2.3. Let $A \in \mathbb{C}^{m\times m}$ be hermitian. An eigenvector of A is a nonzero vector $x \in \mathbb{C}^m$ such that $Ax = \lambda x$ for some $\lambda \in \mathbb{C}$, the corresponding eigenvalue.

(a) Prove that all eigenvalues of A are real.

A - hermitian $\Rightarrow$ e.g. $\rightarrow$ $\begin{pmatrix} a_{11}+b_{11}i & a_{12}+b_{12}i \\ a_{21}+b_{21}i & a_{22}+b_{22}i \end{pmatrix} = \begin{pmatrix} a_{11}-b_{11}i & a_{21}-b_{21}i \\ a_{12}-b_{12}i & a_{22}-b_{22}i \end{pmatrix}$.

So, $a_{11}+b_{11}i = a_{11}-b_{11}i \Rightarrow$
$\Rightarrow b_{11} = 0$. Same for b_{22} $\Rightarrow$
$\Rightarrow$ we have real numbers on the diagonal

$\begin{pmatrix} a_{11} & a_{12}+b_{12}i \\ a_{21}+b_{21}i & a_{22} \end{pmatrix} X = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} X \Rightarrow$

real $\mathbb{R} \Rightarrow \lambda_1 \in \mathbb{R}$
$a_{11} = \lambda_1$
$a_{22} = \lambda_2$
real $\mathbb{R} \Rightarrow \lambda_2 \in \mathbb{R}$

$\Rightarrow \lambda = (\lambda_1, \lambda_2) \in \mathbb{R}^2$

Same for more dimensions i.e. $\mathbb{R}^n$