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CALCULUS I - DIFFERENTIATION FORMULAS
QUIZ 1

Question 1: If $f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$ then $f'(x)$ is ?

Solution

Let $\frac{1}{x} + \frac{1}{x^2} = x^{-1} + x^{-2}$

Using product rule, $\frac{\delta y}{\delta x} = U dV + V du$

where $U = x^{-1} + x^{-2}$, $V = (3x^3 + 27)$

$$\therefore \frac{\delta y}{\delta x} = (x^{-1} + x^{-2})(9x^2) + (3x^3 + 27)(-x^{-2} - 2x^{-3})$$

$$= (9x + 9)(3x^3 + 27)(-x^{-2} - 2x^{-3})$$

$$= \frac{1}{-54x^3} - \frac{1}{27x^2} + 6x + 3$$

Rearrange in proper form to obtain,

$$= 3 + 6x - 27x^{-2} - 54x^{-3}$$

Ans

(2) If $y = (2x^7 - x^2)\left(\frac{x-1}{x+1}\right)$ then $y'(1)$ is

recall from product rule $\frac{\delta y}{\delta x} = U du + V du$

Let $U = (2x^7 - x^2)$

$V = \left(\frac{x-1}{x+1}\right)$ where $V = \frac{x-1}{x+1}$ is a quotient rule

$$\therefore \frac{\delta V}{dx} = \frac{V du - U dv}{V^2}$$

Question 2 Cont'd

$$= \left(2x^7 - x^2\right) \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2} + \frac{(x-1)}{(x+1)}\left(14x^6 - 2x\right)$$

$$= \left(2x^7 - x^2\right) \frac{(x+1) - x + 1}{(x+1)^2} + \frac{14x^7 - 2x^2 - 14x^6 + 2x}{x+1}$$

Also $f'(1) = \frac{2(1)^7 - (1)^2 (1+1) - 1 + 1}{(1+1)^2} + \frac{14(1)^7 - 2(1)^2 - 14(1)^6 + 2(1)}{(1+1)}$

$$= \frac{1}{2}$$

$$\therefore \frac{\delta y}{\delta x}(1) = \frac{1}{2}$$

Ans

Question 3 : If $f(x) = (x^2+1)\sec x$ then $f'(x)$ is?

This is also a product rule, Let $\frac{dy}{\delta x} = Udv + Vdu$

where $U = (x^2+1)$, $\delta u = 2x$

$V = \sec x$ $\quad \delta v = \sec x \tan x$

$$\therefore \frac{dy}{\delta x} = (x^2+1)(\sec x \tan x) + (\sec x)(2x)$$

$$= \sec x\left(x^2+1 \tan x + 2x\right)$$

Ans

Question 4: If $f(x) = \frac{1}{\cot x}$ then $f'(x)$ is?

<u>Solution</u>

Recall that $\tan x = \frac{1}{\cot s}$

and $\frac{dy}{d x}(\tan x) = \sec^2 x$

$\therefore \quad f'(x) = \sec^2 x$

Ans.

Question 5: If $y = \tan x$ then $\frac{d^2y}{dx^2}$ is?

Solution:

$$\frac{d^2y}{dx^2} = \text{Second derivative of } y \text{ with respect to } x.$$

Recall that $\frac{dy}{dx}(\tan x) = \sec^2 x$

$$\therefore \frac{dy}{dx}(\sec^2 x) = 2\sec^2 x \cdot \tan x$$

$$\frac{d^2y}{dx^2} = 2\sec^2 x \cdot \tan x$$ Ans.

Question 6: If $f(x) = \sin^3 x$, then $f'(x)$ is?

Solution

$$\text{Let } \frac{dy}{dx} = f'(x) = f'(\sin^3 x)$$

$$\therefore \text{ using } \frac{dy}{dx} = \frac{\delta y}{du} \cdot \frac{\delta u}{\delta x}$$

Let $u = \sin^3 x$

$\delta u = 3\sin^2 \delta x$, $\quad \frac{\delta u}{\delta x} = 3\sin^2 x$

Also $\frac{\delta y}{}(\sin x) = \cos x$

Question 7: If $f(x) = \cos 3\left(\frac{x}{x+1}\right)$ then $f'(x)$ is?

Solution

Using $\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} \cdot \frac{\delta u}{dx}$

$u = \left(\frac{x}{x+1}\right)$

Rewrite $f(x) = \cos^3 u$

$\cancel{u = \cos^3}$

$\cancel{\delta u = 3\cos^2}$

$$\delta u\left(\frac{x}{x+1}\right) = \frac{(x+1)(1)-(x)(1)}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$$

$$= 3\cos^2\left(\frac{x}{x+1}\right)\left[-\sin\frac{x}{x+1}\right]\left[\frac{1}{(x+1)^2}\right]$$

$$= -3\cos^2\left(\frac{x}{(x+1)^2}\right)\ \sin\frac{x}{(x+1)} * \frac{1}{x+1}$$

$$= \frac{-3}{(x+1)^2}\cos^2\left(\frac{x}{x+1}\right)\sin\left(\frac{x}{x+1}\right)$$

Ans

Question 8: If $f(x) = \left[x^4 - \sec(4x^2-2)\right]^{-4}$ then $f'(x)$ is?

Solution

Let $u = (4x^2-2)^{-4}$

$\frac{\delta y}{\delta x} =$

Question 9: If $y = \dfrac{\sin x}{\sec(3x+1)}$ then y' is?

Solution: using $\dfrac{\delta y}{\delta x} = \dfrac{V du - U dv}{V^2}$ and $\dfrac{\delta y}{\delta x} = \dfrac{\delta y}{\delta u} \cdot \dfrac{\delta u}{\delta x}$

Let $V = \sec(3x+1)$

$$\frac{\delta v}{\delta x} = 3 \sec(3+1) \tan(3x+1)$$

$$\therefore \quad \frac{\delta y}{\delta x} = \frac{\sec(3x+1)(\cos x) - (\sin x)(3\sec(3x+1)\tan(3x+1))}{\sec^2(3x+1)}$$

$$= \frac{\sec(3x+1)[(\cos x) - (3 \sin x \tan(3x+1))]}{\sec^2(3x+1)}$$

$$= \cos x \cos(3x+1) - 3 \sin x \sin(3x+1)$$ Ans

Question 10: If $y = \left(\dfrac{1+x^2}{1-x^2}\right)^{17}$ then y' is ?

Solution: Using quotient rule $\dfrac{\delta y}{\delta x} = \dfrac{V du - U dv}{V^2}$ and $\dfrac{\delta y}{\delta x} = \dfrac{\delta y}{\delta u} \cdot \dfrac{du}{\delta x}$

Let $U = \left(\dfrac{1+x^2}{1-x^2}\right) \quad \therefore \; y = u^{17}$

$$\frac{\delta y}{\delta u} = 17 u^{16}$$

CALCULUS I - DERIVATIVES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS - QUIZ 1

Question 1: If $y = \ln\left(\dfrac{(x^2+1)^5}{\sqrt{1-x}}\right)$, then y' is?

Solution:

Let $y = 5\ln(x^2+1) - \frac{1}{2}(1-x)$

$$\frac{dy}{dx} = Vdu - Udv$$

where $V = x^2+1$ $\qquad$ $U = 1-x$

$\delta V = 2x$ $\qquad$ $\delta u = -1$

$$\therefore \frac{dy}{dx} = \frac{5(2x)}{x^2+1} - \frac{1}{2}^{(-1)}\left(\frac{1}{1-x}\right)$$

$$= \frac{10x}{x^2+1} + \frac{1}{2(1-x)}$$

Ans

Question 2: If $y = \ln(\ln x)$, then y' is?

Solution:

Let $u = \ln x$ $\qquad$ $\frac{\delta y}{\delta u} = \frac{1}{u}$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\therefore \frac{dy}{dx} = \frac{\delta y}{\delta u} \cdot \frac{\delta u}{\delta x} = \frac{1}{u} \cdot \frac{1}{x}$$

Question 3: If $y = \dfrac{x \ln x}{1 + \ln x}$, then y' is?

Solution: ~~Recall from Question~~

$$\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x\,(1) = 1 + \ln x$$

and $\dfrac{dy}{dx} = \dfrac{V du - U dv}{V^2}$, $V = (1 + \ln x)$, $U = x \ln x$

$$\therefore \frac{dy}{dx}\left(\frac{x \ln x}{1 + \ln x}\right) = \frac{(1+\ln x)\left(\ln x + \frac{x}{x}\right) - x \ln x\left(\frac{1}{x}\right)}{(1+\ln x)^2}$$

$$= \frac{(1+\ln x)(\ln x + 1) - \frac{x}{x}\ln x}{(1+\ln x)^2}$$

$$= \frac{(1+\ln x)^2 - \ln x}{(1+\ln x)^2}$$

$$= 1 - \frac{\ln x}{(1+\ln x)^2}$$

Ans.

Question 4: If $y = \dfrac{1 + \ln t}{t}$, then y' is?

Solution using quotient rule, $\dfrac{dy}{dt} = \dfrac{V du - U dv}{V^2}$, $V = t$, $U = 1 + \ln t$

$dv = 1$

$du = 1/t$

Question 3: If $y = \frac{x \ln x}{1 + \ln x}$, then y' is?

Solution: ~~Recall from Question 2, $y'(x \ln x) = \frac{1}{x \ln x}$~~

and $\frac{dy}{dx} = \frac{Vdu - Udv}{V^2}$, $V = (1 + \ln x)$, $U = x \ln x$

$\frac{\delta u}{dx} = x\left(\frac{1}{x}\right) + \ln x (1)$
$= 1 + \ln x$

$$\therefore \frac{dy}{dx}\left(\frac{x \ln x}{1 + \ln x}\right) = \frac{(1+\ln x)(\ln x + \frac{x}{x}) - x \ln x\left(\frac{1}{x}\right)}{(1+\ln x)^2}$$

$$= \frac{(1+\ln x)(\ln x + 1) - \frac{x}{x}\ln x}{(1+\ln x)^2}$$

$$= \frac{(1+\ln x)^2 - \ln x}{(1+\ln x)^2}$$

$$= 1 - \frac{\ln x}{(1+\ln x)^2}$$

Ans.

Question 4: If $y = \frac{1 + \ln t}{t}$, then y' is?

Solution using quotient rule, $\frac{\delta y}{dt} = \frac{Vdu - Udv}{V^2}$, $V = t$, $U = 1 + \ln t$

$dv = 1$
$du = 1/t$

$$\therefore \frac{dy}{dt} = \frac{\cancel{t}\left(\frac{1}{\cancel{t}}\right) - (1+\ln t)(1)}{t^2} = \frac{1-(1+\ln t)}{t^2} = \frac{1-1-\ln t}{t^2}$$

$$= \frac{-\ln t}{t^2}$$ Ans.

Question 5: If $y = t\sqrt{\ln t}$, then y' is ?

Solution:

$$\text{Let } \sqrt{\ln t} = (\ln t)^{1/2}$$

Also, using $\cancel{\frac{dy}{\delta x} = \frac{\delta y}{\delta u} \cdot \frac{\delta u}{\delta x}}$ $\quad \frac{dy}{\delta x} = V du + u dv$

~~Let u=~~ $V = t, \quad u = (\ln t)^{1/2}$

$$dv = 1, \quad du = \tfrac{1}{2}(\ln t)^{-1/2} \cdot \tfrac{1}{t}$$

$$\frac{\delta y}{dx} = t\left(\tfrac{1}{2}(\ln t)^{-1/2}\right) + (\ln t)^{t}(1)$$

$$= \ln t + \frac{\frac{d}{dx}(t)}{2\sqrt{\ln t}}$$

$$= \sqrt{\ln t} + + \frac{1}{2(\ln t)^{1/2}}$$

$$= (\ln t)^{1/2} + \frac{1}{2(\ln t)^{1/2}}$$

Ans

Question 6: If $y = \ln \frac{10}{x}$, then y' is?

Solution Let $\frac{10}{x} = 10x^{-1}$

$$\frac{dy}{dx} = \frac{\delta y}{du} \cdot \frac{du}{dx}$$

$y = \ln 10x^{-1}$

Question 8: If $f(x) = e^{3x-1}$, then $f'(x)$ is?

Solution:

Let $3x-1 = u$ $\qquad y = e^u$

$u = 3x-1$

$\frac{dy}{dx} = 3$ $\qquad \frac{dy}{du} = e^u$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 3e^u$ (but $u = 3x-1$)

$= 3e^{3x-1}$ Ans.

SECTION 3.7 INDETERMINATE FORMS & L'HOSPITAL RULE

Question 1: find the $\lim\limits_{\theta \to 0} \frac{\tan\theta}{\theta}$

Solution:

$$\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = \lim_{\theta \to 0} \frac{(\tan\theta)}{(\theta)}$$

$$= \lim_{\theta \to 0} \frac{\sec^2\theta}{1} = \underline{\underline{1}}\ \text{Ans}$$

Question 3: $\lim\limits_{x \to 0} \frac{\sin^{-1} 2x}{x}$

Solution

$$\lim_{x \to 0} \frac{d'}{dx}\left(\frac{\sin^{-1} 2x}{x}\right)$$

Let $u = 2x$ $\qquad y = \sin^{-1} u$

$\frac{du}{dx} = 2$ $\qquad \frac{dy}{du} = \sqrt{1-2^2x^2}$

$\qquad\qquad\qquad = \sqrt{1-4x^2}$

$$\therefore \lim \frac{d'}{dx}\left(\frac{\sin^{-1} 2x}{x}\right) = \lim_{x \to 0} \frac{2}{\sqrt{1-4x^2}}$$

Question 4 : $\lim\limits_{x \to \pi/2} \sec 3x \cos 5x$

Solution

$$= \lim_{x \to (\pi/2)} \frac{\cos 5x}{\cos 3x}$$

$$= \lim_{x \to (\pi/2)} \frac{-5 \,\mathrm{Sln}\, 5x}{-3 \,\mathrm{Sln}\, 3x}$$

$$= \frac{-5(1)}{(-3)(-1)}$$

$$= \frac{-5}{3} \text{ ans}$$

Question 6 : $\lim (2-x)^{\tan(\pi/2)x}$

Solution

$y = (2-x)^{\tan(\pi/2)x}$

In logarithm of both side,

$\ln y = \ln(2-x)^{\tan(\pi/2)x}$

$= \tan\left(\frac{\pi}{2}x\right) \ln(2-x)$

$$= \lim_{x \to 1} \frac{(2-x)^{-1} \cdot (-1)}{-\mathrm{Cse}^2\left(\frac{A}{2}x\right) \cdot \frac{\pi}{2}}$$

CALCULUS II - INTEGRATION

Question 1: $\int_{-2}^{0} (2x+5)\,dx$

Solution

$$\int_{-2}^{0} (2x+5)\,dx = \left[\frac{2x^2}{2} + 5x + c\right]_{-2}^{0}$$

$$= \left[\frac{2(0)^2}{2} + 5(0)\right] - \left[\frac{\cancel{2}(-2)^2}{\cancel{2}} + 5(-2)\right]$$

$$= [0+0] - [4-10]$$

$$= -(-6)$$

$$= 6 \text{ ans}$$

Question 2: $\int_{0}^{\pi} \sin x\,dx$

Solution

$$\int_{0}^{\pi} \sin x\,dx = \left[-\cos x\right]_{0}^{\pi}$$

$$= (-\cos\pi) - (-\cos 0)$$

$$= -\cos 180 - (-\cos 0)$$

Question 3 $\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|)\,dx$

Solution

$$\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|)\,dx$$

$$= \int_0^{\pi/2} \frac{1}{2}(\cos x + \cos x)\,dx + \int_{\pi/2}^{\pi} \frac{1}{2}(\cos x - \cos x)\,dx$$

$$= \left[\sin x\right]_0^{\pi/2} = \sin \pi/2 - \sin 0$$

$$= \underline{\underline{1}}$$ ans

Question 4 : $\int \frac{9r^2\,dr}{\sqrt{1-r^3}}$

Solution : let $u = 1-r^3$

$$\frac{du}{dr} = -3r^2,\quad du = -3r^2\,dr$$

Let $v = (1-r^3)^{1/2}$

$$\int -3u^{-1/2}\,du = -3(2)u^{1/2} + c$$

$$= \underline{\underline{-6(1-r^3)^{1/2} + C}}$$ ans

Question 5 : $\int \frac{dx}{\sqrt{5x+8}}$

Solution

Let $u = 5x+8$ $du = 5\,dx$, $dx = \frac{du}{5}$

$$= \int (5x-8)^{-1/2}\,dx$$

$$= \int u^{-1/2}\,dx$$

Question 6 $\int \tan^7\left(\frac{x}{2}\right) \sec^2\left(\frac{x}{2}\right) dx$

Solution

Let $u = \tan\left(x/2\right)$ $\quad \frac{du}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$

$2du = \sec^2\left(x/2\right)dx$

$2\int u^7\, du = 2\left(\frac{u^8}{8}\right) + C$

$= \cancel{2}\left(\frac{(\tan x/2)^8}{\cancel{8}\,4}\right) + c$

$= \frac{\tan^8 x/2}{4} + c$ Ans

Question 7: $\int xe^{2x}dx$

Solution

This is integration by part,

using $\int u\,dv = uv - \int v\,du$

Let $u = x$, $v = \frac{1}{2}e^{2x}$, $du = dx$, $dv = e^{2x}dx$

$\int u\,dv = \int xe^{2x}dx = x\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}dx$

$= \frac{xe^{2x}}{2} - \frac{1}{2}\left[\int e^{2x}dx\right]$

$= \frac{xe^{2x}}{2} - \frac{1}{2}\left[\frac{1}{2}e^{2x}\right]$

$= xe^{2x} \quad \frac{1}{4}e^{2x}$

Question 8 : $\int \frac{x^2}{x+1}\,dx$

Solution

$$\int \frac{x^2}{x+1}\,dx = \int \left((x-1) + \frac{1}{x-1} \right) dx$$

$$= \frac{x^2}{2} - x + 1 \ln(x-1) + C$$

$$= \frac{x^2}{2} - x + \ln(x+1) + C$$

Ans

Question 9 : $\int \frac{1}{\sqrt{16-x^2}}\,dx$

Question 10 : $\int_0^\infty x^{-3/2}\,dx$

$$= \lim_{0 \to \infty} \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} \Big]_2^0$$

$$= \lim_{0 \to \infty} -2x^{-1/2} \Big]_2^0$$

$$= \sqrt{2}$$

Ans