An Elegant Method for Incorporating the Wellbore Storage Effect in the Laplace Domain

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There is an interesting technique for incorporating the wellbore storage effect into the constant-rate analytical solution. This is one of my favorite topics in pressure transient analysis.

If we have a constant (sandface) rate solution $p_{wDc}(t_D)$ (dimensionless pressure), which may or may not include skin effect, we can derive an analytical solution for a variable-rate problem by applying the principle of superposition.

First, we discretize the sand face rate:

$$p_{wD} \approx \frac{1}{q} \sum_{i=1}^{n} (q_{sf,i} - q_{sf,i-1}) p_{wDc} \left(t_D - t_{D,i-1}\right)$$

where p_{wD} is the dimensionless pressure that accounts for wellbore storage, q is the (constant) surface rate, and q_{sf} is the sand face rate. Here, the subscript i denotes denotes discretized time steps.

We define the dimensionless rate as $q_D = \dfrac{q_{sf}}{q}$, so the expression becomes:

$$p_{wD} \approx \sum_{i=1}^{n} (q_{D,i} - q_{D,i-1}) p_{wDc} \left(t_D - t_{D,i-1}\right)$$

$$p_{wD} \approx \sum_{i=1}^{n} \frac{q_{D,i} - q_{D,i-1}}{t_{D,i} - t_{D,i-1}} \left[p_{wDc} \left(t_D - t_{D,i-1}\right)\right] \left(t_{D,i} - t_{D,i-1}\right)$$

If we take the limit $n \to \infty$ (divide the interval so that the $q_{D,i} - q_{D,i-1}$ becomes infinitesimaly small),

$$p_{wD} = \int_0^{t_D} \frac{d}{d\tau} q_D(\tau) p_{wDc}(t_D - \tau) \, d\tau$$

This is a convolution integral. Therefore, in the Laplace domain:

$$\mathcal{L}\left[p_{wD}\right] = s\mathcal{L}\left[q_D\right] \mathcal{L}\left[p_{wDc}\right] \tag{1}$$

where s is Laplace variable.

From the definition of the wellbore storage effect:

$$q_D + C_D \frac{dp_{wD}}{dt_D} = 1$$

Take the Laplace transform:

$$\mathcal{L}[q_D] + sC_D\mathcal{L}[p_{wD}] = \frac{1}{s}$$

Rearranging:

$$\mathcal{L}[q_D] = \frac{1}{s} - sC_D\mathcal{L}[p_{wD}] \tag{2}$$

Substituting Equation 2 into Equation 1, we eliminate $\mathcal{L}[q_D]$:

$$\mathcal{L}[p_{wD}] = s\left(\frac{1}{s} - sC_D\mathcal{L}[p_{wD}]\right)\mathcal{L}[p_{wDc}]$$

$$\left(1 + s^2C_D\mathcal{L}[p_{wDc}]\right)\mathcal{L}[p_{wD}] = \mathcal{L}[p_{wDc}]$$

$$\mathcal{L}[p_{wD}] = \frac{\mathcal{L}[p_{wDc}]}{1 + s^2C_D\mathcal{L}[p_{wDc}]}$$

This is a very interesting and elegant result. It shows that if we have the constant-rate solution, we can modify it to account for wellbore storage under a variable-rate condition.