Optimal monetary and fiscal policy without fiscal backing *

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Abstract

This paper studies optimal monetary and fiscal policy when the Treasury is unable to provide fiscal backing to the central bank. The Treasury levies taxes and issues bonds. The central bank issues reserves for its liquidity value and pays interest expenses on reserves, but transfers from the Treasury are constrained. This lack of fiscal backing has implications for monetary and fiscal policy. On the monetary side, the central bank tolerates higher inflation in response to negative productivity shocks, because the central bank optimally chooses a lower nominal interest rate. On the fiscal side, the volatility of the distortionary tax on sales is higher over the business cycle, because the central bank retains its earnings and transfers less to the Treasury. Moreover, this difference due to the lack of fiscal backing is exacerbated by the level of reserves and the size of the shock.

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1 Introduction

The conventional assumption in macroeconomic models of monetary-fiscal policy is that the monetary and fiscal authorities are part of a "government" with a single (consolidated) budget constraint (Sargent and Wallace, 1981). The starting point of this paper is a departure from the consolidated government budget. Under the Federal Reserve Act, when income exceeds expenses, the Federal Reserve is required to transfer these profits to the Treasury. However, when expenses exceed income, the Treasury does not compensate the Federal Reserve for these losses. For example, from September 2022 until now, the Federal Reserve has been incurring significant interest expenses for large reserves (5% per annum on more than $3 trillion), and transfers from the Treasury have been zero. The Fed's cumulative net loss over this time period is $210 billion; 0.7% of GDP. If the Treasury had to offer fiscal backing for these losses, it would have to transfer the equivalent of 4.7% of the Treasury's annual tax revenue. These facts motivate my interest in situations in which the budgets of the monetary and fiscal authorities are unconsolidated in the sense that transfers from the central bank to the Treasury are restricted to be nonnegative (or negative but small in absolute value).

This paper studies the implications of the lack of fiscal backing for the central bank on the optimal monetary and fiscal policy. I develop the optimal monetary-fiscal policy problem under discretion where the government chooses policies to maximize household utility subject to implementable equilibrium conditions. The novelties are that 1) the government can issue two types of interest-bearing liabilities: bonds as in the conventional model, and central bank's reserves as a new ingredient, which provide liquidity; 2) transfers from the Treasury to the central bank are constrained by a lower bound.

Section 2 presents the model: Households consume, work, and trade bonds and reserves. Bonds and reserves provide liquidity value. Firms are modeled as in the standard New Keynesian framework with adjustment costs in changing prices, using labor for production, and facing cost-push and productivity shocks. Treasury finances public expenditures through bonds and distortionary taxes on firms' sales. In addition to this standard tax smoothing, remittances from the central bank help the Treasury finance public expenditures. The central bank supplies the reserves to the market and pays interest. A high nominal interest rate on large reserves leads to central bank's loss, making transfers from the Treasury optimal.

The goal of this research is to compare the optimal monetary and fiscal policies between the two models: a "consolidated model" and an "unconsolidated model". The former assumes no constraint on transfers; a fully consolidated government budget. The central bank is fiscally backed by the Treasury, allowing interest payments by the central bank to be financed through transfers from the Treasury. The second model imposes a constraint on transfers that prevents the central bank from reducing large reserves and paying interest expenses.

The first finding is that in response to a negative productivity shock, the optimal policy response for the central bank without fiscal backing is to tolerate higher inflation. The logic is as follows: a negative productivity shock raises marginal costs, leading to a higher inflation rate. In the consolidated model, the central bank raises the nominal interest rate to align the policy rate with the natural rate. As a result, both the output gap and the inflation response shrink to zero. In the unconsolidated model, however, the central bank does not raise the interest rate enough due to the lack of fiscal backing to cover interest expenses. Therefore, consumption falls less than in the consolidated model, and the inflation responds more.

The second finding is that in the case of a cost-push shock, the lack of *fiscal* backing constrains optimal *fiscal* policy. This is in sharp contrast to a productivity shock, where optimal *monetary* policy is constrained. The logic is as follows: A cost-push shock raises the marginal costs of labor. To bring the marginal product of labor closer to the higher marginal cost, the government lowers the sales tax rate. In the consolidated model, this reduction in tax revenue motivates the central bank to support the Treasury by issuing more reserves, raising funds from the private sector, and transferring them to the Treasury. These increased reserves must later be reduced by receiving funds from the Treasury. In the unconsolidated model, however, the central bank anticipates that it will not be able to reduce large reserves through remittances in the future, which limits its willingness to issue additional reserves. Consequently, the reduced central bank support to the Treasury in the unconsolidated model leads to a smaller tax cut, resulting in higher marginal costs and a higher inflation rate.

The third finding is that the higher inflation due to lack of fiscal backing is exacerbated by (i) larger shocks and (ii) higher initial reserves. When the shock is small or the initial reserves are low, the difference in inflation between the consolidated and unconsolidated models is small. This difference increases when the shock or reserves are large. For example, when initial reserves are

10% below the steady state, the inflation rate in the unconsolidated model is 0.01 percentage points higher than in the consolidated model. This difference increases to 0.08 percentage points when reserves are 10% higher than the steady state. The logic is, first, that transfers do not reach the lower bound with a smaller shock, but they do with a larger shock. Second, higher reserves exacerbate inflation because interest expenditures increase and additional funds are required from the Treasury. This nonlinear response inherits the nonlinearity of the inequality constraint on remittances.

Section 6 examines the business cycle properties of inflation (Chari, Christiano, and Kehoe, 1991; Schmitt-Grohé and Uribe, 2004). While inflation can reduce the real value of liabilities, it imposes costs due to price stickiness. I find that the lack of fiscal backing increases the volatility of the optimal inflation rate by 3%. Without fiscal backing, the government's ability to use monetary and fiscal policy to stabilize shocks is limited, resulting in a more volatile inflation rate. As for the average inflation rate, it is only 0.05 percentage points higher in the unconsolidated model than in the consolidated model. This slight increase is optimal because in the absence of remittances, the government reduces the real value of reserves through higher inflation. However, as Schmitt-Grohé and Uribe (2004) found, the benefits of reducing liabilities via inflation are much smaller than the costs of sticky prices. Thus, its impact on the average inflation rate is minimal.

Section 7 evaluates the welfare gain of fiscal backing. I find that the conditional welfare gain after a large shock is substantial. For example, following a shock that increases the wage mark-up by 10%, the welfare loss from the shock is 20% lower in the unconsolidated model compared to the consolidated model, as measured in consumption equivalence. Although the conditional welfare gain is significant, the unconditional welfare gain over the business cycle is small: less than 0.01% in terms of consumption equivalence, as is usually the case in Lucas (1987). While fiscal backing impacts the volatility of variables, it has a minimal effect on the mean, leading to a quantitatively small welfare gain over the business cycle.

I also explore the implications of the model for practical policy on remittances. In practice, central banks such as the Bank of Japan and the Bank of England have a policy of retaining a portion of their profits rather than remitting the full amount to the Treasury. My model predicts that the central bank has an incentive to reduce remittances in the unconsolidated model than in the consolidated model. This incentive to reduce remittances is consistent with the policy implemented in practice. In the unconsolidated model, the central bank expects that larger reserves due to higher remittances will not be reduced by future transfers from the Treasury, leading it to reduce current remittances in the unconsolidated model.

Related literature First, this paper is related to the large literature on monetary-fiscal policy interactions (Sargent and Wallace (1981); Leeper (1991); Sims (1994); Woodford (2001), among others). Sargent and Wallace (1981) is the seminal paper that highlights the consolidated government budget, showing that the Treasury's budget affects the central bank's ability to control inflation. Since then, the assumption of a consolidated government budget constraint has become a common framework in the literature. This paper departs from this assumption, motivated by the recent fact that transfers from the Treasury to the central bank have been zero since 2022.

Some recent studies have considered the unconsolidated budgets of the Treasury and the central bank. These papers study various issues, including central bank solvency (Hall and Reis, 2015; Bassetto and Messer, 2013; Bassetto and Sargent, 2020; Niwa, 2024), price determinacy (Del Negro and Sims, 2015; Benigno and Nisticò, 2020), the value of fiat money (Bolt, Frost, Shin, and Wierts, 2024), the effectiveness of central bank asset purchase policies (Benigno, 2020), the impact of helicopter drops (Amador and Bianchi, 2023), monetary union with separate budget constraints of each member country (Bassetto and Caracciolo, 2021), the projected path of central bank net worth and earnings (Christensen, Lopez, and Rudebusch, 2015; Carpenter, Ihrig, Klee, Quinn, and Boote, 2018; Cavallo, Del Negro, Frame, Grasing, Malin, and Rosa, 2023), and the empirical relationship between central bank losses or negative net worth and monetary policy conduct (Stella, 2005, 2008; Goncharov, Ioannidou, and Schmalz, 2023).

This paper is the first to focus on the lack of fiscal backing for the central bank. Moreover, this paper addresses the normative implications, while the existing literature has examined the positive implications. A novel finding of this paper is that the lack of fiscal backing constrains optimal fiscal policy, although it is often simplified as a lump sum tax and is therefore less studied in the literature (Del Negro and Sims, 2015; Benigno and Nisticò, 2020; Benigno, 2020; Amador and Bianchi, 2023).

Previous literature has focused on estimating how much seigniorage would be required to balance the central bank's budget, finding that the required inflation rate is substantial (Del Negro and Sims, 2015). In contrast, this paper examines a situation in which the government can optimally choose not only inflation, but also the nominal interest rate, asset purchase policy, and transfers to satisfy central bank's budget. The results show that while the welfare cost of not having fiscal support is small during a typical business cycle, it becomes significantly larger in the face of a larger shock or larger reserves. This nonlinearity inherits the nonlinear property of the inequality constraint on remittances.

Moreover, the literature on unconsolidated government lacks a comprehensive analysis of distortionary taxes within the optimal policy framework. While existing studies keep the fiscal side simple, such as lump-sum tax (Del Negro and Sims, 2015; Benigno and Nisticò, 2020; Benigno, 2020; Amador and Bianchi, 2023), this paper models the tax smoothing motive by introducing distortionary taxes. I show that central bank policy influences optimal fiscal policy, such as taxation and bond issuance. In this respect, this paper follows the conclusions of the traditional monetary-fiscal policy literature, which emphasizes that monetary and fiscal policies constrain each other (Sargent and Wallace (1981); Leeper (1991), and more recently Bianchi (2013); Bianchi and Ilut (2017); Bassetto and Sargent (2020)).

The closest papers are Berriel and Bhattarai (2009) and Amador and Bianchi (2023). Berriel and Bhattarai (2009) study optimal policy in a New Keynesian framework. Berriel and Bhattarai (2009) assume that the government's objective function directly includes central bank's losses. This paper does not directly incorporate the welfare loss from a destabilized central bank net worth. Instead, the government in my model is concerned with the central bank's balance sheet because the price of reserves must be lower to incentivize households to hold excess reserves, but lower prices affecting the real allocation are not always the optimal policy. Additionally, the focus of Berriel and Bhattarai (2009) is not a constraint on transfers from the Treasury to the central bank.

Amador and Bianchi (2023) is the only other paper that models the constraint on transfers from the Treasury to the central bank but it focuses on helicopter drops during a liquidity trap from a positive perspective. It finds that helicopter drops can be an effective stabilization policy in a liquidity trap when the central bank faces constraints on resource allocation. In contrast, my paper approaches monetary-fiscal policy from a normative perspective.

Second, this paper is also related to the literature on central bank balance sheet policies. The literature has extensively examined central banks' asset policies.[1] In contrast, this paper emphasizes the liability side.

[1] For theoretical papers, see Gertler and Karadi (2011); Curdia and Woodford (2011), among others; for empirical studies, see Krishnamurthy and Vissing-Jorgensen (2011); D'Amico and King (2013), among others.

In terms of liability policies, this paper also relates to studies on the macroeconomic implications of large reserves (Poole, 1968; Cochrane, 2014; Reis, 2016; Ennis, 2018; Williamson, 2019; Arce, Nuno, Thaler, and Thomas, 2020; Bianchi and Bigio, 2022). Ennis (2018) finds that price level indeterminacy may arise when banks hold excess reserves. Reis (2016) finds that only the interest on reserves and not the size of the balance sheet has an effect on inflation. Arce et al. (2020) proposes a New Keynesian equilibrium model to compare the pre-crisis lean balance sheet regime and the post-crisis floor system with a large balance sheet. Bianchi and Bigio (2022) study the credit channel of monetary policy in a model where banks manage liquidity facing frictions in the interbank market.

The contribution of this paper lies in examining the implications of excess reserves within the standard New Keynesian framework. Unlike papers which delve into the microstructure of interbank lending and payments (Afonso and Lagos, 2015; Lagos and Navarro, 2023), this paper simplifies the demand for reserves, but models in detail a general equilibrium New Keynesian model, with a particular focus on (i) fiscal policy, including distortionary taxes and bonds, and (ii) transfers between the Treasury and the central bank. While much of the literature on large reserves overlooks fiscal considerations, this paper emphasizes the importance of the central bank's fiscal backing for fiscal policy.

The closest works in this literature are Berentsen, Marchesiani, and Waller (2014) and Berentsen, Kraenzlin, and Müller (2018), as both highlight that the implications of excess reserves critically depend on the level of fiscal backing provided to the central bank. Berentsen, Kraenzlin, and Müller (2018) finds that when the central bank lacks fiscal backing, reducing reserves is the optimal policy. They compare the floor system and the channel system from a normative perspective and conclude that the floor system is superior when there is fiscal backing. While Berentsen, Kraenzlin, and Müller (2018) explores regimes of full fiscal backing and no fiscal backing, this paper assumes a lack of *optimal* fiscal backing, where there is no constraint on transfers to the Treasury, but transfers to the central bank are constrained. This assumption is more closely aligned with the current policy of the Federal Reserve.

Finally, this paper contributes to the literature on optimal monetary and fiscal policy. The literature focuses on (i) the optimal volatility and persistence of inflation (Chari, Christiano, and Kehoe, 1991; Schmitt-Grohé and Uribe, 2004; Siu, 2004; Chugh, 2006, 2007; Correia, Nicolini, and

Teles, 2008), (ii) the temptation to erode government nominal liabilities through inflation (Schmitt-Grohé and Uribe, 2004; Niemann, Pichler, and Sorger, 2013; Faraglia, Marcet, Oikonomou, and Scott, 2013; Leeper, Leith, and Liu, 2021), and (iii) the stochastic behavior of debt (Barro, 1979; Lucas and Stokey, 1983; Aiyagari, Marcet, Sargent, and Seppälä, 2002; Schmitt-Grohé and Uribe, 2004; Benigno and Woodford, 2003), particularly examining whether debt follows a random-walk property regardless of the shock process.

The existing literature abstracts from the issues of fiscal backing and reserves, which are another form of interest-bearing government liabilities. This paper incorporates both and examines whether the conventional results in the literature change. The findings indicate that, without fiscal backing: (i) the optimal volatility of the inflation rate is higher, (ii) the government uses inflation to erode reserves, and (iii) the stock of reserves becomes more persistent while the stock of bonds is less persistent.

Layout The rest of the paper is organized as follows. Section 2 describes the model and defines the private sector equilibrium. Section 3 formulates the government's problem. Section 4 discusses parametrization and the solution method. Sections 5 and 6 show numerical results. Section 7 shows the welfare analysis. Section 8 concludes.

2 Model

This section describes the model and defines the equilibrium. The model is a standard New Keynesian framework incorporating both the Treasury's and the central bank's budget constraints. Before presenting the model, I define key terminology. "Government" refers to an agent that chooses both monetary and fiscal policies, with the objective of maximizing household utility. The problem for the government is defined in section 3. The "Treasury" and "central bank" can be considered to be subsets of the government, each with its own budget constraint. The Treasury collects taxes, provides public expenditure, and issues liabilities: "bonds". The central bank issues another form of liability: "reserves". These two types of liabilities, reserves and bonds, are distinguished by issuers.

2.1 Households

Representative households choose consumption, labor supply, and the holdings of one-period risk-free nominal reserves and long-duration risk free nominal bonds to maximize the expected discounted sum of the future period utilities. The households get a convenience yield from trading both reserves and bonds. Households maximize

$$\max_{\{C_t,N_t,D_t,B_t^H\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\nu} N_t^{1+\nu} + \frac{\chi_1}{1-\gamma_1} \left(Q_t^C \frac{D_t}{P_t} \right)^{1-\gamma_1} + \frac{\chi_2}{1-\gamma_2} \left(Q_t^T \frac{B_t^H}{P_t} \right)^{1-\gamma_2} \right], \tag{1}$$

where C_t is a Dixit-Stiglitz aggregate consumption of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}} di \right]^{\frac{\theta-1}{\theta}} \tag{2}$$

with an elasticity of substitution equal to $\theta > 1$. Also, N_t is the labor supply, Q^C is the price of the reserves, D is the quantity of reserves in nominal, Q^T is the price of bonds, B^H is the quantity of bonds in nominal held by the households, P_t is the price of consumption goods.

Each differentiated good is supplied by a monopolistically competitive producer. The goods in each industry i are produced using labor that is specific to that industry. The representative households supply all types of labor as

$$N_t = \int_0^1 N_t(i) di. \tag{3}$$

Households receive a convenience yield from holding reserves and bonds. The convenience yield from reserves can be interpreted as the expected savings in transaction costs for commercial banks; they do not need to sell loans to manage deposit outflows. Assuming that households own these commercial banks, they value the reduction in transaction costs that benefits the households. An example of the convenience yield from holding bonds is that Treasury securities lower the costs associated with transacting in less liquid securities, such as corporate bonds.[2] The convenience yield terms enable the model to match the steady-state values for both reserves and bonds, which

[2]For more discussion of convenience yield, see Lopez-Salido and Vissing-Jorgensen (2023) for reserves and Krishnamurthy and Vissing-Jorgensen (2012) for bonds among others.

critically influences interest expenses and is essential for quantitative analysis.

The budget constraint is

$$P_t C_t + Q_t^C D_t + Q_t^T B_t^H = D_{t-1} + (1 + \rho^T Q_t^T) B_{t-1}^H + w_t N_t + P_t \Phi_t, \tag{4}$$

where Φ_t is the profit from the goods producers. In period t, the households buy bonds B^H at price Q_t^b that pay a declining coupon of ρ^{Tj} dollars in period $j+1$, where $0 \leq \rho \leq \beta^{-1}$ (Woodford, 2001). A measure of the duration of the bond is given by $(1 - \beta\rho^T)^{-1}$. The households bring nominal bonds of $(1 + \rho^T Q_t^b) B_{t-1}^H$ into period t.

The prices of reserves, Q_t^C, and bonds , Q_t^T, are the policy instrument for the government.

2.2 Firms

There is a continuum of goods producers indexed by $i \in [0, 1]$. Firms use labor as an input and produce imperfectly substitutable goods according to a linear production function. Each firm sets the price of its own good to maximize the expected discounted sum of future profits.

Firms i face three constraints. A linear production function,

$$y_t(i) = A_t N_t(i), \tag{5}$$

where A_t is the exogenous aggregate productivity, a demand curve of the form

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}, \tag{6}$$

where the aggregate demand is defined as

$$y_t(i) = \left(\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{7}$$

and quadratic adjustment costs in changing prices, as in Rotemberg (1982), that are defined as

$$\frac{\varphi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}-1\right)^2 P_t Y_t, \tag{8}$$

where φ is the degree of nominal price rigidity.

Firm i sets its price $p_t(i)$ in period t to maximize the expected discounted sum of future profits,

$$\max_{p_t(i)} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1-\tau_t) p_t(i) y_t(i) - \mu_t^w w_t N_t(i) - \frac{\varphi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}-1\right)^2 P_t Y_t \right), \tag{9}$$

where Λ_t is the stochastic discount factor given by

$$\Lambda_t \equiv \frac{C_t^{-\sigma}}{P_t}. \tag{10}$$

The stochastic discount factor measures the marginal value of an additional unit of profits to the household. τ_t is the tax on sales levied by the Treasury. μ_t^w captures the shock to wage mark-up. One possible interpretation of the wage markup shock is that it represents the bargaining power of labor unions. There is no heterogeneity in the time-zero prices across firms. That is $p_{-1}(i) = P_{-1}$ for a given constant P_{-1}.

The cost-push shock and productivity follow an AR(1) process.

$$\mu_t^w = \bar{\mu}^w + \rho^w(\mu_{t-1}^w - \bar{\mu}^w) + \varepsilon_t^w. \tag{11}$$

$$A_t = \bar{A} + \rho^a(A_{t-1} - \bar{A}) + \varepsilon_t^a, \tag{12}$$

where ε_t^w and ε_t^a are shocks and are distributed with mean zero and standard deviation σ_w and σ_a. Parameters $\bar{\mu}^w$ and $\bar{A}$ are the mean value of wage mark-up and productivity that are equal to one. The first-order condition implies the non-linear Phillips curve

$$N_t C_t^{-\sigma}\left(\varphi(\pi_t - 1)\pi_t A_t - (1-\theta)(1-\tau_t)A_t - \theta\mu_t^w N_t^{\nu} C_t^{\sigma}\right) = \beta E_t\left[A_{t+1} N_{t+1} C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right]. \tag{13}$$

2.3 Treasury and central bank

The government's problem is introduced in the next section. This section describes three constraints that the government faces: Treasury's budget, central bank's budget, and a constraint on transfers from Treasury to central bank.

The Treasury finances exogenous, stochastic, and useless public expenditures G_t. Aggregate public expenditure is

$$G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{14}$$

such that the government demand for goods is given by

$$G_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} G_t \tag{15}$$

The exogenous public expenditure follows an AR(1) process,

$$G_t = \bar{G} + \rho^g (G_{t-1} - \bar{G}^w) + \varepsilon_t^g. \tag{16}$$

The expenditures are financed by a sales tax, issuance of bonds, and a remittance from the central bank. The Treasury's budget constraint is given by

$$Q_t^T B_t^T + \tau_t P_t Y_t + P_t H_t = P_t G_t + (1 + \rho^T Q_t^T) B_{t-1}^T. \tag{17}$$

$\tau_t Y_t$ is the distortionary tax on sales. H_t is the remittance from central bank to Treasury. A positive H_t implies that the central bank transfers to the Treasury. B^T is the total supply of bonds. The bonds are held by households and central bank. They are denoted by B_t^H and B_t^C.

The central bank's role is to supply reserves that households appreciate. In addition to reserves, the central bank also trades bonds. The central bank budget constraint is

$$Q_t^C D_t + (1 + \rho^T Q_t^T) B_{t-1}^C = D_{t-1} + Q_t^T B_t^C + P_t H_t, \tag{18}$$

where D_t are reserves and H_t is the remittance from central bank to Treasury. B_t^C are bonds held the central bank.

The left-hand side of the budget constraint represents the income for the central bank. The central bank generates revenue by issuing more reserves ($Q_t^C D_t$), through income gains from bonds holdings (B_{t-1}^C), and through capital gains from bonds holdings ($\rho^T Q_t^T B_{t-1}^C$). As bonds are long-duration bonds, revenues from them are separated into income gain and capital gain. This models a capital loss from central bank's asset holding. When the central bank raises the nominal interest rate, the price of reserves (Q_t^C) falls, leading to a capital loss.

The right-hand side of the budget constraint represents expenditures for central bank. Expenditures consist of the redemption of reserves (D_{t-1}), purchase of assets ($Q_t^T B_t^C$), and remittance to Treasury ($P_t H_t$). A positive H_t means that Treasury receives and central bank sends. If the nominal interest rate increases (the price of reserves drops), $Q_t^C D_t$ drops and requires remittance from Treasury (decline in $P_t H_t$).

Note that this budget constraint is based on mark-to-market valuation, meaning that the price of long-duration bonds reflects their market price rather than their book value. The model assumes that the central bank trades assets every period, which contrasts with holding assets until maturity, where market prices do not affect the central bank's budget. In practice, while the Bank of England adopts mark-to-market valuation for central bank assets, the Federal Reserve and other central banks, including the Bank of Japan, use book-value accounting to calculate asset values. This paper follows a mark-to-market valuation.

For the central bank's asset purchase policy, B_t^C, it is assumed that a constant fraction of bonds is held by the central bank.

$$B_t^C = \alpha B_t^T, \tag{19}$$

where α is a parameter, and B^T is the total supply of bonds. This assumption helps reduce the size of the state space. As I will describe in the next section, the model has four state variables: exogenous shocks, reserves, total supply of bonds, and bonds held by the central bank. This complexity introduces computational difficulties. By assuming (19), the model is reduced to three state variables, allowing for a global solution.

The justification for this assumption is that the focus of this paper is on the central bank's liability side, not its asset side. While the central bank manages both liabilities and assets, this paper emphasizes the liability side and endogenizes the liability policy. In contrast, one of the main objectives of asset purchase policies is to stabilize financial markets (Gertler and Karadi, 2011), which is beyond the scope of this model. Therefore, I treat the asset policy as exogenous.

Under this assumption, the central bank's asset purchases remain smooth over time. A sudden change in the amount of asset purchases could destabilize the asset market.[3] This assumption aligns with the central bank's objective of minimizing disruptions in the financial market.

Finally, the most important feature of this model is the constraint on optimal transfers from the Treasury to the central bank. This constraint is one-sided: While the Treasury can receive as many resources as needed from the central bank, Treasury is limited in its ability to transfer to central bank. The inequality constraint on the remittance is

$$H_t \geq H^*, \tag{20}$$

where H^* is an exogenous parameter. The model aligns with the Federal Reserve's actual policy. Transfers from the central bank to the Treasury have a lower bound of zero. When the central bank's costs exceed its earnings, a so-called "deferred asset" is accumulated. A deferred asset is a negative liability that represents the cumulative value of the shortfall in earnings. Once the Fed returns to positive net income, it will use those earnings to pay down the deferred asset. During this period, the Fed does not transfer funds to the Treasury, even if it earns profits; those profits are used to reduce the deferred asset. Once the deferred asset reaches zero, the Fed will resume sending remittances to the Treasury. As of October 2024, the deferred asset stands at \$210 billion, approximately 0.7% of annual GDP.

The motivation for this constraint stems from "political reasons" from the Treasury's perspective. To cover the central bank's losses, the Treasury would need to raise taxes or issue additional debt, both of which are politically costly. Moreover, the central bank pays interest expenses to commercial banks, which are not perfectly competitive. If the Treasury decides to cover these interest expenses through taxation, it effectively redistributes income from households, including poorer ones, to

[3] See the literature on the Taper Tantrum (Eichengreen and Gupta, 2015) for a discussion of the costs associated with sudden changes in the central bank's asset purchase policy.

commercial banks. This type of redistribution is difficult for the Treasury to justify politically.

From the central bank's perspective, requesting capital injections could help maintain a strong financial position and avoid a weakened balance sheet. However, there is concern that relying on parliamentary decisions for funding could threaten central bank independence.[4] A motivation for this constraint is to maintain central bank independence in the sense that parliamentary decisions do not impact the conduct of monetary policy.

The literature on the unconsolidated government budget already models remittance (Bassetto and Messer, 2013; Del Negro and Sims, 2015; Benigno and Nisticò, 2020; Benigno, 2020). However, no model has ever studied the constraint on remittance.

Before 2022, when the Federal Reserve began incurring losses, it transferred funds to the Treasury due to profits from its asset holdings. Since the central bank's assets are primarily long-duration bonds with term premia, the income from holding these bonds exceeded expenses on interest paid on short-duration reserves. These profits are substantial; in practice, the transfers from the Federal Reserve finances 2% of federal expenditure on average between 2009 and 2021. Details on data related to remittances are provided in Appendix A. I discuss the fiscal implications of central bank support to the Treasury in this model.

2.4 Market Clearing

Clearing of the goods i market requires the following.

$$y_t(i) = C_t(i) + G_t(i) + \frac{\varphi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - 1\right)^2 y_t(i) \tag{21}$$

$$Y_t(i) = A_t N_t(i). \tag{22}$$

In a symmetric equilibrium,

$$Y_t = C_t + G_t + \frac{\varphi}{2}(\pi_t - 1)^2 Y_t \tag{23}$$

$$Y_t = A_t N_t. \tag{24}$$

[4]For actual comments from central bankers, see Floden (2022), where the Deputy Governor of the Riksbank expresses these concerns

The clearing conditions for bonds are

$$B_t^H + B_t^C = B_t^T, \tag{25}$$

where bonds are held by households and the central bank. Note that I impose an assumption that central bank holds α fraction of total supply of bonds.

$$B_t^H = (1-\alpha)B_t^T \tag{26}$$

$$B_t^C = \alpha B_t^T \tag{27}$$

There is also market clearing in reserves markets where reserves held by households evolves according to the central bank's budget constraint.

The price index is defined as

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{28}$$

$$\pi_t \equiv \frac{P_t}{P_{t-1}}. \tag{29}$$

2.5 Equilibrium

Given the distribution of initial prices, $p_{-1}(i)$, stochastic processes, and the initial level of liabilities B_{-1}, D_{-1}, a competitive equilibrium of this economy consists of allocations $\{C_t, N_t, Y_{i,t}\}_{t=0}^{\infty}$, prices $\{p_t(i)\}_{t=0}^{\infty}$, and policy instruments $\{Q^C, Q^T, \tau, B_t, D_t\}_{t=0}^{\infty}$, such that

- Allocations solve the problem of the household given prices and policies.
- $p_t(i)$ solves the problem of firm i.
- $p_t(i) = p_t(j)$.
- The Treasury, central bank's budgets, and a constraint on remittance are satisfied.
- Goods markets, reserve market, and bond market clear.

Implementable equilibrium conditions are summarized in section 3.

3 The Optimal Policy Problem under discretion

This section outlines the optimal policy problem under discretion. The government makes decisions sequentially. Each period t, the government maximizes the utility of households by choosing $\{C_t, N_t, \Pi_t, Q_t^C, Q_t^T, b_t, d_t, \tau_t, H_t\}$ and the Lagrangian multipliers associated with the equilibrium conditions, taking as given the next period value and next policy functions. I define real value of reserves and bonds, $b_t \equiv \frac{B_t}{P_t}$ and $d_t \equiv \frac{D_t}{P_t}$.

$$V_t(s_t) = \underset{a_t}{max} \quad \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\nu} N_t^{1+\nu} + \frac{\chi_1}{1-\gamma_1} (Q_t^C d_t)^{1-\gamma_1} + \frac{\chi_2}{1-\gamma_2} ((1-\alpha) Q_t^T b_t)^{1-\gamma_2} + \beta E_t V_{t+1}(s_{t+1}), \tag{30}$$

where $s_t \equiv \{\mu_t^w, A_t, G_t, b_{t-1}, b_{t-1}\}$ and $a_t \equiv \{C_t, N_t, \Pi_t, Q_t^C, Q_t^T, b_t, d_t, \tau_t, H_t\}$.

Equilibrium conditions are

$$\text{(Euler for reserves)} \quad C_t^{-\sigma} Q_t^C = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] + \chi_1 (Q_t^C d_t)^{-\gamma_1} Q_t^C \tag{31}$$

$$\text{(Euler for Treasury bond)} \quad C_t^{-\sigma} Q_t^T = \beta E_t \left[C_{t+1}^{-\sigma} \frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}} \right] + \chi_2 ((1-\alpha) Q_t^T b_t)^{-\gamma_2} Q_t^T \tag{32}$$

$$\text{(Firm FOC)} \quad N_t C_t^{-\sigma} \left(\varphi(\pi_t - 1)\pi_t A_t - (1-\theta)(1-\tau_t) A_t - \theta \mu_t^w N_t^\nu C_t^\sigma \right) = \beta E_t \left[A_{t+1} N_{t+1} C_{t+1}^{-\sigma} \varphi(\pi_{t+1} - 1)\pi_{t+1} \right]. \tag{33}$$

$$\text{(Market Clearing)} \quad A_t N_t = C_t + \frac{\varphi}{2}(\pi_t - 1)^2 A_t N_t + G_t \tag{34}$$

$$\text{(Treasury Budget)} \quad Q_t^T b_t + \tau_t Y_t + H_t = G_t + (1 + \rho^T Q_t^T) \frac{b_{t-1}}{\pi_t}. \tag{35}$$

$$\text{(central bank's Budget)} \quad Q_t^C d_t + (1 + \rho^T Q_t^T) \alpha \frac{b_{t-1}}{\pi_t} = \frac{d_{t-1}}{\pi_t} + Q_t^T \alpha b_t + H_t \tag{36}$$

$$\text{(Remittance)} \quad H_t \geq H^* \tag{37}$$

$$\text{(Shocks)} \quad \mu_t^w = \bar{\mu}^w + \rho^w (\mu_{t-1}^w - \bar{\mu}^w) + \varepsilon_t^w. \tag{38}$$

$$A_t = \bar{A} + \rho^a(A_{t-1} - \bar{A}) + \varepsilon_t^a. \tag{39}$$

$$G_t = \bar{G} + \rho^g(G_{t-1} - \bar{G}) + \varepsilon_t^g. \tag{40}$$

The discretionary equilibrium is determined by the first-order conditions, expectations that are consistent with policy functions, and the exogenous process for shocks. The solution is the time-invariant Markov-perfect equilibrium policy rules that map states $\{\mu_t^w, A_t, G_t, b_{t-1}, d_{t-1}\}$ to optimal decisions for $\{C_t, N_t, \Pi_t, Q_t^T, Q_t^C, b_t, d_t, \tau_t, H_t\}$.

The government is constrained to behave in a time-consistent manner. The economic agents anticipate that the government faces this constraint and form expectations. However, the government can change the expectations by choosing state variables, bonds and reserves. In rational expectation equilibrium, the expectations are formed based on the mapping that map endogenous variables to the state-space.

I impose that the central bank holds an α fraction of the total supply of bonds. The government only chooses the total supply of bonds; it does not need to separately decide on the total supply of bonds and the portion held by the central bank. The allocation of bonds between households and the central bank is determined by equations (26) and (27). The state variables are shocks, reserves, and the total supply of bonds.

Define the consolidated model and the unconsolidated model I define two models; the consolidated model and the unconsolidated model. The unconsolidated model is defined as the model where the government maximizes the utility of households subject to (31) - (40). The remittance from the Treasury to the central bank is constrained, so central bank's and Treasury's budgets are not fully consolidated.

"The consolidated model" is defined as the model where H^* is low enough that the equation (37) never binds. Later, H^* is set to be $-\infty$ in quantitative exercises. The consolidated government budget is given by substituting H_t in equation (35) into equation (36).

$$Q_t^C d_t + (1-\alpha)Q_t^T b_t + \tau_t Y_t = \frac{d_{t-1}}{\pi_t} + (1 + \rho^T Q_t^T)(1-\alpha)\frac{b_{t-1}}{\pi_t} + G_t. \tag{41}$$

In the consolidated model, the government maximizes household utility subject to equations

(31)-(34), (38)-(40), and (41). The government faces only the consolidated government budget constraint, instead of separate Treasury's and central bank's budget, and the constraint on transfers. Optimal transfers between the central bank and the Treasury are always achievable. It is important to note that the consolidated model represents the optimal monetary-fiscal policy problem within a standard New Keynesian framework (Benigno and Woodford, 2003; Schmitt-Grohé and Uribe, 2004). The key differences from the literature and the consolidated model in this paper are as follows: (1) while my model considers optimal policy under discretion, those papers analyze policy under commitment; (2) my model includes reserves as interest-bearing liabilities, whereas those papers include money.

4 Solution method and calibration

4.1 Solution method

Two reasons necessitate the use of global solution methods. The first is the presence of an occasionally binding constraint on remittance. Stochastic shocks cause the remittance to occasionally bind the lower bound. The policy problem is highly nonlinear, and a perturbation technique is not suitable. Second, under discretionary policies, the model's steady-state which local dynamics should be approximated around depends on the derivative of expectations with respect to reserves and government liabilities. This is because the state variables in the model include reserves and government liabilities, and the expectations in Euler equations and a New Keynesian Phillips Curve are evaluated at each reserves and bonds. This derivative of expectations with respect to reserves and bond is endogenously determined as a part of model solution, making the steady-state a priori unknown. This is a common approach in the literature (Niemann and Pichler, 2011; Niemann, Pichler, and Sorger, 2013; Leeper, Leith, and Liu, 2021).

The model is solved using the time-iteration method of Coleman (1991). I begin by guessing initial policy functions and then compute the associated expectations. Assuming that the guessed policy functions apply to the next period, I solve the first-order necessary conditions of the government's problem on a discrete set of grids to find the policy functions for the current period. This process is repeated until the policy functions converge, such that the difference between today's and tomorrow's policy functions becomes arbitrarily small. Details are described in Appendix C.

4.2 Calibration

I calibrate my model to the US economy. The time unit is a quarter. The calibration table is summarized in table 1. Parameters that are commonly used in the New Keynesian literature are taken from the literature. I set $\beta = 0.995$, which implies a 2% annual real interest rate. The intertemporal elasticity of substitution is set to $\frac{1}{2}$, a value commonly found in the literature. The Frisch elasticity of labor supply is set at $\frac{1}{7}$, while the elasticity of substitution between intermediate goods is 10, implying a monopolistic markup of 10%. The price adjustment cost parameter is set at 100. The coupon decay parameter, $\rho^T = 0.95$, corresponds to an average debt maturity of around 5 years, consistent with U.S. data (see Table 2 in Leeper and Zhou (2021)).

Other parameters are calibrated in my model. I describe my calibration strategy. The parameters, χ_1, χ_2, that decide the convenience of reserves and bonds, are chosen to match the steady-state value of reserves and bonds. Steady-state reserves are matched to 15% of GDP. The recent level of government liabilities in the data is 120% of GDP, and the convenience yield is not enough to match the high level of government liabilities. I match the steady-state bonds as the 30% of GDP that is the average of 1970-2020 in US data.

The fraction of bonds held by the central bank, α, is calibrated at 0.4 to match the central bank's asset-to-liability ratio. A higher α leads to a higher asset-to-liability ratio. In the data, the Federal Reserve's asset-to-liability ratio is approximately 1. Setting α at 0.4 in the model achieves an asset-to-liability ratio of one. In practice, the observed α from the data is lower than the model's value (peaking at 21% in 2022).[5] This discrepancy between model and data arises because, in reality, the Federal Reserve holds assets beyond bonds, such as Mortgage-Backed Securities and government-sponsored enterprise fixed assets, which are not included in this model. If I set $\alpha = 0.21$ to match the data and align the model's reserve size with observed values, the central bank's assets would fall significantly below its liabilities. This mismatch would alter the quantitative results, particularly since the model needs to capture both capital loss on the asset side and the interest expense on reserves. To avoid this issue, I set $\alpha = 0.4$, ensuring that the asset-to-liability ratio remains approximately 1, consistent with the data.

[5] I show the ratio of U.S. Treasury Securities held by the Federal Reserves to Federal Debt. The data is from FRED.

Table 1: Calibration

Variable	Value	Description	Target	Model	Data
β	0.995	Discount factor	-	-	-
σ	2	Risk aversion	-	-	-
ν	7	Frisch Elasticity	Frisch Elasticity	1/7	-
θ	10	Elasticity of substitution among goods	Markup	7%	-
ϕ	100	Price adjustment cost	Slope of NKPC	0.05	-
ρ^T	0.95	Duration of Treasury	Average Maturity	5 years	5 years
χ_1	0.0012	Utility from reserve	Steady-state reserves	15% of GDP	15% of GDP
χ_2	0.002	Utility from Treasury bond	Steady-state bonds	30% of GDP	120% of GDP
γ_1	1.7	Curvature of utility from reserve	Elasticity of price to reserve supply	-0.2	0.2
γ_2	1.5	Curvature of utility from bonds	Elasticity of price to bonds supply	-0.1	-0.1
α	0.4	CB's asset holding	Asset/liability	1	1
H^*	-0.0025	Lower bound on remittance	-	-	-

The parameters describing the curvature of the utility of reserves and bonds, γ_1, γ_2, are calibrated so that the elasticity of price to quantity in the data matches that in the model following Krishnamurthy and Vissing-Jorgensen (2012). The details are discussed in the Appendix B.

Lastly, the lower bound on remittance, H^*, is -0.0025, that is 0.06% of annual GDP. In practice, H^* is zero implying that central bank does not receive any amounts of funds from Treasury. I tried to set $H^* = 0$, but under this parameter, the model does not converge. The possible reason is that in a state-space with large reserves and an exogenous shock leading to higher nominal interest rate, the reserves keep increasing because central bank cannot receive any funds from Treasury, but central bank needs to pay high interest rate on reserves. Then, reserves keep increasing, and there is no equilibrium. As a result, I set * slightly negative so that the model converges. I set $H^* = -0.0025$ that is the highest and closes value to zero such that the model converges.

5 Responding to shocks

This section describes the main exercises: How policies respond to shocks, a positive cost-push shock, a negative productivity shock, and a positive public expenditure shock under the consolidated model and unconsolidated model. The goal is to show a difference between the consolidated and the unconsolidated models.

For all types of shocks, the procedure is as follows: at time $t = 1$, an exogenous shock occurs. The state variables include reserves and bonds, which I give as an initial condition. The exogenous shock then follows its own AR(1) process as described by equations (38)-(40). The innovations in the AR(1) process are randomly drawn from a normal distribution. I generate 5000 paths of the exogenous shock, each with a different sequence of innovations randomly drawn from normal distribution. I then calculate the dynamics of variables for each sequence of exogenous shocks and the policy functions. I take the average of the 5000 simulated dynamics.

5.1 Productivity shock

This section shows the transition dynamics of the consolidated and unconsolidated models following a negative productivity shock. The dynamics of this type of shock are of interest in their own right (Blanchard and Galí, 2007). Furthermore, this exercise is motivated by the fact that the central bank raises the nominal interest rate after a negative productivity shock, leading to higher interest expenditures.

Figure 1 shows the results after a decrease in productivity, A_t, by 5%. The productivity, A_t, falls from 1 to 0.95. The blue line represents the results in the consolidated model, while the red line represents the results in the unconsolidated model. To simulate the dynamics, initial state variables are provided for reserves and bonds. Reserves are set at a higher value, corresponding to the 90th percentile of the simulated economy. This is motivated by the fact that, in practice, the central bank holds large reserves. For example, the Federal Reserve holds more than $3 trillion in reserves, and a large shock, such as a COVID shock or an inflationary shock, hits the economy.

The intuition behind the consolidated model is as follows: In response to the shock, households optimally reduce both labor supply and consumption. Consequently, marginal costs increase, prompting firms to raise prices, which results in higher inflation. To counter this, the government increases the nominal interest rate to reduce demand and mitigate inflation. This leads to a decline in the price of reserves, that is an increase in the nominal interest rate, as shown in Figure 1-(c). Lower aggregate output reduces tax revenues. The government raises the tax rate to finance public expenditures (Figure 1-(g)) and issues bonds to spread tax distortions over time (Figure 1-(f)).

With large initial reserves, the central bank continues to reduce reserves in figure 1-(e). The central bank must pay interest expenses on reserves, so it receives transfers from the Treasury in figure 1-(h).

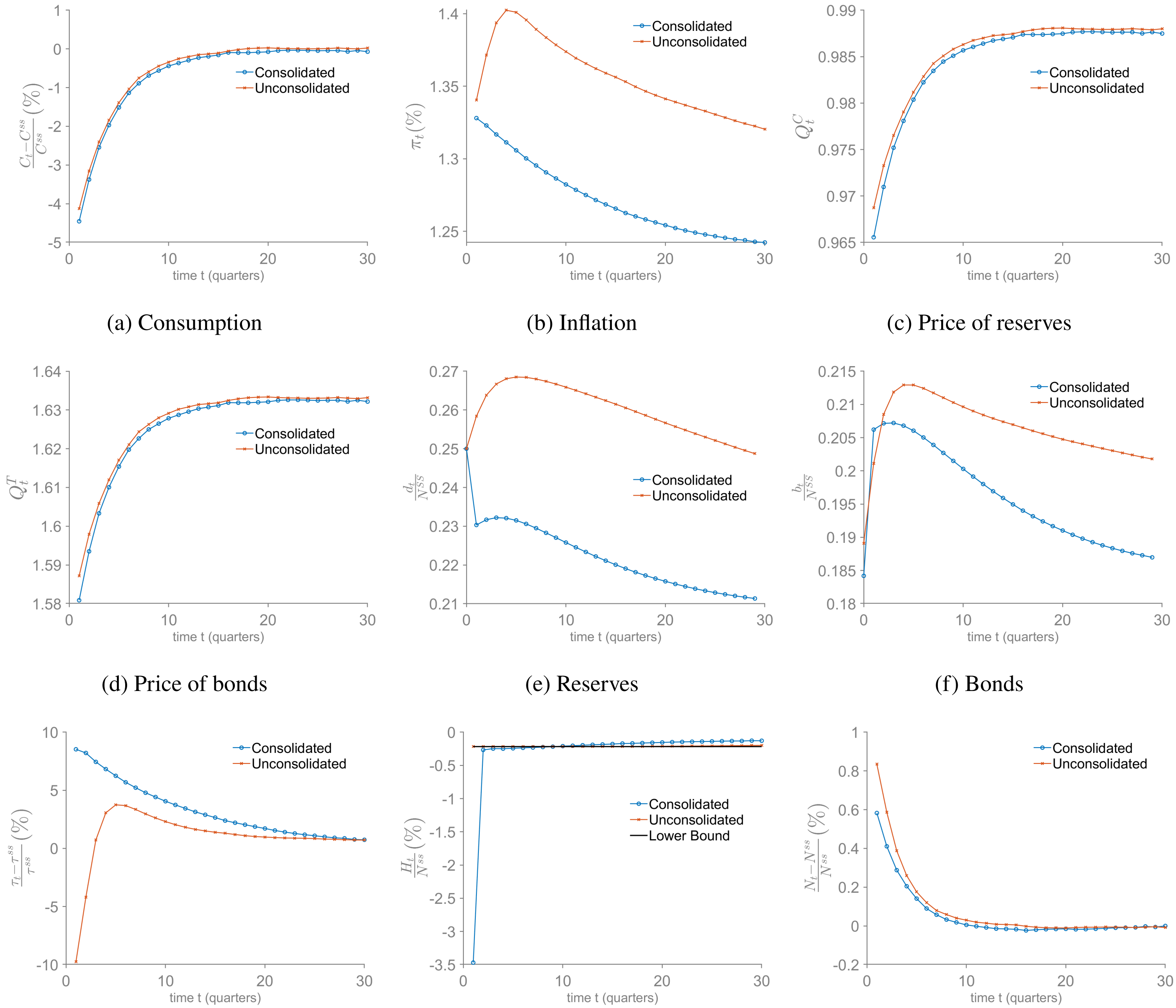
I then explain the intuition behind the unconsolidated model. A key result is that the central bank is limited to raise the nominal interest rate sufficiently; as a result, the price of reserves does not decrease as much in the unconsolidated model compared to the consolidated model, as illustrated in Figure 1-(c). In the consolidated model, the price of the reserve falls to 0.965, while in the unconsolidated model it is 0.969. Paying high interest on reserves leads to an accumulation of reserves, yet a constraint on remittances limits the central bank's ability to reduce reserves. As the marginal utility of reserves diminishes with their increased levels, the price must decline for households to be willing to hold more reserves. This instability in the price of reserves, reflected by the nominal interest rate, disrupts real allocations, motivating the government to set a lower nominal interest rate to prevent excessive reserve accumulation.

Due to the lower nominal interest rate, consumption does not decline sufficiently in the unconsolidated model. In the consolidated model, consumption falls by 4.5%, while in the unconsolidated model it falls by 4.1%. Labor supply increases more in the unconsolidated model because households consume more. The response of labor supply in the consolidated model is 0.6%, while that in the unconsolidated model is 0.8%. In addition, the inflation rate is higher in the unconsolidated model because the interest rate cannot be raised enough to offset the inflationary effect of a negative productivity shock. The peak inflation rate in the consolidated model is 1.34%, while the peak inflation rate in the unconsolidated model is 1.4%.

In summary, the lack of fiscal backing limits the central bank to increase nominal interest rate.

5.2 Cost-push shock

This section examines the transition dynamics following a cost-push shock. Similar to the productivity shock analysis, I simulate the economy's response to a positive cost-push shock, assuming



(a) Consumption (b) Inflation (c) Price of reserves

(d) Price of bonds (e) Reserves (f) Bonds

(g) Sales tax (h) Remittance (i) Labor supply

Figure 1: Productivity shock

Note: This figure shows the dynamics of key variables upon the negative productivity shock by 5%. A productivity drops from 1 to 0.95. The horizontal axis is quarters. The vertical axis for consumption, tax rate, and labor supply are the steady-state deviation represented by percent. The vertical axis for inflation rate is annualized inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance, bonds, and reserves are ratio to GDP. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

larger reserves. The initial conditions for the state variables are reserves and bonds. The initial level of reserves is high, at the 90th percentile of the simulated economy. The level of bonds is at the steady-state level. Note that the economy does not start from the steady state, so the variable moves even without the cost-push shock. I have not netted out the dynamics when the same size shock hits, but the economy starts from the steady state.

As with the productivity shock, a positive cost-push shock prompts the central bank to increase the nominal interest rate. However, this section underscores a key difference: fiscal policy is constrained by the lack of fiscal backing, while monetary policy is constrained after a productivity shock.

Figure 2 plots the results after increasing the wage mark-up μ^w by 8%. The wage mark-up increases from 1 to 1.08. The horizontal axis is in quarters. The blue line represents the result in the consolidated model. The red line represents the result in the unconsolidated model.

First, I describe the policy chosen by the government in the consolidated model. Under the optimal policy, the government reduces the sales tax rate to offset the increase in the marginal cost of labor. By reducing the sales tax rate, the government can increase the marginal product of labor, which helps it to track the marginal cost even after a shock. According to the first-order conditions for firms shown in equation (13), changes in the wage mark-up can be offset by adjusting the tax rate. This reduction in the tax rate is illustrated in figure 2-(g).

As a result of the lower tax rate, tax revenue falls, but the Treasury still needs to finance public expenditure. To finance this, the Treasury issues bonds in figure 2-(f). From the central bank's perspective, the central bank helps the Treasury's budget by issuing reserves, borrowing from the private sector, and sending funds to the Treasury. The central bank issues more reserves after $t = 2$. However, at $t = 1$, the central bank holds large reserves. The central bank receives funds to reduce the level of reserves at $t = 1$. Therefore, reserves decrease and then increase to raise funds. This is shown in figure 2-(e).

For the transfer, it takes a negative value at $t = 1$, i.e. the Treasury sends and the central bank receives. This is to reduce large reserves. After $t = 1$, the transfer is close to zero and flat. This is because the two effects cancel each other out. The first is that the central bank issues more reserves and sends funds to the Treasury. The second is that the central bank has to buy more bonds as the Treasury issues more. In order for the central bank to purchase assets, the central bank must receive funds from the Treasury. These two effects cancel each other out. As a result, the transfer in 2-(h) is flat after $t = 1$.

Consumption falls and the inflation rate rises. To reduce demand, the central bank raises the nominal interest rate, causing the price of reserves and bonds to fall. Notably, the nominal interest rate responds only modestly, while the Treasury uses tax cuts more aggressively. The price of reserves falls by only 0.004 percentage points, while the consumption tax falls by 6% from its steady-state level. This quantitative difference suggests that tax rebates serve as a more efficient policy tool in response to a cost-push shock.

I then explain the intuition behind the transition dynamics in the unconsolidated model. The main results are that the inflation rate is higher and consumption falls more. This is because the government cannot lower the tax rate sufficiently, so the high marginal cost reduces output and feeds through to the inflation rate. Figure 2-(g) shows that the tax rate decreases by 6.1% in the consolidated model, while it is 3.6% in the unconsolidated model. Why can the government not reduce the tax rate enough? From the central bank's point of view, the issue of new reserves is smaller. Figure 2-(e) shows that reserves in the consolidated model increase after $t = 2$ to send resources to the Treasury, while reserves in the unconsolidated model do not increase much. This is because the central bank expects that it will not be able to reduce large reserves through transfers in the future, which limits its willingness to issue additional reserves. Less reserve issuance means that transfers from the central bank to the Treasury are smaller in the unconsolidated model. As a result, the government cannot reduce the tax rate sufficiently to offset the cost-push shock. The central bank tolerates a higher inflation rate. The inflation rate responds more in the unconsolidated model, by 1.3%. Consumption falls by more than 0.07%. Since consumption falls by 0.8% and inflation rises by 0.4% in the consolidated model, the difference between the consolidated and unconsolidated models is significant.

In summary, the central bank does not help the Treasury's budget as much in the unconsolidated model as in the consolidated model, because the central bank is limited in its ability to raise funds from the private sector. The lack of *fiscal* support affects *fiscal* policy; tax break.

An important point to note is that the main mechanism works through fiscal policy (the sales tax rate) rather than the nominal interest rate. If the Treasury could finance a tax break through lump-sum taxation, the cost-push shock could be fully offset. However, because the Treasury does not have access to lump-sum tax, tax break is supported by the central bank through remittances in the consolidated model. This support is limited in the unconsolidated model.

A cost-push shock contrasts sharply with a productivity shock, where optimal monetary policy is constrained. Why is the difference so large? The answer lies in the different effectiveness of policy instruments for each type of shock. Following a negative productivity shock, the most effective policy instrument is to adjust the nominal interest rate to its natural rate. In response to a cost-push shock, however, a tax break on sales is more effective. The former policy affects the central bank's budget more, thereby constraining monetary policy, while the latter affects the Treasury's budget more, thereby constraining fiscal policy.

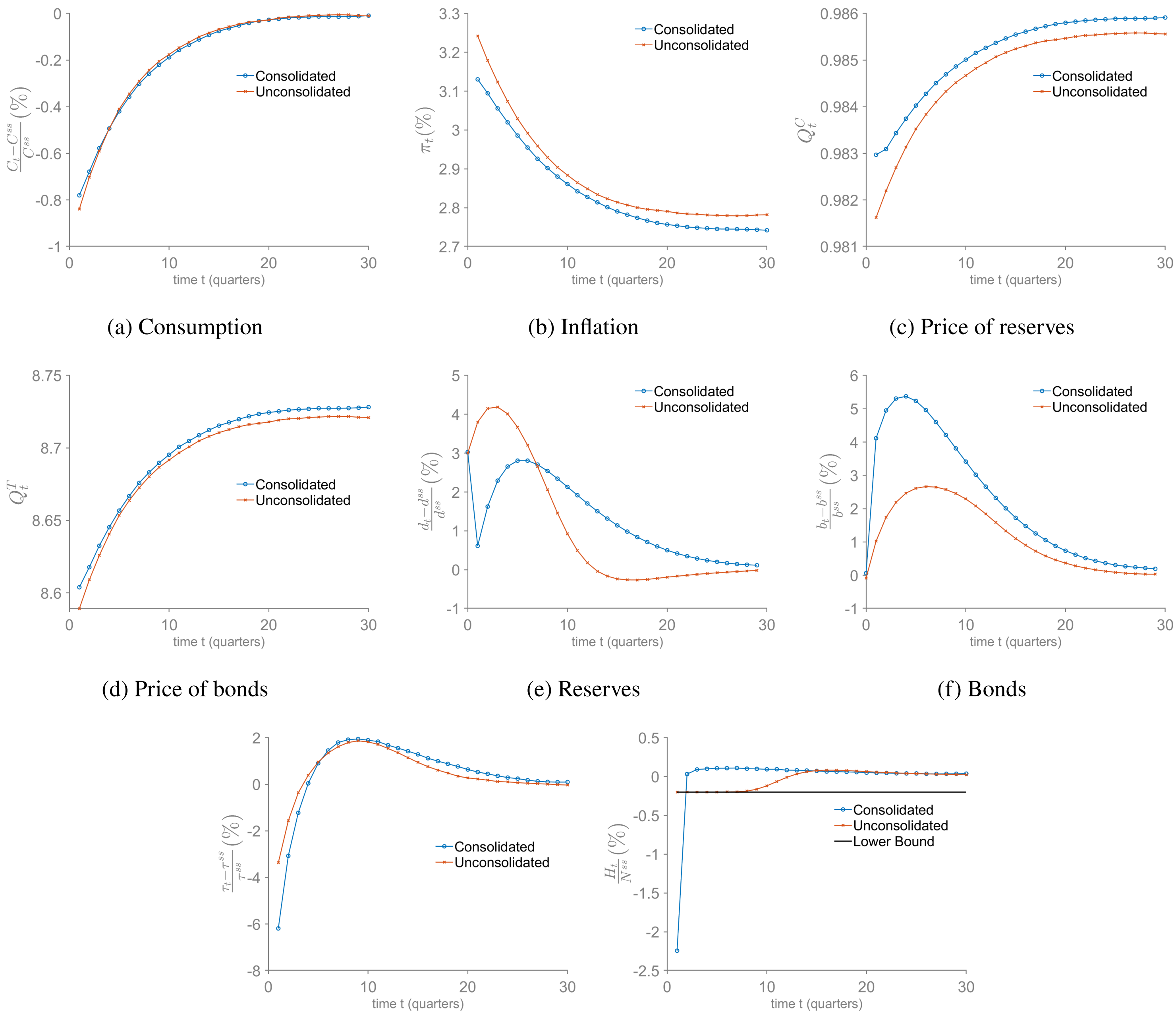
5.3 Public expenditure shock

This section describes the transition dynamics following a public expenditure shock. Motivated by the significant increase in public spending observed during events like the COVID-19 pandemic, I examine the role of the central bank in responding to a positive public expenditure shock. My results highlight the central bank's ability to raise funds from the private sector and support the Treasury.

The initial condition assumes a larger stock of bonds than the steady-state, while the level of reserves is set at the steady-state. Upon the shock, larger bonds limits Treasury's capacity to issue additional bonds. In contrast, the central bank holds reserves at their steady-state level, allowing it to raise more funds by issuing additional reserves. This leads to a stark contrast between the consolidated and unconsolidated models.

Figure 3 shows the results after increasing G_t by 2% of GDP. Public spending increases from 15% to 17% of GDP at time $t = 1$. The dynamics in the consolidated model are represented by the blue line, while the red line represents the unconsolidated model. I will first explain the intuition behind the consolidated model.

The Treasury finances its expenditures by raising the tax rate and issuing bonds to smooth out tax distortions. The central bank issues reserves and makes transfers to support the Treasury's budget. At the time of the shock, the Treasury holds a substantial amount of bonds, so bond prices are low (i.e., bond interest rates are high). In contrast, the supply of reserves is relatively small, so the price of reserves is higher than that of bonds. To raise funds, the central bank increases



(a) Consumption

(b) Inflation

(c) Price of reserves

(d) Price of bonds

(e) Reserves

(f) Bonds

(g) Sales tax

(h) Remittance

Figure 2: Cost-push shock

Note: This figure shows the dynamics of key variables upon the positive wage markup shock by 8%. The wage mark-up increases from 1 to 1.08. The horizontal axis is quarters. The vertical axis for consumption, prices of reserves, prices of bonds, reserves, bonds are the steady-state deviation represented by percent. The vertical axis for inflation rate is annualized inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

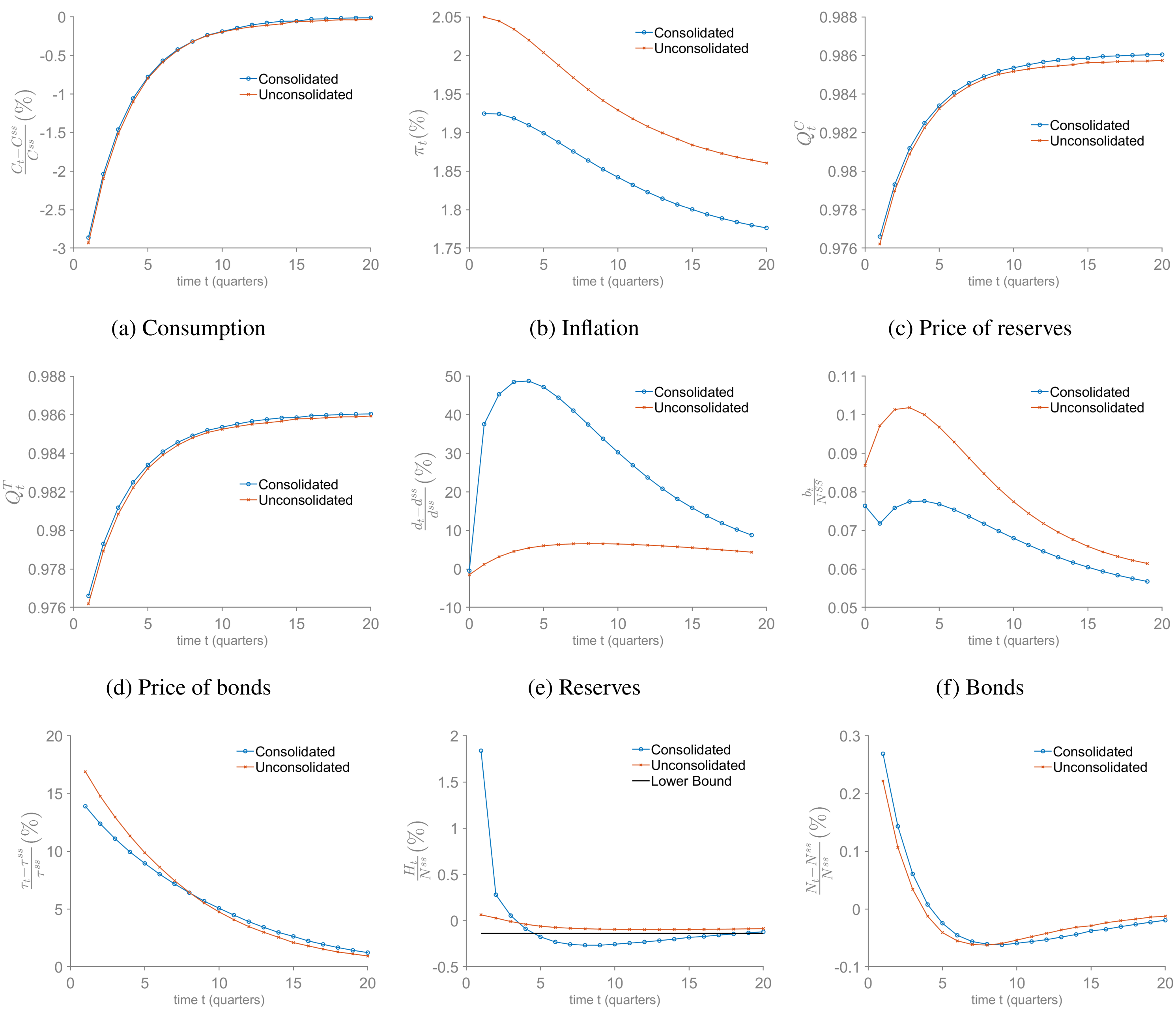
reserves, which have a lower nominal interest rate. When $t = 1$, remittances become positive, i.e. the Treasury receives funds from the central bank. Later, remittances fall below their steady-state level as the central bank receives payments to reduce the expanded reserves.

In terms of real allocations and inflation, labor supply increases to meet the demand for high public expenditure, while consumption falls due to crowding out. The inflation rate rises as firms increase production, leading to higher marginal costs. Additionally, the government has an incentive to inflate nominal liabilities because of higher level of liabilities. Regarding prices, the price of reserves is low and the nominal interest rate is high, which contracts consumption.

Next, I outline the intuition behind the unconsolidated model. In summary, the lack of fiscal backing limits the government's ability to smooth taxes. In the consolidated model, the central bank can reduce reserves by receiving transfers from the Treasury, but this is not possible in the unconsolidated model. Anticipating that large reserves cannot be reduced by future transfers, the central bank becomes less willing to issue reserves without fiscal backing. Consequently, the central bank provides less support to the Treasury. This effect is reflected in the path of remittances: as shown in figure 3-(h), at t=1 remittances are 1.8% of GDP in the consolidated model, but nearly zero in the unconsolidated model. The Treasury must issue more bonds to finance the increase in public spending due to the reduction in central bank remittances. Bonds increase more in the unconsolidated model, but given that the Treasury already holds a substantial stock of bonds with high nominal interest rates, it becomes difficult to issue enough bonds to fully smooth the fiscal distortions. As a result, the increase in government spending is financed more through taxes than through bonds, leading to a larger tax rate response in the unconsolidated model than in the consolidated model, as shown in figure 3-(g). Specifically, the sales tax increases by 14.7% in the consolidated model and by 16.5% in the unconsolidated model at $t = 1$.

In terms of real allocations, a higher tax rate on sales leads to lower production, i.e., lower labor supply. The difference in consumption between the two models is not significant. However, the inflation rate remains persistently higher in the unconsolidated model as a way to reduce reserves by inflating nominal liabilities, that is, inflating reserves.

In summary, in a situation where support from central bank for Treasury is important, the lack of fiscal backing limits the government's ability to smooth taxes.



(a) Consumption (b) Inflation (c) Price of reserves

(d) Price of bonds (e) Reserves (f) Bonds

(g) Sales tax (h) Remittance (i) Labor supply

Figure 3: Public expenditure shock

Note: This figure shows the dynamics of key variables upon the positive public expenditure shock by 2% of GDP. The public expenditure increases from 15% of GDP to 17% of GDP. The horizontal axis is quarters. The vertical axis for consumption, prices of reserves, prices of bonds, reserves, bonds are the steady-state deviation represented by percent. The vertical axis for inflation rate is annualized inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

5.4 The size of shock

This section studies the relationship between the size of the shock and fiscal backing. Given that the lower bound on remittances is a fixed parameter, it is reasonable to expect that the differences in inflation dynamics between the consolidated and unconsolidated models depend on the size of the shock. For example, with a small shock, the inflation difference between the models is minimal, suggesting that the absence of fiscal backing has a limited impact. However, for a larger shock, the lack of fiscal backing may have a significant impact.

Figure 4 shows the dynamics of inflation differential between the consolidated and unconsolidated models for various sizes of productivity shocks. The horizontal axis represents time in quarters and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$. The procedure is as follows: I compute $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) following a shock for the unconsolidated model and π_t^{con} is the corresponding rate for the consolidated model. For example, a 5% positive productivity shock occurs at time $t = 1$ and follows an AR(1) process. I simulate the dynamics of the inflation rate following this exogenous shock for both models and then calculate the difference between the two. This exercise is repeated for shocks of different sizes: 5%, 2%, -2%, and -5%. The initial conditions for reserves and bonds are set to their steady-state values.

Figure 4 illustrates that as the absolute size of a negative shock increases (e.g., -5%), the inflation differential between the two models increases relative to a smaller shock (e.g., -2%). For example, a 5% decline in productivity results in a peak inflation rate that is 0.032% higher in the unconsolidated model than in the consolidated model, while a 2% decline in productivity results in a 0.022% difference in peak inflation rates. This suggests that larger shocks lead to greater divergence between the models. In contrast, for a positive shock, the inflation differential is less sensitive to the size of the shock. For example, for a 5% increase in productivity, the peak inflation rate in the unconsolidated model is only 0.007% higher than in the consolidated model, and for a 2% increase in productivity, the difference is 0.011%. The reason why a negative shock has a significant impact on inflation is that it leads to an increase in the nominal interest rate, which requires additional Treasury funds and thus increases the difference. Conversely, a positive shock lowers the nominal interest rate and does not require additional Treasury support, resulting in a smaller differential.

Figure 5 shows the inflation differential between the consolidated and unconsolidated models for different sizes of productivity shocks. The horizontal axis represents the size of the exogenous

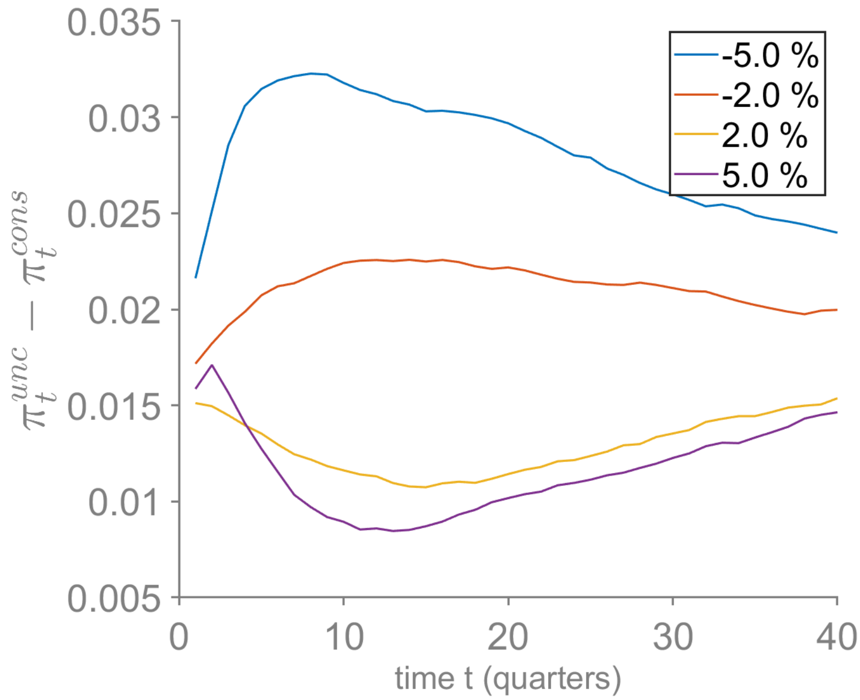


Figure 4: Difference in inflation between consolidated and unconsolidated model for productivity shock

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various shock sizes. The horizontal axis represents time, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. Different lines represent different shock sizes. For example, a 5% positive productivity shock occurs at time $t = 1$ and follows an AR(1) process. I simulate the inflation rate based on this exogenous shock. The same exercise is conducted for shocks of 5%, 2%, -2%, and -5%.

shock, while the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$ at $t = 4$, where the inflation differential peaks. For each shock size on the horizontal axis, I compute the transition dynamics of the inflation rates, π_t^{unc} and π_t^{con}, and plot $\pi_t^{unc} - \pi_t^{con}$ at $t = 4$ on the vertical axis.

Figure 5 shows a nonlinearity in the inflation differential with respect to the size of the shock. For positive shocks, the inflation differential remains flat at 0.014% regardless of the size of the shock. However, for negative shocks below a certain threshold, the inflation differential increases monotonically as the size of the shock decreases. For example, when productivity falls to 0.96, $\pi_t^{unc} - \pi_t^{con}$ reaches 0.028% at $t = 4$.

Intuitively, this result follows from an inequality constraint and the response of the nominal interest rate. For larger negative productivity shocks, the central bank raises the nominal interest

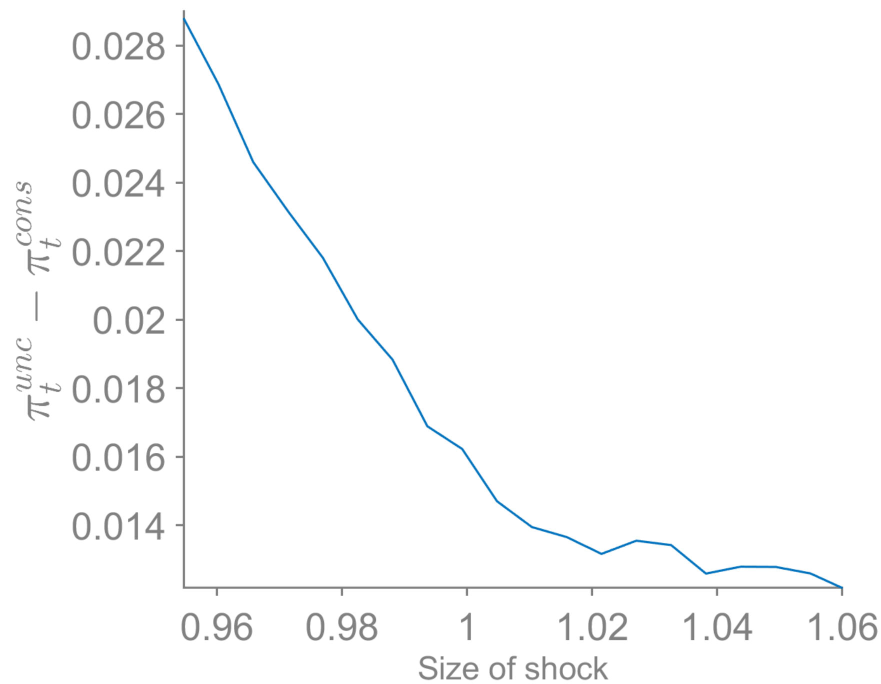


Figure 5: Difference in inflation between consolidated and unconsolidated model for productivity shock

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various shock sizes. The horizontal axis represents the size of an exogenous shock, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$ at $t = 4$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. For example, when the productivity drops to 0.96, $\pi_t^{unc} - \pi_t^{con}$ at $t = 4$ is 0.028%.

rate, leading to higher interest expenditures. Since the constraint on transfers prevents the central bank from reducing reserves, higher interest expenses lead to a higher level of reserves. This constraint motivates the central bank to lower the nominal interest rate to manage reserves, thus tolerating a higher rate of inflation as the size of the shock increases. The same exercise for other types of shock is discussed in the Appendix E.

5.5 The initial condition of reserves

The purpose of this section is to understand the relationship between initial reserve levels and the impact of fiscal support. The initial level of reserves affects the amount of transfers required. For example, if the initial level of reserves is high and a shock raises the nominal interest rate, the central bank will incur more interest expense and the lack of fiscal backing will amplify the shock's impact on inflation.

This section examines the transition dynamics following a 5% decline in the productivity shock for various initial levels of reserves. The procedure is as follows: I compute $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation dynamics (in percentage points) after a shock for the unconsolidated model, and π_t^{con} is the same for the consolidated model. At time $t = 1$, a 5% decrease in productivity occurs, following an AR(1) process. The initial conditions include reserves and bonds, with different levels of reserves provided: 10% above the steady state, 5% above the steady state, 5% below the steady state, and 10% below the steady state (all values are expressed as deviations from the steady state). I then simulate the inflation rate dynamics for both models, π_t^{unc} and π_t^{con}. The initial level of bonds is set at the steady-state level.

Figure 6 plots $\pi_t^{unc} - \pi_t^{con}$ for different initial levels of reserves. It shows that the difference in inflation rates between the two models increases with the initial reserve level. When reserves are 10% above the steady-state level, inflation in the unconsolidated model is 0.09% higher than in the consolidated model. When reserves are 5% higher, inflation in the unconsolidated model is 0.05% higher. In contrast, results for steady-state reserves and reserves below the steady state show similar inflation dynamics. The impact of no fiscal backing is amplified with larger reserves,

The reasoning is as follows: Higher initial reserves affect remittances in two ways. First, reserves are above the steady-state level after the shock, leading the central bank to reduce reserves, which requires funds from the Treasury. Second, a negative productivity shock raises the nominal interest rate, and a higher nominal interest rate on larger reserves results in increased interest expenses, further requiring transfers. In the unconsolidated model, the central bank is constrained in reducing larger reserves, and households have limited capacity to absorb reserves. To manage this, the planner sets a lower interest rate to prevent an excessive increase in reserves.

Figure 7 plots $\pi_t^{unc} - \pi_t^{con}$ for different initial reserve levels at $t = 4$. The horizontal axis represents the initial reserve levels as a percentage deviation from the steady state. The vertical axis shows $\pi_t^{unc} - \pi_t^{con}$ at $t = 4$, where the inflation difference is largest. The procedure is as follows: I compute $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) after a shock in the unconsolidated model, and π_t^{con} is the same in the consolidated model. At time $t = 1$ there is a 5% decline in productivity, following an AR(1) process. The initial reserve levels are set

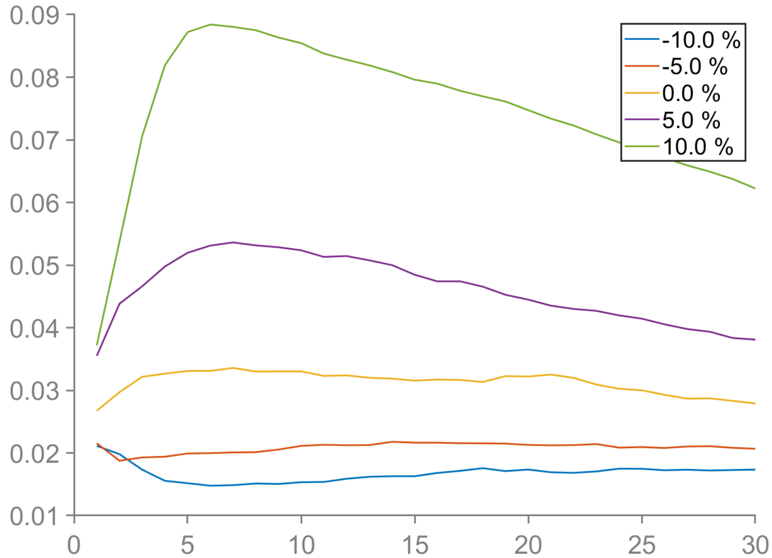


Figure 6: Difference in inflation between consolidated and unconsolidated model for productivity shock

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various initial reserves. The horizontal axis represents time, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. Different lines represent different initial reserves. The exercise is conducted for shocks of 10% above the steady state, 5% above the steady state, 5% below the steady state, and 10% below the steady state (all values are expressed as deviations from the steady state)

according to the values shown on the horizontal axis.

Figure 7 shows a nonlinear relationship between the inflation differential and the initial level of reserves. For example, as initial reserves increase from 10% below the steady state to 5% below the steady state, the inflation differential increases from 0.012% to 0.015%. However, as reserves increase from 5% above the steady state to 10% above the steady state, the inflation differential rises more sharply, from 0.042% to 0.072%.

The logic is as follows: if initial reserves are below the steady-state level, the central bank must supply more reserves and transfers the funds created to the Treasury. A negative productivity shock raises the nominal interest rate, increasing interest costs and requiring additional funds from the Treasury. However, this need for funds is largely offset by the central bank's incentive to issue more reserves and transfer additional funds to the Treasury. As a result, the lack of fiscal backing does not make a significant difference.

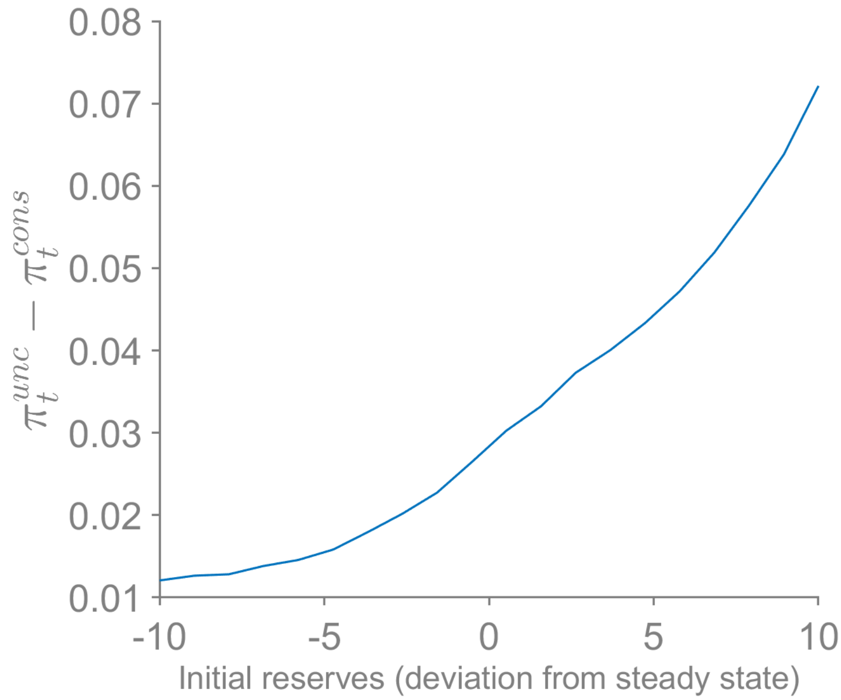


Figure 7: Difference in inflation between consolidated and unconsolidated model for productivity shock

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various initial reserves. The horizontal axis represents time, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. Different lines represent different initial reserves. The exercise is conducted for shocks of 10% above the steady state, 5% above the steady state, 5% below the steady state, and 10% below the steady state (all values are expressed as deviations from the steady state)

In contrast, if initial reserves are above steady state, the central bank must reduce reserves by receiving funds from the Treasury. In addition, a negative productivity shock further increases interest costs, so the lack of fiscal backing has a significant impact.

6 Business cycle properties

This section is motivated by the question of how monetary and fiscal policies should be set during the business cycle, with particular focus on two questions: (i) the optimal volatility of inflation: how does the government use inflation to stabilize the real value of debt? and (ii) the persistence of taxes and debt: how does the government smooth tax distortions by issuing liabilities? This paper addresses these questions while sharing key elements with earlier literature, including: (a) taxes are distortionary, (b) the government issues only nominal and non-state-contingent debt, and (c) prices are sticky. A key deviation from previous studies is (i) the constraint on transfers from the Treasury to the central bank, and (ii) reserves that the other type of interest-bearing liabilities.

This section presents moments for key variables under both unconsolidated and consolidated models. The moments are computed as follows: I first generate simulated time series of length T for the variables and calculate the corresponding moments. This procedure is repeated J times, and the average of the moments is computed. In the table, T equals 1000 periods, and J equals 1000 repetitions. The economy is simulated with one shock, while other shocks are excluded due to the large size of the state-space.

6.1 Monetary-fiscal policy and consumption

Table 2 presents the moments for inflation, consumption, and the tax rate in both the consolidated and unconsolidated models. Each panel displays the moments for each shock; productivity, cost-push, or public expenditure shock.

The main results are as follows: (i) inflation is more volatile in the unconsolidated model across all shocks; (ii) the average inflation rate increases, though the rise is small; (iii) while the tax rate is more volatile in response to productivity and government expenditure shocks in the unconsolidated model, it is less volatile for cost-push shocks; and (iv) the volatility of consumption is minimally affected.

First, inflation is more volatile in the unconsolidated model than in the consolidated model, with volatility increasing by 4.1% for cost-push shocks, 8.3% for productivity shocks, and 28% for public expenditure shocks. Overall, the inflation rate volatility remains low relative to observed data for all types of shocks. For example, with a productivity shock, the standard deviation of inflation in the consolidated model is 0.012 percentage points, as I examine equilibrium under optimal policy. This volatility increases in the unconsolidated model.

The mechanism leading to higher inflation volatility is discussed in detail in section 5. Here I briefly summarize the mechanism for each of the three types of shocks. For cost-push shocks, the government aims to align marginal cost with marginal product by adjusting the sales tax rate. This mechanism is reflected in the strong negative correlation between taxes and the shock, with a correlation of -0.78. Changes in tax revenue prompt the central bank to support the Treasury through transfers, but this support is limited in the unconsolidated model. The limited ability to adjust taxes leads to a higher inflation rate. This mechanism is further supported by the third result, which shows that the tax rate is less volatile in the unconsolidated model.

The logic of the public expenditure shock is similar to that of the cost-push shock. In response to an increase in public spending, the central bank partially finances it through remittances, but this support is limited in the unconsolidated model. As a result, public spending is financed primarily through taxes rather than remittances, leading to a more volatile tax rate. The resulting fluctuations in the marginal product, driven by the volatile tax rate, lead to higher inflation volatility. This mechanism is further supported by the third result, which shows that the tax rate is more volatile in response to public expenditure shocks.

The logic for the productivity shock differs from the previous two shocks in that monetary policy is constrained. The higher nominal interest rate is not supported by remittances, giving the central bank an incentive to lower the nominal interest rate. As a result, the productivity shock cannot be stabilized by tracking the natural rate, leading to greater inflation volatility. Since the tax rate plays a smaller role in responding to productivity shocks, changes in tax rate volatility are less pronounced.

The second finding is that although the average inflation rate is higher in the unconsolidated model, the difference is minor. The average inflation is 0.06 percentage points higher for cost-push shocks, 0.008 percentage points for productivity shocks, and 0.07 percentage points for public expenditure shocks.

The higher inflation rate in the unconsolidated model is due to the central bank's incentive to reduce the real value of reserves through inflation, given that remittances to reduce reserves are constrained. This mechanism is similar to the one found in the literature examining the government's incentive to reduce liabilities through inflation (Schmitt-Grohé and Uribe, 2004). However, this incentive is quantitatively small due to price stickiness. As shown in Schmitt-Grohé and Uribe (2004), the costs of inflation arising from price stickiness outweigh the benefits of reducing the real value of liabilities. This paper shows that the same mechanism applies when the central bank issues liabilities without the option of remittances to reduce reserves.

Table 2: Dynamic Properties of the Optimal Policy

Variable	Mean	Std. Dev.	Auto. Corr.	Corr w/ Shock
Consolidated Model with Cost-Push Shock				
π	1.741	0.0862	0.93	0.81
c_t	88.5	0.14	0.84	-0.94
τ_t	10.2	0.52	0.57	-0.78
Unconsolidated Model with Cost-Push Shock				
π	1.803	0.0898	0.90	0.82
c_t	88.5	0.14	0.79	-0.96
τ_t	10.3	0.43	0.49	-0.80
Consolidated Model with Productivity Shock				
π	1.759	0.0120	0.93	-0.84
c_t	88.5	0.010	0.73	1.00
τ_t	10.2	0.094	0.90	-0.89
Unconsolidated Model with Productivity Shock				
π	1.767	0.0130	0.91	-0.85
c_t	88.5	0.010	0.74	1.00
τ_t	10.3	0.097	0.87	-0.94
Consolidated Model with Public Expenditure Shock				
π	1.765	0.0615	0.93	0.80
c_t	88.5	0.643	0.72	-1.00
τ_t	10.3	0.473	0.88	0.88
Unconsolidated Model with Public Expenditure Shock				
π	1.839	0.0791	0.93	0.76
c_t	88.5	0.647	0.71	-1.00
τ_t	10.3	0.482	0.86	0.91

Notes: π and τ are expressed in percentage points, and c is in levels.

6.2 Persistency of liabilities and taxes

An important result in the public finance literature is the idea of smoothing tax distortions over time. Lucas and Stokey (1983) show that bonds inherit the stochastic process of exogenous shocks. This implies that if shocks are serially uncorrelated, bonds should also follow an uncorrelated process. However, Barro (1979) and Aiyagari et al. (2002) demonstrate that Lucas and Stokey's results depend on the assumption that the government can issue state-contingent debt. They show that, in the absence of state-contingent debt, bonds follow a near random walk behavior, regardless of the process assumed for the shocks. This paper examines the process of liabilities and taxes in a setting with non-state-contingent nominal debt and sticky prices, similar to Schmitt-Grohé and Uribe (2004).

The main departure from the existing literature is the inclusion of reserves. In the data, reserves are substantial-over $3 trillion and accounting for more than 15% of GDP, making their behavior an important area of study. This section examines whether both reserves and bonds exhibit near-random walk behavior under discretionary policy. In addition, the paper examines the implications of the absence of fiscal backing: does the absence of fiscal backing alter the stochastic behavior of reserves and bonds? Furthermore, does it affect the properties of taxation?

Table 3 presents the autocorrelations for the reserves, bonds, and tax rates. In the consolidated model, both reserves and bonds exhibit persistent behavior across all types of shocks, consistent with the findings of Schmitt-Grohé and Uribe (2004). The autocorrelations of reserves and bonds are close to one, indicating their highly persistent nature.

The government uses both reserves and bonds to smooth taxes. Thus, an economy in which reserves are significantly more persistent than bonds cannot be equilibrium. Households receive a convenience return from reserves and bonds, which are imperfect substitutes. If the government relies on only one of these liabilities to smooth taxes, the marginal utility of that liability becomes higher than the other, resulting in a suboptimal policy. Thus, the government raises funds proportionally by issuing both reserves and bonds.

For taxes, the autocorrelation is high for productivity and public expenditure shocks, but lower for cost-push shocks. This difference reflects the government's tax-smoothing to productivity and public expenditure shocks. In the case of a public expenditure shock, taxes are used primarily to finance the increase in public spending. For a productivity shock, taxes are adjusted to compensate for changes in tax revenue due to fluctuations in output. In both cases, the government uses taxes to finance public expenditures, resulting in a strong tax-smoothing motive. In contrast, for cost-push shocks, the government adjusts sales taxes to offset the impact of the shock on marginal costs, leading to a tax process that closely follows the process of the shock.

In the unconsolidated model, the absence of fiscal backing causes reserves to become more persistent and bonds to become less persistent across all types of shocks. For example, under a cost-push shock, the autocorrelation of reserves is 0.97 in the consolidated model but increases to 0.99 in the unconsolidated model. This persistence in reserves follows a similar logic across all shocks: once the central bank issues reserves, its ability to reduce them through remittances is constrained. Consequently, reserves decrease slowly, resulting in higher persistence.

In the unconsolidated model, bonds have lower persistence. This is because bonds are more responsive to shocks in the unconsolidated model than in the consolidated model. The demand for government liabilities is split between reserves and bonds, as both are used to finance public expenditure. In the unconsolidated model, however, the central bank's ability to use reserves for fiscal smoothing is limited. As a result, the government relies more heavily on bonds to generate resources, making bonds both more responsive and more volatile to shocks. After this larger initial response, bonds quickly return to their steady state, leading to increased volatility and reduced persistence in the unconsolidated model.

6.3 Implications on retained earnings

This section examines the relationship between the model and the practical policies of central banks. A key aspect of my model is the transfers between the central bank and the Treasury, which raises the question of how central banks determine their transfers in practice. An important observation for practical policy is that many central banks retain some of their profits rather than transferring all of their profits to the Treasury.

For example, in the UK, the remittance policy depends on the central bank's net worth. If the Bank of England's net worth is below a target level, all profits are retained to boost net worth. If net worth is above the target but below a ceiling level, half of the profits go to the Treasury, while the other half is added to capital. If net worth exceeds the ceiling, all profits are transferred to

Table 3: Autocorrelation of Reserves, Bonds, and Tax

Reserves	Bonds	Tax on Sales
Consolidated Model with Cost-Push Shock		
0.97	0.97	0.57
Unconsolidated Model with Cost-Push Shock		
0.99	0.96	0.49
Consolidated Model with Productivity Shock		
0.98	0.98	0.90
Unconsolidated Model with Productivity Shock		
0.99	0.94	0.87
Consolidated Model with Public Expenditure Shock		
0.97	0.97	0.88
Unconsolidated Model with Public Expenditure Shock		
0.99	0.96	0.86

Notes: Reserves, bonds, and tax rate are in levels.

the Treasury[6]. In Japan, 5% of the Bank of Japan's profits are retained, with the remaining 95% transferred to the Treasury. Given the large BOJ's balance sheet, the retained earnings at BOJ is significant.[7]

Can the unconsolidated model replicate the fact that central banks do not transfer their entire profit? Is the unconsolidated model more consistent with actual practice than the consolidated model? The answer is yes.

Figure 8 presents a histogram of remittances over the business cycle in both the consolidated and unconsolidated models. I simulate the economy for 1000 periods and generate a sequence of remittances for both models. The histograms display remittances in the consolidated model (blue) and the unconsolidated model (red). The left side shows cases where the central bank receives transfers, while the right side shows cases where the Treasury receives remittances.

In the unconsolidated model, there is a lower bound on remittances so that remittances do not take a value below a threshold. Notably, remittances take on higher values less frequently in the unconsolidated model compared to the consolidated model, particularly at the right tail of the

[6] See here for details on the BoE's remittance policy.

[7] In 2023, the ratio of profit earned by BOJ to tax revenues collected by Japanese Treasury is 7%, showing a significant fiscal support from BOJ to Treasury.

histogram. While the unconsolidated model imposes a lower bound on remittances, this constraint also affects the upper tail of the remittance distribution, making it thinner. Intuitively, if the central bank prints more reserves and transfers more to the Treasury, the excess reserves cannot be reduced through remittances due to the constraint. As a result, the central bank chooses not to transfer more. This behavior aligns with the actual practice of central banks, where remittances are often lower due to retained earnings.

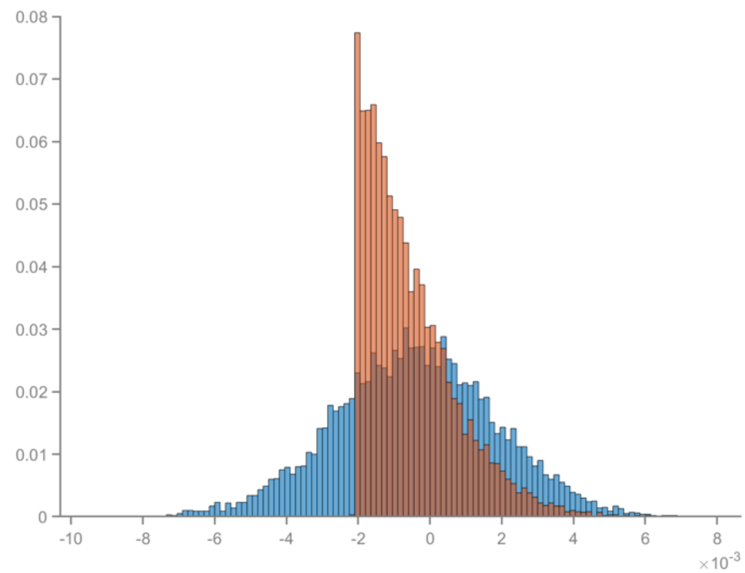


Figure 8: The histogram of remittance in the consolidated and unconsolidated model.

Note: This figure shows a histogram remittance in the consolidated and unconsolidated model. I simulate an economy for 1000 periods and obtain a sequence of remittance for both models. An underlying exogenous shock is cost-push shock. I show it as a histogram after normalization so that an area is equal to one.

7 Welfare analysis

This section aims to analyze the welfare gain of fiscal backing for central bank. Interestingly, different government choose different stance for fiscal backing. In UK, there is agreement between central bank and Treasury that any losses caused by central bank's asset purchase policy is compensated by Treasury. Since the loss created by Bank of England is large, there is discussion about if Treasury should compensate central bank's loss as central bank's loss is essentially compensated by household's tax. The model in this paper includes the cost and benefit of fiscal backing. The cost is that the increase of distortionary tax: the central bank's loss is compensated by distotionary tax. The loss is the impact of monetary and fiscal policy. This section quantitatively evaluates the welfare gain.

7.1 Unconditional welfare loss

I compute unconditional welfare losses under optimal policy, evaluating the utility of households (1), both in the unconsolidated and consolidated model. Welfare losses are obtained averaging the discounted losses across 10000 simulations, of the initial states (s_0, d_0, b_0) from their steady-state values, each 1000 periods long.[8] I obtained welfare loss for two models compared to their own steady-state.

Table 4 shows the welfare cost of business cycle in the consolidated and unconsolidated model. Welfare losses are expressed in terms of their welfare equivalent permanent consumption reduction in percent. The welfare losses associated with cyclical fluctuation is fairly small in absolute size as is usually the case in New Keynesian models.

The difference of welfare loss between the consolidated and unconsolidated model is fairly small. The largest difference is productivity shock, but it is 0.003$\%$ of permanent consumption. The difference is much smaller for public expenditure and cost-push shock.

A good comparison of this model is model with zero lower bound on nominal interest rate. Adam and Billi (2007) shows that the existence of zero lower bound lowers the welfare loss by 0.003$\%$ under the discretionary policy and 0.0001$\%$ under the commitmment policy.

7.2 Conditional welfare loss

This section presents the welfare loss conditional on the size of the shock. This section aims to demonstrate that the larger the shock, the greater the welfare loss from the lack of fiscal backing: the difference in welfare between the consolidated and unconsolidated models.

The procedure is as follows: First, I feed an exogenous path of cost-push shocks into the model. A positive cost-push shock reduces household utility. I then compute the perpetual consumption

[8]To exclude the impact of initial state on welfare calculation, I through away the first 100 periods from the simulated 1000 periods. Also, (s_0, d_0, b_0) is the stochastic steady-state.

Table 4: Unconditional Welfare Loss

	Consolidated	Unconsolidated
Cost-push shock		
	-0.000698	-0.000864
Productivity shock		
	-0.021491	-0.024720
Public expenditure shock		
	-0.002692	-0.003323

Notes: I compute unconditional welfare losses under optimal policy, evaluating the utility of households in both the unconsolidated and consolidated models. Welfare losses are calculated by averaging the discounted losses across 10,000 simulations, with the initial states (s_0, d_0, b_0) set to their steady-state values. Each simulation lasts for 1,000 periods.

transfer necessary to make household utility after the shock equivalent to that of households in the steady state. For instance, with no shock, there is zero welfare loss. If a cost-push shock increases by 2%, households are worse off by 0.02% of consumption. For each shock size, I compute the welfare cost of the shock. Second, I perform the same exercise for both the consolidated and unconsolidated models. The initial conditions for each model are set to the steady-state values of reserves and bonds.

Figure 9 presents the results. The horizontal axis represents the size of the shock, measured as a percentage increase in the wage mark-up, ranging from zero to 9%. The vertical axis shows the welfare cost of the cost-push shock, expressed as a percentage of consumption equivalence. The red line represents the welfare cost in the unconsolidated model, while the blue line represents the cost in the consolidated model.

First, the welfare cost increases with the size of the shock. This occurs because a positive cost-push shock leads to a drop in consumption and an increase in the inflation rate. Second, the welfare cost of the cost-push shock is consistently higher in the unconsolidated model compared to the consolidated model. This is by construction, as the equilibrium in the unconsolidated model is always feasible in the consolidated model, leading to higher welfare in the latter. Third, the gap between the two models widens as the size of the shock increases. For example, when the cost-push shock is a 9% increase, the welfare cost is 0.11% of consumption in the unconsolidated model and 0.9% in the consolidated model. This is because, as the shock grows larger, the constraint on remittances binds for a longer period.

The welfare loss in the unconsolidated model arises from an insufficient tax break due to the lack of fiscal backing. When a cost-push shock occurs, the Treasury seeks to implement a tax break, and the central bank supports this by transferring resources to the Treasury. However, in the unconsolidated model, this support is limited, resulting in an insufficient tax break. As a result, the cost-push shock is not fully mitigated, leading to lower consumption and higher inflation, both of which are costly in terms of welfare.

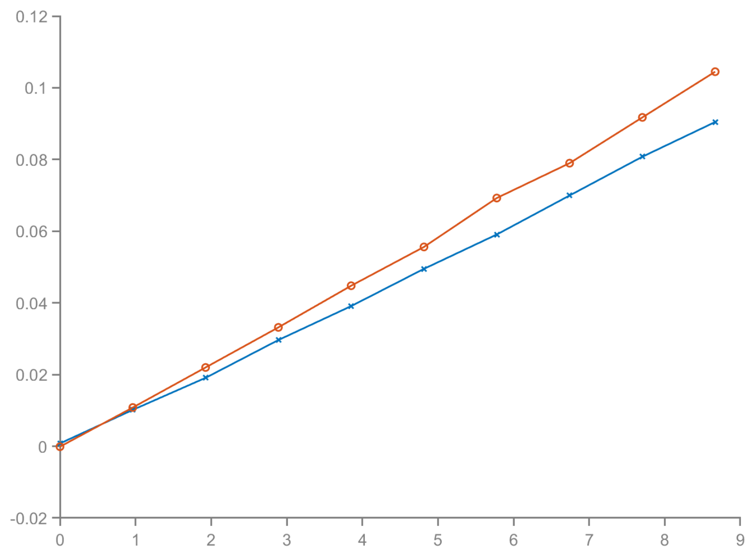


Figure 9: The welfare gain of fiscal backing and size of shock.

Note: This figure shows The welfare gain of fiscal backing and size of shock. The horizontal axis is the size of shock represented by percentage point. The vertical axis is the welfare cost of cost-push shock represented by consumption equivalence in percentage point. The red is the welfare cost in the unconsolidated mode. The blue is that in consolidated model.

8 Conclusion

A common assumption in conventional macroeconomic models of monetary-fiscal policy is a consolidated government budget. However, in practice, the Federal Reserve does not receive funds from Treasury when it incurs a loss. This paper studies optimal monetary and fiscal policy when Treasury is unable to provide optimal fiscal backing to the central bank: the unconsolidated government budget.

I analyze the optimal monetary and fiscal policy in a New Keynesian model with central bank and Treasury, where the transfer of resources from Treasury to the central bank is constrained. This lack of fiscal backing implies: (i) the central bank, without fiscal backing, tolerates higher inflation in response to cost-push shocks, and this inflation response increases with the level of reserves; (ii) while the lack of fiscal backing increases the volatility of the optimal inflation rate by 3%, the average optimal inflation rate is minimally affected; (iii) the welfare gains from fiscal backing are small over the business cycle, but in the case of large shocks, a fiscal backstop reduces the welfare cost of the shock by 20%.

Finally, I discuss several avenues for future research. First, in this model, a single agent, "the government", jointly sets both monetary and fiscal policy. However, my model distinguishes between the central bank and the Treasury, so a natural extension would be to allow the central bank to set monetary policy and the Treasury to set fiscal policy. To implement this, each authority would need its own policy objectives and a specified timing protocol. A plausible assumption is that the monetary authority prioritizes inflation stabilization more than the fiscal authority (Rogoff, 1985).

Second, while this model imposes a lower bound on remittances, my model cannot explain why such a constraint exists. One possible reason could be that the central bank values its independence and seeks to avoid reliance on fiscal backing. Since the benefits of maintaining central bank independence lie outside the model, an extension could involve endogenizing the constraint on remittances.

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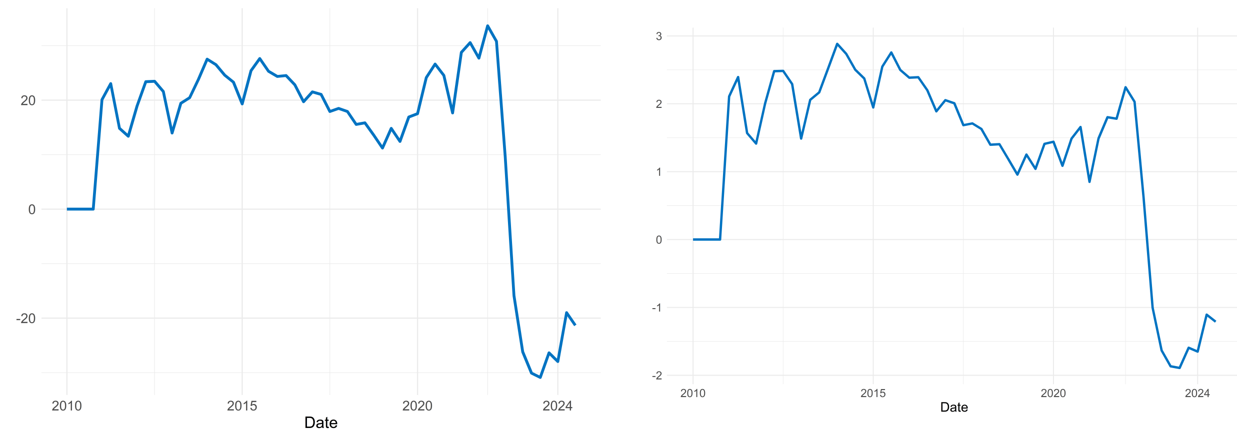
Appendix A Data on Remittance

This section describes the data on remittances between the Treasury and the Federal Reserve. The data are available in FRED as “Earnings Remittances Due to the U.S. Treasury.”. This dataset records weekly remittances to the U.S. Treasury through September 2022. After that date, it reflects the Federal Reserve’s deferred asset, which represents its cumulative loss.

Figure 10 illustrates the transfers between the U.S. Treasury and the Federal Reserve. Before September 2022, I plot the direct “transfers to the U.S. Treasury” as shown in FRED. After September 2022, since FRED shows cumulative losses, I calculate the loss flow by taking the difference between successive data points. In fact, there are no transfers between the Treasury and the Federal Reserve after 2022, so the data for this period reflect the Federal Reserve’s losses. The time horizon is quarterly.

Figure 10-(a) shows the transfers in billions of dollars. Prior to 2022, the Federal Reserve earned an average of about $20 billion per quarter and transferred it to the Treasury. The cumulative transfers from January 2011 to September 2022 are $996 billion, or 5% of annual GDP. After 2022, the losses are about $20 billion. The cumulative loss from September 2022 to October 2024 is $210 billion. To examine the fiscal impact, figure 10-(b) shows the ratio of remittances to federal government spending as a percentage. Before 2022, this ratio averages 2%, indicating that Federal Reserve profits financed about 2% of federal government spending. After 2022, this ratio shifts to -2%, meaning that the Federal Reserve’s losses effectively increase federal government spending by an additional 2%.



(a) In billion dollars.

(b) Divided by Federal Expenditure.

Figure 10: Remittance between Treasury and Federal Reserves

Note: This figure illustrates the remittances between the U.S. Treasury and the Federal Reserve. Before September 2022, I plot "Earnings Remittances Due to the U.S. Treasury" directly, as shown in FRED. After September 2022, because FRED shows cumulative losses, I calculate and plot the flow of losses by taking the difference between successive data points. Figure (a) is in billions of dollars. Figure (b) is remittances divided by federal spending, expressed in percent. The time horizon is quarters.

Appendix B Details on calibration

This section describes details on calibration. Key parameters are γ_1 and γ_2 that deciplines the curvature of utility in convenience yield term.

In the Euler equation for reserves, a price of reserves (Q^C in the left hand side) depends on the quantity of reserves (d_t in the right hand side).

$$\text{(Euler for reserve)} \quad C_t^{-\sigma} Q_t^C = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] + \chi_1 (Q_t^C d_t)^{-\gamma_1} Q_t^C \tag{42}$$

When I take the left-hand side derivetive with respect to d_t, this derivative depends on γ_1. That suggests that the elasticity of price to quantity is deciplined by γ_1. High γ_1 means that the elasticity is higher i.e, the slope of the demand curve is steep. Low γ_1 means that the elasticity is lower i.e, the slope of the demand curve is flat. Although this is not mathematically very precise, I confirmed that in equilibrium, the elasticity of price to quantitiy decreases with γ_1.

The purpose of this calibration is to match the price elasticity in data to that in model. I run regression both in model and data.

In the model, I simulate the economy and obtain the sequence of the left-hand side and $\frac{b_t^C}{Y_t}$ in equation (44) and $\frac{b_t^T}{Y_t}$ in equation (48). Then, I obtain estimated β_1 and β_3 by running time-series regression of (44) and (48). The β_1 and β_3 are negative and decreases with γ_1 and γ_2.

In data, I also estimate equations (46) and (49). For reserves, I control deposits following Vissing-Jorgensen (2023). For bonds, I follow Krishnamurthy and Vissing-Jorgensen (2012). I use data from 2009 to 2020. The frequency of data is quarterly. EFR represents the effective fed fund rate. IOR represents interest rate on reserves. Reserves are 'Reserve Balances with Federal Reserve Banks' in FRED. All data is available on FRED.

For reserves and γ_1

$$\text{model} \quad r_t - i_t = \beta_0 + \beta_1 \log(\frac{d_t}{Y_t}) + \epsilon_t \tag{43}$$

$$r_t \equiv \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right] \frac{1}{C_t^{-\sigma}} \tag{44}$$

$$i_t \equiv \frac{1}{Q_t^d}. \tag{45}$$

$$\text{data} \quad \text{EFR}_t - \text{IOR}_t = \tilde{\beta}_0 + \tilde{\beta}_1 \log\left(\frac{\text{Reserve}_t}{\text{GDP}_t}\right) + \log(\text{Deposit}_t) + \epsilon_t. \tag{46}$$

I proxy the efffective fed fund rate in data as r_t that is the hypothetical nominal interest rate obtained if there is no convenience value for reserves. i_t in the model is the price of reserves when there is a convenience yiled for reserves.

For γ_2 and bonds. For Treasury yield, I use yield of 1 month, 3 month, and 6 month maturity's bonds. Both Treasury yield and effective federal funds rate are annualized. Federal liability is Federal government's liability in FRED.

$$\text{model} \quad r_t - i_t^T = \beta_2 + \beta_3 \log(\frac{b_t}{Y_t}) + \epsilon_t \tag{47}$$

$$i_t^T \equiv \frac{1}{Q_t^b}. \tag{48}$$

$$\text{data} \quad \text{EFR}_t - \text{Teasury yield}_t = \tilde{\beta}_2 + \tilde{\beta}_3 \log\left(\frac{\text{Federal liability}_t}{\text{GDP}_t}\right) + \epsilon_t. \tag{49}$$

Table 5 and 6 show estimated results. In data, the estimated $\tilde{\beta}_1$ are significantly negative, that is a demand curve is downward-sloping. I target $\tilde{\beta}_1 = -0.2$. For bonds, I target $\tilde{\beta}_3 = -0.1$. I choose

Table 5

	Dependent variable:	
	$\text{EFR}_t - \text{IOR}_t$	
	(1)	(2)
log(res_gdp)	-0.165^{***}	-0.297^{***}
	(0.026)	(0.024)
log(dep_gdp)		0.020^{***}
		(0.003)
Constant	0.297^{***}	0.667^{***}
	(0.062)	(0.064)
Observations	36	36
R^2	0.545	0.827
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 6

	Dependent variable:		
	$\text{EFR}_t - 1\ \text{month}_t$	$\text{EFR}_t - 3\ \text{month}_t$	$\text{EFR}_t - 6\ \text{month}_t$
	(1)	(2)	(3)
log(debt_gdp)	0.057	-0.105	-0.321
	(0.102)	(0.136)	(0.220)
Constant	-0.178	0.495	1.361
	(0.447)	(0.596)	(0.963)
Observations	40	40	40
R^2	0.008	0.015	0.053

Notes: ”1 month”, ”3 month”, and ”6 month” represents the yield of each maturities Treasury bonds. *p<0.1; **p<0.05; ***p<0.01

γ_1 and γ_2 so that I have $\tilde{\beta}_1 = -0.2$. and $\tilde{\beta}_3 = -0.1$.

Appendix C Solution methods

I use the collocation method with time iteration. A grid of N interpolation nodes is defined over the state space $S_t \equiv (s_t, b_{t-1}, d_{t-1})$, where s_t represents an exogenous shock out of three exogenous shock, $s_t \in \{\mu_t^w, A_t, G_t\}$. Due to the size of the state space, I include one shock at a time, chosen from cost-push, productivity, or public expenditure shocks. This model has three state variables.

The Markov Perfect Equilibrium consists of policy functions $X(S_t) \equiv \{C(S_t), N(S_t), \pi(S_t), Q^C(S_t), Q^T(S_t), b(S_t), d(S_t), \tau(S_t), H(S_t)\}$ that solves the government's problem when the expectation terms in the Euler equations for reserves and bonds, and New Keynesian Philips Curve are given by

$$E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right] = \int \frac{C(S_{t+1}(\varepsilon^s))^{-\sigma}}{\pi(S_{t+1}(\varepsilon^s))} f(\epsilon^s)d(\epsilon^s) \tag{50}$$

$$E_t\left[C_{t+1}^{-\sigma}\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right] = \int C(S_{t+1}(\varepsilon^s))^{-\sigma}\frac{1+\rho^T Q^T(S_{t+1}(\varepsilon^s))}{\pi(S_{t+1}(\varepsilon^s))} f(\epsilon^s)d(\epsilon^s) \tag{51}$$

$$E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right] = \int N(S_{t+1}(\varepsilon^s))C(S_{t+1}(\varepsilon^s))^{-\sigma}\varphi(\pi(S_{t+1}(\varepsilon^s))-1)\pi(S_{t+1}(\varepsilon^s))f(\epsilon^s)d(\epsilon^s), \tag{52}$$

where

$$S_{t+1}(\varepsilon^s) \equiv (\bar{s} + \rho^s(s_t - \bar{s}) + \varepsilon^s, b_t(S_t), d_t(S_t)), \tag{53}$$

and $f(\epsilon^s)$ is the probability density distribution of innovations.

The expectations are evaluated at each interpolation N nodes using M node Gaussian-Hermite quadrature. My numerical algorithm is as follows:

Step1: Guess initial policy functions for choice variables at the interpolation nodes N. Create policy functions for two cases: binding policy functions and non-binding policy functions. Denote them $X^{bind}(S_t)$ and $X^{non-bind}(S_t)$

Step2: (a) At each interpolation nodes N, compute the expectations (50)-(52) implied by the current guessed policy functions. If $H(s_{t+1}, d_t, b_t) > H^*$, the policy functions for non-binding case, $X^{non-bind}(S_t)$, are used to evaluate the expectation. Otherwise,the policy functions for binding case, $X^{bind}(S_t)$, are used to evaluate the expectation.

(b) Given the expectations for the Euler equations and New Keynesian Phillips Curve, I derive the first-order conditions for the optimal policy problem consisting of (30)-(40). The first-order conditions with respect to b_t and d_t are given by

$$\chi_1(Q_t^C d_t)^{-\gamma_1}Q_t^C - \phi_t^1\chi_1(-\gamma_1)(Q_t^c)^2(Q_t^C d_t)^{-\gamma_1-1} + \phi_t^6 Q_t^C - E_t\left[\frac{\phi_{t+1}^6}{\pi_{t+1}}\right]$$
$$+\frac{\partial E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right]}{\partial d_t} + \frac{\partial E_t\left[C_{t+1}^{-\sigma}\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right]}{\partial d_t} + \frac{\partial E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right]}{\partial d_t} = 0, \quad (54)$$

$$\chi_2((1-\alpha)Q_t^T b_t)^{-\gamma_2}(1-\alpha)Q_t^T - \phi_t^2\chi_2(-\gamma_2)((1-\alpha)Q_t^T)^2((1-\alpha)Q_t^T b_t)^{-\gamma_2-1}$$
$$+\phi_t^5 Q_t^T - E_t\left[\phi_{t+1}^5\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right]$$
$$+\frac{\partial E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right]}{\partial b_t} + \frac{\partial E_t\left[C_{t+1}^{-\sigma}\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right]}{\partial b_t} + \frac{\partial E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right]}{\partial b_t} = 0, \quad (55)$$

where ϕ_t^1 is the Lagrange multiplier on equation (31), ϕ_t^2 is that on (32), ϕ_t^5 is that on (35), and ϕ_t^6 is that on (36). Those numerical derivatives,

$$\frac{\partial E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right]}{\partial b_t}, \quad \frac{\partial E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right]}{\partial d_t},$$
$$\frac{\partial E_t\left[C_{t+1}^{-\sigma}\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right]}{\partial b_t}, \quad \frac{\partial E_t\left[C_{t+1}^{-\sigma}\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right]}{\partial d_t},$$
$$\frac{\partial E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right]}{\partial b_t}, \quad \frac{\partial E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right]}{\partial d_t}., \quad (56)$$

can be directly computed given the guessed policy functions for C_t, N_t, π_t, and Q_t^T. However, to avoid the numerical inaccuracy, I use

$$\frac{\partial E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right]}{\partial b_t} = E_t\left[-\sigma C_{t+1}^{-\sigma-1}\pi_{t+1}^{-1}\frac{\partial C_{t+1}}{\partial b_t}\right] + E_t\left[C_{t+1}^{-\sigma}(-1)\pi_{t+1}^{-2}\frac{\partial \pi_{t+1}}{\partial b_t}\right]$$
$$\frac{\partial E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right]}{\partial d_t} = E_t\left[-\sigma C_{t+1}^{-\sigma-1}\pi_{t+1}^{-1}\frac{\partial C_{t+1}}{\partial d_t}\right] + E_t\left[C_{t+1}^{-\sigma}(-1)\pi_{t+1}^{-2}\frac{\partial \pi_{t+1}}{\partial d_t}\right]$$

$$
\begin{aligned}
&\frac{\partial E_t\left[C_{t+1}^{-\sigma}\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right]}{\partial b_t} \\
&= E_t\left[-\sigma C_{t+1}^{-\sigma-1}\pi_{t+1}^{-1}(1+\rho^T Q_{t+1}^T)\frac{\partial C_{t+1}}{\partial b_t}\right] \\
&+ E_t\left[C_{t+1}^{-\sigma}(-1)\pi_{t+1}^{-2}(1+\rho^T Q_{t+1}^T)\frac{\partial \pi_{t+1}}{\partial b_t}\right] + E_t\left[C_{t+1}^{-\sigma}\pi_{t+1}^{-1}\rho^T\frac{\partial Q_{t+1}^T}{\partial b_t}\right] \\
&\frac{\partial E_t\left[C_{t+1}^{-\sigma}\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right]}{\partial d_t} \\
&= E_t\left[-\sigma C_{t+1}^{-\sigma-1}\pi_{t+1}^{-1}(1+\rho^T Q_{t+1}^T)\frac{\partial C_{t+1}}{\partial d_t}\right] \\
&+ E_t\left[C_{t+1}^{-\sigma}(-1)\pi_{t+1}^{-2}(1+\rho^T Q_{t+1}^T)\frac{\partial \pi_{t+1}}{\partial d_t}\right] + E_t\left[C_{t+1}^{-\sigma}\pi_{t+1}^{-1}\rho^T\frac{\partial Q_{t+1}^T}{\partial d_t}\right] \\
&\frac{\partial E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right]}{\partial b_t} \\
&= E_t\left[C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\frac{\partial N_{t+1}}{\partial b_t}\right] \\
&+ E_t\left[N_{t+1}(-\sigma)C_{t+1}^{-\sigma-1}\varphi(\pi_{t+1}-1)\pi_{t+1}\frac{\partial C_{t+1}}{\partial b_t}\right] + E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(2\pi_{t+1}-1)\frac{\partial \pi_{t+1}}{\partial d_t}\right] \\
&\frac{\partial E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right]}{\partial d_t} \\
&= E_t\left[C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\frac{\partial N_{t+1}}{\partial d_t}\right] \\
&+ E_t\left[N_{t+1}(-\sigma)C_{t+1}^{-\sigma-1}\varphi(\pi_{t+1}-1)\pi_{t+1}\frac{\partial C_{t+1}}{\partial d_t}\right] + E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(2\pi_{t+1}-1)\frac{\partial \pi_{t+1}}{\partial d_t}\right]
\end{aligned}
$$

I assume that $E_t[\frac{\partial f(s_{t+1})}{\partial b_t}] = \partial E_t[f(s_{t+1})]/\partial b_t$, which is valid due to the Interchange of Integration and Differention Theorem. Those are substituted into the first-order conditions with respect to b_t and d_t, equations (54) and (55).

(c) Given those numerical derivatives, find choice variables that satisfies the first-order conditions. The first-order conditions are obtained for two cases

   (i) H_t is not constrained.

   (ii) Substitute $H_t = H^*$ into the implementable budget constraint, and take the first-order condition except for H_t.

The first-order conditions with respect to b_t and d_t include the forward-looking terms for governments,

$$E_t\left[\frac{\phi^6_{t+1}}{\pi_{t+1}}\right] \tag{57}$$

$$E_t\left[\phi^5_{t+1}\frac{1+\rho^T Q^T_{t+1}}{\pi_{t+1}}\right], \tag{58}$$

Those expectations are evaluated using the guessed policy functions

$$\phi^5_{t+1}(s_{t+1}, d_t, b_t), \quad \phi^6_{t+1}(s_{t+1}, d_t, b_t), \quad \pi(s_{t+1}, d_t, b_t), \quad Q^T(s_{t+1}, d_t, b_t)$$

at the points of $\{s_{t+1}, d_t, b_t\}$. If $H(s_{t+1}, d_t, b_t) > H^*$, the policy functions for non-binding case, $X^{non-bind}(S_t)$, are used to evaluate the expectation. Otherwise,the policy functions for binding case, $X^{bind}(S_t)$, are used to evaluate the expectation.

Given the numerical derivatives, (57), and (58), the choice variables are obtained by solving the non-linear system of equations using *fsolve*.

Choice variables for the case (i) is used for the updated non-binding policy functions, $X^{non-bind}(S_t)$. Choice variables for the case (ii) is used for the updated binding policy functions, $X^{bind}(S_t)$.

Step3: Compute the expectations given the new policy functions. Repeat step 2 until the guessed policy functions and updated policy functions are close enough.

This paper focuses on optimal policy under discretion. Although it would be interesting to solve for the optimal policy under commitment, doing so would involve solving a model with six state variables: the shock, reserves, bonds, and three Lagrange multipliers for the forward-looking equations (Marcet and Marimon, 2019). This significantly increases computational complexity, making it highly challenging.

Appendix D Initial conditions and transition dynamics after shocks

The goal of this section is to understand how the initial levels of reserves and bonds affect the dynamics after a shock. In section 5, the economy does not start with steady-state levels of reserves and bonds. For the cost-push and productivity shocks, the economy starts with reserves that motivate the central bank to reduce reserves and require transfers from the Treasury. For the government spending shock, the economy starts with a large amount of bonds, in which case the Treasury does not smooth taxes by issuing additional bonds.

Section D.1 examines the dynamics following a shock when the economy starts from a steady state. In this scenario, the difference between the consolidated and unconsolidated models is smaller than in the results presented in section 5.

Section D.2 examines the impact of the central bank's assets. While the exercise in section 5 focuses on the impact of the central bank's liabilities, section D.2 examines the impact of the central bank's assets on fiscal backing. The effect of assets is ambiguous. The central bank's assets consist of long-dated assets, so when the nominal interest rate rises, the price of these assets falls significantly, resulting in capital losses. If the central bank holds large assets, it will incur larger capital losses, which may require transfers from the Treasury. However, holding larger assets also means that the central bank could sell assets rather than rely on transfers from the Treasury. In addition, while a lower price for long-term bonds results in capital losses, it also allows the central bank to purchase assets at reduced prices, which benefits the central bank's asset position. Section D.2 examines each of these mechanisms.

D.1 Steady-state as an initial condition

D.1.1 Productivity shock

This section describes the dynamics after a positive public expenditure shock. The initial condition for reserves and bonds are stead-state level, unlike the exercise in section 5.1 where the economy starts from large reserves. The goal of this section is to show the difference between the unconsolidated and consolidated model depends on the initial conditions.

Figure 11 shows results. The key difference between 1 and 11 are how long remittance is binding in the unconsolidated model. In figure 11, reserves are smaller and resulting interest payment is smaller. The central bank does not require funds from Treasury a lot, so the lack of fiscal backing does not impact the dynamics a lot. In figure 1, initial reserves are large. Central bank needs funds to reduce it. In addition, a shock happens and nominal interest rate increases. That requires additional funds from Treasury. To balance the budget, central bank cannot raise nominal interest rate in the unconsolidated model. Consumption does not contract enough and inflation rate is not stabilized. As a result, the lack of fiscal backing makes a difference.

D.1.2 Public expenditure shock

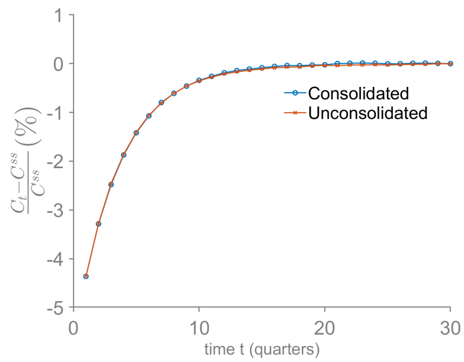
This section describes the dynamics after a positive public expenditure shock. The initial condition for reserves and bonds are stead-state level, unlike the exercise in section 5.3 where the economy starts from large bonds. The goal of this section is to show the difference between the unconsolidated and consolidated model depends on the initial conditions.

Figure 12 shows results. The key different from figure 12 and figure 3 is the dynamics for sales tax. In figure 12, the unconsolidated model shows that the tax smoothing is limited; the initial response of tax rate in the unconsolidated model is larger than that in the consolidated model. However, the overshooting of tax rate in the unconsolidated model is smaller in figure 12 than that in figure 3. This is because Treasury holds relatively smaller bonds in exercise of figure 12. Treasury has more capacity to issue bonds more, and can smooth tax even though support from central bank is limited.

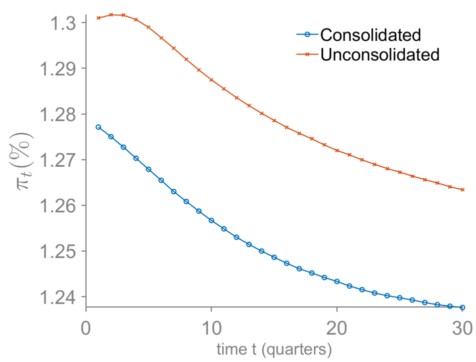
D.2 Role of central bank's assets

D.2.1 Productivity shock

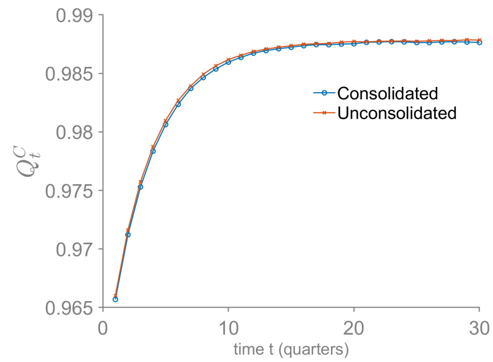
This section aims to understand the role of central bank's assets as an initial condition. The central bank's asset has an ambiguous impact on the budget upon the shock. Suppose central bank has larger assets than steady-state value. When the nominal interest rate increases, the price of long-duration bonds drops more. This leads to capital loss for central bank. As central bank holds larger assets, the capital loss is larger. In contrast, larger asset means that central bank can sell assets when its



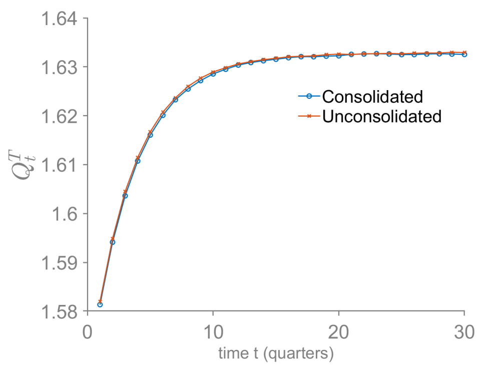
(a) Consumption



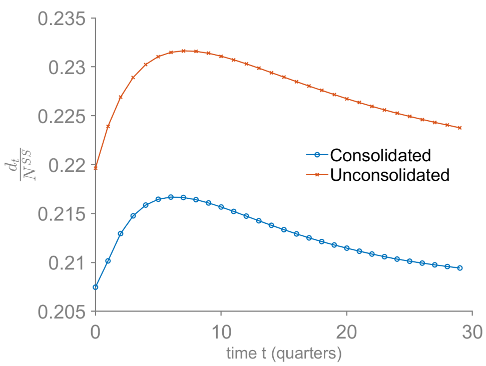
(b) Inflation



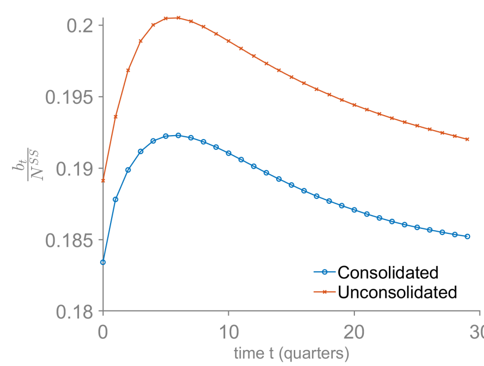
(c) Price of reserves



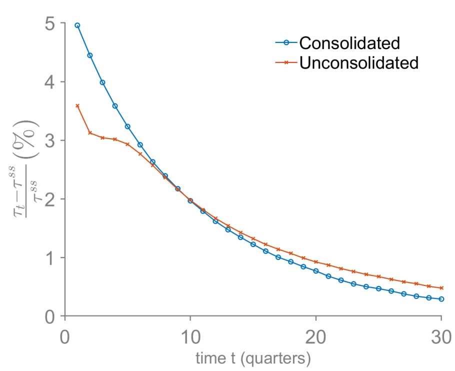
(d) Price of bonds



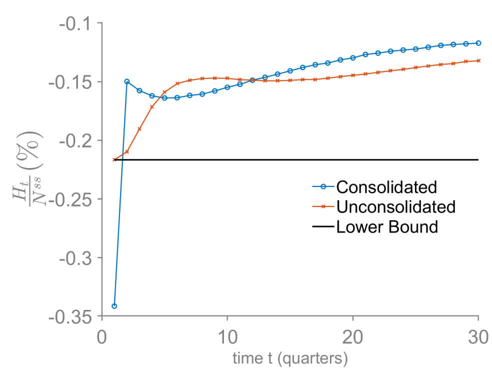
(e) Reserves



(f) Bonds



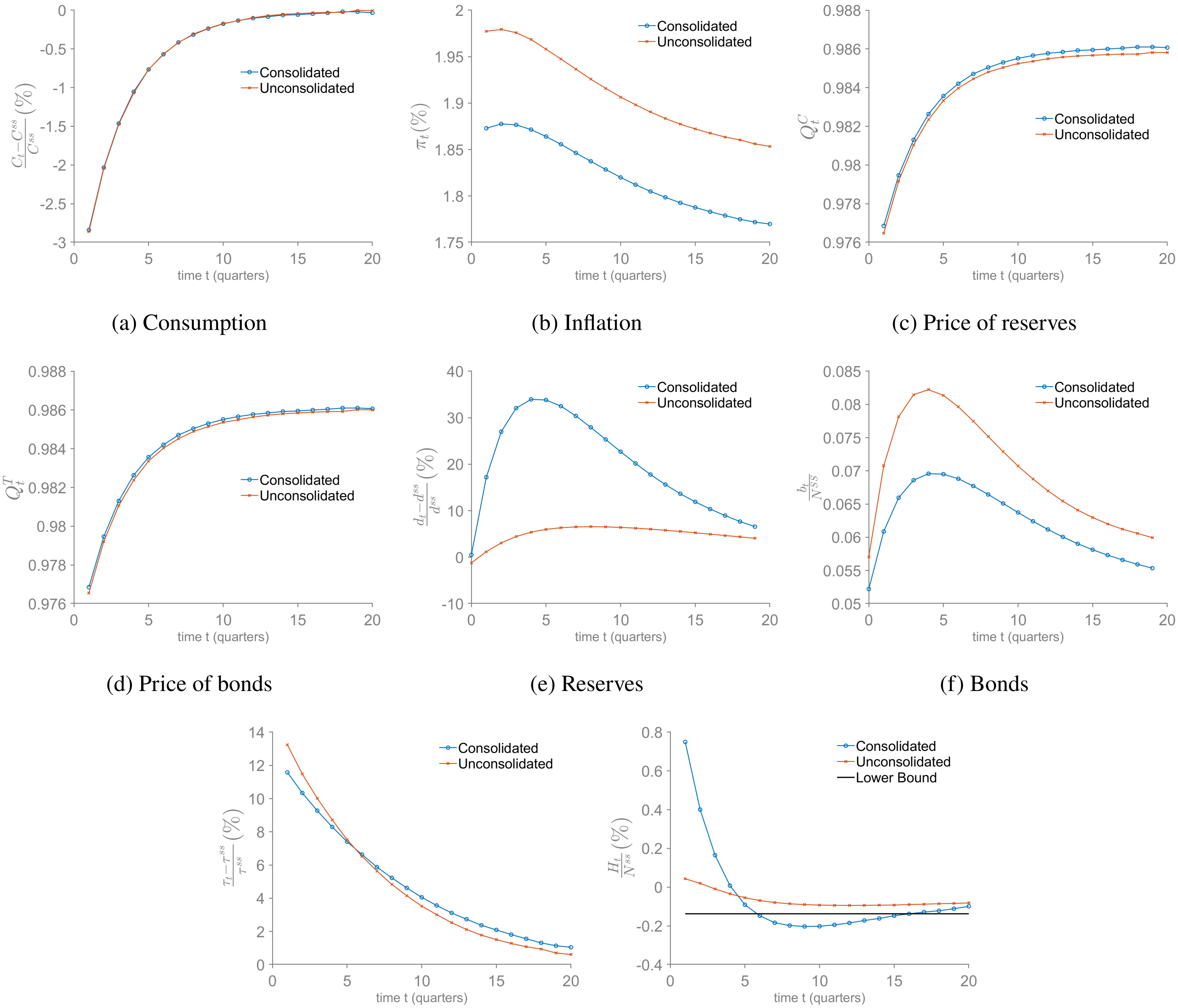
(g) Sales tax



(h) Remittance

Figure 11: Productivity shock

Note: This figure shows the dynamics of key variables upon the negative productivity shock by 5%. A productivity drops from 1 to 0.95. The horizontal axis is quarters. The vertical axis for consumption, prices of reserves, prices of bonds, reserves, bonds are the steady-state deviation represented by percent. The vertical axis for inflation rate is quarterly inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.



(a) Consumption (b) Inflation (c) Price of reserves

(d) Price of bonds (e) Reserves (f) Bonds

(g) Sales tax (h) Remittance

Figure 12: Public expenditure shock

Note: This figure shows the dynamics of key variables upon the positive public expenditure shock by 2%. The horizontal axis is quarters. The vertical axis for consumption, prices of reserves, prices of bonds, reserves, bonds are the steady-state deviation represented by percent. The vertical axis for inflation rate is annualized inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

budget is tightened. That help central bank's budget.

Figure 13 shows the transition dynamics following a negative productivity shock, with initial conditions of larger reserves and fewer bonds. After the shock, the central bank holds fewer bonds as assets. The difference between the consolidated and unconsolidated models is more pronounced here than in Figure 1. With fewer assets, the central bank needs to increase its asset holdings. To purchase assets, transfers from the Treasury are required, but these are constrained. Consequently, the central bank issues more reserves to finance asset purchases, resulting in excess reserves. Since households are unwilling to absorb the excess reserves, the central bank aims to lower the nominal interest rate, thereby amplifying the difference between the two models.

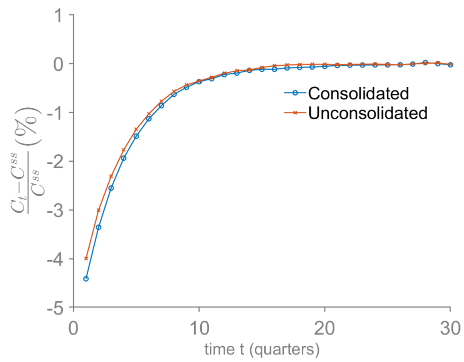
Appendix E Size of shock and reserves as an initial condition

E.1 Cost-push shock

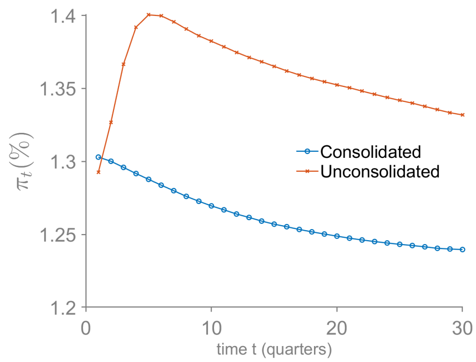
Size of shock This paragraph does the same exercise as in section 5.4 and replace the shock by cost-push shock. Figure 14 illustrates the difference in inflation between the consolidated and unconsolidated models for various shock sizes. I plot $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. The inflation rate is simulated based on this exogenous shock, with the same exercise conducted for shocks of 8%, 4%, 0%, -4%, and -6%.

Figure 14 also shows that the difference of inflation between the two models is large when the size of shock is large. When the shock increases wage mark-up by 8%, the difference of inflation is 0.1% at $t = 1$. In contrast, the shock increases wage mark-up by 4%, the difference is 0.06% t $t = 1$. As in the results of section 5.4, the larger shock leads to larger difference of inflation between the two models.

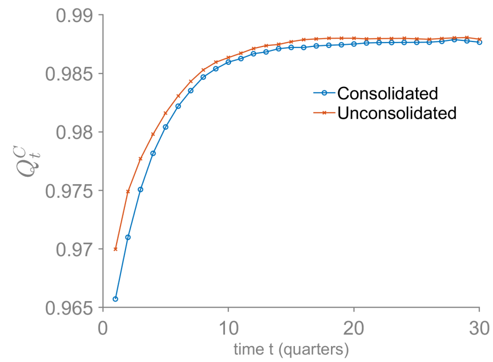
Figure 15 shows the inflation differential between the consolidated and unconsolidated models for different sizes of productivity shocks. The horizontal axis represents the size of the exogenous shock, while the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$ at $t = 1$, where the inflation differential peaks. For each shock size on the horizontal axis, I compute the transition dynamics of the inflation rates, π_t^{unc} and π_t^{con}, and plot $\pi_t^{unc} - \pi_t^{con}$ at $t = 1$ on the vertical axis.



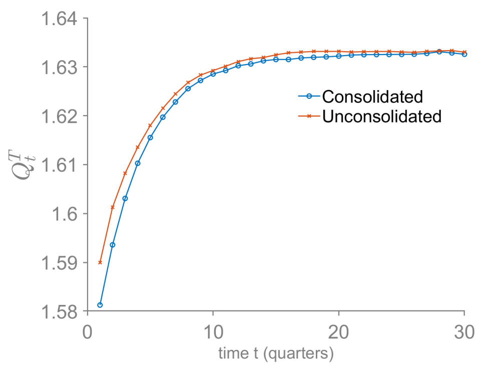
(a) Consumption



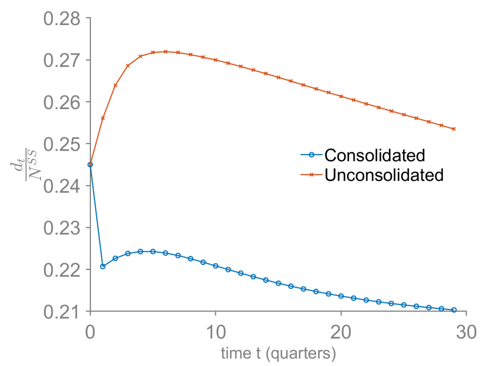
(b) Inflation



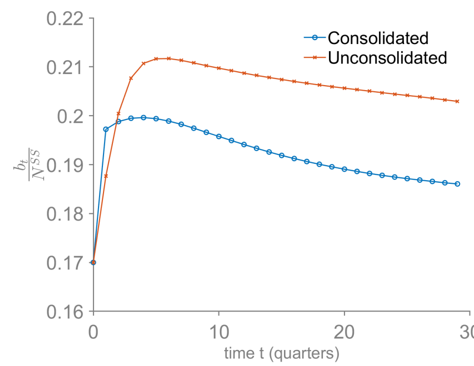
(c) Price of reserves



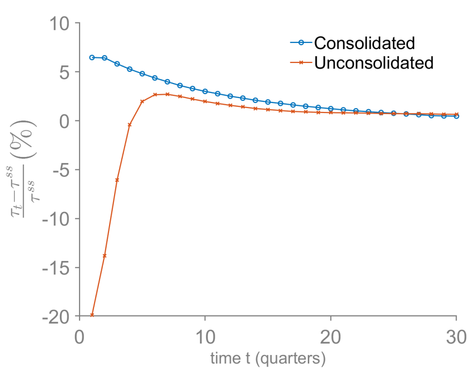
(d) Price of bonds



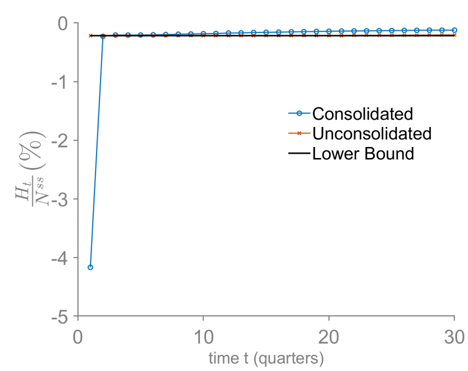
(e) Reserves



(f) Bonds



(g) Sales tax



(h) Remittance

Figure 13: Productivity shock

Note: This figure shows the dynamics of key variables upon the negative productivity shock by 5%. A productivity drops from 1 to 0.95. The horizontal axis is quarters. The vertical axis for consumption, prices of reserves, prices of bonds, reserves, bonds are the steady-state deviation represented by percent. The vertical axis for inflation rate is annualized inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

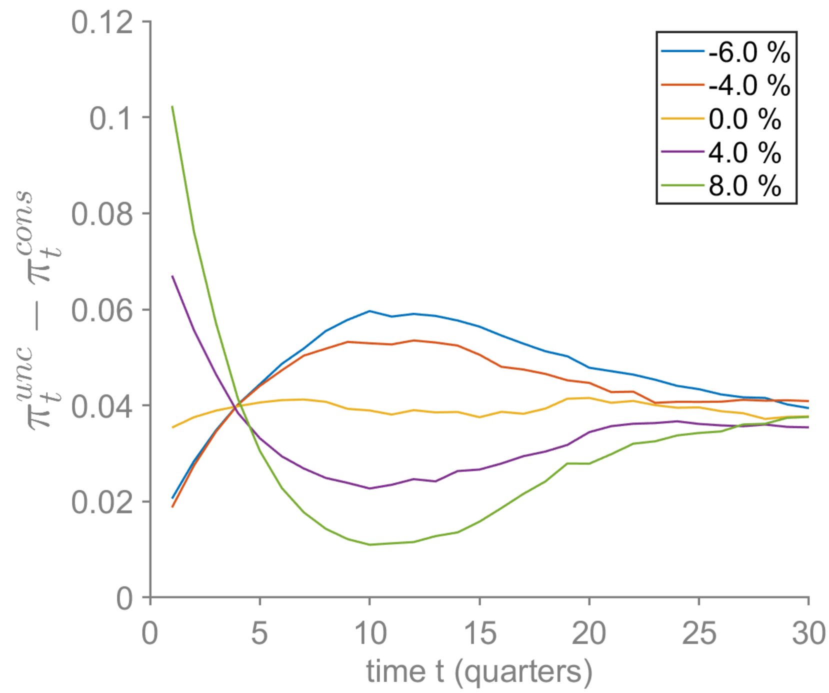


Figure 14: Difference in inflation between consolidated and unconsolidated model for cost-push shock

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various shock sizes. The horizontal axis represents time, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. Different lines represent different shock sizes. For example, a 8% positive cost-push shock occurs at time $t = 1$ and follows an AR(1) process. I simulate the inflation rate based on this exogenous shock. The same exercise is conducted for shocks of 8%, 4%, 0%, -4%, and -6%.

Reserves as an initial condition This section performs the same exercise as in section 5.5 and replaces the shock with a cost-push shock. Figure 16 illustrates the difference in inflation between the consolidated and unconsolidated models for different levels of reserves as initial condition. I plot $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. The inflation rate is simulated based on the positive cost-push shock that increases the wage premium by 8%, with the same exercise performed for different levels of reserves. The initial conditions include reserves and bonds, with different levels of reserves: 10% above steady state, 5% above steady state, 5% below steady state, and 10% below steady state (all values are expressed as deviations from steady state).

Figure 16 shows that larger reserves in the initial condition lead to larger inflation differences

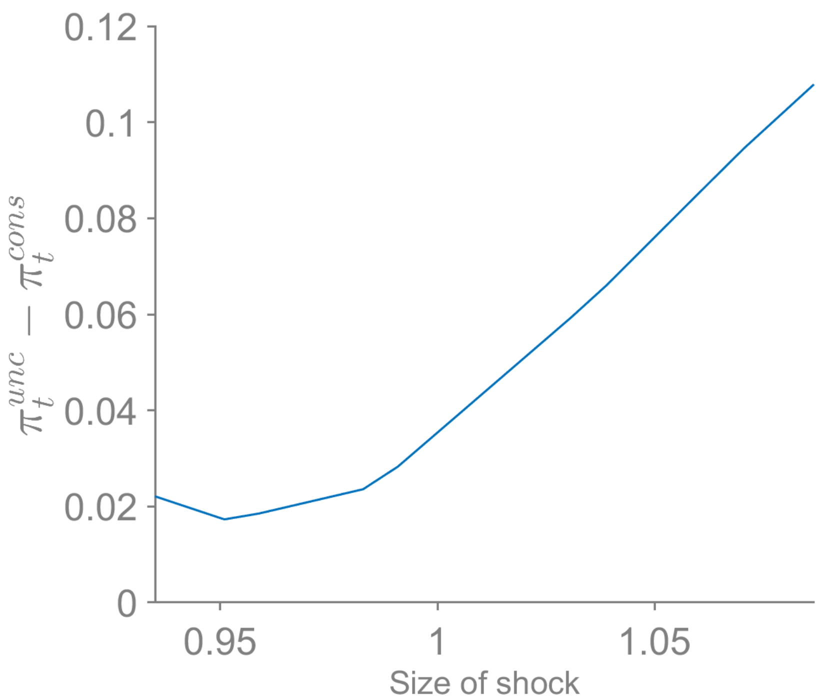


Figure 15: Difference in inflation between consolidated and unconsolidated model for cost-push shock

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various shock sizes. The horizontal axis represents the size of an exogenous shock, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$ at $t = 4$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. For example, when the productivity drops to 0.96, $\pi_t^{unc} - \pi_t^{con}$ at $t = 4$ is 0.028%.

between the two models. For example, if initial reserves are 10% higher than the steady state, $\pi_t^{unc} - \pi_t^{con}$ at $t = 1$ is 0.14%. In contrast, if the initial reserves are at the steady state level, $\pi_t^{unc} - \pi_t^{con}$ at $t = 1$ is 0.11%.

In particular, when the initial reserves are larger, the difference between the two models is larger. For example, if initial reserves are at steady state, $\pi_t^{unc} - \pi_t^{con}$ at $t = 20$ is 0.02%, which means the difference is smaller. In contrast, if initial reserves are 10% higher than steady state, $\pi_t^{unc} - \pi_t^{con}$ at $t = 20$ is still 0.06%. As initial reserves increase, the difference between the two models persists.

Figure 17 plots $\pi_t^{unc} - \pi_t^{con}$ for different initial reserve levels at $t = 1$ and $t = 4$. The horizontal axis represents the initial reserve levels as a percentage deviation from the steady state. The vertical axis shows $\pi_t^{unc} - \pi_t^{con}$ at $t = 4$, where the inflation difference is largest. The procedure is as follows: I compute $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points)

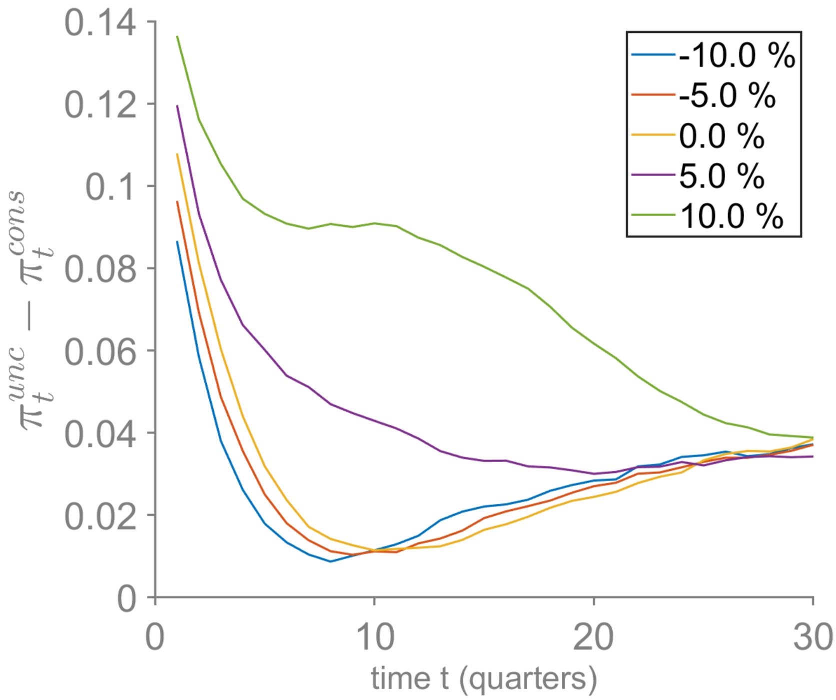


Figure 16: Difference in inflation between consolidated and unconsolidated model for productivity shock

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various initial reserves. The horizontal axis represents time, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. Different lines represent different initial reserves. The exercise is conducted for shocks of 10% above the steady state, 5% above the steady state, 5% below the steady state, and 10% below the steady state (all values are expressed as deviations from the steady state)

after a shock in the unconsolidated model, and π_t^{con} is the same in the consolidated model.

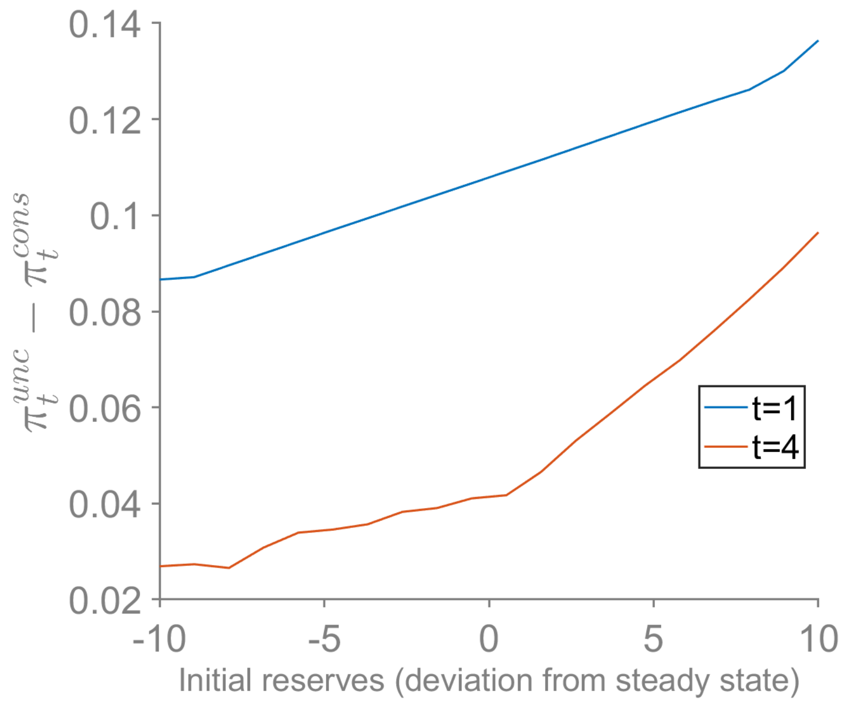


Figure 17: Difference in inflation between consolidated and unconsolidated model for cost-push shock

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various initial reserves. The horizontal axis represents time, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. Different lines represent different initial reserves. The exercise is conducted for shocks of 10% above the steady state, 5% above the steady state, 5% below the steady state, and 10% below the steady state (all values are expressed as deviations from the steady state)