4. $\cancel{\text{SUSY}}$

4-1. $\cancel{\text{SUSY}}$ deformation

At E, critical pt.,

$$\delta L = \phi_I^a M^a + D_I^{ai} \mu^{ai}$$

$$\text{w/ } \phi_I^a = \phi_I \hat{v}^a, \quad D_I^{ai} = \hat{m}^i \hat{v}^a$$

In IR,

$$D_I^{ai} \mu^{ai} \longrightarrow m^i \, \mathrm{tr}\left(\frac{i}{4} \bar{\lambda} \sigma^i \lambda + \phi D^i \right) \quad (m^i \propto \hat{m}^i)$$

$$\sim \mathrm{tr}\left(\frac{i}{4} \underbrace{m^i \bar{\lambda} \sigma^i \lambda}_{\downarrow \, m_\lambda} - \frac{1}{2} \underbrace{m^i m^i \phi^2}_{\text{uplift Coulomb branch} \, \rightarrow \, a=0} \right)$$

Symmetries :

- $\phi^a_I = 0 = \hat{m}^i$ (E. SCFT)

  SUSY , $SU(2)_I$, $SU(2)_R$

- $\phi^a_I \neq 0$, $\hat{m}^i = 0$ ($\to$ SYM)

  SUSY , $U(1)_I$, $SU(2)_R$

- generic pt.

  ~~SUSY~~ , $U(1)_I$, $U(1)_R$

- $\phi^a_I = 0$, $\hat{m}^i \neq 0$

  ~~SUSY~~ , $U(1)_I$, $U(1)_R$, $\mathrm{diag}(\mathbb{Z}_2^R \times \mathbb{Z}_2^I)$

$$\left(\begin{array}{l} \mathbb{Z}_2^R : M^a \to M^a , \ \mu^{ai} \to -\mu^{ai} \\ \mathbb{Z}_2^I : M^a \to -M^a , \ \mu^{ai} \to -\mu^{ai} \end{array} \right)$$

4-2. Discrete spacetime symmetries (34)

$$\mathcal{L}_{fer} = -i\bar{\lambda}\gamma^\mu D_\mu \lambda + \frac{i}{4} m^i \bar{\lambda}\sigma^i \lambda$$

$$P: \lambda \to i\gamma^1 \lambda(Px) \qquad (C: \lambda^m \to -\varepsilon_{mn}\lambda^n)$$

$$T: \lambda \to -i\gamma^{04}\lambda(Tx) \qquad (\gamma^4 = -i\gamma^{0123})$$

$m^i = 0$

fermion #

$$P^2,\ T^2,\ (PT)^2 : \lambda \to -\lambda$$

$$\therefore\ P^2 = (-1)^F,\quad T^2 = (-1)^F,\quad \underline{PTPT = (-1)^F}$$

$PT \neq TP$!

This forms

$$Q_8 = \{x, y \mid x^4 = 1,\ x^2 = y^2,\ y^{-1}xy = x^{-1}\}$$

"quaternion group"

$m^i \neq 0$

$$P: \bar{\lambda}\sigma^i\lambda \to -\bar{\lambda}\sigma^i\lambda$$

$$T: i\bar{\lambda}\sigma^i\lambda \to i\bar{\lambda}(\sigma^i)^*\lambda$$

$$(C: \bar{\lambda}\sigma^i\lambda \to -\bar{\lambda}(\sigma^i)^*\lambda)$$

naively

$$\begin{cases} \cancel{C} \quad \cancel{P} \quad T & \text{for } m^i = (m^1, 0, m^3) \\ \cancel{C} \quad \cancel{P} \quad \cancel{T} & \text{for generic } m^i \\ C \quad \cancel{P} \quad \cancel{T} & \text{for } m^i = (0, m, 0) \end{cases}$$

But we can redefine P & T using "broken $SU(2)_R$"

$$P' \equiv U_n P, \quad T' \equiv U_n T$$

$$(U_n \equiv i n^i \sigma^i \text{ w/ } m^i n^i = 0,\ n^i n^i = 1)$$

and preserve P' & T' for $^\forall m^i$

We can show

$$P'^2 = 1, \quad T'^2 = (-1)^F$$

$$P'T' = (-1)^F T'P'$$

This forms

$$D_8 = \{ x, y \mid x^4 = 1, y^2 = 1, yxy = x^{-1} \}$$

"dihedral group"

Another relation:

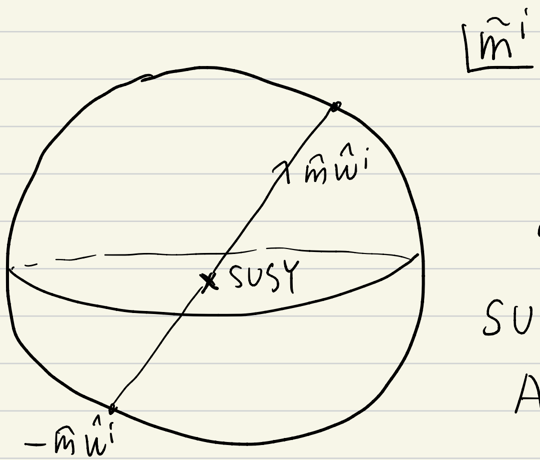
$$P' e^{i\alpha m^i \sigma^i} P' = e^{-i\alpha m^i \sigma^i}$$

$$T' e^{i\alpha m^i \sigma^i} T' = (-1)^F e^{-i\alpha m^i \sigma^i}$$

$\rightarrow$ $U(1)_R$ & D_8 are not direct product!

$$U(1)_R \rtimes D_8$$

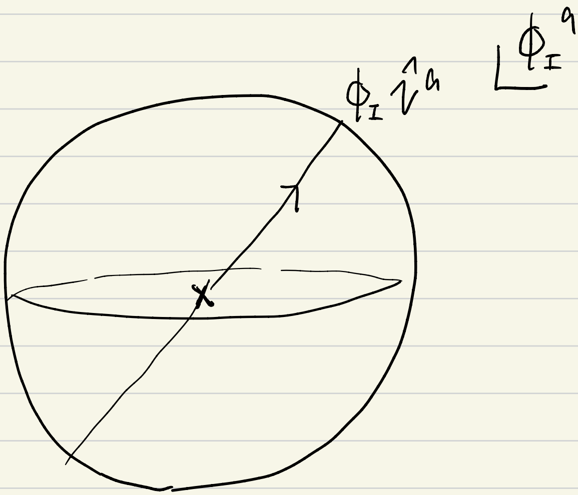
4-3. Putative phase diagram



pt's on the sphere describe the same physics

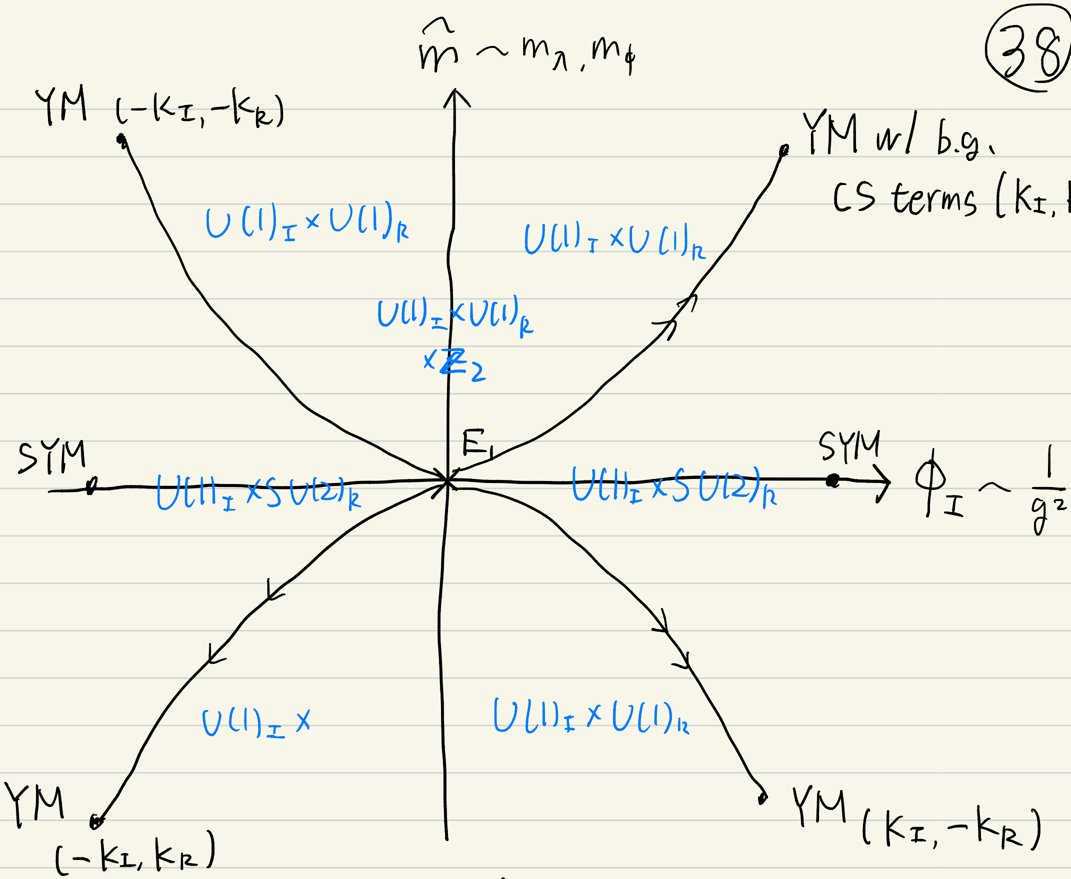
$$SU(2)_R \longrightarrow U(1)_R$$

$$A^i_R\, I^i \qquad\qquad A^i_R\, \hat{w}^i$$



Let's fix $(\hat{r}^a, \hat{w}^i)$ and vary $(\phi_I, \hat{m})$

choose particular $U(1)_I$ & $U(1)_R$



Constraints from "duality"

- $\mathbb{Z}_2^R$:

$$\phi_I \to \phi_I, \quad \hat{m} \to -\hat{m}, \quad A_I \to A_I, \quad A_R \to -A_R$$

$$(k_I \to k_I, \quad k_R \to -k_R)$$

- $\mathbb{Z}_2^I$:

$$\phi_I \to -\phi_I, \quad \hat{m} \to -\hat{m}, \quad A_I \to -A_I, \quad A_R \to A_R$$

$$(k_I \to -k_I, \quad k_R \to k_R)$$

Determination of k_R

take $\phi_I \gg 1$ $(g \sim 0)$

$$\delta h \supset \frac{i}{4} \mathrm{tr}(\bar\lambda m^i \sigma^i \lambda) = \frac{i}{4} m\, \underbrace{\mathrm{tr}(\bar\lambda \hat{w}^i \sigma^i \lambda)}$$

massive fermion
charged under $U(1)_R$
$\rightarrow$ CS level shift

adj. of $SU(2)_g$

$$\Rightarrow \quad k_R = -\frac{3}{2}\ \mathrm{sign}(m)$$

- sign(m) reflects massless gaugino at $m=0$

- $k_R \notin \mathbb{Z}$ $\rightarrow$ obstruction to gauge $U(1)_R$

  consistent w/ 5d analog. of Witten anomaly
  coming from $\pi_5(SU(2)) = \mathbb{Z}_2$
  ($\because$ we have 3 $SU(2)_R$ doublets)

Determination of k_I

another derivation WANTED

First consider SUSY regime ($\hat{m}^i = 0$)

$$F_{1\text{-loop}}(\phi_I, a) = 2\phi_I a^2 + \frac{4}{3} a^3$$

not duality inv.!

$$F_{1\text{-loop}}(-\phi_I, a+\phi_I) - F_{1\text{-loop}}(\phi_I, a) = -\frac{2}{3}\phi_I^3$$

However,

$$F(\phi_I, a) \equiv F_{1\text{-loop}}(\phi_I, a) \underline{-\frac{1}{3}\phi_I^3}$$

is invariant

$$k_I = -2$$

this can be extended to $\hat{m}^i \neq 0$

$\because$ $\nexists$ fermions charged under $U(1)_I$ changing sign(m) as varying $\hat{m}^i$

$\exists$ jump of CS levels along $h=0$!

$\longrightarrow$ phase transition

Simple possibilities :

1. SSB of $\mathbb{Z}_2$ (typically 1st order)

2. $\exists$ new gapless modes & flowing to new CFT

   flip of (k_I, k_R)

   $\longrightarrow$ $\exists$ massless d.o.f. charged under $\underbrace{U(1)_I}_{\text{non-pert. object}}$ & $U(1)_R$?

   (tensionless monopole string?)

   $\longrightarrow$ interacting CFT ?

   but might be free fermion

4-3. mixed CS terms

$U(1)_I - [U(1)_R]^2$

$$U(1)_g - [SU(2)_R]^2 \xrightarrow{\text{SUSY}} U(1)_I - [SU(2)_R]^2 \xrightarrow{\hat{m} \neq 0} U(1)_I - [U(1)_R]^2$$

$$k_{IRR} = \mathrm{sign}(\phi_I)$$

$U(1)_R - [\text{gravity}]^2$

$$\frac{1}{192\pi^2} \int A \wedge \mathrm{Tr}(R \wedge R)$$

$$k_{RGG} = -\frac{3}{2}\,\mathrm{sign}(m)$$

$U(1)_I - [\text{grav}]^2$

$$k_{IGG} = \mathrm{sign}(\phi_I)$$

4-4. Outlook

- further evidence
  (other anomalies, bootstrap, etc...)
  [cf. higher form sym.: Genolini-Tizzano]
- generalization
  (e.g. other gauge groups, adding matters)