1 Lecture 3

1.1 Naked planet

The naked planet (or bare-rock) model is the simplest of global climate model. It assume a planet behaves as a perfect black body ($\epsilon = 1$), in steady-state radiating thermally at a rate equivalent to solar irradiation. The only "atmospheric" effect is determined by an albedo (α). To begin, assume steady-state:

$$R_{\text{in}} = R_{\text{out}}$$

where R is the radiation flux [W]. Incomming radiation (R_{in}) is determined by:

$$R_{\text{in}} = L(1-\alpha)A_{\text{sh}}$$

and outgoing:

$$R_{\text{out}} = \epsilon\sigma T^4 A_{\text{sp}}$$

where L is the solar constant ($= 1360.8 \pm 0.5$ W/m^2 at earth), α is the planetary albedo (≈ 0.3 for earth), A_{sh} is the area of a planet's shadow ($= \pi R^2$), A_{sp} is the area of a planet's surface ($= 4\pi R^2$), ϵ is the emissivity of the planet's surface, σ is the Stefan Boltzmann constant ($\approx 5.67 \times 10^{-8}$ Wm^{-2}K^{-4}), and T is temperature [K].

Combining the above and solving for T yeilds the planetary temperature of the "naked planet":

$$T = \sqrt[4]{\frac{L(1-\alpha)}{4\epsilon\sigma}}.$$

Plugging-in values for earth says that it should be at an equlibrium temperature of $\approx 255\,\text{K}$ (or $-18.6°$C).

1.2 Greenhouse effect

1.2.1 single layered atmosphere

The next model builds on the naked model by adding an atmosphere. At steady-state, two temperatures have to be accounted for: the temperature of the ground (T_g) and the temperature of the atmosphere (T_a). Again, at steady state $R_{\text{in}} = R_{\text{out}}$, and thus from the ground's perspective:

$$\frac{L(1-\alpha)}{4} + \epsilon\sigma T_a^4 = \epsilon\sigma T_g^4$$

and from the atmosphere:

$$\epsilon\sigma T_g^4 = 2\epsilon\sigma T_a^4$$

Overall, the "skin layer" energy balance is equivalent to the naked planet model, except now, the temperature returned is that of the upper atmosphere:

$$\frac{L(1-\alpha)}{4} = \epsilon\sigma T_a^4$$

And from this, ground temperature can be determined by re-arranging the above:

$$T_g = \sqrt[4]{2} \cdot T_a$$

(It's important to note that it is implicitly assumed that all bodies/layers behave as a black-body.)

1.2.2 2-Layered system

To enhance the "greenhouse effect," the atmosphere can be subdivided. For instance, by imposing a 2-layered atmospheric system, temperatures of both the top (T_{at}) and bottom (T_{ab}) atmospheric layers need accounting for. The energy balance for the top atmosphere is:

$$\epsilon\sigma T_{ab}^4 = 2\epsilon\sigma T_{at}^4$$

the bottom atmospheric layer:

$$\epsilon\sigma T_g^4 + \epsilon\sigma T_{at}^4 = 2\epsilon\sigma T_{ab}^4$$

Again, we can simplify with the skin-layer steady-state energy balance (and again, this assumes all surfaces act like a black-body):

$$\frac{L(1-\alpha)}{4} = \epsilon\sigma T_{at}^4$$

The bottom atmospheric layer can then be determined by:

$$T_{ab} = \sqrt[4]{2} \cdot T_{at}$$

and the ground temperature by:

$$T_g = \sqrt[4]{3} \cdot T_{at}$$

1.2.3 Nuclear winter

Lastly, we can consider a model in which the "dirty" atmosphere completely absorbs the solar radiation. In this case, the ground temperature equilibrates with the dirty atmosphere, so:

$$\frac{L(1-\alpha)}{4} = \epsilon\sigma T_a^4$$

and,

$$T_g = T_a.$$