Predicate Logic

Variables

- $E(x)$ = x is even.
- $G(x,y)$ = x is greater than y. — This is not a statement. — open — No truth values.

closed:

- $G(2,1)$ = 2 is greater than 1. True
- $G(3,6)$ = 3 is greater than 6. False

Quantifiers

$\forall x\ P(x)$: "For all x, x is P" — $\forall$: universal

$\exists x\ P(x)$: For some x, x is P" — $\exists$: Existential; At least one exists

$\forall x\ P(x) = P(1) \wedge P(2) \wedge P(3) \ldots P(n)$

$\exists x\ P(x) = P(1) \vee P(2) \vee P(3) \ldots P(n)$

Negating Quantifiers

$\neg \forall x\ P(x) = \neg (P(1) \wedge P(2) \wedge P(3) \ldots P(n))$

Using DeMorgan's

$= \neg P(1) \vee \neg P(2) \vee \neg P(3) \ldots \neg P(n)$

$= \exists x [\neg P(x)]$

Examples

1) $\forall x\ P(x) \iff P(1) \wedge P(2) \ldots$

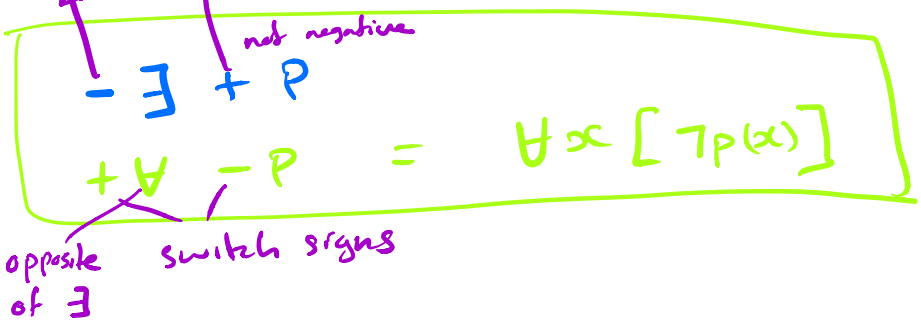
$= \neg \exists x (\neg P(x))$

2) $\exists x\ P(x) \iff P(1) \vee P(2) \ldots$

$= \neg \forall x [\neg P(x)]$

3) $\neg \forall x\ P(x) \iff = \exists x [\neg P(x)]$

4) $\neg \exists x\ P(x) = \forall x [\neg P(x)]$



eg. $\exists x\ P(x)$ $\neg \forall x [\neg P(x)]$

$+\exists\ +P$

$-\forall\ -P$

5) $\forall x\ \exists y\ [P(x,y) \wedge Q(y)]$

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① $\exists x\ (P(x) \wedge Q(x))$

$\forall (x)\ (\neg P(x) \vee \neg Q(x))$

② $\forall x\ (Px \rightarrow Qx)$

$\neg [\forall x (Px \rightarrow Qx)$

$$\exists x [\neg (Px \rightarrow Qx)]$$

$$\exists x [\neg (\neg Px \vee Qx)]$$

$$\exists x (Px \wedge \neg Qx)$$