Γ(z)は階乗の定義を
複素平面に拡張した関数.

実軸上で対数凸となる
解析関数

$$\frac{1}{\sin z}$$

Γ関数は零点をもたず
原点と負の整数に一位の極を持つ

$$\mathrm{Res}(\Gamma, -n) = \frac{(-1)^n}{n!}$$

? $\int z^{k-1} e^{-z}\, dz$

定義域を
複素数へ拡張

$$\Gamma(z) = \lim_{n\to\infty} \frac{n^z \cdot n!}{\prod_{k=0}^{n} (z+k)}$$

t=nuとして

▷ $G_n(z) = \int_0^n t^{z-1} \left(1 - \frac{t}{n}\right)^n dt = n^z \int_0^1 u^{z-1}(1-u)^n du$

$$g_0(z) = \int_0^1 (u^{z-1})\, du = \left[\frac{u^z}{z}\right]_{u=0}^{1} = \frac{1}{z}$$

$$\lim_{n\to\infty} G_n(z) = \int_0^\infty t^{z-1} e^{-t}\, dt$$

$$g_n(z) = \int_0^1 \left(\frac{u^z}{z}\right)' (1-u)^n du$$

$$= \frac{n}{z} \int_0^1 u^z (1-u)^{n-1} du$$

$$= \frac{n}{z} g_{n-1}(z+1)$$

$$G_n(z) = \frac{n^z n!}{\prod_{k=0}^{n} (z+k)}$$

$\prod \frac{\alpha}{z+k}$

$-1, \cdots, -n$ まで零点

$\Gamma(z)$

▷ $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt \quad (\mathrm{Re}(z) > 0)$

▷ $\Gamma(z) = \lim_{n\to\infty} \dfrac{n^z \cdot n!}{\prod_{k=0}^{n} (z+k)} \quad$ (一般の $z$)

$$\underline{\Gamma(z+1) = z \cdot \Gamma(z)}$$

$$F(z+1) = \int_0^\infty t^z \cdot e^{-t} dt$$

$$= \left[ -t^z \cdot e^{-t} \right]_0^\infty + z \int_0^\infty t^{z-1} e^{-t} dt$$

$$\therefore F(z+1) = z\Gamma(z)$$

$$\underline{\Gamma(1)} = \int_0^\infty e^{-t} dt = \left[ -e^{-t} \right]_0^\infty = 1$$

$$\Gamma(n+1) = n!$$

$$\Gamma(k) = \int_0^{\infty} \underline{y^{k-1} \cdot e^{-y}}\, dy$$

$$y^{k-1}(-e^{-y})'$$

$$= \cancel{(k-1)}\left[y^{k-1} \cdot -e^{-y}\right]_0^{\infty} + \int (k-1) y^{k-2} \cdot e^{-y}\, dy$$

$$0 \qquad \infty \quad 0$$

$$= \overbrace{(k-1)\cdots(k-r)} \underbrace{\int y^{k-r} \cdot e^{-y}\, dy}_{\Gamma(k-r)}$$

$$0 < k-r < 1$$

$$\Gamma\left(\frac{1}{n}\right) = \int_0^{\infty} y^{\frac{1}{n}-1} \cdot e^{-y}\, dy = \int_0^{\infty} \cancel{z^{1-n}} \cdot e^{-z^n}\, d\cancel{z^{n-1}} \cdot n\, dz$$

$$= \int_0^{\infty} e^{-z^n} \cdot n\, dz$$

$$y^{\frac{1}{n}} = z \qquad y = z^n$$

$$dy = n z^{n-1} \cdot dz$$

Γ-func

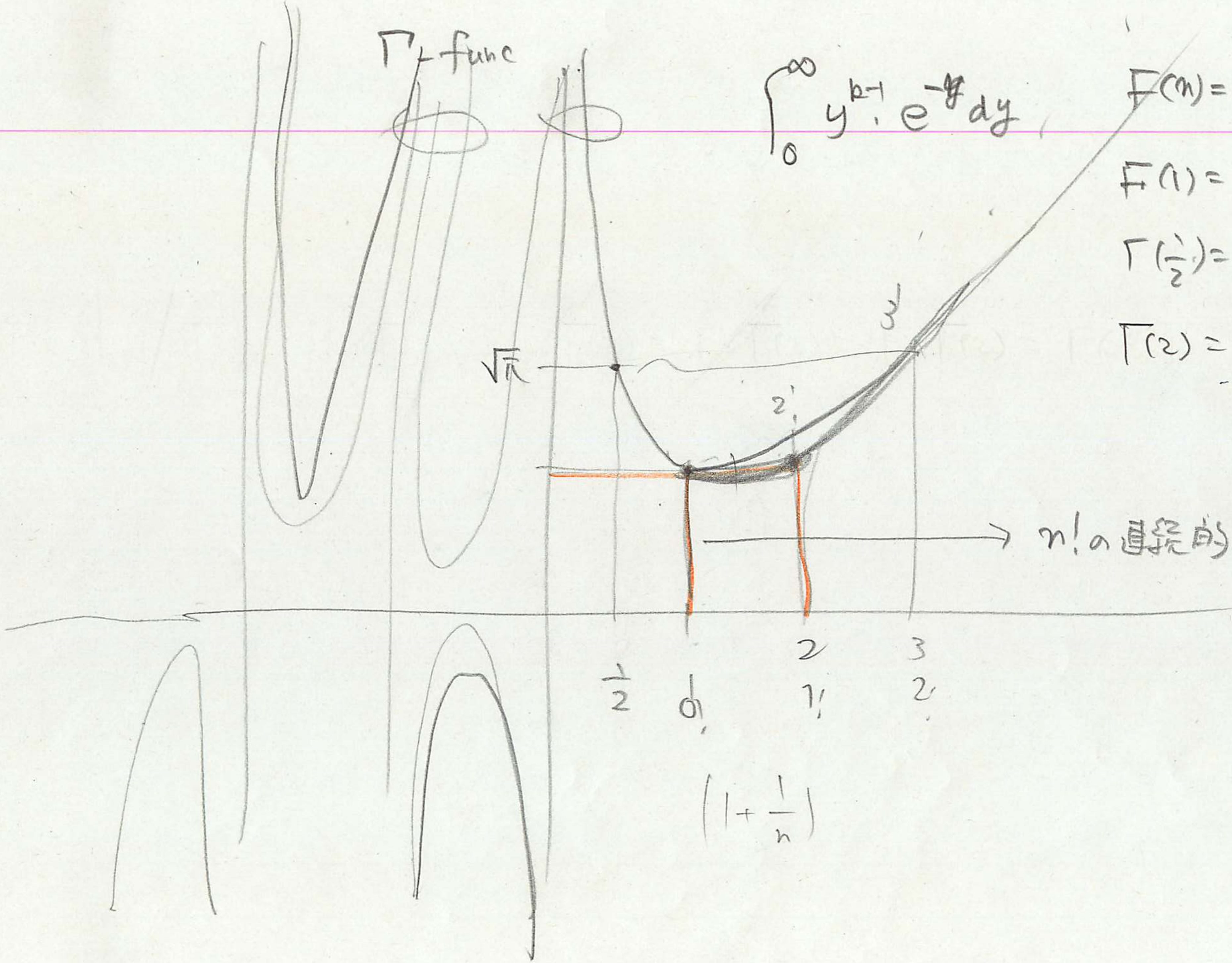
$$\int_0^{\infty} y^{k-1} \cdot e^{-y}\, dy$$

$\Gamma(n) = n-1 \cdot \Gamma(n-1)$

$\Gamma(1) = 1 \quad (0!)$

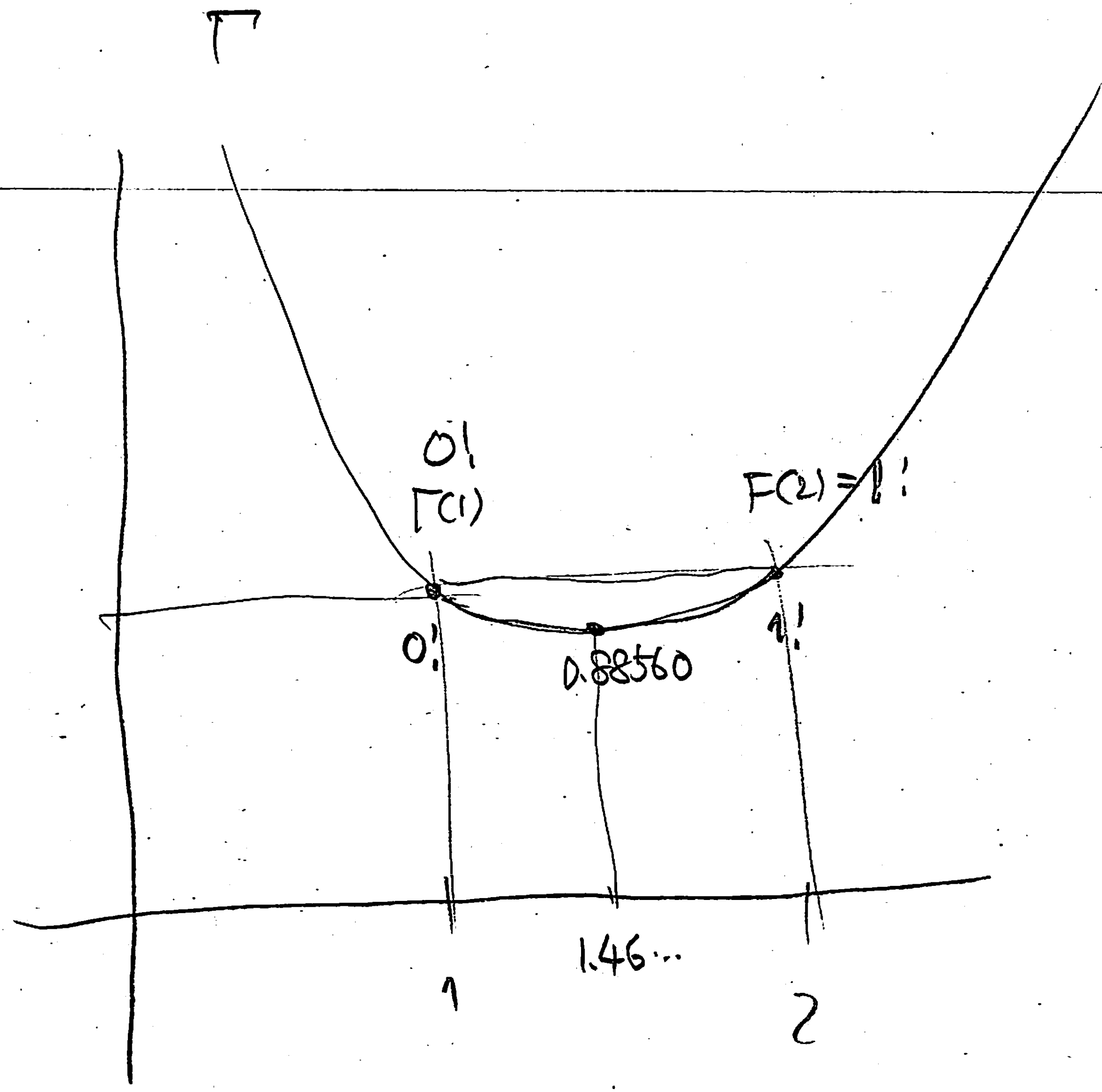
$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$

$\Gamma(2) = 1 \quad (1!)$




$n!$の連続的な拡張.

$\left(1 + \frac{1}{n}\right)$


Γ
0!
Γ(1)
Γ(2) = 1!
0!
0.88560
1!
1.46...
1
2

$\left[-y^{k-1}(e^{-y})'\right]_0^\infty = 0$

$\Gamma(k)$

$= \int_0^\infty y^{k-1} e^{-y} dy \quad (k>0)$

$+\int (k-1) y^{k-2} e^{-y}$

$\vdots$

$(k-1)(k-2)\cdots \int_0^\infty e^{-y} dy$

$(-1)^k (k-1)! \left[e^{-y}\right]_0^\infty$

$k \in \mathbb{Z}$

$(k-1)!$ になる訳.

$\sum_0^\infty \frac{y^{\cancel{k}}}{n!}(-1)^{\cancel{k}}$

$\cancel{n<k}$

$\cancel{n=k}$

$\cancel{n>k}$

$\int_0^\infty \frac{\quad}{\quad}$

$$\Gamma(r) = (r-1)\cdots\underbrace{(r-n)}_{0<r-n<1}\cdot\Gamma(r-n)$$

$$\Gamma(1) = 1$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{2} \quad \leftarrow \text{ガウス}$$

$$\Gamma\left(\frac{1}{3}\right)$$

$$-(x^2+y^2)$$

$$-(x^3+y^3)$$

$$-\underbrace{(x+y)}_{\substack{r(\cos\theta+\sin\theta) \\ \sqrt{2}\,r(\bigcirc)}}\underbrace{(x^2-xy+y^2)}_{(1-\sin\theta\cdot\cos\theta)}$$

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty t^{-\frac{1}{2}} \cdot e^{-t} dt$$

$t^{\frac{1}{2}} = u$

$\frac{1}{\sqrt{t}}$ $\frac{\frac{1}{2} t^{-\frac{1}{2}} dt = du}{}$

$t^{-\frac{1}{2}} = u$

$e^{u^2}$

$-\frac{1}{2} t^{-\frac{3}{2}} dt - du$ $\quad \left(t^{\frac{1}{2}}\right)' = \frac{1}{2} t^{-\frac{1}{2}}$ $\quad \frac{1}{\sqrt{t}}$ $\quad \frac{1}{2u} dt = du$

$tu^3 \; dt = \frac{2}{u^3} du$

$dt = 2u\,du$

$$= \int_0^\infty (\sqrt{t})' \cdot e^{-(\sqrt{t})^2} dt \qquad \int_0^\infty e^{-t^2} du$$

$$= \left[\cancel{\sqrt{t}} \cdot e^{-(\sqrt{t})^2}\right]_0^\infty - \int_0^\infty \sqrt{t} \cdot 2\sqrt{t}$$

$$\Gamma(n) = \int_0^\infty t^{n-1} \cdot e^{-t} dt$$

$$\cancel{\left(\frac{1}{n} t^n \cdot e^{-t}\right) - \frac{1}{n} \int t}$$

$$\left(-t^{n-1} e^{-t}\right) +$$

$y$

$z^{n-1},\ e^{nz},\ \frac{1}{n}\frac{y^{\frac{1-n}{n}}}{z^{1-n}}.$

$$\int_0^\infty \frac{1}{n} e^{z^n}\, dz = \left[e^{nz}\right]_0^\infty$$

$$\left[e^{nz}\right]'$$

$$\frac{1}{n}\int_0^\infty e^{z^n}\, dz \qquad e^{-z^n}$$

$$\left(e^{-z^n}\right) =$$

$$e^{-x} = 1 + \frac{1}{1!}(-x) + \frac{1}{2!}(-x)^2 + \cdots$$

$e^{-z^n}$

$z = r\cos$

$\cancel{x}^n + y^n|$

$r^n \cdot \cos^n\theta + \sin^n\theta$

$x^4 + y^4$

$n = 2m$

$n = 2m+1$

$\underline{(x+y)}(x^2 - xy + y^2)$

$x^3 - x^2y + xy^2$

$yx^2 - xy^2 + y^3$

$$\Gamma\left(\frac{1}{n}\right) = n\int_0^\infty e^{-z^n}\,dz$$

$x^2+y^2$

$$\cancel{\int_0^\infty\int_0^\infty e^{-(x^n+y^n)}\,dx\,dy}$$

$$x^3+y^3 = (x+y)(x^2-xy+y^2) \qquad \frac{\sqrt{2}}{2}\sin\left(\theta+\frac{\pi}{4}\right)$$

$(x+y)^{-2}$

$r$

$$\cos\theta\sin\theta = \frac{1}{2}\sin 2\theta$$

$2\theta$

$$\int_0^\infty e^{-z^r}\,dz = \int_0^\infty \sum \frac{1}{n!}\left[-z^r\right]^n dx$$

$$= \sum_{n=} \frac{1}{(nr+1)\,n!}(-1)^n x^{rn+1}\Big|_0^\infty \quad \forall r$$

[illegible]

1.1. 1.2

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$$

$$= \left[-x^{n-1} \cdot e^{-x}\right]_0^\infty + \int_0^\infty (n-1) x^{n-2} e^{-x} dx$$

$$= (n-1)\int_0^\infty x^{n-2} \cdot e^{-x} dx$$

$$= (n-1) \cdot \Gamma(n-1)$$

$\Gamma(0.1) = 9.5135\cdots$

$\Gamma(0.01) = 99.43258\cdots$

$\Gamma(1.01) = 0.994325\cdots$

$1.1 = 0.95135$

$$\Gamma(1.1) = \int_0^\infty x^{0.1} e^{-x} dx$$

$$3.1 = \cancel{10}\int_0^\infty t^{10} \cdot e^{-t^{10}} dt$$

$x^{\frac{1}{10}} e^{-x} dx$

$x^{\frac{1}{10}} = t$

$\frac{1}{10} x^{-\frac{9}{10}} dx = dt$

$dx = 10 x^{\frac{9}{10}} dt$

$e^{-10t}$

$$\Gamma(k) = \int_0^\infty y^{k-1} \cdot e^{-y}\, dy$$

$$(y^{k-1})'$$

$$y^{k-1} \cdot e^{-y} = -(y^{k-1} \cdot (e^{-y})') - \underline{(k-1)\, y^{k-2} \cdot e^{-y}}$$

$$\Gamma(1.1) = \int_0^\infty \underbrace{x^{\frac{1}{10}}}_{t} \cdot e^{-x} dx$$

$$x = t^{10}$$

$$= \int_0^\infty t\, e^{-t^{10}} \cdot 10t^9 dt = \int_0^\infty \underline{10t^{10} \cdot e^{-t^{10}}} \cdot dt$$

$$\int_0^\infty t^{10} e^{-t^{10}} dt \qquad t \cdot \left(\frac{e^{-t^{10}}}{10}\right)' \qquad \boxed{\frac{1}{10}\int e^{-t^{10}} \cdot dt}$$

$$(e^{-t^{10}})' = (-t^{10})' \cdot e^{-t^{10}}$$

$$= -10t^9 \cdot e^{-t^{10}}$$

$$\left(\frac{e^{-t^{10}}}{10}\right)' = \qquad t^9 e^{-t^{10}} \qquad \frac{e^{-t^{10}}}{10}$$

$$\Gamma(3.1) = \int_0^\infty x^{2.1} \cdot e^{-x}\, dx$$

$$= \left[-x^{2.1} \cdot e^{-x}\right]_0^\infty + \int_0^\infty 2.1 \cdot x^{1.1} e^{-x}\, dx$$

$$= 2.1 \cdot \left\{\left[-x^{1.1} \cdot e^{-x}\right]_0^\infty + 1.1 \int_0^\infty x^{0.1} \cdot e^{-x}\, dx\right\}$$

$$= 2.1 \times 1.1 \times \Gamma(1.1)$$

$$\Gamma(1.1) = \int_0^\infty x^{\frac{1}{10}} e^{-x} dx \qquad x = t^{10}$$

$$dx = 10t^9 \cdot dt$$

$$= \int_0^\infty t \cdot e^{-t^{10}} \cdot (10t^9)\, dt$$

$$= 10 \int_0^\infty (t^{10}) \cdot \underline{e^{-t^{10}}} \cdot dt$$

$$= 1 \int_0^\infty t \cdot \underbrace{10t^9 \cdot e^{-t^{10}}}_{(e^{-t^{10}})'} dt$$

$$= -\left[t\, e^{-t^{10}}\right]_0^\infty + \int_0^\infty e^{-t^{10}} dt$$

$$= \int_0^\infty e^{-t^{10}} dt$$

$$\iint e^{-(x^{10} + y^{10})}\, dx\, dy$$