積分変換

領域 $t \to$ 領域 u へ

$$\underbrace{(Tf)}_{\text{出力}}(u) = \int_{t_1}^{t_2} \underbrace{K(t,u)}_{\text{核}} f(t)\,dt$$

入力

フーリエ変換

$$\frac{e^{-iut}}{\sqrt{2\pi}},\ \frac{e^{+iut}}{\sqrt{2\pi}} \quad -\infty \sim +\infty$$

ラプラス。

$$e^{-ut},\ \frac{e^{+ut}}{2\pi i} \quad 0 \sim \infty$$

$K^{-1}(u,t)$ が存在し、

反転公式

$$f(t) = \int_{u_1}^{u_2} K^{-1}(u,t)\,(Tf(u))\,du$$

Laplace 変換

$f(t)$, $t \geq 0$ に対し

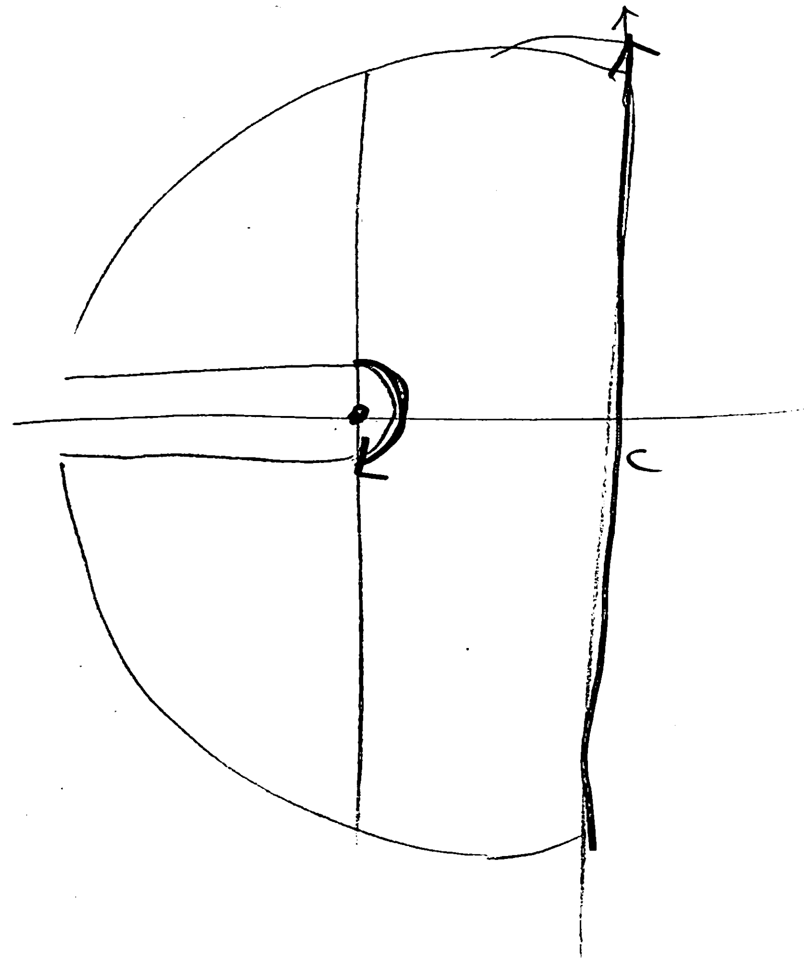
ラプラス積分

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

時間領域 → 複素平面

$c > 0$

$$f(t) = \lim_{p \to \infty} \frac{1}{2\pi i} \int_{c-ip}^{c+ip} F(s) e^{st} ds$$



$$\frac{1}{\sqrt{s}+c} = \frac{\sqrt{s}-c}{s-c^2} = \frac{-c}{s-c^2} + \boxed{\frac{\sqrt{s}}{s-c^2}}$$

$$\frac{\sqrt{s}}{s-c^2} = \frac{\alpha}{\sqrt{s}} + \frac{\beta\sqrt{s} + \cancel{c}}{(\sqrt{s}-c)(\sqrt{s}+c)} \qquad \cancel{\sqrt{s}-c}$$

$$= (s-c^2)\alpha + \beta s \cancel{+c\sqrt{s}}$$

$$\frac{1}{\sqrt{s}} + \frac{c\sqrt{s}+1}{s-c^2}$$

$$\sqrt{s} = \cancel{\sqrt{s}} \cdot (s-c^2) - s + \sqrt{s}$$

$$\alpha = \beta$$

$$\frac{\alpha}{\sqrt{s}} + \frac{\beta}{\sqrt{s}-c} + \frac{\gamma}{\sqrt{s}+c} = \frac{\beta\sqrt{s} + \gamma\sqrt{s}}{s-c^2}$$

$$\frac{\alpha(s-c^2) + \beta(s+c\sqrt{s}) + \gamma(s-c\sqrt{s})}{}$$

$$\alpha s + \beta s + \gamma s = 0$$

$$-\alpha c^2 + \beta c\sqrt{s} - \gamma c\sqrt{s}$$

$$\parallel$$

$$0 \qquad c\sqrt{s}(\beta - \gamma)$$

$$\frac{1}{2c}\left(\frac{1}{\sqrt{s}-c} + \frac{1}{\sqrt{s}+c}\right)$$

$$\frac{\sqrt{s}}{s-c^2} \quad \begin{matrix} F \\ \frac{1}{G} \end{matrix}$$

$f(u)\cdots$

$\sqrt{s}$ $\quad \delta = -\frac{1}{2}$

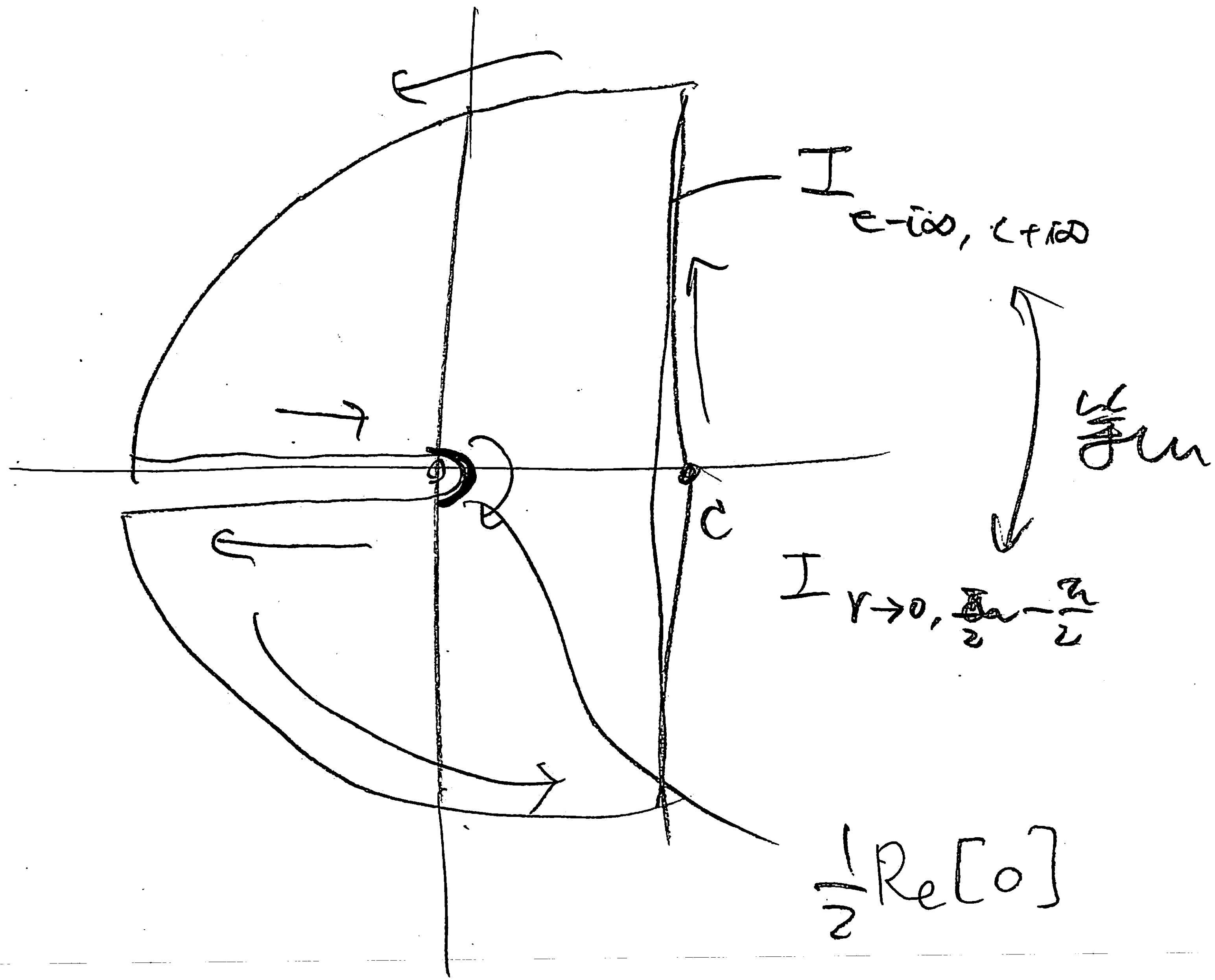
$$\frac{\dfrac{t^{-\frac{1}{2}}}{\Gamma(\frac{1}{2})}}{\sqrt{\pi}} \cdot \underline{u(t)} \qquad (t>0)$$

$$\frac{1}{s+\alpha} = e^{-\alpha t} \cdot u(t)$$

$$e^{c^2 t} \cdot u(t)$$

$$\frac{1}{s-c^2} = e^{+c^2 t} \cdot u(t)$$

$$\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{2}{\sqrt{\pi}} \, t^{-1/2} * e^{c^2(s-\sigma)}$$



$I_{c-i\infty,\, c+i\infty}$
等しい.
c
$I_{r\to 0}$
$\frac{1}{2}Re[0]$