Section 1

Introduction

Regionally Distributed Runoff Recharge model is a large-scale high-resolution grid-based numerical model optimized for implementation on parallel computer architectures. No process of the model is in any way novel, rather a suite of existing model structures have been combined into one platform chosen specifically for their ease of implementation, and their computational scalability.

The model's primary intention is to interpolate watershed moisture conditions for the purposes of estimating reagional groundwater recharge, given all available data. While the model can project streamflow discharge and overland runoff at any point in space, the model is not optimized, nor intended to serve as your typical rainfall runoff model. Outputs from this model will most often be used to constrain regional groundwater models withing the ORMGP jurisdiction and southern Ontario.

This model is currently in beta mode, and its use for practical application should proceed with caution. Users should be aware that results posted online is subject to change. Ultimately, the intent of this model is to produced ranges of long-term (monthly and greater) water budgeting as a hydrological reference for the partners of the ORMGP.

Model components:

1. regional climate fields (model inputs—precipitation, temperature, etc.)
2. explicit soil moisture accounting scheme
3. cold content energy balance snowpack model
4. local inertial approximation of the shallow water equation for lateral movement of water
5. distribution function-based shallow groundwater model

1.1 Computational Elements

Model grids in the model represent a homoginized area on the land surface, similar to the concept of a Hydrologic Response Unit, or HRU. The term HRU will be avoided here as the concept itself is not well defined and the term "computational element" (CE) will be used herein to avoid abiguity. All processes within the computational element are assumed to be zero-dimensional, that is processes

are modelled at the point-scale, which effectively represents the "average" condition within the grid cell. The spatial proximity of each computational element is maintained, meaning that runoff occurring from an upgradient CE will runon to an adjacent downgradient CE, thus preserving the spatial distribution of land surface moisture.

1.2 Time-stepping

The model runs using a variable time stepping approach. The soil moisture accounting portion of the model runs on a 6-hourly basis in order to capture hydrological dependence on the diurnal fluctuation of energy flux at the ground surface, and the local inertial approximation SWE simulation time step varies to a maximum of 6-hours based on its computational requirements (see §2.4).

Section 2

Theory

2.1 Soil moisture accounting

The explicit soil moisture accounting (SMA—Todini and Wallis, 1977; O'Connell, 1991) algorithm used is intentionally simplified. Where this conceptualization differs from typical SMA schemes is that here there is no distinction made among a variety of storage reservoirs commonly conceptualized in established models. For example, all of interception storage, depression storage and soil storage (i.e., soil moisture below field capacity) is assumed representative by a single aggregate store, thus significantly reducing model parameterization. This conceptualization is allowed by the fact that this model is intended for regional-scale application ($\gg$1000 km^2) with large computational elements (>50×50 m^2 cells). Users must consider the results of the model from a "bird's-eye view," where water retained at surface is indistinguishable from the many stores typically conceptualized in hydrological models. The SMA overall water balance is given as:

$$\Delta S = P - E - R - G, \tag{2.1}$$

where ΔS is the change in (moisture) storage, P is "active" precipitation (which includes direct inputs rainfall + snowmelt, otherwise termed snowpack drainage—P_{drn}, equation 2.15, and indirectect inputs such as runoff originating from upslope areas), which is the source of water to the computational element. E is total evaporation (transpiration + evaporation from soil and interception stores), G is groundwater recharge, R is excess water that is allowed to translate laterally. All of E, R, and G are dependent on moisture storage (S) and $S_{\max}$, the water storage capacity of the computational element. Both E and G have functional relationships to S and $S_{\max}$, and are dependent on other variables, whereas R only occurs when there is excees water remaining (i.e., $S > S_{\max}$), strictly speaking:

$$\Delta S = \underbrace{P}_{\text{sources}} - \underbrace{f\left(E_p, K_{\text{sat}}, S, S_{\max}\right)}_{\text{sinks}} - \underbrace{R|_{S>S_{\max}}}_{\text{excess}} \tag{2.2}$$

2.1.1 Total Evaporation

Total evaporation, i.e., loss from the computational element to the atmosphere, including plant transpiration, evaporation from land surface, soil pores and interception stores, is dependent on the

current soil moisture storage (S), soil moisture capacity ($S_{\max}$) and potential evapotranspiration (E_p), which is interpreted here as the capacity for the atmosphere to remove moisture from a saturated and replenishable surface.

Potential evaporation is computed based on the empirical Makkink (1957) method, which is functionally identical to the more common Priestly-Taylor (1972) approach. The main difference is that the Mikkink method utilizes global radiation (R_g) which can be computed from solar irradiation thoery as opposed to net radiation (R_n), used in the Priestly-Taylor approach, that requires observational measurements from sources that are rare and not adequately distributed at the regional scale the model is applied to. The Makkink (1957) equation is given by:

$$\lambda E_p = \alpha \frac{\Delta}{\Delta + \gamma} R_g + \beta, \tag{2.3}$$

where λ is the latent heat of vapouration for water (MJ/m^3), Δ is the slope of the saturation vapour pressure vs. temperature curve (kPa/K), γ is the psychrometic constant (kPa/K), and α is an empircal coefficient (similar to Priestly-Taylor) and β the background energy flux typically set to 0.61 and -1.2$\times$10^{-4} MJ/m^2, respectively (Makkink, 1957). λ and Δ are dependent on air temperature alone and γ on air temperature and atmospheric pressure, which is assumed constant at 101.3kPa.

While it is understood great simplification is made by choosing such an empirical model (i.e., neglecting the impacts from wind speed, surface roughness, atmospheric moisture capacity, etc.) it is deemed approriate for the purpose of this regional recharge model on the basis that in southern Ontario climates, E_p is negligeable during the later fall, winter, and early spring seasons, whereas the remaining season, actual evaporation is limited by moisture availability and not by evaporative potential, meaning that evaporative flux has a greater dependence on soil moisture accounting rather than the method chosen to estimate E_p.

2.2 Snowmelt model

Snowmelt is computed using a temperature-index approach that closely follows the methodology of DeWalle and Range (2008). The method's complexity lies somewhere in between the empirical degree-day approach and the physics-based energy-balance approach. The method has the same data requirements as the degree-day approach (namely, temperature and precipitation), yet has additional snowpack metamorphism processes simulated that enables the modelled snowpack to account for the potential to refreeze liquid water contained within its pore spaces (on the basis cold-content accounting) and thus gains the ability to determine snowpack density, depth, and temperature.

The model represents the snowpack as a single layer with uniform temperature with depth. The model represents processes at the point (i.e., zero-dimensional) scale and does not consider spatial distribution within a computational element. That said, by virtue of the model being applied in a spatially distibuted manner, large scale snowpack patterns will be realized on the basis of local slope and aspect which effectively adjusts the duration of sunshine exposure.

The water balance within the snowpack is accounted on the basis of snowpack snow water equivalent (P_{swe}), which is the total mass of water in both ice and liquid form. The liquid portion of P_{swe} is accounted for as the snowpack liquid water content (P_{lwc}). While the cold-content

dictates the energetic state of the snowpack, differences in P_{swe} and P_{lwc} dictate the mass state of the snowpack; for example, the fraction of ice within the snowpack can be determined by:

$$f_{\text{ice}} = 1 - \frac{P_{\text{lwc}}}{P_{\text{swe}}}. \tag{2.4}$$

2.2.1 Model description

The model begins first by assessing whether an existing snowpack exists. Air temperature (T_a) for this model is set to daily maximum temperature. If snowfall is observed, then it is added to the existing pack, if it exists, or a new snowpack is created. The density of fresh snowfall (ρ_{SF}) is assumed to be linearly correlated to air temperature as demonstrated in Judson and Doesken (2000), and is given by:

$$\rho_{SF} = \begin{cases} \rho_{0^\circ\text{C}} & T_a \geq 0^\circ\text{C} \\ \max\left(m_\rho T_a + \rho_{0^\circ\text{C}},\, \rho_{SF_{min}}\right) & \text{otherwise} \end{cases} \tag{2.5}$$

where $\rho_{0^\circ\text{C}}$ is the density of snowfall warmed to 0°C, $\rho_{SF_{min}}$ is the lower limit to snowfall density (assumed at $25\,\text{kg/m}^3$), and m_ρ is the slope of the temperature-density relationship.

Next, snowfall is added to the snowpack, adjusting pack density (ρ_p) by:

$$\rho_p = \frac{P_{\text{swe}} \cdot \rho_{p_\ell} + SF_{\text{swe}} \cdot \rho_{SF}}{P_{\text{swe}} + SF_{\text{swe}}} \tag{2.6}$$

where P_{swe} is the pack snow water equivalent, ρ_{p_ℓ} is the snowpack density prior to the addition of snow, and SF_{swe} is the snowfall water equivalent. SF_{swe} is also added to P_{swe}.

Temperature of the snowpack surface (T_s) is set to T_a if the pack is new or SF_{swe} exceeds 0.5 cm, otherwise:

$$T_s = \min\left(C_{\text{TSF}}\left(T_a - T_{s_\ell}\right),\, 0\right) \tag{2.7}$$

where T_{s_ℓ} is the snow surface temperature prior to adjustment, and C_{TSF} is the surface temperature adjustment factor. T_s is restricted to an upper-limit of 0°C.

The degree-day factor (DDF) is then computed from snowpack density following the approach given by Martinec (1960). The idea hear is that over time, as the snowpack settles, its albedo also decreases causing the pack to more susceptible to melt. This provides a convenient and indirect means of relating snow melt potential to snow pack age. DDF is adjusted by (Martinec, 1960):

$$\text{DDF} = C_{\text{DDF}} \frac{\rho_p}{\rho_w}, \tag{2.8}$$

where ρ_w is the density of liquid water at 0°C ($\approx 999.84\,\text{kg/m}^3$), and C_{DDF} is the density-DDF adjustment factor. Next, potential snowmelt (M_{swe}) is computed by:

$$M_{\text{swe}} = \min\left(\max\left(\text{DDF} \cdot \left(T_a - T_b\right),\, 0\right),\, P_{\text{swe}}\right), \tag{2.9}$$

where T_b is a base (or critical) temperature from which melt can occur. Any potential melt is added to the pack liquid water content store (P_{lwc}), and pack density is then adjusted by:

$$\rho_p = f_{\text{lw}}\left(\rho_w - \rho_{\text{froz}}\right) + \rho_{\text{froz}} \tag{2.10}$$

where ρ_{froz} is the density of the dry snowpack, if all of the pack's liquid water (P_{lwc}) were to be removed, and is given by:

$$\rho_{\text{froz}} = \frac{P_{\text{swe}} \cdot \rho_p - P_{\text{lwc}} \cdot \rho_w}{P_{\text{swe}} - P_{\text{lwc}}}. \tag{2.11}$$

If rainfall (R) occurs its amount is added to both P_{swe} and P_{lwc} and the pack density is adjusted by:

$$\rho_p = \frac{P_{\text{swe}} \cdot \rho_{p_\ell} + R\rho_w}{P_{\text{swe}} + R} \tag{2.12}$$

Next, should liquid water be contained within the snowpack, cold content of the pack (CC) must be satisfied before drainage from the pack can ensue. Liquid water freezing internally to the snowpack causes its density to be adjusted by:

$$\rho_p = f_{\text{lw}} \left(\rho_i - \rho_w\right) + \rho_{p_\ell} \tag{2.13}$$

where ρ_i is the density of ice at 0°C ($\approx 917\,\text{kg/m}^3$), and the fraction of the snowpack mass in liquid form is given by:

$$f_{\text{lw}} = \begin{cases} \frac{P_{\text{lwc}}}{P_{\text{swe}}} & P_{\text{lwc}} \leq \text{CC} \\ \frac{\text{CC}}{P_{\text{swe}}} & \text{otherwise} \end{cases}. \tag{2.14}$$

The amount frozen to the pack is subtracted from P_{lwc}.

In the final step, liquid water exceeding the snowpack liquid water holding capacity is allowed to drain from the base of the pack. When $P_{\text{swe}} = P_{\text{lwc}}$, i.e., the snowpack is composed solely of liquid water, the entirety of the snowpack water equivalent is allowed to drain and the snow pack layer is eliminated. Pack drainage is given by:

$$P_{\text{drn}} = \begin{cases} P_{\text{lwc}} - P_{\text{lwcap}} & P_{\text{lwc}} \geq P_{\text{lwc}} \\ 0 & \text{otherwise} \end{cases}. \tag{2.15}$$

Snowpack liquid water holding capacity (P_{lwcap}) is defined by:

$$P_{\text{lwcap}} = \phi S_{cap} d_p \tag{2.16}$$

where S_{cap} is the snowpack liquid water retention capacity as a degree of pore space saturation (≈ 0.05; DeWalle and Rango, 2008), d_p is pack depth, and ϕ is pack porosity, defined by:

$$\phi = 1 - \frac{\rho_p - \rho_w \theta_p}{\rho_i} \tag{2.17}$$

All liquid water above P_{lwcap} is added to pack drainage (P_{drn}) and removed from P_{swe}, P_{lwc}. Pack density is adjusted according to:

$$\rho_p = \frac{\left(P_{\text{lwc}} - P_{\text{drn}}\right) \rho_w + \left(P_{\text{swe}} - P_{\text{lwc}}\right) \rho_{\text{froz}}}{P_{\text{swe}} - P_{\text{drn}}} \tag{2.18}$$

Pack densification (i.e., settlement) is caused by two processes: internal redistribution of water through phase changes and consolidation though the weight of the snowpack. The former process is

implicitly handled by the model described above, while the latter is handled by a settlement factor given by:

$$\rho_p = \begin{cases} \rho_{p_\ell} \left(\frac{\rho_i}{\rho_{\text{froz}}} \right)^{C_{\text{dens}}} & p_\ell < \rho_i \\ \rho_{p_\ell} & \text{otherwise} \end{cases} \tag{2.19}$$

2.2.2 Model validation

Data from 537 current and historical EC climate stations were collected and screened for snow depth data. Of these stations, the data were further screened for winter seasons (October through May) for complete climate data reported for more than 100 consecutive days. Based on the screening, 100 stations were selected, some having multiple season meeting the screening criteria. In all, 233 snow seasons were collected and were reordered into a continuous 233-year dataset for model validation.

TODO

2.3 Shallow groundwater model

Typically, the handling of groundwater in rainfall-runoff models involves the use of a groundwater storage reservoir with a linear/exponential decay function. When calibrating these models to long-term stream flow records, this model structure can be quite successful at matching baseflow recession. Given its success and it relative simplicity, the linear decay groundwater storage model is widely used and preferred for model calibration to stream flow gauging stations. However, implied in the linear decay groundwater storage model is the assumption that groundwater recharge is uniformly distributed basin-wide, or at most among common surficial material types. Consequently, catchment topography is neglected, which has a large role in distributing recharge, both in terms of focused recharge at overland flow convergences and rejected recharge at low-lying areas where the groundwater table is close to surface. For the purpose of this model, which is primarily intended for distributed recharge estimation, the linear decay groundwater storage model on its own will not suffice.

The groundwater model structure employed here follows the TOPMODEL structure of Beven and Quinn (1979). TOPMODEL employs a distribution function that projects predicted groundwater discharge in streams to groundwater recharge distributed in space using a "soil-topological index" that considers both distributed surficial material properties and topography. TOPMODEL does not explicitly model integrated groundwater/surface water processes, however it does provide the ability, albeit an indirect one, to include the influence of near-surface watertables. Most importantly, low-lying areas close to drainage features will be prevented from accepting recharge resulting in saturated overland flow conditions. Assumptions to TOPMODEL are that the groundwater reservoir is lumped meaning it is equivalently and instantaneously connected to all locations of the catchment and are constrained by surface water divides. Consequently "potential" recharge computed in the distributed sense is aggregated when added to the lumped reservoir. For additional discussion on theory and assumptions, please refer to Beven et. al.(1995) and Beven (2012).

At the regional scale, multiple TOPMODEL reservoirs have been established to represent groundwater dynamics in sub-watersheds smaller than a pre-specified threshold area. Groundwater discharge estimates from each TOPMODEL reservoir instantaneously contributes to the outflow of the sub-watershed.

TOPMODEL establishes that the sub-surface storage deficit at a given point i in the sub-watershed (D_i) can be approximated by:

$$D_i = \overline{D} + m\left[\gamma - \ln\frac{a}{T_o \tan\beta}\right], \tag{2.20}$$

where, T_o is the lateral transmissivity at model cell i (i.e., the computational element) when soils are saturated $[m^2/s]$, $\tan\beta$ is the cell slope angle, and a is the unit contributing area to cell i, defined here as the total contributing area to cell i divided by cell width $[m]$. The expression $\ln(a/T_o \tan\beta)$ is referred to as the soil-topologic index. γ is the catchment average soil-topologic index and is given by:

$$\gamma = \frac{1}{A}\sum_i A_i \ln\left(\frac{a}{T_o \tan\beta}\right)_i, \tag{2.21}$$

where A and A_i is the sub-watershed and computational element areas, respectively. Before every time step, basin-wide moisture deficit is updated by:

$$\overline{D}_t = \frac{1}{A}\sum_i A_i D_i + Q_{b,t-1} - G_{t-1}, \tag{2.22}$$

where G_{t-1} is the sub-watershed-wide groundwater recharge computed during the previous time step, and $Q_{b,t-1}$ is the baseflow computed during the previous time step, where

$$Q_b = Q_o e^{-\overline{D}/m}. \tag{2.23}$$

It should be noted that the above equation is the equivalent to the linear groundwater storage model decay function, except here, TOPMODEL allows for an approximation of spatial soil moisture distribution, which will, in turn, determine spatial recharge patterns as $D_i \leq 0$ will prevent recharge from occurring at cell i. Q_o, defined as baseflow discharge when the sub-watershed is fully saturated (i.e., $\overline{D} = 0$) and is to be pre-specified as a model parameter. The other parameter (m) can be determined from baseflow recession analysis (Beven et. al., 1995; Beven, 2012).

Initially, watershed average soil moisture deficit can also be determined by stream flow records by:

$$\overline{D} = -m \ln\left(\frac{Q_{t=0}}{Q_o}\right), \tag{2.24}$$

where $Q_{t=0}$ is the measured stream flow known at the beginning of the model run.

Although plausible (and argued for), TOPMODEL can be allowed to have negative soil moisture deficits (i.e., $D_i < 0$), meaning that water has ponded and overland flow is being considered. For the current model, ponded water and overland flow is being handled by the explicit soil moisture accounting (SMA) system and the lateral shallow water movement module discussed below. Conditions where $D_i < 0$ for any computational element will have the excess water removed and added to the SMA system and D_i will be set to zero. Under these conditions, no recharge will occur at this cell and the TOPMODEL portion of the model will only be used to represent groundwater discharge to streams.

2.4 Lateral movement of land surface moisture

The simulation of lateral water movement is approximated by the local inertial approximation (LIA) of the shallow water equations (SWE) of (de Almeda et. al., 2012; 2013).

It is important to note that the interpretation of the use of the LIA to the SWE does not include the physical movement of sheet flow, rather, the LIA is being interpreted as a means of moving water laterally, either thought a network of micropores and rill structures. That being said, roughness coefficients may be higher than typically associated with flood forecasting. only when water depths achieve appreciable depths (say greater than 1 cm) will the interpretation of the LIA as representing depth-averaged shallow surface water movement hold. Under most conditions, however, flow depths, on average, will report depths much less than 1 cm. As flow depths receded beyond this (albeit arbitrary) threshold, flow paths through marcopore networks and rill structures are expected to lengthen relative to the grid cell dimension and is assumed to be subcritical. Fortunately, the solution of de Almeda et. al.(2012) is best applicable to low Froude numbers, $Fr < 0.5$, thus suitable for the interpretation made here.

2.4.1 Governing Equations

The shallow water (i.e., depth-averaged) approximation of incompressible fluid flow in two-dimension can be solved using the following system of equations (de Almeda et. al.2013):

Conservation of mass:

$$\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0$$

Conservation of momentum:

$$\underbrace{\frac{\partial q_x}{\partial t}}_{\substack{\text{local}\\ \text{acceleration}}} + \underbrace{\frac{\partial}{\partial x}\left(uq_x\right) + \frac{\partial}{\partial y}\left(vq_x\right)}_{\substack{\text{convective}\\ \text{acceleration}}} + \underbrace{gh\frac{\partial\left(h+z\right)}{\partial x}}_{\substack{\text{pressure +}\\ \text{bed gradients}}} + \underbrace{ghS_{f_x}}_{\substack{\text{friction}}} = 0$$

$$\underbrace{\frac{\partial q_y}{\partial t}}_{\substack{\text{local}\\ \text{acceleration}}} + \underbrace{\frac{\partial}{\partial y}\left(vq_y\right) + \frac{\partial}{\partial x}\left(uq_y\right)}_{\substack{\text{convective}\\ \text{acceleration}}} + \underbrace{gh\frac{\partial\left(h+z\right)}{\partial y}}_{\substack{\text{pressure +}\\ \text{bed gradients}}} + \underbrace{ghS_{f_y}}_{\substack{\text{friction}}} = 0$$

in de Almeda et. al.(2012), friction slope (S_f) is approximated using the Manning-Strickler equation and assuming wide shallow cross-sectional area normal flow (i.e., $R \approx h$), where:

$$\sqrt{S_f} = \frac{nq}{h^{5/3}}$$

thus yielding:

$$\frac{\partial q_x}{\partial t} + \frac{\partial}{\partial x}\left(uq_x\right) + \frac{\partial}{\partial y}\left(vq_x\right) + gh\frac{\partial\left(h+z\right)}{\partial x} + \frac{gn^2\|\mathbf{q}\|q_x}{h^{7/3}} = 0$$

$$\frac{\partial q_y}{\partial t} + \frac{\partial}{\partial y}\left(vq_y\right) + \frac{\partial}{\partial x}\left(uq_y\right) + gh\frac{\partial\left(h+z\right)}{\partial y} + \frac{gn^2\|\mathbf{q}\|q_y}{h^{7/3}} = 0$$

2.4.2 Numerical Scheme

$$q_{i-1/2}^{n+1} = \frac{\theta q_{i-1/2}^{n} + \frac{1-\theta}{2}\left(q_{i-3/2}^{n} + q_{i+1/2}^{n}\right) - g h_f^n \frac{\Delta t}{\Delta x}\left(\eta_i^n - \eta_{i-1}^n\right)}{1 + g\Delta t n^2 ||\mathbf{q}_{i-1/2}^n|| / h_f^{7/3}}$$

where

$$||\mathbf{q}_{i-1/2}^n|| = \sqrt{\left(q_{x,i-1/2}^n\right)^2 + \left(q_{y,i-1/2}^n\right)^2},$$

Courant-Friedrichs-Lewy condition:

$$\Delta t = \alpha \frac{\Delta x}{\sqrt{g h_{\max}}}, \qquad 0 < \alpha \leq 1$$

after updating fluxes for time step $n+1$, cell heads are updated by:

$$\eta_{i,j}^{n+1} = \eta_{i,j}^n + \frac{\Delta t}{\Delta x}\left(q_x|_{i-1/2,j}^{n+1} - q_x|_{i+1/2,j}^{n+1} + q_y|_{i,j-1/2}^{n+1} - q_y|_{i,j+1/2}^{n+1}\right)$$

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