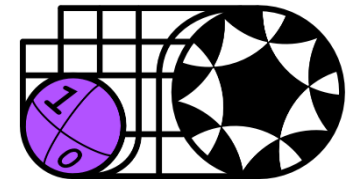
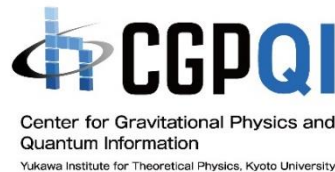


Application of Quantum Computation to High Energy Physics

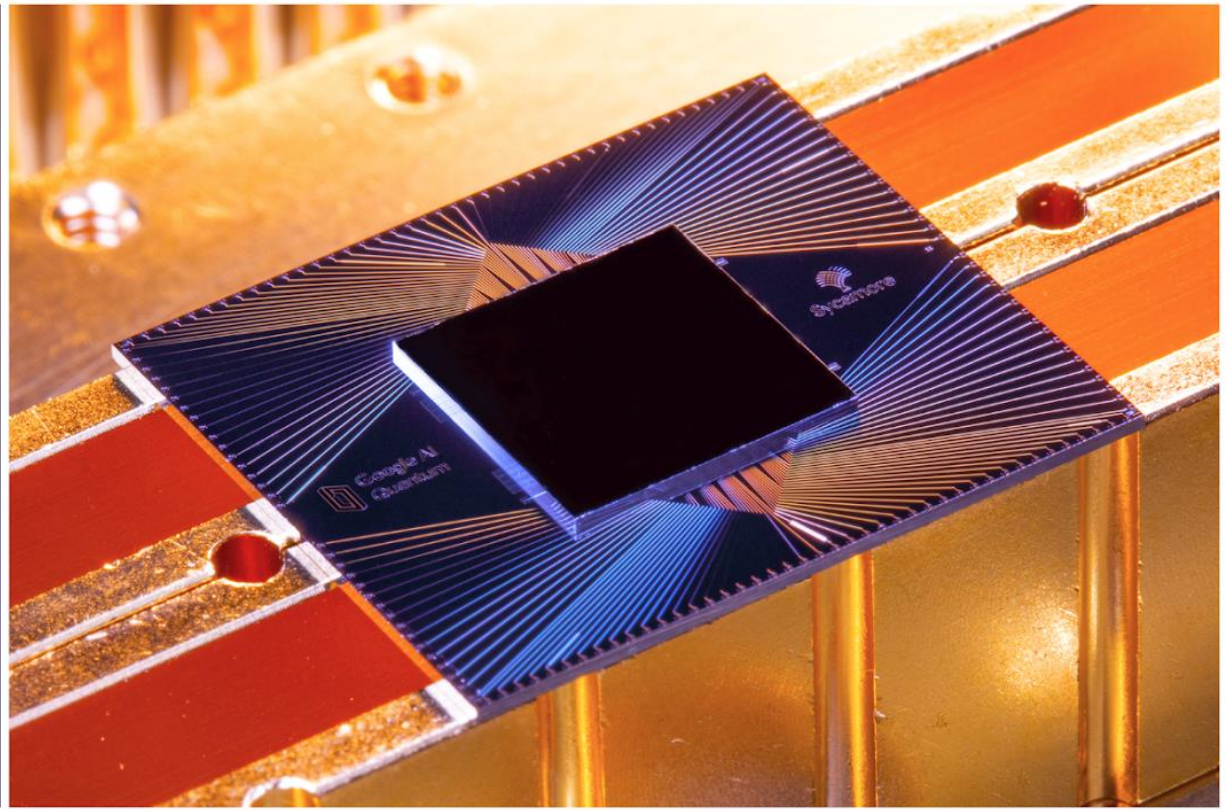
– Basics & Spin system –

Masazumi Honda

(本多正純)



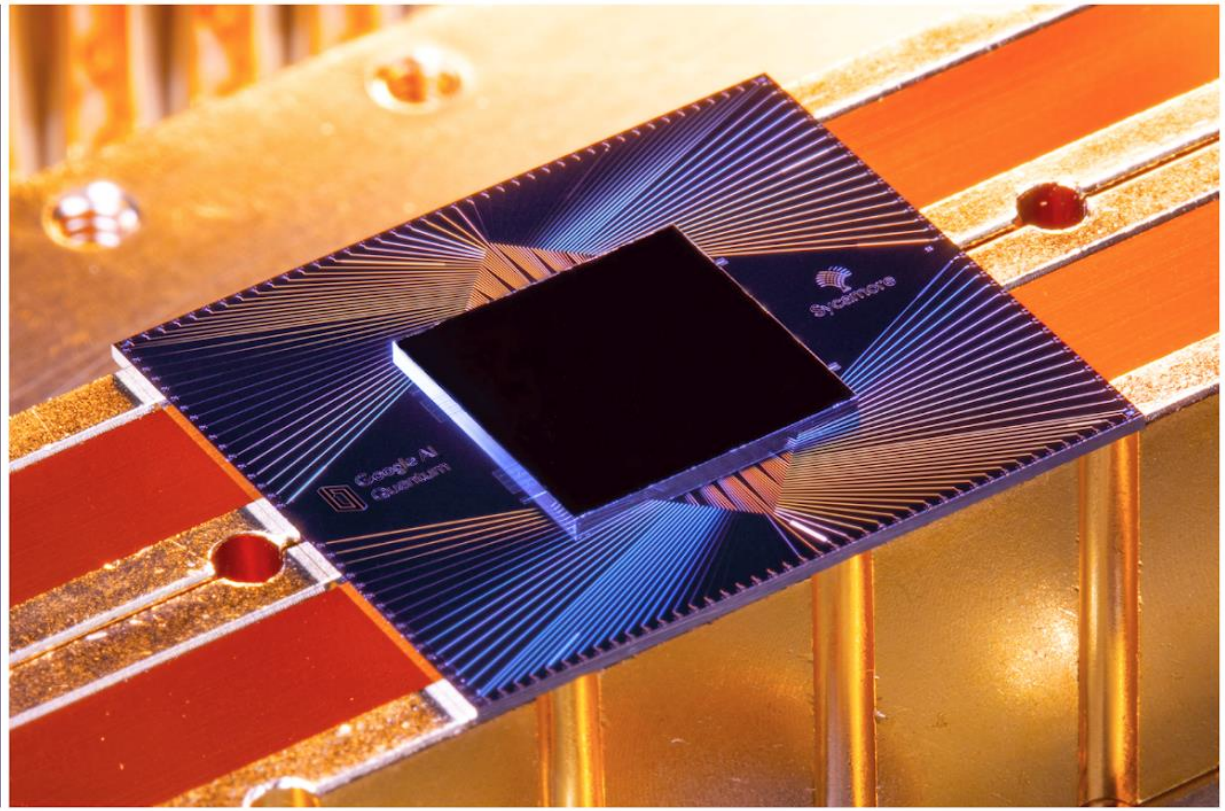
Quantum computer sounds growing well...



Article

Quantum supremacy using a programmable superconducting processor

Quantum computer sounds growing well...

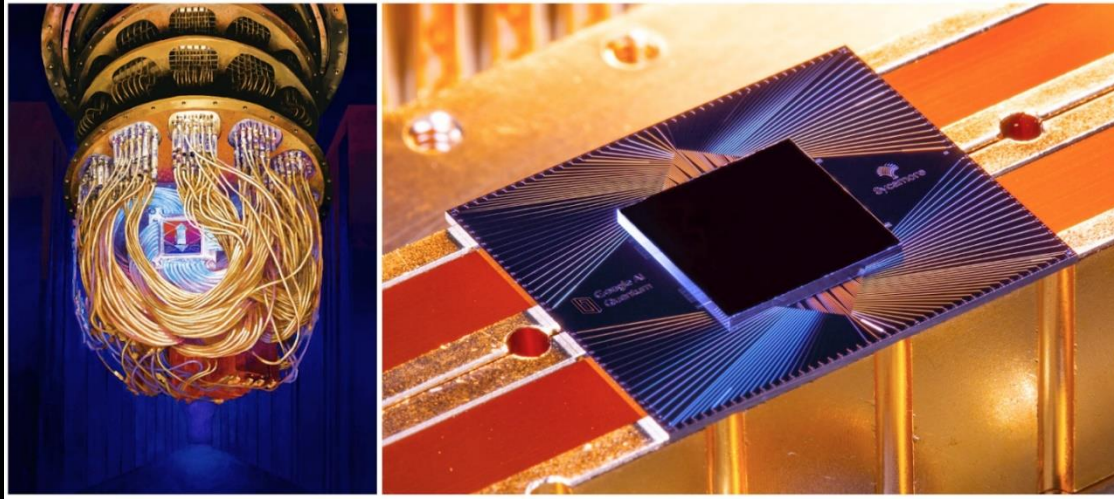


Article

Quantum supremacy using a programmable superconducting processor

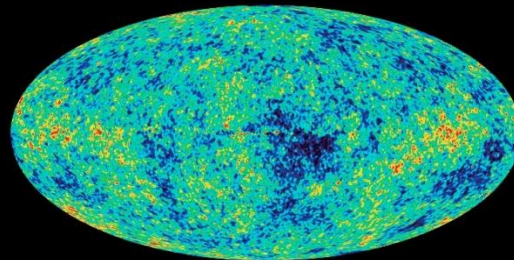
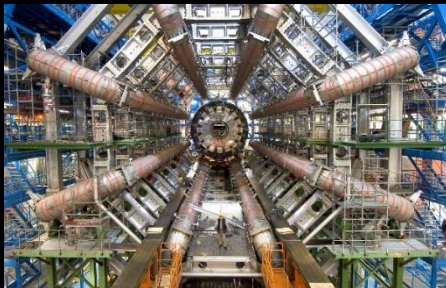
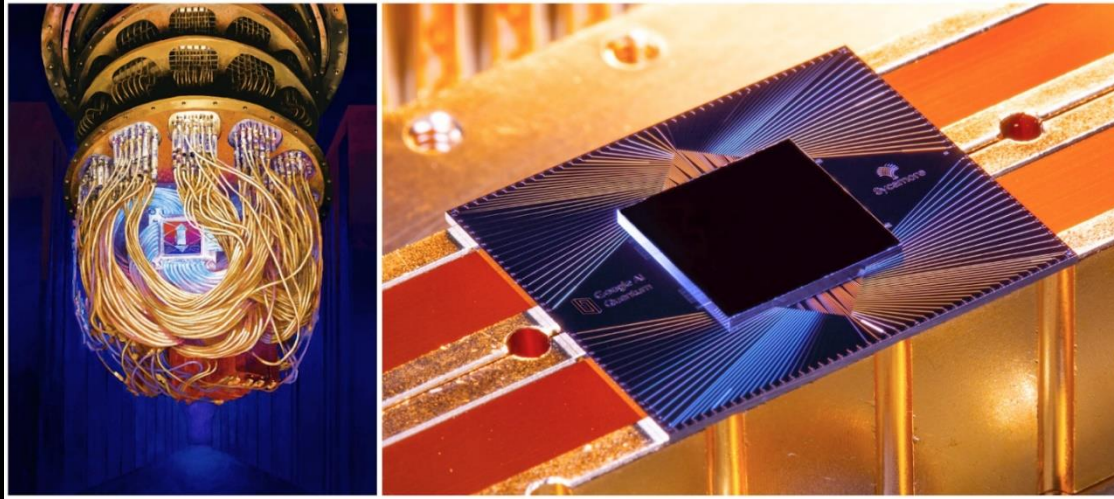
What can we do?

Applications mentioned in media ?



etc...

In my mind...



etc...

This lecture is on

Application of Quantum Computation to Quantum Field Theory (QFT)

- Generic motivation:

simply would like to use powerful computers?

- Specific motivation:

This lecture is on

Application of Quantum Computation to Quantum Field Theory (QFT)

- Generic motivation:

simply would like to use powerful computers?

- Specific motivation:

Quantum computation is suitable for **operator** formalism

→ Liberation from infamous **sign problem** in Monte Carlo?

(next slide)

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT: (this point will be elaborated tomorrow)

① Discretize **Euclidean** spacetime by lattice:



& make **path integral** finite dimensional:

$$\int D\phi \mathcal{O}(\phi) e^{-S[\phi]} \quad \longrightarrow \quad \int d\phi \mathcal{O}(\phi) e^{-S(\phi)}$$

②

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT: (this point will be elaborated tomorrow)

① Discretize **Euclidean** spacetime by lattice:



& make **path integral** finite dimensional:

$$\int D\phi \mathcal{O}(\phi) e^{-S[\phi]} \quad \longrightarrow \quad \int d\phi \mathcal{O}(\phi) e^{-S(\phi)}$$

② Numerically Evaluate it by (Markov Chain) Monte Carlo method regarding the Boltzmann factor as a **probability**:

$$\langle \mathcal{O}(\phi) \rangle \simeq \frac{1}{\#(\text{samples})} \sum_{i \in \text{samples}} \mathcal{O}(\phi_i)$$

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

problematic when Boltzmann factor **isn't** $R_{\geq 0}$ & is highly oscillating

Examples w/ sign problem:

- topological term ——— complex action
- chemical potential ——— indefinite sign of fermion determinant
- real time ——— “ $e^{iS(\phi)}$ ” *much worse*

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

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Examples w/ sign problem:

- topological term ——— complex action
- chemical potential ——— indefinite sign of fermion determinant
- real time ——— “ $e^{iS(\phi)}$ ” *much worse*

In **operator formalism**,

sign problem is absent from the beginning

(\exists various approaches within framework of path integral formalism but I'll skip it)

Cost of operator formalism

We have to play with huge vector space
since QFT typically has ∞ -dim. Hilbert space
regularization needed!

Technically, computers have to
memorize huge vector & multiply huge matrices

Cost of operator formalism

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Quantum computers do this job?

Should we care now as “users”?

Quantum computers don't have sufficient powers yet.

Shouldn't we start to care after quantum supremacy comes?

Should we care now as “users”?

Quantum computers don't have sufficient powers yet.

Shouldn't we start to care after quantum supremacy comes?

I personally think:

∃ **Many things to do even now in various contexts**

(numerical/analytic/purely algorithmic/lat/th/ph/astro/cosmo)

For instance,

- we haven't established

- how to put QCD efficiently on quantum computers
 - how to efficiently pick up various real time physics

(e.g. scattering/dynamical hadronization)

- ∃ only few examples so far to take a serious continuum limit

[cf. Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

Plan of the intensive lectures

Day 1

(If 2nd lecture of each day ends early, then we start hands-on early)

- Lecture 1: introduction, basics of quantum computation
- Lecture 2: quantum simulation of spin system
- Hands-on 1: Basics on IBM's qiskit

Day 2

- Lecture 3: quantum field theory (QFT)
- Lecture 4: QFT on quantum computer
- Hands-on 2: Time evolution of spin system

Day 3

- Lecture 5: quantum error correction
- Lecture 6: some advanced topics, future prospects
- Hands-on 3: Constructing ground state of spin system

Plan of lecture 1

0. Introduction

1. Qubits and gates

2. Some demonstrations in IBM Q Experience

3. Quantum simulation of Spin system

4. Summary

Qubit = Quantum Bit

Qubit = Quantum system w/ 2 dim. Hilbert space

Basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{“computational basis”}$$

Generic state:

$$\alpha|0\rangle + \beta|1\rangle \quad \text{w/} \quad |\alpha|^2 + |\beta|^2 = 1$$

Ex.) Spin 1/2 system:

$$|0\rangle = |\uparrow\rangle, \quad |1\rangle = |\downarrow\rangle$$

(We don't need to mind how it is realized as “users”)

Single qubit operations

- Acting unitary operator: $|\psi\rangle \rightarrow U|\psi\rangle$ (multiplying 2x2 unitary matrix)

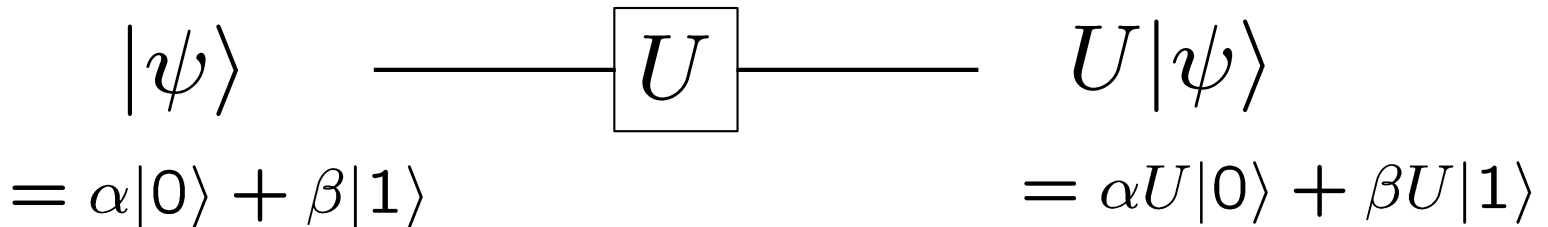
In quantum circuit notation,

$$\begin{array}{ccc} |\psi\rangle & \text{---} \boxed{U} \text{---} & U|\psi\rangle \\ = \alpha|0\rangle + \beta|1\rangle & & = \alpha U|0\rangle + \beta U|1\rangle \end{array}$$

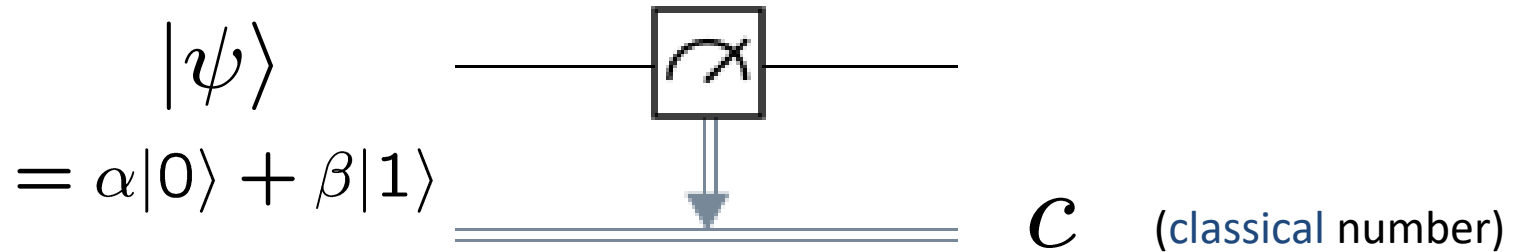
Single qubit operations

- Acting unitary operator: $|\psi\rangle \rightarrow U|\psi\rangle$ (multiplying 2x2 unitary matrix)

In **quantum circuit** notation,



- Measurement:



$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

Single qubit gates used here

X, Y, Z gates : (just Pauli matrices)

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

X is “**NOT**”: $X|0\rangle = |1\rangle, X|1\rangle = |0\rangle$

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$$R_X(\theta) = e^{-\frac{i\theta}{2}X}, \quad R_Y(\theta) = e^{-\frac{i\theta}{2}Y}, \quad R_Z(\theta) = e^{-\frac{i\theta}{2}Z}$$

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Hadamard gate :

$$H = \frac{1}{\sqrt{2}}(X + Z) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \equiv |+\rangle, \quad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \equiv |-\rangle$$

Single qubit gates used here

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T gate :

$$T = e^{\frac{\pi i}{8}} R_Z\left(\frac{\pi}{4}\right) = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

Multiple qubits

2 qubits – 4 dim. Hilbert space:

$$|\psi\rangle = \sum_{i,j=0,1} c_{ij} |ij\rangle, \quad |ij\rangle \equiv |i\rangle \otimes |j\rangle$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

N qubits – 2^N dim. Hilbert space:

$$|\psi\rangle = \sum_{i_1, \dots, i_N=0,1} c_{i_1 \dots i_N} |i_1 \dots i_N\rangle,$$

$$|i_1 i_2 \dots i_N\rangle \equiv |i_1\rangle \otimes |i_2\rangle \otimes \dots \otimes |i_N\rangle$$

Two qubit gates used here

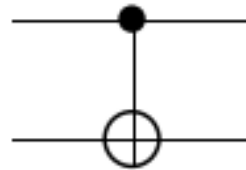
Controlled X (NOT) gate:

$$\begin{cases} CX|00\rangle = |00\rangle, & CX|01\rangle = |01\rangle, \\ CX|10\rangle = |11\rangle, & CX|11\rangle = |10\rangle \end{cases}$$

or equivalently

$$CX|0\rangle \otimes |\psi\rangle = |0\rangle \otimes |\psi\rangle, \quad CX|1\rangle \otimes |\psi\rangle = |1\rangle \otimes X|\psi\rangle$$

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$



Two qubit gates used here

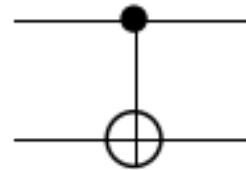
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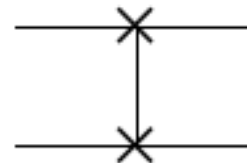
$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$



SWAP gate:

$$\text{SWAP}|\psi\rangle \otimes |\phi\rangle = |\phi\rangle \otimes |\psi\rangle$$

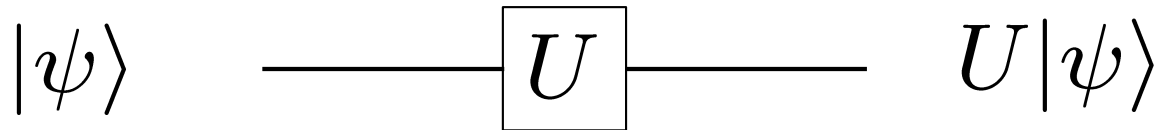
$$\text{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$



We'll see this is useful to compute Renyi entropy

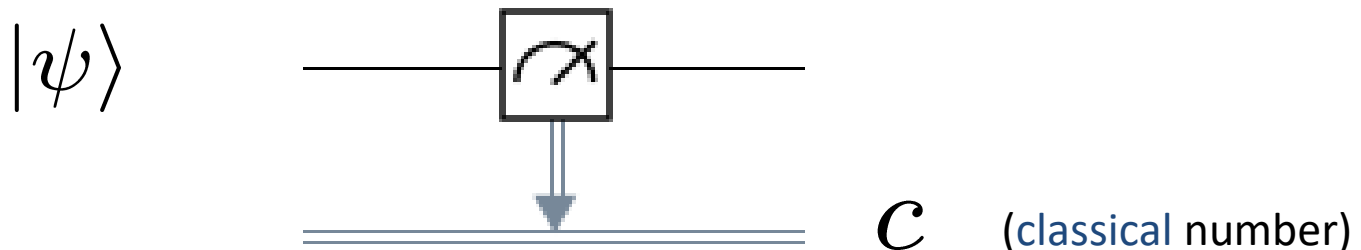
Rule of the game

Do something interesting by a combination of
action of Unitary operators:



&

measurements:

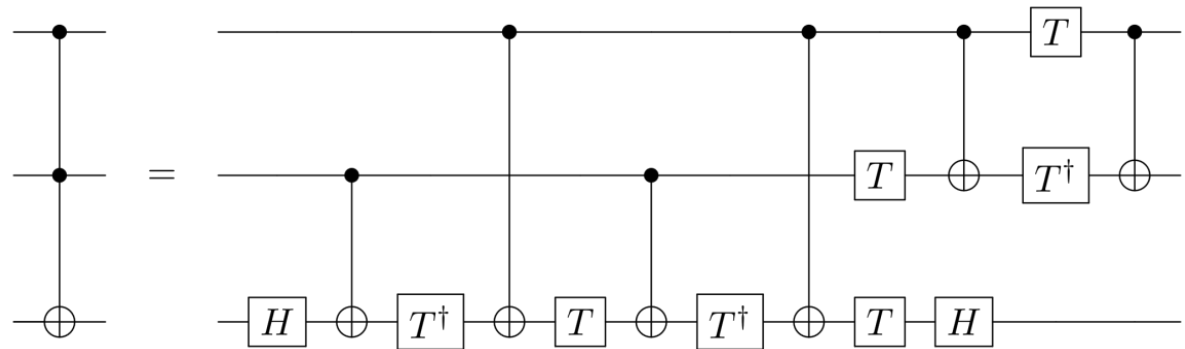


Universality

- Any unitary gate is a combination of single qubit gates & CX (“Single qubit gates & CX are **universal**”)

Ex.) Toffoli

(controlled-controlled-NOT)

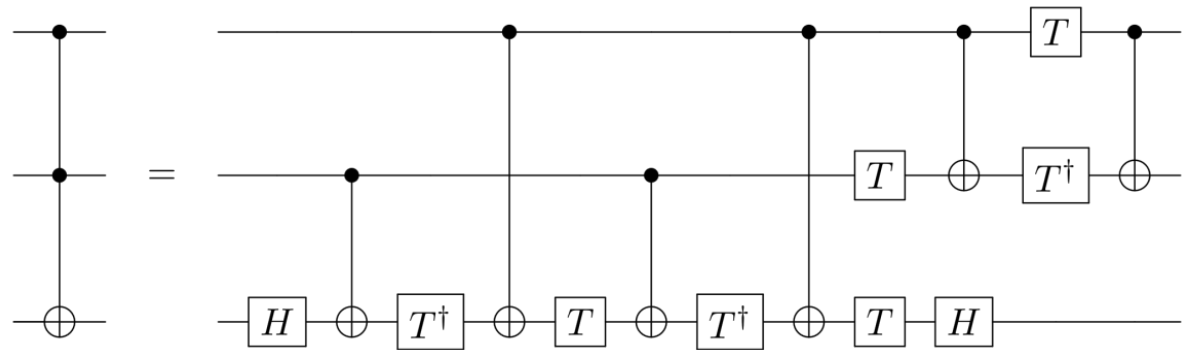


Universality

- Any unitary gate is a combination of single qubit gates & CX (“Single qubit gates & CX are **universal**”)

Ex.) Toffoli

(controlled-controlled-NOT)



- Any single qubit gate is approximated by a combination of H & T in arbitrary precision (next slide)

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

- H, T & CX are universal**

Approximation of single qubit gate by H & T

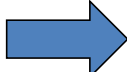
- ① Get a rotation with angle $2\pi \times$ (irrational):

$$THTH = e^{\frac{i\pi}{4}} R_{\vec{n}}(\theta) \quad \text{with } R_{\vec{n}}(\theta) \equiv e^{-\frac{i}{2}\vec{n} \cdot \vec{\sigma}}$$

where

$$\vec{n} = \frac{1}{\sqrt{1 + \cos^2(\pi/8)}} \begin{pmatrix} \cos(\pi/8) \\ \sin(\pi/8) \\ \cos(\pi/8) \end{pmatrix} \quad \& \quad \underbrace{\cos(\theta/2)}_{2\pi \times (\text{irrational})!} \equiv \cos^2(\pi/8)$$

- ② Use Weyl's uniform distribution theorem:

$\frac{\theta}{2\pi} \mathbf{Z}$ is uniformly distributed mod 1  approximate $R_{\vec{n}}(\alpha)$ for $\forall \alpha$

- ③ Construct rotation around another axis:

$$HR_{\vec{n}}(\alpha)H = R_{\vec{m}}(\alpha) \quad \text{with } \vec{m} = \frac{1}{\sqrt{1 + \cos^2(\pi/8)}} \begin{pmatrix} \cos(\pi/8) \\ -\sin(\pi/8) \\ \cos(\pi/8) \end{pmatrix}$$

- ④ Approximate \forall single qubit gate: $R_{\vec{n}}(\alpha)R_{\vec{m}}(\beta)R_{\vec{n}}(\gamma)$

(To achieve accuracy ϵ , it requires $\mathcal{O}(\log^c(1/\epsilon))$ gates w/ $c \sim 2$) [Solovay '95, Kitaev '97]

Errors in classical computers

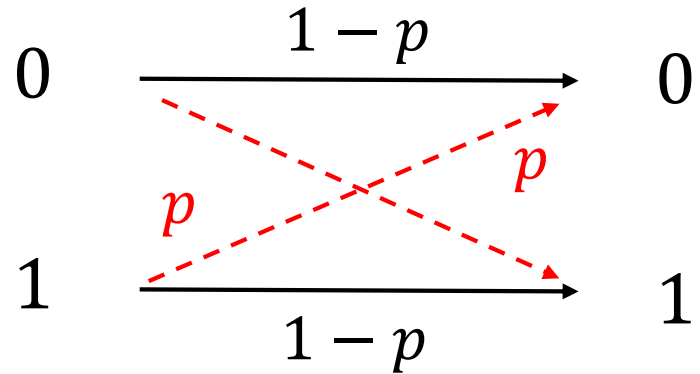
Computer interacts w/ environment → error/noise

Errors in classical computers

Computer interacts w/ environment \Rightarrow error/noise



one bit



Suppose we send a bit but have “error” in probability p

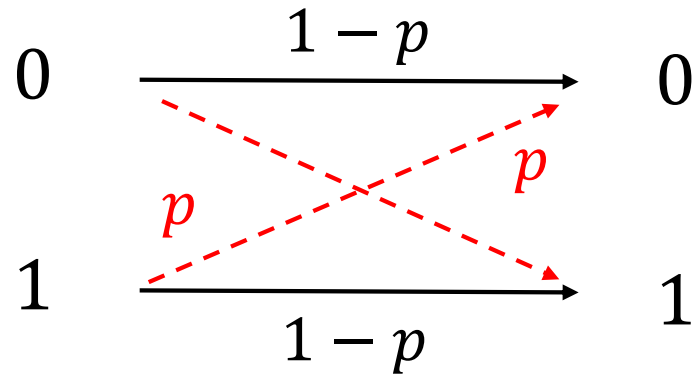
A simple way to correct errors:

Errors in classical computers

Computer interacts w/ environment \Rightarrow error/noise



one bit



Suppose we send a bit but have “error” in probability p

A simple way to correct errors:

① Duplicate the bit (**encoding**): $0 \rightarrow 000$, $1 \rightarrow 111$

② Error detection & correction by “**majority voting**”:

$001 \rightarrow 000$, $011 \rightarrow 111$, etc...

$\Rightarrow P_{\text{failed}} = 3p^2(1-p) + p^3$ (improved if $p < 1/2$)

Errors in quantum computers

Computer interacts w/ environment  error/noise

(In addition to decoherence & measurement errors)

Unknown unitary operators are multiplied:

$$|\psi\rangle \xrightarrow[\text{error!}]{\text{error!}} U|\psi\rangle$$

not only bit flip!

We need to include “quantum error corrections”
but it seems to require a huge number of qubits

~ major obstruction of the development

(We will come back to this point on the 3rd day)

(Classical) simulator for Quantum computer

Quantum computation \subset Linear algebra

The same algorithm can be implemented in classical computer but w/o speed-up (1 quantum step = many classical steps)

Simulator = Tool to simulate **quantum** computer
by **classical** computer

- Doesn't have errors \rightarrow ideal answers
(More precisely, classical computer also has errors but its error correction is established)
- The same code can be run in quantum computer w/ speed-up

Useful to test algorithm & estimate computational resources
(\sim # of qubits, gates)

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Short summary

- Qubit = Quantum bit

- Important gates:

$$R_X(\theta) = e^{-\frac{i\theta}{2}X}, \quad R_Y(\theta) = e^{-\frac{i\theta}{2}Y}, \quad R_Z(\theta) = e^{-\frac{i\theta}{2}Z}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \equiv |+\rangle, \quad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \equiv |-\rangle$$

$$CX|0\rangle \otimes |\psi\rangle = |0\rangle \otimes |\psi\rangle, \quad CX|1\rangle \otimes |\psi\rangle = |1\rangle \otimes X|\psi\rangle$$

- Do something interesting by a combination of acting unitary op. & measurement

- H, T & CX are universal

$$T = e^{\frac{\pi i}{8}} R_Z\left(\frac{\pi}{4}\right) = \begin{pmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{pmatrix}$$

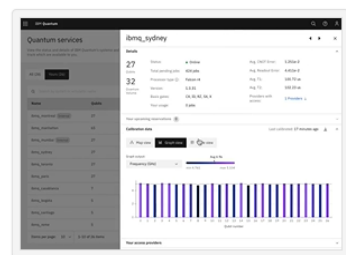
- Real quantum computer has errors

- Simulator = Tool to simulate quantum computer by classical computer

Some demonstrations in IBM Quantum

Real quantum computers. Right at your fingertips.

IBM offers cloud access to the most advanced quantum computers available.
Learn, develop, and run programs with our quantum applications and systems.



View quantum system details

Check out the status, topology, calibration data, and access details of your IBM quantum systems.

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Welcome, Honda Masazumi



Graphically build circuits with
IBM Quantum Composer

[Launch Composer](#)



Develop quantum experiments in
IBM Quantum Lab

[Launch Lab](#)

Jump back in:

[Untitled circuit](#)
[Untitled circuit](#)
[Untitled circuit](#)
[Untitled circuit](#)

API token ⓘ



[View account details](#)

Optimize circuit execution with
Qiskit Runtime programs

2

Primitive
programs

5

Prototype
programs



Run on circuits & programs via
**IBM Quantum compute
resources**

[View all](#)

9

Your
systems

5

Your
simulators

0

Reservable
systems

Recent jobs

[View all](#)

0

Pending

274

Completed

No pending jobs

A trivial problem: measure $|0\rangle$

Untitled circuit *Saved* | File Edit View

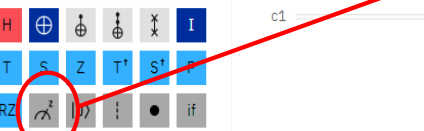
Visualizations seed 800 Setup and run

Operations

Search

q[0]

c1



The image shows a quantum circuit editor interface. On the left is a panel titled 'Operations' containing a grid of quantum gates and measurement symbols. A red circle highlights the measurement symbol (a meter with a circle and a dot) in the third row, second column. A red arrow points from this measurement symbol to the 'q[0]' register line in the circuit diagram on the right. The circuit diagram shows two horizontal lines representing qubits, labeled 'q[0]' and 'c1'. The 'q[0]' line is currently empty, while the 'c1' line has a small blue circle at its end. The interface also includes a menu bar at the top with 'File', 'Edit', and 'View' options, and a status bar at the bottom showing 'Visualizations seed 800' and a 'Setup and run' button.

A trivial problem: measure $|0\rangle$ (Cont'd)

Untitled circuit *Saved* | File Edit View

Visualizations seed 800 Setup and run

Operations Left alignment Inspect

Search

q[0] $|0\rangle$

c1

Operations panel:

- H, \oplus , \otimes , \otimes , \otimes , \otimes , \otimes , \otimes
- T, S, Z, T^\dagger , S^\dagger , P
- RZ, $|0\rangle$, $|1\rangle$, $|2\rangle$, $|3\rangle$, if
- \sqrt{X} , \sqrt{X}^\dagger , Y, RX, RY, RXX
- RZZ, U, RCCX, RC3X, \otimes

Measure 1024 times in simulator

View

Left alignment

Inspect

[0]

c1

Set up and run your circuit

Step 1

Choose a system or simulator

Search by system or simulator name

5000 Qubits

☐ simulator_mps [See details](#)

Simulator status Online

Total pending jobs 0

100 Qubits

☐ simulator_extended_stabilizer [See details](#)

Simulator status Online

Total pending jobs 0

63 Qubits

☒ ibmq_qasm_simulator [See details](#)

Simulator status Online

Total pending jobs 0

32 Qubits

☐ simulator_statevector [See details](#)

Simulator status Online

Total pending jobs 0

32 Qubits

Step 2

Choose your settings

Instance

ibm-q/open/main

Shots *

1024

Job limit: 5 remaining

Tags (optional)

Add tags

Close

Run on ibmq_qasm_simulator

Trivial result

Details

582ms

Total completion time

[ibmq_qasm_simulator](#)

Compute resource

Sent from: [Untitled circuit](#)

Status:  Completed

Instance: [ibmq-q/open/main](#)

Program: [circuit-runner](#)

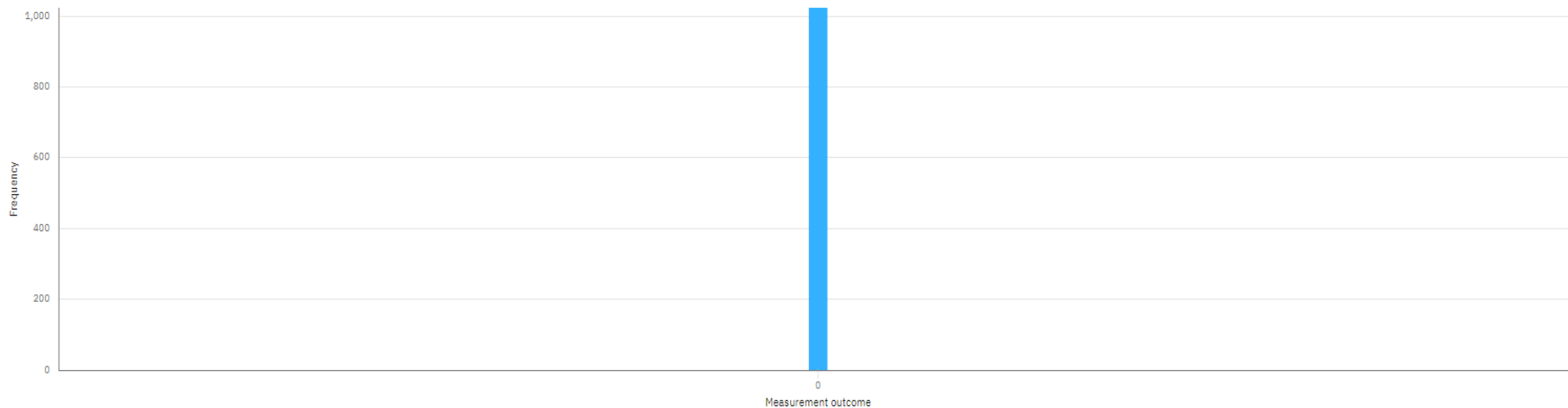
of shots: 1024

of circuits: 1

Status Timeline

- Created: Apr 16, 2023 12:10 AM
- In queue: less than a second
- Running: Apr 16, 2023 12:10 AM
time in classical and quantum computation 1s
- Completed: Apr 16, 2023 12:10 AM

Histogram



Of Course!

Measure 1024 times in quantum computer

View

Left alignment

Inspect

q[0]

c1

Set up and run your circuit

Step 1

Choose a system or simulator

Search by system or simulator name

- ☒ **ibmq_belem** [See details](#)
System status: Online
Total pending jobs: 0
5 Qubits 16 QV 2.5K CLOPS
- ☐ **ibmq_lima** [See details](#)
System status: Online
Total pending jobs: 5
5 Qubits 8 QV 2.7K CLOPS
- ☐ **simulator_stabilizer** [See details](#)
Simulator status: Online
Total pending jobs: 0
5000 Qubits
- ☐ **simulator_mps** [See details](#)
Simulator status: Online
Total pending jobs: 0
100 Qubits
- ☐ **simulator_extended_stabilizer** [See details](#)

Close

Step 2

Choose your settings

Instance

ibm-q/open/main

Shots *

1024

Job limit: 5 remaining

Tags (optional)

Add tags

Run on ibmq_belem

Result of quantum computer?

Details

6.8s

Total completion time

[ibmq_belem](#)

Compute resource

Sent from: [Untitled circuit](#)

Status: Completed

Instance: [ibmq-q/open/main](#)

Program: [circuit-runner](#)

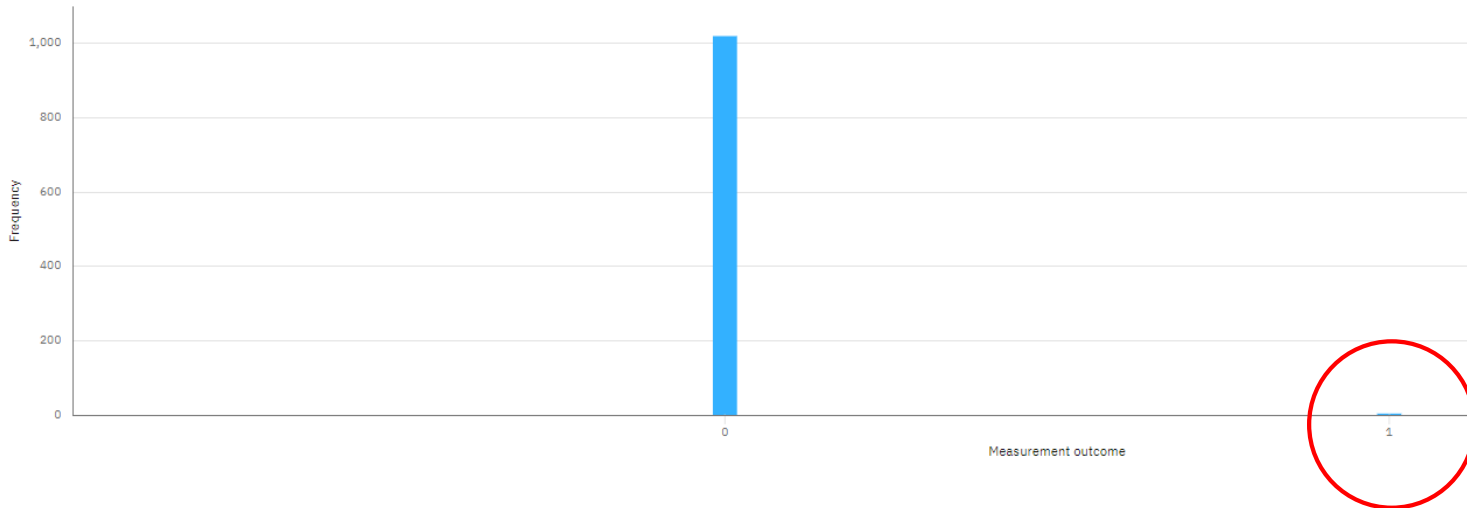
of shots: 1024

of circuits: 1

Status Timeline

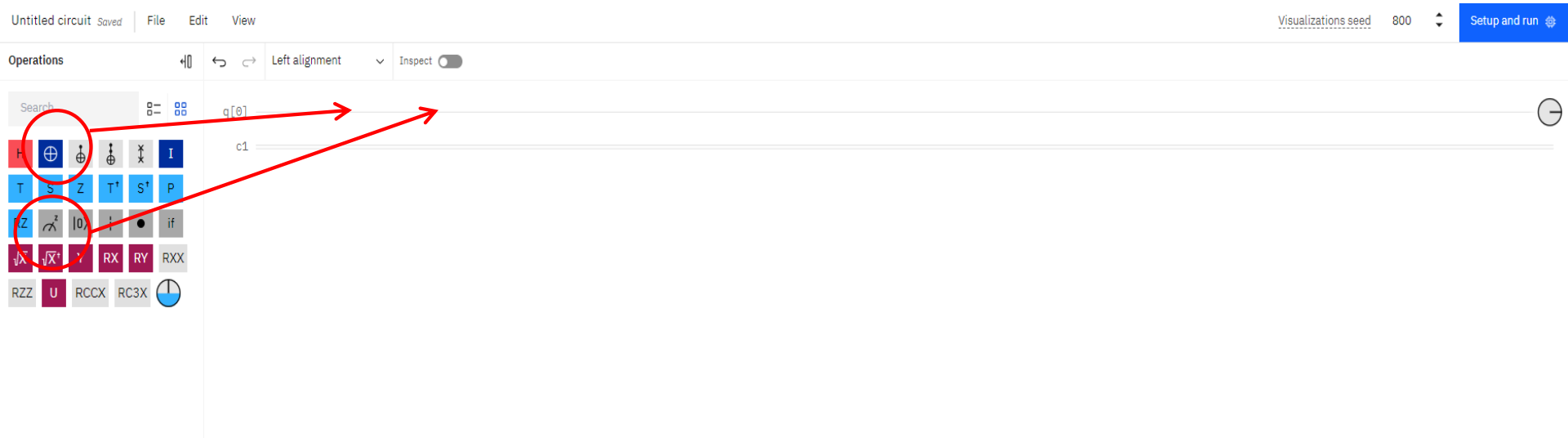
- Created: Apr 16, 2023 12:14 AM
- In queue: less than a second
- Running: Apr 16, 2023 12:14 AM
time in classical and quantum computation 5s
- Completed: Apr 16, 2023 12:14 AM

Histogram



This is the error!

Another trivial problem: measure $|1\rangle$



$$X|0\rangle = |1\rangle$$

Another trivial problem: measure $|1\rangle$ (Cont'd)

Untitled circuit Saved | File | Edit | View

Visualizations seed 800 Setup and run

Operations Left alignment Inspect

Search

q[0] + ?

c1 0

Operations palette:

- H, \oplus , \otimes , \otimes , \otimes , \otimes , I
- T, S, Z, T', S', P
- RZ, \sqrt{X} , $|0\rangle$, $|1\rangle$, $|0\rangle$, if
- \sqrt{X} , \sqrt{X}^\dagger , Y, RX, RY, RXX
- RZZ, U, RCCX, RC3X, $\frac{\pi}{2}$

Result of simulator (1024 shots)

Details

730ms

Total completion time

[ibmq_qasm_simulator](#)

Compute resource

Sent from: [Untitled circuit](#)

Status:  Completed

Instance: [ibmq-open/main](#)

Program: [circuit-runner](#)

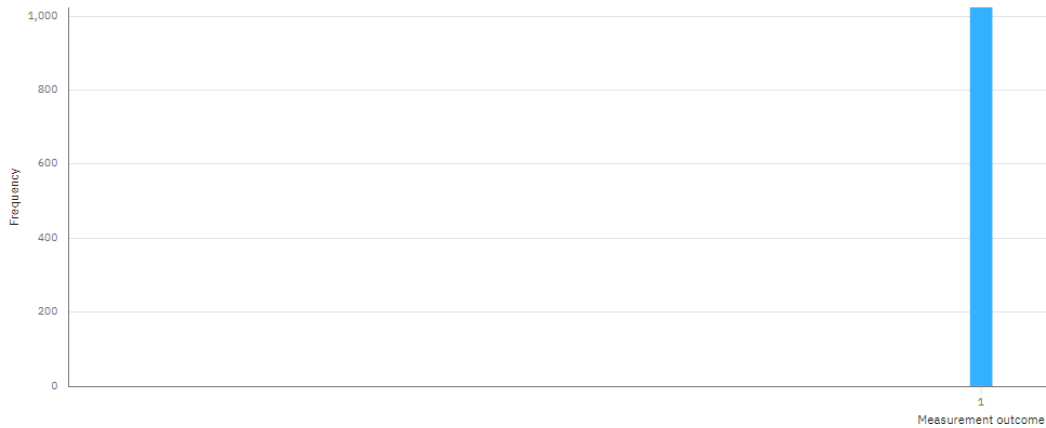
of shots: 1024

of circuits: 1

Status Timeline

- Created: Apr 16, 2023 12:24 AM
- In queue: less than a second
- Running: Apr 16, 2023 12:24 AM
time in classical and quantum computation 1s
- Completed: Apr 16, 2023 12:24 AM

Histogram



Result of quantum computer (1024shots)

Details

3.5s
Total completion time

Sent from: [Untitled circuit](#)

Status: Completed

Instance: [ibmq-belem](#)
Compute resource

Program: [circuit-runner](#)

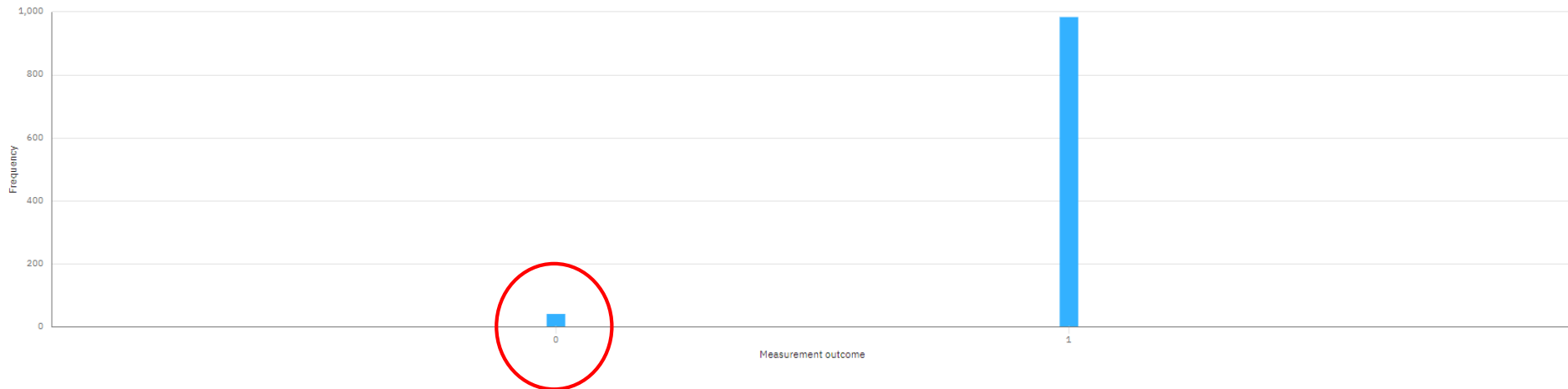
of shots: 1024

of circuits: 1

Status Timeline

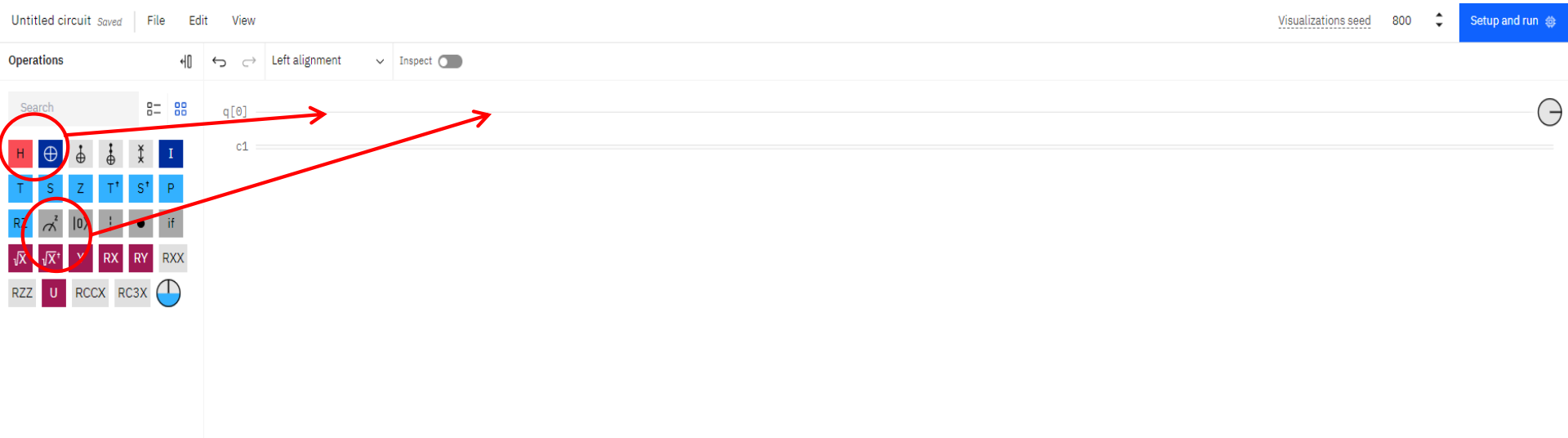
- Created: Apr 16, 2023 12:24 AM
- In queue: less than a second
- Running: Apr 16, 2023 12:24 AM
time in classical and quantum computation 2s
- Completed: Apr 16, 2023 12:24 AM

Histogram



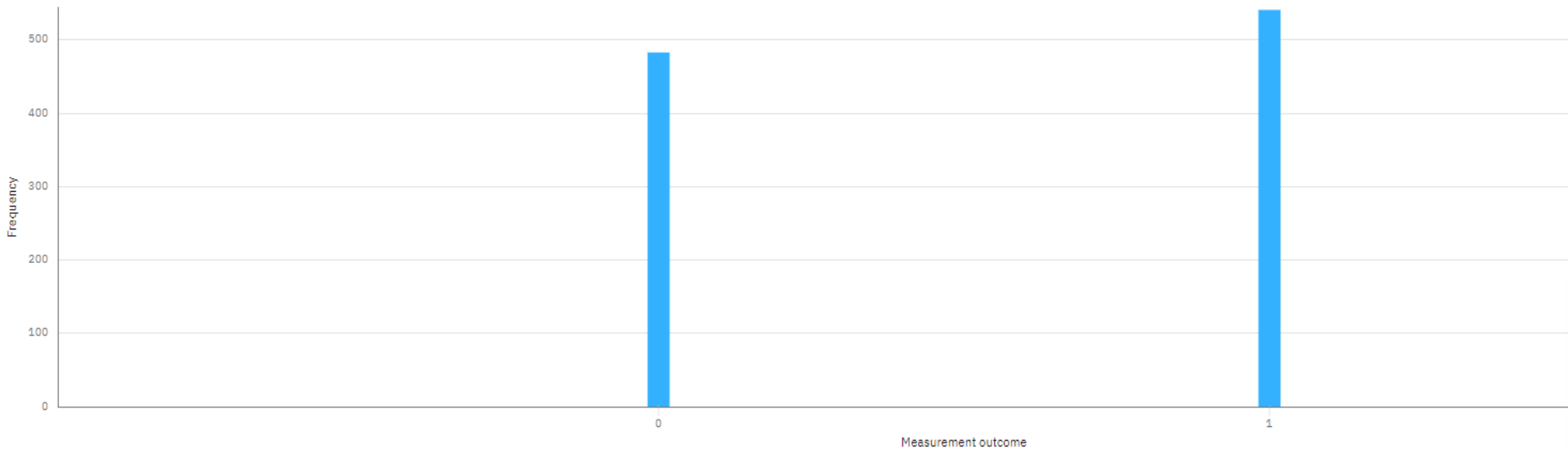
Error again

The simplest nontrivial problem: Hadamard gate



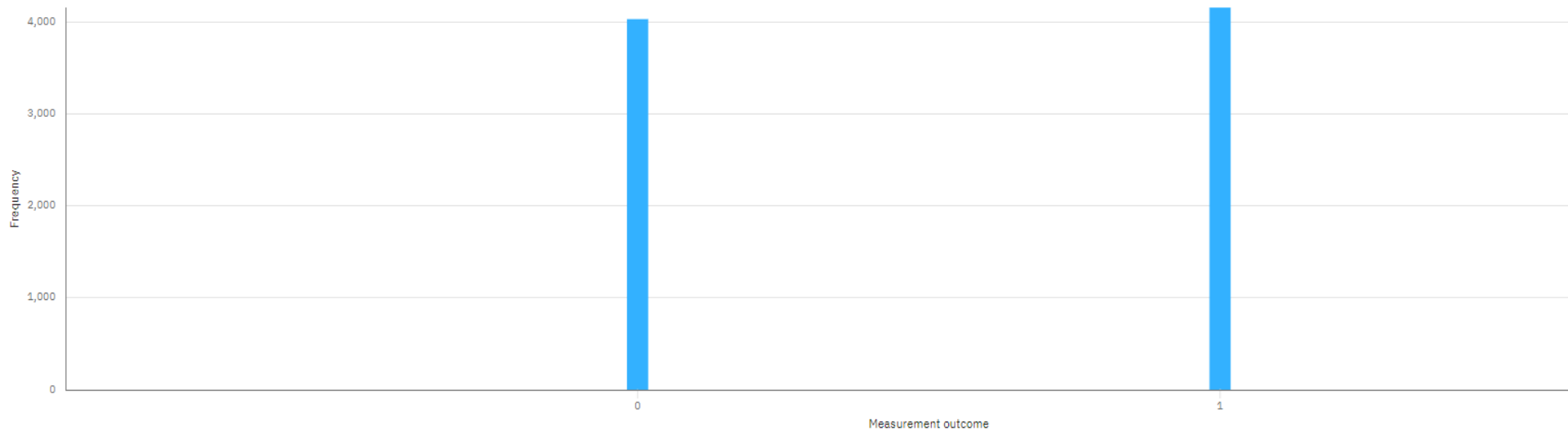
$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Result of **simulator** (1024 shots)



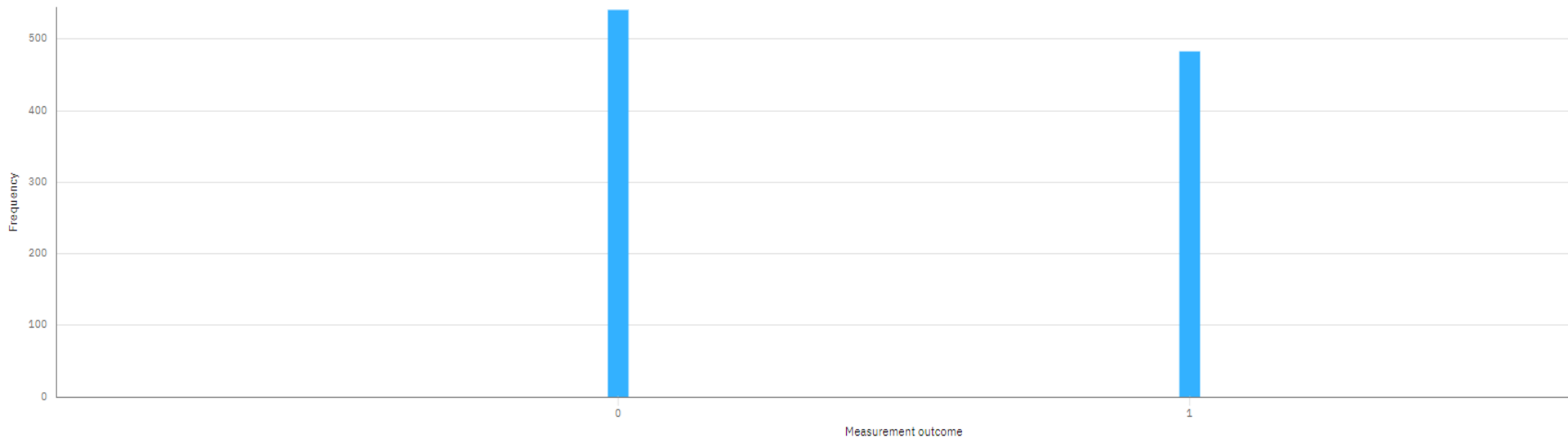
Not exactly 50:50 because of **statistical errors**

Result of simulator (8192 shots)



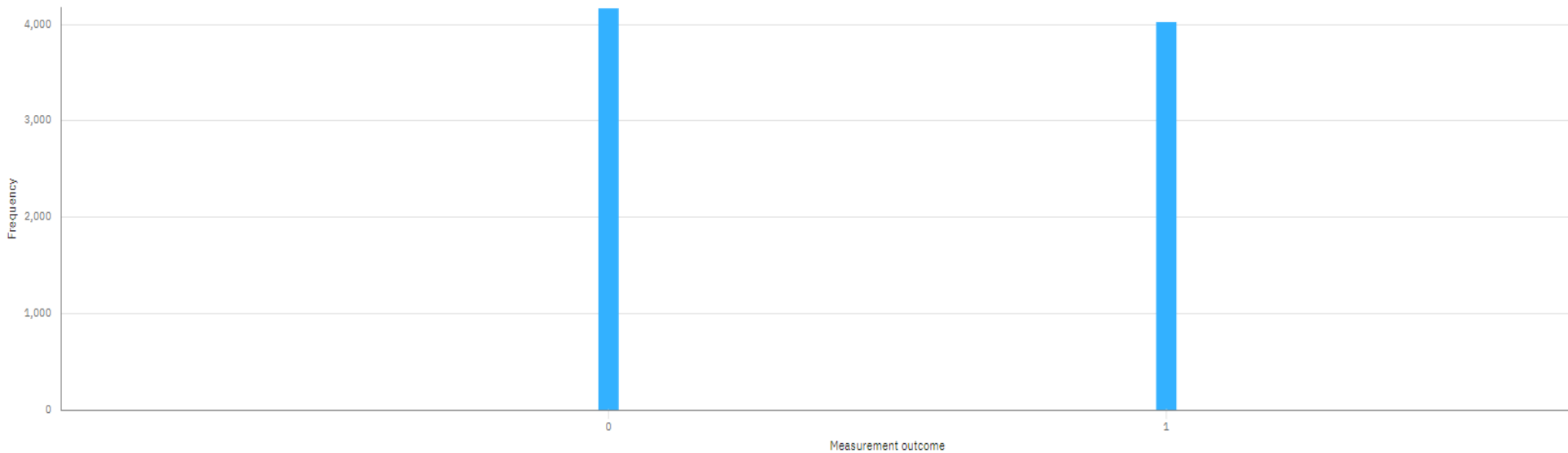
Improved!

Result of quantum computer (1024 shots)



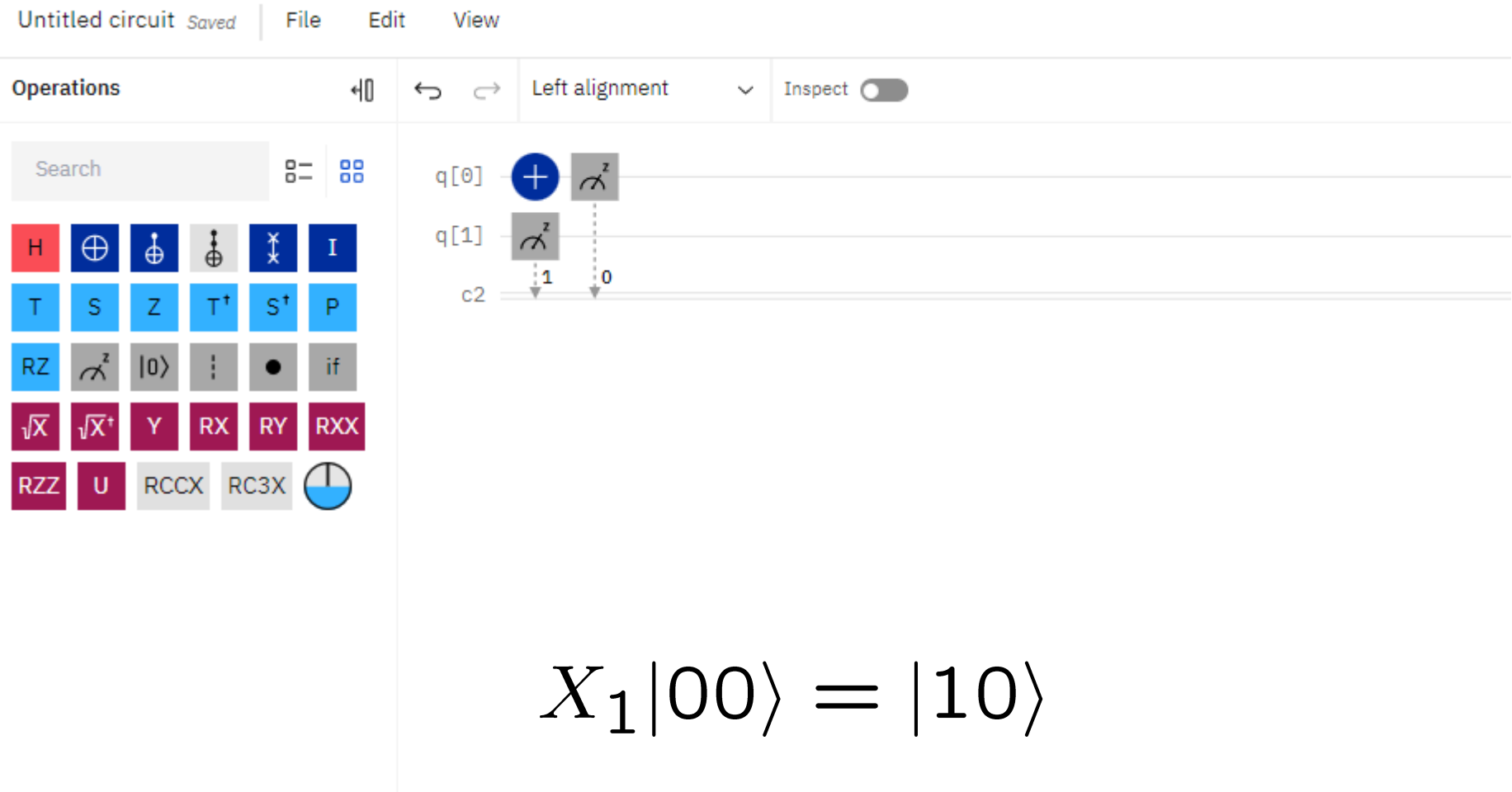
≡ Both errors & statistical errors

Result of quantum computer (8192 shots)



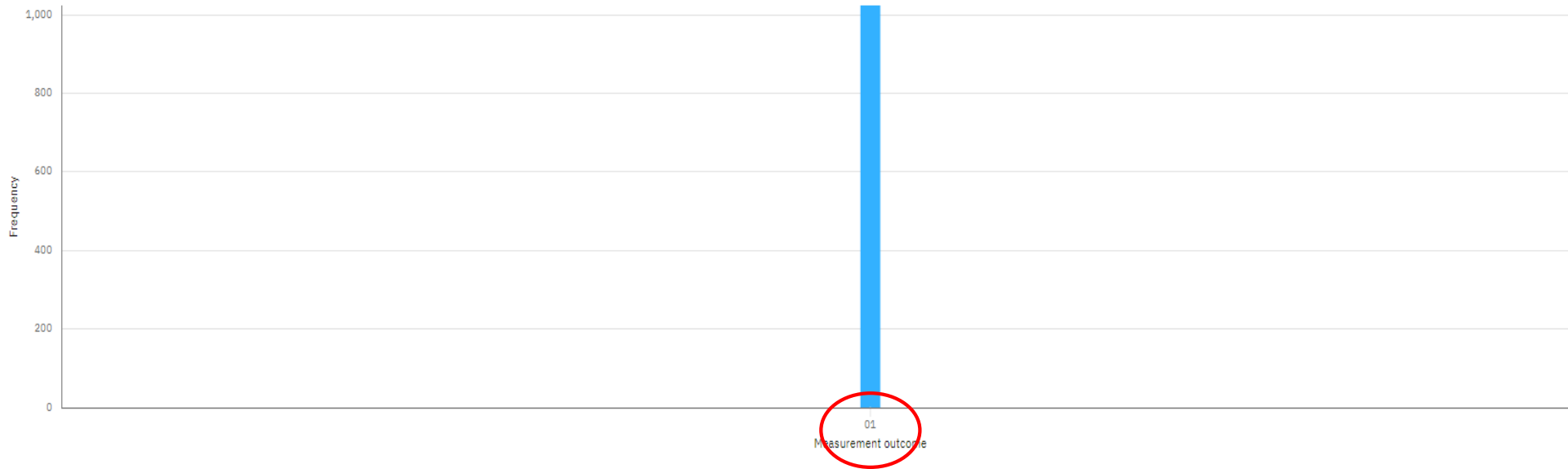
Statistical errors are improved

A trivial problem for 2 qubits



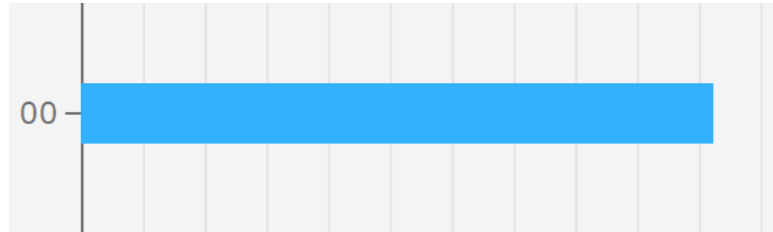
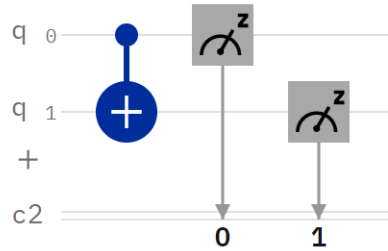
Result of simulator (1024 shots)

$$X_1|00\rangle = |10\rangle$$

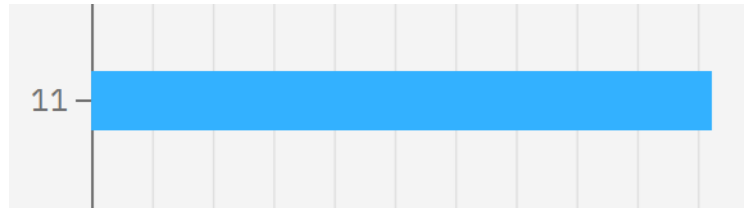
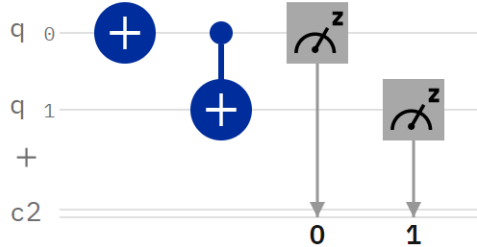


Note that notation is different from the ket notation

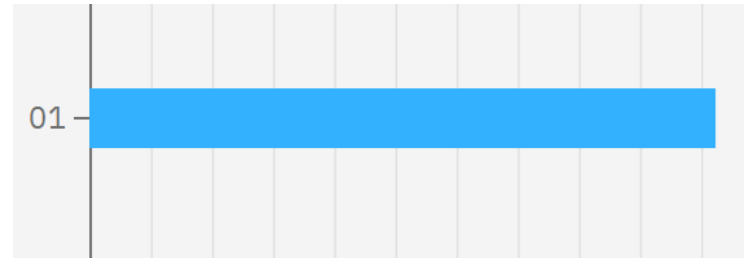
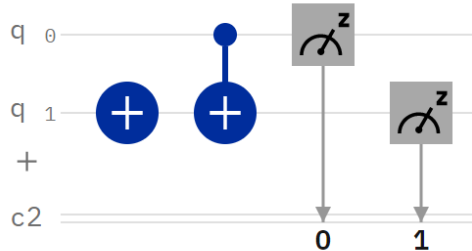
2 qubit operation by simulator



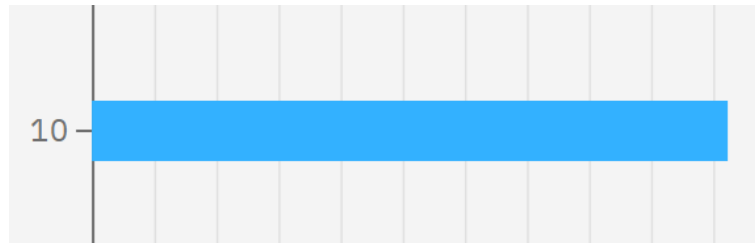
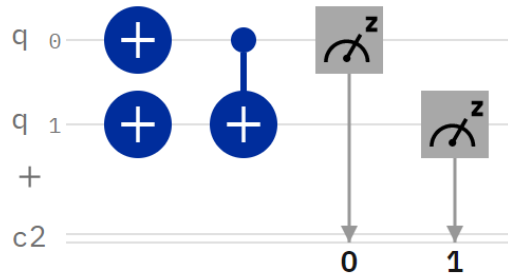
$$CX|00\rangle = |00\rangle$$



$$CX|10\rangle = |11\rangle$$

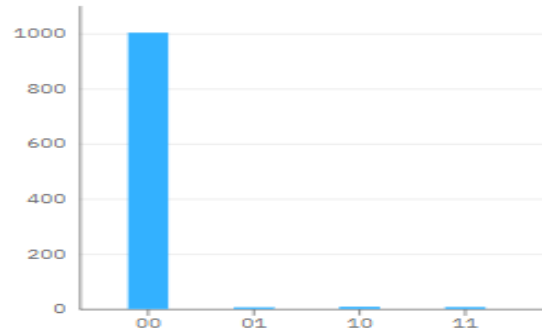
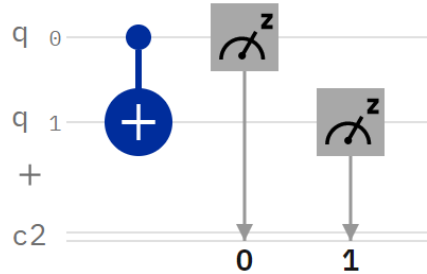


$$CX|01\rangle = |01\rangle$$

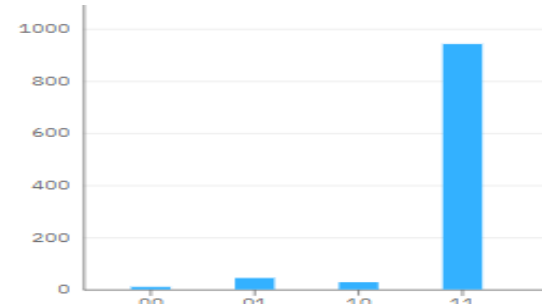
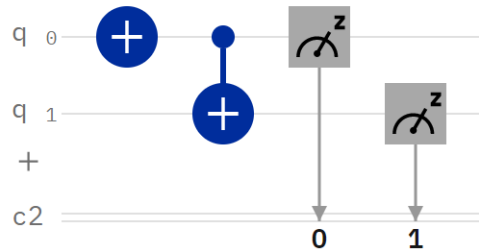


$$CX|11\rangle = |10\rangle$$

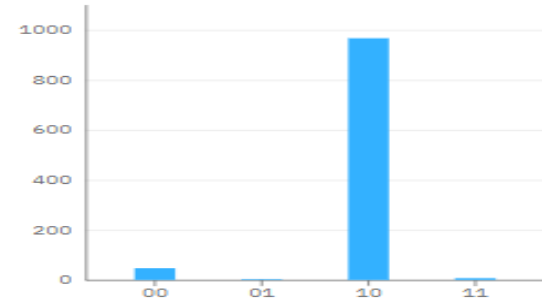
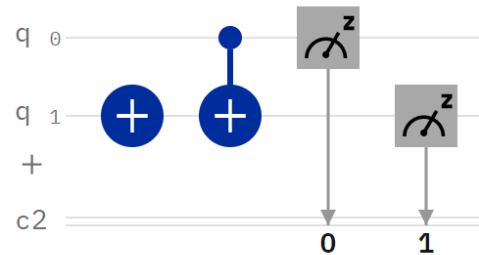
2 qubit operation by quantum computer (1024 shots)



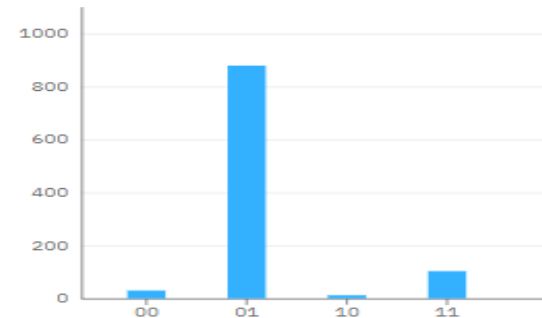
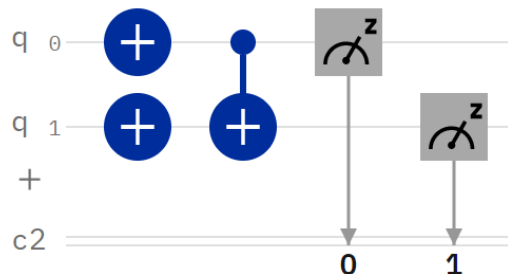
$$CX|00\rangle = |00\rangle$$



$$CX|10\rangle = |11\rangle$$



$$CX|01\rangle = |01\rangle$$



$$CX|11\rangle = |10\rangle$$

Tutorial 0: Play with IBMQ

(until the end of this class)

(Here is the expected end of the lecture part of the 1st class)

Plan of the intensive lectures

Day 1

(If 2nd lecture of each day ends early, then we start hands-on early)

- Lecture 1: introduction, basics of quantum computation
- **Lecture 2: quantum simulation of spin system**
- Hands-on 1: Basics on IBM's qiskit

Day 2

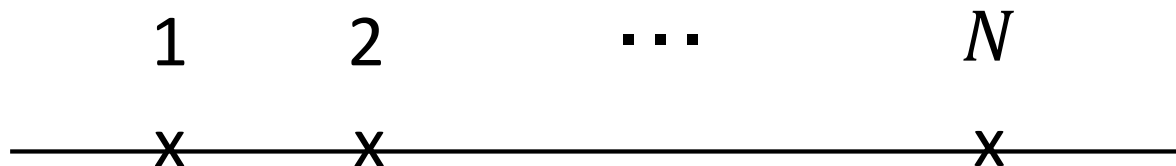
- Lecture 3: quantum field theory (QFT)
- Lecture 4: QFT on quantum computer
- Hands-on 2: Time evolution of spin system

Day 3

- Lecture 5: quantum error correction
- Lecture 6: some advanced topics, future prospects
- Hands-on 3: Constructing ground state of spin system

Quantum simulation of Spin system

The (1+1)d transverse Ising model



Hamiltonian (w/ open b.c.):

(X_n, Y_n, Z_n : $\sigma_{1,2,3}$ at site n)

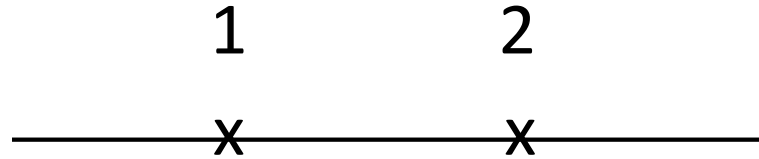
$$\hat{H} = -J \sum_{n=1}^{N-1} Z_n Z_{n+1} - h \sum_{n=1}^N X_n - m \sum_{n=1}^N Z_n$$

For simplicity, take $N = 2$ & $m = 0$ for a while:

$$\hat{H} = -J Z_1 Z_2 - h(X_1 + X_2)$$

Let's construct the time evolution op. $e^{-i\hat{H}t}$

Warm up: 2-site transverse Ising model



$$\hat{H} = -JZ_1Z_2 - h(X_1 + X_2)$$

We are going to

- construct time evolution operator
- obtain vacuum state
- compute vacuum expectation values
- compute Renyi entropy

Time evolution operator

Time evolution of any state is studied by acting the operator

$$e^{-i\hat{H}t} = e^{-i(H_X + H_{ZZ})t}$$

where

$$H_X = -h(X_1 + X_2), \quad H_{ZZ} = -JZ_1Z_2$$

How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc...)

Time evolution operator

Time evolution of any state is studied by acting the operator

$$e^{-i\hat{H}t} = e^{-i(H_X + H_{ZZ})t}$$

where

$$H_X = -h(X_1 + X_2), \quad H_{ZZ} = -JZ_1Z_2$$

How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc...)

Step 1: Suzuki-Trotter decomposition:

(\exists higher order improvements)

$$\begin{aligned} e^{-i\hat{H}t} &= \left(e^{-i\hat{H}\frac{t}{M}} \right)^M \\ &\simeq \left(e^{-iH_X\frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}} \right)^M + \mathcal{O}(1/M) \end{aligned} \quad (M: \text{large positive integer})$$

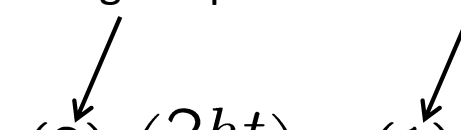
Time evolution operator (Cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X \frac{t}{M}} e^{-iH_{ZZ} \frac{t}{M}} \right)^M$$

The **1st** one is trivial:

$$e^{-iH_X \frac{t}{M}} = e^{-i\frac{ht}{M}X_2} e^{-i\frac{ht}{M}X_1} = R_X^{(2)}\left(\frac{2ht}{M}\right) R_X^{(1)}\left(\frac{2ht}{M}\right)$$

acting on qubit 2 acting on qubit 1



Time evolution operator (Cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X \frac{t}{M}} e^{-iH_{ZZ} \frac{t}{M}} \right)^M$$

The **1st** one is trivial:

$$e^{-iH_X \frac{t}{M}} = e^{-i\frac{ht}{M} X_2} e^{-i\frac{ht}{M} X_1} = R_X^{(2)} \left(\frac{2ht}{M} \right) R_X^{(1)} \left(\frac{2ht}{M} \right)$$

acting on qubit 2 acting on qubit 1

The **2nd** one is nontrivial:

$$e^{-iH_{ZZ} \frac{t}{M}} = e^{-i\frac{Jt}{M} Z_1 Z_2} = \cos \frac{Jt}{M} - i Z_1 Z_2 \sin \frac{Jt}{M}$$

One can show (see next slide)

$$e^{-i\frac{Jt}{M} Z_1 Z_2} = CX R_Z^{(2)} \left(\frac{2Jt}{M} \right) CX$$

Time evolution operator (Cont'd)

$$e^{-icZ_1Z_2} = CXR_Z^{(2)}(2c)CX$$

Proof:

$$CXR_Z^{(2)}(2c)CX|0\rangle \otimes |\psi\rangle$$

$$= CXR_Z^{(2)}(2c)|0\rangle \otimes |\psi\rangle = CX|0\rangle \otimes R_Z(2c)|\psi\rangle$$

$$= |0\rangle \otimes R_Z(2c)|\psi\rangle = \cos c|0\rangle \otimes |\psi\rangle - i \sin c Z|0\rangle \otimes Z|\psi\rangle$$

$$CXR_Z^{(2)}(2c)CX|1\rangle \otimes |\psi\rangle$$

$$= CXR_Z^{(2)}(2c)|1\rangle \otimes X|\psi\rangle = CX|1\rangle \otimes R_Z(2c)X|\psi\rangle = |1\rangle \otimes XR_Z(2c)X|\psi\rangle$$

$$= \cos c|1\rangle \otimes XX|\psi\rangle - i \sin c |1\rangle \otimes XZX|\psi\rangle$$

$$= \cos c|1\rangle \otimes |\psi\rangle - i \sin c Z|1\rangle \otimes Z|\psi\rangle$$

Thus,

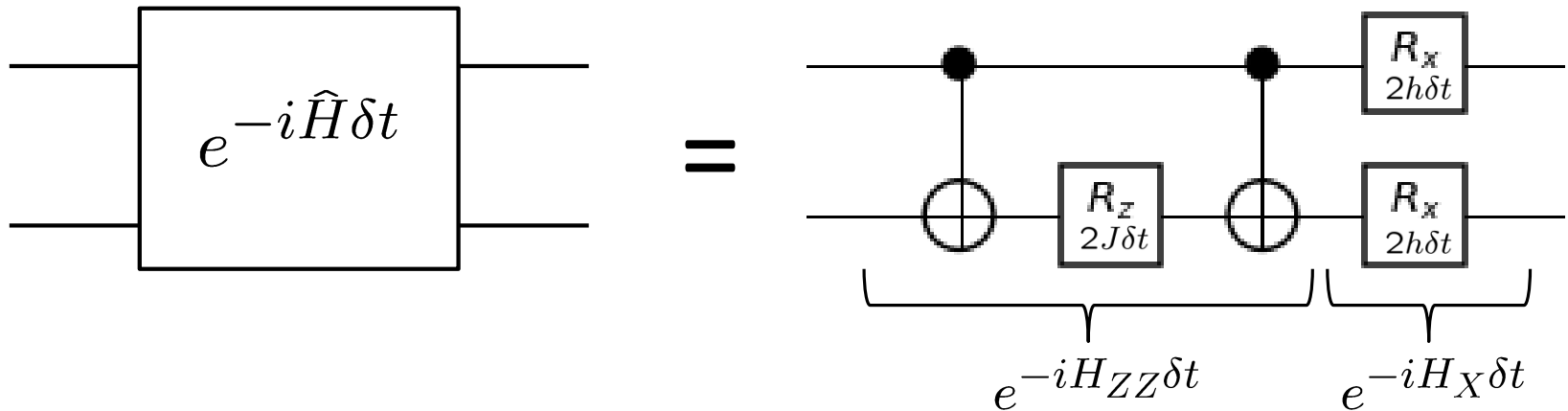
$$CXR_Z^{(2)}(2c)CX|\varphi\rangle \otimes |\psi\rangle = \cos c|\varphi\rangle \otimes |\psi\rangle - i \sin c Z|\varphi\rangle \otimes Z|\psi\rangle$$

$$= e^{-icZ_1Z_2}|\varphi\rangle \otimes |\psi\rangle$$

Quantum circuit for time evolution op.

$$H_X = -h(X_1 + X_2), \quad H_{ZZ} = -JZ_1Z_2$$

$$\delta t = \frac{t}{M} \ll 1$$



$$+\mathcal{O}(\delta t)$$

Survival probability of free vacuum

For $J = 0$, the ground state is

$$\hat{H}|_{J=0} = -h(X_1 + X_2)$$

$$|++\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}} = H^{(2)} H^{(1)} |00\rangle$$

We can compute survival probability of the free vacuum:

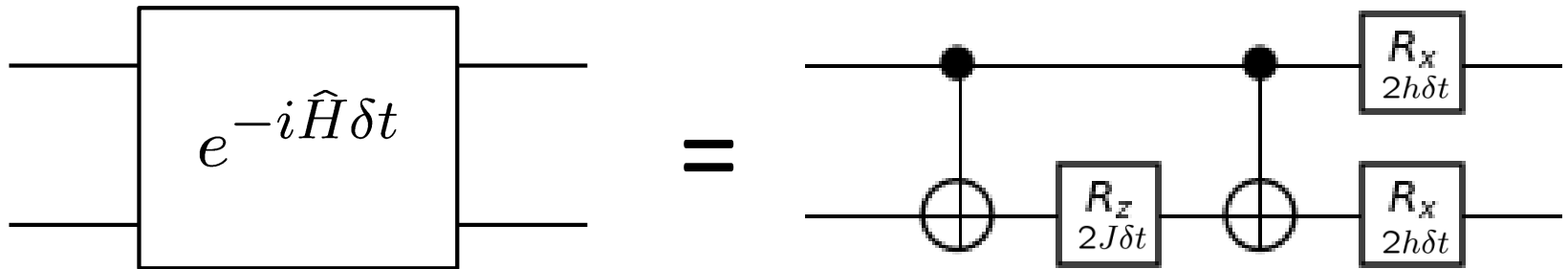
$$\begin{aligned} P(t) &= \left| \langle ++ | e^{-i\hat{H}t} | ++ \rangle \right|^2 && \text{toy version of} \\ & && \text{Schwinger effect} \\ &= \left| \langle 00 | H^{(2)} H^{(1)} e^{-i\hat{H}t} H^{(2)} H^{(1)} | 00 \rangle \right|^2 \end{aligned}$$

Measure the probability having $|00\rangle$ inside the state

$$H^{(2)} H^{(1)} e^{-i\hat{H}t} H^{(2)} H^{(1)} |00\rangle$$

Demonstration for the survival probability

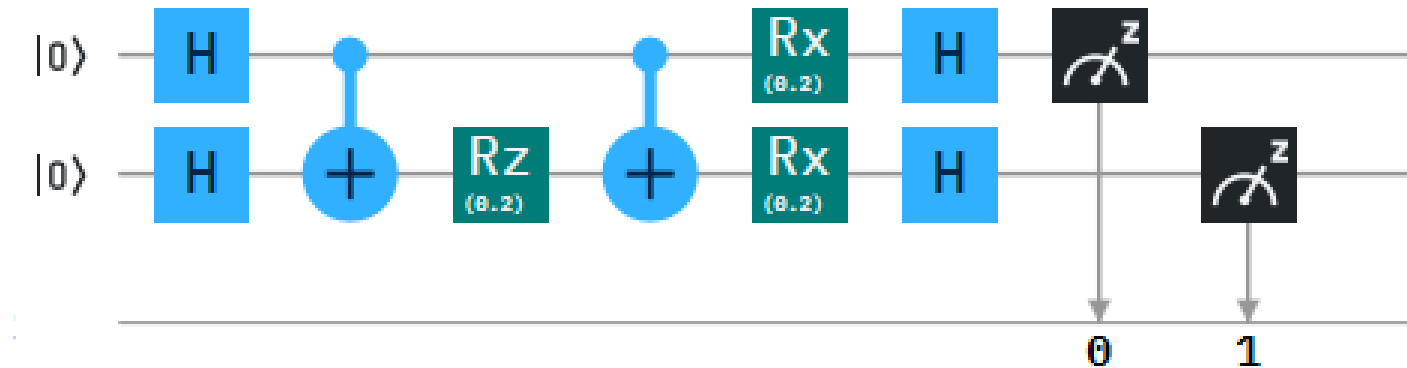
$$P(t) = \left| \langle ++ | e^{-i\hat{H}t} | ++ \rangle \right|^2 = \left| \langle 00 | H^{(2)} H^{(1)} e^{-i\hat{H}t} H^{(2)} H^{(1)} | 00 \rangle \right|^2$$



Let's compute it for $J = 1, h = 1, t = 0.1, M = 1$

$$\delta t = \frac{t}{M}$$

Demonstration for the survival probability (Cont'd)



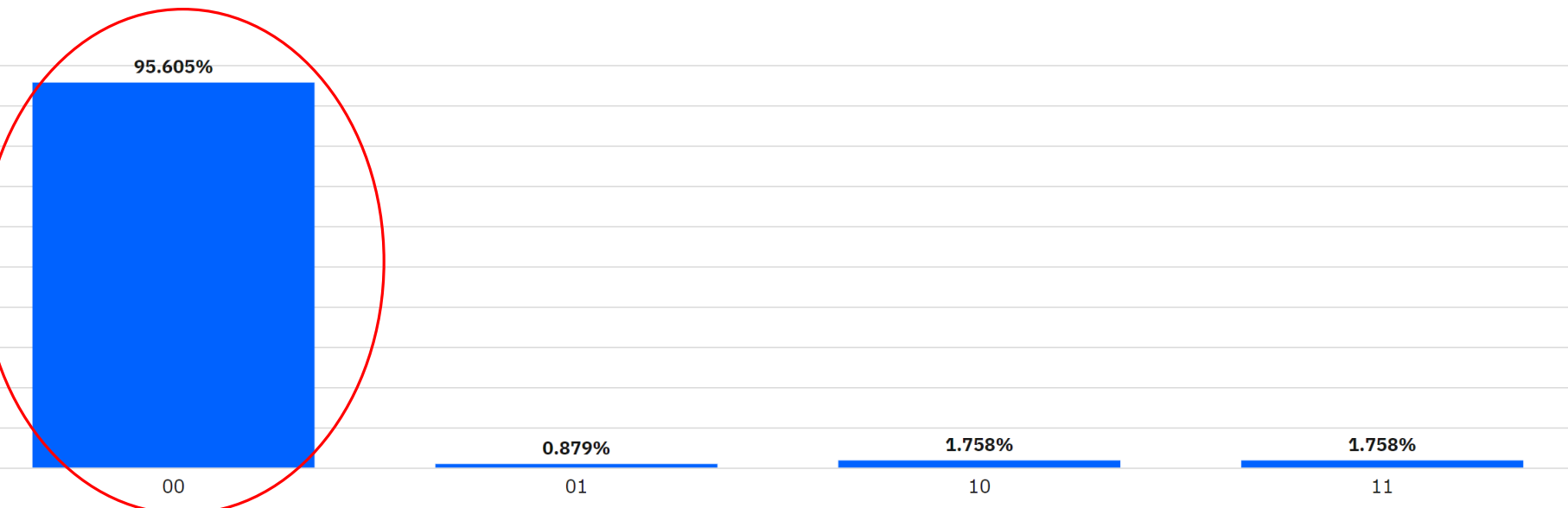
Result by simulator w/ 1024 shots:



Result of simulator (1024 shots):

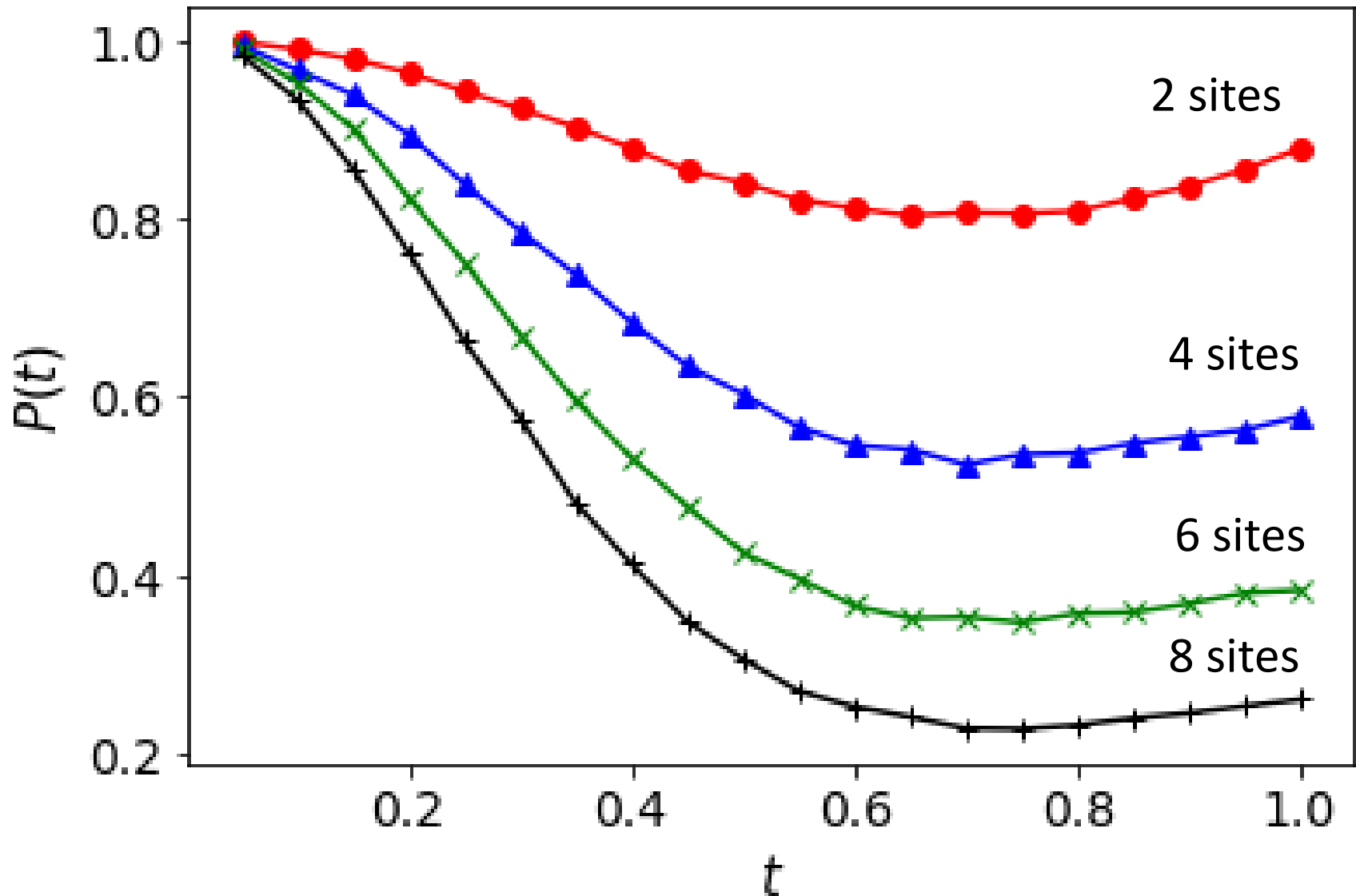


Result of quantum computer (1024 shots):



More serious computation

$J = 1, h = 1, t = 1, M = 100, 10000$ shots



Computational costs for large size system

$$P(t) = |\langle + \cdots + | e^{-i\hat{H}t} | + \cdots + \rangle|^2$$

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X \frac{t}{M}} e^{-iH_{ZZ} \frac{t}{M}} \right)^M$$

Classical computer

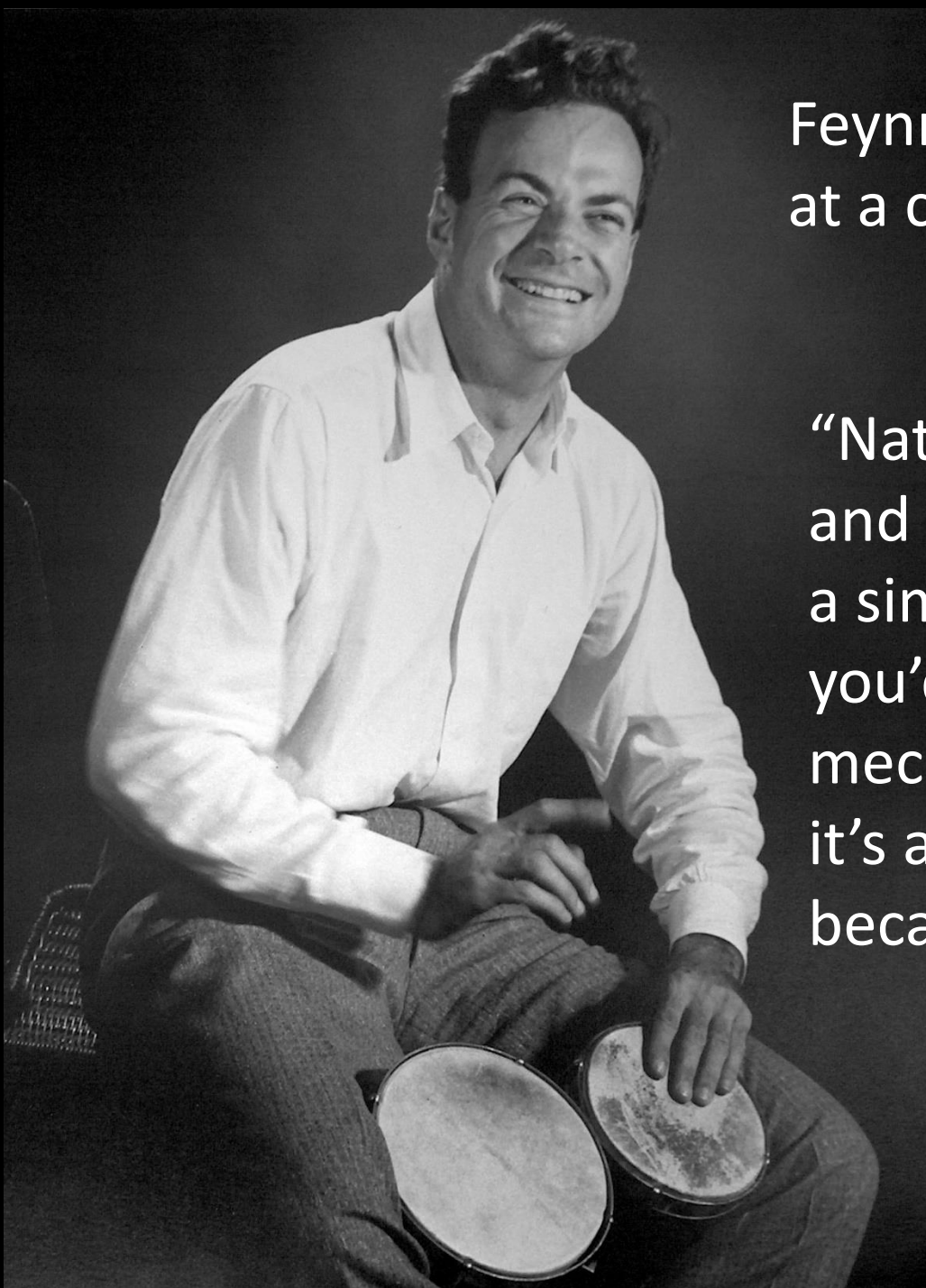
multiplications of matrices to vectors w/ sizes = 2^N

exponentially large steps

Quantum computer

- time evolution = $\mathcal{O}(NM)$ experimental operations
- taking the inner product is done by acting N gates & a measurement

polynomial steps

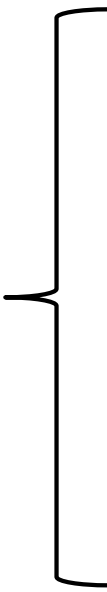


Feynman as a keynote speaker
at a conference in MIT (1981):

“Nature isn’t classical, dammit,
and if you want to make
a simulation of Nature,
you’d better make it quantum
mechanical, and by golly
it’s a wonderful problem
because it doesn’t look so easy.”

Constructing vacuum (ground state)

∃ various quantum algorithms to construct vacuum:

- 
- adiabatic state preparation
 - algorithms based on variational method
 - imaginary time evolution etc...

Here, let's apply

adiabatic state preparation

Adiabatic state preparation of vacuum

Step 1: Choose an **initial** Hamiltonian H_0 of a simple system whose ground state $|\text{vac}_0\rangle$ is known and unique

Step 2:

Step 3:

Adiabatic state preparation of vacuum

Step 1: Choose an **initial** Hamiltonian H_0 of a simple system whose ground state $|\text{vac}_0\rangle$ is known and unique

Step 2: Introduce **adiabatic** Hamiltonian $H_A(t)$ s.t.

$$\left\{ \begin{array}{l} \cdot H_A(0) = H_0, H_A(T) = H_{\text{target}} \\ \cdot \left| \frac{dH_A}{dt} \right| \ll 1 \text{ for } T \gg 1 \end{array} \right.$$

Step 3:

Adiabatic state preparation of vacuum

Step 1: Choose an **initial** Hamiltonian H_0 of a simple system whose ground state $|\text{vac}_0\rangle$ is known and unique

Step 2: Introduce **adiabatic** Hamiltonian $H_A(t)$ s.t.

$$\left\{ \begin{array}{l} \bullet H_A(0) = H_0, H_A(T) = H_{\text{target}} \\ \bullet \left| \frac{dH_A}{dt} \right| \ll 1 \text{ for } T \gg 1 \end{array} \right.$$

Step 3: Use the **adiabatic theorem**

If $H_A(t)$ has a **unique** ground state w/ a finite **gap** for $\forall t$, then the ground state of H_{target} is obtained by

$$|\text{vac}\rangle = \lim_{T \rightarrow \infty} \mathcal{T} \exp \left(-i \int_0^T dt H_A(t) \right) |\text{vac}_0\rangle$$

For transverse Ising model

$$\hat{H} = -J \sum_{n=1}^{N-1} Z_n Z_{n+1} - h \sum_{n=1}^N X_n - m \sum_{n=1}^N Z_n$$

Choose

$$\left\{ \begin{array}{l} H_0 = -h \sum_{n=1}^N X_n \quad \longrightarrow \quad |\text{vac}_0\rangle = |+\cdots+\rangle \\ H_A(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} \hat{H} \end{array} \right.$$

Discretize the integral:

$$\mathcal{T} \exp \left(-i \int_0^T dt H_A(t) \right) |\text{vac}_0\rangle \simeq U(T) U(T-\delta t) \cdots U(2\delta t) U(\delta t) |\text{vac}_0\rangle$$

where

$$U(t) = e^{-iH_A(t)\delta t}, \quad \delta t = \frac{T}{M} \ll 1$$

Magnetization

Once we get the vacuum, we can compute VEV of operators:

$$\langle \text{vac} | \mathcal{O} | \text{vac} \rangle$$

It is easiest to compute **magnetization**:

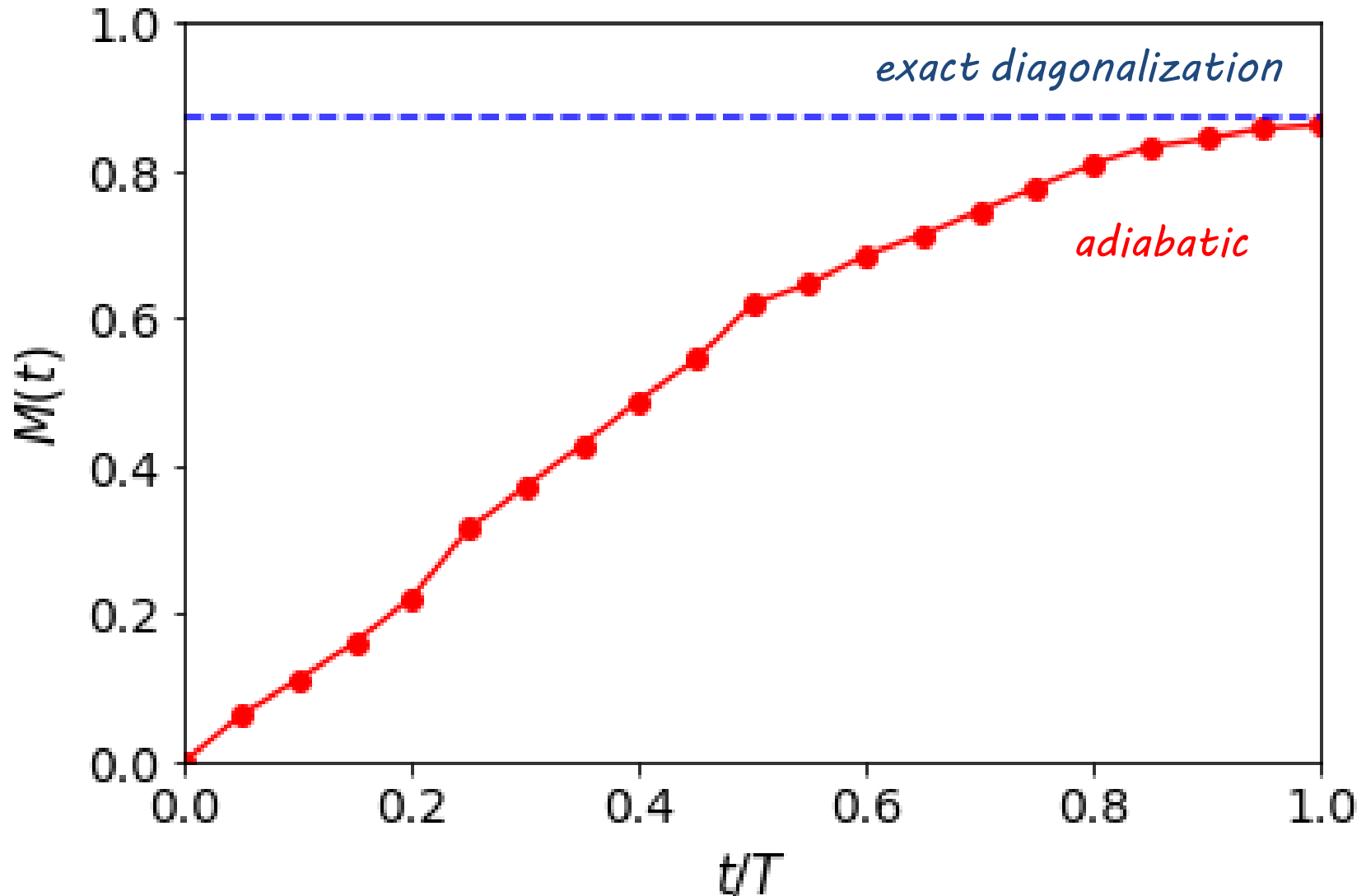
$$\begin{aligned} \frac{1}{N} \langle \text{vac} | \sum_{n=1}^N Z_n | \text{vac} \rangle &= \frac{1}{N} \sum_{n=1}^N \sum_{i_1 \dots i_N = 0,1} \langle \text{vac} | Z_n | i_1 \dots i_N \rangle \langle i_1 \dots i_N | \text{vac} \rangle \\ &= \frac{1}{N} \sum_{n=1}^N \sum_{i_1 \dots i_N = 0,1} (-1)^{i_n} |\langle i_1 \dots i_N | \text{vac} \rangle|^2 \end{aligned}$$

Transverse one is a bit more tricky:

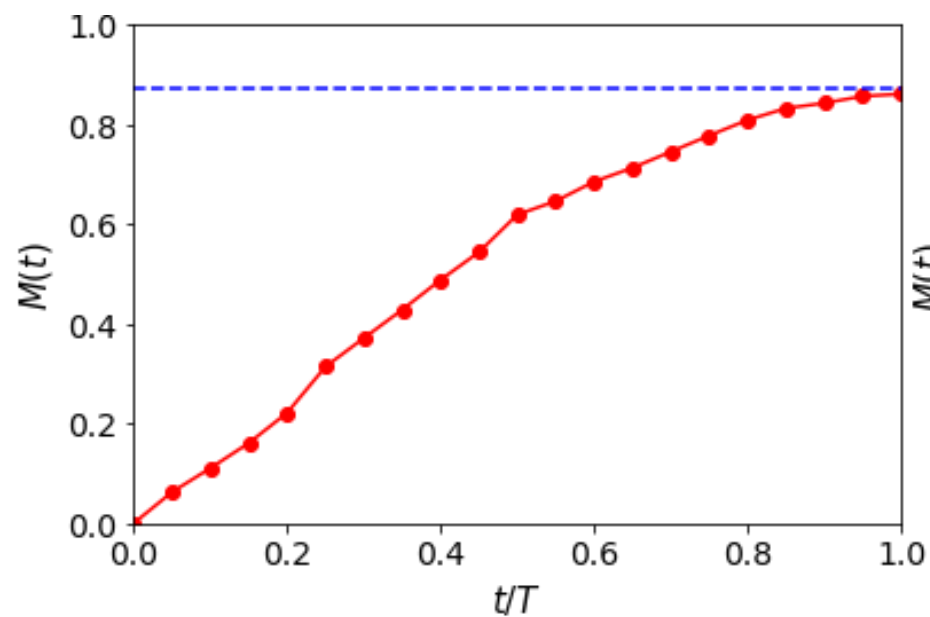
$$\begin{aligned} \frac{1}{N} \langle \text{vac} | \sum_{n=1}^N X_n | \text{vac} \rangle &= \frac{1}{N} \langle \text{vac} | \sum_{n=1}^N H^{(n)} Z_n H^{(n)} | \text{vac} \rangle \\ &= \frac{1}{N} \sum_{n=1}^N \sum_{i_1 \dots i_N = 0,1} (-1)^{i_n} |\langle i_1 \dots i_N | H^{(n)} | \text{vac} \rangle|^2 \end{aligned}$$

Result by simulator (10000 shots)

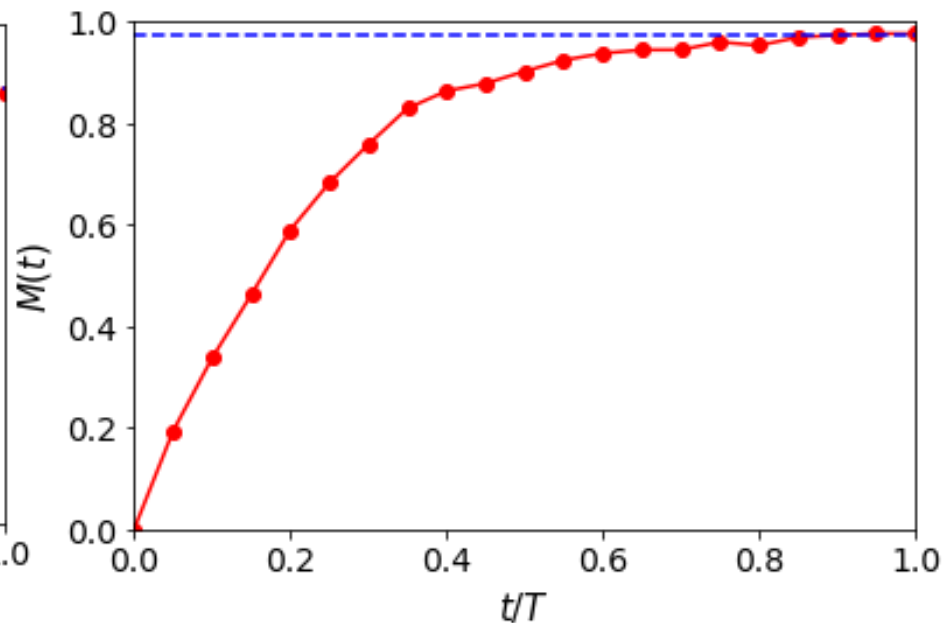
2 sites, $J = 1, h = 1, m = 1, T = 100, \delta t = 0.05$, 2000 time steps



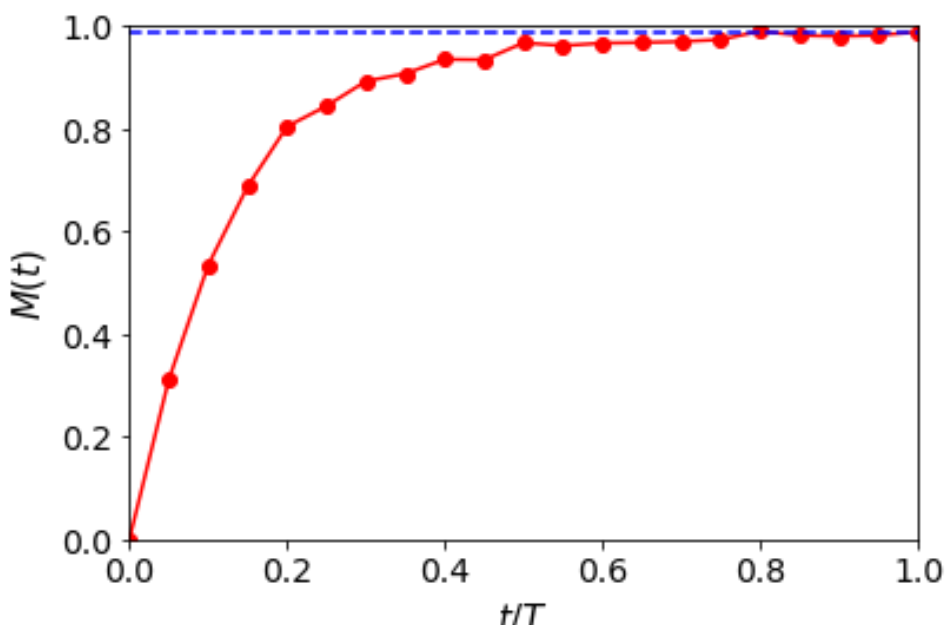
2 sites



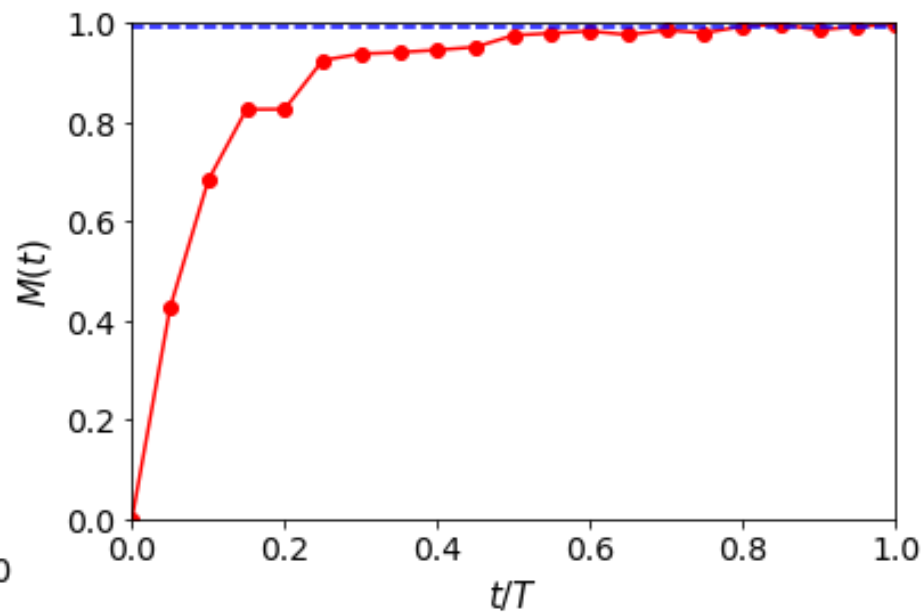
4 sites



6 sites



8 sites



Renyi entropy

Dividing total Hilbert space as

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_A \otimes \mathcal{H}_B,$$

reduced density matrix is defined as

$$\rho_A = \text{tr}_{\mathcal{H}_B} (\rho_{\text{tot}})$$

Entanglement entropy:

$$S_A = -\text{tr}_{\mathcal{H}_A} (\rho_A \log \rho_A)$$

n-th Renyi entropy:

$$S_n = \frac{1}{1-n} \log \text{tr}_{\mathcal{H}_A} (\rho_A^n) \quad \left(S_A = \lim_{n \rightarrow 1} S_n \right)$$

Quantum algorithm for 2nd Renyi entropy

Consider $(N_A + N_B)$ -qubit system and the density matrix

$$\rho_{N_A+N_B} = |\Psi\rangle\langle\Psi|$$

Let's divide the system into two systems: $\mathcal{H}_{N_A+N_B} = \mathcal{H}_{N_A} \otimes \mathcal{H}_{N_B}$
& consider the 2nd Renyi entropy

$$S_2 = -\log \text{tr}_{\mathcal{H}_{N_A}} (\rho_A^2), \quad \rho_A = \text{tr}_{\mathcal{H}_{N_B}} (\rho_{N_A+N_B})$$

Quantum algorithm for 2nd Renyi entropy

Consider $(N_A + N_B)$ -qubit system and the density matrix

$$\rho_{N_A+N_B} = |\Psi\rangle\langle\Psi|$$

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& consider the 2nd Renyi entropy

$$S_2 = -\log \text{tr}_{\mathcal{H}_{N_A}} (\rho_A^2), \quad \rho_A = \text{tr}_{\mathcal{H}_{N_B}} (\rho_{N_A+N_B})$$

One can show (next slide)

[Hastings-Gonzalez-Kallin-Melko'10]

$$\text{tr}_{\mathcal{H}_{N_A}} (\rho_A^2) = \langle\Psi| \otimes \langle\Psi| \text{SWAP}_A |\Psi\rangle \otimes |\Psi\rangle$$

SWAP_A : Exchange of A – part in $|\Psi\rangle \otimes |\Psi\rangle$

$$\left(\begin{array}{l} \text{For } |\Psi\rangle = \sum_{i,j} c_{ij} |i_1 \cdots i_{N_A} j_1 \cdots j_{N_B}\rangle, \\ \text{SWAP}_A |\Psi\rangle \otimes |\Psi\rangle \equiv \sum_{i,j,i',j'} c_{ij} c_{i'j'} |i'_1 \cdots i'_{N_A} j_1 \cdots j_{N_B}\rangle \otimes |i_1 \cdots i_{N_A} j'_1 \cdots j'_{N_B}\rangle \end{array} \right)$$

Quantum algorithm for 2nd Renyi entropy (Cont'd)

$$\text{tr}_{\mathcal{H}_{N_A}} (\rho_A^2) = \langle \Psi | \otimes \langle \Psi | \text{SWAP}_A | \Psi \rangle \otimes | \Psi \rangle$$

Proof:

$$\langle \Psi | \otimes \langle \Psi | \text{SWAP}_A | \Psi \rangle \otimes | \Psi \rangle$$

$$= \sum_{k,\ell,k',\ell'} \bar{c}_{k\ell} \bar{c}_{k'\ell'} \langle \{k'\}\{\ell'\} | \otimes \langle \{k\}\{\ell\} | \sum_{i,j,i',j'} c_{ij} c_{i'j'} | \{i'\}\{j\} \rangle \otimes | \{i\}\{j'\} \rangle$$

$$= \sum_{i,j,i',j'} c_{ij} \bar{c}_{i'j} c_{i'j'} \bar{c}_{ij'}$$



$$(\rho_A)_{ii'} = \sum_j \langle \{i\}\{j\} | \rho_{N_A+N_B} | \{i'\}\{j\} \rangle = \sum_j c_{ij} \bar{c}_{i'j}$$

$$= \sum_{i,i'} (\rho_A)_{ii'} (\rho_A)_{i'i} = \text{tr}_{\mathcal{H}_{N_A}} (\rho_A^2)$$

Demonstration: 2nd Renyi entropy of Bell state

Bell state:

$$|B\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

Reduced density matrix:

$$\rho_{\text{red}} = \text{tr}_2 |B\rangle\langle B| = \frac{1}{2} (|0\rangle\langle 0| + |1\rangle\langle 1|)$$

2nd Renyi entropy:

$$\text{tr} \rho_{\text{red}}^2 = \text{tr} \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix} = \frac{1}{2}$$

$$S_2 = -\log \text{tr} \rho_{\text{red}}^2 = \log 2$$

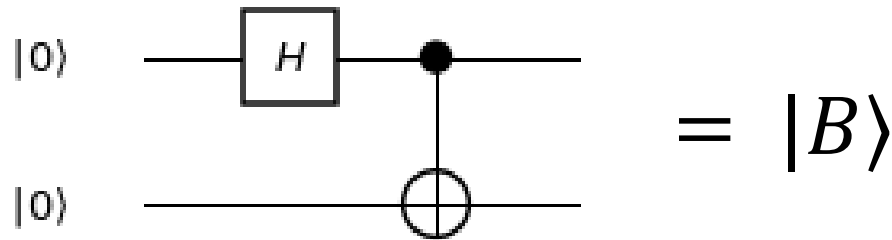
Let's reproduce it in IBM Quantum!

Demonstration: 2nd Renyi entropy of Bell state (Cont'd)

We know

$$\text{tr} \rho_{\text{red}}^2 = \langle B| \otimes \langle B| \text{SWAP}^{(1,3)} |B\rangle \otimes |B\rangle \quad |B\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

The Bell state is written as

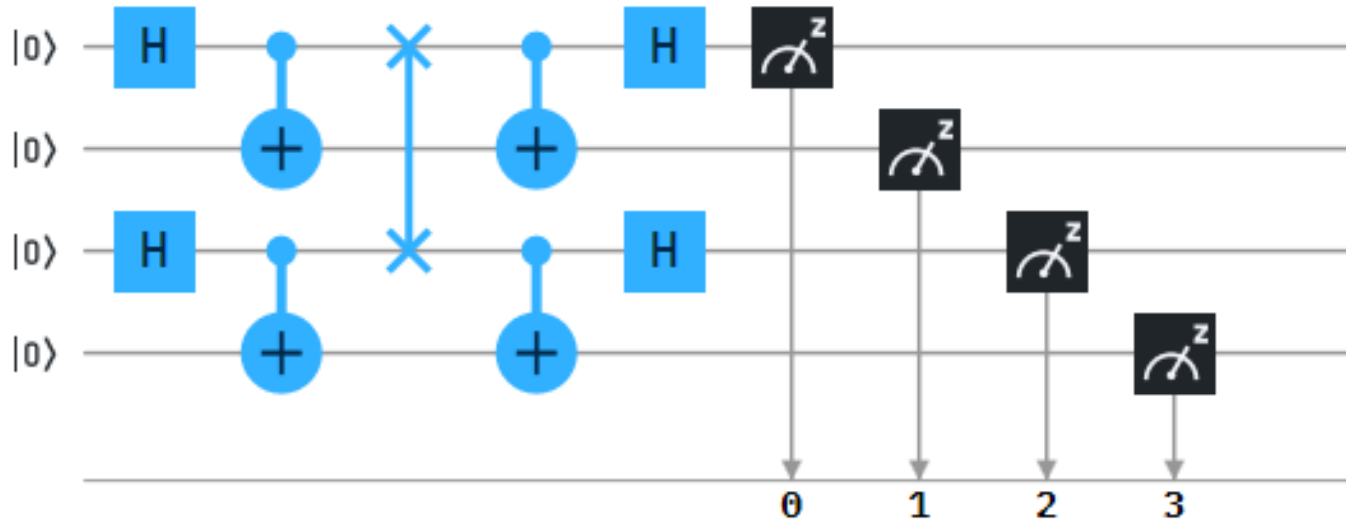


Therefore,

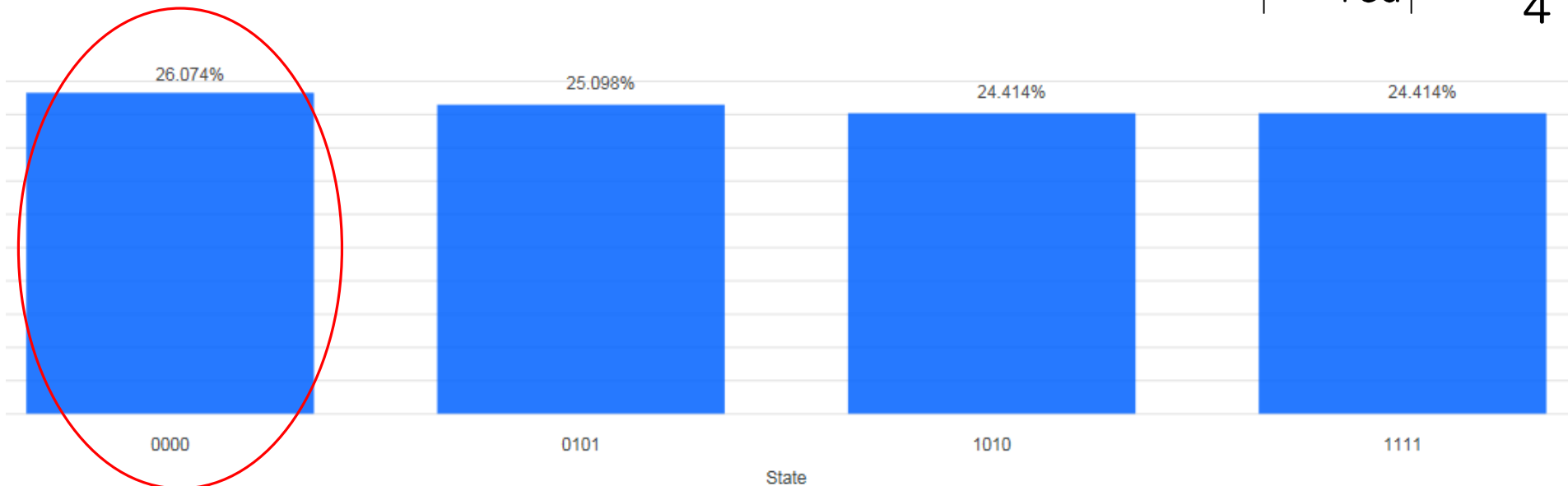
$$\text{tr} \rho_{\text{red}}^2 = \langle 0000| U^\dagger \text{SWAP}^{(1,3)} U |0000\rangle \quad (|B\rangle \otimes |B\rangle \equiv U|0000\rangle)$$

$$|\text{tr} \rho_{\text{red}}^2|^2 = |\langle 0000| U^\dagger \text{SWAP}^{(1,3)} U |0000\rangle|^2$$

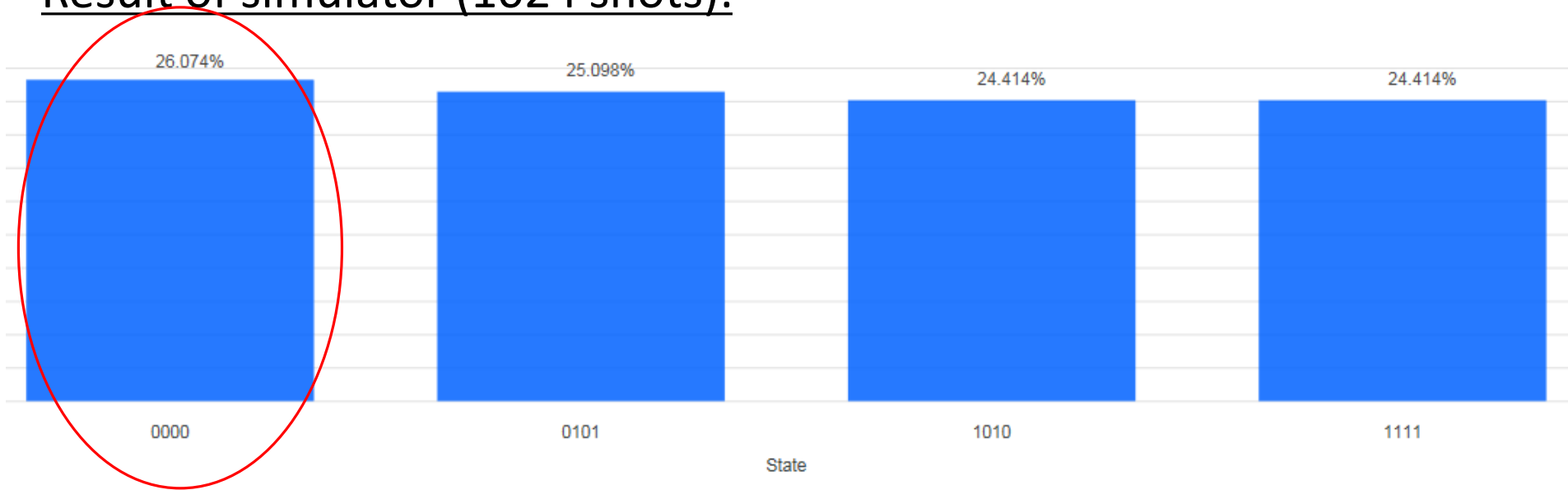
Demonstration: 2nd Renyi entropy of Bell state (Cont'd)


$$|\mathrm{tr} \rho_{\mathrm{red}}^2|^2 = \frac{1}{4}$$

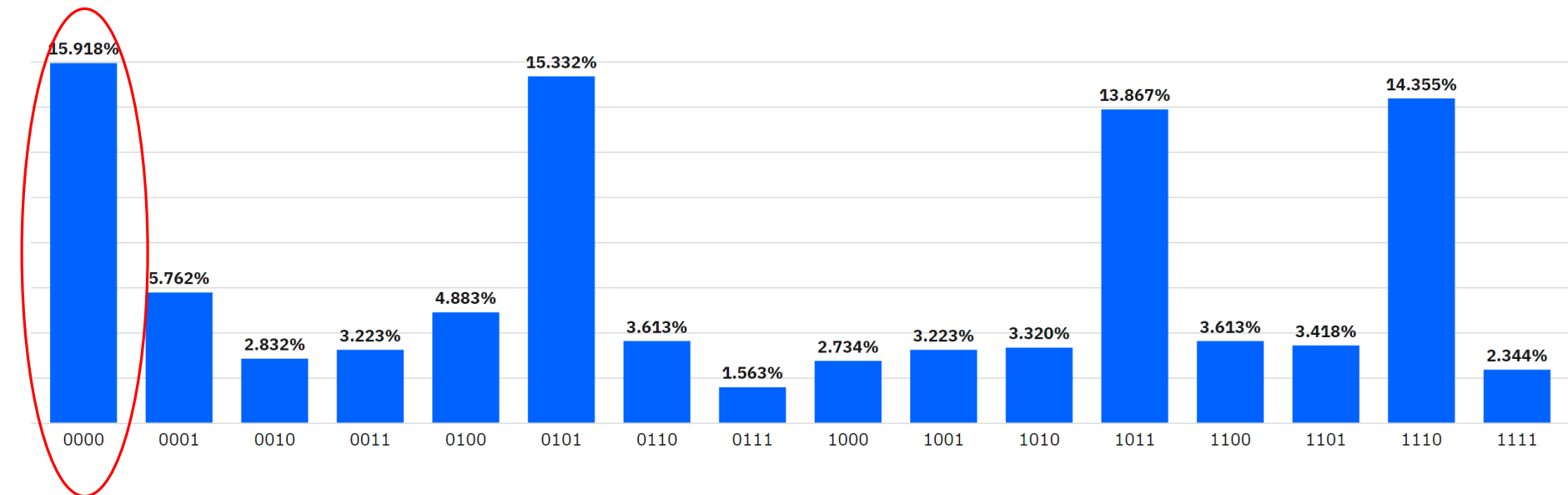
Result of simulator (1024 shots):



Result of simulator (1024 shots):



Result of quantum computer (1024 shots):



More direct way?

We've directly computed

$$|\mathrm{tr} \rho_{\mathrm{red}}^2|^2 = |\langle 0000 | U^\dagger \mathrm{SWAP}^{(1,3)} U | 0000 \rangle|^2$$

rather than itself:

$$\mathrm{tr} \rho_{\mathrm{red}}^2 = \langle 0000 | U^\dagger \mathrm{SWAP}^{(1,3)} U | 0000 \rangle$$

Can we directly compute it?

—— Yes, there is a way to compute expectation value of unitary op. under any state: (next slide)

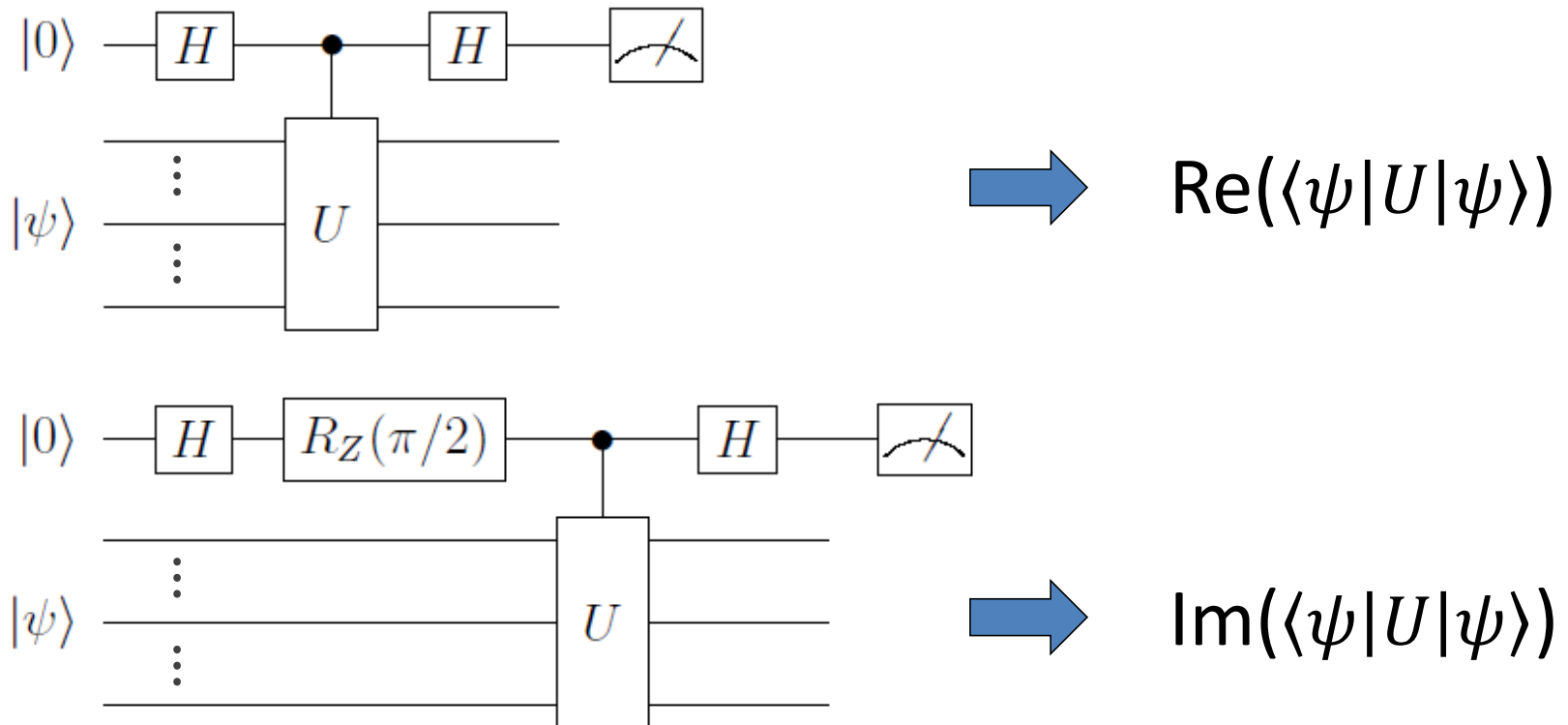
$$\langle \psi | U | \psi \rangle$$

“Hadamard test”: standard way to compute $\langle \psi | U | \psi \rangle$

① Extend Hilbert space & consider the state

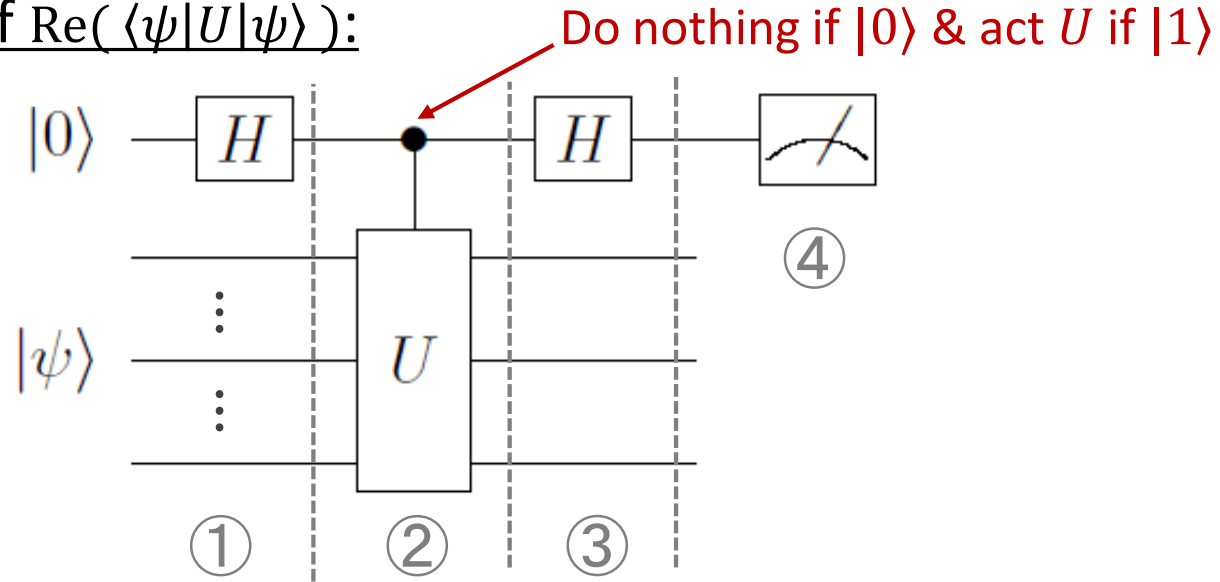
$$\underbrace{|0\rangle}_{\text{“ancillary qubit”}} \otimes |\psi\rangle$$

② We can compute $\langle \psi | U | \psi \rangle$ by using the 2 circuits: (next slide)



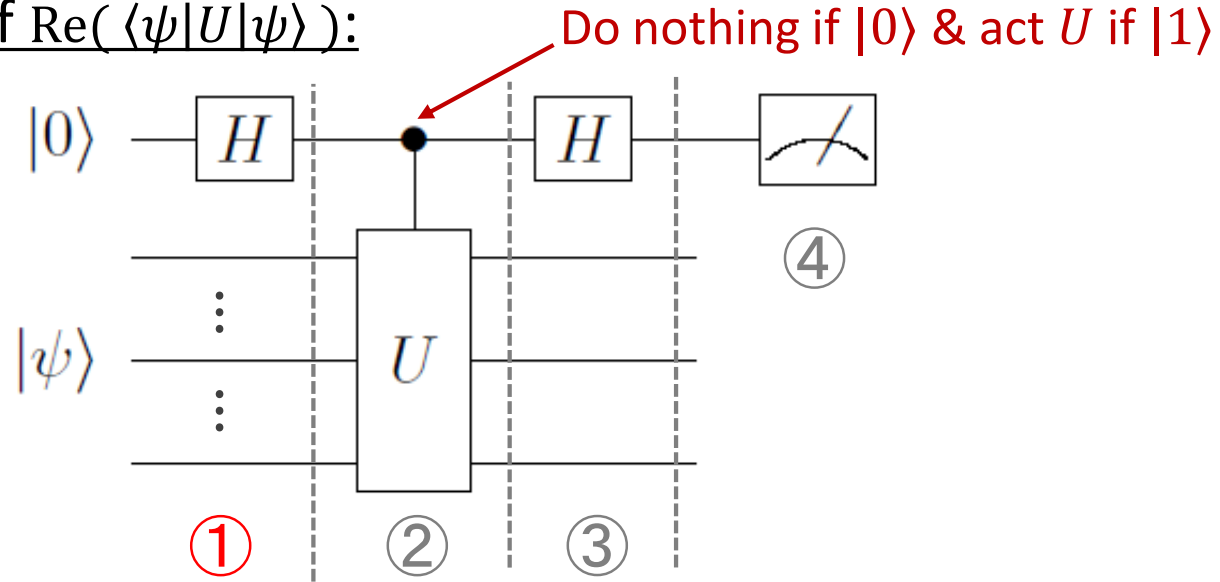
“Hadamard test”: standard way to compute $\langle \psi | U | \psi \rangle$ (Cont'd)

Computation of $\text{Re}(\langle \psi | U | \psi \rangle)$:



“Hadamard test”: standard way to compute $\langle \psi | U | \psi \rangle$ (Cont'd)

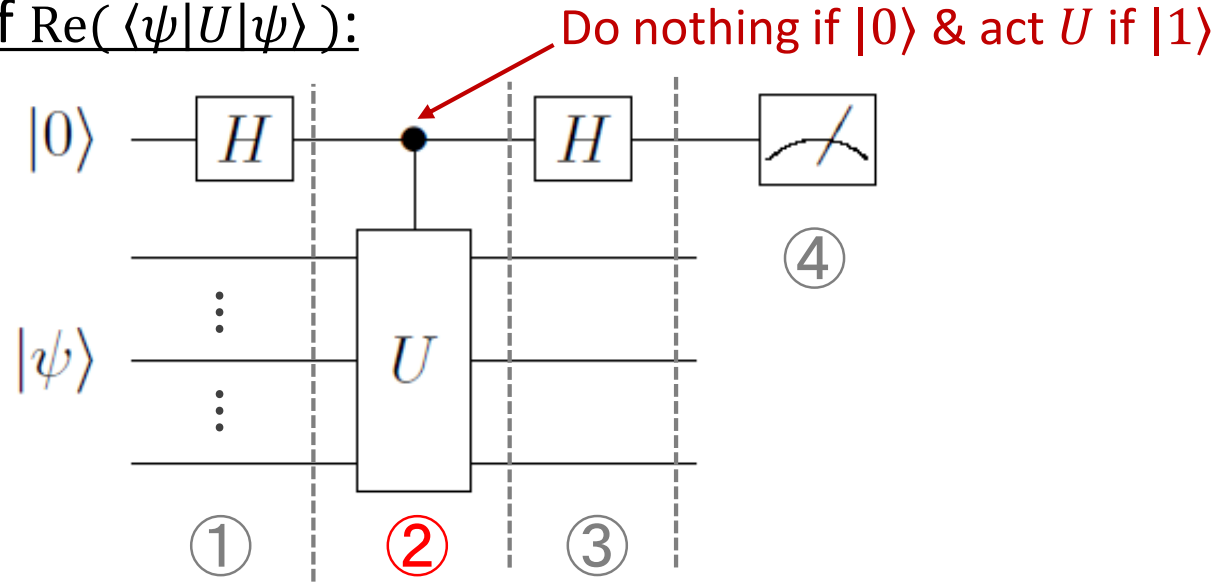
Computation of $\text{Re}(\langle \psi | U | \psi \rangle)$:



① $H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$

“Hadamard test”: standard way to compute $\langle \psi | U | \psi \rangle$ (Cont'd)

Computation of $\text{Re}(\langle \psi | U | \psi \rangle)$:

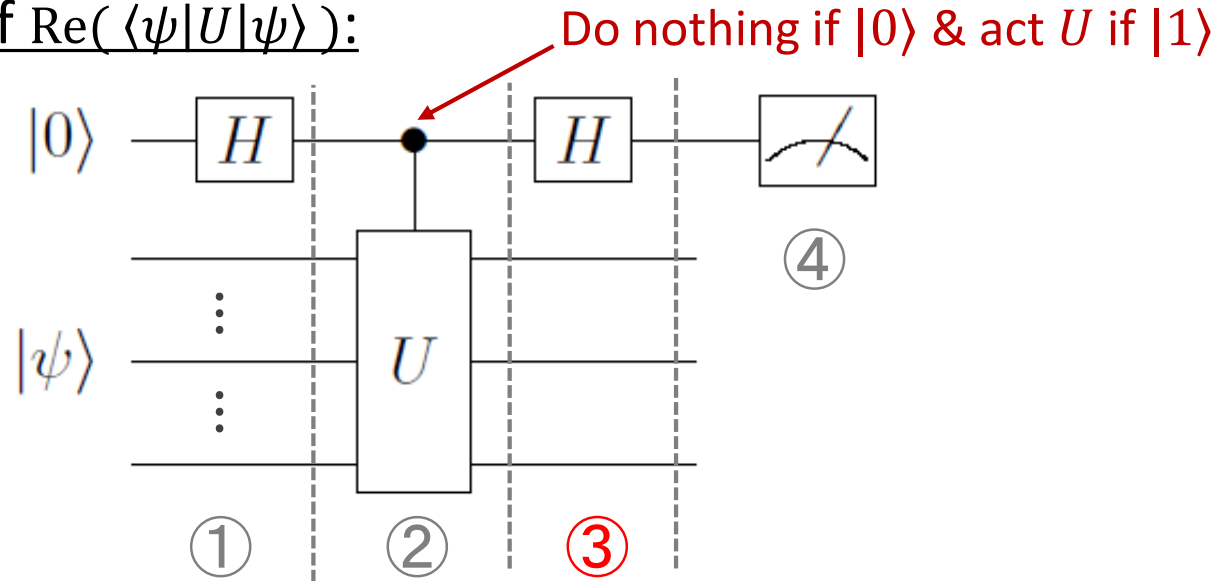


$$\textcircled{1} \quad H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$$

$$\textcircled{2} \quad \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$$

“Hadamard test”: standard way to compute $\langle \psi | U | \psi \rangle$ (Cont'd)

Computation of $\text{Re}(\langle \psi | U | \psi \rangle)$:



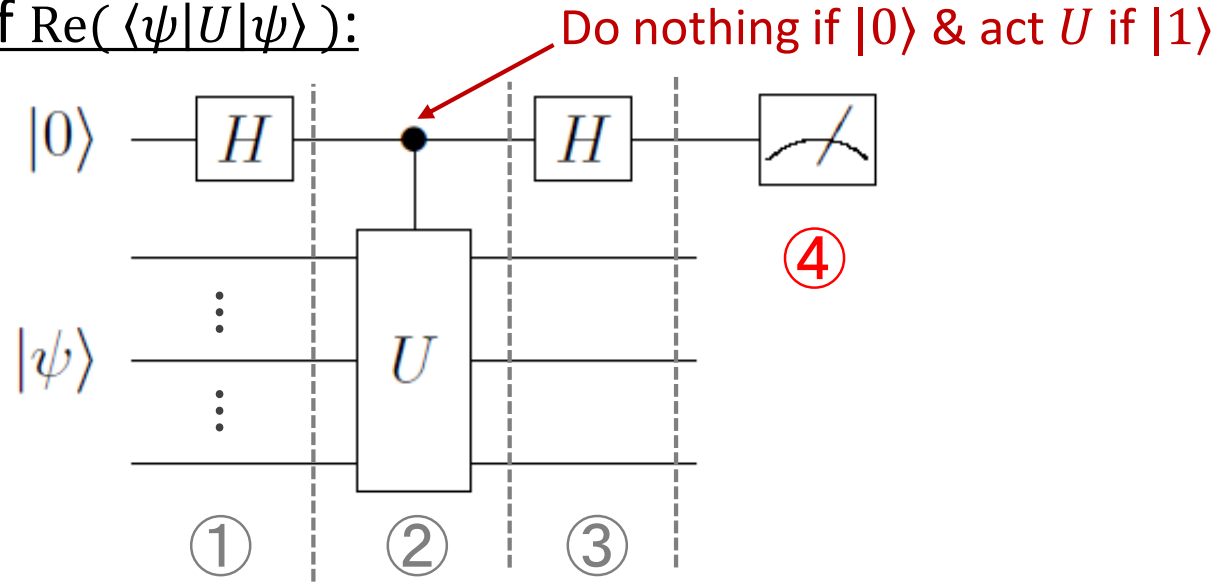
$$\textcircled{1} \quad H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$$

$$\textcircled{2} \quad \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$$

$$\textcircled{3} \quad \frac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle + \frac{1}{2}(|0\rangle - |1\rangle) \otimes U|\psi\rangle$$
$$= \frac{1}{2}|0\rangle \otimes (1 + U)|\psi\rangle + \frac{1}{2}|1\rangle \otimes (1 - U)|\psi\rangle$$

“Hadamard test”: standard way to compute $\langle \psi | U | \psi \rangle$ (Cont'd)

Computation of $\text{Re}(\langle \psi | U | \psi \rangle)$:



$$\textcircled{1} \quad H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$$

$$\textcircled{2} \quad \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$$

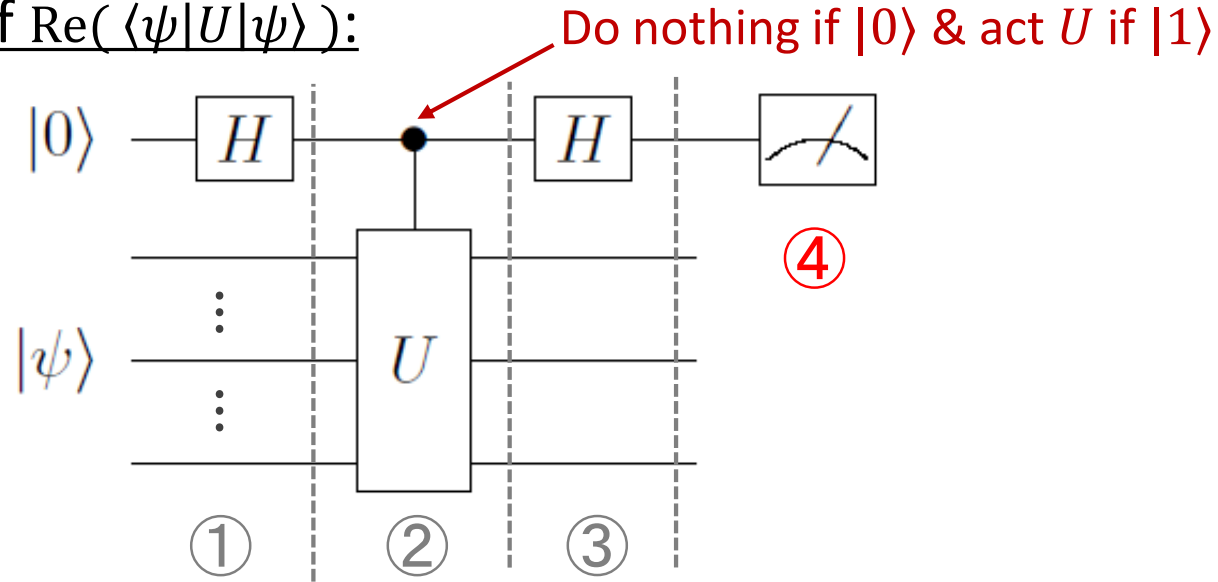
$$\begin{aligned} \textcircled{3} \quad & \frac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle + \frac{1}{2}(|0\rangle - |1\rangle) \otimes U|\psi\rangle \\ &= \frac{1}{2}|0\rangle \otimes (1 + U)|\psi\rangle + \frac{1}{2}|1\rangle \otimes (1 - U)|\psi\rangle \end{aligned}$$

$$\textcircled{4} \quad P_0 = \frac{1}{4} |(1 + U)|\psi\rangle|^2 = \frac{1}{2} (1 + \text{Re}\langle \psi | U | \psi \rangle)$$

$$P_1 = \frac{1}{4} |(1 - U)|\psi\rangle|^2 = \frac{1}{2} (1 - \text{Re}\langle \psi | U | \psi \rangle)$$

“Hadamard test”: standard way to compute $\langle \psi | U | \psi \rangle$ (Cont'd)

Computation of $\text{Re}(\langle \psi | U | \psi \rangle)$:

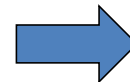


$$\textcircled{1} H|0\rangle \otimes |\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$$

$$\textcircled{2} \frac{1}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$$

$$\begin{aligned} \textcircled{3} \frac{1}{2}(|0\rangle + |1\rangle) \otimes |\psi\rangle + \frac{1}{2}(|0\rangle - |1\rangle) \otimes U|\psi\rangle \\ = \frac{1}{2}|0\rangle \otimes (1 + U)|\psi\rangle + \frac{1}{2}|1\rangle \otimes (1 - U)|\psi\rangle \end{aligned}$$

$$\begin{aligned} \textcircled{4} P_0 &= \frac{1}{4} |(1 + U)|\psi\rangle|^2 = \frac{1}{2} (1 + \text{Re}\langle \psi | U | \psi \rangle) \\ P_1 &= \frac{1}{4} |(1 - U)|\psi\rangle|^2 = \frac{1}{2} (1 - \text{Re}\langle \psi | U | \psi \rangle) \end{aligned}$$

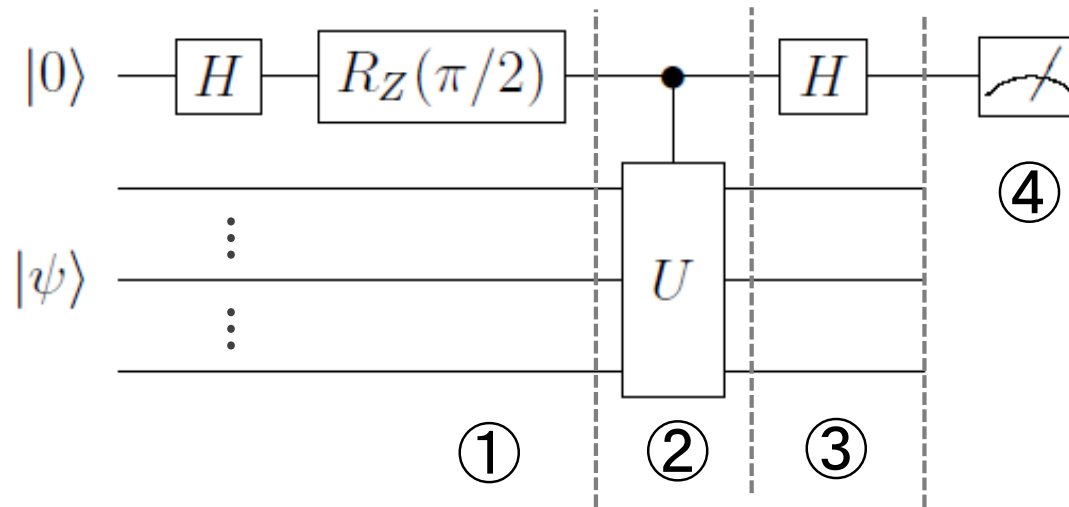


$$\text{Re}\langle \psi | U | \psi \rangle = P_0 - P_1$$

“Hadamard test”: standard way to compute $\langle \psi | U | \psi \rangle$ (Cont'd)

Computation of $\text{Im}(\langle \psi | U | \psi \rangle)$:

$$\left[R_Z(\theta) = e^{-\frac{i\theta}{2}Z} \right]$$



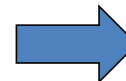
$$\textcircled{1} \quad R_Z(\pi/2)H|0\rangle \otimes |\psi\rangle = \frac{e^{-\frac{\pi i}{4}}}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{e^{+\frac{\pi i}{4}}}{\sqrt{2}}|1\rangle \otimes |\psi\rangle$$

$$\textcircled{2} \quad \frac{e^{-\frac{\pi i}{4}}}{\sqrt{2}}|0\rangle \otimes |\psi\rangle + \frac{e^{+\frac{\pi i}{4}}}{\sqrt{2}}|1\rangle \otimes U|\psi\rangle$$

$$\textcircled{3} \quad \frac{e^{-\frac{\pi i}{4}}}{2}|0\rangle \otimes (1 + iU)|\psi\rangle + \frac{e^{-\frac{\pi i}{4}}}{2}|1\rangle \otimes (1 - iU)|\psi\rangle$$

$$\textcircled{4} \quad P_0 = \frac{1}{4} |(1 + iU)|\psi\rangle|^2 = \frac{1}{2} (1 - \text{Im}\langle \psi | U | \psi \rangle)$$

$$P_1 = \frac{1}{4} |(1 - iU)|\psi\rangle|^2 = \frac{1}{2} (1 + \text{Im}\langle \psi | U | \psi \rangle)$$



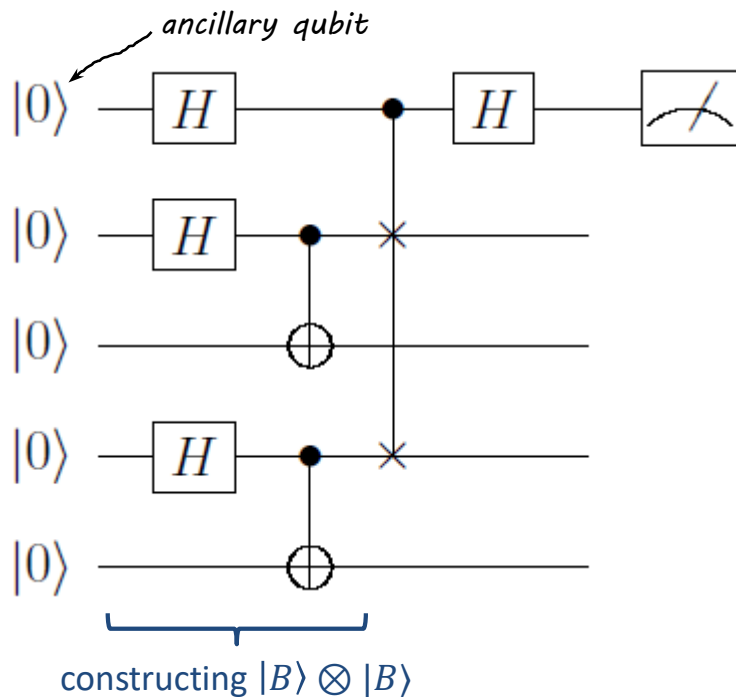
$$\text{Im}\langle \psi | U | \psi \rangle = P_1 - P_0$$

Coming back to the Renyi entropy of Bell state

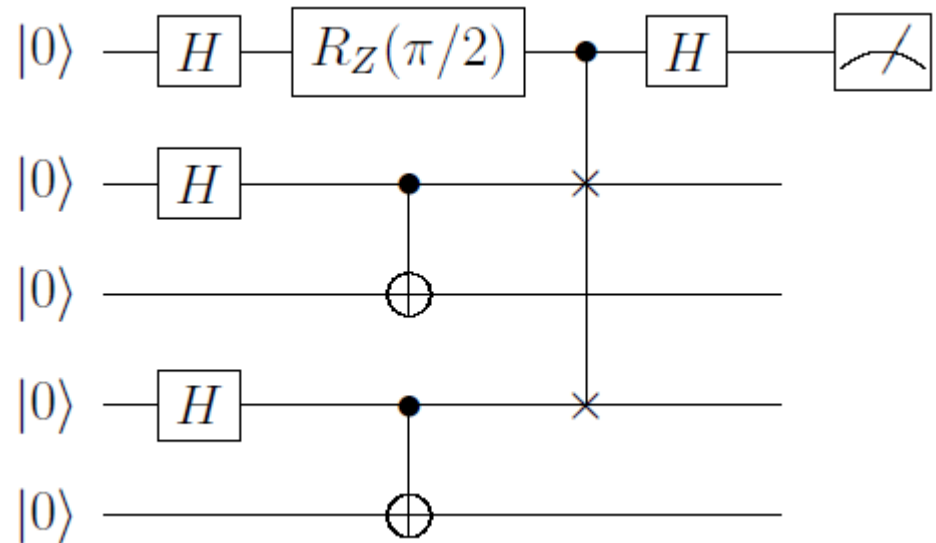
Taking $|\psi\rangle = |B\rangle \otimes |B\rangle$ & $U = \text{SWAP}^{(1,3)}$, we can directly compute

$$\text{tr} \rho_{\text{red}}^2 = \langle B| \otimes \langle B| \text{SWAP}^{(1,3)} |B\rangle \otimes |B\rangle$$

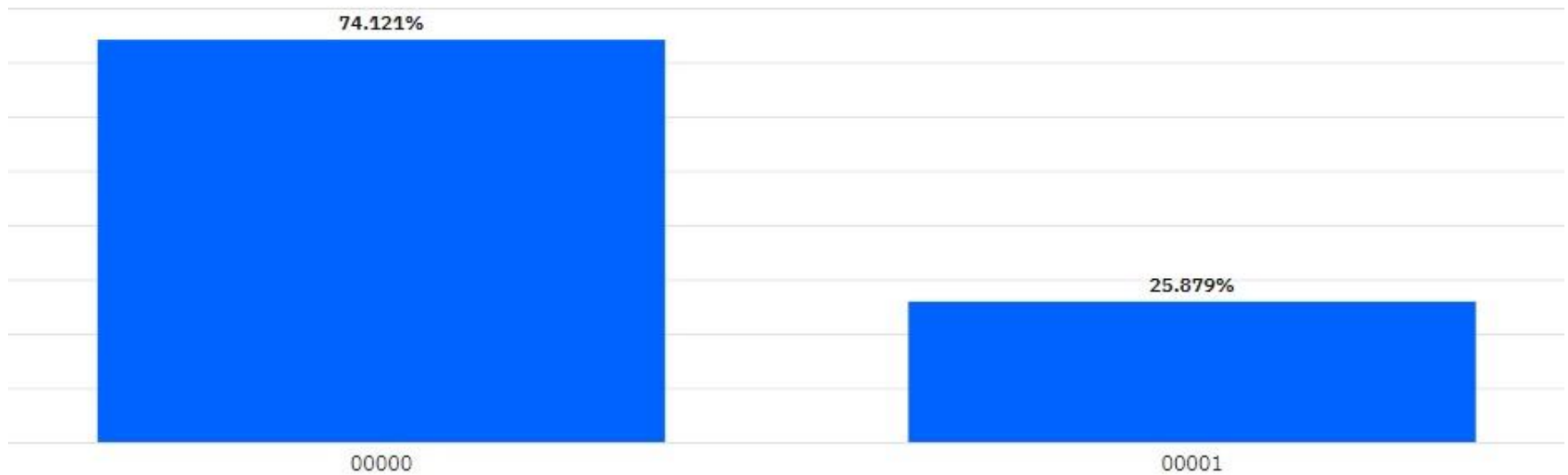
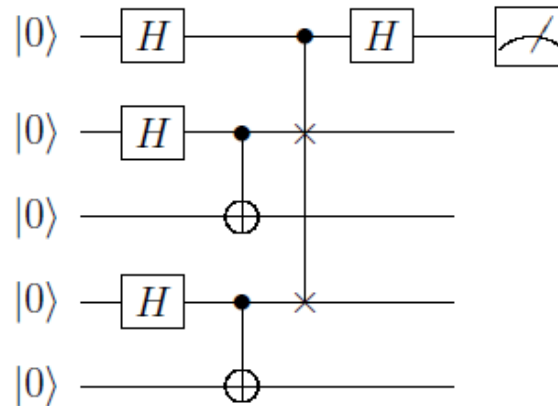
Real part:



Imaginary part:

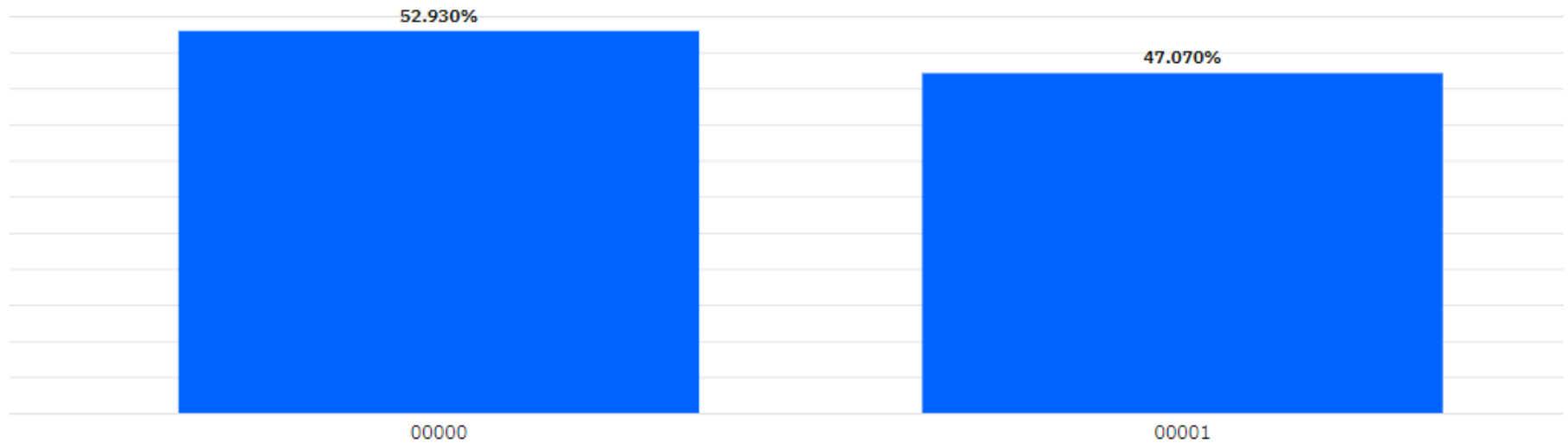
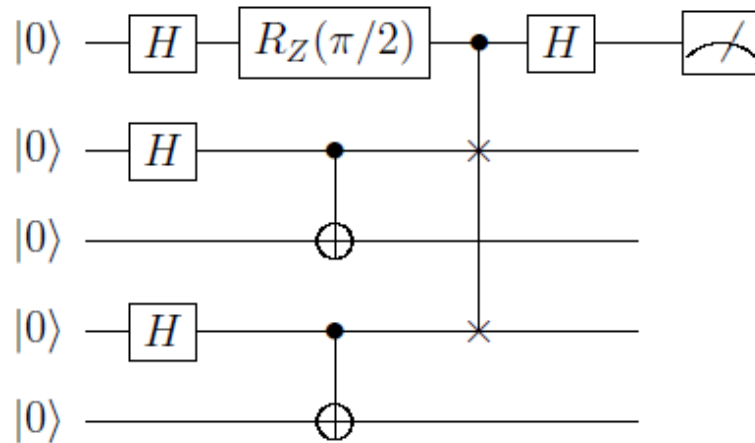


Result of simulator (real part, 1024 shots)



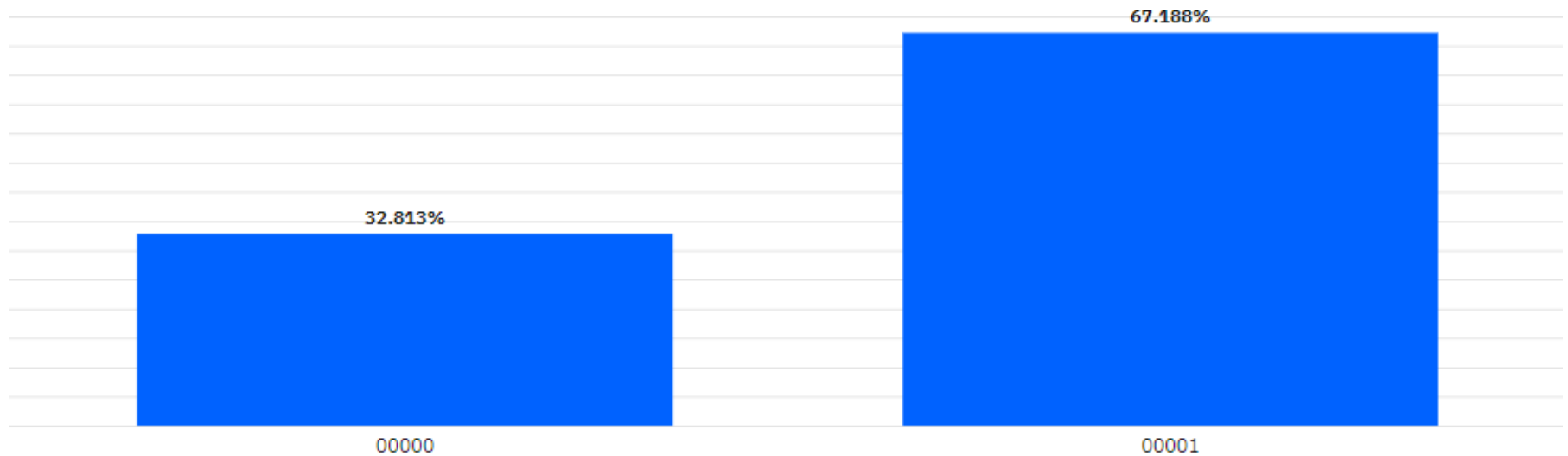
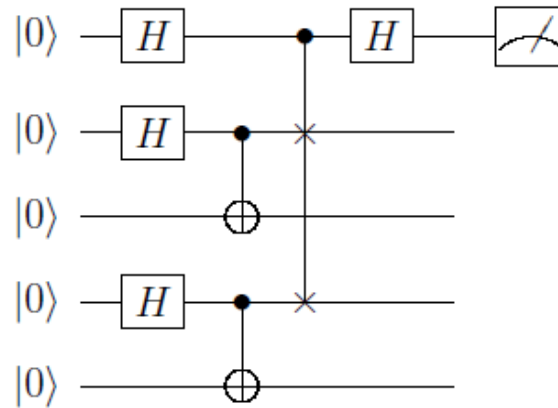
Expectation: $P_0 - P_1 = \text{Re tr} \rho_{\text{red}}^2 = \frac{1}{2}$

Result of simulator (imaginary part, 1024 shots)



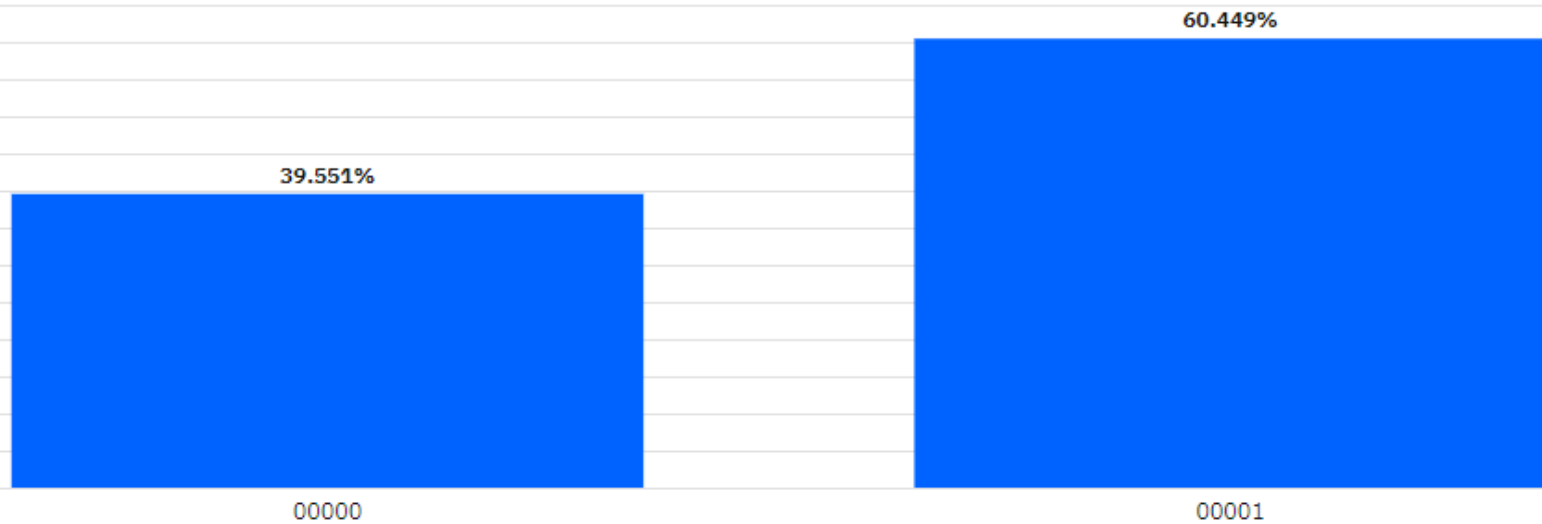
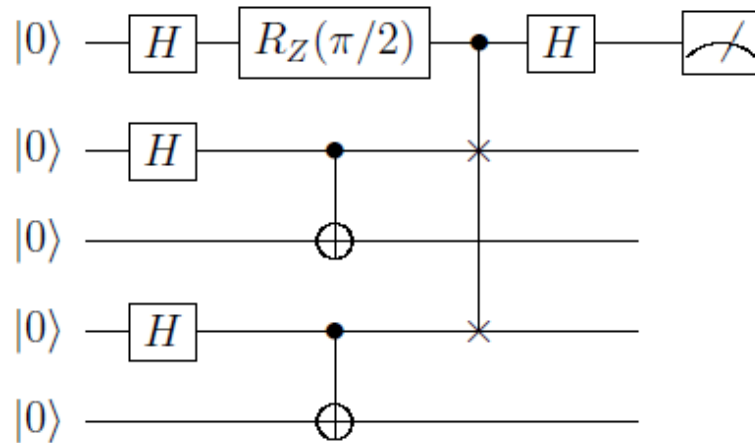
Expectation: $P_1 - P_0 = \text{Im tr} \rho_{\text{red}}^2 = 0$

Result of quantum computer (real part, 1024 shots)



Expectation: $P_0 - P_1 = \text{Re tr} \rho_{\text{red}}^2 = \frac{1}{2}$

Result of quantum computer (imaginary part, 1024 shots)



Expectation: $P_1 - P_0 = \text{Im tr} \rho_{\text{red}}^2 = 0$

Other similar quantities

Renyi entropy:

$$\rho = |\Psi\rangle\langle\Psi|, \sigma = |\Phi\rangle\langle\Phi|$$

$$S_n = \frac{1}{1-n} \log \text{tr}_{\mathcal{H}_{N_A}}(\rho_A^n)$$

$$\text{tr}_{\mathcal{H}_{N_A}}(\rho_A^n) = \langle\Psi|^{\otimes n} \text{SWAP}_{A_1, A_2} \cdots \text{SWAP}_{A_{n-1}, A_n} |\Psi\rangle^{\otimes n}$$

Renyi relative entropy:

$$S_n(\rho||\sigma) = \frac{1}{1-n} \log \frac{\text{tr}_{\mathcal{H}_{N_A}}(\rho_A \sigma_A^{n-1})}{\text{tr}_{\mathcal{H}_{N_A}}(\rho_A^n)}$$

$$\text{tr}_{\mathcal{H}_{N_A}}(\rho_A \sigma_A^{n-1}) = \langle\Phi|^{\otimes n-1} \otimes \langle\Psi| \text{SWAP}_{A_1, A_2} \cdots \text{SWAP}_{A_{n-1}, A_n} |\Psi\rangle \otimes |\Phi\rangle^{\otimes n-1}$$

Renyi Pseudo entropy:

[Nakata-Takayanagi-Taki-Tamaoka-Wei '20]

$$S^{(n)}(\mathcal{J}_A^{\Psi|\Phi}) = \frac{1}{1-n} \log \text{tr}_{\mathcal{H}_{N_A}}(\mathcal{J}_A^{\Psi|\Phi})^n, \quad \mathcal{J}_A^{\Psi|\Phi} = \text{tr}_{\mathcal{H}_{N_B}} \frac{|\Psi\rangle\langle\Phi|}{\langle\Phi|\Psi\rangle}$$

$$\text{tr}_{\mathcal{H}_{N_A}}(\mathcal{J}_A^{\Psi|\Phi})^n \propto \langle\Phi|^{\otimes n} \text{SWAP}_{A_1, A_2} \cdots \text{SWAP}_{A_{n-1}, A_n} |\Psi\rangle^{\otimes n}$$

Summary of the lecture part of the 1st day

Summary

- Quantum computation is suitable for operator formalism which is free from sign problem
- Instead we have to deal with huge vector space. Quantum computers in future may do this job.
- "Rule" of quantum computation
= Do something interesting by a combination of acting unitary op. & measurement
- Real quantum computer has errors
- Quantum error correction is important

(Here is the expected end of the lecture 2)

Hands-on 1

Download a file from my webpage:

<https://www2.yukawa.kyoto-u.ac.jp/~masazumi.honda/QC/QC.html>

Appendix

What if we replace T by something else?

$$T = e^{\frac{i\pi}{8}} R_Z(\pi/4) \quad \Rightarrow \quad T' \equiv R_Z(\phi) ??$$

We have the identity:

$$T'HT'H = R_{\vec{n}}(\theta)$$

where

$$\vec{n} = \frac{1}{\sqrt{1 + \cos^2(\phi/2)}} \begin{pmatrix} \cos(\phi/2) \\ \sin(\phi/2) \\ \cos(\phi/2) \end{pmatrix} \quad \& \quad \cos(\theta/2) \equiv \cos^2(\phi/2)$$

We can approximate any single qubit gate
by combining H & T' if $\theta/2\pi$ is irrational