Active Sensing with a UAV-mounted Chemical Sensor

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Abstract

Modelling a spatio-temporal phenomena, eg. weather temperature across large area, is often a challenging task due to the constraint of limited number of possible measurements, as well as their accuracy. This poses the question of active sensing - how to most efficiently take measurements in order to decrease the overall uncertainty in the model and predict unobserved states based on those limited measurements.

This proposal describes a framework for using a novel UAV-mounted chemical for detecting hazardous gases and rapidly identify the source of the emission. For the purpose, we plan to use Gaussian Processes as to model the concentration of the gas over the observed area. This gives us a way to explicitly model the uncertainty in our model as well as identify places of potential high concentration.

Expected results from this project include: integrating an UAV, onboard computer and spectrometer; obtaining an array of measurements from the spectrometer in a representative environment; implement active sensing capabilities using Gaussian Processes.

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1. Purpose

In this proposal, we aim to tackle the problem of active sensing using mobile robot-mounted sensors. We plan to use a UAV with a novel Fourier-transform spectrometer for broadband detection and characterization of gases. The goal is to actively collect data as to interpolate the field and then to improve the quality of the predictive modelling. Deciding where to take measurements from can be formalized as an exploration/exploitation problem. From one side we want to explore areas of high uncertainty, places where we have sparse measurements about the observed phenomena. However, once we have found areas of interest we want to take more samples from there, or as in our case - head towards the source of contamination.

For the purpose, we will implement a Gaussian Processes regression. Using this model comes with some highly desirable properties which can be exploited in the specified problem. First of all, we can specify useful priors, eg. close locations have similar concentration of gases. Secondly, we can explicitly model the uncertainty of the field, based on the smooth change of concentration of gases as well as the noise from our sensor. Finally, the model gives us a formal framework where we can either pick a place with high uncertainty (exploration) or high concentration (exploitation). Moreover, we can do all these calculations and measurements in a lazy fashion - make a measurement, update our model, pick a new place to make another measurement, update our model again, etc.

Detecting and finding the source of hazardous gases is already a useful application for military and government purposes. However, once the system is developed, hardware-wise and software wise, the UAV could be as easily used for other applications. This includes measuring wind speeds, temperature, atmospheric pressure and so on which can be done by a sensor substitution. Analyzing different weather aspects has major practical implications. Example include renewable energy integration, tracking dynamic spatio-temporal phenomena such as vortices in the atmospheric boundary level, etc. Successful implementation can lead to they system being used as a scientific tool for measurement in a variety of different fields.

2. Background

2.1 Spectroscopy

Previous work has built the foundations towards our proposed multidisciplinary project. Research by our collaborators from Heriot-Watt [1] has led to the development of mid-infrared Fourier-transform spectroscopy for standoff detection of liquids on surfaces [17] and the detection of aerolized airborne hazards [10]. However, in both of those cases, they relied on powerful and large femtosecond laser system, incompatible for mounting and using on a UAV system. In our sys-

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tem they plan to replace it with conventional thermal light source and investigate what level of performance can be maintained.

The purpose of a spectrometer is to measure the emission spectrum of a light source. The beam is focused through a slit, dispersed and a sensor on the other side detects the different wavelengths. The infrared spectra of more than 60 explosive compounds was analyzed in [12]. In this study, it was shown that explosives can be distinguished by their absorption in different wavelengths regions - alcohols showing a strong band near $2.9\mu m$ and amines near $3.0\mu m$. It can also be seen that it might be possible to detect common explosives like TNT, PETN, TATP and RDX from their absorption spectra in the $2.6\mu m$ to $3.6\mu m$ band [14].

2.2 Gaussian Processes for modelling spatio-temporal phenomena

Using Gaussian Processes ($\mathcal{GP}$) regression for modelling spatio-temporal phenomena has been an active area of research in recent years. An example of monitoring a spatio-temporal phenomena and their dynamics is discussed in [15]. The phenomenon discussed in the paper is water quality in rivers and lakes. The goal specified is to maximize the information collected, while taking into account the limitation of the sensor devices and robots being used. Hence $\mathcal{GP}$ are a good choice since they can quantify the amount of information they have collected. Moreover, they can pick locations they want to go to next, based on the places with higher uncertainty.

In modeling spatio-temporal phenomena a common problem seems to be few incorrect extreme measurements leading to inaccurate model [8]. $\mathcal{GP}$ tend to reproduce a field around those few extreme measurements, while predictions being low in desirable locations. However, using log-measurements mitigates the issue as it removes extremity and skewness.

Another key idea in conducting inference with $\mathcal{GP}$ is lazy evaluation [5]. We continuously pick a place with high uncertainty - conduct measurements - make inference and update our model - pick a place with high uncertainty - etc. Usually correlation decreases exponentially as the distance between points increases. However, often variables apart from each other are dependent. By exploiting locality in kernel functions we can achieve significant speedup of the algorithm [5]. $\mathcal{GP}$ also prove to be effective in modeling spatio-temporal phenomena over long period of time [7].

There have also been interest in using UAV mounted sensors [1] [6] [11]. Examples of this include coastal wetland mapping, flood and wildfire surveillance, tracking oil spills, urban studies, and Arctic ice investigations [6]. However, most of the work is focused around data collection with the processing down offline after the flight is finished. Currently, not a lot of work looks into the problem of active sensing, where the UAV is processing the data online, updating an underlying model and actively steer to achieve a certain goal.

3. Methods

3.1 Hardware Setup

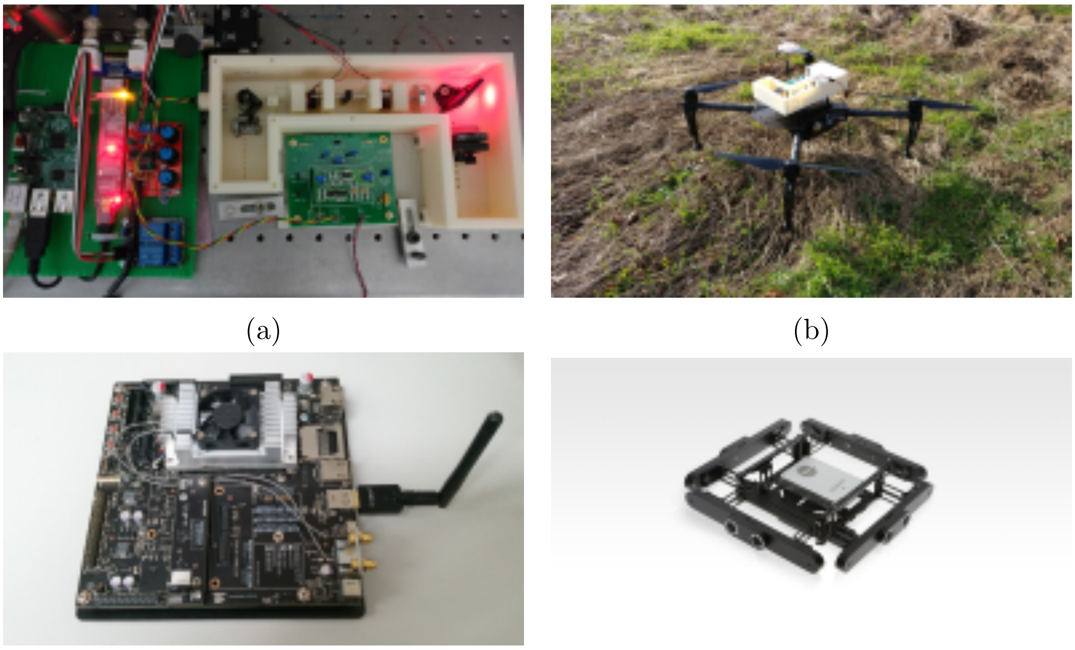
The first part of the project will involve the integration of all the necessary tools on a single UAV platform. The spectrometer sensor is in final stages of development with a few modules left to replaced by more portable equivalents and design being improved as to fit on the quadcopter.

There are three challenges that need to be addressed in order to fit all the necessary equipment on the UAV - weight, power consumption and footprint. With two batteries attached the quadcopter has expected flight time of 40 minutes (20 minutes per battery). In order to reduce the weight of the rest of the components, we plan to remove all the unnecessary parts, eg. fan of the computer, ethernet port, etc., reduce the footprint of the 3D-printed case of the spectrometer. Currently, extra modules are being attached to the quadcopter using square aluminum cases, which utilize a small fraction of the space available on top of the quadcopter. In order to accommodate the larger footprint of the electronics we plan to use, we will be printing a custom 3D case following the contour of the base plate. The battery on the UAV comes with an extension for powering outside electronics. Due to the increased weight and power consumption from the onboard computer and sensor, we expect the flight time to be reduced more than a half, having expected flight time of 5-10 minutes with one battery and up to 15 minutes with two. This should be enough to perform necessary experiments.

The spectroscopy package will be developed by collaborators at Heriot-Watt university. The sensor will be innovative 3D-printed Fourier-transform infrared spectrometer. The main goal is to developed a lightweight and compact embodiment, so it can easily be deployed to UAV. For the purpose, our collaborators plan to use mid-infrared diode for emitting light replacing the large femtosecond laser system. The beam will then be altered using an off-axis parabolic mirror and directed into miniature scanning Michelson interferometer. A PbSe infrared detector will be placed either along the frame of the quadcopter or directly below sensing atmospheric absorption or surface contamination respectively.

By replacing the laser with a thermal light source we will inevitably experience drop in accuracy of the measurements made. However by using the methods outlined below we can explicitly model the noise from our sensor and still construct an accurate model within our noise measurements.

An interesting path of exploration would be to use the error signal from the UAV's autopilot algorithm to sense wind speed and direction. This could provide useful information about the dynamics of the nearby wind, which can further help in modelling the concentration of the observed gases.



(a) (b) (c) (d)

Figure 3.1: **Hardware stack of the project** (a) Spectrometer developed at HWU. Mounting a high-resolution Fourier-transform spectroscopy on a UAV has not been previously reported, thus being one of the novel aspects of the project (b) Matrice 100 quadcopter platform with the spectrometer (c) NVidia TX1 mobile computer with wifi module attached (d) Guidance system. Five ultrasonic sensors monitor the four directions around the quadcopter and below and provide obstacle avoidance functionality

3.2 Active sensing using Gaussian Processes

In order to describe our proposed solution of finding the underlying structure of the concentration of gases and how to fly in order to take the most efficient measurements, we are going to start with a definition of active sensing and spatio-temporal phenomena.

Definition 3.2.1 *Spatio-temporal phenomena is an event relating to, or existing in both space and time.*

Definition 3.2.2 *Active sensing, or active information gathering, is collecting* *the most useful information about a problem and then using the gathered information to do inference with the goal of maximizing the accuracy of the inference while minimizing the quantity of information gathered.*

Using spectroscopy on a UAV can be formulated as an active sensing of a spatio-temporal phenomena. We can also think about the problem as regression - we try to find the underlying model based on samples we take. However, there are some key differences from standard regression. First of all, we don't have the ground truth values. Secondly, we can take only limited number of measurements and each measurement has an associated cost (eg. battery discharge to fly to the location). Thirdly, in our scenario, there are some useful prior we can exploit - smooth change in the concentration of gases with respect to time and space as well as known noise in our sensor readings. Those are some of the reasons which make $\mathcal{GP}$ useful for our project. In order to give an intuition of how they work, we are again going to start with some formal definitions.

Definition 3.2.3 *A Gaussian Process is a collection of random variables with the property that the joint distribution of any finite subset is a Gaussian.*

Definition 3.2.4 *A Gaussian process is fully specified by a mean and a covariance function.*

In order to model the smooth concentration of gases mentioned earlier, our $\mathcal{GP}$ will need to generate smooth functions. As such we are going to be using the squared exponential kernel defined in eq.3.1 and eq.3.2.

$$k(x_n, x_{n'}) = \sigma_f^2 exp\left(-\frac{1}{2l^2}(x_n - x_{n'})^2\right) \tag{3.1}$$

$$cov(y_n, y_{n'}) = k(x_n, x_{n'}) + \sigma_v^2 \delta_{nn'} \tag{3.2}$$

We can also think of $\mathcal{GP}$ as a way of sampling functions with certain properties, defined by a mean function (in this case always 0) and kernel function (squared exponential).

$$f(x) \sim \mathcal{GP}(m(x), k(x, x')) \tag{3.3}$$

In our case, the function we will be generating will have smooth properties. Examples of the generated smooth functions for different hyper parameters can be seen in fig.3.2, 3.3 and 3.4. Each of the three parameters can be setup according to our sensor/gas we want to model. The noise hyperparameter will be the noise our sensor is giving for a fixed concentration. The horizontal and vertical lengthscale define the expected change in concentration of gases across the space and then minimum and maximum concentration accordingly.

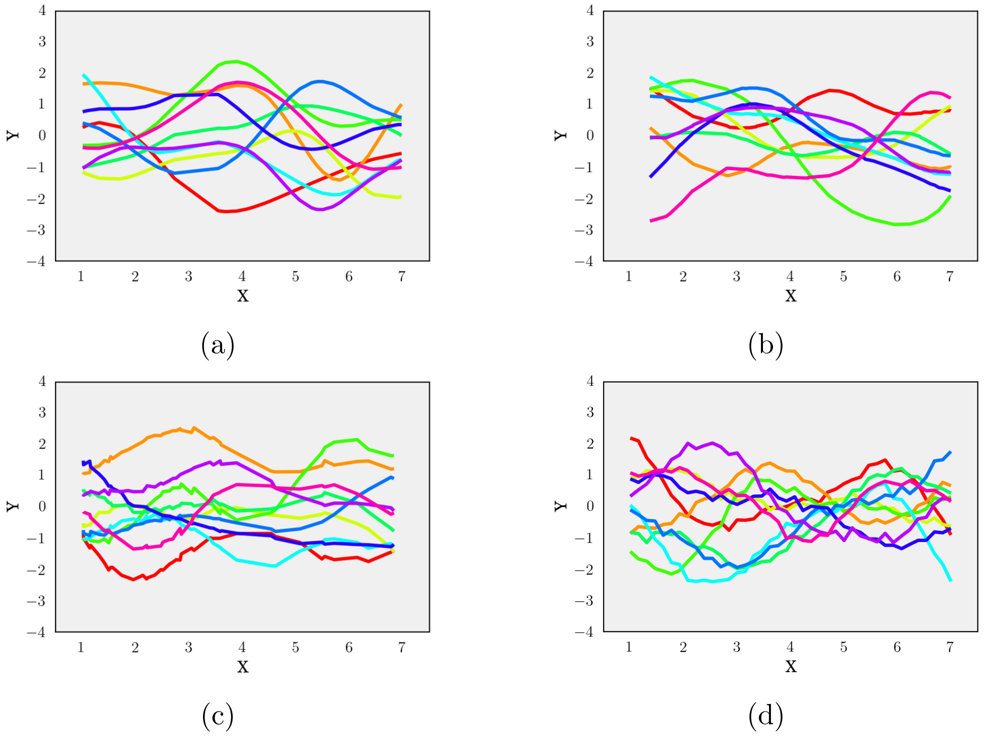


Figure 3.2: **Effect of noise hyperparameter σ_v.** We fix $\sigma_f = 1$ and $l = 1$ and vary σ_v. (a) $\sigma_v = 0$ (b) $\sigma_v = 0.01$ (c) $\sigma_v = 0.05$ (d) $\sigma_v = 0.1$

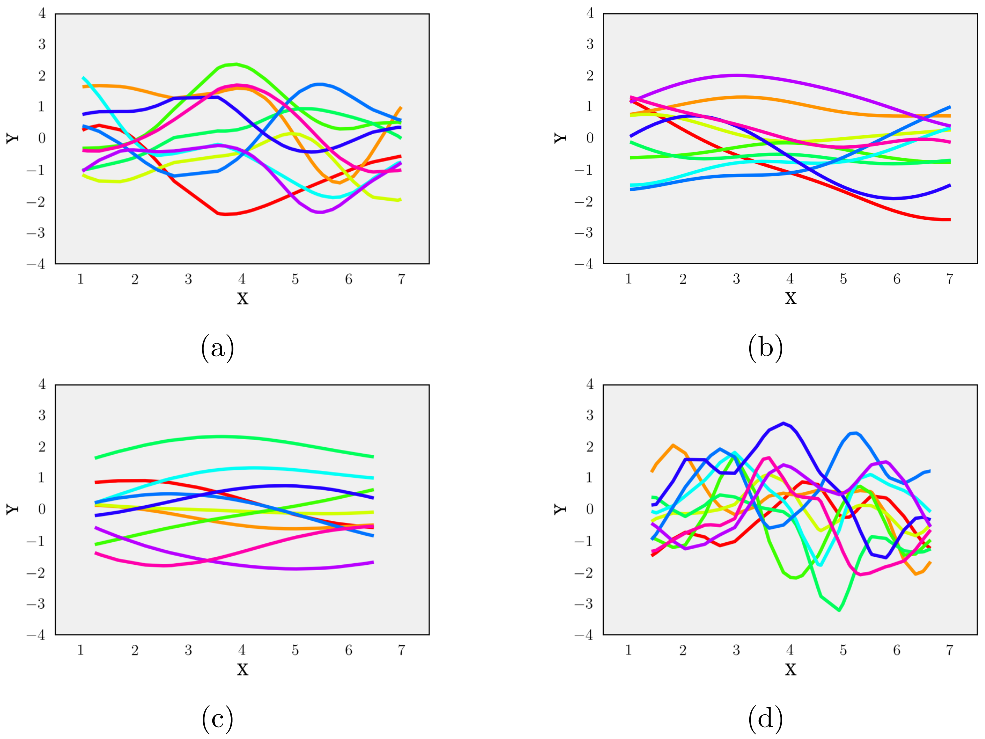
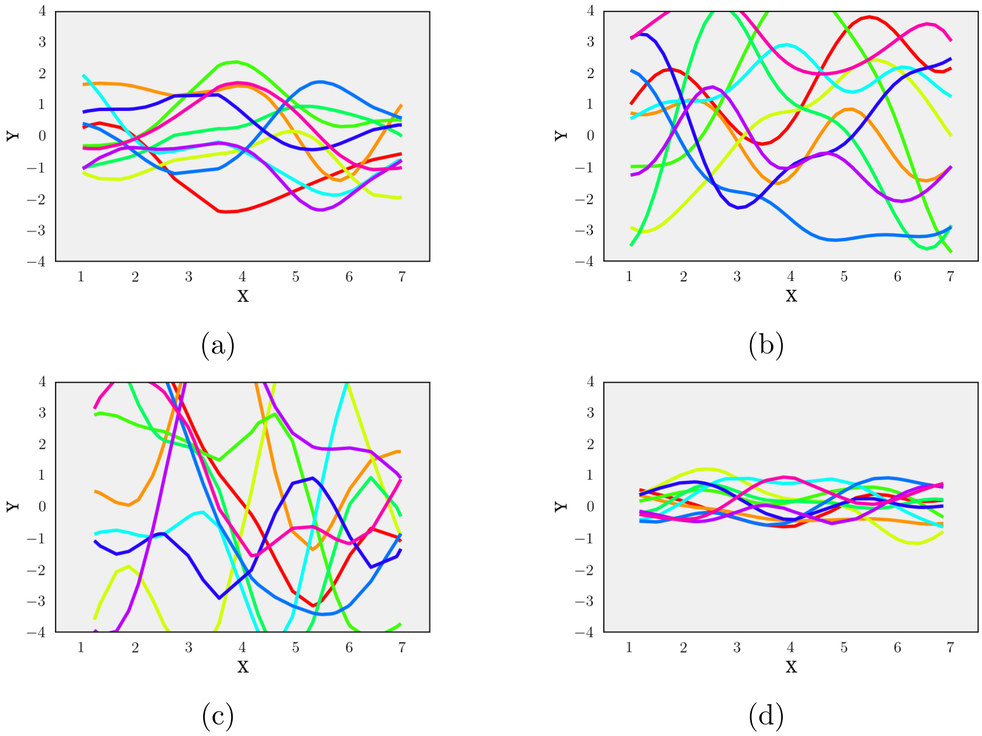


Figure 3.3: **Effect of noise hyperparameter l.** We fix $\sigma_v = 0$ and $\sigma_v = 1$ and vary l. (a) $l = 1$ (b) $l = 3$ (c) $l = 5$ (d) $l = 0.5$



(a) (b) (c) (d)

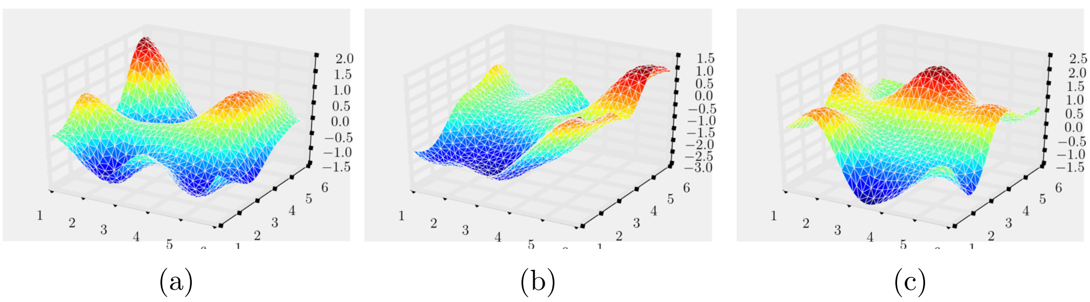
Figure 3.4: **Effect of noise hyperparameter σ_f.** We fix $\sigma_v = 0$ and $l = 1$ and vary σ_f. (a) $\sigma_f = 1$ (b) $\sigma_f = 2$ (c) $\sigma_f = 3$ (d) $\sigma_f = 0.5$

So far we discussed 1D function. However, $\mathcal{GP}$ can easily be extended to higher dimensions by calculating the covariance matrix according to eq.3.4 and 3.5.

$$cov(y_n, y_{n'}) = k(x_n, x_{n'}) + \sigma_v^2 \delta_{nn'} \tag{3.4}$$

$$k(x_n, x_{n'}) = \sigma_f^2 exp\left(-\sum_{d=1}^{D} \frac{1}{2l^2}(x_{dn} - x_{dn'})^2\right) \tag{3.5}$$

In the 2D case we can see that this generates smooth planes.



(a) (b) (c)

Figure 3.5: **Sample 2D functions (surfaces) generated from $\mathcal{GP}$ with squared exponential kernel**

For our purposes of modelling concentration we have to go one dimension further and use 3D hyperplanes.

3.2.1 Inference in Gaussian Processes

So far we described what useful properties $\mathcal{GP}$ have for modelling concentration of gases, however, we didn't discuss how we can use actual data reading from the sensor and incorporate them in our model. Taking eq.3.3 and substituting the mean function, with 0 in our case, and the generated covariance function from our square exponential kernel we can rewrite the equation as in eq.3.6.

$$f_* \sim N(0, \Sigma(X_*, X_*)) \tag{3.6}$$

Now we introduce a set of data points $\{x_n, y_n\}_{n=1}^N$. In a vector form we can represent them as $\{X, f\}$. We can use X_* from our prior and X from our data to calculate covariance matrices using the kernel from eq.3.2. Next we can define the joint Gaussian distribution as in eq.3.7.

$$\begin{bmatrix} f \\ f_* \end{bmatrix} \sim N\left(0, \begin{bmatrix} \Sigma(X, X) & \Sigma(X, X_*) \\ \Sigma(X_*, X) & \Sigma(X_*, X_*) \end{bmatrix}\right) \tag{3.7}$$

Finally, we can calculate the conditional distribution of functions, given our prior and data in eq.3.8. Since we are conditioning two Gaussian distributions, we know that the posterior will also be a Gaussian distribution.

$$f_* | X_*, X, f \sim N(\mu_{f_*}, \Sigma_{f_*}) \tag{3.8}$$

Calculating the mean vector μ_{f_*} and covariance matrix Σ_{f_*} is derived in great details in [2] (sec. 2.3.1). We can see the results in eq.3.9, 3.10.

$$\mu_{f_*} = \Sigma(X_*, X)\Sigma(X, X)^{-1} f \tag{3.9}$$

$$\Sigma_{f_*} = \Sigma(X_*, X_*) - \Sigma(X_*, X)\Sigma(X, X)^{-1}\Sigma(X, X_*) \tag{3.10}$$

Finally, we can implement a simple example of nonlinear regression with $\mathcal{GP}$ as in fig.3.6.

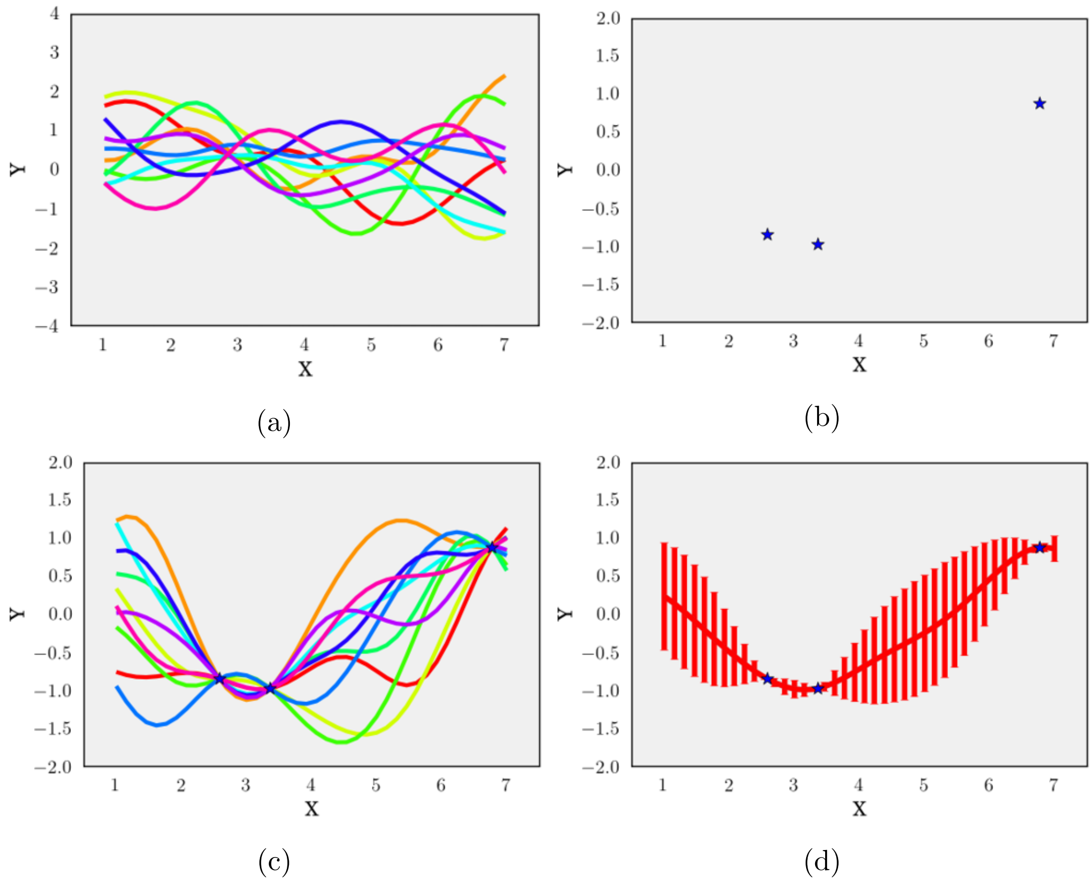
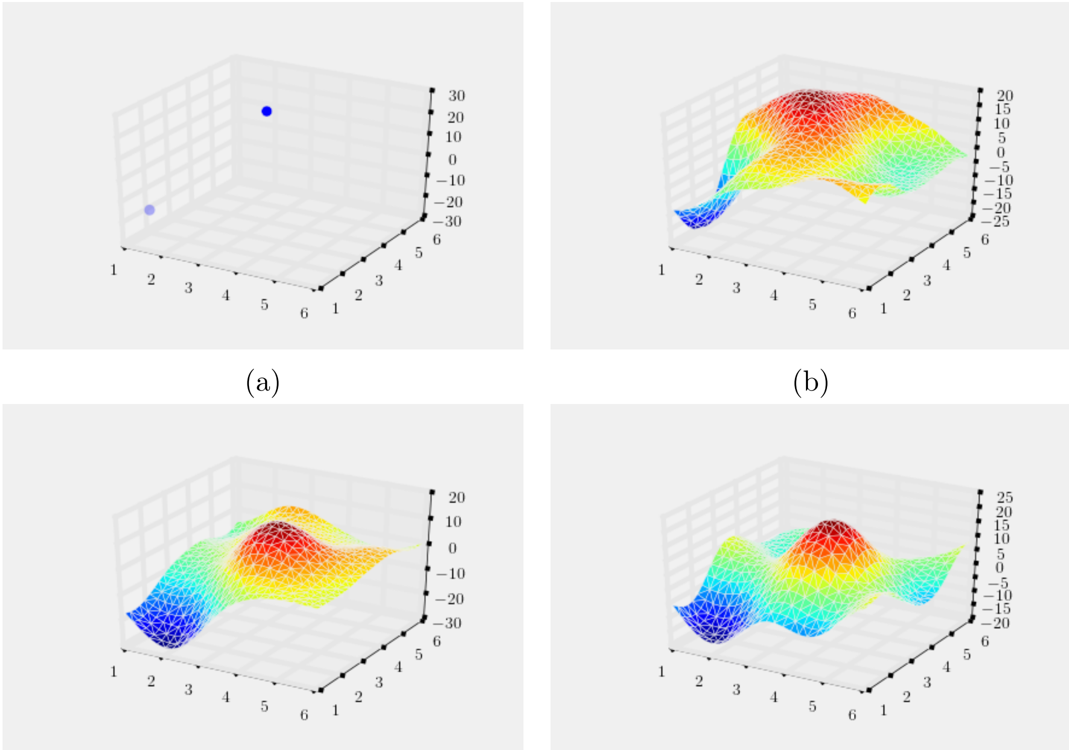


Figure 3.6: **Inference in 1D $\mathcal{GP}$.** (a) Prior (b) Data (c) Posterior (d) Error bars

Again this can be easily extended to higher dimensional data. In fig.3.7 we can see an example of 2D regression. All three of the surfaces go through the two data points while preserving their smooth properties. For this the purpose of this project we have to add one more dimension and do inference in 3D hyperplanes.



(a) (b) (c) (d)

Figure 3.7: **Inference in 2D $\mathcal{GP}$.** (a) Two data points (b), (c), (d) Sample planes going through the data points. We notice how all three of the planes go through the two data points, while preserving their smooth properties as defined by the squared exponential kernel

3.2.2 Benefits and comparison with other methods

Doing regression with $\mathcal{GP}$ has a couple of useful properties. First of all, unlike neural nets for example, $\mathcal{GP}$ is a non-parametric model. We don't have to do apply many iteration of a learning algorithm like backpropagation. Learning a model consists of only setting the hyperparameters and calculating a conditional probability distribution of two Gaussians. Moreover, $\mathcal{GP}$ provides a framework for quantifying uncertainty. This proves to be of great importance in different fields where we can only make a limited number of noisy measurements.

In this proposal we discuss only the squared exponential kernel. However, there are a wide variety of other kernels, which can generate and model different types of functions. In fact we can also combine different functions to compose a kernel, the only requirement being that the kernel has to generate positive definite covariance matrix [13]. We can use $\mathcal{GP}$ for classification similarly to neural network by applying sigmoid nonlinearity [9]. An interesting theoretical result is that $\mathcal{GP}$ can simulate shallow neural networks with one hidden layer with an infinite number of neurons [16]. This combines with the fact that we can represent any function using a shallow neural network, suggests that $\mathcal{GP}$ is indeed a powerful model[3].

They've been some recent advances to scale $\mathcal{GP}$ similar to neural networks by introducing hierarchical structure to improve learning [4].

4. Evaluation

The evaluation of the project will consist of multiple stages. The quadcopter used for the project (DJI Matrice 100) has a built in flight controller and an application developed, where you can specify a flight plan and do a raster scan over selected area. The first milestone will be integrating the UAV system with the onboard TX1 Nvidia computer. The second major milestone will be integrating the HWU-developed spectrometer on the quadcopter using the onboard power and storing the information on the TX1. The third milestone will be to fly the quadcopter with the sensor on board, and fly to pre-specified points while storing the sensor data on the onboard computer. The final milestone would be collecting data and while flying using $\mathcal{GP}$ to pick a new place to flight to, in order to maximize the information collected and find the source of contamination.

A challenging part of the evaluation process is the limitation that we don't have access to the underlying ground truth of the concentration of gases we are trying to measure. However, this can be overcome by at least two ways. First of all, we can introduce a baseline of flying to random locations, measure different concentrations and report the highest concentration found. This can be compared to our active sensing using $\mathcal{GP}$ where we expect to see improvements both in reduced time needed to find a high concentration, as well as finding a place with higher concentration.

Another possible way to tackle the problem of limited information about the phenomena we are trying to model could be done as follows - we take a few measurements, update our predictions, fly to a new location, make measurements and compare how they relate to the prediction we've made before making the measurements. This will give us the certainty that when we later go into active sensing mode, the prediction we make will be accurate (within some defined error).

5. Challenges

The project involves multiple challenges, both technical, but also safety, administrative and ethical. Integrating onboard computer, GPS, sonar avoidance system together with a FTS spectrometer poses numerous challenges involving power management, data flow and in general fitting all the equipment on the actual UAV. Flying a quadcopter requires acquiring a permission when tested in rural areas. Currently, we plan to do the preliminary testing at Heriot-Watt, because of the more isolated location, giving us freedom to fly to further distances. Final testing of the project will involve flying around sources of dangerous gases which will be a challenging task from an administrative perspective.

Another challenge will be the limitation of our hardware. Without any equipment onboard the UAV has a predicted fly time of 20 minutes. With the added weight from all the equipment, the draw of power from the TX1 and spectrometer, we expect a flight time of no more than 5 minutes. This combined with the physical time required for a battery to charge, give us about half an hour of test time per day.

Due to the interdisciplinary nature of the project, possible problems and delays could arise from multiple reasons - faulty hardware, software bugs, delays in hardware deliveries, etc. The main approach to overcome the possible problems arising would be to start working early and plan ahead for possible problems. The sensor from HWU has been in active development since the beginning of the academic year. We also already have all the required hardware - DJI's Matrice 100, Nvidia TX1, GPS module and sonar system, necessary cables and other tools, etc. Preliminary stages have already been developed like flying the quadcopter, setup of the board. Currently, we are trying to connect the quadcopter with the board with a pending meeting with our collaborators from HWU where we will discuss powering and connecting all the necessary modules together.

Another critical part of the project will be the spectrometer sensor developed at HWU. Since this itself is a novel research we expect possible problem ranging from low accuracy to the sensor malfunctioning in the worst case scenario. Low accuracy should not be an issue in our model since by using $\mathcal{GP}$ we can explicitly specify the noise we believe we have in our sensor. Therefore even if we have high error in our measurements, by making multiple of them, with time we still decrease the overall uncertainty in our model. In the more extreme case of the sensor completely malfunctioning, we are considering using an alternative sensor and apply the same principles of active sensing. An example would be attaching a thermometer and actively seeking a source of heat.

In the unlikely event of bad weather over extended periods of time during the summer, we have considered using simulation as proof of concept using data from the sensor manually moved by a person or a substitute robot. We have also considered possible crashes of the equipment. However, from our preliminary tests the quadcopter behaves good under strong winds. Moreover, the API provided from DJI gives us the ability to fly to a given GPS coordinate and height, where the controller automatically flies the UAV. As a further safety feature we plan to install an obstacle avoidance system to avoid any collisions.

6. Outputs

Integration of all the discussed instruments on a UAV - onboard Nvidia Jetson TX1 computer, mid-infrared Fourier-transform spectroscopy, controller, GPS and 5 sonars - is in itself a very new area. In fact, integrating a high-resolution FTS on board of a UAV has not been previously reported. The data from the UAV-mounted FTS used in a $\mathcal{GP}$ model could provide concentration maps of the field as well as gradients of different hazardous gases as to detect sources of highest contamination.

The first deliverable of the project will be an integrated system where all the equipment - onboard computer, quadcopter and spectrometer sensor are connected and powered. Secondly we want to demonstrate sensing under UAV power on predefined points. The final outcome of the project will be an autonomous system actively flying and steering as to maximize the information collected and find the place with highest concentration.

In this proposal, we focused on detecting explosive gases, however none of the modelling techniques we discussed having restriction about the type of sensor used for doing measurements. The long-term aim of the project is to develop a scientific tool, which can be used in different fields as a way of intelligent gathering of data.

7. Workplan

Some of the preliminary work on the project has already been done. The spectrometer sensor has been in active development by our collaborators at Heriot-Watt University since October 2016 and we expect that they will have a final product by end of May. We've already done a toy example of $\mathcal{GP}$ for modelling spatio-temporal phenomena and plan to incorporate the code on the UAV system. Moreover, we have already purchased most of the necessary equipment, including the quadcopter and onboard computer. In the coming months until the end of semester two the focus will on integrating on the necessary parts together. We plan to have the integrated system by the end of June at latest, so we can have the necessary time to test the system and gather data.

As already discussed number of complications can arise due to the nature of the project. However as we have already started doing initial work on the project and have planned for any unexpected delays, we expect that the project will conclude successfully by the deadline in August. More details of the planned and completed work can be seen in the gantt chart in fig 7.1.

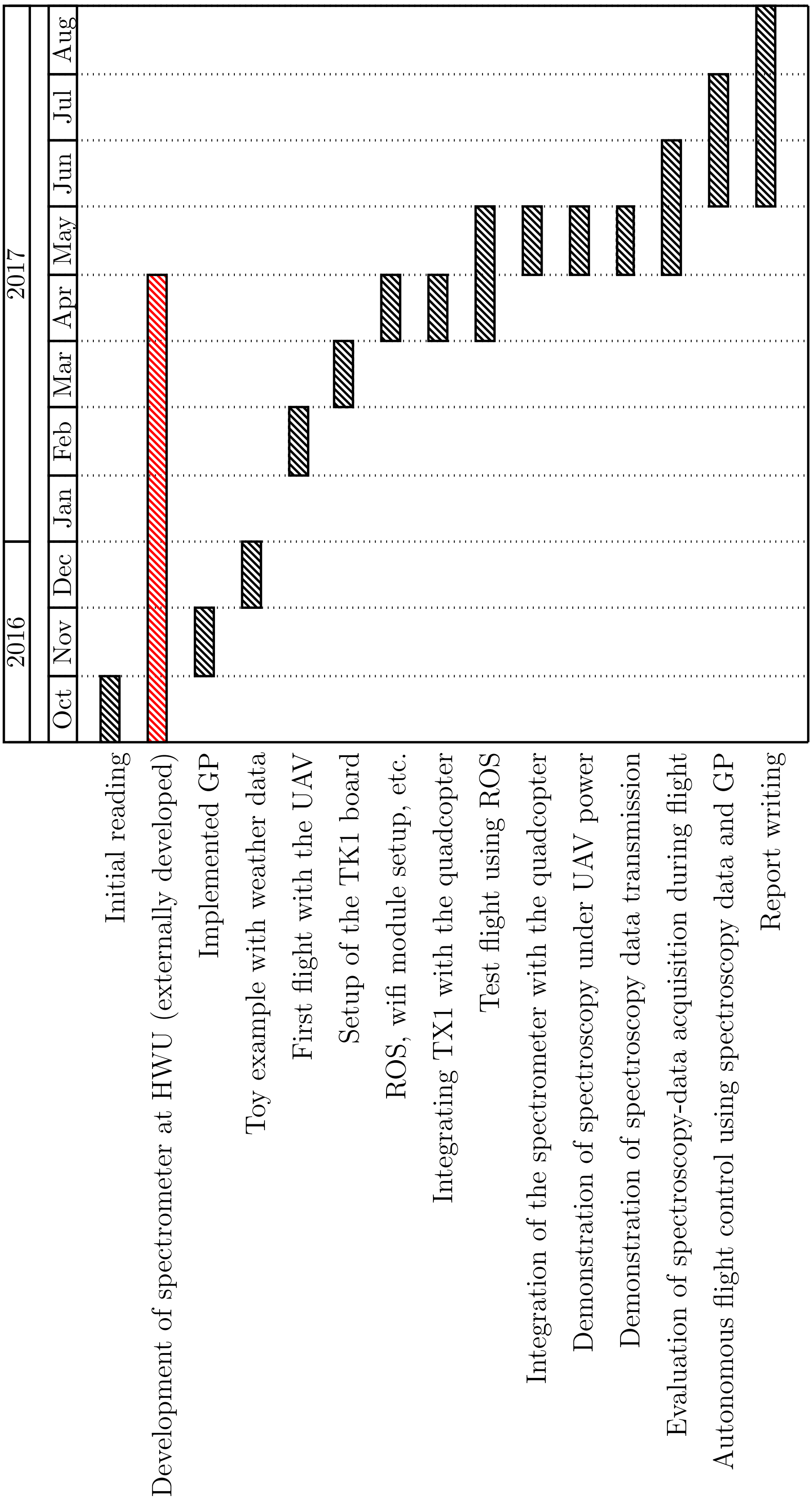


Figure 7.1: Project timeline with milestones

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