

①

$$(A \rightarrow (B \rightarrow C)) \wedge A \rightarrow (B \rightarrow C)$$

①

$$A \rightarrow (B \rightarrow C) \quad A$$

$$B \rightarrow C$$

$$\textcircled{1} \rightarrow (B \rightarrow C)$$

$$[A \rightarrow (B \rightarrow C) \wedge A]$$

$$A \wedge A \rightarrow (B \rightarrow C)$$

$$B \rightarrow C$$

$$(A \rightarrow (B \rightarrow C)) \wedge A \rightarrow (B \rightarrow C)$$

②

$$\underline{(A \wedge B) \wedge C} \rightarrow B \wedge (C \wedge A)$$

①  $(A \wedge B) \wedge C$  は  $A \wedge B$  と  $C$  から証明される

$$\begin{array}{c} \underline{A \wedge B} \\ B \end{array}, \quad \underline{C}, \quad \begin{array}{c} \underline{A \wedge B} \\ A \end{array}$$

$$\underline{C \wedge A}$$

$$\underline{B \wedge (C \wedge A)}$$

$$\begin{array}{c} \underline{[(A \wedge B) \wedge C]} \\ A \wedge B \quad C \\ \hline A \quad B \quad C \end{array}$$

$$\begin{array}{c} \underline{B \quad (C \wedge A)} \quad \uparrow \\ \underline{B \wedge (C \wedge A)} \quad \uparrow \\ \hline \end{array}$$

0

3

$$(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$$

①  $\exists$  仮定しから  $B$  を導くための $A$  を仮定 $[A]$  ② $B$  $\perp$  $\neg A$  ③

$$(\neg B \rightarrow \neg A)$$

$$(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$$

 $(\neg B$  を仮定し  $\neg A$  を導く $\neg B$  $[\neg B]$  ③

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$$(A \vee B) \vee C \rightarrow B \vee (C \vee A)$$

$$\text{分解} \left\{ \begin{array}{l} (A \vee B) \\ C \end{array} \right. \text{ or } \rightarrow B \vee (C \vee A)$$

$$\text{分解} \left\{ \begin{array}{l} (A \vee B) \rightarrow B \vee (C \vee A) \\ A \rightarrow B \vee (C \vee A) \\ B \rightarrow B \vee (C \vee A) \end{array} \right. \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{が導かれる。}$$

$$\begin{array}{rcl} \frac{[A]^3}{C \vee A} \text{ V-I} & \frac{[B]^3}{\downarrow} & \frac{[C]^2}{C \vee A} \\ \frac{C \vee A}{B \vee (C \vee A)} \text{ V-I} & \frac{B \vee (C \vee A)}{B \vee (C \vee A)} & \frac{B \vee (C \vee A)}{B \vee (C \vee A)} \\ \hline [A \vee B]^2 \cdot B \vee (C \vee A) & & [C]^2 \cdot B \vee (C \vee A) \end{array}$$

$$[A \vee B] \vee [C] \cdot B \vee (C \vee A)$$

前提  
必要条件  
充足条件

$$(A \vee B) \vee C \rightarrow B \vee (C \vee A)$$

$$5 \quad (A \wedge B) \vee C \rightarrow (A \vee C) \wedge (B \vee C)$$

$$\begin{array}{c}
 \frac{[(A \wedge B)]^{\neg}}{\begin{array}{cc} \downarrow & \searrow \\ A & B \end{array}} \quad \vee\text{-I} \quad \frac{[C]}{A \vee C} \quad \vee\text{-I} \quad B \vee C \\
 \hline
 \frac{A \vee C \quad B \vee C}{(A \vee C) \wedge (B \vee C)} \quad \vee\text{-I} \quad (A \vee C) \vee (B \vee C) \\
 \hline
 \frac{[(A \wedge B) \vee C]^{\neg} \quad (A \vee C) \wedge (B \vee C)}{(A \wedge B) \vee C \rightarrow (A \vee C) \wedge (B \vee C)}
 \end{array}$$

6 - ①

$$[1] [(A \vee C) \wedge (B \vee C)]$$

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$$(A \wedge B) \vee C$$

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$$[1] \rightarrow (A \wedge B) \vee C$$

⑥ - ②

$$\begin{array}{ccc} & [A] & [C] \\ & [B] & [C] \\ & \vdots & \vdots \\ [1] & & \\ \vdots & & \\ [1] & B \vee C & (A \wedge B) \vee C \\ \vdots & & \\ [1] & A \vee C & (A \wedge B) \vee C \\ \vdots & & \\ & (A \wedge B) \vee C & \\ \hline & [1] \rightarrow (A \wedge B) \vee C & \end{array}$$





$$8 \quad (A \wedge C) \vee (B \wedge C) \rightarrow (A \vee B) \wedge C$$

$$\begin{array}{c}
 \begin{array}{cc}
 \frac{[A \wedge C]^2}{A} & \frac{[A \wedge C]^2}{C} \\
 \hline
 A \vee B & C \\
 \hline
 \end{array}
 \quad
 \begin{array}{cc}
 \frac{[B \wedge C]^2}{B} & \frac{[B \wedge C]^2}{C} \\
 \hline
 A \vee B & C \\
 \hline
 \end{array} \\
 \hline
 \begin{array}{c}
 [A \wedge C] \vee [B \wedge C] \quad (A \vee B) \wedge C \\
 \hline
 (A \vee B) \wedge C \\
 \hline
 \end{array} \\
 \hline
 (A \wedge C) \vee (B \wedge C) \rightarrow (A \vee B) \wedge C
 \end{array}$$

9

$$A \rightarrow (\neg\neg A)$$

$$\begin{array}{rcl}
 [A]^{(2)} & [ \neg A ]^{(1)} & \nearrow \text{消去の仮定 (2)} \\
 \hline
 \downarrow & & \\
 \hline
 \neg\neg A & \neg\text{-E (1)} & \leftarrow \text{証明 (仮定)} \\
 \hline
 A \rightarrow \neg\neg A & \rightarrow\text{-I, (2)} & 
 \end{array}$$

$$10 \quad (\neg\neg A) \rightarrow A$$

$$\frac{\frac{[\neg\neg A]^{\circ}}{A} \neg\neg\text{-E}}{\neg\neg A \rightarrow A} \rightarrow\text{-I } \circ$$

11'  $\neg(A \vee B) \rightarrow (\neg A \wedge \neg B)$

$\frac{\{A\}^{\textcircled{1}}}{\frac{[\neg(A \vee B)]^{\textcircled{2}} \quad A \vee B}{\perp}}$	$\frac{[B]^{\textcircled{1}}}{\frac{[\neg(A \vee B)] \quad A \vee B}{\perp}}$
$\frac{\neg A \quad \neg B}{\neg A \wedge \neg B}$	
$\frac{\neg A \wedge \neg B}{\neg(A \vee B) \rightarrow (\neg A \wedge \neg B)}$	

[A] &

$\neg(A \vee B)$  o

$\neg \neg \perp$

$$12. (\neg A \wedge \neg B) \rightarrow \neg(A \vee B)$$

A, B 对称

$$\begin{array}{c}
 \begin{array}{c}
 \text{c1} \quad \text{a1} \\
 \hline
 \boxed{\neg A \wedge \neg B} \quad A \\
 \hline
 \neg A
 \end{array}
 \qquad
 \begin{array}{c}
 \text{b1} \quad \text{c1} \\
 \hline
 \boxed{B} \quad \boxed{\neg A \wedge \neg B} \\
 \hline
 B \quad \neg B \\
 \hline
 \perp
 \end{array} \\
 \hline
 \begin{array}{c}
 \text{a1, b1} \\
 \boxed{A \vee B} \quad \perp \quad \perp \\
 \hline
 \perp \\
 \hline
 \neg(A \vee B) \\
 \hline
 (\neg A \wedge \neg B) \rightarrow \neg(A \vee B)
 \end{array}
 \end{array}$$

13  $(A \rightarrow B) \rightarrow (\neg A \vee B)$

$$\frac{\frac{\frac{\perp}{\neg A}}{(\neg A \vee B)}}{[\neg(\neg A \vee B)]}$$

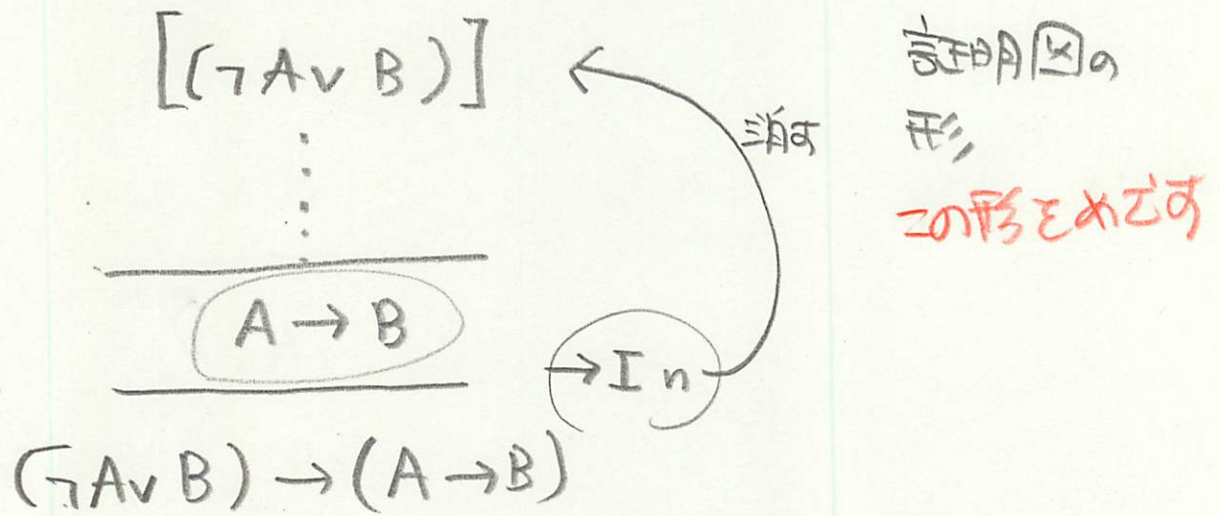
$$\frac{\frac{\frac{\perp}{\neg \neg(\neg A \vee B)}}{\neg A \vee B}}$$

$$\frac{[A]^{(1)} [A \rightarrow B]^{(3)}}{B} \rightarrow E$$

$$\frac{\neg A \vee B}{[\neg(\neg A \vee B)]^{(2)}}$$

$\oplus$

$$14 \quad (\neg A \vee B) \rightarrow (A \rightarrow B)$$



$$A \rightarrow B \text{ を導くには } \frac{[A]}{\vdots} B$$

$(\neg A \vee B)$  の基で  $A \rightarrow B$  を導くには

$$\frac{[ \neg A \vee B ]}{\vdots} \Downarrow \frac{[A]}{\vdots} B \Rightarrow \frac{\cancel{[A]} \vdots}{A \rightarrow B}$$