Unpacking Aggregate Welfare in a Spatial Economy [*]

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Abstract

How do regional productivity shocks or transportation infrastructure affect aggregate welfare? In a general class of spatial equilibrium models, we provide an exact additive decomposition of aggregate welfare changes into (i) technology effects à la Hulten (1978), (ii) spatial dispersion in marginal utility, (iii) fiscal externalities, (iv) technological externalities, and (v) redistribution. We provide a non-parametric formula for second-best spatial transfers and show that Hulten's characterization is recovered whenever they are in place. In an application to the U.S. economy, we find a substantial deviation from Hulten's characterization for the period of 2010-2019 and for counterfactual improvements in transportation infrastructure.

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1 Introduction

How do regional productivity shocks or transportation infrastructure affect aggregate welfare? To answer these questions, there has been significant progress in the development of quantitative spatial general equilibrium models. These frameworks allow researchers to fit the model to geographically disaggregated data and to compute the aggregate welfare of a particular shock or policy. At the same time, these frameworks are highly complex and parameterized, obscuring which forces or parameters in the model govern the aggregate welfare effects.

An alternative approach is to appeal to first-order approximations. Hulten (1978) showed that, in a frictionless economy, the impact on aggregate TFP of microeconomic TFP shocks is equal to the shocked producer's sales as a share of GDP (i.e., Domar weights). In the evaluation of transportation infrastructure, a popular approach has been the "social saving" approach (Fogel 1964), where the benefit of transportation infrastructure is calculated based on the shipment cost saved relative to the next best alternative. Underlying these approaches is a macro-envelope condition resulting from the first welfare theorem. These approaches have the advantage of being agnostic about the details of the underlying disaggregated equilibrium system. However, whether or how these approaches extend to spatial equilibrium models remains an open question.

This paper fills this gap by providing a theory to unpack the first-order aggregate welfare effects of spatially disaggregated shocks in a general class of spatial equilibrium models. We provide an exact additive decomposition of aggregate welfare changes that depends on a minimal set of nonparametric sufficient statistics. Our decomposition clarifies how and why first-order aggregate welfare gains and losses depart from Hulten's characterization. We apply our decomposition to the U.S. economy to assess the aggregate welfare changes during the period of 2010-2019 and in response to counterfactual improvements in transportation infrastructure.

We consider a general class of spatial equilibrium models. Our framework accommodates flexible location-specific utility functions as well as local amenities, production functions, input-output linkages, trade frictions, agglomeration and congestion externalities, ex-ante heterogeneous households, and government transfers across locations and household types. We also introduce idiosyncratic preference shocks to households' location choices that follow arbitrary distributions. By accommodating a flexible correlation of preference shocks across alternatives, we capture general substitution patterns of location choice decisions. Special cases include those without preference shocks (Rosen 1979, Roback 1982, Allen and Arkolakis 2014), i.i.d. extreme value distribution (Redding 2016, Diamond 2016), and the generalized extreme value distribution with arbitrary correlations (McFadden 1978).

We start by observing that the competitive equilibrium allocation is suboptimal from the perspective of maximizing households' expected utility. This suboptimality arises regardless of the Pareto weights associated with ex-ante household types. Equilibrium suboptimality arises for three reasons. First, agents do not internalize technological externalities such as agglomeration or congestion. Second, whenever there are spatial transfers, agents do not internalize how their location choices affect the government budget, thereby generating fiscal externalities. Third, in the competitive equilibrium, the marginal utility of income is not equalized across locations.

The first two sources of suboptimality are perhaps not surprising. The third source of suboptimality is subtle and warrants a discussion. In equilibrium, agents make location decisions based on utility *levels* (inclusive of preference shocks). This implies that *marginal* utility from a dollar is not necessarily equalized across locations. One way to interpret this dispersion of marginal utility is the lack of insurance for the uncertainty associated with location choice. Another interpretation is the lack of redistribution across agents within ex-ante identical households with differing location choices. The observation that spatial equilibrium models involve suboptimality due to the dispersion of marginal utility is reminiscent of Mirrlees (1972), who points out this issue in a stylized residential location choice model within a city without preference shocks.

This equilibrium suboptimality implies that Hulten's analysis does not extend to spatial equilibrium models. The main contribution of this paper is to provide a theory to unpack aggregate welfare changes in spatial equilibrium models and how they depart from Hulten's characterization. We show that first-order aggregate welfare changes are exactly, additively decomposed into five terms. The first term, (i) technology, is the percentage change in productivity multiplied by the revenue of the region or sector receiving a shock, resonating the characterization by Hulten (1978). The remaining four terms, jointly constituting the reallocation effects, are (ii) marginal utility (MU) dispersion, (iii) fiscal externalities (in the presence of spatial transfers), (iv) technological externalities (such as agglomeration or congestion externalities), and (v) redistribution across ex-ante heterogeneous households. The second term is positive if a shock induces a relative increase in consumption where the marginal utility net of resource cost is high. The third term is positive if a shock induces the reallocation of people to locations that are net taxpayers. The fourth term is positive if a shock induces the reallocation of people to locations that generate positive technological externalities. The fifth term is positive if a shock induces the reallocation of consumption toward the types of households with higher welfare weights.

We provide several stylized examples to illustrate how model specifications affect the various components of our welfare decomposition. For example, the (ii) MU dispersion term is zero whenever utility is linear and consumer prices are equalized across locations, in which case the spatial dispersion in marginal utility is absent. Interestingly, this term is also zero whenever there are no preference shocks, as in the tradition of Rosen (1979) and Roback (1982). This is not because there is no spatial dispersion in marginal utility in this environment. Rather it is because reallocation of consumption across locations with different marginal utility of income fails to close the gap in differences in marginal utility across locations. If households are immobile across locations, terms (ii)-(iv) disappear.

We also study how prevailing spatial transfer policies shape the welfare changes from disaggregated shocks. To address this question formally, we first provide a non-parametric formula for optimal spatial transfers, generalizing Fajgelbaum and Gaubert's (2020) results by dispensing with any parametric assumptions over preference shocks. We then show that, if optimal transfer policies are implemented in the pre-shock economy, terms (ii)-(v) add up to zero. This result clarifies that terms (ii)-(v) reflect deviations of the competitive equilibrium from the second-best allocation. This result shows that the economy being suboptimal does not necessarily lead to systematic deviations from Hulten's characterization. Accordingly, whether and how much Hulten's characterization over- or under-predicts aggregate welfare changes depends on the deviation of observed spatial transfers from the optimal ones.

A key advantage of our decomposition is that it provides a minimal set of nonparametric sufficient statistics to identify aggregate welfare changes. In particular, given a minimal set of information about the baseline equilibrium allocation (prices, consumption, transfers, and the population distribution) and the changes in population and consumption, aggregate welfare changes are uniquely pinned down by externality estimates and the spatial dispersion of marginal utility of income. Relying on the econometric literature on discrete choice models (Berry and Haile 2014, Allen and Rehbeck 2019), we argue that the dispersion of marginal utility is nonparametrically identified from location choice data as long as preference shocks are additively separable. In some contexts, researchers are also interested in the counterfactual changes in welfare, without observing the changes in population and consumption in response to shocks. Together with existing nonparametric identification results for factor demand systems (Adao, Costinot, and Donaldson 2017), we argue that these objects are also nonparametrically identified, thereby establishing nonparametric identification of welfare changes for a counterfactual shock.

For our baseline analysis, we assume that preference shocks are additively separable. When we depart from the additively separable specification, preference shocks directly affect the spatial dispersion of marginal utility. While extending our results to such a case is straightforward in theory, it poses an identification challenge, as a monotone transformation of utility function changes marginal utility without affecting the location choice decisions. Nonetheless, if preference shocks are multiplicatively separable and follow a max-stable multivariate Fréchet distribution with an arbitrary correlation, – a predominantly common alternative specification in the literature – aggregate welfare changes are isomorphic to the additively separable specification by taking a log transformation. Therefore, aggregate welfare changes are nonparametrically identified within this class as well as the additively separable class mentioned above.

We show that our approach can be further extended to an environment with idiosyncratic shocks to household productivity, in addition to preferences; general agglomeration externalities that depend on the population in surrounding regions and producers' inputs and outputs; shocks to amenities that are not traded in the market; non-welfarist social welfare functions involving paternalistic motives; and models with cross-regional commuting.

We conclude with two applications to the U.S. economy. In our first application, we analyze the aggregate welfare change for the period 2010-2019 using the observed spatial distribution of economic activities across Metropolitan Statistical Areas (MSAs). We find that Hulten's characterization underpredicts the aggregate (utilitarian) welfare increase during this period. From 2010 to 2015, aggregate welfare (measured in units of GDP) increased by 2.24 percentage points annually, while Hulten's characterization would have predicted 2.04 percentage points. This gap is primarily explained by the (ii) MU dispersion term, arising from the fact that consumption levels converged across MSAs during this period. From 2015 to 2019, aggregate welfare increased by 1.84 percentage points, while Hulten's characterization would have predicted 1.35 percentage points. This gap is primarily explained by the (v) redistribution term (under equal welfare weights across skill types), arising from the fact that consumption levels converged across skill groups.

In our second application, we apply our method to evaluate counterfactual transportation infrastructure improvements and localized productivity shocks. In particular, we apply our decomposition to Allen and Arkolakis (2022), who develop a quantitative general equilibrium model that features endogenous transportation costs and traffic congestion. We find substantial differences between aggregate welfare changes and Hulten's characterization in both counterfactual exercises. For transportation infrastructure improvements, the difference arises primarily because of congestion externalities in shipment routes. For localized productivity shocks, the difference stems primarily from marginal utility dispersion but also from both agglomeration and congestion externalities.

Related Literature Our paper contributes to the literature on spatial equilibrium models. Building on seminal models of location choice which trade off wages, amenities, and cost of living (Rosen 1979, Roback 1982) and models with increasing returns to scale in production (Krugman 1991, Fujita, Krugman, and Venables 2001), recent developments in quantitative spatial equilibrium models incorporate rich geographic heterogeneity in production, amenities, and trade frictions. A growing body of research uses these frameworks to study the aggregate welfare effects of regional productivity shocks or transportation infrastructure.[1] Our contribution is to provide a nonparametric formula to unpack the aggregate welfare effects of disaggregated shocks in this class of models.

Our analysis of the aggregate welfare effects of shocks builds on Hulten (1978), who shows that in a perfectly competitive frictionless economy, the first-order effects of disaggregated shock on aggregate welfare are summarized by Domar weights. We show that this characterization does not generally extend to spatial equilibrium models because spatial equilibria are suboptimal. Researchers have recognized that externalities lead to equilibrium suboptimality and hence affect the first-order welfare effects of disaggregated shocks.[2] The equilibrium suboptimality due to the dispersion of marginal utility of income

[1] See Redding and Rossi-Hansberg (2017) and Redding (2022) for recent surveys on quantitative spatial equilibrium models and Redding and Turner (2015) for the use of these models to study the aggregate impact of transportation infrastructure. Donaldson and Hornbeck (2016) uses a general equilibrium model to explore the deviation from Fogel (1964) of the aggregate effects of U.S. railways. Tsivanidis (2019) and Zárate (2022) use parameterized quantitative spatial equilibrium models to study the impacts of urban transportation infrastructure and provide a quantitative comparison with Hulten's characterization. Caliendo, Parro, Rossi-Hansberg, and Sarte (2018) use these frameworks to study the propagation of region- and sector-specific productivity shocks.

[2] See Lebergott (1966) for an early criticism of Fogel (1964) due to an omission of technological externalities. Tsivanidis (2019) argues that agglomeration externalities affect the welfare gains from urban transport infrastructure beyond the value of travel time saved (VTTS) (i.e., Small and Verhoef 2007).

has been pointed out by Mirrlees (1972) in a stylized model of location decisions within a city without preference shocks. However, this point has been less highlighted in the recent quantitative spatial equilibrium literature.[3] Our contribution is to connect these sources of equilibrium suboptimality to the effects of disaggregated shocks on aggregate welfare.

Our analysis of how equilibrium suboptimality shapes the effects of disaggregated shocks connects with Baqaee and Farhi (2020), who study this question in an economy with a representative agent and thereby abstract from location choice decisions of households. Our paper also relates to Dávila and Schaab (2022) and Dávila and Schaab (2023), who provide welfare decompositions in general equilibrium models with heterogeneous agents. Our work is distinct in that we explicitly derive the deviation from Hulten's characterization in spatial equilibrium models using measurable sufficient statistics and study its relationship to optimal spatial transfer policies.

We also contribute to the literature that aims to measure aggregate welfare. While existing works along this line (Jones and Klenow 2016, Basu, Pascali, Schiantarelli, and Serven 2022) typically abstract from spatial considerations, our application to the U.S. economy in Section 5 shows spatial considerations can be quantitatively important in understanding a nation's aggregate welfare.

2 Spatial Equilibrium Framework

In this section, we set up the general spatial equilibrium framework for our baseline analysis.

2.1 General Setup

There are N locations indexed by $i, j \in \mathcal{N} \equiv \{1, \ldots, N\}$. There are S types of households indexed by $\theta \in \Theta \equiv \{\theta_1, \ldots, \theta_S\}$. The mass of each type is ℓ^θ, and we normalize the total measure to one: $\sum_\theta \ell^\theta = 1$. Each household decides its residential location at the beginning. Households who decide to live in location j consume the location-specific final good aggregator specific to household type θ produced using intermediate goods.

[3]See also Wildasin (1986), who explicitly points out that equilibrium suboptimality is related to the dispersion of marginal utility of income. Mongey and Waugh (2024) discuss this suboptimality in the broader context of discrete choice models.

There are K intermediate goods, some of which can be potentially traded across locations subject to a cost (e.g., food or manufacturing) and some of which are not traded across locations (e.g., housing or nontradable services). Intermediate goods are produced using local labor, intermediate goods, and local fixed factors (e.g., land). Households have ownership of these local fixed factors and earn factor income depending on their type θ, irrespective of their location.

Households of type θ in location j inelastically supply one unit of labor and consume final non-traded goods. Their preferences are given by

$$U_j^\theta(C_j^\theta, \epsilon_j^\theta). \tag{1}$$

Here, the utility function is indexed by j and θ to capture differences in type- and location-specific amenities. ϵ_j^θ is an idiosyncratic household-specific preference shock associated with location j, which we describe further below.

The household's budget constraint is

$$P_j^\theta C_j^\theta = w_j^\theta + T_j^\theta + \Pi^\theta, \tag{2}$$

where P_j^θ is the price of final goods for type θ households in location j, and w_j^θ is the wage for type θ households in location j. T_j^θ is the net government transfer for type θ households in location j. In reality, T_j^θ includes both taxes and transfers explicitly tagged to each location (such as state taxes and transfers in the U.S.) and those set at the national level (such as federal taxes and transfers in the U.S.). We do not impose any additional assumptions about T_j^θ beyond the condition that the net supply of these transfers is zero. Π^θ is the income from fixed factors for type θ households.

Households choose a location that maximizes their utility. The households' optimal location choice conditional on their preference shock draw $\boldsymbol{\epsilon}^\theta = (\epsilon_1^\theta, \ldots, \epsilon_N^\theta)$ solves

$$m^\theta(\boldsymbol{\epsilon}^\theta) \in \arg\max_{m \in \mathcal{N}} U_m(C_m^\theta, \epsilon_m^\theta). \tag{3}$$

Importantly, we do not make any parametric assumptions for the distribution of $\boldsymbol{\epsilon}^\theta$ beyond the regularity condition that they have a strictly positive density everywhere in $\mathbb{R}^N$ or are degenerate. This specification nests different assumptions about location decisions in the literature. For example, Rosen (1979), Roback (1982), and Allen and Arkolakis (2014) consider a case without preference shocks, i.e., where ϵ_m^θ is degenerate for all m; Diamond

(2016) considers a case where ϵ_m^θ is distributed according to an i.i.d. type-I extreme value distribution across locations m; and McFadden (1978) considers a case where $\boldsymbol{\epsilon}^\theta$ is distributed according to a generalized extreme value distribution with arbitrary correlation across alternatives. By aggregating across the draws of idiosyncratic preference shocks, the population size in location j of type θ is given by

$$l_j^\theta = \ell^\theta \mu_j^\theta, \qquad \mu_j^\theta = \int_{\boldsymbol{\epsilon}^\theta} \mathbb{I}\left[j = m^\theta(\boldsymbol{\epsilon}^\theta)\right] dG^\theta(\boldsymbol{\epsilon}^\theta), \tag{4}$$

where μ_j^θ is the probability that type θ households choose location j, $\mathbb{I}\left[j = m^\theta(\boldsymbol{\epsilon}^\theta)\right]$ is an indicator function signifying if the households with preference shocks $\boldsymbol{\epsilon}^\theta$ choose location j, and $G^\theta(\boldsymbol{\epsilon}^\theta)$ is the distribution function of preference shocks $\boldsymbol{\epsilon}^\theta$.

Final goods for type θ households in location j are produced using a constant returns to scale technology over intermediate goods

$$C_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta),$$

where $\mathbf{c}_j^\theta \equiv \{c_{ij,k}^\theta\}_{i,k}$ denotes a vector of intermediate goods used for final goods production, where k indexes intermediate goods and i indexes the origin location of these intermediate goods.

Intermediate good k produced in location i and sold in location j is produced using the following technology

$$y_{ij,k} = \mathcal{A}_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}),$$

where $\mathbf{l}_{ij,k} \equiv \{l_{ij,k}^\theta\}_\theta$ denotes labor inputs, $h_{ij,k}$ denotes the local fixed factor input, $\mathcal{A}_{ij,k}$ is Hicks-neutral productivity (including iceberg trade costs), $f_{ij,k}$ is a production function (which we assume to be strictly increasing, concave, differentiable, and constant returns), and $\mathbf{x}_{ij,k} \equiv \{x_{ij,k}^{l,m}\}_{l,m}$ denotes a vector of intermediate inputs, where m indexes the intermediate goods for inputs and l indexes the location of origin.

We assume that the supply of the local fixed factor at location j is given exogenously by $\bar{h}_j$. We assume that each type θ household owns α^θ share of fixed factors, where $\sum_\theta \ell^\theta \alpha^\theta = 1$. We also denote the price of the local fixed factor by r_j. Then, the aggregate per-capita return from the fixed factor for a type θ household is given by

$$\Pi^{\theta} = \alpha^{\theta} \sum_{j} r_j \bar{h}_j. \tag{5}$$

The government budget constraint is

$$\sum_{\theta} \sum_{j} T_j^{\theta} l_j^{\theta} = 0. \tag{6}$$

Finally, we assume that productivity $\{\mathcal{A}_{ij,k}\}$ is subject to agglomeration spillovers depending on the local population of various household types:[4]

$$\mathcal{A}_{ij,k} = A_{ij,k} g_{ij,k}(\{l_i^{\theta}\}_{\theta}), \tag{7}$$

where $g_{ij,k}(\cdot)$ are the spillover functions, and $A_{ij,k}$ is the fundamental component of productivity. Note that we allow for a flexible functional form for spillovers arising from the population size of different household types θ for different locations and goods i, j, k. For notational convenience, we denote the elasticity of agglomeration spillovers as

$$\gamma_{ij,k}^{\theta} \equiv \frac{\partial \ln g_{ij,k}(\{l_i^{\theta}\}_{\theta})}{\partial \ln l_i^{\theta}}. \tag{8}$$

We define the decentralized equilibrium of this economy as follows.

Definition 1 (Equilibrium)**.** The decentralized equilibrium consists of prices $\{\{P_j^{\theta}, w_j^{\theta}\}, \{p_{ij,k}\}, r_j\}$, quantities $\{\{C_j^{\theta}, \mathbf{c}_j^{\theta}, \mu_j^{\theta}, l_j^{\theta}\}, \{\mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}\}\}$, transfers $\{T_j^{\theta}\}$, and productivities $\{\mathcal{A}_{ij,k}\}$ such that:

(i) $\{C_j^{\theta}\}$ satisfies households' budget constraint (2), and $\{\mu_j^{\theta}, l_j^{\theta}\}$ solves households' optimal location choice problem (3) and (4);

[4]By interpreting some intermediate goods k as type θ's labor services, this specification nests general agglomeration spillovers from type θ to another type $\tilde{\theta}$'s labor productivity, nesting the framework of Fajgelbaum and Gaubert (2020). In Section 4.5, we show that it is straightforward to extend the agglomeration externalities beyond local population size, e.g., introducing cross-region productivity spillovers (e.g., Ahlfeldt, Redding, Sturm, and Wolf 2015) or agglomeration/congestion externality specific to a sector's inputs and outputs (e.g., Allen and Arkolakis 2022).

(ii) Firms maximize profits

$$\mathbf{c}_j^\theta \in \arg\max_{\tilde{\mathbf{c}}_j^\theta} P_j^\theta \mathcal{C}_j^\theta(\tilde{\mathbf{c}}_j^\theta) - \sum_{i,k} p_{ij,k}\tilde{c}_{ij,k}^\theta \tag{9}$$

and

$$(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \in \arg\max_{\tilde{\mathbf{l}}_{ij,k}, \tilde{h}_{ij,k}, \tilde{\mathbf{x}}_{ij,k}} p_{ij,k}\mathcal{A}_{ij,k} f_{ij,k}(\tilde{\mathbf{l}}_{ij,k}, \tilde{h}_{ij,k}, \tilde{\mathbf{x}}_{ij,k}) - \sum_\theta w_i^\theta \tilde{l}_{ij,k}^\theta - r_i \tilde{h}_{ij,k} - \sum_{l,m} p_{li,m}\tilde{x}_{ij,k}^{l,m}; \tag{10}$$

(iii) Goods markets clear

$$\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{jl,m}^{i,k} = \mathcal{A}_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) \tag{11}$$

$$C_j^\theta \ell^\theta \mu_j^\theta = \mathcal{C}_j^\theta(\mathbf{c}_j^\theta); \tag{12}$$

(iv) Labor markets clear

$$\sum_{i,k} l_{ji,k}^\theta = \ell^\theta \mu_j^\theta; \tag{13}$$

(iv) Fixed factor markets clear

$$\sum_{i,k} h_{ji,k} = \bar{h}_j; \tag{14}$$

(v) Aggregate factor payments Π^θ satisfy (5);

(vi) The government budget constraint (6) holds;

(vii) Productivity $\{\mathcal{A}_{ij,k}\}$ is subject to agglomeration spillovers given by (7).

We also define aggregate welfare of this economy as follows:

Definition 2 (Welfare)**.** Aggregate welfare W is given by a social welfare function that depends on the expected utility of each type θ household:

$$W = \mathcal{W}(\{W^\theta\}), \qquad W^\theta \equiv \mathbb{E}[\max_j\{U_j^\theta(C_j^\theta, \epsilon_j^\theta)\}]. \tag{15}$$

In a special case with ex-ante homogeneous household types ($S = 1$), the objective function is simply the expected utility of the household, or equivalently, a utilitarian social welfare function with respect to preference shocks. One restriction of Definition 2 is that aggregate welfare only depends on the expected utility of households, rather than directly on allocations. In Section 4.4, we show that our results straightforwardly extend to alternative welfare criteria, capturing cases where the social welfare function involves a paternalistic motive.

We refer to the welfare weight of type θ households Λ^θ as the marginal value of their expected utility to aggregate welfare, relative to their population size:

$$\Lambda^\theta \equiv \frac{\partial \mathcal{W}(\{W^\theta\})}{\partial W^\theta} \frac{1}{\ell^\theta}. \tag{16}$$

With a linear social welfare function, $\{\Lambda^\theta\}$ corresponds to what are often referred to as "Pareto weights". Under a utilitarian social welfare function, we would have $\Lambda^\theta = 1$. Without loss of generality, we will normalize $\mathcal{W}$ such that $\sum_\theta \ell^\theta \Lambda^\theta = 1$ at the equilibrium we consider.

We first focus on the case where the utility function is additively separable between the common location-specific component and the idiosyncratic component:

$$U_j^\theta(C_j^\theta, \epsilon_j^\theta) = u_j^\theta(C_j^\theta) + \epsilon_j^\theta. \tag{17}$$

The key implication of this assumption is that the marginal utility of consumption is not affected by idiosyncratic preference shocks. In Section 4.1, we describe how the departure from this assumption influences our results. There, we show that for a common alternative specification in the literature where the preference shocks enter multiplicatively and follow a max-stable multi-variate Fréchet distribution, all of our results remain isomorphic to the additively separable specification.

For expositional purposes, we choose numeraire so that the population-weighted average of the inverse of the marginal utility of income $u_j^{\theta\prime}(C_j^\theta)$ is one:

$$\sum_\theta \sum_j l_j^\theta \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} = 1. \tag{18}$$

The left-hand side is the dollar cost of increasing utility by one unit in all locations for all types, which is the numeraire in this economy.

We also focus on the case where the decentralized equilibrium is unique and interior ($l_j^\theta > 0$ for all j and θ). Since our approach relies on a first-order approximation, this assumption avoids dealing with the case where equilibrium outcomes are non-differentiable with respect to the shock. That is, when a small shock can induce a switch to a different equilibrium.[5]

2.2 Useful Representational Lemmas

We present two lemmas that will be useful later. First, we introduce a convenient alternative representation of location choice decisions. Following Hofbauer and Sandholm (2002), the discrete location choice decision under additive preference shocks (17) can be isomorphically represented by households jointly choosing the population size subject to a cost function, as summarized in the following lemma:

Lemma 1 (Hofbauer and Sandholm 2002)**.** Under an additively separable utility function (17), the share of type θ households living in each location $\{\mu_j^\theta\}_j$ can be represented as the solution to the following problem given a vector of equilibrium consumption $\{C_j^\theta\}_j$:

$$\begin{aligned} W^\theta = \max_{\{\mu_j^\theta\}_j} & \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \\ \text{s.t.} \quad & \sum_j \mu_j^\theta = 1 \end{aligned} \tag{19}$$

for some function $\psi^\theta(\{\mu_j^\theta\}_j)$, which we provide an explicit expression for in Appendix A.1. Moreover, W^θ coincides with the expected utility in (15), i.e., $W^\theta = \mathbb{E}[\max_j\{u_j^\theta(C_j^\theta)+\epsilon_j^\theta\}]$.

The proofs of this lemma and the subsequent propositions of this paper are found in Appendix A. Importantly, $\psi^\theta(\{\mu_j^\theta\}_j)$ summarizes the influence of preference shocks on households' location decisions. If there are no preference shocks, we have $\psi^\theta(\{\mu_j^\theta\}_j) = 0$. If preference shocks follow an i.i.d. type-I extreme value distribution with shape parameter ν, then $\psi^\theta(\{\mu_j^\theta\}_j) = \frac{1}{\nu}\sum_j \mu_j^\theta \ln \mu_j^\theta$ (Anderson, De Palma, and Thisse 1988). When $\{\epsilon_j^\theta\}_j$ follow a type-I generalized extreme value (GEV) with arbitrary correlations (i.e., McFadden 1978), we show in Appendix C that $\psi^\theta(\{\mu_j^\theta\}_j) = \frac{1}{\nu}\sum_j \mu_j^\theta \ln S_j^\theta(\{\mu_i^\theta\}_i)$, where

[5] See Allen and Arkolakis (2014) and Allen, Arkolakis, and Li (2020) for sufficient conditions for equilibrium uniqueness in spatial equilibrium models.

function $S_j^\theta(\cdot)$ depends on the correlation function of $\{\epsilon_j^\theta\}_j$ across alternatives j.[6]

Second, the following lemma shows that the competitive equilibrium allocation can be represented as the solution to a "pseudo-planning" problem.

Lemma 2. Any decentralized equilibrium allocations $\{\check{C}_j^\theta, \check{\mathbf{c}}_j^\theta, \check{\mathbf{x}}_{ij,k}, \check{\mathbf{l}}_{ij,k}, \check{h}_{ij,k}, \check{\mu}_j^\theta, \check{l}_j^\theta, \check{\mathcal{A}}_{ij,k}\}$ solves the following pseudo-planning problem

$$W = \max_{\{W^\theta, C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mu_j^\theta, \mathcal{A}_{ij,k}\}} \mathcal{W}(\{W^\theta\}_{\theta\in\Theta}) \tag{20}$$

$$\text{subject to} \qquad (7), (11)\text{-}(14)$$

$$W^\theta = \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \tag{21}$$

$$\{\mu_j^\theta\}_j \in \arg\max_{\{\tilde{\mu}_j\}:\sum_j \tilde{\mu}_j = 1} \sum_j \tilde{\mu}_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}_j) \tag{22}$$

$$C_j^\theta = \check{C}_j^\theta. \tag{23}$$

The objective function is what we define as aggregate welfare. Constraint (7) stipulates the spillover functions, and constraints (11)-(14) correspond to resource constraints. Constraint (22) imposes that location choices are incentive compatible, and (23) restricts consumption to be equal to its equilibrium value. If constraints (22) and (23) were slack, then the problem coincides with the first-best planning problem, where the planner maximizes aggregate welfare subject to resource constraints. In this case, Hulten's theorem applies. However, as we show below, constraints (22) and (23) are not slack in general. Therefore, Hulten's theorem cannot be applied to spatial equilibrium models.

3 Unpacking Welfare Effects of Disaggregated Shocks

How do regional productivity shocks or transportation infrastructure improvements affect aggregate welfare? This section provides our main theoretical result which decomposes the first-order effects of disaggregated regional shocks. Section 3.1 provides our decomposition, where the first term corresponds to Hulten's characterization and the remaining terms correspond to reallocation effects. We explain each term through several

[6] An alternative interpretation of $\psi^\theta(\cdot)$ is that it captures congestion externalities. For example, the model with preference shocks following an i.i.d. type-I extreme value distribution with shape parameter ν is isomorphic to a model without preference shocks where utility is given by $u_j^\theta(C_j^\theta) - \frac{1}{\nu}\ln \mu_j^\theta$. See Appendix D.4 for further discussion about this isomorphism.

special cases of our model. Section 3.2 shows that Hulten's characterization is recovered if second-best location-specific transfers are in place, implying that the sign and magnitude of the reallocation effects reflect a deviation from second-best policies. Section 3.3 uses our decomposition to discuss the nonparametric identification of aggregate welfare changes in spatial equilibrium models.

For expositional purposes, we introduce the following expectation and covariance operators. The first set of operators takes the expectation and covariance of statistics associated with location j for a given type θ household, weighted by location share μ_j^θ:

$$\mathbb{E}_{j|\theta}[X_j^\theta] \equiv \sum_j \mu_j^\theta X_j^\theta, \qquad \mathrm{Cov}_{j|\theta}(X_j^\theta, Y_j^\theta) \equiv \mathbb{E}_{j|\theta}[X_j^\theta Y_j^\theta] - \mathbb{E}_{j|\theta}[X_j^\theta]\mathbb{E}_{j|\theta}[Y_j^\theta]. \tag{24}$$

The second set of operators takes the expectation and covariance of statistics associated with type θ households, weighted by population share ℓ^θ:

$$\mathbb{E}_\theta[X^\theta] \equiv \sum_\theta \ell^\theta X^\theta, \qquad \mathrm{Cov}_\theta(X^\theta, Y^\theta) \equiv \mathbb{E}_\theta[X^\theta Y^\theta] - \mathbb{E}_\theta[X^\theta]\mathbb{E}_\theta[Y^\theta]. \tag{25}$$

3.1 Main Results

Consider small changes in the exogenous components of productivity specific to origin location, destination location, and sector: $\{d \ln A_{ij,k}\}$. These shocks can represent region-sector TFP shocks (e.g., Caliendo et al. 2018) or transportation infrastructure changes (e.g., Allen and Arkolakis 2014, Donaldson and Hornbeck 2016).[7] We also allow for the possibility that the structure of transfers may change simultaneously, denoted by $\{dT_j^\theta\}$, either because of exogenous policy changes or as an endogenous response to the productivity shocks.

By applying the envelope theorem to the pseudo-planning problem in Lemma 2, we obtain the following expression for welfare changes:

Proposition 1. *Consider an arbitrary set of small shocks to the exogenous components of productivity $\{d \ln A_{ij,k}\}$, as well as changes in transfers $\{dT_j^\theta\}$, in a decentralized equilib-*

[7] In some context, researchers are interested in the shocks to amenities instead of productivity. Our analysis includes those cases by interpreting some intermediate goods as local amenities. From a measurement perspective, applying Proposition 1 requires knowledge of prices of the amenities, which is often unobserved and needs to be calibrated or estimated. For example, if transportation infrastructure also brings amenity benefits by shortening commuting time, one can use the value of time for $p_{ij,k}$ and commuting time for $y_{ij,k}$ (i.e., Small and Verhoef 2007). In Section 4.2, we provide an alternative expression for Proposition 1 without using amenity prices.

rium. The first-order impact on welfare can be expressed as

$$dW = \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k}}_{\text{(i) Technology } (\Omega_T)} + \underbrace{\mathbb{E}_\theta\left[Cov_{j|\theta}\left(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta)dC_j^\theta\right)\right]}_{\text{(ii) MU Dispersion } (\Omega_{MU})}$$
$$+ \underbrace{\mathbb{E}_\theta\left[Cov_{j|\theta}(-T_j^\theta, d\ln l_j^\theta)\right]}_{\text{(iii) Fiscal Externality } (\Omega_{FE})} + \underbrace{\mathbb{E}_\theta\left[Cov_{j|\theta}(\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta, d\ln l_j^\theta)\right]}_{\text{(iv) Technological Externality } (\Omega_{TE})} \tag{26}$$
$$+ \underbrace{Cov_\theta\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right], \mathbb{E}_{j|\theta}\left[u_j^{\theta\prime}(C_j^\theta)dC_j^\theta\right]\right)}_{\text{(v) Redistribution } (\Omega_R)}.$$

Below, we explain each term of Proposition 1 and illustrate them through special cases. We summarize our decomposition in those special cases in Table 1.

Technology, Ω_T. The first term of Proposition 1, which we refer to as (i) technology (Ω_T), captures the effects of productivity changes absent the reallocation of resources. The coefficient in front of $d\ln A_{ij,k}$, $p_{ij,k}y_{ij,k}$, corresponds to the total sales of intermediate inputs k produced in i and sold in j. The observation that total sales summarize the aggregate effects of a shock reflects the celebrated result of Hulten (1978). If the equilibrium maximizes aggregate welfare W, the first term is sufficient for the welfare consequence of disaggregated shocks, to a first-order. However, since the equilibrium is generally suboptimal, the reallocation of resources has a first-order effect on welfare.

MU Dispersion, Ω_{MU}. The second term, which we refer to as (ii) MU (marginal utility) dispersion (Ω_{MU}), captures the fact that shocks reallocate resources across locations that potentially differ in their marginal utility of income. A shock leads to an increase in utility of $u_j^{\theta\prime}(C_j^\theta)dC_j^\theta$ in each location j for type θ households. The covariance is positive if utility changes are higher in locations with higher marginal utility of income $u_j^{\theta\prime}(C_j)/P_j^\theta$. The expectation $\mathbb{E}_\theta[\cdot]$ takes the weighted average of this covariance across household types θ.

This term is generally non-zero if the marginal utility of income is not equalized across locations for each of the household types θ, and this is what happens in spatial equilibrium. In equilibrium, agents make location decisions based on utility *levels* (inclusive of preference shocks). This implies that *marginal* utility of income is not necessarily equalized across these locations given household type θ. In fact, the marginal utility $u_j^{\theta\prime}(C_j^\theta)$ never shows up in any of the equilibrium conditions.

There are two ways to interpret this equilibrium suboptimality due to the dispersion of marginal utility of income. The first interpretation is the lack of insurance for the uncertainty associated with location choice. Depending on the preference draws, or depending on the random location assignment in the absence of preference shocks, individual households may end up in a variety of locations that differ in terms of their associated marginal utility of income. Ex-ante, households can benefit by committing to making transfers from a state where they end up in a location with a low marginal utility of income to a state with a high marginal utility of income. However, there is no security that allows for such a transfer. The second interpretation is the lack of redistribution across households with differing residential locations within household type θ.

In certain special cases, the MU dispersions are absent in the spatial equilibrium. For example, this case arises under linear utility (i.e., $u_j^\theta(C_j^\theta) = C_j^\theta + B_j^\theta$ for some B_j^θ) *and* no trade frictions such that final good prices P_j are equalized across locations j. A set of primitive assumptions that delivers the equalization of final good prices is $\mathcal{A}_{ij,k} = \mathcal{A}_i^k$, $f_{ij,k}(\cdot) = f_i^k(\cdot)$, and $\mathcal{C}_j^\theta(\cdot) = \mathcal{C}^\theta(\cdot)$. Kline and Moretti (2014) consider this special case under ex-ante homogeneous households and argue that expected utility is maximized in the competitive equilibrium without technological externalities. Alternatively, if the utility function is log, $u_j^\theta(C_j^\theta) = \ln C_j^\theta + B_j^\theta$, there is no spatial transfers, $T_j^\theta = 0$, *and* nominal wages are equalized across locations, $w_j^\theta = w^\theta$ for all j, spatial dispersion in marginal utility of income is absent, $u_j^{\theta\prime}(\frac{w_j+\Pi^\theta}{P_j^\theta})/P_j^\theta = \frac{1}{w^\theta+\Pi^\theta}$. This is the case highlighted in recent work by Mongey and Waugh (2024).

Importantly, the MU dispersion is present even without preference shocks as in the tradition of Rosen (1979) and Roback (1982) (free mobility). This is highlighted by Mirrlees (1972). Despite this observation, the MU dispersion term, Ω_{MU}, is always zero without preference shocks. To see this, note that utility levels across locations are always equalized in spatial equilibrium, $u_j^\theta(C_j^\theta) = W^\theta$. This implies that the shift in utility levels across locations in response to any shocks are always the same, $u_j^{\theta\prime}(C_j^\theta)dC_j^\theta = dW^\theta$, which implies the covariance inside Ω_{MU} is zero. The reason that the MU dispersion term is absent under free mobility is not because there is no MU dispersion in equilibrium. Rather it is because reallocation fails to close the gap in marginal utility across locations.

Fiscal Externality, Ω_{FE}. The third term, which we refer to as (iii) fiscal externality (Ω_{FE}), comes from the fact that the shock affects the government's budget. If the shock induces population movement toward a location that pays taxes on net (higher $-T_j^\theta$), this term has a positive effect on welfare.[8] This term is absent whenever there are no transfers ($T_j^\theta = 0$ for all j and θ) or the shock does not induce any labor reallocation ($d \ln l_j^\theta = 0$ for all j and θ).

Technological Externality, Ω_{TE}. The fourth term, which we refer to as (iv) technological externality (Ω_{TE}), captures agglomeration externalities in productivity. If the shock induces the population to move toward a location with a higher agglomeration externality $\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta$, this term has a positive effect on welfare. Clearly, this term becomes zero if there are no technological externalities in the pre-shock equilibrium, i.e., $\gamma_{ij,k}^\theta = 0$ for all i, j, k, and θ.

Importantly, assuming constant elasticity agglomeration externality ($\gamma_{ij,k}^\theta = \gamma$) alone does not ensure that the technological externality term is zero. To see why, consider a special case with a single sector $K = 1$, single type $S = 1$, and no fixed factor $\bar{h}_j = 0$ for all j. We further assume that there are no intermediate inputs used in production ($y_{ij} = \mathcal{A}_{ij} f_{ij}(l_{ij})$, dropping subscript k and θ). In this case, from profit maximization and labor market clearing condition, $\sum_l p_{jl} y_{jl} = w_j l_j$, we have that the term $\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{ij,k}^\theta$ simplifies to $w_j \gamma$. Therefore, the reallocation of the population toward a location with a higher nominal marginal product of labor generates positive effects on aggregate welfare. This result is consistent with the observation of Fajgelbaum and Gaubert (2020), who show that spatial equilibria involve misallocation of the population even under constant elasticity of agglomeration externality as long as the marginal product of labor is not equalized (e.g., due to compensating differentials).

Redistribution, Ω_R. The fifth term, which we refer to as (v) redistribution (Ω_R), is the covariance between the marginal increase in expected utility of type θ households $\mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta]$ and the utility weight on those household $\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right]$. The first term of the utility weight is the welfare weight defined by Equation (16) and the second

[8] In some existing models, researchers assume that some fraction of fixed factor income is rebated to local households directly (such as through local governments' ownership of local fixed factors), which implies that Π_i depends on i (e.g., Caliendo et al. 2018). In such cases, the fiscal externality term is simply modified to capture these local rebates, i.e., $w_j^\theta - P_j^\theta C_j^\theta = -(T_j^\theta + \Pi_j^\theta)$.

Table 1: Decomposition in Special Cases

	Ω_T	Ω_{MU}	Ω_{FE}	Ω_{TE}	Ω_R
1. Linear utility and no trade frictions	✓		✓	✓	✓
2. No preference shocks	✓		✓	✓	✓
3. No location-specific transfers	✓	✓		✓	✓
4. No technological externalities	✓	✓	✓		✓
5. Single type	✓	✓	✓	✓	
6. No population mobility	✓				✓
7. Second-best transfers	✓				✓
8. ...with redistribution	✓				

term is the expected inverse marginal utility of income. If all households are ex-ante homogenous ($S = 1$), the (v) redistribution term becomes zero. Moreover, all the expectation operators with respect to household types $\mathbb{E}_\theta[\cdot]$ drop out from terms (ii)-(iv).

Remarks. In some cases, researchers model locations without allowing for population mobility. This is nested in our framework by setting $S = N$ and preference shocks are such that type θ_i households always locate themselves in location i: $\mu_i^{\theta_i} = 1$. Examples of such specifications arise in international trade models where researchers typically abstract from international migration or when studying the short-run effects of an acute shock.[9] If the population is immobile ($d \ln l_j^\theta = 0$ for all j and θ), (ii) MU dispersion term, (iii) fiscal externality and (iv) technological externality terms all become zero.

Proposition 1 provides a characterization of the changes in aggregate welfare. In many contexts, researchers may wish to convert the welfare changes into alternative measurable units. For example, one may wish to compute how much uniform labor productivity changes induce equivalent changes in aggregate welfare. To answer this question, one can again use Proposition 1 to ask what uniform productivity changes $d \ln \tilde{A}$ achieve the same dW.

[9] For example, Fajgelbaum, Goldberg, Kennedy, and Khandelwal (2020) study the short-run welfare effects of the 2018 trade war in the U.S. by restricting population mobility across U.S. states.

3.2 Welfare Changes if Optimal Spatial Transfers are in Place

So far, we have remained agnostic about how transfers T_j^θ are determined in equilibrium. In reality, national governments may set spatial transfers T_j^θ to correct for agglomeration externalities (Fajgelbaum and Gaubert 2020, Rossi-Hansberg, Sarte, and Schwartzman 2019) or to address spatial inequalities (Gaubert, Kline, and Yagan 2021). While Proposition 1 embraces any endogenous response of T_j^θ to shocks, it is instructive to consider how each term in Proposition 1 is affected in those cases. We also argue below that understanding the government's incentive facilitates the interpretation of Proposition 1.

Specifically, we consider a scenario where the government sets spatial transfers T_j^θ to trace the Pareto frontier subject to incentive compatibility constraints. The government's (constrained) Pareto efficient transfer policy solves

$$\max_{\{C_j^\theta,\mathbf{c}_j^\theta,\mathbf{x}_{ij,k},\mathbf{l}_{ij,k},h_{ij,k},l_j^\theta,\mu_j^\theta,\mathcal{A}_{ij,k},w_j^\theta,r_j,p_{ij,k},P_j^\theta,T_j^\theta\}} \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \tag{27}$$

subject to (2)-(14) and the constraint that

$$\sum_j \mu_j^{\tilde{\theta}} u_j^{\tilde{\theta}}(C_j^{\tilde{\theta}}) - \psi^{\tilde{\theta}}(\{\mu_j^{\tilde{\theta}}\}_j) \geq \underline{W}^{\tilde{\theta}} \text{ for all } \tilde{\theta} \neq \theta. \tag{28}$$

Fixing θ and tracing the frontier for all feasible values of $\underline{W}^{\tilde{\theta}}$ for all $\tilde{\theta}$ defines the set of efficient transfers.

To solve this problem, we follow the primal approach in the public finance literature. That is, we focus on a relaxed planning problem where the Planner picks an incentive-compatible population allocation and later confirms that the solution to the relaxed problem is also a solution to the original one. The relaxed planning problem is defined as follows.

$$\max_{\{C_j^\theta,\mathbf{c}_j^\theta,\mathbf{x}_{ij,k},l_{ij,k}^\theta,h_{ij,k},l_j^\theta,\mathcal{A}_{ij,k}\}} \sum_j \mu_j^{\bar{\theta}} u_j^\theta(C_j^{\bar{\theta}}) - \psi^{\bar{\theta}}(\{\mu_j^{\bar{\theta}}\}_j) \tag{29}$$

$$\text{subject to} \qquad (7), (11)\text{-}(14)$$

$$\{\mu_j^\theta\}_j \in \arg\max_{\{\tilde{\mu}_j\}:\sum_j \tilde{\mu}_j=1} \sum_j \tilde{\mu}_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}_j) \tag{30}$$

$$\sum_j \mu_j^{\tilde{\theta}} u_j^{\tilde{\theta}}(C_j^{\tilde{\theta}}) - \psi^{\tilde{\theta}}(\{\mu_j^{\tilde{\theta}}\}_j) \geq \underline{W}^{\tilde{\theta}}, \quad \text{for all } \tilde{\theta} \neq \bar{\theta} \tag{31}$$

Compared to the pseudo-planning problem in Lemma 2, the relaxed planning problem chooses consumption instead of taking the equilibrium allocation as given (equation (23)). We also assume that the government traces the Pareto frontier under constraints (31), instead of maximizing the social welfare function defined by Definition 2. At the same time, this problem is different from the first-best planning problem as considered in Appendix B, as the Planner must choose an incentive-compatible population allocation (30). For this reason, we refer to these policies as the second-best policies. The following proposition provides our key characterization of the second-best transfer policy.

We let $\{\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)\}$ denote the location choice function that maps a vector of consumption in each location to location choice probabilities as the solution to (30).

Proposition 2. *Assume that preference shocks are non-degenerate. If the second-best transfer policy is implemented, the allocation* $\{C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mu_j^\theta\}$ *must satisfy* (7), (11)-(14), (30) *and*

$$\mu_j^\theta\left[\tilde{\Lambda}^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta\right] = \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta}\left[T_i^\theta - \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta}\gamma_{il,k}^\theta\right], \quad \textit{for all } j, \theta, \tag{32}$$

for some $\tilde{\Lambda}^\theta > 0$ *that satisfies* $\sum_\theta \ell^\theta \tilde{\Lambda}^\theta = 1$, *and* $\frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta}$ *is the location choice response to consumption, and* $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$. *Furthermore, this allocation can be implemented with transfers* T_j^θ.

This proposition summarizes the key trade-off associated with optimal spatial transfer policy. The left-hand side of this expression summarizes the marginal benefit from transferring one unit of consumption to location j for type θ. In particular, if the marginal utility $u_j^{\theta\prime}(C_j^\theta)$ is high and the associated price P_j^θ is low in location j relative to other locations, the net benefit of transfers to location j tends to be high. On the right-hand side of this equation, we summarize the marginal cost of this transfer through fiscal and technological externalities. In particular, a unit increase of consumption in location j increases population by $\frac{\partial \hat{\mu}_i^\theta}{\partial C_j^\theta}$ in location i. Notice that this relocation happens in all locations, not only in location j. This population relocation is associated with fiscal externality T_i^θ and technological externality $\sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta}\gamma_{il,k}^\theta$.

The above formula has a strong connection to optimal unemployment insurance literature (Baily 1978, Chetty 2006). In fact, our formula (32) resembles what is often called as Baily-Chetty formula, which balances the trade-off between insurance against job loss and fiscal externality from discouraging job search. Relative to the optimal unemployment insurance formula, aside from obvious differences in the contexts, our formula differs in that it incorporates many location choice decisions and the cost includes technological externality in addition to fiscal externality.

Proposition 2 is a strict generalization of Fajgelbaum and Gaubert (2020), who study the same problem in the special cases where there are no preference shocks.[10] In particular, if we take the limit where the variance of preference shocks goes to zero, the elasticity of population with respect to consumption diverges to infinity, i.e., $|\frac{\partial \mu_i^\theta}{\partial C_j^\theta}| \to \infty$. By noting that $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$, the only way to satisfy Equation (32) is to set $w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta = E^\theta$ for some constant E^θ,[11] corresponding to the formula in Fajgelbaum and Gaubert (2020). Therefore, the cross-location component of transfers only addresses technological externalities, and the cross-type component of transfers addresses redistribution concerns. Our formula generalizes their cases by dispensing with any parametric assumptions and thereby highlights key nonparametric statistics for optimal transfers.

Our formula can be used to assess Pareto inefficiency of the current transfer schemes. Since our formula requires the existence of positive welfare weights, $\tilde{\Lambda}^\theta > 0$, equilibrium allocations that lead to negative inferred welfare weights are Pareto inefficient.

Corollary 1. *If there exists j and θ such that*

$$\mu_j^\theta P_j^\theta < \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[-T_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right], \tag{33}$$

then there exists an alternative transfer scheme that Pareto improves the original one.

The underlying idea is the same as Werning (2007) in the context of optimal non-linear income taxation. Importantly, the test of Pareto inefficiency does not require researchers to take a stance on welfare weights or the marginal utility of consumption in each location. The test only requires a handful of sufficient statistics such as price indices, migration elasticities, and agglomeration elasticities.

[10] They also consider a case where preferences take the form of $U_j(C_j, \epsilon_j) = \tilde{\epsilon}_j C_j$ and $\tilde{\epsilon}_j$ follow independent Frechét distribution. As we show in Section 4.1, this specification is isomorphic to its log-transformation: $\ln C_j + \epsilon_j$, where ϵ_j follows an i.i.d. type-I extreme value distribution. Consequently, our formula nests this case as well.

[11] See Appendix A.4 for a more formal treatment of this limit case.

We now consider how the implementation of these second-best policies affects the aggregate welfare effects of disaggregated shocks in Proposition 1. Note that, if we multiply Equation (32) by $\ell^\theta dC_j^\theta$ and sum up across j and θ, the second term of the left-hand side of Equation (32) coincides with the (ii) MU dispersion term in Proposition 1, and the right-hand side of this equation coincides with the negative of the (iii) fiscal externality and (iv) technological externality terms. Therefore, optimal spatial transfer policy offsets these three distortions, and welfare changes are summarized solely by the (i) technology and (v) redistribution terms.

Proposition 3. *Suppose that second-best transfers $\{T_j^\theta\}_{j,\theta}$ are implemented according to Proposition 2 in the pre-shock equilibrium. Then, Proposition 1 comes down to*

$$dW = \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k}}_{\text{(i) Technology } (\Omega_T)} + \underbrace{Cov_\theta\left(\Lambda^\theta - \tilde{\Lambda}^\theta, \mathbb{E}_{j|\theta}\left[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta\right]\right)}_{\text{(v) redistribution } (\Omega_R)} . \tag{34}$$

Furthermore, if the implied Pareto weights of the second-best policy $\tilde{\Lambda}^\theta$ coincide with the welfare weights in the social welfare function Λ^θ, then the (v) redistribution term also disappears, thereby obtaining Hulten's characterization in a spatial economy.

Corollary 2. *Suppose that transfers $\{T_j^\theta\}_{j,\theta}$ are set so that Proposition 2 holds with $\tilde{\Lambda}^\theta = \Lambda^\theta$ for all θ in the pre-shock equilibrium. Then, Proposition 1 comes down to*

$$dW = \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k}}_{\text{(i) Technology } (\Omega_T)} . \tag{35}$$

Interestingly, despite the policy being the second-best but not the first-best, the reallocation effects are absent, as they add up to zero: $\Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R = 0$. This is precisely because transfers are set optimally. If the reallocation terms were not zero, then a small change in transfers could have improved welfare. Note that the underlying reason why Hulten's characterization applies is distinct from that in the first-best environment. There, each of the reallocation terms is zero: $\Omega_{MU} = \Omega_{FE} = \Omega_{TE} = \Omega_R = 0$. In a world with second-best transfers, although each term is non-zero, they sum up to zero. This observation resonates with Costinot and Werning (2018), who show that Hulten's characterization holds under the second-best policies, although the environment they consider is very different from ours.

In reality, it is unlikely the case that the government implements optimal transfers. Nevertheless, Proposition 3 and Corollary 2 are careful reminders that the spatial equilibrium models being suboptimal do not necessarily imply systematic deviations from Hulten (1978) and thereby provide an important benchmark case. Moreover, Proposition 3 and Corollary 2 are helpful to facilitate the assessment of how the aggregate welfare changes depart from Hulten's characterization. In particular, by assessing which of the left-hand side and right-hand side of Proposition 2 is greater under observed transfers $\{T_j\}$, one can conclude whether Hulten's characterization over- or under-predict the aggregate welfare changes.

3.3 Nonparametric Identification of Welfare Changes

A key advantage of Proposition 1 is that it clarifies the minimal set of sufficient statistics to uniquely identify the aggregate welfare changes. In this section, we discuss the nonparametric identification of these sufficient statistics, and hence the aggregate welfare changes.

We first consider the case where the changes in productivity, consumption, and population $\{d\ln A_{ij,k}, dC_j^\theta, d\ln l_j^\theta\}$ are observed, corresponding to the case of *ex-post* welfare evaluation. Suppose we also observe the subset of baseline equilibrium prices $\{P_j^\theta, p_{ij,k}, w_j^\theta\}$, quantities $\{C_j^\theta, l_j^\theta, y_{ij,k}\}$, and transfers $\{T_j^\theta\}$. Suppose we also take a stance on welfare weights $\{\Lambda^\theta\}$. The only remaining two statistics are the agglomeration externality elasticities $\{\gamma_{ij,k}^\theta\}$ and the spatial dispersion of marginal utility $\{u_j^{\theta\prime}(C_j^\theta)\}$ evaluated around the baseline equilibrium. For the former, identification requires the causal effect of exogenous population changes on productivity. The long-standing literature on agglomeration economies provides plausible values for these parameters.[12]

The identification of the spatial dispersion of marginal utility is highlighted less in the literature on spatial equilibrium models. Fortunately, the existing econometrics literature on discrete choice models provides a way to nonparametrically identify these objects from location choice data. Let us focus on the case where preference shocks are additively separable and not degenerate.[13] In this case, location choice decisions are summarized by the function $\{\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)\}_i = \arg\max_{\{\mu_j^\theta\}:\sum_j \mu_j^\theta=1} \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j)$. Suppose that

[12]For example, see Melo, Graham, and Noland (2009) for a meta-analysis of the agglomeration externality.

[13]If the preference shocks are degenerate, the (ii) MU dispersion term is zero as discussed in Section 3.1. Therefore, the relative marginal utility does not directly influence aggregate welfare changes in Proposition 1.

we have a sufficiently long period of data observations. Suppose also that we have an exogenous variation of consumption in each location, so that we can credibly identify the response of the population size in i to consumption change in j, $\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)/\partial C_j^\theta$, for all location pairs i, j and household types θ. Berry and Haile (2014) establish sufficient conditions for the nonparametric identification of such a discrete choice system (see Appendix E for further detail).

Once we have the discrete choice system, Allen and Rehbeck (2019) show that the dispersion of marginal utility is obtained using Lemma 1. Namely, denoting the expected utility of type θ household as a function of location-specific utility $\hat{W}^\theta(\boldsymbol{u}^\theta) = \max_{\{\mu_j^\theta\}_j:\sum_j \mu_j^\theta=1} \mu_j^\theta u_j^\theta - \psi^\theta(\{\mu_j^\theta\}_j)$, from the envelope theorem and the chain rule,

$$\frac{\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} = \frac{\partial^2\hat{W}^\theta(\boldsymbol{u}^\theta)}{\partial u_i^\theta \partial u_j^\theta} u_j^{\theta\prime}(C_j^\theta). \tag{36}$$

Taking the ratio between arbitrary pair (i, j), we have

$$\frac{u_j^{\theta\prime}(C_j^\theta)}{u_i^{\theta\prime}(C_i^\theta)} = \frac{\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} / \frac{\partial\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta}. \tag{37}$$

Intuitively, if the marginal utility is higher in j than i, a marginal increase of consumption in location j induces a larger population reallocation away from i, compared to the other way around (consumption increase in i on population reallocation away from j).

We next consider the case where we only know the changes in productivity $\{d\ln A_{ij,k}\}$, corresponding to the *ex-ante* welfare evaluation in the previous section. In this case, we additionally need to identify the changes in consumption $\{dC_j^\theta\}$ and population size $\{d\ln l_j^\theta\}$ as a response to counterfactual shocks $\{d\ln A_{ij,k}\}$. These equilibrium responses are uniquely determined by the factor supply and demand systems. The factor supply system, i.e., how population $\{d\ln l_j^\theta\}$ responds to the vector of consumption $\{dC_j^\theta\}$, can be nonparametrically identified following Berry and Haile (2014) as discussed above. The factor demand system, i.e., how the changes in consumption $\{dC_j^\theta\}$ affect each location's labor demand $\{d\ln l_j^\theta\}$, can be nonparametrically identified following Adao et al. (2017), who establish the nonparametric identification of factor demand system in general equilibrium trade models. Together, $\{dC_j^\theta, d\ln l_j^\theta\}$ can be nonparametrically identified for a counterfactual shock $\{d\ln A_{ij,k}\}$.

While it is reassuring that the welfare changes are in principle nonparametrically identified, the data requirement for the nonparametric identification is unrealistic in most applications. For example, identifying the factor supply system, $\{\partial\hat{\mu}_i^\theta(\boldsymbol{C}^\theta)/\partial C_j^\theta\}_{i,j}$ for all i and j, requires a long period of data and exogenous variation of consumption at every location. Therefore, the purpose of this section is not to suggest a practical non-parametric estimation procedure. Instead, this discussion aims to establish a clear mapping between nonparametric welfare-relevant sufficient statistics and data moments. Such results are useful because they point toward the data moments that discipline the welfare conclusions drawn from spatial equilibrium models.

4 Extensions

We discuss the scope of our results and argue that our framework can accommodate further extensions and generalizations of our baseline environment.

4.1 Beyond Additively Separable Preference Shocks

So far, we have focused on specifications where preference shocks are additively separable. This section relaxes this assumption. We first discuss the general case, and next discuss a special case where preference shocks are multiplicatively separable and follow a max-stable multivariate Fréchet distribution.

General Case. We now assume that utility in location i is given by $U_i^\theta(C_i^\theta, \epsilon_i^\theta)$. Compared to the additively separable specification, the critical difference from additively separable preference is that marginal utility in each location depends on the preference shock draws. Appendix D.1 shows that our theoretical results remain unchanged by appropriately redefining marginal utility.

While this extension is straightforward in theory, it poses a challenge to the identification of aggregate welfare. To understand this challenge, consider a monotone transformation of the utility function from the additively separable class: $U_j^\theta(C_j^\theta, \epsilon_j^\theta) = m(u_j^\theta(C_j^\theta) + \epsilon_j^\theta)$ for some strictly increasing function $m(\cdot)$. This transformation does not affect the model's *positive* prediction because of the ordinal nature of the utility function for location choice decisions. However, the expected marginal utility in each location becomes

$$\mathcal{MU}_j^\theta = u_j^{\theta\prime}(C_j^\theta)\mathbb{E}\left[m'(u_j^\theta(C_j^\theta) + \epsilon_j^\theta)|j = \arg\max_l m(u_l^\theta(C_l^\theta) + \epsilon_l^\theta)\right]. \tag{38}$$

Therefore, the function $m(\cdot)$ generally affects the marginal utility in each location. This discussion implies that the *normative* prediction, i.e., aggregate welfare, generically depends on the choice of $m(\cdot)$. Since $m(\cdot)$ cannot be identified from data, aggregate welfare also cannot be identified from location choice data alone.

This lack of identification is worrisome, as it indicates that welfare predictions are not uniquely pinned down from data. Even when the two models match the same data moments, the welfare conclusions drawn from these two models can be arbitrarily different. However, we show below that, under a common parametric assumption in the existing literature, such a concern is not warranted.

Multiplicative Shocks with Multivariate Fréchet Distribution. We now focus on a special case of nonseparable preference shocks. Specifically, we assume that preference shocks are multiplicatively separable and follow a max-stable multivariate Fréchet distribution. Formally, we assume that preferences for households living in location j of type θ are given by

$$\begin{aligned} &\tilde{U}_j^\theta(C_j^\theta, \tilde{\epsilon}_j^\theta) = \tilde{u}_j^\theta(C_j^\theta)\tilde{\epsilon}_j^\theta, \\ \text{with} \quad &\mathbb{P}(\tilde{\epsilon}_1^\theta \le \bar{\epsilon}_1, \dots, \tilde{\epsilon}_N^\theta \le \bar{\epsilon}_N) = \exp(-G^\theta(K_1^\theta(\bar{\epsilon}_1)^{-\nu^\theta}, \dots, K_N^\theta(\bar{\epsilon}_N)^{-\nu^\theta})), \end{aligned} \tag{39}$$

where G^θ is a function that is homogeneous of degree one, which we call the "correlation function." The key implication of this specification is the max-stability property, where the distribution of the maximum is Fréchet with shape parameter ν^θ.[14]

Assumption (39) covers almost all specifications that appear in the previous literature besides the additively separable one. For example, Redding (2016) is a special case with i.i.d. Fréchet distribution, which corresponds to the case with $G^\theta(x_1, \dots, x_N) = \sum_{j=1}^N x_j$. More generally, this preference specification delivers a generalized extreme value (GEV) demand system with flexible substitution patterns as introduced by McFadden (1978). Dagsvik (1995) shows that GEV demand systems can approximate arbitrary demand systems generated by random utility models.

To consider the property of this specification, consider the log transformation of this utility specification: $u_j^\theta(C_j^\theta) = \ln(\tilde{u}_j^\theta(C_j^\theta))$ and $\epsilon_j^\theta = \ln(\tilde{\epsilon}_j^\theta)$. It is straightforward to show that ϵ_j^θ follows a multivariate Gumbel distribution with the same correlation function

[14] See McFadden (1978) for further properties of this demand system and the correlation function. See also Lind and Ramondo (2023) for the application of this demand system to Ricardian trade models.

$G^{\theta}(\cdot)$ such that

$$U_j^{\theta}(C_j, \epsilon_j) = u_j^{\theta}(C_j^{\theta}) + \epsilon_j^{\theta}, \tag{40}$$
$$\text{with} \quad \mathbb{P}(\epsilon_1 \leq \bar{\epsilon}_1^{\theta}, \ldots, \epsilon_N^{\theta} \leq \bar{\epsilon}_N) = \exp(-G^{\theta}(K_1^{\theta}(\exp(-\nu^{\theta}\bar{\epsilon}_1)), \ldots, K_N^{\theta}(\exp(-\nu^{\theta}\bar{\epsilon}_N)))).$$

Since $\ln(\cdot)$ is a monotone transformation, the systems (39) and (40) have isomorphic *positive* predictions. The following proposition shows that these two models also deliver isomorphic *normative* predictions.

Proposition 4. *Consider the spatial equilibrium with multivariate Fréchet preference shocks with arbitrary correlation (39). Let $\tilde{W} \equiv \tilde{\mathcal{W}}(\{\tilde{W}^{\theta}\}_{\theta\in\Theta})$ be welfare in this economy. Consider another economy with the log transformation of utility specification (40) without changing remaining equilibrium conditions in Definition 1. Let $W \equiv \mathcal{W}(\{W^{\theta}\}_{\theta\in\Theta})$ be welfare in this economy where $\mathcal{W}(\{W^{\theta}\}_{\theta\in\Theta}) \equiv \ln \tilde{\mathcal{W}}(\{\exp(W^{\theta})\}_{\theta\in\Theta})$ is the social welfare function.*

1. *Equilibrium allocations are identical in both economies.*

2. *The welfare decomposition of Proposition 1 is identical in both economies up to a multiplicative constant. Formally, let $d\tilde{W} = \tilde{\Omega}_T + \tilde{\Omega}_{MU} + \tilde{\Omega}_{FE} + \tilde{\Omega}_{TE} + \tilde{\Omega}_R$ be the decomposition in the economy with multiplicative Fréchet preference shocks, and let $dW = \Omega_T + \Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R$ be the decomposition in an economy with the additively separable preference shocks counterpart. Then,*

$$d\tilde{W} = \tilde{W}dW, \quad \tilde{\Omega}_c = \tilde{W}\Omega_c, \quad \text{for } c \in \{T, MU, FE, TE, R\}. \tag{41}$$

Proposition 4 establishes that an economy with multiplicative Fréchet shocks is isomorphic to an economy to its additively separable counterpart for both positive *and* normative implications. An important corollary of Proposition 4 is that all the welfare relevant sufficient statistics of an economy with multiplicative Fréchet shocks are identified, provided that they are identified in an economy with additively separable preference shocks, as in Section 3.3. The possibility of identification is enlightening. As discussed earlier, outside of additively separable preference shocks, it is generally not possible to identify the marginal utility of consumption from location choice data, and therefore our decomposition lacks any empirical content. However, Proposition 4 shows such a concern is not warranted under a class of models with nonseparable preference shocks that covers almost all of the applications in the literature.

What is the reason behind the equivalence under multiplicative Fréchet? As discussed in the previous section, the transformation of the utility function matters only through the differences in marginal utility. The marginal utility of consumption of households in location j in the system (39) is given by

$$\begin{aligned}\mathcal{MU}_j^\theta &= u_j^{\theta\prime}(C_j^\theta)\mathbb{E}\big[\tilde{u}_j^\theta(C_j^\theta)\tilde{\epsilon}_j^\theta | j = \arg\max_l \tilde{u}_l^\theta(C_l^\theta)\tilde{\epsilon}_l^\theta\big] \\ &= u_j^{\theta\prime}(C_j^\theta)\tilde{W}^\theta,\end{aligned} \tag{42}$$

where the first transformation uses the fact that $u_j(C_j) = \ln(\tilde{u}_j^\theta(C_j^\theta))$ and $\epsilon_j^\theta = \ln(\tilde{\epsilon}_j^\theta)$. The second transformation of Equation (42) follows from the max-stable property of $\tilde{\epsilon}_j$: that the distribution of the maximum follows the same distribution irrespective of the chosen option (McFadden 1978, Lind and Ramondo 2023). Therefore, the marginal utility under this specification is identical to its log transformation (40) up to scale $\tilde{W}$. Given that all terms in our welfare decomposition in Proposition 1 scale up by marginal utility under price normalization (18), we have the isomorphism in aggregate welfare.[15]

4.2 Shocks to Amenity and Amenity Externalities

In Section 3, we analyzed the effects of productivity shocks on aggregate welfare. In some contexts, researchers are interested in the shocks to amenities instead of productivity. The analysis in Section 3 embraces this possibility by interpreting some intermediate goods as local amenities. From a measurement perspective, applying Proposition 1 requires knowledge of prices associated with amenities, which is often unobserved. Below, we provide an alternative expression for Proposition 1 without using prices for the amenities.

To consider this extension, we explicitly introduce amenity as an argument in the utility function as follows:

$$U_j^\theta(C_j^\theta, B_j^\theta, \epsilon_j^\theta) = u_j^\theta(C_j^\theta, B_j^\theta) + \epsilon_j^\theta, \tag{43}$$

where B_j^θ is the amenity in region j. Furthermore, we assume that these amenities take

[15] Another way to interpret this result is through a particular property of the Fréchet distribution: the expectation of the log coincides with the log of the expectation.

the following form:

$$B_i^\theta = \tilde{B}_i^\theta g_i^{B,\theta}(\{l_i^\theta\}_\theta), \qquad \gamma_i^{B,\tilde{\theta}\theta} = \frac{\partial \ln g_i^{B,\tilde{\theta}\theta}(\{l_i^\theta\}_\theta)}{\partial \ln l_i^{\tilde{\theta}}}, \tag{44}$$

$\tilde{B}_i^\theta$ is the fundamental component of amenity, $g_i^{B,\theta}(\{l_i^\theta\}_\theta)$ is the spillover function, and $\gamma_i^{B,\tilde{\theta}\theta}$ is the amenity spillover elasticity from type $\tilde{\theta}$ to type θ in location i.

Under this extension, we consider an arbitrary set of small shocks to the exogenous components of productivity, $\{d\ln A_{ij,k}\}$, amenities, $\{d\ln \tilde{B}_i^\theta\}$, and $\{dT_j^\theta\}$. Proposition 1 is modified as follows. First, the technology term now includes the shocks to amenities:

$$\Omega_T = \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k} + \sum_{i,\theta} l_i^\theta \partial_b u_i^\theta B_i^\theta d\ln \tilde{B}_i^\theta, \tag{45}$$

where $\partial_b u_j^\theta \equiv \frac{\partial u_j^\theta}{\partial B_j^\theta}$. The second term captures the effects of exogenous amenity shocks absent reallocation effects. Second, the technological externality term now includes the spillover in amenities:

$$\Omega_{TE} = \mathbb{E}_\theta\left[\text{Cov}_{j|\theta}\left(\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta}\gamma_{jl,k}^\theta + \sum_{\tilde{\theta}} \partial_b u_j^{\tilde{\theta}} B_j^{\tilde{\theta}} \gamma_j^{B,\theta\tilde{\theta}}, d\ln l_j^\theta\right)\right]. \tag{46}$$

The second component has the same feature: if the amenity is traded, the term reflecting changes in amenities from externalities collapses to the same form as the productivity externality term. All the other terms are unaffected.

In quantitative spatial equilibrium literature, researchers often argue that amenity congestion externalities are isomorphic to preference shocks and use these specifications interchangeably. We show in Appendix D.4 that a model without preference shocks but with amenity congestion externality of a particular form is in fact isomorphic to a model without amenity congestion externality but with preference shocks with GEV distribution.

4.3 Idiosyncratic Productivity Shocks

In the baseline analysis, we have considered idiosyncratic shocks to preferences only. Some existing works (e.g., Bryan and Morten 2019) consider instead idiosyncratic shocks to household productivity. In Appendix D.5, we show under some additional assumptions, we can tractably incorporate idiosyncratic productivity shock in addition to idiosyncratic preference shocks.

We consider an environment that households of type θ draw both idiosyncratic productivity shocks in each location, $\boldsymbol{z}^\theta \equiv (z_1^\theta, \ldots, z_N^\theta)$, in addition to idiosyncratic preference shocks, $\boldsymbol{\epsilon}^\theta$. The idiosyncratic productivity shocks determine households' endowment of efficiency units of labor in each location. We impose three additional assumptions relative to the baseline model. First, we restrict attention to the case of log utility, $u_j^\theta(c) = B_j^\theta \ln c$. Second, we assume away the presence of fixed factors. Third, we assume that location-specific transfers are linear in labor income.[16] With these assumptions, the budget constraint of the household living in location j is $P_j^\theta c_j^\theta = (1+\tau_j^\theta) w_j^\theta z_j^\theta$, where τ_j^θ denotes the transfer rate. We let $C_j^\theta \equiv w_j^\theta(1+\tau_j^\theta)/P_j^\theta$ denote the consumption per efficiency unit of labor.

Appendix D.5 shows that, in the above environment, Proposition 1 continues to hold with two modifications. First, C_j^θ now denotes consumption per efficiency unit of labor. Second, the fiscal externality term, Ω_{FE}, contains an additional term that takes into account the changes in the composition of households of different productivity across locations induced by the shock.

4.4 Non-Welfarist Social Welfare Function

Our baseline analysis focuses on the welfarist approach. In some contexts, researchers may want to consider an alternative welfare criterion. For example, consider a scenario where researchers alternatively interpret idiosyncratic preference shocks $\{\epsilon_j^\theta\}$ as "mistakes". Under this interpretation, one may desire to exclude this component of ϵ_j from aggregate welfare.

In Appendix D.6, we consider a general non-welfarist objective

$$W = \mathcal{W}\left(\{\mathcal{U}^{SP,\theta}(\{C_j^\theta\}_j, \{\mu_j^\theta\}_j)\}_\theta\right), \tag{47}$$

where $\mathcal{U}^{SP,\theta}$ is defined arbitrarily on the distribution of consumption and population of household type θ. Appendix D.6 shows that our decomposition in Proposition 1 includes one additional term. The term captures the potential misalignment between the social Planner's welfare assessment of the marginal value of consumption with that of private agents.

[16] This modification is inconsequential in the baseline model, as we can always rewrite $T_j^\theta = \tau_j^\theta w_j^\theta$.

Such an approach is useful also in considering welfare criteria and optimal policies that are exclusively based on subcomponents of our decompositions, as in Dávila and Schaab (2022). In Appendix D.6, we explicitly construct social welfare functions that exclusively target subcomponents of our decompositions and derive optimal spatial transfer formula in each case.

4.5 General Spillovers

In our main model, we assumed that agglomeration externalities are purely a function of local population size (7). In some contexts, researchers specify that a higher population size in the surrounding regions also generates agglomeration spillovers (e.g., Ahlfeldt et al. 2015). In other contexts, researchers also specify that the externality arises from the specific producers' input use (e.g., free entry model with labor fixed cost such as Krugman (1991)) or the producers' output (e.g., congestion cost from shipment, as in Allen and Arkolakis (2022)). In Appendix D.3, we generalize our results by allowing the spillover function (7) to depend on the population in all locations as well as the distribution of production in each location.

4.6 Commuting

Our baseline model assumes that households supply labor at the same location as their residential location. In the urban economics literature, it is often assumed that households make separate decisions about their residential and employment location decisions (e.g., Ahlfeldt et al. 2015, Tsivanidis 2019, Zárate 2022). Our framework can be straightforwardly extended to such a framework by reinterpreting household's location decisions j as a combination of residential and work locations, (j_1, j_2), where the first index captures the residential location and the second index captures the work location. For example, the utility of agents deciding home location j_1 and work location j_2 is given by $U^{\theta}_{j_1 j_2}(C^{\theta}_{j_1 j_2}, \epsilon^{\theta}_{j_1 j_2})$, where $\epsilon^{\theta}_{j_1 j_2}$ is home-and-work-specific preference shocks.[17] Consequently, Proposition 1 remains unchanged by simply replacing j with (j_1, j_2) combinations.

[17] This extension accommodates the specification where households consume different consumption bundles depending on the home-work combination, as studied by Miyauchi, Nakajima, and Redding (2021).

5 Applications

We demonstrate the usefulness of our formula through two applications. The first application takes an "ex-post" approach. We study the welfare changes of the U.S. economy implied by the observed changes in allocation during 2010-2019 while being agnostic about the details of underlying economic structure. The second application takes an "ex-ante" approach. We assess the welfare consequences of counterfactual transportation infrastructure improvement in the U.S. economy using a fully specified model.

5.1 Welfare Changes in the U.S. 2010-2019

What can we learn about the aggregate welfare from the observed spatial distribution of economic activity? We study this question in the context of the U.S. 2010-2019. To answer this question, consider a dataset generated by the model of Section 2 at different dates indexed by t. We assume that the preferences satisfy

$$U^{\theta}_{j,t}(C^{\theta}_{j,t}, \epsilon^{\theta}_{j,t}) = \frac{(C^{\theta}_{j,t})^{1-\rho^{\theta}}}{1-\rho^{\theta}} + \xi^{\theta}_{j,t} + \epsilon^{\theta}_{j,t}, \tag{48}$$

and $\epsilon^{\theta}_{j,t}$ follows Type-I extreme value distribution with shape parameter ν^{θ}. $\xi^{\theta}_{j,t}$ captures unobserved amenity shifters specific to each skill types. As discussed earlier in Section 3.3, while non-parametric identification is possible in theory, we proceed with the above parametric assumptions for practical purposes. The parameter $\rho^{\theta} > 0$ captures the degree to which the marginal utility of consumption varies depending on real consumption.

For any type θ at any location i at time t, suppose we observe population distribution, $l^{\theta}_{j,t}$, pre-tax income $w^{\theta}_{j,t} + \Pi^{\theta}_{t}$, net transfers, T^{θ}_{j}, and price index $P^{\theta}_{j,t}$, as well as total sales of each location, $\sum_{l,k} p_{jl,k} y_{jl,k}$. Also suppose that the researchers know the utility function, $\{\rho^{\theta}\}_{\theta}$, and agglomeration functions, $\{g_{ij,k}(\cdot)\}_{i,j,k}$.

Under these assumptions, Proposition 1 readily gives the first-order welfare changes between any dates that are attributed to (ii) MU dispersion (Ω_{MU}), (iii) fiscal externality (Ω_{FE}), and (iv) technological externality (Ω_{TE}). In Appendix F.1, we show that if we assume away from the input-output linkages and have data on producer price indices, the technology term, (i) Ω_T can be read off from the data. If the researchers are further willing to take a stand on welfare weights across different households, $\{\Lambda^{\theta}\}$, then the first-order welfare changes that are attributed to (v) Redistribution (Ω_R) are recovered as well.

Data. We implement the above approach in the context of Metropolitan Statistical Areas (MSAs) in the United States during the periods of 2010-2019. Our sample consists of 214 MSAs. Following Diamond (2016) and Fajgelbaum and Gaubert (2020), we consider two ex-ante types based on educational attainments: high-skill (more than 4-years of college education) and low-skill (less than 4-years of college education). We also restrict our analysis to the working-age (age 18-64) population. We construct our dataset using a combination of BEA's regional economic accounts, the American Community Survey (ACS) (IPUMS-USA, Ruggles, Flood, Goeken, Grover, Meyer, Pacas, and Sobek 2023), March supplement of the Current Population Survey (March CPS) (IPUMS-CPS, Flood, King, Rodgers, Ruggles, Warren, and Westberry 2023), Consumer Expenditure Survey (CEX), and IRS Statistics of Income (SOI) data.

We first obtain population, pre-tax income, and transfer receipts by MSAs from BEA. We allocate them to each skill group based on their shares in each MSA using 5-years sample of ACS. We obtain tax payments by county from IRS SOI, and then aggregate them to the MSA level using the crosswalk provided by NBER. We further allocate them to each skill group based on the aggregate shares in tax payments by each skill group using March CPS. The net transfer, T_j^θ, is constructed as the difference between transfer receipts and tax payments, but we adjust them by adding a common constant so as to ensure the government budget balance. Finally, we construct price indexes for each MSA and skill group as follows. BEA provides price indexes for four broad categories at the MSA level: goods, housing, utilities, and other services. We compute the expenditure share on these four categories for each skill group using CEX. We then construct price indexes for each MSA and skill group by weighting the price level of four categories using the expenditure weight for each skill group.

Estimation of Utility Function. The key statistic that governs the welfare consequence of regional growth is marginal utility in each location. As our discussion in 3.3 highlights, they can be identified from the location choice data. To demonstrate the feasibility, we estimate $(\rho^\theta, \nu^\theta)$ through the generalized method of moments (GMM). To build instrument variables, we construct a shift-share instrument that interacts with local industry composition with the national industry employment growth for each skill type θ, similarly to Diamond (2016). We focus on long changes from 2010 to 2019. We describe the detailed estimation procedure in Appendix F.2.

Table 2 shows the estimation results. We find that the low-skill households have a

	Low-skill	High-skill
ρ^θ	1.52	1.29
	(0.40)	(0.80)
ν^θ	0.23	0.42
	(0.23)	(0.48)
Observations	214	214

Table 2: GMM Estimates of Utility Function Parameters

Note: The table reports estimates of $(\rho^\theta, \nu^\theta)$ for each skill type. The standard error in parenthesis is computed using a consistent estimator of the asymptotic covariance matrix.

	dW	Ω_T	Ω_{MU}	Ω_{FE}	Ω_{TE}	Ω_R
2010-2015	2.247%	2.043%	0.136%	0.022%	0.014%	0.033%
2015-2019	1.843%	1.354%	-0.003%	0.009%	-0.073%	0.556%

Table 3: Welfare Changes in the U.S. 2010-2019

Note: The table reports welfare decomposition based on Proposition 1 based on observed changes in allocation across U.S. MSAs during the periods on each row. The welfare is expressed in the units of GDP in the initial period. All reported numbers are annualized changes.

higher curvature (ρ^θ) and lower migration elasticity (ν^θ) than high-skill households, although the estimates show fairly a wide range of uncertainty. Note that for our welfare decompositions, the parameters ν^θ are not relevant.

Rest of Parameterization. We assume away the input-output linkages and set the total sales of each skill type in each location as their pre-tax income. We take the values of productivity spillover elasticity from Fajgelbaum and Gaubert (2020) and set them to $(\gamma^l_{ij,l}, \gamma^h_{ij,l}, \gamma^l_{ij,h}, \gamma^h_{ij,h}) = (0.003, 0.044, 0.02, 0.053)$, where $\gamma^\theta_{ij,\theta'}$ corresponds to the productivity spillover from type θ to θ' for the goods shipped from i to j. The skill type h denotes high-skill and l denotes low-skill. For welfare weights, we proceed with assuming utilitarian welfare, $\Lambda^\theta = 1$ for all θ.

Results. Table 3 shows the welfare decomposition based on observed changes in allocation during 2010-2015 and 2015-2019. All these numbers are annualized changes and

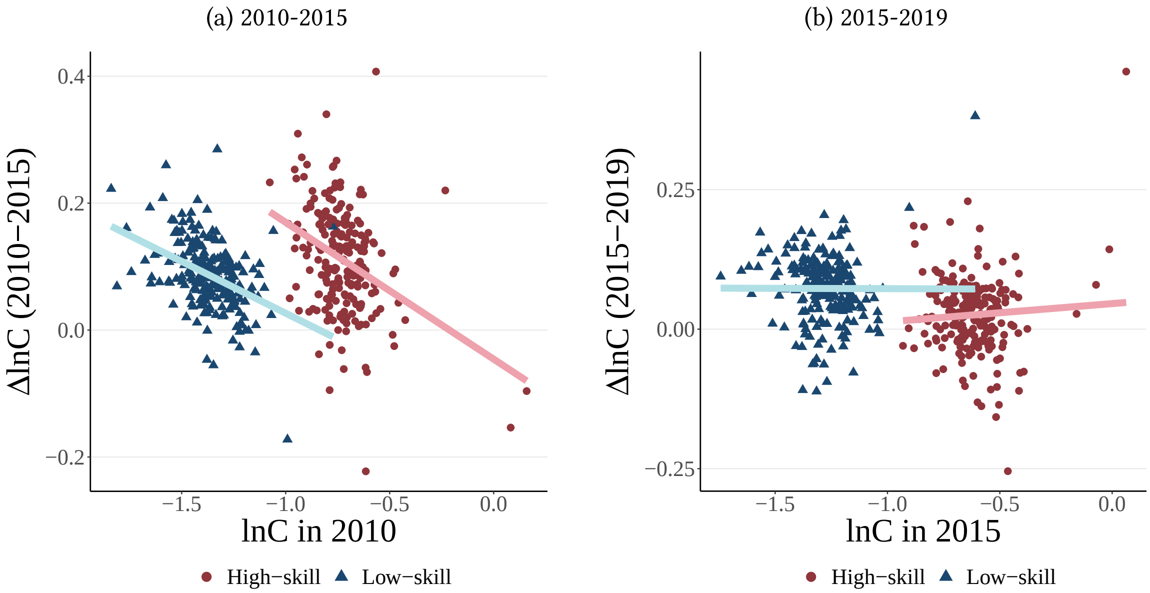

Figure 1: Convergence in Consumption Across MSAs and Skill Groups

Note: The figure plots consumption changes for each MSA and skill group against the initial consumption level. The solid line is the best linear fit within each skill group.

expressed in the units of GDP in the initial period. Combined with our choice of numeraire (18), these numbers answer the following questions: "if we were to achieve the same welfare changes by increasing utility level in all locations for all household types by increasing consumption, how much percentage of GDP do we need?"

In 2010-2015, aggregate welfare increased by 2.2% of GDP, and the largest contributor is technology. The MU dispersion plays a non-trivial role in amplifying aggregate welfare increase. This is driven by the reduction in spatial consumption inequality within a skill group, the left panel of Figure 1 shows. All other three components, fiscal externality, technological externality, and redistribution, also amplify the aggregate welfare increase, but the magnitudes are small. During 2015-2019, the MU dispersion term is close to zero, while the redistribution term substantially amplifies the welfare gains. The right panel of Figure 1 explains why. During this period, the spatial distribution of consumption stopped converging, but there was a convergence of consumption across skill groups.

Sensitivity Analysis. In Appendix F.3, we present sensitivity analysis to parameter values. While the value of ρ^θ, the parameter governing the marginal utility, does not affect $\Omega_T, \Omega_{TE}, \Omega_{FE}$, while Ω_{MU} and Ω_R appear to be sensitive to the value of ρ^θ.

5.2 Transportation Infrastructure

What are the aggregate welfare effects of improving a leg of transportation infrastructure? To answer this question, Allen and Arkolakis (2022) developed a quantitative parametric general equilibrium model that features endogenous transportation costs and traffic congestion. We apply our framework to their model to unpack the welfare gains from counterfactual transportation infrastructure improvement and productivity shocks.

5.2.1 Allen and Arkolakis (2022) Model.

We summarize the overview of Allen and Arkolakis (2022) model here. We describe the details of their setup in Appendix G.1. consider an environment with a homogeneous population, therefore we drop superscript θ. They assume log utility with type-I extreme value preference shocks. Each location produces location-specific goods using labor. The local productivity is subject to agglomeration externality with an elasticity of α. The location-specific goods can be traded subject to shipment costs and are then combined in a CES manner to produce final consumption goods. They do not have spatial transfers.

The critical feature of Allen and Arkolakis (2022) model is that the shipment cost $\tau_{ij,k}$ is endogenously determined through a route choice problem. Furthermore, the shipment cost is subject to congestion externality with an elasticity of λ.

Allen and Arkolakis (2022) use the above model to study the welfare impacts of a marginal reduction in shipment technology. We show that applying Proposition 1 into this environment implies that the aggregate welfare impacts from arbitrary changes in exogenous component of shipment costs, $\tilde{t}_{kl}$, and location-specific productivity, A_i, can be expressed as $dW = \Omega_T + \Omega_{MU} + \Omega_{TE}$, where

$$\Omega_T = -\sum_{k,l} \Xi_{k,l} d\ln \tilde{t}_{kl} + \sum_i Y_i d\ln A_i, \tag{49}$$

$$\Omega_{MU} = \mathrm{Cov}_j\left(-w_j, d\ln C_j\right), \tag{50}$$

$$\Omega_{TE} = \Omega_{TE,S} + \Omega_{TE,A}, \quad \Omega_{TE,S} = -\lambda \sum_{k,l} \Xi_{k,l} d\ln \Xi_{kl}, \quad \Omega_{TE,A} = \alpha \sum_i Y_i d\ln l_i, \tag{51}$$

and $\Omega_{TE,S}$ and $\Omega_{TE,A}$ correspond to the technological externality arising from shipment congestion externality and productivity agglomeration externality, respectively. In the above expression, $\Xi_{k,l}$ is the total value of shipments from k to l, and Y_i is the income in location i

Transportation Infrastructure Improvement. We follow Allen and Arkolakis (2022) to analyze the aggregate welfare effects of highway networks across United States. Using 2012 Highway Performance Monitoring System (HPMS) dataset by the Federal Highway Administration, they create the infrastructure network across core-based statistical areas (CBSAs). The resulting network consists of 228 locations and 704 links between adjacent nodes. We follow the same calibration procedure to obtain the baseline equilibrium population allocation and trade flows and the same calibrated parameters at $\alpha = 0.1$ and $\lambda = 0.092$. See Allen and Arkolakis (2022) for the details of their calibration procedure.

Using the calibrated model, Allen and Arkolakis (2022) undertakes the counterfactual simulation to decrease the shipment cost by 1 percent for each of 704 links and documents substantial heterogeneity in gains from the transportation infrastructure improvement. Here, we undertake the same counterfactual simulation and obtain the welfare gains ΔW for each link. We then obtain each term of our decomposition Ω_T, Ω_{MU}, $\Omega_{TE,S}$, $\Omega_{TE,A}$ using Equations (49), (50), and (51). We also obtain the residual $\Omega_{Resid} \equiv \Delta W - \Omega_T - \Omega_{MU} - \Omega_{TE,S} - \Omega_{TE,A}$, which can be interpreted as the second-order effect.

Table 4a provides the results of the welfare decomposition. In Panel (A), we present the results of the linear regression of each term of our decomposition on the overall welfare gains ΔW, where each observation corresponds to each of the experiments to decrease $\tilde{t}_{km}$ by 1 percent. These coefficients add up to 1 by construction. The coefficients are equivalent to a variance decomposition in which the covariance terms are split equally.

There are several notable findings. First, the coefficient of the technology term Ω_T is around 2 (Column 1). This implies that one would substantially overestimate the dispersion of the heterogeneous welfare gains by solely relying on the calculation proposed by Hulten (1978). Second, the coefficient of the shipment congestion externality term $\Omega_{TE,S}$ is around minus one (Column 2). This finding indicates that the shipment congestion externality has a large offsetting effect for the heterogeneous welfare gains of transportation infrastructure. These results are consistent with the results of the counterfactual simulation by Allen and Arkolakis (2022), who reach the same conclusion by comparing the simulation results with and without shipment costs ($\lambda = 0.092$ vs $\lambda = 0$). Third, the terms Ω_T and $\Omega_{TE,S}$ jointly account for a major source of the variation of heterogeneous welfare gains.

Productivity Shocks. To further illustrate our approach, we use Allen and Arkolakis (2022) to study how regional productivity shocks give rise to welfare gains. More con-

(a) Transportation Infrastructure Improvement

	Dependent variable:				
	Ω_T	Ω_{MU}	$\Omega_{TE,S}$	$\Omega_{TE,A}$	Ω_{Resid}
	(1)	(2)	(3)	(4)	(5)
ΔW	2.04	-0.03	-1.03	0.01	0.02
	(0.04)	(0.02)	(0.04)	(0.01)	(0.0005)
Observations	703	703	703	703	703
R^2	0.77	0.004	0.48	0.004	0.66

(b) Productivity Shock

	Dependent variable:				
	Ω_T	Ω_{MU}	$\Omega_{TE,S}$	$\Omega_{TE,A}$	Ω_{Resid}
	(1)	(2)	(3)	(4)	(5)
ΔW	1.08	-0.30	0.10	0.10	0.02
	(0.01)	(0.01)	(0.004)	(0.005)	(0.0001)
Observations	227	227	227	227	227
R^2	0.99	0.65	0.77	0.65	1.00

Table 4: Welfare Decomposition in Allen and Arkolakis (2022) Model

Note: The table reports variance decomposition of aggregate welfare changes for each of the counterfactual experiments in Allen and Arkolakis (2022) model. Appendix Figure G.1a and G.1b show the scatter plot that visualizes the above relationships.

cretely, we conduct a counterfactual simulation to increase the regional productivity A_i by one percent for each of 227 CBSA within our sample.

Table 4b presents our results. The coefficient of the technology term Ω_T is 1.08 (Column 1), which is close to (yet significantly larger than) one. The R-squared is high at 0.99, indicating that the calculation based on Hulten (1978) has a reasonable approximation to the heterogeneous welfare gains from regional productivity shocks. However, it does not imply that other terms are relevant. We find that the coefficient of the MU dispersion term Ω_{MU} is -0.3 (Column 2), offsetting the overall dispersion of heterogeneous welfare gains. This pattern arises because a location with large revenue share tend to have high real income in the data. Therefore, in a low revenue share region, productivity shocks have a larger welfare effect because of a higher marginal utility under log utility. We also find that the coefficients of the technological externality from shipping congestion cost ($\Omega_{TE,S}$) and agglomeration externality ($\Omega_{TE,A}$) is both positive at 0.1 (Columns 3 and 4), substantially contributing the hetherogeneity of overall welfare gains.

6 Concluding Remarks

In a general class of spatial equilibrium models, we have developed a theory to unpack the sources of welfare gains from changes in technology. The starting point of our analysis is the observation that the spatial equilibrium does not maximize aggregate welfare not only because of agglomeration externality but also because of spatial dispersions in marginal utility. This implies that Hulten's theorem does not apply to spatial equilibrium models. We provide a non-parametric sufficient statistics formula that characterizes the departure from Hulten's characterization. The formula shows the first-order changes in aggregate welfare can be exactly decomposed into five terms: (i) technology effects á la Hulten, (ii) spatial dispersion in marginal utility, (iii) fiscal externalities, (iv) technological externalities, and (v) redistribution. We then show that Hulten's characterization is recovered in the presence of the second-best spatial transfers, which we provide a non-parametric formula for. We have demonstrated the usefulness and the relevance of our decompositions through two applications to the US economy. The natural next step is to incorporate dynamics into our framework, which we pursue in an ongoing work (Donald, Fukui, and Miyauchi 2023).

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Online Appendix for "Unpacking Aggregate Welfare in a Spatial Economy"

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A Proofs

A.1 Proof of Lemma 1

Economy with Heterogeneous Preferences. Consider the problem of households of type θ deciding where to live. We index each individual by $\omega \in [0, \ell^\theta]$, and $\{\epsilon_k^\theta(\omega)\}_k$ denote the preference draw of individual ω. Each individual solves the following problem:

$$\begin{aligned} v^\theta(\omega) = \max_{\{\mathbb{I}_j^\theta(\omega)\}_j} \sum_j \mathbb{I}_j^\theta(\omega) \left[u_j^\theta(C_j^\theta) + \epsilon_j^\theta(\omega) \right] \\ \text{s.t.} \quad \sum_j \mathbb{I}_j^\theta(\omega) = 1, \end{aligned} \tag{A.1}$$

where $\mathbb{I}_j^\theta(\omega) \in \{0, 1\}$ is an indicator function for location choice of individual ω, and $C_j^\theta = w_j^\theta / P_j^\theta$. The fraction of individuals living in location j is given by

$$\mu_j^\theta = \frac{1}{\ell^\theta} \int_0^{\ell^\theta} \mathbb{I}_j^\theta(\omega) d\omega. \tag{A.2}$$

Economy with Representative Agent. Define the following function:

$$\begin{aligned} \psi^\theta(\{\mu_j^\theta\}_j) = - \max_{\{\mathbb{I}_j^\theta(\omega)\}_{\omega,j}} \frac{1}{\ell^\theta} \int_0^{\ell^\theta} \sum_j \epsilon_j^\theta(\omega) \mathbb{I}_j^\theta(\omega) d\omega \\ \text{s.t.} \quad \frac{1}{\ell^\theta} \int_0^{\ell^\theta} \mathbb{I}_j^\theta(\omega) d\omega = \mu_j^\theta \\ \sum_j \mathbb{I}_j^\theta(\omega) = 1. \end{aligned} \tag{A.3}$$

The representative agent solves

$$W^\theta = \max_{\{\mu_j^\theta\}_j : \sum_j l_j = 1} \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \tag{A.4}$$

Equivalence Result. We formally restate the equivalence result of Lemma 1 as follows.

Lemma. *Suppose $\{\mathbb{I}_j^\theta(\omega)\}_j$ solves (A.1) for all ω. Then, $\{\mu_j^\theta\}_j$, given by (A.2), solves (A.4). Conversely, suppose $\{\mu_j^\theta\}_j$ solves (A.4). Then $\{\mathbb{I}_j^\theta(\omega)\}_{\omega,j}$, given by the solution to (A.3) associated with $\{\mu_j^\theta\}_j$, solves (A.1) for almost all ω. Moreover, the expected utility in the economy* *with heterogeneous preferences equals the utility of the representative agent:*

$$\frac{1}{\ell^\theta}\int_0^{\ell^\theta} v^\theta(\omega)d\omega = W^\theta$$

Proof. We prove the first part. Suppose to the contrary, there exists $\{\tilde{\mu}_j^\theta\}_j$ such that

$$\sum_j \tilde{\mu}_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}_j) > \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j). \tag{A.5}$$

Let $\{\tilde{\mathbb{I}}_j^\theta(\omega)\}_{\omega,j}$ denote the solution to (A.3) associated with $\{\tilde{\mu}_j^\theta\}_j$. Plugging into (A.5),

$$\frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta)+\epsilon_j^\theta(\omega)\right]d\omega > \frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_j \mathbb{I}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta)+\epsilon_j^\theta(\omega)\right]d\omega, \tag{A.6}$$

where $\sum_j \tilde{\mathbb{I}}_j^\theta(\omega) = 1$ and $\sum_j \mathbb{I}_j^\theta(\omega) = 1$ for all ω. However, this is a contradiction because by our presumption, for any ω,

$$\sum_j \mathbb{I}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta)+\epsilon_j^\theta(\omega)\right] \geq \sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta)+\epsilon_j^\theta(\omega)\right]$$

for all $\tilde{\mathbb{I}}_j^\theta(\omega)$, which would imply

$$\frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta)+\epsilon_j^\theta(\omega)\right]d\omega \leq \frac{1}{\ell^\theta}\int_0^{\ell^\theta}\sum_k \mathbb{I}_k^\theta(\omega)\left[u_j^\theta(C_j^\theta)+\epsilon_j^\theta(\omega)\right]d\omega. \tag{A.7}$$

Now we prove the converse. Suppose to the contrary, there exists $\{\tilde{\mathbb{I}}_j^\theta(\omega)\}_j$ such that

$$\sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta)+\epsilon_j^\theta(\omega)\right] > \sum_j \mathbb{I}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta)+\epsilon_j^\theta(\omega)\right] \tag{A.8}$$

and $\sum_j \tilde{\mathbb{I}}_j^\theta(\omega) = 1$ hold for all $\omega \in \Omega$, where $\Omega \subset [0, \ell^\theta]$ and $|\Omega| > 0$. Define

$$\tilde{\mu}_j^\theta = \frac{1}{\ell^\theta}\int_0^{\ell^\theta}\tilde{\mathbb{I}}_j^\theta(\omega)d\omega. \tag{A.9}$$

Then

$$\sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi(\{l_j\}_j) = \frac{1}{\ell^\theta}\int_0^{\ell^\theta} \sum_j \mathbb{I}_j(\omega)\left[u_j^\theta(C_j^\theta) + \epsilon_j(\omega)\right] d\omega$$
$$< \frac{1}{\ell^\theta}\int_0^{\ell^\theta} \sum_j \tilde{\mathbb{I}}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta) + \epsilon_j(\omega)\right] d\omega$$
$$\leq \sum_j \tilde{\mu}_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}_j).$$

This is a contradiction that $\{l_j\}_j$ is a solution to (A.4).

We need to show that the expected utility coincides with each other in the two economies. This immediately follows given the above result. Let $\{\mathbb{I}_j^\theta(\omega)\}_{\omega,j}$ be the solution to (A.1) for all ω, and let $\{l_j\}_j$ denote the solution to (A.4). Then

$$\frac{1}{\ell^\theta}\int_0^{\ell^\theta} \sum_j \mathbb{I}_j^\theta(\omega)\left[u_j^\theta(C_j^\theta) + \epsilon_j^\theta(\omega)\right] d\omega = \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j). \tag{A.10}$$

□

A.2 Proof of Lemma 2

The constraints (22) and (23) immediately imply that at the solutions of pseudo-planning problem, $C_j^\theta = \check{C}_j^\theta$ and $\mu_j^\theta = \check{\mu}_j^\theta$, and therefore $l_j^\theta = \check{l}_j^\theta$ and $\mathcal{A}_{ij,k} = \check{\mathcal{A}}_{ij,k}$.

We now show the remaining allocations in the pseudo-planning problem coincide with decentralized equilibrium. In the decentralized equilibrium, quantities $\{\check{\mathbf{c}}_j^\theta, \check{\mathbf{x}}_{ij,k}, \check{\mathbf{l}}_{ij,k}, \check{h}_{ij,k}\}$ and prices $\{P_j^\theta, p_{ij,k}, w_i^\theta, r_i\}$ solve the resource constraints (11)-(14) as well as the following firms' optimality conditions, given by

$$P_j^\theta \frac{\partial \mathcal{C}_j^\theta}{\partial c_{ij,k}^\theta} = p_{ij,k}, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^\theta, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i, \quad p_{ij,k} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}, \tag{A.11}$$

The first-order conditions of the pseudo planner's problem with respect to $c_{ij,k}$, $l_{ij,k}^\theta$, $h_{ij,k}$, $x_{ij,k}^{l,m}$

are

$$P_j^{L,\theta}\frac{\partial \mathcal{C}_j^\theta}{\partial c_{ij,k}^\theta}=p_{ij,k}^L,\quad p_{ij,k}^L A_{ij,k}\frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta}=w_i^{L,\theta},\quad p_{ij,k}^L A_{ij,k}\frac{\partial f_{ij,k}}{\partial h_{ij,k}}=r_i^L,\quad p_{ij,k}^L A_{ij,k}\frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}}=p_{li,m}^L, \tag{A.12}$$

where $\{P_j^{L,\theta},p_{ij,k}^L,w_i^{L,\theta},r_i^L\}$ are Lagrangian multipliers on (11)-(14), respectively. Therefore the decentralized equilibrium allocation satisfies the optimality conditions for the pseudo-planning problem. Moreover, the decentralized equilibrium prices and the Lagrangian multipliers in the pseudo-planning problem coincide up to a multiplicative constant.

A.3 Proof of Proposition 1

The Lagrangian of the pseudo-planning problem is

$$\begin{aligned}
\mathcal{L}=\mathcal{W}&\left(\left\{\sum_j \hat{\mu}_j^\theta(\boldsymbol{C}^\theta)u_j^\theta(C_j^\theta)-\psi^\theta(\{\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)\}_j)\right\}_{\theta\in\Theta}\right)\\
&+\sum_{i,j,k}p_{ij,k}^L\left[A_{ij,k}g_{ij,k}(\{\ell^\theta\mu_j^\theta(\boldsymbol{C}^\theta)\}_\theta)f_{ij,k}(\mathbf{l}_{ij,k},h_{ij,k},\mathbf{x}_{ij,k})-\left(\sum_\theta c_{ij,k}^\theta+\sum_{l,m}x_{jl,m}^{i,k}\right)\right]\\
&+\sum_{j,\theta}P_j^{L,\theta}\left[\mathcal{C}_j^\theta(\mathbf{c}_j^\theta)-C_j^\theta\ell^\theta\mu_j^\theta(\boldsymbol{C}^\theta)\right]\\
&+\sum_{j,\theta}w_j^{L,\theta}\left[\ell^\theta\hat{\mu}_j^\theta(\boldsymbol{C}^\theta)-\sum_{i,k}l_{ji,k}^\theta\right]\\
&+\sum_j r_j^L\left[\bar{h}_j-\sum_{i,k}h_{ji,k}\right]\\
&+\sum_j\eta_j^\theta\left[C_j^\theta-\check{C}_j^\theta\right],
\end{aligned}$$

where we have substituted constraints (7), (21) and (22). Since one of the constraints $C_j^\theta=\check{C}_j^\theta$ is implied by the resource constraints, we normalize the Lagrangian multipliers on them so that $\sum_\theta\sum_j\frac{1}{u_j^{\theta\prime}(\check{C}_j^\theta)}\eta_j^\theta=0$. The first order condition of pseudo-planning problem with respect to C_j^θ is given by

$$\Lambda^\theta\ell^\theta\mu_j^\theta u_j^{\theta\prime}(C_j^\theta)-P_j^{L,\theta}\ell^\theta\mu_j^\theta$$

$$+\sum_i \frac{\partial \hat{\mu}_l(\boldsymbol{C}^\theta)}{\partial C_j^\theta}\left[w_l^{L,\theta}\ell^\theta - P_l^{L,\theta}\ell^\theta + \sum_{i,j,k} p_{ij,k}^L \mathcal{A}_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k})\gamma_{ij,k}^\theta \ell^\theta \frac{1}{l_l^\theta}\right] + \eta_j^\theta = 0.$$

Dividing both sides by $u_j'(C_j^\theta)$ and adding up across j and θ, we have

$$\underbrace{\sum_\theta \Lambda^\theta \ell^\theta}_{=1} - \sum_\theta \sum_j \frac{P_j^{L,\theta}}{u_j^{\theta\prime}(C_j^\theta)} l_j^\theta$$
$$+ \sum_\theta \sum_i \underbrace{\sum_j \frac{\partial \hat{\mu}_l(\boldsymbol{C}^\theta)}{\partial C_j^\theta}\frac{1}{u_j^{\theta\prime}(C_j^\theta)}}_{=0}\left[w_l^{L,\theta}\ell^\theta - P_l^{L,\theta}\ell^\theta + \sum_{i,j,k} p_{ij,k}^L \mathcal{A}_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k})\gamma_{ij,k}^\theta \ell^\theta \frac{1}{l_l^\theta}\right]$$
$$+ \underbrace{\sum_\theta \sum_j \frac{1}{u_j^{\theta\prime}(C_j^\theta)}\eta_j^\theta}_{=0} = 0,$$

which implies

$$\sum_\theta \sum_j \frac{P_j^{L,\theta}}{u_j^{\theta\prime}(C_j^\theta)} l_j^\theta = 1. \tag{A.13}$$

Since this corresponds to how we chose the numeraire, (18), and Lemma 2 implies the Lagrangian multipliers coincide with the decentralized equilibrium prices up to scale, we have $P_j^{L,\theta} = P_j^\theta, p_{ij,k}^L = p_{ij,k}, w_i^{L,\theta} = w_i^\theta, r_i^L = r_i$.

By applying the Envelope theorem,

$$\begin{aligned}
\frac{dW}{d\ln A_{il,k}} =& \frac{d\mathcal{L}}{d\ln A_{il,k}} \\
=& p_{il,k} y_{il,k} + \sum_\theta \sum_j \eta_j^\theta \frac{d\check{C}_j^\theta}{d\ln A_{il,k}} \\
=& p_{il,k} y_{il,k} + \sum_\theta \sum_j l_j^\theta [\Lambda^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta] \frac{dC_j^\theta}{d\ln A_{il,k}} \\
&+ \sum_\theta \sum_j \left[w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta}\gamma_{jl,k}^\theta\right] \ell^\theta \frac{\partial \hat{\mu}_j(\boldsymbol{C}^\theta)}{\partial C_j^\theta}\frac{d\check{C}_j^\theta}{d\ln A_{il,k}}.
\end{aligned}$$

Therefore, noting $dl_j^\theta = \ell^\theta d\mu_j^\theta$

$$\begin{aligned} dW =& \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k} + \sum_\theta \sum_j l_j^\theta [\Lambda^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta] dC_j^\theta \\ &+ \sum_\theta \sum_j [w_j^\theta - P_j^\theta C_j^\theta] dl_j^\theta + \sum_\theta \sum_j \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta dl_j^\theta . \end{aligned} \tag{A.14}$$

Now,

$$\begin{aligned} &\sum_\theta \sum_j l_j^\theta [\Lambda^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta] dC_j^\theta \\ =& \sum_\theta \ell^\theta \sum_j \mu_j^\theta [\Lambda^\theta - \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}] u_j^{\theta\prime}(C_j^\theta) dC_j^\theta \\ =& \sum_\theta \ell^\theta \left[\sum_j Cov_{j|\theta}(\Lambda^\theta - \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta) dC_j^\theta) + \mathbb{E}_{j|\theta}[\Lambda^\theta - \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}] \mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta] \right] \\ =& \sum_\theta \ell^\theta \left[\text{Cov}_{j|\theta}(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta) dC_j^\theta) + \left(\Lambda^\theta - \mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}] \right) \mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta] \right] \\ =& \mathbb{E}_\theta[\text{Cov}_{j|\theta}(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta) dC_j^\theta)] + Cov_\theta \left(\Lambda^\theta - \mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}], \mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta] \right) \\ &+ \mathbb{E}_\theta \left[\Lambda^\theta - \mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}] \right] \mathbb{E}_\theta \left[\mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta] \right] \\ =& \mathbb{E}_\theta[\text{Cov}_{j|\theta}(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta) dC_j^\theta)] + Cov_\theta \left(\Lambda^\theta - \mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}], \mathbb{E}_{j|\theta}[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta] \right), \end{aligned}$$

where the last equation used the fact that $\mathbb{E}_\theta \left[\Lambda^\theta \right] = 1$ under our normalization of welfare weights $\sum_\theta \ell^\theta \Lambda^\theta = 1$ and $\mathbb{E}_\theta \left[\mathbb{E}_{j|\theta}[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}] \right] = \mathbb{E}_\theta \left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} \right] = 1$ under our price normalization (18). The two terms correspond to (ii) MU dispersion and (v) redistribution in Proposition 1.

Similarly,

$$\begin{aligned} &\sum_\theta \sum_j [w_j^\theta - P_j^\theta C_j^\theta] dl_j^\theta \\ =& \sum_\theta \ell^\theta \sum_j \mu_j^\theta [w_j^\theta - P_j^\theta C_j^\theta] d\ln l_j^\theta \end{aligned}$$

$$
\begin{aligned}
&= \sum_{\theta} \ell^{\theta} \left[\mathrm{Cov}_{j|\theta}(w_j^{\theta} - P_j^{\theta} C_j^{\theta}, d\ln l_j^{\theta}) + \mathbb{E}_{j|\theta}[w_j^{\theta} - P_j^{\theta} C_j^{\theta}] \underbrace{\mathbb{E}_{j|\theta}[d\ln l_j^{\theta}]}_{=0} \right] \\
&= \mathbb{E}_{\theta} \left[\mathrm{Cov}_{j|\theta}(w_j^{\theta} - P_j^{\theta} C_j^{\theta}, d\ln l_j^{\theta}) \right] \\
&= \mathbb{E}_{\theta} \left[\mathrm{Cov}_{j|\theta}(-\Pi^{\theta} - T_j^{\theta}, d\ln l_j^{\theta}) \right] \\
&= \mathbb{E}_{\theta} \left[\mathrm{Cov}_{j|\theta}(-T_j^{\theta}, d\ln l_j^{\theta}) \right],
\end{aligned}
$$

which corresponds to (iii) fiscal externality term. Finally,

$$
\begin{aligned}
&\sum_{\theta} \sum_{j} \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^{\theta}} \gamma_{jl,k}^{\theta} dl_j^{\theta} \\
&= \sum_{\theta} \ell^{\theta} \sum_{j} \mu_j^{\theta} \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^{\theta}} \gamma_{jl,k}^{\theta} d\ln l_j^{\theta} \\
&= \mathbb{E}_{\theta} \left[\mathrm{Cov}_{j|\theta}\Big(\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^{\theta}} \gamma_{jl,k}^{\theta}, d\ln l_j^{\theta}\Big) \right],
\end{aligned}
$$

which corresponds to (iv) technological externality term.

A.4 Proof of Proposition 2

We first characterize the first-order condition of the relaxed problem (29). The first-order conditions of Problem with respect to $c_{ij,k}^{\theta}, l_{ij,k}^{\theta}, h_{ij,k}, x_{ij,k}^{l,m}$ are given by

$$
P_j^{SB,\theta} \frac{\partial \mathcal{C}_j^{\theta}}{\partial c_{ij,k}^{\theta}} = p_{ij,k}^{SB}, \quad p_{ij,k}^{SB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}^{\theta}} = w_i^{SB,\theta}, \quad p_{ij,k}^{SB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i^{SB}, \quad p_{ij,k}^{SB} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}^{SB}, \tag{A.15}
$$

where $P_j^{SB,\theta}, p_{ij,k}^{SB}, w_j^{SB,\theta}$, and r_j^{SB} are Lagrangina multipliers on the constraints (11)-(14). These conditions are identical to the equilibrium conditions (A.11), with $P_j^{SB,\theta}, p_{ij,k}^{SB}, w_j^{SB,\theta}$, and r_j^{SB} coinciding with $P_j^{\theta}, p_{ij,k}, w_j^{\theta}$, and r_j up to a multiplicative constant. The first-order condition with respect to C_j^{θ} is given by

$$
\ell^{\theta} \mu_j^{\theta} \left[\hat{\Lambda}^{\theta} u_j^{\theta\prime}(C_j^{\theta}) - P_j^{SB,\theta} \right] = \ell^{\theta} \sum_i \frac{\partial \hat{\mu}_i^{\theta}(\boldsymbol{C}^{\theta})}{\partial C_j^{\theta}} \left[P_i^{SB,\theta} C_i^{\theta} - w_i^{SB,\theta} - \sum_{l,k} p_{il,k}^{SB} y_{il,k} \frac{1}{l_i^{\theta}} \gamma_{il,k}^{\theta} \right], \tag{A.16}
$$

where $\hat{\Lambda}^\theta = 1$ for $\theta = \bar{\theta}$ and $\hat{\Lambda}^\theta$ for $\theta = \tilde{\theta}$ correspond to Lagrangian multipliers on (31). Dividing both sides by $u_j^{\theta\prime}(C_j^\theta)$ and summing across j and θ,

$$
\begin{aligned}
&\sum_\theta \ell^\theta \sum_j \mu_j^\theta \left[\tilde{\Lambda}^\theta - \frac{P_j^{SB,\theta}}{u_j^{\theta\prime}(C_j^\theta)}\right] \\
&= -\sum_\theta \ell^\theta \sum_j \sum_i \frac{1}{u_j^{\theta\prime}(C_j^\theta)} \frac{\partial \mu_i^\theta}{\partial C_j^\theta} \left[w_i^{SB,\theta} - P_i^{SB,\theta} C_i^\theta + \sum_{l,k} p_{il,k}^{SB} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right] \\
&= -\sum_\theta \ell^\theta \sum_i \underbrace{\sum_j \frac{1}{u_j^{\theta\prime}(C_j^\theta)} \frac{\partial \mu_i^\theta}{\partial C_j^\theta}}_{=0} \left[w_i^{SB} - P_i^{SB,\theta} C_i^\theta + \sum_{l,k} p_{il,k}^{SB} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right] \\
&= 0,
\end{aligned}
$$

which implies

$$
\sum_\theta \sum_j l_j^\theta \frac{P_j^{SB,\theta}}{u_j^{\theta\prime}(C_j^\theta)} = \sum_\theta \ell^\theta \hat{\Lambda}^\theta. \tag{A.17}
$$

Comparing (A.17) and (18), and noting that $P_j^{SB,\theta}, p_{ij,k}^{SB}, w_j^{SB,\theta}$, and r_j^{SB} coinciding with $P_j^\theta, p_{ij,k}, w_j^\theta$, and r_j up to a multiplicative constant, we have

$$
P_j^{SB,\theta} = P_j^\theta \sum_\theta \ell^\theta \hat{\Lambda}^\theta, \quad p_{ij,k}^{SB} = p_{ij,k} \sum_\theta \ell^\theta \hat{\Lambda}^\theta, \quad w_j^{SB,\theta} = w_j^\theta \sum_\theta \ell^\theta \hat{\Lambda}^\theta, \quad r_j^{SB} = r_j \sum_\theta \ell^\theta \hat{\Lambda}^\theta.
$$

In turn, we can rewrite (A.16) using equilibrium prices as

$$
\mu_j^\theta \left[\tilde{\Lambda}^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta\right] = \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta} \left[P_i^\theta C_i^\theta - w_i^\theta - \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta\right], \tag{A.18}
$$

where $\tilde{\Lambda}^\theta \equiv \frac{\hat{\Lambda}^\theta}{\sum \ell^\theta \hat{\Lambda}^\theta}$. By noting $T_i^\theta = P_i^\theta C_i^\theta - w_i^\theta - \Pi^\theta$, we obtain (32).

It remains to be shown that all the equilibrium conditions are satisfied under $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$ where C_j^θ satisfies (32) with supporting prices $\{P_j^\theta, p_{ij,k}, w_j^\theta, r_j\}$ that satisfy (A.18). First, it is immediate to see that market clearing conditions are satisfied because (11)-(14) enter as constraints. The constraint (30) implies that the population distribution solves (19). Given prices $\{P_j^\theta, p_{ij,k}, w_j^\theta, r_j\}$, the firm's optimality conditions (A.12) are satisfied because they are identical to (A.15).

Finally, it remains to show that the government budget (6) and price normalization (18) are satisfied. Multiplying $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$ by l_j^θ and summing across j and θ,

$$
\begin{aligned}
&\sum_\theta \sum_j T_j^\theta l_j^\theta \\
&= \sum_\theta \sum_j P_j^\theta C_j^\theta l_j^\theta - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_\theta \sum_{i,j,k} p_{ij,k} c_{ij,k}^\theta l_j^\theta - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_{i,j,k} p_{ij,k} \left[A_{ij,k} f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k} \mathbf{x}_{ij,k}) - \sum_{l,m} p_{ij,k} x_{jl,m}^{i,k} \right] \\
&\qquad - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_{i,j,k} p_{ij,k} \left[\sum_\theta A_{ij,k} \frac{\partial f_{ij,k}}{\partial l_{ij,k}} l_{ij,k}^\theta + A_{ij,k} \frac{\partial f_{ij,k}}{\partial h_{ij,k}} h_{ij,k} + \sum_{l,m} A_{ij,k} \frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} x_{ij,k}^{l,m} - \sum_{l,m} p_{ij,k} x_{jl,m}^{i,k} \right] \\
&\qquad - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_{i,j,k} \left[\sum_\theta w_i^\theta l_{ij,k}^\theta + r_i h_{ij,k} + \sum_{l,m} p_{li,m} x_{ij,k}^{l,m} - \sum_{l,m} p_{ij,k} x_{jl,m}^{i,k} \right] \\
&\qquad - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= \sum_\theta \sum_j w_j^\theta l_j + \sum_j r_j \bar{h}_j - \sum_\theta \sum_j w_j^\theta l_j^\theta - \sum_\theta \ell^\theta \Pi^\theta \\
&= 0.
\end{aligned}
$$

Without preference shocks. We also discuss the case without preference shocks as considered by Fajgelbaum and Gaubert (2020). To do so, we rewrite the second-best problem as follows:

$$
\max_{\{C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, h_{ij,k}, \mu_j^\theta, A_{ij,k}, W^\theta\}} \sum_j \mu_j^{\bar{\theta}} u_j^{\bar{\theta}}(C_j^{\bar{\theta}}) \tag{A.19}
$$

$$
\text{s.t.} \qquad (7), (11)\text{-}(14) \tag{A.20}
$$

$$
\ell^\theta \mu_j^\theta \left[u_j^\theta(C_j^\theta) - W^\theta \right] = 0 \text{ for all } \theta \tag{A.21}
$$

$$
\sum_j \mu_j^\theta = 1 \text{ for all } \theta \tag{A.22}
$$

$$\sum_j \mu_j^{\tilde{\theta}} u_j^{\tilde{\theta}}(C_j^{\tilde{\theta}}) \geq \underline{W}^{\tilde{\theta}} \text{ for all } \tilde{\theta} \neq \bar{\theta} \tag{A.23}$$

Note that we rewrote households' incentive compatibility constraint for location choice (22) with utility equalization (A.21) and adding up constraint (A.22). Note also that $\psi^\theta(\cdot) = 0$ without preference shocks.

The first-order condition for μ_j^θ is given by

$$\ell^\theta \left[w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta \right] + \delta^\theta + \underbrace{l_j^\theta \kappa_j^\theta \left[u_j^\theta(C_j^\theta) - \overline{u} \right]}_{=0} = 0$$

$$\Leftrightarrow \quad w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta = \delta^\theta / \ell^\theta, \tag{A.24}$$

where κ_j^θ and δ^θ denote the Lagrange multipliers for (A.21) and (A.22), respectively. By noting that $T_j^\theta = P_j^\theta C_j^\theta - w_j^\theta - \Pi^\theta$, the cross-location component of transfers only addresses technological externalities, and the cross-type component of transfers addresses redistribution concerns, as highlighted by Fajgelbaum and Gaubert (2020).

A.5 Proof of Proposition 3 and Corollary 2

By multiplying equation (32) by $dC_j^\theta / u_j^{\theta\prime}(C_j^\theta)$ and summing up across j and θ, we have

$$\begin{aligned}
&\sum_j \sum_\theta l_j^\theta \left[\tilde{\Lambda}^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^\theta \right] dC_j^\theta \\
&= -\sum_j \sum_\theta \sum_i \ell^\theta dC_j^\theta \frac{\partial \mu_i^\theta}{\partial C_j^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right] \\
&= -\sum_\theta \sum_i \ell^\theta \underbrace{\sum_j dC_j^\theta \frac{\partial \mu_i^\theta}{\partial C_j^\theta}}_{=d\mu_i^\theta} \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta \right]
\end{aligned} \tag{A.25}$$

By following the same procedure as in the Proof of Proposition 1, we prove the statement in the proposition.

Given Proposition 3, Corollary 2 is immediate. One can also prove Corollary 2 is also obtained by directly applying the envelope theorem to the relaxed planning problem

(29). Despite the presence of incentive compatibility constraint of households' location decisions, there are no reallocation effects because technological effects do not directly affect this constraint.

A.6 Proof of Proposition 4

Since $\ln(\cdot)$ is a monotone transformation, it is immediate that the economy under multiplicative preference shocks (39) and the one under additive preference shocks (40) have the same allocation and relative prices. We now seek the relationship in terms of price levels. We choose the numeraire in both economies such that (18) holds with $u_j^\theta(C_j^\theta) = \ln \tilde{u}_j^\theta(C_j^\theta)$. Given this choice, the price levels coincide as well.

We now prove the second statement. We first note that, given the assumption that $\mathcal{W}(\{W^\theta\}_{\theta\in\Theta}) \equiv \ln \tilde{\mathcal{W}}(\{\exp(W^\theta)\}_{\theta\in\Theta})$, we have

$$dW = \sum_\theta \frac{\partial \mathcal{W}}{\partial W^\theta} dW^\theta, \qquad d\ln\tilde{W} = \sum_\theta \frac{\partial \mathcal{W}}{\partial W^\theta} d\ln\tilde{W}^\theta \tag{A.26}$$

Therefore, to show $dW = d\ln\tilde{W}$, it is sufficient to show the same isomorphism for the expected utility for each type θ, i.e., $dW^\theta = d\ln\tilde{W}^\theta$. The expected households' utility in (39) is given by

$$W^\theta = \frac{1}{\nu^\theta} \ln G^\theta(\exp(\nu^\theta u_1^\theta(C_1^\theta)), \ldots, \exp(\nu^\theta u_N^\theta(C_N^\theta))) \tag{A.27}$$

and that in (40) is given by

$$\tilde{W}^\theta = G^\theta(\tilde{u}_1^\theta(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N^\theta(C_N^\theta)^{\nu^\theta})^{1/\nu^\theta}. \tag{A.28}$$

See Appendix C for detailed mathematical derivation. Therefore, under $u_j^\theta(C_j^\theta) = \ln(\tilde{u}_j^\theta(C_j^\theta))$, and $\epsilon_j^\theta = \ln(\tilde{\epsilon}_j^\theta)$, we have $W^\theta = \ln\tilde{W}^\theta$.

Finally, we prove that the decomposition is also identical. The Lagrangian of the pseudo-planning problem in an economy with multiplicative preference shocks (39) is

$$\begin{aligned}\mathcal{L} = {} & \tilde{\mathcal{W}}\left(\left\{G^\theta(\tilde{u}_1^\theta(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N^\theta(C_N^\theta)^{\nu^\theta})^{1/\nu^\theta}\right\}_{\theta\in\Theta}\right) \\ & + \sum_{i,j,k} p_{ij,k}^L \left[A_{ij,k} g_{ij,k}(\{\ell^\theta \mu_j^\theta(\boldsymbol{C}^\theta)\}_\theta) f_{ij,k}(\mathbf{l}_{ij,k}, h_{ij,k}, \mathbf{x}_{ij,k}) - \left(\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{jl,m}^{i,k}\right)\right]\end{aligned}$$

$$
\begin{aligned}
&+\sum_{j,\theta} P_j^{L,\theta}\left[\mathcal{C}_j^\theta(\mathbf{c}_j^\theta) - C_j^\theta \ell^\theta \mu_j^\theta(\boldsymbol{C}^\theta)\right] \\
&+\sum_{j,\theta} w_j^{L,\theta}\left[\ell^\theta \hat{\mu}_j^\theta(\boldsymbol{C}^\theta) - \sum_{i,k} l_{ji,k}^\theta\right] \\
&+\sum_j r_j^L\left[\bar{h}_j - \sum_{i,k} h_{ji,k}\right] \\
&+\sum_j \eta_j^\theta\left[C_j^\theta - \check{C}_j^\theta\right],
\end{aligned}
$$

where we normalize one of the Lagrangian multipliers so that $\sum_\theta \sum_j \eta_j^\theta / u'(\check{C}_j^\theta) = 0$ as in the proof of Proposition 1. As in Proposition 1, the Lagrangian multiplies, $\{P_j^{L,\theta}, w_j^{L,\theta}, r_j^L, p_{ij,k}^L\}$ coincide with equilibrium prices up to a multiplicative constant. The first-order conditions with respect to C_j^θ are

$$
\ell^\theta \mu_j^\theta\left[\frac{\partial \tilde{\mathcal{W}}}{\partial \tilde{W}^\theta}\tilde{W}^\theta u_j^{\theta\prime}(C_j^\theta) - P_j^{L,\theta}\right] + \eta_j^\theta = \ell^\theta \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta}\left[P_i^{L,\theta} C_i^\theta - w_i^{L,\theta} - \sum_{l,k} p_{il,k}^L y_{il,k} \frac{1}{l_i^\theta}\gamma_{il,k}^\theta\right],
$$

where we used the fact that (see also Appendix C)

$$
\frac{\partial \tilde{W}^\theta}{\partial C_j^\theta} = \tilde{W}^\theta \frac{G_j^\theta(\tilde{u}_1^\theta(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N^\theta(C_N^\theta)^{\nu^\theta})}{G^\theta(\tilde{u}_1^\theta(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N^\theta(C_N^\theta)^{\nu^\theta})}\tilde{u}_j^{\theta\prime}(C_j^\theta) = \tilde{W}^\theta \mu_j^\theta \frac{\tilde{u}_j^{\theta\prime}(C_j^\theta)}{\tilde{u}_j^\theta(C_j^\theta)} = \tilde{W}^\theta \mu_j^\theta u_j^{\theta\prime}(C_j^\theta). \tag{A.29}
$$

Since $\Lambda^\theta = \frac{1}{\tilde{\mathcal{W}}}\frac{\partial \tilde{\mathcal{W}}}{\partial \tilde{W}^\theta}\tilde{W}^\theta$ under our assumption, we can rewrite the above expression as

$$
\ell^\theta \mu_j^\theta\left[\Lambda^\theta \tilde{\mathcal{W}} u_j^{\theta\prime}(C_j^\theta) - P_j^{L,\theta}\right] + \eta_j^\theta = \ell^\theta \sum_i \frac{\partial \hat{\mu}_i^\theta(\boldsymbol{C}^\theta)}{\partial C_j^\theta}\left[P_i^{L,\theta} C_i^\theta - w_i^{L,\theta} - \sum_{l,k} p_{il,k}^L y_{il,k} \frac{1}{l_i^\theta}\gamma_{il,k}^\theta\right].
$$

Dividing both sides by $u_j^{\theta\prime}(C_j^\theta)$ and adding up across j and θ,

$$
\sum_\theta \sum_j \frac{P_j^{L,\theta}}{u_j^{\theta\prime}(C_j^\theta)} = \tilde{\mathcal{W}} \tag{A.30}
$$

Comparing (18) and (A.30), the Lagrangian multipliers are multiplicative of equilibrium prices by a factor of $\tilde{\mathcal{W}}$.

Applying envelope theorem as in the Proof of Proposition 1,

$$
\begin{aligned}
d\tilde{W} =& \sum_{i,j,k} p^L_{ij,k} y_{ij,k} d\ln A_{ij,k} + \sum_\theta \sum_j l^\theta_j [\tilde{\mathcal{W}}\Lambda^\theta u^{\theta\prime}_j(C^\theta_j) - P^{L,\theta}_j] dC^\theta_j \\
&+ \sum_\theta \sum_j [w^{L,\theta}_j - P^{L,\theta}_j C^\theta_j] dl^\theta_j + \sum_\theta \sum_j \sum_{l,k} p^L_{jl,k} y_{jl,k} \frac{1}{l^\theta_j} \gamma^\theta_{jl,k} dl^\theta_j \qquad \text{(A.31)} \\
=& \tilde{\mathcal{W}} \Bigg[\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k} + \sum_\theta \sum_j l^\theta_j [\Lambda^\theta u^{\theta\prime}_j(C^\theta_j) - P^\theta_j] dC^\theta_j \\
&+ \sum_\theta \sum_j [w^\theta_j - P^\theta_j C^\theta_j] dl^\theta_j + \sum_\theta \sum_j \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l^\theta_j} \gamma^\theta_{jl,k} dl^\theta_j \Bigg]. \qquad \text{(A.32)}
\end{aligned}
$$

Since the term inside the square bracket is identical to what we obtain in the proof of Proposition 1, the decompositions remain identical up to a multiplicative constant $\tilde{\mathcal{W}}$.

A.7 Proof of Proposition D.1

As shown in Appendix D.6, the welfare decomposition with non-welfarist welfare criteria is given by

$$
dW = \Omega_T + \Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R + \Omega_{PM}, \qquad \text{(A.33)}
$$

where

$$
\Omega_{PM} = \mathbb{E}_\theta \left[\Lambda^\theta \mathbb{E}_{j|\theta} \left[\left(\omega^\theta_j - 1 \right) u^{\theta\prime}_j(C_j) dC^\theta_j \right] \right] \qquad \text{(A.34)}
$$

with our assumption (D.38). Suppose that $\omega^\theta_j = \frac{1}{\Lambda^\theta} \left[\left(1 + \frac{P^\theta_j}{u^{\theta\prime}_j(C^\theta_j)} \right) - \left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P^\theta_j}{u^{\theta\prime}_j(C^\theta_j)} \right] \right) \right]$. Then

$$
\begin{aligned}
\Omega_{MU} + \Omega_R + \Omega_{PM} =& \mathbb{E}_\theta \left[Cov_{j|\theta} \left(-\frac{P^\theta_j}{u^{\theta\prime}_j(C^\theta_j)}, u^{\theta\prime}_j(C^\theta_j) dC^\theta_j \right) \right] \qquad \text{(A.35)} \\
&+ Cov_\theta \left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P^\theta_j}{u^{\theta\prime}_j(C^\theta_j)} \right], \mathbb{E}_{j|\theta} \left[u'(C^\theta_j) dC^\theta_j \right] \right) \qquad \text{(A.36)} \\
&+ \mathbb{E}_\theta \left[\Lambda^\theta \mathbb{E}_{j|\theta} \left[\left(\omega^\theta_j - 1 \right) u'(C^\theta_j) dC^\theta_j \right] \right] \qquad \text{(A.37)} \\
=& \mathbb{E}_\theta \left[\mathbb{E}_{j|\theta} \left(1 - \frac{P^\theta_j}{u^{\theta\prime}_j(C^\theta_j)} u^{\theta\prime}_j(C^\theta_j) dC^\theta_j \right) \right] \qquad \text{(A.38)}
\end{aligned}
$$

$$+\mathbb{E}_\theta\left(\left(\Lambda^\theta-\mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right]\right)\mathbb{E}_{j|\theta}\left[u'(C_j^\theta)dC_j^\theta\right]\right) \tag{A.39}$$

$$-\mathbb{E}_\theta\left[\mathbb{E}_{j|\theta}\left(1-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}u_j^{\theta\prime}(C_j^\theta)dC_j^\theta\right)\right] \tag{A.40}$$

$$-\mathbb{E}_\theta\left(\left(\Lambda^\theta-\mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right]\right)\mathbb{E}_{j|\theta}\left[u'(C_j^\theta)dC_j^\theta\right]\right) \tag{A.41}$$

$$=0. \tag{A.42}$$

This proves the first claim. Likewise, if $\omega_j^\theta = -\frac{1}{\Lambda^\theta}\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right]\right)$, then $\Omega_R + \Omega_{PM} = 0$, and if $\omega_j^\theta = \frac{1}{\Lambda^\theta}\left(1+\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}\right)$, then $\Omega_{MU}+\Omega_{PM}=0$.

B First-Best Allocation

In this section, we discuss the first-best planning problem, where the Planner can specify the consumption allocation and location decisions based on the draw of preference shocks $\boldsymbol{\epsilon}$. The problem is given by follows:

$$W = \max_{\{W^\theta, C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \boldsymbol{\mu}_{ij,k}, h_{ij,k}, \mu_j^\theta, A_{ij,k}\}} \mathcal{W}(\{W^\theta\}_{\theta\in\Theta}) \tag{B.1}$$

$$\text{s.t.} \qquad (7), (11)\text{-}(14) \tag{B.2}$$

$$W^\theta = \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \tag{B.3}$$

$$\sum_j \mu_j^\theta = 1 \tag{B.4}$$

In the first-best planning problem, the only constraints that the planner faces are the resource constraints. Unlike the second-best planning problem, the incentive compatibility constraints of the households are absent, as the planner can directly control location choice and consumption decisions directly.

The first-order conditions with respect to $c_{ij,k}, l_{ij,k}^\theta, h_{ij,k}, x_{ij,k}^{l,m}$ are

$$P_j^{FB,\theta}\frac{\partial \mathcal{C}_j^\theta}{\partial c_{ij,k}} = p_{ij,k}^{FB}, \quad p_{ij,k}^{FB}A_{ij,k}\frac{\partial f_{ij,k}}{\partial l_{ij,k}^\theta} = w_i^{FB,\theta}, \quad p_{ij,k}^{FB}A_{ij,k}\frac{\partial f_{ij,k}}{\partial h_{ij,k}} = r_i^{FB}, \quad p_{ij,k}^{FB}A_{ij,k}\frac{\partial f_{ij,k}}{\partial x_{ij,k}^{l,m}} = p_{li,m}^{FB}, \tag{B.5}$$

where $P_j^{FB,\theta}, p_{ij,k}^{FB}, w_i^{FB,\theta}, r_i^{FB}$ are the Lagrangian multipliers on (12), (11), and (14), respectively. We let all the variables with FB superscript denote those of the planner's solution. Therefore, the relative quantities of inputs are not distorted in equilibrium.

The Planner's solution deviates from the equilibrium when we consider optimality conditions for C_j^θ and l_j^θ. The first-order condition with respect to C_j^θ gives

$$\Lambda^\theta u_j'(C_j^{FB,\theta})/P_j^{FB,\theta} = 1, \tag{B.6}$$

i.e., the marginal utility of income is equalized across locations conditional on type θ. The optimality condition for μ_j^θ is

$$\begin{aligned}&\Lambda^\theta u_j^\theta\left(C_j^{FB,\theta}\right) + w_j^{FB,\theta} + \sum_{i,k} p_{ji}^{k,FB}\frac{\partial}{\partial l_j^\theta} g_{ji,k}(\{l_j^\theta\}_\theta) f_{ji}^k(\boldsymbol{\mu}_{ji}^k, h_{ji}^k, \mathbf{x}_{ji}^k) - P_j^{FB,\theta}C_j^{FB,\theta}\\ =&\Lambda^\theta \frac{\partial\psi^\theta(\{\mu_k^\theta\}_k)}{\partial\mu_j^\theta} + u^{FB,\theta},\end{aligned} \tag{B.7}$$

where $u^{FB,\theta}$ is a Lagrangian multiplier on (B.4). Now, with $\{P_j^{FB,\theta}, p_{ij,k}^{FB}, w_i^{FB,\theta}, r_i^{FB}\}$ coinciding with $\{P_j^\theta, p_{ij,k}, w_i^\theta, r_i\}$ up to scale, the only way in which Equation (B.7) is satisfied in the equilibrium is that there is no transfer so that $C_j^\theta = w_j^\theta/P_j^\theta$ and there are no agglomeration externalities, i.e., $\frac{\partial}{\partial l_j^\theta} g_{ji,k}(\{l_j^\theta\}_\theta) = 0$. However, there is no guarantee in general that (B.6) is satisfied for this C_j^θ, except for the knife-edge case where the marginal utility is equalized across all locations. We summarize the results as follows.

Proposition B.1. *Decentralized equilibrium is suboptimal for any Pareto weights Λ^θ except for the case that marigal utility of income is equalized across locations ($\frac{1}{P_j^\theta} u_j^{\theta\prime}\left(w_j^\theta/P_j^\theta\right)$ is equalized across j for all θ), there are no agglomeration externalities ($\gamma_{ij,k}^\theta = 0$ for all i,j,k,θ), and there is no transfer ($T_j^\theta = 0$ for all j and θ).*

What is the reason for the suboptimality of the equilibrium? There are two reasons. First, market does not internalize the agglomeration externalities in productivity (**??**). Second, the market does not equalize the marginal utility of income across locations.

The first source of suboptimality is perhaps not surprising; it is simply an externality that the market does not internalize. The second source of suboptimality is subtle and warrants a discussion. In equilibrium, agents make location decisions based on utility *levels* (inclusive of preference shocks). This implies that *marginal* utility of income is not necessarily equalized across these locations. In contrast, in the first-best allocation, the Planner equalizes the marginal utility of income across locations, while simultaneously controlling for population movement by breaking the incentive compatibility constraint of households' location decisions.

There are two ways to interpret this suboptimality of the dispersion of marginal utility of income. The first interpretation is the lack of insurance for the uncertainty associated with location choice. Depending on the preference draws, or depending on the random sunspot process of location assignment in the absence of preference shocks, individual households may end up in a variety of locations that differ in terms of their associated marginal utility of income. Ex-ante, households can benefit by committing to making transfers from a state where they end up in a location with a low marginal utility of income to that with a high marginal utility of income, but there is not security that allows for such a transfer. The second interpretation is the lack of redistribution across agents depending on location preference and where they reside. By taking the expected utility as a welfare criterion, we effectively attach equal social marginal weight to individuals with different preference draws. The observation that spatial equilibrium models involve suboptimality due to dispersion of marginal utility is reminiscent of Mirrlees (1972), who show this issue in the context of location decisions within a city.

Our result is also related to a recent work by Mongey and Waugh (2024). They show that the discrete choice model where the choice differs only in terms of prices, P_j^θ, is efficient under log utility. We can see their results through the lens of Proposition B.1: if $u_j^\theta(C) = \ln(C)$ and $w_j^\theta = w^\theta$, then the marginal utility of income is equalized across locations. In the context of the spatial equilibrium model, such a condition is unlikely to be met because it requires the nominal wages to be the same across all locations (i.e., absence of compensating differential).

C Location Choice under Generalized Extreme Value (GEV) Preference Shocks

We describe the isomorphic representative agent formulation of location choice under GEV preference shocks.

C.1 Additively Separable Case

Consider the additively separable utility function of the form

$$U_j^\theta(C_j^\theta, \epsilon_j^\theta) = u_j^\theta(C_j^\theta) + \epsilon_j^\theta, \tag{C.1}$$

and ϵ_j^θ follows type-I generalized extreme value distribution:

$$\mathbb{P}[\epsilon_1^\theta \leq \bar{\epsilon}_1, \ldots, \epsilon_N^\theta \leq \bar{\epsilon}_N] = \exp(-G^\theta(\exp(-\nu^\theta \bar{\epsilon}_1), \ldots, \exp(-\nu^\theta \bar{\epsilon}_N)), \tag{C.2}$$

and G is a correlation function for its definition), and we normalize the location so that the unconditional mean is zero. As is well known since McFadden (1978), this yields the following location choice probability.

$$\mu_j^\theta = \frac{G_j^\theta(V_1^\theta, \ldots, V_N^\theta)V_j^\theta}{\sum_l G_l^\theta(V_1^\theta, \ldots, V_N^\theta)V_l^\theta}, \quad \text{where} \quad V_j^\theta \equiv \exp(\nu^\theta u^\theta(C_j^\theta)). \tag{C.3}$$

and

$$G_j^\theta = \frac{\partial G^\theta(V_1^\theta, \ldots, V_N^\theta)}{\partial V_j^\theta}. \tag{C.4}$$

Now we construct a representative agent formulation that is isomorphic to the above model. Let $\boldsymbol{\mu}^\theta = [\mu_j^\theta]_j$ Define a mapping $V_j^\theta = S_j^\theta(\boldsymbol{\mu}^\theta)$ that satisfies the following condition for all j:

$$G_j^\theta(V_1^\theta, \ldots, V_N^\theta)V_j^\theta = \mu_j^\theta. \tag{C.5}$$

The representative agent solves

$$W^\theta = \max_{\{\mu_j^\theta\}: \sum_k \mu_k^\theta = 1} \sum_j u_j^\theta(C_j^\theta)\mu_j^\theta - \frac{1}{\nu^\theta} \sum_j \mu_j^\theta \ln S_j^\theta(\boldsymbol{\mu}). \tag{C.6}$$

The first-order optimality condition is given by

$$u_i^\theta(C_i^\theta) - \frac{1}{\nu^\theta} \ln S_i^\theta(\boldsymbol{\mu}) - \frac{1}{\nu^\theta} \sum_j \mu_j^\theta \frac{\partial \ln S_j^\theta(\boldsymbol{\mu})}{\partial \mu_i^\theta} - \bar{u}^\theta = 0, \tag{C.7}$$

where $\bar{u}^\theta$ is the Lagrangian multiplier on the adding up constraint, $\sum_k \mu_k^\theta = 1$. Note that

$$\sum_j \mu_j^\theta \frac{\partial \ln S_j^\theta(\boldsymbol{\mu}^\theta)}{\partial \mu_i^\theta} = 1, \tag{C.8}$$

for all j. To see this, we add up (C.5) across j to have $G(S_1(\boldsymbol{\mu}^\theta), \ldots, S_N(\boldsymbol{\mu}^\theta)) = \sum_j \mu_j^\theta$. Taking the derivative with respect to μ_i gives

$$\sum_j G_j(S_1(\boldsymbol{\mu}^\theta), \ldots, S_N(\boldsymbol{\mu}^\theta)) S_j(\boldsymbol{\mu}) \frac{\partial \ln S_j^\theta(\boldsymbol{\mu}^\theta)}{\partial \mu_i^\theta} = 1 \tag{C.9}$$

$$\Leftrightarrow \quad \sum_j \mu_j^\theta \frac{\partial \ln S_j^\theta(\boldsymbol{\mu}^\theta)}{\partial \mu_i^\theta} = 1, \tag{C.10}$$

where we used (C.5) in the second line. Therefore the optimality condition collapses to

$$S_i^\theta(\boldsymbol{\mu}^\theta) = \exp(-\nu \bar{u}^\theta - 1 + \nu u_i^\theta(C_i^\theta)). \tag{C.11}$$

Since $V_i^\theta = S_i^\theta(\boldsymbol{\mu}^\theta)$ satisfies (C.5) by its definition, we plug back $V_i^\theta = S_i^\theta(\boldsymbol{\mu}^\theta)$ into (C.5) to obtain

$$\mu_i^\theta = \exp(-\nu \bar{u}^\theta - 1) G_i^\theta\big(\exp(\nu u^\theta(C_1^\theta)), \ldots, \exp(\nu u_N^\theta(C_N^\theta))\big) \exp(\nu u_i(C_i)). \tag{C.12}$$

The adding up constraint, $\sum_i \mu_i^\theta = 1$, implies that

$$\exp(\nu \bar{u}^\theta + 1) = \sum_j G_j^\theta\big(\exp(\nu u(C_1^\theta)), \ldots, \exp(\nu u_N^\theta(C_N^\theta))\big) \exp(\nu^\theta u_j^\theta(C_j^\theta)). \tag{C.13}$$

Therefore we obtain

$$\mu_j = \frac{G_j^\theta(V_1^\theta, \ldots, V_N^\theta) V_j^\theta}{\sum_l G_l^\theta(V_1^\theta, \ldots, V_N^\theta) V_l^\theta}, \quad \text{where} \quad V_j^\theta \equiv \exp(\nu^\theta u^\theta(C_j^\theta)), \tag{C.14}$$

coinciding with the solution to the discrete choice problem (C.20).

Finally, we confirm that the indirect utility coincides with each other. In the discrete choice problem, the indirect utility is given by (see McFadden (1978))

$$W^\theta \equiv \mathbb{E}\left[\max_j \left\{u_j^\theta(C_j^\theta) + \epsilon_j^\theta\right\}\right] \tag{C.15}$$

$$= \frac{1}{\nu^\theta} \ln G^\theta(\exp(\nu^\theta u_1^\theta(C_1^\theta)), \ldots, \exp(\nu^\theta u_N^\theta(C_N^\theta))). \quad \text{(C.16)}$$

In the representative agent model, substituting (C.7) and (C.13) into (C.6), we obtain

$$W^\theta = \frac{1}{\nu^\theta} \ln G^\theta(\exp(\nu^\theta u_1^\theta(C_1^\theta)), \ldots, \exp(\nu u_N^\theta(C_N^\theta))), \quad \text{(C.17)}$$

verifying that the indirect utility also coincides with the original discrete choice formulation.

C.2 Multiplicatively Separable Case

Consider the multiplicatively separable utility function of the form

$$\tilde{U}_j^\theta(C_j^\theta, \epsilon_j^\theta) = \tilde{\epsilon}_j^\theta \tilde{u}_j^\theta(C_j^\theta) \quad \text{(C.18)}$$

and $\tilde{\epsilon}_j$ follows type-II generalized extreme value distribution (multi-variate Fréchet):

$$\mathbb{P}[\tilde{\epsilon}_1^\theta \leq \bar{\epsilon}_1, \ldots, \tilde{\epsilon}_N^\theta \leq \bar{\epsilon}_N] = \exp(-G^\theta((\bar{\epsilon}_1)^{-\nu^\theta}, \ldots, (\bar{\epsilon}_N)^{-\nu^\theta})), \quad \text{(C.19)}$$

and G is a correlation function. This yields the following location choice probability.

$$\mu_j^\theta = \frac{G_j^\theta(V_1^\theta, \ldots, V_N^\theta)V_j^\theta}{\sum_l G_l^\theta(V_1^\theta, \ldots, V_N^\theta)V_l^\theta} = \frac{G_j^\theta(V_1^\theta, \ldots, V_N^\theta)V_j^\theta}{G^\theta(V_1^\theta, \ldots, V_N^\theta)}, \quad \text{where} \quad V_j^\theta \equiv \tilde{u}^\theta(C_j^\theta)^\nu. \quad \text{(C.20)}$$

where

$$G_j^\theta(V_1^\theta, \ldots, V_N^\theta) = \frac{\partial G^\theta(V_1^\theta, \ldots, V_N^\theta)}{\partial V_j^\theta}. \quad \text{(C.21)}$$

The indirect utility is

$$\tilde{W}^\theta = G^\theta(V_1^\theta, \ldots, V_N^\theta)^{1/\nu^\theta} \text{where} \quad V_j^\theta \equiv \tilde{u}^\theta(C_j^\theta)^{\nu^\theta}. \quad \text{(C.22)}$$

Now we construct a representative agent formulation that is isomorphic to the above model. Define the utility function of the representive agent as

$$\mathcal{U}^\theta(\{C_j^\theta\}_j, \{\mu_j^\theta\}) = \sum_j (\mu_j^\theta)^{\frac{\nu^\theta - 1}{\nu^\theta}} G_j^\theta\Big(\tilde{u}_1^\theta(C_1^\theta)^{\nu^\theta}, \ldots, \tilde{u}_N^\theta(C_N^\theta)^{\nu^\theta}\Big)^{1/\nu^\theta} \tilde{u}_j^\theta(C_j^\theta). \quad \text{(C.23)}$$

The representative agent solves

$$\tilde{W}^{\theta} = \max_{\{\mu_j^{\theta}\}: \sum_k \mu_k^{\theta}=1} \mathcal{U}^{\theta}(\{C_j^{\theta}\}_j, \{\mu_j^{\theta}\}). \tag{C.24}$$

The first-order condition is

$$(\mu_j^{\theta})^{-1/\nu^{\theta}} G_j^{\theta}\Big(\tilde{u}_1^{\theta}(C_1^{\theta})^{\nu}, \ldots, \tilde{u}_N^{\theta}(C_N^{\theta})^{\nu\theta}\Big)^{1/\nu^{\theta}} \tilde{u}_j^{\theta}(C_j^{\theta}) - \bar{u}^{\theta} = 0, \tag{C.25}$$

where $\bar{u}$ is the Lagrangian multiplier on the adding up constraint, $\sum_k \mu_k = 1$. Solving the set of first-order conditions together with the adding up constraint, we have

$$\mu_j^{\theta} = \frac{G_j^{\theta}(V_1^{\theta}, \ldots, V_N^{\theta})V_j^{\theta}}{\sum_l G_l^{\theta}(V_1^{\theta}, \ldots, V_N^{\theta})V_l^{\theta}} = \frac{G_j^{\theta}(V_1^{\theta}, \ldots, V_N^{\theta})V_j^{\theta}}{G^{\theta}(V_1^{\theta}, \ldots, V_N^{\theta})}, \quad \text{where} \quad V_j^{\theta} \equiv \tilde{u}_j^{\theta}(C_j)^{\nu^{\theta}}, \tag{C.26}$$

as desired. We can plug the above expression into the objective to confirm that the indirect utility also coincides with the original discrete choice formulation:

$$\tilde{W}^{\theta} = G^{\theta}(V_1^{\theta}, \ldots, V_N^{\theta})^{1/\nu^{\theta}} \text{where} \quad V_j^{\theta} \equiv \tilde{u}_j^{\theta}(C_j^{\theta})^{\nu^{\theta}}. \tag{C.27}$$

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D Details on Extensions

D.1 Non-Separable Utility

In the baseline model, we have focused on specifications where preference shocks are additively separable. This section relaxes this assumption.

We now assume that utility in location i is given by $U_i^\theta(C_i^\theta, \epsilon_i^\theta)$. Compared to the additively separable specification, marginal utility in each location now depends on the preference shock draws. To see this, the average marginal utility for households deciding to live in location j is given by

$$\mathbb{E}\left[\frac{\partial}{\partial C_j^\theta} U_j^\theta(C_j^\theta, \epsilon_j^\theta) | j = \arg\max_l U_l^\theta(C_l^\theta, \epsilon_l^\theta)\right]. \tag{D.1}$$

Unlike the additively separable specification, i.e., $\frac{\partial}{\partial C_j^\theta} U_j^\theta(C_j^\theta, \epsilon_j^\theta) = u_j^{\theta\prime}(C_j^\theta)$, the selection of preference shocks influences the marginal utility of consumption in each location.

Under this general preference specification, the isomorphic representation of households' location decisions in Lemma 1 is modified as

$$\max_{\{\mu_j^\theta\}: \sum_j \mu_j^\theta = 1} \mathcal{U}^\theta(\{C_j^\theta\}, \{\mu_j^\theta\}), \tag{D.2}$$

where

$$\begin{aligned} \mathcal{U}^\theta(\{C_j^\theta\}_j, \{\mu_j^\theta\}_j) = & \max_{\{\mathbb{I}_j^\theta(\omega)\}_{\omega,j}} \int_0^1 \sum_j u_j^\theta(C_j^\theta, \epsilon_j^\theta(\omega)) \mathbb{I}_j^\theta(\omega) d\omega \\ \text{s.t.} \quad & \int_0^1 \mathbb{I}_j^\theta(\omega) d\omega = \mu_j^\theta \\ & \sum_j \mathbb{I}_j^\theta(\omega) = 1. \end{aligned} \tag{D.3}$$

We followed the same notation and setup as in Appendix A.1.

Under additively separable specification, $\mathcal{U}^\theta(\{C_j^\theta\}, \{\mu_j^\theta\}) = \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j)$, and $\partial \mathcal{U}^\theta(\{C_j^\theta\}, \{\mu_j^\theta\}) / \partial C_j^\theta = \mu_j^\theta u_j^{\theta\prime}(C_j^\theta)$, i.e., marginal expected utility only depends on j's population and consumption. In the general case, it is affected by the entire vector of population distribution $\{\mu_j^\theta\}_j$ and consumption $\{C_j^\theta\}_j$ beyond location j through the selection of preference draws.

In this generalized environment, Proposition 1 is simply modified by replacing the marginal utility per household $u_j^{\theta\prime}(C_j^\theta)$ with the one under this general specification. In particular, (ii) MU dispersion term becomes

$$\Omega_{MU} = \mathbb{E}_\theta \left[\mathrm{Cov}_{j|\theta} \left(-\frac{P_j^\theta}{\mathcal{MU}_j^\theta}, \mathcal{MU}_j^\theta dC_j^\theta \right) \right], \qquad \mathcal{MU}_j^\theta = \frac{1}{\mu_j^\theta} \frac{\partial \mathcal{U}^\theta(\{C_j^\theta\}, \{\mu_j^\theta\})}{\partial C_j^\theta}. \quad \text{(D.4)}$$

Conditional on the price normalization using this marginal utility (18), all other terms are unaffected.

While this extension is straightforward in theory, it poses a challenge to the identification of aggregate welfare. To understand this challenge, consider a monotone transformation of the utility function from the additively separable class: $U_j^\theta(C_j^\theta, \epsilon_j^\theta) = m(u_j^\theta(C_j^\theta) + \epsilon_j^\theta)$ for some strictly increasing function $m(\cdot)$. This transformation does not affect the model's *positive* prediction because of the ordinal nature of the utility function for location choice decisions. However, the expected marginal utility in each location becomes

$$\mathcal{MU}_j^\theta = u_j^{\theta\prime}(C_j^\theta) \mathbb{E} \left[m'(u_j^\theta(C_j^\theta) + \epsilon_j^\theta) | j = \arg\max_l m(u_l^\theta(C_l^\theta) + \epsilon_l^\theta) \right]. \quad \text{(D.5)}$$

Therefore, the function $m(\cdot)$ generally affects the marginal utility in each location. This discussion implies that the *normative* prediction, i.e., aggregate welfare, generically depends on the choice of $m(\cdot)$. Since $m(\cdot)$ cannot be identified from data, aggregate welfare also cannot be identified from location choice data alone.

This lack of identification is worrisome, as it indicates that welfare predictions are not uniquely pinned down from data. Even when the two models match the same data moments, the welfare conclusions drawn from these two models can be arbitrarily different. However, we show below that, under a common parametric assumption in the existing literature, such a concern is not warranted.

D.2 Shocks to Amenity and Amenity Externalities

In Section 3, we analyzed the effects of productivity shocks on aggregate welfare. In some contexts, researchers are interested in the shocks to amenities instead of productivity. The analysis in Section 3 embraces this possibility by interpreting some intermediate goods as local amenities. From a measurement perspective, applying Proposition 1 requires knowledge of prices associated with amenities, which is often unobserved. Below, we provide an alternative expression for Proposition 1 without using prices for the amenities.

To consider this extension, we explicitly introduce amenity as an argument in the utility function as follows:

$$U_j^\theta(C_j^\theta, B_j^\theta, \epsilon_j^\theta) = u_j^\theta(C_j^\theta, B_j^\theta) + \epsilon_j^\theta, \tag{D.6}$$

where B_j^θ is the amenity in region j. Furthermore, we assume that these amenities take the following form:

$$B_i^\theta = \tilde{B}_i^\theta g_i^{B,\theta}(\{l_i^\theta\}_\theta), \qquad \gamma_i^{B,\tilde{\theta}\theta} = \frac{\partial \ln g_i^{B,\tilde{\theta}\theta}(\{l_i^\theta\}_\theta)}{\partial \ln l_i^{\tilde{\theta}}}, \tag{D.7}$$

$\tilde{B}_i^\theta$ is the fundamental component of amenity, $g_i^{B,\theta}(\{l_i^\theta\}_\theta)$ is the spillover function, and $\gamma_i^{B,\tilde{\theta}\theta}$ is the amenity spillover elasticity from type $\tilde{\theta}$ to type θ in location i.

Under this extension, Proposition 1 is modified as follows. Consider an arbitrary set of small shocks to the exogenous components of productivity, $\{d\ln A_{ij,k}\}$, and amenities, $\{d\ln \tilde{B}_i^\theta\}$. The first-order impact of microeconomic shocks on welfare in utility terms can be expressed as

$$\begin{aligned} dW = & \underbrace{\sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln A_{ij,k} + \sum_{i,\theta} l_i^\theta \partial_b u_i^\theta B_i^\theta d\ln \tilde{B}_i^\theta}_{\text{(i) Technology } (\Omega_T)} + \underbrace{\mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}\left(-\frac{P_j^\theta}{\partial_c u_j^\theta}, \partial_c u_j^\theta dC_j^\theta\right)\right]}_{\text{(ii) MU Dispersion } (\Omega_{MU})} \\ & + \underbrace{\mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}(-T_j^\theta, d\ln l_j^\theta)\right]}_{\text{(iii) Fiscal Externality } (\Omega_{FE})} + \underbrace{\mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}(\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta + \sum_{\tilde{\theta}} \partial_b u_j^{\tilde{\theta}} B_j^{\tilde{\theta}} \gamma_j^{B,\theta\tilde{\theta}}, d\ln l_j^\theta)\right]}_{\text{(iv) Technological Externality } (\Omega_{TE})} \\ & + \underbrace{\mathrm{Cov}_\theta\left(\Lambda^\theta - \mathbb{E}_{j|\theta}\left[\frac{P_j^\theta}{\partial_c u_j^\theta(C_j^\theta)}\right], \mathbb{E}_{j|\theta}\left[\partial_c u_j^\theta dC_j^\theta\right]\right)}_{\text{(v) Redistribution } (\Omega_R)}. \end{aligned} \tag{D.8}$$

where $\partial_b u_j^\theta \equiv \frac{\partial u_j^\theta}{\partial B_j^\theta}$ and $\partial_c u_j^\theta \equiv \frac{\partial u_j^\theta}{\partial C_j^\theta}$. The main difference from Proposition 1 is the additional components in the (i) technology and (iv) technological externality terms. The second component inside the (i) technology term captures the effects of exogenous amenity shocks absent reallocation effects. The coefficient in front of $d\ln \tilde{B}_i^\theta$, $l_i \partial_b u_i^\theta B_i^\theta$, is the population-weighted sum of the marginal utility of amenity. This term strongly resembles the technology effect from productivity (the first term). In particular, if the amenity is traded and priced in the market, $\partial_b u_i$ corresponds to the competitive price of the amenity, and hence $l_i \partial_b u_i B_i$ is the total sales of the amenity, corresponding to $p_{ij,k} y_{ij,k}$. The second component inside the (iv) technological externality term has the same feature: if the amenity is traded, the term reflecting changes in amenities from externalities collapses to the same form as the productivity externality term.

To obtain (D.8), we follow the same steps in the proof of Proposition 1. The first order condition of pseudo planning problem with respect to C_j^θ is given by

$$\chi_j^\theta = l_j^\theta [\Lambda^\theta \partial_C u_j^\theta - P_j^\theta] \tag{D.9}$$

where χ_j^θ and P_j^θ correspond to the Lagrange multipliers for constraints (22) and (12), respectively. Furthermore, the first order condition with respect to μ_j^θ is given by

$$\eta_i^\theta = \ell^\theta \left[w_i^\theta - P_i^\theta C_i^\theta + \sum_{l,k} p_{il,k} y_{il,k} \frac{1}{l_i^\theta} \gamma_{il,k}^\theta + \sum_{\tilde{\theta}} \partial_B u_i^{\tilde{\theta}} B_i^{\tilde{\theta}} \gamma_i^{B,\theta\tilde{\theta}} \right] \tag{D.10}$$

where η_j^θ, w_j^θ and $p_{il,k}$ correspond to the Lagrange multipliers for constraints (12), (14) and (11), respectively.

By applying the Envelope theorem to pseudo planning problem (20),

$$\begin{aligned}
dW =& \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_{i,\theta} l_i^\theta \partial_B u_i^\theta B_i^\theta d\ln \tilde{B}_i^\theta + \sum_\theta \sum_j \chi_j^\theta P_j^\theta l_j^\theta dC_j^\theta \\
=& \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_{i,\theta} l_i^\theta \partial_B u_i^\theta B_i^\theta d\ln \tilde{B}_i^\theta + \sum_\theta \sum_j l_j^\theta [\Lambda^\theta \partial_C u_j^\theta - P_j^\theta] dC_j^\theta \\
&+ \sum_\theta \sum_j [w_j^\theta - P_j^\theta C_j^\theta] dl_j^\theta + \sum_\theta \sum_j \left[\sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta + \sum_{\tilde{\theta}} \partial_B u_j^{\tilde{\theta}} B_j^{\tilde{\theta}} \gamma_j^{B,\theta\tilde{\theta}} \right] dl_j^\theta,
\end{aligned}$$

which delivers the expression (D.8).

D.3 General Spillovers

In our main model, we assumed that agglomeration externalities are purely a function of local population size (7). In some contexts, researchers specify that a higher population size in the surrounding regions also generates agglomeration spillovers (e.g., Ahlfeldt et al. 2015). In other contexts, researchers also specify that the externality arises from the specific producers' input use (e.g., free entry model with labor fixed cost such as Krugman (1991)) or the producers' output (e.g., congestion cost from shipment, as in Allen and Arkolakis (2022)). To capture these general externalities, we extend the spillover function (7) such that

$$A_{ij,k} = \tilde{A}_{ij,k} g_{ij,k}(\{l_\ell^\theta\}_{\ell,\theta}, \{l_{ij,k}^\theta\}_\theta, y_{ij,k}), \tag{D.11}$$

where the first argument of $g_{ij,k}(\cdot)$ corresponds to the population size across types and locations, the second argument corresponds to labor input in production, and the third argument corresponds to output. We also denote the spillover elasticities such that

$$\gamma_{ij,k}^{P,\ell\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln l_\ell^\theta}, \qquad \gamma_{ij,k}^{L,\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln l_{ij,k}^\theta}, \qquad \gamma_{ij,k}^{Y,\theta} = \frac{\partial \ln g_{ij,k}}{\partial \ln y_{ij,k}}. \tag{D.12}$$

Under this extension, the only modification in Proposition 1 is the (iv) technological externality term, which is modified as:

$$\Omega_{TE} = \sum_{j,l,k} p_{jl,k} y_{jl,k} \left(\sum_{\ell,\theta} \gamma_{\ell l,k}^{P,\ell\theta} d\ln l_j^\theta + \sum_\theta \gamma_{jl,k}^{L,\theta} d\ln l_{ij,k}^\theta + \gamma_{jl,k}^{Y} d\ln y_{ij,k} \right). \tag{D.13}$$

This expression comes down to the (iv) technological externality term in Proposition 1 if the spillover function only depends on local population size. The only difference here is that the reallocation of population in surrounding regions and other quantities may have first-order effects on aggregate welfare through additional technological externalities.

D.4 Isomorphism between Amenity Externalities and Preference Shocks

In quantitative spatial equilibrium literature, researchers often argue that amenity congestion externalities are isomorphic to preference shocks and use these specifications interchangeably.[1] This section discusses this isomorphism through the lens of our framework.

For expositional convenience, we assume a single type and drop superscript θ. This implies $l_j = \mu_j$. Consider the following utility specification with amenity externalities

[1] See, for example, Allen and Arkolakis (2014) and Desmet, Nagy, and Rossi-Hansberg (2018), for papers that mention the isomorphism between the two specifications.

but without preference shocks:

$$U_j(C_j, B_j, \epsilon_j) = u_j(C_j) + B_j, \qquad B_j = g_j(\{l_i\}_i) = -\frac{1}{\nu} \ln S_j(\{l_i\}_i), \tag{D.14}$$

where $S_j(\{l_i\}_i)$ satisfies the following property

$$\frac{1}{\nu} \sum_j l_j \frac{\partial \ln S_j(\{l_i\}_i)}{\partial l_i} = 1. \tag{D.15}$$

Note that this specification accommodates that the population in i generates externalities in other regions. A special case of this example is when $S_j(\{l_i\}_i) = l_j^\nu$, i.e., amenities are iso-elastic to local population size with elasticity $-\nu$.

In an interior equilibrium, utility levels are equalized for all locations:

$$u_j(C_j) + \frac{1}{\nu} \ln S_j(\{l_i\}_i) = \bar{u}, \tag{D.16}$$

for some $\bar{u}$.

Now we show below this is isomorphic to the case where there are no amenity externalities but preference shocks follow a max-stable multivariate Gumbel distribution with shape parameter ν, i.e., $U_j(C_j, B_j, \epsilon_j) = u_j(C_j) + \epsilon_j$ and $\{\epsilon_j\}$ follows Specification (40). As we show in Appendix C, with multivariate Gumbel distribution, $\psi(\{l_j\})$ in Lemma 1 takes the form of $\psi(\{l_j\}) = \frac{1}{\nu} \sum_j l_j \ln S_j(\{l_i\}_i)$, where $S_j(\{l_i\}_i)$ satisfies D.15. The first-order condition for the representative household problem is

$$u_j(C_j) - \frac{1}{\nu} \ln S_j(\{l_i\}_i) - \underbrace{\frac{1}{\nu} \sum_j l_j \frac{\partial \ln S_j(\{l_i\})}{\partial l_i}}_{=1} - \tilde{u} = 0, \tag{D.17}$$

which is the same as (D.16). Therefore the equilibrium allocations will be identical. Moreover, it is also straightforward to see that both specifications deliver the same households' expected utilities, thereby delivering identical normative predictions as well.

This isomorphism arises because this particular form of congestion externality does not induce misallocation. In particular, the amenity component of the (iv) technological externality term in Equation (D.8) comes down to

$$\mathrm{Cov}_j\left(-\sum_i l_i \frac{\partial \ln S_i(\{l_j\}_i)}{\partial \ln l_j}, d\ln l_j\right) = \mathrm{Cov}_j(-\nu, d\ln l_j) = 0, \tag{D.18}$$

where we used (D.15) and $\partial_b u_i = 1$. Given that all other terms in Equation (D.8) are identical between the two specifications, the aggregate welfare predictions are also isomorphic.

This discussion also clarifies this isomorphism holds only when preference shocks follow a max-stable multivariate Gumbel distribution, or equivalently, when the congestion externality takes the specific functional form given by (D.14) and (D.15). Outside these special cases, congestion externalities generate a source of misallocation, and hence the isomorphism does not hold in general.[2]

D.5 Idiosyncratic Productivity Shocks

We generalize our baseline model by allowing households to draw idiosyncratic productivity $z^\theta = (z_1^\theta, z_2^\theta, \ldots, z_N^\theta)$, in addition to preference shocks, ϵ^θ. When households decide to live in location j, the efficiency unit of labor that households supply is z_j^θ.

We make several modifications to our baseline model to make our analysis tractable and transparent. First, we restrict our attention to the case of log utility,

$$u_j^\theta(c) = B_j^\theta \ln c. \tag{D.19}$$

Second, we assume that location-specific transfers are linear in household labor income, which we denote as τ_j^θ. Third, we assume away the presence of fixed factors.

The household's location choice problem with productivity draw $\boldsymbol{z}^\theta$ and preference draw $\boldsymbol{\epsilon}^\theta$ is

$$\max_j u_j^\theta(c_j^\theta) + \epsilon_j^\theta \tag{D.20}$$

[2] Fajgelbaum and Gaubert (2020) show that, under multiplicative utility specification, spatial equilibria involve misallocation even under iso-elastic amenity externality. Through the lens of Equation (D.8), this source of misallocation appears in the (ii) MU dispersion term. The multiplicative amenity without preference shocks implies that the marginal utility of income is not equalized across locations. Furthermore, unlike our baseline model abstracting direct effects of shocks on utility $u_j(\cdot)$, the utility changes from consumption changes dC_j are not equalized because of the changes of utility from amenity. Therefore the term (ii) is not zero. Note that this specification is isomorphic to the specification with multiplicative max-stable Fréchet shocks (without amenity externality) as discussed in Section 4.1. In this case, the dispersion of marginal utility instead arises from preference shock draws.

$$\text{s.t.} \quad P_j^\theta c_j^\theta = z_j^\theta w_j^\theta (1+\tau_j^\theta). \tag{D.21}$$

Let

$$C_j^\theta \equiv \frac{w_j^\theta (1+\tau_j^\theta)}{P_j^\theta} \tag{D.22}$$

denote the consumption of household θ in location j per efficiency unit of labor. With our assumption on the utility function (D.19), we can write the location choice problem as

$$\max_j u_j^\theta (C_j^\theta) + \underbrace{B_j^\theta \ln z_j^\theta + \epsilon_j^\theta}_{\equiv \varepsilon_j^\theta}. \tag{D.23}$$

Viewing ε_j^θ as the convoluted idiosyncratic productivity and amenity shocks, we can apply Lemma 1 to obtain the same location choice characterization as in the baseline model.

$$\max_{\{\mu_j^\theta\}_j} \sum_j \mu_j^\theta u_j^\theta (C_j^\theta) + \psi^\theta (\{\mu_j^\theta\}_j). \tag{D.24}$$

Let $\hat{\mu}_j^\theta (\{C_j^\theta\})$ be the location choice function associated with the solution to the above problem.

It will be convenient to define the average efficiency units of labor in each location-type pair as a function of a vector of $\{C_j^\theta\}$:

$$Z_j^\theta (\{C_j^\theta\}) \equiv \mathbb{E}\left[z_j^\theta \middle| j = \arg\max_m u_m^\theta (C_m^\theta) + B_j^\theta \ln z_m^\theta + \epsilon_m^\theta \right]. \tag{D.25}$$

To the extent that location choice function is invertibleSee Berry, Gandhi, and Haile (2013) for a sufficient condition for invertibility., i.e., the inverse function of $\hat{\mu}(\{C_j^\theta\})$, $C_j^\theta = \hat{C}_j^\theta (\{\mu_j^\theta\})$ exists, we can alternatively define the average efficiency unit of labor as a function of location choice probability:

$$\mathcal{Z}_j^\theta (\{\mu_j^\theta\}) = Z_j^\theta (\{\hat{C}_j^\theta (\{\mu_j^\theta\})\}). \tag{D.26}$$

The goods market clearing conditions are modified as follows

$$\sum_\theta c_{ij,k}^\theta + \sum_{l,m} x_{jl,m}^{i,k} = \mathcal{A}_{ij,k} f_{ij,k} (\mathbf{l}_{ij,k}, \mathbf{x}_{ij,k}) \tag{D.27}$$

$$\mathcal{Z}_j^\theta(\{\mu_j^\theta\})C_j^\theta \ell^\theta \mu_j^\theta = \mathcal{C}_j^\theta(\boldsymbol{c}_j^\theta), \tag{D.28}$$

where the first equation is modified due to the absence of a fixed factor, and the second equation takes into account heterogeneity in consumption within a location-type pair. The labor market clearing condition is

$$\sum_{i,k} l_{ji,k}^\theta = \mathcal{Z}_j^\theta(\{\mu_j^\theta\})\ell^\theta \mu_j^\theta, \tag{D.29}$$

which takes into account heterogeneity in efficiency units of labor within a location-type pair. The rest of the equilibrium conditions remain unchanged.

It would be straightforward to extend Lemma 2 to this environment. Any decentralized equilibrium solves the following pseudo-planning problem is

$$W = \max_{\{W^\theta, C_j^\theta, \mathbf{c}_j^\theta, \mathbf{x}_{ij,k}, \mathbf{l}_{ij,k}, \mu_j^\theta, \mathcal{A}_{ij,k}\}} \mathcal{W}(\{W^\theta\}_{\theta\in\Theta}) \tag{D.30}$$

$$\text{subject to} \qquad (7), \text{(D.27)-(D.29)}$$

$$W^\theta = \sum_j \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}_j) \tag{D.31}$$

$$\{\mu_j^\theta\}_j \in \arg\max_{\{\tilde{\mu}_j\}:\sum_j \tilde{\mu}_j = 1} \sum_j \tilde{\mu}_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\tilde{\mu}_j^\theta\}_j) \tag{D.32}$$

$$C_j^\theta = \check{C}_j^\theta \tag{D.33}$$

Applying the envelope theorem, we obtain

$$\begin{aligned} dW &= \Omega_T + \Omega_{MU} + \Omega_{TE} + \Omega_R \\ &+ \sum_\theta \sum_j \left[w_j^\theta - P_j^\theta C_j^\theta\right] \mathcal{Z}_j^\theta(\{\mu_l^\theta\}_l)\ell^\theta d\mu_j^\theta + \sum_\theta \sum_j \sum_m \left[w_m^\theta - P_m^\theta C_m^\theta\right] \ell^\theta \mu_m^\theta \frac{\partial \mathcal{Z}_m^\theta(\{\mu_j^\theta\})}{\partial \mu_j^\theta} d\mu_j^\theta \end{aligned}$$

Denoting $T_j^\theta = \tau_j^\theta w_j^\theta \mathcal{Z}_j^\theta(\{\mu_l^\theta\}_l)$ as the average transfers that households of type θ in location j receive, we can rewrite the above expression as follows

$$\begin{aligned} dW &= \Omega_T + \Omega_{MU} + \Omega_{TE} + \Omega_R \\ &+ \underbrace{\mathbb{E}_\theta\Big[\mathrm{Cov}_{j|\theta}(-T_j^\theta, d\ln l_j^\theta)\Big] + \mathbb{E}_\theta\left[\mathrm{Cov}_{j|\theta}\left(-\sum_m T_m^\theta l_m^\theta \frac{\partial \ln \mathcal{Z}_m^\theta(\{\mu_j^\theta\})}{\partial \ln \mu_j^\theta}, d\ln l_j^\theta\right)\right]}_{\text{Fiscal Externality, } \Omega_{FE}}. \end{aligned}$$

Therefore the only difference from Proposition 1 is the second term inside fiscal externality, Ω_{FE}. This term arises because migration changes the composition of workers in all locations, which in turn affects the government budget. For example, suppose that migration into location j is associated with an increase in the average productivity of workers living in location j but a decrease in other locations. If location j is a net taxpayer ($\tau_j^\theta < 0$ and thereby $T_j^\theta < 0$), then this will loosen the government budget.

D.6 Non-Welfarist Welfare Criteria

Consider a general non-welfarist welfare criteria

$$W = \mathcal{W}(\{\mathcal{U}^{SP,\theta}(\{C_j^\theta\}_j, \{l_j^\theta\}_j)\}_\theta),$$

where $\mathcal{U}^{SP,\theta}$ is defined arbitrarily on the distribution of consumption and population across locations of household type θ. Then, by applying the envelope theorem to the pseudo planner's problem as in the proof of Proposition 1 yields

$$\begin{aligned} dW =& \sum_{i,j,k} p_{ij,k} y_{ij,k} d\ln \tilde{A}_{ij,k} + \sum_\theta \sum_j \left[\frac{\partial}{\partial C_j^\theta} \mathcal{U}^{SP}(\{l_j^\theta\}, \{C_j^\theta\}) - l_j^\theta P_j^\theta \right] dC_j^\theta \\ &+ \sum_\theta \sum_j \left[w_j^\theta - P_j^\theta C_j^\theta \right] dl_j^\theta + \sum_\theta \sum_j \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta dl_j^\theta \end{aligned} \tag{D.34}$$

The only difference from our main proposition is the second term. By denoting $\mathcal{MU}_j^{SP,\theta} = \frac{\partial}{\partial C_j} \mathcal{U}^{SP,\theta}(\{C_i^\theta\}, \{l_i^\theta\})$,

$$\begin{aligned} &\sum_\theta \sum_j \left[\Lambda^\theta \mathcal{MU}_j^{SP,\theta} - P_j^\theta \right] dC_j^\theta \\ =& \sum_\theta \ell^\theta \sum_j \mu_j^\theta \left[\Lambda^\theta \left(\frac{1}{\mu_j^\theta} \frac{\mathcal{MU}_j^{SP,\theta}}{u_j^{\theta\prime}(C_j^\theta)} - 1 \right) + \Lambda^\theta - \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} \right] u_j^{\theta\prime}(C_j^\theta) dC_j^\theta \\ =& \mathbb{E}_\theta \left[\mathrm{Cov}_{j|\theta} \left(-\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}, u_j^{\theta\prime}(C_j^\theta) dC_j^\theta \right) \right] + Cov_\theta \left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} \right], \mathbb{E}_{j|\theta} \left[u_j^{\theta\prime}(C_j^\theta) dC_j^\theta \right] \right) \\ &+ \mathbb{E}_\theta \left[\Lambda^\theta \mathbb{E}_{j|\theta} \left[\frac{1}{\mu_j^\theta} \left(\mathcal{MU}_j^{SP,\theta} - u_j^{\theta\prime}(C_j^\theta) \right) dC_j^\theta \right] \right]. \end{aligned} \tag{D.35}$$

Consequently, Proposition 1 comes down to

$$dW = \Omega_T + \Omega_{MU} + \Omega_{FE} + \Omega_{TE} + \Omega_R + \Omega_{PM}, \tag{D.36}$$

where

$$\Omega_{PM} = \mathbb{E}_\theta \left[\Lambda^\theta \mathbb{E}_{j|\theta} \left[\left(\mathcal{MU}_j^{SP,\theta} - u_j^{\theta\prime}(C_j) \right) dC_j^\theta \right] \right], \tag{D.37}$$

which captures the potential misalignment between the social Planner's welfare assessment of the marginal value of consumption with that of private agents (marginal utility).

Such an approach is useful also in considering welfare criteria and optimal policies that are exclusively based on subcomponents of our decompositions, as in Dávila and Schaab (2022). Consider the following welfare criteria

$$\mathcal{U}^{SP,\theta}(\{C_j^\theta\}_j, \{\mu_j^\theta\}_j) = \sum_j \omega_j^\theta \mu_j^\theta u_j^\theta(C_j^\theta) - \psi^\theta(\{\mu_j^\theta\}). \tag{D.38}$$

Appropriate choice of the type-location specific weights ω_j^θ leads to the following result.

Proposition D.1. *Consider welfare criteria based on (47) and (D.38).*

1. *If $\omega_j^\theta = \frac{1}{\Lambda^\theta} \left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} - \left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} \right] \right) \right]$, then the decomposition of aggregate welfare as in Proposition 1 consists of Ω_T, Ω_{FE}, and Ω_{TE} only.*

2. *If $\omega_j^\theta = -\frac{1}{\Lambda^\theta} \left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} \right] \right)$, then the decomposition of aggregate welfare as in Proposition 1 consists of $\Omega_T, \Omega_{FE}, \Omega_{TE}$ and Ω_{MU} only.*

3. *If $\omega_j^\theta = \frac{1}{\Lambda^\theta} \frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)}$, then the decomposition of aggregate welfare as in Proposition 1 consists of $\Omega_T, \Omega_{FE}, \Omega_{TE}$ and Ω_R only.*

The first case bases welfare criteria entirely on aggregate efficiency consideration. The second case incorporates spatial MU dispersion, and the third case incorporates redistribution consideration, on top of aggregate efficiency consideration.

Now we derive optimal policies with welfare criteria in each of the three cases discussed in Proposition D.1. In the first case, since Ω_{MU} and Ω_R cancel with Ω_{PM}, in order for the transfer policy to be locally optimal, for all perturbation of C_i^θ

$$0 = -\sum_j \mu_j^\theta T_j^\theta \frac{\partial \ln \hat{\mu}_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta} + \sum_j \mu_j^\theta \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta \frac{\partial \ln \hat{\mu}_j^\theta(\boldsymbol{C}^\theta}{\partial C_i^\theta}. \tag{D.39}$$

We can rewrite the above expression to obtain the optimal spatial policy formula that exclusively targets aggregate efficiency consideration:

$$0 = -\sum_j \frac{\partial \hat{\mu}_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta} \left[w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta \right]. \tag{D.40}$$

Therefore, the left-hand side of our baseline formula in Proposition 2 is modified to be zero. The second case incorporates spatial MU dispersion, and the third case incorporates redistribution consideration, on top of aggregate efficiency consideration.

In the second case, the optimal policy formula is

$$\mu_j^\theta \left[u_j^{\theta\prime}(C_j^\theta) - P_j^\theta \right] = -\sum_j \frac{\partial \hat{\mu}_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta} \left[w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta \right]. \tag{D.41}$$

In the third case, the optimal policy formula is

$$\left(\Lambda^\theta - \mathbb{E}_{j|\theta} \left[\frac{P_j^\theta}{u_j^{\theta\prime}(C_j^\theta)} \right] \right) \mu_j^\theta u'(C_j^\theta) dC_j^\theta = -\sum_j \frac{\partial \hat{\mu}_j^\theta(\boldsymbol{C}^\theta)}{\partial C_i^\theta} \left[w_j^\theta - P_j^\theta C_j^\theta + \sum_{l,k} p_{jl,k} y_{jl,k} \frac{1}{l_j^\theta} \gamma_{jl,k}^\theta \right]. \tag{D.42}$$

E Nonparametric Identification of Location Choice System

We discuss the conditions under which the location choice system, $\{\mu_j(\boldsymbol{C})\}_j$ are nonparametrically identified. To do so, we build on the existing results of the nonparametric identification of discrete choice models (Berry and Haile 2014). We abstract household types and drop superscript θ.

We start by formalizing our econometric environment. Consider a dataset generated by the model of Section 2. We assume that we observe equilibrium configurations under different sets of fundamentals, indexed by $t = 0, 1, \ldots, \mathcal{T}$. A natural interpretation of t is time, while one could also interpret them as types of individuals or demographic groups. We assume that $\{w_{j,t}, l_{j,t}, P_{j,t}, T_{j,t}, \Pi_t\}$ are observed to the econometrician, so that consumption $C_{j,t} = (w_{j,t} + T_{j,t} + \Pi_t)/P_{j,t}$ is observed as well.

We specify the utility of residing in location by $u_j(C_{j,t}, \zeta_{j,t}) + \epsilon_{j,t}(\omega)$, where $\zeta_{j,t}$ is a scalar variable that is unobserved to the econometrician. The unobserved location heterogeneity, $\zeta_{j,t}$, captures amenity that varies over t. Analogous to Assumption 1 of Berry and Haile (2014), we assume that $\zeta_{j,t}$ only affects location choice through the utility index $u_j(C_{j,t}, \zeta_{j,t})$, but it does not affect the distribution of $\{\epsilon_{j,t}\}$.

Assumption E.1. The distribution function of preference shocks $\epsilon_{j,t}$ is independent of $\{\zeta_{j,t}\}$ and t, i.e.,

$$\mathbb{P}(\epsilon_{1,t} \le \bar{\epsilon}_1, \ldots, \epsilon_{N,t} \le \bar{\epsilon}_N | \{\zeta_{j,t}\}) = H(\bar{\epsilon}_1, \ldots, \bar{\epsilon}_N). \tag{E.1}$$

While this assumption is restrictive, we are not imposing any parametric assumption for the distribution function $H(\cdot)$, allowing for flexible correlation of preference shocks across locations.

For the sake of expositional clarity, we also assume in the main text that the unobserved heterogeneity enters into the utility function in as a multiplicative of consumption, $u_j(C_{j,t}, \zeta_{j,t}) = \bar{u}_j(\zeta_{j,t} C_{j,t})$. As is demonstrated by Berry and Haile (2014), this assumption can be relaxed but it requires more technically invovled assumptions.

Importantly, we assume that there are vectors of instruments $\mathbf{z}_t$ that are mean independent of unobserved component of location choice, $\ln \zeta_{j,t}$, for all j and t (Assumption E.2), and there is a sufficient variation of $\mathbf{z}_t$ to induce the changes in consumption vector (Assumption E.3).

Assumption E.2. $\mathbb{E}[\ln \zeta_{j,t} | \mathbf{z}_t] = 0 \quad$ for all j, t.

Assumption E.3. For all functions $B(C_{jt})$ with finite expectation, if $E[B(C_{jt})|\mathbf{z}_t] = 0$ almost surely, then $B(C_{jt}) = 0$ almost surely.

Assumption E.2 is the standard exclusion restriction. Assumption E.3 requires the completeness of the joint distribution $\{C_{jt}, \mathbf{z}_t\}$, capturing the idea that instruments $\mathbf{z}_t$ induces sufficient variation in C_{jt}. Under these assumptions, Berry and Haile (2014) show that the location choice system $\mu_{j,t}(\mathbf{C}_t)$ is identified.

Lemma E.1. (Berry and Haile 2014). Suppose Assumptions E.1, E.2, E.3 hold. Then the location choice system $\mu_{j,t}(\mathbf{C}_t)$ is identified.

Therefore, location choice system $\{\mu_{j,t}(\boldsymbol{C}_t)\}_j$ are, at least in principle, nonparametrically identified. At the same time, the data requirement of the excluded instruments $\mathbf{z}_t$ (Assumptions E.2 and E.3) is substantial. Importantly, to fully identify the flexible substitution patterns for location choice, we need instruments $\mathbf{z}_t$ that induce independent variation in consumption levels in each location $C_{j,t}$. More fundamentally, we need independent observation of equilibrium configurations across different fundamentals (t).

F Details on Application I

F.1 Inferring Technology, Ω_T

We explain the detailed procedure of how we construct productivity growth at the MSA level. Let GDP_i be nominal GDP of MSA i and Y_i be real GDP of MSA i deflated using the GDP deflator in i, P_i^Y. We apply Collorary 1 in Baqaee and Farhi (2019) to express the first-order changes in real GDP at the MSA i:

$$d\ln Y_i = \sum_{j,k} \frac{p_{ij,k}y_{ij,k}}{GDP_i} d\ln \mathcal{A}_{ij,k} + \sum_{\theta} \frac{w_i^\theta l_i^\theta}{GDP_i} d\ln l_i^\theta \tag{F.1}$$

Furthermore, note that

$$d\ln \mathcal{A}_{ij,k} = d\ln A_{ij,k} + \sum_{\theta} \gamma_{ij,k}^{\theta} d\ln l_i^\theta. \tag{F.2}$$

Using these expressions and given the knowledge of $\{\gamma_{ij,k}^\theta\}$, we construct a measure of technological changes at MSA i as follows

$$\sum_{j,k} p_{ij,k}y_{ij,t} d\ln A_{ij,k} = GDP_i \left[d\ln Y_i - \sum_{\theta} \frac{w_i^\theta l_i^\theta}{GDP_i} d\ln l_i^\theta - \sum_{j,k} \frac{p_{ij,k}y_{ij,k}}{GDP_i} \sum_{\theta} \gamma_{ij,k}^\theta d\ln l_i^\theta \right] \tag{F.3}$$

Summing across all MSAs, we obtain the technology term:

$$\Omega_{TE} = \sum_i \sum_{j,k} p_{ij,k}y_{ij,k} d\ln A_{ij,k} \tag{F.4}$$

$$= \sum_i GDP_i \left[d\ln Y_i - \sum_{\theta} \frac{w_i^\theta l_i^\theta}{GDP_i} d\ln l_i^\theta - \sum_{j,k} \frac{p_{ij,k}y_{ij,k}}{GDP_i} \sum_{\theta} \gamma_{ij,k}^\theta d\ln l_i^\theta \right]. \tag{F.5}$$

By definition of real GDP,

$$d\ln Y_i = d\ln GDP_i - d\ln P_i^Y. \tag{F.6}$$

Plugging (F.6) back into F.5, we have

$$\Omega_{TE} = \sum_i \left[dGDP_i - \sum_\theta w_i^\theta l_i^\theta d\ln l_i^\theta - \sum_{j,k} p_{ij,k} y_{ij,k} \sum_\theta \gamma_{ij,k}^\theta d\ln l_i^\theta \right] \tag{F.7}$$

$$- \sum_l GDP_l \sum_i \frac{GDP_i}{\sum_j GDP_j} d\ln P_i^Y. \tag{F.8}$$

Given the lack of producer prices at MSA level that starts in year 2010, we measure the last term $\sum_i \frac{GDP_i}{\sum_j GDP_j} d\ln P_i^Y$ using the GDP deflator at the national level. We assume the only factor of production is labor and assume away the presence of input-output linkages. These assumptions imply that GDP of MSA i can be inferred as the total pre-tax personal income and that sales of skill group θ can be inferred as their pre-tax personal income.

F.2 Estimation of Utility Function

We describe the details of the estimation of utility functions. Recall that we impose the following parametric assumptions:

$$u_{j,t}^\theta(C_{j,t}^\theta) + \epsilon_{j,t}^\theta = \frac{(C_{j,t}^\theta)^{1-\rho_\theta}}{1-\rho_\theta} + \xi_{j,t}^\theta + e_{j,t}^\theta, \tag{F.9}$$

and $e_{j,t}^\theta$ follows independent type-I extreme value distribution with shape parameter ν_θ.This results in the following logit location choice system

$$\mu_{j,t}^\theta = \frac{\exp\left(\nu_\theta \left[\frac{(C_{j,t}^\theta)^{1-\rho_\theta}}{1-\rho_\theta} + \xi_{j,t}^\theta\right]\right)}{\sum_m \exp\left(\nu_\theta \left[\frac{(C_{m,t}^\theta)^{1-\rho}}{1-\rho_\theta} + \xi_{m,t}^\theta\right]\right)} \tag{F.10}$$

Taking log and time-differencing and further differencing out with location 1, we obtain

$$\Delta \ln(\mu_{j,t}^\theta / \mu_{1,t}^\theta) = \nu_\theta \left[\Delta \frac{(C_{j,t}^\theta)^{1-\rho_\theta}}{1-\rho_\theta} - \Delta \frac{(C_{1,t}^\theta)^{1-\rho_\theta}}{1-\rho_\theta}\right] + \nu_\theta \left[\Delta\xi_{j,t}^\theta - \Delta\xi_{1,t}^\theta\right], \tag{F.11}$$

where $\Delta x_t \equiv x_t - x_{t-1}$ denotes the time-difference for any variable x_t. The identification threat in estimating equation (F.11) is that unobserved location-specific amenity shock $\Delta\xi_{j,t}^\theta$ is correlated with changes in consumption. We therefore need instrumental variables, $\boldsymbol{Z}_{j,t}$ that are uncorrelated with the location-specific amenity shock. Define the structural residual as follows.

$$e^{\theta}_{j,t}(\boldsymbol{\beta}_{\theta}) = \Delta \ln(\mu^{\theta}_{j,t}/\mu^{\theta}_{1,t}) - \nu_{\theta}\left[\Delta\frac{(C^{\theta}_{j,t})^{1-\rho_{\theta}}}{1-\rho_{\theta}} - \Delta\frac{(C^{\theta}_{1,t})^{1-\rho_{\theta}}}{1-\rho_{\theta}}\right], \tag{F.12}$$

where $\boldsymbol{\beta}_{\theta} = (\rho_{\theta}, \nu_{\theta})$. Given a vector of $\boldsymbol{Z}_{j,t}$ that satisfies the following moment conditions:

$$\mathbb{E}\left[\left(\Delta\xi^{\theta}_{j,t} - \Delta\xi^{\theta}_{1,t}\right)\boldsymbol{Z}_{j,t}\right] = 0, \tag{F.13}$$

we construct a consistent GMM-estimator of $(\rho_{\theta}, \nu_{\theta})$ that solve

$$\hat{\boldsymbol{\beta}}_{\theta} = \arg\min_{\boldsymbol{\beta}_{\theta}} \boldsymbol{e}^{\theta}(\boldsymbol{\beta}_{\theta})'\boldsymbol{Z}\Phi\boldsymbol{Z}'\boldsymbol{e}^{\theta}(\boldsymbol{\beta}_{\theta}), \tag{F.14}$$

where Φ is a weighting matrix.

To build instrument variables, we construct a shift-share instrument that interacts with local industry composition with the national industry employment growth for each skill type θ, similarly to Diamond (2016). Specifically, we construct the following shift-share instrument:

$$z^{\theta}_{j,t} = \sum_{k}\frac{l^{\theta}_{j,k,t-1}}{\sum_{k} l^{\theta}_{j,k,t-1}}\Delta \ln l^{\theta}_{-j,k,t}, \tag{F.15}$$

where $l^{\theta}_{j,k,t-1}$ denotes the industry k employment of type θ in location j at time $t-1$, and $\Delta \ln l^{\theta}_{-j,k,t}$ is the national industry employment growth of skill θ excluding location j. We construct the above instrument using 5-years sample of ACS for years 2010 and 2019. We construct an additional instrumental variable that interacts $z^{\theta}_{j,t}$ with consumption growth, $z^{\theta}_{j,t} \times \Delta d \ln C^{\theta}_{j,t}$. We set the weighting matrix to be an identity matrix.

We report standard errors based on the consistent estimator of the asymptotic covariance matrix of the GMM estimator, $\hat{V}$:

$$\hat{V} = (\hat{G}'\hat{\Omega}^{-1}\hat{G})', \quad \hat{G} \equiv \frac{\partial}{\partial \boldsymbol{\beta}}\left(\boldsymbol{e}^{\theta}(\hat{\boldsymbol{\beta}}_{\theta})'\boldsymbol{Z}\right), \tag{F.16}$$

and $\hat{\Omega} = \frac{1}{N}\sum_{i=1}^{N}\left(\boldsymbol{e}^{\theta}_{i}(\hat{\boldsymbol{\beta}}_{\theta})'\boldsymbol{Z}_{i}\right)\left(\boldsymbol{e}^{\theta}_{i}(\hat{\boldsymbol{\beta}}_{\theta})'\boldsymbol{Z}_{i}\right)'$.

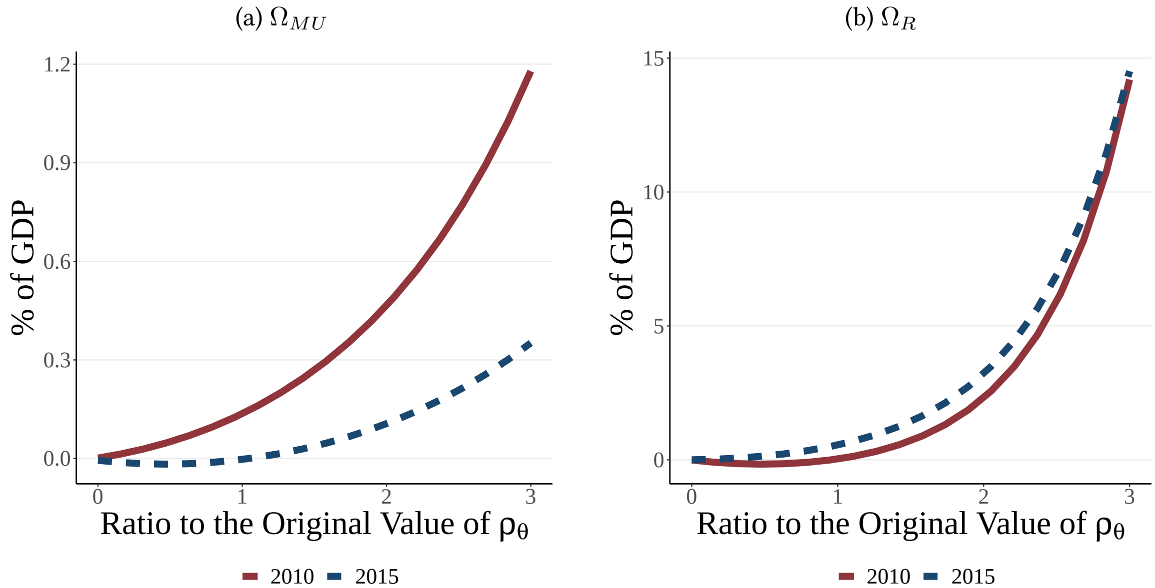

Figure F.1: Sensitivity to the value of ρ_θ

F.3 Sensitivity Analysis

We present the sensitivity of our results to parameter values for the analysis in Section 5.1. We first vary the parameters governing the marginal utility, ρ_θ. We assume $\rho_\theta = \bar{\rho}_\theta \times x$, where $\bar{\rho}_\theta$ is our baseline estimates, and vary x. Figure F.1 shows the results. As the values of ρ_θ increase, we see both Ω_{MU} and Ω_R grow substantially in absolute terms.

G Details on Application II

G.1 Allen and Arkolakis (2022) Model

Allen and Arkolakis (2022) consider an environment with a homogeneous population, therefore we drop superscript θ. They specify the utility function as

$$U_j(C_j, \varepsilon_j) = \ln C_j + \varepsilon_j, \tag{G.1}$$

where ε_j follows type-I extreme value distribution with shape parameter ν.[3] They assume away the possibilities of spatial transfers so that $T_j = 0$ for all j.

[3]As we discuss in Appendix D.4, this specification is isomorphic to assuming an iso-elastic log-linear congestion externality $\nu \ln l_j$ in replace of ε_j. Furthermore, as discussed in Section 4.1, the welfare changes are invariant by also applying an exponential transformation.

The final goods production technology is constant elasticity of substitution (CES), given by

$$C_j = \left(\int_k c_{j,k}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{G.2}$$

where $k \in K = [0,1]$ indexes the industry, and σ is the elasticity of substitution. The intermediate goods production technology is linear in labor, given by

$$y_{ij,k} = \mathcal{A}_i \tau_{ij,k} l_{ij,k}, \tag{G.3}$$

where $\tau_{ij,k}$ is the iceberg shipment cost and $\mathcal{A}_i$ indexes the productivity of region i. The region's productivity is subject to iso-elastic agglomeration externality in local population size given by

$$\mathcal{A}_i = A_i \left(l_i\right)^{\alpha}. \tag{G.4}$$

A key feature of Allen and Arkolakis (2022) is the modeling of the shipment cost $\tau_{ij,k}$ through a route choice problem. Denote by $\mathcal{R}_{ij}$ all possible routes connecting i to j. Formally, $r \in \mathcal{R}_{ij}$ is a sequence of legs (a pair of adjacent locations). Passing through each leg (k,l) incurs iceberg shipment cost t_{mn}. The optimal route choice for producers in region i and sector k implies that the shipment cost $\tau_{ij,k}$ is given by

$$\tau_{ij,k} = \min_{r \in \mathcal{R}_{ij}} \prod_{l=1}^{|r|} t_{r_{l-1} r_l} \epsilon_{ij,k}, \tag{G.5}$$

where $\epsilon_{ij,k}$ is the idiosyncratic cost for each (i,j,k). Finally, they assume that the leg-specific shipment cost may be subject to congestion externality depending on the traffic passing through the leg. In particular, they assume

$$t_{mn} = \tilde{t}_{mn} \left(\Xi_{mn}\right)^{\lambda}, \tag{G.6}$$

where $\tilde{t}_{mn}$ is the exogenous component of the leg-specific shipment cost, which is in part affected by the transportation infrastructure, Ξ_{mn} is the value flows passing the leg (m,n), and λ is the parameter that captures the strength of the congestion externality in shipment cost.

Allen and Arkolakis (2022) use this model to study the aggregate welfare changes from a marginal decrease of $\tilde{t}_{mn}$ to assess the leg-specific improvement of transportation infrastructure. Below, we analyze the same counterfactual experiment, as well as the regional productivity changes A_i. It is straightforward to derive our welfare decomposition in Proposition 1. Since we abstract spatial transfers and multiple types, (iii) fiscal externality and (v) redistribution terms are zero. The remaining three terms, (i) technology, (ii) MU dispersion, and (iv) technological externality come down to

$$\Omega_T = -\sum_{k,l} \Xi_{k,l} d\ln \tilde{t}_{kl} + \sum_i Y_i d\ln A_i, \tag{G.7}$$

$$\Omega_{MU} = \text{Cov}_j \left(-w_j, d\ln C_j\right), \tag{G.8}$$

$$\Omega_{TE} = \Omega_{TE,S} + \Omega_{TE,A}, \qquad \Omega_{TE,S} = -\lambda \sum_{k,l} \Xi_{k,l} d\ln \Xi_{kl}, \quad \Omega_{TE,A} = \alpha \sum_i Y_i d\ln l_i, \tag{G.9}$$

where $\Omega_{TE,S}$ and $\Omega_{TE,A}$ correspond to the technological externality arising from shipment congestion externality and productivity agglomeration externality, respectively.

G.2 Additional Figures

(a) Transportation Infrastructure Improvement

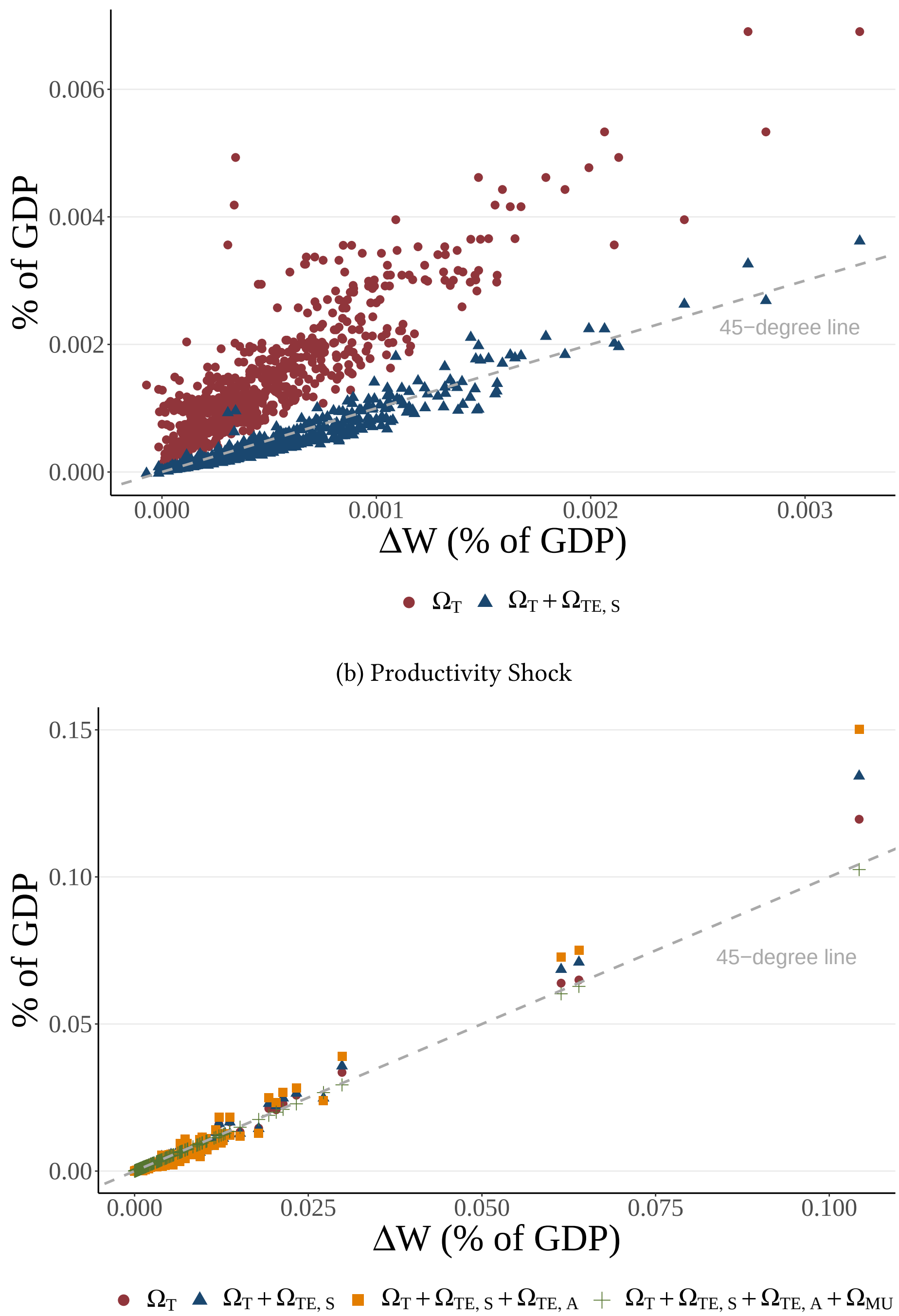

Figure G.1: Welfare Decomposition in Allen and Arkolakis (2022) Model

Note: The figure plots welfare decomposition described in the main text for each of the counterfactual experiments in Allen and Arkolakis (2022) model. Panel G.1a is the counterfactual experiment of reducing the shipment cost by 1 percent for each of 704 links. Panel G.1b is the counterfactual experiment of increasing productivity by 1 percent for each of 227 CBSAs.