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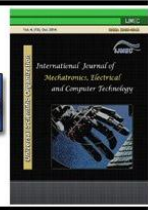
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## General Relativity Search Algorithm for Optimization in Real Numbers Space

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### Abstract

This paper introduces a novel evolutionary optimization algorithm inspired by General Relativity Theory (GRT). This optimization method is called General Relativity Search Algorithm (GRSA). In GRSA, a population of particles (agents) is considered in a space free from all external non-gravitational fields. These agents evolve toward a position with least Action. Based on GRT, a system of particles has conserved mass and each of them moves along geodesic trajectories in a curved spacetime. According to physical action principle, a system of particles goes to a position with minimum action. By inspiring these notions, GRSA will make solution agents move toward the optimal point of an optimization problem. Performance of the proposed optimization algorithm is investigated by using several standard test functions. Effectiveness and abilities of the algorithm to solve optimization problems is shown through a comparative study with two well-known heuristic search methods, Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). Numerical simulation results demonstrate the efficiency, robustness and convergence speed of GRSA in finding the optimal solution of different benchmark functions.

**Keywords:** Evolutionary Algorithms, Optimization, General Relativity Theory, test functions.

### 1. Introduction

The fundamental problem of optimization is to arrive at the best possible decision in any given set of circumstances [1]. Many conventional techniques such as gradient-based search algorithms and various mathematical programming methods have been proposed to deal with optimization problems. However, these techniques have severe limitations in handling nonlinear, discontinuous functions and constraints because exact methods are more likely to get stuck at local optima [2]. Evolutionary Algorithms (EAs) do not rely on gradient information, whilst exact methods depend on. In this situation, EAs are still able to work satisfactorily.

In the last decades, different EAs have been introduced for solving optimization problems. Some of these algorithms have received increased interest such as GA [3], PSO [4] and [5-11]. These algorithms usually start with an initial random population of feasible solutions and then evolve and change the population position to obtain the global optimal point of the objective function. It can be understood that there are two important parameters that control the evolution of a solution [12]. These

parameters are the step length and step direction. While, all of the above optimization algorithms use this process inherently. There is insufficient attention made to these parameters in the optimization algorithms.

The EAs solve different practical optimization problems [2]. However, there is no specific algorithm to achieve the best solution for all optimization problems. Some algorithms result a better solution for some particular problems rather than others. Therefore, searching for new heuristic optimization algorithm is an open problem [13].

In this paper, a novel optimization algorithm inspired by GRT that is named GRSA, is introduced [14]. In the proposed algorithm, an initial population of particles will be generated randomly. This initial population of particles is represented as a tensor. A position where particles find least physical action is named the best position. Based on energy-momentum amount of particles, they move toward the best position on trajectories that are named geodesics. Geodesic trajectories are formed according to geometry of the curved spacetime. These trajectories have minimum resistance against particle motion under gravitational fields. Energy or mass in the space is the source of gravitational fields. A particle with greater energy-momentum yields greater gravitational field. Therefore, it has more effect on creating gravitational field and forming geodesic trajectories in the curved spacetime. The tensor of particles changes in each step of iterations. Any particle obtains different mass and energy from previous stage. After the end of iterations, the best position for particles will be obtained.

The rest of this paper is organized as follows. In section two, the GRT is presented, curtly. In section three, the proposed method, GRSA, is described in detail. In section four, GRSA effectiveness in comparison with some other existing algorithms is evaluated using several benchmark functions. Finally, the conclusion is given in Section five.

## 2. General Relativity Theory

GRT is the geometric theory of gravitation in modern physics [15, 16]. GRT generalizes special relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time, or spacetime. The curvature of spacetime is directly related to the energy and momentum of whatever matter and radiation are present. There is no gravitational force deflecting objects from their natural, straight paths. Instead, gravity corresponds to changes in the properties of space and time, which in turn changes the straightest-possible paths that objects will naturally follow. The curvature is caused by the energy-momentum of matter. The stress-energy tensor is a tensor quantity in physics that describes the density and flux of energy and momentum in spacetime, generalizing the stress tensor of Newtonian physics. The stress-energy tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass density is the source of such a field in Newtonian gravity.

In the general relativity, stress-energy tensor is being studied using Einstein's (field) equations. These equations are general relativity core and can be written in the form of tensor of the following equation:

$$R_{ij} - \frac{1}{2} R g_{ij} = \frac{8\pi G}{c^4} T_{ij}. \quad (1)$$

Where  $R_{ij}$  is Ricci tensor,  $R$  is the curvature scalar,  $g_{ij}$  is the geometry tensor,  $T_{ij}$  is the energy-momentum tensor (Einstein tensor),  $G$  is the gravitational constant and  $c$  is the speed of light. In a curved line coordination system,  $g_{ij}$  components are functions of coordination system and specify all elements and thus they form metric, and show the coordination system and curvature of the space. From Einstein's equations two results can be educed. Firstly Mass is conserved and secondly Particle always moves along a geodesic. Mass and energy form particles. Mass and energy are transmutable to

each other, while they have conserved value. The motion of matter from one place to another place causes an alteration of the gravitational field in the surrounding space [17].

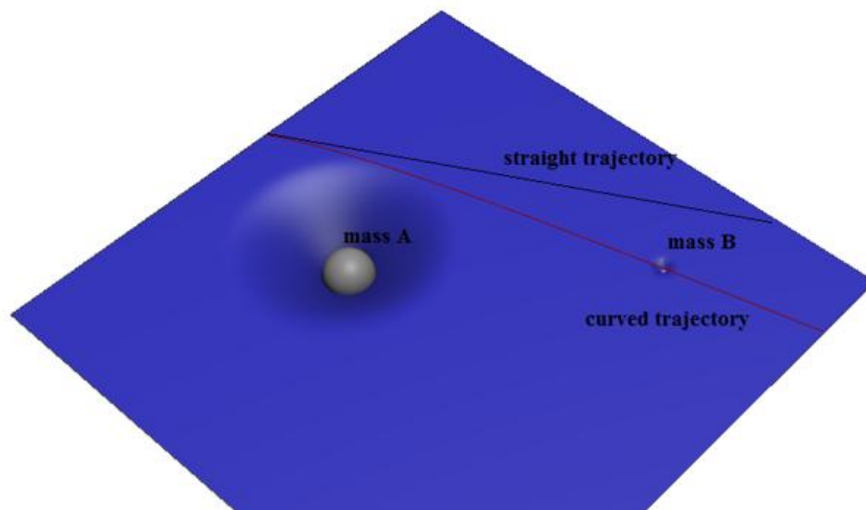
In GRT, a geodesic generalizes the notion of a straight line to curved spacetime. Importantly, the world line of a particle free from all external, non-gravitational forces, is a particular type of geodesic. In general relativity, gravity can be regarded as not a force but a consequence of curved spacetime geometry where the source of curvature is the stress-energy tensor. In a curved spacetime geometry three type of geodesic can be defined: (a) timelike geodesics have a tangent vector whose norm is negative, (b) null geodesics have a tangent vector whose norm is zero and (c) spacelike geodesics have a tangent vector whose norm is positive. An ideal particle whose gravitational field and size are ignored not subject to any non-gravitational field will always follow timelike geodesics. First law of Newton replaces with this premise and motion and acceleration equation obtain from geodesic equations. Massless particles like the photon follow null geodesics and thus light ray follow null geodesics. Spacelike geodesics do not correspond to the path of any physical particle; in a space that has space-sections orthogonal to a timelike Killing vector. In general relativity total energy can be expressed as sum of rest mass energy and kinetic energy as follow:

$$m\gamma c^2 = E_k + mc^2. \quad (2)$$

Where  $m$  is the particle mass,  $c$  is the speed of light,  $E_k$  is kinetic energy and  $\gamma$  is Lorentz transformation coefficient which is defined as:

$$\gamma = \sqrt{\frac{g_{tt}}{g_{tt} + g_{ss}v^2}}. \quad (3)$$

Where  $g_{ss}$  and  $g_{tt}$  are spacetime geometry dependant variables and  $v$  is speed of particle motion. In general relativity, 2-dimensional curved spacetime may imagine by a curved surface as illustrated in Figure 1 in a 3-dimensional Cartesian coordination system.



**Figure 1:** Curvature of 2-dimensional spacetime in presence of mass A (large) and B (small).

### 3. General Relativity Search Algorithm

GRSA starts with an initial population of particles that their positions are generated randomly in the curved spacetime [14]. A particle position is a feasible solution of an optimization problem. Positions of the particles are represented in a tensor. A particle based on its energy-momentum and surrounding gravitational fields tracks a geodesic in curved spacetime. It moves toward the best position where it has minimum action. Objective function of the optimization problem is computed for all particles and

considered as their actions. Step length and step direction for updating position of a particle is determined by its velocity and geodesic tangent. Particles with big gravitational fields have significant effect on forming geodesics for other particles. While, evolution of these particles are mainly depend on their energy-momentum and geodesic tangent.

GRSA evolves to find solution through iterative calculation of step length and step direction for any particle position. Step length and step direction in GRSA are calculated using existing relativistic equations for energy-momentum, geometry of curved spacetime and geodesic tangent vectors. Finally, using Einstein field equation the effect of gravitational field of particles on spacetime is incorporated. Therefore, new position of particles is yielded. In next subsections different stages of the algorithm are presented.

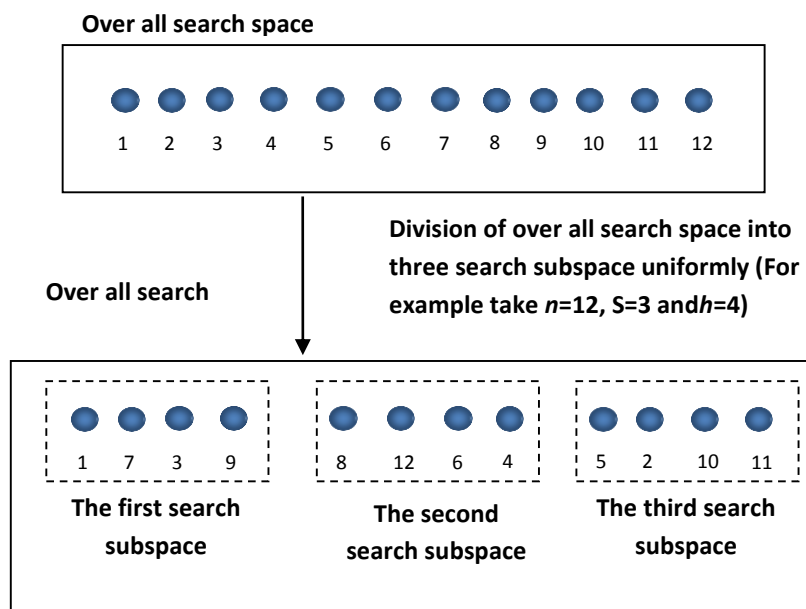
### 3.1. GRSA Initiation

Initial random population is set into a tensor as follows. Each element of tensor  $T$  will be defined as:

$$T_{ij}(1) = T_j^{\min} + rand()(T_j^{\max} - T_j^{\min}).$$

$$i \in \{1, 2, \dots, n\}, \quad j \in \{1, 2, \dots, d\}. \quad (4)$$

Where  $n$  is the number of particles and  $d$  is the particle or spacetime dimension.  $T_j^{\max}$  and  $T_j^{\min}$  are maximum and minimum of the  $j$ th element of any particle  $T_i$ , respectively. Function  $rand()$  provides a uniformly random number in the interval  $[0,1]$ . This initial tensor of search space,  $T$ , is uniformly divided into  $S$  search subspaces such that each search subspaces includes  $h$  particles. Fig. 2 illustrates the notion of initial search space tensor division into  $S$  search subspaces. Objective function of the optimization problems is computed for all particles and minimum action among all of them is selected as the best solution in this stage.



**Figure 2:** Division of over all search space into subspaces uniformly.

### 3.2. Step Length Calculation

In this stage, the search step length is calculated using relativistic total energy-momentum (Equation (2)). Initially, the ratio of kinetic energy to the rest mass energy is defined as:

$$\xi = \frac{E_k}{mc^2}. \quad (5)$$

We introduce an intuitive equation for  $\xi$  at time  $t$  ( $t$ th iteration) for  $i$ th test particle in  $s$ th ( $s \in \{1, 2, \dots, S\}$ ) subspace as:

$$\xi_i(t) = \frac{I_{rand} - n}{n-1} GM_1 + \frac{1 - I_{rand}}{n-1} GM_2. \quad (6)$$

Where  $\xi(t)$  is the energy conversion ratio at  $t$ th iteration ( $t \in \{1, 2, \dots, t_{max}\}$ ) and  $I_{rand}$  is a random selected index of one of the positions in  $s$ th search subspace. For example, according to Figure 3, if the third particle of the second search subspace is selected randomly then  $I_{rand}=7$ . Also,  $GM_1$  and  $GM_2$  are geometry coefficients of spacetime which specify the rate of energy conversion. In order to keep conservation law,  $GM_1$  and  $GM_2$  must satisfy following equation:

$$GM_1 + GM_2 = 1, \quad GM_1, GM_2 \in [GM_{min}, GM_{max}]. \quad (7)$$

We define initial Lorentz transformation coefficient in equation (2) as:

$$\gamma_{0,i}(t) = 1 + \xi_i(t). \quad (8)$$

The quantity  $\gamma_0$  is a scalar value that shows the change in energy-momentum of an agent. In order to transmute  $\gamma_0$  into a vector form,  $\gamma$ , a random vector,  $K_g$ , is defined.  $K_g$  (is a  $1 \times d$  vector) and its components are uniformly random numbers in range  $[0, 1]$ . We define final Lorentz coefficient vector,  $\gamma_i(t)$ , as:

$$\gamma_i(t) = \gamma_{0,i}(t) + (1 - \gamma_{0,i}(t)) K_g. \quad (9)$$

Now, by using Equation (3) particles velocity can be calculated as:

$$V_{ij}(t) = K_{V,ij}(t) \sqrt{\frac{1}{\gamma_{ij}^2(t)} - 1}. \quad (10)$$

Where  $V_{ij}(t)$  is the  $j$ th component of the velocity vector for  $i$ th particle at  $t$ th iteration. Variable  $K_{V,ij}(t)$  that is called spacetime coefficient shows impact of spacetime geometry on particles motion. From equation (2), this coefficient is equal to  $K_V = \sqrt{(g_{tt} / g_{ss})}$ . We define  $K_{V,ij}(t)$  as the distance of a random particle position in the  $s$ th search subspace as:

$$K_{V,ij}(t) = |T_j^{Best,s}(t) - T_j^{rand,s}(t)|. \quad (11)$$

Where  $T_j^{Best,s}$  is the  $j$ th component of the best position in  $s$ th search subspace and  $T_j^{rand,s}$  is the  $j$ th component of a random position in  $s$ th search subspace. Thus, step length is obtained as:

$$\lambda_{ij}(t) = \omega_t V_{ij}(t). \quad (12)$$

$$i \in \{1, 2, \dots, n\}, \quad j \in \{1, 2, \dots, d\}.$$

Where  $\lambda_{ij}(t)$  is step length of the  $j$ th component of the  $i$ th particle at  $t$ th iteration. Variable  $\omega_t$  is a weighted factor which decreased from 0.9 to 0.1 linearly with increasing iterations and will cause

smaller step length in adjacent of optimal point. Similarly, for all particles in subspaces the step length can be calculated.

### 3.3. Step Direction Calculation

Step direction for a particle is obtained from the three existing types of its geodesics. Directions of timelike, spacelike and null geodesics are defined in the following relations, respectively:

$$\delta_{tl,i}(t) = \text{sign}(T_i(t) - T_i(t-1)). \quad (13)$$

$$\delta_{sl,i}(t) = \text{sign}(T_i(t) - TG). \quad (14)$$

$$\delta_{null,i}(t) = \text{sign}(T_i(t) - TB_i). \quad (15)$$

Where  $\delta_{tl}$ ,  $\delta_{sl}$  and  $\delta_{null}$  are the timelike, spacelike and the null geodesic directions of the  $i$ th particle at the  $t$ th iteration, respectively. *Sign* is the signum function and  $\delta_{tl}$ ,  $\delta_{sl}$  and  $\delta_{null} \in \{-1, 0, 1\}$ . Also,  $TG$  is the global best particle position in the tensor.  $TB_i$  is the best position that the particle has obtained in the exploration so far. In order to have a purposive search using different geodesic directions, we define final equation for particle step direction as:

$$\delta_{ij}(t) = -\text{sign}(\delta_{tl,ij}(t) + K_{f,j}\delta_{sl,ij}(t) + (1 - K_{f,j})\delta_{null,ij}(t)). \quad (16)$$

Where  $\delta_{ij}(t)$  represents step direction of  $j$ th component of  $i$ th particle and  $K_f$  is a random vector with  $K_{f,j} \in [0, 1]$ . As it is obvious from (16), if  $K_{f,j}$  becomes one, then the corresponding component of timelike geodesic tangent will be in same direction with spacelike geodesic and else it will be in same direction with null geodesic. Therefore, the tendency to move toward a particle with big energy-momentum increases along a timelike geodesic.

### 3.4. Updating Particles Tensor

By computing step length from equation (12) and step direction from equation (16), new position for particles is:

$$T_{ij}(t+1) = T_{ij}(t) + \lambda_{ij}(t)\delta_{ij}(t). \quad (17)$$

To improve search capabilities, a mutation operation will be performed for particles with worst positions in over all search space. In this process,  $S$  worst positions in over all search space are chosen and the mutation operation will be performed with the best positions in all subspaces. For this purpose, we apply Einstein's field equation to these positions. We can rewrite Einstein's field equation (1) as:

$$T_{ij} = \frac{c^4}{8\pi G} R_{ij} - \frac{1}{2} R \frac{c^4}{8\pi G} g_{ij} = \alpha_{1,j} R_{ij} + \alpha_{2,j} g_{ij}. \quad (18)$$

$$R_i \in \{T_1, T_2, \dots, T_S\} \mid f(T_1) \geq f(T_2) \geq \dots \geq f(T_S). \quad (19)$$

$$g_i \in \{T_1^{(1)}, T_1^{(2)}, \dots, T_1^{(S)}\}. \quad (20)$$

$$f(T_1^{(i)}) = \max_{i \in \{1, 2, \dots, S\}} \left\{ f(T_1^{(i)}), f(T_2^{(i)}), \dots, f(T_h^{(i)}) \right\}.$$

Where  $f(T_i)$  denotes objective function value.  $R$  is the set of the worst positions in over all search space and  $g$  is the set of the best position of all search subspaces. Equations (19) and (20) stand for a minimization problem.

To perform the mutation operation, variables  $\alpha_1$  and  $\alpha_2$  are defined in a way that  $\alpha_1$  becomes a random vector with  $\alpha_{1,j} \in \{0, 1\}$  and  $\alpha_2$  is a complementary vector for  $\alpha_1$  just as follows:

$$\alpha_{1,j} + \alpha_{2,j} = 1. \quad (21)$$

Therefore, the mutation operation is given by:

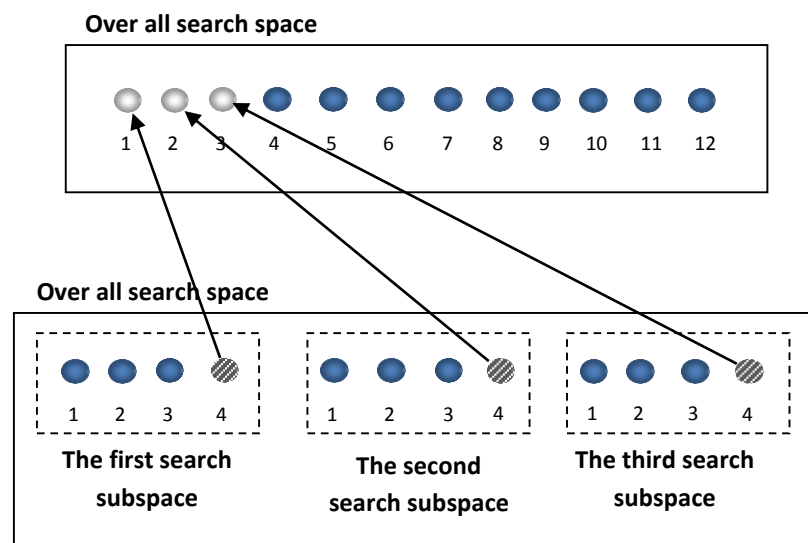
$$T_{qj}(t+1) = \alpha_{1,j} R_{qj}(t) + \alpha_{2,j} g_{qj}(t). \quad (22)$$

$$q \in \{1, 2, \dots, S\}, \quad j \in \{1, 2, \dots, d\}.$$

Finally, the tensor will be update as:

$$T(t+1) = [T_1(t+1), T_2(t+1), \dots, T_n(t+1)]^T. \quad (23)$$

After updating the tensor, a new position for the particles population is obtained. In fact, an iteration of the algorithm has been completed. Figure 3 demonstrates the mutation operation between particles and search subspaces. In this figure, positions of particles are sorted in descending order for a minimization problem. Particles with bright color represent set  $R$  and those with dark color represent set  $g$ .



**Figure 3:** Mutation operation between worst positions in over all search space and the best positions in all search subspaces.

The steps of the proposed GRSA approach are illustrated bellow:

- i. Generate a random initial tensor for population of particles,
- ii. Divide initial tensor to some subspaces uniformly,
- iii. Calculate Action (cost function) for all particles,
- iv. Assign ratio of kinetic energy to the mass energy conversion for each particle,



- v. Calculate velocity of particles,
- vi. Compute step length for particles,
- vii. Compute step direction for particles,
- viii. Update position of particles,
- ix. Perform random element-wise replacement operation for number of  $S$  particles with worst positions using Einstein's equation,
- x. Repeat steps (iii) to (ix) until the stop criteria is satisfied,
- xi. End.

#### 4. Simulation Results and Analysis

A set of 6 well-known benchmark functions is used to perform numerical simulations. These functions simulate a vast spectrum of practical optimization problems with different complexity and dimension. The aim is to show the efficiency of existing or new proposed search algorithms through these benchmark functions which are given in Table 1. In this table,  $n$  is objective function dimension. More detailed description of each function is given in [18].

**Table 1:** Multimodal high dimensional test functions.

Test function	n	Domain	$f_{\min}$
$f_1(x) = \sum_{i=1}^n -x_i \sin(\sqrt{ x_i })$	30	$[-500, 500]^n$	-12569.5
$f_2(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10]$	30	$[-5.12, 5.12]^n$	0
$f_3(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right)$	30	$[-32, 32]^n$	0
$f_4(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	$[-600, 600]^n$	0
$f_5(x) = \frac{\pi}{n} \left\{ 10 \sin^2(\pi y_i) + \sum_{i=1}^{n-1} (y_i - 1)^2 \left[ 1 + 10 \sin^2(\pi y_{i+1}) \right] \right\} + \sum_{i=1}^{n-1} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{1}{4} (x_i + 1), u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a, \\ 0, & -a \leq x_i \leq a, \\ k(-x_i - a)^m, & x_i < -a, \end{cases}$	30	$[-50, 50]^n$	0
$f_6(x) = 0.1 \left\{ \sin^2(3\pi x_i) + \sum_{i=1}^{n-1} (x_i - 1)^2 \left[ 1 + \sin^2(3\pi x_{i+1}) \right] \right\} + \sum_{i=1}^{n-1} u(x_i, 10, 100, 4)$	30	$[-50, 50]^n$	0

To show effectiveness and abilities of the proposed algorithm and a comparative study with some existing optimization algorithms such as basic GA (in the simulations the parameter for linear crossover is 0, the crossover probability is 0.9 and mutation probability is 0.1) and PSO (PSO parameters are set to  $C_1=C_2=2$  and  $w=0.7$  for all case studies), a set of numerical simulations have been performed. The initial population and the total number of iterations are selected 100 and 500, respectively.

Optimization performance of each search algorithm is investigated by using three indexes. These indexes are the Best Function Value (BFV), Average Value (AV) and Standard Deviation (STD) of function values that are obtained at the end of 30 successive runs of these optimization algorithms.

Multimodal high dimension functions have many local optimum points and almost are the most difficult functions to optimize. For multimodal functions, the final results are more important because they reflect ability of search algorithm in escaping from local optima and finding global optimum or

reach near it. The functions listed in Table 1 are multimodal high dimensional functions (dimension30). For 30 runs of the search optimization algorithms, BFV, AV and STD are obtained. Results are shown in Table 2. Results show a great superiority of the proposed GRSA rather than GA and PSO.

Moreover, convergence curve of GA, PSO and GRSA algorithms are depicted in figure 4. This figure illustrates the BFV index versus iteration. BFV index of GRSA descends faster than GA and PSO. Therefore, figure 2 shows the effectiveness and convergence speed of the proposed GRSA.

**Table2:** Simulation results of test functions.

function	Algorithms	BFV	AVB	STD
$f_1$	GA	-1.1121e+004	-9.7111e+003	796.2669
	PSO	-7.8333e+003	-6.3254e+003	673.6822
	<b>GRSA</b>	<b>-1.2569e+004</b>	<b>-1.1998e+004</b>	<b>806.3384</b>
$f_2$	GA	28.5587	48.0958	11.8329
	PSO	27.9103	43.8791	12.0843
	<b>GRSA</b>	<b>1.9899</b>	<b>10.1609</b>	<b>5.2855</b>
$f_3$	GA	1.5019	2.4706	0.3613
	PSO	3.0150	4.9310	1.2307
	<b>GRSA</b>	<b>2.0512e-008</b>	<b>2.2951e-006</b>	<b>3.9401e-006</b>
$f_4$	GA	1.0550	1.1365	0.0341
	PSO	0.7077	1.0053	0.0914
	<b>GRSA</b>	<b>1.2212e-015</b>	<b>0.0021</b>	<b>0.0069</b>
$f_5$	GA	1.5019	2.4706	0.3613
	PSO	3.0150	4.9310	1.2307
	<b>GRSA</b>	<b>2.0512e-008</b>	<b>2.2951e-006</b>	<b>3.9401e-006</b>
$f_6$	GA	0.7148	1.3284	0.4445
	PSO	6.3180	33.3441	17.4949
	<b>GRSA</b>	<b>1.2477e-019</b>	<b>0.0011</b>	<b>0.0034</b>

GRSA is investigated by using 6 selected benchmark functions. BFV, AV and STD indexes are employed to evaluate the proposed algorithm in comparison with GA and PSO. BFV, AV and STD are criteria for getting the global optimum, good convergence and robustness, respectively. Obtained results verify that GRSA is an effective method in finding the optimal solution of multimodal large dimensional functions.

Previous discussions illustrate GRSA performance in solving different optimization problems and also verify GRSA robustness in finding near global optimal solutions. Based on numerical simulation results and comparison with two existing search algorithms, we can conclude that GRSA can be applied to a vast group of optimization problems. Specially, it can be apply to multimodal large dimensional functions in a large search space for obtaining near global optimal solutions, satisfactorily. In other word, GRSA has good performance in finding near global optimal solutions for nonlinear, complex and multimodal large dimensional functions in a large search space.

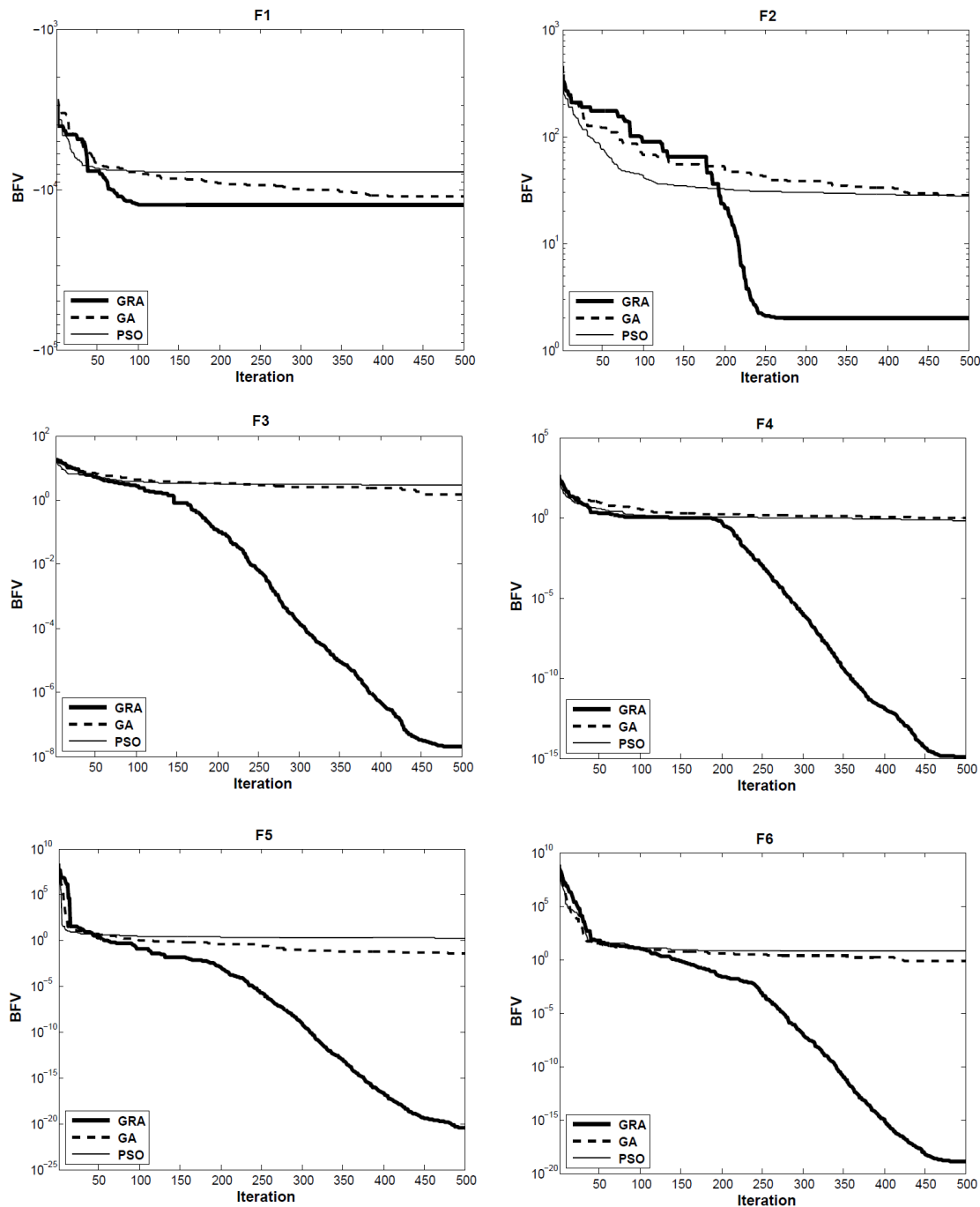


Figure 1: Convergence curve of GA, PSO and GRSA for the test functions.

## 5. CONCLUSION

In this paper, a new optimization algorithm which is called General Relativity Search Algorithm (GRSA) is introduced. GRSA is inspired by general relativity theory (GRT). GRSA searcher agents update their positions by calculating the step length and step direction, separately. Velocity determines the step length and the three types of geodesic determine the step direction. The spacetime geometry properties and also particle energy-momentum together determine the velocity of the particle. Accordingly, the particles evolve to a position with minimum action step by step. In the proposed algorithm, particles are solution candidates for the optimization problem. Particles actions are

determined by values of objective function. Numerical simulation results show that GRSA has major advantages in solving multimodal large dimensional optimization problems, especially in finding near global optimal results.

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