

2.3 命題論理の証明論の説明

$$\frac{\Gamma(A) \vdash \text{--- } A \text{ 含む}}{\vdash A}$$

①

$$\frac{A \quad B}{A \wedge B}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B}$$

②, ③

$$\frac{A \wedge B}{\underline{A} \quad \underline{B}}$$

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \quad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B}$$

\Rightarrow に分解

$$A \wedge B$$

④

$$\begin{array}{l} \text{前提} \\ \text{を} \\ \text{仮定} \end{array} \left\{ \begin{array}{l} [A]^n \\ \vdots \\ B \end{array} \right. \quad \begin{array}{l} \text{消去} \\ \text{E} \end{array} \quad \begin{array}{l} \text{前提} \\ \text{を} \\ \text{仮定} \end{array} \quad \frac{A \rightarrow B}{\text{消去}} \quad \text{と} \text{して} \text{と} \text{して} \text{と} \text{して}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B}$$

前提を消す。

$$\frac{A, A \rightarrow B}{B}$$

$$\frac{\Gamma \vdash A, \Gamma \vdash A \rightarrow B}{\Gamma \vdash B}$$

⑥

$$\frac{\vdots}{\perp} \quad \frac{\perp}{\neg A} \quad \left. \begin{array}{l} [A]^n \\ \vdots \\ \perp \end{array} \right\} \begin{array}{l} \text{前提} \\ \text{消す} \\ \neg I_n \end{array}$$

$$\frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A}$$

⑦

$$\frac{A, \neg A}{\perp} \quad \neg E$$

$$\frac{\Gamma \vdash A, \Gamma \vdash \neg A}{\Gamma \vdash \perp} \quad (B)$$

~~$$\begin{array}{c} [A]^n \quad [B]^n \\ \vdots \quad \vdots \\ A \vee B \quad C \quad C \\ \hline C \end{array} \quad \begin{array}{c} \Gamma \vdash A \vee B \\ \hline \Gamma, A \vdash C, \Gamma, B \vdash C \\ \hline \Gamma \vdash C \end{array}$$~~

⑩

$$\begin{array}{c} [A]^n \quad [B]^n \\ \vdots \quad \vdots \\ A \vee B \quad C \quad C \\ \hline C \end{array}$$

$$\frac{\Gamma, A \vdash C, \Gamma, B \vdash C, \Gamma \vdash A \vee B}{\Gamma \vdash C}$$

8.9

$$\frac{A}{A \vee B} \quad \vee\text{-I}$$

$$\frac{B}{A \vee B} \quad \vee\text{-I}$$

(11)

$$\frac{\perp}{A} \quad \perp\text{-E}$$

$$\frac{\Gamma \vdash \perp}{\Gamma \vdash A}$$

(12)

$$\frac{\neg\neg A}{A} \quad \neg\neg\text{-E}$$

$$\frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}$$

①

$$(A \rightarrow (B \rightarrow C)) \wedge A \rightarrow (B \rightarrow C)$$

①

$$A \rightarrow (B \rightarrow C) \quad A$$

$$B \rightarrow C$$

$$\textcircled{1} \rightarrow (B \rightarrow C)$$

$$[A \rightarrow (B \rightarrow C) \wedge A]$$

$$A \wedge A \rightarrow (B \rightarrow C)$$

$$B \rightarrow C$$

$$(A \rightarrow (B \rightarrow C)) \wedge A \rightarrow (B \rightarrow C)$$

② $(A \wedge B) \wedge C \rightarrow B \wedge (C \wedge A)$

① $(A \wedge B) \wedge C$ は ~~決定子~~ か証明できる

$$\frac{\frac{A \wedge B}{B} \quad \frac{C}{C} \quad \frac{\frac{A \wedge B}{A}}{C \wedge A}}{B \wedge (C \wedge A)}$$

$$\frac{B \wedge (C \wedge A)}{B \wedge (C \wedge A)}$$

$$\frac{[(A \wedge B) \wedge C]}{A \wedge B \quad C}$$

$$\frac{2 \quad B \quad (C \wedge A) \quad \uparrow}{1 \quad B \wedge (C \wedge A) \quad \uparrow}$$

$$0$$

3

$$(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$$

① 仮定

から B を導くための

A を仮定

[A] ②

B

⊥

 $\neg A$ ③ $(\neg B \rightarrow \neg A)$

$$(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)$$

(⊥ は仮定より $\neg A$ を導く)

[⊥] ③

⊥

4

$$(A \vee B) \vee C \rightarrow B \vee (C \vee A)$$

$$\text{分解} \left\{ \begin{array}{l} (A \vee B) \\ C \end{array} \right. \text{ or } \rightarrow B \vee (C \vee A)$$

$$\text{分解} \left\{ \begin{array}{l} (A \vee B) \rightarrow B \vee (C \vee A) \\ A \rightarrow B \vee (C \vee A) \\ B \rightarrow B \vee (C \vee A) \end{array} \right. \left. \vphantom{\begin{array}{l} (A \vee B) \rightarrow B \vee (C \vee A) \\ A \rightarrow B \vee (C \vee A) \\ B \rightarrow B \vee (C \vee A) \end{array}} \right\} \text{が導かれる。}$$

$$\begin{array}{rcl} \frac{[A]^3}{C \vee A} \text{ V-I} & \frac{[B]^3}{\downarrow} & \frac{[C]^2}{C \vee A} \\ \frac{C \vee A}{B \vee (C \vee A)} \text{ V-I} & B \vee (C \vee A) & B \vee (C \vee A) \\ \hline [A \vee B]^2 B \vee (C \vee A) & & [C]^2 B \vee (C \vee A) \end{array}$$

$$[A \vee B] \vee [C] \quad B \vee (C \vee A)$$

前提
必要条件
[A] 2

$$(A \vee B) \vee C \rightarrow B \vee (C \vee A)$$

$$5 \quad (A \wedge B) \vee C \rightarrow (A \vee C) \wedge (B \vee C)$$

$$\begin{array}{c}
 \frac{[(A \wedge B)]^1}{\begin{array}{cc} \downarrow & \downarrow \\ A & B \end{array}} \quad \vee\text{-I} \quad \frac{[C]}{A \vee C} \quad \vee\text{-I} \quad B \vee C \\
 \hline
 \frac{A \vee C \quad B \vee C}{(A \vee C) \wedge (B \vee C)} \quad \vee\text{-I} \quad (A \vee C) \vee (B \vee C) \\
 \hline
 \frac{[(A \wedge B) \vee C]^1 \quad (A \vee C) \wedge (B \vee C)}{(A \wedge B) \vee C \rightarrow (A \vee C) \wedge (B \vee C)}
 \end{array}$$

6 - 0

$$\text{ED}[(A \vee C) \wedge (B \vee C)]$$

$$(A \wedge B) \vee C$$

$$[1] \rightarrow (A \wedge B) \vee C$$

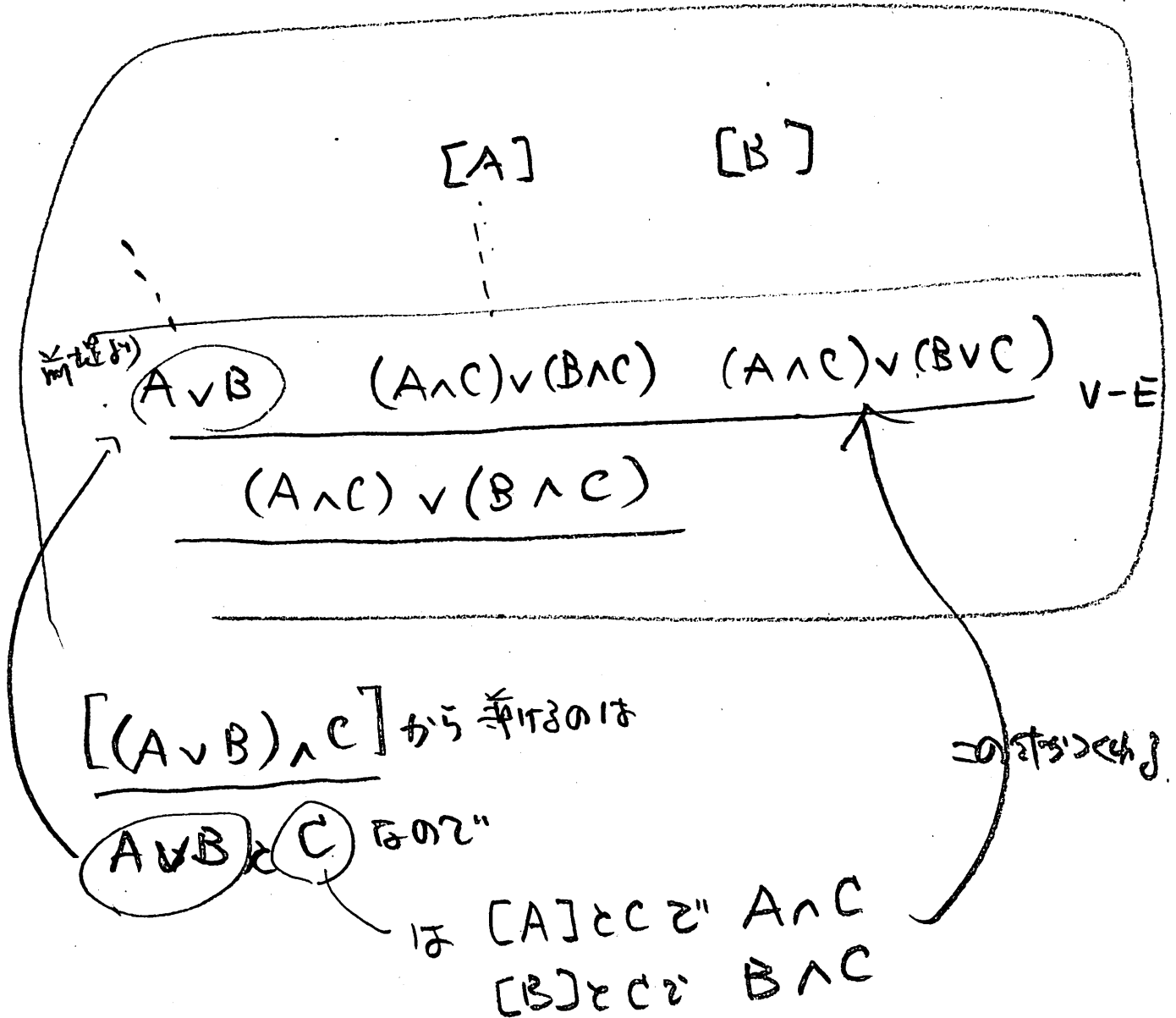
⑥ - ②

$$\begin{array}{ccc} & [A] & [e] \\ & [B] & [c] \\ & \vdots & \vdots \\ [1] & & \\ \vdots & & \\ [1] & B \vee C & (A \wedge B) \vee C \\ & \hline & A \vee C & (A \wedge B) \vee C \\ & \hline & (A \wedge B) \vee C \\ & \hline [1] \rightarrow (A \wedge B) \vee C \end{array}$$

7

$$\frac{(A \vee B) \wedge C}{\text{⑫}} \rightarrow \frac{(A \wedge C) \vee (B \wedge C)}{\text{⑬}}$$

目印です。



$$8 \quad (A \wedge C) \vee (B \wedge C) \rightarrow (A \vee B) \wedge C$$

$$\begin{array}{cc} \frac{[A \wedge C]^2}{A} & \frac{[A \wedge C]^2}{C} \\ \hline A \vee B & C \end{array} \qquad \begin{array}{cc} \frac{[B \wedge C]^2}{B} & \frac{[B \wedge C]^2}{C} \\ \hline A \vee B & C \end{array}$$

$$\begin{array}{c} 1 \quad \frac{[A \wedge C] \vee [B \wedge C] \quad (A \vee B) \wedge C}{(A \vee B) \wedge C} \end{array}$$

$$(A \wedge C) \vee (B \wedge C) \rightarrow (A \vee B) \wedge C$$

9

$$A \rightarrow (\neg\neg A)$$

$$\begin{array}{rcl}
 [A]^{(2)} & [\neg A]^{(1)} & \text{消す 残りの仮定 (2)} \\
 \hline
 \downarrow & & \\
 \hline
 \neg\neg A & \neg\text{-E } (1) & \leftarrow \text{証明 (仮定)} \\
 \hline
 A \rightarrow \neg\neg A & \rightarrow\text{-I, (2)} &
 \end{array}$$

$$10 \quad (\neg\neg A) \rightarrow A$$

$$\frac{\frac{[\neg\neg A]^{\text{O}}}{A} \neg\neg\text{-E}}{\neg\neg A \rightarrow A} \rightarrow\text{-I } \text{O}$$

11

$$\neg(A \vee B) \rightarrow (\neg A \wedge \neg B)$$

$\frac{[A]^{\textcircled{1}}}{\frac{[\neg(A \vee B)]^{\textcircled{2}} \quad A \vee B}{\perp}}$	$\frac{[B]^{\textcircled{1}}}{\frac{[\neg(A \vee B)] \quad A \vee B}{\perp}}$
$\frac{\neg A \quad \neg B}{\neg A \wedge \neg B}$	
$\frac{\neg A \wedge \neg B}{\neg(A \vee B) \rightarrow (\neg A \wedge \neg B)}$	

$[A]^{\textcircled{1}}$

$\neg(A \vee B)$

$\neg(A \vee B) \rightarrow (\neg A \wedge \neg B)$

$$12. (\neg A \wedge \neg B) \rightarrow \neg(A \vee B)$$

A, B 对称

$$\begin{array}{c}
 \begin{array}{c}
 \text{c1} \quad \text{a1} \\
 \frac{[\neg A \wedge \neg B] \quad A}{\neg A}
 \end{array}
 \qquad
 \begin{array}{c}
 \text{b1} \quad \text{c1} \\
 \frac{[B] \quad [\neg A \wedge \neg B]}{B \quad \neg B} \\
 \perp
 \end{array} \\
 \hline
 \begin{array}{c}
 \text{a1, b1} \\
 \frac{[A \vee B] \quad \perp \quad \perp}{\perp} \\
 \neg(A \vee B)
 \end{array} \\
 \hline
 (\neg A \wedge \neg B) \rightarrow \neg(A \vee B)
 \end{array}$$

$$13 \quad (A \rightarrow B) \rightarrow (\neg A \vee B)$$

$$\frac{\frac{\frac{\perp}{\neg A}}{(\neg A \vee B)}}{[\neg(\neg A \vee B)]}$$

$$\frac{\frac{\perp}{\neg \neg(\neg A \vee B)}}{\neg A \vee B}$$

$$\frac{[A]^{(1)} \quad [A \rightarrow B]^{(3)}}{B} \rightarrow E$$

$$\frac{\neg A \vee B \quad [\neg(\neg A \vee B)]^{(2)}}{\perp}$$