

Example:

$$A = \begin{bmatrix} 1 & 4 \\ 9 & 1 \end{bmatrix}$$

$$|A| = 1 - 9 \times 4 = -35$$

$$\begin{aligned} |A - \lambda I| &= \left| \begin{bmatrix} 1 & 4 \\ 9 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| \\ &= \begin{vmatrix} 1-\lambda & 4 \\ 9 & 1-\lambda \end{vmatrix} = 0 \end{aligned}$$

$$\Leftrightarrow (1-\lambda)^2 - 36 = 0 \quad \Leftrightarrow (1-\lambda)^2 = 36$$

$$\Leftrightarrow 1-\lambda = \pm 6$$

$$\lambda_1 = 7 \quad \text{ou} \quad \lambda_2 = -5.$$

Ensemble :

$$\left\{ x \in \mathbb{R}^2 : \text{tel que } (A - \lambda I)x = 0 \right\}$$

$$(A - 7I)x = 0 \Leftrightarrow \begin{bmatrix} -6 & 4 \\ 9 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Leftrightarrow \begin{cases} -6x_1 + 4x_2 = 0 & \dots \textcircled{1} \\ 9x_1 - 6x_2 = 0 & \dots \textcircled{2} \end{cases}$$

①
2

\Leftrightarrow

$$\begin{cases} -3x_1 + 2x_2 = 0 \end{cases}$$

\Leftrightarrow

$$3x_1 = 2x_2$$

②
3

\Leftrightarrow

$$\begin{cases} 3x_1 - 2x_2 = 0 \end{cases} \Leftrightarrow$$

$$3x_1 = 2x_2$$

$$x_1 = \frac{2}{3}x_2$$

$$\begin{aligned} \left\{ x \in \mathbb{R}^2 : (A - \lambda I)x = 0 \right\} &= \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 : 3x_1 - 2x_2 = 0 \right\} = \left\{ \alpha \in \mathbb{R} : \alpha \begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} \right\} \\ &= \left\{ \alpha \in \mathbb{R} : \alpha \begin{pmatrix} 3 \\ 2 \end{pmatrix} \right\} \end{aligned}$$

$$\{x \in \mathbb{R}^2: (A + 5I)x = 0\}$$

$$(A + 5I)x = \begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Leftrightarrow \begin{cases} 6x_1 + 4x_2 = 0 \\ 9x_1 + 6x_2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 = -\frac{2}{3}x_2 \\ x_1 = -\frac{2}{3}x_2 \end{cases}$$

$$\left\{ x \in \mathbb{R}^2; (A + 5I)x = 0 \right\} = \left\{ x \in \mathbb{R}^2; x = \alpha \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right\}$$

$$\alpha \in \mathbb{R}$$

$$\lambda_1 = 7 \quad \text{et} \quad \lambda_2 = -5$$

$$v_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \text{et} \quad v_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$