$\Gamma(z)$は階乗の定義を
複素平面に拡張した関数.

実軸上で対数凸となる
解析関数

$\frac{1}{\sin z}$

Γ関数は零点をもたず
原点と負の整数に一位の極を持つ

$$\mathrm{Res}(\Gamma, -n) = \frac{(-1)^n}{n!}$$

$$? \int z^{k-1} e^{-z} dz \qquad \Gamma(z) = \lim_{n\to\infty} \frac{n^z \cdot n!}{\prod_{k=0}^{n} (z+k)}$$

定義域を
複素数へ拡張

$t = nu$ とくと

$$\triangleright \quad G_n(z) = \int_0^n t^{z-1} \left(1 - \frac{t}{n}\right)^n dt = n^z \int_0^1 u^{z-1} (1-u)^n du$$

$$g_0(z) = \int_0^1 (u^{z-1}) du = \left[\frac{u^z}{z}\right]_{u=0}^{1} = \frac{1}{z}$$

$$\lim_{n\to\infty} G_n(z) = \int_0^\infty t^{z-1} e^{-t} dt$$

$$g_n(z) = \int_0^1 \left(\frac{u^z}{z}\right)' (1-u)^n du$$

$$= \frac{n}{z} \int_0^1 u^z (1-u)^{n-1} du$$

$$= \frac{n}{z} g_{n-1}(z+1)$$

$$G_n(z) = \frac{n^z n!}{\prod_{k=0}^{n} (z+k)} \qquad \prod \frac{\alpha}{z+k}$$

$-1, \cdots, -n$ まで拡張

$\Gamma(z)$

▷ $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad (\mathrm{Re}(z) > 0)$

▷ $\Gamma(z) = \lim_{n\to\infty} \frac{n^z \cdot n!}{\prod_{k=0}^{n} (z+k)}$ （一般のZ）

$\underline{\Gamma(z+1) = z \cdot \Gamma(z)}$

$$\Gamma(z+1) = \int_0^\infty t^z \cdot e^{-t} dt$$
$$= \left[-t^z \cdot e^{-t}\right]_0^\infty + z\int_0^\infty t^{z-1} e^{-t} dt$$

$\therefore \Gamma(z+1) = z\Gamma(z)$

$\underline{\Gamma(1)} = \int_0^\infty e^{-t} dt = \left[-e^{-t}\right]_0^\infty = 1$

$\Gamma(n+1) = n!$

$$\Gamma(k)=\int_0^\infty \underline{y^{k-1}\cdot e^{-y}}\,dy$$

$$y^{k-1}\left(-e^{-y}\right)'$$

$$=\cancel{(k-1)}\left[\underset{\uparrow\atop 0}{y^{k-1}}\cdot -\underset{\uparrow\atop \infty}{e^{-y}}\right]_0^\infty+\int(k-1)y^{k-2}\cdot e^{-y}dy$$

$0 \quad \infty \quad 0$

$$=\overbrace{(k-1)\cdots(k-r)}\int \overset{\Gamma(k-r)}{y^{k-r}\cdot e^{-y}dy}$$

$$0<k-r<1$$

$$\Gamma\left(\tfrac{1}{n}\right)=\int_0^\infty y^{\frac{1}{n}-1}\cdot e^{-y}dy=\int_0^\infty \cancel{z^{1-n}}\cdot e^{-z^n}\cancel{dz}\,\cancel{z^{n-1}}\cdot ndz$$

$$=\int_0^\infty e^{-z^n}\cdot ndz$$

$y^{\frac{1}{n}}=z \quad y=z^n$

$dy=nz^{n-1}\cdot dz$

Γ func

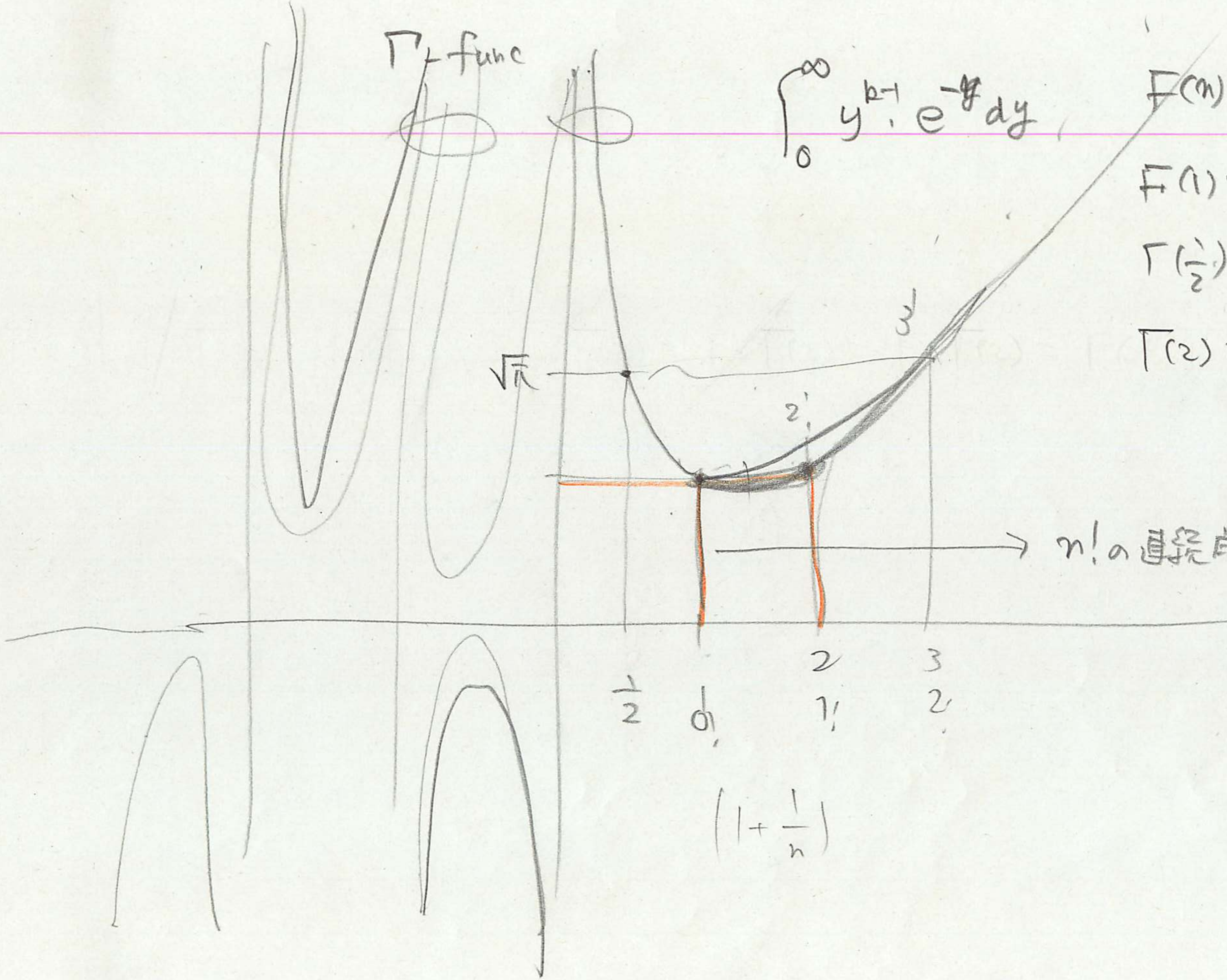
$$\int_0^{\infty} y^{k-1} \cdot e^{-y}\, dy$$

$\Gamma(n) = n-1 \cdot \Gamma(n-1)$

$\Gamma(1) = 1 \quad (0!)$

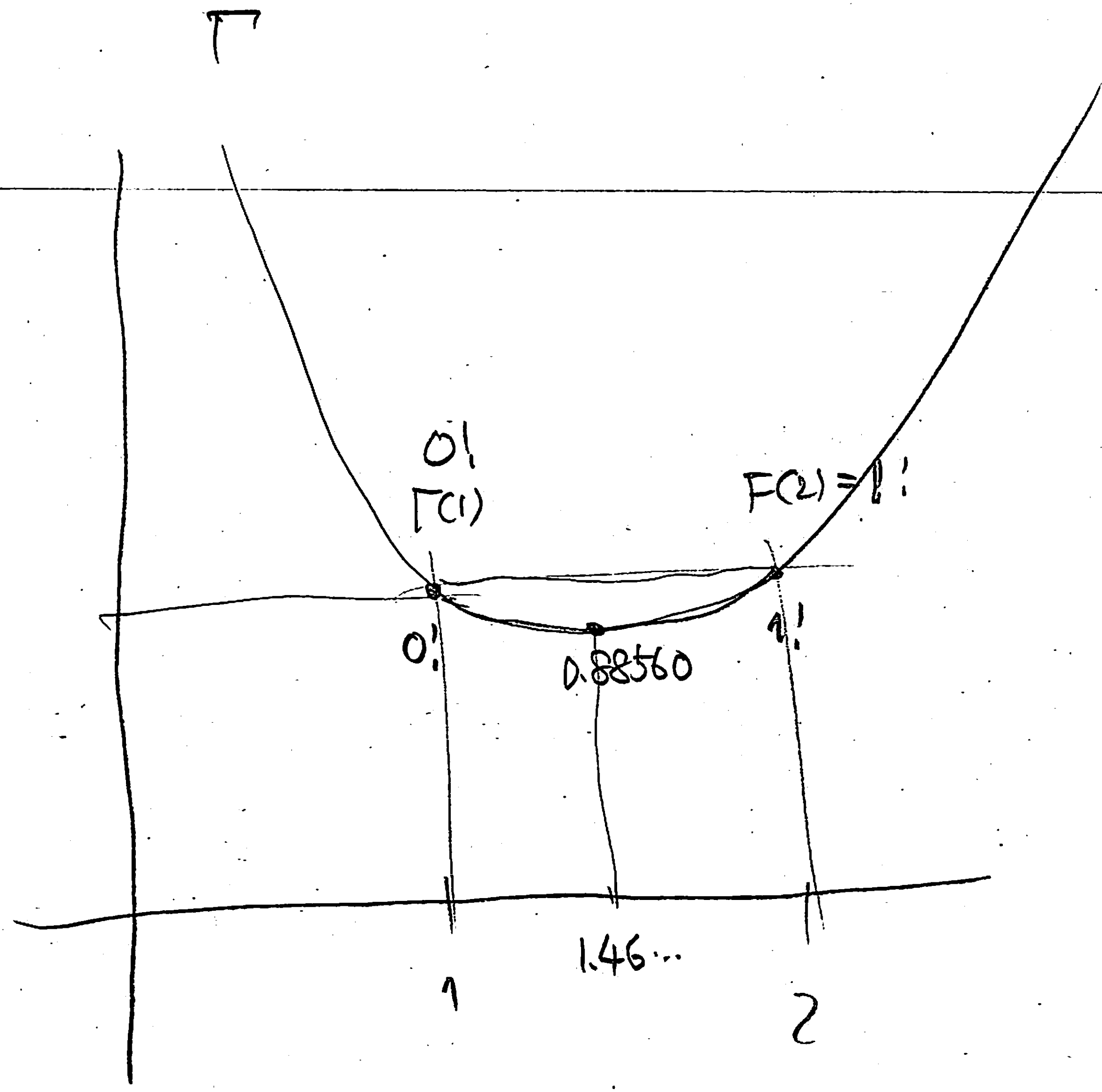
$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$\Gamma(2) = 1 \quad (1!)$



→ n!の連続的な拡張.

$\left(1 + \frac{1}{n}\right)$


Γ
0!
Γ(1)
F(2) = 1!
0!
0.88560
1!
1
1.46...
2

$$\left[-y^{k-1}(e^{-y})'\right]_0^{\infty} = 0$$

$$\Gamma(k)$$

$$= \int_0^{\infty} y^{k-1} e^{-y} dy \qquad (k>0)$$

$$-\int (k-1) y^{k-2} e^{-y}$$

$$\vdots$$

$$(k-1)(k-2)\cdots \int_0^{\infty} e^{-y} dy$$

$$(-1)^{k} \ (k-1)! \left[e^{-y}\right]_0^{\infty}$$

$k \in \mathbb{Z}$

$(k-1)!$ になる訳.

$$\sum_0^{\infty} \frac{y^{\cancel{k}}}{n!} (-1)^{\cancel{k}}$$

$n<k$

$n=k$

$n>k$

$$\int_0^{\infty} \frac{\quad}{\quad}$$

$$\Gamma(r) = (r-1)\cdots\underset{0<r-n<1}{(r-n)}\cdot\Gamma(r-n)$$

$$\Gamma(1) = 1$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{2} \leftarrow \text{カウス}$$

$$\Gamma\left(\frac{1}{3}\right)$$

$$-(x^2+y^2)$$

$$-(x^3+y^3)$$

$$-(x+y)\underbrace{(x^2-xy+y^2)}_{(1-\sin\theta\cdot\cos\theta)}$$

$$r(\cos\theta+\sin\theta)$$

$$\sqrt{2}\,r(\bigcirc)$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^{\infty} t^{-\frac{1}{2}} \cdot e^{-t} dt$$

$t^{\frac{1}{2}} = u$

$\frac{1}{\sqrt{t}}$ $\frac{1}{2} t^{-\frac{1}{2}} dt = du$

$t^{-\frac{1}{2}} = u$ e^{u^2}

$-\frac{1}{2} t^{-\frac{3}{2}} dt - du$ $(t^{\frac{1}{2}})' = \frac{1}{2} t^{-\frac{1}{2}}$ $\frac{1}{\sqrt{t}}$ $\frac{1}{2u} dt = du$

tu^3 $dt = \frac{2}{u^3} du$ $dt = 2u\,du$

$$= \int_0^{\infty} (\sqrt{t})' \cdot e^{-(\sqrt{t})^2} dt$$ $\int_0^{\infty} e^{-t^2} du$

$$= \left[\cancel{\sqrt{t}} \cdot e^{-(\sqrt{t})^2}\right]_0^{\infty} - \int_0^{\infty} \sqrt{t} \cdot 2\sqrt{t}$$

$$\Gamma(n) = \int_0^{\infty} t^{n-1} e^{-t} dt$$

$\cancel{\left(\frac{1}{n} t^n \cdot e^{-t}\right) - \frac{1}{n} \int t}$

$\left(-t^{n-1} e^{-t}\right) +$

y

$$z^{n-1},\ e^{nz},\ \frac{1}{n}\frac{y^{\frac{1-n}{n}}}{z^{1-n}}$$

$$\int_0^\infty \frac{1}{n} e^{z^n} dz = \left[e^{nz}\right]_0^\infty$$

$$\boxed{e^{nz}}'$$

$$\frac{1}{n}\int_0^\infty e^{z^n} dz \qquad e^{-z^n}$$

$$\left(e^{-z^n}\right) =$$

$$e^{-x} = 1 + \frac{1}{1!}(-x) + \frac{1}{2!}(-x)^2 + \cdots$$

e^{-z^n}

$z = r\cos$

$\cancel{x^n} + y^n|$

$r^n \cdot \cos^n \theta + \sin^n \theta$

$x^4 + y^4$

$n = 2m$

$n = 2m+1$

$(x+y)(x^2 - xy + y^2)$

$x^3 - \cancel{x^2 y} + x \cancel{y^2}$

$y \cancel{x^2} - xy^2 + y^3$

$$\Gamma\left(\frac{1}{n}\right) = n\int_0^\infty e^{-z^n}\,dz \qquad x^2+y^2$$

$$\cancel{\int_0^\infty\int_0^\infty e^{-(x^n+y^n)}\,dx\,dy}$$

$$x^3+y^3 = (x+y)(x^2-xy+y^2) \qquad \frac{\sqrt{2}}{2}\sin\left(\theta+\frac{\pi}{4}\right)$$

$$(x+y)^{-2}$$

$$\cos\theta\sin\theta = \frac{1}{2}\sin 2\theta$$

$$2\theta$$

$$\int_0^\infty e^{-z^r}\,dz = \int_0^\infty \sum \frac{1}{n!}\left[-z^r\right]^n dx$$

$$= \sum_{n=} \frac{1}{(nr+1)\,n!}(-1)^n x^{rn+1}\Big|_0^\infty \quad \forall r$$

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$$

$$= \left[-x^{n-1} \cdot e^{-x} \right]_0^\infty + \int_0^\infty (n-1) x^{n-2} e^{-x} dx$$

$$= (n-1) \int_0^\infty x^{n-2} \cdot e^{-x} dx$$

$$= (n-1) \cdot \Gamma(n-1)$$

1.1 : 1.2

$\Gamma(0.1) = 9.5135 \cdots$

$\Gamma(0.01) = 99.43258 \cdots$

$\Gamma(1.01) = 0.994325 \cdots$

$1.1 \quad = 0.95135$

$$\Gamma(1.1) = \int_0^\infty x^{0.1} e^{-x} dx$$

$$3.1 = \cancel{10} \int_0^\infty t^{10} \cdot e^{-t^{10}} dt$$

$x^{\frac{1}{10}} e^{-x} dx$

$dx = 10 x^{9} dt$

e^{10t}

$$\Gamma(k) = \int_0^\infty y^{k-1} \cdot e^{-y} \, dy$$

$$(y^{k-1})'$$

$$y^{k-1} \cdot e^{-y} = -(y^{k-1} \cdot (e^{-y})') - \underline{(k-1) y^{k-2} \cdot e^{-y}}$$

$$\Gamma(1.1) = \int_0^\infty \underbrace{x^{\frac{1}{10}}}_{t} \cdot e^{-x} dx$$

$x = t^{10}$

$$= \int_0^\infty t \, e^{-t^{10}} \cdot 10 t^9 dt = \int_0^\infty \underline{10 t^{10} \cdot e^{-t^{10}}} \cdot dt$$

$$\int_0^\infty t^{10} e^{-t^{10}} dt \qquad t \cdot \left(\frac{e^{-t^{10}}}{10}\right)' \qquad \boxed{\frac{1}{10} \int e^{-t^{10}} \cdot dt}$$

$$(e^{-t^{10}})' = -(t^{10})' \cdot e^{-t^{10}}$$

$$= -10 t^9 \cdot e^{-t^{10}}$$

$$\left(\frac{e^{-t^{10}}}{10}\right)' = \qquad t^9 e^{-t^{10}} \qquad \frac{e^{-t^{10}}}{10}$$

$$\Gamma(3.1) = \int_0^{\infty} x^{2.1} \cdot e^{-x}\, dx$$

$$= \left[-x^{2.1} e^{-x}\right]_0^{\infty} + \int_0^{\infty} 2.1 \cdot x^{1.1} e^{-x}\, dx$$

$$= 2.1 \cdot \left\{\left[-x^{1.1} e^{-x}\right]_0^{\infty} + 1.1 \int_0^{\infty} x^{0.1} e^{-x}\, dx\right\}$$

$$= 2.1 \times 1.1 \times \Gamma(1.1)$$

$$\Gamma(1.1) = \int_0^{\infty} x^{\frac{1}{10}} e^{-x} dx \qquad \begin{aligned} x &= t^{10} \\ dx &= 10t^9 \cdot dt \end{aligned}$$

$$= \int_0^{\infty} t \cdot e^{-t^{10}} (10t^9)\, dt$$

$$= 10 \int_0^{\infty} (t^{10}) \cdot \underline{e^{-t^{10}}} \cdot dt$$

$$= 1 \int_0^{\infty} t \cdot \underbrace{10t^9 \cdot e^{-t^{10}}}_{(e^{-t^{10}})'} dt$$

$$= -\left[t\, e^{-t^{10}}\right]_0^{\infty} + \int_0^{\infty} e^{-t^{10}} dt$$

$$= \int_0^{\infty} e^{-t^{10}} dt$$

$$\iint e^{-(x^{10} + y^{10})}\, dxdy$$