# Unique Equilibrium in a Model of Secular Stagnation*

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**Abstract**

In secular-stagnation models, multiple equilibria can arise, implying that monetary policy may not be able to achieve an inflation target even when the target is sufficiently high and the government is perfectly credible. Under perfect information about fundamentals, a unique equilibrium cannot be pinned down because agents' beliefs are perfectly coordinated. I relax the assumption of perfect information and develop an endogenous equilibrium-selection mechanism by integrating a global-game approach into a secular-stagnation model, which generates strategic complementarity in households' decision-making,"*my action depends on my belief of your action*", resulting in a unique equilibrium choice. In contrast to the existing literature on Secular Stagnation, I find that given an inflation target, a temporary fiscal expansion or an average-inflation-targeting policy (AIT) can raise the likelihood that a better equilibrium is chosen by reinforcing strategic complementarity, which promotes households' ability to coordinate to resolve the demand shortage. In a calibrating example, I find that the selection probability of a secular-stagnation equilibrium was high in the US after the Great Recession but more fiscal expansion and AIT reduced it.

**Keywords:** Secular Stagnation; Global Game; Monetary Policy; Fiscal Policy; Determinacy; Equilibrium Selection; Debt Sustainability

**JEL Classification:** E6, E7, E12, E23, E31, E32, E43, E52, E62, H3, J1

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# 1 Introduction

The post-Great Recession experience has taught us that the zero lower bound (ZLB) on short-term nominal interest rates can be binding for much longer than initially expected. In the US, the ZLB was binding from 2008 until the Fed raised interest rates in 2016. In Japan, interest rates have been zero or negative since the mid-1990s. However, despite research on how to model the ZLB, early generations of ZLB models could not address these long-lasting ZLB episodes because the nominal interest rate was assumed to rise back above zero in a reasonably short period [1].

The secular-stagnation literature presents the latest generation of ZLB models. It suggests that these arbitrarily long ZLB episodes are caused by slow-moving secular forces that are difficult to reverse, such as aging and rising inequality. For example, Eggertsson et al. (2019) introduce the ZLB into the overlapping generations (OLG) model and find that ZLB episodes can be arbitrarily long. Caballero and Farhi (2017) show that long-lasting ZLB episodes arise from a shortage of safe assets. Michaillat and Saez (2021, 2022) extend the New Keynesian model by incorporating relative wealth into households' utility function and show that under this assumption, the model can capture long-lasting slumps. Since it is unlikely that the secular downward trend in the natural rate of interest will reverse in the future, even given the current high inflation rate, the ZLB will be likely to bind again when adverse shocks hit the economy.

However, secular-stagnation models also raise new challenges, even if the central bank chooses sufficiently high inflation targets, but that choice was the key policy proposal suggested in the earlier ZLB literature (see, for example, Krugman 1998). Incorporating into the model slow-moving secular forces such as demographics creates two determinate equilibria: a (good) inflation-targeting equilibrium and a (bad) secular-stagnation one. In the first equilibrium, the inflation target and full-employment are achieved. In the other, inflation persistently remains below the inflation target and output below the full-employment level. Existing literature, however, provides little guidance about which equilibrium is chosen.

Many authors, such as DeLong and Summers (2012) and Eggertsson et al. (2019), propose a large, permanent debt expansion as a possible solution to secular stagnation. Permanent debt expansion raises the natural rate of interest. A large-enough fiscal expansion causes the inflation-targeting equilibrium to be the unique equilibrium because it raises the natural rate permanently. However, the plausibility of this solution has recently been challenged

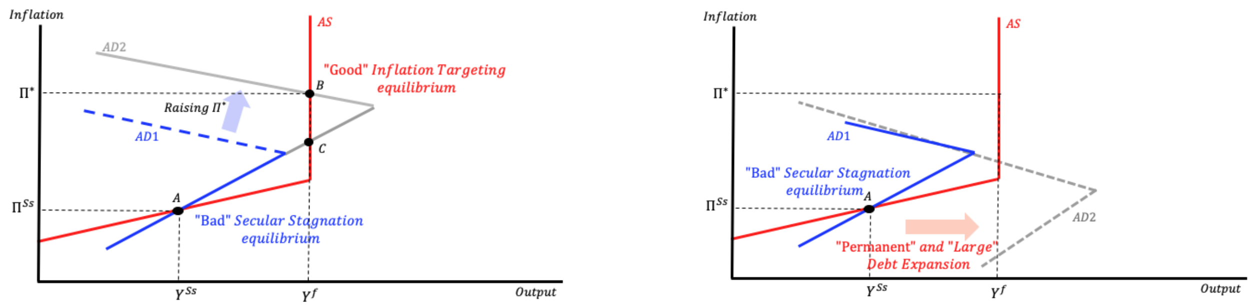
[1] The first generation of ZLB models describe ZLB episodes as resulting from temporary exogenous shocks such as preference shocks–for example, Krugman (1998) and Eggertsson and Woodford (2003). The second generation introduces into the model the nature of underlying shocks such as financial disturbances–for example, Gertler and Kiyotaki (2010), Eggertsson and Krugman (2012), and Gurrieri and Lorenzoni (2017).

(see, for example, Garga 2020) because the magnitudes involved may imply very high ratios of government debt to output. Eggertsson et al. (2019), for example, report that raising the natural rate to 1% would require a permanent increase in the ratio to 215%. Such a drastic measure could threaten debt unsustainability, which could adversely affect demand and undermine the original purpose of fiscal expansion, as shown by Garga (2020).

My first contribution is to resolve the difficulty of pinning down a unique equilibrium in secular-stagnation models. I argue that the multiplicity of equilibria is the result of assuming perfect information about fundamentals. I assume instead that agents have imperfect common knowledge about the natural rate of interest. The natural rate is a prominent indicator of fundamentals, but it is also an uncertain indicator (Williams 2018, Powell 2018). So it is a natural way to introduce imperfect information into the model. Once households observe private signals about fundamentals, the assumption of common knowledge of fundamentals is no longer valid. Applying the global-game technique, as found in Morris and Shin (1998, 2003) and Angeltos and Lian (2022), I show that this is enough to select a unique equilibrium in the secular-stagnation model. I show that in a certain range of underlying natural rates in the model, the secular-stagnation equilibrium is chosen, while in another range, the inflation-targeting equilibrium is chosen.

My second contribution is to show how being able to pin down a unique equilibrium has distinct policy implications. For example, existing secular-stagnation models include a thought experiment in which a temporary increase in public debt is irrelevant in equilibrium (regardless of which one is chosen). In contrast, I show that such an increase can significantly increase the likelihood of the good inflation-targeting equilibrium being chosen. Similarly, existing literature shows that when the inflation target is higher than the (negative of) the natural rate, increasing it further has no effect. In contrast, I show that increasing it further will generally increase the probability of achieving the target. Finally, in existing secular-stagnation models, policies such as the recently adopted average-inflation-targeting policy (AIT), implemented by the Federal Reserve in 2020, and the Bank of Japanâs Inflation Overshooting Commitment, adopted in 2021, yield the same results as a regular inflation targeting regime. In contrast, I show that both policies increase the probability of selecting the inflation-targeting equilibrium. At the heart of these results is the assumption that these policy choices interact with the strategic complementarity of household decisions and thus influence the probability of each equilibrium being selected. A related contribution is that I show how various exogenous factors, such as labor-market structure and demographics, affect the equilibrium-selection probability.

My third contribution is to offer a simple calibrated example to capture the state of the US economy prior to the COVID-19 pandemic. I find that the selection probability of the




(a) Multiple Equilibria (b) Suggested Resolution

Figure 1: Perceived AD-AS Diagram under Perfect Information

Note: The left panel shows multiple equilibria in the Secular Stagnation model given sufficiently high inflation target. The right panel shows that "*large*" and "*permanent*" debt expansion is required to obtain a unique "*good*" equilibrium.

secular-stagnation equilibrium was around 90% in 2015 in the US but fiscal expansion could have effectively reduced the likelihood to 25%. However, I also find that the likelihood of secular stagnation can easily increase without appropriate monetary and fiscal policies if relatively minor changes in the underlying secular forces occur.

The key to my results is the strategic complementarity of households' choices; that is, their actions depend on their beliefs about others' actions. There are many ways of formalizing the choice structure that gives rise to this force. I assume that households make a discrete decision to work either part time or full time at the beginning of each period[2]. Households make the decision based on their private signals, which matter for equilibrium choice because it increases aggregate borrowing. The more full-time workers, the greater the total labor income, which is the source of aggregate borrowing. If a sufficient number of households coordinate to work full time, the coordination pushes the economy toward the inflation-targeting equilibrium. Strategic complementarity can be introduced via various different mechanisms that I highlight in the body of the paper. In the appendix, I consider an alternative example in which households invest in human capital. Other examples, which I leave for future research, may include irreversible-investment and asset-portfolio choices.

# 2 Challenges in the Secular Stagnation Models

Secular-stagnation models raise new challenges while allowing us to address long-lasting ZLB episodes: (1) determinate multiple equilibria and (2) concerns about fiscal sustainability. I informally discuss these problems here, leaving formal discussion to the later section. The left and right panels of figure 1 show an aggregate-demand (AD) / aggregate-supply (AS)

[2]Since indivisibility of choice is important, we can differently interpret this assumption as applying the same kind of economic decision and obtain the same result. Appendix B shows a human capital investment.

diagram of the secular-stagnation model (see Eggertsson et al. 2019 and Caballero and Farhi 2017). The kink in the AD curve arises at the inflation rate at which monetary policy is constrained by the ZLB. In other words, the AD curve will become upward sloping when the inflation rate is low so that the implied nominal rate the central bank wants to set is below zero. As for the AS curve, the kink occurs at the inflation rate at which full-employment is achieved. That is, the AS curve will become a vertical line when the inflation is sufficiently high so that the full-employment is achieved because real wages fall and firms hire more labor as inflation increases.

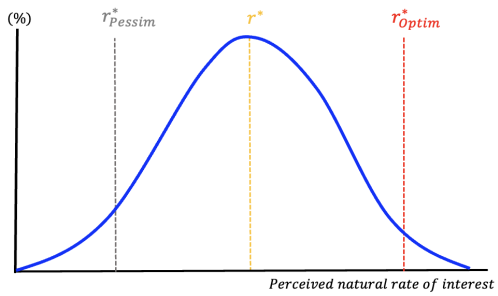
In both panels in figure 1, suppose that some large enough shock occurs that moves the natural rate of interest negative, which decreases output for any given inflation rate[3]. This fall in output is mainly caused by the decrease in consumption by the households who cannot borrow as much as before to finance their consumption. In the "good" equilibrium, this reduction in spending would be compensated by a fall in the real interest rate, which restores their spending to its pre-shock level. However, the ZLB prevents this adjustment to revert the "good" equilibrium. Therefore, the shock moves the economy off the vertical segment of the AS curve to the "bad" secular-stagnation equilibrium (point $A$) where the ZLB is binding. Here, the equilibrium deflation raises the equilibrium real wages above their market-clearing level, thereby contracting output while depressing demand for labor.

In the left panel, there are three equilibria given a sufficient inflation target: the secular-sta gnation equilibrium (point $A$), at which below-target inflation and low output are realized; the inflation-targeting equilibrium (point $B$), at which the inflation target and full employment are realized; and between these two (point $C$), an equilibrium that is indeterminate and not learnable. Because it is not learnable, the third equilibrium can be excluded, according to some learning criteria.[4] In contrast, since the equilibria $A$ and $B$ satisfy the standard determinacy conditions, neither one can be ruled out.

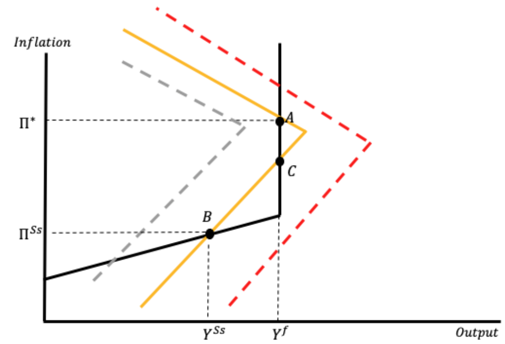
There is no obvious way to determine which equilibrium arises. Due to this multiple equilibria problem, monetary policy loses its ability to control inflation in equilibrium $A$ because it is unclear when the ZLB will no longer be binding. Thus, many authors, such as Summers and De Long (2012) and Eggertsson et al. (2019), propose a large, permanent debt expansion as a possible solution to secular stagnation. As the right panel of figure 1 shows, permanent debt expansion raises the natural rate of interest, thus shifting the entire AD curve out.

[3] We can interpret this shock as for example the deb deleveraging shock in Eggertsson and Krugman (2012) and Eggertsson et al. (2019).

[4] For example, Christiano et al. (2018) discuss how indeterminate ZLB episodes driven by self-fulfilling expectations can be ruled out since they are not learnable. Gibbs (2018) stresses that indeterminate equilibria cannot justify coordination of expectations.




(a) Fundamental




(b) Perceived AD/AS Diagram

Figure 2: Perceived AD/AS Diagram under Perfect Information

Note: Panel (a) shows the state of the fundamental. Panel (b) shows the perceived AD and AS curves; for example, a household whose signal is $r^*$ forms the perceived AD curve, which is depicted by the yellow line. The AS curve is independent of the fundamental.

However, its plausibility is questionable because the fiscal expansion must be permanent. Eggertsson et al. find that raising the natural rate to 1% would require a permanent increase in the ratio of US government debt to GDP to 215%. Such a drastic measure could raise concerns about debt sustainability as shown by Garga (2020). High government debt to GDP ratio is typically associated with expectation of future tax increases and potential unsustainability of government debt. If investors perceive the possibility of a sovereign debt crisis as high, government debt will be perceived to be risky, and thus the real interest rate will rise, which is necessary to compensate for the risk of holding government debt. In such case, debt unsustainability adversely affects demand and undermine the original purpose of fiscal expansion. This paper discusses how we can resolve the challenges arise in secular-stagnation models

# 3 Baseline Model

I start by developing the model, which is an extension of Samuelson (1958). To understand the role of coordination, I first assume perfect information about the fundamentals. The baseline structure is a simple three-generation OLG model with nominal rigidities. The only modification of the perfect-information assumption concerns households' labor choice between full- and part-time work. Introducing a binary choice creates room for coordinated household behavior once the perfect-information assumption is relaxed. I discuss an alternative mechanism that gives room for coordination.

As Eggertsson et al. (2019) argue, the key factors in multiple equilibria are ZLB and demographics, which are incorporated in my model. So, the same multiple equilibria occur in my model, as in figure 1. Since the equilibrium $C$ can be ruled out because of its nonlearnability, I focus on the determinate equilibria $A$ and $B$.

### 3.1 Perfect Information

The economy is characterized by households' belief about a fundamental, $s_t \sim N(\mu_s, \sigma_s^2)$—namely, their common perception of the natural rate of interest. For simplicity, I assume that the underlying fundamental is fixed at some $r^*$. It is a straightforward generalization of allowing $r^*$ distributes but isolates the role of beliefs. $s_t$ represents the fundamentals of how optimistic households are about the natural rate of interest. Under perfect information, they know $s_t$.

Households are biased to be optimistic at times and pessimistic at others. Survey data show that household forecasts of macroeconomic outcomes are biased and that both optimistic and pessimistic biases fluctuate over the business cycle (for example, Bianchi et al. 2021 and Bhandari et al. 2022). In secular stagnation, they can be more biased because coordination is difficult at such times. As noted, the natural rate of interest is a prominent guiding indicator of the state (Del Negro et al. 2017), but estimates of it tend to be highly uncertain (Williams 2018 and Powell 2018). So the common perception of it is that it is a natural candidate as a state of uncertain fundamentals.

Figure 2 provides an intuitive understanding of this fundamental. Suppose that $r^*$ is the true fundamental. At $s_t = r^*_{optim} > r^*$ in panel (a), households are optimistic in such a way that the perceived AD curve (the dotted red line in panel [b]) intersects the upper segment of the AS curve at a unique point. At $s_t = r^*_{pssim} < r^*$, households are pessimistic such that the perceived AD curve (the dotted gray line) intersects the bottom segment of the AS curve at a unique point. At $s_t = r^*$, households are neutral, as it equals $r^*$. But this state of affairs leads to multiple equilibria. Let us now consider the microfoundations of this state of affairs.

### 3.2 Households

Households are born in period 1 (young age), become middle-aged in period 2, and retire in period 3 when they enter old age. There is no physical capital, so intergenerational borrowing/lending is the only way to save. The young borrow from the middle-aged, and the old live on accumulated savings. The young live hand-to-mouth, as their borrowing constraint is always binding.

Household $j$ of a cohort born at time $t$ maximizes the following problem:

$$\max_{C_{t,j}, C_{t,j+1}, C_{t,j+2}, L_{t+1,j}} E_t \left\{ u\left(C^y_{t,j}\right) + \beta \left\{ u\left(C^m_{t+1,j}, L_{t+1,j}\right)\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right\}, \tag{1}$$

s.t.

$$C_{t,j}^{y}=B_{t+1,j}^{y}, \tag{2}$$

$$C_{t+1,j}^{m}=w_{t+1}L_{t+1,j}-T_{t+1}^{m}+B_{t+1,j}^{m}-\frac{1+i_t}{\Pi_{t+1}}B_{t+1,j}^{y}+z_{t+1}, \tag{3}$$

$$C_{t+2,j}^{o}=-T_{t+2,j}^{o}-\frac{1+i_{t+1}}{\Pi_{t+2}}B_{t+1,j}^{m}, \tag{4}$$

$$\frac{1+i_t}{E_t\Pi_{t+1}}B_{t,j}^{i}\leq D_t, i=\{y,m,o\}, \tag{5}$$

$$i_t\geq 0 \tag{6}$$

Here the first constraint is the budget constraint for the young: consumption $C_{t,j}^{y}$ is financed by borrowing $B_{t,j}^{y}$. The second constraint is the budget constraint for the middle-aged, who receive real labor income $w_{t+1}L_{t+1,j}$, pay tax $T_{t+1}^{m}$ (or receive a subsidy), save $-B_{t+1,j}^{m}$, receive profits $z_t$, and repay, at nominal interest rate $i_t$, what they borrowed while young. The labor supply is determined by the household's labor choice between full-time and part-time employment, though labor rationing is possible because of downward nominal wage rigidity. The third constraint is the budget constraint for the old, who consume their savings and interest and pay tax $T_{t+2}^{o}$. The first inequality is the exogenous borrowing limit $D_t$, and the second inequality is the ZLB of the nominal interest rate. The simplest utility function is given as follows:

$$u\left(C_{t+1,j}^{m},L_{t+1,j}\right)=lnC_{t+1,j}^{m}-\varphi L_{t+1,j} \tag{7}$$

Here $\varphi$ denotes a parameter that governs the size of labor disutility.

We assume the simplest form of nominal rigidity, in which households never accept lower wages than a wage norm:

$$W_t=max\left\{\tilde{W}_t,W_t^{flex}\right\} \tag{8}$$

Here $\tilde{W}_t=W_{t-1}^{\gamma}\left(W_t^{flex}\right)^{1-\gamma}$ and $W_t^{flex}=P_t\alpha L_t^{\alpha-1}$. That is, if the nominal wages necessary for market clearing are below $\tilde{W}_t$, labor rationing occurs.

## 3.3 Indivisible Choice: Labor Supply

I introduce an indivisible labor choice. This assumption is common in the macroeconomic labor literature. Labor choice is highly indivisible in the US in the sense that most variation in total hours worked is due to variation in the number of employees, as opposed to variation in hours worked per worker, as discussed by Hansen (1985) and Rogerson and Shimer (2010).

Households choose their labor type between $\left\{L^F,L^P\right\}$, where $L^F$ is the constant labor supply of full-time work and $L^P$ is that of part-time work:[5]

$$L_{t+1,j} = \begin{cases} L^F \\ L^P \end{cases}, \tag{9}$$
$$L^P = \delta L^F$$

Here $\delta < 1$; that is, a full-time worker works more than a part-time worker.

Each period $t$ is divided into two stages: in the first stage, each household receives common knowledge about the fundamentals and decides on its optimal labor choice by comparing the utility gain of each choice by taking other households' optimal decision-making on labor, consumption, and savings.

In the second stage, household decisions are aggregated, which determines other variables such as inflation. Because of the downward wage rigidity, labor rationing can occur under deflation. Suppose that a proportion $\eta$ of households work full time and the rest of them work part time. Once deflation begins, labor rationing constrains the total labor supply $L_t^R = \left\{ \Pi_t^{\frac{1-\alpha}{\alpha(1-\gamma)}} Y^f \right\}^{\frac{1}{\alpha}}$, which is derived from the wage norm. Total labor is allocated to full-time and part-time workers.[6] Since the indivisibility of the decision is key, this assumption can be interpreted differently, for example, human capital investment.

## 3.4 Benefit and Cost of Labor Choice

The labor choice can yield different outcomes because of labor rationing. Households take this into account when choosing their labor type. To see this, let us focus on the benefits of full-time work. What is the incentive to choose full-time work? There are three benefits arising from the inflation target achievement. The consumption/saving plan is determined by the Euler equation:

$$\frac{1}{C_{t+1,j}^m} = \beta E_t \left[ \frac{1}{C_{t+2}^o} \frac{1 + i_{t+1}}{\Pi_{t+2}} \right] \tag{10}$$

First, a rise in the expected inflation rate will ease the borrowing constraints of the young:

$$C_{j,t}^y = \frac{\Pi^*}{1 + i^*} D > \Pi_t^{Ss} D = C_{j,t}^y \big|_{Ss}, \tag{11}$$

Here $\Pi^{Ss}$ is a realized level of deflation and subscript $Ss$ means that deflation is realized.

[5] The original indivisible-labor model, in Hansen (1985) and Rogerson (1988), assumes that workers choose between working and not working.

[6] Suppose that proportion $\eta$ of households choose full-time work and the rest choose part-time work; the total labor supply is $\eta L^F + (1 - \eta) L^P = L^R$. So $L^P = \frac{L^R}{\eta \delta^{-1} + 1 - \eta}$ and $L^F = \frac{\delta^{-1} L^R}{\eta \delta^{-1} + 1 - \eta}$.

Second, the middle-aged households can consume more under inflation than under deflation because their labor incomes increase more:

$$C^m_{j,t} = \frac{\alpha \bar{L}^{\alpha-1} L}{1+\beta} - \frac{D}{1+\beta} > \frac{L_{Ss}}{1+\beta} \frac{(1-\gamma)\, \alpha \bar{L}^{\alpha-1}}{1-\gamma \left(\Pi^{Ss}_t\right)^{-1}} - \frac{D}{1+\beta} = C^m_{j,t}\big|_{Ss} \tag{12}$$

Third, the old households can consume more under inflation than under deflation because their accumulated savings increase:

$$C^o_{j,t} = \frac{1+i^*}{\Pi^*}\, B^m_{j,t}\big| > \frac{1}{\Pi^{Ss}}\, B^m_{j,t}\big|_{Ss} = C^o_{j,t}\big|_{Ss} \tag{13}$$

There is a potential cost of full-time work. Even if a household chooses full-time work, nominal wages may be binding at the wage norm, resulting in labor rationing. Under labor rationing, labor supply $L_{Ss}$ is small even for a full-time worker. As a result, though labor income increases only slightly, the worker suffers higher labor disutility because labor disutility is linear as $\varphi L^F > \varphi L^P$.

## 3.5 Firms

Firms are perfectly competitive and take prices as given. Their profit-maximization problem is as follows:

$$\max_{L_t} P_t Y_t - W_t L_t \tag{14}$$

s.t.

$$Y_t = L_t^\alpha \tag{15}$$

$L_t$ is the sum of the labor supply of full-time and part-time workers. The firms' labor-demand condition is given as follows:

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1} \tag{16}$$

## 3.6 Central Bank

The central bank declares that nominal interest rates are controlled by the Taylor rule:

$$1 + i_t = max\left(1, (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right) \tag{17}$$

Here $\Pi^*$ is the inflation target and $i^*$ is the target nominal interest rate. Since prices are perfectly flexible, we can interpret this rule as saying that the central bank will set inflation equal to $\Pi^*$ as long as the ZLB is not binding. We can compute the inflation rate corresponding to the zero nominal interest rate $\Pi^{zero}$ by rearranging the Taylor rule:

$$\Pi^{zero} = \left(\frac{1}{1+i^*}\right)^{\frac{1}{\phi_\pi}} \Pi^* \tag{18}$$

To escape from the ZLB, we need $\Pi_t > \Pi^{zero}$.

## 3.7 Government

The government budget constraint is as follows:

$$B_t^g + T_t^y (1+g) + T_t^m + \frac{T_t^o}{1+g} = G_t + \frac{1}{1+g} \frac{1+i_{t-1}}{\Pi_t} B_{t-1}^g \tag{19}$$

Here $B_t^g$ denotes the government debt (normalized to the size of the middle-aged generation), $T_t^i, i \in \{y, m, o\}$ denotes taxes on each generation, $g$ is the population growth rate, and $G_t$ denotes government expenditure. Before introducing fiscal policy, I discuss how a unique equilibrium is endogenously selected in the simplest setting—one without fiscal policy. That is, $B_t^g = 0$, $\{T_t^i\}_{i=\{y,m,o\}} = 0$, and $G_t = 0$.

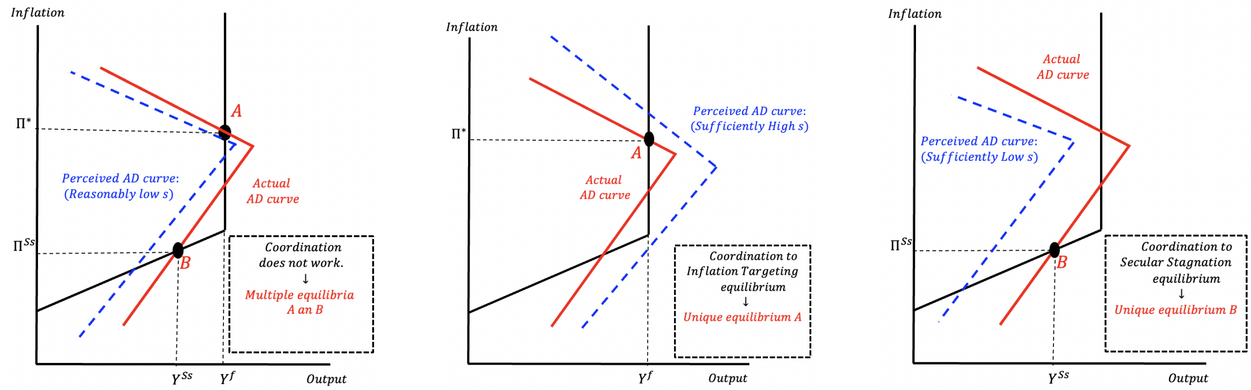
## 3.8 Multiple Equilibria with Common Belief

Let us first see how the model works under full information when all agents' beliefs are coordinated at some $s_{t,p}$ and their perceptions are fixed in future periods so that $s_{t+j,p} = s_{t,p} = s, \forall j$. This will clarify why a lack of coordination of beliefs is an issue for equilibrium selection by partitioning the fundamental space into three intervals. Importantly, aggregate borrowing varies with $s$. I derive the steady-state AD and AS curves and their forms given a common belief about fundamental $s$.

The AS curve consists of two regimes: full employment and labor rationing. The form of the curve is not affected by $s$ because it is independent of the natural rate of interest. Consider first the case of positive inflation: $\Pi > 1$. Positive inflation implies that the real wage is set so that the labor market clears. Given the proportion of part-time workers $\bar{\Phi}$, labor demand is $\bar{L} = \left\{\bar{\Phi} L^P + \left(1 - \bar{\Phi}\right) L^F\right\}^\alpha$, and the AS curve is defined as follows:

$$Y = \left(\bar{L}\right)^\alpha = Y^f \quad for \;\; \Pi \geq 1 \tag{20}$$

Consider now deflation. In this case, the wage norm is binding, resulting in labor




(a) Multiple equilibria

(b) Coordination: Inflation Targeting

(c) Coordination: Secular Stagnation

Figure 3: Perceived and Actual AD/AS Diagram under Perfect Information

Note: Solid lines are actual AD and AS curves, and dotted lines are perceived AD curves, which can deviate from the actual AD curve depending on $s_t$. Panel (a) shows how multiple equilibria remain as a result of coordination failure. Panel (b) shows how the inflation-targeting equilibrium is coordinately selected. Panel (c) shows how the secular-stagnation equilibrium is coordinately chosen.

rationing. The AS curve becomes the following:

$$\Pi = \left(\frac{Y}{Y^f}\right)^{\frac{\alpha(1-\gamma)}{1-\alpha}} \qquad for \;\; \Pi < 1 \tag{21}$$

Next, since the AD curve is affected by the natural rate of interest and the common perception $s$ can deviate from the true fundamental $r^*$, we need to distinguish between *perceived* and *actual* AD curves. Let us derive the two curves consisting of two regimes: one in which ZLB is binding, and one in which it is not. The essential difference between the two curves concerns how they are used: the perceived AD curve generates households' decision rules, but the actual AD curve determines the equilibrium outcome based on the households' decision rules.

The form of the perceived AD curve depends on $s$ because the perceived natural rate of interest matters. Since $\frac{\partial 1+r_t^*}{\partial D} > 0$, $\frac{\partial 1+r_t^*}{\partial g} > 0$, and $\frac{\partial 1+i_t^*}{\partial 1+r_t^*} > 0$, the higher $s$ is converted as the higher $D$, $g$, and $1+i^*$.[7] This implies that the higher the $s$, the higher the natural rate of interest and the more the AD curve shifts out. Given common beliefs about fundamentals, and combining the equilibrium real interest rate with the Fisher equation and Taylor rule, we get the perceived AD curve:

$$Y(s) = D(s) + \frac{(1+\beta)(1+g(s))D(s)\Gamma^*(s)}{\beta}\frac{1}{\Pi^{\phi_\pi - 1}} \quad for \, i > 0 \tag{22}$$

[7] The natural rate of interest is given by $1+r^* = \frac{1+\beta}{\beta}\frac{(1+g)D}{(Y^f)^\alpha - D}$.

Here $\Gamma^*(s) \equiv (1+i^*(s))^{-1} (\Pi^*)^{\phi_\pi}$ and $\phi^\pi > 1$. The following is also true:

$$Y(s) = D(s) + \frac{(1+\beta)(1+g(s)) D(s)}{\beta} \Pi \quad for\, i = 0 \tag{23}$$

In contrast, the actual AD curve is not affected by $s$, so its functional forms are equivalent to equations (22) and (23) except for excluding $s$ so that $s$ is consistent with $r^*$. Importantly, the perceived AD curve can deviate from the actual AD curve depending on $s$.

Figure 3 plots both perceived and actual AD and AS curves. In each panel, I assume a different level of $s$. The dotted lines represent the perceived AD curves, the solid red lines represent the actual AD curves, and the solid black lines represent the AS curves. In panel (b), the perceived AD curve interects the AS curve at a unique point in the vertical part of the AS curve. In this case, households coordinate to choose their decision rule so that it is consistent with the inflation-targeting equilibrium. Based on their decision rule, the actual AD and AS curves pin down a unique inflation-targeting equilibrium. Similarly, in panel (c), the perceived AD curve intersects the AS curve at a unique point in the lower part of the AS curve. Thus, coordination among households can select a unique secular-stagnation equilibrium in the diagram with actual AD and AS curves.

However, as panel (a) illustrates, the perceived AD curve only slightly deviates from the actual AD curve, resulting in intersections in both parts of the AS curve. In this case, it is not obvious to households which labor choice is rational, as equilibrium selection depends on what they believe. If all households believe in the inflation-target realization, the inflation-targeting equilibrium is chosen, while if all think that deflation will occur, another equilibrium is chosen. Under perfect information, they cannot coordinate because they have no way of predicting others' behavior. Thus, multiple equilibria always exist if the perception of $s$ is near the natural rate of interest. This corresponds to the multiple equilibria discussed by Eggertsson et al. (2019).

# 4 Unique Equilibrium

In this section, I relax the assumption of perfect information about common beliefs about fundamentals and, by applying the global-game approach of Morris and Shin (1998, 2003), show how strategic complementarity among households pins down a unique equilibrium.

## 4.1 Imperfect Information

To introduce ex ante uncertainty, suppose that household $j$ receives a private signal $s_{j,t}$ about the belief of fundamental $s_t$:

$$s_{j,t} = s_t + \epsilon_{t,j}, \tag{24}$$
$$\epsilon_{t,j} \sim N\left(0, \sigma^2\right), s_t \sim N\left(\mu_s, \sigma_s^2\right)$$

I consider the case $1 + r^*_{j,t}\left(s_{j,t}\right) = 1 + s_{j,t}$. The ex ante uncertainty in the model concerns the ex ante belief dispersion about the natural rate of interest.[8] In contrast to the common-belief case, once $s_{j,t}$ is drawn, each household forecasts the distribution of other households' perceptions of the natural rate of interest and computes their own lifetime expected utility based on the same fundamental forecast for their own decision-making. The realization of $s_t$ allows all households to know the true distribution of perceived natural rates of interest. (I provide details on how this difference affects households' decisions in the coming sections.) In the aggregate, a unique equilibrium can be pinned down by affecting the AD and AS curves.

This is a natural way to introduce ex ante uncertainty as belief dispersion because (as noted) the natural rate of interest has become a prominent indicator used for monetary policy (Del Negro et al. 2017) but its estimates are highly uncertain (Williams 2018; Powell 2018).

## 4.2 Optimal Threshold Strategy

### 4.2.1 Timing Structure and Threshold Strategy on Labor Choice

The timing of events is important for incorporating the global-game method into the secular -stagnation model. Let us assume that the natural rate of interest falls into negative territory for some reason and stays negative because of demographic transitions. This implies that multiple equilibria can arise if there is a common belief among households about fundamentals, as we have seen. However, once we introduce belief dispersion, each household forecasts other households' perceptions. That households take others' beliefs into account generates a unique equilibrium.

Each period is divided into two stages: (1) a stage in which households receive private signals about the natural rate of interest and make a labor choice, taking beliefs about

[8] As a special case, we can assume that the mean of the private-signal distribution is fixed but its dispersion is uncertain; that is, ex ante uncertainty about $\sigma^2$ makes the perceived distribution of the natural rate of interest heterogeneous among households.

others' behavior into account; and (2) a stage in which the true state (that is, the true labor-choice distribution) is revealed. Then, households decide on their savings/consumption plans, production takes place, and all markets clear.

In the beginning of the first stage, when each household $j$ receives a private signal $s_{j,t}$, it computes the expected utility of each labor choice. Households are assumed to adopt a threshold strategy $s^*_{j,t}$ for labor choice using the private signals. $s^*_{j,t}$ is a cut-off value that indicates that households with signals above $s^*_{j,t}$ work full time and those below $s^*_{j,t}$ work part time. That is,

$$L_{t,j} = \begin{cases} L^P \;\; if \;\; s_{j,t} < s^*_{j,t} \\ L^F \;\; if \;\; s_{j,t} \geq s^*_{j,t}. \end{cases} \tag{25}$$

To compute their expected utility, they derive the subjective probability of realizing the inflation-targeting equilibrium and the secular-stagnation equilibrium. Agents optimally incorporate others' decision-making into their own decision-making. Hence agents' beliefs about other agents' beliefs matter. Similarly, how agents predict other agents' beliefs is also important. In this way, optimal decisions are made by repeatedly considering each other's beliefs. In other words, higher-order beliefs matter (for example, Angeltos and La'O 2009, Farhi and Werning 2019, Coibion et al. 2021).

In the second stage, the realized true state reveals the labor-choice distribution. The realization of aggregate borrowing/savings then determines whether the inflation-target or secular-stagnation equilibrium is realized. Given the labor-choice distribution, consumption /saving, production, and inflation are simultaneously determined. Households back out the achievable inflation rate from their consumption/saving decision as we will now see.

### 4.2.2 Solving Backward from the Second Stage

I solve the model backward from the second stage, taking as given the labor-choice distribution from the first stage. I show that in contrast to the case of common beliefs, dispersion in beliefs pins down a unique equilibrium in the second stage. To show this, I first characterize a set of variables in each equilibrium and discuss equilibrium choice by verifying whether an equilibrium inflation rate is consistent with the inflation-targeting equilibrium. I show that if it is consistent, the secular-stagnation equilibrium cannot occur. The same steps apply for the opposite case. Then a unique equilibrium is obtained.[9]

In the second stage, I take as given the labor-choice distribution, which has been made

[9] I have not come across examples of nonexistence of equilibria, although I do not provide a general proof that equilibrium always exists.

through households' threshold strategy in the first stage of the game (see equation [25] and derivation in section 4.2.6). It is captured by the function $\Phi\left(s_t, s^*_{j,t}\right)$, which is the fraction of part-time workers out of all workers. Total labor supply is then given as follows:

$$L_t = \Phi L^P + (1-\Phi)\, L^F \tag{26}$$

Given the labor supply, households decide on their consumption/saving plans, which are aggregated and determine the equilibrium.

### 4.2.3 Characterization of Inflation-Targeting Equilibrium: Heuristic Proof of Equilibrium (Part 1)

I now characterize the inflation-targeting equilibrium and the secular-stagnation equilibrium and derive the endogenous equilibrium-selection probability for a given $\Phi$. As we will see in section 3.3, this case corresponds when $s_t$ exceeds the threshold $\Omega\left(s^*\right)$. Suppose we are in the inflation-targeting equilibrium. The equations satisfied in this equilibrium are as follows. Full-time workers' labor supply is $L^F$, and part-time workers' labor supply is $L^P$. Under the assumption that the inflation target is realized, the wage norm is not binding, so all available labor supply is employed. Then the real wage is given as follows:

$$w_t = \alpha\left\{\Phi L^P + (1-\Phi)\, L^F\right\}^{\alpha-1} \tag{27}$$

To characterize the equilibrium, we need to take the labor-choice distribution into account. Using equations (10) and (26) and the budget constraints, I find the consumption of full-time workers in each generation. For households whose signal is $s_{j,t} \geq s^*_t$, choosing full-time work, I obtain the following:

$$\begin{cases} C^o_{t+2,F} = \frac{\alpha\beta(1+i^*)\left\{\Phi L^P+(1-\Phi)L^F\right\}^{\alpha-1}L^F}{\Pi^*(1+\beta)^2} - \frac{(1+i^*)\beta D}{\Pi^*(1+\beta)}, \\ C^m_{t+1,F} = \frac{\alpha\left\{\Phi L^P+(1-\Phi)L^F\right\}^{\alpha-1}L^F}{1+\beta} - \frac{D}{1+\beta}, \\ C^y_{t,F} = \frac{\Pi^*}{1+i^*}D \end{cases} \tag{28}$$

Similarly, for a household with $s_{j,t} < s^*_t$ that chooses part-time work, I obtain the following consumption plan:

$$
\begin{cases}
C^{o}_{t+2,P} = \frac{\alpha\beta(1+i^*)\left\{\Phi L^P+(1-\Phi)L^F\right\}^{\alpha-1}L^P}{\Pi^*(1+\beta)^2} - \frac{(1+i^*)\beta D}{\Pi^*(1+\beta)}, \\
C^{m}_{t+1,P} = \frac{\alpha\left\{\Phi L^P+(1-\Phi)L^F\right\}^{\alpha-1}L^P}{1+\beta} - \frac{D}{1+\beta}, \\
C^{y}_{t,P} = \frac{\Pi^*}{1+i^*}D
\end{cases}
\tag{29}
$$

Aggregate demand is given as follows:

$$
\begin{aligned}
Y_t = &\left\{\Phi\left(1+g\right)C^{y}_{t,P} + \left(1-\Phi\right)\left(1+g\right)C^{y}_{t,F}\right\} \\
&+ \left\{\Phi C^{m}_{t,P} + \left(1-\Phi\right)C^{m}_{t,F}\right\} + \left\{\frac{\Phi C^{o}_{t,P}}{\left(1+g\right)} + \frac{\left(1-\Phi\right)C^{o}_{t,F}}{\left(1+g\right)}\right\}
\end{aligned}
\tag{30}
$$

Using these equations, we can characterize the upper segments of the AD and AS curves. Combining equations (27) to (30) yields the upper segment of the AD curve under dispersed beliefs about fundamentals:

$$
\begin{aligned}
Y(s) = &\frac{D}{\left\{\Phi\left(s,s^*\right)\delta + \left(1-\Phi\left(s,s^*\right)\right)\right\}^{-\alpha}\bar{\chi}} \\
&+ \frac{\left(1+g\right)D\Gamma^*}{\left\{\Phi\left(s,s^*\right)\delta + \left(1-\Phi\left(s,s^*\right)\right)\right\}^{-\alpha}\bar{\chi}\Pi\left(s\right)^{\phi_\pi-1}}
\end{aligned}
\tag{31}
$$

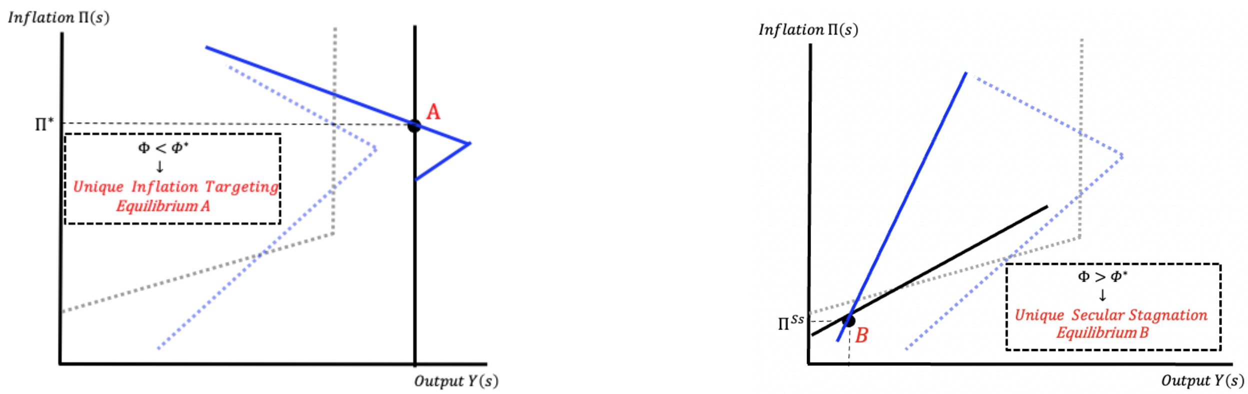
Here $\bar{\chi} = \left\{\bar{\Phi}\delta + \left(1-\bar{\Phi}\right)\right\}^{\alpha}$. One difference between the AD curves with and without common beliefs concerns the terms associated with the proportion of part-time workers, $\Phi$. The lower $\Phi$, the more full-time workers, which leads to an increase in aggregate labor income. If we set $\Phi = \bar{\Phi}$, the AD curve takes the same form as the AD curve with common beliefs (equation [22]).

Since there is full employment, the upper segment of the Phillips curve is as follows:

$$
Y\left(s\right) = Y^f\left(s\right) = \left\{\Phi\left(s,s^*\right)L^P + \left(1-\Phi\left(s,s^*\right)\right)L^F\right\}^{\alpha}
\tag{32}
$$

If we set $\Phi = \bar{\Phi}$, equation (32) converges on equation (20), which is the Phillips curve with common beliefs.

As in figure 4(a), the lower $\Phi$, the more full-time workers, which shifts out the AS curve (solid black line) because of an increase in aggregate labor supply. Since labor income increases, the AD curve also shifts out (solid blue line). Importantly, as long as $\Phi < \Phi^*$, the lower parts of the AD and AS curves with common belief (light dotted lines) disappear;




(a) Inflation-Targeting Equilibrium: $(\Phi < \Phi^*)$

(b) Secular-Stagnation Equilibrium: $(\Phi > \Phi^*)$

Figure 4: AD/AS Diagram under Imperfect Information

Panel (a) shows how AD and AS curves pin down the inflation-targeting equilibrium $A$ if $\Phi < \Phi^*$. Panel (b) shows how AD and AS curves pin down the secular-stagnation equilibrium $B$ if $\Phi > \Phi^*$.

hence equilibrium $A$ is pinned down. Recall that another intersection of the AD curve with the upper segment of the AS curve is excluded because of learning criteria as discussed by Eggertsson et al. (2019).

#### 4.2.4 Characterization of Secular-Stagnation Equilibrium: Heuristic Proof of Equilibrium (Part 2)

Let us now suppose that we are in the secular-stagnation equilibrium. As we will see in section 3.3, this case corresponds when $s_t$ is below some threshold $\Omega(s^*)$. The equations satisfied in this equilibrium are as follows. A full-time worker obtains $L^F_{Ss} < L^F$, and a part-time worker obtains $L^P_{Ss} < L^P$ because of labor rationing. With a binding wage norm, the real wage is as follows:

$$w_t = \frac{(1-\gamma)\,\alpha\left\{\Phi L^P + (1-\Phi)\,L^F\right\}^{\alpha-1}}{1-\gamma\,(\Pi_t)^{-1}} \tag{33}$$

Using the Euler equation (10), the labor-choice distribution (equation [26]), the wage, and the budget constraints, we can get full-time workers' consumption in each generation in this equilibrium. Households whose signal is $s_{j,t} \geq s_t^*$ choose full-time work:

$$\begin{cases} C^o_{t+2,F} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{(1-\gamma)\alpha\left\{\Phi L^P+(1-\Phi)L^F\right\}^{\alpha-1}}{1-\gamma(\Pi_{Ss})^{-1}}\right)\frac{L^F_{SS}}{\Pi_{Ss}} - \left(\frac{\beta}{1+\beta}\right)\frac{D}{\Pi_{Ss}}, \\ C^m_{t+1,F} = \left(\frac{1}{1+\beta}\right)\left(\frac{(1-\gamma)\alpha\left\{\Phi L^P+(1-\Phi)L^F\right\}^{\alpha-1}}{1-\gamma(\Pi_{Ss})^{-1}}\right)\frac{L^F_{SS}}{\Pi_{Ss}} - \left(\frac{1}{1+\beta}\right)\frac{D}{\Pi_{Ss}}, \\ C^y_{t,F} = \Pi_{Ss} D \end{cases} \tag{34}$$

Similarly, for a household with $s_{j,t} < s_t^*$ that chooses part-time work, consumption is as follows:

$$\begin{cases} C^o_{t+2,P} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{(1-\gamma)\alpha\left\{\Phi L^P+(1-\Phi)L^F\right\}^{\alpha-1}}{1-\gamma(\Pi_{Ss})^{-1}}\right)\frac{L^P_{SS}}{\Pi_{Ss}} - \left(\frac{\beta}{1+\beta}\right)\frac{D}{\Pi_{Ss}}, \\ C^m_{t+1,P} = \left(\frac{1}{1+\beta}\right)\left(\frac{(1-\gamma)\alpha\left\{\Phi L^P+(1-\Phi)L^F\right\}^{\alpha-1}}{1-\gamma(\Pi_{Ss})^{-1}}\right)\frac{L^P_{SS}}{\Pi_{Ss}} - \left(\frac{1}{1+\beta}\right)\frac{D}{\Pi_{Ss}}, \\ C^y_{t,P} = \Pi_{Ss} D \end{cases} \tag{35}$$

Aggregate demand is thus given as follows:

$$\begin{aligned} Y_t = &\left\{\Phi\left(1+g\right)C^y_{t,P} + \left(1-\Phi\right)\left(1+g\right)C^y_{t,F}\right\} \\ &+ \left\{\Phi C^m_{t,P} + \left(1-\Phi\right)C^m_{t,F}\right\} + \left\{\frac{\Phi C^o_{t,P}}{(1+g)} + \frac{\left(1-\Phi\right)C^o_{t,F}}{(1+g)}\right\} \end{aligned} \tag{36}$$

Using these equations, we characterize the lower segments of the curves. Combining equations (33) to (36) yields the lower segment of the AD curve under belief dispersion, which is analogous to the AD curve in equation (23):

$$\begin{aligned} Y(s) = &\frac{D}{\left\{\Phi\left(s,s^*\right)\delta + \left(1-\Phi\left(s,s^*\right)\right)\right\}^{-\alpha}\bar{\chi}} \\ &+ \frac{\left(1+g\right)D\left\{\Phi\left(s,s^*\right)L^P_{Ss} + \left(1-\Phi\left(s,s^*\right)\right)L^F_{Ss}\right\}\Pi\left(s\right)}{\left\{\Phi\left(s,s^*\right)\delta + \left(1-\Phi\left(s,s^*\right)\right)\right\}^{1-\alpha}\bar{\chi}} \end{aligned} \tag{37}$$

One difference between the AD curves with and without a common belief about fundamentals is a term associated with the proportion of part-time workers $\Phi$. The higher $\Phi$, the more part-time workers, leading to a decrease in aggregate labor income. As a result, inflation becomes less responsive to changes in aggregate demand. Once we set $\Phi = \bar{\Phi}$, the AD curve with belief dispersion takes the same form as the AD curve with common beliefs (equation [23]).

The lower segment of the Phillips curve is given as follows:

$$\Pi(s) = \left( \frac{Y(s)}{\left\{ \Phi(s, s^*) L^P + (1 - \Phi(s, s^*)) L^F \right\}^{\alpha} Y^f} \right)^{\frac{\alpha(1-\gamma)}{1-\alpha}} \tag{38}$$

Similarly, a difference from the case with common beliefs concerns terms associated with $\Phi(s_t, s_t^*)$. Once $\Phi = \bar{\Phi}$, it takes the same form as the AS curve with common beliefs (equation [21]).

As in figure 4(b), the higher $\Phi$, the more part-time workers, which shifts inward the AS curve (solid black line) because of a decrease in aggregate labor supply. Since labor income decreases, the AD curve shifts inward (solid blue line). Importantly, when $\Phi > \Phi^*$, the upper parts of the AD and AS curves disappear and equilibrium $B$ is pinned down.

### 4.2.5 Endogenous Equilibrium Selection

This section gives a formal proof of a unique equilibrium, using a guess-and-verify approach, given the labor-choice distribution and the optimal threshold strategy $s_t^*$. Below is a central proposition of the paper.

**Proposition 1.** Suppose that all households adopt the threshold strategy on labor choice given by equation (25) and the optimal strategy is given by $s_t^*$. The inflation-targeting equilibrium is uniquely selected if the following holds:

$$\Phi^* > \Phi(s_t, s_t^*) \tag{39}$$

The secular-stagnation equilibrium is the unique equilibrium if the following holds:

$$\Phi^* < \Phi(s_t, s_t^*) \tag{40}$$

Here $\Phi^* = \Phi\left(\Pi^*, r^*, \{L^i, L^i_{Ss}\}^{i=\{F,P\}} | \tilde{\Pi} = \Pi^{zero}\right)$ is a cut-off level of the proportion of part-time workers and $\Pi^{zero}$ is an equilibrium inflation rate $\tilde{\Pi}$ if the implied nominal interest rate $1 + \tilde{i}_t = (1 + i^*) \left(\frac{\tilde{\Pi}_t}{\Pi^*}\right)^{\phi_\pi}$ equals 1.

**Proof.**

See appendix A.

□

Condition (39) states that once the proportion of part-time workers falls below $\Phi^* = \Phi\left(\Pi^*, r^*, \{L^i, L^i_{Ss}\}^{i=\{F,P\}} \,|\tilde{\Pi} = \Pi^{zero}\right)$, the unique inflation-targeting equilibrium is pinned down. This is because a sufficient number of households choose full-time work, thereby yielding sufficient aggregate labor income to generate the needed aggregate borrowing. As a result, the inflation target is achieved, which also excludes secular stagnation because households know that the ZLB will not be binding in the future.

Condition (40) states that if the proportion of part-time workers exceeds $\Phi^*$, the unique secular-stagnation equilibrium is selected. This is because the number of full-time workers is insufficient to yield enough aggregate borrowing to achieve the inflation target, leading to labor rationing, which also excludes the inflation-targeting equilibrium because households know that the ZLB will be permanently binding. If equality holds in this inequality condition—that is, if $\Phi^* = \Phi(s_t, s_t^*)$—there are multiple equilibria; however, this event has zero probability.

Formal discussion of pinning down a unique equilibrium is provided in a proof in appendix A. Here I briefly discuss how I obtain proposition 1. First, I check whether the inflation target can be achieved. Given the labor-choice distribution, I compute the candidate equilibrium inflation rate $\tilde{\Pi}_t$ using aggregate demand (equation [31]) and the Phillips curve (equation [32]). Then, plugging $\tilde{\Pi}_t$ into the Taylor rule yields the implied nominal interest rate for achieving the inflation target. It is given as follows:

$$1 + \tilde{i}_t = (1 + i^*)\left(\frac{\tilde{\Pi}_t}{\Pi^*}\right)^{\phi_\pi} \tag{41}$$

If the implied nominal interest rate $\tilde{i}_t$ is negative, this implies that the ZLB is binding and the central bank cannot achieve the inflation target, and thus is inconsistent with the inflation-targeting equilibrium. If the implied nominal interest rate is positive, the inflation target is achieved, and I establish condition (39) by rearranging equation (41). I can show that the condition (39) is equivalent to the condition that guarantees the existence of the intersection of the upper segments of the AD and AS curves in the sense that the AD curve kinks at a larger output level than the full-employment level.

All that remains is verifying that secular stagnation can occur after we find that the inflation-targeting equilibrium does not exist. Given the same labor-choice distribution, I compute the candidate equilibrium inflation rate using aggregate demand (equation [37]) and the Phillips curve (equation [38]) and show that it is lower than one, so labor rationing occurs. Rearranging the condition $\tilde{\Pi}_t < 1$ yields the cut-off level of the proportion of part-time workers for checking for the occurrence of secular stagnation:

$$\Phi^{Ss} = \Phi\left(\Pi^*, r^*, \left\{L^i, L^i_{Ss}\right\}^{i=\{F,P\}} \,|\tilde{\Pi} = 1\right) \tag{42}$$

Here $\Phi^{Ss}$ is the value of $\Phi(\cdot)$ if $\tilde{\Pi} = 1$. This is the cut-off proportion of part-time workers that divide between labor rationing and full employment. Once $\Phi(s_t, s_t^*) > \Phi^{Ss}$, the shortage of aggregate borrowing results in labor rationing, yielding the secular-stagnation equilibrium.

Since I can show that $\Phi^{Ss} > \Phi^*$, if $\Phi \in (\Phi^*, \Phi^{Ss})$, the candidate inflation rate may be consistent with another equilibrium in between the secular-stagnation and inflation-targeting equilibria. However, this type of equilibrium (point $C$ in figure 2[b]) is indeterminate and is excluded by some learning criteria, as discussed by Eggertsson et al. (2019). Thus, I assume that the secular-stagnation equilibrium satisfies its consistency criteria if $\Phi(s_t, s_t^*)$ is higher than $\Phi^*$. Condition (40) guarantees the intersection of the lower segments of the AD and AS curves.

In contrast, if $\tilde{\Pi}_t$ is consistent with the inflation-targeting equilibrium in the first step, I follow the same steps as above to determine whether secular stagnation can occur. Then I get the conditions for a unique equilibrium, (39) and (40). Since $\Phi(s_t, s_t^*)$ is determined by the optimal threshold strategy $s_t^*$, which is set in the first stage, I go back to the first stage to complete this proposition in the next section.

#### 4.2.6 First Stage: Optimal Labor Choice

Let us go back to the first stage to obtain the final form of the equilibrium-selection rule, given the optimal consumption/saving plan (equations [28] to [35]) and the equilibrium-selection conditions (39) and (48). In the beginning of the first stage, household $j$ receives a private signal $s_{j,t}$ and calculates the subjective expected utility of each labor choice.

Suppose that each household believes that $s_{j,t}$ is the best estimate of $s_t$ and adopts threshold strategy $s_{j,t}^*$ for forecasting the labor-choice distribution, which is governed by $\Phi\left(s_{j,t}^*, s_{j,t}\right)$ with $\Phi$ part-time workers and $1 - \Phi$ full-time workers. The worker's forecast labor-choice distribution is featured with the following aggregate labor supply:

$$L_{t,j} = \Phi\left(\frac{s_{j,t}^* - s_{j,t}}{\sigma}\right) L^P + \left(1 - \Phi\left(\frac{s_{j,t}^* - s_{j,t}}{\sigma}\right)\right) L^F \tag{43}$$

In contrast to the common-belief case, the size of $L_{t,j}$ can differ from the aggregate labor supply forecast by other households.

Given $s_{j,t}$, I recompute the consumption/saving plan (equations [28] to [35]). Applying proposition 1, I obtain the expected utility of full-time work as follows:

$$
\begin{aligned}
E_t U_{j,t}\left(C_j, L^F\right) = & \int_{-\infty}^{\Pi^{zero}} \left(u\left(C^y_{t,j}\right) + \beta\left\{u\left(C^m_{t+1,j}\right) - \varphi v\left(L^F_{Ss}\right)\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\left(\tilde{\Pi}_{j,t+1}\left|s_{j,t}\right.\right)\Bigg|_{Ss\, eqm} \\
& + \int_{\Pi^{zero}}^{\infty} \left(u\left(C^y_{t,j}\right) + \beta\left\{u\left(C^m_{t+1,j}\right) - \varphi v\left(L^F\right)\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\left(\tilde{\Pi}_{j,t+1}\left|s_{j,t}\right.\right)\Bigg|_{IT\, eqm}
\end{aligned} \tag{44}
$$

And the expected utility of part-time work is as follows:

$$
\begin{aligned}
E_t U_{j,t}\left(C_j, L^P\right) = & \int_{-\infty}^{\Pi^{zero}} \left(u\left(C^y_{t,j}\right) + \beta\left\{u\left(C^m_{t+1,j}\right) - \varphi v\left(L^P_{Ss}\right)\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\left(\tilde{\Pi}_{j,t+1}\left|s_{j,t}\right.\right)\Bigg|_{Ss\, eqm} \\
& + \int_{\Pi^{zero}}^{\infty} \left(u\left(C^y_{t,j}\right) + \beta\left\{u\left(C^m_{t+1,j}\right) - \varphi v\left(L^P\right)\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\left(\tilde{\Pi}_{j,t+1}\left|s_{j,t}\right.\right)\Bigg|_{IT\, eqm}
\end{aligned} \tag{45}
$$

Here $1-F_{j,t}$ is the selection probability of the inflation-targeting equilibrium and $\tilde{\Pi}_{j,t+1}$ is a candidate equilibrium inflation rate, both as perceived by household $j$. $\Pi^{zero}$ is the inflation rate that corresponds to the implied nominal interest rate of zero. The first term in equation (44) is the lifetime utility conditional on realizing the secular-stagnation equilibrium, and the second term is conditional on the realization of the inflation-targeting equilibrium.

First, I discuss when strategic complementarity comes into play. In this paper, strategic complementarity is defined as household $j$ has more incentive to choose full-time work if they believe more other households will choose full-time work. The proposition below shows a range of $\delta \equiv \frac{L^P}{L^F}$ between $\underline{\delta}$ and $\bar{\delta}$ in which strategic complementarity is strong enough that $\Phi$ is not 0 or 1.

**Proposition 2.** If $\delta \leq \underline{\delta}$, $\Phi = 0$, and if $\delta > \bar{\delta}$, $\Phi = 1$. If $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$, $\Phi \in (0, 1)$.

**Proof.**

See appendix A.

□

Proposition 2 states that if the difference in labor supply between the full-time and the part-time worker is too small, $U_j\left(C_t, L^P\right) > U_j\left(C_t, L^F\right)$ holds for any household $j$ and any $s_t$. Thus, all households choose part-time work without coordination because they know that coordination does not yield any additional benefits. Similarly, if the difference in labor supply between the full-time and part-time workers is too large, then $U_j\left(C_t, L^F\right) > U_j\left(C_t, L^P\right)$

holds for any household $j$ and any $s_t$. Thus, all households choose full-time work without coordination.

However, if $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$, the size of $\Phi$ matters because as $\Phi$ becomes smaller, it is more likely that $U_j\left(C_t, L^F\right) > U_j\left(C_t, L^P\right)$. That is, strategic complementarity works large enough, resulting in $\Phi \in (0,1)$. Hence, the discussion below assumes $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$ so we can examine how the strategic complementarity works to pin down a unique equilibrium.

Given that $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$ as in proposition 2, the optimal labor-choice rule for household $j$ is derived by comparing equations (44) and (45).

**Lemma 1.** Household $j$ chooses full-time work if and only if

$$Pr\left(\Phi^* > \Phi\left(\frac{s_{j,t}^* - s_t}{\sigma}\right)\right) \geq \Psi_j\left(\left\{\Lambda_i^k\right\}_{i=\{Y,M,O\}}^{k=\{Gain,Cost\}}\right), \tag{46}$$

where $\Lambda_i^{Gain}$ is the utility gain from full-time work in the i-aged and $\Lambda_i^{Cost}$ is the utility cost of full-time work in the i-aged.

**Proof.**

See appendix A.

$\square$

Recall that $Pr\left(\Phi^* > \Phi\left(\frac{s_{j,t}^* - s_t}{\sigma}\right)\right)$ is the selection probability of the inflation-targeting equilibrium as perceived by household $j$, as noted in proposition 1. Lemma 1 states that household $j$ works full time if their perceived selection probability of the inflation-targeting equilibrium exceeds threshold $\Psi_j$. In other words, household $j$ believes that the choice of full-time work gives higher expected utility if they predict a sufficient number of full-time workers.

$\Psi_j$ increases if the utility gain $\Lambda_i^{Gain}$ increases. The utility gain consists of the benefit of full-time work to households of each generation in the inflation-targeting equilibrium: $(a)$ the benefit from easing the borrowing constraint by increasing inflation expectations, $(b)$ the increase in consumption for middle-aged households due to a reduction in the real interest rate, and $(c)$ the increase in consumption for old-aged households due to an increase in accumulated savings, as stated in equations (11) to (13).

$\Psi_j$ is a decreasing function of the utility cost $\Lambda_i^{Cost}$; that is, full-time work has larger disutility in the secular-stagnation equilibrium. Since the candidate equilibrium inflation rate turns out to be a function of the labor-choice distribution, even at the individual level, optimal labor choice takes others' decision-making into account.

#### 4.2.7 Uniqueness of Optimal Threshold Strategy on Labor Choice

The previous section derived an optimal labor-choice decision at the individual level, so its uniqueness was not guaranteed. This section proves the existence of a unique optimal equilibrium threshold strategy on labor choice.

**Proposition 3.** There is a unique equilibrium strategy in which for any $j$, household $j$ chooses full-time work if and only if

$$s_{j,t} \geq s_t^* \left( \Pi^*, r^*, \left\{ L^i, L^i_{Ss} \right\}^{i=\{F,P\}} \right), \tag{47}$$

$$s_t^* \left( \Pi^*, r^*, \left\{ L^i, L^i_{Ss} \right\}^{i=\{F,P\}} \right) = \xi_{\Phi^*} \Theta^{-1} \left( \Phi^* \right) + \xi_{\Psi} \Theta^{-1} \left( 1 - \Psi \right) + r^*.$$

Here $\Theta$ denotes the cumulative distribution function of the standard normal distribution.

**Proof.**
See appendix A.

□

This proposition states that $s_t^*$ are affected by the distributional property of $s_t$, the inflation target, and the labor-market structure. Recall that $\Phi^*$ is a cut-off of the proportion of part-time workers below which the inflation-targeting equilibrium is selected and above which the secular-stagnation equilibrium is selected. As discussed in section 4.2.5, $\Phi^*$ is tightly linked to the utility gain that full-time workers obtain once the inflation-targeting equilibrium is realized. In other words, if the benefit of realizing the inflation-targeting equilibrium is larger, $\Phi^*$ needs to decrease since more households want to choose full-time work. This implies that the lower $\Phi^*$, the stronger the incentive to prefer the inflation-targeting equilibrium. In this sense, $1 - \Phi^*$ is a proxy that represents the benefit of realizing the inflation-targeting equilibrium.

$1 - \Psi$ represents the relative cost of full-time work; that is, the smaller $1 - \Psi$, the stronger households' incentive to prefer full-time work. This effect mainly comes from the potential cost of full-time work as discussed in section 3.4. Once the secular-stagnation equilibrium is realized, full-time workers can suffer from larger utility loss than part-time workers because labor income increases only slightly while labor disutility increases more.

## 4.3 Equilibrium Definition under Belief Dispersion

This section completes the formal definition of a unique equilibrium by combining the discussions in sections 4.1 and 4.2. In contrast to the common-belief case, a unique equilibrium is pinned down, and which equilibrium is selected depends on the optimal threshold strategy of households. Thus, the equilibrium definition needs to explicitly take into account the endogenous equilibrium-selection mechanism discussed in section 4.2.

**Definition.** An equilibrium is a set of allocations $\left\{Y_t, \{C_t^i\}_{i=\{y,m,o\}}, L_t\right\}_{t=0}^{\infty}$, a proportion of part-time workers $\{\Phi_t\}_{t=0}^{\infty}$, prices$\{i_t, \Pi_t, w_t\}_{t=0}^{\infty}$, and an exogenous process $\{s_t\}_{t=0}^{\infty}$ that jointly satisfy equations (2), (3), (4), (5), (6), (25), (16), (17), (19), and (47).

The equilibrium definition is similar to that in Eggertsson et al. (2019), except for the labor choice and the optimal threshold strategy $s_t^*$. More importantly, either the inflation-targeting equilibrium or the secular-stagnation equilibrium is endogenously determined. In other words, by substituting (47) into (39) and (40), we can show that the inflation-targeting equilibrium is selected if

$$s_t \geq \Omega_t \left(\Pi^*, r^*, \left\{L^i, L_{Ss}^i\right\}^{i=\{F,P\}}\right), \tag{48}$$

where $\Omega_t \equiv \xi_\Psi \Theta^{-1}(1-\Psi) + \xi_{\tilde{\Phi}^*}\Theta^{-1}(\Phi^*) + r^*$ and $\xi_{\tilde{\Phi}^*} = \left(\frac{\sigma_s^3}{\sigma_\epsilon^2}\right)$. Otherwise, the secular-stagnation equilibrium is pinned down.

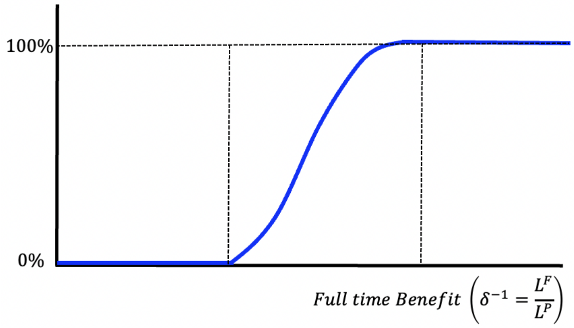
Condition (48) states that once the realization of $s_t$ is above $\Omega$, the inflation-targeting equilibrium is uniquely selected. This is because given $s_t^*$, a sufficiently high $s_t$ leads to a decrease in the number of part-time workers $\Phi$, resulting in a sufficiently large amount of aggregate borrowing. The expectation of an increase in aggregate borrowing reinforces strategic complementarity so that households coordinate on choosing full-time work to achieve the inflation-targeting equilibrium. In contrast, if $s_t$ is below $\Omega$, it leads to an increase in $\Phi$, resulting in a large shortage of aggregate borrowing. The expectation of the shortage of aggregate borrowing generates a coordinated choice of part-time work reinforced by strategic complementarity. So the secular-stagnation equilibrium is chosen.

# 5 Property of the Equilibrium-Selection Mechanism

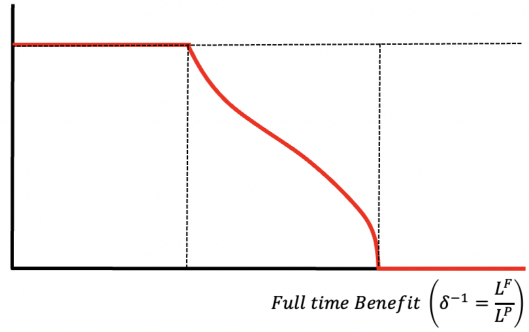
## 5.1 Endogenous Equilibrium-Selection Probability

Once we know how to pin down a unique equilibrium, the first question that arises is how likely the inflation-targeting equilibrium or the secular-stagnation equilibrium is to be uniquely selected. To answer this question, this section discusses the probabilistic nature of equilibrium selection. Since the inflation-targeting equilibrium is uniquely selected if $s_t \geq \Omega_t$, its equilibrium-selection probability is as follows:

$$Pr\left(s_t \geq \Omega_t\left(\Pi^*, r^*, \left\{L^i, L_{Ss}^i\right\}^{i=\{F,P\}}, \varphi\right)\right) \tag{49}$$




(a) Probability of the Inflation-Targeting Equilibrium: $\Pr(s_t \geq \Omega(s^*))$




(b) Optimal Threshold: $(\Omega(s^*))$

Figure 5: Selection Probability of the Inflation-Targeting Equilibrium

Panel (a) plots the selection probability of the inflation-targeting equilibrium as a function of full-time-work benefit $\delta^{-1}$. Panel (b) plots the optimal threshold strategy $s^*$ as a function of $\delta^{-1}$.

Condition (49) shows that $\Omega_t$ is affected by the inflation target, natural rate of interest, and labor-choice factors. So the selection probability of the inflation-targeting equilibrium is affected by these factors. For example, let us define the benefit of full-time work as $\delta^{-1} \equiv \frac{L^F}{L^P}$.

Figure 5(a) plots the selection probability of the inflation-targeting equilibrium as a function of $\delta^{-1}$. The larger the $\delta^{-1}$, the higher the selection probability of the inflation-targeting equilibrium. This is because the optimal threshold $\Omega(s^*)$ is lowered as $\delta^{-1}$ becomes larger as in figure 5(b), which translates to an increase in the fraction of full-time workers. Indeed, since $\frac{\partial 1-\Psi}{\partial \delta^{-1}} < 0$ and $\frac{\partial \Phi^*}{\partial \delta^{-1}} < 0$, we obtain the following:

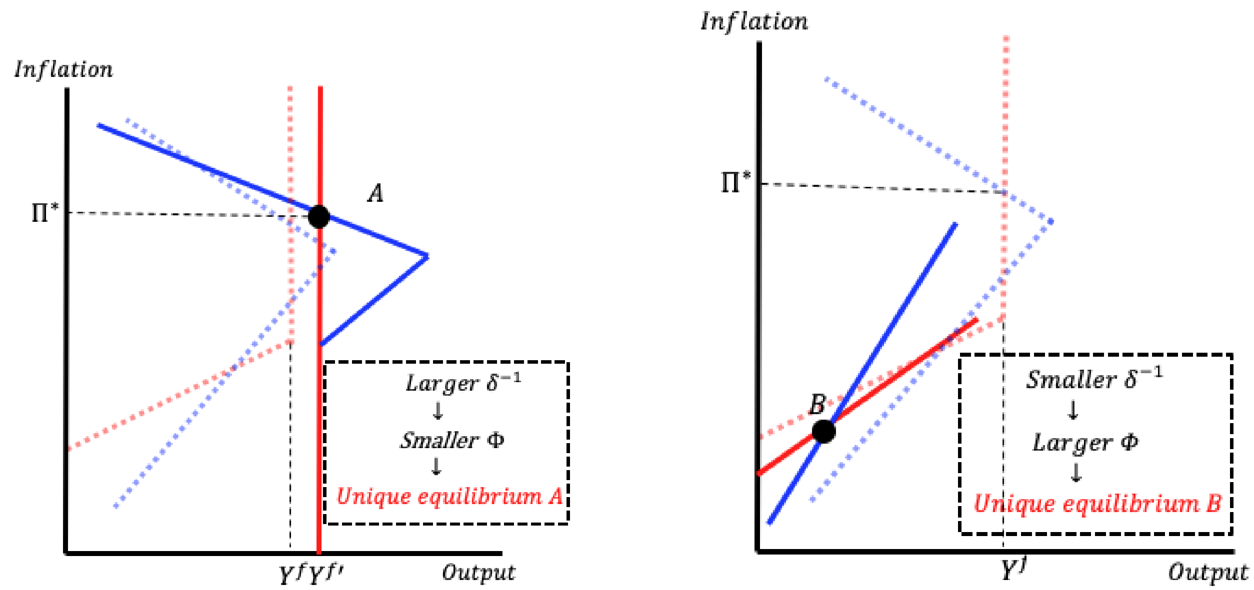
$$\frac{\partial \Omega\left(\Pi^*, r^*, \{L^i, L^i_{Ss}\}^{i=\{F,P\}}, \varphi\right)}{\partial \delta^{-1}} < 0, \tag{50}$$

$$\lim_{\delta^{-1}\to\infty} Pr\left(s_t \geq \Omega_t\left(\Pi^*, r^*, \left\{L^i, L^i_{Ss}\right\}^{i=\{F,P\}}, \varphi\right)\right) = 1, \tag{51}$$

$$\lim_{\delta^{-1}\to 1} Pr\left(s_t \geq \Omega_t\left(\Pi^*, r^*, \left\{L^i, L^i_{Ss}\right\}^{i=\{F,P\}}, \varphi\right)\right) = 0 \tag{52}$$

These equations imply that the larger the benefit to full-time labor, the higher the selection probability of the inflation-targeting equilibrium. This is because an increase in that benefit implies an increase in the utility gain that households can obtain if the inflation-targeting equilibrium is achieved, which makes full-time work more attractive.

However, as discussed in section 4.2.6, if that benefit is too small, more households will choose part-time work, resulting in the secular-stagnation equilibrium. An increase in the benefit of full-time work can increase the proportion of full-time workers via strengthening




(a) Larger Full-Time Benefit and the Inflation-Targeting Equilibrium

(b) Smaller Full-Time Benefit and the Secular-Stagnation Equilibrium

Figure 6: Full-Time Benefit and a Unique Equilibrium
Suppose that the same level of $s_t$ occurs in both panels. Panel (a) shows how AD and AS curves pin down the inflation-targeting equilibrium $A$ if $\delta^{-1}$ is large. Panel (b) shows how AD and AS curves pin down the secular-stagnation equilibrium $B$ if $\delta^{-1}$ is small.

strategic complementarity. This means that with sufficiently high full-time-work benefits, which implies the potential for large-enough labor income, households will have a strong incentive to increase the likelihood that the inflation-targeting equilibrium will be realized, which allows more households to choose full-time work while reducing the expected potential cost of full-time work.

The property of the equilibrium-selection probability affects how the AD and AS curves respond to the same level of $s_t$. Figure 6(a) shows that given a large-enough $\delta^{-1}$, the inflation-targeting equilibrium is realized through a smaller $\Phi$ by shifting out the AD curve while eliminating the lower parts of the AD and AS curves. In figure 6(b), given a small $\delta^{-1}$, the secular-stagnation equilibrium is realized since a larger $\Phi$ shifts the AD and AS curves inward while eliminating the upper parts of the AD and AS curves.

## 5.2 Neutrality of Inflation Target on $r^*$

This section examines whether the inflation target affects the natural rate of interest, which can determine whether this equilibrium selection is driven purely by strategic complementarity and whether its effect on the natural rate is irrelevant.

As Eggertsson et al. (2019) and Summers (2014) argue, a main cause of secular stagnation is the persistent decline in the natural rate of interest. Thus, the obvious way to eliminate it is to raise the natural rate. Eggertsson et al. conclude that raising the inflation target cannot solve this problem because it does not affect the natural rate. I examine whether raising the inflation target affects the natural rate even if the endogenous equilibrium-selection mechanism is taken into account.

Let us first derive the natural rate of interest. Recall that the equilibrium real interest rate is determined in the loan market, which is given as follows:

$$1+r_t = \frac{(1+g)\,D}{\left(\frac{\alpha\beta}{1+\beta}\right)\left(\Phi\left(s_t, s_t^*\right)L^P + \left(1-\Phi\left(s_t, s_t^*\right)\right)L^F\right)^\alpha - \left(\frac{\beta}{1+\beta}\right)D} \tag{53}$$

In this economy, the natural rate of interest is consistent with full labor force participation, so the natural rate is as follows:

$$1+r_t^* = \frac{(1+g)\,D}{\left(\frac{\alpha\beta}{1+\beta}\right)\left(L^F\right)^\alpha - \left(\frac{\beta}{1+\beta}\right)D} \tag{54}$$

Since $\frac{\partial 1+r_t^*}{\partial \Pi^*} = 0$, even with an endogenous equilibrium-selection mechanism in place, raising the inflation target is independent of the determination of the natural rate. So an important contribution of the inflation target is to create the possibility of another, better equilibrium, even in the presence of a downward trend in the natural rate.[10]

# 6 Policy Implications

In the previous sections, by relaxing the perfect-information assumption and showing the probability of each occurring, I showed that an equilibrium is uniquely pinned down in the secular-stagnation model. This finding raises another question: can monetary and fiscal policy affect the probability of an equilibrium being selected?

In contrast to the existing literature on Secular Stagnation, I show that appropriately decided fiscal policy, even if temporary, can increase the probability of achieving the inflation target through strengthening strategic complementarity. I further show that the interaction between inflation targeting and fiscal policy and AIT enhances strategic complementarity, which promotes households' ability to coordinate to resolve the demand shortage.

[10] As discussed below, if the inflation target is too low, the AD and AS curves will not intersect in the upper segment, eliminating the possibility of inflation-target equilibrium.

## 6.1 Temporary Fiscal Expansion

This section introduces fiscal intervention to examine how it affects equilibrium selection. Many authors such as DeLong and Summers (2012) and Eggertsson et al. (2019) argue that permanent debt expansion, if large, would eliminate the secular-stagnation equilibrium. However, a large, permanent debt expansion raises concerns about debt sustainability as shown by Garga (2020). Also, since Eggertsson et al. (2019) assume permanent debt expansion, their results arise purely from raising the natural rate of interest. Thus, multiple equilibria do not exist a priori.

This paper discusses whether a unique equilibrium can be selected in the presence of multiple equilibria. Hence I focus on *temporary* fiscal expansion to keep the natural rate constant as it does not affect the natural rate. The simplest way to do this is to suppose a one-time fiscal expansion that the government finances by issuing debt $B_t^g = \bar{B}$ and uses this for a transfer to the middle-aged ($T_t^m = -B_t^g$) with the following tax policy:

$$T_{t+1}^o = \left(\frac{1+i_t}{E_t\Pi_{t+1}}\right)\psi B_t^g \tag{55}$$

Here $\psi$ is a parameter that governs how temporary the fiscal expansion is by taking advantage of a low natural rate. Since the natural rate is permanently negative in the secular-stagnation equilibrium, the real interest rate is positive because of permanently binding ZLB and deflation. To keep fiscal expansion temporary, we must set $\psi = 1$.

In contrast, in the inflation-targeting equilibrium, the equilibrium real interest rate is negative, and thus fiscal expansion can be temporary even if $\psi < 1$. One way to model $\psi$ is as follows:

$$\psi = \begin{cases} \tilde{\psi} & \text{if } -\text{inflation targeting equilibrium} \\ 1 & \text{otherwise} \end{cases}$$

Here $\tilde{\psi} < 1$. One way of modeling $\tilde{\psi}$ is to model it as a decreasing function of $\Pi^*$. This modeling approach is consistent with those of Blanchard (2019, 2022) and Furman and Summers (2020), who discuss how taking advantage of a low real-interest-rate environment to create fiscal space is desirable from both a policy-management perspective and a welfare perspective. As Blanchard (2019) and Mehrotra and Sergeyev (2021) show, the cost of servicing the government debt has been negative in many advanced economies including the US, in the sense that the real interest rate has been below the growth rate of the economy. In this environment, even without raising the tax rate, the ratio of government debt to GDP will shrink to zero as long as the debt-servicing cost is negative.

### 6.1.1 Optimal Threshold Strategy with Fiscal Policy

I first derive a unique optimal equilibrium strategy on labor choice with fiscal expansion.

**Proposition 4.** There is a unique optimal equilibrium strategy in which for any $j$, household $j$ chooses full-time work if and only if

$$s_{j,t} \geq s_t^* \left( \Pi^*, r^*, \left\{ L^i, L^i_{Ss} \right\}^{i=\{F,P\}}, \varphi, \bar{B}^g \right),$$

where

$$s_t^* \left( \Pi^*, r^*, \left\{ L^i, L^i_{Ss} \right\}^{i=\{F,P\}}, \varphi, \bar{B}^g \right) = \xi_{\Phi^*} \Theta^{-1} \left( \Phi^{*,B^g} \right) + \xi_{\Psi} \Theta^{-1} \left( 1 - \Psi^{B^g} \right) + r^*.$$

**Proof.**
See appendix A.

□

This condition implies that an optimal equilibrium strategy takes a similar form to that in proposition 3 in the sense that household $j$ works full time if their private signal $s_{j,t}$ exceeds $s_t^*$. As discussed in lemma 1 and proposition 3, its economic background is that household $j$ works full time if they believe that full-time work gives higher expected utility and that a sufficient number of others will work full time. One difference from proposition 3 is that a unique optimal threshold strategy additionally depends on the size of the fiscal expansion. We have the following derivatives:

$$\frac{\partial s_t^*}{\partial \Phi^{*,B^g} \left( \Pi^*, r^*, \{ L^i, L^i_{Ss} \}^{i=\{F,P\}}, \bar{B}^g \right)} \underbrace{\frac{\partial \Phi^{*,B^g} \left( \Pi^*, r^*, \{ L^i, L^i_{Ss} \}^{i=\{F,P\}}, \bar{B}^g \right)}{\partial \bar{B}^g}}_{Reinforcement\ \ Effect} < 0, \tag{56}$$

$$\frac{\partial s_t^*}{\partial 1 - \Psi^{B^g} \left( \Pi^*, r^*, \{ L^i, L^i_{Ss} \}^{i=\{F,P\}}, \bar{B}^g \right)} \underbrace{\frac{\partial 1 - \Psi^{B^g} \left( \Pi^*, r^*, \{ L^i, L^i_{Ss} \}^{i=\{F,P\}}, \bar{B}^g \right)}{\partial \bar{B}^g}}_{Reinforcement\ \ Effect} < 0 \tag{57}$$

These derivatives imply that given structural factors such as the natural rate of interest and labor-market factors fixed, fiscal expansion lowers the optimal threshold strategy $s_t^*$, reinforcing strategic complementarity. Derivatives (56) and (57) indicate that reinforcement effects (that is, further reduction in $\Phi^{*,B^g}$ and further decrease in $\Psi^{B^g}$) convince more pessimistic households to coordinate on working full time to achieve the inflation-targeting equilibrium. The reinforcement effects arise because (1) increasing the utility gain of full-time work once the inflation-targeting equilibrium is realized induces households to lower $s_t^*$

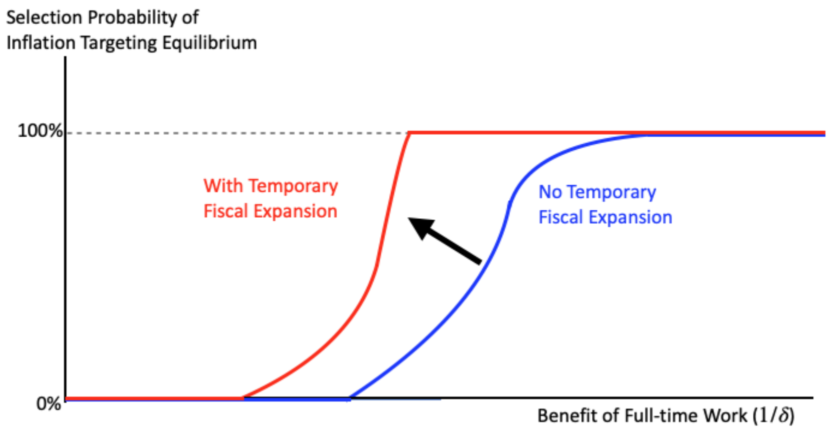



Figure 7: Full-Time-Work Benefit and Temporary Fiscal Expansion

Note: The solid blue line represents the selection probability of the inflation-targeting equilibrium without fiscal expansion. The solid red line represents the selection probability of the inflation-targeting equilibrium with temporary fiscal expansion. The benefit of full-time work is defined as $\delta^{-1} = \frac{L^F}{L^P}$; that is, the higher the benefit is, the more labor supply is available to full-time workers in the inflation-targeting equilibrium. The temporary fiscal expansion can increase the likelihood of realizing the inflation target by steepening the probability curve.

and (2) reducing the relative cost of full-time work through decreasing the chance of labor rationing also induces households to lower $s_t^*$.

In other words, since future tax payment depends on actual equilibrium realization, a further reduction in tax payment in the inflation-targeting equilibrium will increase the expected utility of full-time work compared to that obtained in the secular-stagnation equilibrium; that is, though temporary fiscal expansion does not affect the natural rate, the old-aged households are expected to benefit from tax reduction via inflation, $T^o_{t+1,j} = \frac{1+i^*}{\Pi^*}\psi B^g_t < \frac{B^g_t}{\Pi^{Ss}}$. This additional effect reinforces other potential benefits—(1) easing borrowing constraints for the young and (2) increasing consumption by the middle-aged—which gives further incentive to raise the likelihood of realizing the inflation-targeting equilibrium. Through these channels, an increase in expected aggregate demand mitigates the expected adverse effects of labor rationing by reducing the likelihood of its occurring because expected aggregate borrowing increases. As a result, households have a stronger incentive to coordinate on choosing full-time work by lowering $s^*$ than in the case without the fiscal expansion.

### 6.1.2 Endogenous Equilibrium-Selection Probability with Fiscal Policy

This section revisits the probabilistic property of the equilibrium selection to examine how fiscal expansion works. We can derive the selection probability of the inflation-targeting equilibrium as follows:

$$Pr\left(s_t \geq \Omega^{B^g}\left(\Pi^*, r^*, \left\{L^i, L^i_{Ss}\right\}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)\right) \tag{58}$$

Equation (58) shows that the threshold $\Omega^{B^g}$ is additionally affected by the government debt, so the equilibrium-selection probability is also affected by fiscal expansion.

Figure 7 plots the selection probability of the inflation-targeting equilibrium as a function of the full-time-work benefit $\delta^{-1}$. The solid blue line is the case with no temporary fiscal expansion, and the solid red line indicates the case with temporary fiscal expansion. As this figure shows, temporary fiscal expansion can steepen the probability curve, making the probability of achieving the inflation-targeting equilibrium higher when full-time-labor benefits are smaller. Indeed, we have the following derivatives:

$$\frac{\partial\Theta^{-1}\left(\Phi^{*,B^g}\left(\cdot\right)\right)}{\partial\delta^{-1}} < 0,\ \ \frac{\partial^2\Theta^{-1}\left(\Phi^{*,B^g}\left(\cdot\right)\right)}{\partial\delta^{-1}\partial\bar{B}^g} > 0 \tag{59}$$

The second condition in (59) implies that a temporary fiscal expansion allows the full-time-work benefit to generate a further reduction in $\Omega^{B^g}$ in (58). These imply that the greater the temporary fiscal expansion, the larger the utility gain once the inflation-targeting equilibrium is realized as discussed in section 6.1.1. That is, the attractiveness of full-time work further increases, leading households to aim for raising the likelihood of achieving the inflation target, which promotes households' coordinated decisions to choose full-time work, reinforced by strategic complementarity.

We also have the following derivatives:

$$\frac{\partial\Theta^{-1}\left(1-\Psi^{B^g}\left(\cdot\right)\right)}{\partial\delta^{-1}} < 0,\ \ \frac{\partial\Theta^{-1}\left(1-\Psi^{B^g}\left(\cdot\right)\right)}{\partial\delta^{-1}\partial\bar{B}^g} > 0 \tag{60}$$

The second condition in equation (60) implies that a temporary fiscal expansion allows the full-time-work benefit to generate a further reduction in $\Omega^{B^g}$ in equation (58). These imply that the greater the temporary fiscal expansion, the lesser the incentive to work part time. Since the fiscal expansion increases expected aggregate demand through reducing future tax payments because of inflation, the relative cost of full-time work decreases more than in the case without fiscal expansion, further strengthening the incentive to lower $s_t^*$.

In sum, a temporary fiscal expansion can increase the selection probability of the inflation-targeting equilibrium by inducing households to more strongly coordinate on working full time, reinforced by strategic complementarity through equations (59) and (60). These reinforcing effects arising from temporary fiscal expansion steepen the probability curve, making the probability of achieving the inflation target higher when full-time-labor benefits are smaller.

Next, to see how fiscal expansion endogenously affects the equilibrium selection, let us revisit the AD/AS diagram. The AD curve is given by the following equations. If ZLB is not binding,

$$Y(s) = \frac{D}{\{\Phi(s,s^*,B^g)\,\delta + (1-\Phi(s,s^*,B^g))\}^{-\alpha}} + \frac{(1+g)\,D\Gamma^*}{\{\Phi(s,s^*,B^g)\,\delta + (1-\Phi(s,s^*,B^g))\}^{-\alpha}\,\Pi^{\phi_\pi-1}}, \quad if\ \ i>0. \tag{61}$$

And if ZLB is binding,

$$Y(s) = \frac{D}{\{\Phi(s,s^*,B^g)\,\delta + (1-\Phi(s,s^*,B^g))\}^{-\alpha}} + \frac{(1+g)\,D\left\{\Phi(s,s^*,B^g)\,L^P_{Ss} + (1-\Phi(s,s^*,B^g))\,L^F_{Ss}\right\}\Pi}{\{\Phi(s,s^*,B^g)\,\delta + (1-\Phi(s,s^*,B^g))\}^{1-\alpha}}, \quad if\ \ i=0. \tag{62}$$

The AS curve is given as follows:

$$Y = \left\{\Phi(s,s^*,B^g)\,L^P + (1-\Phi(s,s^*,B^g))\,L^F\right\}^{\alpha}, \quad if\ \ \Pi \geq 1 \tag{63}$$

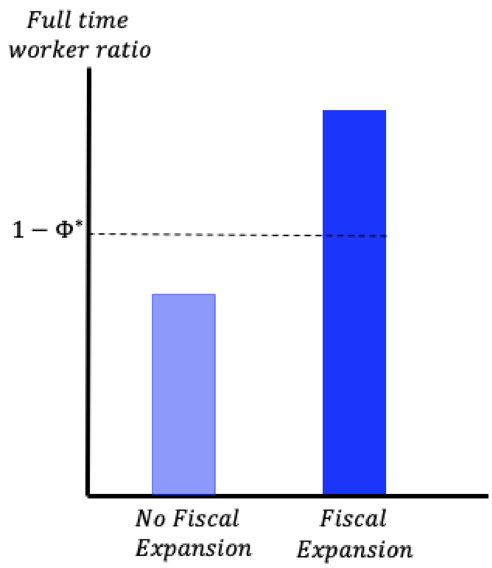
And

$$\Pi_t = \left(\frac{Y_t}{\{\Phi(s,s^*,B^g)\,L^P + (1-\Phi(s,s^*,B^g))\,L^F\}^{\alpha}}\right)^{\frac{\alpha(1-\gamma)}{1-\alpha}}, \quad if\ \ \Pi<1. \tag{64}$$

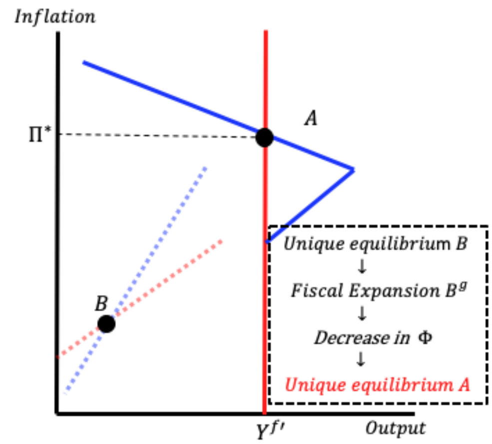
These equations are summarized in the AD/AS diagram in figure 8. One difference from the no-fiscal-intervention case is that the government-debt level affects the shape of the AD and AS curves via affecting the proportion of part-time workers, $\Phi$. The solid lines are AD and AS curves with fiscal expansion, and the dotted lines are those without fiscal expansion in figure 8(b).

Suppose that $s_t$ is small and so the resulting strategic complementarity uniquely pins down the secular-stagnation equilibrium $B$ via a shortage of full-time workers below $1-\Phi^*$ (the light blue bar in panel [a] and the dotted blue and red lines in panel [b]). However, once a sufficiently large fiscal expansion is introduced, the additional benefit of full-time work (that is, the expected tax reduction from inflation) emerges and reinforces the other two potential benefits for the middle-aged and the young, leading to an increase in full-time workers $\frac{\partial 1-\Phi}{\partial B^g} > 0$, as in panel (a). Then, as the solid blue lines show, the movement of the AD curve becomes sufficiently large that it eliminates their intersection, resulting in the unique equilibrium $A$ (solid blue and red lines in panel [b]). Thus, temporary fiscal expansion can reinforce the effects of strategic complementarity so that more strongly promotes households' coordinated choice of full-time work.

By focusing on the size of the fiscal expansion, we can also examine how temporary fiscal expansion affects equilibrium-choice probability. Given the inflation target and the structure




(a) Full-Time Worker Ratio




(b) Equilibrium Selection on AD/AS Diagram

Figure 8: Effects of Temporary Fiscal Expansion

Note: Suppose that the same level of $s_t$ occurs in both panels. Panel (a) shows the proportion of full-time workers with and without fiscal expansion. $1-\Phi^*$ denotes a cut-off level that divides the inflation-targeting equilibrium and the secular-stagnation equilibrium. Panel (b) shows how AD and AS curves move in response to fiscal expansion (from dotted lines to solid lines).

of the labor market, let us suppose that the selection probability of the inflation-targeting equilibrium is zero in the absence of fiscal expansion. Figure 9 depicts how this selection probability changes with different sizes of fiscal expansion. The selection probability of the inflation-targeting equilibrium increases with the size of the fiscal expansion. Indeed, the following relationship can be derived:

$$\frac{\partial \Omega^{B^g}\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)}{\partial \bar{B}^g} < 0, \tag{65}$$

$$\lim_{\bar{B}^g \to \infty} Pr\left(s_t \geq \Omega_t\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)\right) = 1 \tag{66}$$

Equations (65) and (66) show that fiscal expansion can raise the selection probability of the inflation-targeting equilibrium and that the sufficiently large fiscal expansion uniquely selects the inflation-targeting equilibrium. The underlying mechanism is that the larger the fiscal expansion, the larger the reinforcement effects on strategic complementarity to increase the full-time workers arise through equations (59) and (60).

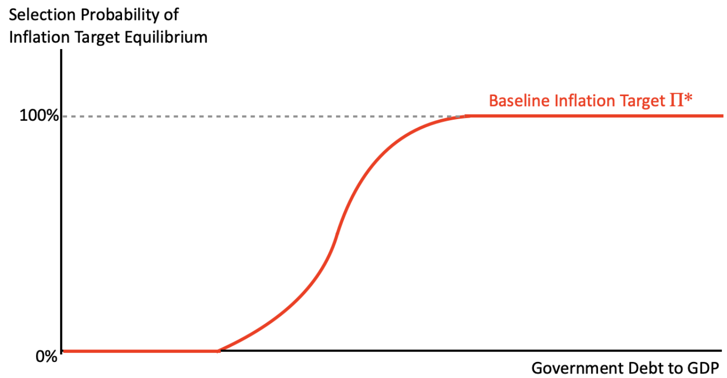



Figure 9: Equilibrium-Selection Probability and Temporary Fiscal Expansion
Note: The solid red line represents the selection probability of the inflation-targeting equilibrium with temporary fiscal expansion. The inflation target is set at $\Pi^*$. The $x$-axis denotes the ratio of government debt to GDP, instead of the full-time-work benefit.

## 6.2 Inflation Target under Fiscal Expansion

Given the equilibrium-selection probability (58), monetary policy may recover potentials because the optimal threshold is a function of both monetary and fiscal policy measures. This section revisits whether the inflation target matters for equilibrium selection once a temporary fiscal expansion is introduced. The equilibrium-selection probability (58) is depicted in figure 10.

I assume different levels of the inflation target, $\Pi_H^* > \Pi^* > \Pi_L^{*'} > \Pi_L^{*''} > \Pi_L^{*'''}$. The solid red line is the baseline case with $\Pi^*$. The higher inflation target ($\Pi_H^*$) steepens the probability curve that allows the smaller government debt to have higher chance of realizing the inflation target. In contrast, the lower the inflation target ($\Pi_L^{*'} > \Pi_L^{*''} > \Pi_L^{*'''}$), the flatter the probability curve is. Once the inflation target is lowered to $\Pi_L^{*'''}$, the size of the fiscal expansion does not affect the equilibrium-selection probability.

Given these observations, we can see how the inflation target interacts with fiscal policy, resulting in stronger strategic complementarity. I first obtain the following:

$$\frac{\partial \Theta^{-1}\left(\Phi^{*,B^g}\left(\cdot\right)\right)}{\partial \bar{B}^g} < 0, \ \frac{\partial^2 \Theta^{-1}\left(\Phi^{*,B^g}\left(\cdot\right)\right)}{\partial \bar{B}^g \partial \Pi^*} < 0 \tag{67}$$

This shows that raising the inflation target leads to a further reduction in $\Omega^{B^g}$ than the case with the baseline inflation target. Equation (67) implies that the higher the inflation target, the more strongly the effect of temporary fiscal expansion in reinforcing strategic complementarity. This is because given the government-debt level, a future larger reduction of the tax payment by realizing higher inflation is expected if the inflation target is higher than the baseline level. The larger reduction of tax payment further amplifies the stimulative effects on young and middle-aged households. In addition, the higher inflation target can

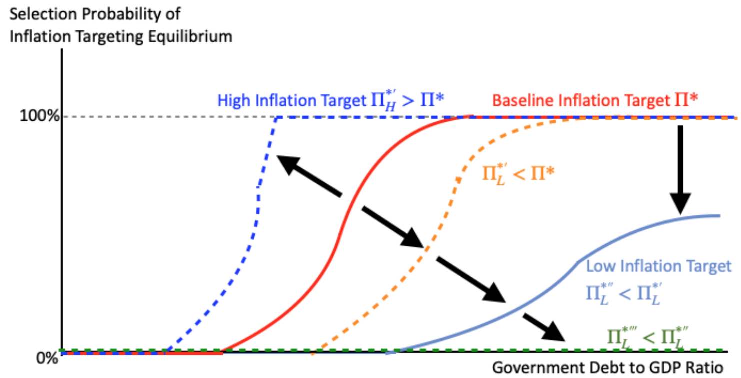



Figure 10: Equilibrium-Selection Probability and Different Levels of Inflation Target

Note: The red line plots the probability of selecting the inflation-targeting equilibrium in the baseline in which the inflation target is set at $\Pi^*$. The dotted blue line represents the case in which the inflation target is increased to $\Pi_H^{*'}$. The dotted orange line assumes that the inflation target is set at a $\Pi_L^{*'}$ that is slightly lower than $\Pi^*$. The light blue line represents the case in which the inflation target is lowered to $\Pi_L^{*''} < \Pi_L^{*'}$. The dotted green line assumes that the inflation target is set at $\Pi_L^{*'''} < \Pi_L^{*''}$.

increase the attractiveness of full-time work by raising the likelihood of realizing the inflation target. As the AD curve shifts out while flattening its slope, the marginal increase in aggregate demand from raising inflation becomes larger, which more strongly motivates households to aim for achieving the inflation target.

We also have the following derivatives:

$$\frac{\partial \Theta^{-1}\left(1-\Psi^{B^g}(\cdot)\right)}{\partial \bar{B}^g} < 0, \quad \frac{\partial \Theta^{-1}\left(1-\Psi^{B^g}(\cdot)\right)}{\partial \bar{B}^g \partial \Pi^*} < 0 \tag{68}$$

These show that raising the inflation target leads to a greater reduction in $\Omega^{B^g}$ than the case with the baseline inflation target. This implies that the higher the inflation target, the weaker the potential incentive to prefer part-time work. Given the government-debt level, this effect also originally arises from further tax reduction from higher realized inflation, which stimulates aggregate demand beyond that with the baseline inflation target. A further increase in aggregate demand decreases the likelihood of labor rationing. Thus, this stimulus effect reduces the relative cost of full-time work more than under the baseline inflation target since labor rationing is less likely to occur. In sum, with a higher inflation target, fiscal expansion can further strengthen strategic complementarity to achieve the inflation target through these two interaction forces ([67] and [68]) between the inflation target and the fiscal expansion.

Once the inflation target is lowered to $\Pi_L^{*'''}$, even the extremely large government debt has no impact on the equilibrium-selection probability. This is because the intersection of the upper segment of the AD and AS curves does not exist if the inflation target is low so that $\Pi^{*'''} < \frac{1}{1+r^*}$.

## 6.3 What If the Natural Rate of Interest Falls Further?

This section discusses how a further decline in the natural rate of interest would affect the selection probability of the inflation-targeting equilibrium. The US Census Bureau estimates that in the US, the aging problem is difficult to reverse. For example, the proportion of the population aged over 65 will increase from about 8% in 1980 to about 20% in 2050. As previous studies have shown, the long-term downward trend in the natural rate of interest is difficult to reverse because the aging of the population is one of its main causes. Thus, it may be natural to expect that this trend will continue.

Given this observation, the first problem that comes to mind is how the equilibrium-selection probability changes if the natural rate falls further. To address this issue, suppose the population growth rate $g$ declines further because $\frac{\partial 1+r_t^*}{\partial g} < 0$, which is obtained from equation (54). With the equilibrium-selection probability (58), we have the following derivatives:

$$\frac{\partial \Theta^{-1}\left(1-\Psi^{B^g}\left(\cdot\right)\right)}{\partial g} < 0, \; \frac{\partial \Theta^{-1}\left(\Phi^{*,B^g}\left(\cdot\right)\right)}{\partial g} < 0 \tag{69}$$

These imply that a further fall in $r^*$ creates upward pressure on $s^*$ as seen in the following:

$$\frac{\partial s_t^*}{\partial \Phi^{*,B^g}\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \bar{B}^g\right)} \underbrace{\frac{\partial \Phi^{*,B^g}\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \bar{B}^g\right)}{\partial g}}_{Attenuation\ Effect} < 0, \tag{70}$$

$$\frac{\partial s_t^*}{\partial 1-\Psi^{B^g}\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \bar{B}^g\right)} \underbrace{\frac{\partial 1-\Psi^{B^g}\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \bar{B}^g\right)}{\partial g}}_{Attenuation\ Effect} < 0 \tag{71}$$

Hence, a fall in the natural rate leads to an increase in households who work part time because of a fall in the relative utility gain of full-time work that can be obtained when the inflation-targeting equilibrium is realized. This is because a lower natural rate induces larger shortages of aggregate borrowing, which implies that more households have to coordinate to achieve the inflation target. Such an expectation can make households more pessimistic than before. Thus, the further fall in the natural rate induces households to coordinate on choosing part-time work, reinforced by strategic complementarity, which leads to the following inequality:

$$\frac{\partial Pr\left(s_t \geq \Omega^{B^g}\left(\Pi^*, r^*, \{L^i, L_{Ss}^i\}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)\right)}{\partial g} > 0 \tag{72}$$

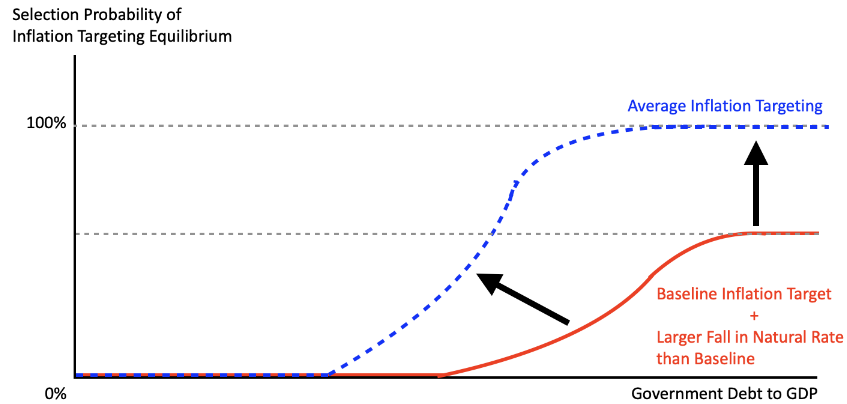



Figure 11: The Effects of Average Inflation Targeting

Note: The solid red line plots the selection probability of the inflation-targeting equilibrium when the natural rate of interest falls further below $r^*$. The dotted blue line plots the selection probability of the inflation-targeting equilibrium when average inflation targeting is introduced.

As a result, a fall in the natural rate of interest makes it more difficult to pin down the inflation-targeting equilibrium, as in the solid red line in figure 11. Instead, the secular-stagnation equilibrium is more likely to be the unique selection.

## 6.4 How Does Average Inflation Targeting Work?

The previous section showed that once the natural rate of interest further falls, it becomes more difficult to determine a unique inflation-targeting equilibrium because the expectation of the shortage of aggregate borrowing generates a more strongly coordinated choice of part-time work, reinforced by strategic complementarity. In this case, even when concern about debt sustainability increases, should we further increase the fiscal expansion?

Perhaps one way to address this issue is to introduce AIT, which was adopted by the Federal Reserve when amending its policy framework in August 2020. The Bank of Japan introduced a similar policy framework, the Inflation-Overshooting Commitment, in 2021.

In both policy frameworks, after periods of inflation below 2%, an appropriate monetary policy would aim to achieve inflation moderately above 2% for some extended time. Blanchard (2022) points out that the potential advantage over the case without temporary inflation overshooting is (1) an implied decrease in the real value of debt, (2) the effect of a lower real interest rate for some time on debt dynamics, and (3) an increase in the inflation rate to compensate for an inflation rate below the target in the past.

Following Eggertsson et al. (2021), I model AIT as follows:

$$1 + i_t = max\left(1, (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\left(\frac{\Pi_t^{NN}}{\Pi^*}\right)^{\phi_{AIT}}\right) \tag{73}$$

Here $\Pi_t^{NN}$ is average inflation over the last $NN$ periods. Given that the inflation rate was below the target for the last $NN$ periods, this formulation implies that the central bank commits to keep the nominal interest rate below the target to temporarily overshoot inflation above the target. The equilibrium-selection probability of the inflation-targeting equilibrium is as follows:

$$Pr\left(s_t \geq \Omega^{B^g}\left(\left\{\Pi^*, \Pi_t^{NN}\right\}, r^*, \left\{L^i, L_{Ss}^i\right\}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)\right) \tag{74}$$

Figure 11 plots the selection probability of the inflation-targeting equilibrium as a function of the ratio of government debt to GDP. The solid red line shows the case in which the natural rate of interest further falls. The blue dotted line plots the case with AIT. Once we introduce AIT, the probability curve becomes steeper, making the probability of achieving the inflation-targeting equilibrium higher for smaller amount of government debt. Indeed, we derive the following conditions by using equation (74):

$$\frac{\partial^2 \Theta^{-1}\left(\Phi^{*,B^g}(\cdot)\right)}{\partial g \partial \tilde{\Pi}^*} > 0, \quad \frac{\partial^2 \Theta^{-1}\left(1 - \Psi^{B^g}(\cdot)\right)}{\partial g \partial \tilde{\Pi}_t^*} > 0 \tag{75}$$

The first condition of equation (75) implies that the more inflation temporarily overshoots because of AIT, the more strongly it reinforces households' coordinated decision to choose full-time work because it further increases the utility gain from full-time work once the inflation-targeting equilibrium is realized. The second condition of equation (75) means that the more inflation temporarily overshoots because of AIT, the more strongly it weakens the potential incentive to prefer part-time work because the relative cost of full-time work is mitigated.

In sum, AIT can recover the effect of the strategic complementarity to pin down a unique inflation-targeting equilibrium even if the natural rate of interest further falls, as seen in the following:

$$\frac{\partial^2 Pr\left(s_t \geq \Omega^{B^g}\left(\left\{\Pi^*, \epsilon_t^{\Pi^*}\right\}, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)\right)}{\partial g \partial \tilde{\Pi}_t^*} > 0 \tag{76}$$

Let us examine in more detail why the selection probability of the inflation-targeting equilibrium further increases once AIT is introduced. In particular, does AIT have an additional propagation channel compared to inflation targeting? Inflation overshooting due to AIT more strongly strengthens the interaction between the inflation target and the fiscal expansion than the inflation targeting because a larger fall in real interest rate allows households to reduce their tax payment, as discussed in section 6.2.

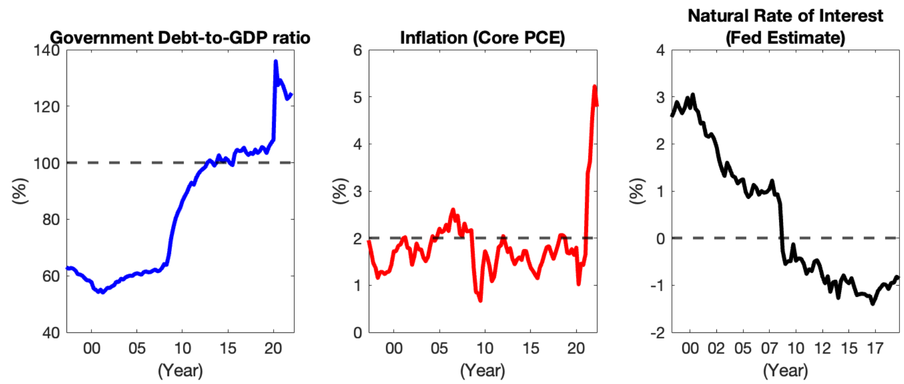



Figure 12: Government Debt, Inflation, and Natural Rate of Interest
Sources: FRED, Federal Reserve, Lopez-Salido et al. (2020).

In addition to the interaction channel between monetary and fiscal policy, another channel arises from the temporary overshooting. Since the young household lives hand-to-mouth, if inflation overshoots, they can temporarily borrow more than before as seen in the following:

$$B^y_{t,j} = \frac{\tilde{\Pi}^*_t}{1+i^*}D > \frac{\Pi^*}{1+i^*}D \tag{77}$$

Suppose inflation converges to the original inflation target next period. Then, the household's debt payment increases compared to before:

$$\left(\frac{1+i^*}{\Pi^*}\right)\left(\frac{\tilde{\Pi}^*_t}{1+i^*}D\right) > D \tag{78}$$

And if the secular-stagnation equilibrium is selected next period, then the debt payment further increases compared to the inflation-targeting equilibrium:

$$\left(\frac{1}{\Pi^{Ss}}\right)\left(\frac{\tilde{\Pi}^*_t}{1+i^*}D\right) > \left(\frac{1+i^*}{\Pi^*}\right)\left(\frac{\tilde{\Pi}^*_t}{1+i^*}D\right) \tag{79}$$

Thus, in comparison to the case without the inflation overshooting, an additional incentive to strengthen the strategic complementarity arises to avoid a rise in the future debt payment, which increases the relative utility gain of choosing the full-time work and leads to equation (75).

| | Symbol | Value | Source |
|---|---|---|---|
| Natural Rate of Interest | $r^*$ | $-1.47\%$ | Federal Reserve |
| Inflation Target | $\Pi^*$ | 2.00% | Federal Reserve |
| Policy coefficient in Taylor rule | $\phi_\pi$ | 1.50 | Federal Reserve |
| Rate of Time Preference | $\beta$ | 0.96 | Eggertsson et al. (2019) |
| Borrowing limit | $D$ | 26.0% | Eggertsson et al. (2019) |
| Labor Share | $\alpha$ | 0.70 | Eggertsson et al. (2019) |
| Population growth rate | $g$ | 0.70% | World Bank |
| Wage Adjustment | $\gamma$ | 0.94 | Eggertsson et al. (2019) |
| Labor supply difference | $\delta$ | 1.60 | Bureau of Labor Statistics |
| Labor disutility | $\varphi$ | 2.00 | Hansen (1985) |
| Temporality of Fiscal Expansion | $\tilde{\psi}$ | 0.18 | Chen et al. (2012) |
| Std of mean perception of $r^*$ | $\sigma_\epsilon$ | 0.50 | Del Negro et al. (2017) |

Table 1: Calibrated Parameters

# 7 Application: Selection Probability of Inflation-Targeting Equilibrium in the US

This section calibrates the endogenous equilibrium-selection mechanism for the US. I first quantitatively derive the equilibrium-selection probabilities for the US in 2015 as a baseline. Then I discuss counterfactual policies: (1) further fiscal expansion and (2) AIT.

In the US, as the left and the middle panels in figure 12 show, from the Great Recession until the COVID-19 pandemic, inflation was below the inflation target and economic growth was sluggish. Accordingly, both monetary and fiscal policy were quite expansionary. As seen in the right panel, the natural rate of interest is in a secular downtrend. Previous literature suggests that the downtrend could be a main cause of the weak inflation and economic growth (Summers 2020 and Eggertsson et al. 2019).

However, existing ZLB models have difficulty addressing these issues, as discussed in section 1, and I found that an endogenous equilibrium-selection mechanism can address these issues without suffering from multiple equilibria. This section calibrates the model for the US and examines how an equilibrium is selected.

## 7.1 Calibration Strategy

First, I calibrate the model to the US economy in 2015, as summarized in table 1. I choose 2015 because, although seven years had passed since the Great Recession, the economy was still suffering from inflation below target and sluggish economic growth. It also makes it easier to derive policy implications by enabling a comparison with Eggertsson et al. (2019). Thus,

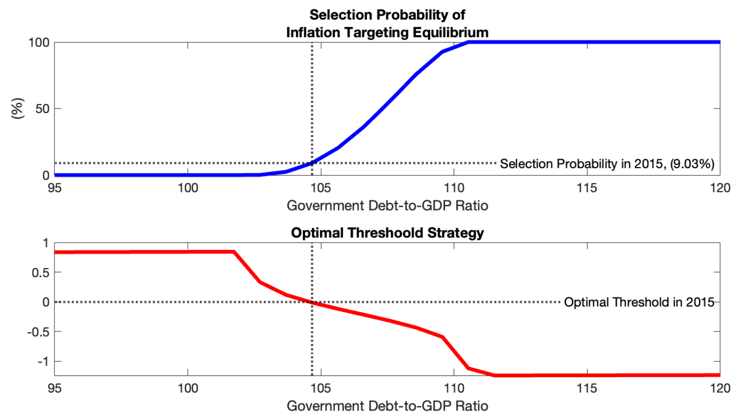



Figure 13: Equilibrium-Selection Probability in the US

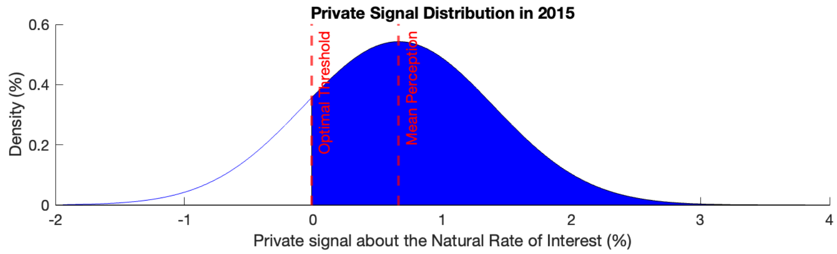
Note: In the upper panel, the solid blue line indicates the selection-probability curve of the inflation-targeting equilibrium $Prob\left(s_t > \Omega\left(s_t^*\right)\right)$ in the US. The intersection of the dotted lines shows that the selection probability is 9.03% in 2015. The bottom panel reports the corresponding levels of the optimal threshold strategy. The selection probability of the inflation-targeting equilibrium can increase if the fiscal expansion increases, through further reducing the optimal threshold. The selection probability of the secular-stagnation equilibrium is $1 - Prob\left(s_t > \Omega\left(s_t^*\right)\right) = 90.97\%$.

it is a suitable period to examine to see how likely it is to return to the inflation-targeting equilibrium. Given the three-period OLG-model structure, each period is set to 20 years, so the values in table 1 should be converted to 20 years in the simulation.

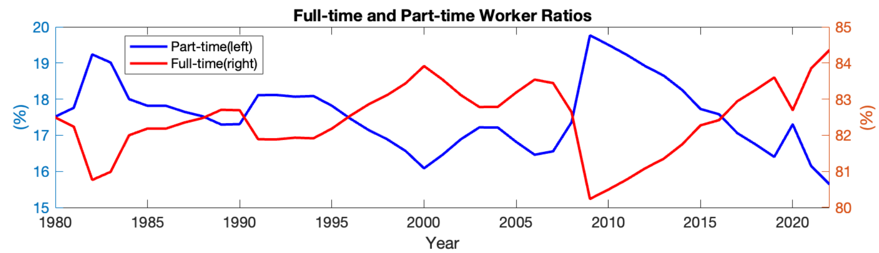
Following Eggertsson et al. (2019), I calibrate the natural rate of interest to $-1.47\%$ in 2015. As in the right panel of figure 12, the natural rate in recent periods has been slightly below $-1\%$ according to the Fed's estimates. Following the Fed, the inflation target is calibrated at 2%. Following Eggertsson et al. (2019), the borrowing limit is set at 23.4% of income, and the wage-adjustment parameter $\gamma$ is set at 0.94, which is consistent with the estimate of Schmitt-Grohe and Uribe (2016). Labor supply for full-time workers is assumed to be 1.6 times as high as that of part-time workers, a figure obtained from Bureau of Labor Statistics data. The labor-disutility parameter, $\varphi$, is set to 2 following Hansen (1985). The parameter $\tilde{\psi}$, which governs the temporality of fiscal expansion, is set at 0.18. This value is consistent with the parameter value estimated by Chen et al. (2012). My baseline uses 0.5 as the standard deviation of private signal $\sigma_\epsilon$ (Del Negro et al. 2017). Labor-supply parameters are chosen to normalize full-employment output to one. The rest of the parameter values have been widely used in previous literature.

## 7.2 Selection Probability of the Inflation-Targeting Equilibrium

Figure 13 plots the selection-probability curve of the inflation-targeting equilibrium and the optimal threshold strategy for the US. Note that it is an ex-ante selection probability of the inflation-targeting equilibrium that is computed in the beginning of 2015 before the true belief distribution realizes, assuming different levels of the government Debt-to-GDP ratio.




(a) Realized Private-Signal Distribution of $r^*$




(b) Full-Time- and Part-Time-Worker Ratios in the US

Figure 14: Realized Private Signal Distribution of $r^*$

Sources: Author's calculation, Bureau of Labor Statistics (BLS)

Note: Panel (a) plots the realized distribution of private signals backed out from the optimal threshold strategy $s^*$ and the full-time- and part-time-worker ratios, which are taken from the BLS data in panel (b). As the dotted red line shows, the realized mean perception of $r^*$ was 0.66%.

This figure shows that (1) the larger the ratio of government debt to GDP is, the higher the probability of realizing the inflation-targeting equilibrium is; (2) the larger that ratio is, the lower the optimal threshold strategy $s^*$ is; and (3) the selection probability has nonlinearity in the sense that a debt-to-GDP ratio below 100% has no impact on the selection probability of the inflation-targeting equilibrium.

Given a debt-to-GDP ratio of 103% in 2015, the inflation-targeting equilibrium could be chosen with a probability of around 9%, meaning there was a 91% chance that the economy was headed toward the secular-stagnation equilibrium in the US even though it had been seven years since the Great Recession. This is consistent with the fact that below-target inflation and sluggish growth plagued policy makers at that time.

This is because, as the lower panel of figure 13 shows, the optimal threshold strategy $s_t^*$ was set higher, slightly below zero, making it less attractive to achieve the inflation-targeting equilibrium and thus increasing the likelihood that more households would work part time.

By using full-time- and part-time-worker ratios, we can back out households' belief distribution. Panel (a) of figure 14 reports the private-signal distribution in the US. The vertical dotted lines indicate the optimal threshold level and the mean perception level. The shaded area plots the fraction of full-time workers.

In panel (b), the solid blue line shows the full-time-worker ratio and the part-time-worker ratio, the latter of which was around 18% in 2015. Combining the latter ratio and the optimal threshold strategy reveals that the mean perception of the natural rate of interest is 0.66%. Households' mean perceived level of the natural rate was thus still quite low, though slightly above the Fed's estimate.

## 7.3 Counterfactual 1: Further Fiscal Expansion

A natural counterfactual experiment would be to examine what would happen if the government had undertaken a larger fiscal expansion. The ratio of government debt to GDP has been growing since the Great Recession and reached 109% right before the pandemic in 2020.

Suppose that the government undertook further fiscal expansion to reach its prepandemic peak. The intersection of red dotted lines in figure 15 represents the probability of selecting the inflation-targeting equilibrium in this counterfactual scenario. We can see that the choice probability of equilibrium nonlinearly increases from 9% to 75%. In other words, since the selection probability of the secular-stagnation equilibrium falls to 25%, secular stagnation becomes less likely because of further fiscal expansion. This is caused by a reduction in the optimal threshold strategy, as shown in the lower panel, which is driven by (1) increasing the utility gain from full-time work once the inflation-targeting equilibrium is realized and (2) reducing the relative cost of full-time work through decreasing the chance of labor rationing, as discussed in sections 6.1 and 6.2. This result is also intuitively consistent with the data, which show that inflation remained slightly below target but approached the inflation target at the end of the prepandemic period (indeed, the inflation rate reached 1.9% in February 2020).

Importantly, the equilibrium-selection probability has a nonlinear feature: if the fiscal expansion shrinks below 100%, the probability of secular stagnation jumps to around 100%. This result implies that as long as underlying secular drivers such as demographics are resolved, without an appropriately designed monetary and fiscal policy it is still likely that the US will fall into secular stagnation once the current high inflation subsides.

The results of the counterfactual also suggest that the selection probability of the inflation -targeting equilibrium reaches 100% when the ratio of government debt to GDP increases to 115%. Eggertsson et al. (2019) report that a permanent increase in that ratio to 215% would be required to eliminate the secular-stagnation equilibrium. Given that my model assumes a temporary fiscal expansion, with the inflation target, the scale of the fiscal expansion needed to achieve the inflation target with certainty is much smaller than that discussed by Eggertsson et al. The difference arises because of the nonlinearity of $s^*$, which arises from belief dispersion and the strategic-complementarity channel.

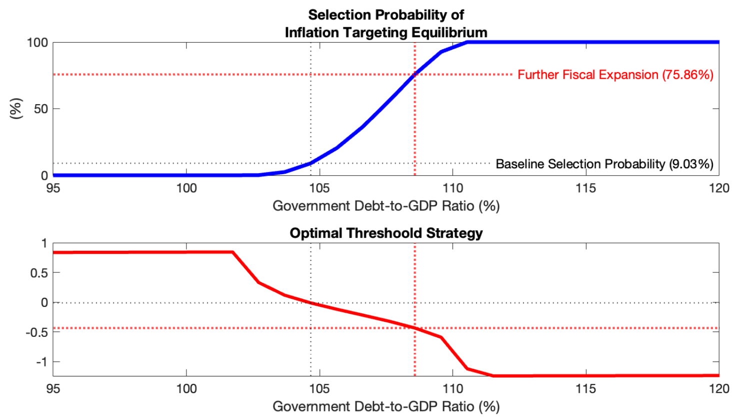



Figure 15: Counterfactual 1: Further Fiscal Expansion

Note: The solid blue line plots the selection-probability curve of the inflation-targeting equilibrium in the US. The intersections of the dotted lines show that the selection probabilities are 9.03% for the baseline and 75.86% for the counterfactual in which the debt-to-GDP ratio increases to the pre-COVID-19 peak. The bottom panel plots the corresponding levels of the optimal threshold strategy.

## 7.4 Counterfactual 2: Average Inflation Targeting

The Fed adopted AIT in 2020. In this framework, inflation is allowed to temporarily overshoot the target. By applying the discussion in section 6.4, we can examine how the policy change affects the selection probability of the inflation-targeting equilibrium.

In figure 16, the dotted blue line represents the baseline probability curve for the US in 2015, and the solid red line represents the counterfactual hypothetical case in which AIT is introduced. Since inflation had been below the target for several years, inflation is permitted to overshoot the target for some further periods. Here I assume that inflation can overshoot the target by 1% on average for two years once it exceeds the target. Because AIT allows overshooting, the probability curve shifts upward and to the left from the dotted blue line to the solid red line. The probability curve also becomes slightly steeper. In sum, the marginal benefit of fiscal expansion is strengthened.

The intersection of the dotted red line represents the counterfactual's equilibrium-selection probability, which increased to 61% from 9% in the US. As in the previous section, the underlying reason is that an additional incentive to strengthen strategic complementarity arises from (1) the interaction between monetary and fiscal policy (section 6.2) and (2) the need to avoid a rise in future debt payments (section 6.4), which increases the relative utility gain of choosing full-time work and leads to equation (75). As a result, as the bottom panel shows, the optimal threshold strategy falls further than the baseline. This result may imply that my model has new potential for effective forward guidance via the strategic-complementarity channel even in secular stagnation while avoiding the forward-guidance puzzle (Del Negro et al. 2012 and Mckay et al. 2016).

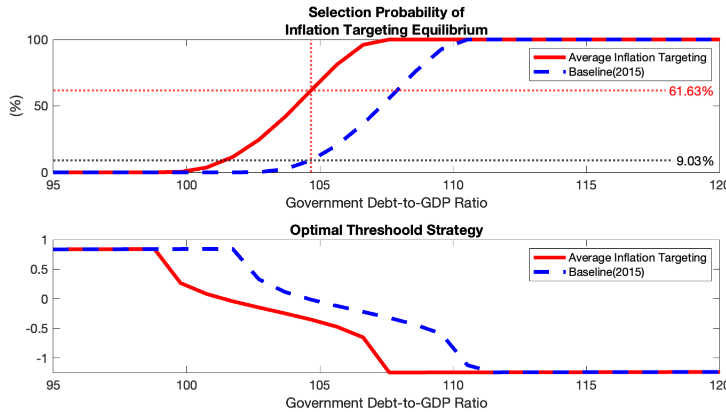



Figure 16: Counterfactual 2: Average Inflation Targeting

Note: The dotted blue line plots the baseline selection-probability curve of the inflation-targeting equilibrium in the US, and the solid red line is the same selection probability if average inflation targeting is introduced. The intersections of the dotted lines show that the selection probabilities are 9.03% for the baseline and 61.63% for the counterfactual. The bottom panel reports the corresponding levels of the optimal threshold strategy.

# 8 Conclusion

This paper developed an endogenous equilibrium-selection mechanism that pins down a unique equilibrium in the secular-stagnation model. In the secular-stagnation literature, ZLB models can address long ZLB episodes. However, these models leave a crucial question unanswered: how to obtain a unique equilibrium. In addition, the policy prescription suggested in the previous literature is a massive permanent debt expansion, which raises the concern of debt sustainability.

To resolve this issue, I relax the assumption of perfect information about the natural rate of interest and tie the model to the global-game framework. I then show how strategic complementarity among households can lead to a unique equilibrium.

One main message of this paper is that imperfect information is enough to determine a unique equilibrium in secular-stagnation models. Once strategic complementarity comes into play, even a temporary fiscal expansion together with AIT gains power to endogenously raise the selection probability of the better equilibrium by reinforcing the strategic-complementarity channel, which promotes households' ability to coordinate on a decision to resolve the structural shortage of aggregate demand.

In a calibrating example, I found that the selection probability of a secular-stagnation equilibrium will be high once inflation pressure subsides in the US. I then showed that further fiscal expansion, even if temporary, and AIT can increase the likelihood of realizing the inflation-targeting equilibrium.

However, I also find that the likelihood of secular stagnation can easily increase without appropriate monetary and fiscal policies if relatively minor changes in the underlying secular forces occur. Thus, even with the current high inflation rate, the possibility of a secular-stagnation equilibrium in the future is difficult to rule out since the underlying causes of the long-run downward trend in the natural rate of interest remain. Thus, it may be necessary to add to secular-stagnation models equilibria in which above-target, high inflation persists and to reexamine its endogenous equilibrium selection. This point is not addressed in this paper, but I hope it will be explored in future studies.

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# Appendix

## A Proofs of Proposition and Lemma

### A.1 Proof of Proposition 1

First, we check if the inflation target can be achieved. Given the labor choice distribution, the candidate of equilibrium inflation rate $\tilde{\Pi}_t$ can be computed using the aggregate demand (31) and the Phillips curve (32).

$$\tilde{\Pi}_t = \left\{ \frac{(1+g)\, D\Gamma^*}{D - \left(\Phi\left(s_t, s_t^*\right) L^P + \left(1 - \Phi\left(s_t, s_t^*\right)\right)\right)^{-\alpha} \bar{\chi}^2} \right\}^{\frac{1}{\phi^\pi - 1}} \tag{A.1}$$

Then plugging $\tilde{\Pi}_t$ into the Taylor rule yields the implied nominal interest rate for achieving the inflation target. It is given by:

$$1 + \tilde{i}_t = (1 + i^*) \left( \frac{\tilde{\Pi}_t}{\Pi^*} \right)^{\phi_\pi}. \tag{A.2}$$

If the implied nominal interest rate $\tilde{i}_t$ is negative, this implies that the ZLB is binding and the central bank cannot achieve the inflation target, and thus is inconsistent with the inflation targeting equilibrium. If the implied nominal interest rate is positive, the inflation target is achieved. Thus rearranging equation (A.2) yields that the inflation target is achieved if:

$$\tilde{\Pi}_t \geq \Pi^{zero} = \left( \frac{1}{1 + i^*} \right)^{\frac{1}{\phi_\pi}} \Pi^*, \tag{A.3}$$

, and then establish the condition (39) by rearranging equation (41).

$$\Phi^* > \Phi\left(s_t, s_t^*\right), \tag{A.4}$$

where

$$\Phi^* = \frac{L^F}{L^F - L^P} + \frac{\left(\Pi^{zero}\right)^{\phi_\pi - 1} \bar{\chi}^2}{\left((1+g)\,\Gamma^* - \left(\Pi^{zero}\right)^{\phi_\pi - 1}\right) D \left(L^F - L^P\right)}, \tag{A.5}$$

where $\Phi^*$ is a cut-off level of proportion of part-time worker and $\Pi^{zero}$ is a level of equilibrium inflation rate when an implied nominal interest rate $1 + \tilde{i}_t = (1 + i^*) \left( \frac{\tilde{\Pi}_t}{\Pi^*} \right)^{\phi_\pi}$ equals 1.

Recall that $\tilde{\Pi}_t$ is a candidate for an equilibrium inflation rate with full employment, which means that the level of inflation where is an intersection of the vertical segment of AS curve and the upper segment of the AD curve. Recall that the upper segment of the AD curve is a downward sloping curve since:

$$\frac{\partial \Pi_t}{\partial Y_t} < 0, \quad i \geq 0, \tag{A.6}$$

Thus, if $\tilde{\Pi}_t > \Pi^{zero}$, there is an intersection of the upper segment of the AD curve and the upper segment of the AS curve, which proves the existence of the inflation targeting equilibrium. On the other hand, if $\tilde{\Pi}_t > \Pi^{zero}$, there is no intersection between the AD and AS curves because the AD curve kinks at a smaller output level than the full-employment output level. Thus, in this case, the inflation targeting equilibrium does not exist.

Next, let us move on to check if secular stagnation occurs. Given the labor choice distribution, the candidate of equilibrium inflation rate can be computed using the aggregate demand (37) and the Phillips curve (38). For simplicity, let us assume $\alpha = aaaa$, then we obtain:

$$\tilde{\Pi}_{t,Ss} = \frac{D}{\bar{\chi}} \left\{ \frac{\bar{\chi}^{\alpha}}{\left(\Phi\left(s_t, s_t^*\right) L^P + \left(1 - \Phi\left(s_t, s_t^*\right)\right)\right)^{\alpha}} - \frac{\left(1+g\right)\left(\Phi\left(s_t, s_t^*\right) L_{Ss}^P + \left(1 - \Phi\left(s_t, s_t^*\right)\right) L_{Ss}^F\right)}{\left(\Phi\left(s_t, s_t^*\right) L^P + \left(1 - \Phi\left(s_t, s_t^*\right)\right)\right) \bar{\chi}} \right\}^{-1} \tag{A.7}$$

Since we have:

$$\frac{\partial \Pi_t}{\partial Y_t} > 0, \quad i = 0, \tag{A.8}$$

when $\tilde{\Pi}_t < 1$, there is an intersection of the lower part of the AD curve and the lower part of the AS curve, which ensures the existence of the secular stagnation equilibrium. Rearranging the condition $\tilde{\Pi}_t < 1$, we get:

$$\Phi > \Phi^{Ss} \tag{A.9}$$

where

$$\Phi^{Ss} = \frac{\bar{\chi}^2}{\left(D + \left(1+g\right)\xi\right)\left(L^F - L^P\right)} + \frac{L^F}{L^F - L^P}, \tag{A.10}$$

where $\Phi^{Ss}$ is a cut-off level of proportion of part-time worker and a level of equilibrium inflation equals 1. Taking the partial derivative with respect to $\Phi$, we get:

$$\frac{\partial \tilde{\Pi}_{t,Ss}}{\partial \Phi} < 0 \tag{A.11}$$

Notice that when $\tilde{\Pi}_{t,Ss} = \Pi^{zero}$, we have:

$$\Phi = \Phi^* \tag{A.12}$$

Thus we have:

$$\Phi^{Ss} > \Phi^* \tag{A.13}$$

The conditions (A.4) and (A.13) imply that if $\Phi > \Phi^*$, the inflation targeting equilibrium is excluded. If $\Phi < \Phi^{Ss}$, the secular stagnation equilibrium is uniquely selected while the inflation targeting equilibrium is excluded.

If $\Phi \in \left(\Phi^*, \Phi^{Ss}\right)$, the candidate inflation rate may be consistent with another equilibrium in between the secular stagnation and the inflation targeting equilibrium. However, this type of equilibrium (the point $C$ in figure 2.b) is indeterminate and is excluded by some learning criteria, as discussed in Eggertsson et al. (2019). Thus, I assume that the secular stagnation equilibrium satisfies its consistency criteria if $\Phi\left(s_t, s_t^*\right)$ is higher than $\Phi^*$. Notice that the condition (40) can be derived as the condition that guarantees an existence for intersection of lower segments of AD/AS curves.

$\square$

## A.2 Proof of Proposition 2

First, we derive the condition that there is no room strategic complementarity to exist. Given $\bar{\Phi}$ and $\delta$, if the candidate of equilibrium inflation rate exceeds $\Pi^{zero}$, then Proposition 1 implies that the inflation target is surely achieved without any coordination, i.e.,

$$\left\{\frac{(1+g)\,D\Gamma^*}{D-\left(\bar{\Phi}\delta+\left(1-\bar{\Phi}\right)\right)^{\alpha}}\right\}^{\frac{1}{\phi_\pi-1}} \geq \Pi^{zero}, \tag{A.14}$$

Rearranging equation (A.14) yields that the inflation targeting equilibrium is surely achieved if:

$$\delta \geq \bar{\delta}, \tag{A.15}$$

where

$$\bar{\delta} = \bar{\Phi}\left\{\left(D-\frac{(1+g)\,D\Gamma^*}{\left(\pi^{zero}\right)^{\phi_\pi-1}}\right)^{\frac{1}{\alpha}} - 1 + \bar{\Phi}\right\}^{-1}, \tag{A.16}$$

In this case, the selection probability of the inflation targeting equilibrium perceived by each household is 1, for all households. Thus all households choose full-time work since $U_j\left(C_j, L^F\right) > U_j\left(C_j, L^P\right)$ for all $j$.

Next, we derive another condition that there is no room for the strategic comeplementarity to exist. Given $\bar{\Phi}$ and $\delta$, if the candidate of equilibrium inflation rate is below 1, then Proposition 1 implies that the secular stagnation is surely achieved without any coordination, i.e.,

$$\frac{D}{\bar{\chi}}\left\{1+\frac{1+g}{\left(\bar{\Phi}\delta+\left(1-\bar{\Phi}\right)\right)^{\alpha}}\right\}^{-1} \leq 1, \tag{A.17}$$

Rearranging equation (A.17) yields that the secular stagnation equilibrium is surely achieved if:

$$\delta \leq \underline{\delta}, \tag{A.18}$$

where

$$\underline{\delta} = \bar{\Phi}\left\{(D-1-g)^{\frac{1}{\alpha}}+\bar{\Phi}-1\right\}^{-1}, \tag{A.19}$$

In this case, the selection probability of the secular stagnation equilibrium perceived by each household is 1, for all households. Thus all households choose part-time work since $U_j\left(C_j, L^P\right) > U_j\left(C_j, L^F\right)$ for all $j$.

Therefore, there is a room for the strategic compelmentarity to work strong enough so that $\Phi \in \left(\underline{\delta}, \bar{\delta}\right)$ if $\delta \in \left(\underline{\delta}, \bar{\delta}\right)$.

□

## A.3 Proof of Lemma 1

The proof of Lemma 1 is as follows. Using equations from (27) to (44), we can rewrite the expected utility of full-time work as:

$$
\begin{aligned}
E_t U_{j,t}\left(C_j, L^F\right) =& \int_{-\infty}^{\Pi^{zero}} \left(u\left(C^y_{t,j}\right) + \beta\left\{u\left(C^m_{t+1,j}\right) - \varphi L_{t+1,j}\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\Bigg|_{Ss\,eqm} \\
&+ \int_{\Pi^{zero}}^{\infty} \left(u\left(C^y_{t,j}\right) + \beta\left\{u\left(C^m_{t+1,j}\right) - \varphi L^j_{t+1}\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\Bigg|_{IT\,eqm} \\
=& \left( ln\left(\frac{\Pi^*}{1+i^*}D\right) + \beta ln\left[\left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^F - \frac{D}{1+\beta}\right] - \beta\varphi L^F \right. \\
&\left. +\beta^2 ln\left[\frac{1+i^*}{\Pi^*}\left(\frac{\beta}{1+\beta}\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^F - \frac{1+i^*}{\Pi^*}\frac{\beta D}{1+\beta}\right]\right) \\
&+ \left[ ln\left(\frac{(1+i^*)\,\Pi^{Ss}}{\Pi^*}\right) + \beta ln\left\{\frac{\left(\frac{(1-\gamma)\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma(\Pi^{Ss})^{-1}}\right)L^F_{Ss} - \frac{D}{1+\beta}}{\left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^F - \frac{D}{1+\beta}}\right\} - \beta\varphi\left(L^F_{Ss} - L^F\right) \right. \\
&\left. +\beta^2 ln\left\{\frac{\left(\frac{\beta}{1+\beta}\right)\left(\frac{(1-\gamma)\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma(\Pi^{Ss})^{-1}}\right)\frac{L^F_{Ss}}{\Pi^{Ss}} - \left(\frac{\beta}{1+\beta}\right)\frac{D}{\Pi^{Ss}}}{\frac{1+i^*}{\Pi^*}\left(\frac{\beta}{1+\beta}\right)\left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^F - \frac{1+i^*}{\Pi^*}\frac{\beta D}{1+\beta}}\right\}\right] F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero}\left|s_{j,t}\right.\right)
\end{aligned}
$$

where $1 - F_{j,t}$ denotes the selection probability of the inflation targeting equilibrium perceived by household $j$ and $\tilde{\Pi}_{j,t}$ denotes the achievable inflation rate perceived by household $j$. The above rearranged form of the expected utility can be simplified as follows.

$$
E_t U_{j,t}\left(C_j, L^F\right) = \Gamma^F + \left(\Gamma^F_{Ss} - \Gamma^F\right) F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero}\left|s_{j,t}\right.\right) \tag{A.20}
$$

where

$$
\begin{aligned}
\Gamma^F =& ln\left(\frac{\Pi^*}{1+i^*}D\right) + \beta ln\left[\left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^F - \frac{D}{1+\beta}\right] - \beta\varphi L^F \\
&+ \beta^2 ln\left[\frac{1+i^*}{\Pi^*}\left(\frac{\beta}{1+\beta}\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^F - \frac{1+i^*}{\Pi^*}\frac{\beta D}{1+\beta}\right]
\end{aligned}
$$

$$
\Gamma^F_{Ss} = ln\left(\frac{\Pi^{Ss}}{1}D\right) + \beta ln\left[\left(\frac{\left(1-\gamma\right)\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma(\Pi^{Ss})^{-1}}\right)L^F_{Ss} - \frac{D}{1+\beta}\right] - \beta\varphi L^F_{Ss}
$$

$$
+ \beta^2 ln\left[\left(\frac{\beta}{1+\beta}\frac{\left(1-\gamma\right)\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma\left(\Pi^{Ss}\right)^{-1}}\right)\frac{L^F_{Ss}}{\Pi^{Ss}} - \frac{\beta}{1+\beta}\frac{\beta D}{\Pi^{Ss}}\right]
$$

$\Gamma^F$ denotes the utility gain obtained when the inflation targeting equilibrium is realized with the choice of full-time work, and $\Gamma^F_{Ss}$ denotes the utility gain when the secular stagnation equilibrium is realized with the choice of full time work.

Rearranging equations from (27) to (**??**), we can rewrite the expected utility of part-time work as:

$$
\begin{aligned}
E_t U_{j,t}\left(C_j, L^P\right) = & \int_{-\infty}^{\Pi^{zero}} \left(u\left(C^y_{t,j}\right) + \beta u\left(C^m_{t+1,j}\right) + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\Bigg|_{Ss\, eqm} \\
& + \int_{\Pi^{zero}}^{\infty} \left(u\left(C^y_{t,j}\right) + \beta u\left(C^m_{t+1,j}\right) + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\Bigg|_{IFT\, eqm} \\
& = ln\left(\frac{\Pi^*}{1+i^*}D\right) + \beta ln\left(\left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \left(\frac{1}{1+\beta}\right)D\right) - \beta\varphi L^P \\
& + \beta^2 ln\left(\frac{1+i^*}{\Pi^*}\left(\left(\frac{\beta\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \left(\frac{1+i^*}{\Pi^*}\frac{\beta}{1+\beta}\right)D\right)\right) \\
& + \left[ln\left(\frac{\left(1+i^*\right)\Pi^{SS}}{\Pi^*}\right) + \beta ln\left\{\frac{\left(\frac{(1-\gamma)\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma(\Pi^{Ss})^{-1}}\right)L^P_{Ss} - \frac{D}{1+\beta}}{\left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \frac{D}{1+\beta}}\right\} - \beta\varphi\left(L^P_{Ss} - L^P\right)\right. \\
& \left. + \beta^2 ln\left\{\frac{\left(\frac{\beta\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)\frac{L^P_{Ss}}{\Pi^{Ss}} - \frac{\beta}{1+\beta}\frac{D}{\Pi^{Ss}}}{\frac{1+i^*}{\Pi^*}\left(\left(\frac{\beta\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \frac{\beta D}{1+\beta}\right)}\right\}\right] F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero}\left|s_{j,t}\right.\right)
\end{aligned}
$$

We can then rewrite this expected utility as simpler form.

$$E_t U_{j,t}\left(C_j, L^P\right) = \Gamma^P + \left(\Gamma^P_{Ss} - \Gamma^P\right) F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero}\left| s_{j,t}\right.\right) \tag{A.21}$$

where

$$\Gamma^P = ln\left(\frac{\Pi^*}{1+i^*}D\right) + \beta ln\left[\left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \frac{D}{1+\beta}\right] - \beta\varphi L^P$$
$$+\beta^2 ln\left[\frac{1+i^*}{\Pi^*}\left(\frac{\beta}{1+\beta}\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \frac{1+i^*}{\Pi^*}\frac{\beta D}{1+\beta}\right]$$

$$\Gamma^P_{Ss} = ln\left(\frac{\Pi^{Ss}}{1}D\right) + \beta ln\left[\left(\frac{(1-\gamma)\,\alpha\left(L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)\right)^{\alpha-1}}{1-\gamma\left(\Pi^{Ss}\right)^{-1}}\right)L^P_{Ss} - \frac{D}{1+\beta}\right] - \beta\varphi L^P_{Ss}$$
$$+\beta^2 ln\left[\left(\frac{\beta}{1+\beta}\frac{(1-\gamma)\,\alpha\left(L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)\right)^{\alpha-1}}{1-\gamma\left(\Pi^{Ss}\right)^{-1}}\right)\frac{L^P_{Ss}}{\Pi^{Ss}} - \frac{\beta}{1+\beta}\frac{\beta D}{\Pi^{Ss}}\right]$$

$\Gamma^P$ denotes an utility gain of choosing part time work in the inflation targeting equilibrium and $\Gamma^P_{Ss}$ denotes an utility gain of choosing part time work in the secular stagnation equilibrium.

Since household $j$ chooses full time work if $E_t U_{j,t}\left(C_j, L^F\right) \geq E_t U_{j,t}\left(C_j, L^P\right)$, we can derive the following condition by using this relation. Household j chooses the full time work if and only if:

$$1 - F_{j,t} \geq \Psi_j\left(\Pi^*, r^*, L^F, L^P, L^F_{Ss}, L^P_{Ss}, \varphi\right) \tag{A.22}$$

where

$$1 - F_{j,t} = Pr\left(\Upsilon \geq G_{j,t}\left| s_{j,t}\right.\right),$$
$$\Psi_j = 1 - \frac{\sum_{i\in\{Young,Middle,Old\}} \Lambda^{Gain}_i\left(\Pi^*, r^*, L^F, L^P, \varphi\right)}{\sum_{i\in\{Young,Middle,Old\}} \Lambda^{Cost}_i\left(\Pi^*, r^*, L^F, L^P, L^F_{Ss}, L^P_{Ss}.\varphi\right)},$$

Note that $\Lambda^{Gain}_i$ is the relative utility gain of "Full time work" in the i-aged and $\Lambda^{Cost}_i$ is the relative utility cost of "Full time work" in the i-aged. The relative utility gain of “Full time work” is given by:

$$\sum_{i\in\{Young,Middle,Old\}} \Lambda_i^{Gain} = \Gamma^F\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right) - \Gamma^P\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right) \tag{A.23}$$

The relative utility cost of "Full time work" is:

$$\sum_{i\in\{Young,Middle,Old\}} \Lambda_i^{Cost} = \left\{\Gamma^F\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right) - \Gamma^P\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right)\right\}$$

$$+\left\{\Gamma_{Ss}^P\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right) - \Gamma_{Ss}^F\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right)\right\} \tag{A.24}$$

□

## A.4 Proof of Proposition 3

The proof of proposition 3 is presented in three steps. Let us define the payoff function of household $j$:

$$V_{j,t}\left(s_{j,t}, \Phi\right) \equiv E_t U_{j,t}\left(C_j, L^F\right) - E_t U_{j,t}\left(C_j, L^P\right), \tag{A.25}$$

where $E_t U_{j,t}\left(C_j, L^F\right)$ is an expected utility of choosing the full-time work and $E_t U_{j,t}\left(C_j, L^F\right)$ is an expected utility of choosing the part-time work and $\Phi$ is a proportion of households who chose part-time work. We first show the following condition.

**Condition 1:**

$If\ 1-\Phi \geq 1-\Phi'\ for\ all\ s_{j,t},\ then\ V\left(s_{j,t}, \Phi\right) \geq V\left(s_{j,t}, \Phi'\right)\ for\ all\ s_{j,t}.$

**Proof.**

Since achievable aggregate consumption satisfies: $\tilde{C}_t\left(\Phi\right) \geq \tilde{C}_t\left(\Phi'\right)$, achievable inflation rate also has $\tilde{\Pi}_{t+1}\left(\Phi\right) \geq \tilde{\Pi}_{t+1}\left(\Phi'\right), \forall s_{j,t}$. Thus we have:

$$1 - F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero} \left| s_{j,t}, \Phi\right.\right) \geq 1 - F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero} \left| s_{j,t}, \Phi'\right.\right). \tag{A.26}$$

The payoff function can be rewritten as:

$$V\left(s_{j,t},\Phi\right)=\left\{\Gamma^{F}-\Gamma^{P}\right\}+\left\{\left(\Gamma^{F}_{Ss}-\Gamma^{F}\right)-\left(\Gamma^{P}_{Ss}-\Gamma^{P}\right)\right\}F_{j,t}\left(\tilde{\Pi}_{j,t+1}<\Pi^{zero}\left|s_{j,t},\Phi\right.\right). \tag{A.27}$$

Since $\left\{\left(\Gamma^{F}_{Ss}-\Gamma^{F}\right)-\left(\Gamma^{P}_{Ss}-\Gamma^{P}\right)\right\}<0$ and $1-F_{j,t}\left(\tilde{\Pi}_{j,t}<\Pi^{zero}\left|s_{j,t},\Phi\right.\right)\geq 1-F_{j,t}\left(\tilde{\Pi}_{j,t+1}<\Pi^{zero}\left|s_{j,t},\Phi'\right.\right)$, we have:

$$V\left(s_{j,t},\Phi\right)\geq V\left(s_{j,t},\Phi'\right),\ \ \forall s_{j,t}. \tag{A.28}$$

□

The condition 1 implies that the household's decisions to choose full-time work are strategic complements. If a strategy profile $s^*$ entails a larger proportion of households who choose the full-time work for any private signals than another strategy profile $s^{*'}$, then the payoff to work full-time is larger given $s^*$ than that given by $s^{*'}$.

Before we proceed the following step, let us define the threshold strategy profile as follows.

$$I_{s^*}\left(s_{j,t}\right)=\begin{cases}1 & \text{if } \mathrm{s_{j,t}<s^*}\\ 0 & \text{if } \mathrm{s_{j,t}\geq s^*}\end{cases} \tag{A.29}$$

$$\tag{A.30}$$

This threshold strategy states that every household chooses the part-time work if and only if $s_{j,t}<s^*$ and chooses the full-time work, otherwise.

Next we show the following condition.

**Condition 2:**

$V\left(s^*,I_{s^*}\left(s_{j,t}\right)\right)$ *is continuous and strictly decreasing in* $s^*$.

**Proof.**

Given the threshold strategy profile $I_{s^*}\left(s_{j,t}\right)$, let us define $\Theta\left(\cdot\right)$ a distribution function of households. Then, a proportion of households who choose to work part-time is written as $\Theta\left(\frac{s^*-s_t}{\sigma}\right)$. Taking partial derivative of $\Theta\left(\cdot\right)$ with respect to $s^*$ yields $\frac{\partial\Theta\left(\frac{s^*-s_t}{\sigma}\right)}{\partial s^*}>0$.

Recall that we have from the discussion in the main text,

$$1-F_{j,t}\left(\tilde{\Pi}_{j,t+1}<\Pi^{zero}\left|s_{j,t}\right.\right)=Pr\left(\Phi^*\geq\Theta\left(\frac{s^*-s_t}{\sigma}\right)\right). \tag{A.31}$$

So we have:

$$\frac{\partial 1 - F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero}\left|s_{j,t}\right.\right)}{\partial s^*} = \frac{\partial Pr\left(\Phi^* \geq \Theta\left(\frac{s^*-s_t}{\sigma}\right)\right)}{\partial s^*} < 0. \tag{A.32}$$

Rearranging the payoff function in terms of $\Theta\left(\frac{s^*-s_t}{\sigma}\right)$ yields:

$$\begin{aligned} V\left(s^*, \Phi\left(\frac{s^*-s_t}{\sigma}\right)\right) &= \left\{\Gamma^F - \Gamma^P\right\} + \left\{\left(\Gamma^F_{Ss} - \Gamma^F\right) - \left(\Gamma^P_{Ss} - \Gamma^P\right)\right\} F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero}\left|s_{j,t}\right.\right), \\ &= \left\{\Gamma^F - \Gamma^P\right\} + \left\{\left(\Gamma^F_{Ss} - \Gamma^F\right) - \left(\Gamma^P_{Ss} - \Gamma^P\right)\right\}\left(1 - Pr\left(\Phi^* \geq \Theta\left(\frac{s^*-s_t}{\sigma}\right)\right)\right). \end{aligned} \tag{A.33}$$

We then get:

$$\frac{\partial V\left(s^*, \Theta\left(\frac{s^*-s_t}{\sigma}\right)\right)}{\partial s^*} = -\left\{\left(\Gamma^F_{Ss} - \Gamma^F\right) - \left(\Gamma^P_{Ss} - \Gamma^P\right)\right\}\frac{\partial Pr\left(\Phi^* \geq \Theta\left(\frac{s^*-s_t}{\sigma}\right)\right)}{\partial s^*} < 0. \tag{A.34}$$

Therefore, $V\left(s^*, I_{s^*}\left(s_{j,t}\right)\right)$ is strictly decreasing in $s^*$. The continuity of $V\left(s^*, I_{s^*}\left(s_{j,t}\right)\right)$ follows from the fact that it is an integral in which the limits of integration are themselves continuous.

□

Intuitively, this condition implies that the stronger fundamentals, the lower payoff to choose to work part-time for a household on the margin of switching from full-time work to part-time work.

Finally, we prove the following condition.

**Condition 3:**

*There is a unique $s_t^*$ such that in any equilibrium of the game with imperfect information of the fundamentals, a household with signal$s_{j,t}$ choose to work full time if and only if $s_{j,t} \geq s^*_{j,t}$.*

**Proof.** Condition 2 ensures that $V\left(s^*, I_{s^*}\left(s_{j,t}\right)\right)$ is continuous and strictly decreasing in $s^*$. We begin by guaranteeing that $V\left(s^*, I_{s^*}\left(s_{j,t}\right)\right) = 0$ for some $s^*$. If a private signal of marginal household is sufficiently small such as $s_{j,t} < \underline{s} - \epsilon$, the marginal household with a private signal $s_{j,t}$ knows that $s_t \leq \underline{s}$. Since the payoff of part-time job is positive and the payoff to full-time job is negative at any $s_t$ in this region, we can get $V\left(s^*, I_{s^*}\left(s_{j,t}\right)\right) < 0$.

Similarly, if a private signal of marginal household is sufficiently large such as $s_{j,t} > \bar{s} + \epsilon$, he knows that $s_t \geq s$. Since the payoff of part-time job is negative and the payoff to full-time job is positive at any $s_t$ in this region, we can get $V\left(s^*, I_{s^*}\left(s_{j,t}\right)\right) > 0$. Thus, there is a unique level of $s^*$ that satisfies Since the payoff of part-time job is positive and the payoff to full-time job is negative at any $s_t$ in this region, we can get $V\left(s^*, I_{s^*}\left(s_{j,t}\right)\right) = 0$.

Next, we show that the equilibrium is given by the step function $I_{s^*}$. To do this, let us first define:

$$\underline{s}_{j,t} = inf\left\{s_{j,t} | 1 - \Theta\left(s_{j,t}\right) < 1\right\}, \tag{A.35}$$

$$\bar{s}_{j,t} = sup\left\{s_{j,t} | 1 - \Theta\left(s_{j,t}\right) > 0\right\}. \tag{A.36}$$

The following relation is satisfied:

$$\bar{s}_{j,t} \geq sup\left\{s_{j,t} | 1 > 1 - \Theta\left(s_{j,t}\right) > 0\right\} \geq \underline{s}_{j,t} = inf\left\{s_{j,t} | 0 < 1 - \Theta\left(s_{j,t}\right) < 1\right\} \geq \underline{s}_{j,t}. \tag{A.37}$$

Thus we have : $\underline{s}_{j,t} \leq \bar{s}_{j,t}$. If $1 - \Theta\left(s_{j,t}\right) < 1$, there are some households who chooses part-time work, which means that this case is only consistent with an equilibrium if the payoff to work part-time is at least as high as the payoff to work full-time given the private signal $s_{j,t}$. This is applicable for $\underline{s}_{j,t}$ because of continuity. Therefore we have $V\left(\underline{s}_{j,t}, 1 - \Theta\right) \leq 0$. Since it is obvious that $I_{\underline{s}_{j,t}} \leq 1 - \Theta$, combining condition 1 and this relation yields:

$$V\left(\underline{s}_{j,t}, I_{\underline{s}_{j,t}}\right) \leq V\left(\underline{s}_{j,t}, 1 - \Theta\right) \leq 0. \tag{A.38}$$

,which implies that $V\left(\underline{s}_{j,t}, I_{\underline{s}_{j,t}}\right) \leq 0$.

Condition 2 shows that $V\left(s_{j,t}, I_{s_{j,t}}\right)$ is decreasing in $s_{j,t}$ and $s^*$ is the unique value of $s_{j,t}$ that solves $V\left(s_{j,t}, I_{s_{j,t}}\right) = 0$, we then have $\underline{s}_{j,t} \geq s^*$. Similarly, we have $\bar{s}_{j,t} \leq s^*$. Therefore, we have:

$$\underline{s}_{j,t} \geq s^* \geq \bar{s}_{j,t}. \tag{A.39}$$

Since we also have $\underline{s}_{j,t} \leq s^* \leq \bar{s}_{j,t}$, we have:

$$\underline{s}_{j,t} = s^* = \bar{s}_{j,t}. \tag{A.40}$$

, which means that the equilibrium $1 - \Theta$ is given by $I_{s^*}$ with optimal threshold $s^*$ that solves $V\left(s^*, I_{s^*}\right) = 0$.

Applying this result for Lemma 1 indicates that the optimal threshold strategy is given by considering the marginal household whose private signal is equivalent to $s_t^*$ that is a solution to the following equality condition.

$$1 - F_{j,t} = Pr\left(\Phi^* \geq \Theta\left(\frac{s_t^* - s_t}{\sigma}\right)|s_{j,t} = s_t^*\right) = \Psi. \tag{A.41}$$

After observing the private signal and the central bank estimates of ther natural rate of interest, household $j$ use Bayes' rule to update his belief to:

$$s_t|r^*, s_{j,t} \sim N\left(\frac{\sigma_s^{-1}r^* + \sigma_\epsilon^{-1}s_{j,t}}{\sigma_s^{-1} + \sigma_\epsilon^{-1}}, \frac{1}{\sigma_s^{-1} + \sigma_\epsilon^{-1}}\right) \tag{A.42}$$

Recall that the condition for the inflation targeting equilibrium in the second stage is:

$$\Phi^* \geq \Theta\left(\frac{s_{t,j}^* - s_t}{\sigma}\right) \tag{A.43}$$

, which implies that:

$$s_t \geq s_t^* - \sigma_s\Theta^{-1}\left(\Upsilon\right) \tag{A.44}$$

Therefore, the selection probability of the inflation targeting equilibrium can rewritten as:

$$1 - F_{j,t} = 1 - \Theta\left(\frac{s_t^* - \sigma_s\Theta^{-1}\left(\Phi^*\right) - \frac{\frac{1}{\sigma_s^2}r^* + \frac{1}{\sigma_\epsilon^2}s_{j,t}}{\frac{1}{\sigma_s^2} + \frac{1}{\sigma_\epsilon^2}}}{\sqrt{\frac{1}{\frac{1}{\sigma_s^2} + \frac{1}{\sigma_\epsilon^2}}}}\right). \tag{A.45}$$

Thus the selection probability of the inflation targeting equilibrium perceived by the marginal household is given by:

$$1 - F_{*,t} = 1 - \Theta\left(\frac{\left(\frac{1}{\sigma_s^2} + \frac{1}{\sigma_\epsilon^2}\right)\left(s_t^* - \sigma_s\Theta^{-1}\left(\Phi^*\right)\right) - \left(\frac{1}{\sigma_s^2}r^* + \frac{1}{\sigma_\epsilon^2}s_t^*\right)}{\sqrt{\frac{1}{\sigma_s^2} + \frac{1}{\sigma_\epsilon^2}}}\right). \tag{A.46}$$

Applying proposition 1 implies that a unique optimal threshold strategy $s_t^*$ satisfies:

$$1 - \Theta\left(\frac{\left(\frac{1}{\sigma_s^2} + \frac{1}{\sigma_\epsilon^2}\right)\left(s_t^* - \sigma_s\Theta^{-1}\left(\Phi^*\right)\right) - \left(\frac{1}{\sigma_s^2}r^* + \frac{1}{\sigma_\epsilon^2}s_t^*\right)}{\sqrt{\frac{1}{\sigma_s^2} + \frac{1}{\sigma_\epsilon^2}}}\right) = \Psi \tag{A.47}$$

By solving this equality condition, we get:

$$s_t^*\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right) = \xi_\Psi\Theta^{-1}\left(1 - \Psi\right) + \xi_{\Phi^*}\Theta^{-1}\left(\Phi^*\right) + r^* \tag{A.48}$$

where $\xi_\Psi = \sigma_s^2 \sigma_{s,\epsilon}$ and $\xi_{\Phi^*} = \sigma_s \left(1 + \frac{\sigma_s^2}{\sigma_\epsilon^2}\right)$.

$\square$

## A.5 Proof of Proposition 4

The proof of proposition 4 is similar to the proof of proposition 3. First, let us prove the following lemma.

**Lemma 2.** Household $j$ chooses the full time work if and only if:

$$1 - F_{j,t} \geq \Psi^{B^g} \left(\Pi^*, r^*, \left\{L^i, L^i_{Ss}\right\}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)$$

where

$$1 - F_{j,t} = Pr\left(\Upsilon^{B^g}\left(\Pi^*, r^*, \left\{L^i, L^i_{Ss}\right\}^{i=\{F,P\}}, \bar{B}^g\right) \geq \Theta\left(\frac{s^*_{t,j} - s_t}{\sigma}\right) | s_{j,t}\right)$$

$$\Psi^{B^g} = 1 - \frac{\sum_{i \in \{Young, Middle, Old\}} \Lambda_i^{Gain}\left(\Pi^*, r^*, \{L^i, L^i_{Ss}\}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)}{\sum_{i \in \{Young, Middle, Old\}} \Lambda_i^{Cost}\left(\Pi^*, r^*, \{L^i, L^i_{Ss}\}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)},$$

Note that $\Lambda_i^{Gain}$ is the relative utility gain of "Full time work" in the i-aged and $\Lambda_i^{Cost}$ denotes the relative utility cost of "Full time work" in the i-aged.

Proof.

Recall that we assume that the fiscal expansion is temporary. So its effect comes into play only if the inflation targeting equilibrium is selected. So, I briefly review equilibrium behavior focusing on the inflation targeting equilibrium.

**Full-time work in Inflation Targeting Equilibrium** Recall that under flexible prices and the Taylor rule, the equilibrium inflation rate is $\Pi^*$ and the nominal interest rate is at the target level $1 + i^*$. If household j decides to choose to work full time, then the consumption of the middle-aged household is given as follows. Recall that the Euler equation between middle-aged and old-age households is given by:

$$\frac{1}{C^m_{t+1,j}} = \beta E_t \left[\frac{1}{C^o_{t+1}} \frac{1 + i_t}{\Pi_{t+1}}\right] \tag{A.49}$$

Substituting the budget constraints and the FOC of the firms into the Euler equation between middle-aged and old-age households yields the loan supply for household $j$.

$$-B^{m}_{t+1,j} = \left(\frac{\psi+\beta}{1+\beta}\right)\bar{B}^{g} + \left(\frac{\beta}{1+\beta}\right)\left(\alpha L_{t,j}\left(\frac{s^{*}_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}\right)L^{F} - \left(\frac{\beta}{1+\beta}\right)D \tag{A.50}$$

the middle-aged household's, consumption is given by:

$$C^{m}_{t+1,j} = \left(\frac{1-\psi}{1+\beta}\right)\bar{B}^{g} + \left(\frac{\alpha L_{t,j}\left(\frac{s^{*}_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^{F} - \frac{D}{1+\beta}. \tag{A.51}$$

Using the Euler equation and consumption in middle age, we obtain the consumption of households in old age.

$$\begin{aligned} C^{o}_{t+2,j} &= -T^{o}_{t+2,j} - \frac{1+i_{t+1}}{\Pi_{t+2}}B^{m}_{t+1,j} \\ &= \frac{1+i^{*}}{\Pi^{*}}\left(\frac{(1-\psi)\,\beta}{1+\beta}\right)\bar{B}^{g} + \frac{1+i^{*}}{\Pi^{*}}\left(\frac{\beta}{1+\beta}\right)\left(\frac{\alpha L_{t,j}\left(\frac{s^{*}_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^{F} \\ &\quad - \frac{1+i^{*}}{\Pi^{*}}\frac{\beta D}{1+\beta} \end{aligned} \tag{A.52}$$

Households of the young generation are assumed to be "Hand-to-mouth". So their consumption is given by

$$C^{y}_{t,j} = \frac{\Pi^{*}}{1+i^{*}}D \tag{A.53}$$

Using equations(A.51) to (A.53), we can derive the expected utility of full-time work with fiscal policy as:

$$\begin{aligned} E_{t}U_{j,t}\left(C_{j},L^{F}\right) = &\int_{-\infty}^{\Pi^{zero}} \left(u\left(C^{y}_{t,j}\right) + \beta\left\{u\left(C^{m}_{t+1,j}\right) - \varphi L_{t+1,j}\right\} + \beta^{2}u\left(C^{o}_{t+2,j}\right)\right)dF_{j,t}\Big|_{Ss\,eqm} \\ &+ \int_{\Pi^{zero}}^{\infty} \left(u\left(C^{y}_{t,j}\right) + \beta\left\{u\left(C^{m}_{t+1,j}\right) - \varphi L_{t+1,j}\right\} + \beta^{2}u\left(C^{o}_{t+2,j}\right)\right)dF_{j,t}\Big|_{IT\,eqm} \end{aligned}$$

We can rewrite the expected utility of full time work as follows.

$$E_{t}U_{j,t}\left(C_{j},L^{F}\right) = \Gamma^{F}\left(\bar{B}^{g}\right) + ln\frac{\Gamma^{F}_{Ss}}{\Gamma^{F}\left(\bar{B}^{g}\right)}F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero}\,|s_{j,t}\right) \tag{A.54}$$

where

$$\Gamma^F = ln\left(\frac{\Pi^*}{1+i^*}D\right) + \beta ln\left[\left(\frac{1-\psi}{1+\beta}\right)\bar{B}^g + \left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^F - \frac{D}{1+\beta}\right] - \beta\varphi L^F$$
$$+\beta^2 ln\left[\frac{1+i^*}{\Pi^*}\left(\left(\frac{(1-\psi)\beta}{1+\beta}\right)\bar{B}^g + \frac{\beta}{1+\beta}\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}L^F\right) - \frac{1+i^*}{\Pi^*}\frac{\beta D}{1+\beta}\right]$$

$$\Gamma^F_{Ss} = ln\left(\frac{\Pi^{Ss}}{1}D\right) + \beta ln\left[\left(\frac{(1-\gamma)\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma\left(\Pi^{Ss}\right)^{-1}}\right)L^F_{Ss} - \frac{D}{1+\beta}\right] - \beta\varphi L^F_{Ss}$$
$$+\beta^2 ln\left[\left(\frac{\beta}{1+\beta}\frac{(1-\gamma)\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma\left(\Pi^{Ss}\right)^{-1}}\right)\frac{L^F_{Ss}}{\Pi^{Ss}} - \frac{\beta}{1+\beta}\frac{\beta D}{\Pi^{Ss}}\right]$$

$\Gamma^F\left(\bar{B}^g\right)$ denotes the utility gain obtained when the inflation targeting equilibrium is realized with the choice of full-time work, and government debt additionally affects its value. $\Gamma^F_{Ss}$ denotes the utility gain when the secular stagnation equilibrium is realized with the choice of full time work, but the government debt does not affect its value in the secular stagnation equilibrium.

**Part-time Work in the Inflation Targeting Equilibrium** Using the Euler equation between middle-aged and old-age households, the loan supply for middle-aged household j is given by:

$$-B^m_{t+1,j} = \left(\frac{\psi+\beta}{1+\beta}\right)\bar{B}^g + \left(\frac{\beta\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \frac{\beta D}{1+\beta} \tag{A.55}$$

So, middle-age consumption is:

$$C^m_{t+1,j} = \left(\frac{1-\psi}{1+\beta}\right)\bar{B}^g + \left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \left(\frac{1}{1+\beta}\right)D \tag{A.56}$$

the old-age households' consumption is:

$$C^o_{t+2,j} = -T^o_{t+2,j} - \frac{1+i_{t+1}}{\Pi_{t+2}} B^m_{t+1,j}$$
$$= \frac{1+i^*}{\Pi^*}\left(\left(\frac{(1-\psi)\,\beta}{1+\beta}\right)\bar{B}^g + \left(\frac{\beta\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \left(\frac{\beta}{1+\beta}\right)D\right) \quad \text{(A.57)}$$

The young household's consumption is given by:

$$C^y_{t,j} = \frac{\Pi^*}{1+i^*}D \quad \text{(A.58)}$$

Using equations(A.5) to (A.5) instead of equations from (??) to (??), we can derive the expected utility of full-time work with fiscal policy as:

$$E_t U_{j,t}\left(C_j, L^P\right) = \int_{-\infty}^{\Pi^{zero}} \left(u\left(C^y_{t,j}\right) + \beta\left\{u\left(C^m_{t+1,j}\right) - \varphi L_{t+1,j}\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\Big|_{Ss\,eqm}$$
$$+ \int_{\Pi^{zero}}^{\infty} \left(u\left(C^y_{t,j}\right) + \beta\left\{u\left(C^m_{t+1,j}\right) - \varphi L^j_{t+1}\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right) dF_{j,t}\Big|_{IT\,eqm}$$

We can rewrite the expected utility of full time work as follows.

$$E_t U_{j,t}\left(C_j, L^P\right) = \Gamma^P\left(\bar{B}^g\right) + ln\frac{\Gamma^P_{Ss}}{\Gamma^P\left(\bar{B}^g\right)} F_{j,t}\left(\tilde{\Pi}_{j,t+1} < \Pi^{zero}\,|s_{j,t}\right) \quad \text{(A.59)}$$

where

$$\Gamma^P = ln\left(\frac{\Pi^*}{1+i^*}D\right) + \beta ln\left[\left(\frac{1-\psi}{1+\beta}\right)\bar{B}^g + \left(\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \frac{D}{1+\beta}\right] - \beta\varphi L^P$$
$$+ \beta^2 ln\left[\frac{1+i^*}{\Pi^*}\left(\left(\frac{(1-\psi)\,\beta}{1+\beta}\right)\bar{B}^g + \frac{\beta}{1+\beta}\frac{\alpha L_{t,j}\left(\frac{s^*_{t,j}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1+\beta}\right)L^P - \frac{1+i^*}{\Pi^*}\frac{\beta D}{1+\beta}\right]$$

$$\Gamma_{Ss}^{P} = ln\left(\frac{\Pi^{Ss}}{1}D\right) + \beta ln\left[\left(\frac{(1-\gamma)\,\alpha L_{t,j}\left(\frac{s_{t,j}^{*}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma\Pi^{-1}}\right)L_{Ss}^{F} - \frac{D}{1+\beta}\right] - \beta\varphi L_{Ss}^{P}$$

$$+\beta^{2} ln\left[\left(\frac{\beta}{1+\beta}\frac{(1-\gamma)\,\alpha L_{t,j}\left(\frac{s_{t,j}^{*}-s_{t,j}}{\sigma}\right)^{\alpha-1}}{1-\gamma\left(\Pi^{Ss}\right)^{-1}}\right)\frac{L_{Ss}^{P}}{\Pi^{Ss}} - \frac{\beta}{1+\beta}\frac{\beta D}{\Pi^{Ss}}\right]$$

$\Gamma^{P}\left(\bar{B}^{g}\right)$ denotes the utility gain obtained when the inflation targeting equilibrium is realized with the choice of part-time work, and government debt additionally affects its value. $\Gamma_{Ss}^{P}$ denotes the utility gain when the secular stagnation equilibrium is realized with the choice of part-time work, but the government debt does not affect its value in the secular stagnation equilibrium.

Since household $j$ chooses full time work if $E_t U_{j,t}\left(C_j, L^F\right) \geq E_t U_{j,t}\left(C_j, L^P\right)$, we can derive the following condition by using this relation. Household j chooses the full time work if and only if:

$$1 - F_{j,t} \geq \Psi_{j}^{B^g}\left(\Pi^{*}, r^{*}, L^{F}, L^{P}, L_{Ss}^{F}, L_{Ss}^{P}, \varphi, \bar{B}^{g}\right) \tag{A.60}$$

where

$$1 - F_{j,t} = Pr\left(\Upsilon\left(\Pi^{*}, r^{*}, \left\{L_{k}^{i}\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \bar{B}^{g}\right) \geq \Theta\left(\frac{s_{t,j}^{*}-s_{t,j}}{\sigma}\right)|s_{j,t}\right)$$

$$\Psi^{B^g} = 1 - \frac{\sum_{i\in\{Young,Middle,Old\}}\Lambda_{i}^{Gain}\left(\Pi^{*}, r^{*}, L^{F}, L^{P}, \varphi, \bar{B}^{g}\right)}{\sum_{i\in\{Young,Middle,Old\}}\Lambda_{i}^{Cost}\left(\Pi^{*}, r^{*}, L^{F}, L^{P}, L_{Ss}^{F}, L_{Ss}^{P}, \varphi, \bar{B}^{g}\right)},$$

where $\Lambda_{i}^{Gain}$ is the relative utility gain of "Full time work" in the i-aged and $\Lambda_{i}^{Cost}$ denotes the relative utility cost of "Full time work" in the i-aged. The relative utility gain of "Full time work" is given by:

$$\sum_{i\in\{Young,Middle,Old\}}\Lambda_{i}^{Gain} = \Gamma^{F}\left(\Pi^{*}, r^{*}, \left\{L_{k}^{i}\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi, \bar{B}^{g}\right) - \Gamma^{P}\left(\Pi^{*}, r^{*}, \left\{L_{k}^{i}\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi, \bar{B}^{g}\right) \tag{A.61}$$

The relative utility cost of "Full time work" is:

$$\sum_{i\in\{Young,Middle,Old\}} \Lambda_i^{Cost} = \left\{\Gamma^F\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi, \bar{B}^g\right) - \Gamma^P\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi, \bar{B}^g\right)\right\}$$

$$+\left\{\Gamma_{Ss}^P\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right) - \Gamma_{Ss}^F\left(\Pi^*, r^*, \{L_k^i\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi\right)\right\} \quad (A.62)$$

$\square$

Next, let us define the payoff function of household $j$:

$$V_{j,t}\left(s_{j,t}, \Phi, \bar{B}^g\right) \equiv E_t U_{j,t}\left(C_j, L^F, \bar{B}^g\right) - E_t U_{j,t}\left(C_j, L^P, \bar{B}^g\right). \quad (A.63)$$

In comparison to equation (A.25), payoff function is affected be the size of the government debt. This difference does not affect validity of three conditions necessary to have a unique equilibrium optimal strategy.

So,simply applying the result of proposition 1 for this indicates that the optimal threshold strategy is given by considering the marginal household whose private signal is equivalent to $s_t^*$ that is a solution to the following equality condition.

$$1 - F_{j,t} = Pr\left(\Phi^* \geq \Theta\left(\frac{s_t^* - s_t}{\sigma}\right) | s_{j,t} = s_t^*\right) = \Psi. \quad (A.64)$$

By solving this equality condition, we get:

$$s_t^*\left(\Pi^*, r^*, \left\{L_k^i\right\}_{k=\{IT,Ss\}}^{i=\{F,P\}}, \varphi, \bar{B}^g\right) = \xi_\Psi \Theta^{-1}\left(1 - \Psi^{B^g}\right) + \xi_{\Phi^*}\Theta^{-1}\left(\Phi^{*,B^g}\right) + r^* \quad (A.65)$$

where $\xi_\Psi = \sigma_s^2 \sigma_{s,\epsilon}$ and $\xi_{\Phi^*} = \sigma_s\left(1 + \frac{\sigma_s^2}{\sigma_\epsilon^2}\right)$.

$\square$

# B Alternative Specification: Binary Investment Choice in Human Capital

In this section, I discuss alternative specification of household's binary choice that allows us to uniquely pin down a equilibrium in the model of secular stagnation. One example way to assume that households choose whether they invest in human capital or not, instead of labor type choice. If households want to invest in human capital, then they need to pay some investment cost $\nu$. I assume that the outcome of investment in human capital is state-dependent as:

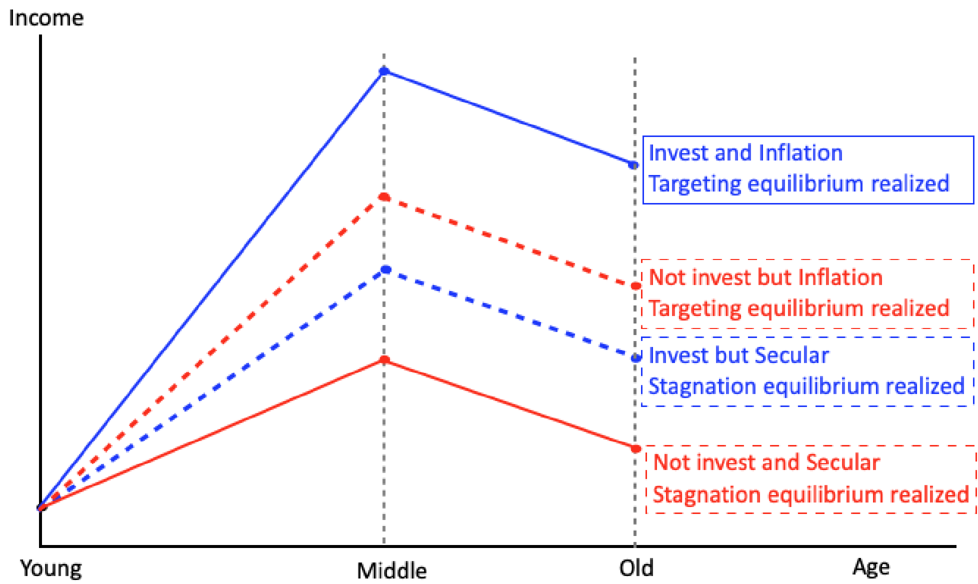


Figure 17: Life-cycle income and Human Capital Investment

$$L^I = \begin{cases} L^H \left(> L^{L'}\right) & \text{if Inflation Targeting equilibrium} \\ L^{L'} \left(> L^L\right) & \text{Otherwise} \end{cases}$$

$$L^{NI} = \begin{cases} L^L & \text{if Inflation Targeting equilibrium} \\ L^L & \text{Otherwise} \end{cases}$$

Figure 13 allows us to intuitively understand what state-dependent life-cycle income. $L^I$ denotes a labor opportunity if households choose to invest in human capital. Since both equilibria realizations are possible, outcomes of investment are state-dependent in the sense that $L^H > L^{L'} > L^L$. In this sense, as a blue solid line shows, if the inflation targeting equilibrium is realized, the net income level at the middle age becomes the highest because of the outcome of investment in human capital and also the income level at the old age is the highest because of the largest interest income if households invest in human capital. Though he invests in human capital, if the secular stagnation equilibrium is realized, as the blue dotted line shows, then increases in income will be suppressed since a reduction of labor opportunity and payment of investment cost.

On the other hand, if households decide not to invest in human capital, then labor opportunity is stably low in both equilibria realizations but they do not need to pay the investment cost. In the inflation targeting equilibrium, there is full employment and so even if they did not invest in human capital, their net income increase because they did not pay the investment cost, as a red dotted line shows.

We can then reformulate the model as follows. Household $j$ of a cohort born at time $t$ maximizes the following utility maximization problem.

$$\max_{C_{t,j}, C_{t,j+1}, C_{t,j+2}, i=I,NI} E_t \left\{ u\left(C^y_{t,j}\right) + \beta \left\{ u\left(C^m_{t+1,j}\right)\right\} + \beta^2 u\left(C^o_{t+2,j}\right)\right\}, \tag{A.66}$$

s.t.

$$C^y_{t,j} = B^y_{t+1,j}, \tag{A.67}$$

$$C^m_{t+1,j} = w_{t+1}L_{t+1,j} - \nu_j - T^m_{t+1} + B^m_{t+1,j} - \frac{1+i_t}{\Pi_{t+1}}B^y_{t+1,j} + z_{t+1}, \tag{A.68}$$

$$C^o_{t+2,j} = -T^o_{t+2,j} - \frac{1+i_{t+1}}{\Pi_{t+2}}B^m_{t+1,j}, \tag{A.69}$$

$$\frac{1+i_t}{E_t\Pi_{t+1}}B^i_{t,j} \leq D_t, i = \{y, m, o\}, \tag{A.70}$$

$$i_t \geq 0, \tag{A.71}$$

Since this formulation does not change the rest of the model structure, we can obtain the same results as the original discussion with a binary labor choice.