Appendix A
Complete Proof of Theorem 1

Theorem 1 (NP-hardness). *AIRM problem is NP-hard under HyLT model*

Proof. Consider a special case for a hypergraph G where all of the hyperedges include exactly two vertices. Then, G can be regarded as a non-hyper graph, and each type of restriction measures can be regarded as a deletion of an edge. Let $e_{v,u}$ be an edge from v to u and $w_{v,u}$ be its weight so that $w_{v,u} := IF(v) \cdot IF(e_{v,u})$. Then, the HyLT model can be regarded as the LT model where each edge's weight is represented by $w_{v,u}$.

Thus, the influence blocking maximization when applied to edges and the LT model are the special case of AIRM and HyLT, respectively. According to [10], the influence blocking maximization when applied to edges is NP-hard under the LT model. Therefore, the AIRM problem is also NP-hard under HyLT model. □

Appendix B
Complete Proof of Theorem 2

Theorem 2 (Submodularity). *For a hypergraph $G(V, H)$ and a measure m, $O_m(X)$ is submodular over $X \subseteq H$ under HyLT model. That is, for $S \subseteq T \subseteq H$ and $h \in H \setminus T$*

$$O_m(S \cup \{h\}) - O_m(S) \geq O_m(T \cup \{h\}) - O_m(T)$$

Proof. Given a hypergraph $G(V, H)$, we consider its transformation to a non-hyper directed graph $G^{non}(V, E_H)$ generated as follows:

- For each $h \in H$, $|V(h)|(|V(h)| - 1)$ directed non-hyper edges are generated by connecting each vertex in $V(h)$ to the rest ones (thus, $V(h)$ becomes a clique).
- An edge $e_{v,u}$ generated from h has a weight $w_{v,u}$ which is equal to $IF(h) \cdot IF(v)$ ($e_{v,u}$ is a directed edge from a vertex v to u).

Then, we can obtain an one-to-one correspondence from $G(V, H)$ to $G^{non}(V, E_H)$. The behavior of LT model on $G^{non}(V, E_H)$ is equivalent to that of HyLT model on $G(V, H)$.

Let G_x^{non} be a *live-edge* graph [23] generated from $G^{non}(V, E_H)$ as discussed in Section IV-A. Let also the realization graph of $G(V, H)$ be G_x. Due to the aforementioned one-to-one correspondence, G_x is equivalent to G_x^{non}, i.e.,

$$Pr[G_x | G^{non}(V, E_H)] = Pr[G_x | G(V, H)] \tag{7}$$

Next, we discuss the correspondence of each restriction measure m_{can}, m_{sh}, m_{sp} on the non-hyper graph $G^{non}(V, E_H)$. For $m_{can(h)}$, the corresponding operation that can be applied to $G^{non}(V, E_H)$ is to delete a set of non-hyper edges $E_{can} \subseteq E_H$ s.t. $E_{can} = \{e_{v,u} | \ \forall v, u \in V(h)\}$. For $m_{sh(h)}$, the corresponding operation that can be applied to $G^{non}(V, E_H)$ is to delete a set of non-hyper edges E_{sh}. Let h_0 be the hyperedge after shrinking h. Then, $E_{sh} = \{e_{v,u} | \ \forall v, u \in V(h)\} \setminus \{e_{v,u} | \ \forall v, u \in V(h_0)\}$. Finally, for $m_{spl(h)}$, the corresponding operation that can be applied to $G^{non}(V, E_H)$ is to delete a set of non-hyper edges E_{sp}. Let $\{h_0, h_1, ..., h_{n-1}\}$ be the set of hyperedges after splitting h. Then, $E_{sp} = \{e_{v,u} | \ \forall v, u \in V(h)\} \setminus \left(\bigcup_{i \in [0,n)} \{e_{v,u} | \ \forall v, u \in V(h_i)\}\right)$. Therefore, any restriction measure applied to a non-hyper graph corresponds to the deletion of a certain set of edges.

Let $I'(v, E)$ be the influence speed from a vertex v under LT model on the graph $G^{non}(V, E)$. Let also $E_{H,m}(X)$ be a set of non-hyper edges to be deleted when applying $m(X)$ ($X \subseteq H$ and $E_{H,m}(X) \subseteq E_H$). Then, according to Eg. 7,

$$\begin{aligned} &I(v, H \setminus m(X) \cup X) \\ &\quad = \sum_{G_x \in \mathcal{G}} \Pr[G_x | G(V, H \setminus m(X) \cup X)] \cdot r(v, G_x) \\ &\quad = \sum_{G_x \in \mathcal{G}^{non}} \Pr[G_x | G^{non}(V, E_H \setminus E_{H,m}(X)))] \cdot r(v, G_x) \\ &\quad = I'(v, E_H \setminus E_{H,m}(X)) \end{aligned} \tag{8}$$

where $\mathcal{G}$ and $\mathcal{G}^{non}$ are a set of all realization graphs for $G(V, H \setminus m(X) \cup X)$ and $G^{non}(V, E_H \setminus E_{H,m}(X))$, respectively; $r(v, G_x)$ is the number of reachable vertices from v in G_x.

According to [10], the influence speed function of LT model is supermodular when considering edge deletions. Thus, for a non-hyper graph $G^{non}(V, E)$, any set of non-hyper edges $S^{non} \subseteq T^{non} \subseteq E$, and $e \in E \setminus T^{non}$, the following is established:

$$\begin{aligned} &I'(v, E \setminus (S^{non} \cup \{e\})) - I'(v, E \setminus S^{non}) \\ &\leq I'(v, E \setminus (T^{non} \cup \{e\})) - I'(v, E \setminus T^{non}) \end{aligned}$$

Therefore, for any $X^{non} \subseteq E \setminus T^{non}$,

$$\begin{aligned} &I'(v, E \setminus (S^{non} \cup X^{non})) - I'(v, E \setminus S^{non}) \\ &\leq I'(v, E \setminus (T^{non} \cup X^{non})) - I'(v, E \setminus T^{non}) \end{aligned} \tag{9}$$

On the other hand, based on Eq.(8), for any $S \subseteq T \subset H$ and $h \in H \setminus T$,

$$\begin{aligned} &O_m(S \cup \{h\}) - O_m(S) \\ &= \tfrac{1}{n} \sum_{v \in V} \{I(v, H \setminus S \cup m(S)) - I(v, H \setminus (S \cup \{h\}) \cup m(S \cup \{h\}))\} \\ &= \tfrac{1}{n} \sum_{v \in V} \{I'(v, E_H \setminus E_{H,m}(S)) - I'(v, E_H \setminus E_{H,m}(S \cup \{h\}))\} \quad (\because (8)) \\ &= \tfrac{1}{n} \sum_{v \in V} \{I'(v, E_H \setminus E_{H,m}(S)) - I'(v, E_H \setminus (E_{H,m}(S) \cup E_{H,m}(\{h\})))\} \\ &\qquad (\because E_{H,m}(S \cup \{h\}) = E_{H,m}(S) \cup E_{H,m}(\{h\})) \\ &\geq \tfrac{1}{n} \sum_{v \in V} \{I'(v, E_H \setminus E_{H,m}(T)) - I'(v, E_H \setminus (E_{H,m}(T) \cup E_{H,m}(\{h\})))\} \\ &\qquad (\because (9) \text{ and } S \subseteq T \Rightarrow E_{H,m}(S) \subseteq E_{H,m}(T)) \\ &= \tfrac{1}{n} \sum_{v \in V} \{I'(v, E_H \setminus E_{H,m}(T)) - I'(v, E_H \setminus E_{H,m}(T \cup \{h\}))\} \\ &= O_m(T \cup \{h\}) - O_m(T) \end{aligned}$$

Therefore,

$$O_m(S \cup \{h\}) - O_m(S) \geq O_m(T \cup \{h\}) - O_m(T),$$

which completes the proof of Theorem 2. □

Appendix C
Proof of Corollary 1

Corollary 1. Suppose $\widehat{X}$ is a solution obtained by Algorithm 1, and X^{OPT} is the optimum solution. Then,

$$O_m(\widehat{X}) \geq \left(1 - \frac{1}{e}\right) O_m(X^{OPT})$$

where e is Napier's constant.

Proof. According to [38], a general optimization problem for a non-decreasing function $z(\cdot)$,

$$\max_{S \subseteq N} \{z(S) : |S| \leq k, z(S) \text{ is submodular}, z(\varnothing) = 0\},$$

can approximately be solved by a greedy algorithm within a factor of $(1 - 1/e)$ from the optimum solution. Therefore, the approximate solution $\widehat{X}$ obtained by Algorithm 1 results in a $(1 - 1/e)$ approximation ratio for the maximization problem of $O_m(X)$, which is a non-decreasing submodular function and $O_m(\varnothing) = 0$ due to Theorem 2. □

Appendix D
Complete Proof of Lemma 1

Lemma 1. *For a set of θ random RR paths R,*

$$\mathbb{E}[O_m(X)] \approx \frac{1}{\theta} \sum_{r \in R} \Psi_X(r)$$

when θ is large enough.

Proof. Let r be a random RR path generated from a hypergraph $G(V, H)$ via HyLT model. Then, according to [24], the following equation holds true:

$$\mathbb{E}[I(v, H)] = n \cdot \Pr[r \text{ includes } v]$$

Let X be a set of restriction hyperedges and m be its restriction measure. r'_X is r's sub-path from the origin to the closest restriction hyperedge. Similarly,

$$\mathbb{E}\left[I(v, H \setminus X \cup m(X))\right] = n \cdot \Pr[r'_X \text{ includes } v]$$

Therefore,

$$\begin{aligned}
\mathbb{E}[O_m(X)] &= \mathbb{E}\left[\frac{1}{n} \sum_{v \in V} \{I(v, H) - I\left(v, H \setminus X \cup m(X)\right)\}\right] \\
&= \frac{1}{n} \sum_{v \in V} \{\mathbb{E}\left[I(v, H)\right] - \mathbb{E}[I\left(v, H \setminus X \cup m(X)\right)]\} \\
&= \sum_{v \in V} \{\Pr[r \text{ includes } v] - \Pr[r' \text{ includes } v]\} \\
&= \sum_{v \in V} \Pr[r \text{ includes } v \wedge r' \text{ does not include } v]
\end{aligned}$$

We define a random variable $\psi_{v,X}(r)$ as follows:

$$\psi_{v,X}(r) := \begin{cases} 1 & \text{if } r \text{ includes } v \wedge r'_X \text{ does not include } v \\ 0 & \text{otherwise} \end{cases}$$

For θ random RR paths R, the approximate value for $\Pr[\psi_{v,X}(r) = 1]$ is represented as follows:

$$\Pr[\psi_{v,X}(r) = 1] \approx \frac{\sum_{r \in R} \psi_{v,X}(r)}{\theta},$$

when θ is large enough.

Since $\sum_{v \in V} \psi_{v,X}(r) = \Psi_X(r)$, the following is established:

$$\begin{aligned}
\mathbb{E}[O_m(X)] &= \sum_{v \in V} \Pr[\psi_{v,X}(r) = 1] \\
&\approx \sum_{v \in V} \frac{\sum_{r \in R} \psi_{v,X}(r)}{\theta} \\
&= \frac{1}{\theta} \sum_{r \in R} \sum_{v \in V} \psi_{v,X}(r) \\
&= \frac{1}{\theta} \sum_{r \in R} \Psi_X(r)
\end{aligned}$$

Thus, the lemma holds true. □

Appendix E
Complete Proof of Lemma 2

Lemma 2. *Efficiency of Algorithm 3 is $O(k(|H| + \theta|V|))$.*

Proof. The dominant part of Algorithm 3 is the greedy selection (Lines 8–11). For each iteration of the loop, $O(|H|)$ time is required to obtain the arguments of maxima (Line 9). Scanning θ samples requires $O(\theta)$ time and each sample has $O(|V|)$ vertices (Line 11). Thus, the total time complexity is $O(k(|H| + \theta|V|))$. □

Appendix F
Complete Proof of Corollary 2

Corollary 2. Algorithm 4 returns $(1 - 1/e - \epsilon)$-approximate solution with at least $1 - 1/n^l$ probability.

Proof. Basic steps of Algorithm 4 is the same as IMM [15], [25] . There are two key differences compared with IMM: (i) search space and (ii) objective function. The search space of our algorithm is different than that of IMM because our algorithm selects edges while IMM vertices. Therefore, k-combination of a vertex set in IMM parameters is modified into that of an edge set in our algorithm, i.e., $\binom{n}{k}$ is into $\binom{|H|}{k}$. Therefore, the parameters of IMM,

$$\begin{aligned}
\alpha_{IMM} &= \sqrt{l \ln n + \ln 2} \\
\beta_{IMM} &= \sqrt{(1 - 1/e)(\ln \tbinom{n}{k} + l \ln n + \ln 2)} \\
\lambda^*_{IMM} &= 2n((1 - 1/e)) \cdot \alpha + \beta)^2 \cdot \epsilon^{-2} \\
\lambda'_{IMM} &= (2 + 2\epsilon'/3)(\ln \tbinom{n}{k} + l \ln n + \ln \log_2 n) \cdot n \cdot \epsilon'^2
\end{aligned}$$

are respectively modified into

$$\begin{aligned}
\alpha &= \sqrt{l \ln n + \ln 2} \quad \text{(no difference)} \\
\beta &= \sqrt{(1 - 1/e)(\ln \tbinom{|H|}{k} + l \ln n + \ln 2)} \\
\lambda^* &= 2n((1 - 1/e)) \cdot \alpha + \beta)^2 \cdot \epsilon^{-2} \quad \text{(no difference)} \\
\lambda' &= (2 + 2\epsilon'/3)(\ln \tbinom{|H|}{k} + l \ln n + \ln \log_2 n) \cdot n \cdot \epsilon'^2
\end{aligned}$$

Second, the objective function $I()$ in IMM is changed to $O_m()$ in our algorithm. However, since $O_m()$ holds submodularity and RR path is based on a concept of RR set,

it does not affect the frame of the algorithm.

Ref.. results in the fact that IMM achieves an $(1 - 1/e - \epsilon)$ approximation ratio with at least $1 - 1/n^l$ probability. The above is based on the following: (i) the speed function of IMM is submodular; and (ii) the aforementioned IMM's parameters. Because we have already proved the submodularity of Om (see....) and that the IMM's parameters (see above) do not change in our context, then we conclude that Algorithm 4 achieves an $(1 - 1/e - \epsilon)$ approximation ratio with at least $1 - 1/n^l$ probability.

Therefore, since these two modifications of IMM do not affect the approximation quality, Algorithm 4 also returns an $(1 - 1/e - \epsilon)$-approximate solution with at least $1 - 1/n^l$ probability in the same way as IMM. $\square$