Finance Cheatsheet (Basic)

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Abstract

A collection of sad formulas that, despite hating rote memorization, we must be able to state reflexively and unconsciously for our profession.

I will add more as I realize they are necessary.

1. BS Model?
2. BS Equation?
3. Solution for European Call Option?
4. Solution for European Call Option using Fut?
5. Local Volatility Model?
6. Dupire's Local Volatility?
7. The inverse of Girsanov's Theorem?
8. Exponential Martingale SDE and its solution?

1 BS Model

$$\begin{cases} \dfrac{dS}{S} &= \mu dt + \sigma dW \\ \dfrac{dB}{B} &= r dt \end{cases}$$

2 BS Equation

$$rf = \frac{\partial f}{\partial t} + (r-q)S\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}$$

3 Solution for European Call Option

$$C = e^{-q(T-t)} S\Phi(d_+) - e^{-r(T-t)} K\Phi(d_-)$$

Here, $d_\pm$ and Φ are, respectively,

$$d_\pm = \frac{\log\dfrac{S}{K} + (r - q \pm \dfrac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy$$

4 Solution for European Call Option using Fut

Using Fut not only provides a simpler expression but also a formula that can be used even if the interest rate is not constant.

$$C = e^{-\int_0^T r(s)ds} \Big(F\Phi(d_+) - K\Phi(d_-) \Big)$$

Here, F and $d_\pm$ are, respectively,

$$F = Se^{\int_0^T r(s)ds}$$

$$d_\pm = \frac{\log\dfrac{F}{K} \pm \dfrac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$$

5 Dupire's Local Volatility

$$\sigma_{\mathrm{LV}}^2(T,K) = \frac{\dfrac{\partial C}{\partial T} + r(T)K\dfrac{\partial C}{\partial K}}{\dfrac{1}{2}K^2\dfrac{\partial^2 C}{\partial K^2}}$$

This resembles the BS equation, so you can recall it by mentally transforming the following.

$$\iff \qquad \frac{\partial C}{\partial T} + r(T)K\frac{\partial C}{\partial K} - \frac{1}{2}K^2\sigma_{\mathrm{LV}}^2(T,K)\frac{\partial^2 C}{\partial K^2} = 0$$

$$c.f., \text{ BS eqn:} \qquad rf = \frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 f}{\partial S^2}$$

6 The inverse of Girsanov's Theorem

$$W_t^{\mathbb{Q}} = W_t^{\mathbb{R}} + \int_0^t \gamma_s ds$$

$$\iff \qquad \frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left(-\int_0^T \gamma_t dW_t - \frac{1}{2}\int_0^T \gamma_t^2 dt\right)$$

Pay attention to the signs.

7 Exponential Martingale SDE and its solution

$$\frac{dX_t}{dt} = \sigma_t dW_t$$

$$\iff \qquad X_t = X_0 \exp\left(\int_0^t \sigma_s dW_s - \frac{1}{2}\int_0^T \sigma_s^2 dt\right)$$

Pay attention to the signs.