Application of Quantum Computation to High Energy Physics

– Some advanced topics –

Masazumi Honda

(本多正純)

19th Apr., 2023

Intensive lectures @RESCEU（day 3）

Plan of the intensive lectures

Day 1

- Lecture 1: introduction, basics of quantum computation
- Lecture 2: quantum simulation of spin system
- Hands-on 1: Basics on IBM’s qiskit

Day 2

- Lecture 3: quantum field theory (QFT)
- Lecture 4: QFT on quantum computer
- Hands-on 2: Time evolution of spin system

Day 3

- Lecture 5: quantum error correction
- Lecture 6: some advanced topics, future prospects
- Hands-on 3: Constructing ground state of spin system

Plan

1. Screening, confinement & negative tension in higher charge Schwinger model

[MH-Itou-Kikuchi-Nagano-Okuda '21]
[MH-Itou-Kikuchi-Tanizaki '21]

2. String/M-theory on quantum computer

3. Summary & Future prospect

Let's consider charge-q Schwinger model:

$$L = \frac{1}{2g^2}F_{01}^2 + \frac{\theta_0}{2\pi}F_{01} + \overline{\psi}\,\mathrm{i}\,\gamma^\mu(\partial_\mu + \mathrm{i}qA_\mu)\psi - m\,\overline{\psi}\psi$$

Field content:

- $U(1)$ gauge field
- charge-q Dirac fermion

Let's explore

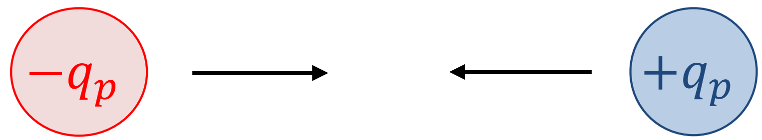
screening vs confinement problem

(next slide)

Screening versus Confinement

Let's consider

potential between 2 heavy charged particles



Classical picture:

$$V(x) = \frac{q_p^2\, g^2}{2} x \ ?$$

Coulomb law in 1+1d
||
confinement

too naive in the presence of dynamical fermions

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama ’88, Gross-Klebanov-Matytsin-Smilga ’95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right)$$

screening

- massive case:

Expectations from previous analyzes

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- massless case:

$$\mu \equiv g/\sqrt{\pi}$$

$$V(x) = \frac{q_p^2 \, g^2}{2\mu}\left(1 - e^{-q\mu x}\right) \qquad \textit{screening}$$

- massive case: [cf. Misumi-Tanizaki-Unsal '19]

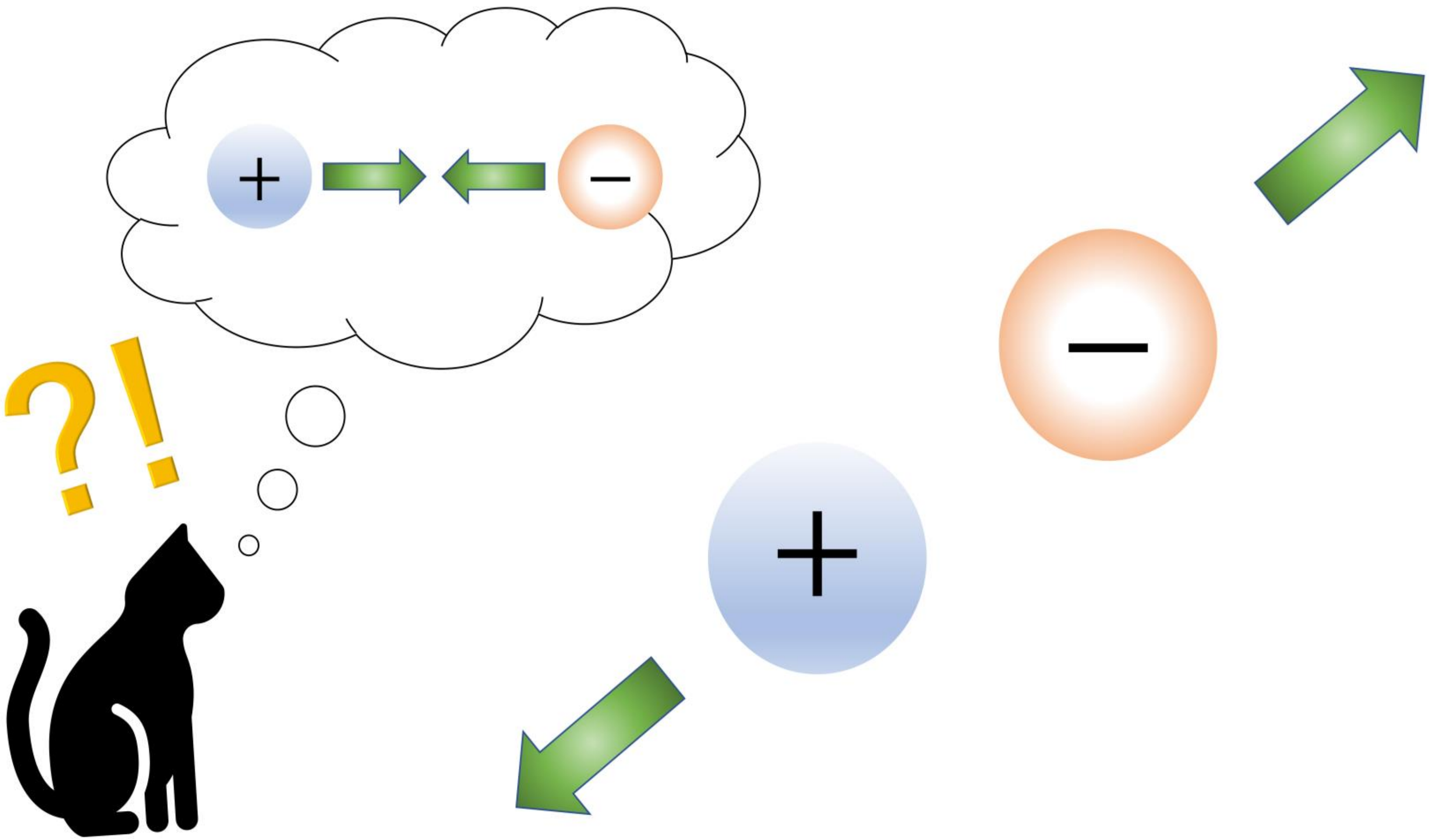
$$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right) x \qquad (\mathrm{m} \ll g,\ |x| \gg 1/g\ )$$

$$\begin{cases} = \text{Const.} & \text{for } \mathrm{q_p/q} = \mathbf{Z} \quad \textit{screening} \\ \propto x & \text{for } \mathrm{q_p/q} \neq \mathbf{Z} \quad \textit{confinement?} \end{cases}$$

*but sometimes **negative** slope!*

That is, as changing the parameters…



Let’s explore this aspect by quantum simulation!

UT me! MARKET

UTme!トップ | 新規登録/ログイン | よくある質問 | カート

おすすめ | ランキング | スペシャル | 新着 | 検索

DETAIL アイテムの詳細

DESIGNED BY
masazumi318

アイテム

¥1,990

- ベーシックTシャツ
- ベーシックTシャツ（2021年モデル）
- Uクルーネック T(半袖)
- ベーシック T シャツ（長袖）
- Uクルーネック T （ﾊﾞｯｸﾌﾟﾘﾝﾄ、長袖）
- KIDS カラークルーネックT(半袖)
- KIDS Uクルーネック T （ﾊﾞｯｸﾌﾟﾘﾝﾄ、長袖）
- トートバッグ

Charge-q Schwinger model

Continuum:

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i}\, q\, A_\mu)\psi - m\, \overline{\psi}\psi$$

Taking temporal gauge $A_0 = 0$, (Π: conjugate momentum of A_1)

$$H(x) = \frac{g^2}{2}\left(\Pi - \frac{\theta_0}{2\pi}\right)^2 - \bar{\psi}\, \mathrm{i}\, \gamma^1 (\partial_1 + \mathrm{i}\, q A_1)\psi + m\bar{\psi}\psi,$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1 \Pi - qg\bar{\psi}\gamma^0\psi$$

Symmetries in charge-q Schwinger model

$$L=\frac{1}{2g^2}F_{01}^2+\frac{\theta_0}{2\pi}F_{01}+\overline{\psi}\,\mathrm{i}\,\gamma^\mu(\partial_\mu+\mathrm{i}\,q\,A_\mu)\psi-m\,\overline{\psi}\psi$$

- $\boldsymbol{Z_q}$ chiral symmetry for $m=0$

  —— ABJ anomaly: $U(1)_A\to\boldsymbol{Z_q}$

  —— known to be spontaneously broken

- $\boldsymbol{Z_q}$ 1-form symmetry

  —— remnant of $U(1)$ 1-form sym. in pure Maxwell

  —— Hilbert sp. is decomposed into q-superselect. sectors

  (cf. common for $(d-1)$-form sym. in d dimensions) *"universe"*

Put the theory on lattice

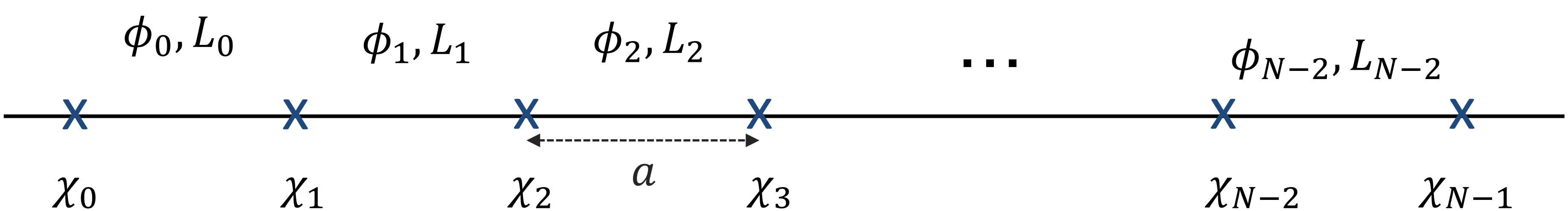
▪Fermion (on site):

"Staggered fermion" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{\underbrace{a}_{\text{lattice spacing}}{}^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \begin{array}{l} \rightarrow \text{odd site} \\ \rightarrow \text{even site} \end{array}$$

▪Gauge field (on link):

$$\phi_n \leftrightarrow -agA^1(x), \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$



Lattice theory w/ staggered fermion

Hamiltonian:

$$H = J\sum_{n=0}^{N-2}\left(L_n + \frac{\theta_0}{2\pi}\right)^2 - \mathrm{i}w\sum_{n=0}^{N-2}\left[\chi_n^\dagger (U_n)^q \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=0}^{N-1}(-1)^n \chi_n^\dagger \chi_n$$

$$\left(w = \frac{1}{2a}, J = \frac{g^2 a}{2}\right)$$

Commutation relation:

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

Gauss law:

$$L_n - L_{n-1} = q\left[\chi_n^\dagger \chi_n - \frac{1-(-1)^n}{2}\right]$$

Eliminate gauge d.o.f.

1. Take open b.c. & solve Gauss law:

$$L_n = L_{-1} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right) \qquad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

Then,

$$H = -\mathrm{i}w\sum_{n=1}^{N-1}\left[\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=1}^{N}(-1)^n \chi_n^\dagger \chi_n$$
$$+ J\sum_{n=1}^{N}\left[\frac{\theta_0}{2\pi} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right)\right]^2.$$

This acts on finite dimensional Hilbert space

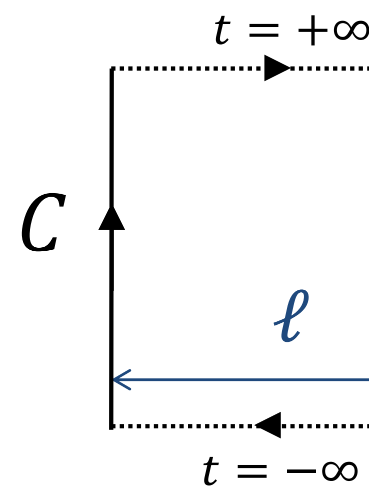
Insertion of the probe charges

① Introduce the probe charges $\pm q_p$:

$$e^{iq_p \int_C A}$$

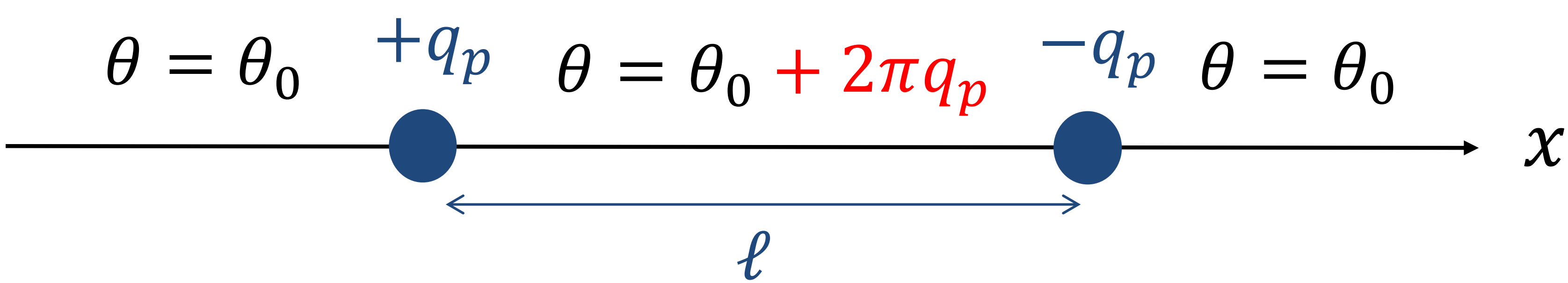
$$||$$

$$e^{iq_p \int_{S,\partial S=C} F}$$



local θ-term w/ $\theta = 2\pi q_p$!!

② Include it to the action & switch to Hamilton formalism



③ Compute the ground state energy (in the presence of the probes)

Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \ \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator:

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

Now the system is purely a spin system:

$$H = -\mathrm{i}w\sum_{n=1}^{N-1}\left[\chi_n^\dagger\chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n + J\sum_{n=1}^{N}\left[\frac{\vartheta_n}{2\pi} + q\sum_{j=1}^{n}\left(\chi_j^\dagger\chi_j - \frac{1-(-1)^j}{2}\right)\right]^2$$

$$\Downarrow$$

$$H = J\sum_{n=0}^{N-2}\left[q\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi}\right]^2 + \frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1} + Y_nY_{n+1}\right] + \frac{m}{2}\sum_{n=0}^{N-1}(-1)^n Z_n$$

Qubit description of the Schwinger model !!

Expectations from previous analyzes (repeated)

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right) \qquad \textit{screening}$$

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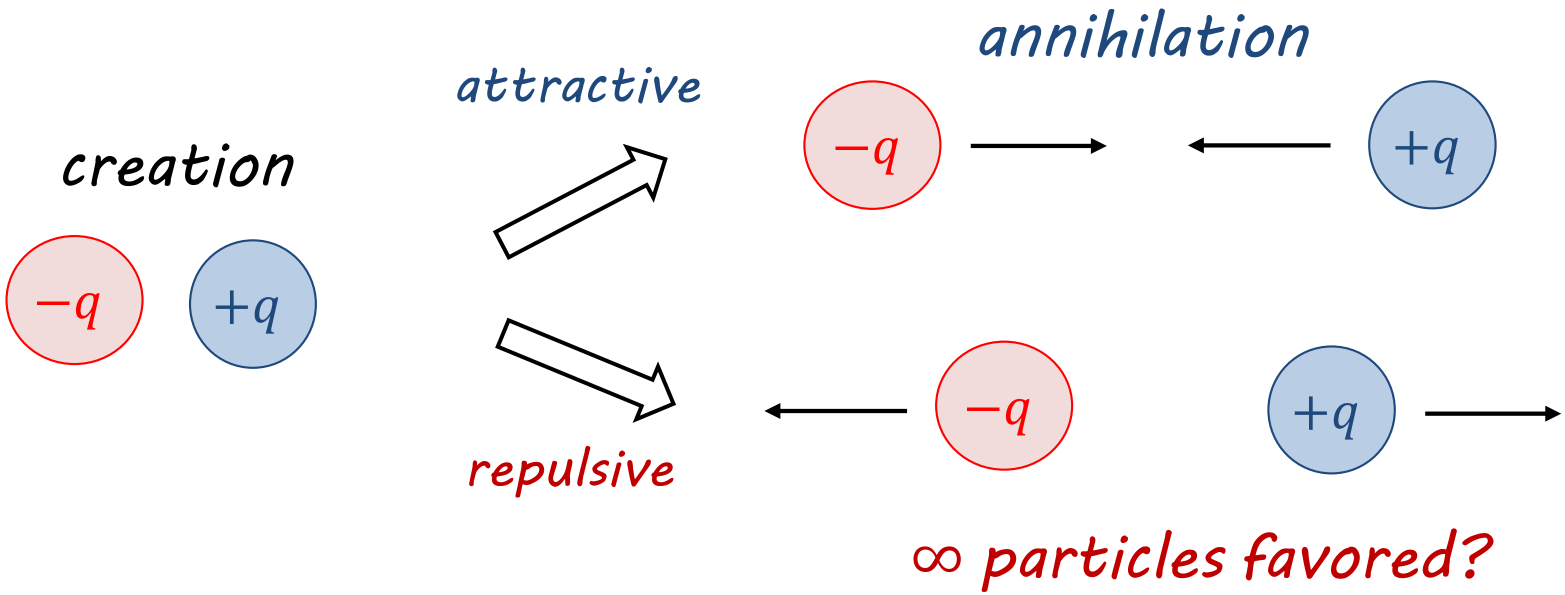
but sometimes negative slope!

Let's explore this aspect by quantum simulation!

FAQs on negative tension behavior

Q1. It sounds that many pair creations are favored.
Is the theory unstable?

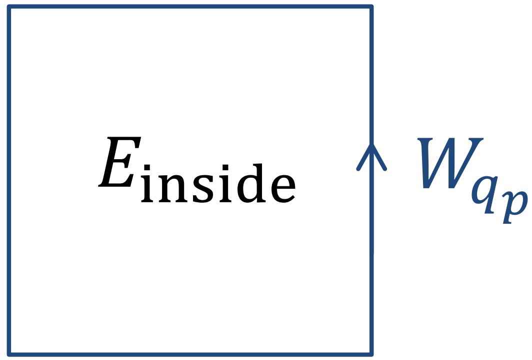


— **No.** Negative tension appears only for $q_p \neq q\boldsymbol{Z}$.

So, such unstable pair creations do not occur.

FAQs on negative tension behavior (cont'd)

[cf. MH-Itou-Kikuchi-Tanizaki '21]



$E_{\text{outside}}\ (= E_0?\,)$

Q2. It sounds $E_{\text{inside}} < E_{\text{outside}}$. Strange?

—— Inside & outside are in different superselect. sectors decomposed by Z_q 1-form sym.

$$\mathcal{H} = \bigoplus_{\ell=0}^{q-1} \mathcal{H}_\ell \qquad \text{"universe"}$$

E_{inside} & E_{outside} are lowest in each universe:

$$E_{\text{inside}} = \min_{\mathcal{H}_{\ell+q_p}}(E)\,, \qquad E_{\text{outside}} = \min_{\mathcal{H}_\ell}(E)$$

Adiabatic state preparation of vacuum (repeated)

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2: Introduce adiabatic Hamiltonian $H_A(t)$ s.t.

$$\begin{cases} \cdot\ H_A(0) = H_0,\ H_A(T) = H_{\mathrm{target}} \\ \cdot\ \left|\frac{dH_A}{dt}\right| \ll 1 \ \text{ for } \ T \gg 1 \end{cases}$$

Step 3: Use the adiabatic theorem

If $H_A(t)$ has a unique ground state w/ a finite gap for $\forall t$, then the ground state of H_{target} is obtained by

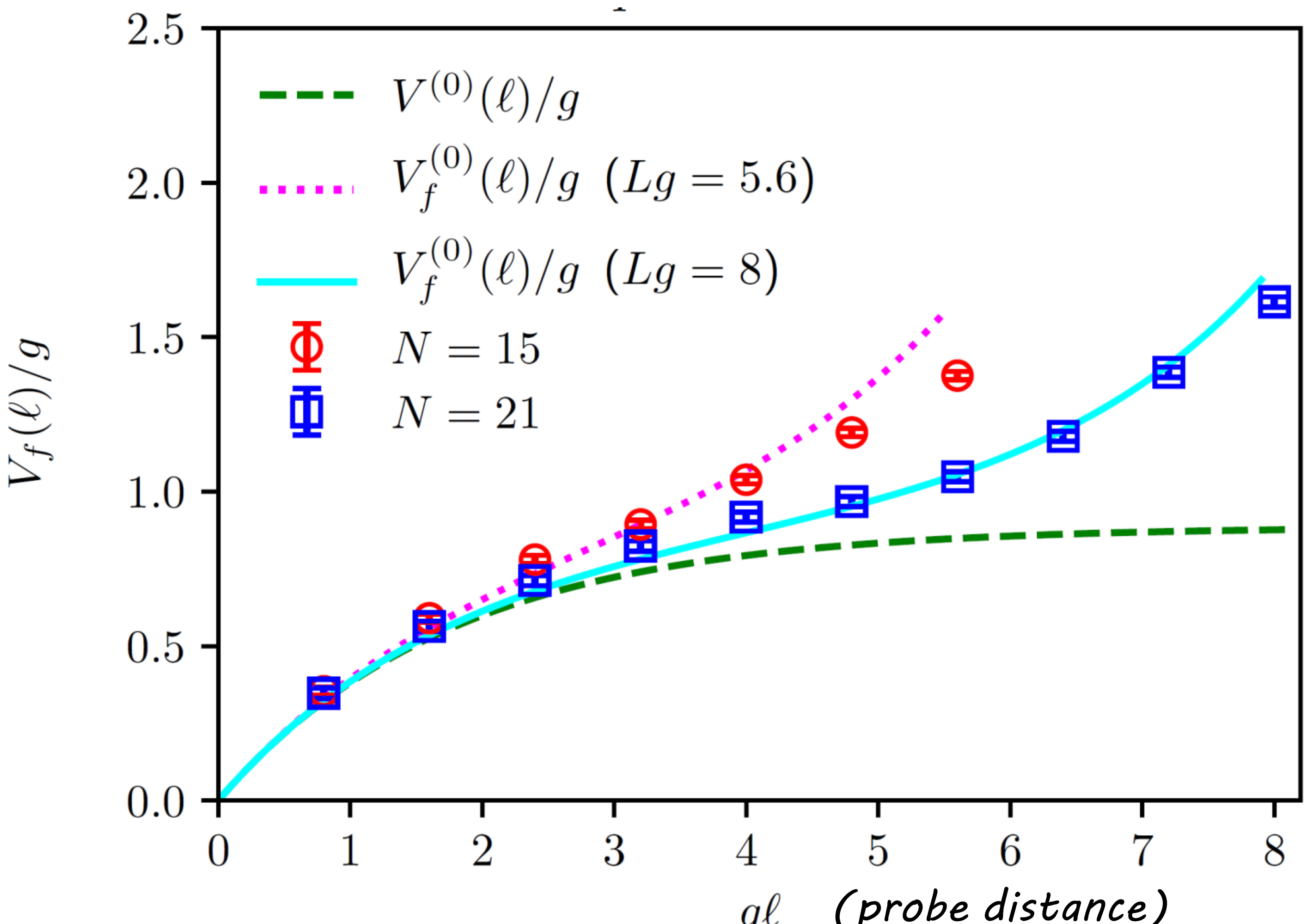
$$\boxed{|\mathrm{vac}\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\, H_A(t)\right) |\mathrm{vac}_0\rangle}$$

Results for massless, $\theta_0 = 0$ & $q_p/q \in \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q_p/q = 1, m = 0$

Lines: analytical results in the continuum limit (finite & ∞ vols.)



Results for massive, $\theta_0 = 0$ & $q_p/q \in \boldsymbol{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda ’21]

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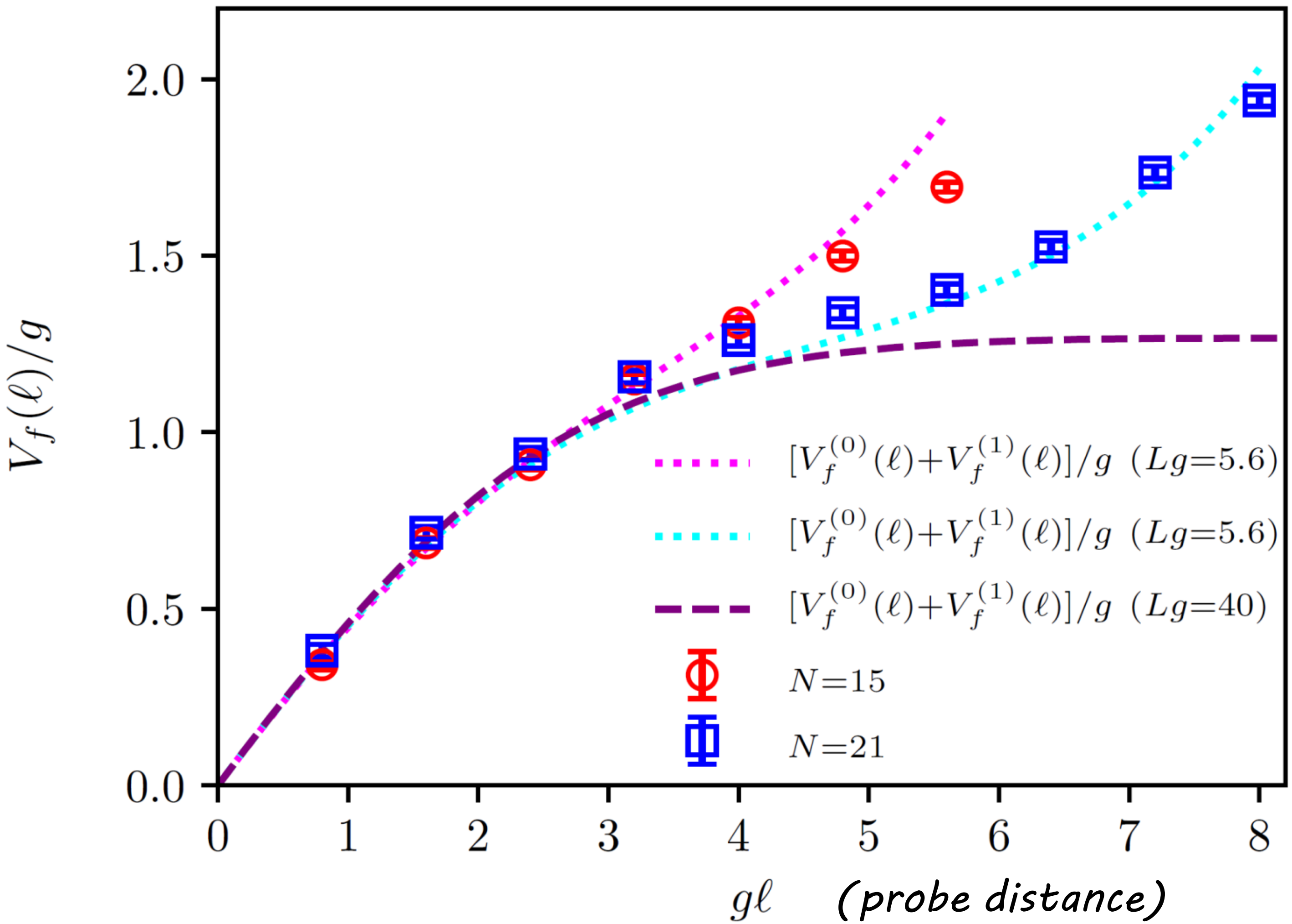

Massless vs massive for $\theta_0 = 0$ & $q_p/q \in \mathbb{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q_p/q = 1$

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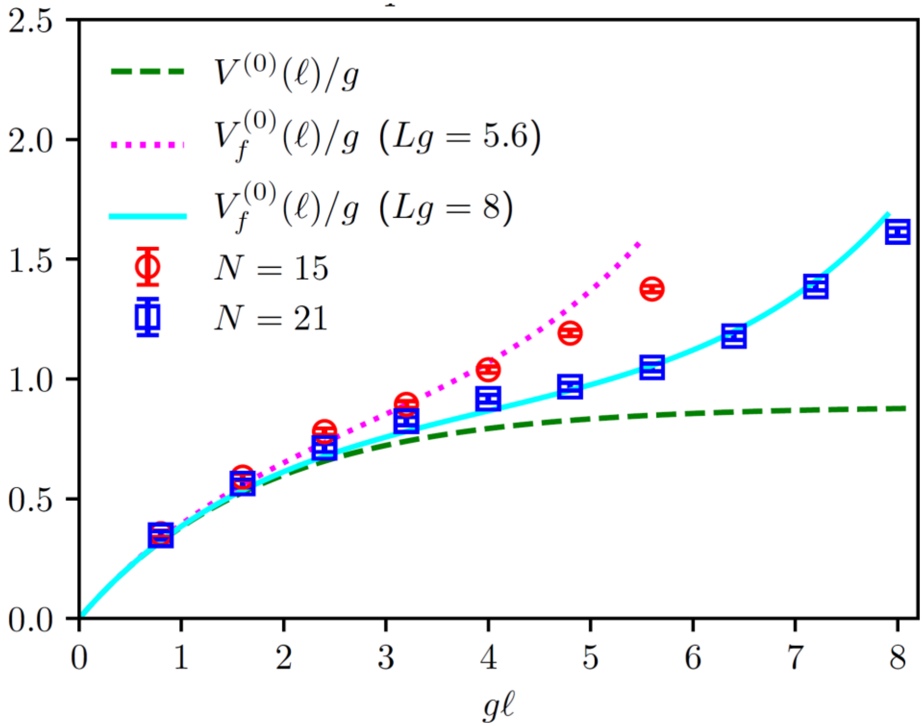
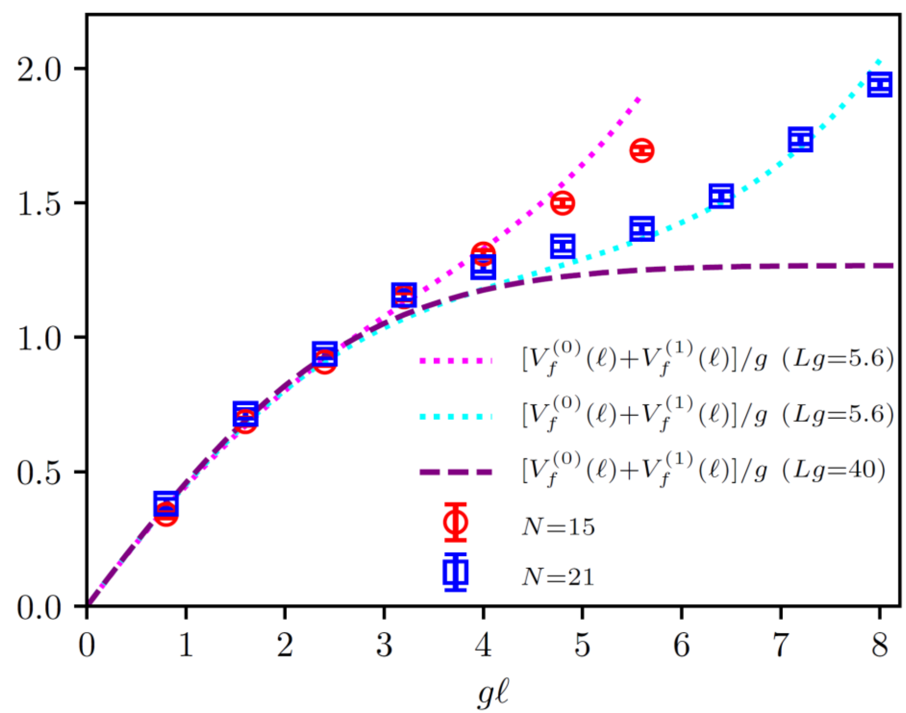
$q_p = 1, m = 0$



$q_p = 1, m/g = 0.2$



Consistent w/ expected screening behavior

Results for $\theta_0 = 0$ & $q_p/q \notin \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

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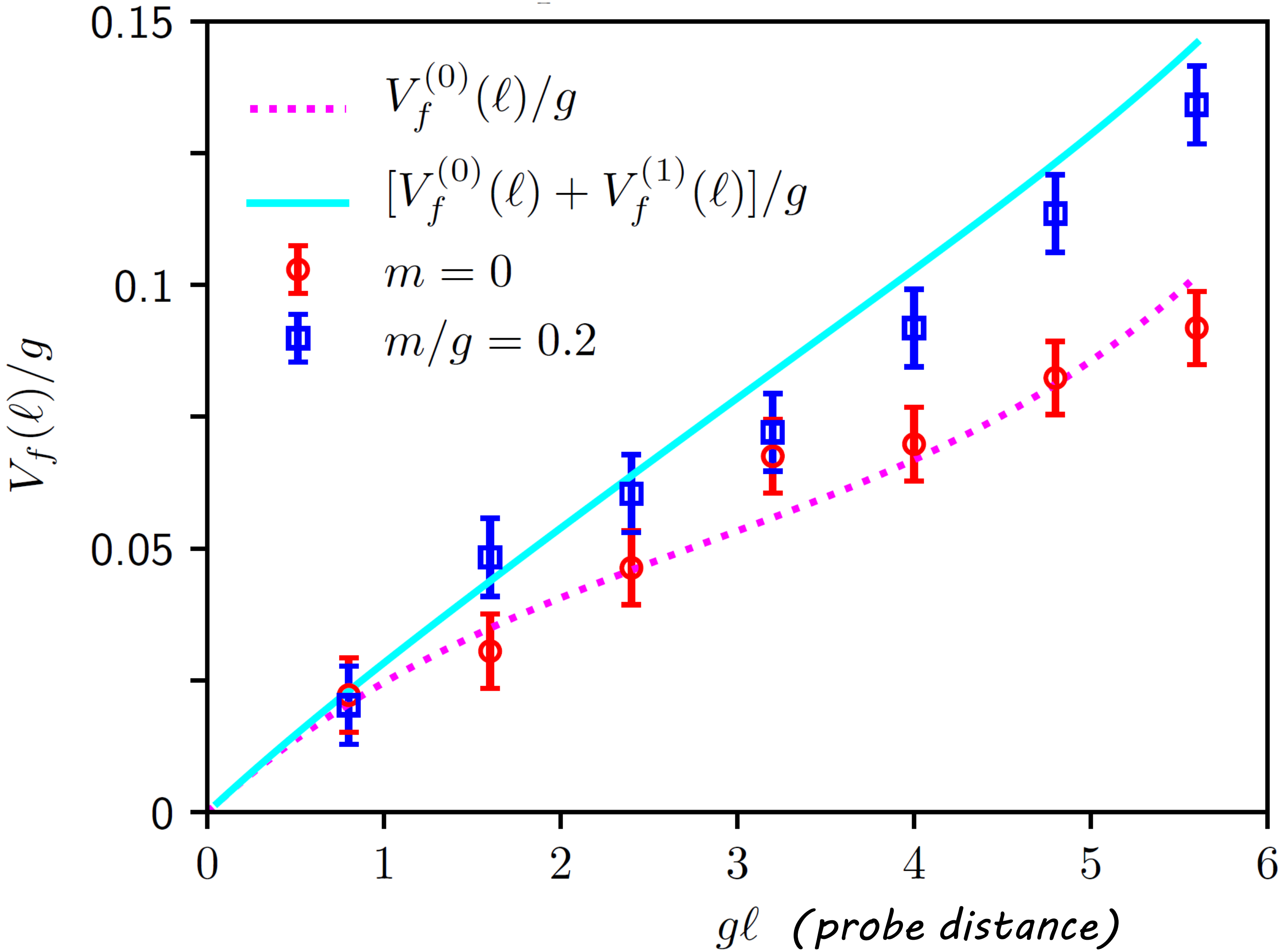

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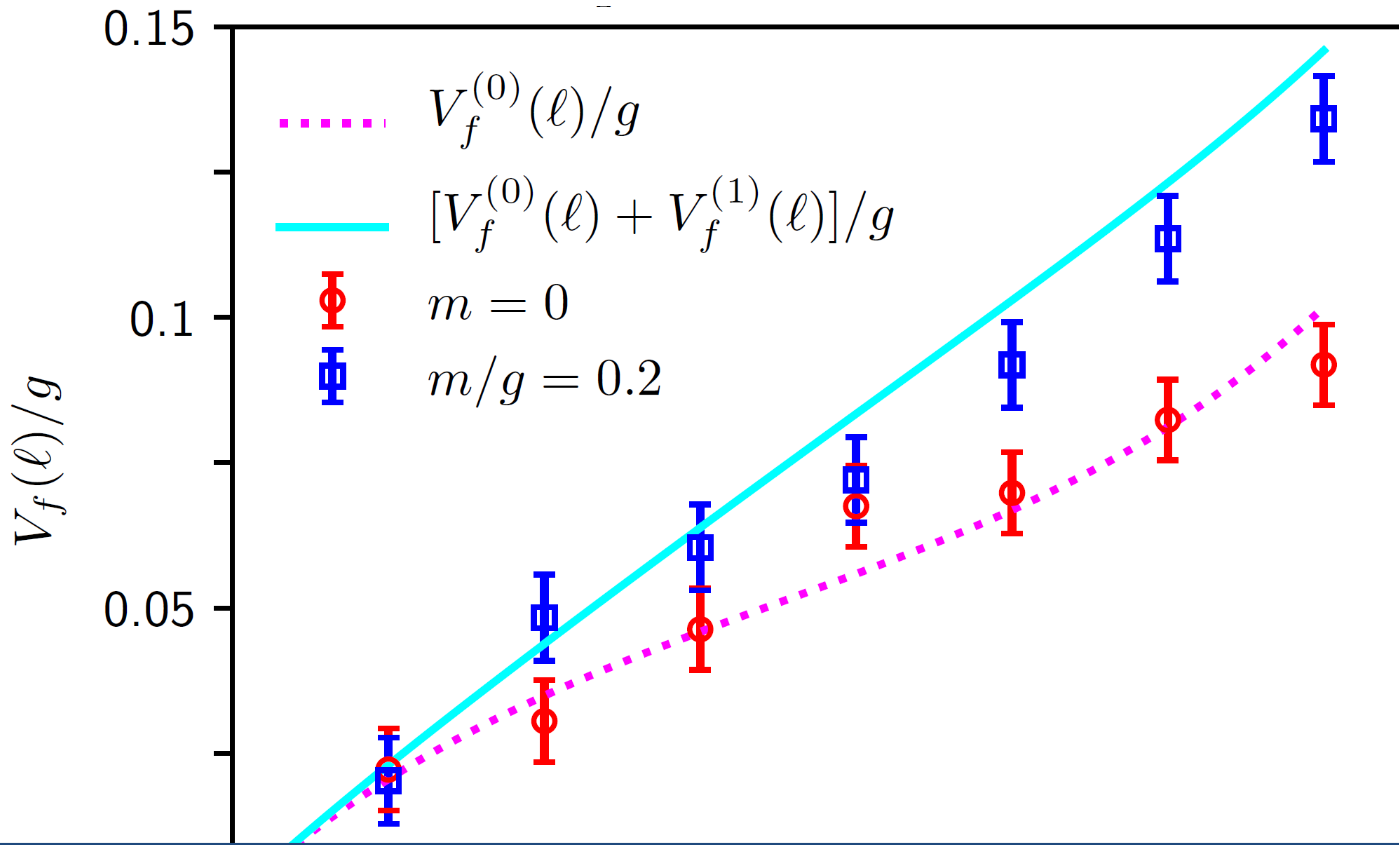


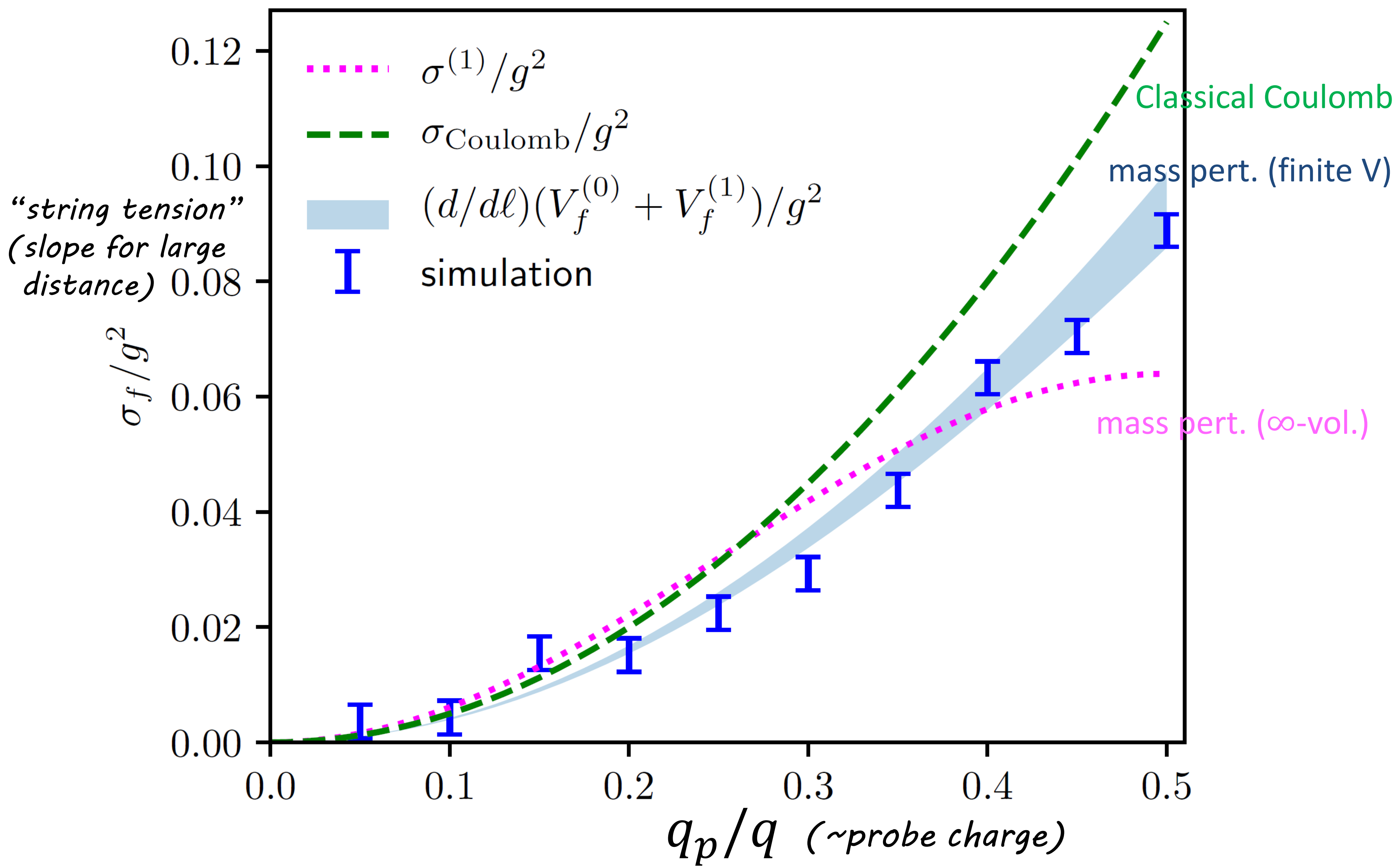
Consistent w/ expected confinement behavior

-> interesting to estimate string tension for various q_p?

“String tension” for $\theta_0 = 0$

[MH-Itou-Kikuchi-Nagano-Okuda ’21]

Parameters: $g = 1, a = 0.4, N = 15, T = 99, m/g = 0.2$



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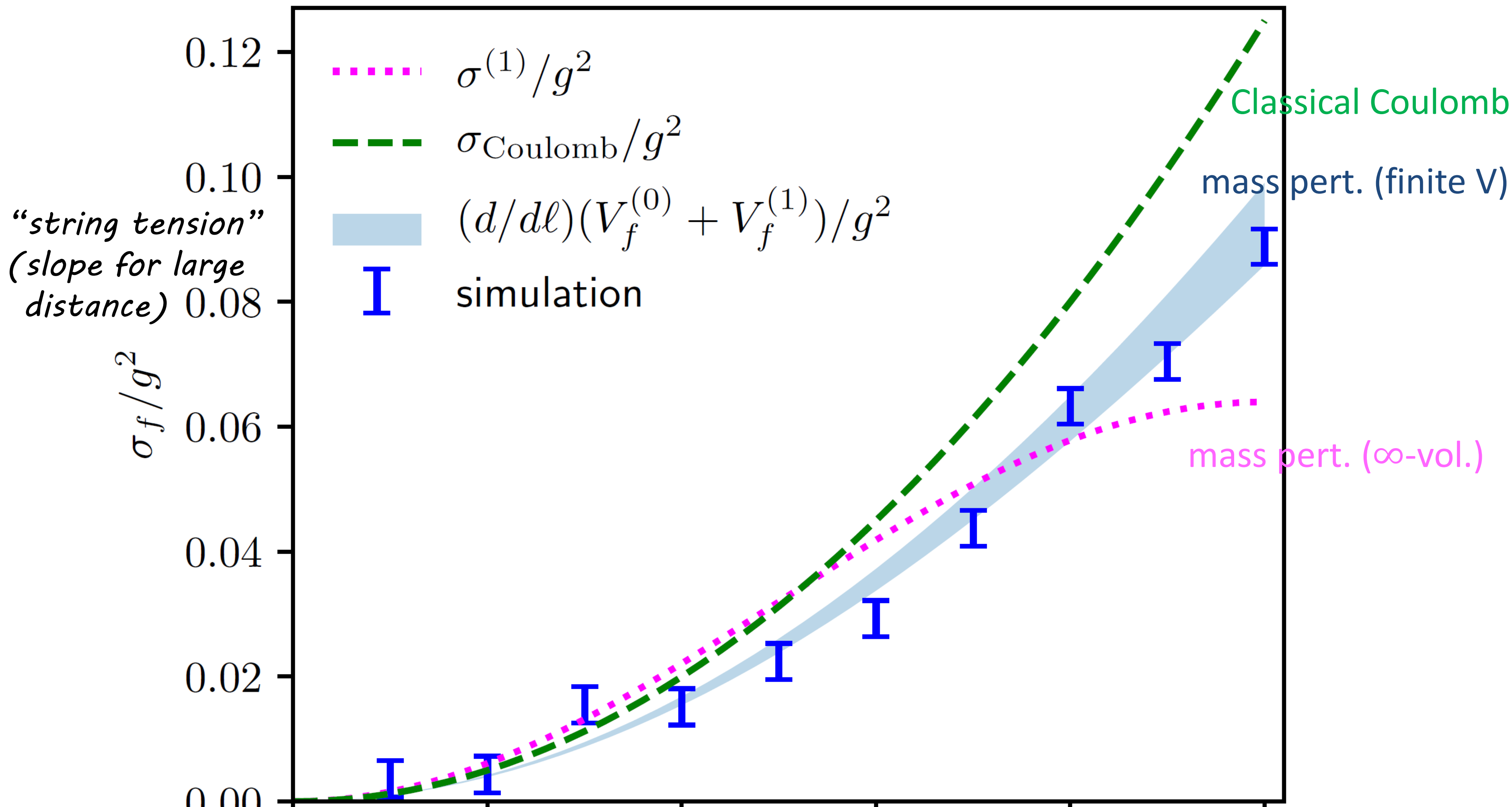
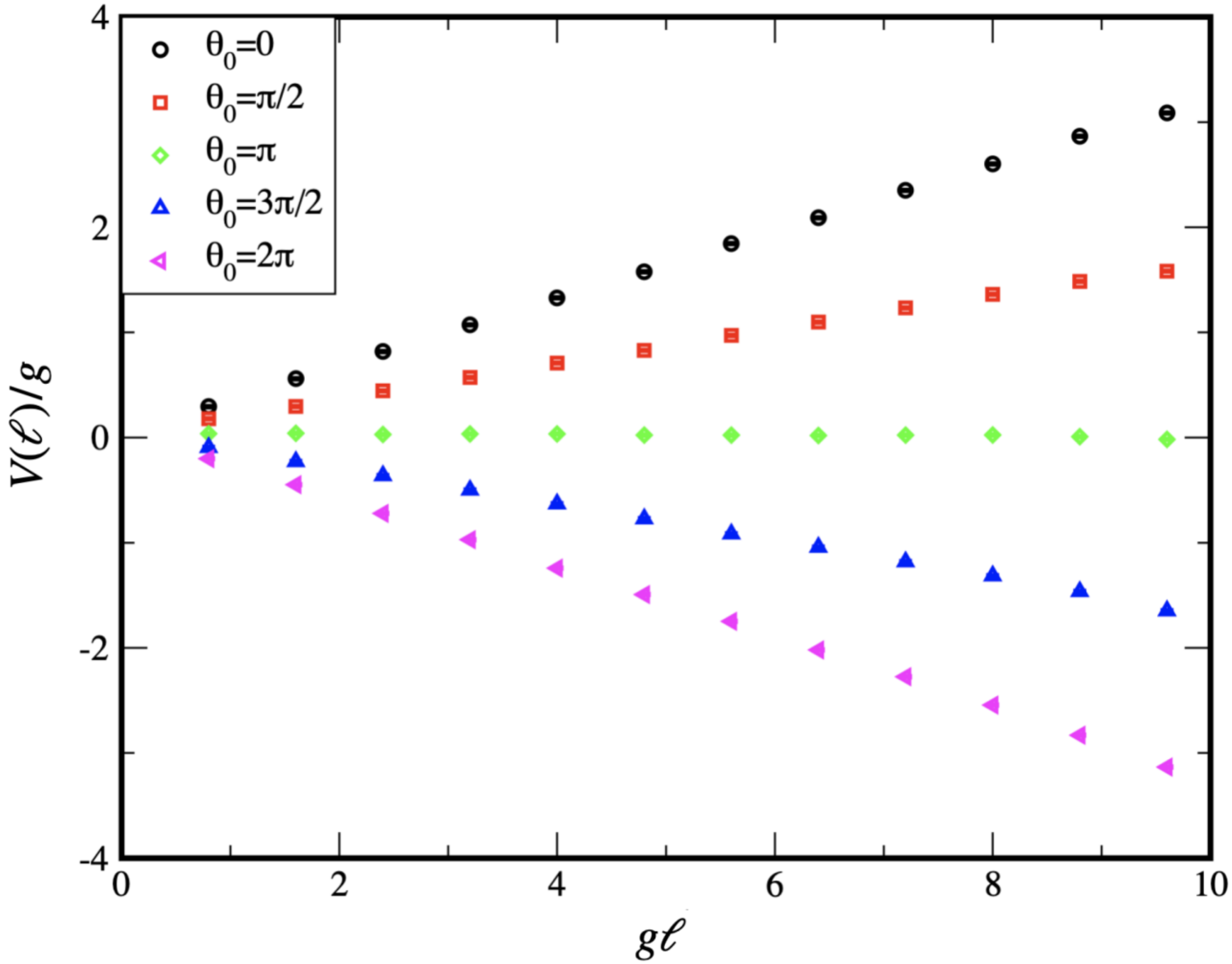


confinement by nontrivial dynamics!

Positive / negative string tension

[MH-Itou-Kikuchi-Tanizaki ’21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$

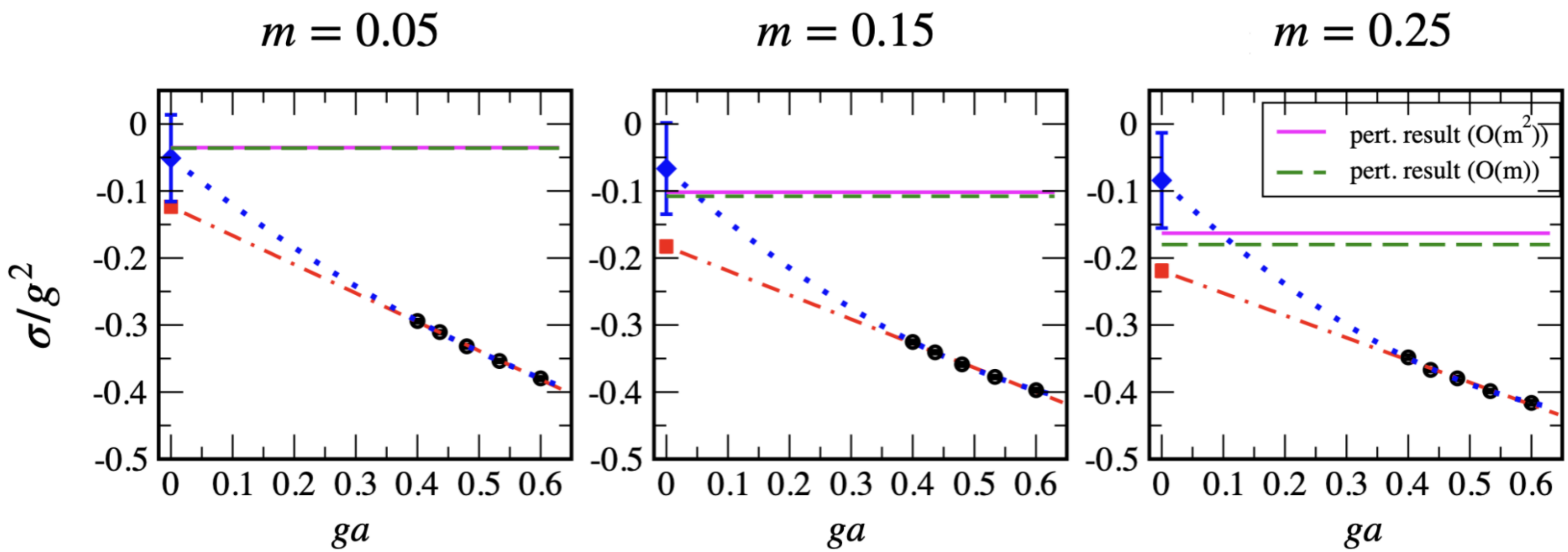


Sign(tension) changes as changing θ-angle!!

Continuum limit of string tension

[MH-Itou-Kikuchi-Tanizaki '21]

$$g = 1,\ (\mathrm{Vol.}) = 9.6/g, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$

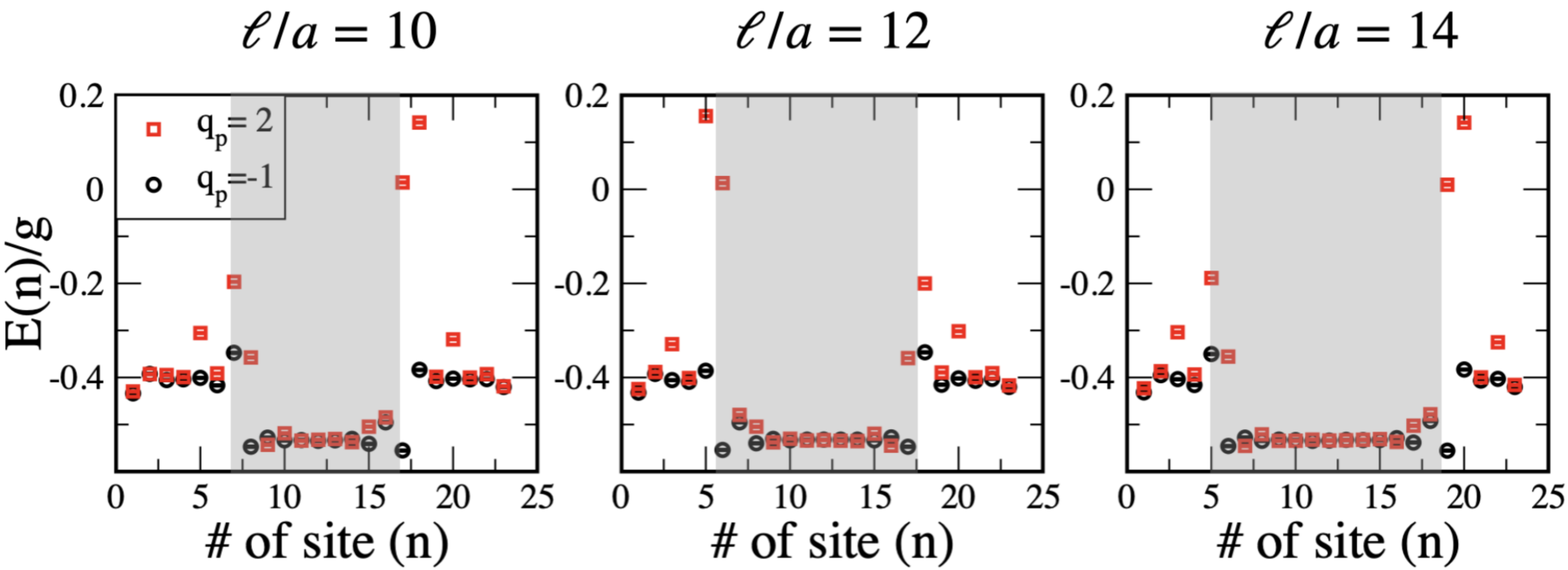


basically agrees with mass perturbation theory

Energy density @ negative tension regime

[MH-Itou-Kikuchi-Tanizaki '21]

$$g = 1, a = 0.4, N = 25, T = 99, \ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$



Lower energy inside the probes!!

Comment

The problem here involves only

ground state in 1+1D gapped system

→ Tensor Network (TN) approach is better in this situation

Result of Density Matrix Renormalization Group:

Comment

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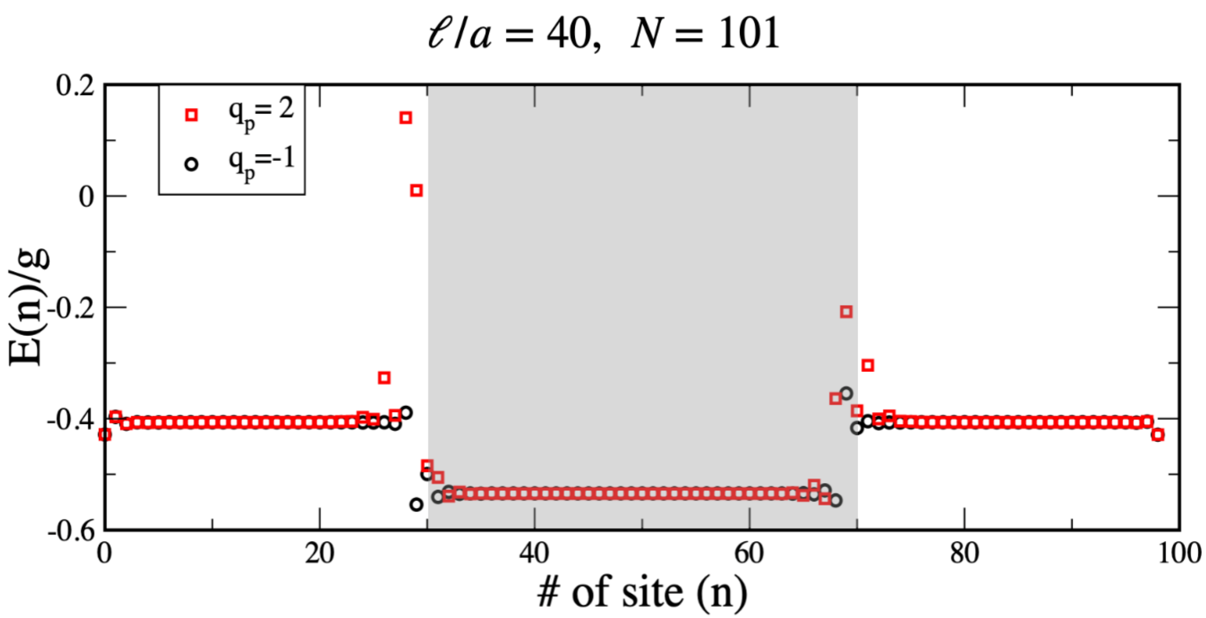
Result of Density Matrix Renormalization Group: [MH-Itou-Kikuchi-Tanizaki '22]



should consider prob. not efficiently solved by TN in future

Plan

1. Screening, confinement & negative tension in higher charge Schwinger model

[MH-Itou-Kikuchi-Nagano-Okuda '21]
[MH-Itou-Kikuchi-Tanizaki '21]

2. String/M-theory on quantum computer

3. Summary & Future prospect

Attempts to put "QGs" on quantum computer

- SYK model [Garcia-Alvarez-Egusquiza-Lamata-Campo-Sonner-Solano '17, etc…]

  —— dual to Jackiw-Teitelboim gravity (1+1d)

- BMN matrix model (gauged matrix QM) [Gharibyan-Hanada-MH-Liu '20]

  —— related to various setups of string/M-theory

- Loop quantum gravity (?) [cf. Cohen-Brady-Huang-Liu-Qu-Dowling-Han '20]

  —— another candidate of QG

(I haven't seen direct argument for string theory yet)

Attempts to put "QGs" on quantum computer

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High Energy Physics - Theory

[Submitted on 21 Mar 2023]

A simple quantum system that describes a black hole

Juan Maldacena

During the past decades, theorists have been studying quantum mechanical systems that are belie
Majorana fermions. It is conjectured to describe a black hole in an emergent universe governed by
necessary to see some black hole features.

Comments: 12 pages, 1 figure

Subjects: **High Energy Physics - Theory (hep-th)**; General Relativity and Quantum Cosmology (gr-qc); Quantum

Cite as: arXiv:2303.11534 **[hep-th]**

(or arXiv:2303.11534v1 **[hep-th]** for this version)

https://doi.org/10.48550/arXiv.2303.11534

Matrix Quantum Mechanics (QM)

|| literally!

QM of matrices

Ex.)

Matrix Quantum Mechanics (QM)

$||$ literally!

QM of matrices

Ex.) One Hermitian matrix QM: ($X(t)$: Hermitian matrix)

- Path integral formalism

$$L = \mathrm{Tr}\left[\frac{1}{2}\dot{X}^2 - V(X)\right], \qquad Z = \int DX\, e^{i\int dt\, L}$$

- Operator formalism

$$H = \mathrm{Tr}\left[\frac{1}{2}P^2 + V(X)\right], \qquad \left[X_{ij}\,, P_{k\ell}\right] = i\delta_{ik}\delta_{j\ell}$$

Technically,

special case of many particle QM

BMN matrix model ($U(N)$ gauged matrix QM)

[Berenstein-Maldacena-Nastase '02]

$$L = \frac{1}{g^2}\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{i\mu}{6}\epsilon^{ijk}X_i X_j X_k \right.$$
$$\left. + \frac{i}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_I [X_I, \Psi] - \frac{i\mu}{8}\Psi^\dagger \gamma_{123}\Psi \right\},$$

- (0+1) dim. $U(N)$ gauge theory
- all the fields are $N \times N$ Hermitian matrices
- X_I: bosonic matrices ($I = 1, \cdots, 9$)
- Ψ: 16 component Majorana-Weyl fermion
- $i = 1,2,3, a = 4, \cdots, 9$

BMN matrix model (cont'd)

[Berenstein-Maldacena-Nastase '02]

$$L = \frac{1}{g^2}\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{i\mu}{6}\epsilon^{ijk}X_i X_j X_k \right.$$
$$\left. + \frac{i}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_I [X_I, \Psi] - \frac{i\mu}{8}\Psi^\dagger \gamma_{123}\Psi \right\},$$

related to various interesting "stringy" theories:

- M-theory on pp-wave spacetime
- 3d $\mathcal{N} = 8$ SYM on $\boldsymbol{R} \times S^2 \sim$ D2-branes in IIA string theory
- 4d $\mathcal{N} = 4$ SYM on $\boldsymbol{R} \times S^3 \sim$ D3-branes in IIB string theory

[Ishii-Ishiki-Shimasaki-Tsuchiya '08, etc…]

- 6d $\mathcal{N} = (2{,}0)$ theory on $\boldsymbol{R} \times S^5 \sim$ M5-branes in M-theory

[Maldacena-Sheikh-Jabbari-Van Raamsdonk '02]

- holographic duals


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SUSY QFTs from BMN matrix model

X_i part:

$$L|_{A_t=X_a=\Psi=0} = \mathrm{Tr}\left\{\frac{1}{2}(\partial_t X_i)^2 + \frac{g^2}{4}[X_i, X_j]^2 - \frac{\mu^2}{18}X_i^2 - \frac{i\mu g}{3}\epsilon^{ijk}X_iX_jX_k\right\}$$
$$= \mathrm{Tr}\left\{\frac{1}{2}(\partial_t X_i)^2 + \frac{g^2}{4}\left([X_i, X_j] - \frac{i\mu}{3g}\epsilon_{ijk}X_k\right)^2\right\}.$$

SUSY vacua:

"*Fuzzy sphere*"

$$X_i = \frac{\mu}{3g}J_i, \qquad [J_i, J_j] = i\epsilon^{ijk}J_k$$

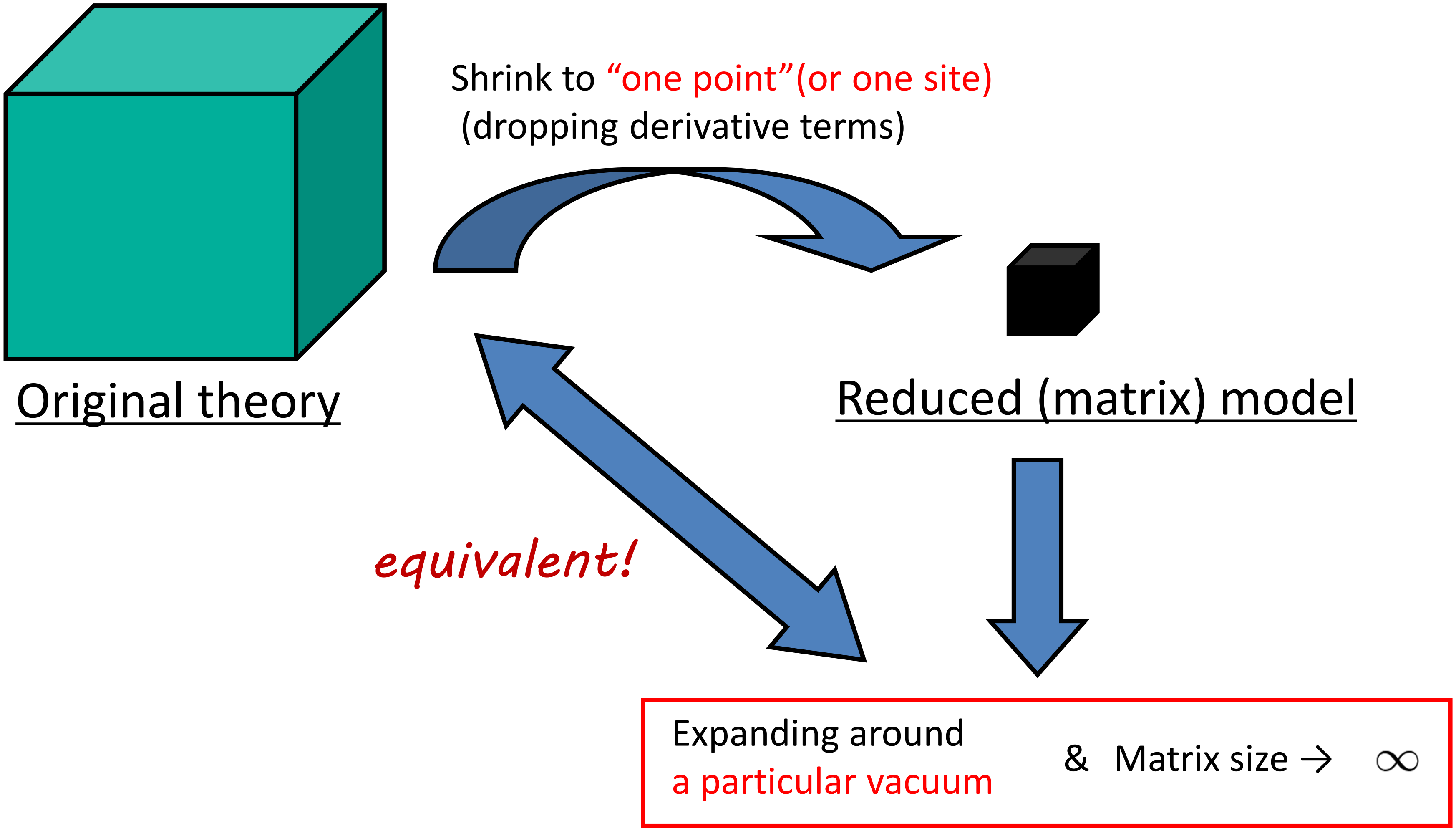
J_i: $SU(2)$ generator in N-dim. (ir)reducible rep.

Expanding the theory around fuzzy sphere sols. w/ appropriate reps., we can obtain SUSY QFTs in the large-N limit via "large-N reduction"

[Maldacena-Sheikh-Jabbari-Van Raamsdonk '02, Ishii-Ishiki-Shimasaki-Tsuchiya '08, etc…]

Concept of Large N reduction

[Eguchi-Kawai, Bhanot-Heller-Neuberger,
Gonzalez-Arroyo-Okawa, Gross-Kitazawa, etc.]



Here we apply it only for space and leave time continuous

SUSY QFTs from BMN matrix model

$$X_i = \frac{\mu}{3g} J_i$$ fuzzy sphere

- 3d $\mathcal{N} = 8$ SYM on $\boldsymbol{R} \times S^2$: ($J_i^{(s)}$: $SU(2)$ generator of $2s+1$ dim. representation)

$$J_i = J_i^{(s)} \otimes \mathbf{1}_{N_2} \qquad s = \frac{N_5 - 1}{2},\ N_5 \to \infty$$

-

-

SUSY QFTs from BMN matrix model

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$$J_i = J_i^{(s)} \otimes \mathbf{1}_{\boldsymbol{N_2}}$$

$s = \frac{N_5-1}{2}$, $N_2 \to \infty$

- 4d $\mathcal{N} = 4$ SYM on $\boldsymbol{R} \times S^3$:

$$J_i = \bigoplus_{s=n-T}^{n+T} (J_i^{(s)} \otimes \mathbf{1}_{\boldsymbol{k}})$$

$k, n, T, n-T \to \infty$

Operator formalism

$$\Psi = \begin{pmatrix} \psi_{Ip} \\ \epsilon_{pq}\psi^{\dagger Iq} \end{pmatrix}$$

$$\hat{H} = \mathrm{Tr}\left\{\frac{1}{2}(\hat{P}_I)^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2 + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k \right.$$
$$\left. + g\hat{\psi}^{\dagger Ip}\sigma_p^{iq}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip}g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip}\right\}$$

Commutation relations:

(α, β: gauge indices)

$$\left[\hat{X}_{I\alpha}, \hat{P}_{J\beta}\right] = i\delta_{IJ}\delta_{\alpha\beta}, \qquad \left\{\hat{\psi}^{\dagger Ip\alpha}, \hat{\psi}^{\beta}_{Jq}\right\} = \delta_{IJ}\delta^{pq}\delta^{\alpha\beta}$$

Gauss law:

$$\hat{G}_\alpha|\mathrm{phys}\rangle = 0 \quad \text{w/} \quad \hat{G}_\alpha = \sum_{\beta,\gamma=1}^{N^2}\left(\sum_{I=1}^{9}\hat{X}_I^\beta\hat{P}_I^\gamma - i\sum_{I,p}\hat{\psi}^{\dagger Ip\alpha}\hat{\psi}^\gamma_{Ip}\right)$$

We can regularize it as in scalar field theory

[Gharibyan-Hanada-MH-Liu '20]

The essence is common w/ single particle QM

$$\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{\omega^2}{2}\hat{x}^2 + V_{\mathrm{int}}(\hat{x})$$

Most naïve approach = truncation in harmonic osc. basis:

$$\hat{a} = \sqrt{\frac{\omega}{2}}\,\hat{x} + \frac{i}{\sqrt{2\omega}}\,\hat{p} \;=\; \sum_{n=0}^{\infty} \sqrt{n+1}\,|n\rangle\langle n+1|$$

$$\xRightarrow[\text{regularize!}]{} \;\sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

Then replace $\hat{p}$ & $\hat{x}$ by

$$\hat{x}\Big|_{\mathrm{regularized}} \equiv \frac{1}{\sqrt{2\omega}}(\hat{a}+\hat{a}^\dagger)\Big|_{\mathrm{regularized}}$$

$$\hat{p}\Big|_{\mathrm{regularized}} \equiv \frac{1}{i}\sqrt{\frac{\omega}{2}}\,(\hat{a}-\hat{a}^\dagger)\Big|_{\mathrm{regularized}}$$

The essence is common w/ single particle QM (Cont'd)

$$\hat{a}\Big|_{\mathrm{regularized}} = \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

We can rewrite the Fock basis in terms of qubits:

$$|n\rangle = |b_{K-1}\rangle|b_{K-2}\rangle\cdots|b_0\rangle \qquad K \equiv \log_2 \Lambda$$

$$n = \mathrm{b}_{\mathrm{K}-1}2^{\mathrm{K}-1} + \mathrm{b}_{\mathrm{K}-2}2^{\mathrm{K}-2} + \cdots + b_0 2^0$$ (binary representation)

Then,

$$|n\rangle\langle n+1| = \bigotimes_{\ell=0}^{K-1} \underbrace{(|b'_\ell\rangle\langle b_\ell|)}_{\textit{either one of}}$$

$$\left(\begin{aligned} |0\rangle\langle 0| &= \frac{1_2 - \sigma_z}{2}, & |1\rangle\langle 1| &= \frac{1_2 + \sigma_z}{2}, \\ |0\rangle\langle 1| &= \frac{\sigma_x + i\sigma_y}{2}, & |1\rangle\langle 0| &= \frac{\sigma_x - i\sigma_y}{2} \end{aligned} \right)$$

Computational costs

of qubits:

- Single particle QM w/ truncation Λ requires $\log_2 \Lambda$ qubits
- The BMN model has 9 scalars & 16 component real fermion which are $N \times N$ matrices

$$\Rightarrow \quad 9N^2 \log_2 \Lambda + 8N^2 \ \text{qubits}$$

of spin ops. in Hamiltonian:

- each annihilation/creation op. has less than $\mathcal{O}(\Lambda^2)$ spin ops.
- we have 4-pt. interaction at most
- ${}^{\exists}\mathcal{O}(N^4)$ combinations regarding the color indices

$$\Rightarrow \quad < \mathcal{O}(\Lambda^8 N^4) \ \text{spin ops.}$$

Possible applications

various real time physics such as

- Testing holography for real time
- Out of time order correlator
- Black hole thermalization
- decay of fuzzy sphere for non-SUSY cases

etc...

of qubits to simulate black hole

BMN w/ truncation has [Maldacena '23]

$$\boxed{9N^2 \log_2 \Lambda + 8N^2 \ \text{ qubits}}$$

What N & Λ needed to simulate black hole?

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- MC study suggests BH entropy is (approximately) reproduced at

$$N = 16, \frac{T}{(g^2 N)^{1/3}} = 0.3, \frac{\mu}{T} = 1.6$$

[Patelpudis-Bergner-Hanada-Rinaldi-Schafer -Vranas-Watanabe-Bpdendorfer '22]

- Important energy levels should satisfy about $E_n < \mathcal{O}(T)$

$$\longrightarrow \quad \Lambda \sim 4$$

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- Important energy levels should satisfy about $E_n < \mathcal{O}(T)$

$$\longrightarrow \quad \Lambda \sim 4$$

Totally, we need

$$\boxed{\sim 7000 \text{ qubits}}$$

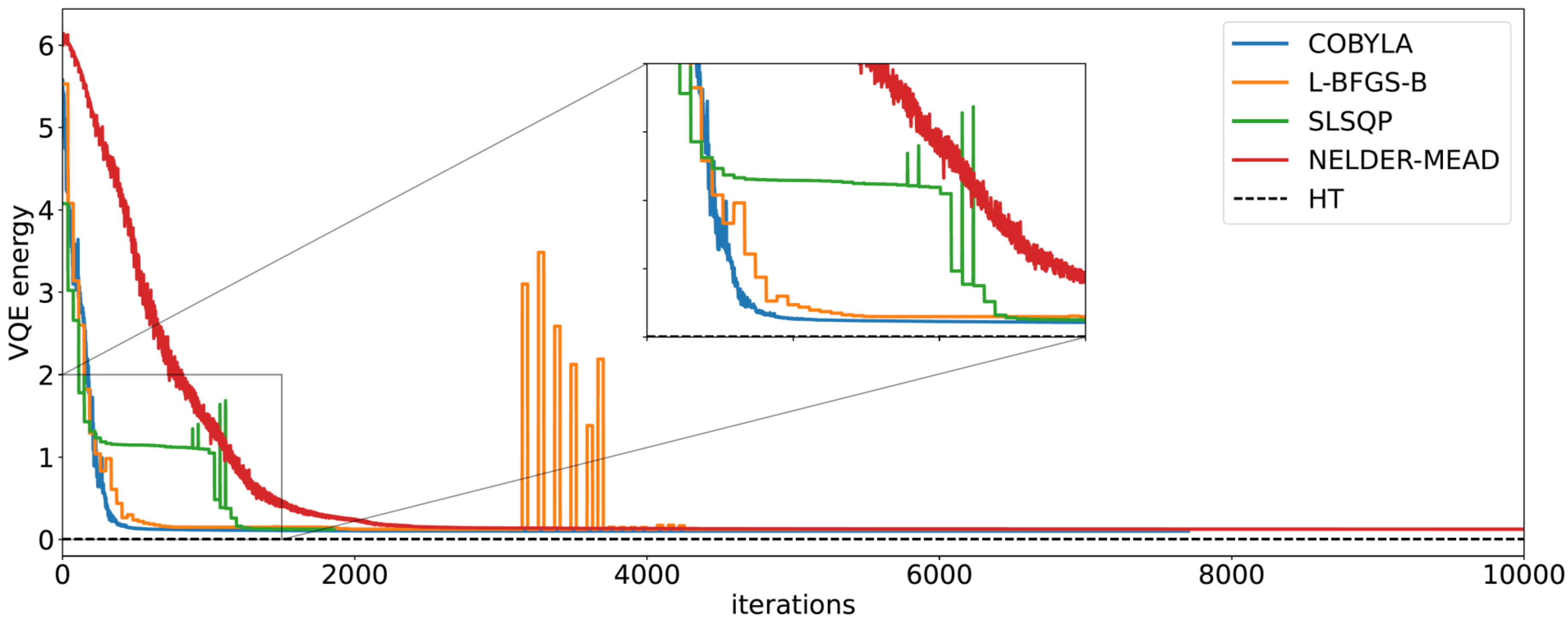
(similar to the condition for "quantum supremacy" in factoring integer)

An implementation for “$SU(2)$ mini-BMN”

[Rinaldi-Han-Hassan-Feng-Nori-McGuigan-Hanada ’21]

$$\hat{H} = \mathrm{Tr}\left(\frac{1}{2}\hat{P}_I^2 - \frac{g^2}{4}[\hat{X}_I,\hat{X}_J]^2 + \frac{g}{2}\hat{\bar{\psi}}\Gamma^I[\hat{X}_I,\hat{\psi}] - \frac{3i\mu}{4}\hat{\bar{\psi}}\hat{\psi} + \frac{\mu^2}{2}\hat{X}_I^2\right) - (N^2-1)\mu$$

Ground state energy by VQE on simulator $(\Lambda = 2)$



Summary & Future prospect

Near future prospect

In near future, available device is so-called [Preskill '18]

Noisy intermediate-scale quantum device (NISQ)

w/ limited number of qubits & non-negligible errors

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$\Rightarrow$ nice if $^{\exists}$a way to reduce errors w/o increasing qubits

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On such device,

- quantum error correction can't be enough

  $\Rightarrow$ nice if $^{\exists}$a way to reduce errors w/o increasing qubits

  $\Rightarrow$ "quantum error mitigation"

- algorithms w/ less gates are preferred

  $\Rightarrow$ Hybrid quantum-classical algorithm

  (Popular one for finding vacuum: "variational method")

Quantum Error mitigation

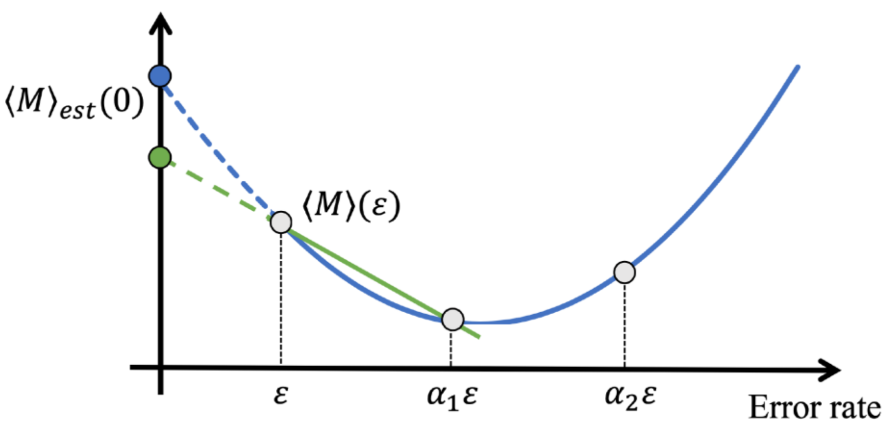
[Figs. are from Endo-Cai-Benjamin-Yuan ’20]

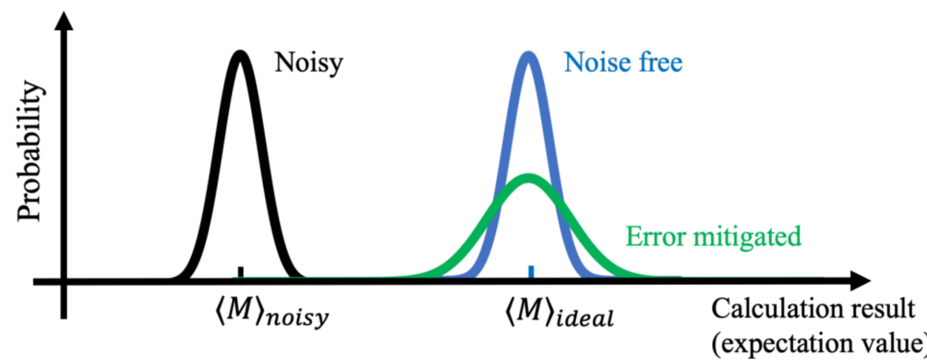
the simplest way = extrapolation

In general,

difficult to decrease errors but possible to increase them

⇨ error-free result by fitting as a function of error rate





This doesn’t need to increase qubits but needs more shots

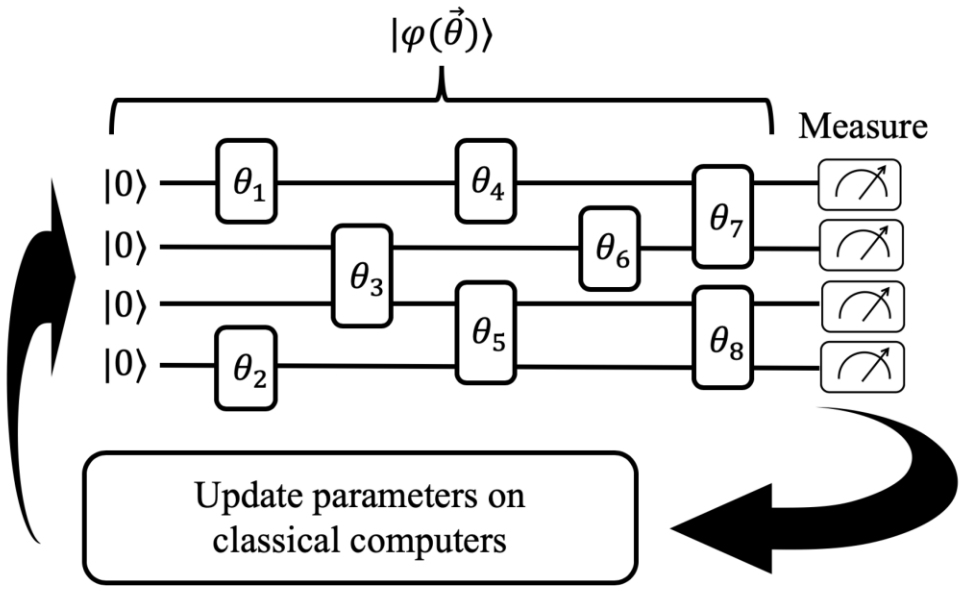
Variational quantum algorithm

Idea:

[Fig. is from Endo-Cai-Benjamin-Yuan '20]

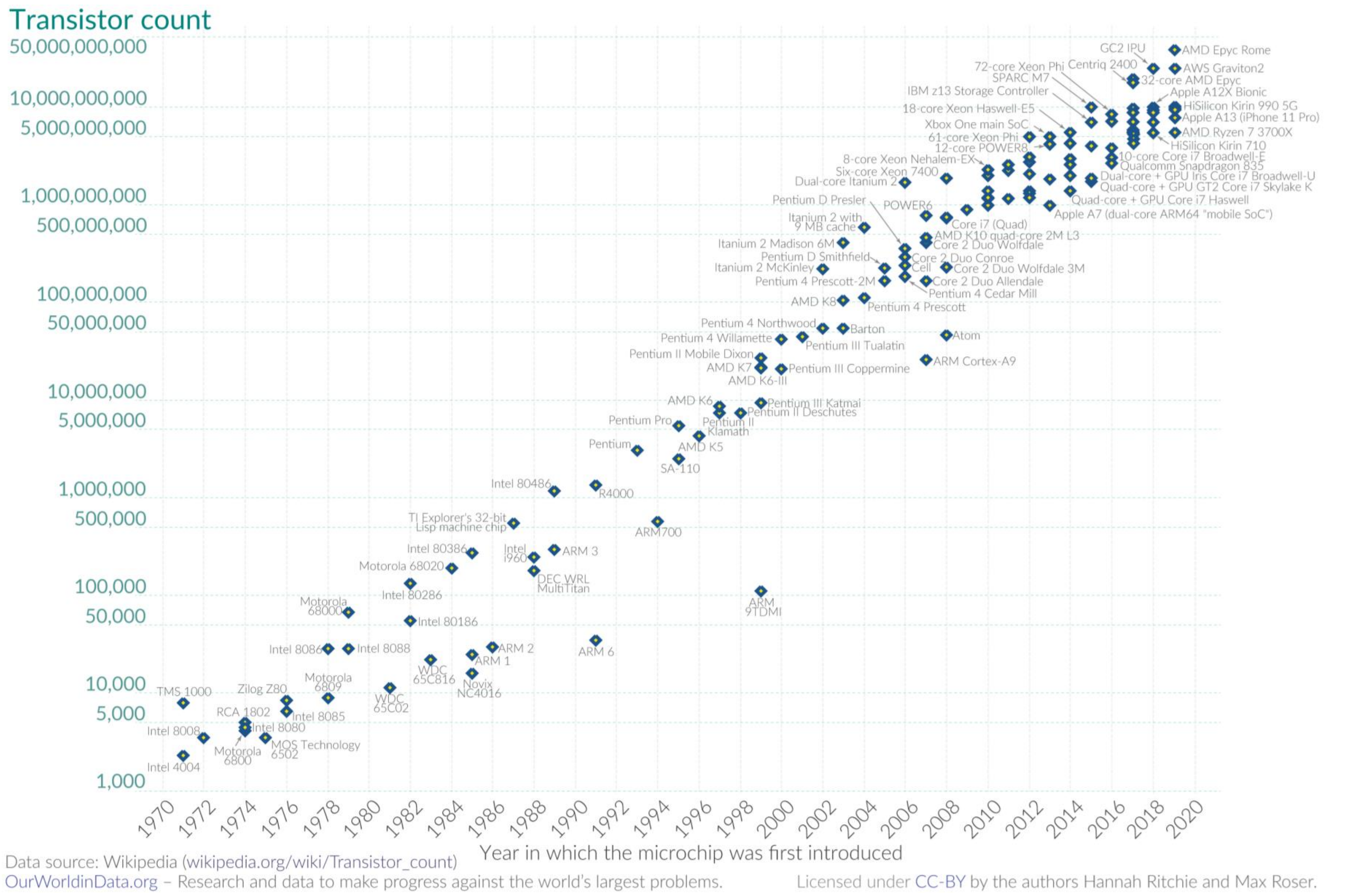
Acting gates & measurements $\Rightarrow$ Quantum computer

Parameter optimization $\Rightarrow$ Classical computer



This method needs much less gates than adiabatic state preparation but it's not guaranteed to get true ground state

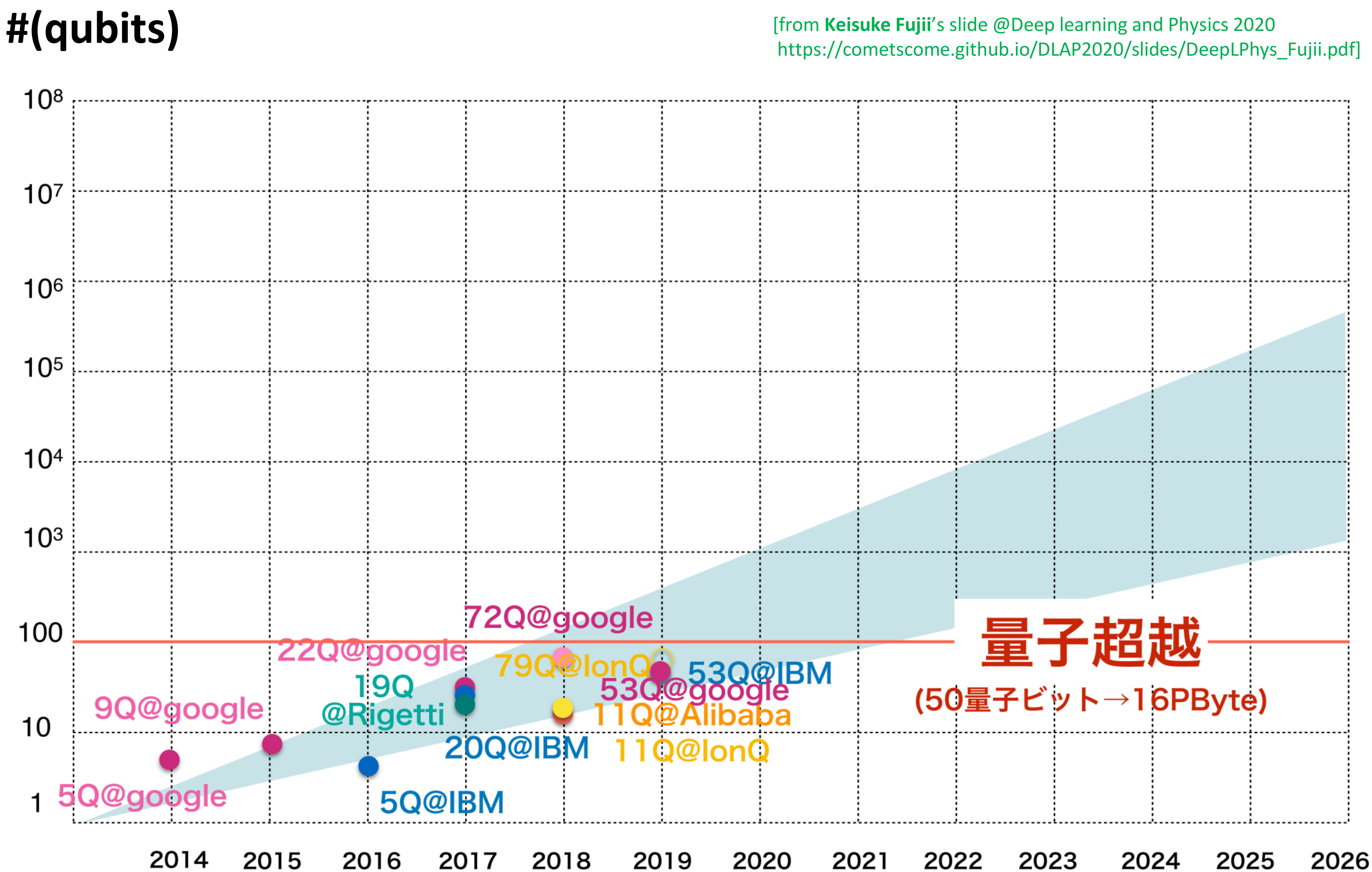
Moore’s law (classical computer)



[by Max Roser, Hannah Ritchie, https://ourworldindata.org/uploads/2020/11/Transistor-Count-over-time.png]

“Quantum” Moore’s law?

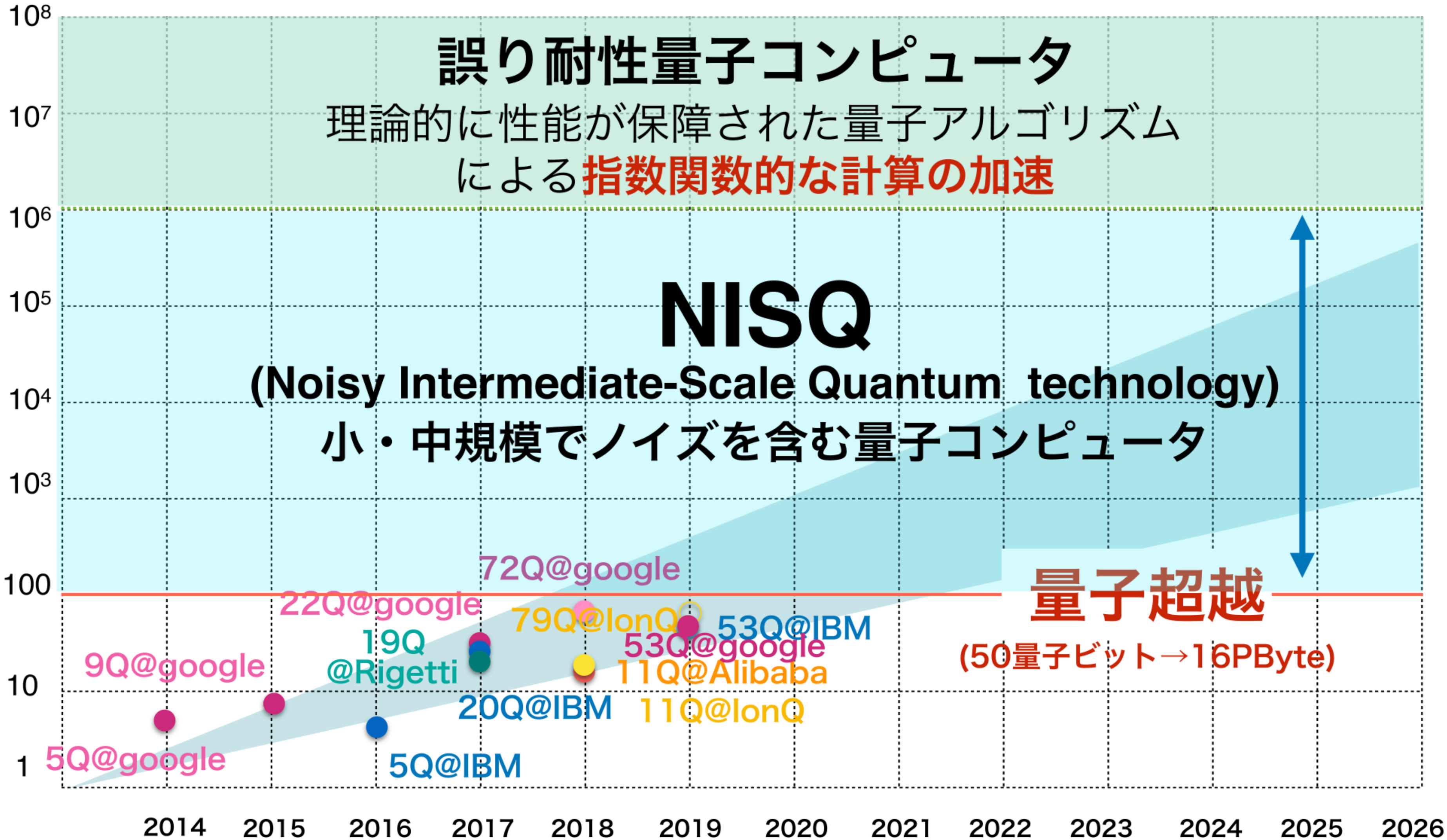
[from **Keisuke Fujii**’s slide @Deep learning and Physics 2020
https://cometscome.github.io/DLAP2020/slides/DeepLPhys_Fujii.pdf]



“Quantum” Moore’s law?

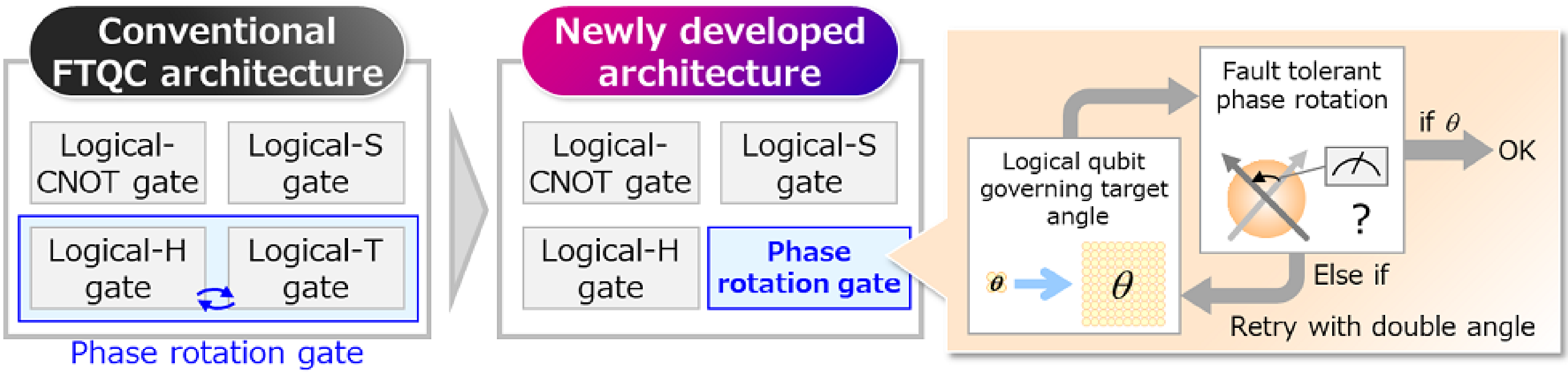
#(qubits)

[from **Keisuke Fujii**’s slide @Deep learning and Physics 2020
https://cometscome.github.io/DLAP2020/slides/DeepLPhys_Fujii.pdf]



Game change? - new architecture -

[Akahoshi-Maruyama-Oshima-Sato-Fujii 2023 March(!)]



Big saving of computational cost is claimed:

- # of qubits $\sim \dfrac{1}{10} \times$ (usual) !!
- # of gates $\sim \dfrac{1}{20} \times$ (usual) !!

Unexplored region in QFT

- Taking continuum limit in simulation w/ noises
- Taking continuum limit in scalar field theory
- Non-abelian gauge theory
- Higher dimensions ($\geq$ (2+1)-dim.)
- Finite temperature, chemical potential
- Curved space
- Supersymmetric theories
- Other lattice fermions
- Phase transitions
- Scattering

etc...

Applications to astro./cosmo. so far

- Inflation (scalar in curved spacetime) [Liu-Li '20]
- Wheeler-De Witt eq. [Joseph-Varela-Watts-White-Feng-Hassan-McGuigan, Czelusta-Mielczarek '21]
- QM in black hole b.g. [Joseph-Whilte-Chandra-McGuigan '21, Chandra-McGuigan '22]
- Dark sector showers [Chigusa-Yamazaki '22]
- Matrix Big Bang (???) [Chandra-Feng-McGuigan '22]
- Boltzmann eq. [Yamazaki-Uchida-Fujisawa-Yoshida '23]

Summary of the whole lectures

fun & $^\exists$many things to do even now

- Quantum computation is suitable for operator formalism which is free from sign problem

- Instead we have to deal with huge vector space. Quantum computers in future may do this job.

- "Rule" of quantum computation
  = Do something interesting by a combination of
  acting unitary op. & measurement

- Real quantum computer has errors

- Handling quantum error is very important

Thanks!

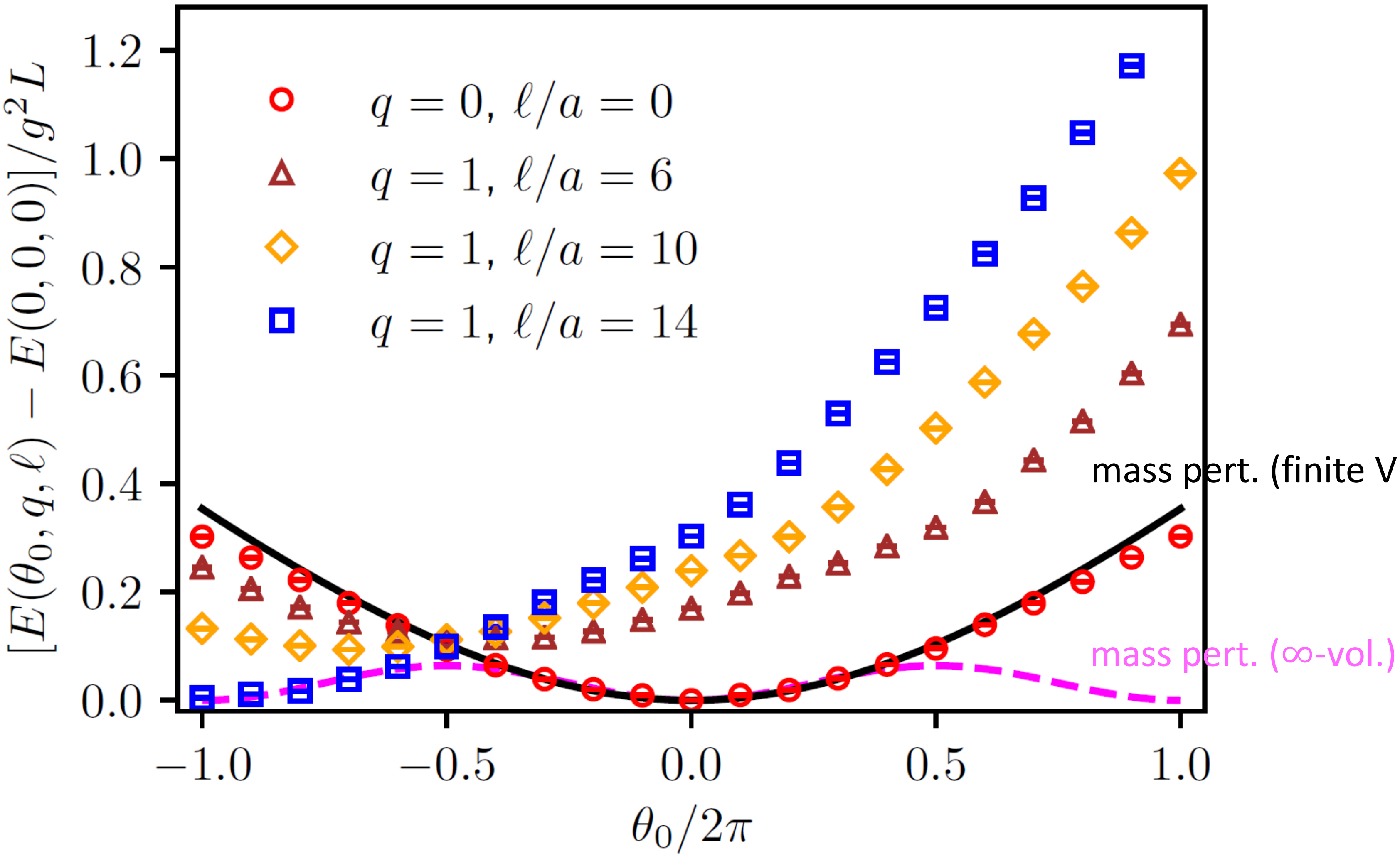
Appendix

Results for $\theta_0 \neq 0$

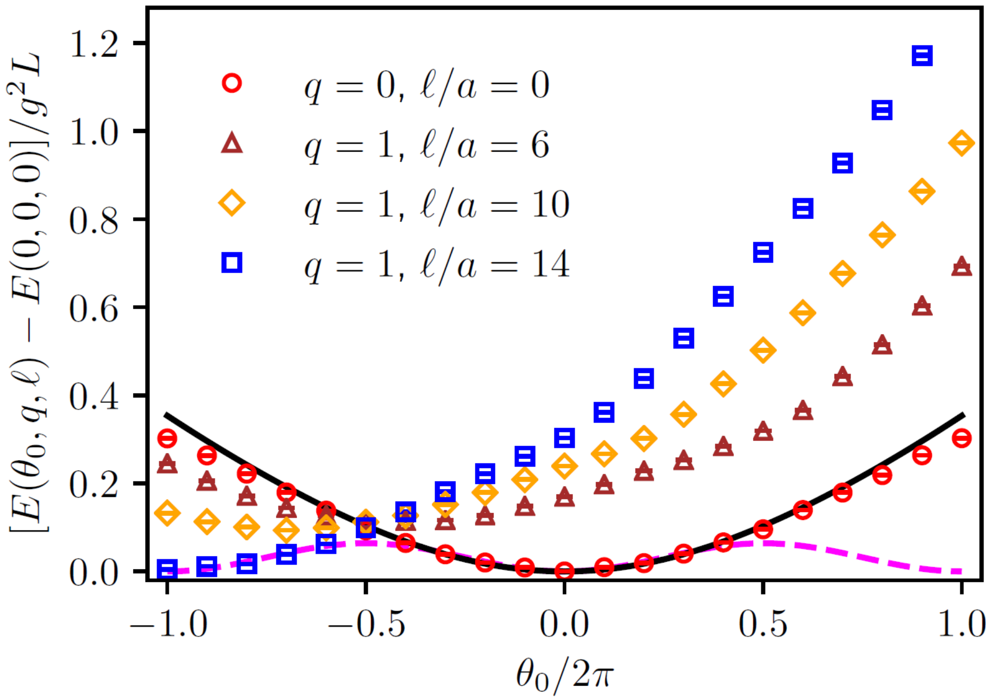
[MH-Itou-Kikuchi-Nagano-Okuda '21]

(difficult to explore by the conventional Monte Carlo approach)

Parameters: $g = 1, a = 0.4, N = 15, T = 99, q_p/q = 1, m/g = 0.2$



Comment on theta angle periodicity



Absence of the periodicity: $\theta_0 \sim \theta_0 + 2\pi$?

This is expected because we're taking open b.c.

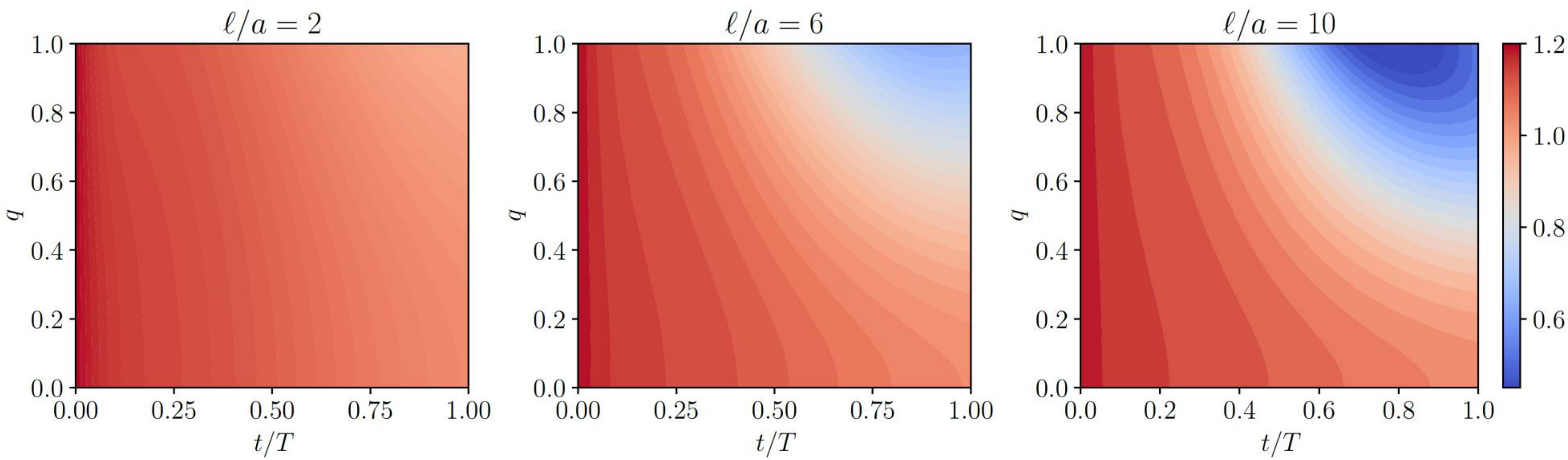
To get the periodicity back, we need to take ∞-vol. limit

[detailed study: MH-Itou-Kikuchi-Tanizaki '22]

Comment: density plots of energy gap

(known as “Tuna slice plot” inside the collaboration) [MH-Itou-Kikuchi-Nagano-Okuda ’21]

Parameters: $g = 1, a = 0.4, N = 15, q_p/q = 1, m/g = 0.15$

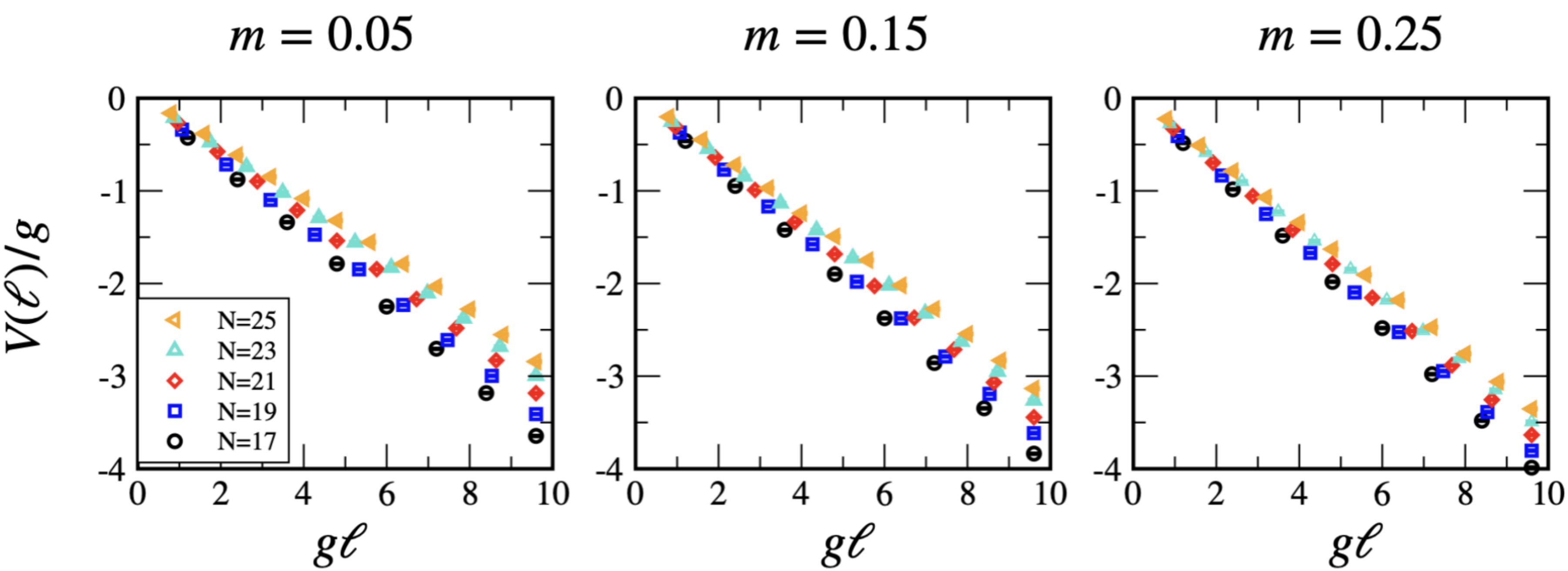


smaller gap for larger ℓ

larger systematic error for larger ℓ

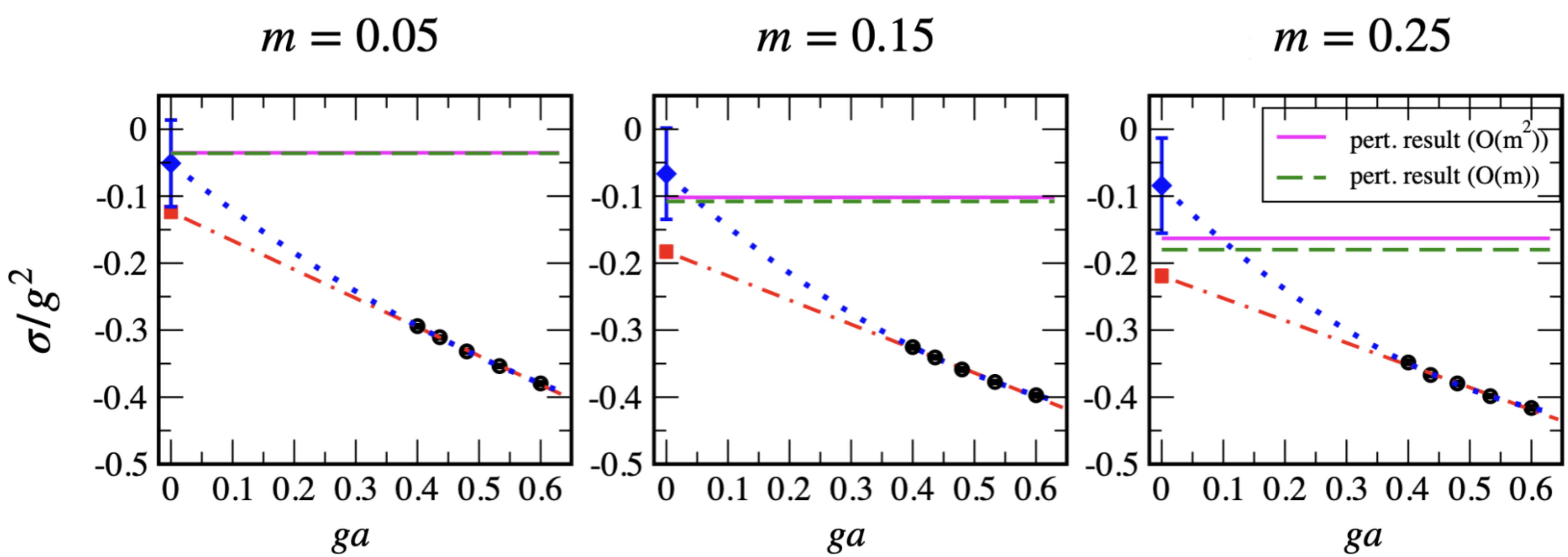
N-dependence of V w/ fixed physical volume

[MH-Itou-Kikuchi-Tanizaki '21]



Continuum limit of string tension

[MH-Itou-Kikuchi-Tanizaki '21]

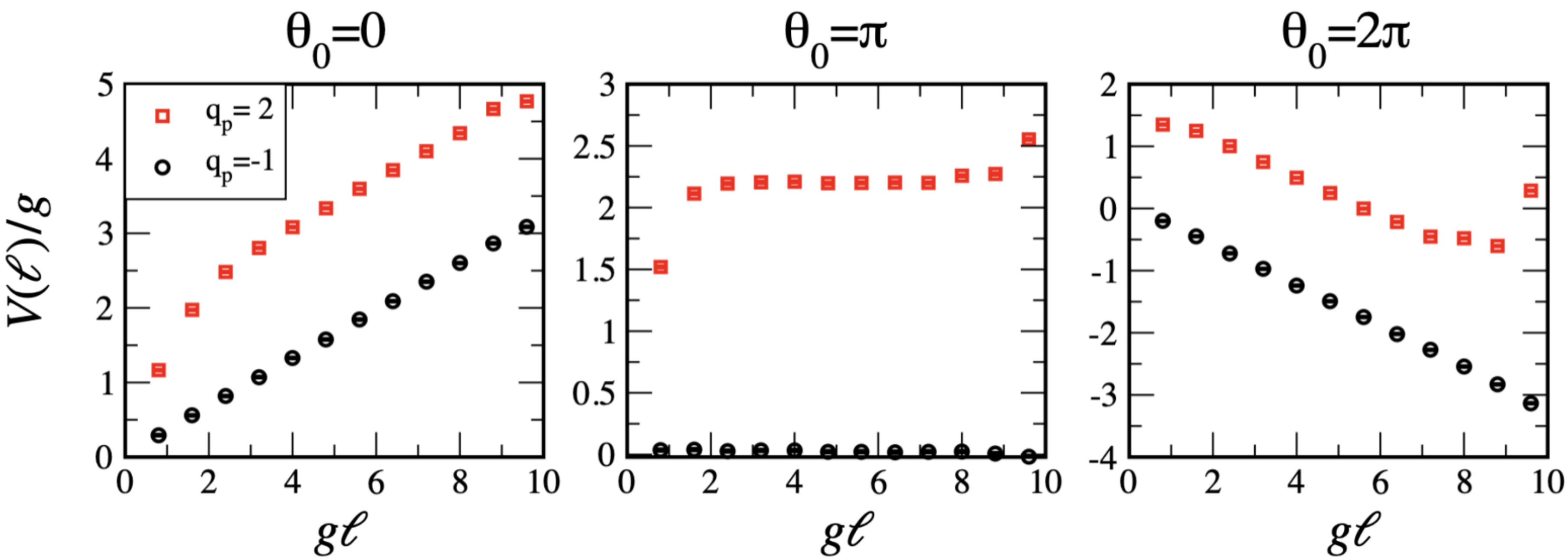


N	ga	$\sigma(m=0.05)/g^2$	$\sigma(m=0.15)/g^2$	$\sigma(m=0.25)/g^2$
17	0.60000	-0.380(3)	-0.397(3)	-0.416(3)
19	0.53333	-0.354(2)	-0.377(2)	-0.399(2)
21	0.48000	-0.332(3)	-0.359(3)	-0.380(3)
23	0.43636	-0.311(2)	-0.341(3)	-0.367(2)
25	0.40000	-0.294(3)	-0.325(3)	-0.348(3)

Comparison of $q_p/q = -1/3$ & $q_p/q = 2/3$

[MH-Itou-Kikuchi-Tanizaki '21]

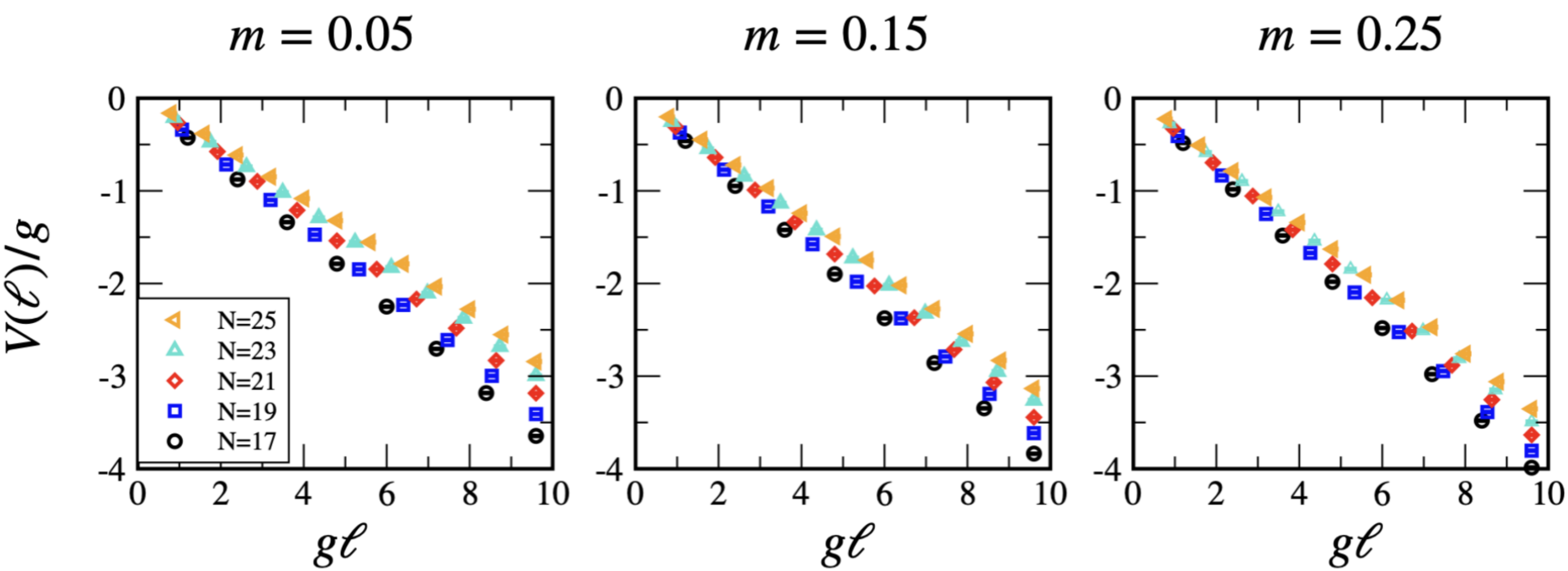
Parameters: $\mathrm{q} = 3, g = 1, a = 0.4, N = 25, T = 99, m = 0.15$



Similar slopes → (approximate) Z_3 symmetry

N-dependence of V w/ fixed physical volume

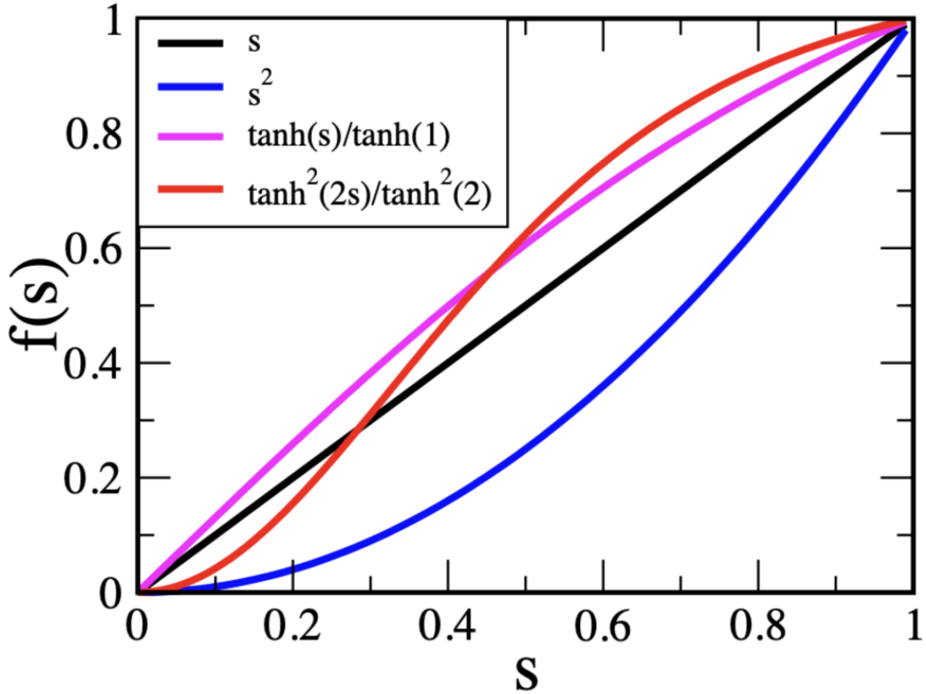
[MH-Itou-Kikuchi-Tanizaki ’21]

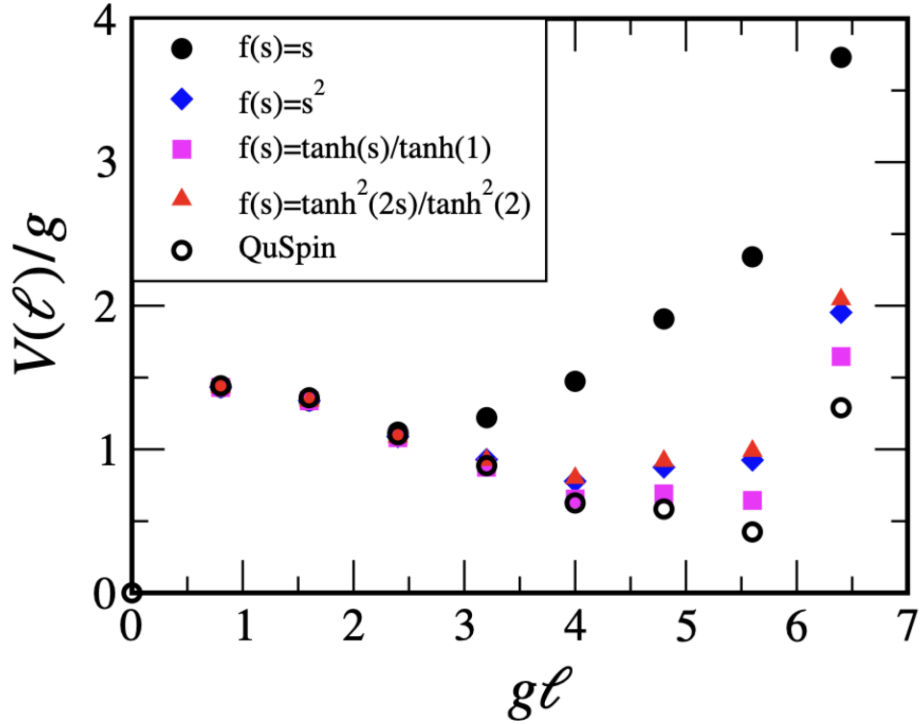


Adiabatic scheduling

[MH-Itou-Kikuchi-Tanizaki '21]

$$N = 17, ga = 0.40, m = 0.20, q_p = 2, \theta_0 = 2\pi,$$





Preparation of fuzzy sphere state

Regularization of Hilbert sp. can done in similar ways.

To get the SUSY QFTs., we need to construct states corresponding to the fuzzy spheres at finite coupling

Steps:

① Expand the theory around the fuzzy sphere

② Take its Fock vacuum at weak coupling limit

$$|J_i\rangle_{g\to 0}$$

③ Starting w/ the Fock vacuum, adiabatically turn on the coupling & apply the adiabatic time evolution

$$|J_i\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\ H_A(t)\right) |J_i\rangle_{g\to 0}$$