① 変数変換

$$t^{(r)} = \frac{r-\mu}{\sigma} = \frac{r-Np}{\sqrt{Npq}} \quad \text{①}$$

逆関数

$$r^{(t)} = \sigma t + \mu = \sqrt{Npq}\cdot t + Np \quad \text{②}$$

$$\frac{r}{N} = \sqrt{\frac{pq}{N}}\cdot t + p \quad \text{③}$$

$$\frac{N-r}{N} = 1 - \frac{r}{N} = (1-p) - \sqrt{\frac{pq}{N}}\,t = q - \sqrt{\frac{pq}{N}}\cdot t \quad \text{④}$$

② 確率密度fの変換

$$F(t) = B(r)\frac{dr}{dt} = \sqrt{Npq}\cdot B(r) = \sqrt{Npq}\,\frac{N!}{r!(N-r)!}\,p^r q^{N-r}$$

③ スターリングの公式によるべき関数化

$$n! \simeq \sqrt{2\pi n}\cdot\left(\frac{n}{e}\right)^n$$

$$\frac{N!}{r!(N-r)!} \simeq \frac{1}{\sqrt{2\pi}}\sqrt{\frac{N}{r(N-r)}}\left(\frac{N}{r}\right)^r\left(\frac{N}{N-r}\right)^{N-r}$$

④ 変数変換①

$$\sqrt{Npq}\cdot B(r) \simeq \sqrt{Npq}\cdot\frac{1}{\sqrt{2\pi}}\sqrt{\frac{N}{r(N-r)}}\cdot\left(\frac{Np}{r}\right)^r\cdot\left(\frac{Nq}{N-r}\right)^{N-r}$$

$$= \frac{1}{\sqrt{2\pi}}\sqrt{\frac{Np}{r}\cdot\frac{Nq}{N-r}}\cdot\left(\frac{Np}{r}\right)^r\left(\frac{Nq}{N-r}\right)^{N-r}$$

⑤ log化

$$-\ln\left(\sqrt{2\pi}\cdot\sqrt{Npq}\cdot B(r)\right) = \left(r+\frac{1}{2}\right)\ln\left(\frac{r}{Np}\right) + \left(N-r+\frac{1}{2}\right)\ln\left(\frac{N-r}{Nq}\right)$$

⑥ 変数変換②

$$\frac{r}{Np} = 1 + \sqrt{\frac{q}{Np}}\cdot t \quad \cdots ③/p$$

$$\frac{N-r}{Nq} = 1 - \sqrt{\frac{p}{Nq}}\,t \quad \cdots ④/q$$

⑦ 変数変換③

$$-\ln\left(\sqrt{2\pi}\sqrt{Npq}\,B(r(t))\right)$$

$$=\left(r(t)+\frac{1}{2}\right)\ln\left(1+\sqrt{\frac{q}{Np}}t\right)+$$

$$\left(N-r(t)+\frac{1}{2}\right)\ln\left(1-\sqrt{\frac{p}{Nq}}t\right)$$

$$=\left(\sqrt{Npq}\cdot t+N\cdot p+\frac{1}{2}\right)\ln\left(1+\sqrt{\frac{q}{Np}}\cdot t\right)+ \quad \cdots (7.1)$$

$$\left(-\sqrt{Npq}t+\underbrace{N-Np}_{Nq}+\frac{1}{2}\right)\ln\left(1-\sqrt{\frac{p}{Nq}}\cdot t\right) \quad — (7.2)$$

⑧ $x \ll 1$ の時 $\ln(1\pm x)=\pm x-\frac{1}{2}x^2$

テイラー展開

$$\ln\left(1+\sqrt{\frac{q}{Np}}t\right)=\sqrt{\frac{q}{Np}}t-\frac{1}{2}\left(\sqrt{\frac{q}{Np}}t\right)^2 \quad — (8.1)$$

$$\ln\left(1-\sqrt{\frac{p}{Nq}}t\right)=-\sqrt{\frac{p}{Nq}}t-\frac{1}{2}\left(\sqrt{\frac{p}{Nq}}t\right)^2 \quad — (8.2)$$

テイラー展開と微少項の無視

⑨ ＊ 8.1と8.2を7.1と7.2に代入して、

$N, N^{\frac{1}{2}}, N^0$の項を計算する

$N^{-\frac{1}{2}}, N^{-1}\ldots$の項は$N\to\infty$で0に

$$\left(\sqrt{Npq}\cdot t+Np+\frac{1}{2}\right)\left(\sqrt{\frac{q}{Np}}\cdot t-\frac{1}{2}\frac{q}{Np}\cdot t^2\right)+$$

$$\left(-\sqrt{Npq}\cdot t+Nq+\frac{1}{2}\right)\left(-\sqrt{\frac{p}{Nq}}t-\frac{1}{2}\frac{p}{Nq}t^2\right)$$

$$=\left(qt^2+\sqrt{Npq}t-\frac{1}{2}qt^2\right)$$

$$+\left(pt^2-\sqrt{Npq}t-\frac{1}{2}pt^2\right)+O\left(N^{-\frac{1}{2}}\right)$$

$$=\frac{1}{2}(p+q)t^2$$

$$=\frac{1}{2}t^2.$$

$$\therefore F(t)\simeq\frac{1}{\sqrt{2\pi}}e^{-t^2}$$