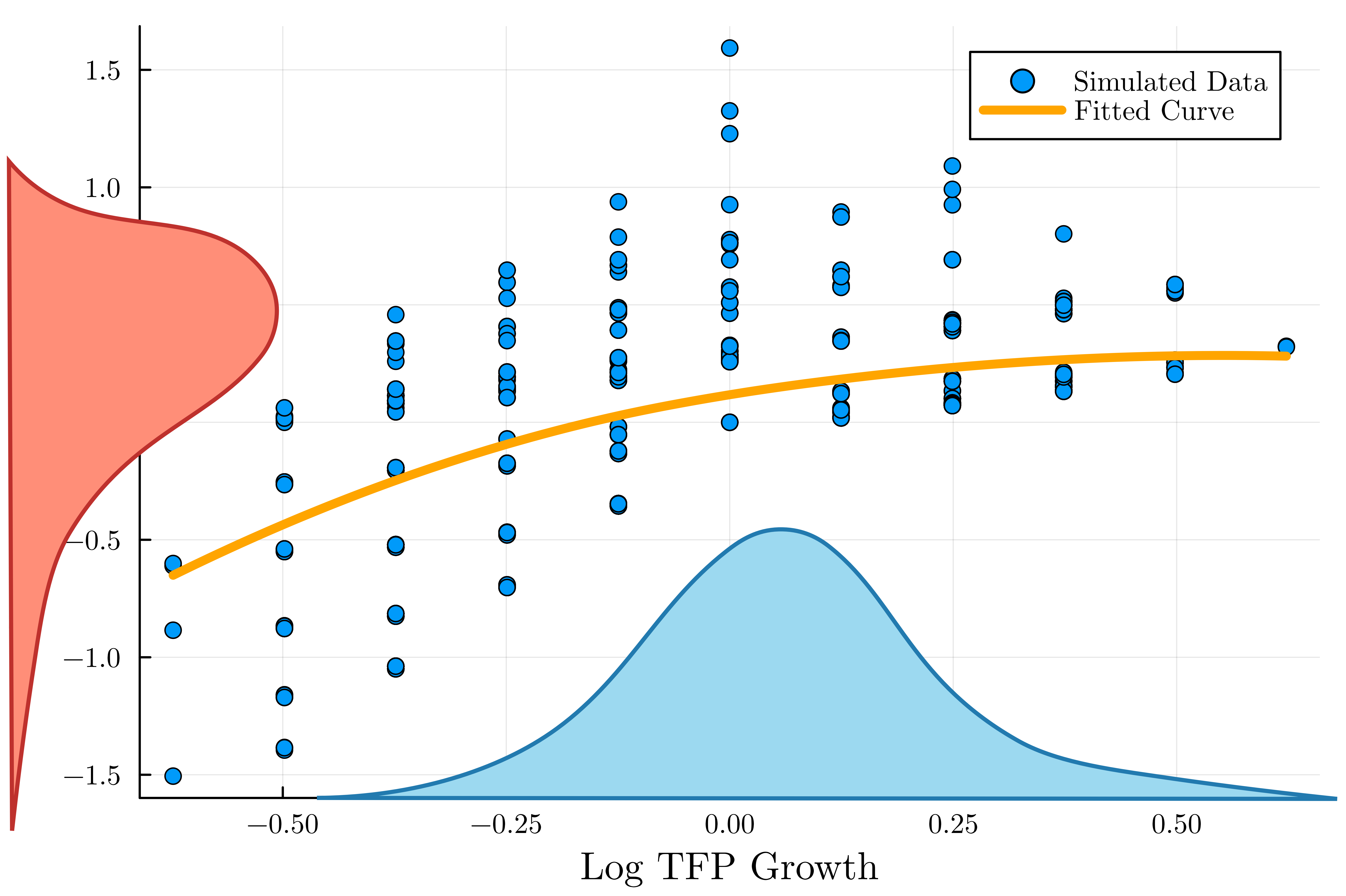
Positive Aggregate Shock



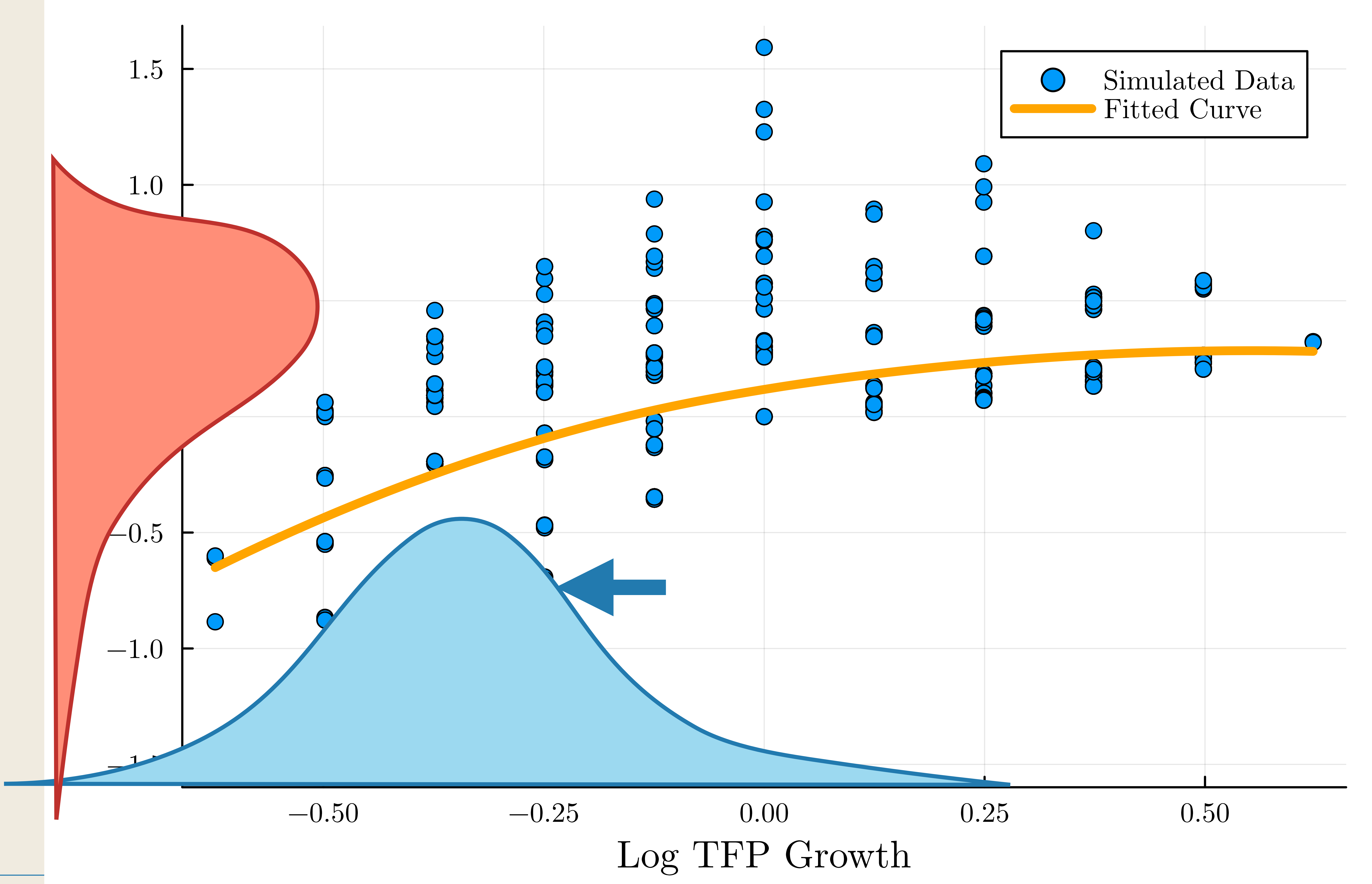
Positive Aggregate Shock



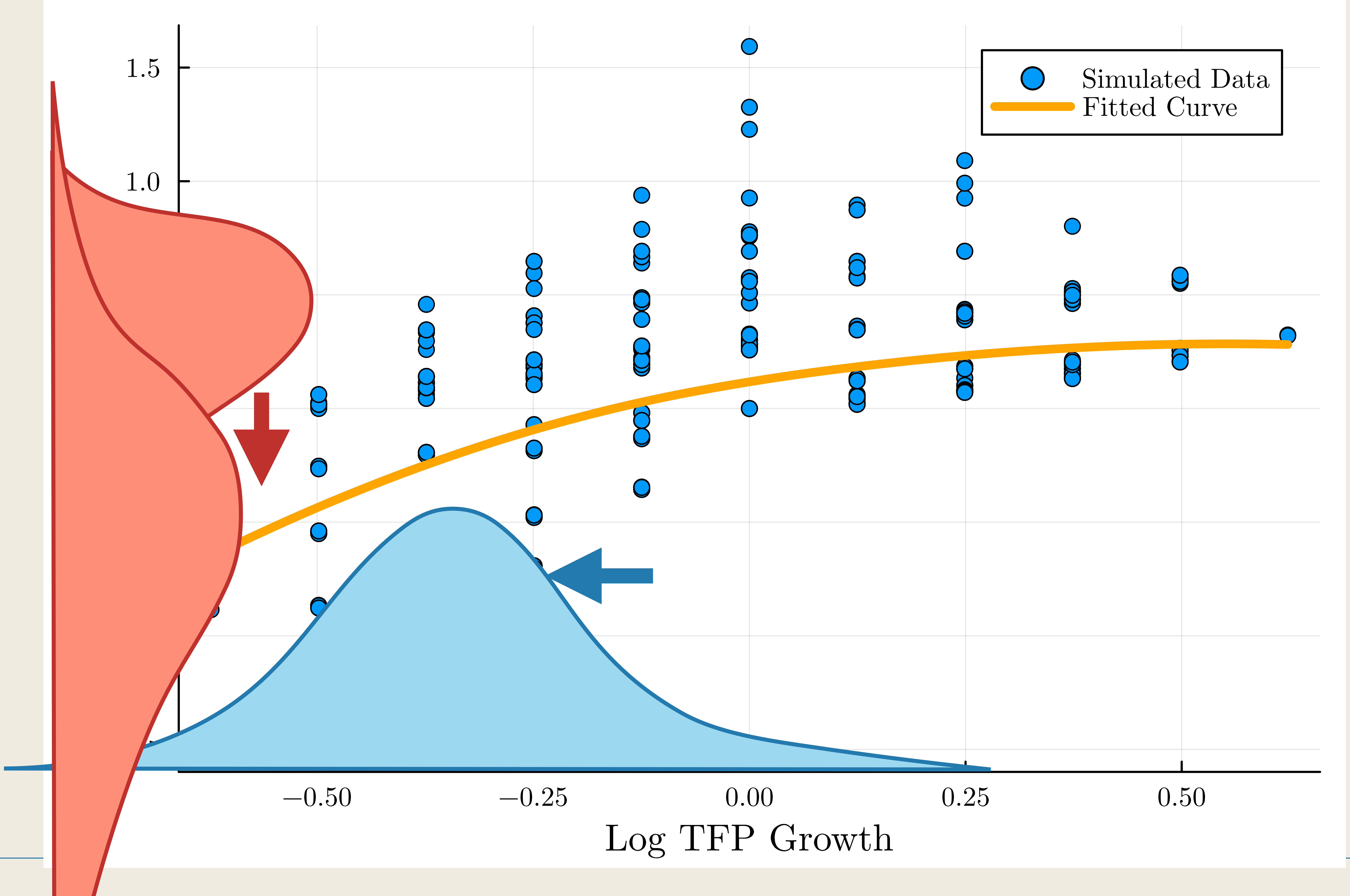
Negative Aggregate Shock



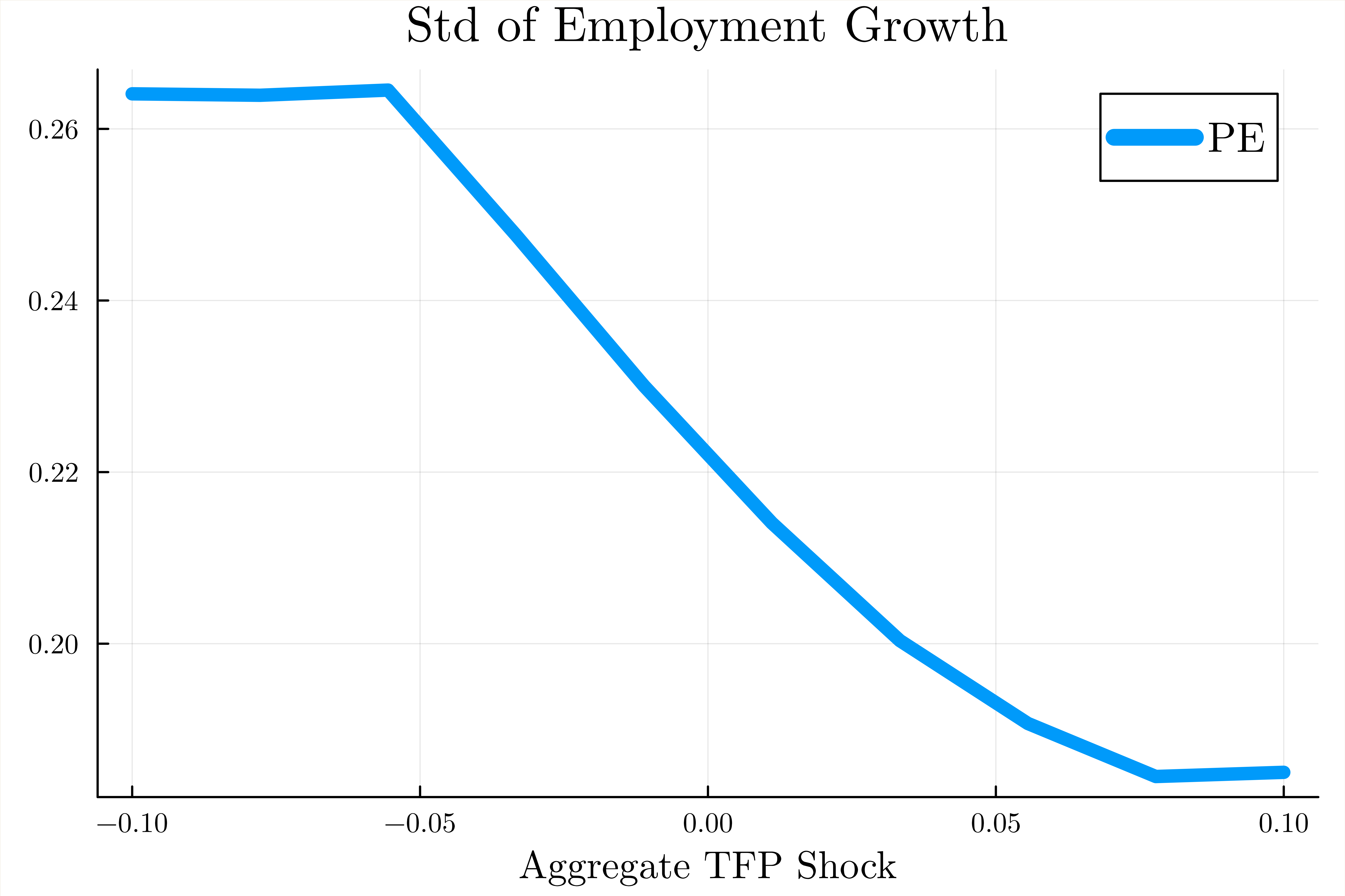
Negative Aggregate Shock



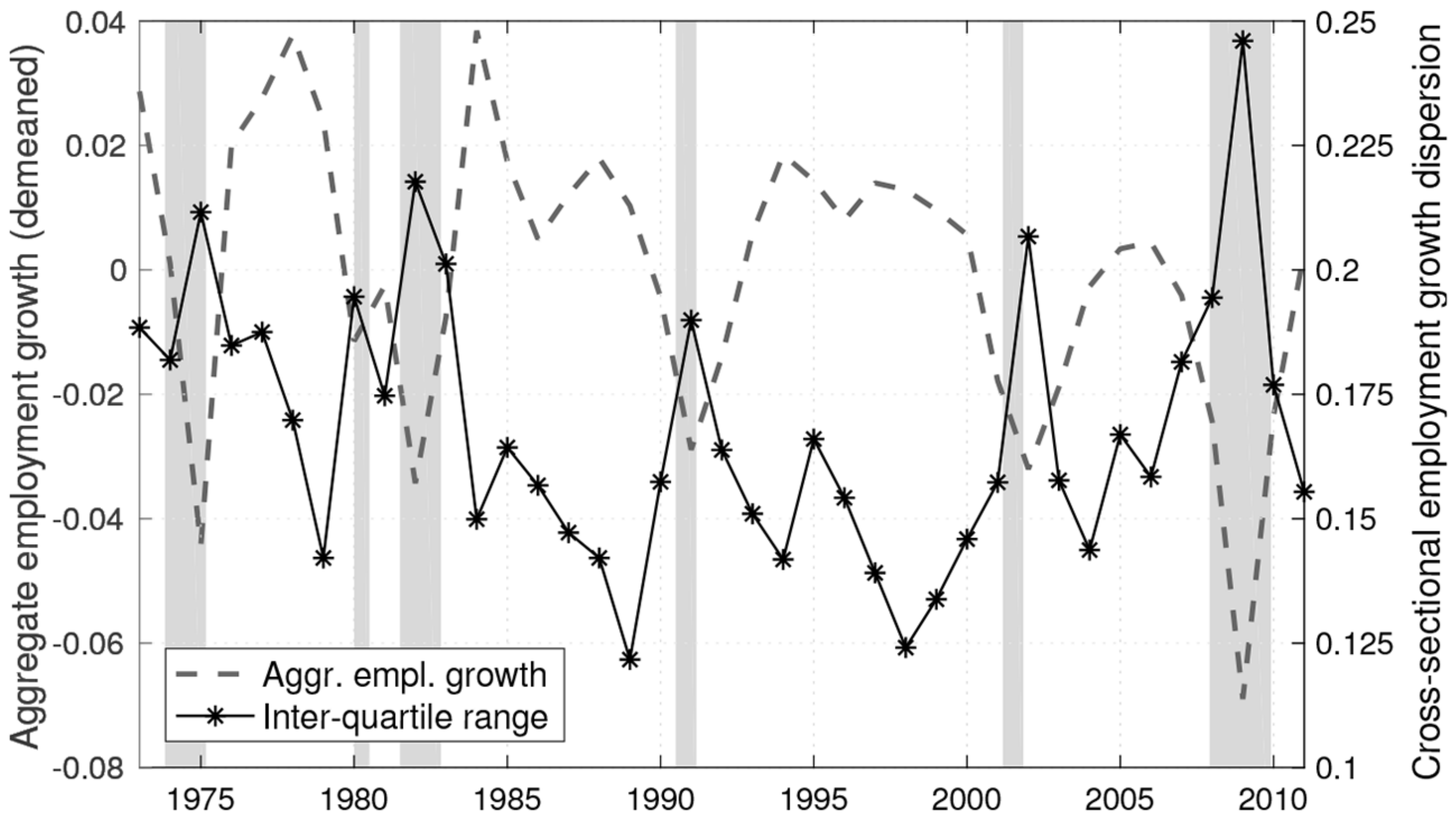
Negative Aggregate Shock



Partial Equilibrium

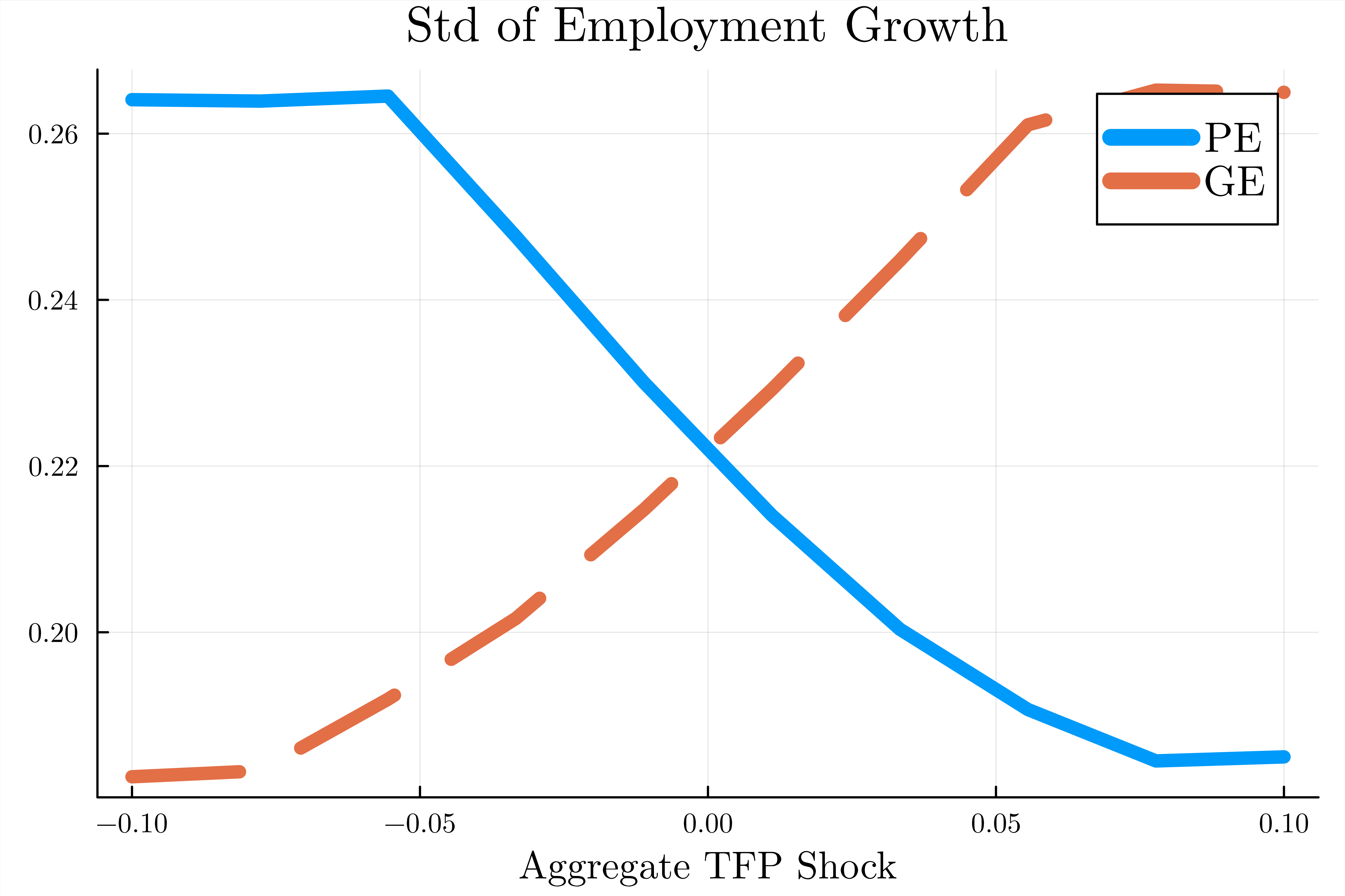


Countercyclical Volatility

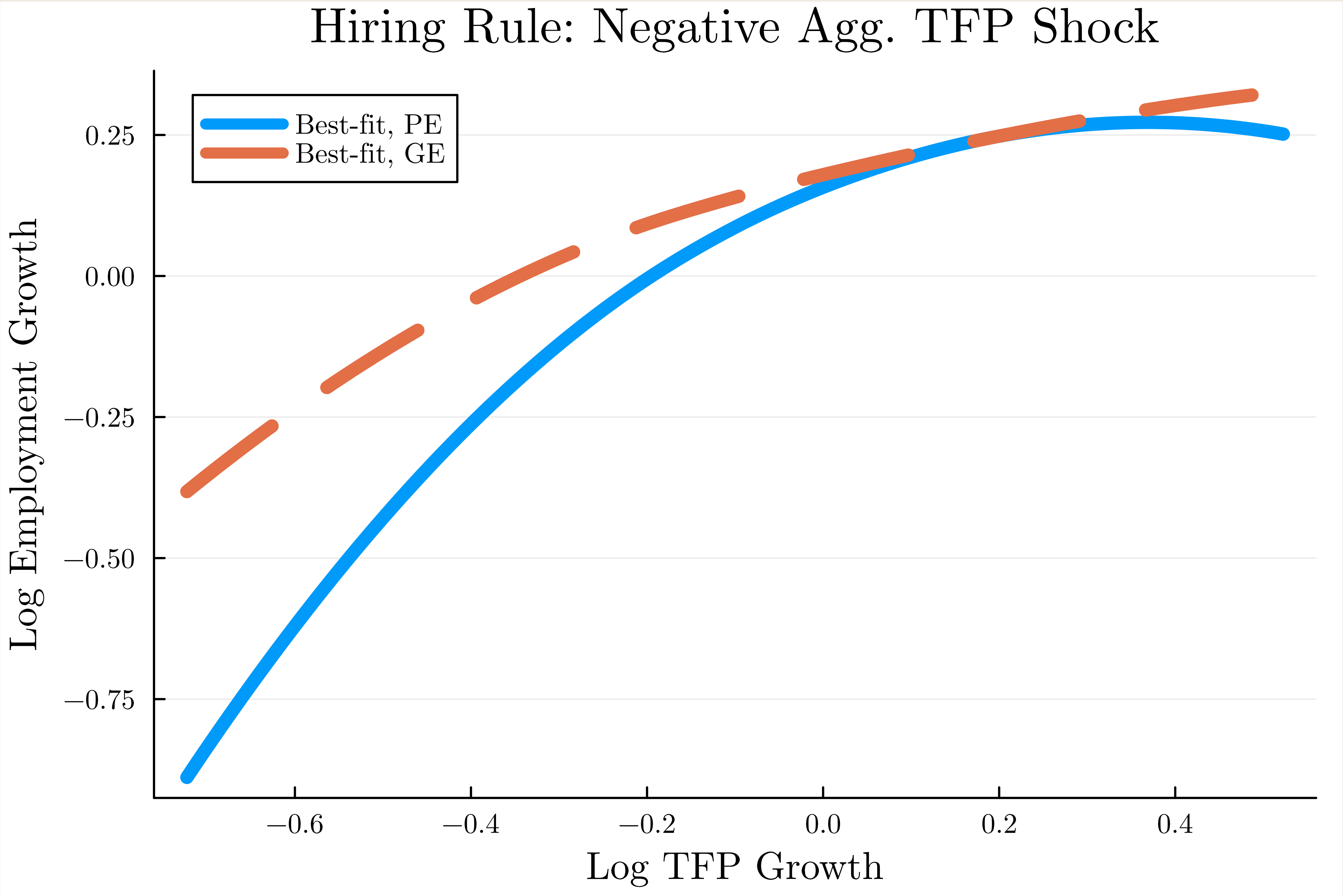


Source: Ilut, Kehrig & Schneider (2018)

Results Flip in GE...



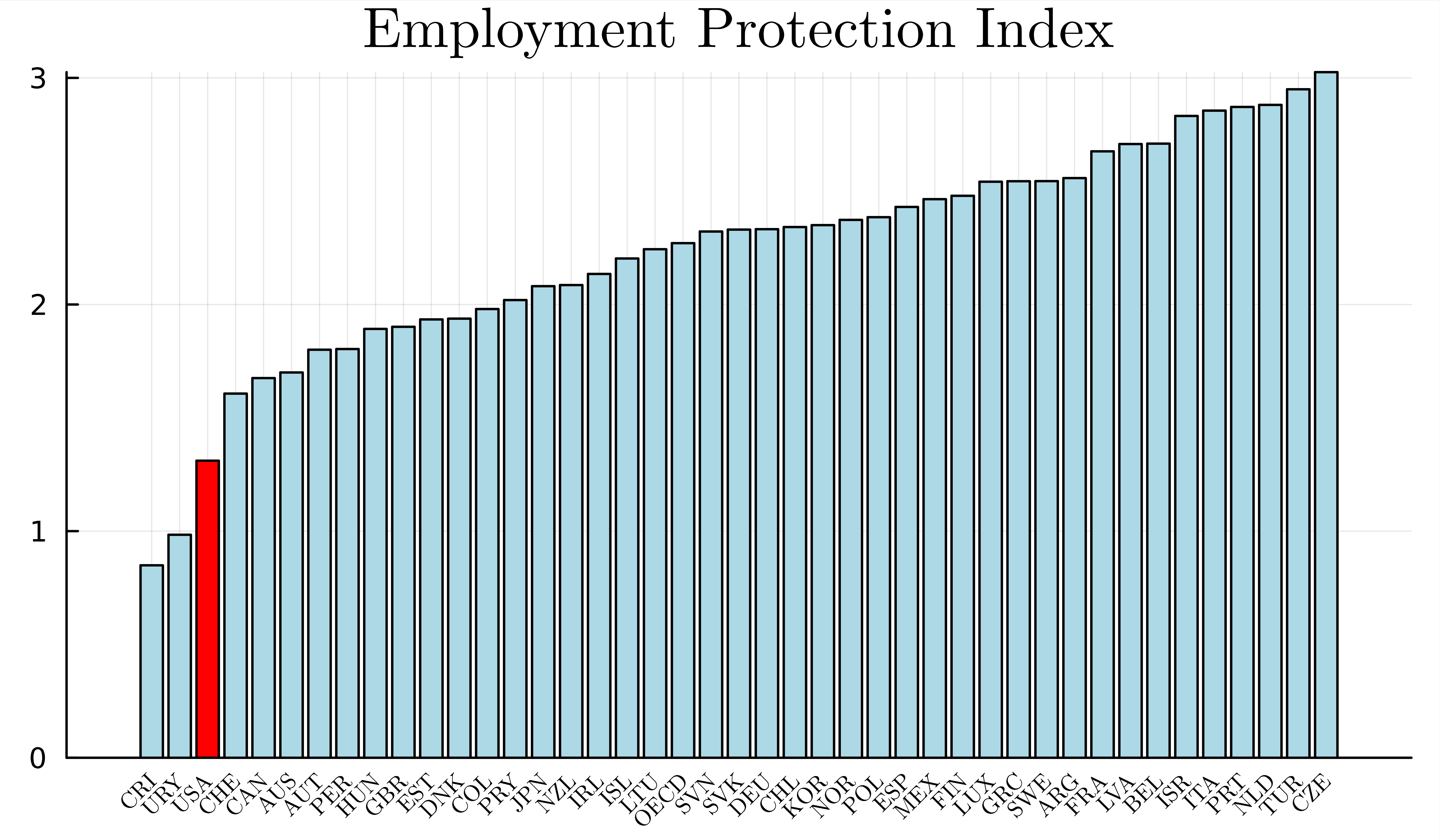
...because Hiring Rule Changes



- GE adjustment in wages:
  1. The flat part of the hiring rule is steeper in booms
  2. The steep part of the hiring rule is flatter in recessions

Firing Cost and Misallocation – Hopenhayn & Rogerson (1993)

Employment Protection Index



Data source: OECD

Question

- What is the cost of strict firing regulations?
- Suppose that in order to fire a worker, firms have to pay $\tau \times$ annual wage salary
  - US: $\tau = 0$
  - Europe: high τ
- Firing costs take the form of taxes
  - Distinct from adjsutment cost Φ, which is a part of the technology
- The collected tax revenue is rebated back to households as lump-sum transfers

Bellman Equation

- Bellman equation:

$$v(n_{-1}, z) = \max \left\{ v^*(n_{-1}, z),\ -\Phi(-(1-\delta), n_{-1}) - \tau(1-\delta)n \right\}$$

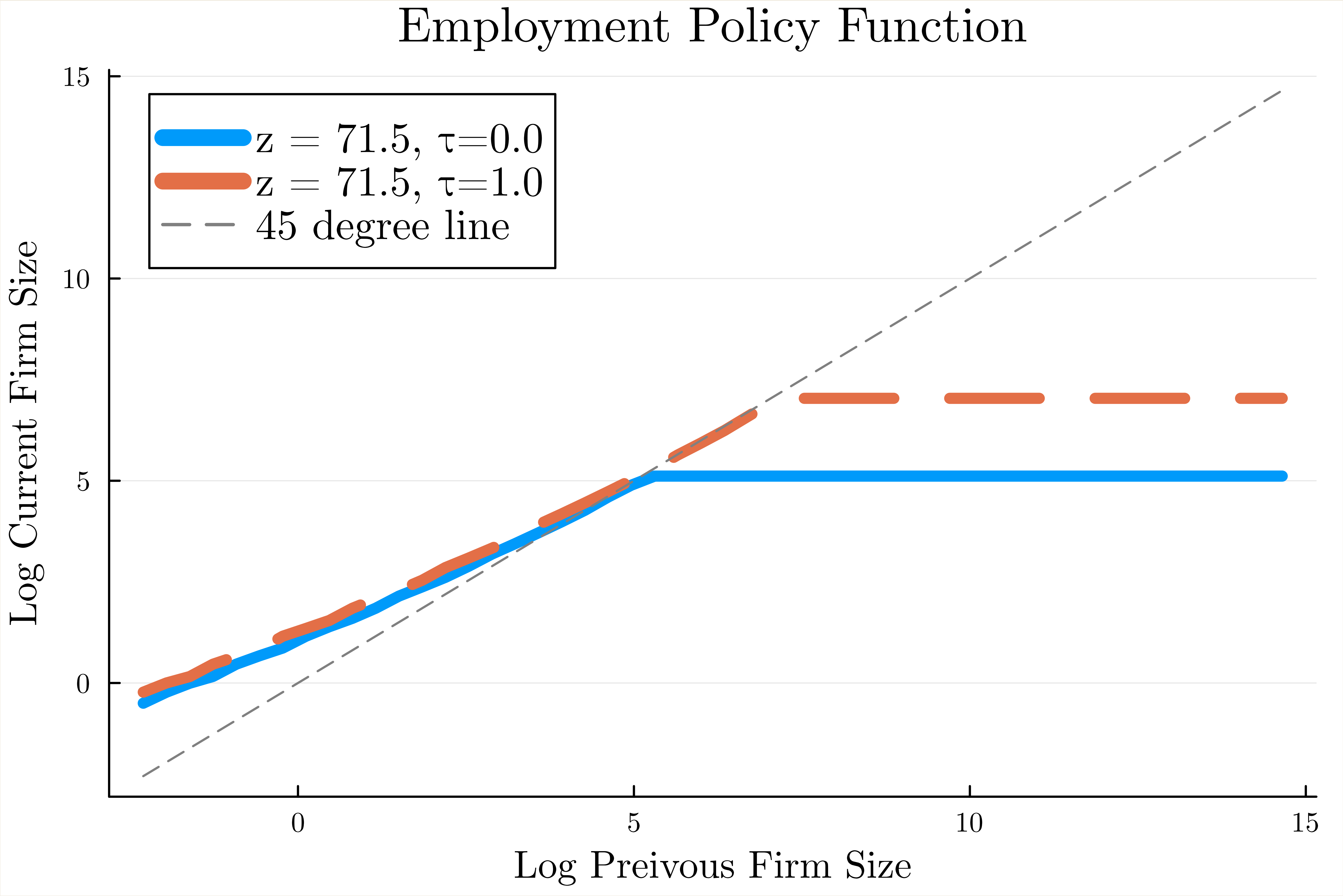
where v^* is the continuation value

$$v^*(n_{-1}, z) = \max_{h,n}\ f(n, z) - wn - c_f - \Phi(h, n_{-1}) + \tau w h \mathbb{I}[h < 0]\ + \beta \mathbb{E} v(n, z')$$

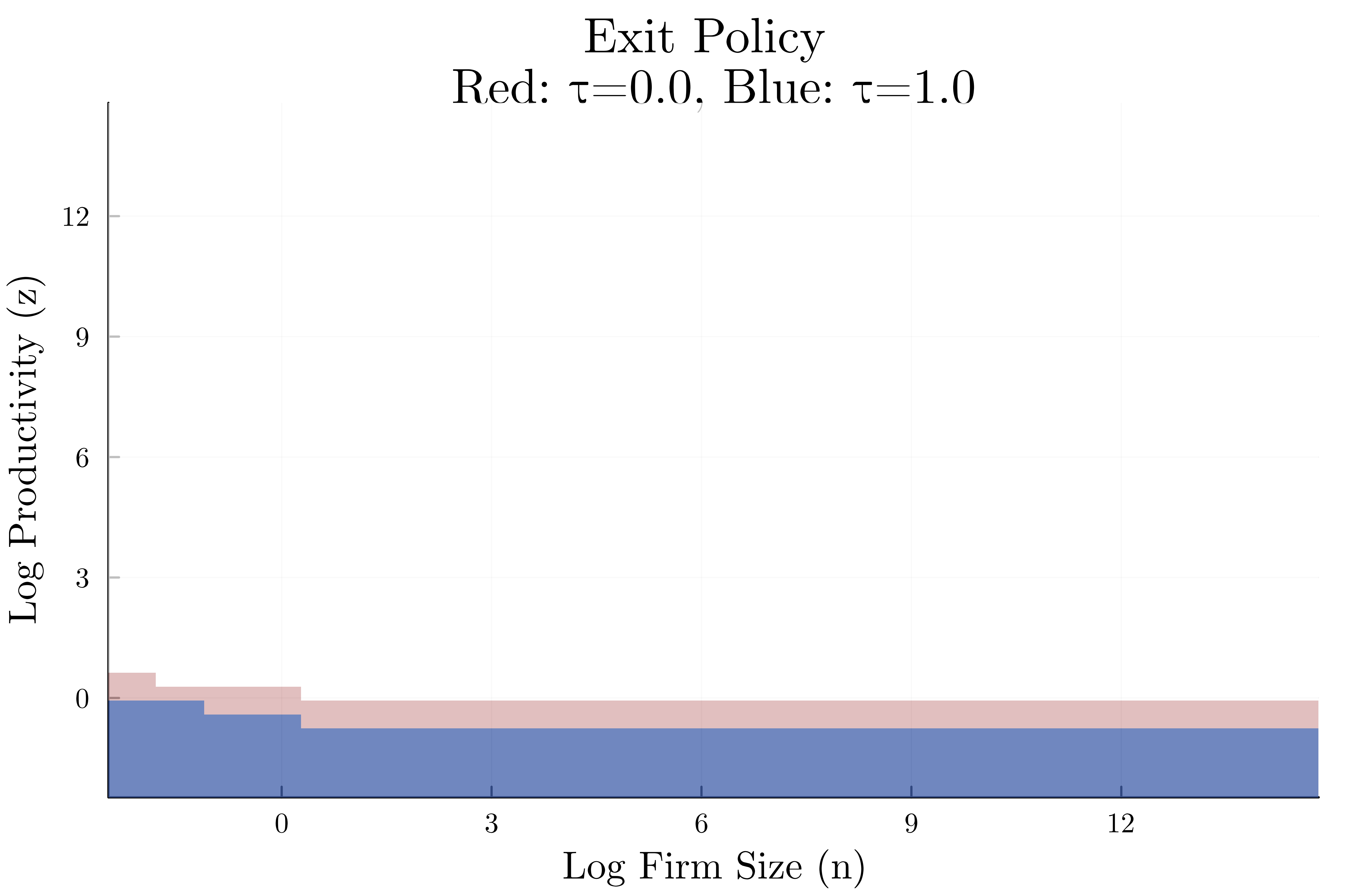
$$\text{s.t.} \quad n = n_{-1}(1 - \delta + h)$$

- The rest of the model is unchanged

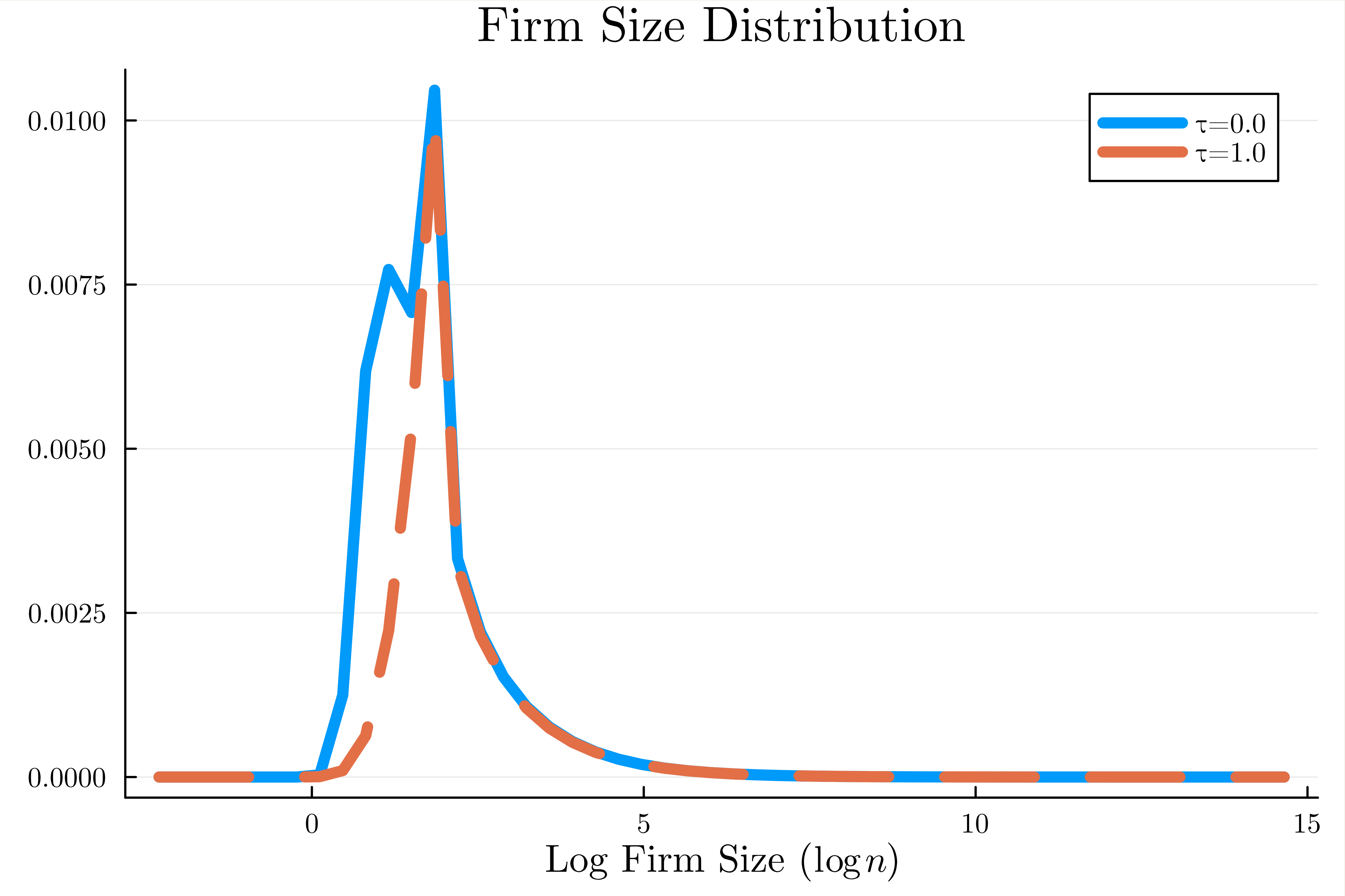
Firms Fire Less

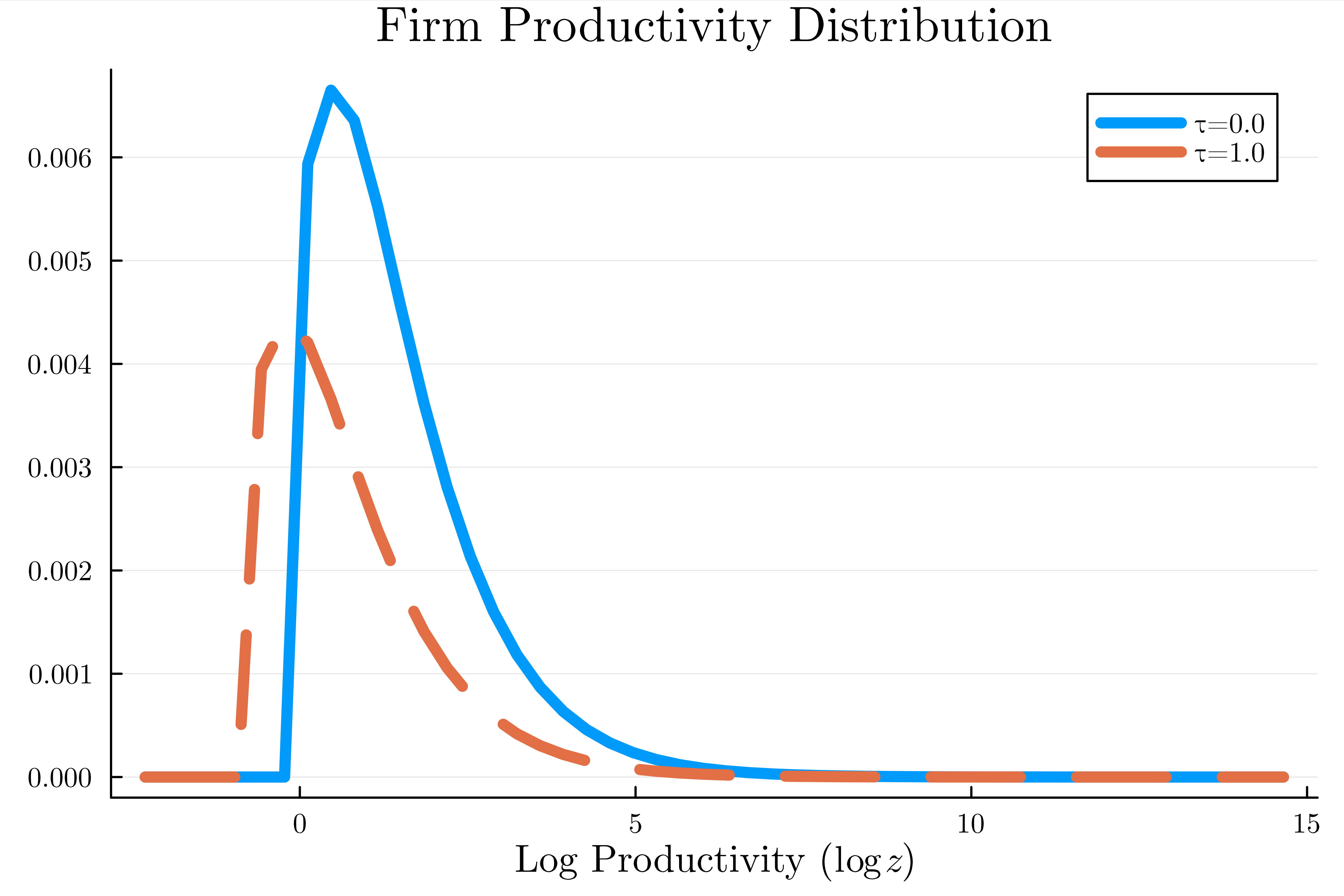


Firms Exit Less

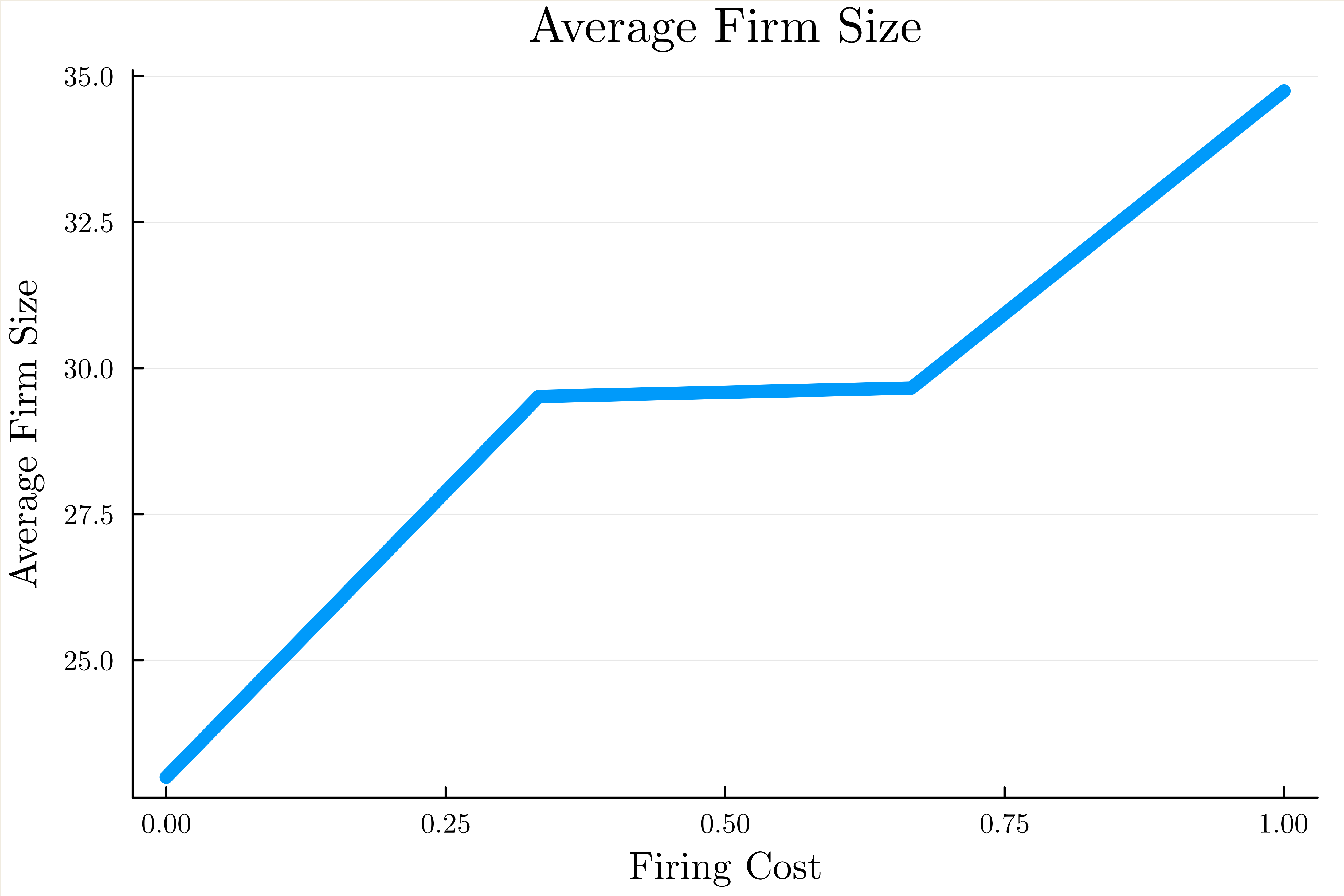


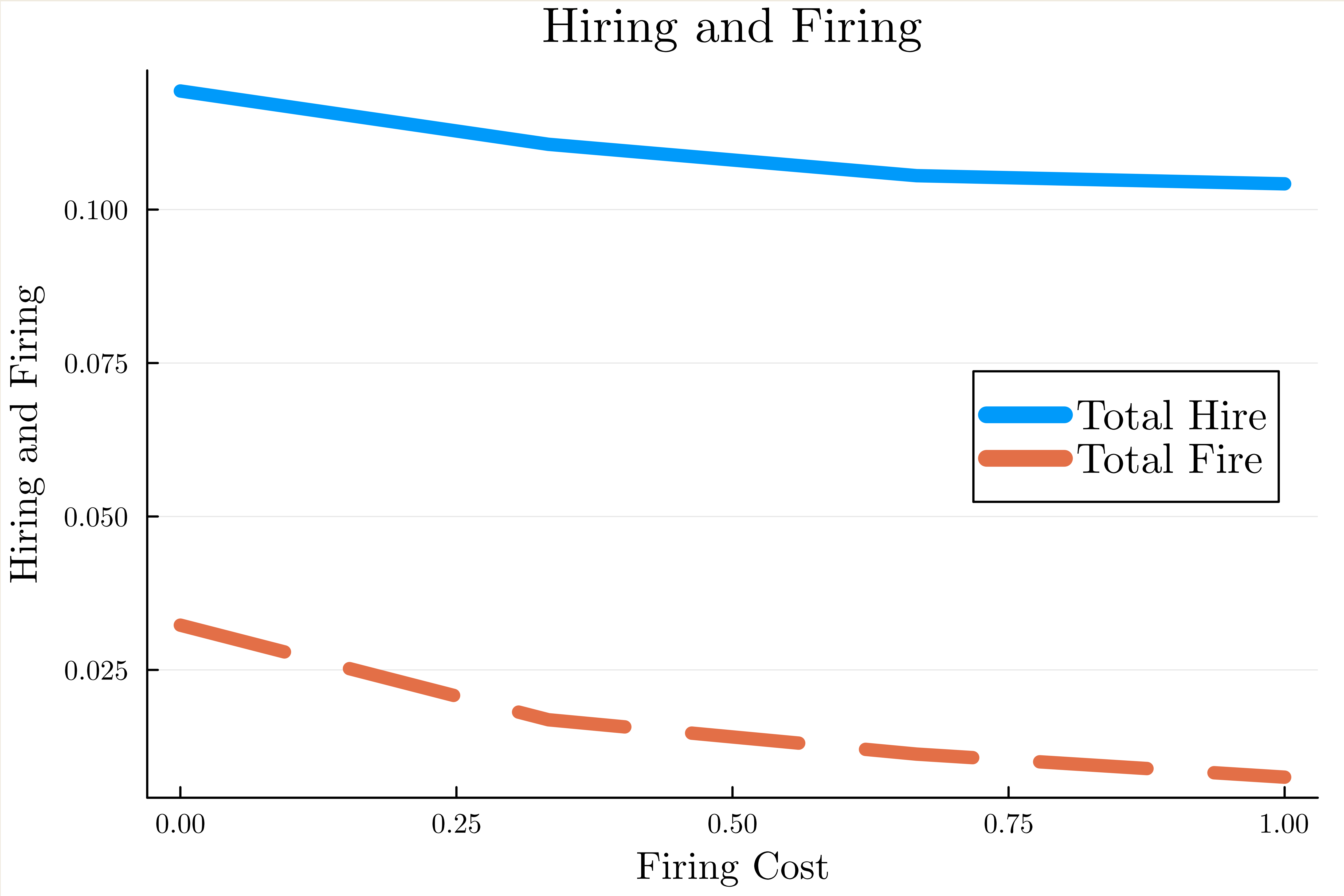
Firms Are Larger and Less Productive



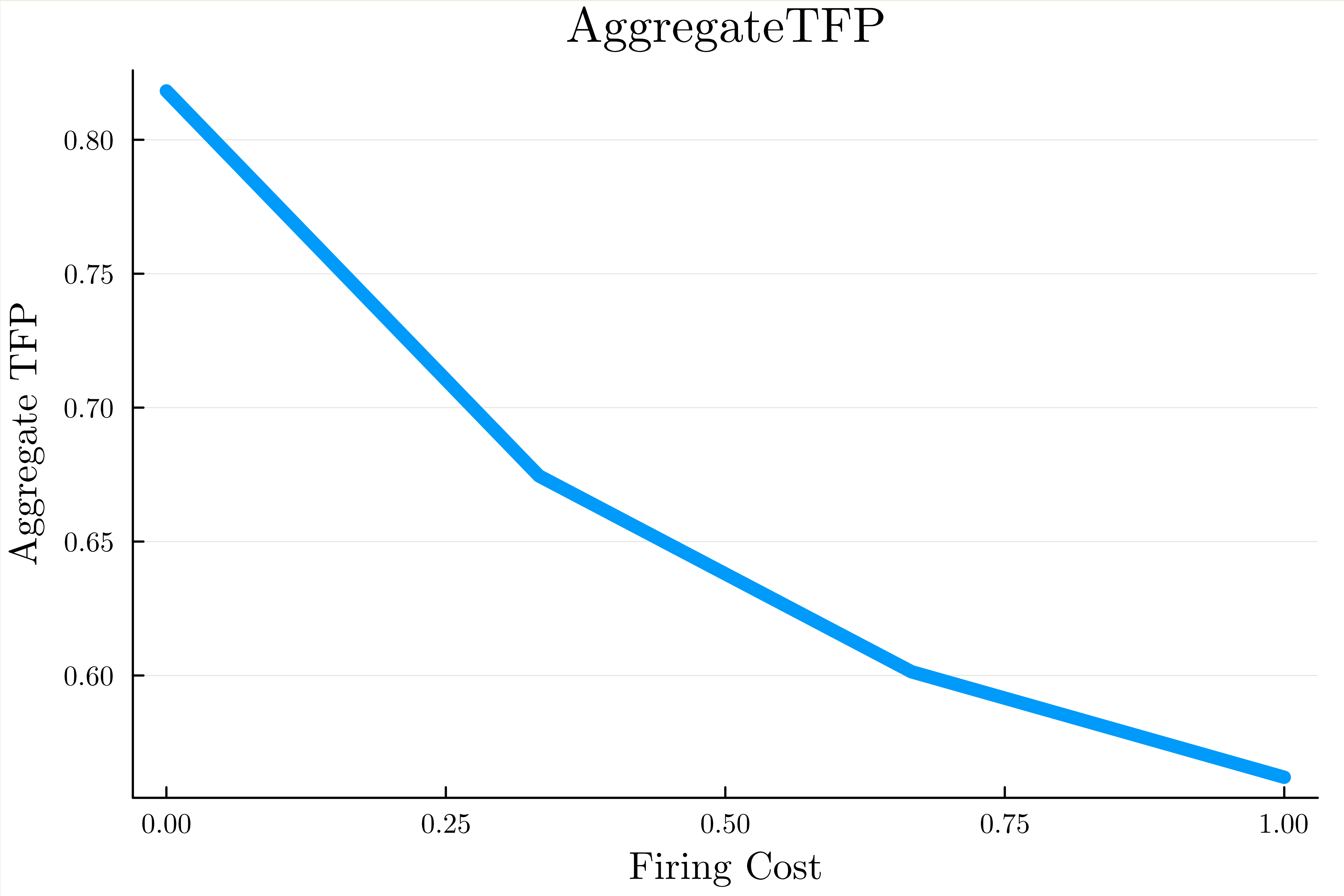


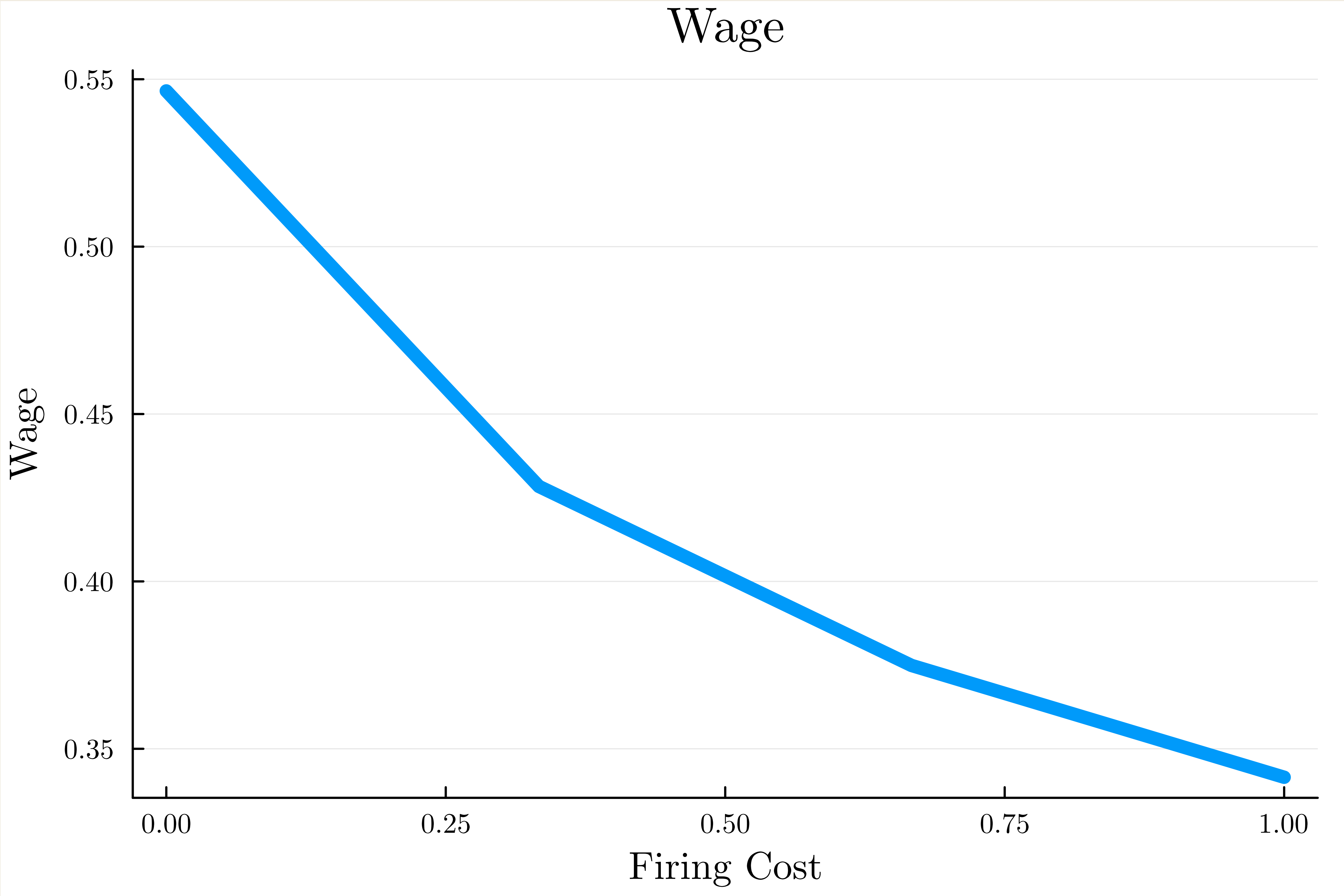
Firing Cost $\uparrow \Rightarrow$ Labor Reallocation$\downarrow$



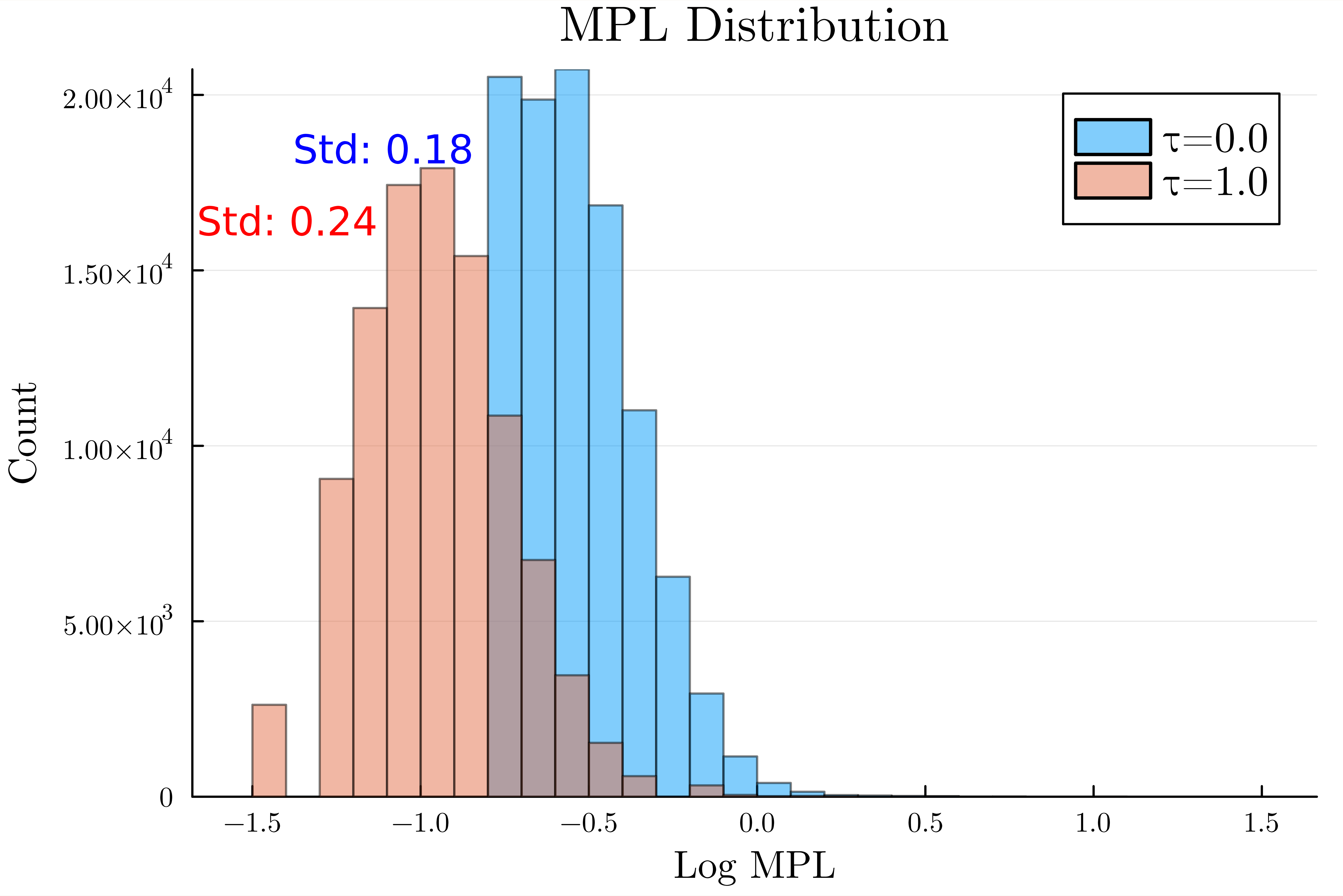


Firing Cost ↑ ⇒ TFP↓ & Wage↓





More Misallocation



Question

- Why, then, do so many countries regulate firing?
- An interesting idea is that firing cost could be a commitment device for firms (Karabay & McLaren, 2011; Créchet, 2024; Souchier, 2023)
- Normative aspects of labor market institutions seem underexplored

Broader Questions on Hopenhayn-Rogerson

- What is z?
  - Many argue z relates to the customer base
    (e.g., Einav, Klenow, Levin & Murciano-Goroff, 2022; Foster, Haltiwanger, Syverson, 2015; Argente, Fitzgerald, Moreira & Priolo, 2021)
- Is the entry really free?
  - Cagetti and De Nardi (2006): a model of entrepreneurship with financial friction
- Does the model get the age distribution right?
  - The average age of Walmart/Amazon size class in the model is 100 years
  - Walmart is 60 years old, and Amazon is 30 years old
- In the model, large firms are large just by luck (ex-ante homogenous). Are they?
  - Hurst & Pugsley (2011) and Pugsley, Sedláček & Sterk (2020) argue not

Non-Parametric Identification of Misallocation

– Carrillo, Donaldson, Pomeranz, & Singhal (2023)

What is the Cost of Misallocation?

- How large is the cost of misallocation in the data?
- Let us step back and consider a **static** model with a **fixed mass** of firms
- Each firm i produces using

$$y_i = f_i(n_i) \tag{7}$$

- The efficient allocation solves

$$Y^* \equiv \max_{\{n_i\}} \int f_i(n_i)di$$

$$\text{s.t.} \quad \int n_i di = L$$

- The solution features equalization of MPL:

$$f_i'(n_i) = w \quad \text{for all } i$$

Variance of MPL as a Sufficient Statistics

- Take arbitrary allocation $\{n_i\}$. Up to a second order around the efficient allocation

$$\log Y - \log Y^* \approx -\frac{1}{2}\mathrm{Var}_{\lambda/\epsilon}\left[\log(MPL_i)\right]$$

where $\mathrm{Var}_{\lambda/\epsilon}[X_i] = \sum_{i=1}^N \frac{\lambda_i^*}{\epsilon_i^*}\left(X_i - \mathbb{E}_{\lambda/\epsilon}[X_i]\right)^2$, $\mathbb{E}_{\lambda/\epsilon}[X_i] = \sum_{i=1}^N \frac{\lambda_i^*}{\epsilon_i^*}X_i$, $MPL_i = f_i'(n_i)$,

$\lambda_i = w_i^* n_i^* / Y^*$, and $\epsilon_i \equiv -\frac{d \log MPL_i}{d \log n_i}$.

- (Weighted) variance of MPL is the key moment for the cost of misallocation
- Testing the presence of misallocation $\Leftrightarrow$ testing $\mathrm{Var}(MPL_i) = 0$
- How do we get the distribution of MPL?

1. Assume $f_i(n_i) = Z_i n_i^{\alpha}$, and then $MPL_i = \alpha \frac{y_i}{n_i}$ (Hsieh & Klenow, 2009)
2. Nonparametrically identify the distribution of MPL (Carrillo et al. 2023)

Nonparametric Identification

- Taking the first-order approximation of equation (7),

$$\Delta y_i = \beta_i \Delta n_i + \varepsilon_i$$

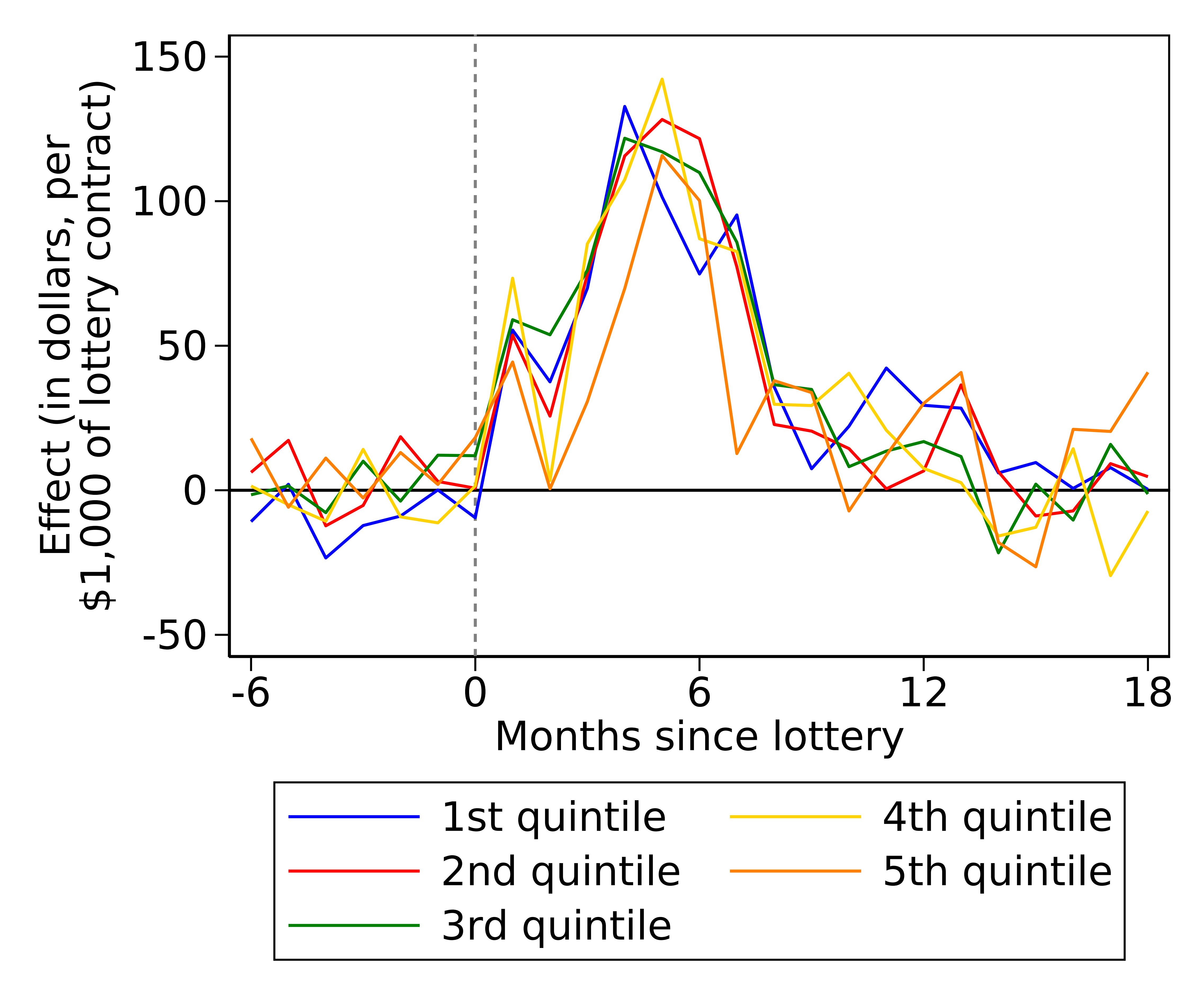
  - ε_i: technology shocks (i.e., changes in $f_i(\cdot)$)
  - $\beta_i = f_i'(n_i) = MPL_i$: treatment effect of exogenously increasing n_i on y_i
- With suitable instruments Z_i that exogenously shift n_i, $\mathbb{E}[\beta_i^k]$ ($k = 1,2,\ldots$) are identified (Masten & Torgovitsky, 2016)

Empirical Implementation

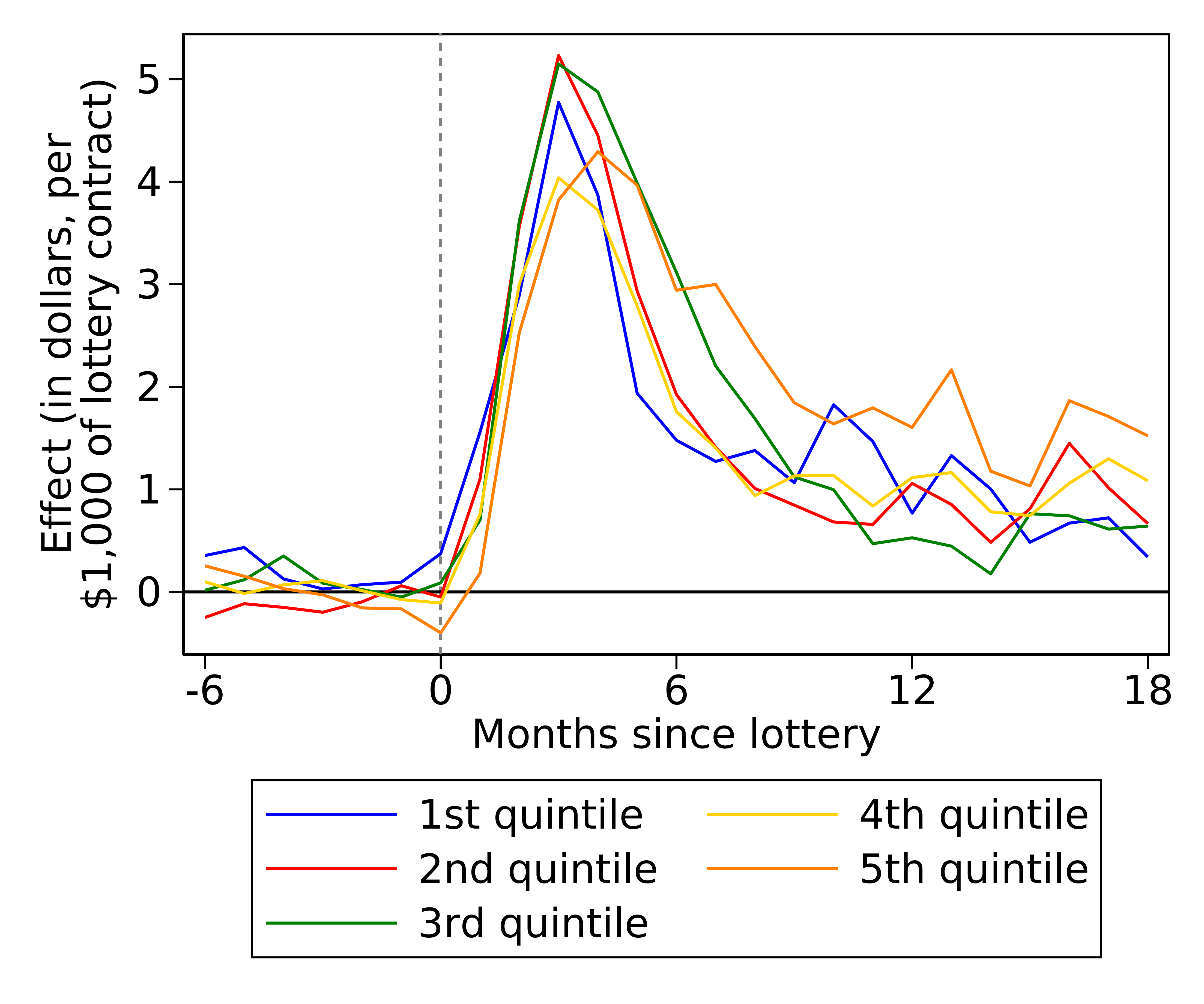
- Construction sector in Ecuador, 2009-2014
- Public construction projects were allocated through a randomized lottery
- Lottery serves as an ideal instrument
  - **exogeneity**: orthogonal to technology shocks ε_i or MPL_i
  - **relevance**: winning a lottery does shift n_i

Heterogenous Treatment Effects by Firm Size?

Sales



Labor Inputs



Small Cost of Misallocation

Table 4: Estimated Cost of Misallocation

	$\mathbb{E}_{\bar{\lambda}}[\bar{\mu}]$ (1)	$\mathbb{V}ar_{\bar{\lambda}}[\bar{\mu}]$ (2)	$\frac{\Delta W}{W}$ (3)
Panel (a): IVCRC estimates			
Baseline	1.126	0.014	0.016
	[1.093, 1.161]	[0, 0.341]	[0, 0.261]
Panel (b): Alternative procedure assuming common scale elasticities			
Constant returns-to-scale ($\gamma = 1$)	1.240	0.611	0.479
	[1.223, 1.257]	[0.544, 0.730]	[0.427, 0.572]

- Assume $\epsilon_i = 3$ for all i
- The welfare cost of misallocation is 1.6%
- Hsieh-Klenow type calculation implies 48% of welfare loss in the same dataset

Question

Does this approach capture the full story of misallocation in Hopenhayn-Rogerson?

– No, for two reasons:

1. Entry & exit
2. Dynamics

Two Open Questions

1. Entry & exit
   - In HR, distortion $\Rightarrow$ fewer firms enter
     - This reduces the mean of *MPL* (not just about variance!)
   - In HR, exit threshold changes $\Rightarrow$ the selection of the active firms change
     - This again shifts the mean of *MPL*
2. Dynamics
   - Laissez-faire of Hopenhayn-Rogersion with labor adjustment costs is efficient
   - But, *MPL* is not equalized in a static sense
   - Firms hire workers until
     (present discounted value of hiring a worker) = (hiring cost today)
   - Hiring a worker is an investment

How do we incorporate these two issues without imposing strong assumptions?

Hopenhayn-Rogerson with Search Frictions

– Based on McCrary (2022)

Search Friction

- Only unemployed workers search for a job
- Firms need to post vacancies to hire workers
- Assume firing is costless
  - Can easily extend to the case with costly firing
- Each vacancy meets an unemployed worker with probability λ^F

Value and Policy Functions

- Policy functions:
  $\chi(n,z) \in \{0,1\}$: exit, $w(n,z)$: wage, $s(n,z)$: firing, $v(n,z)$: vacancy
- The value function of the firm:

$$J(n,z) = (1-\chi(n,z))J^*(n,z)$$

$$J^*(n,z) = f(n,z) - w(n,z)n - c_f - \Phi(v(n,z),n) + \beta \mathbb{E} J(n',z')$$

$$\text{s.t.} \quad n' = (1-\delta - s(n,z))n + \lambda^F v(n,z)n$$

- The value function of the workers:

$$W(n,z) = (1-\chi(n,z))\left[w(n,z) + \beta(1-\delta-s(n,z))\mathbb{E}W(n',z') + \beta(\delta + s(n,z))U\right] + \chi(n,z)U$$

Wage Barganing

- In each period, a coalition of workers and a firm bargains to determine w, v, s, χ
- We assume Nash bargaining with worker bargaining power γ:

$$\max_{\chi(n,z),v(n,z),w(n,z),s(n,z)} \left(W(n,z)n - Un\right)^{\gamma} J(n,z)^{1-\gamma} \tag{1}$$

- Noting $\frac{\partial W(n,z)}{\partial w} = -\frac{\partial J(n,z)}{\partial w}$, FOC w.r.t. w gives

$$\left(W(n,z)n - Un\right) = \gamma S(n,z), \quad J(n,z) = (1-\gamma)S(n,z) \tag{2}$$

where $S(n,z) \equiv J(n,z) + (W(n,z) - U)n$ is the joint match surplus

- Substituting (2) back into (1), we have

$$\max_{\chi(n,z),v(n,z),s(n,z)} \gamma^{\gamma}(1-\gamma)^{1-\gamma} S(n,z)$$

- **Result**: vacancy, firing, and exit policies maximize joint match surplus

Bellman Equation for Surplus

- The joint match surplus solves

$$S(n,z) = \max\{S^*(n,z), 0\} \tag{3}$$

where

$$S^*(n,z) = \max_{v,s,n'} f(n,z) - c_f - \Phi(v,n) - (1-\beta)Un + \beta\left(1 - \frac{\gamma\lambda^F v}{(1-\delta-s)+\lambda^F v}\right)\mathbb{E}S(n',z')$$

$$\text{s.t.} \quad n' = (1-\delta-s)n + \lambda^F vn$$

- Why is there extra discounting $1 - \frac{\gamma\lambda^F v}{(1-\delta-s)+\lambda^F v}$?
  - New hires will "steal" a portion of next-period surplus $S(n', z')$

Mathing

- The total of number of mathces are dictated by a CRS matching function:

$$\mathscr{M}(u, v)$$

- The meeting prob. are

$$\lambda^U = \frac{\mathscr{M}(u, v)}{u} \equiv \mathscr{M}(1,\theta), \quad \lambda^F = \frac{\mathscr{M}(u, v)}{v} \equiv \mathscr{M}(1/\theta,1) \tag{4}$$

where $\theta \equiv v/u$ is the market tightness and

$$u = 1 - \int\int ng(n, z)dndz$$

$$v = \int\int v(n, z)g(n, z)dndz$$

Rest of the Model

- The value of unemployment is

$$U = b + \beta\lambda^U \int\int \frac{v(n,z)}{v} \left[\gamma S(n'(n,z),z') \frac{1}{n'(n,z)} + U \right] g(n,z)dndz \tag{5}$$

where b is the UI benfit, and $g(n,z)$ is the mass of firms with (n,z)

- The free-entry condition is

$$\int (1-\gamma)S(n_0,z)\psi_0(z)dz = c_e \tag{6}$$

- The steady-state distribution satisfies

$$g(n',z') = \int\int (1-\chi(n,z))\Pi(z'|z)\mathbb{I}[n'(n,z) = n']g(n,z)dndz + m\mathbb{I}[n' = n_0]\psi_0(z') \tag{7}$$

Equilibrium Definition

A steady-state recursive equilibrium consists of

- value and policy functions: $\{S(n,z), n'(n,z), v(n,z), s(n,z), \chi(n,z)\}$
- the market tightness and meeting probabilities: $\{\theta, \lambda^U, \lambda^F\}$
- the value of unemployment: U
- the steady state distribution: $g(n,z)$

such that

1. value and policy functions solve the Bellman equation (3)
2. the market tightness and the meeting probabilities satisfy (4)-(5)
3. the value of unemployment satisfies (6)
4. the steady state distribution satisfies (7)

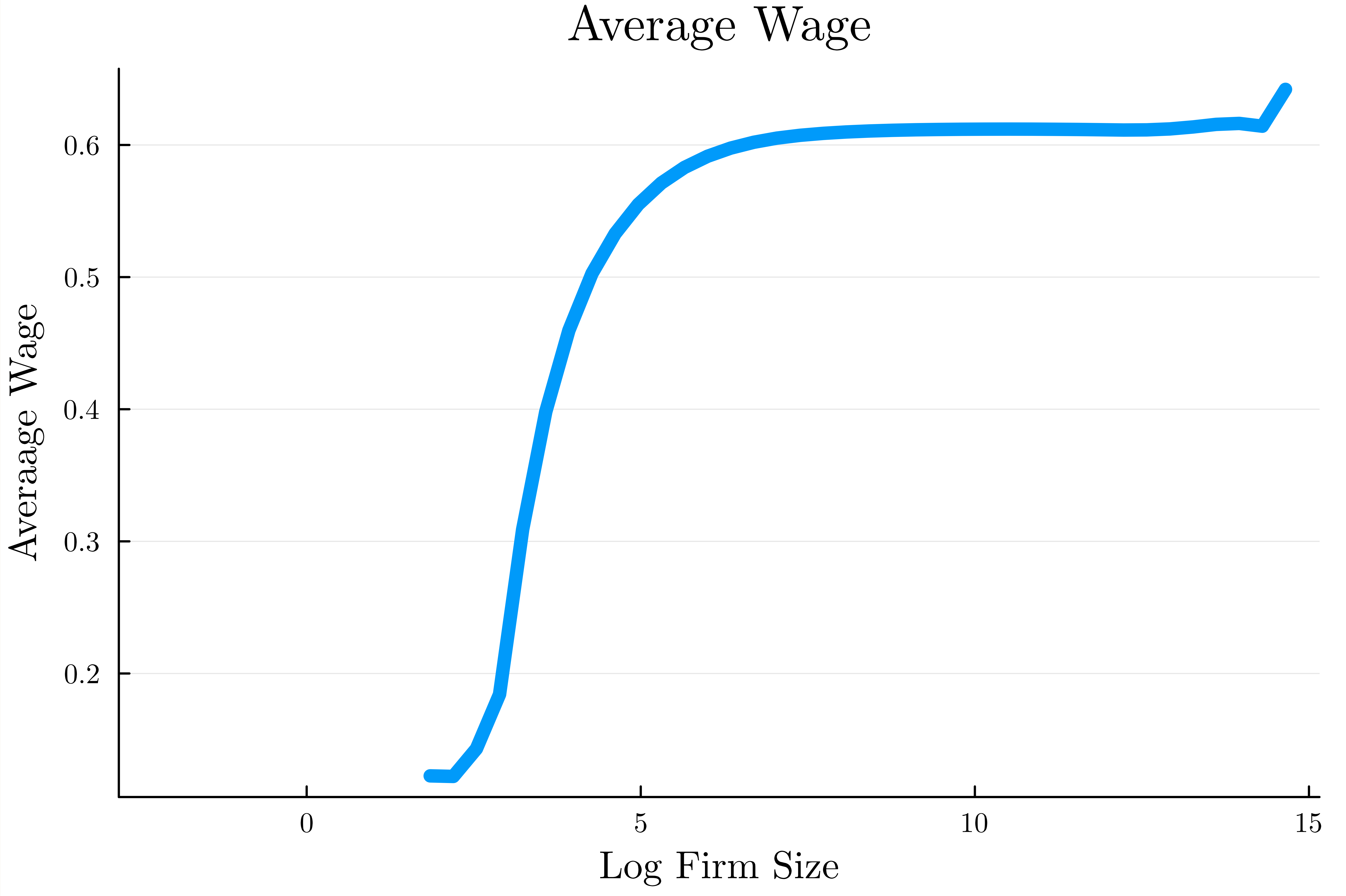
Implications for Firm Wage

- Search frictions allow us to talk about the firm wage
  - This wasn't possible in Hopenhayn-Rogerson
- The wage of firm (n, z) is given by

$$w(n,z) = \gamma \frac{1}{n} \left\{ f(n,z) - c_f - \Phi(v,n) \right\} + \beta(1-\gamma)\frac{\gamma \lambda^F v}{n'} \mathbb{E}S(n', z') + (1-\gamma)(1-\beta)U$$

- Recall, in Burdett-Mortensen, wage and firm size were related monotonically
- Is it possible to break it?

Low Adjustment Cost



High Adjustment Cost