Download the template Jupyter notebook `HW5_Template.ipynb` from Canvas and work from that template.

1. **CODING:** Students in PHYS0150 were tasked to measure the local acceleration of gravity g by dropping an object from height H, measuring the time t it takes to reach the ground, and using the formula,
$$H = \frac{1}{2} g t^2. \tag{1}$$
The datafile `GravityMeasurements.dat` contains measurements of the value of H in meters, its uncertainty σ_{H_i}, time t in seconds, and its uncertainty σ_{t_i} made independently by $N = 350$ students. The file has five columns – 1) index $i = 0, ..., N-1$, 2) measurement $\{H_i\}$, 3) its corresponding uncertainty $\{\sigma_{H_i}\}$, 4) measurement $\{t_i\}$, and 5) its corresponding uncertainty $\{\sigma_{t_i}\}$ from each student.

```
0   20.066  1.120   2.092  0.084
1   20.363  0.389   2.023  0.050
2   21.498  0.706   1.939  0.115
3   19.709  0.791   1.963  0.118
4   21.192  1.225   2.132  0.097
5   16.631  1.479   1.877  0.112
...
```

   (a) (1 pts) Calculate the values of acceleration of gravity $\{g_i\}$.

   (b) (2 pts) Calculate the corresponding uncertainties $\{\sigma_{g_i}\}$ using the proper error propagation formulae.

   (c) (2 pts) In this part and the next one, ignore the uncertainties $\{\sigma_{g_i}\}$, but rather estimate it directly from the measurements by computing the standard deviation of $\{g_i\}$. What is the best estimate of this value $\tilde{\sigma}_g$?

   (d) (2 pts) Using this value of $\tilde{\sigma}_g$, calculate the maximum-likelihood estimate (MLE) of the mean of $\{g_i\}$ and the uncertainty in the mean from the measurements.

   (e) (2 pts) Now compute the inverse-variance weighted MLE of the mean of $\{g_i\}$ using the actual uncertainty $\{\sigma_{g_i}\}$ on each measurement. Also compute the uncertainty in the mean to show that it is smaller than the value $\tilde{\sigma}_g$ estimated in a).

2. **CODING:** Using the same dataset as above:

   (a) (3 pts) Calculate the χ^2 value defined as:
$$\chi^2 = \sum_{i=0}^{N-1} \left(\frac{g_i - \mu'}{\sigma_{g_i}} \right)^2 \tag{2}$$
   where μ' is the inverse-variance weighted MLE of the mean of $\{g_i\}$ from 1c).

   (b) (4 pts) Now calculate the χ^2 value as a function of μ' from $\mu' = 9.600$ to 10.000 in steps of 0.001 and show that the value of μ' that minimizes the χ^2 is indeed given by the answer from 1c).

(c) (4 pts) Finally, determine the lower and upper values of μ' at which the χ^2 is larger than the minimum value by 1.00. (NOTE: These values should match the $\pm 1\sigma$ range from the MLE in part 1e) and is another way to perform parameter estimation using the χ^2 statistic.)

(d) **EXTRA CREDIT:** (2 pts) Make a single plot that shows all of these facts with aappropriate labels to show the important features.