
Structural Interpretation of AKM

Morchio & Moser (2025)

741 Macroeconomics
Topic 2

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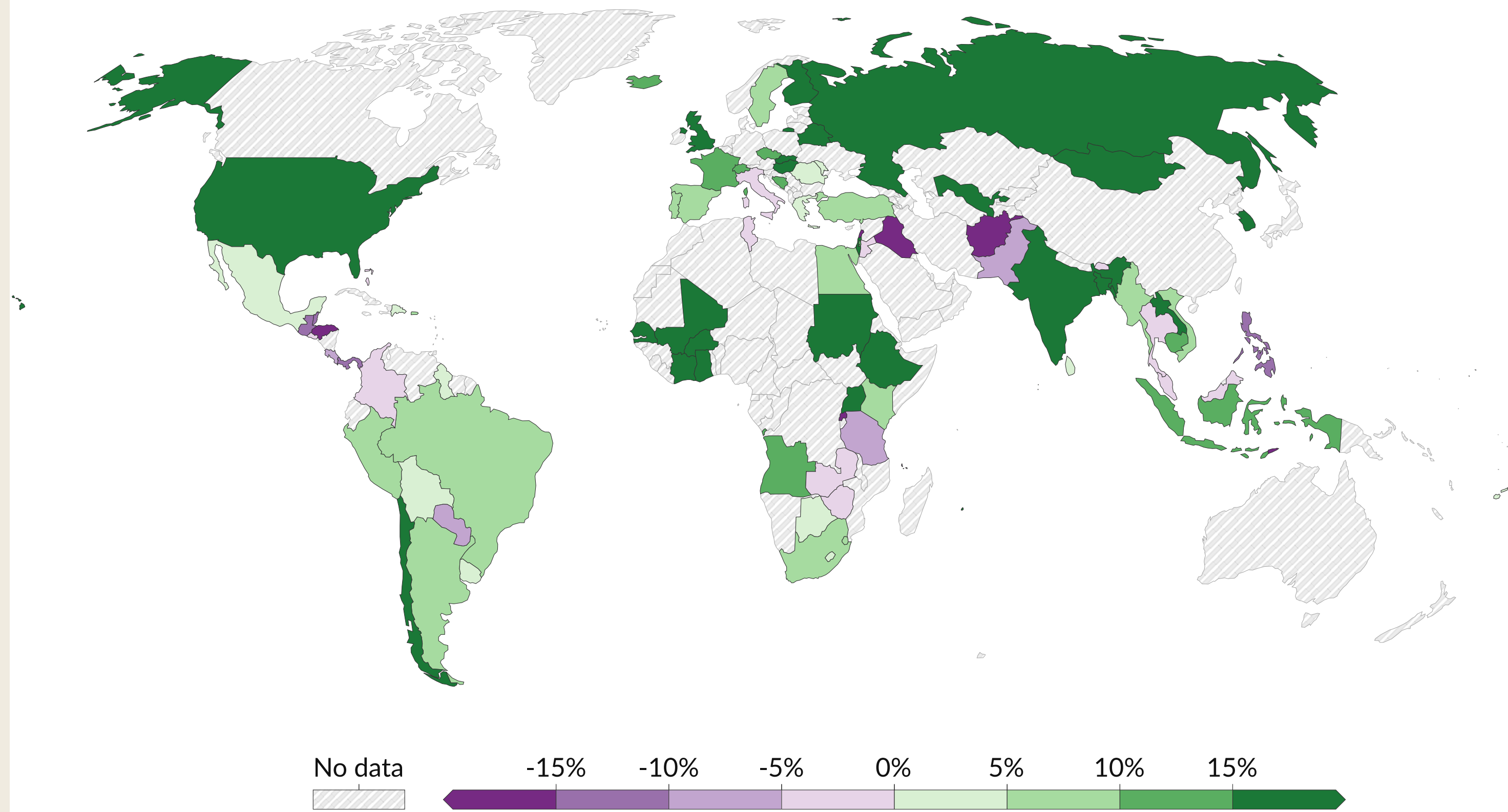
2025 fall

Gender Wage Gap

Unadjusted gender gap in average hourly wages, 2024

Our World
in Data

Gender wage gap, unadjusted for worker characteristics. Estimates correspond to the difference between average earnings of men and women, expressed as a percentage of average earnings of men.



Data source: International Labour Organization (2025)

OurWorldinData.org/economic-inequality-by-gender | CC BY

Note: The data corresponds to gross hourly earnings and includes both full-time and part-time workers.

Adjusted Gender Wage Gap

Female-to-male wage ratio in the US

Our World
in Data

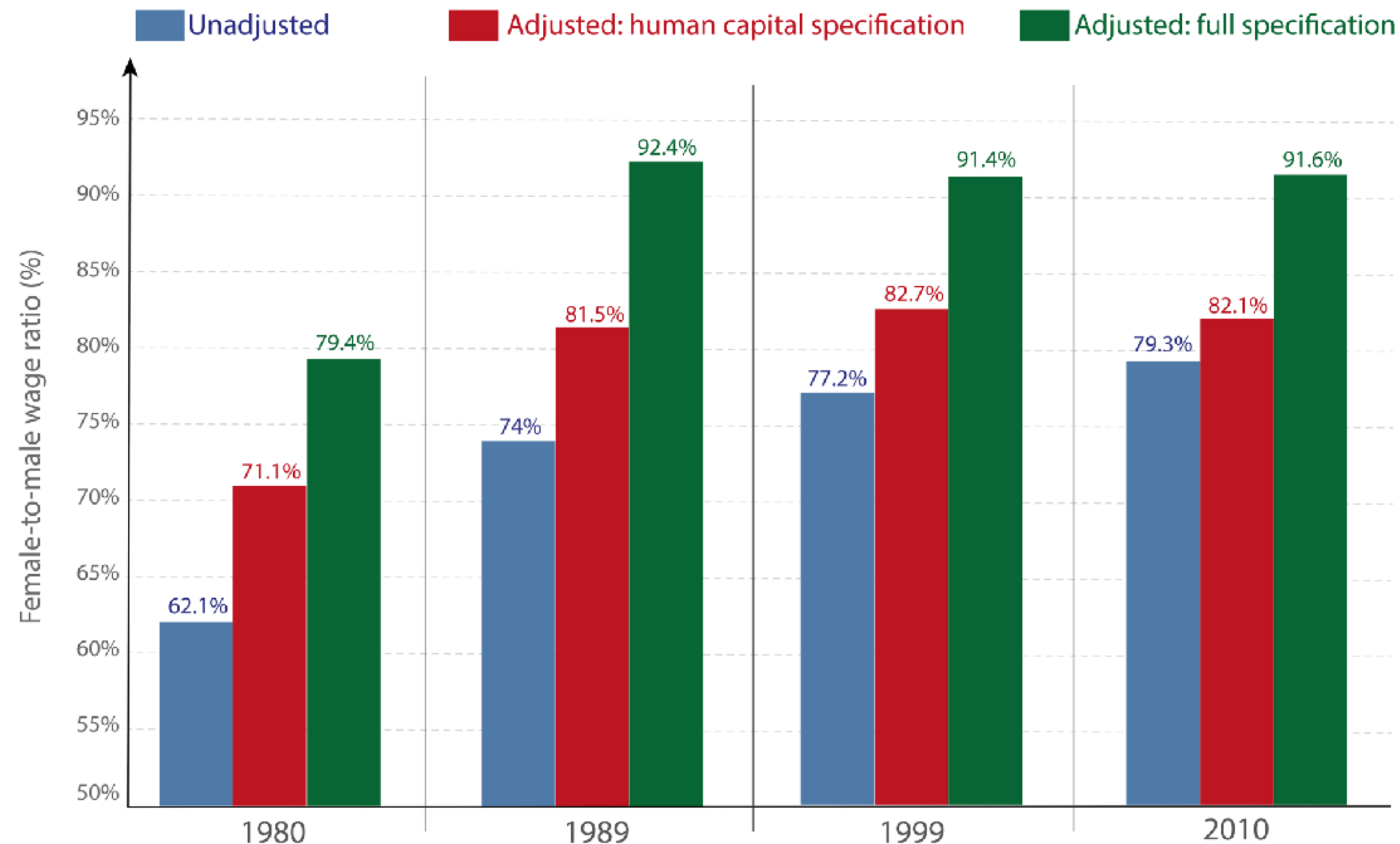
Shown is the evolution of the female to male wage ratios from the years 1980 to 2010 under three scenarios:

(i) Unadjusted for co-variates (blue);

(ii) Adjusted, controlling for gender differences in human capital, i.e. education and experience (red);

(iii) Adjusted, controlling for a full range of covariates, including human capital, occupation, region, race etc. (green).

The difference between 100% and the full specification (shown in green) is the “unexplained” residual.



What Explains the Gender Wage Gap?

- What is the role of firms in shaping the gender wage gap?
 - Do females sort into low-wage firms?
 - Do females receive lower wages than males within a firm?
- What drives these patterns?
 - compensating differential? discrimination? labor market friction?
- Three steps:
 1. Using Brazilian data, estimate AKM by gender
 2. Develop an equilibrium search model that exactly maps into AKM equations
 3. Estimate the model and conduct policy counterfactuals

Role of Firms in Gender Wage Gap

Data

- Brazilian employer-employee data covering all tax-registered employers
 - 2007-2014
 - Focus on age 18-54 and employers with enough mobility flows
 - "firm" = establishment
 - "wage" = monthly earnings

Summary Statistics

| | Overall | Men | Women |
|--------------------------------------------|----------------|----------------|----------------|
| Mean log real monthly earnings (std. dev.) | 7.211 (0.693) | 7.262 (0.697) | 7.129 (0.679) |
| Mean years of education (std. dev.) | 11.1 (3.3) | 10.4 (3.3) | 12.1 (2.9) |
| Mean years of age (std. dev.) | 33.6 (9.4) | 33.5 (9.4) | 33.8 (9.4) |
| Mean employer size (std. dev.) | 2,815 (16,418) | 1,774 (11,509) | 4,497 (22,059) |
| Mean contractual work hours (std. dev.) | 41.7 (5.1) | 42.6 (3.9) | 40.3 (6.4) |
| Mean years of tenure (std. dev.) | 3.9 (5.6) | 3.6 (5.2) | 4.5 (6.1) |
| Share Nonwhite | 0.378 | 0.409 | 0.327 |
| Share female | 0.382 | | |
| Mean log gender earnings gap | 0.133 | | |
| Number of worker-years | 267,318,328 | 165,149,632 | 102,168,696 |
| Number of unique workers | 56,297,308 | 33,761,656 | 22,535,652 |
| Number of unique employers | 607,029 | 403,585 | 203,444 |

AKM with Gender

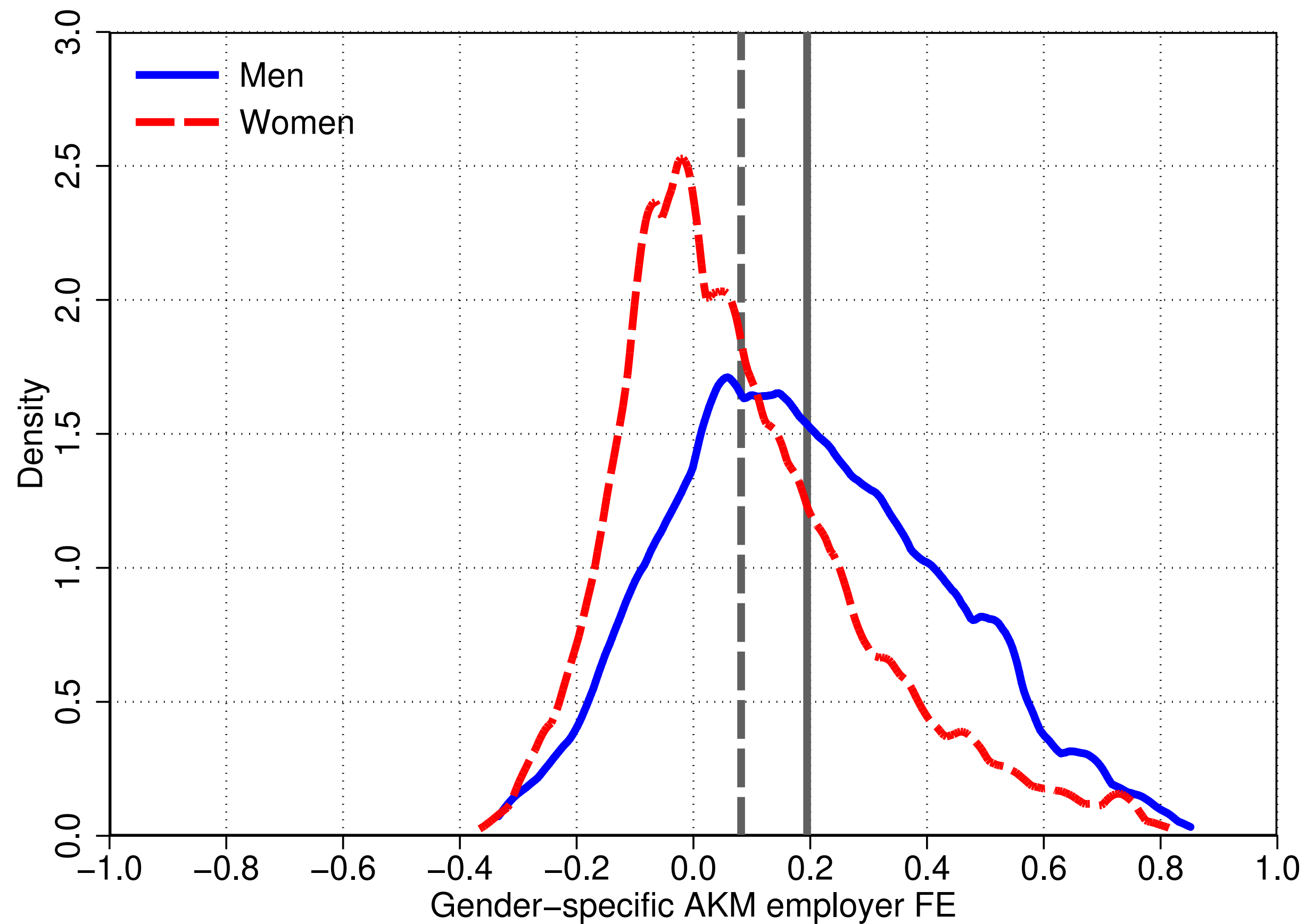
- Estimate AKM augmented with gender: (Card-Cardoso-Kline, 2016)

$$\ln w_{it} = \alpha_i + \psi_{G(i),j(i,t)} + X_{it}\beta_{G(i)} + \epsilon_{it}$$

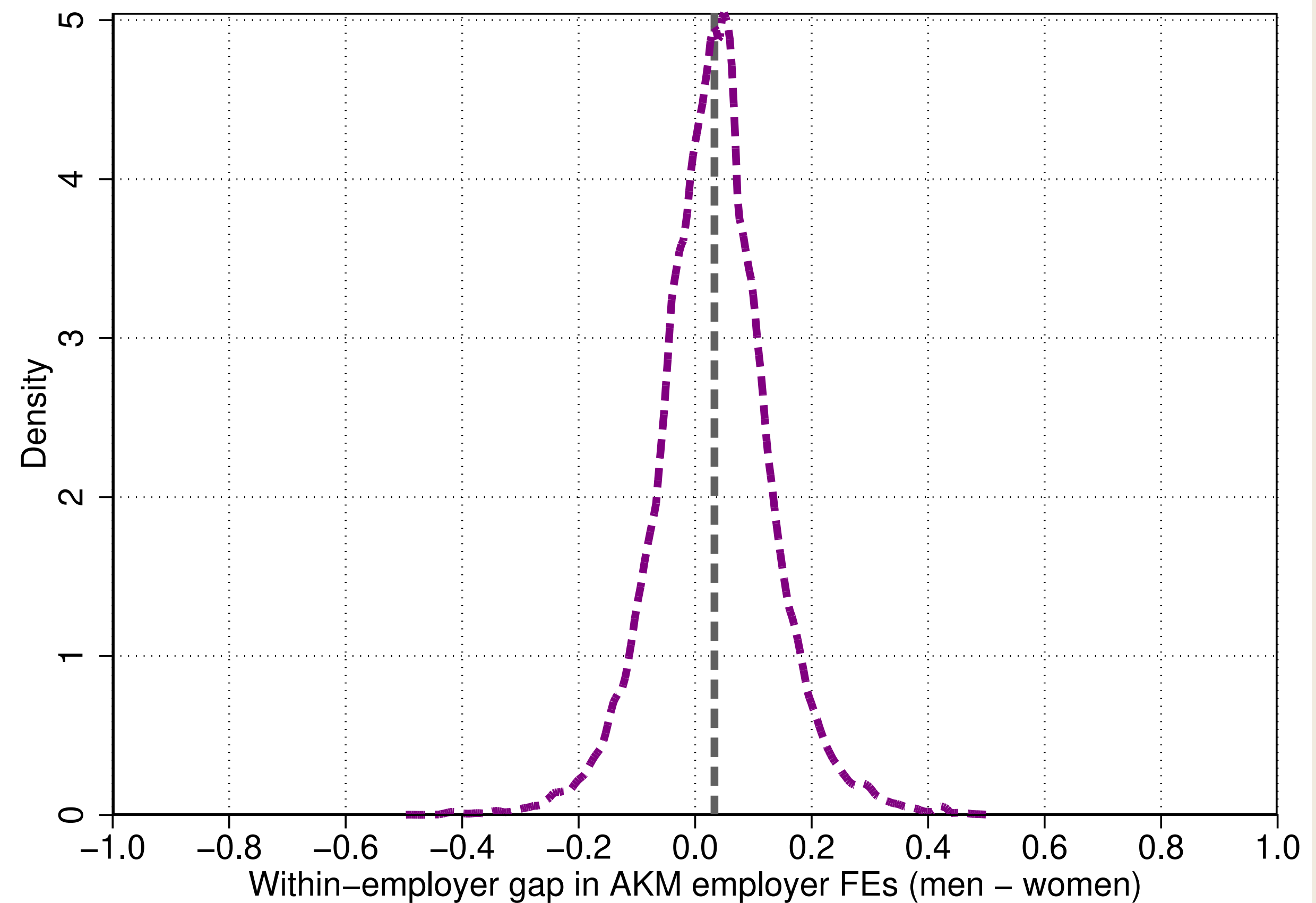
- $G(i)$: gender of worker i
- $\psi_{G,j}$: firm j 's wage effect of gender $G \in \{M, F\}$
 - Set $\psi_{M,j} = \psi_{F,j}$ for j near the bottom of the job-ladder in restaurant & fast-food
- X_{it} : occupation, education-year, age, hours, tenure, experience, etc

Firm FE by Gender

A. Gender-specific employer FE distributions



B. Distribution of within-employer FE differences



Men & Women Sort into Different Firms

| | Men | | Women | |
|--------------------------|---------------|---------------|---------------|---------------|
| | Plug-in | Leave-out | Plug-in | Leave-out |
| Variance of log earnings | 0.497 | 0.258 | 0.482 | 0.250 |
| Variance components: | | | | |
| Employer FEs (%) | 0.065 (13.0%) | 0.064 (25.0%) | 0.056 (11.5%) | 0.055 (22.2%) |
| Person FEs (%) | 0.116 (23.3%) | 0.097 (37.7%) | 0.117 (24.3%) | 0.099 (39.6%) |
| Correlation | 0.212 | 0.245 | 0.255 | 0.297 |
| R^2 | 0.921 | 0.777 | 0.929 | 0.793 |
| Mean employer FE | 0.197 | 0.197 | 0.081 | 0.081 |

- Difference in firm FE (≈ 0.11) accounts for 85% of gender wage gap!
- Decomposition:

$$\mathbb{E}[\psi_{Mj} | M] - \mathbb{E}[\psi_{Fj} | F] = \underbrace{\mathbb{E}[\psi_{Mj} | M] - \mathbb{E}[\psi_{Mj} | F]}_{\text{between} = 78.7\%} + \underbrace{\mathbb{E}[\psi_{Mj} - \psi_{Fj} | F]}_{\text{within} = 21.3\%}$$

Equilibrium Model of Wage, Amenity, and Sizes

Environment

An extension of Burdett-Mortensen model featuring

- Heterogeneous workers
- Heterogeneous firms
- Firms create jobs with endogenous wages and amenities
- Search friction dictates the matching of workers and jobs

Workers

- Infinitely lived, risk-neutral, and discount rate ρ
- Heterogeneous w.r.t. gender $g \in \{M, F\}$ and ability z with associated measure μ_{gz}
- Job search
 - voluntary job offers at rate: λ_{gz}^U for unemployed & $\lambda_{gz}^E \equiv s_g^E \lambda_{gz}^U$ for employed
 - involuntary job offers at rate $\lambda_{gz}^G \equiv s_g^G \lambda_{gz}^U$
 - exogenous separation at rate δ_g
- Job at firm j offers flow utility (fixed over time)

$$x = w + a$$

 - w : wage, a : workplace amenity
- Non-employed receive flow utility $x = b_g z$

Value Functions

- Employed (imposing rank-preserving property):

$$\begin{aligned} \rho S_{gz}(x) = & x + \lambda_{gz}^E \int_x^\infty [S_{gz}(x') - S_{gz}(x)] dF_{gz}(x') + \lambda_{gz}^G \int_{\underline{x}_{gz}}^\infty [S_{gz}(x') - S_{gz}(x)] dF_{gz}(x') \\ & + \delta_g [W_{gz} - S_{gz}(x)] \end{aligned}$$

- $F_{gz}(x)$: utility offer distribution (endogenous), \underline{x}_{gz} : reservation utility offer

- Nonemployed:

$$\rho W_{gz} = b_{gz} + (\lambda_{gz}^U + \lambda_{gz}^G) \int_{\underline{x}_{gz}}^\infty [S_{gz}(x') - W_{gz}] dF_{gz}(x')$$

- Nonemployed accepts the job offer with $x \geq \underline{x}_{gz}$, where \underline{x}_{gz} solves

$$\underline{x}_{gz} = b_{gz} + (\lambda_{gz}^U - \lambda_{gz}^E) \int_{\underline{x}_{gz}}^\infty \frac{1 - F_{gz}(x')}{\rho + \delta_g + \lambda_{gz}^G + \lambda_{gz}^E(1 - F_{gz}(x'))} dx'$$

(R-x)

Firms

- Firms differ in three dimensions

1. productivity p
2. gender wedge τ_g with $\tau_M \equiv 0$: τ_F is an implicit tax on women relative to men
3. amenity cost shifter $c_g^{a,0}$

- Production technology:

$$y_j(\{l_{gz}\}_{gz}) = p \sum_{g \in \{M, F\}} \int z l_{gz} dz$$

- Endogenous amenity provision with cost per worker

$$c_{gzj}^a(a) = c_{gj}^{a,0} \frac{(a/z)^{\eta^a}}{\eta^a} z$$

- Endogenous vacancy creation with cost

$$c_{gz}^v(v) = c_g^{v,0} \frac{(v/\mu_{gz})^{\eta^v}}{\eta^v} z \mu_{gz}$$

Firm Value Function

$$\rho \Pi_j(\{l_{gz}\}) = \max_{\{w_{gz}, a_{gz}, x_{gz}, v_{gz}\}} \sum_g \int \left\{ [(1 - \tau_{gj})p_j z - w_{gz} - c_{gzj}^a(a_{gz})]l_{gz} - c_{gz}^v(v_{gz}) + \partial_t l_{gz}(x_{gz}, v_{gz}) \partial_{l_{gz}} \Pi_j(\{l_{gz}\}) \right\} dz$$

$$\text{s.t.} \quad \partial_t l_{gz}(x, v) = - \left[\delta_g + \lambda_{gz}^G + \lambda_{gz}^E (1 - F_{gz}(x)) \right] l_{gz} + \frac{u_{gz} \lambda_{gz}^U + (1 - u_{gz}) \lambda_{gz}^E G_{gz}(x) + \lambda_{gz}^G}{u_{gz} \lambda_{gz}^U + (1 - u_{gz}) \lambda_{gz}^E + \lambda_{gz}^G} v q_{gz}$$

$$x_{gz} = w_{gz} + a_{gz}$$

- $G_{gz}(x)$: fraction of employed workers with flow utility below x
- From the stock-flow equation (see notes),

$$G_{gz}(x) = \frac{F_{gz}(x)}{1 + \frac{\lambda_{gz}^E}{\delta_g + \lambda_{gz}^G} (1 - F_{gz}(x))}$$

Matching

- The matching is segmented across (g, z) but random within (g, z)
- The number of matches in submarket (g, z) is given by

$$\mathcal{M}(U_{gz}, V_{gz}) = U_{gz}^\alpha V_{gz}^{1-\alpha}$$

where

$$U_{gz} = \mu_{gz}[u_{gz} + s_g^E(1 - u_{gz}) + s_g^G], \quad V_{gz} = \int v_{gz}(j) dj$$

- The meeting rates are given by

$$\lambda_{gz}^U = \theta_{gz}^\alpha, \quad \lambda_{gz}^E = s_g^E \lambda_{gz}^E, \quad \lambda_{gz}^G = s_g^G \lambda_{gz}^U, \quad q_{gz} = \theta_{gz}^{\alpha-1}$$

where $\theta_{gz} \equiv V_{gz}/U_{gz}$ denotes market tightness

Equilibrium Solution

Rewriting Firm's Problem

- Guess and verify the firm's value function takes the form:

$$\Pi_j(\{l_{gz}\}) = \sum_g \int \pi_{gzj}(l_{gz}) dz$$

- Rewrite the firm's subproblem for (g, z) worker as

$$\rho \pi_{gzj}(l_{gz}) = \max_{x,v} [\tilde{p}_{gzj} - x] l_{gz} - c_{gz}^v(v_{gz}) + \partial_t l_{gz}(x, v) \pi_j'(l_{gz})$$

where \tilde{p}_{gzj} is the composite productivity defined by

$$\begin{aligned} \tilde{p}_{gzj} &\equiv \max_a (1 - \tau_{gj}) p_j z + a - c_{gzj}^a(a) \\ &= \underbrace{\left\{ (1 - \tau_{gj}) p_j + (c_{gj}^0)^{\frac{1}{1-\eta^a}} [1 - 1/\eta^a] \right\}}_{\equiv \hat{p}_{gj}} z \end{aligned} \quad (\text{def-p})$$

Back to Burdett-Mortensen

- We look for an eqm where labor market objects are homogenous in z :

- $F_{gz}(x) = F_g(x/z), G_{gz}(x) = G_g(x/z), \lambda_{gz}^x = \lambda_g^x, q_{gz} = q_g, v_{gjz} = \hat{v}_{gj}\mu_{gz}$

- In such an equilibrium,

$$\pi_{gzj}(l_{gz}) = \hat{\pi}_{gj}(\hat{l}_g) z \mu_{gz}$$

where $l_{gz} = \hat{l}_g \mu_{gz}$ and $\pi_{gj}(\hat{l}_g)$ solves

$$\rho \hat{\pi}_{gj}(\hat{l}_g) = \max_{\hat{x}, \hat{v}} [\hat{p}_{gj} - \hat{x}] \hat{l}_g - c_g^v(\hat{v}_g) + \partial_t \hat{l}_g(\hat{x}, \hat{v}) \hat{\pi}'_{gj}(\hat{l}_g)$$

$$\text{with } \partial_t \hat{l}_g(\hat{x}, \hat{v}) \equiv - \left[\delta_g + \lambda_g^G + \lambda_g^E (1 - F_g(\hat{x})) \right] \hat{l}_{gz} + \frac{u_g \lambda_g^U + (1 - u_g) \lambda_g^E G_g(\hat{x}) + \lambda_g^G}{u_g \lambda_g^U + (1 - u_g) \lambda_g^E + \lambda_g^G} \hat{v} q_g$$

where $\hat{x} \equiv x/z, c_g^v(v) = c_0^v \hat{v}^{\eta^v} / \eta^v$

Property of Firm Policies

- For each submarket (g, z) ,
 1. Firms with higher composite productivity \hat{p}_{gj} offer higher flow utility x_{gjz}
 2. Firms with higher composite productivity \hat{p}_{gj} post more vacancies v_{gzj}
- Follows from Topkis' monotonicity theorem
- Firm employment size for each gender is increasing in \hat{p}_{gj}

Firm's Optimality

- First-order optimality conditions with respect to (\hat{x}, \hat{v}) :

$$\partial_{\hat{x}}[\partial_t \hat{l}_g(\hat{x}_{gj}, \hat{v}_{gj})] \hat{\pi}'_{gj}(\hat{l}_{gj}) = \hat{l}_{gj} \quad (\text{FOC-x})$$

$$\partial_{\hat{v}}[\partial_t \hat{l}_g(\hat{x}_{gj}, \hat{v}_{gj})] \hat{\pi}'_{gj}(\hat{l}_{gj}) = c_g^{v'}(\hat{v}_{gj}) \quad (\text{FOC-v})$$

which confirms \hat{v}_{gj} does not depend on z

- Envelope condition evaluated at the steady state (see notes):

$$\pi'_{gj}(\hat{l}_{gj}) = \frac{\hat{p}_{gj} - \hat{x}_{gj}}{\rho + \delta_g + \lambda_g^G + \lambda_g^E (1 - F_g(\hat{x}_{gj}))}.$$

- The steady state firm size (relative to μ_{gz}) is

$$\hat{l}_{gj} = \frac{1}{\left(\delta_g + \lambda_g^G + \lambda_g^E (1 - F_g(\hat{x}_{gj}))\right)^2} \frac{\left(u_g \lambda_g^U + \lambda_g^G\right)}{u_g \lambda_g^U + (1 - u_g) \lambda_g^E + \lambda_g^G} \left(\delta_g + \lambda_g^G + \lambda_g^E\right) \hat{v}_{gj} q_g$$

Equilibrium Utility Offer

- Combining the expressions, (FOC-x) can be rewritten as

$$(\hat{p} - \hat{x}_g(\hat{p})) \frac{2\lambda_g^E F'_g(\hat{x}_g(\hat{p}))}{\rho + \delta_g + \lambda_g^G + \lambda_g^E(1 - F_g(\hat{x}_g(\hat{p})))} = 1, \quad (\text{x-p})$$

$\hat{x}_g(\hat{p})$: normalized utility offer of a firm with normalized composite productivity \hat{p}

- By rank-preserving property,

$$F_g(\hat{x}_g(\hat{p})) = \int_{\underline{x}_{gz}/z}^{\hat{p}} \frac{v_g(\hat{p})}{V_g} \gamma(\hat{p}) d\hat{p} = \int_{\underline{\hat{x}}_{gz}}^{\hat{p}} \frac{v_g(\hat{p})}{V_g} \gamma(\hat{p}) d\hat{p} \equiv H_g(\hat{p})$$

- $\gamma(\hat{p})$: measure of firms with normalized productivity \hat{p}
- the second equality uses $\underline{x}_{gz} = \underline{\hat{x}}_g z$ which follows from (R-x) with our presumption

Equilibrium Utility Offer

- Differentiating both sides of $F_g(\hat{x}_g(\hat{p})) = H_g(\hat{p})$,

$$F'_g(\hat{x}_g(\hat{p}))\hat{x}'_g(\hat{p}) = H'_g(\hat{p})$$

- Plugging back to (x-p), we have a linear ODE in terms of $\hat{x}_g(\hat{p})$:

$$(\hat{p}_g - \hat{x}_g(\hat{p})) \frac{2\lambda_g^E H'_g(\hat{p})}{\rho + \delta_g + \lambda_g^G + \lambda_g^E(1 - H_g(\hat{p}))} = \hat{x}'_g(\hat{p}),$$

- With a boundary condition, $\hat{x}_g(\underline{\hat{x}}_g) = \underline{\hat{x}}_g$, the solution is

$$\hat{x}_g(\hat{p}) = \hat{p} - \int_{\underline{\hat{x}}_g}^{\hat{p}} \left[\frac{\rho + \delta_g + \lambda_g^G + \lambda_g^E(1 - H_g(\hat{p}'))}{\rho + \delta_g + \lambda_g^G + \lambda_g^E(1 - H_g(\hat{p}))} \right]^2 d\hat{p}' \quad (\text{ODE-x})$$

just as in EC704!

Endogenous Vacancy Distribution

- Unlike EC704, $H_g(\hat{p})$ is endogenous due to endogenous vacancy postings
- Using (FOC-v), vacancy posting of firm \hat{p} is given by

$$\hat{v}_g(\hat{p}) = \left[\frac{1}{c_0^v} \underbrace{\frac{\hat{p} - \hat{x}_g(\hat{p})}{\rho + \delta_g + \lambda_g^G + \lambda_g^E (1 - H_g(\hat{p}))}}_{\pi'_{gj}(\hat{l})} \underbrace{\frac{u_g \lambda_g^U + (1 - u_g) \lambda_g^E \frac{(\delta_g + \lambda_g^G) H_g(\hat{p})}{\lambda_g^E (1 - H_g(\hat{p})) + \delta_g + \lambda_g^G} + \lambda_g^G}{u_g \lambda_g^U + (1 - u_g) \lambda_g^E + \lambda_g^G} q_g}_{\partial_{\hat{v}}[\partial_t \hat{l}(\hat{x}, \hat{v})]} \right]^{\frac{1}{\eta^v - 1}}$$

- By definition,

$$H_g(\hat{p}) = \int_{\underline{\hat{x}}_g}^{\hat{p}} \frac{\hat{v}_g(\hat{p}')}{V_g} \gamma(\hat{p}') d\hat{p} \quad \Rightarrow \quad H'_g(\hat{p}) = \frac{\hat{v}_g(\hat{p})}{V_g} \gamma(\hat{p}) \quad (\text{ODE-v})$$

with boundary conditions $H_g(\underline{\hat{x}}_g) = 0$, $\lim_{\hat{p} \rightarrow \infty} H_g(\hat{p}) = 1$

Verifying Our Presumption

- We have already shown that $v_{gjz} = \hat{v}_{gj}\mu_{gz}$, $F_{gz}(x) = F_g(x/z)$, $G_{gz}(x) = G_g(x/z)$
- Finally,
 - vacancies in the submarket (g, z) , V_{gz} , scale with μ_{gz}
 - employed/unemployed workers acceptance prob. do not on z
 \Rightarrow nonemployment in submarket (g, z) , U_{gz} , scale with μ_{gz}

Consequently, $\theta_{gz} = \theta_g$, so that $\lambda_{gz}^x = \lambda_g^x$ (for $x \in \{U, E, G\}$) and $q_{gz} = q_g$

Computational Algorithm

Given (p_j, τ_j, c_{gj}^a) and their distribution,
construct $(\hat{p}, \gamma(\hat{p}))$ using (def-p)

Guess θ_g and compute $(\lambda_g^U, \lambda_g^E, \lambda_g^G, q_g)$ and u_g

Solve (ODE-x) and (ODE-v) to obtain
 $\{\hat{x}_g(\hat{p}), \hat{v}_g(\hat{p}), \hat{V}_g\}$

Compute U_g and thereby $\theta_g^{new} = V_g/U_g$

$\theta_g^{new} \approx \theta_g?$

Done!

Lower θ_g

no

Raise θ_g

yes

$\theta_g^{new} > \theta_g$

no

yes

Bringing the Model to the Data

Equilibrium Properties

1. Pay differences do not necessarily reflect utility differences because of amenity
2. The model generates job-to-job transitions with wage cuts through
 - compensating differential ($x' > x$ but $w' < w$)
 - involuntary job offer
3. Rich sources of the gender pay gap:
 - within- and between-firm gender pay gap through
 - differences in τ_{Fj} and a_{gzj}
 - gender-specific search frictions and vacancies (δ_g, λ_g)
 - monopsony
 - even a non-discriminatory firm treats women differently through eqm forces!

Structural AKM Equation

There exists an equilibrium in which wage of a worker (g, z) employed at firm j takes the form of

$$\ln w_{gzj} = \alpha_z + \psi_{gj}$$

where $\alpha_z = \ln z$, and

$$\psi_{gj} = \ln \left(\hat{p}_{gj} - (c_{gj}^0)^{\frac{1}{1-\eta^a}} - \int_{\hat{x}_g}^{\hat{p}_{gj}} \left[\frac{\rho + \delta_g + \lambda_g^G + \lambda_g^E(1 - H_g(\hat{p}_{gj}))}{\rho + \delta_g + \lambda_g^G + \lambda_g^E(1 - H_g(\hat{p}'))} \right]^2 d\hat{p}' \right)$$

- The model provides an exact map between AKM FEs and structural parameters!

Known Parameters

- Throughout, we assume the following parameters are known
 - discount rate, ρ (standard, annual 5.2%)
 - matching elasticity, α (standard, 0.5)
 - vacancy cost shifter $c_g^{0,v}$ (can be inferred from labor share)
 - vacancy cost elasticity η^v (can be inferred from the profit-vacancy relationship)
 - amenity cost elasticity η^a (can be inferred from cost share of amenities)

Identification Step 1: Ranking Firms

Step 1: Ranking firms using revealed preferences

- For each $g \in \{M, F\}$, firm size is increasing in normalized composite productivity \hat{p}
- Ranking of firm size \Rightarrow ranking of productivity $r \in [0,1]$
 - $\hat{p}_g(r)$: productivity of a firm with rank r , $G_g^r(r) \equiv G_g(\hat{x}(\hat{p}(r)))$, $H_g^r(r) \equiv H_g(\hat{p}(r))$
- Emp-weighted ranking $G_g^r(r) \Rightarrow$ vacancy-weighted ranking $H_g^r(r)$ using

$$G_g^r(r) = \frac{H_g^r(r)}{1 + \frac{\lambda_g^E}{\delta_g + \lambda_g^G}(1 - H_g^r(r))} \quad \dots \text{conditional on the knowledge of } \lambda_g^E, \delta_g, \lambda_g^G$$

- Vacancy-weighted ranking $H_g^r(r) \Rightarrow$ vacancy by ranking, $\hat{v}_g(r)/\hat{V}_g$

Identification Step 2: Labor Market Flows

Step 2: Identifying labor market flow parameters $(\delta_g, \lambda_g^U, \lambda_g^G, \lambda_g^E, \hat{V}_g)$

- EN rate $\Rightarrow \delta_g$
- EE rate that moves up and down the ranking $\Rightarrow \lambda_g^E, \lambda_g^G$
- NE rate $\Rightarrow \lambda_g^U + \lambda_g^G \Rightarrow \lambda_g^U$
- λ_g^U & $[u_{gz} + s_g^E(1 - u_{gz}) + s_g^G] \Rightarrow \hat{V}_g$

Identification Step 3: Firm-Level Parameters

Step 3: Identifying firm-level parameters $(\hat{x}_g(r), \hat{p}_g(r), a_g(r), c_g^{0,a}(r))$

- Use FOC w.r.t. \hat{v} to recover profitability of firm r , $\hat{p}_g(r) - \hat{x}_g(r)$:

$$\frac{\hat{p}_g(r) - \hat{x}_g(r)}{\rho + \delta_g + \lambda_g^G + \lambda_g^E(1 - H_g(r))} \frac{u_g + (1 - u_g)s_g^E G_g(r) + s_g^G}{u_g + (1 - u_g)s_g^E + s_g^G} q_g = c_g^{v'}(\hat{v}_g(r))$$

- Use FOC w.r.t. \hat{x} to recover utility offer of firm r , $\hat{x}_g(r)$, up to a constant:

$$(\hat{p}_g(r) - \hat{x}_g(r)) \frac{2\lambda_g^E H'_g(r)}{\rho + \delta_g + \lambda_g^G + \lambda_g^E(1 - H_g(r))} = \hat{x}'_g(r),$$

- An assumption about the scale of $\hat{x}_g(r) \Rightarrow \hat{x}_g(r)$ and $\hat{p}_g(r)$

- $\psi_g(r)$ from AKM regression + $\hat{x}_g(r) \Rightarrow a_g(r) = \hat{x}_g(r) - \psi_g(r) \Rightarrow c_g^{0,a}(r) = a_g(r)^{1-\eta_a}$

Identification Step 4: Discrimination

Step 4: Identifying discrimination and (non-composite) productivity $(\tau_g(r), p(r))$

- Since $\tau_M(r) = 0$, we have

$$\hat{p}_M(r) = p(r) + (c_M^0(r))^{\frac{1}{1-\eta^a}} [1 - 1/\eta^a]$$

⇒ can infer $p(r)$

- For women,

$$\hat{p}_F(r) = (1 - \tau_F(r)) p(r) + (c_F^0(r))^{\frac{1}{1-\eta^a}} [1 - 1/\eta^a]$$

⇒ can infer firm-level gender discrimination $\tau_F(r)$!

Estimation Results

Estimates of Labor Market Flows

| Parameter | Description | Men | Women |
|---------------|--------------------------------------------------------|-------|-------|
| μ_g | Population shares | 0.599 | 0.401 |
| λ_g^U | Offer arrival rate from nonemployment | 0.104 | 0.091 |
| δ_g | Job destruction rate | 0.035 | 0.028 |
| s_g^E | Relative arrival rate of voluntary on-the-job offers | 0.090 | 0.075 |
| s_g^G | Relative arrival rate of involuntary on-the-job offers | 0.101 | 0.081 |
| b_g | Flow value of nonemployment | 2.282 | 2.223 |

Correlates of Gender Discrimination

$$\ln(1 - \tau_{Fj}) = \gamma' X_j + \epsilon_j$$

| | Coefficient | (std. err.) |
|-----------------------------------------|-------------|-------------|
| Female manager | 0.006*** | (0.002) |
| Nonroutine manual task intensity | −0.001 | (0.007) |
| Nonroutine interpersonal task intensity | −0.002 | (0.006) |
| Mean working hours | −0.010*** | (0.004) |
| No major financial stakeholders | −0.010*** | (0.002) |
| Log size | −0.155*** | (0.007) |
| R^2 | 0.632 | |
| Within- R^2 | 0.089 | |

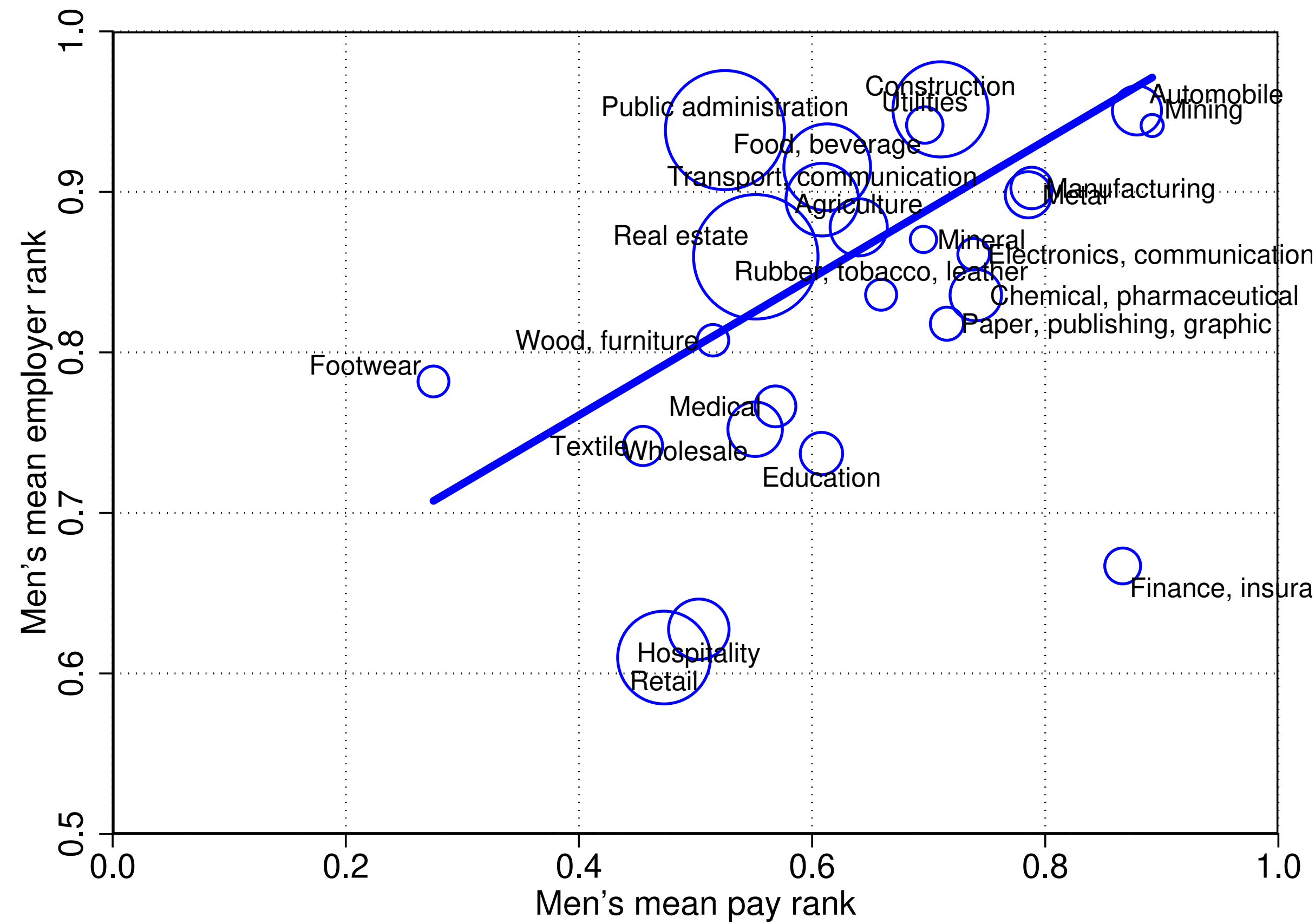
Correlates of Amenity

$$\ln a_{gj} = \gamma'_g X_{gj} + \epsilon_{gj}$$

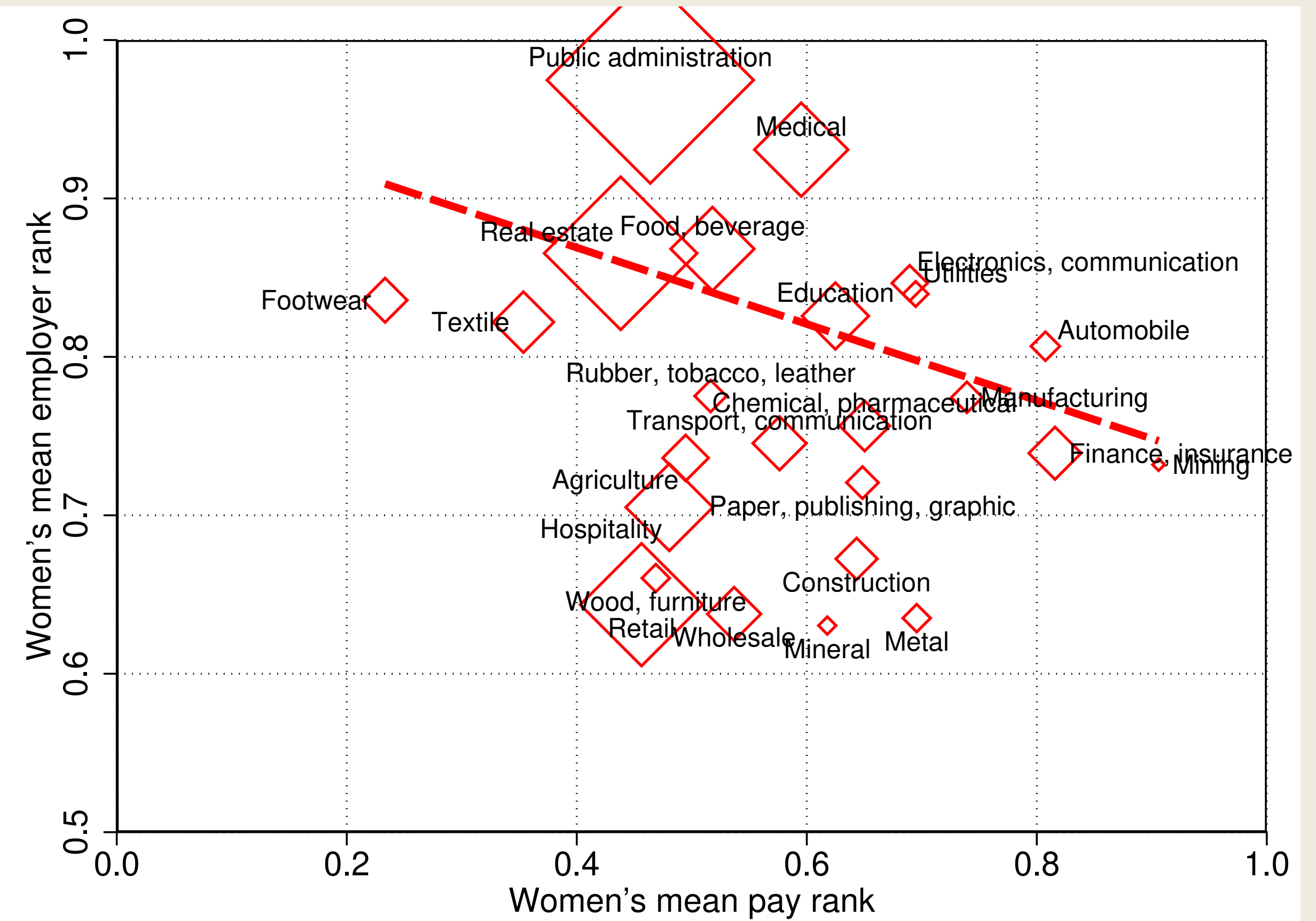
| | Men | | Women | |
|-------------------------------|-------------|-------------|-------------|-------------|
| | Coefficient | (std. err.) | Coefficient | (std. err.) |
| Part-time work incidence | −0.006 | (0.012) | 0.010 | (0.007) |
| Working hours flexibility | 0.008 | (0.013) | 0.020*** | (0.006) |
| Parental leave generosity | 0.093*** | (0.024) | 0.023*** | (0.007) |
| Income fluctuations | −0.034 | (0.032) | −0.002 | (0.007) |
| Workplace hazards | 0.016 | (0.015) | −0.002 | (0.005) |
| Incidence of unjust firings | −0.028** | (0.014) | −0.020** | (0.009) |
| Incidence of workplace deaths | −0.034*** | (0.011) | −0.047*** | (0.010) |
| Log size | 0.201*** | (0.018) | 0.139*** | (0.021) |
| R^2 | 0.704 | | 0.440 | |
| Within- R^2 | 0.238 | | 0.090 | |

Firm Rank vs. Pay Rank

A. Men

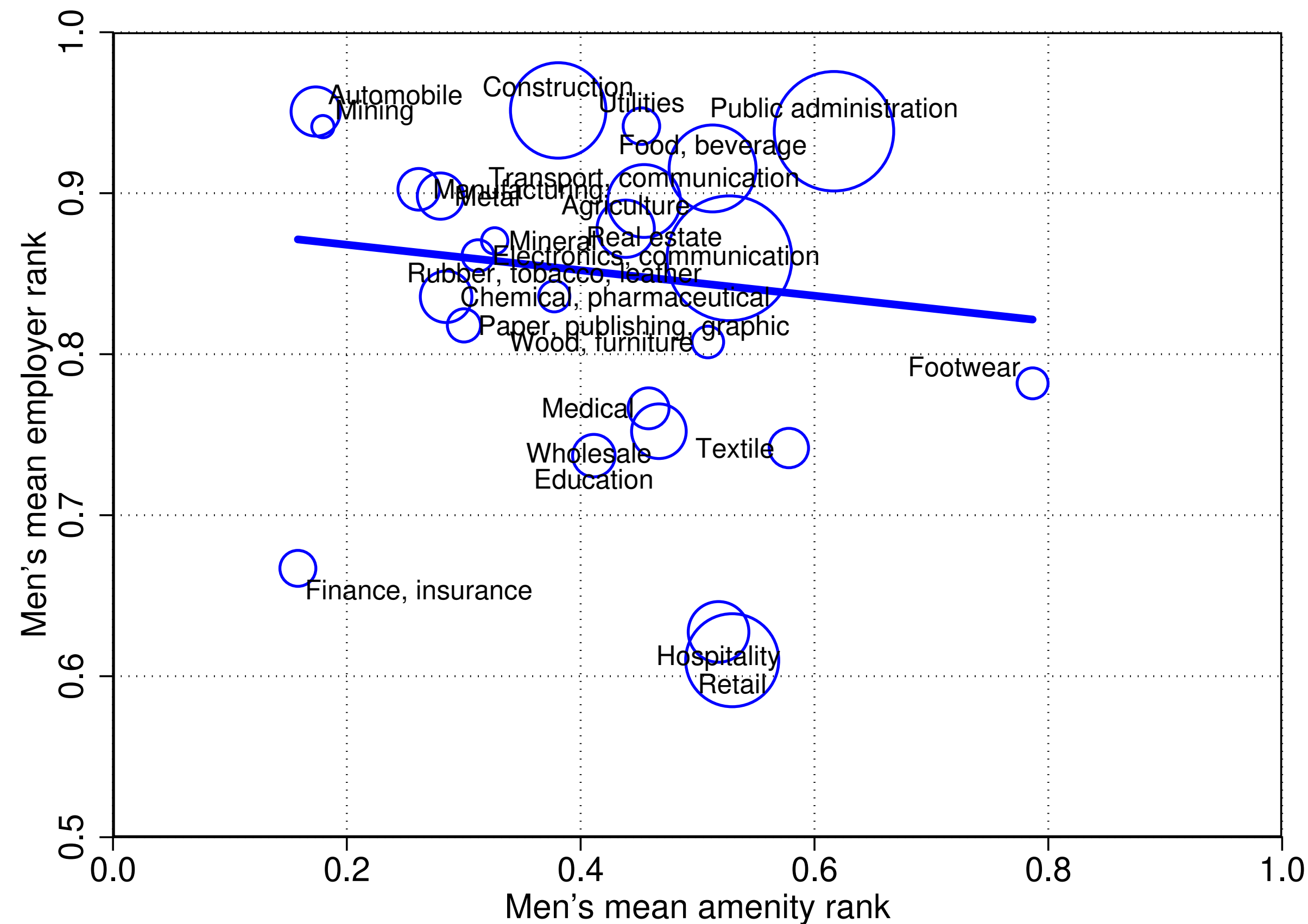


B. Women

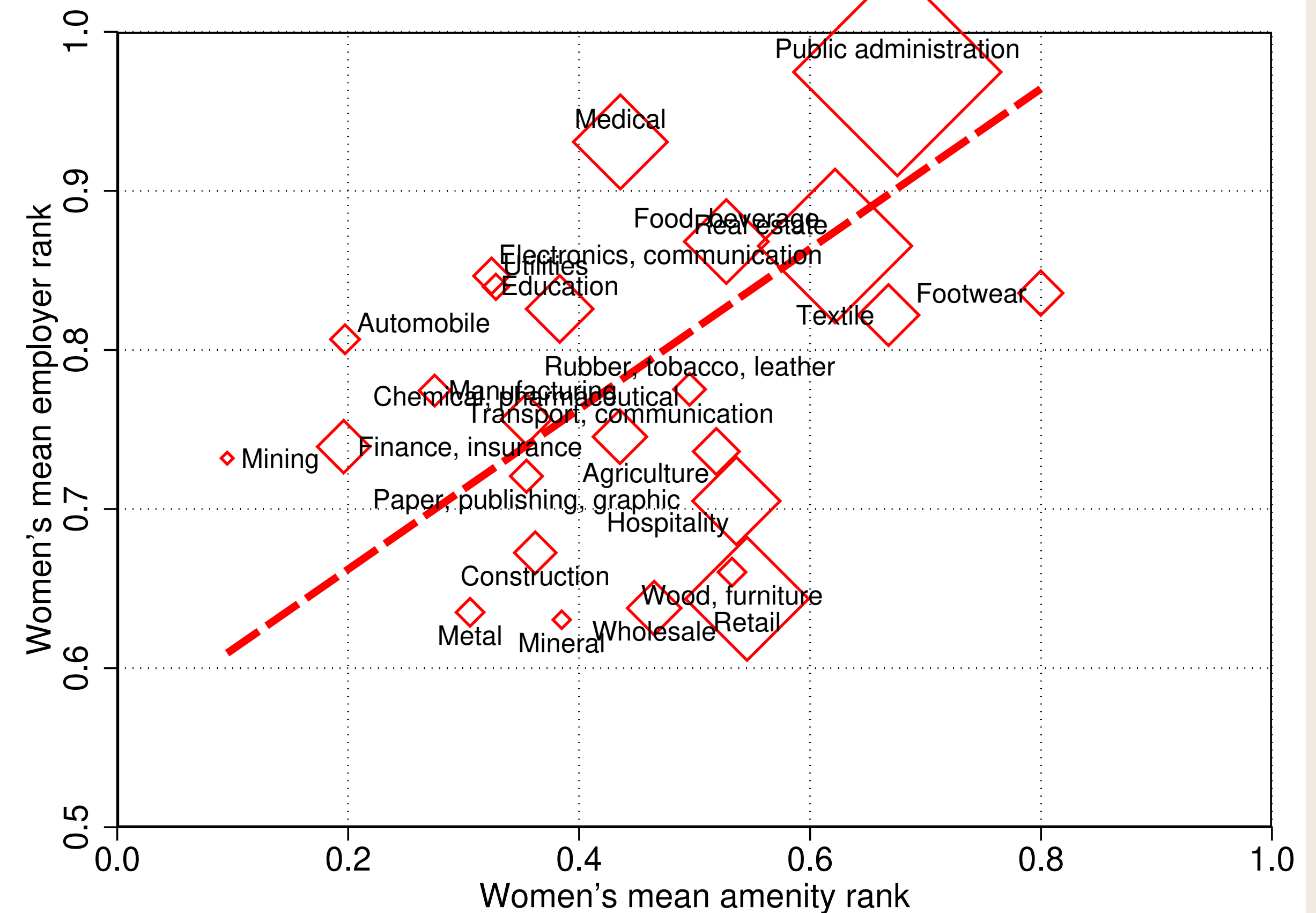


Firm Rank vs. Amenity Rank

A. Men

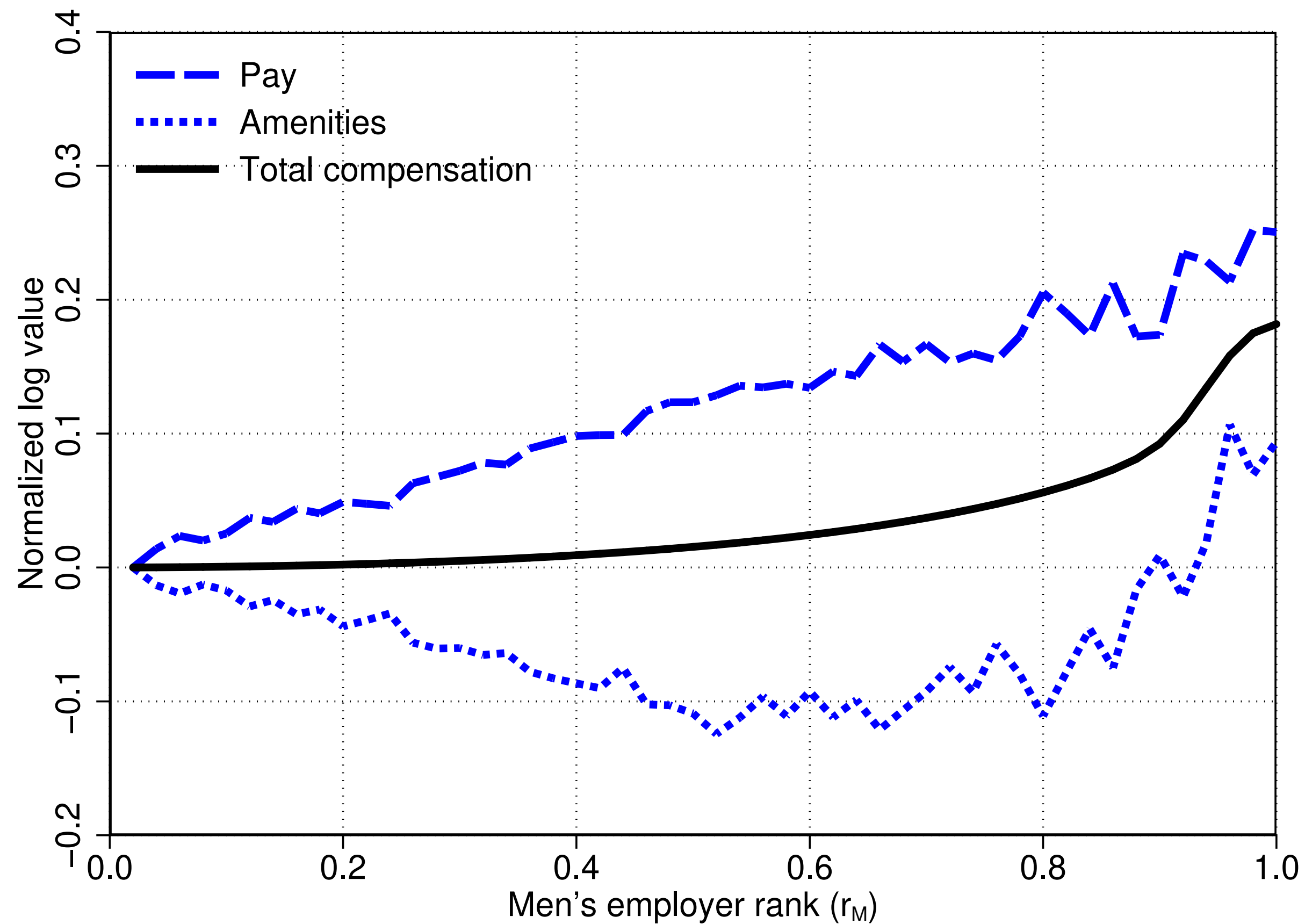


B. Women

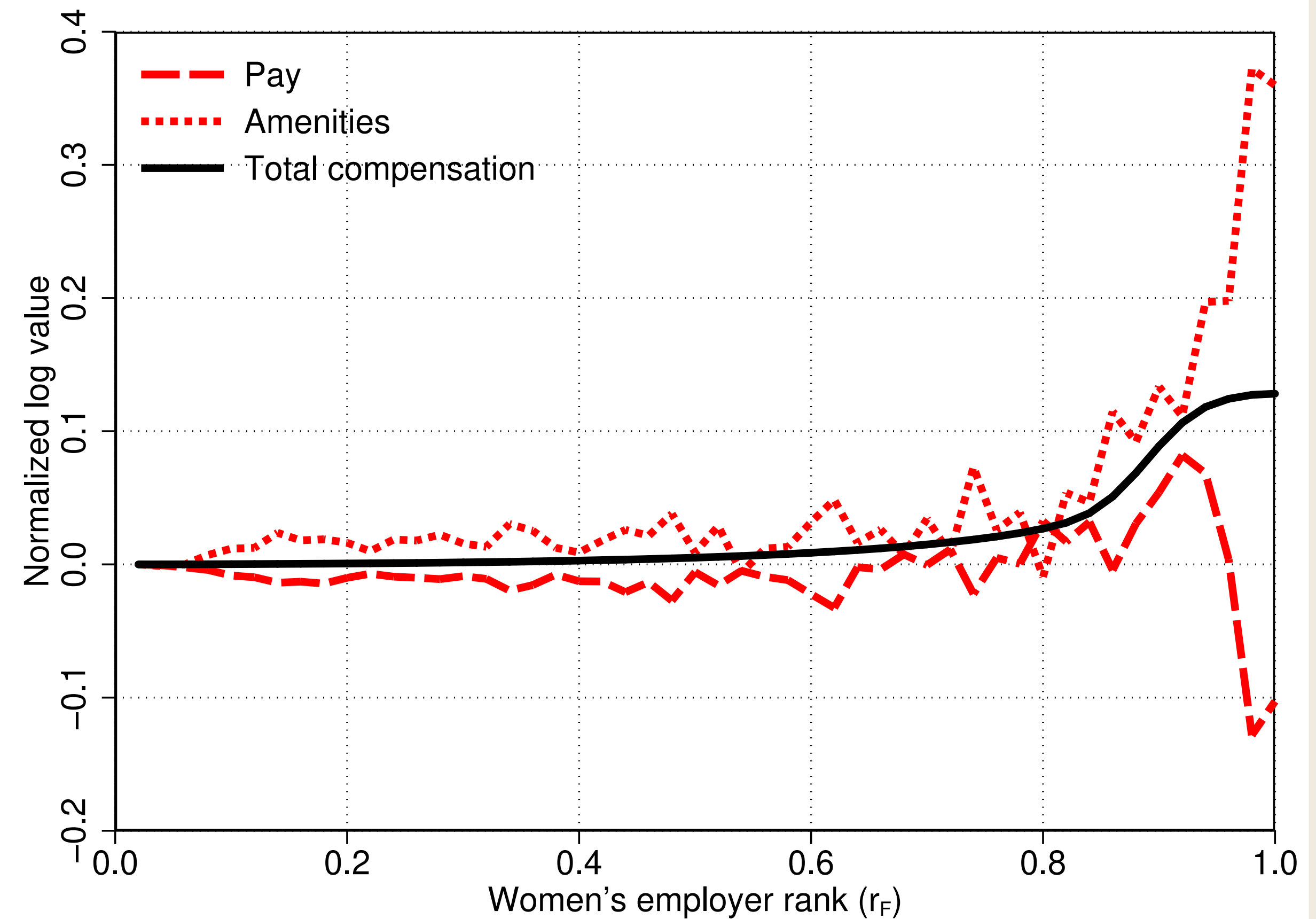


Pay & Amenity by Firm Rank

A. Men



B. Women



Decomposing Pay Inequality

| Variances | Men | | Women | |
|--------------------------------------------------|-------|-----------|--------|-----------|
| | Level | Share (%) | Level | Share (%) |
| Variance of log pay | 0.054 | | 0.044 | |
| Variance components of log pay: | | | | |
| Log utility | 0.002 | 4.4 | 0.002 | 3.6 |
| Log amenities | 0.051 | 94.3 | 0.045 | 102.8 |
| Covariance between log utility and log amenities | 0.001 | 1.3 | −0.003 | −6.4 |
| Covariance components of log pay: | | | | |
| Covariance between log utility and log pay | 0.003 | 5.1 | 0.000 | 0.4 |
| Covariance between log amenities and log pay | 0.052 | 94.9 | 0.044 | 99.6 |

Decomposing Gender Wage Gap

| | Gender gap | Between-employer gap | | Within-employer gap | |
|--------------------|------------|----------------------|-----------|---------------------|-----------|
| | | Level | Share (%) | Level | Share (%) |
| Pay | 0.113 | 0.089 | 78.7 | 0.024 | 21.3 |
| Amenity-valuation | −0.067 | −0.087 | 130.0 | 0.020 | −30.0 |
| Total compensation | 0.046 | 0.002 | 4.6 | 0.044 | 95.4 |

Counterfactuals

Equal-Treatment Policies

| | Baseline (0) | Equal-pay policy (1) | Equal-hiring policy (2) |
|--------------------------|-----------------|-------------------------|----------------------------|
| Gender log pay gap | 0.109 | 0.028 | 0.034 |
| between employers | 0.082 | 0.028 | 0.006 |
| within employers | 0.027 | 0.000 | 0.028 |
| Gender log amenities gap | −0.066 | 0.003 | 0.011 |
| between employers | −0.075 | −0.027 | −0.006 |
| within employers | 0.009 | 0.030 | 0.017 |
| Gender log utility gap | 0.042 | 0.031 | 0.045 |
| between employers | 0.007 | 0.000 | 0.000 |
| within employers | 0.035 | 0.030 | 0.045 |
| Output | 1.000 | 0.986 | 0.997 |
| Worker welfare | 1.000 | 0.996 | 0.992 |
| for men | 1.000 | 0.996 | 0.991 |
| for women | 1.000 | 0.996 | 0.993 |
| Total employment | 0.771 | 0.763 | 0.764 |
| for men | 0.764 | 0.760 | 0.722 |
| for women | 0.781 | 0.767 | 0.825 |

What's Next?

Error Term in AKM?

- Beautiful framework that bridges empirics and theory
 - should have many applications beyond the gender wage gap
- One of the first to give an exact meaning to AKM equation:

$$\ln w_{it} = \alpha_i + \psi_{gj(i,t)}$$

- At the same time, the model predicts there is no error term ϵ_{gzj}
- What is the error term ϵ_{gzj} we always see in the data? – a question we tackle next