# Application of Quantum Computation to Quantum Field Theory

# – QFT, Error Correction & Future –

**Masazumi Honda**

(本多正純)

# Plan of the lectures

(If 2nd lecture in each day ends early, then we start hands-on early)

## Day 1

- Lecture 1: introduction, basics of quantum computation
- Lecture 2: Spin system on quantum computer (QC)
- Hands-on 1: Basics on IBM's qiskit, time evolution of Ising

## Day 2

- Lecture 3: Quantum field theory (QFT) on QC
- Lecture 4:  QFT on QC, error correction & future prospects
- Hands-on 2: vacuum of Ising, Renyi entropy

# Plan of lecture 4

1. Screening, confinement & negative tension in higher charge Schwinger model

[MH-Itou-Kikuchi-Nagano-Okuda '21]
[MH-Itou-Kikuchi-Tanizaki '21]

2. String/M-theory on quantum computer (?)

3. Quantum Error Correction

4. Summary & Future prospect

# Let's consider charge-$q$ Schwinger model:

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i} q A_\mu)\psi - m\, \overline{\psi}\psi$$

Field content:

- $U(1)$ gauge field
- charge-q Dirac fermion

Let's explore

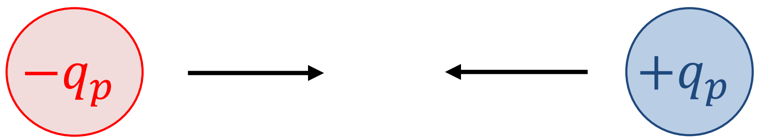
screening vs confinement problem

(next slide)

# Screening versus Confinement

Let's consider

potential between 2 heavy charged particles




Classical picture:

$$V(x) = \frac{q_p^2\, g^2}{2} x \;?$$

*Coulomb law in 1+1d*

||

*confinement*

too naive in the presence of dynamical fermions

# Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95 ]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2 \, g^2}{2\mu}(1 - e^{-q\mu x})$$

*screening*

- massive case:

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[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95 ]

- massless case:

$$\mu \equiv g/\sqrt{\pi}$$

$$V(x) = \frac{q_p^2 \, g^2}{2\mu}\left(1 - e^{-q\mu x}\right) \qquad \textit{screening}$$

- massive case: [cf. Misumi-Tanizaki-Unsal '19 ]

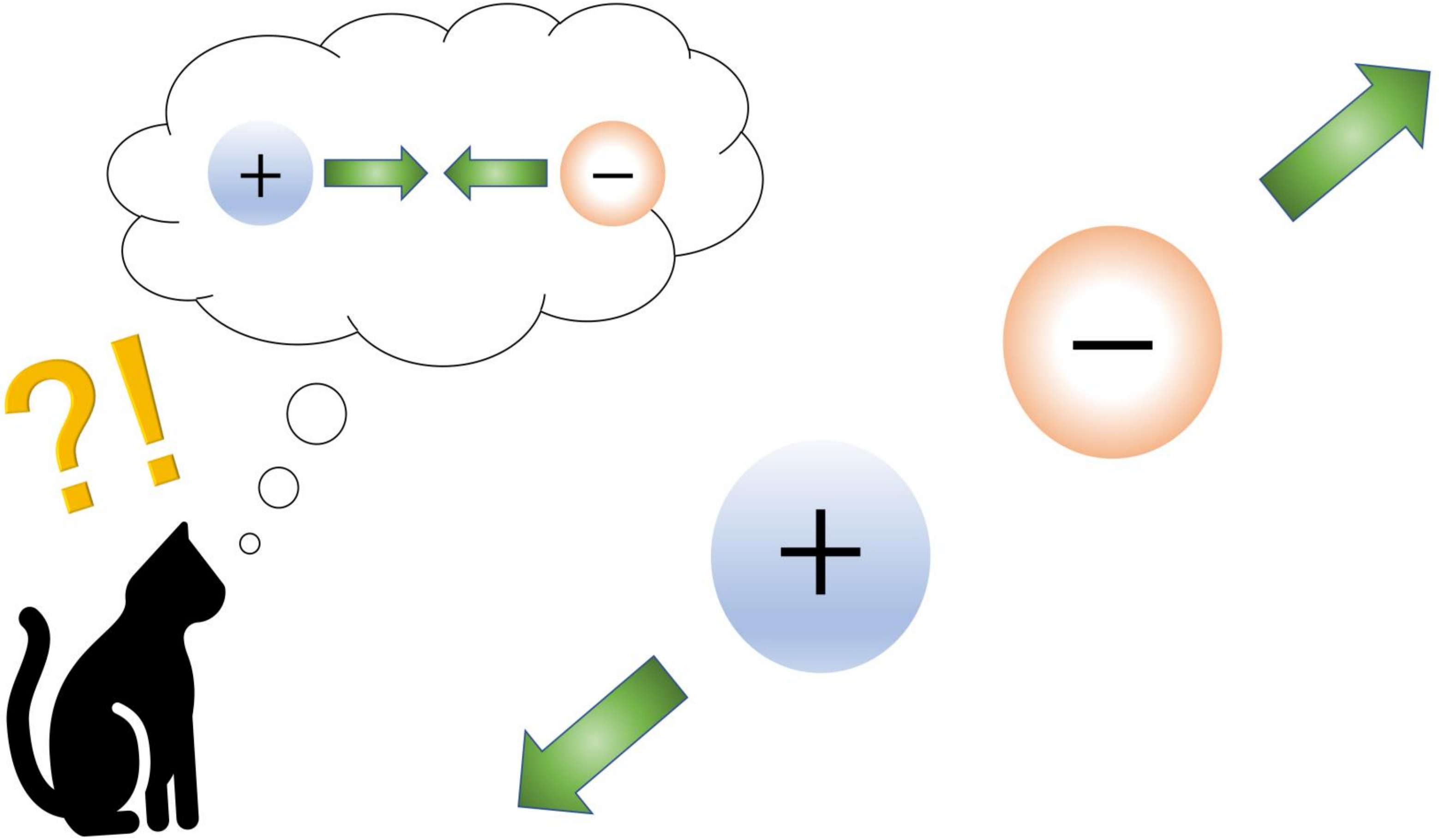
$$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right) x \qquad (\mathrm{m} \ll g,\ |x| \gg 1/g\ )$$

$$\begin{cases} = \text{Const.} & \text{for } \mathrm{q_p/q} = \mathbf{Z} \quad \textit{screening} \\ \propto x & \text{for } \mathrm{q_p/q} \neq \mathbf{Z} \quad \textit{confinement?} \end{cases}$$

*but sometimes **negative** slope!*

# That is, as changing the parameters…




Let's explore this aspect by quantum simulation!

# DETAIL アイテムの詳細

DESIGNED BY

masazumi318

## アイテム

¥1,990

- ベーシックTシャツ
- ベーシックTシャツ (2021年モデル)
- Uクルーネック T(半袖)
- ベーシックＴシャツ（長袖）
- UクルーネックＴ（ﾊﾞｯｸﾌﾟﾘﾝﾄ、長袖）
- KIDS カラークルーネックT(半袖)
- KIDS UクルーネックＴ（ﾊﾞｯｸﾌﾟﾘﾝﾄ、長袖）
- トートバッグ

# Charge-$q$ Schwinger model

## Continuum:

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\, \mathrm{i}\, \gamma^\mu (\partial_\mu + \mathrm{i}\, q\, A_\mu)\psi - m\, \overline{\psi}\psi$$

Taking temporal gauge $A_0 = 0$, (Π: conjugate momentum of $A_1$ )

$$H(x) = \frac{g^2}{2}\left(\Pi - \frac{\theta_0}{2\pi}\right)^2 - \bar{\psi}\, \mathrm{i}\, \gamma^1 (\partial_1 + \mathrm{i}\, q A_1)\psi + m\bar{\psi}\psi,$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1 \Pi - qg\bar{\psi}\gamma^0\psi$$

# Symmetries in charge-$q$ Schwinger model

$$L = \frac{1}{2g^2} F_{01}^2 + \frac{\theta_0}{2\pi} F_{01} + \overline{\psi}\,\mathrm{i}\,\gamma^\mu (\partial_\mu + \mathrm{i}\, q\, A_\mu)\psi - m\,\overline{\psi}\psi$$

- $\boldsymbol{Z_q}$ chiral symmetry for $m = 0$

—— ABJ anomaly: $U(1)_A \to \boldsymbol{Z_q}$

—— known to be spontaneously broken

- $\boldsymbol{Z_q}$ 1-form symmetry

—— remnant of $U(1)$ 1-form sym. in pure Maxwell

—— Hilbert sp. is decomposed into $q$-sectors *"universe"*

(cf. common for $(d-1)$-form sym. in $d$ dimensions)

# Put the theory on lattice

## ▪Fermion (on site):

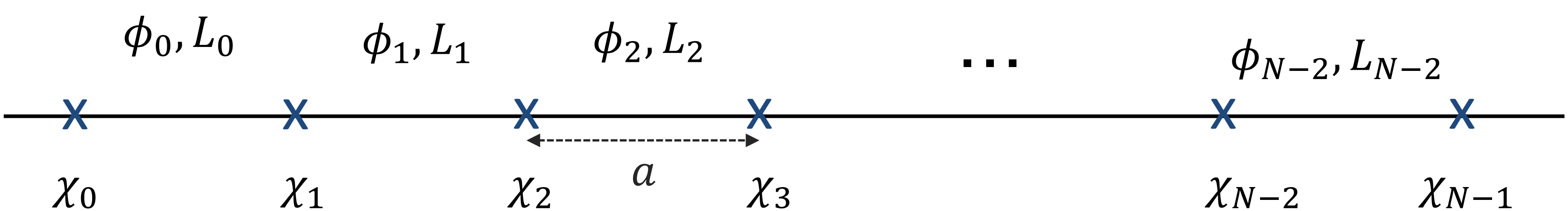
"*Staggered fermion*" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \begin{matrix} \longrightarrow \text{odd site} \\ \longrightarrow \text{even site} \end{matrix}$$

*lattice spacing*

## ▪Gauge field (on link):

$$\phi_n \leftrightarrow -agA^1(x), \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$



# Lattice theory w/ staggered fermion

## Hamiltonian:

$$H = J \sum_{n=0}^{N-2} \left( L_n + \frac{\theta_0}{2\pi} \right)^2 - \mathrm{i}w \sum_{n=0}^{N-2} \left[ \chi_n^\dagger (U_n)^q \chi_{n+1} - \mathrm{h.c.} \right] + m \sum_{n=0}^{N-1} (-1)^n \chi_n^\dagger \chi_n$$

$$\left( w = \frac{1}{2a}, J = \frac{g^2 a}{2} \right)$$

## Commutation relation:

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

## Gauss law:

$$L_n - L_{n-1} = q \left[ \chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2} \right]$$

# Eliminate gauge d.o.f.

1. Take open b.c. & solve Gauss law:

$$L_n = L_{-1} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right) \qquad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

Then,

$$H = -\mathrm{i}w\sum_{n=1}^{N-1}\left[\chi_n^\dagger \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=1}^{N}(-1)^n \chi_n^\dagger \chi_n$$
$$+ J\sum_{n=1}^{N}\left[\frac{\theta_0}{2\pi} + q\sum_{j=1}^{n}\left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2}\right)\right]^2.$$

This acts on finite dimensional Hilbert space

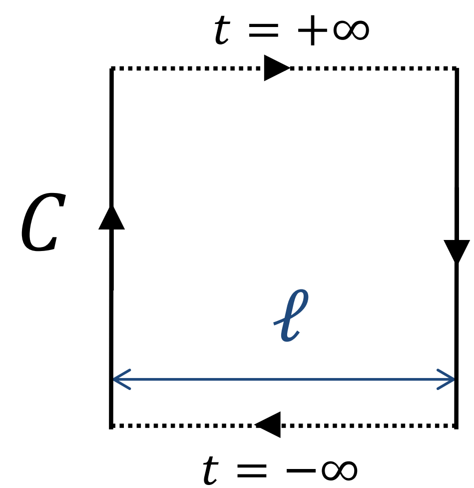
# Insertion of the probe charges

① Introduce the probe charges $\pm q_p$:

$$e^{iq_p \int_C A}$$

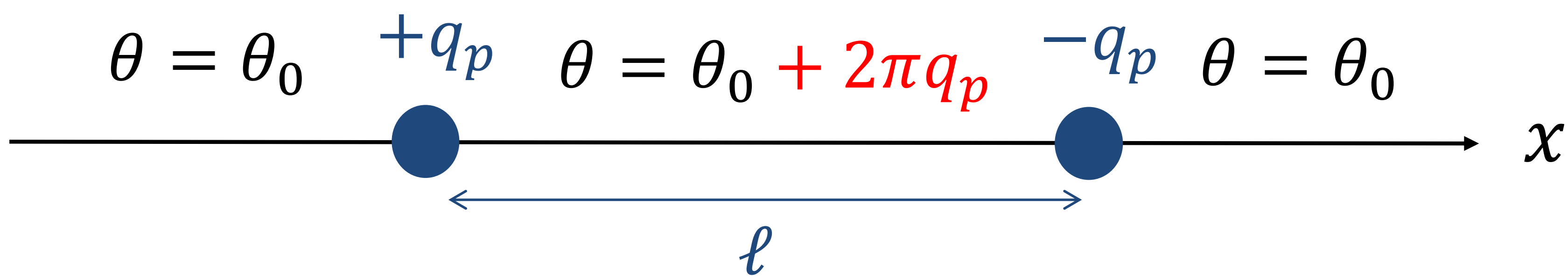
$$||$$

$$e^{iq_p \int_{S,\partial S=C} F}$$




local $\theta$-term w/ $\theta = 2\pi q_p$!!

② Include it to the action & switch to Hamilton formalism




③ Compute the ground state energy (in the presence of the probes)

# Going to spin system

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \ \{\chi_n, \chi_m\} = 0$$

This is satisfied by the operator: *"Jordan-Wigner transformation"*

[Jordan-Wigner'28]

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

($X_n, Y_n, Z_n$: $\sigma_{1,2,3}$ at site $n$)

Now the system is purely a spin system:

$$H = -\mathrm{i}w\sum_{n=1}^{N-1}\left[\chi_n^\dagger\chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=1}^{N}(-1)^n\chi_n^\dagger\chi_n + J\sum_{n=1}^{N}\left[\frac{\vartheta_n}{2\pi} + q\sum_{j=1}^{n}\left(\chi_j^\dagger\chi_j - \frac{1-(-1)^j}{2}\right)\right]^2$$

$$\Downarrow$$

$$H = J\sum_{n=0}^{N-2}\left[q\sum_{i=0}^{n}\frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi}\right]^2 + \frac{w}{2}\sum_{n=0}^{N-2}\left[X_nX_{n+1} + Y_nY_{n+1}\right] + \frac{m}{2}\sum_{n=0}^{N-1}(-1)^n Z_n$$

*Qubit description of the Schwinger model !!*

# Expectations from previous analyzes (repeated)

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95 ]

- massless case:

$$\mu \equiv g/\sqrt{\pi}$$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right)$$ *screening*

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*screening*

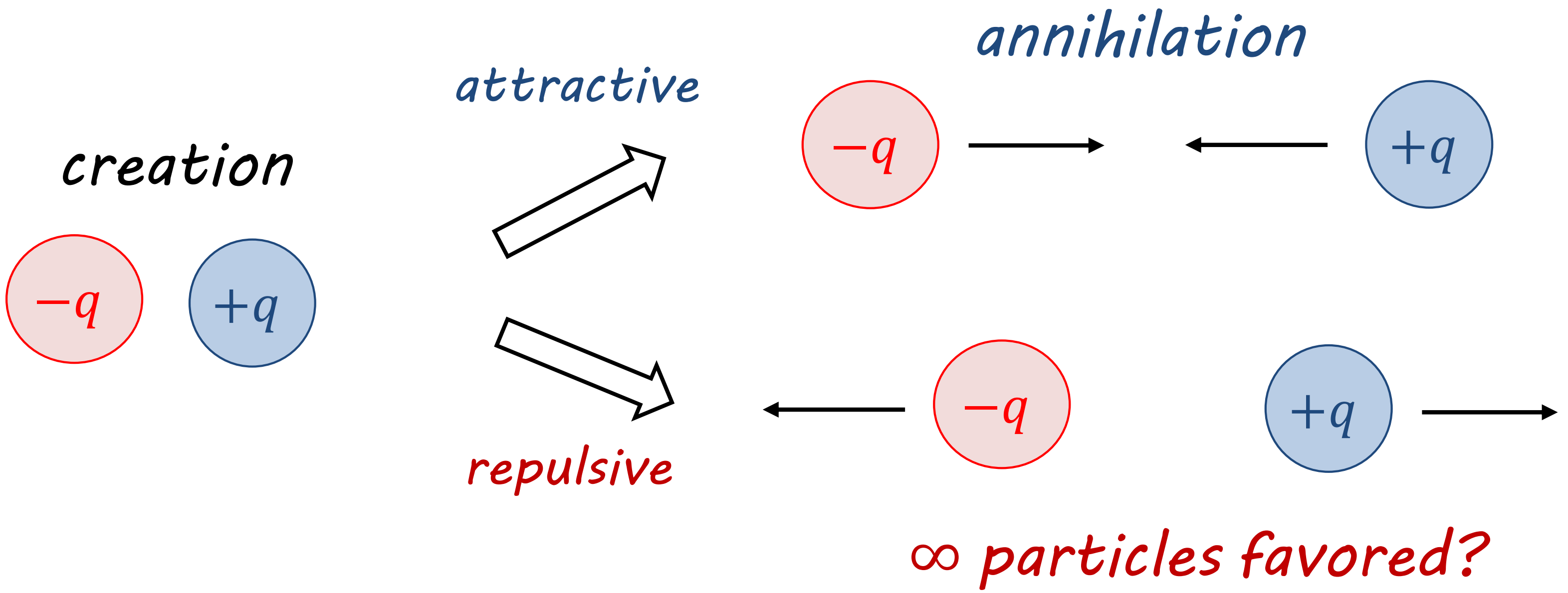
*confinement?*

*but sometimes negative slope!*

Let's explore this aspect by quantum simulation!

# FAQs on negative tension behavior

Q1. It sounds that many pair creations are favored.
Is the theory unstable?

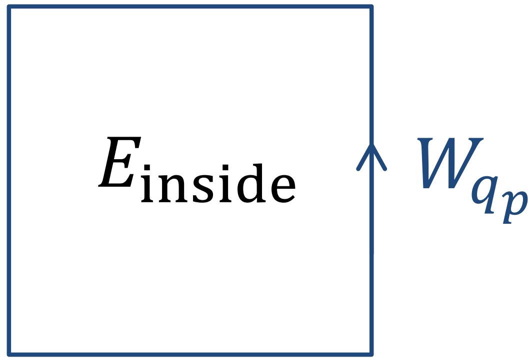



—— No. Negative tension appears only for $q_p \neq q\mathbb{Z}$.

So, such unstable pair creations do not occur.

# FAQs on negative tension behavior (cont'd)

[cf. MH-Itou-Kikuchi-Tanizaki '21]




$E_{\mathrm{outside}}\ (= E_0?\,)$

Q2. It sounds $E_{\mathrm{inside}} < E_{\mathrm{outside}}$. Strange?

—— Inside & outside are in different sectors decomposed by $Z_q$ 1-form sym.

$$\mathcal{H} = \bigoplus_{\ell=0}^{q-1} \mathcal{H}_\ell \qquad \text{"universe"}$$

$E_{\mathrm{inside}}$ & $E_{\mathrm{outside}}$ are lowest in each universe:

$$E_{\mathrm{inside}} = \min_{\mathcal{H}_{\ell+q_p}} (E)\,, \qquad E_{\mathrm{outside}} = \min_{\mathcal{H}_\ell}(E)$$

# Adiabatic state preparation of vacuum (repeated)

Step 1: Choose an initial Hamiltonian $H_0$ of a simple system whose ground state $|\mathrm{vac}_0\rangle$ is known and unique

Step 2: Introduce adiabatic Hamiltonian $H_A(t)$ s.t.

$$\begin{cases} \cdot\ H_A(0) = H_0,\ H_A(T) = H_{\mathrm{target}} \\ \cdot\ \left|\frac{dH_A}{dt}\right| \ll 1 \ \text{for}\ T \gg 1 \end{cases}$$

Step 3: Use the adiabatic theorem

If $H_A(t)$ has a unique ground state w/ a finite gap for $\forall t$, then the ground state of $H_{\mathrm{target}}$ is obtained by

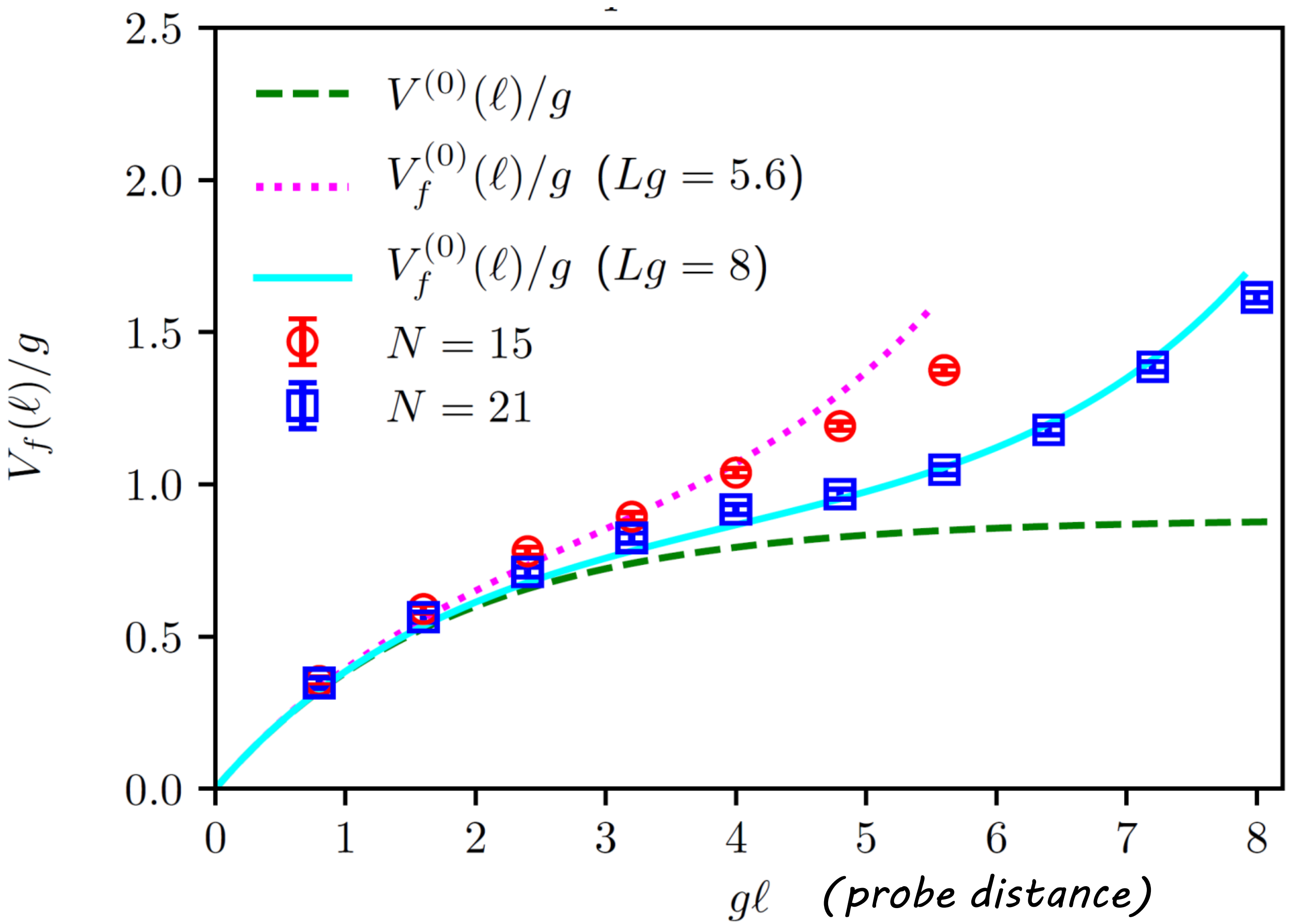
$$\boxed{|\mathrm{vac}\rangle = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\, H_A(t)\right)|\mathrm{vac}_0\rangle}$$

# Results for massless, $\theta_0 = 0$ & $q_p/q \in \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15$ & $21, T = 99, q_p/q = 1, m = 0$

Lines: analytical results in the continuum limit (finite & ∞ vols.)



# Results for massive, $\theta_0 = 0$ & $q_p/q \in \boldsymbol{Z}$

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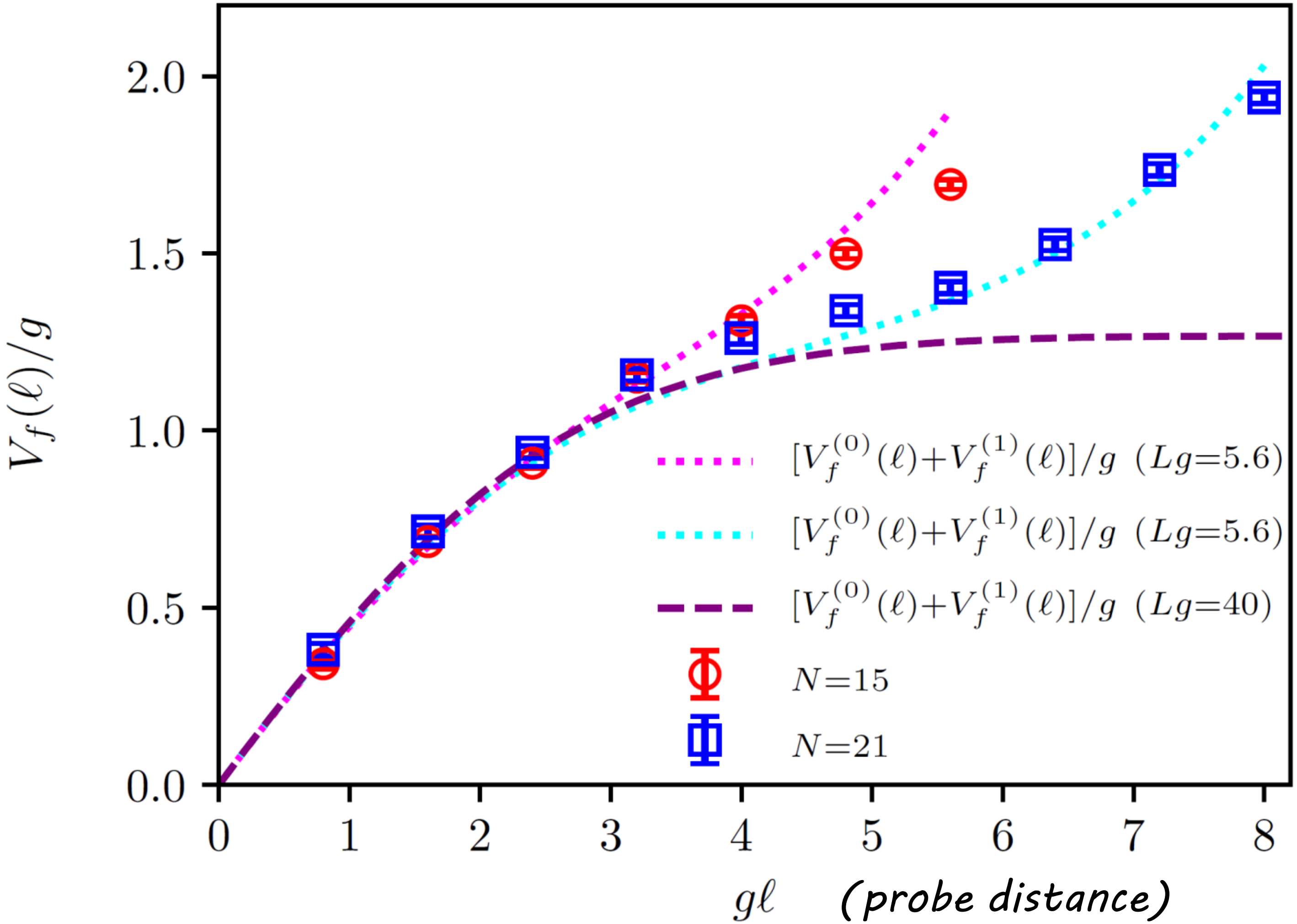

# Massless vs massive for $\theta_0 = 0$ & $q_p/q \in \boldsymbol{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

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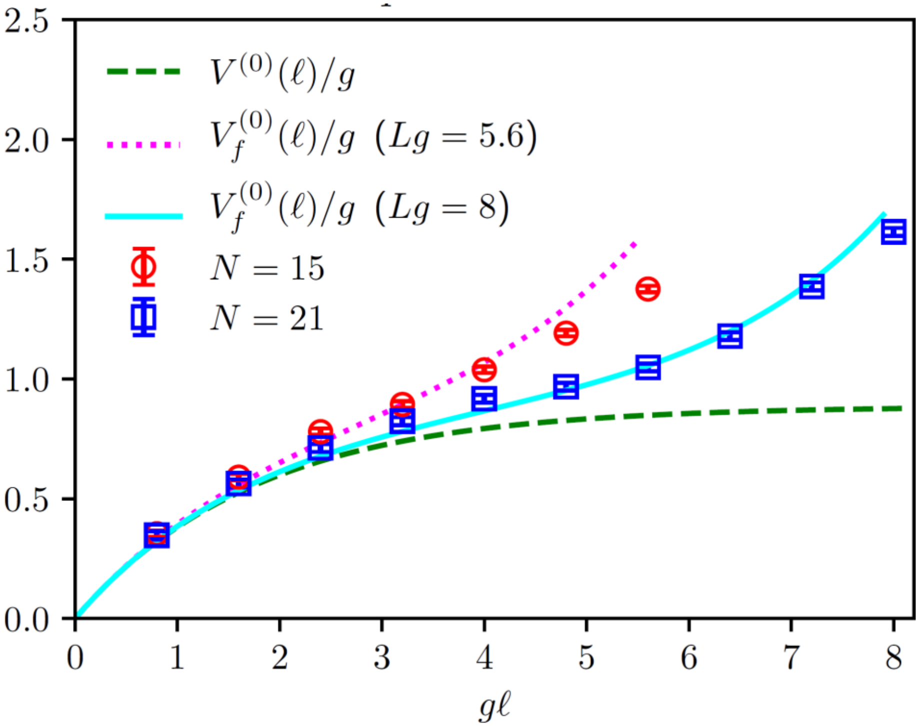
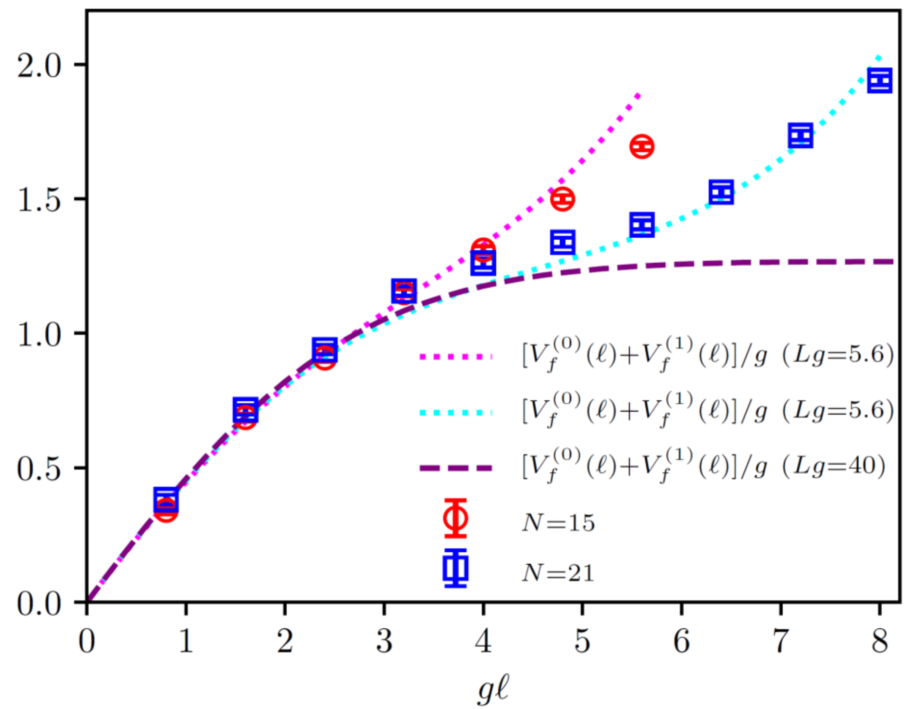
Lines: analytical results in the continuum limit (finite & $\infty$ vols.)

$q_p = 1, m = 0$




$q_p = 1, m/g = 0.2$



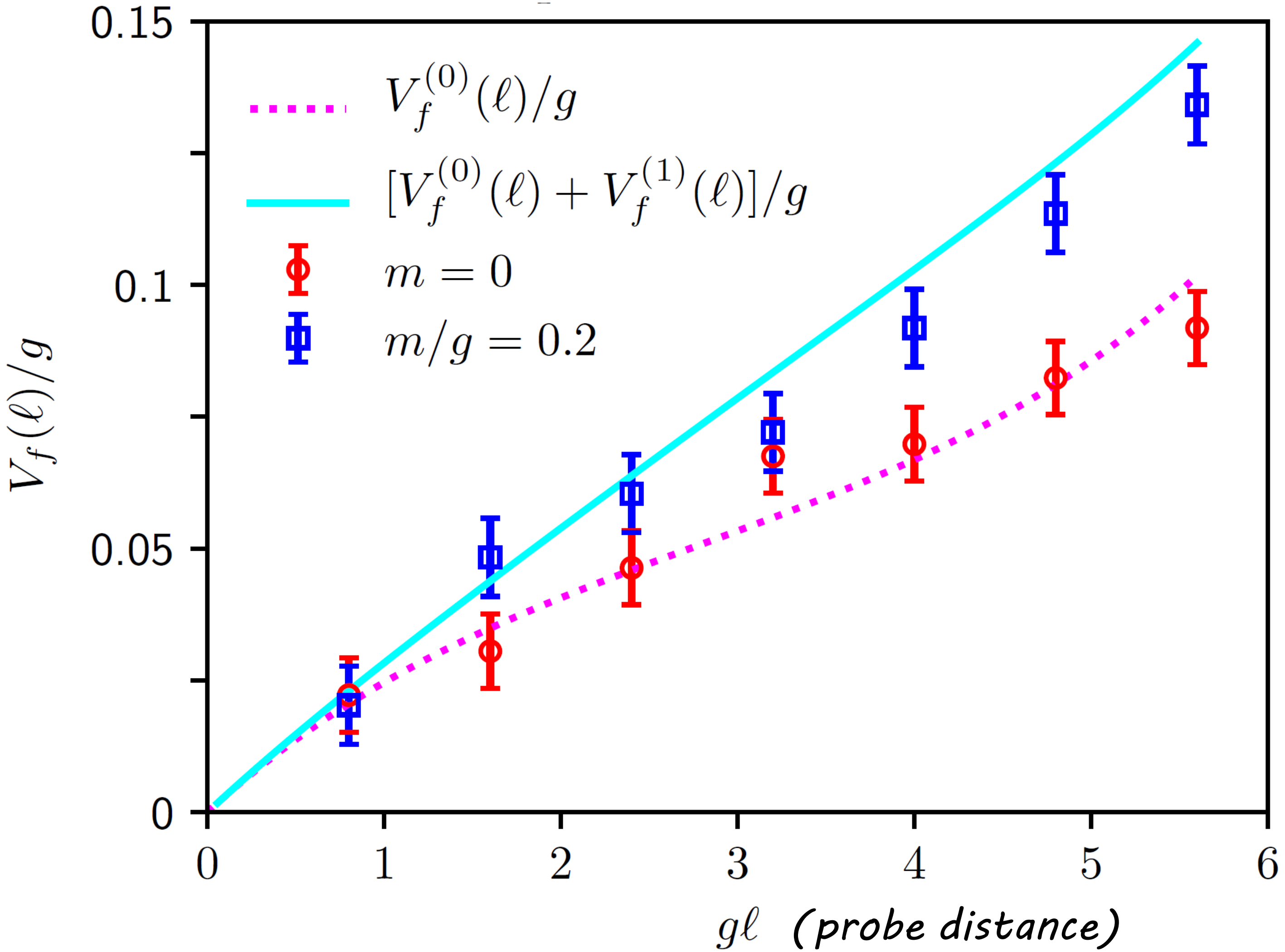

*Consistent w/ expected screening behavior*

# Results for $\theta_0 = 0$ & $q_p/q \notin \mathbf{Z}$

[MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 15, T = 99, q_p/q = 1/4, m = 0$ & $0.2$

Lines: analytical results in the continuum limit (finite & $\infty$ vol.)



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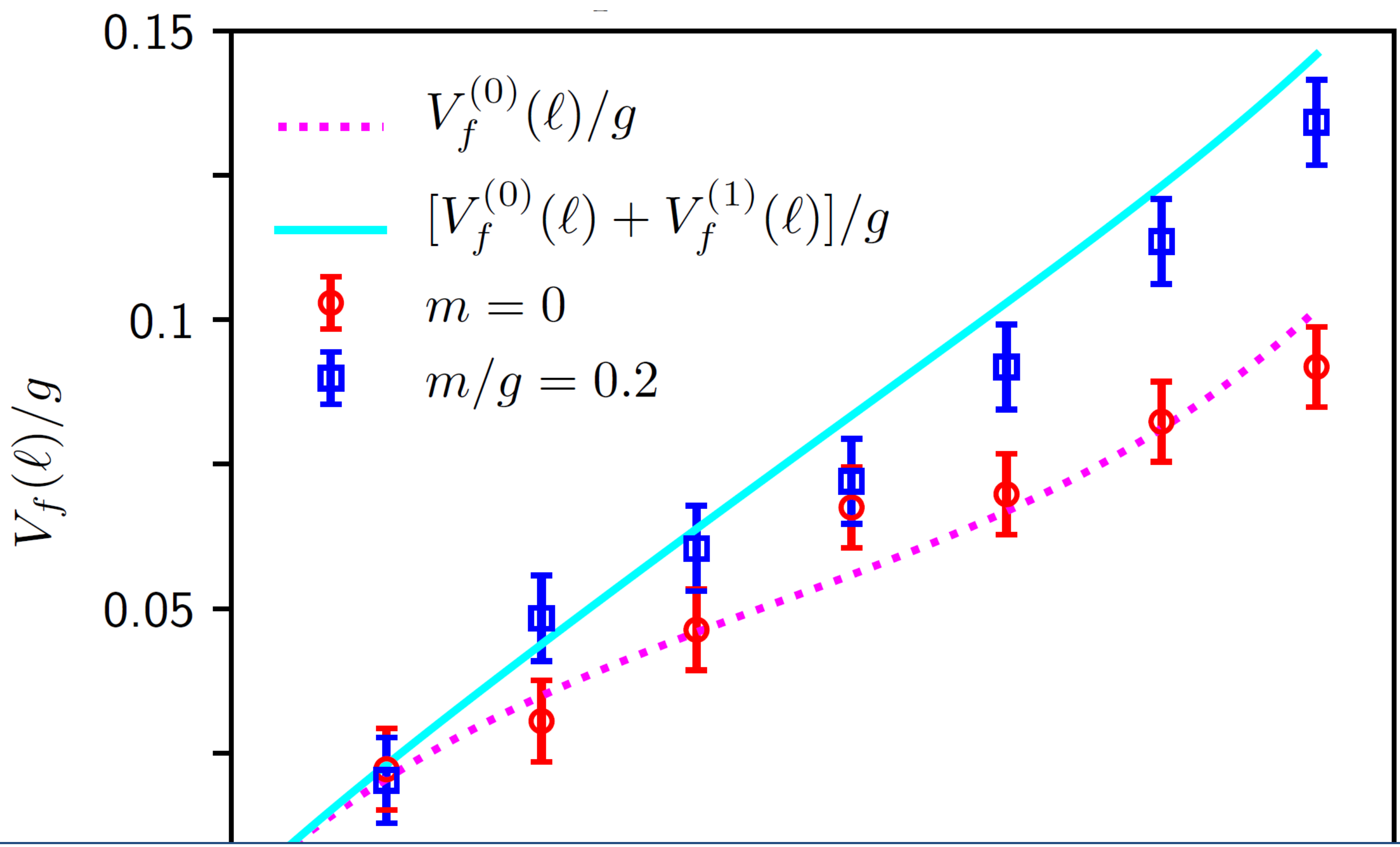


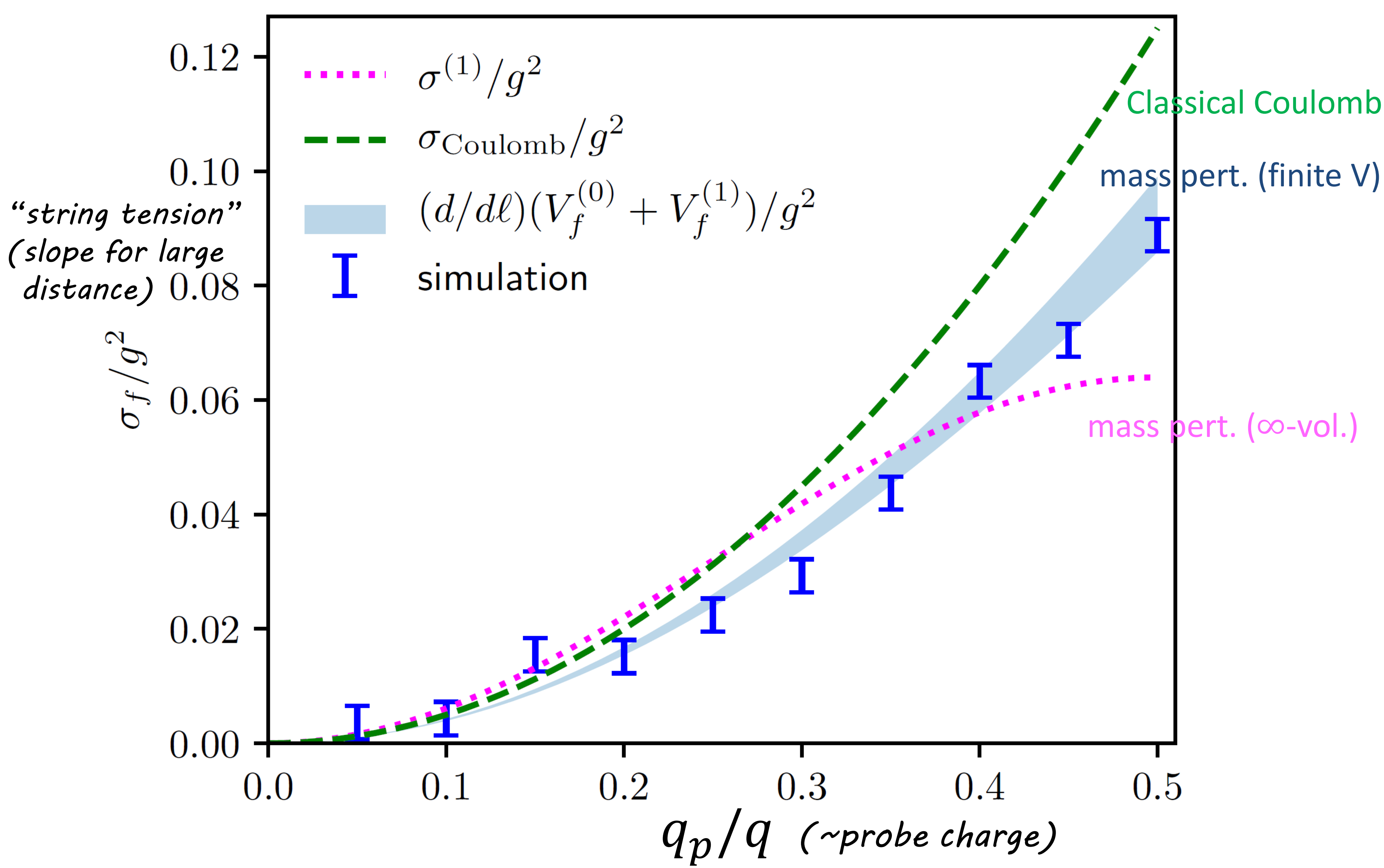

*Consistent w/ expected confinement behavior*

*-> interesting to estimate string tension for various $q_p$?*

# “String tension” for $\theta_0 = 0$

[MH-Itou-Kikuchi-Nagano-Okuda ’21]

Parameters: $g = 1, a = 0.4, N = 15, T = 99, m/g = 0.2$



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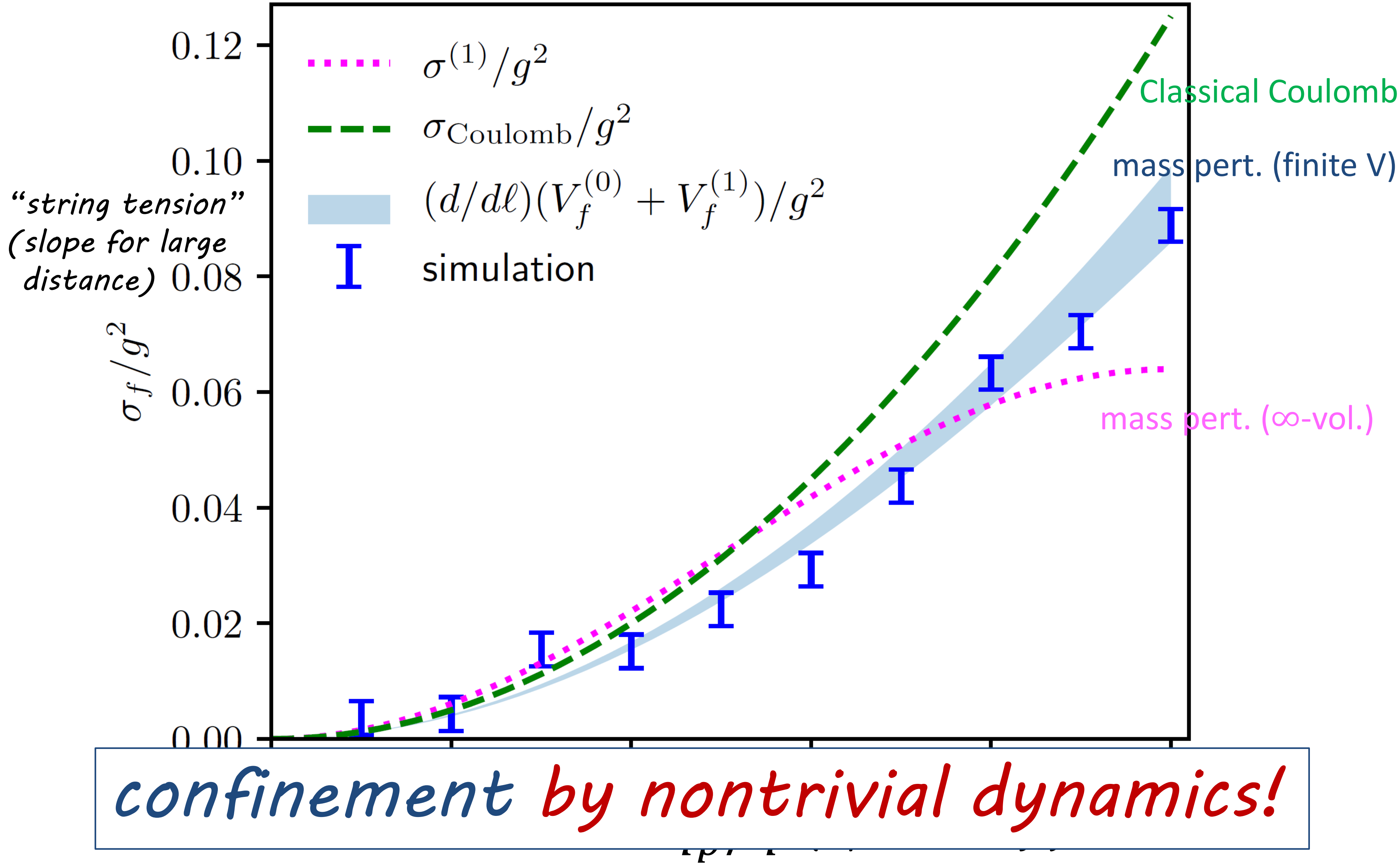
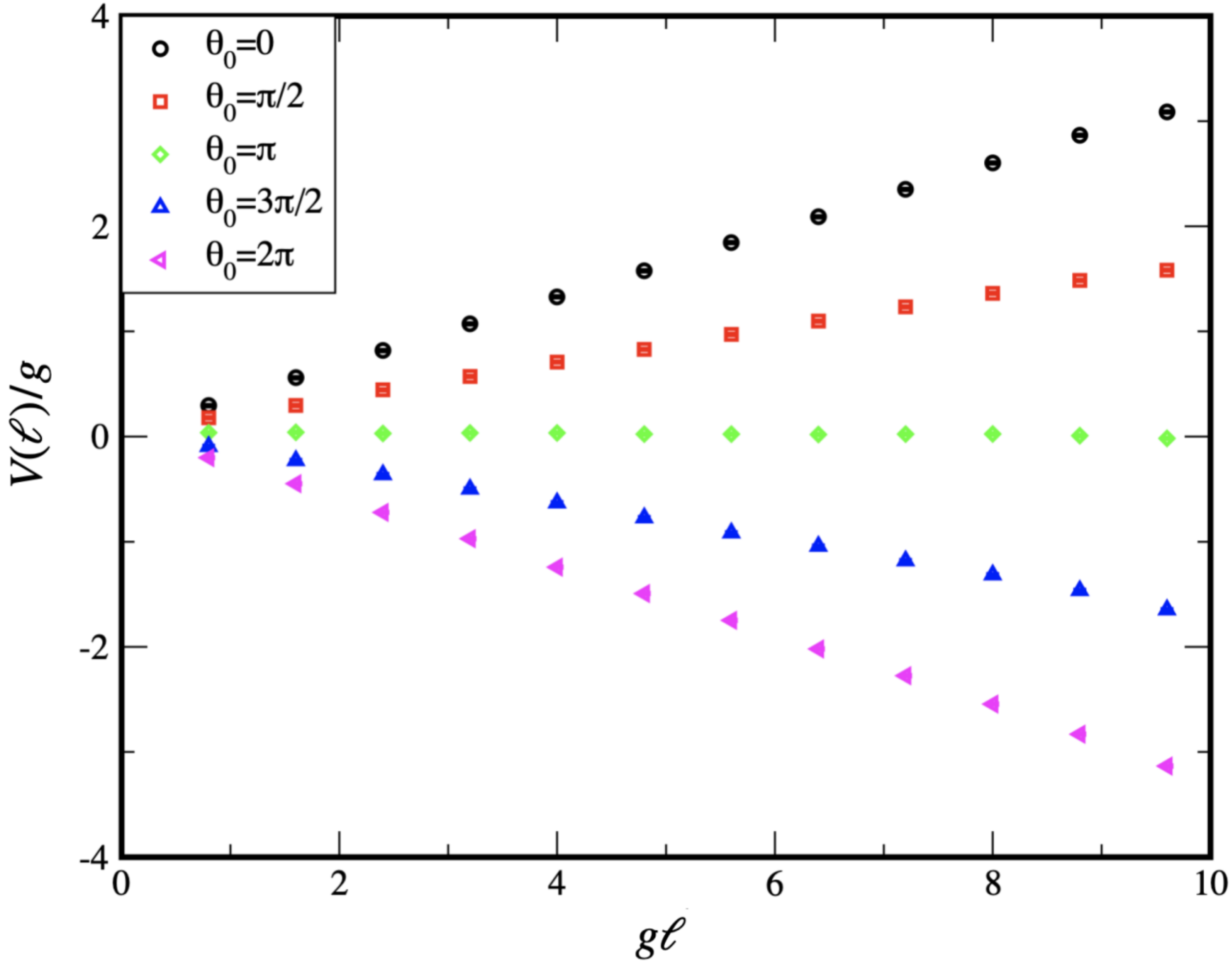



confinement by nontrivial dynamics!

# Positive / negative string tension

[MH-Itou-Kikuchi-Tanizaki ’21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$

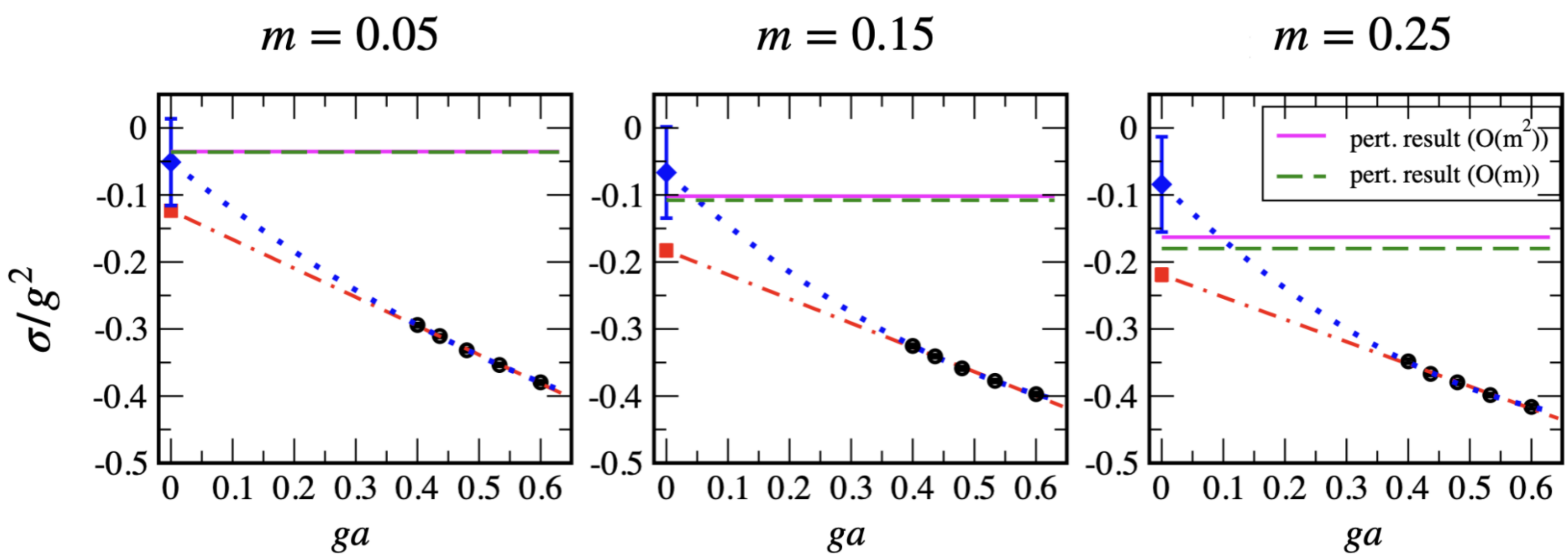



Sign(tension) changes as changing $\theta$-angle!!

# Continuum limit of string tension

[MH-Itou-Kikuchi-Tanizaki '21]

$$g = 1,\ (\mathrm{Vol.}) = 9.6/g, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$

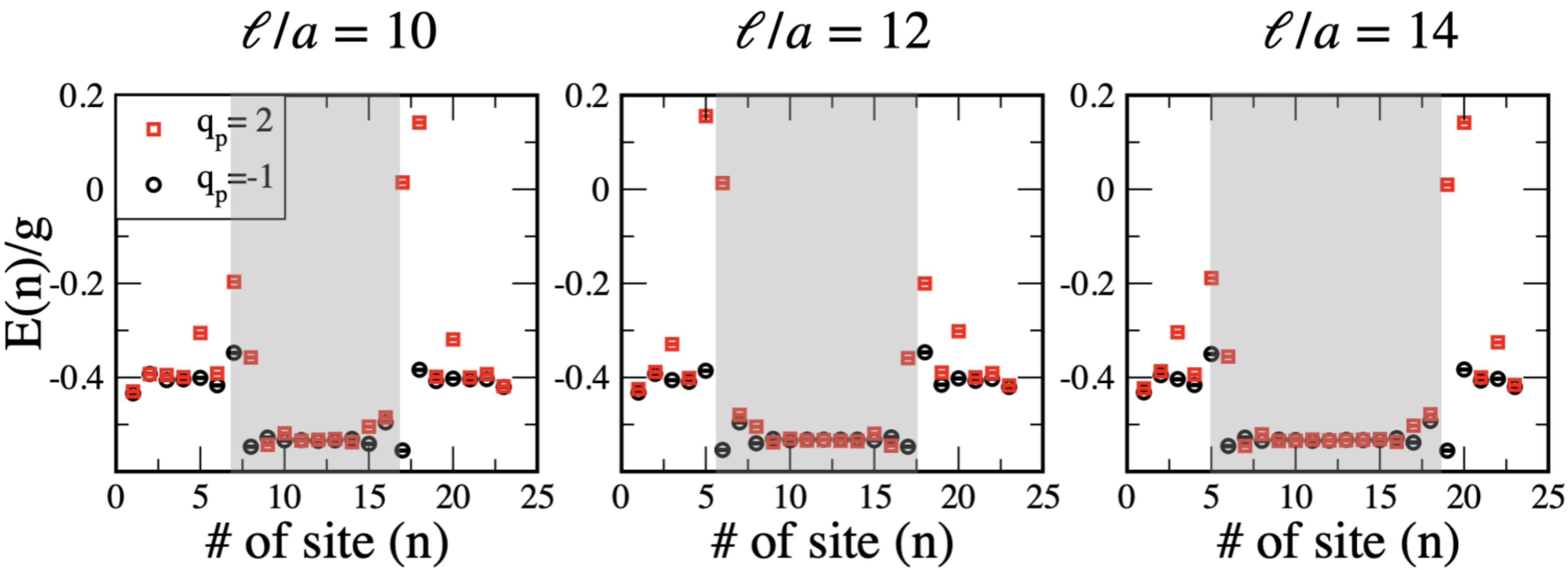



basically agrees with mass perturbation theory

# Energy density @ negative tension regime

[MH-Itou-Kikuchi-Tanizaki '21]

$$g = 1, a = 0.4, N = 25, T = 99,\ q_p/q = -1/3, m = 0.15, \theta_0 = 2\pi$$




Lower energy inside the probes!!

# Comment on adiabatic state preparation

$$\boxed{(\text{"systematic error"}) \sim \frac{1}{T\,(\text{gap})^2}}$$

😄 **Advantage:**

- guaranteed to be correct for $T \gg 1$ & $\delta t \ll 1$
  if $H_A(t)$ has a unique gapped vacuum
- can directly get excited states under some conditions

☹️ **Disadvantage:**

- doesn't work for degenerate vacua
- costly — likely requires many gates

$\Rightarrow$ more appropriate for FTQC than NISQ

# Towards “quantum supremacy”?

The problem involves only ground state in 1+1D

$\rightarrow$ Tensor Network is better $\rightarrow$ able to take $N = \mathcal{O}(100)$

[ MH-Itou-Tanizaki ’22]

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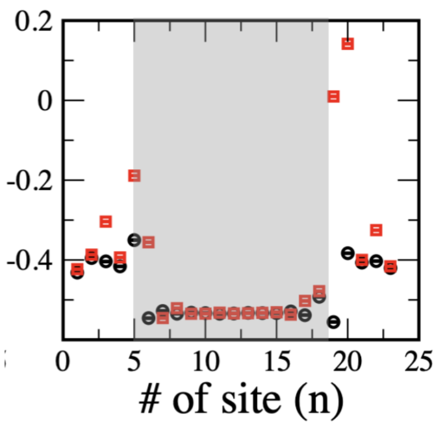
→ Tensor Network is better → able to take $N = \mathcal{O}(100)$

[ MH-Itou-Tanizaki '22]

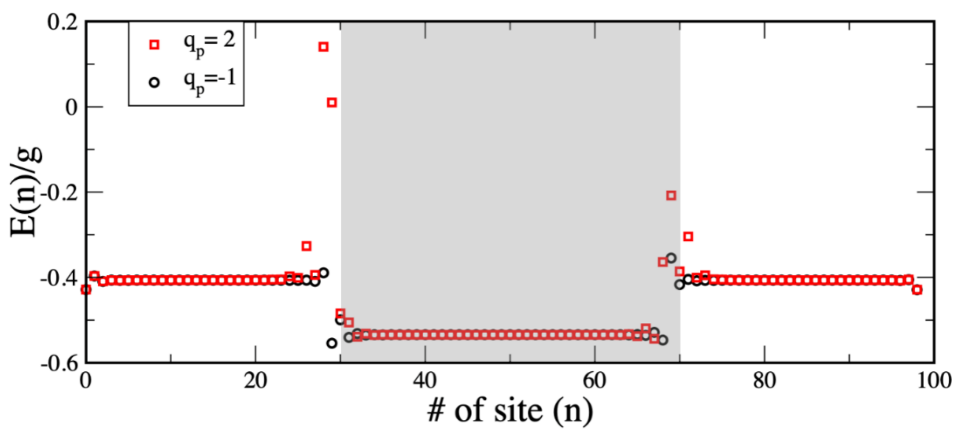
**Adiabatic state preparation:**

$\ell/a = 14$




**Tensor Network (DMRG):**

$\ell/a = 40, \quad N = 101$



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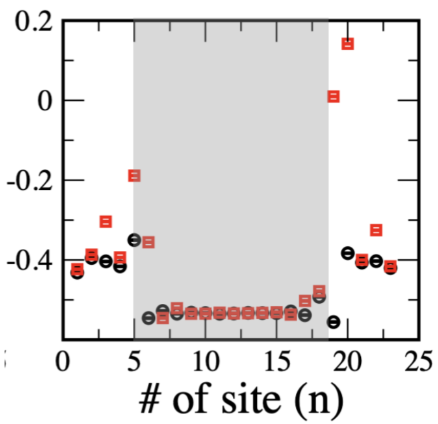


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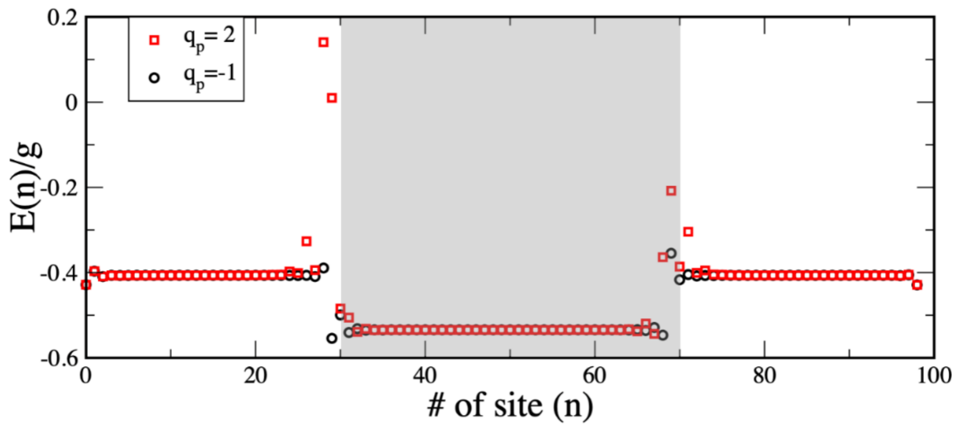


should study problems not efficiently simulated by MC & TN:

- long time evolution, many pt. function, non-local op.
- systems w/ strong entanglement (matrix models?)

# Plan of lecture 4

1. Screening, confinement & negative tension in higher charge Schwinger model

[MH-Itou-Kikuchi-Nagano-Okuda '21]
[MH-Itou-Kikuchi-Tanizaki '21]

2. String/M-theory on quantum computer (?)

3. Quantum Error Correction

4. Summary & Future prospect

# Attempts to put "QGs" on quantum computer

- SYK model [Garcia-Alvarez-Egusquiza-Lamata-Campo-Sonner-Solano '17, etc…]

  —— dual to Jackiw-Teitelboim gravity (1+1d)

- BMN matrix model (gauged matrix QM) [Gharibyan-Hanada-MH-Liu '20]

  —— related to various setups of string/M-theory

- Loop quantum gravity (?) [cf. Cohen-Brady-Huang-Liu-Qu-Dowling-Han '20]

  —— another candidate of QG

(I haven't seen direct argument for string theory yet)

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(I haven't seen direct argument for string theory yet) arXiv > hep-th > arXiv:2303.11534

## High Energy Physics - Theory

[Submitted on 21 Mar 2023]

# A simple quantum system that describes a black hole

Juan Maldacena

During the past decades, theorists have been studying quantum mechanical systems that are belie
Majorana fermions. It is conjectured to describe a black hole in an emergent universe governed by
necessary to see some black hole features.

Comments: 12 pages, 1 figure
Subjects: **High Energy Physics - Theory (hep-th)**; General Relativity and Quantum Cosmology (gr-qc); Quantum
Cite as: arXiv:2303.11534 **[hep-th]**
(or arXiv:2303.11534v1 **[hep-th]** for this version)
https://doi.org/10.48550/arXiv.2303.11534

# Matrix Quantum Mechanics (QM)

|| literally!

# QM of matrices

Ex.)

# Matrix Quantum Mechanics (QM)

$||$ literally!

# QM of matrices

Ex.) One Hermitian matrix QM: ($X(t)$: Hermitian matrix)

- Path integral formalism

$$L = \mathrm{Tr}\left[\frac{1}{2}\dot{X}^2 - V(X)\right], \qquad Z = \int DX\, e^{i\int dt\, L}$$

- Operator formalism

$$H = \mathrm{Tr}\left[\frac{1}{2}P^2 + V(X)\right], \qquad \left[X_{ij}, P_{k\ell}\right] = i\delta_{ik}\delta_{j\ell}$$

Technically,

special case of many particle QM

# BMN matrix model ($U(N)$ gauged matrix QM)

[Berenstein-Maldacena-Nastase '02]

$$L = \frac{1}{g^2}\mathrm{Tr}\Biggl\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{i\mu}{6}\epsilon^{ijk}X_i X_j X_k$$

$$+\frac{i}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_I [X_I, \Psi] - \frac{i\mu}{8}\Psi^\dagger \gamma_{123}\Psi\Biggr\},$$

- (0+1) dim. $U(N)$ gauge theory
- all the fields are $N \times N$ Hermitian matrices
- $X_I$: bosonic matrices ($I = 1, \cdots, 9$)
- $\Psi$: 16 component Majorana-Weyl fermion
- $i = 1,2,3, a = 4, \cdots, 9$

# BMN matrix model (cont'd)

[Berenstein-Maldacena-Nastase '02]

$$L = \frac{1}{g^2}\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{i\mu}{6}\epsilon^{ijk}X_i X_j X_k \right.$$
$$\left. + \frac{i}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_I [X_I, \Psi] - \frac{i\mu}{8}\Psi^\dagger \gamma_{123}\Psi \right\},$$

related to various interesting "stringy" theories:

- M-theory on pp-wave spacetime
- 3d $\mathcal{N} = 8$ SYM on $\boldsymbol{R} \times S^2$ ~ D2-branes in IIA string theory
- 4d $\mathcal{N} = 4$ SYM on $\boldsymbol{R} \times S^3$ ~ D3-branes in IIB string theory

[Ishii-Ishiki-Shimasaki-Tsuchiya '08, etc...]

- 6d $\mathcal{N} = (2,0)$ theory on $\boldsymbol{R} \times S^5$ ~ M5-branes in M-theory

[Maldacena-Sheikh-Jabbari-Van Raamsdonk '02]

- holographic duals

# Operator formalism

$$\Psi = \begin{pmatrix} \psi_{Ip} \\ \epsilon_{pq}\psi^{\dagger Iq} \end{pmatrix}$$

$$\hat{H} = \mathrm{Tr}\left\{ \frac{1}{2}(\hat{P}_I)^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2 + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k \right.$$
$$\left. + g\hat{\psi}^{\dagger Ip}\sigma_p^{iq}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip}g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip} \right\}$$

## Commutation relations:

($\alpha, \beta$: gauge indices)

$$\left[\hat{X}_{I\alpha}, \hat{P}_{J\beta}\right] = i\delta_{IJ}\delta_{\alpha\beta}, \qquad \left\{\hat{\psi}^{\dagger Ip\alpha}, \hat{\psi}^{\beta}_{Jq}\right\} = \delta_{IJ}\delta^{pq}\delta^{\alpha\beta}$$

## Gauss law:

$$\hat{G}_\alpha|\mathrm{phys}\rangle = 0 \quad \text{w/} \quad \hat{G}_\alpha = \sum_{\beta,\gamma=1}^{N^2}\left(\sum_{I=1}^{9}\hat{X}_I^\beta\hat{P}_I^\gamma - i\sum_{I,p}\hat{\psi}^{\dagger Ip\alpha}\hat{\psi}^\gamma_{Ip}\right)$$

We can regularize it as in scalar field theory

[Gharibyan-Hanada-MH-Liu ’20]

# The essence is common w/ single particle QM

(repeated)

$$\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{\omega^2}{2}\hat{x}^2 + V_{\text{int}}(\hat{x})$$

Most naïve approach = truncation in harmonic osc. basis:

$$\hat{a} = \sqrt{\frac{\omega}{2}}\,\hat{x} + \frac{i}{\sqrt{2\omega}}\,\hat{p} \;=\; \sum_{n=0}^{\infty} \sqrt{n+1}\,|n\rangle\langle n+1|$$

$$\xrightarrow[\textit{regularize!}]{} \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

Then replace $\hat{p}$ & $\hat{x}$ by

$$\hat{x}\Big|_{\text{regularized}} \equiv \frac{1}{\sqrt{2\omega}}(\hat{a}+\hat{a}^\dagger)\Big|_{\text{regularized}}$$

$$\hat{p}\Big|_{\text{regularized}} \equiv \frac{1}{i}\sqrt{\frac{\omega}{2}}\,(\hat{a}-\hat{a}^\dagger)\Big|_{\text{regularized}}$$

# The essence is common w/ single particle QM (Cont'd)

(repeated)

$$\hat{a}\Big|_{\text{regularized}} = \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

We can rewrite the Fock basis in terms of qubits:

$$|n\rangle = |b_{K-1}\rangle|b_{K-2}\rangle\cdots|b_0\rangle \qquad K \equiv \log_2\Lambda$$

$$n = \mathrm{b}_{\mathrm{K}-1}2^{\mathrm{K}-1} + \mathrm{b}_{\mathrm{K}-2}2^{\mathrm{K}-2} + \cdots + b_0 2^0$$ (binary representation)

Then,

$$|n\rangle\langle n+1| = \otimes_{\ell=0}^{K-1}\,\underbrace{(|b'_\ell\rangle\langle b_\ell|)}_{\textit{either one of}}$$

$$\left(\begin{array}{ll} |0\rangle\langle 0| = \dfrac{1_2-\sigma_z}{2}, & |1\rangle\langle 1| = \dfrac{1_2+\sigma_z}{2}, \\ |0\rangle\langle 1| = \dfrac{\sigma_x+i\sigma_y}{2}, & |1\rangle\langle 0| = \dfrac{\sigma_x-i\sigma_y}{2} \end{array}\right)$$

# Computational costs

## # of qubits:

- Single particle QM w/ truncation $\Lambda$ requires $\log_2 \Lambda$ qubits
- The BMN model has 9 scalars & 16 component real fermion which are $N \times N$ matrices

$$\Rightarrow \quad 9N^2 \log_2 \Lambda + 8N^2 \ \text{qubits}$$

## # of spin ops. in Hamiltonian:

- each annihilation/creation op. has less than $\mathcal{O}(\Lambda^2)$ spin ops.
- we have 4-pt. interaction at most
- $^\exists \mathcal{O}(N^4)$ combinations regarding the color indices

$$\Rightarrow \quad < \mathcal{O}(\Lambda^8 N^4) \ \text{spin ops.}$$

# # of qubits to simulate black hole

BMN w/ truncation has [Maldacena '23]

$$\boxed{9N^2 \log_2 \Lambda + 8N^2 \text{ qubits}}$$

What $N$ & $\Lambda$ needed to simulate black hole?

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What $N$ & $\Lambda$ needed to simulate black hole?

- MC study suggests BH entropy is (approximately) reproduced at

$$N = 16, \frac{T}{(g^2 N)^{1/3}} = 0.3, \frac{\mu}{T} = 1.6$$

[Patelpudis-Bergner-Hanada-Rinaldi-Schafer -Vranas-Watanabe-Bpdendorfer '22]

- Important energy levels should satisfy about $E_n < \mathcal{O}(T)$

$$\longrightarrow \quad \Lambda \sim 4$$

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- Important energy levels should satisfy about $E_n < \mathcal{O}(T)$

$$\longrightarrow \quad \Lambda \sim 4$$

Totally, we need

$$\boxed{\sim 7000 \text{ qubits}}$$

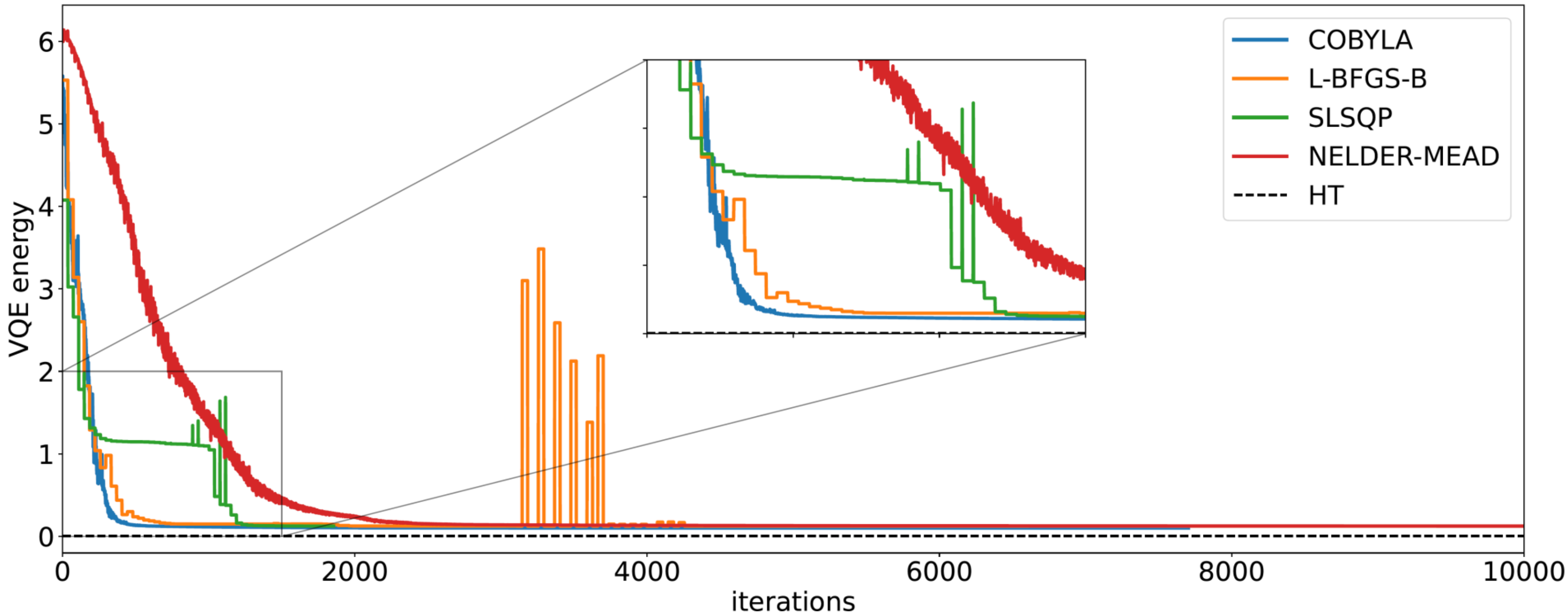
(similar to the condition for "quantum supremacy" in factoring integer )

# An implementation for "$SU(2)$ mini-BMN"

[Rinaldi-Han-Hassan-Feng-Nori-McGuigan-Hanada '21]

$$\hat{H} = \mathrm{Tr}\left(\frac{1}{2}\hat{P}_I^2 - \frac{g^2}{4}[\hat{X}_I,\hat{X}_J]^2 + \frac{g}{2}\hat{\bar{\psi}}\Gamma^I[\hat{X}_I,\hat{\psi}] - \frac{3i\mu}{4}\hat{\bar{\psi}}\hat{\psi} + \frac{\mu^2}{2}\hat{X}_I^2\right) - (N^2-1)\mu$$

## Ground state energy by VQE on simulator $(\Lambda = 2)$



# Plan of lecture 4

1. Screening, confinement & negative tension in higher charge Schwinger model

[MH-Itou-Kikuchi-Nagano-Okuda ’21]
[MH-Itou-Kikuchi-Tanizaki ’21]

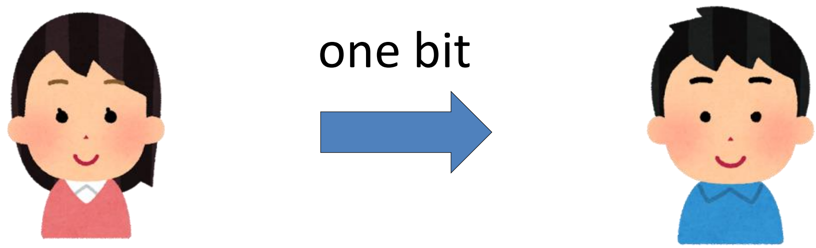
2. String/M-theory on quantum computer (?)

3. Quantum Error Correction

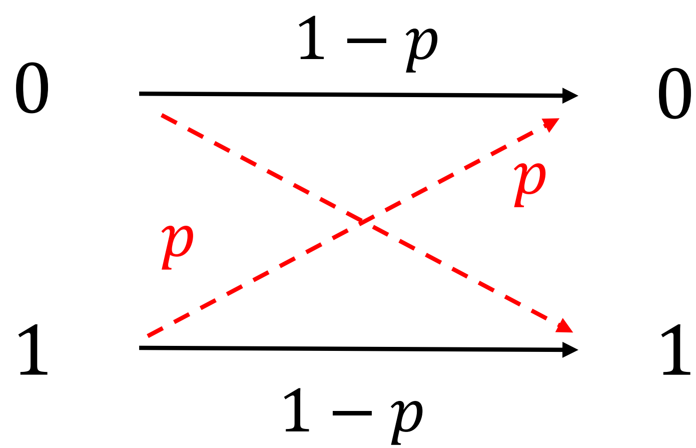
4. Summary & Future prospect

# Classical error

Suppose we’d like to send a bit:

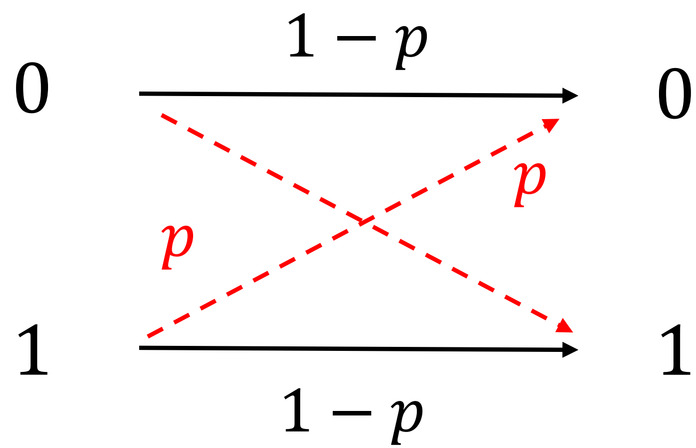



But we have “error” in probability $p$ due to noise:




How can we correct the “error”?

# Classical error correction: “Majority voting”




① Duplicate the bit (encoding):

$$0 \to 000, \qquad\qquad 1 \to 111$$

② Error detection & correction by “majority voting”:

$$001 \to 000, \quad 011 \to 111, \quad \text{etc...}$$

③ But it fails if $^\exists$multiple “errors”:

$$P_{\text{failed}} = 3p^2(1-p) + p^3 \qquad \text{(improved if } p < 1/2\text{)}$$

# Quantum errors

$$|\psi\rangle \;-\!\!\boxed{\text{Error}}\!\!-\; |\psi'\rangle$$

## Differences from classical error:

- Errors are not only bit flips

  —— can be any unitary operators (continuous)

- Measurement destroys states

  —— projected to classical number (or smaller vector)

- No-cloning theorem

  —— impossible to make copies

Nevertheless,

$\exists$ systematic ways to correct errors

# Classification of errors

Let's consider single qubit + environment

Error: 
$$\begin{cases} |0\rangle \otimes |0\rangle_E \rightarrow |0\rangle \otimes |e_{00}\rangle_E + |1\rangle \otimes |e_{01}\rangle_E \\ |1\rangle \otimes |0\rangle_E \rightarrow |0\rangle \otimes |e_{10}\rangle_E + |1\rangle \otimes |e_{11}\rangle_E \end{cases}$$

For any state $|\psi\rangle \equiv c_0|0\rangle + c_1|1\rangle$,

$$|\psi\rangle \otimes |0\rangle_E \rightarrow c_0(|0\rangle \otimes |e_{00}\rangle_E + |1\rangle \otimes |e_{01}\rangle_E) + c_1(|0\rangle \otimes |e_{10}\rangle_E + |1\rangle \otimes |e_{11}\rangle_E)$$

$$= (c_0|0\rangle + c_1|1\rangle) \otimes \frac{|e_{00}\rangle_E + |e_{11}\rangle_E}{2} + (c_0|0\rangle - c_1|1\rangle) \otimes \frac{|e_{00}\rangle_E - |e_{11}\rangle_E}{2}$$
$$+(c_0|1\rangle + c_1|0\rangle) \otimes \frac{|e_{01}\rangle_E + |e_{10}\rangle_E}{2} + (c_0|1\rangle - c_1|0\rangle) \otimes \frac{|e_{01}\rangle_E - |e_{10}\rangle_E}{2}$$

$$\equiv \underbrace{|\psi\rangle \otimes |e_I\rangle_E}_{\text{nothing}} + \underbrace{X|\psi\rangle \otimes |e_X\rangle_E}_{\text{bit flip}} + \underbrace{Z|\psi\rangle \otimes |e_Z\rangle_E}_{\text{phase flip}} + \underbrace{Y|\psi\rangle \otimes |e_I\rangle_E}_{\text{both } (Y = iXZ)}$$

# Classification of errors (cont'd)

## Single qubit case:

$(Y = iXZ)$

$2 \times 2$ unitary matrix spanned by $\{I, X, Z, Y\}$

## 2-qubit case:

$4 \times 4$ unitary matrix spanned by $\{I, X, Z, Y\} \otimes \{I, X, Z, Y\}$

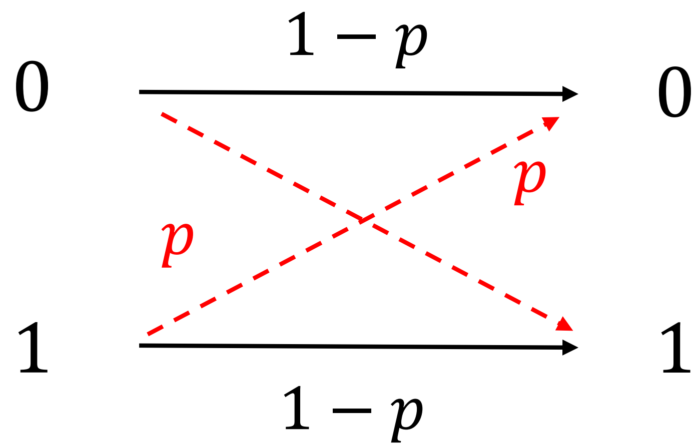
## $n$-qubit case:

$2^n \times 2^n$ unitary matrix spanned by $\{I, X, Z, Y\}^{\otimes n}$

Error = Combination of bit flip & phase flip

# Quantum error correction for bit flip

Classical bit flip:




Quantum bit flip:

$$|\psi\rangle \rightarrow \quad X|\psi\rangle \qquad \text{w/ probability } p$$

$$(c_0|0\rangle + c_1|1\rangle \;\rightarrow\; c_0|1\rangle + c_1|0\rangle)$$

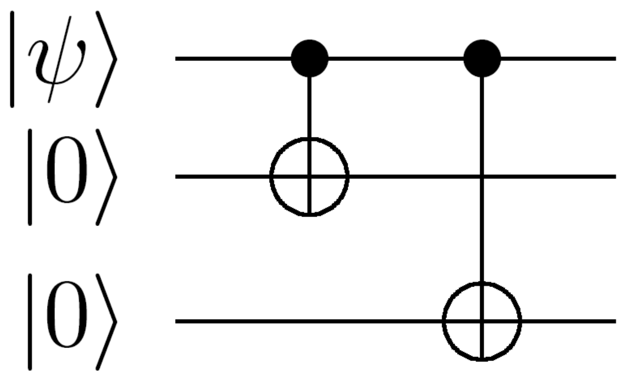
Can we extend the idea of “majority voting”?

# Step 1/3: encoding ("3-qubit bit flip code")

Quantum bit flip:

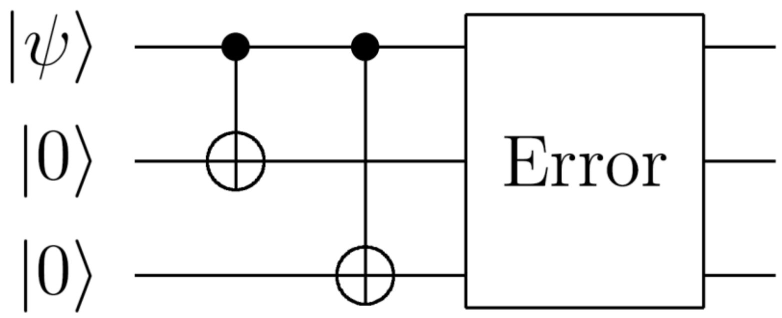
$$|\psi\rangle \to \quad X|\psi\rangle \qquad \text{w/ probability } p$$

Encoding:




$$|\psi\rangle \equiv c_0|0\rangle + c_1|1\rangle \quad \longrightarrow \quad c_0|000\rangle + c_1|111\rangle$$

# Step 2/3: Error detection



## Encoding:

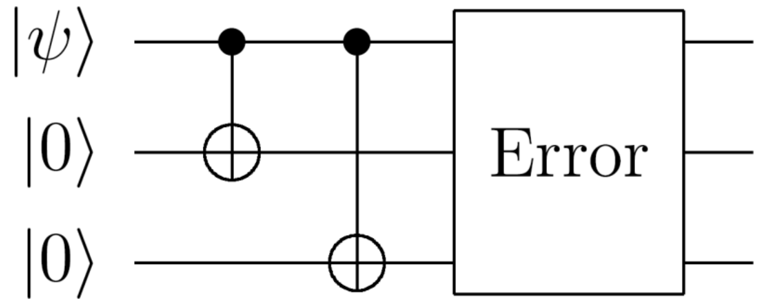
$$|\psi\rangle \rightarrow \; |\psi_E\rangle \equiv c_0|000\rangle + c_1|111\rangle$$

## Bit flip error:

$$|\psi_E\rangle \rightarrow \;\; X_{1,2,3}|\psi_E\rangle \qquad \text{w/ probability } p$$

Can we detect the error w/o destroying the state?

# Step 2/3: Error detection (cont'd)




Errors can be detected by knowing

"Syndrome measurements"

$$Z_1 Z_2 \;\&\; Z_2 Z_3$$

No error: $(Z_1Z_2)|\psi_E\rangle = +|\psi_E\rangle$ , $(Z_2Z_3)|\psi_E\rangle = +|\psi_E\rangle$

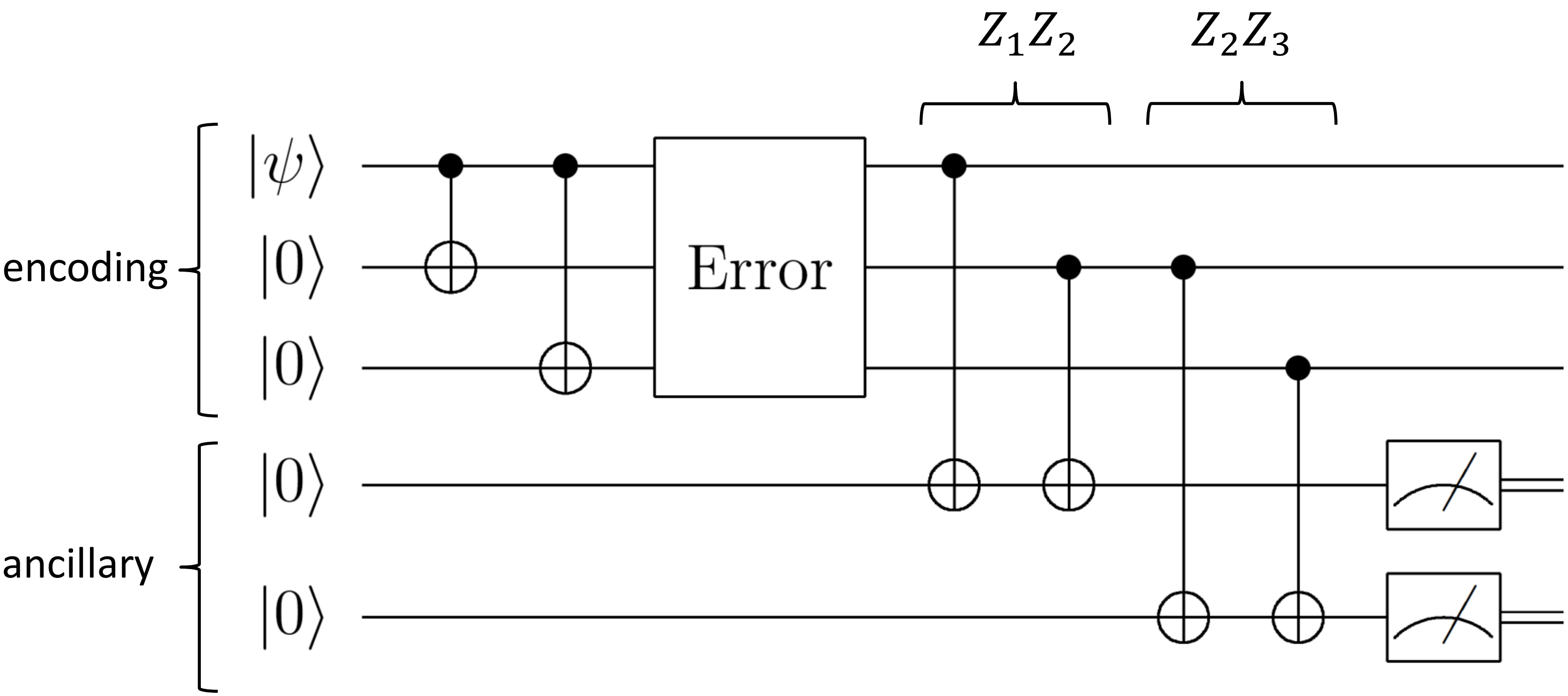
Error on 1st: $(Z_1Z_2)X_1|\psi_E\rangle = -X_1|\psi_E\rangle$, $(Z_2Z_3)X_1|\psi_E\rangle = +X_1|\psi_E\rangle$

Error on 2nd: $(Z_1Z_2)X_2|\psi_E\rangle = -X_2|\psi_E\rangle$, $(Z_2Z_3)X_2|\psi_E\rangle = -X_2|\psi_E\rangle$

Error on 3rd: $(Z_1Z_2)X_3|\psi_E\rangle = +X_3|\psi_E\rangle$, $(Z_2Z_3)X_3|\psi_E\rangle = -X_3|\psi_E\rangle$

But is it possible to know them (w/o destroying the state)?

# Step 2/3: Error detection (cont'd)




<u>Output on the 4th:</u>

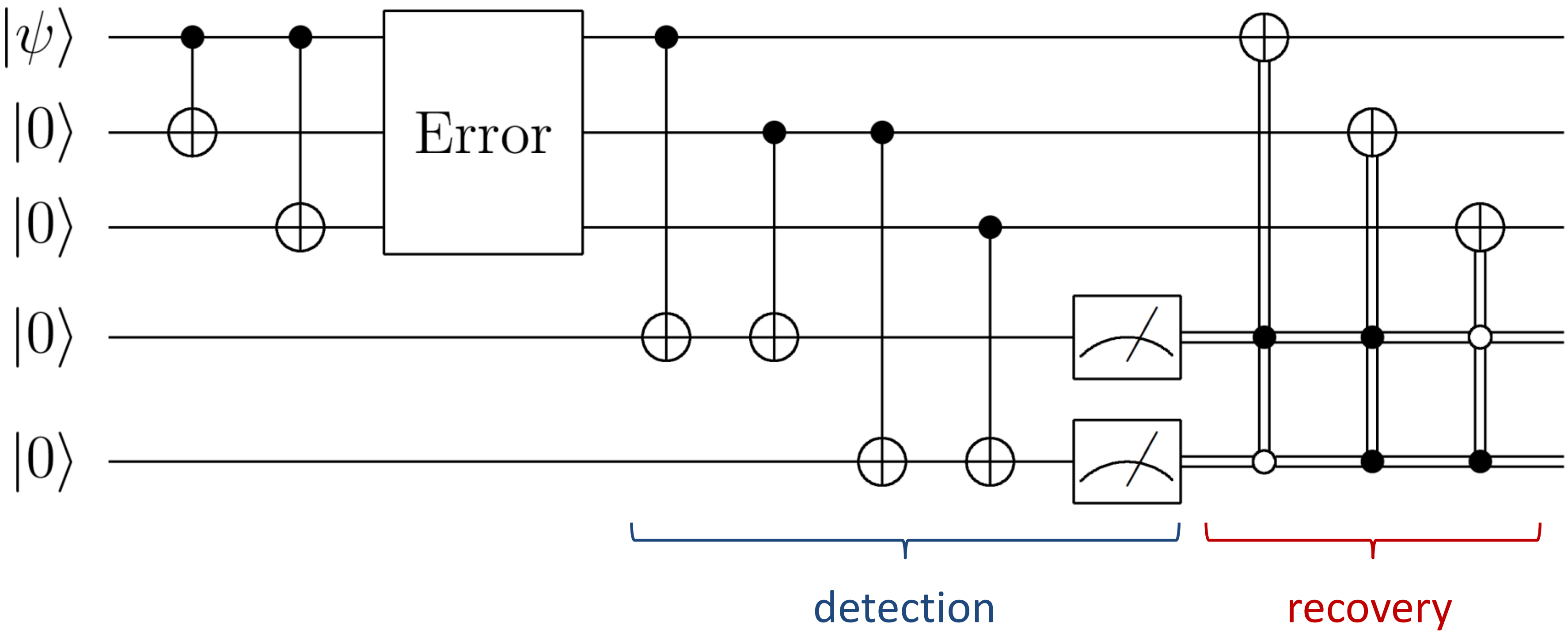
$$0 \text{ if } Z_1Z_2 = +1 \ \ \& \ \ 1 \text{ if } Z_1Z_2 = -1$$

<u>Output on the 5th:</u>

$$0 \text{ if } Z_2Z_3 = +1 \ \ \& \ \ 1 \text{ if } Z_2Z_3 = -1$$

Let's recover the information!!

# Step 3/3: Error recovery




As in the classical case, it fails if $^{\exists}$ multiple "errors":

$$P_{\text{failed}} = 3p^2(1-p) + p^3$$ (improved if $p < 1/2$)

# Quantum error correction for phase flip

Phase flip:

no classical analog

$$|\psi\rangle \rightarrow \; Z|\psi\rangle \quad \text{w/ probability } p$$

$$(c_0|0\rangle + c_1|1\rangle \; \rightarrow \; c_0|0\rangle - c_1|1\rangle)$$

Note:

$(\,|+\rangle \equiv H|0\rangle, \;\; |-\rangle \equiv H|1\rangle\,)$

$$Z|+\rangle = |-\rangle, \qquad Z|-\rangle = |+\rangle$$

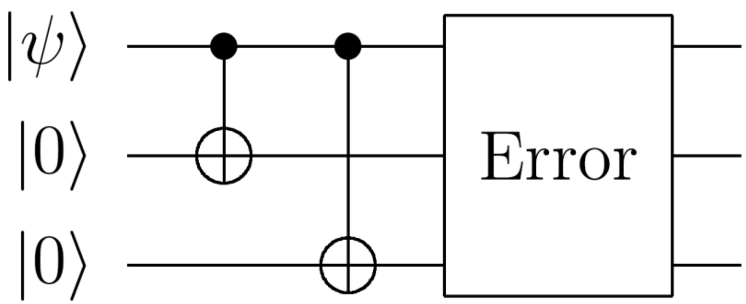
phase flip $=$ bit flip in the basis $|\pm\rangle$

done by a slight modification of the bit flip case

# Step 1/3: encoding

Bit flip

$( |\psi\rangle \rightarrow X|\psi\rangle )$



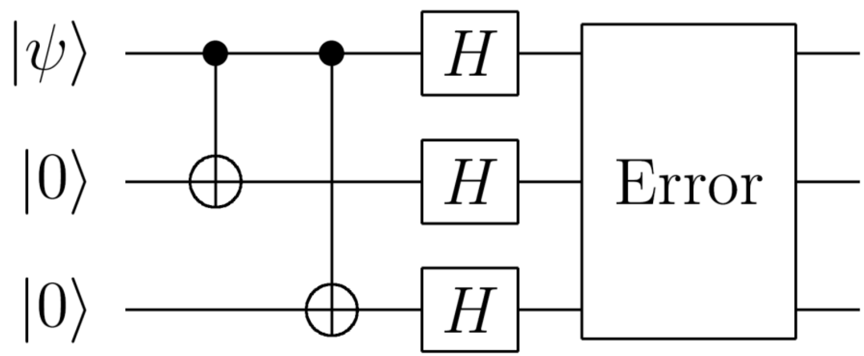

$$|\psi\rangle \equiv c_0|0\rangle + c_1|1\rangle$$

$$\Downarrow$$

$$c_0|000\rangle + c_1|111\rangle$$

Phase flip

$( |\psi\rangle \rightarrow Z|\psi\rangle )$




$$|\psi\rangle \equiv c_0|0\rangle + c_1|1\rangle$$

$$\Downarrow$$

$$c_0|+++\rangle + c_1|---\rangle$$

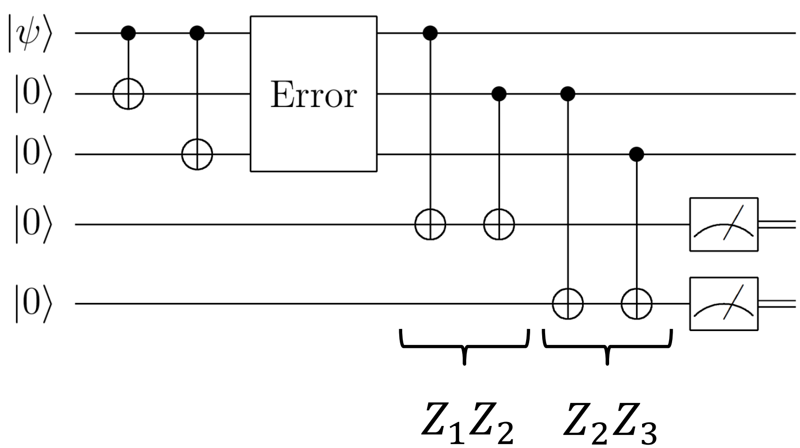
# Step 2/3: error detection

**Bit flip**

( $|\psi\rangle \to X|\psi\rangle$ )

done by knowing

$$Z_1Z_2 \;\; \& \;\; Z_2Z_3$$

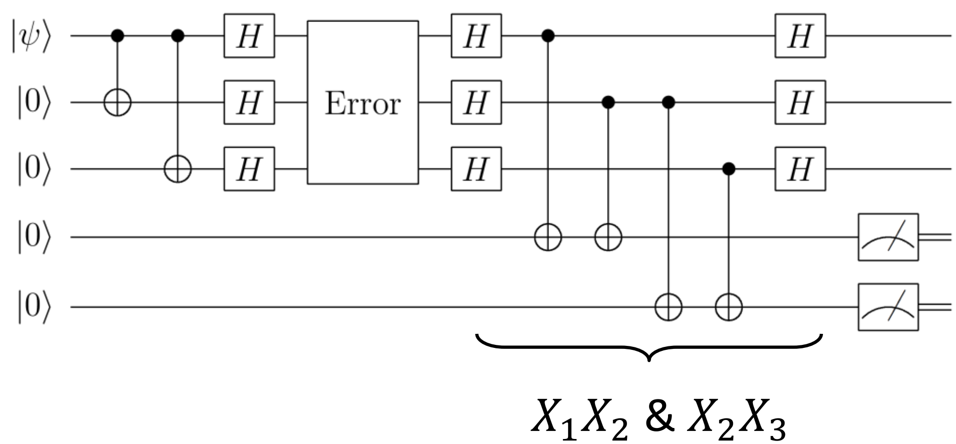



**Phase flip**

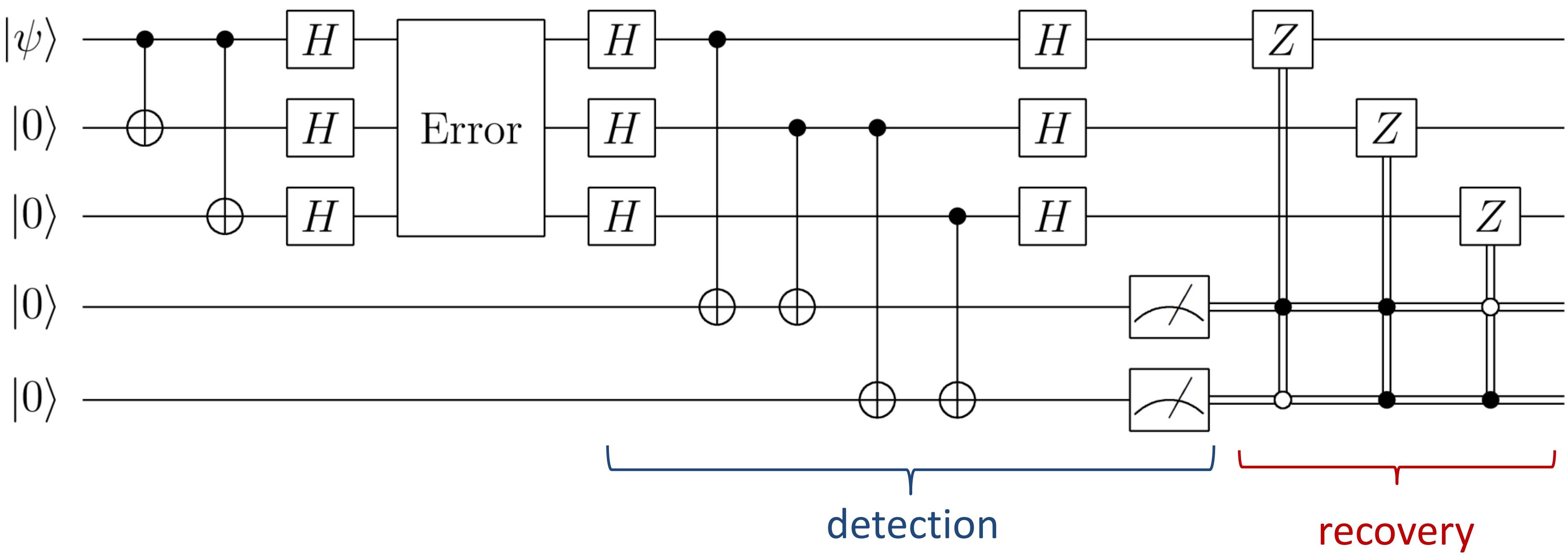
( $|\psi\rangle \to Z|\psi\rangle$ )

done by knowing

$$X_1X_2 \;\; \& \;\; X_2X_3$$



# Step 3/3: Error recovery



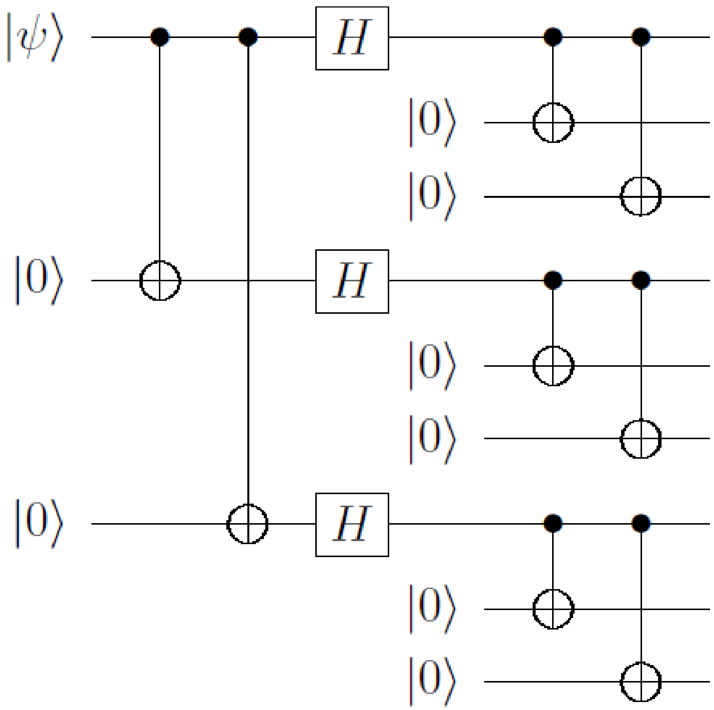

Similarly, it fails if ∃ multiple "errors":

$$P_{\text{failed}} = 3p^2(1-p) + p^3 \qquad \text{(improved if } p < 1/2\text{)}$$

# Correction against arbitrary error on single qubit

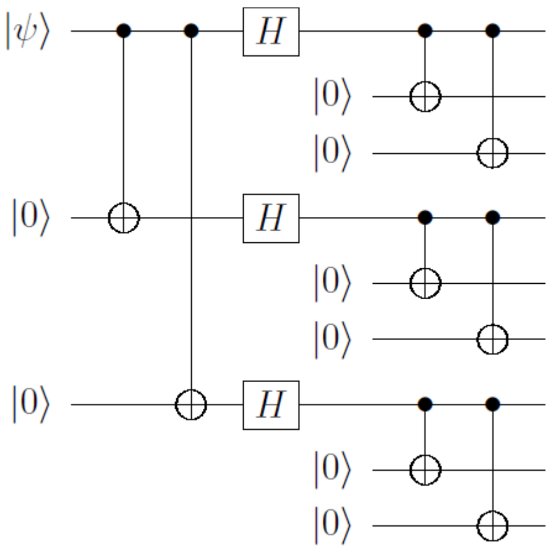
"Shor code" = a combination of the codes against bit flip & phase flip

Encoding:




$$|\psi\rangle \equiv c_0|0\rangle + c_1|1\rangle \to c_0 \frac{(|000\rangle+|111\rangle)^{\otimes 3}}{2\sqrt{2}} + c_1 \frac{(|000\rangle-|111\rangle)^{\otimes 3}}{2\sqrt{2}}$$

# Error detection & recovery in Shor code



Ex.1) Bit flip on qubit 1

- detection by $Z_1Z_2 = -1$, $Z_2Z_3 = +1$
- recovery by applying $X_1$

Ex.2) Phase flip on qubit 1 (the same for qubit 2 & 3 cases)

- detection by $X_1X_2X_3X_4X_5X_6 = -1,\ X_4X_5X_6X_7X_8X_9 = +1$
- recovery by applying $Z_1$, $Z_2$ or $Z_3$

# Prime factorization beyond supercomputer?

- Steane code: (error probability $\epsilon$) $\Rightarrow$ $\mathcal{O}(\epsilon^2)$
- 2-level Steane code: (err. prob. $\epsilon$) $\Rightarrow$ $\mathcal{O}(\epsilon^4)$

  =replacing each qubit in Steane code by Steane code
- $L$-level Steane code: (err. prob. $\epsilon$) $\Rightarrow$ $\mathcal{O}(\epsilon^{2L})$

Suppose

130-digit (=432-bit) prime factorization problem

which takes a few months by (slightly earlier) supercomputer

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Suppose

130-digit (=432-bit) prime factorization problem

which takes a few months by (slightly earlier) supercomputer

- Shor's algorithm requires $5 \times 432$ qubits
- need 3-level Steane code to have small errors (expected)

$$\Longrightarrow \quad 5 \times 432 \times 7^3 + (\text{ancilla}) \sim 1000000 \text{ qubits!}$$

# Summary & Future prospect

# Near future prospect

In near future, available device is so-called [Preskill ’18]

Noisy intermediate-scale quantum device (NISQ)

w/ limited number of qubits & non-negligible errors

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⇨ "quantum error mitigation"

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Noisy intermediate-scale quantum device (NISQ)

w/ limited number of qubits & non-negligible errors

On such device,

▪quantum error correction can't be enough

⇨ nice if $^\exists$a way to reduce errors w/o increasing qubits

⇨ "quantum error mitigation"

▪algorithms w/ less gates are preferred

⇨ Hybrid quantum-classical algorithm

(Popular one for finding vacuum: "variational method")

# Quantum Error mitigation

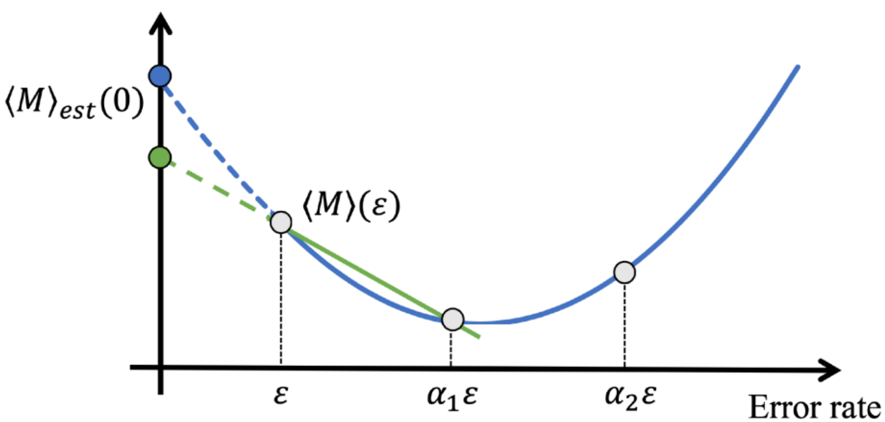
[Figs. are from Endo-Cai-Benjamin-Yuan ’20]

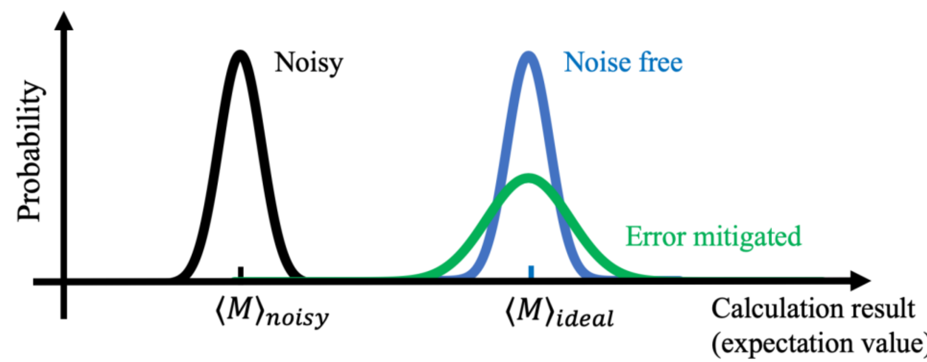
the simplest way = extrapolation

In general,

difficult to decrease errors but possible to increase them

⇨ error-free result by fitting as a function of error rate




This doesn’t need to increase qubits but needs more shots

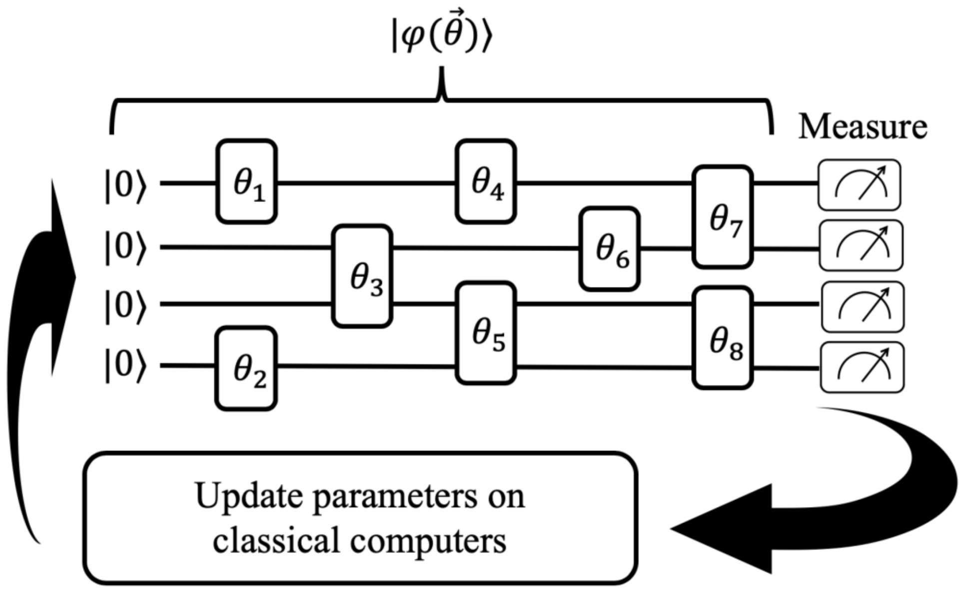
# Variational quantum algorithm

[Fig. is from Endo-Cai-Benjamin-Yuan ’20]

## Idea:

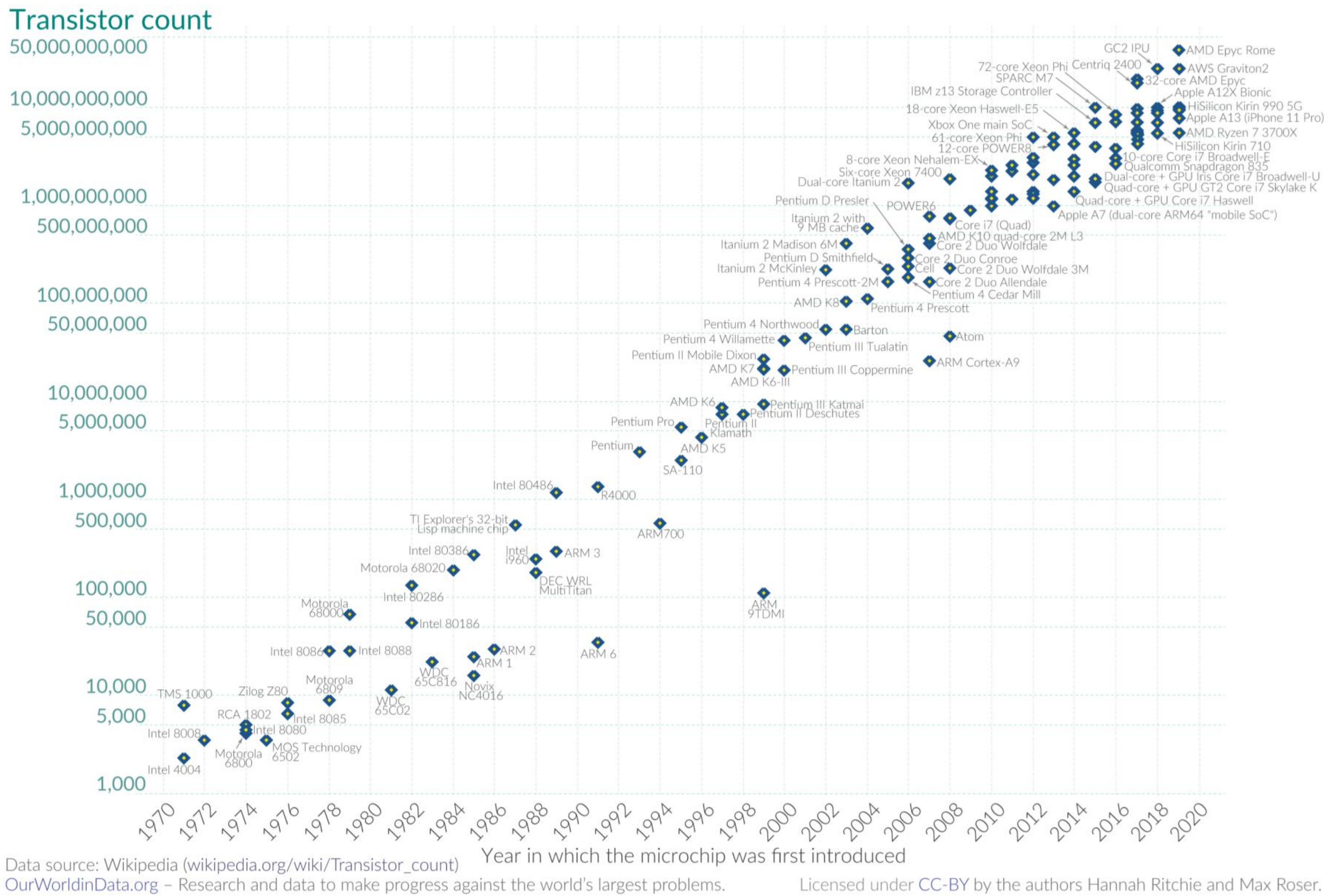
Acting gates & measurements $\Longrightarrow$ Quantum computer

Parameter optimization $\Longrightarrow$ Classical computer




This method needs much less gates than adiabatic state preparation but it’s not guaranteed to get true ground state

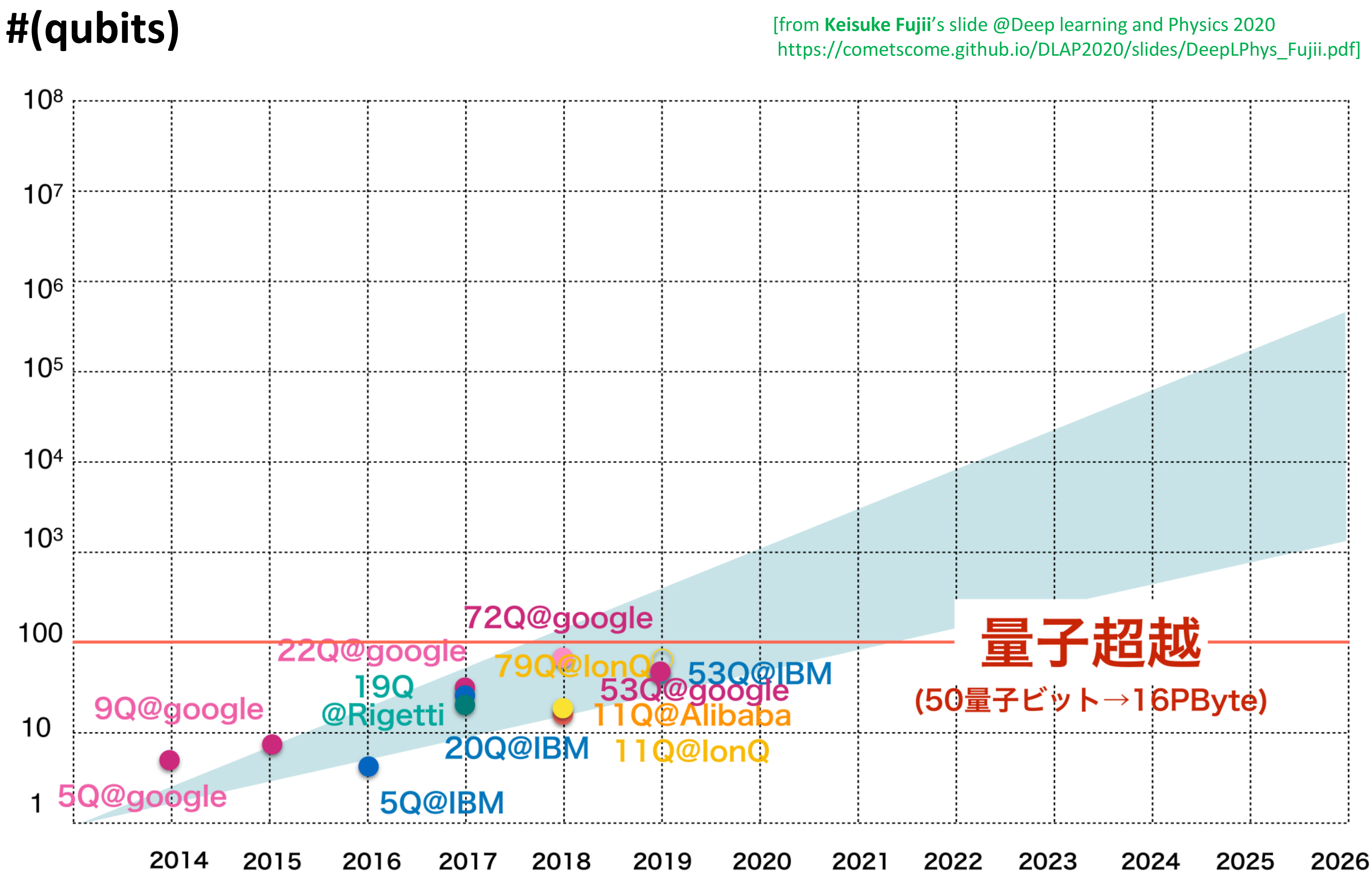
# Moore's law (classical computer)




[by Max Roser, Hannah Ritchie, https://ourworldindata.org/uploads/2020/11/Transistor-Count-over-time.png ]

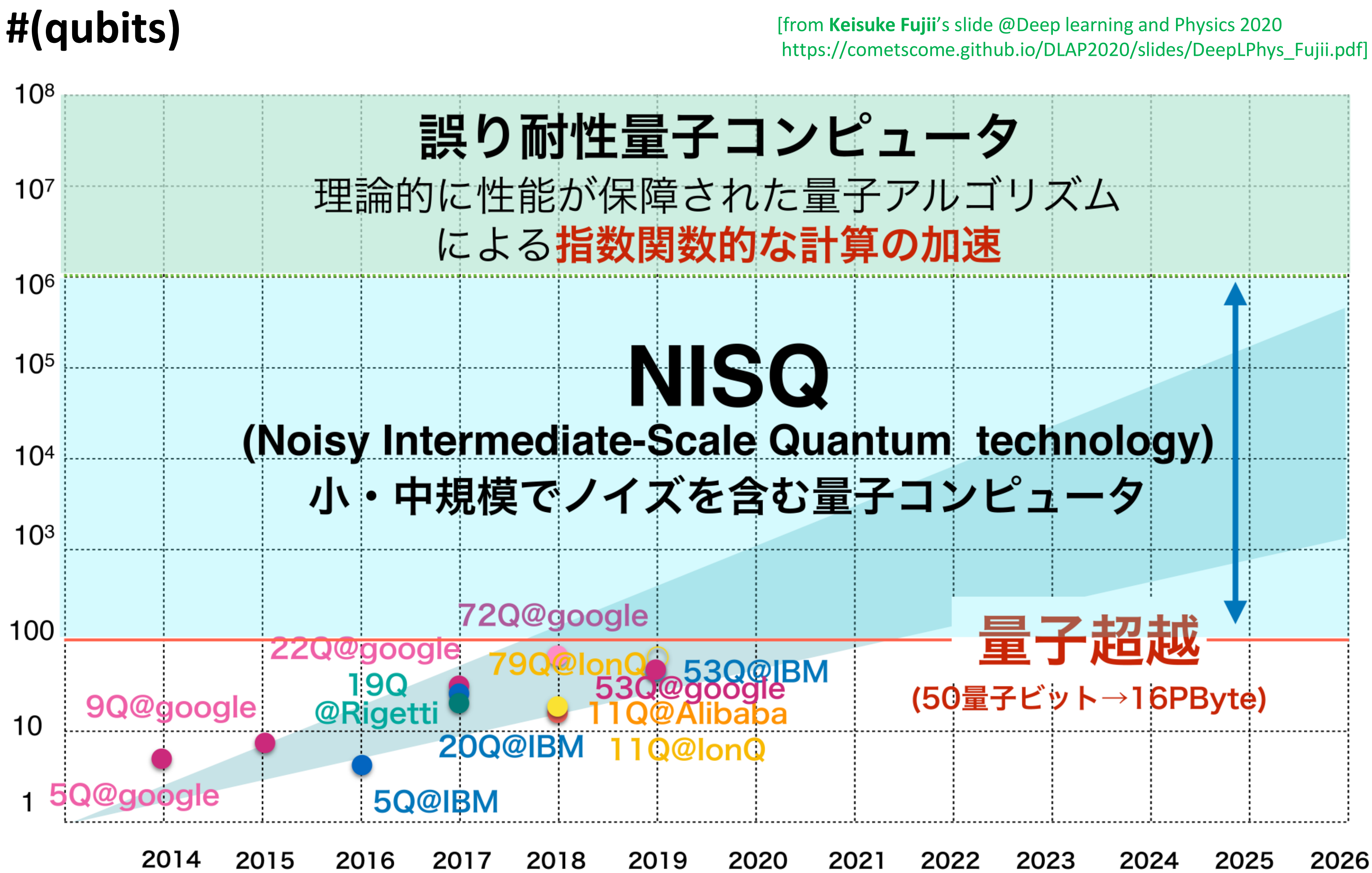
# “Quantum” Moore’s law?

[from **Keisuke Fujii**’s slide @Deep learning and Physics 2020
https://cometscome.github.io/DLAP2020/slides/DeepLPhys_Fujii.pdf]

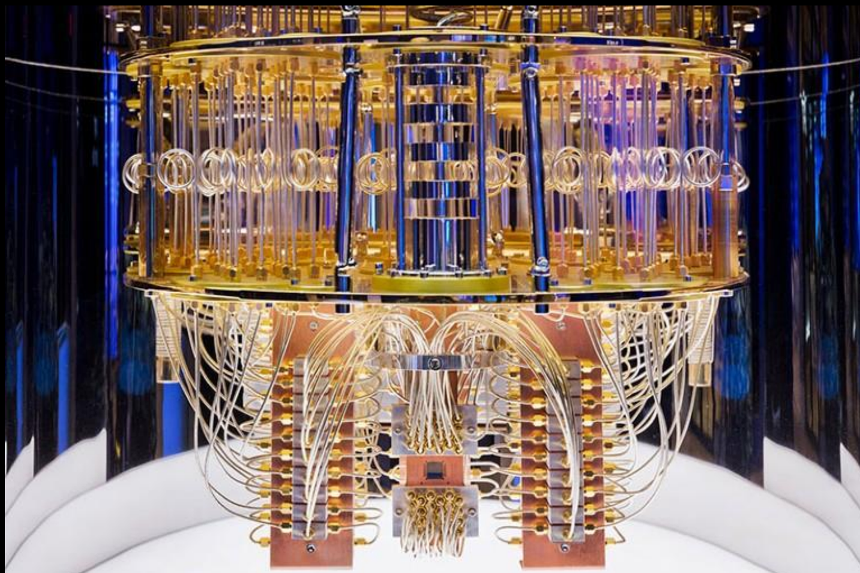


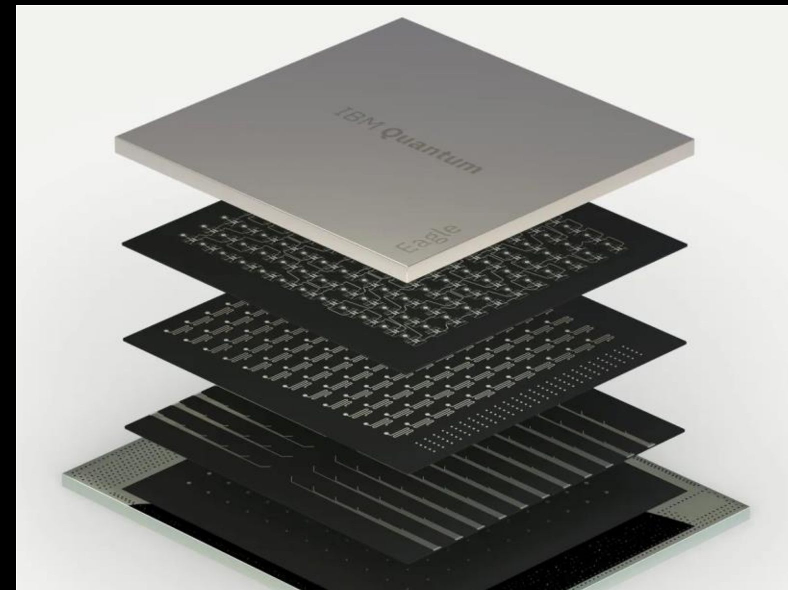
# “Quantum” Moore’s law?

[from **Keisuke Fujii**’s slide @Deep learning and Physics 2020
https://cometscome.github.io/DLAP2020/slides/DeepLPhys_Fujii.pdf]



# The challenge by IBM's 127-qubit device






**Article**

# Evidence for the utility of quantum computing before fault tolerance

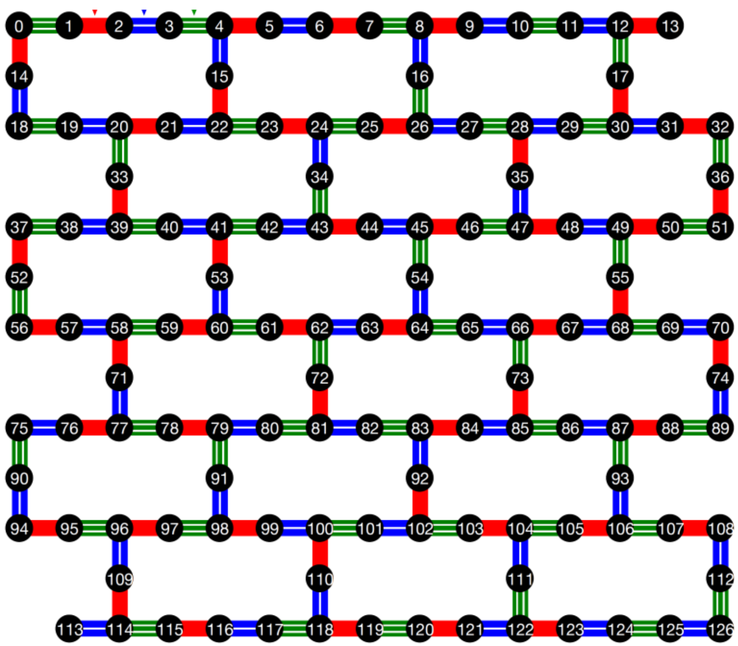
https://doi.org/10.1038/s41586-023-06096-3
Received: 24 February 2023
Accepted: 18 April 2023
Published online: 14 June 2023

Youngseok Kim[1,6]✉, Andrew Eddins[2,6]✉, Sajant Anand[3], Ken Xuan Wei[1], Ewout van den Berg[1], Sami Rosenblatt[1], Hasan Nayfeh[1], Yantao Wu[3,4], Michael Zaletel[3,5], Kristan Temme[1] & Abhinav Kandala[1]✉

Quantum computing promises to offer substantial speed-ups over its classical

# The challenge by IBM's 127-qubit device (cont'd)

Task: time evolution of Ising model on a lattice
w/ shape = the qubit config. of the device



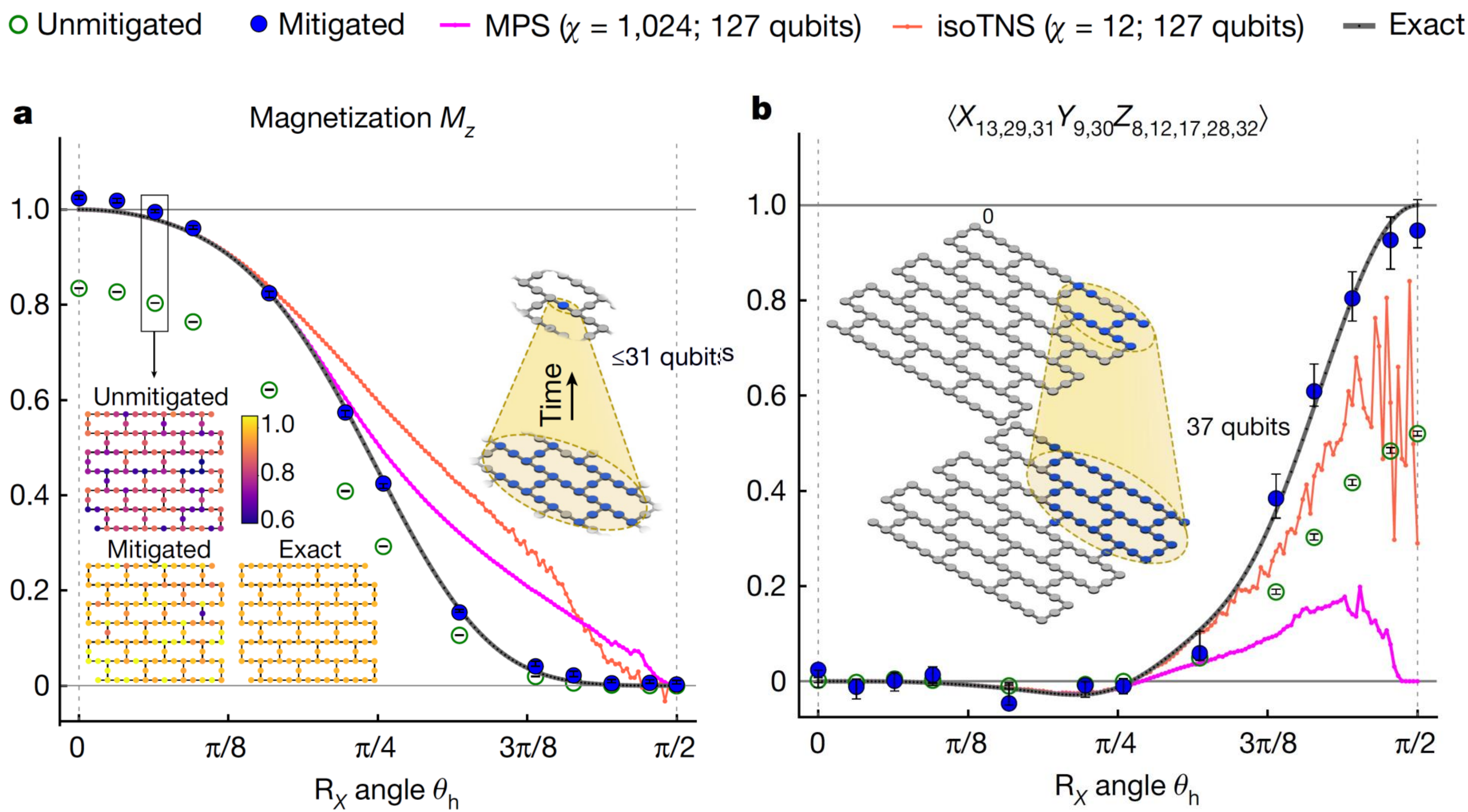

$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j + h \sum_i X_i,$$

$$|\psi(t)\rangle := e^{-iHt}|00\cdots 0\rangle$$

$$\langle \psi(t)|\mathcal{O}|\psi(t)\rangle$$

Strategy: Suzuki-Trotter approximation
+ error mitigation by extrapolation

# The challenge by IBM's 127-qubit device (cont'd)




*"Quantum supremacy"?*

# But…

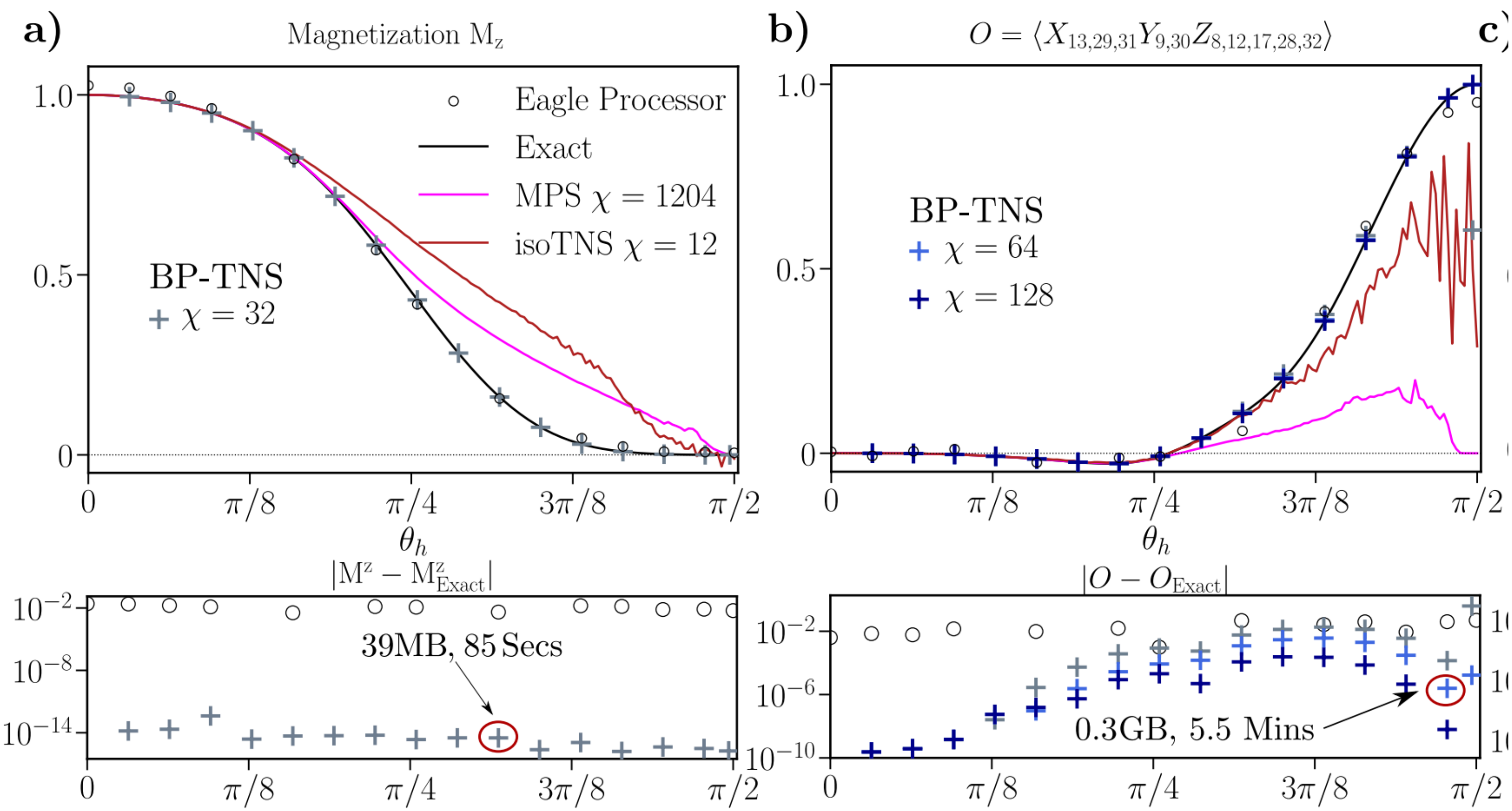
arXiv > quant-ph > arXiv:2306.14887

Quantum Physics

[Submitted on 26 Jun 2023]

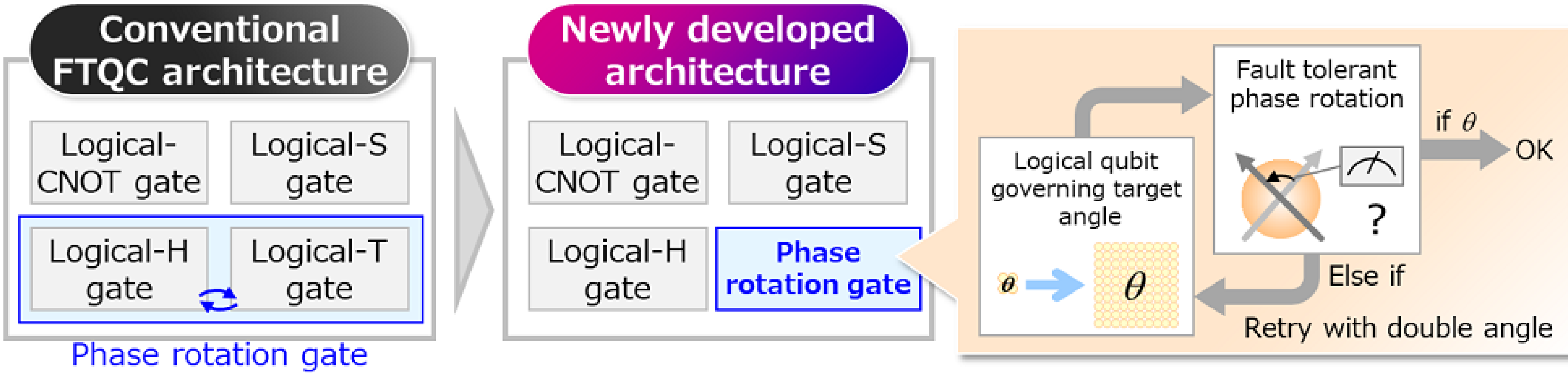
## Efficient tensor network simulation of IBM's kicked Ising experiment

Joseph Tindall, Matt Fishman, Miles Stoudenmire, Dries Sels



# Game change? - new architecture -

[Akahoshi-Maruyama-Oshima-Sato-Fujii 2023 March]




## Big saving of computational cost is claimed:

- # of qubits $\sim \frac{1}{10} \times$ (usual) !!
- # of gates $\sim \frac{1}{20} \times$ (usual) !!

# Unexplored region in QFT

- Taking continuum limit in simulation w/ noises
- Taking continuum limit in scalar field theory
- Non-abelian gauge theory
- Higher dimensions ($\geq$ (2+1)-dim.)
- Finite temperature, chemical potential
- Curved space
- Supersymmetric theories
- Other lattice fermions
- Phase transitions
- Scattering

etc...

# Applications to astro./cosmo. so far

- Inflation (scalar in curved spacetime) [Liu-Li ’20]
- Wheeler-De Witt eq. [Joseph-Varela-Watts-White-Feng-Hassan-McGuigan, Czelusta-Mielczarek ’21]
- QM in black hole b.g. [Joseph-Whilte-Chandra-McGuigan ’21, Chandra-McGuigan ’22]
- Dark sector showers [Chigusa-Yamazaki ’22]
- Matrix Big Bang (???) [Chandra-Feng-McGuigan ’22]
- Boltzmann eq. [Yamazaki-Uchida-Fujisawa-Yoshida ’23, Higuchi-Pedersen-Yoshikawa ’23]

# Summary of the whole lectures

fun & $^{\exists}$many things to do even now

- Quantum computation is suitable for operator formalism which is free from sign problem
- Instead we have to deal with huge vector space. Quantum computers in future may do this job.
- "Rule" of quantum computation
  = Do something interesting by a combination of
  acting unitary op. & measurement
- Real quantum computer has errors
- Handling quantum error is very important

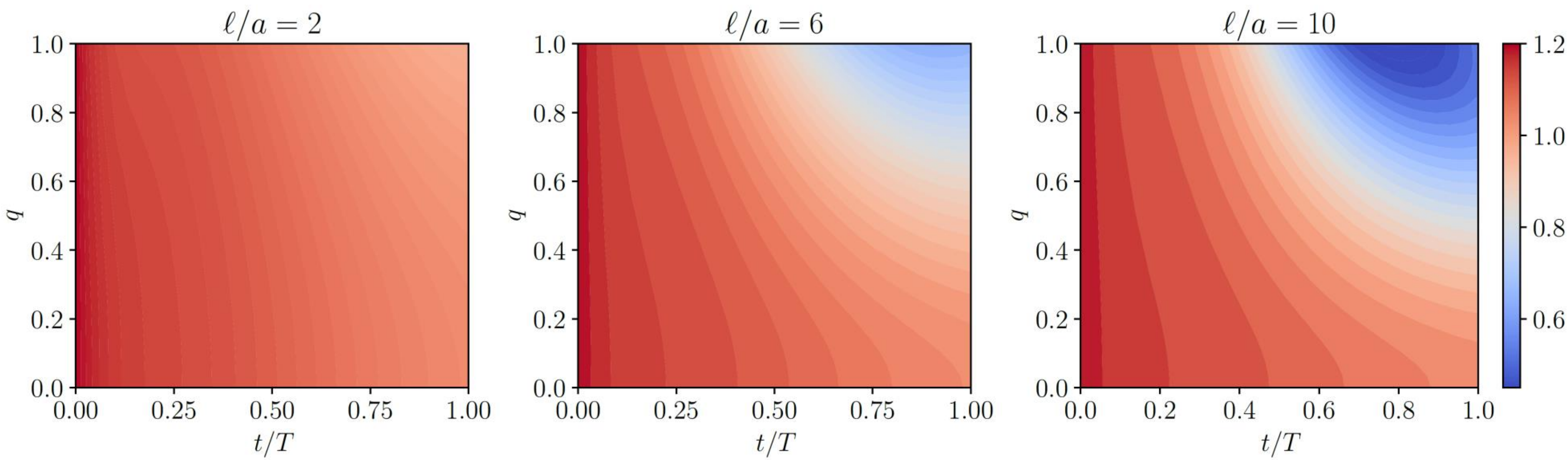
Thanks!

# Appendix

# Comment: density plots of energy gap

(known as “Tuna slice plot” inside the collaboration) [MH-Itou-Kikuchi-Nagano-Okuda ’21]

Parameters: $g = 1, a = 0.4, N = 15, q_p/q = 1, m/g = 0.15$



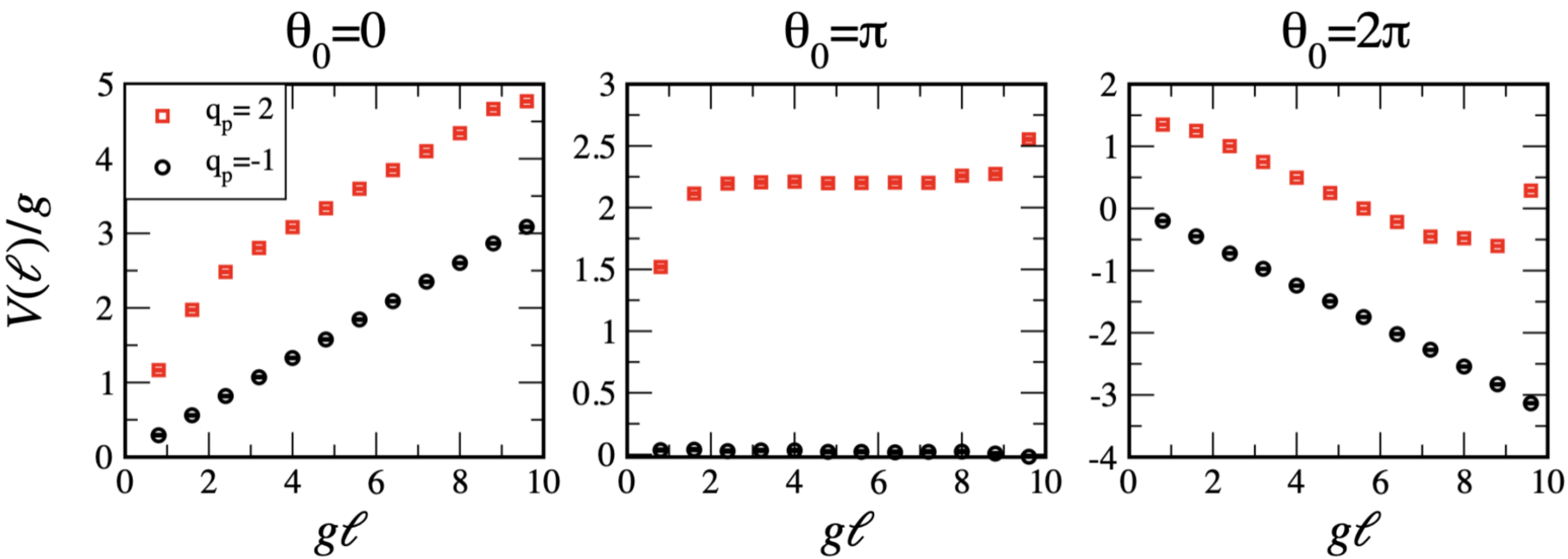

smaller gap for larger $\ell$

larger systematic error for larger $\ell$

# Comparison of $q_p/q = -1/3$ & $q_p/q = 2/3$

[MH-Itou-Kikuchi-Tanizaki '21]

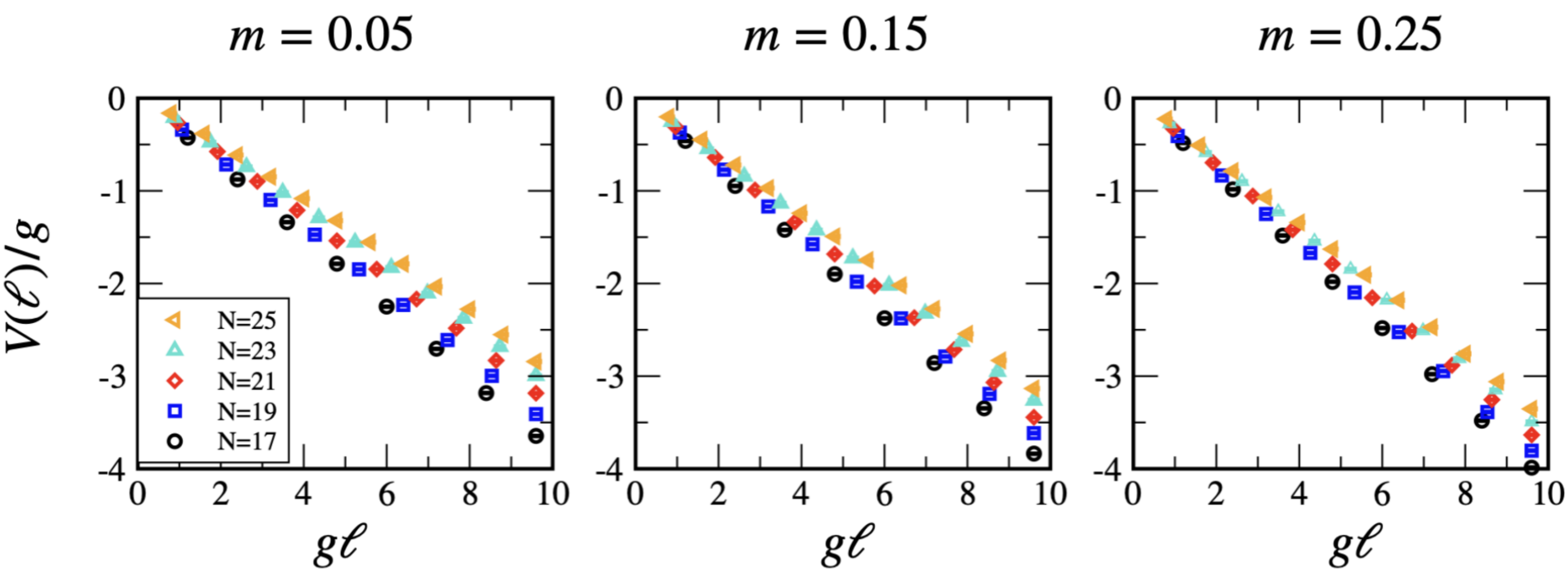
Parameters: $\mathrm{q} = 3, g = 1, a = 0.4, N = 25, T = 99, m = 0.15$




Similar slopes → (approximate) $Z_3$ symmetry

# $N$-dependence of $V$ w/ fixed physical volume

[MH-Itou-Kikuchi-Tanizaki '21]



# Adiabatic scheduling

[MH-Itou-Kikuchi-Tanizaki ’21]

$$N = 17, ga = 0.40, m = 0.20, q_p = 2, \theta_0 = 2\pi,$$