The Impact of Central Bank Stock Purchases:

Evidence from Discontinuities in Policy Rules*

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Abstract

We trace the impact of central bank stock purchases by exploiting discontinuity in the Bank of Japan's policy rule, which triggers purchases when stock market index movements in the morning session fall below a certain threshold. In normal times, the purchases persistently raise the long-term interest rate while leaving no detectable impact on stock prices. After the introduction of yield curve control by the Bank of Japan, which pegs the long-term interest rate to 0%, interest rates stopped responding and stock prices sharply and persistently rise following the purchases. A model with an inelastic stock market and an even more inelastic bond market is fully consistent with these empirical findings.

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1 Introduction

How does a financial flow into the stock market impact asset prices? While textbook models predict almost no effect, the inelastic stock market hypothesis recently proposed by Gabaix and Koijen (2021) argues for a large effect on stock prices. Answering this question goes a long way toward understanding the source of asset price volatility and the effectiveness of central bank asset purchase programs. In fact, as a new form of "quantitative easing," the Bank of Japan (henceforth BoJ) started to purchase stocks in 2010, becoming the largest owner of Japanese stocks worldwide by 2021, as shown in Figure 1. The BoJ explains that the primary goal of this extreme form of quantitative easing is to "reduce the risk premium," but its effectiveness is often subject to policy debates.

The reason why this question is difficult to answer is the endogeneity of financial flows. No hedge fund randomizes when trading financial assets, and the central bank intervenes in the financial market for a reason. For example, the hedge fund might put money into the stock market when it believes the stock market will perform well, or the BoJ might purchase stocks when the stock market performs poorly. Using these kinds of variations in financial flows will give misleading answers to the question because of reverse causality.

In this paper, we address this challenge by exploiting discontinuities in the BoJ's policy rules. Although the BoJ has never made it public, it is widely known that the BoJ tends to purchase stocks precisely on the day when the changes in the stock market index in the morning session fall below a certain threshold. By comparing days when the movements in a stock market index fall slightly below the threshold and slightly above, the discontinuous increase in inflow into the stock market can be viewed as orthogonal to the underlying economic fundamentals. A difficulty in implementing the standard regression discontinuity design in our setup is that the threshold is not necessarily known, as the BoJ has never made the policy rule public. We overcome this problem by estimating the threshold relying on the econometric literature on regression discontinuity design with unknown cutoffs (Porter and Yu, 2015). Indeed, we find a striking discontinuity in the likelihood of the intervention around the estimated cutoffs.

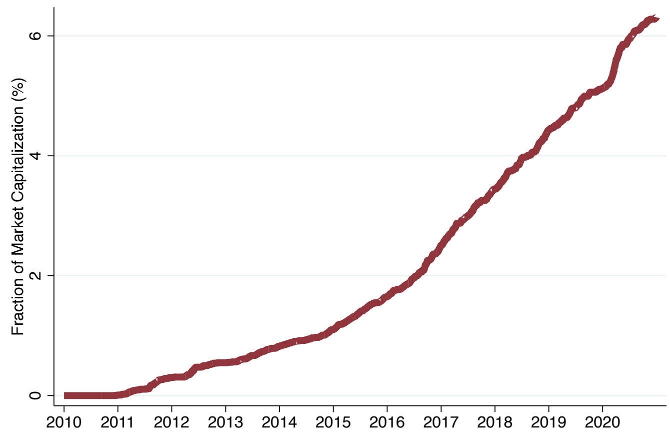

Figure 1: Cumulative ETF Purchases by the BoJ

Notes: Figure 1 plots the cumulative amount of ETF purchases by the BoJ from 2010 to 2020 as a fraction of market capitalization.

Exploiting the discontinuity in the policy rules, we first show that flows into stock markets have a large causal impact on *both* the stock prices and the long-term government bond interest rates. In response to an average size of intervention, which amounts to 0.01% of stock market capitalization, our estimates indicate that the stock prices rise by around 0.1-0.4% following the intervention. Perhaps more surprisingly, the 10-year Japanese government bond (JGB) yield also rises by around 0.5 basis points (b.p.). Although the standard errors increase, these results appear to be persistent, lasting at least several days.

We then argue that the above results mask a strong underlying heterogeneity that depends on the presence of another unconventional monetary policy, yield curve control (YCC). In the middle of 2016, the BoJ pegged the 10-year JGB yield at around 0%. Since then, long-term interest rates have stabilized at around 0%. Given this, it is natural to expect the response of long-term interest rates to the financial flows to be different before and after the introduction of YCC, and we find that this is indeed the case.

Specifically, we find that before the introduction of YCC, in response to the BoJ's stock market purchases, the long-term interest rates rise sharply and persistently following the intervention, while leaving virtually no detectable impact on the stock price. Quantitatively, we find that long-term interest rates rise persistently by 1.0-1.5 b.p. in response to the average size of the intervention. In contrast, we cannot reject the null that the BoJ's stock purchases left no impact on stock prices in the following days, although the standard errors are large.

After the introduction of the YCC, we find that the long-term interest rate entirely stopped responding, and instead, stock prices rise sharply and persistently in response to the BoJ's stock purchases. The effect on the long-term interest rate is precisely estimated at zero. The stock price responds by around 0.2-0.4% in response to the typical size of the intervention, which persists for several days after the intervention.

Our empirical results provide portable statistics that would serve as useful in disciplining theoretical models that incorporate inelastic financial markets. In a nutshell, when interest rates flexibly adjust, a purchase of 1% of stock market capitalization raises the long-term interest rate by around 1 percentage point but leaves no significant impact on stock prices. Whenever the interest rates are fixed, the same flow raises the stock price by around 20%. This number is four times higher than the estimates in Gabaix and Koijen (2021), suggesting that the stock market may be more inelastic than previously thought.

In the final part of the paper, we present a model with inelastic financial markets that is fully consistent with our empirical results. There are two assets in the model: stocks and bonds, which refer to all money-like assets that are liquid and risk-free.[1] Our model nests a case where only the stock market is inelastic, as in Gabaix and Koijen (2021). In this simple case, a central bank stock purchase always raises the stock prices while leaving the bond interest rate unchanged. This is because the bond market is perfectly elastic, where households' Euler equation pins down the interest rate irrespective of the financial flows. Consequently, this simple case fails to account for the response of interest rates to financial flows that we see in the data.

To account for our empirical findings, we consider a general case where the

[1]In the model, we do not make a distinction between money and bonds and call them bonds. This refers to all the money-like assets that are liquid and risk-free. This is a valid assumption if the elasticity of substitution between near-money assets like bonds and money is high, which is empirically the case (Nagel, 2016; Krishnamurthy and Li, 2023).

bond market is also inelastic. Households have a downward-sloping demand for the liquidity services that bonds provide, which breaks the neutrality of financial flows in the interest rate determination. In this environment, a flow from bonds to stocks not only puts upward pressure on stock prices but also on bond interest rates (an inverse of the bond price). The latter effect attenuates or can even reverse the rise in stock prices resulting from the inflow into the stock market. We structurally estimate parameters that govern the inelasticity in financial markets to fit our empirical evidence. Our parameter estimates indicate substantial inelasticity in the stock market but even more inelasticity in the bond market. As a result, when interest rates flexibly adjust, an inflow into the stock raises the interest rate and negatively impacts stock prices. In the presence of yield curve control, which we model as the central bank adjusting the supply of bonds to fix the interest rate, the only consequence of a flow from bonds to stocks is a substantial increase in stock prices, consistent with our empirical evidence.

Collectively, we provide new evidence and theory of the inelasticity in both bond and stock markets. Our results are crucial in understanding the consequences of new forms of unconventional monetary policies – stock purchase programs. Since most central banks purchase only bonds and do not intervene in the stock market, assessing the potential for such a policy option has been difficult. Our results suggest the policy has far-reaching impacts on the financial markets. In particular, the result shows that interventions in the stock market have effects on prices beyond the stock market – the yield curve may react in important ways.

Related Literature

We build most directly on the pioneering work by Gabaix and Koijen (2021) in assessing how a flow from bonds to stocks impacts the financial market. Gabaix and Koijen (2021) and a growing number of studies (e.g., Da et al., 2018; Hartzmark and Solomon, 2021; Li et al., 2021) estimate how the flows from bonds to stocks affect stock prices using various identification strategies. Our contributions to this literature are twofold. First, we argue that it is important to consider jointly how bond prices are affected by such flows both from empirical and theoretical perspectives. On the empirical front, we provide evidence that such flows raise interest rates, which counteracts the upward pressure on stock prices.[2] On the theoretical front, we show that jointly taking into account inelasticity in the bond market is necessary to account for our empirical findings. Second, we propose a novel identification strategy relying on discontinuity in the Bank of Japan's policy rules. An advantage of our approach is that it transparently points toward the source of identification and the underlying assumption.

Our paper is also related to recent literature that seeks to uncover the impact of the BoJ's stock purchases. Since the BoJ purchased a certain stock market index rather than another, many studies exploit the difference in weights in the BoJ's purchase basket to identify the relative price impacts (Barbon and Gianinazzi, 2019; Charoenwong et al., 2021; Harada and Okimoto, 2021; Adachi et al., 2021; Katagiri et al., 2022). In contrast, our empirical strategy allows us to focus on the aggregate effect. In this regard, various studies (Shirota, 2018; Fukuda and Tanaka, 2022; Chung, 2020; Hattori and Yoshida, 2023) assume selection on observables and use the unexplained policy variation that remains after conditioning on observables to identify the aggregate effect.[3] However, it is difficult to rule out the presence of unobservables that simultaneously affect the financial market performance and the likelihood of BoJ intervention. Our identification assumptions, which only require continuity of economic fundamentals with respect to stock price changes in the morning session, are substantially weaker than those of these studies.

More broadly, we contribute to the large literature studying the effect of the central bank asset purchases, so-called "quantitative easing" (e.g., Krishnamurthy and Vissing-Jorgensen, 2011; Chodorow-Reich, 2014; Droste et al., 2021). These studies have focused on central bank purchases of long-term government bonds or mortgage-backed securities, which are swaps between one type of bond (e.g., long-term bonds) with another (e.g., reserves). Our focus is conceptually different from these because central bank stock purchases are swaps between stocks and

[2]Recent work by Caballero et al. (2024) also show that a flow from bonds to stocks has a positive impact on interest rates, consistent with our findings.

[3]Relatedly, Ichiue (2024) estimates how the stock size of the BoJ's holdings affects stock market performance.

bonds in the economy.

While there are many studies to isolate quasi-experimental variation in monetary policy (e.g., Romer and Romer, 1989; Cochrane and Piazzesi, 2002; Angrist et al., 2018; Nakamura and Steinsson, 2018), our approach is unique in exploiting the discontinuity in the policy rule. The closest to our approach is the one in Kuersteiner, Phillips, and Villamizar-Villegas (2018), who also use a discontinuous policy rule to investigate the effectiveness of sterilized foreign exchange interventions in Colombia. Our approach differs not only in terms of the empirical context but also in methodology, as we use the technique of regression discontinuity with unknown discontinuity points (Porter and Yu, 2015).

2 Data

Our primary goal is to measure the impact of an inflow into the stock market induced by central bank stock purchases. We focus on the period starting in October 2010 (the start of the BoJ stock market intervention) toward the end of 2020. The BoJ purchases Exchange Traded Funds (ETFs) indexed to stock market indexes in Japan. We do not distinguish between ETFs and stocks and use them interchangeably. We obtain the dates and amounts of stock purchases for each of the BoJ's interventions from the BoJ website.[4] Figure 1 shows the amount of ETF purchases over time. The BoJ started the stock market purchases in December 2010 as a new form of quantitative easing. By the end of 2020, the BoJ held over 6% of the stock market capitalization in Japan. Below, the amount of ETF purchases is normalized by the stock market capitalization, which we obtained from the Japan Exchange Group Data Cloud.

The BoJ publishes the amount of ETF purchases on the morning of the day following the intervention. Based on the trading volume, it is widely believed that the BoJ submits an order during lunchtime, although the BoJ has never made this practice public.[5] Therefore, investors potentially face uncertainty about whether

[4] https://www3.boj.or.jp/market/jp/menu_etf.htm

[5] See Harada and Okimoto (2021), for example.

the large inflow into the stock market reflects the BoJ's intervention or other factors within the day of the intervention. For this reason, we prefer to use our empirical estimates on the next day as our benchmark estimates.

To measure the response of the stock market, we use tick-by-tick data on the Tokyo Stock Price Index (henceforth, TOPIX), which is the index of the Tokyo Stock Exchange in Japan, tracking all domestic companies of the exchange's first section. We obtain these data from the Japan Exchange Group Data Cloud. To measure the response of the long-term interest rate, we use the tick-by-tick Japanese Government Bond yield data, which we obtained from Refinitiv Japan. Since some observations are missing in the Refinitiv data, we supplement those missing observations with the tick-by-tick data from Bloomberg.[6]

3 Research Design

We consider the following econometric model:

$$\Delta y_{t+l,h} = \beta_{l,h} \times ETF_t + \Gamma'_{l,h}\mathbf{X}_t + \epsilon_{t+l,h}, \tag{1}$$

where $\Delta y_{t+l,h} \equiv y_{t+l,h} - y_{t,0}$ is the change in the variable y (e.g. the log of stock prices) from the end price of the morning session ($h = 0$) on day t to time h on day $t + l$, ETF_t is the amount of stock market purchases by the BoJ relative to the stock market capitalization of Japan, $\mathbf{X}_t$ is the vector of controls, and $\epsilon_{t+l,h}$ contains the unmodeled determinants of the outcome variable. We are interested in estimating $\beta_{l,h}$, which measures the impact of the central bank's stock purchases at time h on l days after day t. We choose this simple linear model for expositional purposes. In Appendix A.1, we consider a non-linear model of (1) and present more technical interpretation of the estimated parameter as in Angrist and Imbens (1995).

An obvious concern with estimating the equation (1) by OLS is reverse causality. For example, one might expect that the central bank is more likely to intervene when the stock market is performing poorly. This leads to a downward bias in the

[6]The results are very similar when we use only the data from either Refinitiv or Bloomberg. The two datasets, when they overlap, are highly correlated with each other, with R^2 exceeding 99.98%.

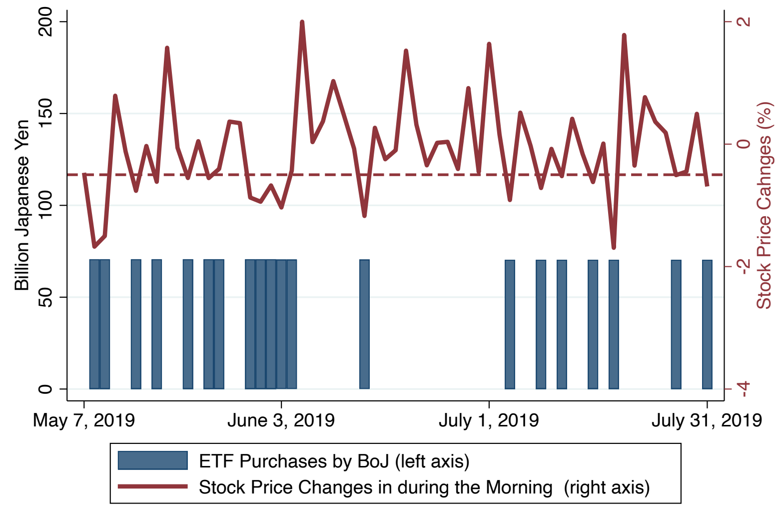

Figure 2: An Example of Cutoff Policy Rule

Notes: Figure 2 illustrates the cut-off policy rule by showing the percentage TOPIX changes and the BoJ (Bank of Japan) purchase amount for each day from May 2019 to July 2019. The solid red line shows the TOPIX changes in the morning session, and the dashed red line is the estimated cutoff of 0.25%. The bar shows the amount of purchases for each intervention in billions of Japanese Yen (approximately 10 million US dollars).

OLS estimates of $\beta_{l,h}$.

To solve this endogeneity problem, we propose an identification strategy based on regression discontinuity design, which builds on the observation that the BoJ's intervention appeared to follow a cut-off rule. It has been widely argued among the media that the BoJ seemed to intervene on the day when the movements in the value of TOPIX fell below a certain threshold in the morning session. For example, the Financial Times writes that "the central bank has tended to step in whenever the TOPIX index has lost more than 0.5 percent in the morning session."[7] In fact, Figure 2 shows that from May to July 2019, the BoJ followed a strict rule to intervene when the stock market index falls more than 0.5% in the morning session. The BoJ intervenes when the index falls slightly below the 0.5% threshold, while it does not intervene when the index falls slightly above the threshold.

Suppose for the moment that such a cut-off is known. Then, we can apply a standard regression discontinuity design. Formally, we assume that the policy

[7]Financial Times, "Bank of Japan backs away from ETF buying scheme," (March 23, 2021), https://www.ft.com/content/a654d1c9-7126-4587-8de6-ed15f567455f.

rule has the following form:

$$ETF_t = ETF_{-,t}(\Delta p_t)\mathbb{I}(\Delta p_t < c_t) + ETF_{+,t}(\Delta p_t)\mathbb{I}(\Delta p_t \geq c_t), \quad (2)$$

where Δp_t is the log-changes in the TOPIX value in the morning, c_t is the cut-off, $ETF_{-,t}$ and $ETF_{+,t}$ are some random functions of the ETF purchase at day t that represent different policy rules depending on whether Δp_t is above or below the cutoff. We assume (i) $\mathbb{E}[\epsilon_{t+l,h}|\Delta p_t, \mathbf{X_t}]$ is continuous at $\Delta p_t = c_t$, (ii) $\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}]$ and $\lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}]$ exist, and (iii) $\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}] \neq \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}]$. Under these assumptions, it follows that

$$\frac{\lim_{\Delta p \uparrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p, \mathbf{X_t}] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p, \mathbf{X_t}]}{\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p, \mathbf{X_t}]} = \beta_{l,h}. \quad (3)$$

As recommended by Hahn, Todd, and Van der Klaauw (2001) and Porter (2003), we can devise local linear regression estimators for the left-hand side to obtain an estimate of $\beta_{l,h}$. Imbens and Lemieux (2008) pointed out that this is numerically equivalent to a two-stage least squares estimator with properly defined instruments and weights. The advantage of their formulation in our context is that it is easy to accommodate heteroskedasticity and auto-correlation. We use the optimal bandwidth proposed by Calonico, Cattaneo, and Titiunik (2014) and estimate $\beta_{l,h}$ using two-stage least squares and report Newey-West standard errors.

The difficulty in implementing the above approach, however, is that the cut-off is not necessarily known. While it was apparently known to the public that the BoJ followed a particular cut-off rule in some periods, it is sometimes not in other periods. In order to formally investigate the hypothesis, we estimate the cut-off with the presumption that the BoJ follows a cut-off rule, following the approach proposed by Porter and Yu (2015). They develop a method to estimate the discontinuity point and show that there is no loss of efficiency with the regression discontinuity estimator using the estimated cutoff. In implementing this approach, we proceed as follows. We first split the sample period to allow time-variation in the policy rule. We assume the cut-off is a constant within the sample split. Then, in each of the sample splits, we consider a set of possible cutoffs, $\mathbb{C} \equiv \{\bar{c}_1, \bar{c}_2, \ldots, \bar{c}_K\}$. For each $\bar{c} \in \mathbb{C}$, we estimate the jump of $\Pr_t(ETF_t > 0|\Delta p)$ around $\bar{c}$, which is

$$J_t(\bar{c}) \equiv \lim_{\Delta p \uparrow \bar{c}} \Pr_t(ETF_t > 0|\Delta p) - \lim_{\Delta p \downarrow \bar{c}} \Pr_t(ETF_t > 0|\Delta p).$$

We select $\bar{c}$ that maximizes square of the jump, $J_t^2(\bar{c})$: $c_t^* \in \arg\max_{\bar{c} \in \mathbb{C}} J_t^2(\bar{c})$.

We implement the above approach with the following specifications. First, we consider the split of the sample period based on the BoJ's announcements regarding the ETF purchases. The BoJ made six announcements that state the changes in the target amount of ETF purchases in March 4, 2013, October 31, 2014, December 18, 2015, July 29, 2016, July 31, 2018, and March 16, 2020. We further divide each period between the two announcements, based on whether the TOPIX closing price falls relative to the opening price for the last two consecutive days. We make this choice based on widespread claims in the media,[8] and we have indeed found that it has a strong explanatory power. Second, we consider the set of potential cutoffs ranging from -1% to 0% with 0.05% intervals. We estimate the jump of $\Pr_t(ETF_t > 0|\Delta p)$ around the potential cutoffs using the local linear regressions with the optimal bandwidth computed from Calonico, Cattaneo, and Titiunik (2014).

Figure 3A shows the path of estimated cutoffs. The estimated cutoffs align well with the widely held consensus. During 2010-2013, it is widely believed that the BoJ followed a so-called "1% rule", in which the BoJ buys ETFs whenever the TOPIX falls more than 1% in the morning session,[9] and our estimates confirm this view. Since April 2013, the BoJ appears to use different cutoffs depending on whether the daily change in the TOPIX has been negative for the past two consecutive days. Since March 2018, the cutoffs appear to be 0.5% when there is no

[8]See, for example, Bloomberg article "The BoJ's ETF Purchase Conditions Likely to Ease if Stocks Continue to Fall" (written in Japanese) (`https://www.bloomberg.co.jp/news/articles/2020-07-22/-0-3-kcwteezj`).

[9]For example, Nikkei Asia writes "the BoJ was widely thought to be following an unwritten rule, dubbed the 1% rule: it would buy ETFs when the Topix index of all issues on the first section of the Tokyo Stock Exchange fell more than 1% in the morning session." (https://asia.nikkei.com/Business/Finance/BOJ-steps-up-REIT-buying-scales-back-ETF-purchases)

(A) Estimated cutoffs over time **(B)** Discontinuities in Policy Intervention

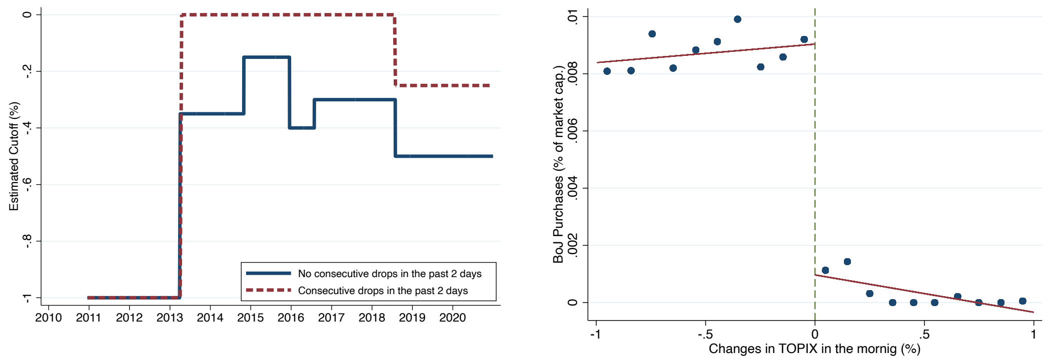

Figure 3: Estimated Cutoffs and Discontinuities around the Cutoffs

Notes: Figure 3A plots the path of estimated cutoffs over our sample period. Figure 3B shows the discontinuity in the amount of the BoJ stock purchases in the range of -1% to 1% around the estimated cutoff. Each dot represents the binned scatter plot with 0.1% bin-width and the red line represents the linear fit on each side of the cutoff.

consecutive fall in the past two days, which is again consistent with the so-called "0.5% rule."

Figure 3B shows the binned scatter plot of the size of the BoJ intervention against the changes in the TOPIX in the morning session of the same day relative to the cutoffs. We confirm that there is a discrete jump in the size of the BoJ interventions around zero. The implied jump in the full sample is 0.83% of the market capitalization with a standard error of 0.0005%. The Cragg-Donald F-statistic is 1821, and the Kleibergen-Paap F-statistic is 261, eliminating weak identification concerns. This discontinuity comes from the discontinuity in the likelihood of intervention, with a jump in the probability of intervention of 86% with a standard error of 0.02%. Importantly, we find strong evidence of discontinuity in any single split of the sample.[10]

A natural concern for discontinuity-based research design is the manipulation around the cutoff. While it is unlikely that investors are able to manipulate the stock price, we formally test the presence of manipulation using the methodology proposed by Cattaneo et al. (2020) in Appendix A.3, and the density plot is shown

[10]We report the discontinuity for each sample split in Appendix A.2 and Figure B.1. .

in Figure B.2. We do not find evidence of manipulation.

Another issue, which is unique to time-series setups like ours, is the autocorrelation in the policy shock. The outcome $y_{t+l,h}$ is affected by the BoJ's ETF purchases up to l days later. Therefore, if falling below the cutoff today is correlated with future and past purchases, our empirical estimates cannot be interpreted as the causal effect of the BoJ's one-time ETF purchases. To address this concern, in Appendix A.4 we test the discontinuity in the amount of ETF purchases around the cutoff across days. Figure B.3 shows the estimates of the discontinuity in the amount of ETF purchases at date $t + l$ around the cutoff at day t. Reassuringly, we find significant discontinuity only at $l = 0$. Thus, the effects we identify are the causal effects of the BoJ's one-time ETF purchases and are not contaminated by future or past ETF purchases.

4 Empirical Results

Armed with the cutoff estimates, we implement the regression discontinuity design to assess the impact of the BoJ's ETF purchases on the financial market. We report the following three main findings. First, the BoJ's stock purchases increase both stock prices and long-term interest rates throughout the sample period. Second, in the periods before the BoJ introduced yield curve control, there is no evidence that the intervention increased the stock price, but it robustly increased the long-term interest rate. Third, after the introduction of yield curve control, the long-term interest rate stopped responding, and the stock price sharply and persistently increased following the intervention.

4.1 Homogenous Effect for the Entire Sample Periods

Figure 4A first assesses whether a discontinuity in policy intervention leads to a discontinuity in stock price changes. It reports the binned scatter plot changes in the TOPIX in the afternoon (from 11AM to 3PM) against the changes in the TOPIX in the morning relative to the estimated cutoff. The figure shows that the stock prices were around 0.2% higher when the TOPIX fell slightly below the cutoff in

(A) Stock Return **(B)** 10 Year JGB Yield

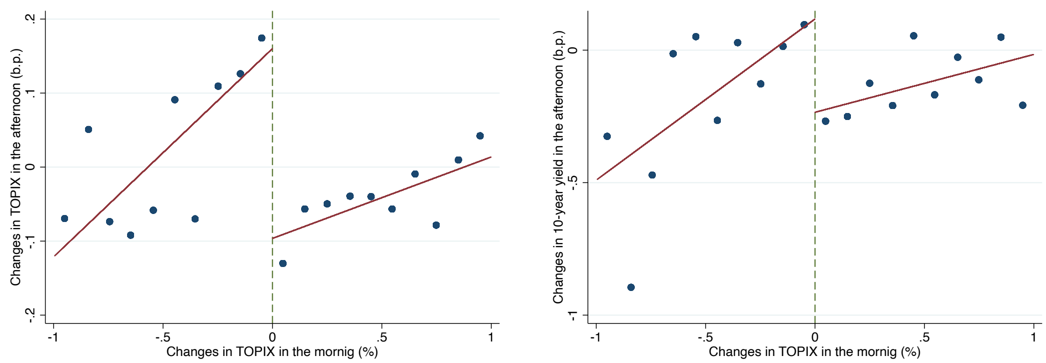

Figure 4: Discontinuities in Stock Returns and Long-Term Interest Rates

Notes: Figure 4A shows the binned scatter plot of the log changes in TOPIX in the afternoon (from 11PM to 3PM) against the changes in TOPIX in the morning session relative to the cutoff. The bin width is 0.1%. The line represents the best fit from the linear regression. Figure 4B is analogous to Figure 4A, with the vertical axis being the changes in the 10-year JGB Yield in basis point (b.p.) in the afternoon (from 11PM to 3PM).

the morning than when it fell slightly above the cutoff. Since the BoJ submitted the order to purchase ETF during the lunch break, this suggests that the BoJ's intervention had a large impact on the stock prices within the day. The magnitude is large considering that the BoJ purchased on average around 0.01% of market capitalization in each of the interventions.

Figure 4B focuses on the 10-year Japanese government bond yields as an outcome variable. Perhaps surprisingly, we also see a discontinuity in the long-term interest rate. The long-term interest rate is 4 basis points higher on the left side of the cutoff than on the right side. Later, we argue that through the lens of the theoretical model, this evidence supports the notion that the bond market is inelastic. Intuitively speaking, central banks swap bonds with stocks. As there are more supplies of bonds in the economy, the bond prices fall when investor demand for bonds is downward sloping. The results so far concern the price changes within a day, and therefore it could be the case that the stock and bond prices revert back to the original level on the following day. Next, we systematically evaluate the price impacts for various horizons.

Figure 5A and 5B plot the impulse response functions of stock prices and bond

(A) Impulse Response of Stock Prices **(B)** Impulse Response of JGB 10 Years Yield

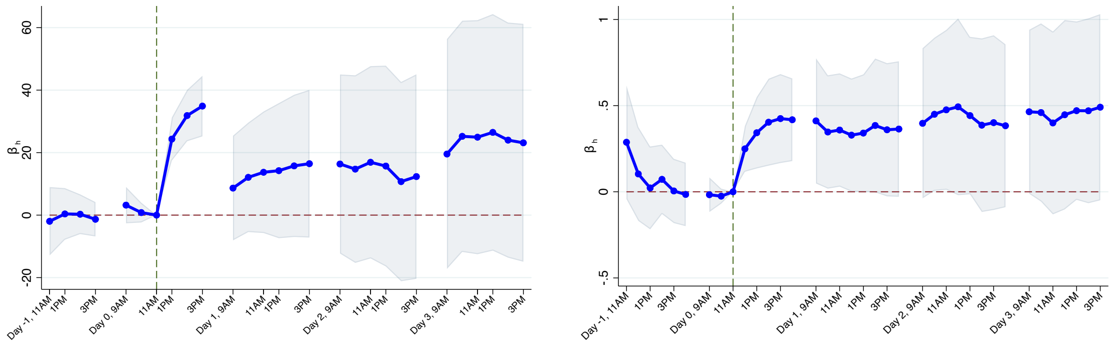

Figure 5: The Impact on Stock Prices and Long-Term Interest Rates

Notes: Figure 5A shows the impulse response function of stock prices by plotting coefficient $\beta_{l,h}$ in equation (1). The coefficient measures the log changes in stock prices in response to stock purchases of 1% of market capitalization. Figure 5B is analogous to Figure 5A and shows the impulse response of the 10-year JGB yield. The coefficient measures the percentage point changes in the yield in response to stock purchases of 1% of market capitalization. In all figures, the shaded areas represent 90% confidence intervals, which account for heteroskedasticity and autocorrelation.

prices. Formally, we plot estimates of $\beta_{l,h}$ in equation (1) for each l and h, where l represents the number of days since the intervention and h represents the time in hours. In Figure 5A, we see an immediate and large stock price response in the afternoon of the intervention. It implies that 1 stock purchase of 0.01% of market capitalization, a typical size of the intervention, increases the stock value by 0.4%. This coefficient is statistically significant. Over the next five days, the coefficient is roughly halved and the standard error is larger, but it does not revert back to zero. Reassuringly, we do not find any evidence of pre-trends, which is consistent with the continuity assumption for the error term.

Figure 5B shows that the 10-year JGB yield also rises sharply after the intervention. The effect appears to be quite persistent, and it remains statistically significant even five days after the intervention. The magnitude is again substantial. In response to a typical size of the purchases (0.01% of market capitalization), the 10-year JGB yield rises by around 0.4-0.5 basis points.

We have shown that the central bank stock purchases have quantitatively large impacts on both the stock and bond markets. In what follows, we argue that these average effects mask an important underlying heterogeneity.

Figure 6: 10 Year JGB Yield

Notes: Figure 6 shows the path of 10-year JGB (Japanese Government Bond) yield over time, where the red vertical the dashed line (September 21, 2016) denotes the start of the yield curve control.

4.2 Heterogenous Effect and Yield Curve Control

Figure 5B showed that the stock market purchases are accompanied by the rise in the long-term interest rate. In standard theoretical models, the rise in interest rates leads to a drop in stock prices. Therefore, we expect that the ability of the interest rate to respond will be critical in determining the stock price responses to the central bank stock purchases.

The BoJ's other unconventional policy, the so-called "yield curve control," provides an ideal laboratory to explore this hypothesis. On September 21, 2016, the BoJ introduced an explicit target for the 10-year Japanese government bond yield at 0%. Figure 6 indeed shows that the long-term rate has stabilized at around 0% since the introduction of yield curve control. The daily standard deviation of the long-term rate is 0.37% before the introduction of yield curve control, but it falls to 0.08% after the introduction. If the BoJ does its best to stabilize the long-term interest rate at 0%, we would expect to see a much smaller response of the long-term rate in response to the stock purchases. To test this, we split our sample periods before and after the introduction of yield curve control and rerun our analysis.

Figures 7A- 8B show the main results of this paper. Figures 7A and 7B show the impulse response of stock prices before and after the introduction of yield curve control, respectively. We find no evidence that the stock market responded positively after the BoJ intervention before the introduction of yield curve control after one day, although the standard error is large. In stark contrast, stock prices

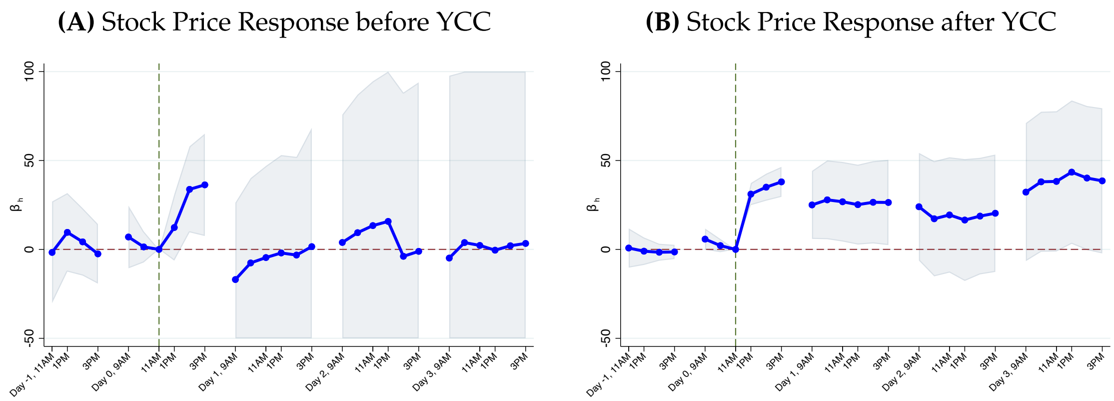

Figure 7: Heterogenous Stock Price Responses before and after Yield Curve Control

Notes: Figures 7A and 7B show the impulse response of stock prices separately estimated before and after yield curve control, which is analogous to Figure 5A.

rise persistently and in a statistically significant manner under yield curve control. Quantitatively, a purchase of 0.01% of stock market capitalization, a typical size of the intervention, by the BoJ causes around a 0.2-0.3% increase in stock prices within at least several days after the intervention.

Figures 8A and 8B explain why. The long-term interest rate responds positively before yield curve control. Quantitatively, a purchase of 0.01% of stock market capitalization by the BoJ causes around one base point increase in the long-term rates, and the effect is statistically significant. However, under the yield curve control, long-term rates stopped responding and the effect is precisely estimated as zero.

4.3 Bond Yield Responses Across Different Maturities

We show that the effect on interest rate is not specific to the 10-year JGB yield, but rather is widespread across different maturities.

Figure 9 shows the point estimates of the effect on the JGB yield across maturities of 1, 2, 5, 10, 20, and 30 years. Before yield curve control, yields rose at all maturities, but the effect was larger at longer maturities. Our preferred interpretation is that the zero lower bound on policy rate has been binding during this

(A) JGB 10-Year Yield Response before YCC **(B)** JGB 10-Year Yield Response after YCC

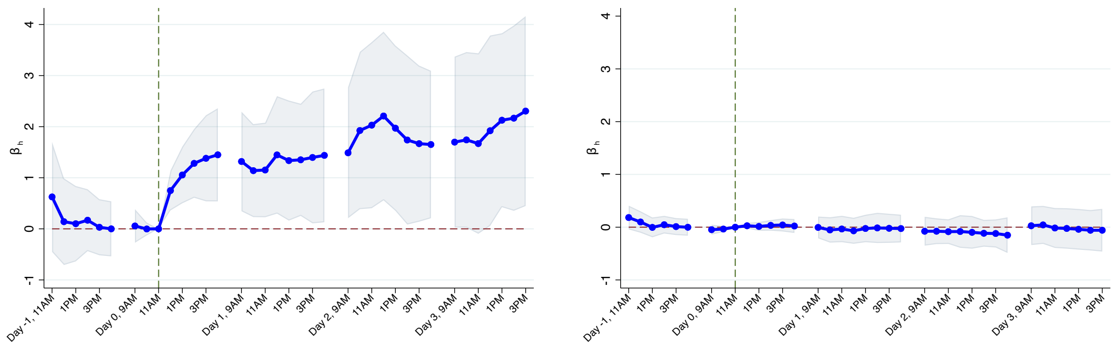

Figure 8: Heterogenous Interest Rate Responses before and after Yield Curve Control

Notes: Figures 8A and 8B show the impulse response of the 10-year JGB yield separately estimated before and after yield curve control, which is analogous to Figure 5B. In all figures, the shaded areas represent the 90% confidence interval, which accounts for heteroskedasticity and autocorrelation.

period, and therefore the shorter-maturity bonds had less room to respond relative to longer-maturity bonds. After yield curve control, all interest rates entirely stopped responding. Although yield curve control was to specifically control the 10-year yield, it can prevent the other maturities from responding because they are interconnected through arbitrage. For example, it is the natural prediction of the preferred habitat model of the term structure by Vayanos and Vila (2021).

4.4 Robustness

Table 1 conducts a battery of robustness checks and shows that our results are robust to various alternatives to the baseline specifications. In rows 1 and 2, we show that the results are not sensitive to changing the bandwidth of the regression discontinuity estimator.[11] Row 3 uses the quadratic local polynomial regression instead of the linear. Row 4 controls the amount of the BoJ's purchases over the past two days. This addresses the concern that if the stock price changes are likely to fall on one side of the cut-off over consecutive days, our estimator confounds

[11] Figure B.5 in the Appendix more systematically explores robustness with respect to the choice of bandwidths. We find that the results are virtually unchanged for a wide variation in the bandwidths.

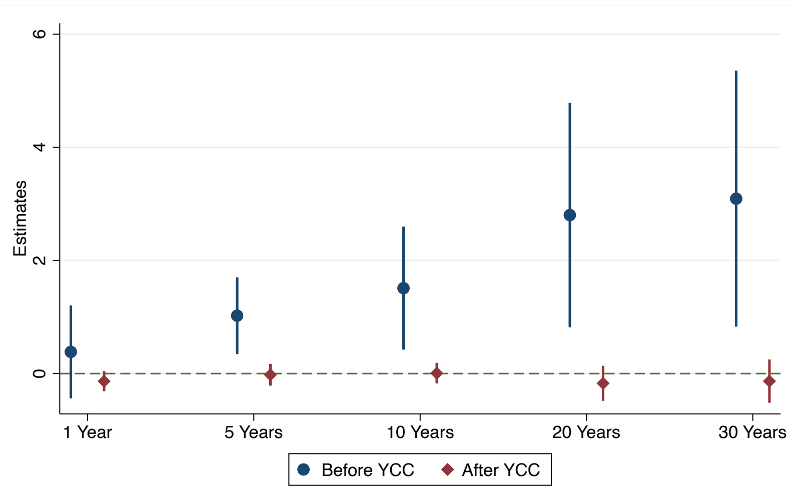

Figure 9: Response of Other Maturities

Notes: Figure 9 shows the response of the JGB yield across different maturities from 11AM of the day of the intervention to 9AM of the next day. The circle dot represents the point estimates before yield curve control, and the diamond dot represents the point estimates after yield curve control. The coefficient measures the percentage point changes in the JGB Yield in response to the purchase of 1% of market capitalization. The vertical lines represent the 90% confidence interval, which accounts for heteroskedasticity and autocorrelation.

the effect of the past and future interventions. Reassuringly, the results are not sensitive to this control. This is not surprising given that there is no evidence that falling below the cutoff today causes future and past ETF purchases, as we formally show in the Appendix A.4. Row 5 and row 6 controls for the past stock market returns and changes in the long-term interest rate over the last two days. Finally, in row 7, we drop observations one week before and after the dates when the cutoff changed. This addresses the concern that the changes in cutoff could be endogenous to the underlying economic fundamentals or the cutoff change may contain signals about future policy stances of the BoJ. Overall, our results appear to be virtually unchanged with various alternative specifications.

4.5 Placebo Tests

One might worry that our results are not driven by the discontinuous changes in BoJ's policy intervention, but rather by some other factors such as investors' sentiments that discontinuously respond to the stock price changes in the morning

Panel A. Stock Price Response

	All		Before YCC		After YCC	
	Same Day	Next Day	Same Day	Next Day	Same Day	Next Day
0. Baseline	34.90	8.65	36.31	-16.95	35.24	22.23
	(5.89)	(10.18)	(17.50)	(26.27)	(4.68)	(11.13)
1. Narrower Bandwidth	41.65	16.22	30.14	-21.25	47.26	33.08
	(7.16)	(16.09)	(22.18)	(39.57)	(7.15)	(16.25)
2. Wider Bandwidth	29.49	6.07	30.63	-2.44	29.41	15.66
	(4.84)	(9.36)	(13.46)	(20.74)	(4.69)	(9.14)
3. Polynominal Order 2	43.40	16.98	41.13	-22.70	47.24	30.63
	(7.47)	(15.64)	(21.64)	(42.42)	(7.14)	(14.39)
4. Control Past Interventions	39.50	12.56	54.57	-16.44	37.32	29.65
	(8.09)	(15.10)	(25.67)	(34.04)	(6.61)	(14.48)
5. Control Past Stock Returns	34.91	8.62	36.15	-17.33	35.18	20.51
	(5.89)	(10.20)	(17.70)	(26.21)	(4.77)	(11.76)
6. Control Past 10-Year Yield	35.96	6.63	40.91	-23.41	35.06	21.90
	(5.84)	(9.94)	(18.33)	(25.42)	(4.67)	(11.13)
7. Drop Around the Cutoff Changes	34.96	8.00	31.21	-20.53	35.07	24.56
	(6.05)	(10.92)	(16.52)	(28.22)	(5.52)	(12.45)

Panel B. JGB 10-Year Yield Response

	All		Before YCC		After YCC	
	Same Day	Next Day	Same Day	Next Day	Same Day	Next Day
0. Baseline	0.47	0.37	1.52	1.41	0.04	-0.00
	(0.18)	(0.23)	(0.59)	(0.68)	(0.07)	(0.13)
1. Half Bandwidth	0.54	0.43	1.96	1.96	-0.00	-0.13
	(0.22)	(0.30)	(0.84)	(0.96)	(0.09)	(0.20)
2. Wider Bandwidth	0.40	0.35	1.28	1.05	0.01	-0.04
	(0.16)	(0.19)	(0.47)	(0.50)	(0.06)	(0.10)
3. Polynominal Order 2	0.52	0.43	1.81	1.75	0.07	0.01
	(0.22)	(0.27)	(0.77)	(0.91)	(0.09)	(0.18)
4. Control Past Interventions	0.47	0.37	1.73	1.79	-0.06	-0.17
	(0.23)	(0.31)	(0.76)	(0.90)	(0.11)	(0.21)
5. Control Past Stock Returns	0.47	0.37	1.53	1.43	0.03	-0.03
	(0.18)	(0.23)	(0.59)	(0.68)	(0.08)	(0.13)
6. Control Past 10-Year Yield	0.46	0.37	1.51	1.42	0.04	-0.01
	(0.17)	(0.23)	(0.60)	(0.69)	(0.07)	(0.13)
7. Drop Around the Cutoff Changes	0.37	0.28	1.12	0.91	0.02	-0.05
	(0.16)	(0.21)	(0.44)	(0.48)	(0.06)	(0.12)

Table 1: Robustness

Notes: Table 1 shows robustness checks against various modifications of our benchmark specifications. Panels A and B show the responses of stock prices and the JGB 10-year yield, respectively. In each panel, row 0 reports the baseline estimates. Row 1 considers bandwidth that is 50% less than the original one. Row 2 considers bandwidth that is 50% larger than the original one. Row 3 considers a local polynomial regression of order 2 instead of 1. Row 4 controls for the BoJ's stock purchases over the past two days. Row 5 controls for stock market returns over the past two days. Row 6 controls for changes in the 10-year yield over the past two days. Row 7 drops observations before and after one week around the cutoff changes. The same-day response indicates the changes in the outcome variable from 11AM to 3PM on the same day of the intervention. The next-day response indicates the changes in the outcome variable from 11AM on the day of the intervention to 9AM on the next day. Standard errors, which account for heteroskedasticity and autocorrelation, are reported in parentheses.

session. For example, investors might speculate that the stock market should see a stronger rebound when stock prices fall below 0% in the morning session.

To address this concern, we conduct placebo tests by testing the presence of discontinuity in outcome variables around an arbitrary cutoff for which we do not expect to find any discontinuity. Specifically, for each value of placebo cutoffs $c^{placebo} \in \{-1\%, -0.95\%, \ldots, -0.05\%, 0\%\}$, we test whether there is a discontinuity in our outcome variables when the stock prices fall below the placebo cutoff $c^{placebo}$. We estimate

$$\Delta y_{t+l,h} = \gamma_{l,h} \times \mathbb{I}(\Delta p_t < c^{placebo}) + e_{t+l,h}, \tag{4}$$

where Δp_t is the percentage change in TOPIX in the morning session, and we focus on within-day changes (from 11AM to 3PM) in the outcome variables. When estimating, we exclude periods for which the actual cutoff is identical to the placebo cutoff $c^{placebo}$ under consideration. We are interested in the estimates of $\gamma_{l,h}$, and we expect that $\gamma_{l,h}$ to be indistinguishable from zero for any value of $c^{placebo}$.

Figures 10A and 10B show the estimated value of $\gamma_{l,h}$, together with the 90% confidence interval. Reassuringly, we find that the estimates of $\gamma_{l,h}$ are indistinguishable from zero in almost all cases. Even if they are significant, the estimates are the opposite sign from our baseline estimates. Moreover, in all cases, the estimates are far enough from our baseline estimates that use the actual cutoff. These results suggest that our results are indeed driven by the policy intervention itself.

4.6 Other Discussion

We discuss several other issues. First, if the BoJ is selling the long-term government bonds at the same time as the stock purchases, then it is not surprising that the long-term interest rate rises in response to the stock purchases. However, during our sample periods, the BoJ sold government bonds only twice, on March 24, 2017, and March 23, 2020, both of which are the periods after yield curve control. Therefore, we can forcefully rule out the concern.

Second, one may wonder whether the difference in the amount of purchases across periods might be driving the heterogeneity before and after yield curve

(A) Placebo Test for Stock Response **(B)** Placebo Test for 10-year JGB Response

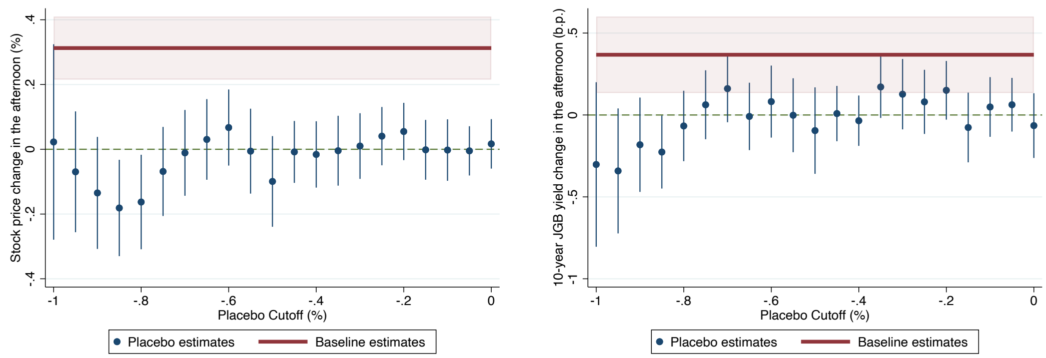

Figure 10: Placebo Tests

Notes: Figure 10A plots the estimates of $\gamma_{l,h}$ in equation (4) as the blue dots for each placebo cutoff, where the outcome variable is the stock price. The estimates are the response within the day of intervention (changes from 11AM to 3PM). We exclude the periods where the placebo cutoff coincides with the actual cutoff. The red line indicates our estimates using the actual cutoff. Figure 10B is analogous to Figure 10A, where the outcome variable is now the 10-year JGB yield. The line and the shaded area represent 90% confidence interval.

control. We first note that from a theoretical perspective, it is unlikely that the size of financial flow matters for whether its effect shows up in the bond market or the stock market. Moreover, Figure B.4 in the appendix shows that, while it is true that the absolute amount of purchases in each intervention is four times larger after the yield curve control than before, the growth is more moderate once expressed as a fraction of market capitalization.

The final issue concerns the interpretation of our results. As discussed by Krishnamurthy and Vissing-Jorgensen (2011), there are broadly two channels through which central bank asset purchases affect asset prices. The first channel is the liquidity channel. This channel operates by changing the aggregate demand and supply of assets. The second channel is the signaling channel. According to this channel, the central bank asset purchases have an effect because they send signals about the central bank's future policy stance. We believe that our empirical results are likely driven by the liquidity channel rather than the signaling channel. The BoJ announces the target amount of stock purchases each year in advance. Therefore, whether or not the BoJ purchases stocks today should not reveal the BoJ's future policy stance.

5 Conceptual Framework

We provide a conceptual framework to provide structural interpretations of our empirical results. We consider a model with two assets: stocks and bonds. We do not make a distinction between money and bonds and call them bonds. This refers to all the money-like assets that are liquid and risk-free.[12] We show that a model with an inelastic stock market and an even more inelastic bond market can qualitatively and quantitatively account for our findings.

5.1 Environment

Time is discrete, and the horizon is infinite, $t = 0, 1, \ldots, \infty$. The economy is populated by a representative household household, a representative firm, investment funds, and a consolidated central bank and government. The only factor of production in the economy is capital in fixed supply, and firms own the capital. The supply of capital is normalized to one, and a unit of capital produces Y units of consumption goods. We assume that Y is constant over time and is exogenously given.

There are two assets in the economy, stocks and bonds. Stocks are claims to capital (firms). Let Q_t denote the ex-dividend price of stocks. Its gross return is given by

$$R^s_{t+1} = \frac{Q_{t+1} + Y}{Q_t}. \tag{5}$$

There is also a risk-free asset in zero net supply. Its objective gross return is denoted as R^b_{t+1}. We assume only the government can issue bonds, and we normalize the supply of capital to be one.

An investment fund manages a part of a household's wealth and invests in

[12]This is a valid assumption if the elasticity of substitution between near-money assets like bonds and money is high, which is empirically the case (Nagel, 2016; Krishnamurthy and Li, 2023).

stocks and bonds. The fund consists of a continuum of members $i \in [0,1]$ with heterogenous beliefs over asset returns. Each member manages an equal amount of assets and invests in assets that they believe to have the highest return. A member i with belief shock $\epsilon_i \equiv (\epsilon_i^b, \epsilon_i^s)$ at time t believes the return from investing in bonds and stocks are given $\epsilon_i^b R_{t+1}^b$ and $\epsilon_i^s R_{t+1}^s$. Therefore the faction of savings invested in asset a, s_t^a, is given by the fraction of fund members that believe asset a has a higher return than the other:

$$s_t^a = \int_0^1 \mathbb{I}\left[a = \arg\max_{\tilde{a}} \epsilon_i^{\tilde{a}} R_{t+1}^{\tilde{a}}\right] di, \tag{6}$$

where $\mathbb{I}[\cdot]$ is an indicator function.

Building on the discrete choice literature, we assume the beliefs are drawn from independent type II independent extreme value (Fréchet) distribution:

$$\text{Prob}\left[\epsilon_t^b(i) \leq \epsilon_t^b, \epsilon_t^s(i) \leq \epsilon_t^s\right] = \exp\left(-\sum_{a \in \{b,s\}} \mu^a (\epsilon_t^a)^{-\theta}\right), \tag{7}$$

where μ^a is the scale parameter, $\theta > 0$ is the shape parameter. Under the above functional form assumptions, the portfolio shares of a fund is given by

$$s_t^a = \frac{\mu^a \left(R_{t+1}^a\right)^\theta}{\sum_{l \in \{s,b\}} \mu^l \left(R_{t+1}^l\right)^\theta} \quad \text{for} \quad a \in \{b,s\}. \tag{8}$$

The asset demand system (8) takes constant elasticity of substitution (CES) form, as in for example Koijen and Yogo (2020), which we micro-found through heterogeneous beliefs. We choose to microfound through heterogenous belief given the empirical evidence on the importance of belief in portfolio allocation (Giglio et al., 2021). However, the underlying microfoundation is not important as long as it delivers an inelastic portfolio choice. Other micro-foundations that lead to the same asset demand system include risks (Okawa and Van Wincoop, 2012), heterogenous returns (Kleinman et al., 2023), and rational inattention (Pellegrino et al., 2021).

The parameter $\theta > 0$ in (8) captures the elasticity of relative asset demand with respect to return differences. With our microfoundation, θ has a structural interpretation as the inverse of belief heterogeneity. As belief heterogeneity vanishes, $\theta \to \infty$, the relative asset demand is infinitely elastic to return differences, a standard assumption in the macroeconomics literature. Given the portfolio share, the portfolio return of the fund is

$$R^p_{t+1} = \sum_a s^a_t R^a_{t+1}. \tag{9}$$

We assume households invest in bonds and the funds, but the funds are illiquid in the sense that households do not actively trade the funds. The households can freely trade bonds and we assume that the bonds provide liquidity services to the households in the form of utility. The household's preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[u(C_t) + v\left(B_t + s^b_t A_t \right) \right], \tag{10}$$

where C_t is the consumption, B_t is the direct bond holdings of the households, $s^b_t A_t$ is the indirect bond holdings by the fund, and $u(\cdot)$ and $v(\cdot)$ are both increasing and concave. Here we assumed that both the direct and indirect bond holding provide liquidity services. Although we do not provide a microfoundation of liquidity service that bond provides, existing literature (Angeletos et al., 2023; Auclert et al., 2024; Di Tella et al., 2024) show that incomplete models with uninsurable income or liquidity risk give representation that is similar to (10).

The household's budget constraint is given by

$$C_t + B_t = R^b_t B_{t-1} + D_t - T_t, \tag{11}$$

where T_t is the lump-sum tax imposed by the government and D_t is the withdrawal from the fund. The evolution of the account in the fund is

$$A_t = R^p_t A_{t-1} - D_t. \tag{12}$$

For simplicity, we assume the value that households maintain in the funds, A_t, follows an exogenous rule of the form

$$A_t = \kappa_0 + \kappa_1 Q_t, \tag{13}$$

where $\kappa_0 > 0$ captures the amount of funds that households always maintain and $\kappa_1 \in [0, 1]$ captures the response of funds value to asset price fluctuations. The household problem is to choose $\{C_t, B_t\}$ to maximize (10) subject to (11). This results in the consumption Euler equation of the following form:

$$u'(C_t) = \beta R^b_{t+1} u'(C_{t+1}) + v'(B_t + s^b_t A_t). \tag{14}$$

We assume the government can issue bonds, invest in stocks, and levy taxes on households. The consolidated government sets the path of stock holdings, S^g_t, bond issuance B^g_t, and lump-sum tax, T_t, that satisfy the government budget constraint:

$$Q_t S^g_t - B^g_t = T_t + (Y + Q_t) S^g_{t-1} - R^b_t B^g_{t-1}. \tag{15}$$

The asset market clearings

$$B^g_t = B_t + s^b_t A_t \tag{16}$$

$$Q_t = Q_t S^g_t + s^s_t A_t. \tag{17}$$

The goods market clearing, $C_t = Y$, is implied by the budget constraints, (11) and (15), and the asset market clearing conditions, (16) and (17).

Given the path of government policies, $\{S^g_t, B^g_t, T_t\}$, that satisfy (15), the equilibrium of this economy consists of $\{\{s^a_t, R^a_{t+1}\}_{a \in \{s,b\}}, C_t, B_t, A_t, D_t, R^p_{t+1}, Q_t\}$ such that (5), (8)-(9), (11)-(14), and (16)-(17) hold. The steady-state equilibrium is the one where all variables are constant over time.

5.2 Central Bank Stock Purchases: Analytical Characterization

We first analytically study central bank stock purchases in the model. We assume before time $t = 0$, the economy is in the steady state, and we let all variables without time subscripts denote the steady-state values before $t = 0$. At time $t = 0$, we consider a shock to central bank stock purchases.

We model the central bank stock purchases as a small permanent increase in S_t^g at time 0. Therefore the path of S_t^g is given by $S_t^g = S^g + dS^g$ for $t \geq 0$. We consider two scenarios that differ in how the stock purchases are financed. In the first experiment, the government finances the stock purchases with an equal amount of bond issuance, $B_t^g = B^g + Q_0 dS^g$ for $t \geq 0$. This experiment aims to replicate the Bank of Japan's stock purchases before the implementation of yield curve control. In the second experiment, the government adjusts the path of $\{B_g^g, T_t\}_{t\geq 0}$ so that the interest rate on the bond is kept constant, $R_t^b = R^b$. This experiment is to mimic the Bank of Japan's stock purchases after the implementation of yield curve control.

Our first characterization of the impact of the central bank stock purchases illustrates a relatively simple case highlighted in Gabaix and Koijen (2021).

Proposition 1. *Assume $v''(B) = 0$. Then, the central bank stock purchase with and without yield curve control has no effect on the bond interest rate, $\frac{d \ln R_{t+1}^b}{dS^g} = 0$, and raises the stock price for all t,*

$$\frac{d \ln Q_t}{dS^g} = \frac{Q}{Q(1 - S^g - (1 - s^b)\kappa_1) + A\theta(1 - s^b)s^b \frac{Y}{Y+Q}} > 0. \tag{18}$$

All the proofs are collected in Appendix C.1. When the liquidity service that the bond provides is constant, $v'(B) = \bar{v}$, the household's consumption Euler equation (14), together with goods market clearing, $C_t = Y$, solely pins down the bond interest rate R^b. That is, the bond market is perfectly elastic. Note that the expression (18) can be rewritten as $\frac{dQ_t}{QdS^g}$, so has an interpretation as the dollar increase in stock market value in the dollar in response to \$1 increase in central bank's stock holdings. Note that $\frac{d \ln Q_t}{dS^g} \to 0$ as the belief heterogeneity vanishes, $\theta \to \infty$. This is the case where the stock market is also perfectly elastic, under which the central bank asset purchases are entirely neutral (Wallace, 1981). With finite θ, the stock market is inelastic. Since the central bank stock purchases create excess demand in the stock, the stock return must fall to clear the stock market, which results in a rise in the stock price. With a perfectly elastic bond market, this is the only consequence of a flow into the stock market, as in Gabaix and Koijen (2021).

Now we consider a more general case where the bonds provide liquidity services, $v''(B) < 0$. The asset returns and prices can be obtained from (14), (16), and (17) together with (8), (5) and (13). Since these equations do not involve any state variable, there are no transition dynamics, and therefore we drop the time subscript. After substituting goods market clearing, $C_t = Y$ and (16) into (14), we obtain

$$u'(Y) = \beta R^b u'(Y) + v'\left(B^g\right), \tag{19}$$

which pins down the bond interest rate given the bond supply, B^g. Substituting (8) into (17), we have

$$Q = QS^g + \left(1 - \tilde{s}^b(R^b, \tilde{R}^s(Q))\right)\left[\kappa_0 + \kappa_1 Q\right]. \tag{20}$$

The two equations (19) and (20) fully characterize two unknowns, Q and R^b. Solving the two equations gives the following results without yield curve control ($dB^g = QdS^g$).

Proposition 2. *Assume $v''(B) < 0$. Without yield curve control, the central bank stock purchases raise both the bond interest rate,*

$$\frac{dR^b}{dS^g} = \frac{-v''(B^g)}{\beta u'(Y)} > 0, \tag{21}$$

and the effect on stock price is ambiguous:

$$\frac{d\ln Q}{dS^g} = Q\frac{1 + \frac{v''(B^g)}{u'(Y)}A\theta s^b(1-s^b)}{\left[(1 - S^g - \kappa_1(1-s^b))Q + A\theta s^b(1-s^b)\frac{Y}{Y+Q}\right]}. \tag{22}$$

The proposition shows that when the demand for bonds is downward sloping, $v''(B) < 0$, the central bank stock purchases financed with the issuance of bonds (central bank reserves) raise the bond interest rate. When the central increases the supply of bonds. Therefore, with an inelastic bond market, a rise in bond interest rate is required to clear the bond market. This result is qualitatively consistent with our empirical finding that central bank stock purchases without yield curve control raise the bond interest rates. Moreover, expression (21) shows that how much the bond interest rate responds relative to the stock price is governed by the degree to which bond demand slopes down, $|v''(B)|$.

The rise in interest rates puts downward pressure on the stock price response because it discourages investment in stocks. Equation (22) shows that a higher value of $|v''(B)|$ dampens or even reverses the stock price response. We say the bond market is more inelastic relative to the stock market whenever $v''(B^g)/u'(Y) < -\frac{1}{A\theta s^b(1-s^b)}$ so that a flow from bonds to stocks lowers the stock price. This is what we will find in our calibration exercise below.

We next turn to the case with yield curve control.

Proposition 3. *Assume $v''(B) < 0$. With yield curve control, the central bank stock purchases have no effect on the bond interest rate and raise the stock price. The size of the stock price response is given by* (18) *and is larger than without yield curve control.*

The part where there is no effect on the bond interest rate is by construction. The next part of the proposition establishes that the size of the stock price response is larger relative to the case without yield curve control. This is because when the interest rate does not rise, a further fall in stock return is required to clear the stock market, which results in a further rise in stock price. This result is qualitatively consistent with our empirical finding that after the introduction of yield curve control, we tend to see a more robust rise in the stock price in response to the central bank stock purchases.

5.3 Central Bank Stock Purchases: Model vs. Data

The previous results highlight that our model is at least qualitatively consistent with the empirical evidence we document. We now explore the model's ability to account for the data quantitatively. Throughout, we work with a first-order approximation around the steady state.

We summarize the calibration of baseline parameters in Table 2. We calibrate our model to the Japanese economy at an annual frequency, although the frequency is irrelevant since our model does not have transitional dynamics. We normalize the output in the economy to one, $Y \equiv 1$. The household discount factor is set to $\beta = 0.86$, a value consistent with the average annual discount rate of 14% reported in Kureishi et al. (2021). We set the steady-state supply of bond to 150% of output, $B^g = 1.5$. This corresponds to the stock of Japanese government bonds averaged over the period 2010-2020, which we obtained from the Ministry of Finance website. The steady-state value of government stock holding is set to zero, $S^g = 0$. The degree to which a fund's value responds to the stock price is set to one, $\kappa_1 = 1$. Since the output is constant, we can normalize $u'(Y) \equiv 1$ without loss of generality. We parameterize the bond in utility as $v(B) = \bar{v}\frac{B^{1-\eta}}{1-\eta}$.

We then choose the three parameters $\{\bar{v}, s^b, \kappa_0\}$ to exactly match the following three moments:[13] (i) the household liquid wealth as a share of the total household wealth ($B/(A+B)$) of 10% obtained from 2014 and 2019 waves of the National Survey of Family Income and Expenditure,[14] (ii) the average net long-term interest rate of 0.4 p.p. during the period 2010-2020, (iii) the average stock return of 11 p.p. during the period 2010-2020. The latter two are obtained from the macro history database (Jordà et al., 2019).

Finally, we choose two parameters $\Theta \equiv (\eta, \theta)$, the parameters that govern the stock market and bond market inelasticity, to best fit our empirical estimates of stock price response to the central bank stock purchases with and without yield curve control and the interest rate response to the central bank stock purchases

[13] Calibrating s^b is equivalent to calibrating μ^b/μ^s.

[14] We define the checking and saving accounts as liquid wealth. We take the average of the 2014 and 2019 waves.

Parameter	Description	Value	Source/Target
β	Discount factor	0.86	Kureishi et al. (2021)
B^g	Supply of bonds	1.5	Debt to GDP ratio
κ_1	Fund's wealth slope	1.0	—
κ_0	Fund's wealth intercept	0.4	Liquid wealth share
s^b	Fund's bond portfolio share	0.05	Stock return
$\bar{v}$	Bond convenience yield coefficient	35.5	Bond interest rate
η	Bond convenience yield curvature	13.7	Match estimates
θ	Fund's portfolio elasticity	2.9	Match estimates

Table 2: Calibration of Baseline Parameters

without yield curve control, which we denote in a vector format as $\boldsymbol{\alpha} \equiv [\alpha_Q, \alpha_Q^{YCC}, \alpha_{R^b}]$, and its model counterpart under the parameter Θ is denoted as $\boldsymbol{\alpha}(\Theta) \equiv [\alpha_Q(\Theta), \alpha_Q^{YCC}(\Theta), \alpha_{R^b}(\Theta)]$. Formally, we set Θ at the solution of the following problem

$$\hat{\Theta} = \arg\min_{\Theta} (\boldsymbol{\alpha}(\Theta) - \boldsymbol{\alpha})' \boldsymbol{\Sigma}^{-1} (\boldsymbol{\alpha}(\Theta) - \boldsymbol{\alpha}), \tag{23}$$

where $\boldsymbol{\Sigma}$ is the weighting matrix. For $\boldsymbol{\alpha}$, we use the next day response reported in the baseline row of Table 1, and we set Σ as a diagonal matrix containing the variance of our estimates in each element. We obtain $\eta = 13.7$ with a standard error of 0.86 and $\theta = 2.9$ with a standard error of 1.8, where standard errors are computed using the asymptotic covariance matrix of Θ, $\frac{\partial \boldsymbol{\alpha}(\Theta)}{\partial \Theta}' \boldsymbol{\Sigma}^{-1} \frac{\partial \boldsymbol{\alpha}(\Theta)}{\partial \Theta}$. Since two parameters are calibrated to fit three moments, the parameters are over-identified.

Table 3 shows the calibrated model can quantitatively replicate the impact of central bank stock purchases we have estimated. The baseline model replicates the rise in interest rate and a fall in stock price in response to the central bank stock purchases without yield curve control as well as the rise in stock price with yield curve control. This is because our calibration features a bond market that is more inelastic than the stock market. As is made precise in Proposition 2, when $v''(B^g) < -\frac{u'(Y)}{A\theta s^b(1-s^b)}$, which is our calibration, the stock price falls in response to the central bank stock purchases without yield curve control. This does not mean

	Stock price response		Interest rate response	
	No YCC	YCC	No YCC	YCC
0. Data	-16.95	22.23	1.41	0.00
	(26.27)	(11.13)	(0.68)	(0.13)
1. Baseline Model	-12.72	16.81	1.45	0.00
2. Elastic bond market ($\eta = 0$)	16.81	16.81	0.00	0.00
3. Elastic stock market ($\theta = \infty$)	-29.52	0.00	1.45	0.00
4. Elastic stock & bond market	0.00	0.00	0.00	0.00

Table 3: Central Bank Stock Purchases: Model vs. Data

Notes: The table reports the response of stock price and the interest rate to the central bank stock purchases, $\frac{dQ}{QdS^g}$ and $\frac{dR^b}{dS^g}$, both in the data and in the model. The data is taken from the baseline rows of Table 1 with standard errors in parenthesis.

the stock market is elastic. In fact, when yield curve control is in place, so that the bond market is effectively elastic, the stock price shows a strong response. Quantitatively our model implies that a flow into the stock market of \$1 leads to a rise in the stock market value of \$16.81, which is roughly in line with our empirical estimates of \$22.23. This is several times larger than the baseline estimates of Gabaix and Koijen (2021), which shows a stock price response of \$5 to \$1 inflow.

In row 2, we contrast the baseline model with a model with an elastic bond market ($\eta = 0$). With an elastic bond market, the interest rate shows no response by construction and the stock price responds exactly in the same way with and without yield curve control. As explained in Proposition 1, this is because the households' Euler equation pins down the bond interest rate, irrespective of financial flows. As a result, the impact of flows is entirely absorbed by the stock prices. In row 3, we show a model with an elastic stock market is unable to explain the substantial positive response of stock price under yield curve control. With a perfectly elastic stock market, the stock return keeps track of the bond interest rate one-for-one. Since the yield curve control fixes the bond interest rate, the stock price does not respond either. Finally, in row 4, a model in which both the bond and the stock market are perfectly elastic predicts the no effect from the central bank stock purchases. As mentioned in Proposition 1, this is the case where the central bank balance sheet is neutral (Wallace, 1981). All these results are inconsistent with our empirical findings, leading us to reject a model in which either the stock market or the bond market, or both, are elastic.

6 Concluding Remarks

How does a flow into the stock market impact the financial market? To answer this question, we exploit the discontinuities in the Bank of Japan's policy rule. We empirically show that central bank stock purchases have a far-reaching impact on the financial market. In normal times, the long-term interest rate is the main absorber of the intervention, while the impact on stock price is noisy zero. When yield curve control – another unconventional monetary policy – is implemented simultaneously, the stock price, rather than the long-term interest rate, becomes the main absorber of the central bank stock purchase. Through the lens of the model, we argue that, while these results support the notion that the stock market is inelastic, taking into account an even more inelastic bond market is necessary to account for our empirical findings.

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Appendix

A Empirical Appendix

A.1 Local Nonlinear Impulse Response Function

In this section, we allow nonlinearity in the impulse response and show that our estimands can be interpreted as dynamic "local average treatment effect" in the spirit of Angrist and Imbens (1995).

We first define a potential outcome framework in our context following Rambachan and Shephard (2021). For each $t \geq 1$, the BoJ decides the amount of ETF purchases ETF_t and let us denote $ETF_{1:t} \equiv (ETF_1, \cdots, ETF_t)$. Let $w_{1:t} \equiv (w_1, \cdots, w_t)$ be a potential assignment path up to t where $w_t \in [0, \bar{w}]$ for all t. Associated with this potential assignment path, the potential outcome at day $t+l$ time h is $Y_{t+l,h}(w_{1:t+l})$.[15] Note that for any different assignment paths, there exist different outcome paths but we only observe $Y_{t+l,h}(ETF_{1:t+l})$. For any day $t+l$ time h, let us denote

$$Y_{t+l,h}(w) \equiv Y_{t+l,h}(ETF_{1:t-1}, \underbrace{w}_{t-th}, ETF_{t+1:t+l}).$$

Using this notation, the observed outcome can be denoted as $Y_{t+l,h}(ETF_t)$ by definition. Now, let the outcome be$\Delta y_{t+l,h} \equiv y_{t+l,h} - y_{t,0}$, which is the change in variable y (e.g. stock prices and interest rates) from the end price of the morning session to at day t to time h at day $t+l$,

$$\Delta y_{t+l,h} = Y_{t+l,h}(ETF_t). \tag{A.1}$$

We assume that the BoJ's ETF purchasing policy rule takes the following form, in

[15] We assume that the potential outcome depends only on past and contemporaneous assignments. Rambachan and Shephard (2021) called this assumption Non-anticipating potential outcomes.

which the amount of ETF purchase at time t, ETF_t, is given by

$$ETF_t = ETF_{-,t}(\Delta p_t)\mathbb{I}(\Delta p_t < c_t) + ETF_{+,t}(\Delta p_t)\mathbb{I}(\Delta p_t \geq c_t), \tag{A.2}$$

where Δp_t is the log-changes in the TOPIX value in the morning, c_t is the cut-off, and $ETF_{-,t}$ and $ETF_{+,t}$ are random functions of Δp_t which represent different policy rules depending on whether Δp_t is above or below the cutoff at time t. The following assumptions guarantee that our estimands identify the dynamic local average treatment effect.

Assumption 1. *(i) $Y_{t+l,h}(w)$ is bounded and continuously differentiable in $w \in [0, \bar{w}]$ with probability one and, (ii) $ETF_{-,t}(\Delta p)$ and $ETF_{+,t}(\Delta p)$ are bounded and continuous at c_t with probability one.*

Assumption 2 (Monotonicity). *$ETF_{-,t}(c_t) \geq ETF_{+,t}(c_t)$ with probability one.*

Assumption 3 (Relevance). $\int \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)|\Delta p_t = c_t)dw > 0.$

Assumption 4 (Local Independence). *For each $t+l$ and h, there exists a neighborhood $N_{t+l,h}$ of c_t such that $\Delta p_t \perp (\{Y_{t+l,h}(w)\}_w, ETF_{-,t}(c_t), ETF_{+,t}(c_t))|\Delta p_t \in N_{t+l,h}$.*

Theorem 1. *If Assumptions 1-4 hold, then*

$$\frac{\lim_{\Delta p \uparrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[\Delta y_{t+l,h}|\Delta p_t = \Delta p]}{\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t|\Delta p_t = \Delta p]}$$
$$= \int \mathbb{E}[\frac{\partial Y_{t+l,h}(w)}{\partial w}|\Delta p_t = c_t, ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)]\bar{\omega} dw,$$

where $\bar{\omega} = \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)|\Delta p_t = c_t) / \int \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)|\Delta p_t = c_t)dw$.

Proof. First, observe that

$$\lim_{\Delta p \uparrow c_t} \mathbb{E}[\Delta y_{t+l,h} | \Delta p_t = \Delta p] = \lim_{\Delta p \downarrow c_t} \mathbb{E}[Y_{t+l,h}(ETF_{-,t}(\Delta p)) | \Delta p_t = \Delta p]$$
$$= \mathbb{E}[Y_{t+l,h}(ETF_{-,t}(c_t)) | \Delta p_t = c_t],$$

where the second equality follows from Assumptions 1 and 4. Therefore,

$$\begin{aligned}
&\lim_{\Delta p \uparrow c_t} \mathbb{E}[\Delta y_{t+l,h} | \Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[\Delta y_{t+l,h} | \Delta p_t = \Delta p] \\
&= \mathbb{E}[Y_{t+l,h}(ETF_{-,t}(c_t)) - Y_{t+l,h}(ETF_{+,t}(c_t)) | \Delta p_t = c_t] \\
&= \mathbb{E}[\int \frac{\partial Y_{t+l,h}(w)}{\partial w} \mathbb{I}\{ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)\} dw | \Delta p_t = c_t] \\
&= \int \mathbb{E}\left[\frac{\partial Y_{t+l,h}(w)}{\partial w} | \Delta p_t = c_t, ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t)\right] \\
&\quad \times \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t) | \Delta p_t = c_t) dw,
\end{aligned}$$

where second equality follows from Assumptions 1 and 3, and the third equality follows from 1. Similarly,

$$\begin{aligned}
&\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_t | \Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_t | \Delta p_t = \Delta p] \\
&= \int \Pr(ETF_{-,t}(c_t) \geq w \geq ETF_{+,t}(c_t) | \Delta p_t = c_t) dw,
\end{aligned}$$

and Assumption 2 guarantees that the denominator is positive. Combining these, we have the stated result. □

Since $Y_{t+l,h}(w) \equiv Y_{t+l,h}(ETF_{1:t-1}, w, ETF_{t+1:t+l})$, the local independence assumption requires that falling below the cutoff at day t is not correlated with the future or past ETF purchases. We test this in Appendix A.4.

A.2 Details on Cutoff Estimation

We first split the sample based on six announcements by the BoJ that publicized changes in the target amount of ETF purchases on April 4, 2013, October 31, 2014, December 18, 2015, July 29, 2016, July 31, 2018, and March 16, 2020. We then divide each sample based on whether TOPIX value falls below zero for the past two consecutive days. For the case with consecutive drops in the past two days, we further split on April 1, 2019, for the reason that we describe below.

In each sample split, we proceed as follows. We take grid points for the cutoff candidates from -1% to 0% with 0.05% interval, $\mathbb{C} = \{-1.0\%, -0.95\%, \ldots, -0.05\%, 0.0\%\}$. For each of $c \in \mathbb{C}$, we estimate the following linear probability model separately on both sides of the candidate cutoff, c:

$$\Pr_{-,t}(ETF_t > 0|\Delta p_t) = \begin{cases} \alpha_- + \beta_- \Delta p_t & \text{for } \Delta p_t \in [c-k, c] \\ \alpha_+ + \beta_+ \Delta p_t & \text{for } \Delta p_t \in [c, c+k] \end{cases}, \tag{A.3}$$

where we take the bandwidth to be 1% around the cutoff, $k = 1\%$. Given the estimates, we can compute the jump around the cutoff as follows:

$$J_t(c) \equiv \lim_{\Delta p \uparrow \bar{c}} \widehat{\Pr}_t(ETF_t > 0|\Delta p) - \lim_{\Delta p \downarrow \bar{c}} \widehat{\Pr}_t(ETF_t > 0|\Delta p),$$

where $\widehat{\Pr}_t$ denote the fitted value of equation (A.3). We select the cutoff that maximizes square of the jump:

$$c_t^* \in \arg\max_{c \in \mathbb{C}} J_t^2(c).$$

Whenever there is a tie, we choose the largest cutoff.

Table B.2 shows the estimated cutoff, and Table B.3 shows the discontinuity in the probability of the Bank of Japan's intervention around the estimated cutoff. As argued in the main text, the estimated cutoffs align well with what is commonly argued among media. The discontinuity around the cutoff is always over 50%, is often over 80%, and they are highly statistically significant. We made a choice to split the sample with consecutive drops in the past two days on April 1, 2019, because there was an apparent change in the cutoff around this period. If we do not split the sample at this point in time, the resulting discontinuity is -0.744 . If we split the sample, the discontinuity is -1.000 in the first half, and it is -0.853 in the second half of the sample. This choice does not materially affect any of our empirical results.

Figure B.1 graphically displays the discontinuity in the probability of intervention for each period. While the magnitude of discontinuity is more apparent in the beginning and the end of the sample period, the sharp discontinuity shows up in all subsamples.

A.3 Manipulation Test

A typical concern in regression discontinuity-based identification strategies is manipulation (McCrary, 2008). We first note that this concern is unlikely in our context since there is little room for investors to precisely manipulate the stock price index. Having said this, we formally test the presence of manipulation by examining the continuity of density function of TOPIX changes in the morning. We estimate the density function using the local polynomial density estimator by Cattaneo et al. (2020) and test the presence of discontinuity around our estimated cutoff.

Figure B.2 shows the estimated density and histogram, and Table B.4 reports the estimates and test statistics for discontinuity. While there is a small mass on the right side of the cutoff, the p-value of testing the discontinuity is 0.447. Therefore, there is no statistical evidence of manipulation.

A.4 (Dis)continuity of ETF Purchases across Days

In this section, we argue that the effects we are identifying are the effects of a one-time shock of ETF purchases. As discussed in A.1, $y_{t+l,h}$ is clearly affected by the BoJ's ETF purchases up to l days later. Therefore, if falling below the cutoff today is correlated with future and past purchases, our empirical estimates cannot be interpreted as the causal effect of one-time BoJ ETF purchases (Rambachan and Shephard, 2021). In order to address this concern, we estimate the discontinuity in the amount of ETF purchases around the cutoff across days. Formally, we estimate the following term,

$$\lim_{\Delta p \uparrow c_t} \mathbb{E}[ETF_{t+l} | \Delta p_t = \Delta p] - \lim_{\Delta p \downarrow c_t} \mathbb{E}[ETF_{t+l} | \Delta p_t = \Delta p]. \tag{A.4}$$

Figure B.3 shows the estimates of discontinuity of the amount of ETF purchases at date $t + l$ around $\Delta p_t = c_t$. Reassuringly, we find significant discontinuity only at $l = 0$. Therefore, our identified effects are the causal effects of one-time BoJ's ETF purchases and are not contaminated by future or past ETF purchases.

B Additional Tables and Figures

Table B.1: Major Announcements by the BoJ

Date	Announcement
October 28, 2010	Intention to purchase 450 billion yen of ETFs
October 30, 2012	Intention to purchase 2.1 trillion yen of ETFs annually
October 31, 2014	Annual purchase target increased to 3 trillion yen
December 18, 2015	Annual purchases target increased to 3.3 trillion yen
July 29, 2016	Annual purchases target increased to 6 trillion yen
March 16, 2020	Annual purchases target increased to 12 trillion yen

Notes: Table B.1 shows the six major announcements by the BoJ regarding the target ETF purchase amounts. Source: Fukuda and Tanaka (2022).

Table B.2: Estimated Cutoff

No Consecutive Drops		Consecutive Drops	
Period	Cutoff	Period	Cutoff
2010/12/15 - 2013/04/03	-1%	2010/12/15 - 2013/04/03	-1%
2013/04/04 - 2014/10/30	-0.35%	2013/04/04 - 2014/10/30	0%
2014/10/31 - 2015/12/17	-0.15%	2014/10/31 - 2015/12/17	0%
2015/12/18 - 2016/07/28	-0.4%	2015/12/18 - 2016/07/28	0%
2016/07/29 - 2018/07/30	-0.3%	2016/07/29 - 2018/07/30	0%
2018/07/31 - 2020/03/15	-0.5%	2018/07/31 - 2020/03/15	-0.25%
2020/03/16 - 2020/12/31	-0.5%	2020/03/16 - 2020/12/31	-0.25%

Notes: Table B.2 shows the estimated cutoff for each of the subsamples.

Table B.3: Discontinuity in Probability of Intervention around the Estimated Cutoff

	No Consecutive Drops			Consecutive Drops		
	Discontinuity estimates	Sample size Left	Sample size Right	Discontinuity estimates	Sample size Left	Sample size Right
2010/12/15 - 2013/04/03	-1.011	43	158	-1.000	15	47
	(0.012)			(0.000)		
2013/04/04 - 2014/10/30	-0.576	60	146	-0.931	25	30
	(0.100)			(0.072)		
2014/10/31 - 2015/12/17	-0.683	72	98	-1.122	11	17
	(0.099)			(0.117)		
2015/12/18 - 2016/07/28	-0.811	20	36	-1.000	8	11
	(0.119)			(0.000)		
2016/07/29 - 2018/07/30	-0.604	78	243	-0.945	40	33
	(0.069)			(0.053)		
2018/07/31 - 2020/03/15	-0.978	49	163	-0.744	30	34
	(0.022)			(0.130)		
2020/03/16 - 2020/12/31	-0.930	29	71	-0.985	13	14
	(0.070)			(0.022)		

Notes: Table B.3 shows the discontinuity in the probability of the BoJ intervention around the estimated cutoff. We estimate the discontinuity using the local linear regression with bandwidth 1% around the cutoff and uniform kernel. The standard errors are reported in parentheses.

Table B.4: Density Discontinuity Test

	Density estimates		Discontinuity test	
	Left	Right	Difference	p-value
	0.423	0.506	0.083	0.373
	(0.065)	(0.067)	(0.093)	
Sample size	667	1790		
Bandwidth	0.512	0.512		
Effective sample size	358	719		

Notes: Table B.4 reports the density estimates on the left and the right of the cutoff and test statistics for the discontinuity test. We use the local polynomial density estimator by Cattaneo et al. (2020) with order 2. Robust standard errors are reported in parenthesis.

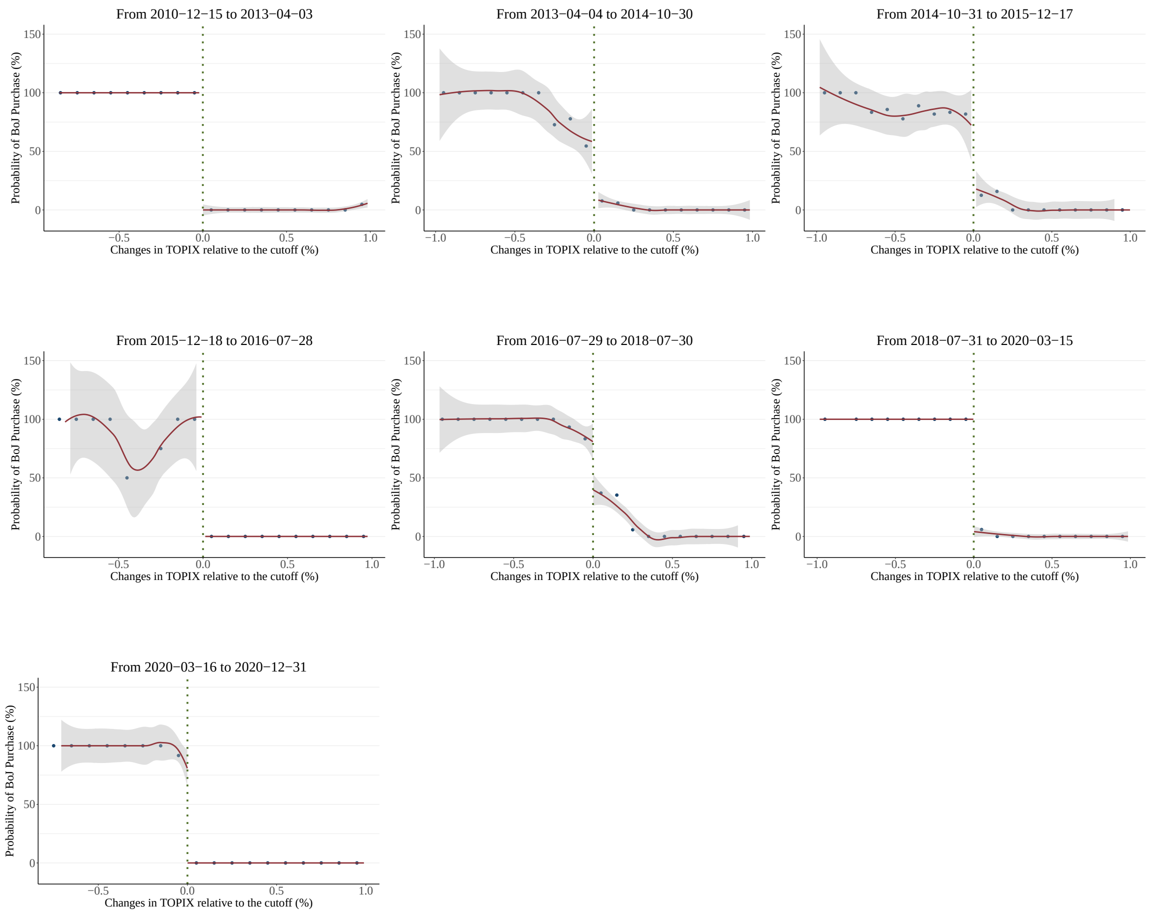


Figure B.1: Discontinuity in the Probability of the BoJ Intervention for each Period
Notes: Figure B.1 shows the discontinuity in the probability of the BoJ intervention around the estimated cutoff for each period. The blue scatter plot is the binned scatter plot with bin width 0.1%, and the red line indicates the LOESS fit with the shaded gray area being the 95% confidence interval.

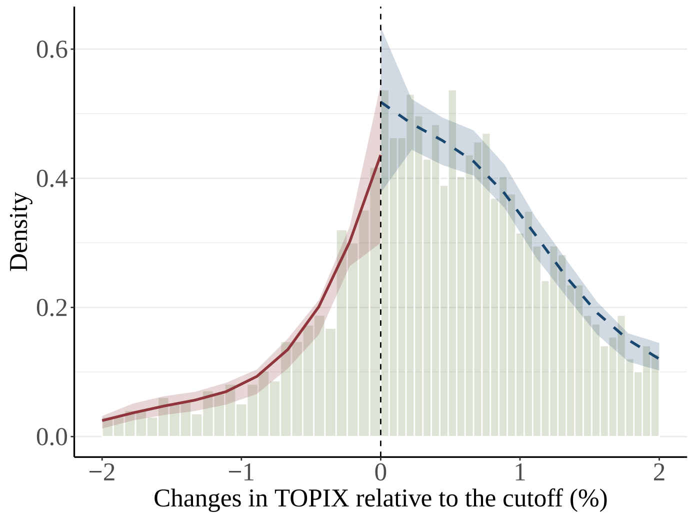


Figure B.2: Density around the cutoff

Notes: Figure B.2 shows the histogram and the density of changes in TOPIX relative to the cutoff. The shaded area is 95% confidence interval. We use the local polynomial density estimator by Cattaneo et al. (2020) with order 2.

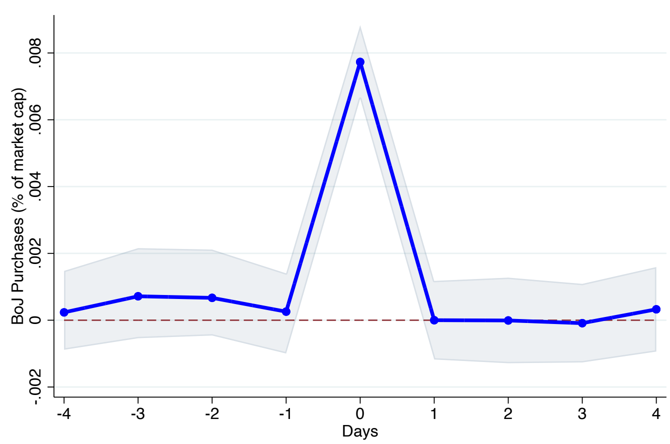
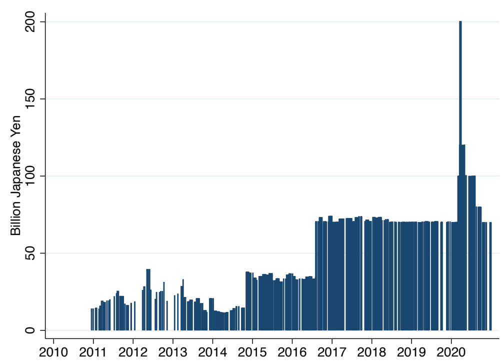


Figure B.3: (Dis)continuity of ETF Purchases across Days

Notes: Figure B.3 shows the estimates of (A.4) across days. The shaded areas represent 90% confidence intervals, which accounts for heteroskedasticity and autocorrelation.

(A) Absolute Amount



(B) Fraction of Market Capitalization

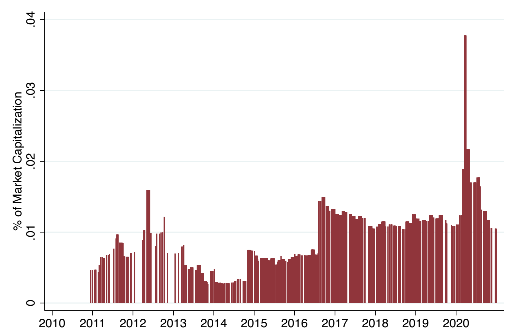
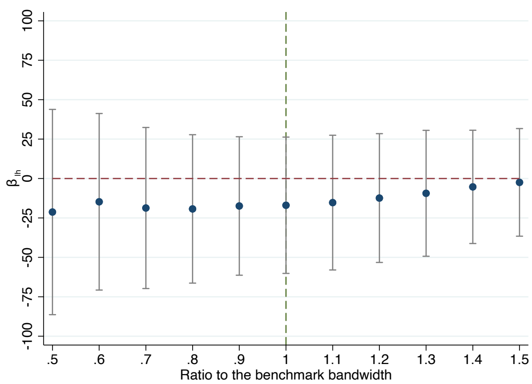


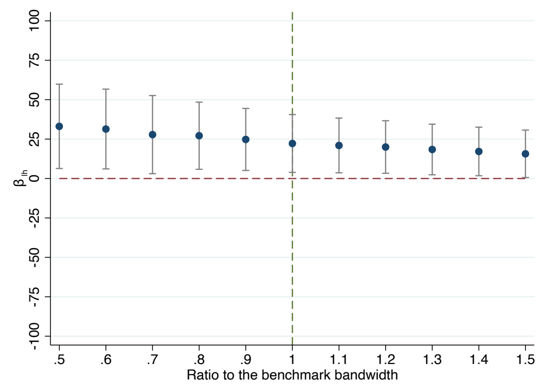
Figure B.4: The Amount of the BoJ Purchases

Notes: Figure B.4 plots the amount of stock purchases by the BoJ in each intervention. Figure B.4A shows the absolute amount of purchases in billion Japanese Yen (approximately 10 million US dollars). B.4B express it as a fraction of market capitalization.

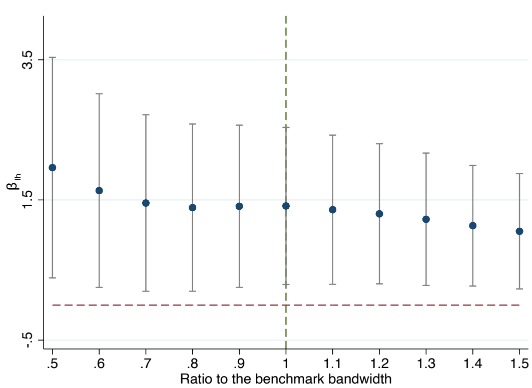
(A) Stock Price before YCC



(B) Stock Price after YCC



(C) JGB 10-year Yield before YCC



(D) JGB 10-year Yield after YCC

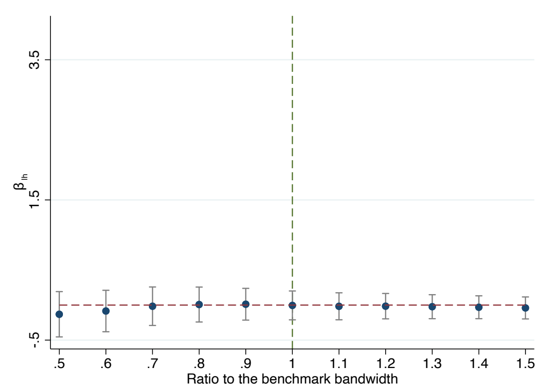


Figure B.5: Robustness to Bandwidth Selection

Notes: Figure B.5 shows the robustness of our estimates with respect to the size of the bandwidth. Each dot represents the point estimates of the response from 11AM of the intervention day to 9AM on the next day. The vertical line represents the 90% confidence interval, which accounts for heteroskedasticity and autocorrelation. The dashed green line is the optimal bandwidth proposed by Calonico, Cattaneo, and Titiunik (2014), which is our benchmark. Figures B.5A and B.5B show the response of stock price before and after YCC, respectively. Figures B.5C and B.5D show the response of 10-year JGB yield before and after YCC, respectively.

C Theory Appendix

C.1 Proofs

C.1.1 Proof of Proposition 1

The household's Euler equation (14) together with goods market clearing conditions $C_t = Y$ and constant $v'(B)$ imply

$$R^b \equiv \left(1 + v'(B)/u'(Y)\right)/\beta \quad \text{for all } t. \tag{C.1}$$

This immediately implies that the bond interest rate is invariant to central bank stock purchases. The stock price Q_t solve (17):

$$Q_t = Q_t S_t^g + \frac{\mu^s (R^s_{t+1})^\theta}{\sum_a \mu^a (R_{t+1})^\theta} [\kappa_0 + \kappa_1 Q_t] \tag{C.2}$$

with $R^b_{t+1} = R^b$ and

$$R^s_{t+1} = (Q_{t+1} + Y)/Q_t. \tag{C.3}$$

Totally differentiating (C.2),

$$Qd\ln Q_t = QS^g d\ln Q_t + QdS^g_0 - s^s(1-s^s)A\theta d\ln R^s_{t+1} + s^s\kappa_1 Qd\ln Q_t, \tag{C.4}$$

and totally differentiating (C.3),

$$d\ln R^s_{t+1} = \frac{1}{R^s} d\ln Q_{t+1} - d\ln Q_t. \tag{C.5}$$

Since these equations do not involve any state variable and the shocks are permanent, there are no transition dynamics, $d\ln Q_{t+1} = d\ln Q_t \equiv d\ln Q$. Imposing it, we can solve (C.4) and (C.5) to obtain

$$\frac{d\ln Q}{dS^g} = \frac{Q}{Q(1 - S^g - s^s\kappa_1) + (1-s^b)s^b \bar{A}\theta \frac{Y}{Y+Q}} > 0, \tag{C.6}$$

as desired. Note that $\frac{d \ln Q}{dS^g} = \frac{dQ}{QdS^g}$

C.1.2 Proof of Proposition 2

Linearizing (19) yields

$$\beta R^b d \ln R^b = -\frac{v''(B^g)}{u'(Y)} dB^g. \tag{C.7}$$

Linearizing (20) yields

$$\left[(1 - S^g - \kappa_1 s^s)Q + \bar{A}\theta s^b(1 - s^b)\frac{Y}{Y+Q}\right] d \ln Q = QdS^g - \bar{A}\theta s^b(1 - s^b) d \ln R^b. \tag{C.8}$$

Plug (C.7) into (C.8) to obtain

$$d \ln Q = \frac{1}{\left[(1 - S^g - \kappa_1(1 - s^b))Q + \bar{A}\theta s^b(1 - s^b)\frac{Y}{Y+Q}\right]} \left(QdS^g + \frac{v''(B^g)}{u'(Y)} \bar{A}\theta s^b(1 - s^b) dB^g \right).$$

The statement in the propositions follows after imposing $dB^g = QdS^g$.

C.1.3 Proof of Proposition 3

The proposition follows from expression (C.8) after imposing $d \ln R^b = 0$.