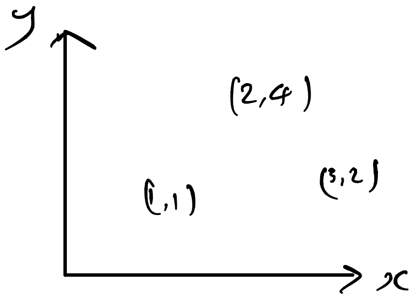

Cartesian product

$A, B \subseteq U$

$(u, v) \in A \times B = \{a, b \mid a \in A, b \in B\}$

$A \times B \neq B \times A$

$(u, v) = (v, x)$

$|A \times B| = |A| * |B|$

Binary Relation

Binary relation from A to B is a subset of $A \times B$

$R \subseteq A \times B$

$(a, b) \in R, \quad aRb$

$\leqslant$

$(2,7) \in \leq$ $\quad 2 \leq 7$

$(7,2) \notin \leq$ $\quad 7 \nleq 2$

Inverse of R, denoted as R^{-1} is a relation from $B \times A$

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① Reflexive if aRa, for every $a \in A$

$(a,a) \in R$

$\leq$ over $\mathbb{Z}$ $\quad a \leq a$, always true

Not reflexive $<$ over $\mathbb{Z}$ $\quad a \not< a$, never true

---

② Symmetric if aRb implies bRa

if $(a,b) \in R$ then $(b,a) \in R$

$2 \leq 3 \Rightarrow 3 \geq 2$

$\leq$ not symmetric

---

③ Transitive

aRb, bRc, then aRc

$a \leq b$, $b \leq c$ then $a \leq c$

eg $2 \leq 7$, $7 \leq 10$ then $2 \leq 10$

(4) Anti-symmetric

If aRb and bRa then $a=b$

If aRb, $a \neq b$ then $b \not R a$.

eg, $1 \leq b$ then $2 \not\leq 1$

$1 \leq 1$ then $1 \leq 1$

Functions

A function $f: A \to B$ is a subset of $A \times B$.

where every $a \in A$ appearing exactly once as the first component of $(a,b) \in A \times B$.

double : $\mathbb{N} \to \mathbb{N}$

double $(x) = 2 * x$

$\{(0,0), (1,2), (2,4), (3,6) \ldots \}$

$f: A \to B$

A domain

B co-domain

$\{f(x) \mid x \in A\} \subseteq B$ range

Every function is a relation

Not every relation is a function.

Composite functions

$f : A \to B$ $g : B \to C$

$(g \circ f)(a) = g(f(a))$

$f(a) = a^2$ $g(a) = 2a$

$g \circ f(a)$, $(g \circ f)(-2) = 8$

Mapping

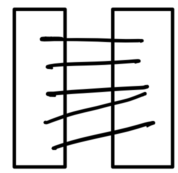
One to one ✓ Injective

one to many ✗

many to one ✓ Injective

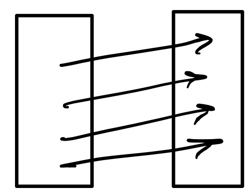
$f: A \to B$, surjective or onto

For every $b \in B$, there is $a \in A$ such that $f(a) = b$.



$f: \mathbb{Z} \to \mathbb{Z}$ $\quad f(a) = a+1$

$g: \mathbb{Z} \to \mathbb{Z}$ $\quad g(a) = a^2$

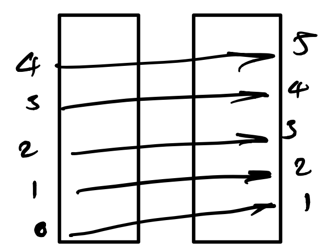


$f: A \to B$ bijective, if f is both injective and surjective.

$f: \mathbb{N} \to \mathbb{N}$, $f(a) = a+1$

Injective ✓

Surjective ✗



no $a \in \mathbb{N}$ such that $f(a) = 0$.

Floor $\lfloor x \rfloor$ integer below x

$\lfloor 2,7 \rfloor = 2$, $\quad \lfloor 3.9 \rfloor = -4$

$\lfloor 3.0 \rfloor = 3$

Ceiling

$\lceil x \rceil$ integer above x.

$\lceil 2.7 \rceil = 3$ $\lceil -3.9 \rceil = -3$

Abs - Modulus

INT (x) Integer part

INT (2.7) = 2 INT (-3.9) = -3

Closer towards 0

Factorial, $x!$

x mod M

% 27 mod 5 = 2 remainder.

-27 mod 5 = 3

-16 mod 2 = 0

-6 mod 8 = 2

Recursive functions