Download the template Jupyter notebook `HW3_Template.ipynb` from Canvas and work from that template. Problems 1a) and 2) are analytical questions.

1. (This is Problem 4.4 from Gregory.) A bottle contains three green balls and three red balls. The bottle is first shaken to mix up the balls.

   (a) (2 pts) Calculate analytically the probability that blindfolded, you will pick a red ball on the third pick, if you learn that at least one red ball was picked on the first two picks.

   (b) **CODING:** (3 pts) Write a Python function that simulates this an instance of drawing three balls from the bottle.

   (c) **CODING:** (3 pts) Using the function from b), empirically calculate the probability from part a).

2. (6 pts; this is Problem 4.7 from Gregory.) In a particular water sample, ten bacteria are found, of which three are of type A. Calculate analytically the probability of obtaining six type A bacteria, in a second independent water sample containing 12 bacteria in total. HINT: You may find the following definite integral to be helpful:

$$\int_0^1 dx\; x^k (1-x)^{n-k} = \frac{1}{(n+1)\binom{n}{k}}. \tag{1}$$

3. **CODING:** This problem demonstrates empirically several properties of counting statistics, which obeys the Poisson distribution. We will approximate a Poisson process by generating random values between 0 and 10, but only looking at the ones between 0 and 1. Technically, this is a multinomial distribution, but for large N and at least moderately large number of bins, it is a good approximation to a Poisson process.

   (a) (3 pts) Generate $N = 10^6$ uniform random deviates between 0 and 10. Count the number of deviates that fall in each of 100 equal-sized bins between 0 and 1 (0.00-0.01, 0.01-0.02, etc). Calculate the mean μ and variance σ^2 of the counts per bin. Make sure that the mean is almost exactly what you expect: $\bar{n} \approx N/100 = 1000$.

   (b) (3 pts) Repeat part a) for 10 and 1000 equally-sized bins. This empirically shows that $\mu = \sigma^2 = \bar{n}$, which is an important property of the Poisson distribution.

   (c) **EXTRA CREDIT:** (2 pts) Now generate $M = 100$ realizations of part a) and calculate the mean and variance of the counts per bin each time using 100 equally-sized bins. Make sure to not use the same random seed each time. You should now have $M = 100$ values of both the mean and variance. Let's call these μ_i and σ_i^2, respectively, where $i = 1, 2, ..., M$. Now calculate the mean and variance of μ_i and σ_i^2. This demonstrates that the general property that the variance (2nd moment) is much more difficult to measure accurately than the mean (1st moment).