Wikipedia

積率母関数 $M_X(t)$

存在するなら

$$M_X(t) := E(e^{tX}),\ t\in\mathbb{R}$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\,dx$$

$$= \int_{-\infty}^{\infty} e^{tx}\,dF(x)$$

収束しない場合あり

テイラー展開

$$= \int_{-\infty}^{\infty} \left(1+tx+\frac{(tx)^2}{2!}+\cdots\right) f(x)\,dx$$

$$= 1 + t m_1 + \frac{t^2 m_2}{2!} + \cdots$$

$$E(X^n) = M_X^{(n)}(0)$$

一般化 $\vec{X} = (X_1, \ldots, X_n)$

$$M_{\vec{X}}(\vec{t}) := E\left(e^{\vec{t}^T \vec{X}}\right)$$

2つの独立prob.変数の和

$$M_{X+Y}(t) = E(e^{t(X+Y)})$$

$$= E(e^{tX})E(e^{tY})$$

$$= M_X(t)\cdot M_Y(t)$$

$$S_n = \sum_{1}^{n} a_i X_i$$

各X_iの分布は異なってもいい

$$M_{S_n}(t) = M_{X_1}(a_1 t)\cdot M_{X_2}(a_2 t) \cdots M_{X_n}(a_n t)$$

確率母関数と積率母関数

$$P(z) = \sum_{k=0}^{\infty} p_k z^k = E\{\underset{e^t}{z}^X\} = \int_{-\infty}^{\infty} (e^t)^x f(x)\,dx$$

$$Y = h(X)$$

xとyの関係

$$y = h(x)$$

$$X = h^{-1}(Y)$$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$P(\underline{h(X) < y}) \Rightarrow P(X < h^{-1}(y))$$

$$G(y) = \int_{-\infty}^{y} g(u)\,du \qquad \int^{\cancel{h^{-1}(y)}} \qquad F(h^{-1}(y))$$

$f(x) = \frac{1}{x}$ f^{-1} $f(x) = \frac{1}{x}$

$x \to \log^{y}$

$y = \log x$ $=$ $x = e^{y}$ なので $g(y) = \frac{1}{e^{y}}$

$f(e^{y}) = \frac{1}{e^{y}}$

$\downarrow$

$g(y)$

y

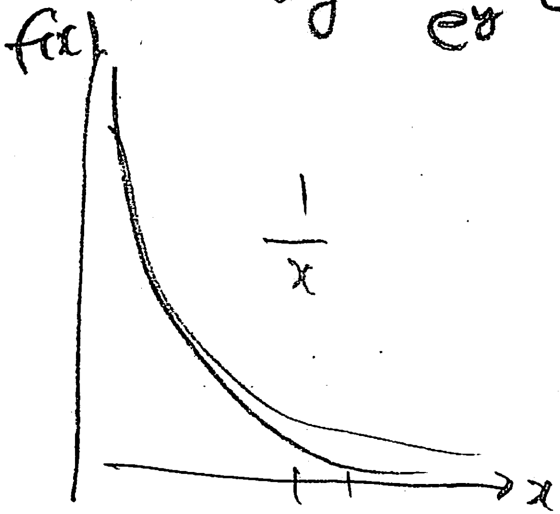
$dy = \frac{1}{x} dx,$ $=$ $\boxed{dx = e^{y} \cdot dy}$ $f'(x)\,dx =$

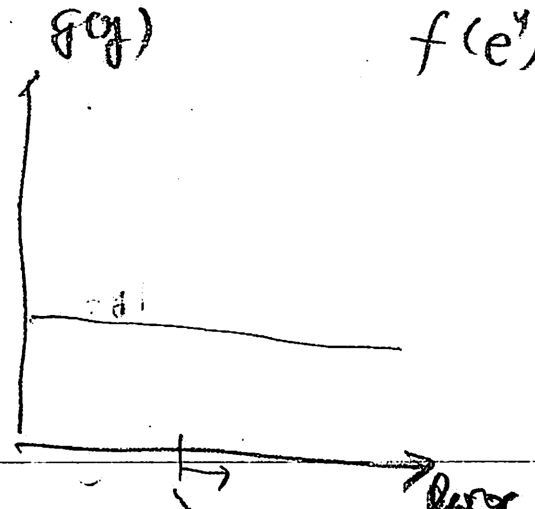
$dy = \frac{1}{e^{y}} e^{y} dy$

$f(x) = \frac{1}{x}$ $\frac{1}{x_{(y)}} = g(\cancel{y})$

$f(e^{y}) = \frac{1}{e^{y}}$

$\frac{1}{x} = g(y)$





$\frac{1}{e^{y}}$

dy

$f(x) = g(y) \frac{dy}{dx}$

$y = \log x \to x = e^{y}$

$dx = e^{y} dy$ $\frac{dx}{dy} = e^{y}$