Download the template Jupyter notebook `HW4_Template.ipynb` from Canvas and work from that template. For Problem 1 and 2a), you can write your solutions down either 1) in Jupyter using markdown and Latex or 2) on handwritten on paper and scanned/submitted to Canvas.

1. (4 pts) For the uniform distribution $f_U(x|a,b)$ over the interval a to b,

$$f_U(x|a,b) = \begin{cases} 1/(b-a) & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

   calculate the expressions for the mean μ, variance σ^2, skewness Σ, and excess kurtosis $\tilde{K}$ in terms of a and b (HINT: the answers are in Section 5.8.2 of Gregory). The skewness and excess kurtosis defined here are:

$$\Sigma = \frac{1}{\sigma^3}\langle (x-\mu)^3 \rangle \tag{2}$$

$$\tilde{K} = \frac{1}{\sigma^4}\langle (x-\mu)^4 \rangle - 3 \tag{3}$$

2. For the binomial distribution,

$$p_B(x|n,p) = \binom{n}{x} p^x\, q^{n-x} = \frac{n!}{x!\,(n-x)!}\, p^x\,(1-p)^{n-x} \tag{4}$$

   for observing x of n items, where p and $q = (1-p)$ are the success and failure probabilities, respectively, using your favorite method, calculate in terms of n, p, and q the expressions for:

   (a) (4 pts) the mean μ and variance σ^2

   (b) **EXTRA CREDIT:** (1 pt) the skewness Σ

   (c) **EXTRA CREDIT:** (2 pts) the excess kurtosis $\tilde{K}$

   HINT: some of the answers are in Section 5.7.1 of Gregory.

3. **CODING:** *Drawing random deviates from arbitrary functions.* Although `np.random` has numerous functions from which you can draw random samples, in real life you will encounter cases that are not in `numpy`. This problem will guide you through the steps of writing your own routine that can sample from an arbitrary distribution function. Consider an asymmetric gaussian distribution function given by the following:

$$f_{2G}(x|\sigma_L,\sigma_R) \propto \begin{cases} e^{-x^2/2\sigma_L^2} & x < 0 \\ e^{-x^2/2\sigma_R^2} & x \ge 0 \end{cases} \tag{5}$$

   The function is continuous across $x = 0$.

   (a) (3 pts) Write a function that returns the value of $f_{2G}(x|\sigma_L,\sigma_R)$ normalized to unity, i.e., such that

$$\int_{-\infty}^{\infty} f_{2G}(x|\sigma_L,\sigma_R)dx = 1 \tag{6}$$

   The function should take three arguments 1) σ_L, 2) σ_R, and 3) `x`, a vector of x values.

(b) (3 pts) Write a function that returns the cumulative distribution function $F(x|\sigma_L, \sigma_R$ of the normalized $p_{2G}(x; \sigma_L, \sigma_R)$, i.e.,

$$F(x|\sigma_L, \sigma_R) = \int_{-\infty}^{x} f_{2G}(x'|\sigma_L, \sigma_R)dx' \tag{7}$$

This function should also take three arguments 1) σ_L, 2) σ_R, and 3) `x`, a vector of x values.

(c) (4 pts) Using the results from above, write a function that draws random deviates from p_{2G}. The sampler should take three arguments: 1) σ_L, 2) σ_R, and 3) `size`, the number of random numbers to return.

(d) (2 pts) Draw 10^6 random numbers from this distribution for $\sigma_L = 0.4$ and $\sigma_R = 1.2$ and plot a histogram of the values from $-2 \leq x \leq 5$. Overplot the analytic function on the same graph.