1) 3 red + 3 green balls in bottle

Let A = red ball on 3^{rd} pick

B = red ball on at least one of first 2 picks

$$\Rightarrow p(A|B) = \frac{p(A)p(B|A)}{p(B)}$$

$\hookrightarrow$ $B_1 = R+G \quad \frac{3}{6}\cdot\frac{3}{5} = \frac{3}{10}$

$B_2 = G+R \quad \frac{3}{6}\cdot\frac{3}{5} = \frac{3}{10}$

$B_3 = R+R \quad \frac{3}{6}\cdot\frac{2}{5} = \frac{1}{5}$

$$p(A) = \frac{1}{2}, \quad p(B|A) = \overset{B_1}{\left(\overset{R}{\frac{2}{5}}\cdot\overset{G}{\frac{3}{4}}\right)} + \overset{B_2}{\left(\overset{G}{\frac{3}{5}}\cdot\overset{R}{\frac{2}{4}}\right)} + \overset{B_3}{\left(\overset{R}{\frac{2}{5}}\cdot\overset{R}{\frac{1}{4}}\right)}$$

$$= \frac{6}{20} + \frac{6}{20} + \frac{2}{20} = \frac{7}{10}$$

$$p(B) = \frac{3}{10} + \frac{3}{10} + \frac{1}{5} = \frac{4}{5}$$

$$\Rightarrow p(A|B) = \frac{\frac{1}{2}\cdot\frac{7}{10}}{\frac{4}{5}} = \frac{7}{20}\cdot\frac{5}{4} = \boxed{\frac{7}{16}} = 0.4375$$

2) D_1 = 3 out of 10 bacteria are type A
D_2 = 6 " " 12 " " " "

This is a binomial process with <u>unknown</u> H = prob. of A.

$$\Rightarrow \quad p(H|D_1) = \frac{p(H)\,p(D_1|H)}{p(D_1)} \qquad ①$$

This will give you $p(H|D_1)$, which you can use as a prior to determine $p(D_2|D_1)$.

$$p(H|D_2, D_1) = \frac{p(H|D_1)\,p(D_2|H, D_1)}{p(D_2|D_1)} \qquad ②$$

what we want.

- For ① $p(H)$ = uniform = 1

$$p(D_1|H) = \binom{10}{3} H^3 (1-H)^7$$

$$p(D_1) = \int_0^1 p(H)\,p(D_1|H)\,dH = \binom{10}{3}\int_0^1 H^3(1-H)^7 dH$$

$$= \binom{10}{3}\frac{1}{11\binom{10}{3}} = \frac{1}{11}$$

$$\Rightarrow p(H|D_1) = 11\binom{10}{3} H^3(1-H)^7 \qquad \binom{10}{3} = \frac{10!}{3!\,7!} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1}$$

$$= 1320\, H^3(1-H)^7 \qquad = 10\cdot 3\cdot 4 = 120$$

- Then, from ②:

$$p(D_2|D_1) = \int_0^1 p(H|D_1)\,p(D_2|H, D_1)\,dH$$

$$\hookrightarrow = \binom{12}{6} H^6 (1-H)^6$$

$$= 1320 \binom{12}{6} \underbrace{\int_0^1 H^9 (1-H)^{13} \, dH}_{\frac{1}{23} \frac{1}{\binom{22}{9}}}$$

$$\binom{12}{6} = \frac{12!}{6! \cdot 6!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 1} = 924$$

$$\binom{22}{9} = \frac{22!}{9! \, 13!} = 497{,}420$$

$$= \frac{1320 \cdot 924}{23 \cdot 497420} = \frac{792}{7429} \approx 0.10661$$

$$\therefore \boxed{p(D_2 | D_1) \approx 0.10661}$$