Optimal monetary and fiscal policy without fiscal backing*

Masayuki Okada [†]
New York University

Job Market Paper

October 31, 2024
Please click here for the most recent version.

Abstract

This paper studies optimal monetary and fiscal policy when Treasury is unable to provide optimal fiscal backing to the central bank. The central bank holds large reserves and incurs interest expenses. In practice, during periods when costs exceed earnings, the Federal Reserve does not receive transfers from Treasury to offset the losses. I analyze the optimal monetary and fiscal policy in a New-Keynesian model with central bank and Treasury, where the transfer of resources from Treasury to the central bank is constrained, but not vice versa. This lack of fiscal support implies: (i) the central bank, without fiscal backing, tolerates higher inflation in response to cost-push shocks, and this inflation response increases with the level of reserves; (ii) while the lack of fiscal support increases the volatility of the optimal inflation rate by 3%, the average optimal inflation rate is minimally affected; (iii) the welfare gains from fiscal support are small over the business cycle, but in the case of large shocks, a fiscal backstop reduces the welfare cost of the shock by 20%.

*I am deeply indebted to my advisor Ricardo Lagos for his invaluable guidance and support. I am grateful to Jaroslav Borovička, Venky Venkateswaran, Kosuke Aoki, Corina Boar, Mark Gertler, Simon Gilchrist, Nobu Kiyotaki, Virgiliu Midrigan, Tom Sargent, and Kenji Wada for insightful discussion.

[†]Email: mo2192@nyu.edu

1 Introduction

A common assumption in conventional models of monetary-fiscal policy is a consolidated government budget (Sargent and Wallace, 1981). Treasury finances public expenditures through taxes and bonds. Central bank issues money and reserves to provide liquidity. Under the consolidated budget assumption, there is no need to separately satisfy the Treasury's and the central bank's budget. Monetary and fiscal policy should satisfy the unified budget incorporating the Treasury and central bank: the consolidated government budget.

The starting point of this paper is to question the assumption of consolidated government budgets. In practice, after the inflationary shock in 2022, the Federal Reserve incurs significant interest costs by paying nominal interest (5.5% in 2023) on large reserves (15% of GDP in 2023). These costs have exceeded income, resulting in losses for the Federal Reserve[1]. Facing a high interest expenses, Treasury does not transfer funds to Federal Reserve to offset its losses. This lack of fiscal support from Treasury requires the Federal Reserve to satisfy its budget by their own income and costs. Conventional models of consolidated government budgets are inappropriate for studying situations in which the central bank's budget is not fully consolidated with Treasury. This paper defines the government budget as unconsolidated if the Treasury does not provide optimal fiscal support to the central bank.

This paper studies the implications of the lack of optimal fiscal support for central bank on the monetary and fiscal policy. The goal is to compare equilibrium in the model with fiscal backing to the model without fiscal backing. I develop the optimal monetary-fiscal policy problem under discretion where the government chooses policies to maximize household utility subject to implementable equilibrium conditions. While the equilibrium conditions are based on the standard New Keynesian models, the novelties are that 1. the government can issue two liabilities: bonds as in the conventional model, reserves as a new ingredient. 2. transfers from Treasury to central bank is constrained, while the reverse is not. In practice, under the Federal Reserve Act, the Federal Reserve is required to transfer profits to the Treasury when income exceeds expenses. The transfer from central bank to Treasury is unconstrained. This contrasts sharply with the lack of transfers from Treasury to central bank during periods when expenses exceed income.

[1]In US, the cumulative net loss from September 2022 through October 2024 is $210 billion; 0.7% of GDP.

Section 2 presents the sticky-price New Keynesian model, which includes Treasury and central bank. Households consume, work, and trade government bonds and reserves. Government bonds and reserves are saving technologies, and they also provide liquidity value. Firms follow the standard New Keynesian framework with adjustment cost, using labor as input, and facing cost push shock and productivity shock. Treasury finances public expenditures through distortionary taxes on firms' sales and government bonds. In addition to this standard tax smoothing mechanism, my model includes the remittance from central bank, allowing the Treasury to rely on transfers. The central bank supplies reserves to the market and pays interest. A high nominal interest rate on large reserves tightens the central bank's budget, making transfers from the Treasury optimal.

Section 3 describes the optimal policy problem under discretion. An agent, called the government, simultaneously decides on fiscal and monetary policy to maximize the utility of households subject to equilibrium conditions. Two models are defined: *consolidated model* and *unconsolidated model*. The difference between the two models is the constraint on transfers. The consolidated model assumes no constraints on remittances and follows a standard New Keynesian framework with the consolidated government budget: optimal transfers are feasible. Central bank is fiscally backed by Treasury, allowing interest payments by the central bank to be financed through transfers from the Treasury. In contrast, the unconsolidated model is a model with a constraint on transfers, meaning the two budgets are not fully consolidated. A constraint on transfers prevents the central bank from reducing large reserves. A high nominal interest rate on large reserves does not satisfy the central bank's budget constraint. The lack of fiscal support limits the central bank to raise the nominal interest rate sufficiently.

Section 4 describes solution methods and calibration. Section 5 examines the dynamics of the economy in response to a negative productivity shock, a positive cost-push shock, and a positive public expenditure shock. The first main finding is that, in response to a negative productivity shock, the "divine coincidence" (Blanchard and Galí, 2007) no longer holds. A negative productivity shock raises marginal costs, leading to a higher inflation rate. The central bank raises the nominal interest rate to counteract this. In the consolidated model, the central bank raises the nominal interest rate sufficiently to align the policy rate with the natural rate. As a result, both the output gap and the inflation response shrink to zero. In the unconsolidated model, however, the central bank cannot raise the interest rate enough because it has limited funds to cover interest expenses. Since the nominal interest rate is not raised enough, consumption falls less than in the consolidated model, and inflation rate responds more.

The second main result is that the central bank in the unconsolidated model tolerates a higher inflation rate following a positive cost-push shock. This mechanism is in sharp contrast to the response after a negative productivity shock, where the lack of fiscal backing limits optimal monetary policy. In the case of a cost-push shock, however, the lack of *fiscal* backing constrains optimal *fiscal* policy. A cost-push shock raises marginal costs, and the most effective policy instrument is the sales tax rate rather than the nominal interest rate. To bring the marginal product closer to the elevated marginal cost, the government lowers the sales tax rate. In the consolidated model, this reduction in tax revenue motivates the central bank to support the Treasury by issuing more reserves, creating resources, and transferring them to the Treasury. In the unconsolidated model, however, the central bank knows that it cannot reduce large reserves through remittances, which limits its ability to create resources by issuing additional reserves. Consequently, the central bank's support for the Treasury is reduced. This leads to a smaller tax cut in the unconsolidated model, resulting in a higher marginal cost and a higher inflation rate. The lack of fiscal support that limits the tax break is the primary source of inefficiency in response to a cost-push shock.

Section 6 examines the dynamic properties of optimal policies over the business cycle. I analyze the volatility of the optimal inflation rate (Chari, Christiano, and Kehoe, 1991; Schmitt-Grohé and Uribe, 2004). A key question addressed is how much inflation the government uses to inflate nominal government liabilities. While inflation volatility is useful to inflate nominal liabilities, the volatility incurs costs due to nominal frictions. I find that the lack of fiscal backing increases the volatility of the optimal inflation rate by 3%. Since the lack of fiscal support limits the optimal monetary and fiscal policy to stabilize the shock, the inflation rate is more volatile.

What happens to average inflation rate? The average optimal inflation rate in the unconsolidated model is 0.05% higher than in the consolidated model. Higher inflation rate is optimal because reserves cannot be reduced by remittance, so the government reduce nominal reserves by higher inflate rate. However, as found in Schmitt-Grohé and Uribe (2004), the benefit of reducing liabilities by inflation is much smaller than the cost of nominal friction. The lack of fiscal backing impacts average inflation rate to a much less though it impacts the volatility.

Section 7 evaluates the welfare gain of fiscal backing for the central bank. The central banks including Bank of England and Reserve Bank of New Zealand incur their loss and Treasury pays to offset the losses. Central bankers show concerns for this policy because of financial independence reason, but others show the advantage because it increases central banks net worth [2]. This paper evaluates the gain of the fiscal backing for the central bank based on only a normative perspective. I find that the welfare gain of fiscal backing for the central bank over the business cycle is small, less than 0.01% of consumption equivalence. Although the volatility of macroeconomics variables are affected by the fiscal backing, the mean is hardly affected. Since the first-moment matters for the welfare analysis, the welfare gain of fiscal backing is small.

Related literature First, this article is closely related to large literature on monetary-fiscal policy interactions (Sargent and Wallace (1981); Leeper (1991); Sims (1994); Woodford (2001), among others). Sargent and Wallace (1981) is a seminal work that highlights the implications of a consolidated government budget, showing that the Treasury's budget constraints the central bank's ability to control inflation. Since then, the assumption of a consolidated government budget constraint has become a common framework in the literature. This paper questions that assumption by considering the current practice, where trasfers between the central bank and the Treasury has effectively stopped.

More recent studies have focused on the unconsolidated budgets of Treasury and central bank. These papers study various issues, including central bank solvency (Hall and Reis, 2015; Bassetto and Messer, 2013; Bassetto and Sargent, 2020), price determinacy (Del Negro and Sims, 2015; Benigno and Nisticò, 2020), the effectiveness of central bank asset purchase policies (Benigno, 2020), the impact of helicopter drops (Amador and Bianchi, 2023), the projected path of central bank net worth and earnings (Christensen, Lopez, and Rudebusch, 2015; Carpenter, Ihrig, Klee, Quinn, and Boote, 2018), and the empirical relationship between central bank losses or negative net worth and monetary policy conduct (Stella, 2005, 2008; Goncharov, Ioannidou, and Schmalz, 2023). This paper is the first to focus on the lack of fiscal backing for central bank. Also, this is the first paper to take a normative perspective, while the literature studies in a positive perspective. A novel finding of this paper is that the lack of fiscal backing limits the optimal fiscal policy, although fiscal policy is overlooked in the literature.

[2]See Floden (2022) andBullock (2022)

Previous literature has focused on estimating how much seigniorage would be required to balance the central bank's budget, finding that the required inflation rate is substantial (Del Negro and Sims, 2015). In contrast, this paper examines a situation in which the government can optimally choose not only inflation, but also the nominal interest rate, asset purchase policy, and tax rate to satisfy central bank's budget. The results show that while the welfare cost of not having a fiscal backing is small during a typical business cycle, it becomes significantly larger in the face of a larger shock.

Moreover, the literature lacks a comprehensive analysis of fiscal policy within the optimal policy framework. While existing studies tend to focus on the monetary side and keep the fiscal side simple, this paper models tax smoothing motive and shows that central bank policies influence optimal fiscal policies, such as taxation and government bond issuance. In this respect, this paper follows the conclusions of the traditional monetary-fiscal policy literature, which emphasizes that monetary and fiscal policies constrain each other (Sargent and Wallace (1981); Leeper (1991), and more recently Bianchi (2013); Bianchi and Ilut (2017)).

The closest papers are Berriel and Bhattarai (2009) and Amador and Bianchi (2023). Berriel and Bhattarai (2009) studies the optimal policy in New-Keynesian framework. Berriel and Bhattarai (2009) assumes that the government's objective function directly includes the central bank's losses. This paper does not directly incorporate the welfare loss from a destabilized central bank net worth. Instead, the government in my model is concerned with the central bank's balance sheet because the price of reserves must be lower to incentivize households to hold excess reserves. Additionally, Berriel and Bhattarai (2009) does not consider a constraint on transfers from the Treasury to the central bank.

Amador and Bianchi (2023) is the only paper that models the constraint on resource allocation from the Treasury to the central bank, focusing on helicopter drops during a liquidity trap from a positive perspective. It finds that helicopter drops can be an effective stabilization policy in a liquidity trap when the central bank faces constraints on resource allocation. In contrast, this paper approaches monetary-fiscal policy from a normative perspective, examining the optimal policy responses under these constraints.

Second, this paper contributes to the literature on central bank balance sheet policies. The literature has extensively examined central banks' asset policies,[3] while studies on liability policies, which are the focus of this paper, are comparatively limited.

In terms of liability policies, this paper relates to studies on the macroeconomic implications of large reserves (Poole, 1968; Cochrane, 2014; Reis, 2016; Ennis, 2018; Williamson, 2019; Arce, Nuno, Thaler, and Thomas, 2020; Bianchi and Bigio, 2022). Ennis (2018) finds that price level indeterminacy may arise when banks hold excess reserves. Reis (2016) finds that only the interest on reserves and not the size of the balance sheet has an effect on inflation. Arce et al. (2020) proposes New-Keynesian equilibrium model to compare the pre-crisis lean balance sheet regime and the post-crisis floor system with a large balance sheet. Bianchi and Bigio (2022) studies credit channel of monetary policy in a model where banks manage liquidity facing frictions in the interbank market.

The contribution of this paper lies in examining the implications of excess reserves within the standard New Keynesian framework. Unlike papers which delve into the microstructure of interbank lending and payments (Afonso and Lagos, 2015; Bianchi and Bigio, 2022), this paper simplifies the demand for reserves by introducing them directly into household utility and models in details a general equilibrium New Keynesian model, with a particular focus on (i) fiscal policy, including distortionary taxes and government bonds, and (ii) transfers between the Treasury and the central bank. While much of the literature on large reserves overlooks fiscal considerations, this paper emphasizes the importance of the central bank's fiscal support for fiscal policy.

The closest works in this literature are Berentsen, Marchesiani, and Waller (2014) and Berentsen, Kraenzlin, and Müller (2018), as both highlight that the implications of excess reserves critically depend on the level of fiscal support provided to the central bank. Berentsen, Kraenzlin, and Müller (2018) finds that when the central bank lacks fiscal support, reducing reserves is the optimal policy. They compare the floor system and the channel system from a normative perspective and conclude that the floor system is superior when there is fiscal backing. While Berentsen, Kraenzlin, and Müller (2018) explores regimes of full fiscal support and no fiscal support, this paper assumes a lack of *optimal* fiscal support, where there is no constraint on transfers to the Treasury, but transfers to the central bank are constrained. This assumption more closely aligns with the current policy of

[3] For theoretical papers, see Gertler and Karadi (2011); Curdia and Woodford (2011), among others; for empirical studies, see Krishnamurthy and Vissing-Jorgensen (2011); D'Amico and King (2013), among others.

the Federal Reserve.

While many empirical literature focuses on asset-induced central bank balance sheet policies, Christensen and Krogstrup (2019) presents empirical evidence of a liability-induced transmission channel of central bank's balance sheet policies to long-term interest rates. Reserve-induced effects are independent of the assets purchased and run through the impact of reserve expansions. They analyze the reaction of Swiss long-term government bond yields to announcements by the Swiss National Bank to expand central bank reserves without acquiring any long-lived securities. They find that long-term yield declines following the announcements. Their findings are consistent with this paper's mechanism that the increase of quantity of reserves by paying interest on reserves change the nominal interest rate even without purchasing assets by central bank.

Third, this paper contributes to the literature on optimal monetary and fiscal policy. The literature focuses on (i) the optimal volatility and persistence of inflation (Chari, Christiano, and Kehoe, 1991; Schmitt-Grohé and Uribe, 2004; Siu, 2004; Chugh, 2006, 2007), (ii) the temptation to erode government nominal liabilities through inflation (Schmitt-Grohé and Uribe, 2004; Niemann, Pichler, and Sorger, 2013; Leeper, Leith, and Liu, 2021), and (iii) the stochastic behavior of debt (Barro, 1979; Lucas and Stokey, 1983; Aiyagari, Marcet, Sargent, and Seppälä, 2002; Schmitt-Grohé and Uribe, 2004; Benigno and Woodford, 2003), particularly examining whether debt follows a random-walk property regardless of the shock process.

The existing literature abstracts from the issues of fiscal backing and reserves, which are another form of interest-bearing government liabilities. This paper incorporates both and examines whether the conventional results in the literature change. The findings indicate that, without fiscal backing: (i) the optimal volatility of the inflation rate is higher, (ii) the government uses inflation to erode reserves, and (iii) reserves become more persistent while government bonds are less persistent.

Layout The rest of the paper is organized as follows. Section 2 describes the model and defines the private sector equilibrium. Section 3 formulates the government's problem. Section 4 discusses parametrization and the solution method. Section 5 and 6 shows numerical results. Section 7 shows welfare analysis. Section 8 concludes.

2 Model

This section describes the model and defines the equilibrium. The model is a standard New Keynesian framework incorporating both the Treasury's and the central bank's budget constraints. Before presenting the model, I define key terminology. ”Government” refers to an agent that chooses both monetary and fiscal policies, with the objective of maximizing household utility. The problem for Government is defined in section 3. The ”Treasury” and ”central bank” can be considered to be subsets of the government, each with its own budget constraint. The Treasury collects taxes, provides public expenditure, and issues liabilities: ”government bonds”. Central bank issues another form of liability: ”reserves”. Two types of liabilities, reserves and government bonds, are distinguished by issuers.

2.1 Households

The representative households choose consumption, labor supply, and the holdings of one period risk free nominal bond (reserves) and long-duration risk free nominal bonds (government bonds) to maximize the expected discounted sum of the future period utilities. The households consume and work. The households get a convenience yield from trading both reserves and government bonds. Households maximizes

$$\max_{\{C_t,N_t,D_t,B_t^H\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\nu} N_t^{1+\nu} + \frac{\chi_1}{1-\gamma_1} \left(Q_t^C \frac{D_t}{P_t} \right)^{1-\gamma_1} + \frac{\chi_2}{1-\gamma_2} \left(Q_t^T \frac{B_t^H}{P_t} \right)^{1-\gamma_2} \right] \tag{1}$$

where C_t is a Dixit-Stiglitz aggregate consumption of a continuum of differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta}{\theta-1}} di \right]^{\frac{\theta-1}{\theta}} \tag{2}$$

with an elasticity of substitution equal to $\theta > 1$. Also, N_t is the labor supply, Q^C is the price of the reserves, D is the quantity of nominal reserves, Q^T is the price of government bonds, B^H is the quantity of nominal government bonds held by the households, P_t is the price of consumption goods.

Each differentiated good is supplied by a monopolistially competitive producer. The goods in each industry i are produced using labor that is specific to that industry. The representative households supplies all types of labor as

$$N_t = \int_0^1 N_t(i)di. \tag{3}$$

Households receive a convenience yield from holding reserves and government bonds. The convenience yield from reserves can be interpreted as the expected savings in transaction costs for commercial banks, as they do not need to sell loans to manage deposit outflows. Under an assumption that households own these commercial banks, they value the reduction in transaction costs that benefits them. An example of the convenience yield from holding government bonds is that Treasury securities lower the costs associated with transacting in less liquid securities, such as corporate bonds. [4]. Convenience yield terms allows me to match the steady-state value of reserves and government bonds, which critically influences interest expenses and is essential for quantitative analysis.

The budget constraint is

$$P_t C_t + Q_t^C D_t + Q_t^T B_t^H = D_{t-1} + (1 + \rho^T Q_t^T) B_{t-1}^H + w_t N_t + P_t \Phi_t, \tag{4}$$

where Φ_t is the the profit. In period t, the households buy government bonds, B^H, at price Q_t^b that pay a declining coupon of ρ^{Tj} dollars in period $j+1$, where $0 \leq \rho \leq \beta^{-1}$ (Woodford, 2001). A measure of the duration of the bond is given by $(1 - \beta\rho^T)^{-1}$. The households bring nominal bonds of $(1 + \rho^T Q_t^b) B_{t-1}^H$ into period t.

The price of reserves, Q_t^C, and government bonds , Q_t^T, are the policy instrument for the government.

2.2 Firms

There is a continuum of goods producers indexed by $i \in [0, 1]$. Firms use labor as inputs and produce imperfectly substitutable goods according to a linear production function. Each firm sets the price of its own good to maximize the expected discounted sum of future profits.

Firms i face three constraints, a linear production function,

$$y_t(i) = A_t N_t(i), \tag{5}$$

[4]For more discussion of convenience yield, see Lopez-Salido and Vissing-Jorgensen (2023) for reserves and Krishnamurthy and Vissing-Jorgensen (2012) for government bonds among others.

where A_t is the exogenous aggregate productivity.

Second, firms face a demand curve of the form

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}, \tag{6}$$

where the aggregate demand is defined as

$$y_t(i) = \left(\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{7}$$

Third, quadratic adjustment costs in changing prices, as in Rotemberg (1982), are defined as

$$\frac{\varphi}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 P_t Y_t, \tag{8}$$

where φ is the degree of nominal price rigidity.

A firm i sets its price $p_t(i)$ in period t to maximize the expected discounted sum of future profits.

$$\max_{p_t(i)} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left((1-\tau_t) p_t(i) y_t(i) - \mu_t^w w_t N_t(i) - \frac{\varphi}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 P_t Y_t \right) \tag{9}$$

where Λ_t is the stochastic discount factor given by

$$\Lambda_t \equiv \frac{C_t^{-\sigma}}{P_t} \tag{10}$$

The stochastic discount factor measures the marginal value of an additional unit of profits to the household. τ_t is the tax on sales levied by the Treasury. μ_t^w captures the shock to wage mark-up. One possible interpretation of the wage markup shock is that it represents the bargaining power of labor unions. There is no heterogeneity in the time-zero prices across firms. That is $p_{-1}(i) = P_{-1}$ for a given constant P_{-1}.

The cost-push shock and productivity follow an AR(1) process.

$$\mu_t^w = \bar{\mu}^w + \rho^w (\mu_{t-1}^w - \bar{\mu}^w) + \varepsilon_t^w. \tag{11}$$

$$A_t = \bar{A} + \rho^a (A_{t-1} - \bar{A}) + \varepsilon_t^a, \tag{12}$$

where ε_t^w and ε_t^a are shocks and are distributed with mean zero and standard deviation σ_w and σ_a. Parameter $\bar{\mu}^w$ and $\bar{A}$ are the mean value of wage mark-up and productivity that are equal to one.

The first-order condition implies the non-linear Phillips curve

$$N_t C_t^{-\sigma} \left(\varphi(\pi_t - 1)\pi_t A_t - (1-\theta)(1-\tau_t)A_t - \theta \mu_t^w N_t^\nu C_t^\sigma\right) = \beta E_t \left[A_{t+1} N_{t+1} C_{t+1}^{-\sigma} \varphi(\pi_{t+1} - 1)\pi_{t+1}\right]. \tag{13}$$

2.3 Treasury and central bank

The government's problem is introduced in the next section. This section describes a set of restrictions that government faces. Government faces three constraints: Treasury's budget, central bank's budget, and a constraint on transfers from Treasury to central bank.

The Treasury plays a role of supplying exogenous, stochastic, and useless public expenditures G_t. Aggregate public expenditure takes

$$G_t \equiv \left(\int_0^1 G_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} \tag{14}$$

such that the government demand for goods is given by

$$G_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} G_t \tag{15}$$

The exogenous public expenditure follows AR(1) process,

$$G_t = \bar{G} + \rho^g (G_{t-1} - \bar{G}^w) + \varepsilon_t^g. \tag{16}$$

The expenditures are financed by sales tax, issuance of the government bonds, and the remittance form the central bank. The Treasury's budget constraint is given by

$$Q_t^T B_t^T + \tau_t P_t Y_t + P_t H_t = P_t G_t + (1 + \rho^T Q_t^T) B_{t-1}^T. \tag{17}$$

$\tau_t Y_t$ is the distortionary tax on sales. H_t is the remittance from central bank to Treasury. A positive H_t implies that the central bank transfers to the Treasury. B_T is the total supply of government bonds. The government bonds are held by households and central bank. They are denoted by B_t^H and B_t^C.

The central bank's role is to supply reserves that households appreciate. In addition to reserves, the central bank also trades government bonds. The central bank budget constraint is

$$Q_t^C D_t + (1 + \rho^T Q_t^T) B_{t-1}^C = D_{t-1} + Q_t^T B_t^C + P_t H_t, \tag{18}$$

where D_t is reserves and H_t is the remittance from central bank to Treasury. B_t^C is the Treasury's bond issued by the central bank's asset.

The left-hand side of the budget constraint represents the income for central bank. Central bank generates revenue by issuing more reserves ($Q_t^C D_t$), income gains from government bonds holdings (B_{t-1}^C), and capital gains from government bonds holdings ($\rho^T Q_t^T B_{t-1}^C$). As government bonds are long-duration bonds, revenues from them are separated into income gain and capital gain. This model includes capital loss from central bank's asset holding. When the central bank raises the nominal interest rate, the price of reserves (Q_t^C) falls, leading to capital loss.

The right-hand side of the budget constraint represents expenditures for central bank. Expenditures consists of redemption of reserved (D_{t-1}), purchase of assets ($Q_t^T B_t^C$), and remittance to Treasury ($P_t H_t$). A remittance can be used for the purchase of assets or the payment of redemption of reserves.

Note that this budget constraint is based on mark-to-market valuation, meaning that the price of long-duration bonds reflects their market price rather than their book value. The model assumes that the central bank trades assets every period, which contrasts with the holding assets until maturity, where market prices do not affect the central bank's budget. In practice, while the Bank of England adopts mark-to-market valuation for central bank assets, the Federal Reserve and other central banks, including the Bank of Japan, use book-value accounting to calculate asset values.

For the central bank's asset purchase policy, B_t^C, it is assumed that the constant fraction of Treasury bond is held by the central bank.

$$B_t^C = \alpha B_t^T \tag{19}$$

, where α is a parameter. This assumption helps reduce the size of the state space. As I will describe in the next section, the model has four state variables: exogenous shocks, reserves, total supply of government bonds, and government bonds held by the central bank. This complexity introduces computational difficulties. By assuming (19), the model is reduced to three state variables, allowing for a global solution.

The justification for this assumption is that the focus of this paper is on the central bank's liability side, not its asset side. The central bank trades both liabilities and assets, but this paper emphasizes the liability side, and I endogenize the liability policy. In contrast, one of the goals of the asset purchase policy is to stabilize financial markets (Gertler and Karadi, 2011), which is beyond the scope of this model. Hence, I treat the asset policy as exogenous. Under this assumption, the central bank's asset purchases are smooth over time. A sudden change in the amount of asset purchases would destabilize the asset market.[5] This assumption aligns with the central bank's objectives.

Finally, the most important part of this model is that the government faces a constraint that prevents optimal transfers from the Treasury to the central bank. This constraint is one-sided: while the Treasury can receive as many resources as needed from the central bank, the central bank is constrained from receiving resources from the Treasury, primarily for political reasons. Inequality constraint on the remittance

$$H_t \geq H^*. \tag{20}$$

H^* is a exogenous parameter. The model aligns with the Federal Reserve's actual policy. Transfers from the central bank to the Treasury have a lower bound of zero. When the central bank's costs exceed its earnings, a so-called "deferred asset" is accumulated. A deferred asset is a negative liability that represents the cumulative value of the shortfall in earnings. Once the Fed returns to a positive net income, it will use those earnings to pay down the deferred asset. During this period, the Fed does not transfer funds to the Treasury, even if it earns profits; those profits are used to reduce the deferred asset. Once the deferred asset reaches zero, the Fed will resume sending remittances to the Treasury. As of October 2024, the deferred asset stands at $210 billion, approximately 0.7% of annual GDP.

The motivation for this constraint stems from "political reasons" from the Treasury's perspective. To cover the central bank's losses, the Treasury would need to raise taxes or issue additional debt, both of which are politically costly. Moreover, the central bank pays interest expenses to commercial banks, which are not perfectly competitive. If the Treasury decides to cover these interest expenses through taxation, it effectively redistributes income from households, including poorer ones, to commercial banks. This type of redistribution is difficult for the Treasury to justify politically.

From the central bank's perspective, requesting capital injections could help maintain a strong financial position and avoid a weakened balance sheet. However, there is concern that relying on

[5] See the literature on the Taper Tantrum (Eichengreen and Gupta, 2015) for a discussion of the costs of sudden changes in the central bank's asset purchase policy.

parliamentary decisions for funding could threaten central bank independence[6]. A motivation for this constraint is to maintain central bank independence.

The literature on the unconsolidated government budget already models remittance (Bassetto and Messer, 2013; Del Negro and Sims, 2015; Benigno and Nisticò, 2020; Benigno, 2020). However, no model has ever studied the constraint on remittance.

2.4 Market Clearing

Clearing of the goods i market requires the following.

$$y_t(i) = C_t(i) + G_t(i) + \frac{\varphi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - 1\right)^2 y_t(i). \tag{21}$$

$$Y_t(i) = A_t N_t(i) \tag{22}$$

In a symmetric equilibrium,

$$Y_t = C_t + G_t + \frac{\varphi}{2}(\pi_t - 1)^2 Y_t. \tag{23}$$

$$Y_t = A_t N_t \tag{24}$$

The clearing conditions for government bonds are

$$B_t^H + B_t^C = B_t^T \tag{25}$$

where government bonds are held by households and the central bank. Note that I impose an assumption that central bank holds α fraction of total supply of government bonds.

$$B_t^H = (1-\alpha)B_t^T \tag{26}$$

$$B_t^C = \alpha B_t^T \tag{27}$$

There is also market clearing in reserves markets where reserves held by households evolves according to the central bank's budget constraint.

[6]For actual comments from central bankers, see Floden (2022), where the Deputy Governor of the Riksbank expresses these concerns

The price index is defined as

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{28}$$

$$\pi_t \equiv \frac{P_t}{P_{t-1}}. \tag{29}$$

2.5 Equilibrium

Given the distribution of initial prices, $p_{-1}(i)$, stochastic processes, and the initial level of liabilities B_{-1}, D_{-1}, a competitive equilibrium of this economy consists of allocations $\{C_t, N_t, Y_{i,t}\}_{t=0}^{\infty}$, prices $\{p_t(i)\}_{t=0}^{\infty}$, and policy instruments $\{Q^C, Q^T, \tau, B_t, D_t\}_{t=0}^{\infty}$, such that

- Allocations solve the problem of the household given prices and policies.
- $p_t(i)$ solves the problem of firm i.
- $p_t(i) = p_t(j)$.
- The Treasury, central bank's budgets, and a constraint on remittance are satisfied.
- Goods market, reserve market, and government bond market clear.

Implementable equilibrium conditions are summarized in section 3.

3 The Optimal Policy Problem under discretion

This section outlines the optimal policy problem under discretion. The government makes decisions sequentially. Each period t, the government maximizes the utility of households by choosing $\{C_t, N_t, \Pi_t, Q_t^C, Q_t^T, b_t, d_t, \tau_t, H_t\}$ and the Lagrangian multipliers associated with the equilibrium conditions, taking as given the next period value and next policy functions. I define real value of reserves and government bonds, $b_t \equiv \frac{B_t}{P_t}$ and $d_t \equiv \frac{D_t}{P_t}$.

$$\begin{aligned} V_t(s_t) = \max_{a_t} \quad & \frac{1}{1-\sigma}C_t^{1-\sigma} - \frac{1}{1+\nu}N_t^{1+\nu} + \frac{\chi_1}{1-\gamma_1}(Q_t^C d_t)^{1-\gamma_1} + \frac{\chi_2}{1-\gamma_2}((1-\alpha)Q_t^T b_t)^{1-\gamma_2} \\ & + \beta E_t V_{t+1}(s_{t+1}), \end{aligned} \tag{30}$$

where $s_t \equiv \{\mu_t^w, A_t, G_t, b_{t-1}^C, b_{t-1}^T\}$ and $a_t \equiv \{C_t, N_t, \Pi_t, Q_t^C, Q_t^T, b_t, d_t, \tau_t, H_t\}$.

Equilibrium conditions are

$$\text{(Euler for reserve)} \quad C_t^{-\sigma} Q_t^C = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] + \chi_1 (Q_t^C d_t)^{-\gamma_1} Q_t^C \tag{31}$$

$$\text{(Euler for Treasury bond)} \quad C_t^{-\sigma} Q_t^T = \beta E_t \left[C_{t+1}^{-\sigma} \frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}} \right] + \chi_2 ((1-\alpha) Q_t^T b_t)^{-\gamma_2} Q_t^T \tag{32}$$

$$\begin{aligned} \text{(Firm FOC)} \quad & N_t C_t^{-\sigma} \left(\varphi(\pi_t - 1)\pi_t A_t - (1-\theta)(1-\tau_t) A_t - \theta \mu_t^w N_t^\nu C_t^\sigma \right) \\ & = \beta E_t \left[A_{t+1} N_{t+1} C_{t+1}^{-\sigma} \varphi(\pi_{t+1} - 1)\pi_{t+1} \right]. \end{aligned} \tag{33}$$

$$\text{(Market Clearing)} \quad A_t N_t = C_t + \frac{\varphi}{2}(\pi_t - 1)^2 A_t N_t + G_t \tag{34}$$

$$\text{(Treasury Budget)} \quad Q_t^T b_t + \tau_t Y_t + H_t = G_t + (1 + \rho^T Q_t T) \frac{b_{t-1}}{\pi_t}. \tag{35}$$

$$\text{(Central Bank's Budget)} \quad Q_t^C d_t + (1 + \rho^T Q_t^T)\alpha \frac{b_{t-1}}{\pi_t} = \frac{d_{t-1}}{\pi_t} + Q_t^T \alpha b_t + H_t \tag{36}$$

$$\text{(Remittance)} \quad H_t \geq H^* \tag{37}$$

$$\text{(Shocks)} \quad \mu_t^w = \bar{\mu}^w + \rho^w(\mu_{t-1}^w - \bar{\mu}^w) + \varepsilon_t^w. \tag{38}$$

$$A_t = \bar{A} + \rho^a (A_{t-1} - \bar{A}) + \varepsilon_t^a. \tag{39}$$

$$G_t = \bar{G} + \rho^g (G_{t-1} - \bar{G}^w) + \varepsilon_t^g. \tag{40}$$

The discretionary equilibrium is determined by the first-order conditions, expectations that are consistent with policy functions, and the exogenous process for shocks. The solution is the time-invariant Markov-perfect equilibrium policy rules that map states $\{\mu_t^w, A_t, G_t, b_{t-1}, d_{t-1}\}$ to optimal decisions for $\{C_t, N_t, \Pi_t, Q_t^T, Q_t^C, b_t, d_t, \tau_t, H_t\}$.

The government is constrained to behave in a time-consistent manner. The economic agents anticipate that the government faces this constraint and form expectations. However, the government can change the expectations by choosing state variables, government bonds and reserves. In rational expectation equilibrium, the expectations are formed based on the mapping that map endogenous variables to the state-space.

Note that I impose that the central bank holds an α fraction of the total supply of government bonds. The government only chooses the total supply of government bonds; it does not need to separately decide on the total supply of bonds and the portion held by the central bank. The allocation of bonds between households and the central bank is determined by equations (26) and (27). The state variables are reserves and the total supply of government bonds.

Define "the consolidated model" and "the unconsolidated model" "The unconsolidated model" is defined as the model where the government maximizes the utility of households subject to (31) - (40). The remittance from the Treasury to the central bank is constrained.

"The consolidated model" is defined as the model where H^* is low enough that the equation (37) never binds. Later, H^* is set to be $-\infty$ in quantitative exercises. The consolidated government budget is given by substituting H_t in equation (35) into equation (36).

$$Q_t^C d_t + (1-\alpha)Q_t^T b_t + \tau_t Y_t = \frac{d_{t-1}}{\pi_t} + (1+\rho^T Q_t^T)(1-\alpha)\frac{b_{t-1}}{\pi_t} + G_t. \tag{41}$$

In the consolidated model, the government maximizes household utility subject to equations (31)-(34), (38)-(40), and (41). The optimal resource allocation between the central bank and the Treasury is always achievable. It is important to note that the consolidated model represents the optimal monetary-fiscal policy problem within a standard New Keynesian framework (Benigno and Woodford, 2003; Schmitt-Grohé and Uribe, 2004). The key differences from Benigno and Woodford (2003); Schmitt-Grohé and Uribe (2004) and the consolidated model are as follows: (1) while my model considers optimal policy under discretion, those papers analyze policy under commitment; (2) my model includes reserves as interest-bearing liabilities, whereas those papers include money.

4 Solution method and calibration

4.1 Solution method

Two reasons necessitate the use of global solution methods. The first is the presence of an occasionally binding constraint on remittance. The stochastic shocks cause the remittance to occasionally bind the lower bound. The policy problem is highly nonlinear, and the purturbation technique is not suitable. Second, under discretionary policies, the model's steady-state which local dynamics should be approximated depends on the derivative of expectations with respect to reserves and government liabilities. This is because the state variables in the model include reserves and government liabilities, and the expectations in Euler equations and New Keynesian Phillips Curve are evaluated at each reserves and government bonds. This derivative of expectations with respect to reserves and bond is endogenously determined as a part of model solution, making the steady-state a priori unknown. This is a common approach in the literature (Niemann and Pichler, 2011; Niemann, Pichler, and Sorger, 2013; Leeper, Leith, and Liu, 2021).

The model is solved using the time-iteration method of Coleman (1991). I begin by guessing initial policy functions and then compute the associated expectations. Assuming that the guessed policy functions apply to the next period, I solve the first-order necessary conditions of the government's problem on a discrete set of grids to find the policy functions for the current period. This process is repeated until the policy functions converge, such that the difference between today's and tomorrow's policy functions becomes arbitrarily small. Details are described in Appendix B.

4.2 Calibration

I calibrate my model to the US economy. The time unit is quarter. The calibration table is summarized in table 1. Parameters that are commonly used in the New Keynesian literature are taken from the literature. I set $\beta = 0.995$, which implies a 2% annual real interest rate. The intertemporal elasticity of substitution is set to $\frac{1}{2}$, a value commonly found in the literature. The Frisch elasticity of labor supply is fixed at $\frac{1}{7}$, while the elasticity of substitution between intermediate goods is 10, implying a monopolistic markup of 10%. The price adjustment cost parameter is set at 100. The coupon decay parameter, $\rho^T = 0.95$, corresponds to an average debt maturity of around 5 years, consistent with U.S. data (See Table 2 in Leeper and Zhou (2021)).

Other parameters are calibrated in my model. I describe my calibration strategy. The parameters, χ_1, χ_2, that decide the convenience of reserves and government bonds are chosen to match the steady-state value of reserves and bonds. Steady-state reserves are matched to 15% of GDP. The recent level of government liabilities in data is 120% of GDP, and the convenience yield is not enough to match high level of government liabilities. I match the steady-state government bonds as the 30% of GDP that is the average of 1970-2020 in US data.

The fraction of government bonds held by the central bank, α, is calibrated at 0.4 to match the central bank's asset-to-liability ratio. A higher α leads to a higher asset-to-liability ratio. In the data, the Federal Reserve's asset-to-liability ratio is approximately 1. Setting α at 0.4 in the model achieves an asset-to-liability ratio of one. In practice, the observed α from data is lower than the model's value (peaking at 21% in 2022)[7]. This discrepancy arises because, in reality, the Federal Reserve holds assets beyond government bonds, such as Mortgage-Backed Securities and government-sponsored enterprise fixed assets, which are not included in this model. If I set $\alpha = 0.21$ to match the data and align the model's reserve size with observed values, the central bank's assets would fall significantly below its liabilities. This mismatch would alter the quantitative results, particularly since the model needs to capture both capital loss on the asset side and the interest expense on reserves. To avoid this issue, I set $\alpha = 0.4$, ensuring that the asset-to-liability ratio remains approximately 1, consistent with the data.

The parameters describing the curvature of the utility of reserves and government bonds, γ_1, γ_2, are calibrated so that the elasticity of price to quantity in the data matches that in the model following Krishnamurthy and Vissing-Jorgensen (2012). The details are discussed in the appendix A.

Lastly, the lower bound on remittance, H^*, is -0.0025, that is 0.06% of annual GDP. In practice, H^* is zero implying that central bank does not receive any amounts of funds from Treasury. I tried to set $H^* = 0$, but under this parameter, the model does not converge. The possible reason is that in a state-space with large reserves and an exogenous shock leading to higher nominal interest rate, the reserves keep increasing because central bank cannot receive any funds from Treasury, but central bank needs to pay high interest rate on reserves. Then, reserves keep increasing, and there is no equilibrium. As a result, I set * slightly negative so that the model converges. I set $H^* = -0.0025$ that is the highest and closes value to zero such that the model converges.

5 Responding to shocks

This section describes the main exercises: how policies respond to shocks, a positive-cost push shock, a negative productivity shock, and a positive public expenditure shock under the consolidated model and unconsolidated model. The goal is to show a difference between the consolidated and the unconsolidated model.

For all types of shocks, the procedure is as follows: at time $t = 1$, an exogenous shock occurs. The state variables include reserves and bonds, which I give as an initial condition. The exogenous shock then follows its own AR(1) process as described by equations (38)-(40). The innovations

[7] I show the ratio of U.S. Treasury Securities held by the Federal Reserves to Federal Debt. The data is from FRED.

Table 1: Calibration.

Variable	Value	Description	Target	Model	Data
β	0.995	Discount factor	-	-	-
σ	2	Risk aversion	-	-	-
ν	7	Frisch Elasticity	Frisch Elasticity	1/7	-
θ	10	Elasticity of substitution among goods	Mark up	7%	-
ϕ	100	Price adjustment cost	Slope of NKPC	0.05	-
ρ^T	0.95	Duration of Treasury	Average Maturity	5 years	5 years
χ_1	0.0012	Utility from reserve	Steady-state reserve	15% of GDP	15% of GDP
χ_2	0.002	Utility from Treasury bond	Steady-state Treasury	30% of GDP	120% of GDP
γ_1	1.7	Curvature of utility from reserve	The elasticity of price to reserve supply	-0.2	0.2
γ_2	1.5	Curvature of utility from government bonds	The elasticity of price to government bonds supply	-0.1	-0.1
α	0.4	CB's asset holding	Asset/liability	1	1
H^*	-0.0025	Lower bound on remittance	-	-	-

in the AR(1) process are randomly drawn from a normal distribution. I generate 5000 paths of the exogenous shock, each with a different sequence of innovations randomly drawn from normal distribution. I then calculate the dynamics of variables given each sequence of exogenous shocks and the policy functions. Finally, I take the average of the 5000 simulated dynamics and plot the result.

5.1 Cost-push shock

The first exercise is to show transition dynamics after the cost-push shock. The purpose of this exercise is to simulate the difference between the consolidated and unconsolidated model, that is the impact of the lack of fiscal backing. This exercise on cost-push shock is motivated by (i). a large inflationary shock in 2022 (ii). large reserves that central bank holds when the shock hits. Corresponding to each motivation, I simulate the dynamics after a positive cost-posh shock and the economy starts from large reserves when the shock hits. The economy does not start from the steady state.

Figure 1 plots the outcomes following a rise in the wage markup μ^w by 8%. The wage mark-up increases to 1.08 from 1 that is the steady-state[8]. The horizontal axis is the quarters. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the

[8]The discussion on the size of the shock is provided in Appendix D. The appendix explains that as the size of the shock increases, the difference between the consolidated and unconsolidated models also becomes larger.

unconsolidated model. The initial conditions for the state-variables are reserves and government bonds. The level of initial reserves are high, 90th percentile of simulated economy. The level of government bonds are the steady-state level. Note that the economy does not start from the steady-state, so the variable moves even without the cost-push shock. I did not net out dynamics when the same size of shock hits but the economy starts from the steady-state. I simply lay out the dynamics.

First, I describe what the government chooses policies in the consolidated model. Under the optimal policy, the government decreases the tax rate on sales to offset the increase of marginal cost. What the government wants to do is to make the marginal product closer to the marginal cost. The marginal cost increases upon the cost-push shock. The government levies taxes on sales, so the government can increase the marginal product by decreasing the tax rate. From the first-order conditions for firms, equation (13), the change in wage mark-up can be offset by changing the tax rate. The decline of tax rate is seen in figure 1-(g).

As a result of low tax rate, the tax revenue drops, but the Treasury needs to finance the public expenditure. The Treasury issues government bonds to finance public expenditure. The Treasury supplies more government bonds. Government bonds increases in figure 1-(f). From the central banks' perspective, the central bank helps Treasury's budget by issuing reserves, creating more resources, and sending resources to the Treasury. The central bank supplies more reserves after $t = 2$. However, upon the shock, the central bank holds large reserves. The central bank receives resources to reduce the level of reserves at $t = 1$. Therefore, reserves decline upon the shock, and then, increase. This is seen in figure 1-(e).

For remittance, at $t = 1$, it takes a negative value, that is, Treasury sends and central bank receives. This is to reduce large reserves. After $t = 1$, the remittance is close to zero and flat. This is because two effects offset each other. The first is that central bank issues more reserves and send resources to Treasury. The second is that central bank needs to purchase more government bonds as Treasury issues more bonds. For central bank to purchase assets, the central bank needs to receive resources from the Treasury. These two effects offset each other. As a result, the remittance is flat in 1-(h) after $t = 1$.

For consumption and inflation, the government cannot fully offset the cost-push shock, so the consumption drops and inflation rate increases. The price of reserves and government bonds drops, that is the increase of nominal inflation rate. This is to alleviate the trade-off between output and consumption. In summary, the role of central bank is to help Treasury to do tax break upon the shock. Central bank can create resources by issuing reserves, so central bank helps Treasury by sending resources to Treasury.

Next, I describe the dynamics in the unconsolidated model. The key results are that the inflation rate is higher and output drops more. This is because the government cannot reduce the tax rate sufficiently, so the high marginal cost reduces the output and pass-trough to the inflation rate. Figure 1-(g) shows that the tax rate does not drop in the unconsolidated model as much as in the consolidated model. Why does the government cannot reduce the tax rate enough? From the central bank perspective, it does issue reserves enough. Figure 1 shows that reserves in consolidated model increases after $t = 2$ to send resources to the Treasury. Reserves in the unconsolidated model does not increase a lot. The issuance of reserves in the unconsolidated model is smaller. This is because the large reserves cannot be reduced by remittance from the Treasury. If central bank issues more reserves, large reserves should be reduced later by remittance. This is not possible in the unconsolidated model. The issuance of reserves in the unconsolidated model is smaller. Fewer issuance of reserves means that the resource allocation from central bank to Treasury is smaller in the unconsolidated model. As a result, the government cannot use tax rate to offset the cost-push shock. The central bank tolerates a higher inflation rate. Inflation rate in the unconsolidated model responds more by 1.3%. Consumption drops more by 0.07%. Given that consumption decreases by 0.8% while inflation rises by 0.4% in consolidated model, the difference between the consolidated and unconsolidated model is significant.

In summary, the central bank does not help Treasury's budget enough in the unconsolidated mode because central bank does not create resources by issuing more reserves. Tax break is not enough in the unconsolidated model. The lack of *fiscal* backing impacts *fiscal* policy.

One important point to note is that the main mechanism operates through fiscal policy (specifically the sales tax rate), rather than the nominal interest rate. This is because a tax break more effectively offsets the shock, while monetary policy is not as effective; monetary policy mitigates the trade-off between output and the inflation rate but does not fully offset the shock. From the firm's first-order condition, equation (13) shows that changes in marginal product can be offset by changes in the tax rate. Full shock offsetting would be possible if the Treasury had access to non-distortionary taxes. If the Treasury could finance a tax break through lump-sum taxation, it could fully offset the shock. However, because the Treasury does not have access to lump-sum taxes, it cannot completely offset the shock. The key observation is that a tax break is more efficient than monetary policy, as it can offset the shock, whereas monetary policy merely alleviates the trade-off between output and inflation.

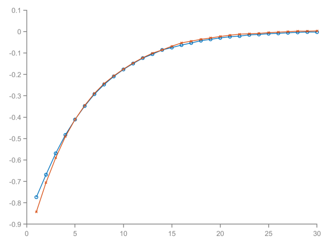
In the unconsolidated model, consumption drops more and inflation rate increases more. This is because tax rate (fiscal policy) is constrained, not because monetary policy is constrained. If monetary policy is constrained and central bank does not increase nominal interest rate enough, consumption drops less and inflation rate increases more. The opposite is true. In figure 1, consumption drops more and inflation rate increases more in the unconsolidated model. This shows that the constraint on tax break is the key mechanism.

5.2 Productivity shock

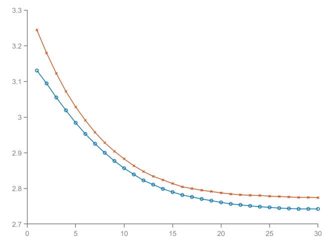
The second exercise compares the dynamics of the two models following a negative productivity shock. The dynamics of this type of shock are of interest in their own right (Blanchard and Galí, 2007). Additionally, this exercise is motivated by the fact that, after such a shock, the central bank typically raises the nominal interest rate, leading to higher interest expenses. Similar to the case of a cost-push shock, this represents another scenario where the central bank must increase the nominal interest rate. However, this section highlights that the mechanism through which the lack of fiscal backing affects policy responses differs from the case of a cost-push shock.

Figure 2 shows the outcomes following a decline in productivity, A_t, by 5%. The blue line represents the outcomes in the consolidated model, while the red line represents the outcomes in the unconsolidated model. To simulate the dynamics, initial conditions for reserves and government bonds are provided. Reserves are set at a higher value, corresponding to the 90th percentile of the simulated economy, consistent with the previous exercise.

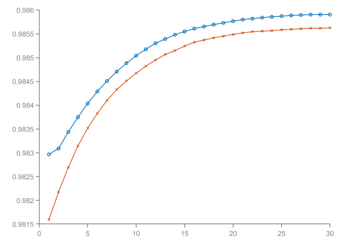
The intuition for the consolidated model is as follows: After a negative productivity shock, the government aims to adjust the policy rate to track the natural interest rate. Following the shock, it is optimal to reduce both labor supply and consumption. The marginal cost also rises, causing an increase in the inflation rate. To contract demand and lower inflation, the government raises the



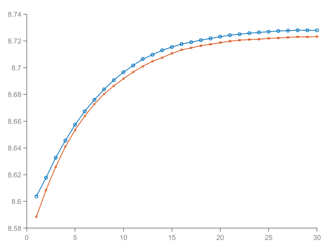
(a) Consumption



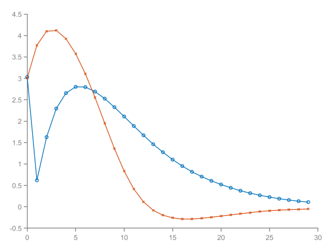
(b) Inflation



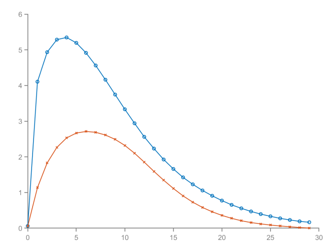
(c) Price of reserves



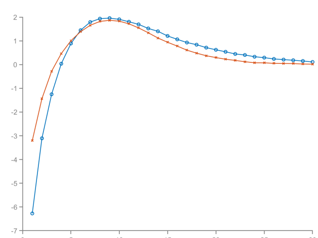
(d) Price of government bonds



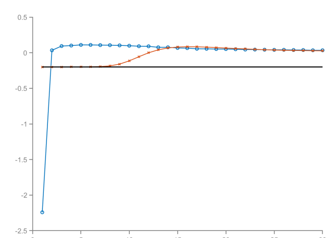
(e) Reserves



(f) Government bonds



(g) Sales tax



(h) Remittance

Figure 1: Cost-push shock

Note: This figure shows the dynamics of key variables upon the positive wage markup shock by 8%. The wage mark-up increases from 1 to 1.08. The horizontal is quarters. The vertical lines for consumption, prices of reserves, prices of government bonds, reserves, government bonds are the steady-state deviation represented by percent. The vertical line for inflation rate is quarterly inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

nominal interest rate. As aggregate output declines, tax revenue decreases. The government raises the tax rate and issues government bonds to smooth tax distortions over time.

For the central bank, with large initial reserves, it receives resources and continues to reduce reserves. Although the inflation rate rises only slightly (by 0.15%), this response is non-zero, which contrasts with the "divine coincidence," which suggests that under optimal policy, the inflation response should be zero. This discrepancy occurs because the government uses inflation to reduce real liabilities and relax the budget constraints of both the Treasury and the central bank. Importantly, the primary policy tool in response to a negative productivity shock is raising the nominal interest rate, which increases by 2.5 percentage points.

Next, I explain the intuition behind the unconsolidated model. A key result is that the central bank cannot raise the nominal interest rate sufficiently. The price of reserves does not decrease as much in the unconsolidated model compared to the consolidated model. This is because paying high interest on reserves leads to larger reserve balances, which cannot be reduced through remittances. When the shock hits, the central bank already holds large reserves, and by paying interest on these reserves, it issues more reserves. However, this does not satisfy the central bank's budget constraint due to the limited remittance. As a result, the government cannot raise the nominal interest rate enough.

Due to the lower nominal interest rate, consumption does not contract sufficiently in the unconsolidated model. Compared to the consolidated model, consumption responds 7% less in the unconsolidated model. Labor supply increases more in the unconsolidated model because households consume more. Additionally, the inflation rate is higher in the unconsolidated model because the interest rate cannot be raised enough to offset the inflationary shock.

On the fiscal side, the sales tax is lower in the unconsolidated model because firms need to produce more, and a lower sales tax leads to a higher marginal product. The Treasury also issues more government bonds because tax revenue is lower, but it still needs to finance public expenditure.

A natural question arises: why does the central bank issue large reserves by paying interest on reserves? The first reason is that large reserves do not satisfy the budget constraint when remittance is limited. Liabilities (reserves) must equal the present discounted value of assets and remittance. Since transfers from the Treasury are constrained, the issuance of reserves is also restricted. More interestingly, the second reason is that large reserves are needed to meet households' demand for reserves. To incentivize households to hold large reserves, the price of reserves must be low. However, this decline in the price of reserves affects real allocations through the households' Euler equation. This effect is costly, so the government does not want to issue excess reserves.

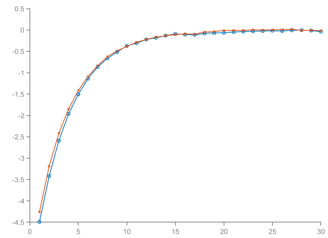
5.3 Public expenditure shock

This section describes the dynamics following a public expenditure shock. Motivated by the significant increase in public spending observed during the COVID-19 pandemic, I examine the role of the central bank in responding to a positive public expenditure shock. My results highlight how the central bank creates resources and supports the Treasury's budget through remittances. To demonstrate the central bank's capacity to generate resources, the initial condition assumes a large stock of government bonds, consistent with U.S. data, where the government liability-to-GDP ratio is 120%. The level of reserves is set at the steady-state, which differs from the exercises involving productivity and cost-push shocks.

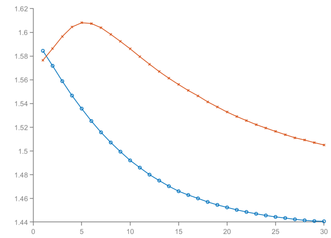
Upon the shock, the Treasury holds a large amount of government bonds, limiting its capacity to issue additional bonds in response to the positive public expenditure shock. In contrast, the central bank holds reserves at their steady-state level, allowing it to create more resources by issuing additional reserves. While the marginal value of government bonds is low, the marginal value of reserves remains relatively high. This leads to a stark contrast between the consolidated and unconsolidated models.

Figure 3 shows the outcomes following an increase in G_t by 3% of GDP. Public expenditure rises from 15% to 18% of GDP at time $t = 1$. The dynamics in the consolidated model are represented by the blue line, while the red line depicts the unconsolidated model. I will first explain the intuition behind the consolidated model.

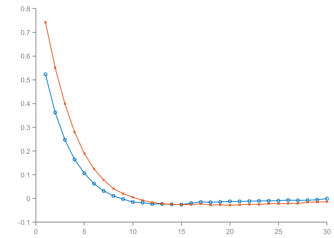
The government experiences a positive public expenditure shock and responds by raising the tax rate, although this is distortionary. To smooth the tax distortion, the Treasury issues more bonds. Central bank issues more reserves. The central bank also transfers resources to support the Treasury's budget. Since the Treasury holds large government bonds at the time of the shock, their price declines to meet the increased demand for bonds. In contrast, reserves are relatively smaller in supply, so the price of reserves is lower than that of government bonds. The central bank uses



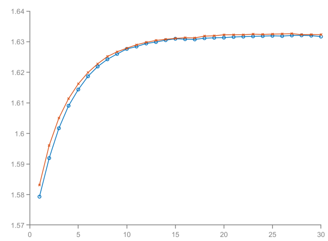
(a) Consumption



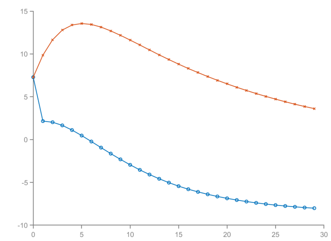
(b) Inflation



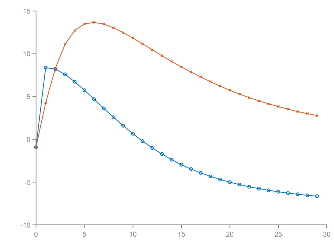
(c) Labor supply



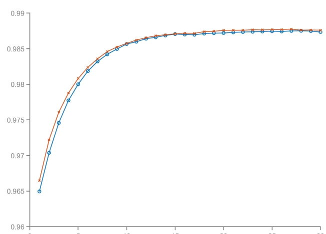
(d) Price of government bonds



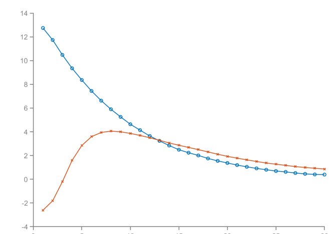
(e) Reserves



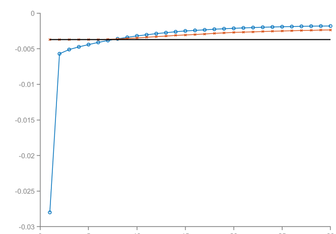
(f) Government bonds



(g) Price of reserves



(h) Sales tax



(i) Remittance

Figure 2: Productivity shock

Note: This figure shows the dynamics of key variables upon the negative productivity shock by 5%. A productivity drops from 1 to 0.95. The horizontal is quarters. The vertical lines for consumption, prices of reserves, prices of government bonds, reserves, government bonds are the steady-state deviation represented by percent. The vertical line for inflation rate is quarterly inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

these cheaper reserves to create additional resources, which are transferred to the Treasury. Upon the shock, remittances take a positive value, meaning that the Treasury receives resources from the central bank. Later, remittances fall below their steady-state level as the central bank receives transfers to reduce the increased reserves.

In terms of real allocations and inflation, labor supply increases to meet the demand for public expenditure, while consumption falls due to crowding out. The inflation rate rises as firms increase production, leading to higher marginal costs. Additionally, the government has an incentive to inflate nominal liabilities because the level of liabilities is higher. Regarding prices, the price of reserves is low and the nominal interest rate is high, which contracts consumption.

Next, I explain the intuition behind the unconsolidated model. In summary, the lack of fiscal backing for the central bank limits its ability to smooth taxes. In the unconsolidated model, remittances cannot fall below a certain threshold. While in the consolidated model, the central bank reduces reserves by receiving transfers from the Treasury, this is not feasible in the unconsolidated model. The central bank understands that large reserves cannot be reduced through remittances later, so it does not issue enough reserves. While the central bank increases reserves in the consolidated model, it does not do so sufficiently in the unconsolidated model. As a result, the central bank does not provide enough support to the Treasury by sending resources. This is shown in the path of remittance. In Figure 3-(h), at time $t = 1$, remittances take a higher value in the consolidated model, while they take a lower value in the unconsolidated model. This indicates that the central bank provides less support to the Treasury's budget in the unconsolidated model. Consequently, the Treasury receives less help from the central bank and must issue more government bonds to finance the increased public expenditure. As a result, government bonds take on a higher value in the unconsolidated model. However, since the Treasury already holds a large quantity of government bonds and their price is high, it becomes difficult for the Treasury to issue enough bonds. This limits the government's ability to smooth taxes. Because the increase in public expenditure is not smoothed through the issuance of liabilities, the higher public expenditure is financed through taxes, resulting in a greater tax rate response in the unconsolidated model compared to the consolidated model.

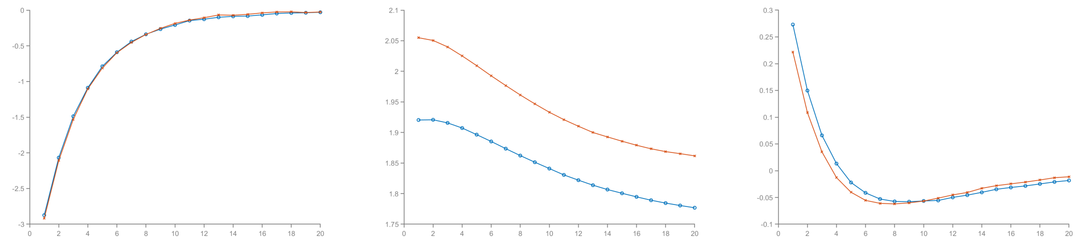
In terms of real allocations, a higher tax rate on sales leads to lower production, i.e., lower labor supply. The difference in consumption between the two models is not significant. However, the inflation rate remains persistently higher in the unconsolidated model as a way to reduce reserves by inflating nominal liabilities, that is, inflating reserves.

Note that these results critically depend on the assumption that the Treasury holds a large amount of government bonds at the time of the shock. Large government bonds limit the Treasury's ability to smooth taxes, making fiscal support from the central bank more crucial when responding to a public expenditure shock. Why is this important? The difference between the unconsolidated model and consolidated model is essentially central bank's ability to fiscally support Treasury. The lack of fiscal backing for the central bank restricts the central bank's ability to provide this support. Since fiscal support from the central bank becomes even more important when the Treasury holds a significant amount of government bonds, the difference between the unconsolidated and consolidated models becomes stark. To show the importance of initial condition, Appendix C.1.2 shows the dynamics when the economy starts from steady-state reserves and government bonds. It shows that the difference between the two models is smaller than that when initial government bonds are small.

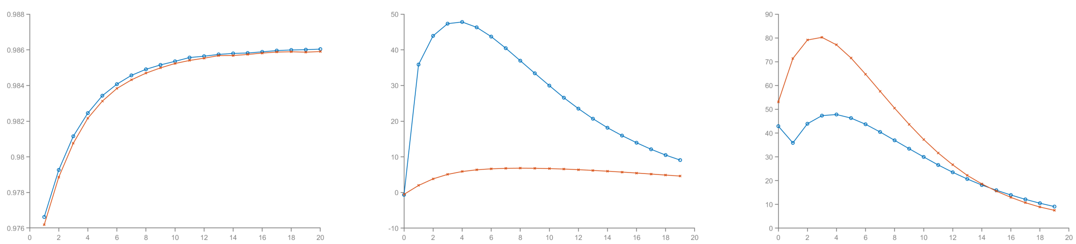
6 Dynamic properties of the policies and allocations

This section is motivated by the question of optimal monetary and fiscal policy over the business cycle. The literature on optimal monetary and fiscal policy addresses how these policies should be set during the business cycle, with particular focus on two questions: (i) the optimal volatility of inflation: how does the government use inflation to stabilize nominal debt? and (ii) the persistence of taxes and debt: how does the government smooth tax distortions by issuing liabilities? This paper addresses these questions while sharing key elements with earlier literature, including: (a) taxes are distortionary, (b) the government issues only nominal and non-state-contingent debt, and (c) product prices are sticky. A key deviation from previous studies is the constraint on transfers from the Treasury to the central bank.

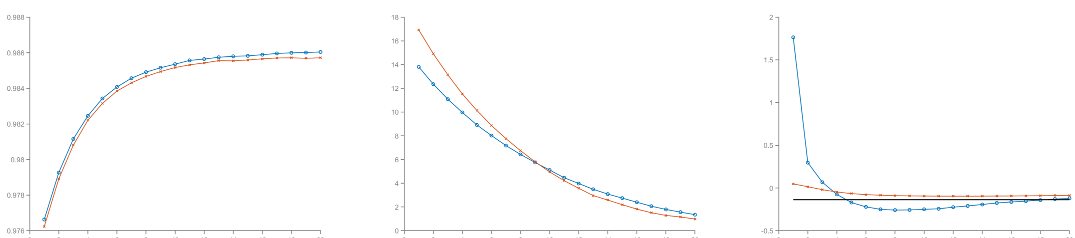
This section presents several sample moments for key macroeconomic variables, as well as monetary and fiscal policy variables, liabilities, and remittances under both the unconsolidated and consolidated models. The goal is to compare the moments between the two models. The moments of interest include the volatility of inflation and tax rates, as well as the persistence of reserves and



(a) Consumption Inflation (b) Labor supply



(c) Price of government bonds (d) Reserves (e) Government bonds



(f) Price of reserves (g) Sales tax (h) Remittance

Figure 3: Public expenditure shock

Note: This figure shows the dynamics of key variables upon the positive public expenditure shock by 3%. The horizontal is quarters. The vertical lines for consumption, prices of reserves, prices of government bonds, reserves, government bonds are the steady-state deviation represented by percent. The vertical line for inflation rate is quarterly inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

government bonds.

The moments are computed as follows: I first generate simulated time series of length T for the variables and calculate the corresponding moments. This procedure is repeated J times, and the average of the moments is computed. In the table, T equals 1000 periods, and J equals 1000 repetitions. The economy is simulated with one shock, while other shocks are excluded due to the large size of the state-space.

6.1 Dynamic property of monetary-fiscal policy and consumption

Table 2 presents the moments for inflation, consumption, and the tax rate in both the consolidated and unconsolidated models. Each panel displays the moments for each shock. Given the size of the state-space, I simulate the model with one shock at a time, excluding the others. The optimal policy is under discretion rather than commitment, as studied in Chari, Christiano, and Kehoe (1991); Schmitt-Grohé and Uribe (2004), which makes direct numerical comparisons with the literature challenging. The volatility of the inflation rate is generally low for all shocks, reflecting the application of optimal monetary and fiscal policy.

The main results are as follows: (i) inflation is more volatile in the unconsolidated model across all shocks; (ii) the average inflation rate increases, though the rise is small; (iii) while the tax rate is more volatile in response to productivity and government expenditure shocks in the unconsolidated model, it is less volatile for cost-push shocks; and (iv) the volatility of consumption is minimally affected.

First, I provide the intuition for the cost-push shock. In the consolidated model, the volatility of inflation and consumption is caused by the cost-push shock, which affects marginal costs. A positive cost-push shock raises inflation and reduces consumption. The volatility of the tax rate arises because the government seeks to align marginal cost with marginal product. When marginal costs rise, it becomes optimal to lower the sales tax rate, as a lower tax rate increases the marginal product. This mechanism is evident in the correlations with the shock: inflation is positively correlated, while consumption and the tax rate are negatively correlated with the shock.

In the unconsolidated model, inflation is more volatile, and the tax rate is less volatile compared to the consolidated model. This is because the issuance of reserves is constrained—large reserves cannot be reduced through remittances. As a result, the central bank does not issue enough reserves and cannot provide sufficient support to the Treasury. The limited ability to reduce taxes in response to the cost-push shock reduces the volatility of the tax rate in the unconsolidated model. Consequently, inflation volatility is higher because changes in the sales tax rate do not sufficiently offset the cost-push shock.

Let me explain the intuition for the consolidated model in response to a public expenditure shock. When a public expenditure shock occurs, the government raises tax rates to finance the increased expenditure. However, since taxes are distortionary, the government smoothes this adjustment over time. This is evident in the high positive correlation between taxes and the public expenditure shock, as well as the high autocorrelation of the tax rate. Consumption is crowded out following a public expenditure shock, and the government increases the inflation rate to reduce the real value of debt, as debt levels rise after the shock. This is reflected in the negative correlation between consumption and the shock, and the positive correlation between inflation and the shock. The volatility of the inflation rate arises from the government's efforts to reduce the real value of debt, while the volatility of the tax rate stems from tax smoothing.

Although the government uses inflation to reduce the real value of debt, this motive is quantitatively small because inflation is costly. In the consolidated model, the volatility of inflation is low, with a standard deviation of less than 0.1 percentage points. While the government has an incentive to use inflation to lower nominal liabilities, the cost of inflation due to price stickiness outweighs the benefits. Schmitt-Grohé and Uribe (2004) find similar results under a Ramsey equilibrium, and while my model operates under discretion, the findings are consistent.

In the unconsolidated model, the volatility of inflation, consumption, and the tax rate is higher than in the consolidated model. The key issue is the government's tax smoothing motive. When faced with a public expenditure shock, the government seeks to smooth tax distortions over time by issuing liabilities. The central bank can issue reserves to support the Treasury's budget by providing resources. However, in the unconsolidated model, the issuance of reserves is constrained, limiting the government's ability to smooth taxes. The volatility of the sales tax leads to volatility in output and inflation through firms' first-order conditions. In summary, the lack of fiscal backing in the unconsolidated model restricts the government's ability to smooth taxes.

Table 2: Dybamic properties of the optimal policy

Variable	Mean	Std. dev.	Auto. corr.	Corr w. shock
Consolidated model with cost-push shock				
π	1.741	0.0862	0.93	0.81
c_t	88.5	0.14	0.84	-0.94
τ_t	10.2	0.52	0.57	-0.78
Unconsolidated model with cost-push shock				
π	1.803	0.0898	0.90	0.82
C_t	88.5	0.00	0.79	-0.96
τ_t	10.3	0.43	0.49	-0.80
Consolidated model with productivity shock				
π	1.759	0.0120	0.93	-0.84
c_t	88.5	0.010	0.73	1.00
τ_t	10.2	0.094	0.90	-0.89
Unconsolidated model with productivity shock				
π	1.767	0.0130	0.91	-0.85
c_t	88.5	0.010	0.74	1.00
τ_t	10.3	0.097	0.87	-0.94
Consolidated model with public expenditure shock				
π	1.765	0.0615	0.93	0.80
c_t	88.5	0.643	0.72	-1.00
τ_t	10.3	0.473	0.88	0.88
Unconsolidated model with public expenditure shock				
π	1.839	0.0791	0.93	0.76
c_t	88.5	0.647	0.71	-1.00
τ_t	10.3	0.482	0.86	0.91

Notes: π, τ are expressed in percentage points, and c are in levels.

6.2 Persistency of liabilities and taxes

An important result in the public finance literature is the idea of smoothing tax distortions over time. Lucas and Stokey (1983) show that government bonds inherit the stochastic process of exogenous shocks. This implies that if shocks are serially uncorrelated, government bonds should also follow an uncorrelated process. However, Barro (1979) and Aiyagari et al. (2002) demonstrate that Lucas and Stokey's results depend on the assumption that the government can issue state-contingent debt. They show that, in the absence of state-contingent debt, government bonds follow a near random walk behavior, regardless of the process assumed for the shocks. This paper examines the process of liabilities and taxes in a setting with non-state-contingent nominal debt and sticky prices, similar to Schmitt-Grohé and Uribe (2004).

The consolidated model in this paper is a standard New Keynesian framework, similar to Schmitt-Grohé and Uribe (2004). The key deviation from the literature is the inclusion of reserves. In the data, reserves are significantly large, making it interesting to study their behavior. The purpose of this section is to investigate whether both reserves and government bonds follow a near random walk under discretion. Additionally, this paper explores the implications of the lack of fiscal support: does the absence of fiscal backing change the stochastic behavior of reserves and government bonds? Moreover, does it alter the properties of taxation?

Table 3 presents the moments for reserves, government bonds, and tax rates. In the consolidated model, both reserves and government bonds exhibit persistent behavior across all types of shocks, consistent with the findings of Schmitt-Grohé and Uribe (2004). The autocorrelations of reserves and government bonds are close to one, indicating their highly persistent nature.

Both reserves and government bonds show near random walk property. Government uses both to smooth tax. Households get convenience yield from reserves and government bonds that are imperfect substitute. If government uses only one of reserves or government bonds to smooth tax, marginal utility of that liability is higher than the other and this is not optimal.

For taxes, while the autocorrelation is high for productivity and public expenditure shocks, it is lower for cost-push shocks. This difference reflects the government's tax-smoothing motive for productivity and public expenditure shocks, where taxes are adjusted over time. For cost-push shocks, however, the government adjusts sales taxes to offset the shock's impact on marginal costs, which leads to a lower autocorrelation.

In the case of a public expenditure shock, taxes are used primarily to finance the increase in public spending. For a productivity shock, taxes are adjusted to compensate for changes in tax revenue due to fluctuations in output. In both cases, the government uses taxes to finance public expenditures, resulting in a strong tax-smoothing motive. In contrast, during a cost-push shock, sales taxes are adjusted to influence marginal costs in response to the shock, leading to a tax process that closely follows the process of the shock, thus explaining the lower autocorrelation.

In the unconsolidated model, the lack of fiscal support causes reserves to become more persistent and government bonds less persistent across all types of shocks. For reserves, the mechanism is similar for all shocks: once the central bank issues reserves, the ability to reduce them through remittance is limited. As a result, reserves decrease slowly, leading to higher persistence.

For government bonds, the persistence is lower in the unconsolidated model. This occurs because the response of government bonds to shocks is larger in the unconsolidated model than in the consolidated model. After this larger response, government bonds return to their steady-state, making them more volatile and less persistent. Why is the response of government bonds larger?

In the consolidated model, the increased demand for liabilities is distributed between reserves and government bonds, as the government utilizes both to generate resources. However, in the unconsolidated model, the central bank's ability to use reserves for tax smoothing is constrained. As a result, the government relies more heavily on government bonds to create resources, making bonds more responsive to shocks and more volatile. Consequently, government bonds exhibit less persistence in the unconsolidated model.

6.3 Implications on retained earnings

This section discusses the relationship between the model and the actual policies of central banks. A key feature of my model is the transfers between the central bank and the Treasury, which naturally raises the question of how central banks decide on their remittances in practice. An important observation is that many central banks retain part of their earnings and do not transfer the entire profit to the Treasury.

In the UK, the remittance policy depends on the central bank's net worth. If the Bank of

Table 3: Autocorrelation of reserves, government bonds, and tax

Reserves	Government bonds	Tax on sales
Consolidated model with cost-push shock		
0.97	0.97	0.57
Unconsolidated model with cost-push shock		
0.99	0.96	0.49
Consolidated model with productivity shock		
0.98	0.98	0.90
Unconsolidated model with productivity shock		
0.99	0.94	0.87
Consolidated model with public expenditure shock		
0.97	0.97	0.88
Unconsolidated model with public expenditure shock		
0.99	0.96	0.86

Notes: Reserves, government bonds, and tax rate are in levels.

England's net worth is below a target level, all profits are retained to boost net worth. If net worth is above the target but below a ceiling level, half of the profits go to the Treasury, while the other half is added to capital. If net worth exceeds the ceiling, all profits are transferred to the Treasury.[9] In Japan, 5% of the Bank of Japan's profits are retained, with the remaining 95% transferred to the Treasury. Given the large BOJ's balance sheet, the retained earnings at BOJ is significant[10].

In summary, BOE and BOJ retain a portion of their profits and do not transfer the entire amount to the Treasury. The Federal Reserve, however, is an exception. In the U.S., the Federal Reserve Act requires the Reserve Banks to remit excess earnings to the U.S. Treasury after covering operating costs. This implies that, during periods when profits exceed costs, remittances are equal to profits, and the Federal Reserve does not accumulate its net worth.

Can the unconsolidated model replicate the fact that central banks do not transfer their entire profit? Is the unconsolidated model more consistent with actual practice than the consolidated model? The answer is yes: the unconsolidated model is more consistent with real-world central bank policies.

Figure 4 presents a histogram of remittances over the business cycle in both the consolidated and unconsolidated models. I simulate the economy for 1000 periods and generate a sequence of remittances for both models. The histograms display remittances in the consolidated model (blue) and the unconsolidated model (red). The left side shows cases where the central bank receives transfers, while the right side shows cases where the Treasury receives remittances.

In the unconsolidated model, there is a lower bound on remittances, meaning the central bank cannot receive more than a certain threshold. Notably, remittances take on higher values less frequently in the unconsolidated model compared to the consolidated model, particularly at the right tail of the histogram. While the unconsolidated model imposes a lower bound on remittances, this constraint also affects the upper tail of the remittance distribution, making it thinner. Intuitively, if the central bank prints more reserves and transfers more to the Treasury, the excess reserves cannot be reduced through remittances due to the constraint. As a result, the central bank chooses not to transfer more. This behavior aligns with the actual practice of central banks, where remittances are

[9]See here for details on the BoE's remittance policy.

[10]In 2023, the ratio of profit earned by BOJ to tax revenues collected by Japanese Treasury is 7%, showing a signiificant fiscal support from BOJ to Treasury.

often lower due to retained earnings.

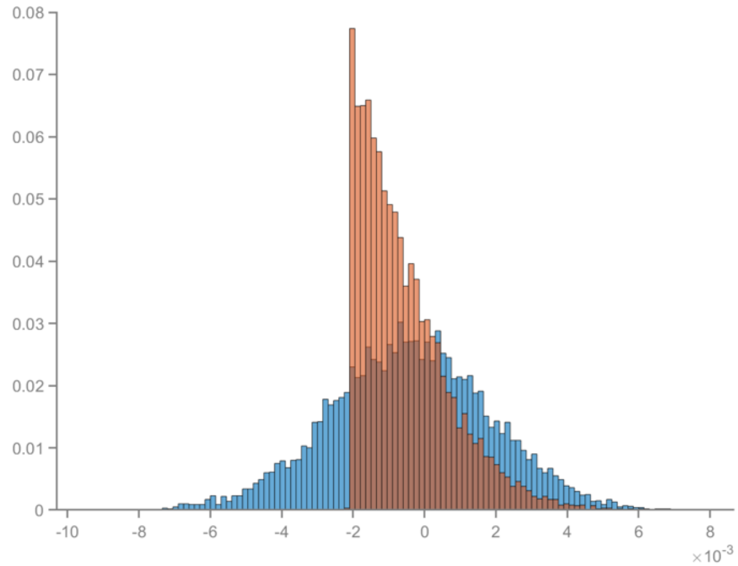


Figure 4: The histogram of remittance in the consolidated and unconsolidated model.

Note: This figure shows a histogram remittance in the consolidated and unconsolidated model. I simulate an economy for 1000 periods and obtain a sequence of remittance for both models. An underlying exogenous shock is cost-push shock. I show it as a histogram after normalization so that an area is equal to one.

7 Welfare analysis

This section aims to analyze the welfare gain of fiscal backing for central bank. Interestingly, different government choose different stance for fiscal backing. In UK, there is agreement between central bank and Treasury that any losses caused by central bank's asset purchase policy is compensated by Treasury. Since the loss created by Bank of England is large, there is discussion about if Treasury should compensate central bank's loss as central bank's loss is essentially compensated by household's tax. The model in this paper includes the cost and benefit of fiscal backing. The cost is that the increase of distortionary tax: the central bank's loss is compensated by distotionary tax. The loss is the impact of monetary and fiscal policy. This section quantitatively evaluates the welfare gain.

Table 4: Unconditional welfare loss

	Consolidated	Unconsolidated
Cost-push shock		
	-0.000698	-0.000864
Productivity shock		
	-0.021491	-0.024720
Public expenditure shock		
	-0.002692	-0.003323

Notes:

7.1 Unconditional welfare loss

I compute unconditional welfare losses under optimal policy, evaluating the utility of households (1), both in the unconsolidated and consolidated model. Welfare losses are obtained averaging the discounted losses across 10000 simulations, of the initial states (s_0, d_0, b_0) from their steady-state values, each 1000 periods long [11]. I obtained welfare loss for two models compared to their own steady-state.

Table 4 shows the welfare cost of business cycle in the consolidated and unconsolidated model. Welfare losses are expressed in terms of their welfare equivalent permanent consumption reduction in percent. The welfare losses associated with cyclical fluctuation is fairly small in absolute size as is usually the case in New Keynesian models.

The difference of welfare loss between the consolidated and unconsolidated model is fairly small. The largest difference is productivity shock, but it is 0.003% of permanent consumption. The difference is much smaller for public expenditure and cost-push shock.

A good comparison of this model is model with zero lower bound on nominal interest rate. Adam and Billi (2007) shows that the existence of zero lower bound lowers the welfare loss by 0.003% under the discretionary policy and 0.0001% under the commitmment policy.

[11] To exclude the impact of initial state on welfare calculation, I through away the first 100 periods from the simulated 1000 periods. Also, (s_0, d_0, b_0) is the stochastic steady-state.

7.2 Conditional welfare loss

This section presents the welfare loss conditional on the size of the shock. This section aims to demonstrate that the larger the shock, the greater the welfare loss from the lack of fiscal backing: the difference in welfare between the consolidated and unconsolidated models.

The procedure is as follows: First, I feed an exogenous path of cost-push shocks into the model. A positive cost-push shock reduces household utility. I then compute the perpetual consumption transfer necessary to make household utility after the shock equivalent to that of households in the steady state. For instance, with no shock, there is zero welfare loss. If a cost-push shock increases by 2%, households are worse off by 0.02% of consumption. For each shock size, I compute the welfare cost of the shock. Second, I perform the same exercise for both the consolidated and unconsolidated models. The initial conditions for each model are set to the steady-state values of reserves and government bonds.

Figure 5 presents the results. The horizontal axis represents the size of the shock, measured as a percentage increase in the wage mark-up, ranging from zero to 9%. The vertical axis shows the welfare cost of the cost-push shock, expressed as a percentage of consumption equivalence. The red line represents the welfare cost in the unconsolidated model, while the blue line represents the cost in the consolidated model.

First, the welfare cost increases with the size of the shock. This occurs because a positive cost-push shock leads to a drop in consumption and an increase in the inflation rate. Second, the welfare cost of the cost-push shock is consistently higher in the unconsolidated model compared to the consolidated model. This is by construction, as the equilibrium in the unconsolidated model is always feasible in the consolidated model, leading to higher welfare in the latter. Third, the gap between the two models widens as the size of the shock increases. For example, when the cost-push shock is a 9% increase, the welfare cost is 0.11% of consumption in the unconsolidated model and 0.9% in the consolidated model. This is because, as the shock grows larger, the constraint on remittances binds for a longer period.

The welfare loss in the unconsolidated model arises from an insufficient tax break due to the lack of fiscal backing. When a cost-push shock occurs, the Treasury seeks to implement a tax break, and the central bank supports this by transferring resources to the Treasury. However, in the unconsolidated model, this support is limited, resulting in an insufficient tax break. As a result, the cost-push shock is not fully mitigated, leading to lower consumption and higher inflation, both of which are costly in terms of welfare.

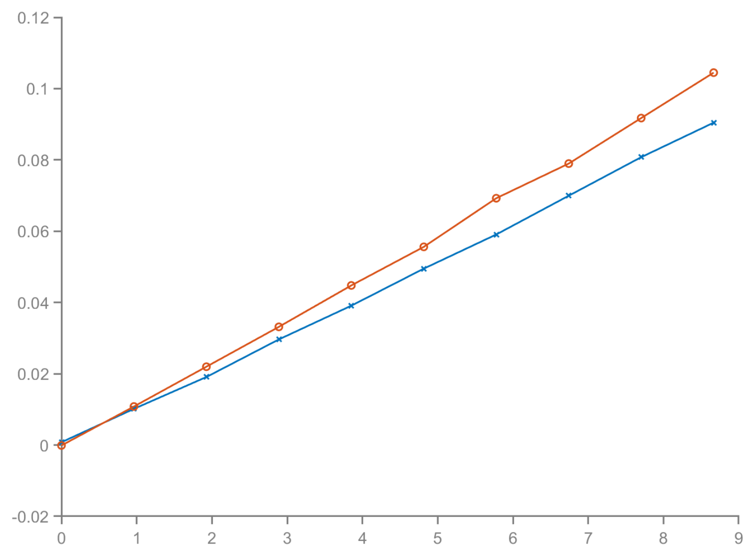


Figure 5: The welfare gain of fiscal backing and size of shock.

Note: This figure shows The welfare gain of fiscal backing and size of shock. The horizontal axis is the size of shock represented by percentage point. The vertical axis is the welfare cost of cost-push shock represented by consumption equivalence in percentage point. The red is the welfare cost in the unconsolidated mode. The blue is that in consolidated model.

8 Conclusion

A common assumption in conventional macroeconomic models of monetary-fiscal policy is a consolidated government budget. However, in practice, the Federal Reserve does not receive funds from Treasury when it incurs a loss. This paper studies optimal monetary and fiscal policy when Treasury is unable to provide optimal fiscal support to the central bank: the unconsolidated government budget.

I analyze the optimal monetary and fiscal policy in a New-Keynesian model with central bank and Treasury, where the transfer of resources from Treasury to the central bank is constrained. This lack of fiscal support implies: (i) the central bank, without fiscal backing, tolerates higher inflation in response to cost-push shocks, and this inflation response increases with the level of reserves; (ii) while the lack of fiscal support increases the volatility of the optimal inflation rate by 3%, the average optimal inflation rate is minimally affected; (iii) the welfare gains from fiscal support are small over the business cycle, but in the case of large shocks, a fiscal backstop reduces the welfare cost of the shock by 20%.

Finally, I discuss several avenues for future research. First, in this model, a single agent, ”the government,” jointly sets both monetary and fiscal policy. However, my model distinguishes between the central bank and the Treasury, so a natural extension would be to allow the central bank to set monetary policy and the Treasury to set fiscal policy. To implement this, each authority would need its own policy objectives and a specified timing protocol. A plausible assumption is that the monetary authority prioritizes inflation stabilization more than the fiscal authority (Rogoff, 1985).

Second, while this model imposes a lower bound on remittances, my model cannot explain why such a constraint exists. One possible reason could be that the central bank values its independence and seeks to avoid reliance on fiscal support. Since the benefits of maintaining central bank independence lie outside the model, an extension could involve endogenizing the constraint on remittances.

References

Adam, Klaus and Roberto M Billi. 2007. “Discretionary monetary policy and the zero lower bound on nominal interest rates.” *Journal of monetary Economics* 54 (3):728–752.

Afonso, Gara and Ricardo Lagos. 2015. “Trade dynamics in the market for federal funds.” *Econometrica* 83 (1):263–313.

Aiyagari, S Rao, Albert Marcet, Thomas J Sargent, and Juha Seppälä. 2002. “Optimal taxation without state-contingent debt.” *Journal of Political Economy* 110 (6):1220–1254.

Amador, Manuel and Javier Bianchi. 2023. "Helicopter Drops and Liquidity Traps." Tech. rep., National Bureau of Economic Research.

Arce, Oscar, Galo Nuno, Dominik Thaler, and Carlos Thomas. 2020. "A large central bank balance sheet? Floor vs corridor systems in a New Keynesian environment." *Journal of monetary economics* 114:350–367.

Barro, Robert J. 1979. "On the determination of the public debt." *Journal of political Economy* 87 (5, Part 1):940–971.

Bassetto, Marco and Todd Messer. 2013. "Fiscal consequences of paying interest on reserves." *Fiscal Studies* 34 (4):413–436.

Bassetto, Marco and Thomas J Sargent. 2020. "Shotgun wedding: Fiscal and monetary policy." *Annual Review of Economics* 12:659–690.

Benigno, Pierpaolo. 2020. "A central bank theory of price level determination." *American Economic Journal: Macroeconomics* 12 (3):258–283.

Benigno, Pierpaolo and Salvatore Nisticò. 2020. "Non-neutrality of open-market operations." *American Economic Journal: Macroeconomics* 12 (3):175–226.

Benigno, Pierpaolo and Michael Woodford. 2003. "Optimal monetary and fiscal policy: A linear-quadratic approach." *NBER macroeconomics annual* 18:271–333.

Berentsen, Aleksander, Sébastien Kraenzlin, and Benjamin Müller. 2018. "Exit strategies for monetary policy." *Journal of Monetary Economics* 99:20–40.

Berentsen, Aleksander, Alessandro Marchesiani, and Christopher J Waller. 2014. "Floor systems for implementing monetary policy: Some unpleasant fiscal arithmetic." *Review of Economic Dynamics* 17 (3):523–542.

Berriel, Tiago C and Saroj Bhattarai. 2009. "Monetary policy and central bank balance sheet concerns." *The BE Journal of Macroeconomics* 9 (1).

Bianchi, Francesco. 2013. "Regime switches, agents' beliefs, and post-World War II US macroeconomic dynamics." *Review of Economic studies* 80 (2):463–490.

Bianchi, Francesco and Cosmin Ilut. 2017. "Monetary/fiscal policy mix and agents' beliefs." *Review of economic Dynamics* 26:113–139.

Bianchi, Javier and Saki Bigio. 2022. "Banks, liquidity management, and monetary policy." *Econometrica* 90 (1):391–454.

Blanchard, Olivier and Jordi Galí. 2007. "Real wage rigidities and the New Keynesian model." *Journal of money, credit and banking* 39:35–65.

Bullock, Michele. 2022. "Review of the Bond Purchase Progra." *Reserve Bank of Australia* .

Carpenter, Seth, Jane Ihrig, Elizabeth Klee, Daniel Quinn, and Alexander Boote. 2018. "The Federal Reserve's balance sheet and earnings: a primer and projections." *39th Issue (March 2015) of the International Journal of Central Banking* .

Chari, Varadarajan V, Lawrence J Christiano, and Patrick J Kehoe. 1991. "Optimal fiscal and monetary policy: Some recent results." *Journal of Money, Credit and Banking* 23 (3):519–539.

Christensen, Jens HE and Signe Krogstrup. 2019. "Transmission of quantitative easing: The role of central bank reserves." *The Economic Journal* 129 (617):249–272.

Christensen, Jens HE, Jose A Lopez, and Glenn D Rudebusch. 2015. "A probability-based stress test of Federal Reserve assets and income." *Journal of Monetary Economics* 73:26–43.

Chugh, Sanjay K. 2006. "Optimal fiscal and monetary policy with sticky wages and sticky prices." *Review of Economic Dynamics* 9 (4):683–714.

———. 2007. "Optimal inflation persistence: Ramsey taxation with capital and habits." *Journal of Monetary Economics* 54 (6):1809–1836.

Cochrane, John H. 2014. "Monetary policy with interest on reserves." *Journal of Economic Dynamics and Control* 49:74–108.

Coleman, Wilbur John. 1991. "Equilibrium in a production economy with an income tax." *Econometrica: Journal of the Econometric Society* :1091–1104.

Curdia, Vasco and Michael Woodford. 2011. “The central-bank balance sheet as an instrument of monetarypolicy.” *Journal of Monetary Economics* 58 (1):54–79.

Del Negro, Marco and Christopher A Sims. 2015. “When does a central bank’s balance sheet require fiscal support?” *Journal of Monetary Economics* 73:1–19.

D’Amico, Stefania and Thomas B King. 2013. “Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply.” *Journal of financial economics* 108 (2):425–448.

Eichengreen, Barry and Poonam Gupta. 2015. “Tapering talk: The impact of expectations of reduced Federal Reserve security purchases on emerging markets.” *Emerging Markets Review* 25:1–15.

Ennis, Huberto M. 2018. “A simple general equilibrium model of large excess reserves.” *Journal of Monetary Economics* 98:50–65.

Floden, Martin. 2022. “My thoughts on the Riksbank’s asset purchases.” *Riksbank* .

Gertler, Mark and Peter Karadi. 2011. “A model of unconventional monetary policy.” *Journal of monetary Economics* 58 (1):17–34.

Goncharov, Igor, Vasso Ioannidou, and Martin C Schmalz. 2023. “(Why) do central banks care about their profits?” *The Journal of Finance* 78 (5):2991–3045.

Hall, Robert E and Ricardo Reis. 2015. “Maintaining central-bank financial stability under new-style central banking.” Tech. rep., National Bureau of Economic Research.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen. 2011. “The effects of quantitative easing on interest rates: channels and implications for policy.” Tech. rep., National Bureau of Economic Research.

———. 2012. “The aggregate demand for treasury debt.” *Journal of Political Economy* 120 (2):233–267.

Leeper, Eric M. 1991. “Equilibria under ‘active’and ‘passive’monetary and fiscal policies.” *Journal of monetary Economics* 27 (1):129–147.

Leeper, Eric M, Campbell Leith, and Ding Liu. 2021. "Optimal time-consistent monetary, fiscal and debt maturity policy." *Journal of Monetary Economics* 117:600–617.

Leeper, Eric M and Xuan Zhou. 2021. "Inflation's role in optimal monetary-fiscal policy." *Journal of Monetary Economics* 124:1–18.

Lopez-Salido, David and Annette Vissing-Jorgensen. 2023. "Reserve demand, interest rate control, and quantitative tightening." *Federal Reserve Board, January* 10.

Lucas, Robert E Jr and Nancy L Stokey. 1983. "Optimal fiscal and monetary policy in an economy without capital." *Journal of monetary Economics* 12 (1):55–93.

Marcet, Albert and Ramon Marimon. 2019. "Recursive contracts." *Econometrica* 87 (5):1589–1631.

Niemann, Stefan and Paul Pichler. 2011. "Optimal fiscal and monetary policies in the face of rare disasters." *European Economic Review* 55 (1):75–92.

Niemann, Stefan, Paul Pichler, and Gerhard Sorger. 2013. "Public debt, discretionary policy, and inflation persistence." *Journal of Economic Dynamics and Control* 37 (6):1097–1109.

Poole, William. 1968. "Commercial bank reserve management in a stochastic model: implications for monetary policy." *The Journal of finance* 23 (5):769–791.

Reis, Ricardo. 2016. "Funding quantitative easing to target inflation." .

Rogoff, Kenneth. 1985. "The optimal degree of commitment to an intermediate monetary target." *The quarterly journal of economics* 100 (4):1169–1189.

Rotemberg, Julio J. 1982. "Sticky prices in the United States." *Journal of political economy* 90 (6):1187–1211.

Sargent, Thomas J and Neil Wallace. 1981. "Some unpleasant monetarist arithmetic." *Federal reserve bank of minneapolis quarterly review* 5 (3):1–17.

Schmitt-Grohé, Stephanie and Martın Uribe. 2004. "Optimal fiscal and monetary policy under sticky prices." *Journal of economic Theory* 114 (2):198–230.

Sims, Christopher A. 1994. "A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy." *Economic theory* 4:381–399.

Siu, Henry E. 2004. "Optimal fiscal and monetary policy with sticky prices." *Journal of Monetary Economics* 51 (3):575–607.

Stella, Peter. 2005. "Central bank financial strength, transparency, and policy credibility." *IMF staff papers* 52 (2):335–365.

———. 2008. "Central bank financial strength, policy constraints and inflation." .

Vissing-Jorgensen, Annette. 2023. "Balance sheet policy above the ELB." *Macroeconomic Stabilisation in a Volatile Inflation Environment* .

Williamson, Stephen D. 2019. "Interest on reserves, interbank lending, and monetary policy." *Journal of Monetary Economics* 101:14–30.

Woodford, Michael. 2001. "Fiscal requirements for price stability."

Appendix A Details on calibration

This section describes details on calibration. Key parameters are γ_1 and γ_2 that deciplines the curvature of utility in convenience yield term.

In the Euler equation for reserves, a price of reserves (Q^C in the left hand side) depends on the quantity of reserves (d_t in the right hand side).

$$\text{(Euler for reserve)} \quad C_t^{-\sigma} Q_t^C = \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] + \chi_1 (Q_t^C d_t)^{-\gamma_1} Q_t^C \tag{42}$$

When I take the left-hand side derivetive with respect to d_t, this derivative depends on γ_1. That suggests that the elasticity of price to quantity is deciplined by γ_1. High γ_1 means that the elasticity is higher i.e, the slope of the demand curve is steep. Low γ_1 means that the elasticity is lower i.e, the slope of the demand curve is flat. Although this is not mathematically very precise, I confirmed that in equilibrium, the elasticity of price to quantitiy decreases with γ_1.

The purpose of this calibration is to match the price elasticity in data to that in model. I run regression both in model and data.

In the model, I simulate the economy and obtain the sequence of the left-hand side and $\frac{b_t^C}{Y_t}$ in equation (44) and $\frac{b_t^T}{Y_t}$ in equation (48). Then, I obtain estimated β_1 and β_3 by running time-series regression of (44) and (48). The β_1 and β_3 are negative and decreases with γ_1 and γ_2.

In data, I also estimate equations (46) and (49). For reserves, I control deposits following Vissing-Jorgensen (2023). For government bonds, I follow Krishnamurthy and Vissing-Jorgensen (2012). I use data from 2009 to 2020. The frequency of data is quarterly. EFR represents the effective fed fund rate. IOR represents interest rate on reserves. Reserves are 'Reserve Balances with Federal Reserve Banks' in FRED. All data is available on FRED.

For reserves and γ_1

$$\text{model} \quad r_t - i_t = \beta_0 + \beta_1 \log(\frac{d_t}{Y_t}) + \epsilon_t \tag{43}$$

$$r_t \equiv \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] \frac{1}{C_t^{-\sigma}} \tag{44}$$

$$i_t \equiv \frac{1}{Q_t^d}. \tag{45}$$

$$\text{data} \quad \text{EFR}_t - \text{IOR}_t = \tilde{\beta}_0 + \tilde{\beta}_1 \log\left(\frac{\text{Reserve}_t}{\text{GDP}_t}\right) + \log(\text{Deposit}_t) + \epsilon_t. \tag{46}$$

I proxy the efffective fed fund rate in data as r_t that is the hypothetical nominal interest rate obtained if there is no convenience value for reserves. i_t in the model is the price of reserves when there is a convenience yiled for reserves.

For γ_2 and government bonds. For Treasury yield, I use yield of 1 month, 3 month, and 6 month maturity's government bonds. Both Treasury yield and effective federal funds rate are annualized. Federal liability is Federal government's liability in FRED.

$$\text{model} \quad r_t - i_t^T = \beta_2 + \beta_3 \log(\frac{b_t}{Y_t}) + \epsilon_t \tag{47}$$

$$i_t^T \equiv \frac{1}{Q_t^b}. \tag{48}$$

$$\text{data} \quad \text{EFR}_t - \text{Teasury yield}_t = \tilde{\beta}_2 + \tilde{\beta}_3 \log\left(\frac{\text{Federal liability}_t}{\text{GDP}_t}\right) + \epsilon_t. \tag{49}$$

Table 5 and 6 show estimated results. In data, the estimated $\tilde{\beta}_1$ are significantly negative, that is a demand curve is downward-sloping. I target $\tilde{\beta}_1 = -0.2$. For government bonds, I target $\tilde{\beta}_3 = -0.1$. I choose γ_1 and γ_2 so that I have $\tilde{\beta}_1 = -0.2$. and $\tilde{\beta}_3 = -0.1$.

Table 5

	Dependent variable:	
	$\text{EFR}_t - \text{IOR}_t$	
	(1)	(2)
log(res_gdp)	-0.165^{***}	-0.297^{***}
	(0.026)	(0.024)
log(dep_gdp)		0.020^{***}
		(0.003)
Constant	0.297^{***}	0.667^{***}
	(0.062)	(0.064)
Observations	36	36
R^2	0.545	0.827
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 6

	Dependent variable:		
	$\text{EFR}_t - 1\ \text{month}_t$	$\text{EFR}_t - 3\ \text{month}_t$	$\text{EFR}_t - 6\ \text{month}_t$
	(1)	(2)	(3)
log(debt_gdp)	0.057	−0.105	−0.321
	(0.102)	(0.136)	(0.220)
Constant	−0.178	0.495	1.361
	(0.447)	(0.596)	(0.963)
Observations	40	40	40
R^2	0.008	0.015	0.053

Notes: ”1 month”, ”3 month”, and ”6 month” represents the yield of each maturities Treasury bonds. *p<0.1; **p<0.05; ***p<0.01

Appendix B Solution methods

I use the collocation method with time iteration. A grid of N interpolation nodes is defined over the state space $S_t \equiv (s_t, b_{t-1}, d_{t-1})$, where s_t represents an exogenous shock out of three exogenous shock, $s_t \in \{\mu_t^w, A_t, G_t\}$. Due to the size of the state space, I include one shock at a time, chosen from cost-push, productivity, or public expenditure shocks. This model has three state variables.

The Markov Perfect Equilibrium consists of policy functions $C(S_t)$, $N(S_t)$, $\pi(S_t)$, $Q^C(S_t)$, $Q^T(S_t)$, $b(S_t)$, $d(S_t)$, $\tau(S_t)$, $H(S_t)$, that solves the government's problem when the expectation terms in the Euler equations for reserves and government bonds, and New Keynesian Philips Curve are given by

$$E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right] = \int \frac{C(S_{t+1}(\varepsilon^s))^{-\sigma}}{\pi(S_{t+1}(\varepsilon^s))} f(\epsilon^s) d(\epsilon^s) \tag{50}$$

$$E_t\left[C_{t+1}^{-\sigma}\frac{1+\rho^T Q_{t+1}^T}{\pi_{t+1}}\right] = \int C(S_{t+1}(\varepsilon^s))^{-\sigma}\frac{1+\rho^T Q^T(S_{t+1}(\varepsilon^s))}{\pi(S_{t+1}(\varepsilon^s))} f(\epsilon^s) d(\epsilon^s) \tag{51}$$

$$E_t\left[N_{t+1}C_{t+1}^{-\sigma}\varphi(\pi_{t+1}-1)\pi_{t+1}\right] = \int N(S_{t+1}(\varepsilon^s))C(S_{t+1}(\varepsilon^s))^{-\sigma}\varphi(\pi(S_{t+1}(\varepsilon^s))-1)\pi(S_{t+1}(\varepsilon^s)) f(\epsilon^s) d(\epsilon^s), \tag{52}$$

where

$$S_{t+1}(\varepsilon^s) \equiv (\bar{s} + \rho^s(s_t - \bar{s}) + \varepsilon^s, b_t(S_t), d_t(S_t)), \tag{53}$$

and $f(\epsilon^s)$ is the probability density distribution of innovations.

The expectations are evaluated at each interpolation N nodes using M node Gaussian-Hermite quadrature. My numerical algorithm is as follows

Step1: Guess initial policy functions for choice variables at the interpolation nodes N.

Step2: At each interpolation nodes N, compute the expectations (50)-(52) implied by the current guessed policy functions. Then, using the first-order conditions of the optimal policy problems, I derive a new guess for the policy functions. The first-order conditions involve the numerical derivative of the expectations (50)-(52) with respect to b_t and d_t. In consolidated model, the new policy functions are obtained. In the unconsolidated model, if $H_t < H^*$, set $H_t = H^*$ and solve for the choice variables. This delivers a new guess.

Step3: Compute the expectations given the new policy functions. Repeat step2 until the guessed policy functions and new policy functions are converged.

This paper focuses on optimal policy under discretion. Although it would be interesting to solve for the optimal policy under commitment, doing so would involve solving a model with six state variables: the shock, reserves, government bonds, and three Lagrange multipliers for the forward-looking equations (Marcet and Marimon, 2019). This significantly increases computational complexity, making it highly challenging.

Appendix C Initial conditions and transition dynamics after shocks

The goal of this section is to understand how the initial levels of reserves and government bonds affect the dynamics after a shock. In section 5, the economy does not start with steady-state levels of reserves and bonds. For the cost-push and productivity shocks, the economy starts with reserves that motivate the central bank to reduce reserves and require transfers from the Treasury. For the government spending shock, the economy starts with a large amount of government bonds, in which case the Treasury does not smooth taxes by issuing additional bonds.

Section C.1 examines the dynamics following a shock when the economy starts from a steady state. In this scenario, the difference between the consolidated and unconsolidated models is smaller than in the results presented in section 5.

Section C.2 examines the impact of the central bank's assets. While the exercise in section 5 focuses on the impact of the central bank's liabilities, section C.2 examines the impact of the central bank's assets on fiscal backing. The effect of assets is ambiguous. The central bank's assets consist of long-dated assets, so when the nominal interest rate rises, the price of these assets falls significantly, resulting in capital losses. If the central bank holds large assets, it will incur larger capital losses, which may require transfers from the Treasury. However, holding larger assets also means that the central bank could sell assets rather than rely on transfers from the Treasury. In addition, while a lower price for long-term bonds results in capital losses, it also allows the central bank to purchase assets at reduced prices, which benefits the central bank's asset position. Section

C.2 examines each of these mechanisms.

C.1 Steady-state as an initial condition

C.1.1 Productivity shock

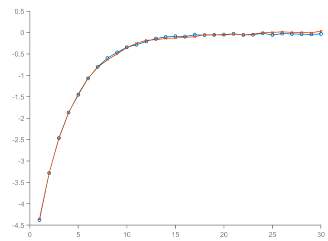
This section describes the dynamics after a positive public expenditure shock. The initial condition for reserves and government bonds are stead-state level, unlike the exercise in section 5.2 where the economy starts from large reserves. The goal of this section is to show the difference between the unconsolidated and consolidated model depends on the initial conditions.

Figure 6 shows results. The key difference between 2 and 6 are how long remittance is binding in the unconsolidated model. In figure 6, reserves are smaller and resulting interest payment is smaller. The central bank does not require funds from Treasury a lot, so the lack of fiscal backing does not impact the dynamics a lot. In figure 2, initial reserves are large. Central bank needs funds to reduce it. In addition, a shock happens and nominal interest rate increases. That requires additional funds from Treasury. To balance the budget, central bank cannot raise nominal interest rate in the unconsolidated model. Consumption does not contract enough and inflation rate is not stabilized. As a result, the lack of fiscal support makes a difference.

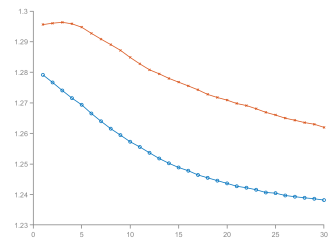
C.1.2 Public expenditure shock

This section describes the dynamics after a positive public expenditure shock. The initial condition for reserves and government bonds are stead-state level, unlike the exercise in section 5.3 where the economy starts from large government bonds. The goal of this section is to show the difference between the unconsolidated and consolidated model depends on the initial conditions.

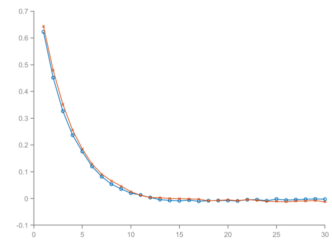
Figure 7 shows results. The key different from figure 7 and figure 3 is the dynamics for sales tax. In figure 7, the unconsolidated model shows that tax smoothing is limited; the initial response of tax rate in the unconsolidated model is larger than that in the consolidated model. However, the overshooting of tax rate in the unconsolidated model is smaller in figure 7 than that in figure 3. This is because Treasury holds relatively smaller government bonds in exercise of figure 7. Treasury has more capacity to issue government bonds more, and can smooth tax even though support from central bank is limited.



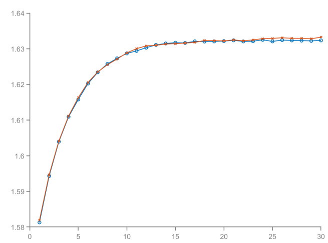
(a) Consumption



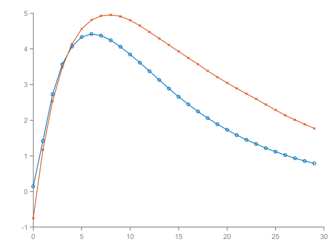
Inflation



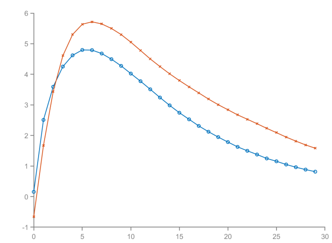
(b) Labor supply



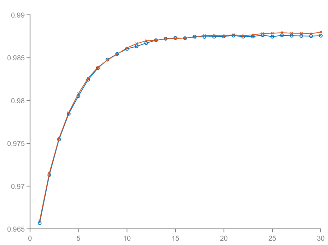
(c) Price of government bonds



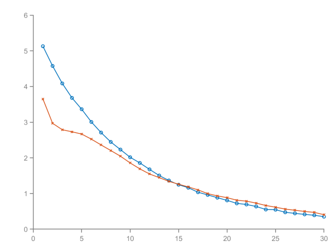
(d) Reserves



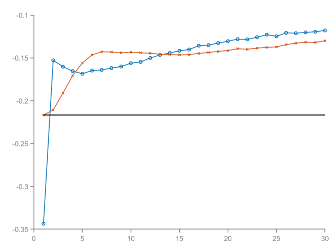
(e) Government bonds



(f) Price of reserves



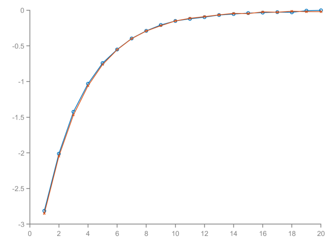
(g) Sales tax



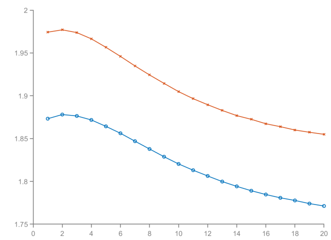
(h) Remittance

Figure 6: Negative productivity shock

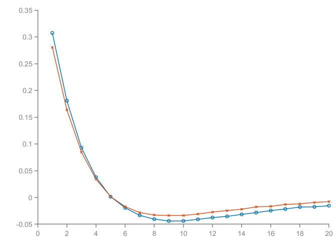
Note: This figure shows the dynamics of key variables upon a negative productivity shock by 5%. The horizontal is quarters. The vertical lines for consumption, prices of reserves, prices of government bonds, reserves, government bonds are the steady-state deviation represented by percent. The vertical line for inflation rate is quarterly inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.



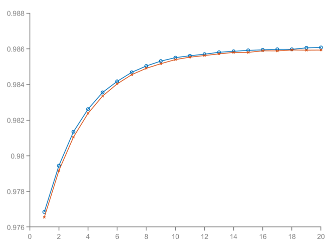
(a) Consumption



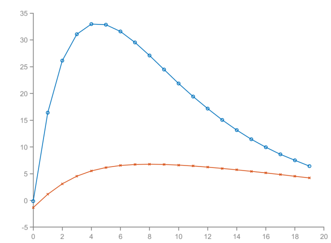
Inflation



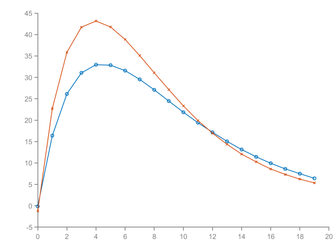
(b) Labor supply



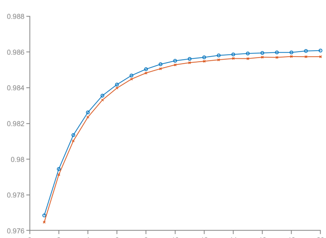
(c) Price of government bonds



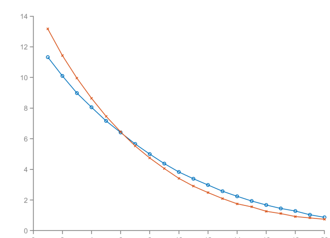
(d) Reserves



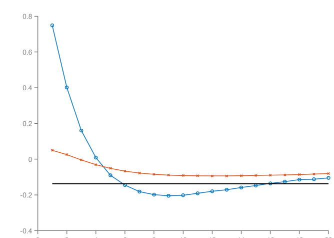
(e) Government bonds



(f) Price of reserves



(g) Sales tax



(h) Remittance

Figure 7: Positive public expenditure shock

Note: This figure shows the dynamics of key variables upon the positive public expenditure shock by 3%. The horizontal is quarters. The vertical lines for consumption, prices of reserves, prices of government bonds, reserves, government bonds are the steady-state deviation represented by percent. The vertical line for inflation rate is quarterly inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

C.2 Role of central bank's assets

C.2.1 Productivity shock

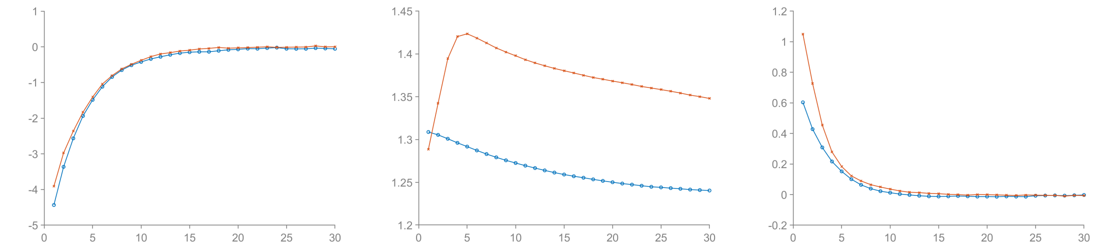
This section aims to understand the role of central bank's assets as an initial condition. The central bank's asset has an ambiguous impact on the budget upon the shock. Suppose central bank has larger assets than steady-state value. When the nominal interest rate increases, the price of long-duration government bonds drops more. This leads to capital loss for central bank. As central bank holds larger assets, the capital loss is larger. In contrast, larger asset means that central bank can sell assets when its budget is tightened. That help central bank's budget.

Figure 8 shows the transition dynamics following a negative productivity shock, with initial conditions of larger reserves and fewer government bonds. After the shock, the central bank holds fewer government bonds as assets. The difference between the consolidated and unconsolidated models is more pronounced here than in Figure 2. With fewer assets, the central bank needs to increase its asset holdings. To purchase assets, transfers from the Treasury are required, but these are constrained. Consequently, the central bank issues more reserves to finance asset purchases, resulting in excess reserves. Since households are unwilling to absorb the excess reserves, the central bank aims to lower the nominal interest rate, thereby amplifying the difference between the two models.

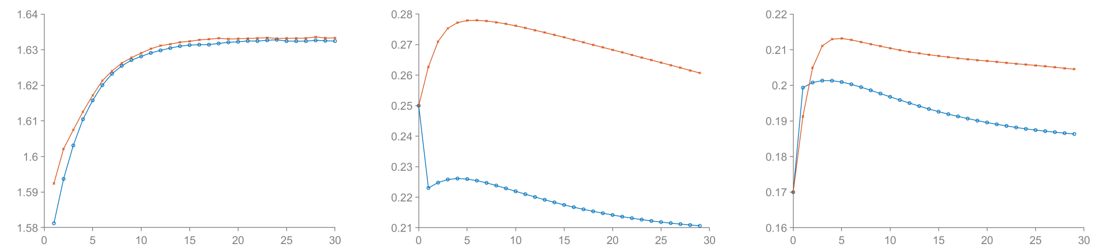
Appendix D Size of shock

This section aims to understand the relation of the size of shock and the difference between the consolidated and unconsolidated model. The difference between the two models are the constraint on remittance. The lower bound on remittance is fixed parameter, so it is reasonable to guess that the difference between the two models depends on the size of shock. For example, when the shock is small, the difference between the two models are small. When the shock is larger, the difference could be larger. I verify this guess in this section.

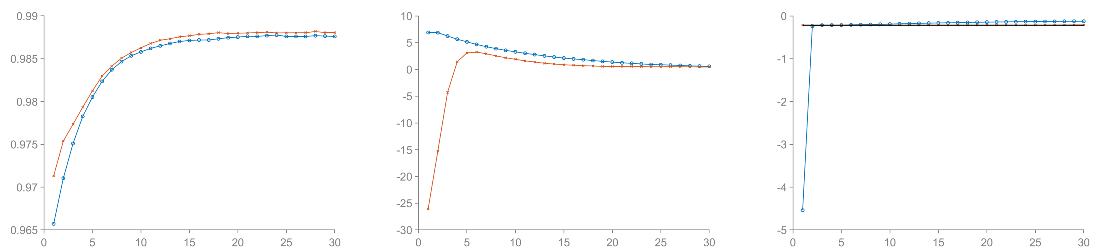
The procedure is as follows; $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. Different lines represent different shock sizes. For example, a 4% positive productivity shock occurs at time



(a) Consumption | Inflation | (b) Labor supply



(c) Price of government bonds | (d) Reserves | (e) Government bonds



(f) Price of reserves | (g) Sales tax | (h) Remittance

Figure 8: Negative productivity shock

Note: This figure shows the dynamics of key variables upon a negative productivity shock by 5%. The horizontal is quarters. The vertical lines for consumption, prices of reserves, prices of government bonds, reserves, government bonds are the steady-state deviation represented by percent. The vertical line for inflation rate is quarterly inflation rate in percent, $\left(\left(\frac{P_t}{P_{t+1}}\right)^4 - 1\right) * 100$. The vertical for remittance is ratio of remittance to GDP in percent $\frac{H_t}{Y_t} * 100$. The blue line represents the outcomes in the consolidated model. The red line represents the outcomes in the unconsolidated model.

$t = 1$ and follows an AR(1) process. I simulate the inflation rate based on this exogenous shock. The same exercise is conducted for different size of shocks.

Productivity shock Figure 9 illustrates the difference in inflation between the consolidated and unconsolidated models for various shock sizes. I plot $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. The inflation rate is simulated based on this exogenous shock, with the same exercise conducted for shocks of 4%, 2%, 0%, -2%, and -4%.

Figure 9 shows that, as the size of the shock increases (e.g., -4%), the difference in inflation between the two models is greater than with a smaller shock (e.g., -2%). For example, when productivity drops by 4%, the inflation rate responds 0.09% more in the unconsolidated model than in the consolidated model. When the productivity shock is a 2% drop, the inflation rate responds 0.075% more in the unconsolidated model. This suggests that larger shocks lead to a greater difference between the two models.

Intuitively, this result comes from the response of nominal interest rate. When a productivity shock is larger, the central bank raises nominal interest rate more, leading to higher interest expenses. Since the transfer from Treasury is bounded by H^*, higher interest expenses leads to higher reserves. Central bank faces larger motive to reduce nominal interest rate in the unconsolidated model. Central bank tolerates higher inflation rate as the size of shock gets larger.

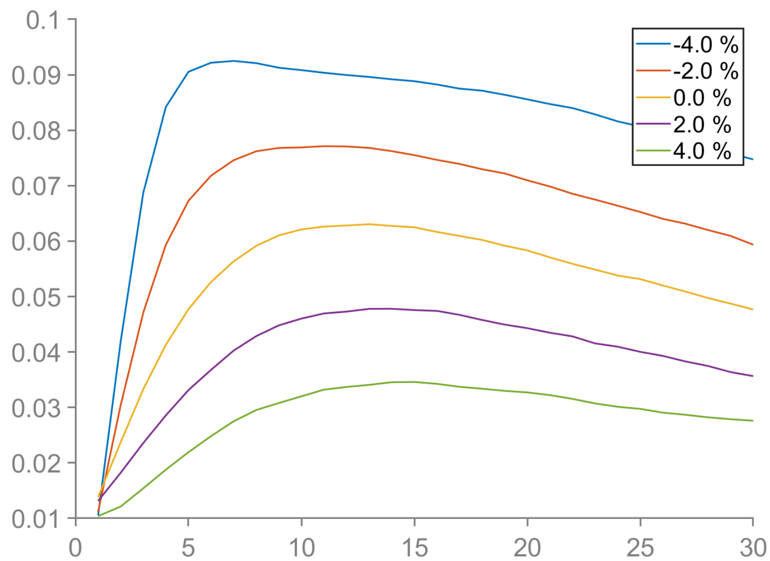


Figure 9: Difference in inflation between consolidated and unconsolidated model

Note: This figure shows the difference in inflation between the consolidated and unconsolidated models for various shock sizes. The horizontal axis represents time, and the vertical axis shows $\pi_t^{unc} - \pi_t^{con}$, where π_t^{unc} is the annualized inflation rate (in percentage points) for the unconsolidated model, and π_t^{con} is the same for the consolidated model. Different lines represent different shock sizes. For example, a 4% positive productivity shock occurs at time $t = 1$ and follows an AR(1) process. I simulate the inflation rate based on this exogenous shock. The same exercise is conducted for shocks of 4%, 2%, 0%, -2%, and -4%.