Download the template Jupyter notebook `HW1_Template.ipynb` from Canvas and work from that template. For Problem 1, you can write your solutions down either 1) in Jupyter using markdown and Latex or 2) handwritten on paper and scanned/submitted to Canvas. You should be able to complete Problem 2 using only basic operations and `numpy`, but feel free to use whatever you are comfortable with (e.g., `pandas`). We will cover `pandas` and make more extensive use of it in the coming weeks.

1. (3 pts; this is Problem 1.3 from Gregory) We saw in class (and in Section 1.4.1) that based on the saliva test result and the prior information, the probability that the person had the unknown disease (UD) was found to be 0.43%. Subsequently, the same person received an independent blood test for UD and again tested positive. If the false-negative rate for this test is 1.4% and the false-positive rate is 0.5%, what is the new probability that the person has UD on the basis of both tests?

2. **CODING:** This problem is meant to provide a warm up in Python.

   (a) (2 pts) Write a function that simulates a sequence of n coin flips with probability of heads H. i.e.,

   ```
   flips = flipOneSet(n, H)
   ```

   where `flips` is an array of "H" and "T" for heads and tails, respectively.

   (b) (2 pts) Write a function that takes the array from above and returns the number of heads and tails. i.e.,

   ```
   nheads, ntails = countFlips(flips)
   ```

   (c) (3 pts) Write a function that takes the array from part a) and counts the number of occurrences of X consecutive heads. For example, in a sequence of 10 flips for which the result is [H, H, H, T, H, T, H, H, H, T], the result for $X = 3$ should be 2; there are 2 instances of 3 consecutive heads (1-2-3 and 7-8-9). The result for $X = 2$ should also be 2 (rather than 4). i.e.,

   ```
   nconsecutive = countConsecutives(flips, X)
   ```

   (d) (3 pts) Using the functions above, write another function that calculates the probability for a sequence of n coin flips with H to result in *at least one* instance of X consecutive heads.

   ```
   prob = probConsecutives(n, H, X)
   ```

   (HINT: You can do this by running `flipOneSet` multiple times (they need to be "sufficiently" large) and measuring the fraction of occurrences.

3. **CODING:** We saw in class that the posterior probability distribution function (PDF) for H = probablity of heads for a sequence of three coin flips $D = h, h, t$ is proportional to $H^2(1 - H)$. i.e.,

   $$p(H|D, I) = AH^2(1 - H).$$

   (a) (2 pts) Calculate the constant A that normalizes the PDF, i.e.,

   $$\int_0^1 p(H|D, I)dH = 1. \tag{1}$$

(b) By calculating the PDF on a grid of H values (e.g., `np.arange(0, 1.01, 0.01)`, calculate the ollowing quantities.

i. (1 pt) posterior mean $\langle H \rangle$ defined as

$$\langle H \rangle = \int_0^1 H p(H|D,I) dH \tag{2}$$

ii. (1 pt) posterior median $\tilde{H}$ such that

$$\int_0^{\tilde{H}} p(H|D,I) dH = 0.5 \tag{3}$$

iii. (1 pt) posterior mode where $p(H|D,I)$ is a maximum,

$$\left. \frac{dp(H|D,I)}{dH} \right|_{H_m} = 0 \tag{4}$$

iv. (2 pts) credible $C = 90\%$ region R as defined in the textbook, i.e., provide the lower and upper bounds H_l and H_u for R such that

$$\int_{H_l}^{H_u} p(H|D,I) dH = 0.90 \tag{5}$$