ABJ Theory in the Higher Spin Limit

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References: arXiv:1504.00365 [hep-th],
JHEP 1508 110 (1506.00781)

based on collaborations with

Shinji Hirano (Witwatersrand U.), Kazumi Okuyama (Shinshu U.)
and Masaki Shigemori (Yukawa Institute)

$$\text{String} \xrightarrow{\text{extremely high energy}} {}^{\exists}\,\text{infinite massless higher spin states}$$

$$\Downarrow$$

Tensionless string has higher spin gauge symmetry?

[Gross '88, etc...]

Known consistent interacting higher spin gauge theory

Vasiliev theory

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However,

(# of spin-s states in typical string theory) $\sim$ e^{s}

(# of spin-s gauge fields in Vasiliev theory) $=$ (Const.)

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(# of spin-s states in typical string theory) $\sim$ e^{s}

(# of spin-s gauge fields in Vasiliev theory) = (Const.)

Is Vasiliev theory related to string theory?

If this is the case, what is the relation?

Today, I will consider

Higher spin AdS/CFT correspondence,

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insights to relation between string and Vasiliev theory

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insights to relation between string and Vasiliev theory

For this purpose,

ABJ theory provides good laboratory

ABJ theory:

[Aharony-Bergman-Jafferis-Maldacena '08,
Aharony-Bergman-Jafferis '08]

3d $\mathcal{N}=6$ $U(N)_k \times U(N+M)_{-k}$ (k: CS level)

superconformal Chern-Simons theory

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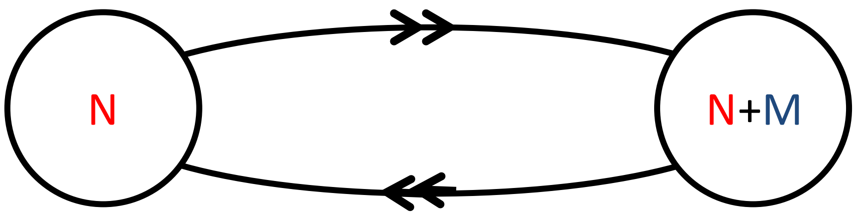
$\supset$
- Vector multiplet (in 3d $\mathcal{N}=2$ language)
- 2 bi-fundamental chiral multiplets
- 2 anti-bi-fundamental chiral multiplets



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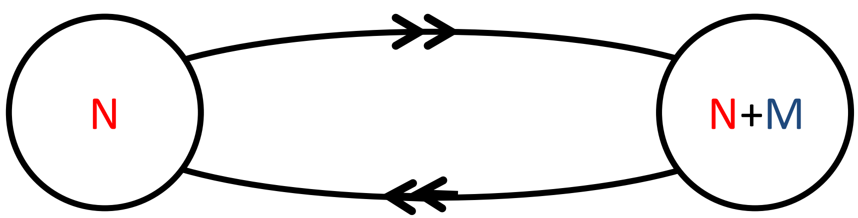

Expected as the low-energy effective theory of

(N M2-branes) + (M fractional M2-branes) on $\mathbf{R^8/Z_k}$

(=M5-branes wrapped on $S^3/Z_k \subset R^8/Z_k$)

CFT_3 / AdS_4

$U(N)_k \times U(N+M)_{-k}$

ABJ theory

CFT_3 / AdS_4

M-theory

on $\mathrm{AdS}_4 \times \mathrm{S}^7/\mathrm{Z}_k$

with $\frac{1}{2\pi}\int_{S^3/Z_k} C_3 = \frac{1}{2} - \frac{M}{k}$

$k \ll N^{1/5}$

$U(N)_k \times U(N+M)_{-k}$

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ABJ theory

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$\lambda = \frac{N}{k} = \text{fixed},\ N \gg 1$

CFT$_3$ / AdS$_4$

U(N)$_k$ x U(N+M)$_{-k}$
ABJ theory

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$\mathcal{N} = 6$ Vasiliev theory
on AdS$_4$

[Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin '11, Chang-Minwalla-Sharma-Yin '12]

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Here we aim to make the dictionary more precise.

1. Identification of "Higher Spin sector" in ABJ

[Hirano-M.H.-Okuyama-Shigemori]

2. Identification of bulk coupling constant

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Ex.) U(N) Chern-Simons theory w/ fundamentals

[Aharony-Gur-Ari-Yacoby,
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$$F_{\text{Higher Spin}} = F_{\text{CFT}} - F_{\text{pureCS}}$$

How about theory w/ bi-fundamentals?

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For this purpose, we compute & compare

- free energy of the ABJ theory on S^3 in the higher spin limit
- 1-loop free energy of the Vasiliev theory

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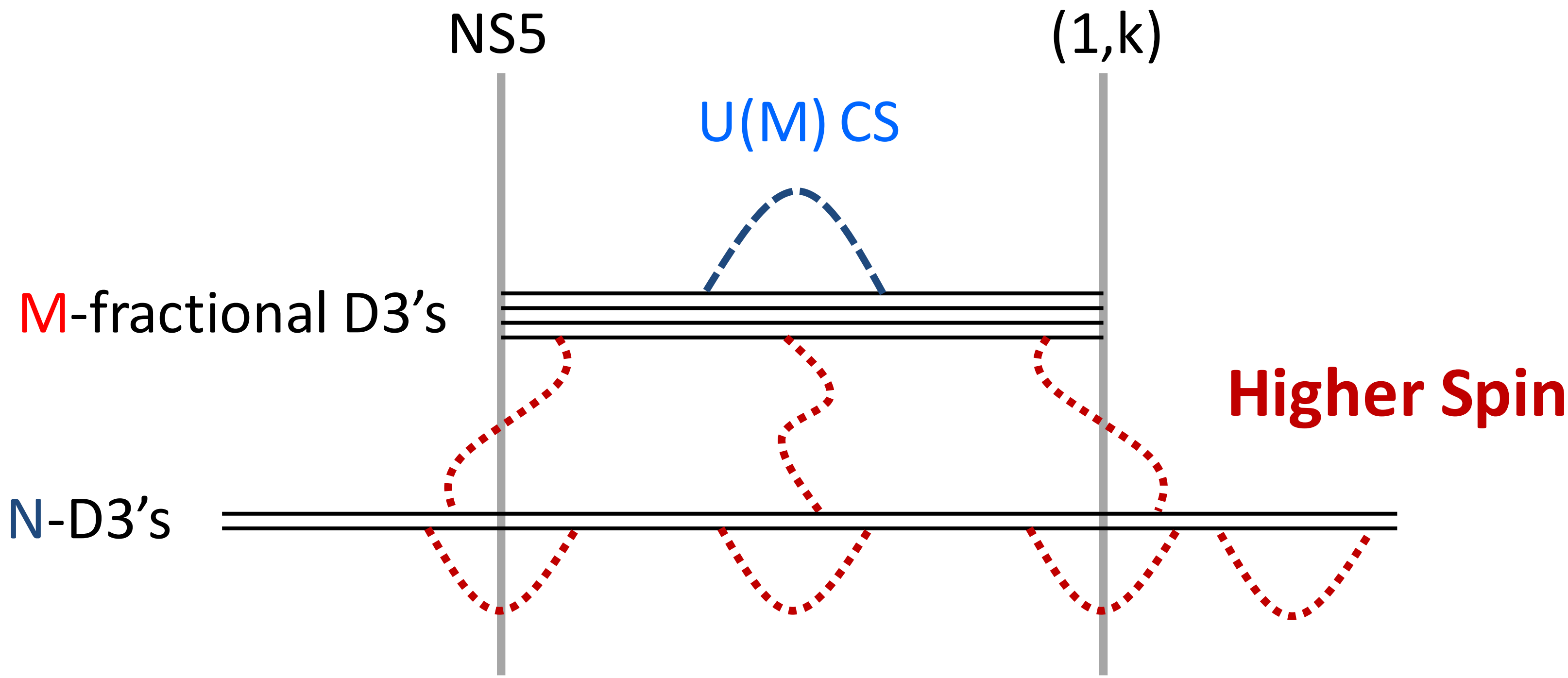
—— Compute stress tensor 2-pt. function by localization

Our proposal



$$\boxed{F_{\mathsf{HS}} = F_{\mathsf{ABJ}} - F_{\mathsf{CS}}}$$

[Hirano-M.H.-Okuyama-Shigemori]

$$\boxed{G_{\mathsf{HS}} = \frac{2}{\pi M}\frac{\pi t}{\sin(\pi t)}}$$

[M.H.]

$$t = \frac{M}{k}$$

Contents

Simple examples of 3d CFT w/ higher spin symmetry

3d U(N) free bosonic vector model:

[Klebanov-Polyakov, etc]

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∃ Conserved higher spin currents for all N:

$$J^{(s)}_{(\mu_1 \cdots \mu_s)} \sim \phi^{a\dagger} \partial_{(\mu_1} \cdots \partial_{\mu_s)} \phi^a$$

∃ operators w/ (spin, dimension) $= \sum_{s=0}^{\infty} (s, s+1)$

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3d U(N) free fermionic vector model:

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Simple SUSY example

3d U(N) SUSY free vector model:

$$S = \int d^3x \;\left[(\partial^\mu \phi^a)^\dagger \cdot (\partial_\mu \phi^a) + \bar{\psi}^a \gamma^\mu \partial_\mu \psi^a\right]$$

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$$J^{(s+\frac{1}{2})}_{(\mu_1 \cdots \mu_s)} \sim \bar{\psi}^a \partial_{(\mu_1} \cdots \partial_{\mu_s)} \phi^a, \;\; \phi^{a\dagger} \partial_{(\mu_1} \cdots \partial_{\mu_s)} \psi^a$$

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$$= (0,1) + (0,2) + 2\sum_{s=1}^{\infty} (s, s+1) + 2\sum_{s=0}^{\infty} \left(s+\frac{1}{2}, s+\frac{3}{2}\right)$$

⟺ Supersymmetric Vasiliev theory

Coupling to Chern-Simons

U(N) Chern-Simons theory w/ vector bosons at fixed point:

[Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin, Aharony-Gur-Ari-Yacoby , etc..]

$$S = S_{\text{CS}} + \int d^3x \, \left[(D^\mu \phi^a)^\dagger \cdot (D_\mu \phi^a) + \frac{\lambda_6}{N} (\phi^a \phi^a)^3 \right]$$

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⇒ Spectrum of Vasiliev theory

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  ⇒ Spectrum of Vasiliev theory

- The Chern-Simons level k breaks parity

  ⇒ 't Hooft coupling λ=N/k corresponds to "parity-violating phase" in Vasiliev theory

CS theory w/ fundamentals and Vasiliev theory

Vasiliev free energy:

$$F_{\mathrm{HS}} = \frac{1}{G_N} F_{\mathrm{HS}}^{(-1)} + F_{\mathrm{HS}}^{(0)} + G_N F_{\mathrm{HS}}^{(1)} + \mathcal{O}\left(G_N^2\right)$$

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CFT free energy:

$$F_{\mathsf{CFT}} = \underbrace{N^2 F^{(-2)}_{\mathsf{CFT}}}_{\text{Coming from self-interaction of gauge field}} + N F^{(-1)}_{\mathsf{CFT}} + F^{(0)}_{\mathsf{CFT}} + \frac{1}{N} F^{(1)}_{\mathsf{CFT}} + \mathcal{O}\left(\frac{1}{N^2}\right)$$

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Coming from self-interaction of gauge field

We should subtract the pure CS free energy:

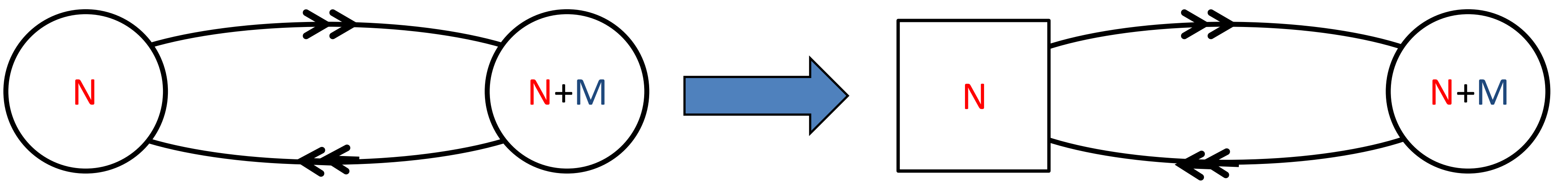
$$F_{\text{CFT}} - F_{\text{pureCS}} = N F_{\text{CFT}}^{\prime(-1)} + F_{\text{CFT}}^{\prime(0)} + \frac{1}{N} F_{\text{CFT}}^{\prime(1)} + \mathcal{O}\left(\frac{1}{N}\right)$$

under the identification:

$$\boxed{G_N \sim \frac{1}{N}}$$

$\mathcal{N}=3$ SQCD w/ CS

Make U(N) vector multiplet non-dynamical in ABJ



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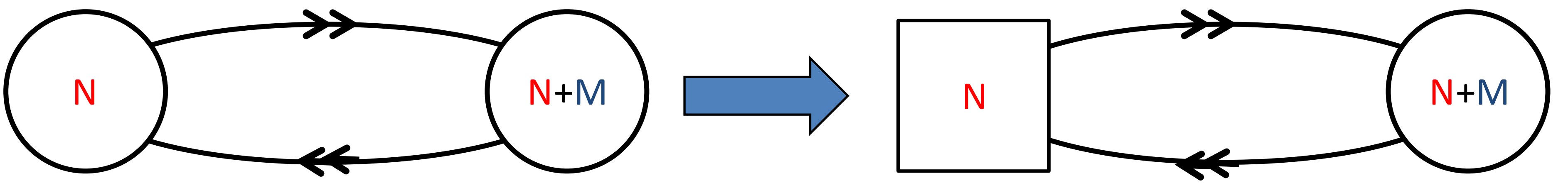

∃ operators w/ (spin, dimension)

$$= N^2 \left[16(0,1) + 16(0,2) + 32 \sum_{s=1}^{\infty} (s, s+1) + 32 \sum_{s=0}^{\infty} \left(s+\frac{1}{2}, s+\frac{3}{2} \right) \right]$$

(up to 1/N correction)

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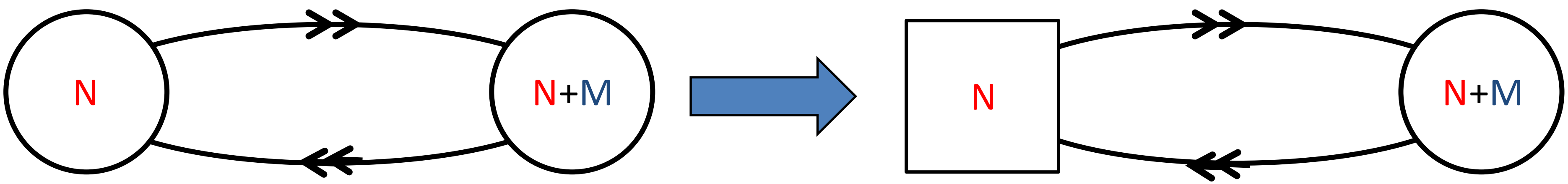

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⟺ $\mathcal{N} = 3$ Vasiliev w/ matrix fields

[Chang-Minwalla-Sharma-Yin'12]

Go back to ABJ…

[Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin '11,
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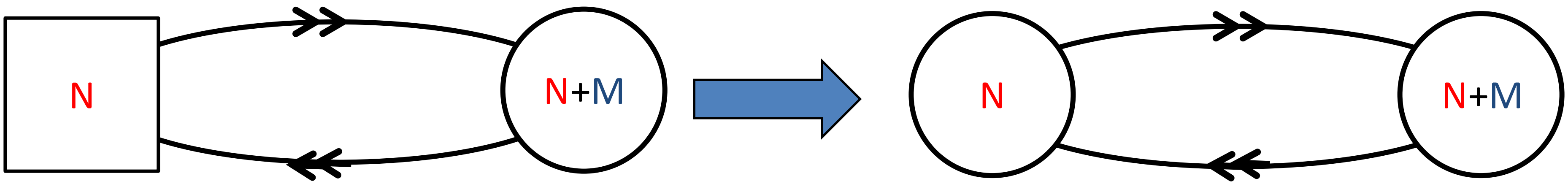
Gauge the U(N) flavor sym. in the SQCD



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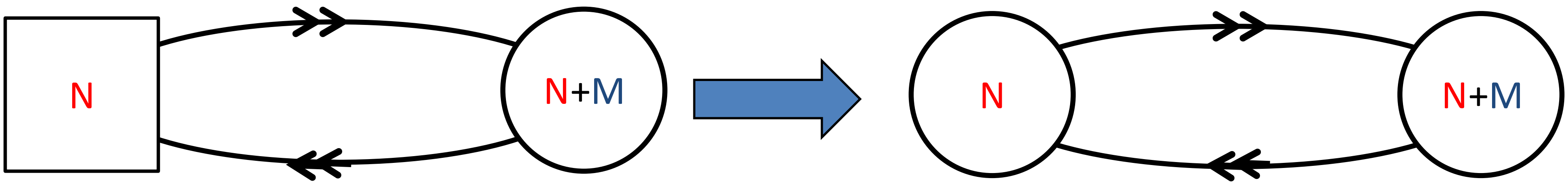

This corresponds to change boundary conditions
for spin-1 fields on the bulk side

[cf. Witten, Leigh-Petkou]

Gauging global sym. at boundary

[cf. Witten, Leigh-Petkou]

Gauge field in AdS_4 near the boundary (on-shell):

$$A_\mu(x,z) = \underbrace{\alpha_\mu(x)}_{\text{gauge field}} + z\underbrace{\beta_\mu(x)}_{\text{Conserved current}} + \mathcal{O}(z^2), \qquad A_z(x,z) = \mathcal{O}(z)$$

Δ+ b.c. (usual):

$$\boxed{F_{ij}\Big|_{z=0} = 0}$$

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Δ- b.c. : $\boxed{F_{zi}\big|_{z=0} = 0}$ Gauged w/o CS level

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Mixed b.c. : $\boxed{\frac{1}{2}\epsilon_{ijk}F_{jk} + i\frac{c_J}{k}F_{zi}\Big|_{z=0} = 0}$ Gauged w/ CS level k

ABJ as a Vasiliev

[Giombi-Minwalla-Prakash-Trivedi-Wadia-Yin '11, Chang-Minwalla-Sharma-Yin '12]

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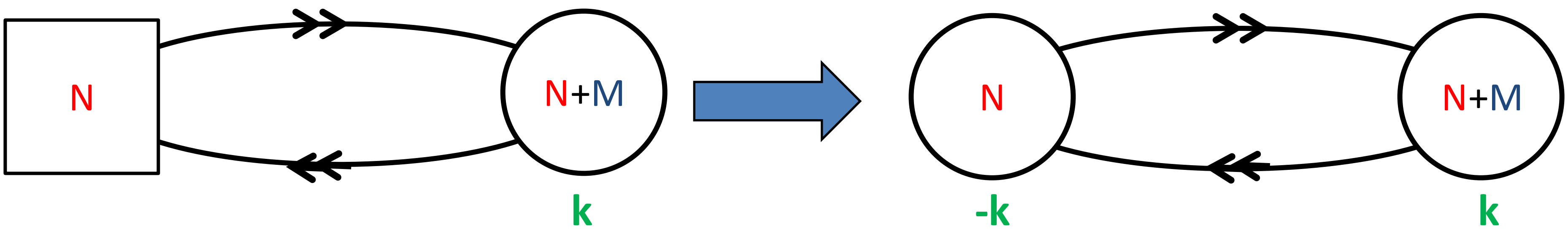


Take mixed b.c. for N^2 spin-1 fields w/ CS level -k

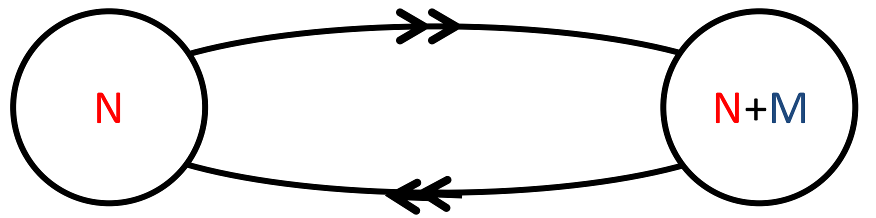
(If the level is different, this becomes Gaiotto-Tomasiello theory)

$\Longleftrightarrow$ $\mathcal{N}=6$ Vasiliev w/ matrix fields

The boundary side: ABJ theory

[Hirano-M.H.-Okuyama-Shigemori]

How do we analyze ABJ theory in HS limit?



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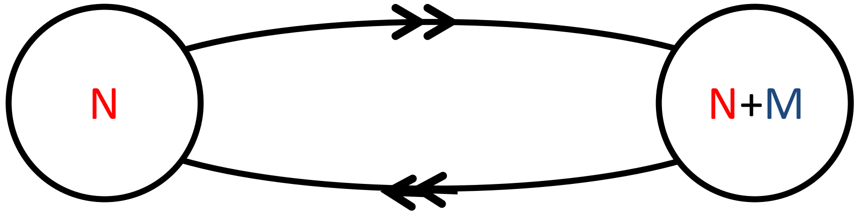

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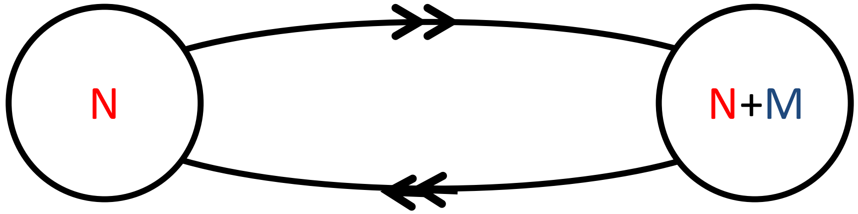

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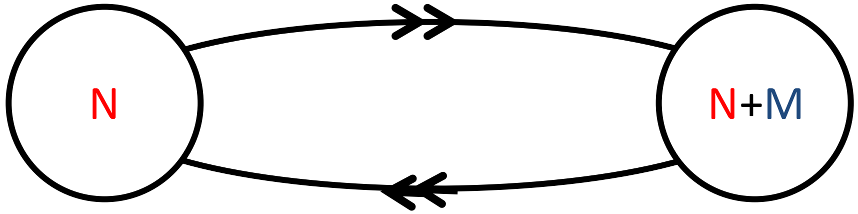

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We'd like to keep λ_2 finite

Use localization!

Computation by localization

Localization formula:

[Kapustin-Willett-Yaakov, Jafferis, Hama-Hosomichi-Lee]

$$Z^{(N,N+M)}(k) = \frac{i^{-\frac{1}{2}(N^2-(N+M)^2)\mathrm{sign}(k)}}{(N+M)!N!} \int_{-\infty}^{\infty} \frac{d^{N+M}\mu}{(2\pi)^{N+M}} \frac{d^N\nu}{(2\pi)^N} e^{-\frac{ik}{4\pi}\left(\sum_{j=1}^{N+M}\mu_j^2 - \sum_{a=1}^{N}\nu_a^2\right)}$$

$$\times \left[\frac{\prod_{1\le j<l\le N+M} 2\sinh\frac{\mu_j-\mu_l}{2} \prod_{1\le a<b\le N} 2\sinh\frac{\nu_a-\nu_b}{2}}{\prod_{j=1}^{N+M}\prod_{b=1}^{N} 2\cosh\frac{\mu_j-\nu_b}{2}} \right]^2,$$

(2N+M)-dim. Integral !

∃ various ways to analyze it in the HS limit

[Drukker-Marino-Putrov, Hirano-M.H.-Okuyama-Shigemori]

(I will explain only one way)

Computation by localization (Cont'd)

[cf. another ways: Drukker-Marino-Putrov]

We can rigorously show

[Awata-Hirano-Shigemori '12, M.H. '13]

$$\boxed{Z^{(N,N+M)}(k) = e^{i\theta(N,M,k)} Z_{\rm CS}^{(M)}(k) \hat{Z}^{(N,N+M)}(k),}$$

$$\begin{cases} \hat{Z}^{(N,N+M)}(k) = \dfrac{1}{N!}\displaystyle\int_{-\infty}^{\infty} \frac{d^N y}{(4\pi|k|)^N} \prod_{a<b} \tanh^2 \frac{y_a - y_b}{2k} \prod_{a=1}^{N} V(y_a), \\ V(x) = \dfrac{1}{e^{\frac{x}{2}} + (-1)^M e^{-\frac{x}{2}}} \displaystyle\prod_{s=-\frac{M-1}{2}}^{\frac{M-1}{2}} \tanh \frac{x + 2\pi i s}{2|k|}. \end{cases}$$

N-dim. Integral !!

M and k appear only in Z_{CS} and the integrand.

Result

$$t = \frac{M}{k}$$

$$\begin{aligned}
-\log \frac{|Z_{\mathrm{ABJ}}|}{Z_{\mathrm{CS}}(M)} &= \frac{2NM}{\pi t}\mathcal{I}\left(\frac{\pi t}{2}\right) + \frac{N^2}{2}\ln\frac{4M}{\pi t\sin(\pi t)} - \frac{N}{2}\ln\frac{2M^2}{\pi t^2} - \ln G_2(N+1) \\
&\quad - \frac{N(2N^2-1)}{48}\left(\frac{\pi t}{M\sin(\pi t)}\right)[3\cos(2\pi t)+1] \\
&\quad - \frac{N^2}{2304}\left(\frac{\pi t}{M\sin(\pi t)}\right)^2[(17N^2+1)\cos(4\pi t)+4(11N^2-29)\cos(2\pi t)-157N^2+211] \\
&\quad - \frac{N}{552960}\left(\frac{\pi t}{M\sin(\pi t)}\right)^3\Big[(674N^4+250N^2+201)\cos(6\pi t) \\
&\qquad -6(442N^4+690N^2-427)\cos(4\pi t)+3(2282N^4+3490N^2-3635)\cos(2\pi t) \\
&\qquad +4348N^4-21940N^2+12750\Big] \\
&\quad - \frac{N^2}{22118400}\left(\frac{\pi t}{M\sin(\pi t)}\right)^4\Big[(6223N^4+8330N^2+2997)\cos(8\pi t) \\
&\qquad -8(3983N^4+6730N^2-363)\cos(6\pi t)+20(3797N^4+1870N^2+1623)\cos(4\pi t) \\
&\qquad -8(22249N^4-44410N^2+37011)\cos(2\pi t)-56627N^4+113630N^2-18753\Big] \\
&\quad + \mathcal{O}(M^{-5}).
\end{aligned}$$

$$\left(\ \mathcal{I}(x) \equiv -\int_0^x dy\,\log\tan y = \mathrm{Im}[\mathrm{Li}_2(i\tan x)] - x\log\tan x = \mathcal{I}\left(\frac{\pi}{2}-x\right)\ \right)$$

The bulk side: Vasiliev theory

[Hirano-M.H.-Okuyama-Shigemori]

Free energy of Vasiliev theory

$$Z_{\mathrm{HS}} \equiv e^{-F_{\mathrm{HS}}}$$

$$F_{\mathrm{HS}} = \frac{1}{G_{\mathrm{HS}}} F_{\mathrm{HS}}^{(-1)} + F_{\mathrm{HS}}^{(0)} + G_{\mathrm{HS}} F_{\mathrm{HS}}^{(1)} + \cdots .$$

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- Leading order needs action but unknown

Free energy of Vasiliev theory

$$Z_{\rm HS} \equiv e^{-F_{\rm HS}}$$

$$F_{\rm HS} = \frac{1}{G_{\rm HS}} F^{(-1)}_{\rm HS} + F^{(0)}_{\rm HS} + G_{\rm HS} F^{(1)}_{\rm HS} + \cdots \, .$$

- Leading order needs action but unknown
- Sub-leading order needs quadratic fluctuation

⇒ We know this when we know spectrum and parity-violating phase is 0

[cf. Giombi-Klebanov, Tseytlin]

(up to boundary CS term associated w/ gauging)

1-loop free energy of Vasiliev theory

Quadratic fluctuation is described by Fronsdal formulation:

$$\begin{cases} Z \sim \int D\phi \; e^{-S[\phi]}, \qquad S[\phi] \sim \sum_s \phi_{\mu_1\cdots\mu_s} \nabla^2 \phi^{\mu_1\cdots\mu_s} + \cdots \\ \delta\phi_{\mu_1\cdots\mu_s} \sim \nabla_{(\mu_1}\xi_{\mu_2\cdots\mu_s)} \end{cases}$$

By appropriate gauge fixing, contribution from spin-s part is

$$Z_{s,\Delta_\pm} = \begin{cases} \left[\dfrac{\det^{\rm STT}_{s-1,\Delta_\pm}[-\nabla^2 + (s+1)(s-1)]}{\det^{\rm STT}_{s,\Delta_\pm}[-\nabla^2 + (s+1)(s-2) - s]} \right]^{1/2} & \text{for } s \in \mathbb{Z}_{\geq 0} \\ \left[\dfrac{\det^{\rm STT}_{s,\Delta_\pm}[-\nabla\!\!\!/^2 + (s-1/2)^2]}{\det^{\rm STT}_{s-1,\Delta_\pm}[-\nabla\!\!\!/^2 + (s+1/2)^2]} \right]^{1/4} & \text{for } s \in \mathbb{Z}_{\geq 0} + \dfrac{1}{2} \end{cases},$$

1-loop free energy of Vasiliev theory(Cont'd)

First we consider the case that
N^2 spin-1 fields at bdy are gauge fields w/o bdy CS term

(Later I will generalize it)

Spectrum of Vasiliev theory w/ (spin, dimension)

$$= N^2 \left[16(0,1) + 16(0,2) + 31(1,2) + (1,1) + 32 \sum_{s=2}^{\infty} (s, s+1) + 32 \sum_{s=0}^{\infty} \left(s + \frac{1}{2}, s + \frac{3}{2} \right) \right]$$

$$\Rightarrow \quad e^{-F_{\rm HS}^{(0)}} = \left[Z_{0,\Delta_+}^{16} Z_{0,\Delta_-}^{16} Z_{1,\Delta_+}^{31} Z_{1,\Delta_-} \prod_{s=2}^{\infty} Z_{s,\Delta_+}^{32} \prod_{s=0}^{\infty} Z_{s+\frac{1}{2},\Delta_+}^{32} \right]^{N^2},$$

1-loop free energy of Vasiliev theory(Cont'd)

It is convenient to introduce

$$F_{(\Delta,s)} = \begin{cases} \frac{1}{2}\log\det{}_s^{\rm STT}\left[-\nabla^2 + \left(\Delta - \frac{3}{2}\right)^2 - s - \frac{9}{4}\right] & \text{for } s \in \mathbb{Z} \\ \frac{1}{2}\log\det{}_s^{\rm STT}\left[-\not{\nabla}^2 + \left(\Delta - \frac{3}{2}\right)^2\right] & \text{for } s \in \mathbb{Z} + \frac{1}{2} \end{cases}$$

This can be computed by spectral zeta function: [Camporesi-Higuchi]

$$\boxed{F_{(\Delta,s)} = -\frac{1}{2}\zeta'_{(\Delta,s)}(0) - \frac{1}{2}\zeta_{(\Delta,s)}(0)\log\Lambda^2}$$

divergent term, should be canceled

[cf. relation to Weyl anomaly: Tseytlin]

where

$$\begin{cases} \zeta_{(\Delta,s)}(z) = \frac{8(2s+1)}{3\pi}\int_0^\infty du \frac{\mu_s(u)}{[u^2 + (\Delta - 3/2)^2]^z}\ , \qquad \zeta'_{(\Delta,s)}(z) = \frac{\partial}{\partial z}\zeta_{(\Delta,s)}(z)\ , \\ \mu_s(u) = \frac{\pi u}{16}\left[u^2 + \left(s + \frac{1}{2}\right)^2\right]\tanh\left(\pi(u + is)\right)\ . \end{cases}$$

Cancellation of log divergence

Bosonic part:

$$F^{(0)}_{\mathrm{HS,B}}\Big|_{\log\ \mathrm{div}} = 32N^2\left[\frac{1}{360}+\lim_{\alpha\to 0}\sum_{s\in\mathbf{Z}_{\geq 0}} s^{-\alpha}\left(\frac{1}{180}-\frac{s^2}{24}+\frac{5s^4}{24}\right)\right]\log\Lambda^2 = 0$$

Fermionic part:

$$F^{(0)}_{\mathrm{HS,F}}\Big|_{\log\ \mathrm{div}} = 32N^2\left[\frac{11}{360}+\lim_{\alpha\to 0}\sum_{s\in\mathbf{Z}_{\geq 0}+1/2} s^{-\alpha}\left(\frac{13}{2880}+\frac{5s^2}{24}-\frac{5s^4}{12}\right)\right]\log\Lambda^2 = 0$$

Nontrivial cancellation among conformal anomalies

Finite part

Almost contributions are finite numerical values
but only spin-1 w/ Δ- b.c. is involved.

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Taking $\Delta_- = 1 + \epsilon,$

$$\log Z_{1,\Delta_-} = \frac{1}{2}\log\epsilon + \mathcal{O}(1)$$

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But we should have 1/M correction: $\Delta_- = 1 + \frac{\sharp}{M} + \mathcal{O}(M^{-2})$

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But we should have 1/M correction: $\Delta_- = 1 + \frac{\sharp}{M} + \mathcal{O}(M^{-2})$

$$\boxed{\log Z_{1,\Delta_-} = -\frac{1}{2}\log M + \mathcal{O}(1)}$$

Thus, the whole 1-loop free energy is

$$\boxed{F_{\mathrm{HS}}^{(0)} = +\frac{N^2}{2}\log M + \mathcal{O}(1)}$$

General CS level?

On CFT side, gauging global sym. is done by double trace deformation:

$$S \;\to\; S + \frac{\lambda_0}{2}\int d^3x\;J^2,$$

(J: conserved current)

[cf. Witten, Leigh-Petkou]

and put it to UV fixed point.

Giombi etal. studied this effect for S^3 free energy for general theory both on CFT and gravity sides

[Giombi-Klebanov-Pufu-Safdi-Tarnopolsky]

For Abelian spin-1 symmetry,

$$\boxed{F_{\rm UV} - F_{\rm IR} = \frac{1}{2}\log M + \mathcal{O}(1)}$$

General CS level? (Cont'd)

$$S \ \rightarrow \ S + \frac{\lambda_0}{2} \int d^3x \ J^2, \qquad F_{\mathrm{UV}} - F_{\mathrm{IR}} = \frac{1}{2} \log M + \mathcal{O}(1)$$

[Giombi-Klebanov-Pufu-Safdi-Tarnopolsky]

We should be careful to apply their result to our case.

General CS level? (Cont'd)

$$S \;\to\; S + \frac{\lambda_0}{2}\int d^3x\ J^2, \qquad F_{\rm UV} - F_{\rm IR} = \frac{1}{2}\log M + \mathcal{O}(1)$$

[Giombi-Klebanov-Pufu-Safdi-Tarnopolsky]

We should be careful to apply their result to our case.

① Their result is on "difference".

—— OK, because we can explicitly show $F^{(0)}_{\rm IR} = F^{(0)}_{\rm SQCD} = \mathcal{O}(1)$

② The deformation is not manifestly supersymmetric.

—— expect difference from "supersymmetrization" is O(1)

Thus, we still expect

$$\boxed{F^{(0)}_{\rm HS} = +\frac{N^2}{2}\log M + \mathcal{O}(1)}$$

Comparison

[Hirano-M.H.-Okuyama-Shigemori]

Identification of "Higher Spin sector" in ABJ

[Hirano-M.H.-Okuyama-Shigemori]

Conditions:

1. Leading order is O(M)

2. "Higher Spin sector" enjoys Seiberg-like duality: [cf. Aharony-Bergman-Jafferis, Giveon-Kutasov, Kapustin-Willett-Yaakov,Willett-Yaakov]

$$U(N)_k \times U(N+M)_{-k} \longleftrightarrow U(N+|k|-M)_k \times U(N)_{-k}$$

3. Matching with the Vasiliev 1-loop free energy

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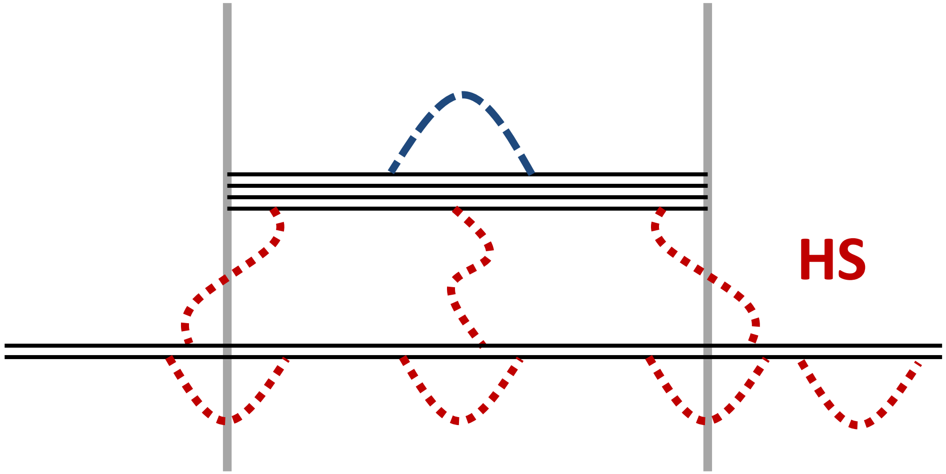
3. Matching with the Vasiliev 1-loop free energy

Our proposal:

$$\boxed{Z_{\mathrm{HS}} = \frac{|Z_{\mathrm{ABJ}}|}{Z_{U(M)\mathrm{CS}}}}$$



U(M) CS versus U(N+M) CS ?

1. Leading order is O(M)
2. Subsector enjoys Seiberg-like duality
3. Matching with the Vasiliev 1-loop free energy

In order to satisfy the condition 1, it is natural to subtract

free energy of U(M+c) pure CS theory with finite c.

(This also satisfies condition 3.)

At first sight, the previous studies might imply c=N
but this is not Seiberg duality invariant.

⇩

Only c=0 is Seiberg duality invariant.

Identification of bulk coupling constant

(Newton constant in the Vasiliev theory)

[M.H.]

Identification of bulk coupling constant

[M.H.]

Most standard way is to compute EM tensor correlator:

$$\boxed{\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = \frac{c_T}{64}(P_{\mu\rho}P_{\nu\sigma} + P_{\nu\rho}P_{\mu\sigma} - P_{\mu\nu}P_{\rho\sigma})\frac{1}{16\pi^2 x^2},}$$

$$\left(P_{\mu\nu} = \delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu,\ x \neq 0,\ c_T|_{\text{free scalar}} = 1\right)$$

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In 3d N=2 SUSY theory with R-symmetry,
there are two ways to compute c_T by localization:

1. To compute squashed S^3 partition function by localization
and consider its derivative with respect to squashing parameter

[Closset-Dumitrescu-Festuccia-Komargodski]

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2-pt. function of flavor symmetry current

Superpotential of ABJ:

bi-fundamental

$$W \sim \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\mathrm{Tr}[A_\alpha B_\gamma A_\beta B_\delta],$$

anti-bi-fundamental

Flavor symmetry current correlator:

$$\boxed{\langle j_a^\mu(x) j_b^\nu(0)\rangle = \frac{\tau_f}{16\pi^2}(\delta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)\frac{1}{x^2},}$$

	A_1	A_2	B_1	B_2
$U(1)_f$	+1	-1	+1	-1

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In ABJ theory, τ_f is related to c_T by

$$\boxed{c_T = 4\tau_f.}$$

[cf. Chester-Lee-Pufu-Yacoby]

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τ_f is generated by partition function of mass deformed ABJ:

$$\boxed{\tau_f = -8\,\mathrm{Re}\frac{1}{Z(0)}\frac{\partial^2 Z(m)}{\partial m^2}\bigg|_{m=0}.}$$

Identification of bulk coupling constant

[M.H.]

c_T is generated by partition function of mass deformed ABJ:

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$$c_T = -32\text{Re}\frac{1}{Z(0)}\frac{\partial^2 Z(m)}{\partial m^2}\bigg|_{m=0}$$

Localization

$$Z(m) = \frac{1}{N_1!N_2!}\int \frac{d^{N_1}\mu}{(2\pi)^{N_1}}\frac{d^{N_2}\nu}{(2\pi)^{N_2}} e^{\frac{ik}{4\pi}\sum_{j=1}^{N_1}\mu_j^2 - \frac{ik}{4\pi}\sum_{a=1}^{N_2}\nu_a^2} \frac{\prod_{1\le i\neq j\le N_1} 2\sinh\frac{\mu_i-\mu_j}{2}\prod_{1\le a\neq b\le N_2} 2\sinh\frac{\nu_a-\nu_b}{2}}{\prod_{j=1}^{N_1}\prod_{a=1}^{N_2} 2\cosh\frac{\mu_j-\nu_a+m}{2}\cdot 2\cosh\frac{\mu_j-\nu_a-m}{2}}$$

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matrix model technique [cf. Drukker-Marino-Putrov]

$$c_T = \frac{16NM\sin(\pi t)}{\pi t} + \mathcal{O}(1)$$

$$G_{\mathrm{HS}} = \frac{2}{\pi M}\frac{\pi t}{\sin(\pi t)}$$

(in canonical normalizations)

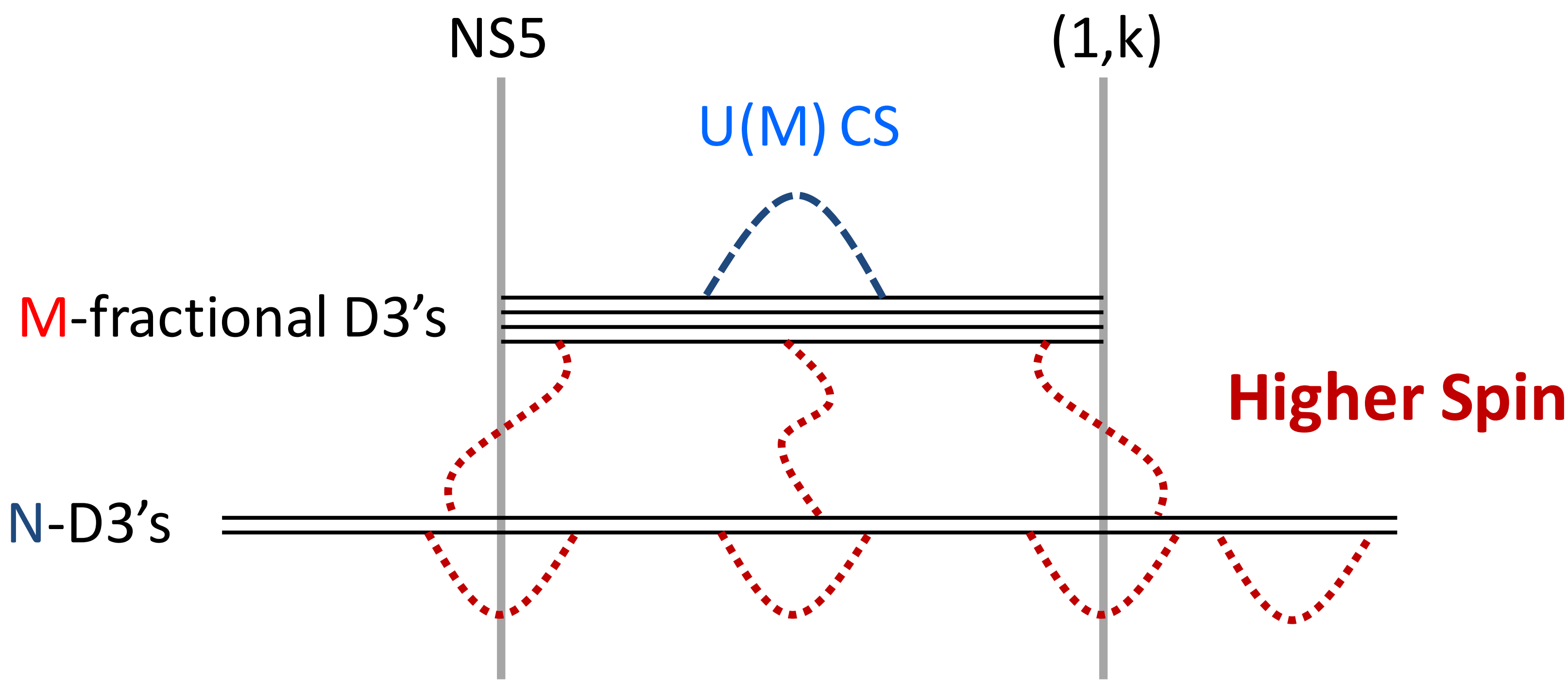
Remark: This is similar to the result in theories w/ fundamentals:

[Aharony-Gur-Ari-Yacoby,Gur-Ari-Yacoby]

$$c_{T,\mathrm{fund.}} = \frac{2M\sin(\pi t)}{\pi t}$$

Summary & Outlook

Our proposal



$$\boxed{F_{\mathrm{HS}} = F_{\mathrm{ABJ}} - F_{\mathrm{CS}}}$$

[Hirano-M.H.-Okuyama-Shigemori]

$$\boxed{G_{\mathrm{HS}} = \frac{2}{\pi M}\frac{\pi t}{\sin(\pi t)}}$$

[M.H.]

Q1. Relation to topological string?

Recently, it has turned out that

the ABJ free energy is described by the refined topological string.

[M.H.-Okuyama, Matsumoto-Moriyama, Hatsuda-Marino-Moriyama-Okuyama]

The topological string has the conifold expansion:

$$\lambda_1 \ll 1, \quad \lambda_2 = \text{finite},$$

which is similar to the Higher spin limit.

This relation might give some useful insights.

Q2. Other quantities?

We know SUSY localization formula also for

Superconformal index, Wilson loop, Vortex loop,
and Renyi entropy, etc…

It is also interesting to compare these in the higher spin limit.

But corresponding observables in the Vasiliev are nontrivial…

Thanks!

Appendix

Localization of 3d N=2 theory on sphere

$\mathcal{N}=2$ Vectormultiplet $\supset \left(A_\mu, \sigma, D, \lambda, \bar{\lambda}\right)$, $\quad \mathcal{N}=2$ Chiralmultiplet $\supset \left(\phi, \bar{\phi}, F, \bar{F}, \psi, \bar{\psi}\right)$

If we choose the deformation term as $QV = S_{\text{YM}} + S_\psi$,

$$S_{\text{YM}} = Q\left[\frac{1}{4g_{\text{YM}}^2}\int d^3x\sqrt{g}\,\text{Tr}\left((Q\lambda)^\dagger\lambda + (Q\bar{\lambda})^\dagger\bar{\lambda}\right)\right], \quad S_\psi = Q\left[\frac{1}{2}\int d^3x\sqrt{g}\,\text{Tr}\left((Q\psi)^\dagger\psi + (Q\bar{\psi})^\dagger\bar{\psi}\right)\right]$$

Saddle point:

(up to gauge trans.)

$$\sigma = \text{const.}, \quad D = -\frac{\sigma}{R}, \quad \text{Others} = 0$$

Coulomb branch

Result:

$$Z = \int_{-\infty}^{\infty} d^{\text{rank}G}x \; Z_{\text{vec}} Z_{\text{chi}}$$

$$Z_{\text{vec}} = \prod_{\alpha \in \text{positive root}} 2\sinh\pi(\alpha\cdot x)\cdot 2\sinh\pi(\alpha\cdot x)$$

$$Z_{\text{chi}} = \prod_{\omega\in\mathbf{R}} s_1\left(i(1-\Delta) - \omega(\hat{\sigma})\right)$$

$$s_b(z) = \prod_{m,n=0}^{\infty} \frac{bm + b^{-1}n + (b+b^{-1})/2 - iz}{bm + b^{-1}n + (b+b^{-1})/2 + iz}$$

Physical origin of the simplification

For (k,M)=(1,0)

$$Z = \frac{1}{2^N N!}\int \frac{d^N x}{(2\pi)^N} \prod_{i<j}\left[\frac{\sinh^2\left(\frac{x_i-x_j}{2}\right)}{\cosh^2\left(\frac{x_i-x_j}{2}\right)}\right]\frac{1}{\prod_i 2\cosh\frac{x_i}{2}}$$

[Kapustin-Willett-Yaakov’10]
[cf. Intrilligator-Seiberg’96
Hanany-Witten 97’]

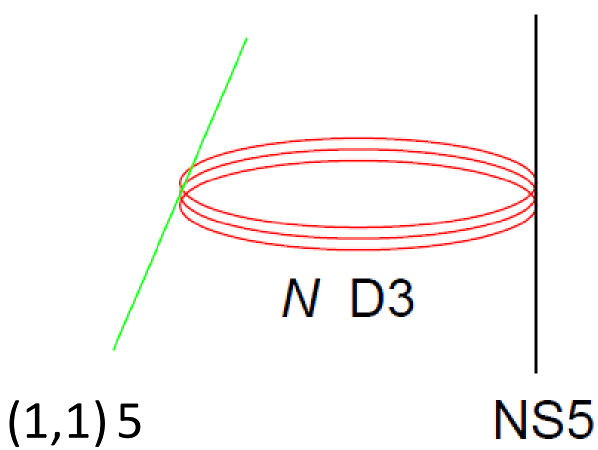
$$\left(\begin{array}{ll} \mathcal{N}=4 \text{ vector multiplet} & + \ \mathcal{N}=4 \text{ hyper multiplet (fundamental)} \\ & + \ \mathcal{N}=4 \text{ hyper multiplet (adjoint)} \end{array}\right)$$

Coulomb branch
↔ Higgs branch

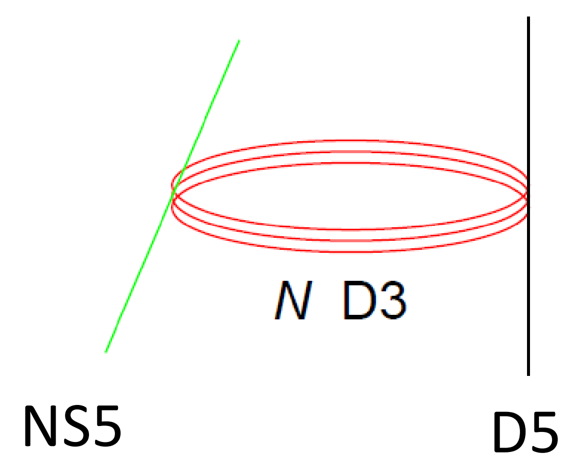
k=1 ABJM ⟺ $\mathcal{N}=4$ theory

AdS/CFT

[ABJM ’08]



SL(2,Z) trans.



A remark on Seiberg-like duality

[Aharony-Bergman-Jafferis '08, Kapustin-Willett-Yaakov, Willett-Yaakov, Awata-Hirano-Shigemori '12]

The ABJ partition function transforms properly under the Seiberg-like duality between the different gauge groups:

$$\boxed{U(N)_k \times U(N+M)_{-k} \quad \text{and} \quad U(N+|k|-M)_k \times U(N)_{-k}.}$$

We can check this duality by using

$$\begin{cases} Z_{\mathrm{CS}}^{(M)}(k) = Z_{\mathrm{CS}}^{(|k|-M)}(-k). \\ \dfrac{1}{e^{\frac{y}{2}} + (-1)^{|k|-M} e^{-\frac{y}{2}}} \displaystyle\prod_{s=-\frac{|k|-M-1}{2}}^{\frac{|k|-M-1}{2}} \tanh \frac{y+2\pi i s}{2|k|} = \dfrac{1}{e^{\frac{y}{2}} + (-1)^{M} e^{-\frac{y}{2}}} \displaystyle\prod_{s=-\frac{M-1}{2}}^{\frac{M-1}{2}} \tanh \frac{y+2\pi i s}{2|k|}. \end{cases}$$

$$\Rightarrow \quad \left| Z_{\mathrm{ABJ}}^{(N,N+M)}(k) \right| = \left| Z_{\mathrm{ABJ}}^{(N,N+|k|-M)}(-k) \right|$$