Propositional logic

- A statement is a declarative sentence which can be either true or false.
- An imperative sentence or questions are not included.
- Propositions are denoted with capital letters, eg. P, Q, R. They will have a translation whereas p, q, r will not.

Connectives

These will change the meaning of a proposition.

P is a well formed formula (wff)

$\neg P$ not p

$P \wedge q$ p and q

$P \vee q$ p or q

$P \rightarrow q$ if p, then q

Logic should be in the affirmative

If there is a variable in a sentence then it is not a statement.

Truth tables

Negation

P	$\neg P$
1	0
0	1

$$\text{Neg } P = 1 - P$$

$$1 - 1 = 0$$

$$1 - 0 = 1$$

Conjunction ($\wedge$, &, $\cdot$)

p	q	$p \wedge q$
0	0	0
1	0	0
1	1	1
0	1	0

$p \wedge q = \min(p, q)$

$2^{\text{number of}}$

Disjunction ($\vee$, +)

p	q	$p \vee q$
0	0	0
1	0	1
1	1	1
0	1	1

$p \vee q = \max(p, q)$

sum

Conditional ($\rightarrow$, $\supset$)

p	q	$p \rightarrow q$
0	0	1
1	0	0
1	1	1
0	1	1

$\boxed{p \rightarrow q = 1 \text{ iff } p \leq q \qquad 1 \nleq 0 = 0}$

Only false when p is true and q is false.

If it's sunny, I will wear sunscreen

when (lying)?

Biconditional ($\leftrightarrow$, $\equiv$)

p	q	$p \leftrightarrow q$

p	q	$p \Leftrightarrow q$
0	0	1
1	0	0
1	1	1
0	1	0

iff $p = q$

Exclusive or ($\oplus$, $\veebar$)

p	q	$p \veebar q$
0	0	0
1	0	1
1	1	0
0	1	1

$p \neq q$

then $p \oplus q = 1$

Logically equivalent.

when two propositions have the same values, there they will be logically equivalent.

eg $p = x$ $q = x$ $p = q$

p	q	$\neg p$	$\neg q$	$p \vee \neg q$		
1	1	0	0	1	0	1
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	1	0	1

p	q	$p \to q$	$p \wedge (p \to q)$	$p \wedge (p \to q) \to q$
1	1	1	1	1
1	0	0	0	1
0	1	1	0	1
0	0	1	0	1

$(p \oplus q) \vee (p \oplus \neg q)$

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	
1	1	0	0	1	1
1	0	1	1	0	1
0	1	0	1	0	1
0	0	1	0	1	1

Sheffer Stroke

↑ Nand   Not   and

$P \uparrow Q$ Not   P and q.

$p \uparrow p = \neg p$

$p \uparrow q \Leftrightarrow \neg (p \wedge q)$

$p \wedge q \Leftrightarrow \neg\neg (p \wedge q)$

$\neg (p \uparrow q)$

$\Leftrightarrow (p \uparrow q) \uparrow (p \uparrow q)$

$p \vee q \Leftrightarrow$

$\neg\neg p \vee \neg\neg q$

$\neg(\neg p \wedge \neg q)$

$(p \uparrow p) \uparrow (q \uparrow q)$

Logic Laws

Tautology : T : (Always 1)

Contradiction : F : (Always 0)

Identity - Will depend on the value of p

$p \wedge T \Leftrightarrow p$

$p \vee F \Leftrightarrow p$

Domination

$p \vee T \Leftrightarrow T$ - As T will always be 1

$p \wedge F \Leftrightarrow F$ - As F will always be 0

$(p \vee F) \wedge (q \vee T)$

$p \wedge T$

Double negation

$\neg\neg p \Leftrightarrow p$

De Morgan's Law

flip

$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

add $\neg$ sign

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

eg. $\neg(\neg p \wedge \neg q)$

$\neg\neg p \vee \neg\neg q$

$p \vee q$

Distributive Law

eg $3 \times (1+2) \Leftrightarrow (3\times 1) + (3\times 2)$

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Absorption Law

Commutative $(\vee, \wedge)$

$p \wedge q \Leftrightarrow q \wedge p$

Associativity $(\vee, \wedge)$

$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

is same

Inverse Law

$p \wedge \neg p \iff F$

$p \vee \neg p \iff T$

Conditional Law

$p \to q \iff \neg p \vee q$

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

$\neg p = 0$

$q = 0$

$\neg p \vee q = 0$

$\neg(p \wedge q) \wedge q)$

$(\neg p \vee \neg q) \wedge q$

$q \wedge (\neg p \vee \neg q)$

$\boxed{\neg p \wedge q} \vee \neg q \wedge q$ impossible

$\neg p \wedge q$

Conditionals

Conditional

$p \to q$

Converse

$q \to p$

switch over

Inverse

$\neg p \rightarrow \neg q$

Opposite signs

logically equivalent

Contra positional

$\neg q \rightarrow \neg p$

Converse, then inverse

q	p	$q \rightarrow p$
0	0	1
0	1	1
1	0	0
1	1	1

$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$
0	0	1
0	1	1
1	0	0
1	1	1

$$p \rightarrow q = \neg p \vee q$$

$$\neg q \rightarrow \neg p = q \vee \neg p$$

$$q \rightarrow p = \neg q \vee p$$

$$\neg p \rightarrow \neg q = p \vee \neg q$$

Examples

1. $2 \times 7 = 14$ only if $6 \times 2 = 18$ False
   T   F

2. If it snows, I will cry.

   $p \rightarrow q$

Converse $q \rightarrow p$
If I cry it will snow

inverse $\neg p \rightarrow \neg q$
If it does not snow, I will not cry

Contra $\neg q \rightarrow \neg p$
If I don't cry, It will not snow

Rules of Inference

- Taking 1 or more premises and getting a conclusion.

① Modus Ponens (MPP)

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

② Modus Tollens (MTT)

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

You can use the contra rule, thereafter apply MPP rule.

③ Hypothetical Syllogism (HS)

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline p \rightarrow r \end{array}$$

$p \rightarrow q \rightarrow r$

taking away the middle man

④ Disjunctive Syllogism (DS)

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

one has to be true

⑤ Addition $\vee I$ (or introduction)

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

⑥ Simplification $\wedge E$

And elimination

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \\ \therefore q \end{array}$$

removal of $\wedge$

⑦ Conjunction $\wedge I$

$\wedge$ Introduction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$