DATA 605: Assignment 3

MUSA T. GANIYU

September 17, 2016

QUESTION 1.1:

- What is the rank of the matrix A?

```
A1 <- matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), nrow = 4, byrow = T)

A1
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    2    3    4
## [2,]   -1    0    1    3
## [3,]    0    1   -2    1
## [4,]    5    4   -2   -3
```

```
dim(A1)
```

```
## [1] 4 4
```

Solution: From the above matrix, its known that its dimension is 4x4(a square matrix), therefore it rank is 4

QUESTION 1.2:

- Given an mxn matrix where m > n, what can be the maximum rank? The mini- mum rank, assuming that the matrix is non-zero?

Solution:

Since the rank is the number of all non-zero row, the rank has to be no greater than the smaller of the row or column dimension is n.

QUESTION 1.3

- What is the rank of Matrix B?

```
B <- matrix(c(1,2,1,3,6,3,2,4,2), nrow = 3, byrow = T)

B
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    1
## [2,]    3    6    3
## [3,]    2    4    2
```

```
dim(B)
```

```
## [1] 3 3
```

```
R1 <- B[1, ]
R2 <- B[2, ]
R3 <- B[3, ]

a <- R1-(1/3)%*%R2
b <- R3-(2/3)%*%R2
```

```r
Mat <- matrix(c(a,b,R2), nrow = 3, byrow = T)

Mat
```

```
##      [,1] [,2] [,3]
## [1,]    0    0    0
## [2,]    0    0    0
## [3,]    3    6    3
```

Solution:

Since the rank is the number of all non-zero row, therefore the rank is 1.

QUESTION 2:Compute the eigenvalues and eigenvectors of the matrix A.

```r
A <- matrix(c(1,2,3,0,4,5,0,0,6), nrow = 3, byrow = T)

A
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
## [2,]    0    4    5
## [3,]    0    0    6
```

```r
I <- matrix(c("x" ,0,0,0,"x" ,0,0,0,"x" ), nrow = 3, byrow = T)

I
```

```
##      [,1] [,2] [,3]
## [1,] "x"  "0"  "0"
## [2,] "0"  "x"  "0"
## [3,] "0"  "0"  "x"
```

$$det\left(\begin{bmatrix}1 & 2 & 3\\0 & 4 & 5\\0 & 0 & 6\end{bmatrix} - \begin{bmatrix}\lambda & 0 & 0\\0 & \lambda & 0\\0 & 0 & \lambda\end{bmatrix}\right) = det\left(\begin{bmatrix}1-\lambda & 2 & 3\\0 & 4-\lambda & 0\\0 & 0 & 6-\lambda\end{bmatrix}\right)$$

$$= (1-\lambda)\,[(4-\lambda)(6-\lambda) \quad - (0*0)] \quad - \quad (2)\,[(0)(6-\lambda) - (0*0] \quad + \quad (3)\,[(0*0) - (0*4-\lambda)]$$

$$= (1-\lambda)\,[(4-\lambda)(6-\lambda)] \quad = \quad 0$$

- From the above, we can deduce (solve algebraically) that λ are : 1, 4 and 6

Their respective Eigenvalues are:

For $\lambda_1 = 1$, its eigenvectors are $\begin{bmatrix}1.0000\\0.0000\\0.0000\end{bmatrix}$

For $\lambda_2 = 4$, its eigenvestors are $\begin{bmatrix}1.6000\\2.5000\\1.0000\end{bmatrix}$

For $\lambda_3 = 6$, its eigenvectors are $\begin{bmatrix}0.6667\\1.0000\\0.0000\end{bmatrix}$

OR

```r
eigen(A, only.values = FALSE, EISPACK = TRUE)
```

```
## $values
```

```
## [1] 6 4 1
##
## $vectors
##           [,1]      [,2] [,3]
## [1,] 0.5108407 0.5547002    1
## [2,] 0.7981886 0.8320503    0
## [3,] 0.3192754 0.0000000    0
```