The Insurance Role of Firms

741 Macroeconomics
Topic 5

Masao Fukui

2025 fall

Firms as Insurance Providers

- Knight (1921) ascribes the very existence of the firm to its role as an insurance provider
  - Businesses are inherently risky and uncertain
  - Agents who can tolerate or diversify risk become the owner of firms
  - They then provide insurance to workers through wage contracts
- The models we have seen so far do not capture this idea
- We make the following modifications:
  - workers are risk-averse
  - firms offer long-term contracts
  - allow for time-varying productivity shocks

A Toy Model of Optimal Contract

Environment

- Two periods, $t = 0,1$
- At $t = 0$, a worker and a firm are matched
- Preferences:
  - Workers are risk-averse: $u(c_0^w) + \beta \mathbb{E}_0[u(c_1^w)]$
  - Firms are risk-neutral: $c_0^f + \beta \mathbb{E}[c_1^f]$
- Firms produce z_t units of output per worker: z_0 is deterministic & z_1 is stochastic
- The firm has to deliver utility of V_0 to the workers (exogenous outside option)
- No financial asset is available

Optimal Contracting Problem

$$\max_{w_0, \{w_1(z_1)\}} \quad z_0 - w_0 + \beta \mathbb{E}\left[z_1 - w_1(z_1)\right]$$

$$\text{s.t.} \quad u(w_0) + \beta \mathbb{E}[u(w_1(z_1))] \geq V_0$$

- Firms write wage contracts contingent on the shocks
- Taking the first-order conditions, (let λ be the Lagrange multiplier)

$$\lambda u'(w_0) = 1$$

$$\lambda u'(w_1(z_1)) = 1$$

$\Rightarrow$ perfect wage/consumption smoothing

- Even in the absence of a financial market, firm can instead act as insurance provider

History-Dependence in Wages

- Eliminating the Lagrange multiplier, we have explicit solutions:

$$w_0 = w_1(z_1) = u^{-1}(V_0/(1+\beta))$$

- A critical aspect, beyond smoothing, is that wage is history-dependent
- Wage at $t = 1$ is a function of outside option of workers at $t = 0$
- This is not a feature of most of the models we have seen
  - There, wage is a function of current and future productivity and outside options
  - The only exception is the sequential auction

Beaudry & DiNardo (1991)

- Beaudry & DiNardo (1991) test the prediction using CPS/PSID 1976-1984
- Do the past labor market conditions predict wages…
  … above and beyond contemporaneous labor market conditions?
- Run the following regression:

$$\ln w_{i,t,t-j} = \beta_1 \text{unemp}_t + \beta_2 \text{unemp}_{t-j} + \beta_3 \text{unemp}^{\min}_{t-j,t} + \gamma' X_{i,t} + \epsilon_{i,t}$$

  - $w_{i,t,t-j}$: wage of worker i at time t hired at time $t-j$
  - unemp_t: unemployment at time t
  - unemp_{t-j}: unemployment when the worker is hired
  - $\text{unemp}^{\min}_{t,t-j}$: the lowest unemployment rate during the tenure

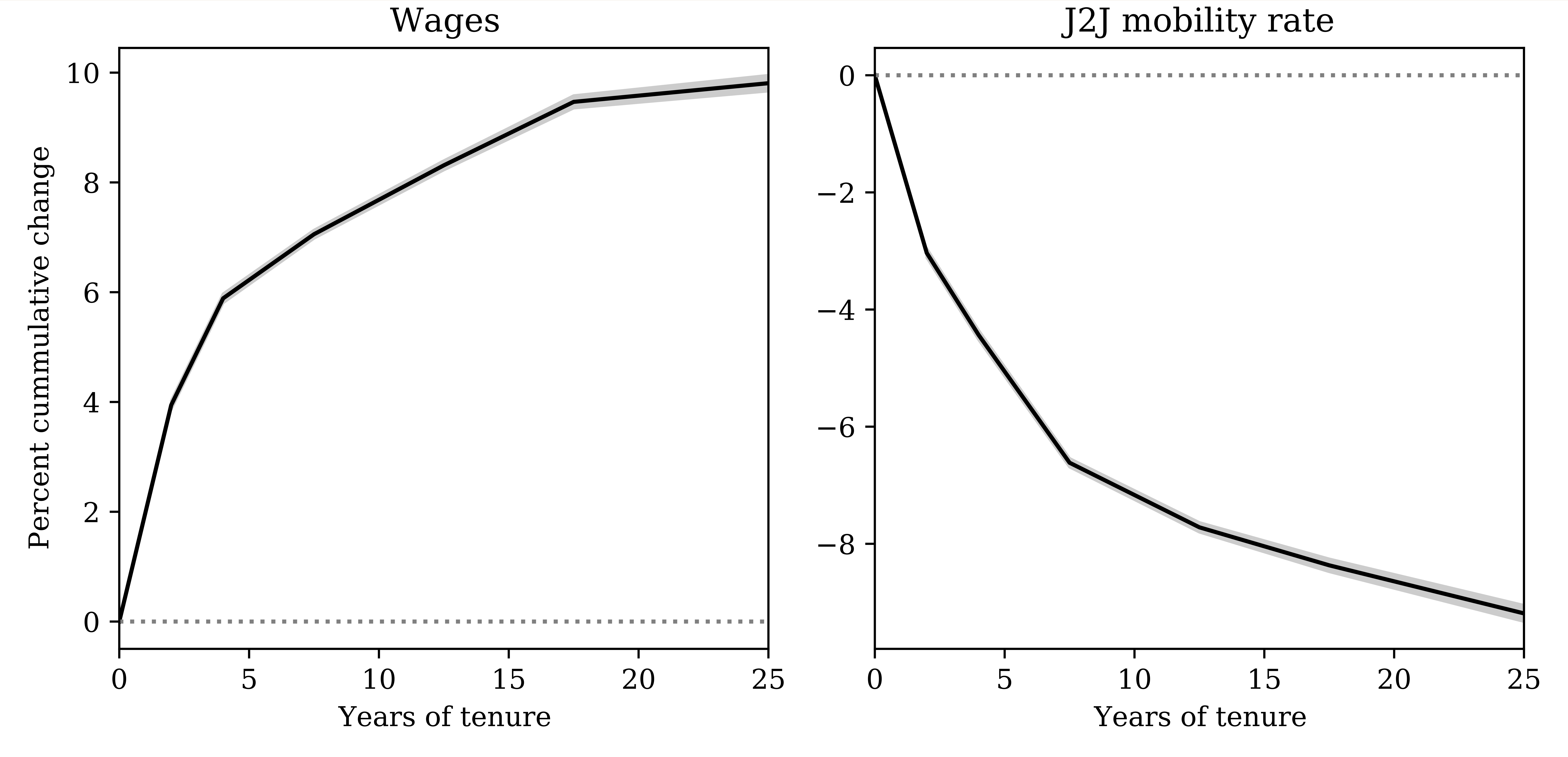
Wages are History Dependent in the Data

	Contemporaneous Unemployment Rate	Unemployment at Start of Job	Minimum Rate since Start of Job	Data
1.	−.020 (.002)	. . .	. . .	PSID (levels)
2.	. . .	−.030 (.002)	. . .	PSID (levels)
3.	. . .	. . .	−.045 (.003)	PSID (levels)
4.	−.010 (.002)	−.025 (.002)	. . .	PSID (levels)
5.	−.001 (.002)	. . .	−.044 (.003)	PSID (levels)
6.	.000 (.002)	.013 (.004)	−.059 (.006)	PSID (levels)
7.	−.014 (.002)	. . .	. . .	PSID (fixed effect)
8.	. . .	−.021 (.003)	. . .	PSID (fixed effect)
9.	. . .	. . .	−.029 (.003)	PSID (fixed effect)
10.	−.007 (.0025)	−.006 (.007)	−.029 (.008)	PSID (fixed effect)

Wages are Not Smooth in the Data

- In the data, wages are rarely perfectly smoothed
  1. Wages rise with tenure on average
  2. Wages respond to idiosyncratic firm-level shocks

Wages Rise and J2J Falls with Tenure



Source: Souchier (2024), French data

Partial Insurance, More So for Risk-Averse Workers

Shock to value added of firm j

$$\Delta \ln w_{ijt} = \beta \, \Delta\epsilon_{j,t} + X'_{ijt}\gamma + \nu_{ijt}$$

	Sensitivity to Permanent Shocks (1)	Sensitivity to Transitory Shocks (2)
$\Delta\epsilon_{j,t}$	.1096	.0151
	(.0324)	(.0144)
	[.0213]	[.1947]
$\Delta\epsilon_{j,t}$ × high risk aversion	−.0832	−.0120
	(.0366)	(.0154)
	[.0157]	[.2468]
$\Delta\epsilon_{j,t}$ × manager	.0778	.0132
	(.1197)	(.0166)
	[.0237]	[.2572]
$\Delta\epsilon_{j,t}$ × s.d.[ln (VA_{jt})]	−.0268	−.0040
	(.0129)	(.0038)
	[.0604]	[.3575]
$\Delta\epsilon_{jt}$ × bankruptcy index	.0327	−.0027
	(.0388)	(.0100)
	[.0118]	[.2474]
Observations	24,956	40,337
J-test (*p*-value)	.3257	.2863

Source: Guiso, Pistaferri & Schivardi (2005), Italian data

Moral Hazard

- Now introduce moral hazard frictions into the optimal contracting problem
- At $t = 1$ (after z_1 realizes), workers receive outside offers
  - Let $F(W_1)$ be the cdf of the offer utility distribution (exogenous)
- Two important assumptions:
  1. Contracts cannot depend on the arrival of the outside offer
     - Either because the outside offer is unverifiable or a fairness concern
  2. Contracts cannot specify worker's job mobility decisions
     - Unconstitutional in many countries: "no slavery"
- If outside offer provides a better utility, the worker leaves for the other firm

Optimal Contracting Problem

$$\max_{w_0, \{w_1(z_1)\}} \quad z_0 - w_0 + \beta \mathbb{E}\left[F(u(w_1(z_1)))\big(z_1 - w_1(z_1)\big) \right]$$

$$\text{s.t.} \quad u(w_0) + \beta \mathbb{E}\left[\int \max\left\{ u(w_1(z_1)), \tilde{W}_1) \right\} dF(\tilde{W}_1) \right] \geq V_0$$

- The FOCs are (let $\tilde{F}(w_1) \equiv F(u(w_1))$

$$\lambda u'(w_0) = 1$$

$$-\tilde{F}(w_1(z_1)) + \tilde{F}'(w_1(z_1))[z_1 - w_1(z_1)] + \lambda \tilde{F}(w_1(z_1)) u_1'(w_1(z_1)) = 0$$

- Getting rid of the Lagrange multiplier,

$$\frac{u'(w_1(z_1))}{u'(w_0)} = 1 - \frac{\tilde{F}'(w_1(z_1))}{\tilde{F}(w_1(z_1))}[z_1 - w_1(z_1)]$$

Backloading and Frontloading

$$\frac{u'(w_1(z_1))}{u'(w_0)} = 1 - \underbrace{\frac{\tilde{F}'(w_1(z_1))}{\tilde{F}(w_1(z_1))}[z_1 - w_1(z_1)]}_{\equiv \xi(z_1)}$$

1. $\xi_1(z_1) > 0 \Leftrightarrow z_1 - w_1(z_1) > 0$
   - Raising wages $\Rightarrow$ increased retention $\Rightarrow$ higher profits (since $z_1 - w_1(z_1) > 0$)
   - Firms therefore **backload** wages $w_1(z_1) > w_0$
2. $\xi_1(z_1) < 0 \Leftrightarrow z_1 - w_1(z_1) < 0$
   - Raising wages $\Rightarrow$ increased retention $\Rightarrow$ lower profits (since $z_1 - w_1(z_1) < 0$)
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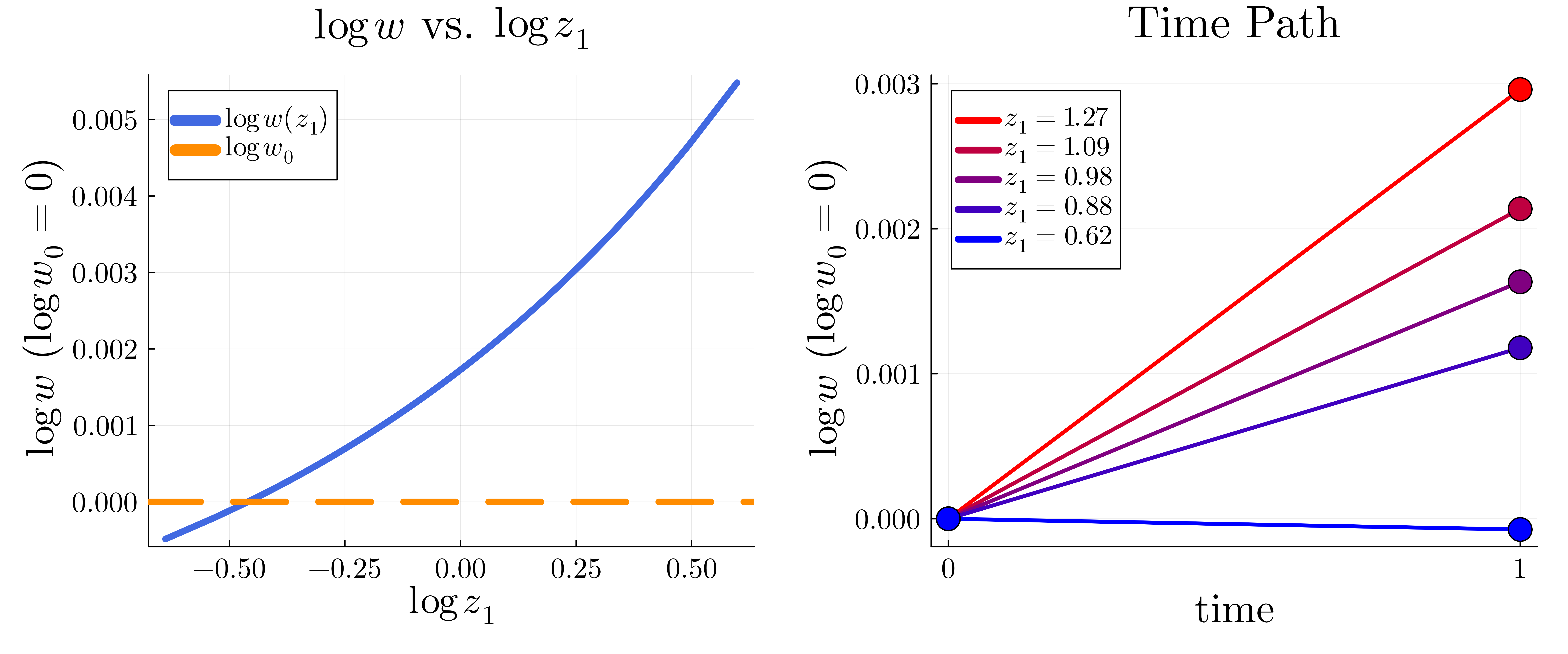
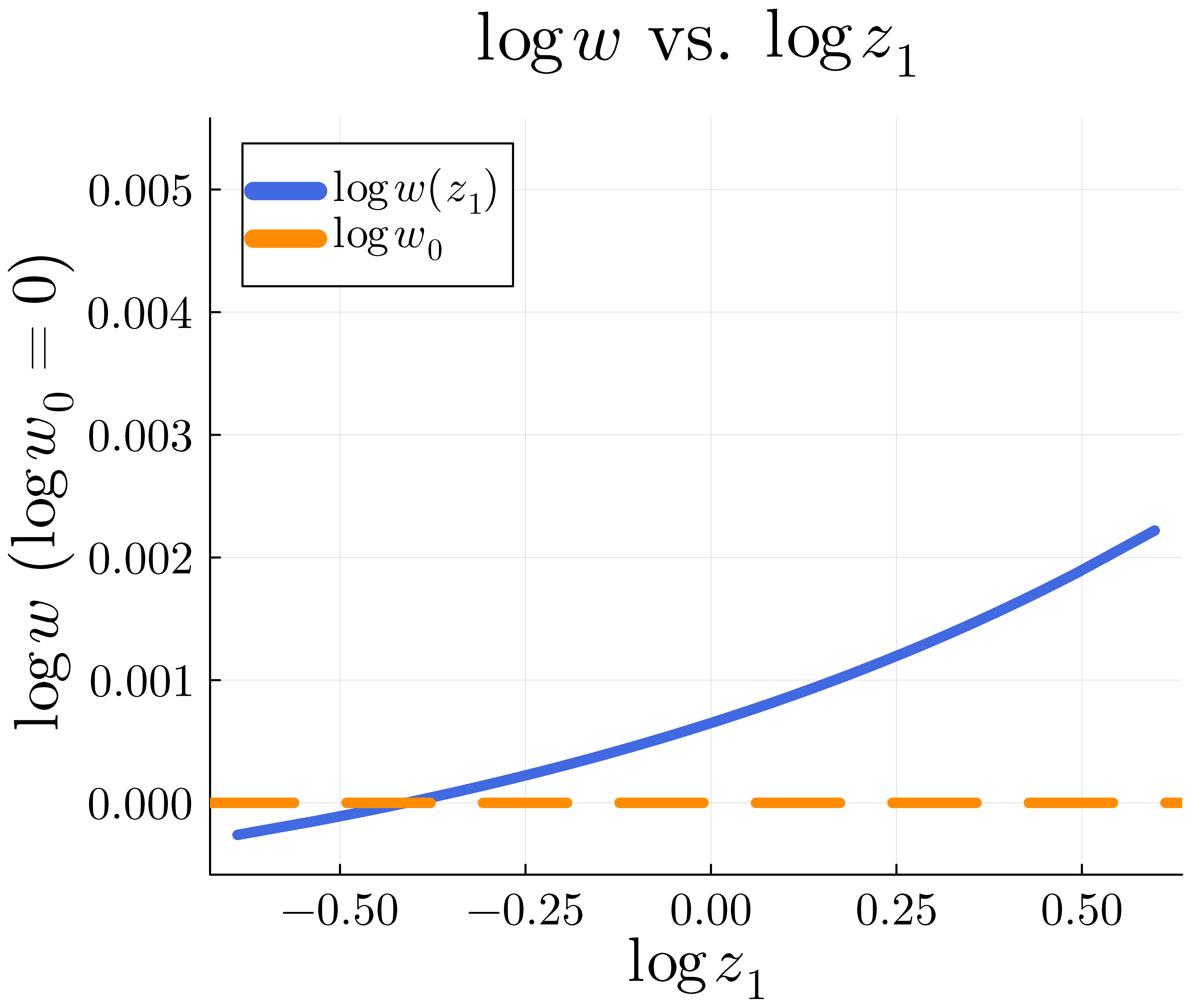
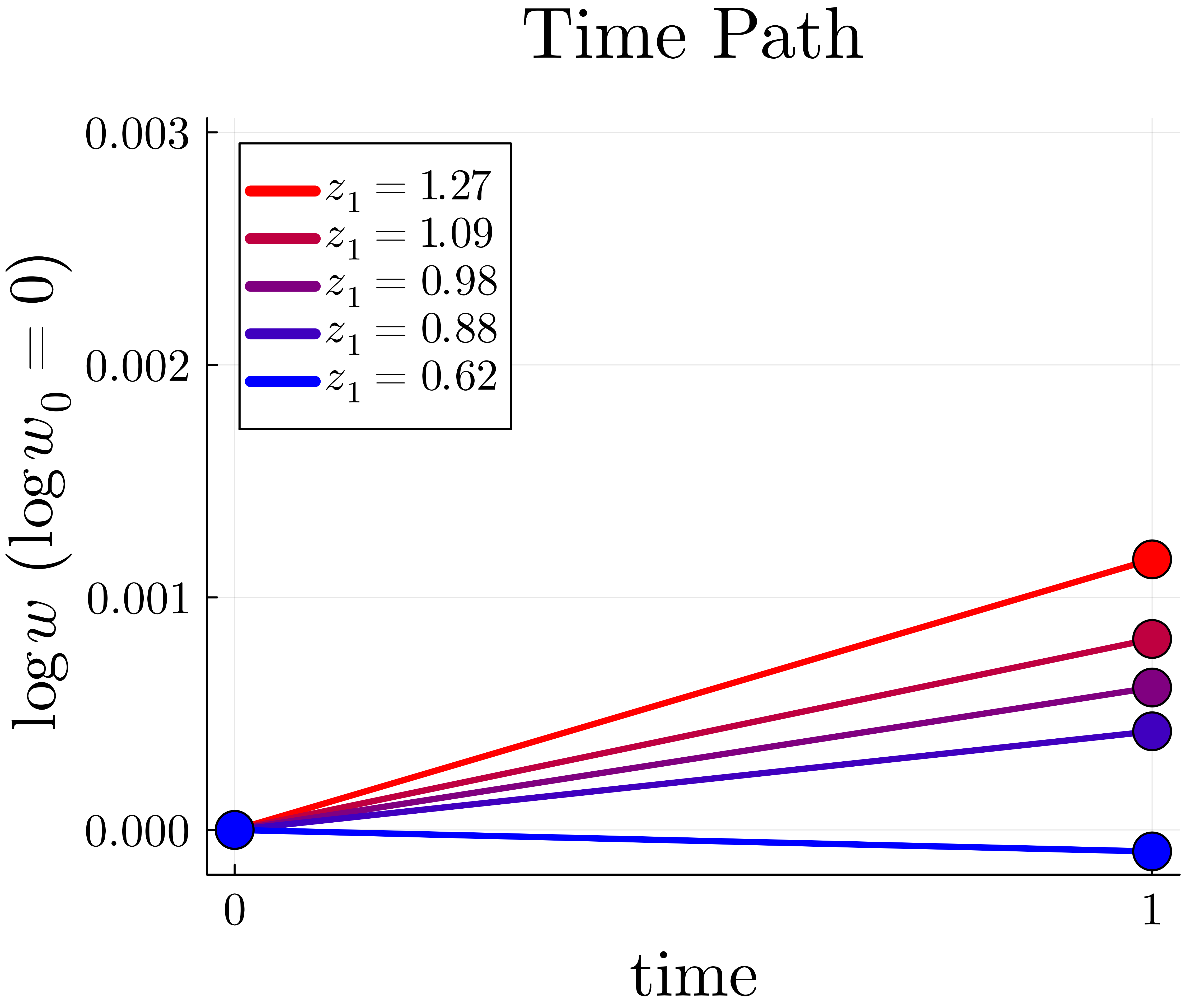
Numerical Examples with Low Risk Aversion



Numerical Examples with High Risk Aversion





Recursive Contract

- We can equivalently rewrite the previous problem in a recursive form

$$\Pi_0(V_0) = \max_{w_0, w_1(z_1)} \quad z_0 - w_0 + \beta \mathbb{E}\left[F(u(w_1(z_1)))\big(z_1 - w_1(z_1)\big)\right]$$

$$\text{s.t.} \quad u(w_0) + \beta \mathbb{E}\left[\int \max\left\{u(w_1(z_1)), u(\tilde{w}_1)\right\} dF(\tilde{w}_1)\right] \geq V_0$$

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$$\text{s.t.} \quad u(w_0) + \beta \mathbb{E}\left[\int \max\Big\{V_1(z_1), u(\tilde{w}_1)\Big\} dF(\tilde{w}_1)\right] \geq V_0$$

- In the next period, firms solve

$$\Pi_1(V_1, z_1) = \max_{w_1} z_1 - w_1$$

$$\text{s.t.} \quad u(w_1) \geq V_1$$

  - V_1 is called **promised utility**
  - Constraints are called **promise-keeping constraints**

Recursive Contracts with Many Periods

- Writing recursively not useful with 2-period, but very useful if more than 2 periods!
- Recursive formulation naturally extends to T-period model:

$$\Pi_t(V_t, z_t) = \max_{w_t, \{V_{t+1}(z_{t+1})\}} z_t - w_t + \beta \mathbb{E}\left[F\left(V_{t+1}(z_{t+1})\right)\Pi_{t+1}(V_{t+1}(z_{t+1}), z_{t+1})\right]$$

$$\text{s.t.} \quad u(w_t) + \beta \mathbb{E}\int \max\{V_{t+1}(z_{t+1}), \tilde{W}\} dF(\tilde{W}) \geq V_t$$

and

$$\Pi_T(V_T, z_T) = \max_{w_T} z_T - w_T$$

$$\text{s.t.} \quad u(w_T) \geq V_T$$

- Can use the standard Bellman technique to solve the optimal contract!

Long-term Wage Contracts in the Frictional Labor Market

Based on
Balke-Lamadon (2022)
Souchier (2024)

Preferences and Technology

- Discrete time, $t = 0, \ldots, \infty$. Focus on the steady state (for now).
- Firms:
  - Risk-neutral with preferences $\sum_{t=0}^{\infty} \beta^t c_t^f$
  - Heterogeneous in their productivity z, which follows Markov process
- Workers:
  - Risk-averse with preferences $\sum_{t=0}^{\infty} \beta^t u(c_t^w)$
  - Fixed and homogenous productivity
- A match produces z units of output
- Unemployed workers produces b at home

Timing

1. Firm-level productivity shocks z_t are realized

2. Firms produce and pay wages

3. All agents search & match
   - Employed and unemployed workers search for jobs
   - Firms post vacancies
   - new matches are formed and new contracts are signed

4. Exogenous separations take place and workers can quit

Directed Search

- Search is directed
  - Random search is a nightmare in this kind of model
- Firms post wage contracts, and workers choose which jobs (submarket) to apply for
- Without loss of generality, submarkets are indexed by worker's continuation value v
  - Continuation value from the job is the only thing that workers care!
- There is a CRS matching function in each submarket v: $M(\phi_u(v) + \zeta\phi_e(v), \phi_f(v))$
  - Define $\lambda^U(v) \equiv M/(\phi_u + \zeta\phi_e)$, $\lambda^E(v) \equiv \zeta\lambda^U(v)$, and $\lambda^F(v) \equiv M/\phi_f$
- Unemployed workers then solve

$$U = u(b) + \beta[\max_v \lambda^U(v)v + (1 - \lambda^U(v))U]$$

Contracts

- Firms offer long-term contracts to workers under full commitment
- Firms specify the wages contingent on the **history** of productivity shocks
  - Again, contracts cannot depend on outside offers or specify mobility decisions
- As before, we can equivalently describe contracts recursively:
  - Given the promised utility V_t and productivity z_t today, firms specify

$$w_t(V_t, z_t), \quad V_{t+1}(z_{t+1}; V_t, z_t)$$

  - subject to promise-keeping constraint:

$$u(w(V_t, z_t)) + \beta \left[\max_v \lambda^E(v)v + (1 - \lambda^E(v))\Big((1-\delta)\max\{\mathbb{E}V_{t+1}(z_{t+1}; V_t, z_t), U\} + \delta U\Big)\right] \geq V_t$$

Bellman Equation

- Value of firm with promised utility V_t and productivity z_t

$$\Pi(V_t, z_t) = \max_{w_t, \{V_{t+1}(z_{t+1})\}} z_t - w_t + \beta(1 - \lambda^E(v))(1 - \delta)(1 - q)\mathbb{E}[\Pi(V_{t+1}(z_{t+1}), z_{t+1})]$$

s.t. $\quad u(w) + \beta \left[\lambda^E(v)v + (1 - \lambda^E(v))W_{t+1}\right] \geq V_t$ (Promise-keeping)

$$v \in \arg\max_{\tilde{v}} \lambda^E(\tilde{v})\tilde{v} + (1 - \lambda^E(\tilde{v}))W_{t+1}$$ (Incentive compatibility for OJS)

$$q = \mathbb{I}\left[\mathbb{E}[V_{t+1}(z_{t+1})] < U\right]$$ (Incentive compatibility for quit)

where W_{t+1} is the continuation value:

$$W_{t+1} \equiv (1 - \delta)(1 - q)\mathbb{E}[V_{t+1}(z_{t+1})] + (\delta + (1 - \delta)q)U$$

Optimal Wage Formula

$$\frac{u'(w_{t+1}(z_{t+1}))}{u'(w_t)} = 1 - \underbrace{\frac{\partial \ln p_{t+1}}{\partial w_{t+1}(z_{t+1})}\mathbb{E}\left[\Pi(V_{t+1}(z_{t+1}), z_{t+1})\right]}_{\equiv \xi(z_{t+1})}$$

- p_{t+1} is the probability that workers stay at the current firm
- Since $\frac{\partial \ln p_{t+1}}{\partial w_{t+1}} \geq 0$, $\xi(z_{t+1}) > 0$ if and only if $\mathbb{E}[\Pi] > 0$
  - If $\xi_{t+1} > 0$, it is optimal to backload $(w_t < w_{t+1})$ so as to incentivize workers to stay
  - If $\xi_{t+1} < 0$, it is optimal to frontload $(w_t > w_{t+1})$ so as to incentivize workers to leave

Free-Entry of Vacancy

- The cost of vacancy posting is κ, and we assume there is a free-entry
- For each submarket v, free entry implies (whenever there is a positive entry)

$$\lambda^F(v)\beta\Pi(v, z_0) = \kappa$$

Recall $\lambda^F(v) = M(1,1/\theta(v))$, where $\theta(v)$ is the market-tightness in submarket v

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    - Low-profits (high wage) are compensated by high vacancy filling rate

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   - Low-profits (high wage) are compensated by high vacancy filling rate
2. $\lambda^F(v)$ increasing in $v \Rightarrow \theta(v)$ is decreasing in v
   - High-wage postings are associated with fewer vacancies relative to applicants
3. Consequently, $\lambda^U(v)$ & $\lambda^E(v)$ are decreasing in v
   - Good jobs are harder to find in equilibrium

Equilibrium Definition

- A recursive equilibrium consists of value functions $\Pi(V, z)$, policy functions $V_{t+1}(z_{t+1}; V, z)$, $w(V, z)$, $v(V, z)$, and $q(V, z)$, as well as meeting rates $\lambda^U(v), \lambda^E(v), \lambda^F(v)$ such that:

1. Value and policy functions solve Bellman equations
2. The free-entry conditions are satisfied
3. Meeting rates are consistent with the matching function

Equilibrium Definition

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- Realize that the definition does **not** involve employment distribution!

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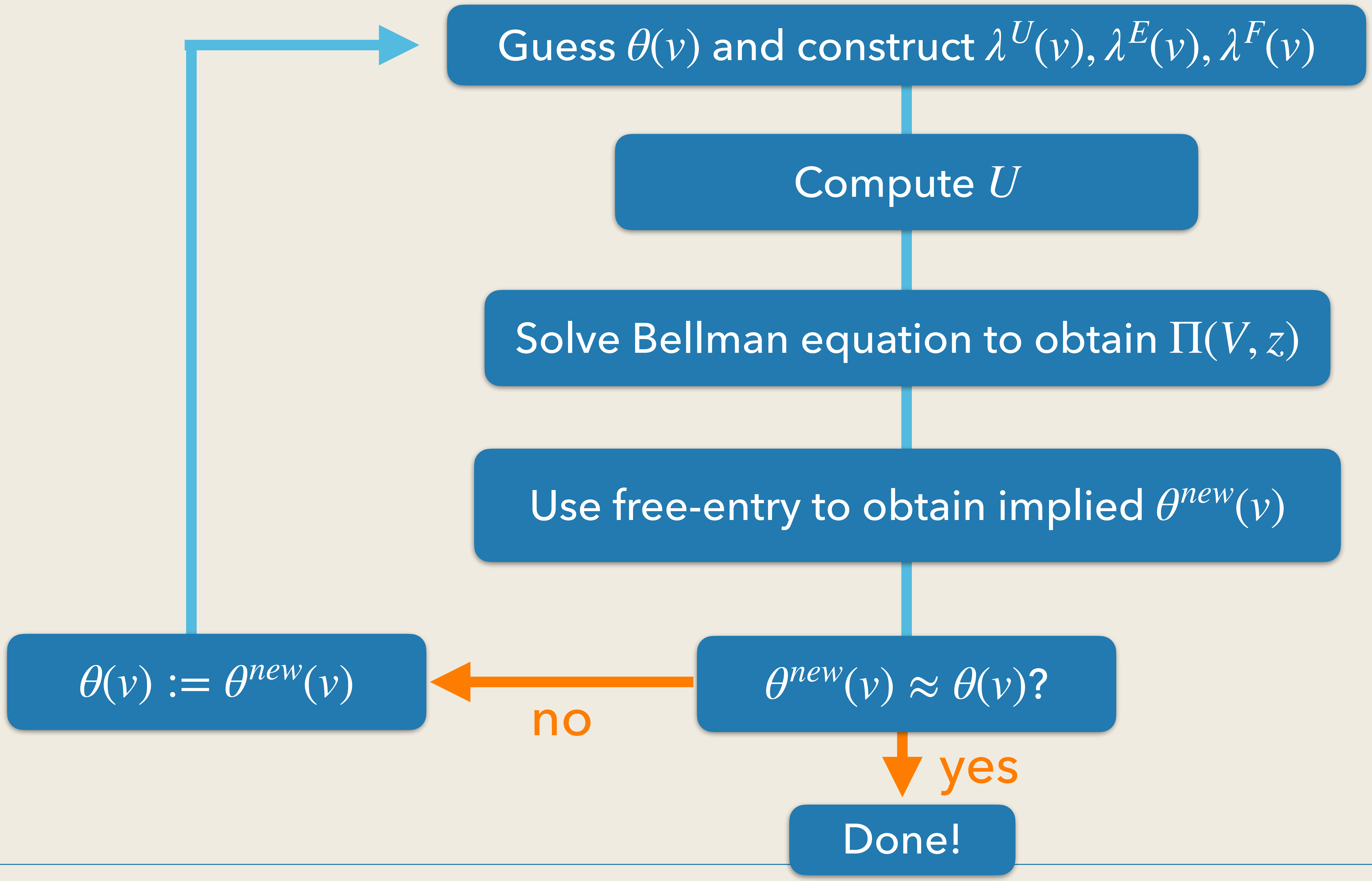
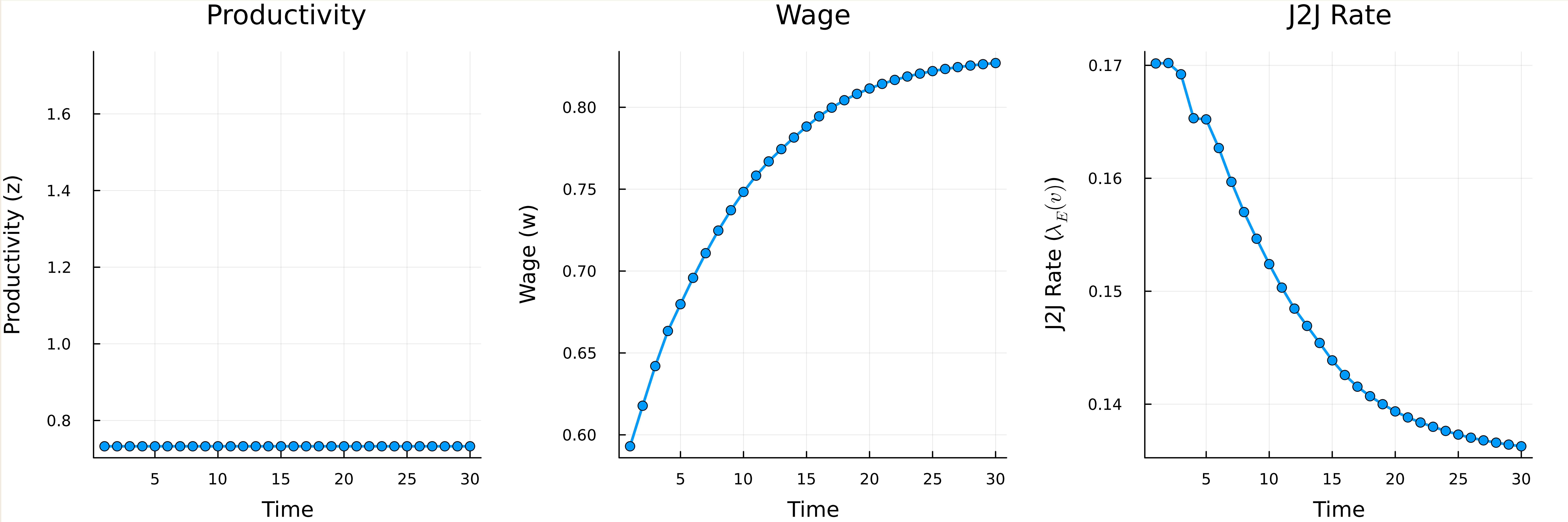
  1. Value and policy functions solve Bellman equations
  2. The free-entry conditions are satisfied
  3. Meeting rates are consistent with the matching function
- Realize that the definition does **not** involve employment distribution!
- This is the so-called **block recursive** property (Shi, 2005; Menzio & Shi, 2011):
  Value and policy functions are indepedent from the distribution.
  - Firms don't need to think about the distribution because of directed search
  - Workers don't need to think about the distribution because of free-entry

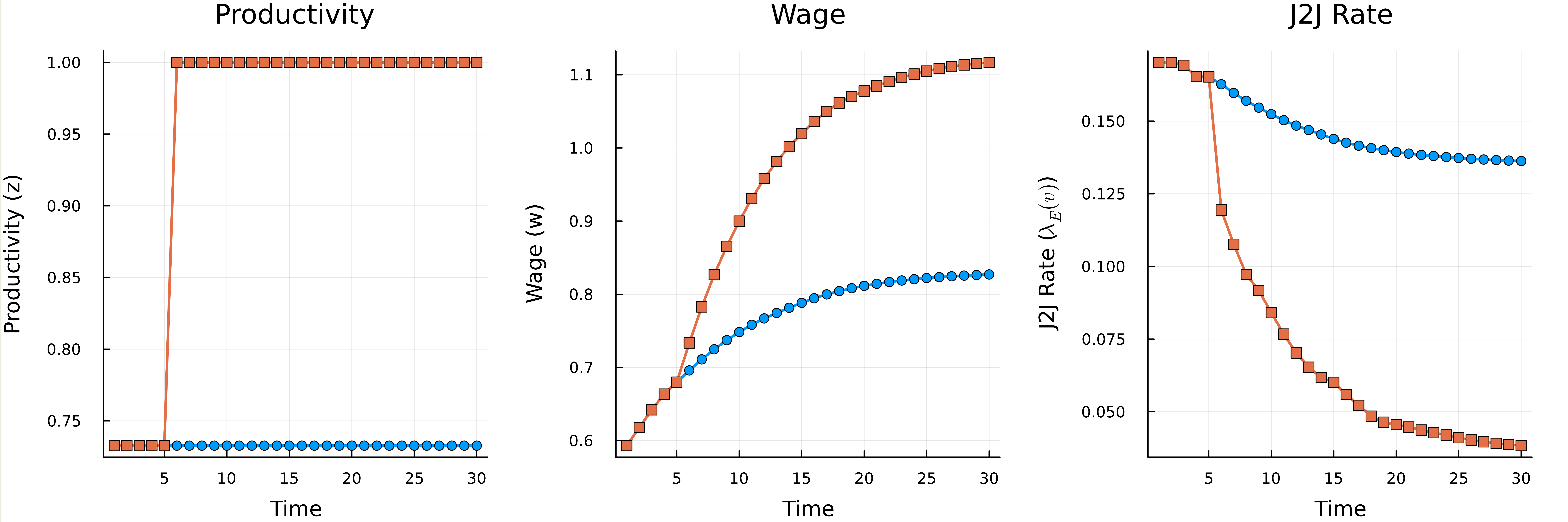
Computational Algorithm



Average Tenure Profile



Positive Shock to Firm Productivity



Negative Shock to Firm Productivity

Which Wage Setting Protocol?

1. Wage posting (Burdett-Mortensen, 1998)
2. Nash bargaining (including sequential auction)
3. Long-term wage contracts

3. Contract

2. Posting

Easy to use

1. Bargaining

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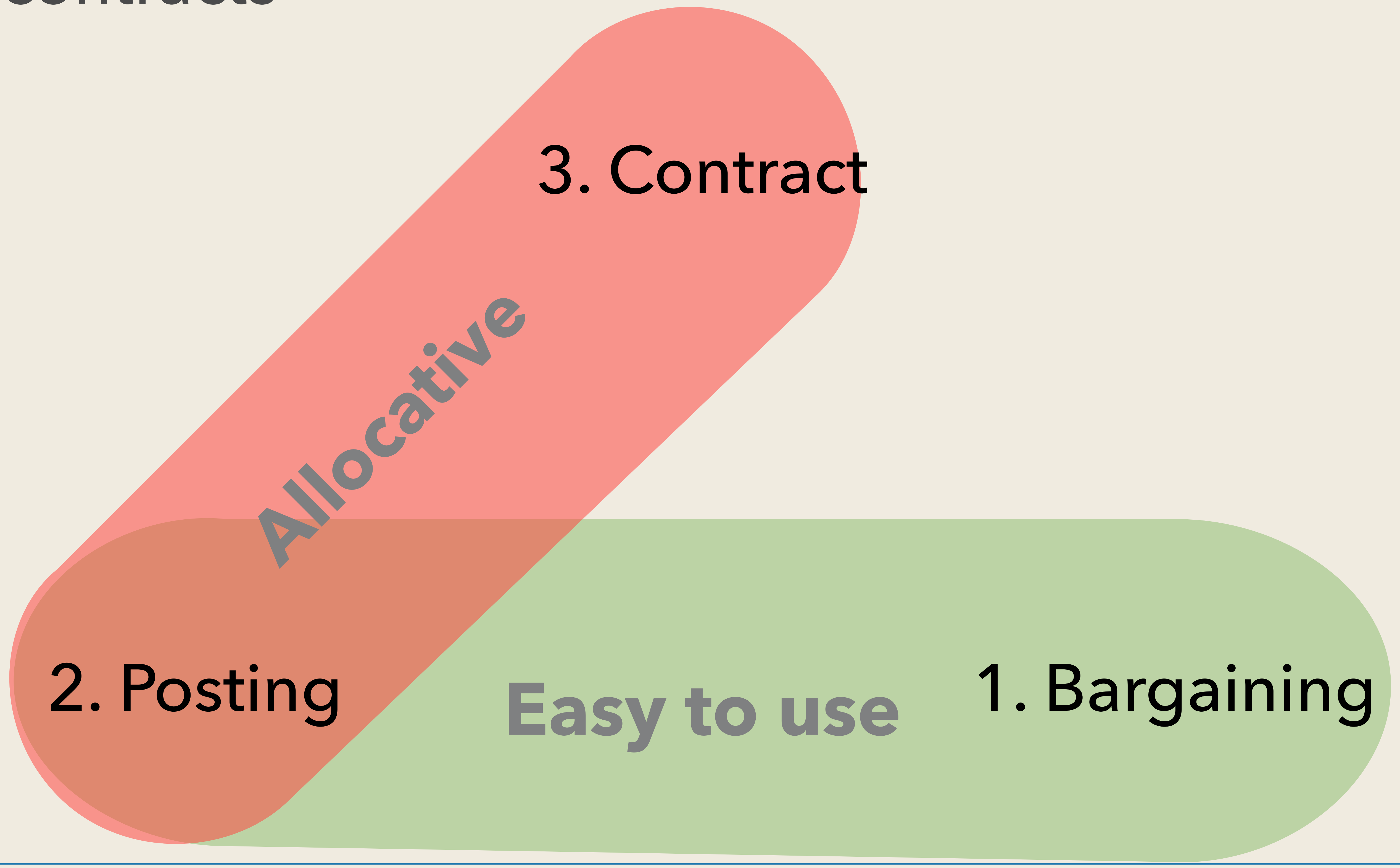

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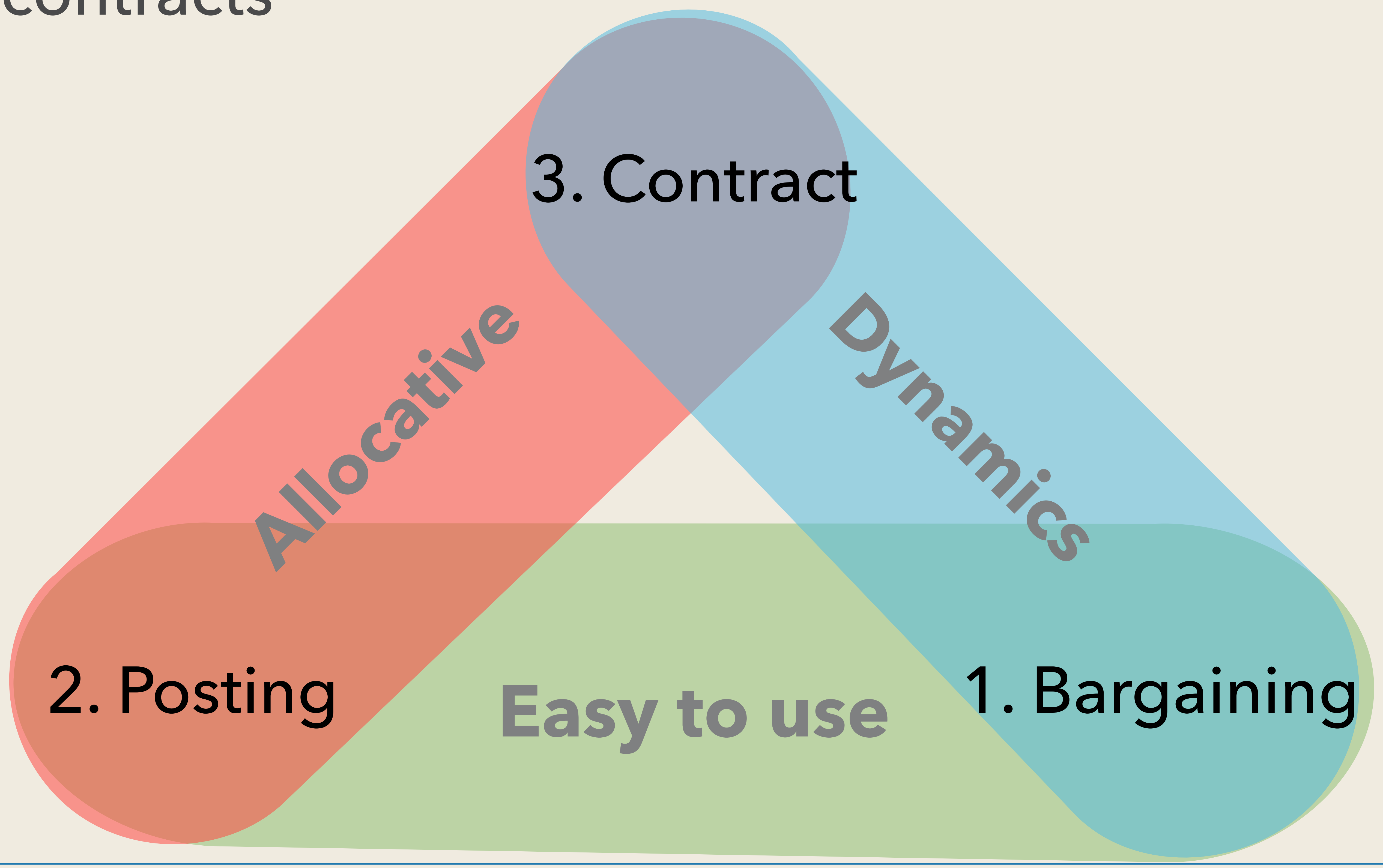

Open Questions

- Can we identify the wage-setting protocol from the data?
  - Long-term contracts imply endogenous persistence and dynamics
  - Beaudry & DiNardo (1991) deserve a modern treatment
- What are we still missing?
  - Typical contracts involve bonuses, benefits, overtime, severance, etc. Why?
  - Do firms really have commitment?
  - Are firms really risk-neutral? Are workers really hand-to-mouth?
  - Aren’t firms facing various constraints on wage setting? (e.g., fairness)

Appendix:
Useful Reformulation
for Numerical Implementation

Redifining State Space

- Define $\tilde{V} \equiv V - U, \tilde{\lambda}^E(\tilde{V}) \equiv \lambda^E(\tilde{V} + U) = \lambda^E(V)$

$$\tilde{\Pi}(\tilde{V}_t, z_t) = \max_{w_t, \{V_{t+1}(z_{t+1})\}} z_t - w_t + \beta(1 - \lambda^E(v))(1 - \delta)(1 - q)\mathbb{E}[\Pi(\tilde{V}_{t+1}(z_{t+1}), z_{t+1})]$$

$$\text{s.t.} \quad u(w) - (1 - \beta)U + \beta\left[\lambda^E(v)\tilde{v} + (1 - \lambda^E(v))\tilde{W}_{t+1}\right] \geq \tilde{V}_t$$

$$\tilde{v} \in \arg\max_{\hat{v}} \tilde{\lambda}^E(\hat{v})\hat{v} + (1 - \lambda^E(\tilde{v}))\tilde{W}_{t+1}$$

$$q = \mathbb{I}\left[\mathbb{E}[\tilde{V}_{t+1}(z_{t+1})] \geq 0\right]$$

$$\tilde{W}_{t+1} \equiv (1 - \delta)(1 - q)\mathbb{E}[\tilde{V}_{t+1}(z_{t+1})]$$

and

$$U = u(b) + \beta[\max_{\hat{v}} \tilde{\lambda}^U(\hat{v})\hat{v} + U]$$

Recursive Lagrangian

- The previous problem is computationally expensive:
  It involves optimizing over $\{V_{t+1}(z_{t+1})\}$, a high dimensional object

- Marcet-Marimon (2019) and Balke-Lamadon (2022) propose an elegant trick

- Define

$$\mathscr{P}(\rho, z) \equiv \max_{\tilde{V}} \tilde{\Pi}(\tilde{V}, z) + \rho \tilde{V}$$

  - Think of this as a Pareto problem with ρ being Pareto weight attached to workers

- We can recover the original value functions as

$$\partial_\rho \mathscr{P}(\rho, z) = \tilde{V}(\rho, z)$$

$$\tilde{\Pi}(\tilde{V}(\rho, z), z) = \mathscr{P}(\rho, z) - \rho \tilde{V}(\rho, z)$$

Recursive Lagrangian

- $\mathscr{P}(\rho, z)$ solve a (version of) Bellman equation:

$$\mathscr{P}(\rho, z) = \min_{\omega} \max_{w,\mathscr{V}\geq 0} z_t - w + \rho \left\{u(w) - (1-\beta)U + r(\mathscr{V})\right\}$$
$$-\beta p(\mathscr{V})\omega\mathscr{V} + \beta p(\mathscr{V})\mathbb{E}_t\left[\mathscr{P}(\omega, z_{t+1}) \,|\, z_t\right]$$

where

$$r(\mathscr{V}) \equiv \beta\left[W(\mathscr{V}) + \lambda^E(v(\mathscr{V}))(v(\mathscr{V}) - W(\mathscr{V}))\right]$$
$$W(\mathscr{V}) \equiv \left[\delta + (1-\delta)q(\mathscr{V})\right]U + (1-\delta)(1-q(\mathscr{V}))\mathscr{V}$$
$$p(\mathscr{V}) \equiv (1-\lambda^E(v(\mathscr{V})))(1-\delta)(1-q(\mathscr{V}))$$
$$v(\mathscr{V}) \in \arg\max_v \lambda^E(v)(v - W(\mathscr{V}))$$
$$q(\mathscr{V}) \equiv \begin{cases} 1 & \text{if } \mathscr{V} \leq 0 \\ 0 & \text{if } \mathscr{V} > 0 \end{cases}$$