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Intuitionistic Fuzzy Sets: New Approach and Applications

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Abstract
This paper presents an extended definition of intuitionistic fuzzy set and some operations on intuitionistic fuzzy sets which will be very useful for solving the real life problems. In fact, the intuitionistic fuzzy set is an extension of the fuzzy set defined in a domain of discourse. In this paper, the intuitionistic fuzzy set has been used as a proper tool for representing the hesitation degree concerning both the membership and non-membership degrees of an element to a set.

Key words: Fuzzy set; Intuitionistic fuzzy set; Domain of discourse

1. Introduction

The IFS (Intuitionistic Fuzzy Set) theory introduced by K. T. Atanassov [1] is interesting and useful to problem solving. The IFS is actually an extension of the fuzzy set characterized by Zadeh [6]. In recent years the IFS theory has been applied in different areas such as logic programming [2,3], decision making problems[5], medical diagnosis [4], etc.

Definition of fuzzy set: a fuzzy set F defined in a space S is a non-empty set of 2-tuple elements:

$$F = \{\langle e, \mu_F(e)\rangle, e \in S\}, \forall e \in S$$

where $\mu_F : S \to [0,1]$ is a membership function of fuzzy set S, which for every element $e \in S$ assigns its membership degree $\mu_F(e) \in [0,1]$ to the fuzzy set F. The set S is called a doamin of discourse and we write $F \subseteq S$.

2. Intuitionistic Fuzzy Sets

A fuzzy set defined in a domain of discourse is called an intuitionistic fuzzy set (IFS) if each element of the fuzzy set is a 4-tuple containing membership degree, hesitation degree, and non-membership degree. The hesitation degree is a part of membership degree or a part of non-membership degree or both. The following examples describe the above three degrees:

Example 1:
Let us consider the set of people in age group 18 years and above is a domain of discourse. The membership degree, hesitation degree and non-membership degree of the intuitionistic fuzzy set is described as follows:

If we denote the young people in age group between 18-40 years by membership degrees, then we can denote the old people in age group 50 years and above by non-membership degree. The people in the age group between 40-50 years may be considered as young or old or both. Thus, we can represent these people (age group between 40-50 years) by hesitation degrees.

Example 2:
Let us consider a set of 24 hours in a single day. The membership degree, hesitation degree and non-membership degree of the intuitionistic fuzzy set is described as follows:
If we denote the day between 7 AM – 6 PM by the membership degrees, then we can denote the night between 7 PM – 5 AM by non-membership degrees. The periods between 5 AM – 6 AM and 6 PM – 7 PM may be considered as day or night or both. Thus, we may denote these periods (5 AM – 6 AM and 6 PM – 7 PM) by hesitation degrees.

Definition:
An intuitionistic fuzzy set F in a domain of discourse S $(F \subseteq S)$ is defined as a non-empty set of 4-tuple elements, that is,

$$F = \{\langle e, \mu_F(e), \pi_F(e), \nu_F(e)\rangle \mid e \in S\},\ \forall e \in S$$

where the notation μ_F, π_F, and ν_F denote the membership function $\nu_F : S \to [0,1]$, hesitation function $\pi_F : S \to [0,1]$, and non-membership function $\nu_F : S \to [0,1]$ respectively. $\mu_F(e)$, $\pi_F(e)$ and $\nu_F(e)$ quantify the membership degree, hesitation degree and non-membership degree of $e \in S$ respectively to the IFS *F*. We can represent μ_G, π_G, and ν_G pictorially (Figure 1).

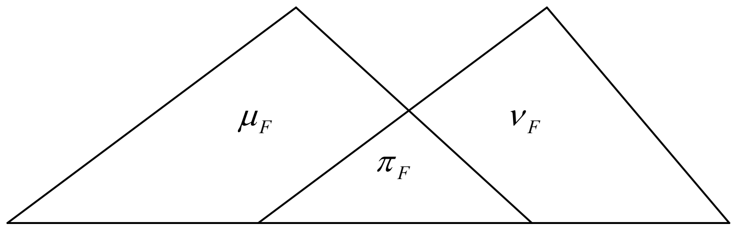

Figure 1. Illustration of the degrees μ_F, π_F, and ν_F

For every $e \in S$, $\mu_F(e) + \pi_F(e) + \nu_F(e) = 1$ and $0 \le \mu_F(e), \pi_F(e), \upsilon_F(e) \le 1$. For instance, if we know the degrees of $\mu_F(e)$ and $\nu_F(e)$, we can calculate the degree of $\pi_F(e)$, that is,

$$\pi_F(e) = 1 - \mu_F(e) - \upsilon_F(e),\ (e \in S)$$

For our convenience, we may denote each element of the IFS *F* as a 3-tuple element, that is, $\langle \mu_F(e), \pi_F(e), \nu_F(e) \rangle$, and also the IFS *F* as follows:

$$F = \{t_i \mid t_i = \langle \mu_F(e_i), \pi_F(e_i), \upsilon_F(e_i) \rangle \,\&\, e_i \in S\}$$

or $F = \{t_i \mid t_i = \langle \mu_i, \pi_i, \upsilon_i \rangle\}$

where $\mu_i = \mu_F(e_i)$, $\pi_i = \pi_F(e_i)$, and $\upsilon_i = \upsilon_F(e_i)$, $\forall e_i \in S$.

For every $t_i, t_j \in F \ (i \ne j)$,

$t_i \le t_j$ if [$\mu_i \le \mu_j$, $\pi_i \circ \pi_j$ and $\upsilon_i \ge \upsilon_j$] OR [$\mu_i \le \mu_j$ and $\upsilon_i \ge \upsilon_j$] where $\circ \in \{<, =, >\}$

$t_i = t_j$ if [$\mu_i = \mu_j$, $\pi_i = \pi_j$ and $\upsilon_i = \upsilon_j$] OR [$\mu_i = \mu_j$ and $\upsilon_i = \upsilon_j$]

If

$$M = \{m_i \mid m_i = \langle \mu_M(e_i), \pi_M(e_i), \upsilon_M(e_i) \rangle \,\&\, e_i \in S\}$$

, $\forall e_i \in S$, and

$$N = \{n_i \mid n_i = \langle \mu_N(e_i), \pi_N(e_i), \upsilon_N(e_i) \rangle \,\&\, e_i \in S\}$$

, $\forall e_i \in S$, are two IFSs defined in the domain of discourse *S*, then

i) $M \subset N$ if and only if $[m_i \le n_i]$, $\forall m_i \in M \,\&\, \forall n_i \in N$

ii) $M = N$ if and only if $[m_i = n_i]$, $\forall m_i \in M \,\&\, \forall n_i \in N$

3. Application of Intuitionistic Fuzzy Set

For our convenience, the weather is divided into three categories: cold, warm, and hot. Let us consider that 'cold' is a membership function, 'hot' is a non-membership function and 'warm' is a hesitation function. Here, the degree of warm may cater to either degree of hot or degree of cold or both. Let us assume that the weather is definitely cold at and below the 20°C, it is definitely hot at and beyond the 32°C and in between the weather is warm, i.e., the level of coldness decreases and the level of hotness increases. We can represent the weather 'cold', 'warm', and 'hot' pictorially (Figure 2).

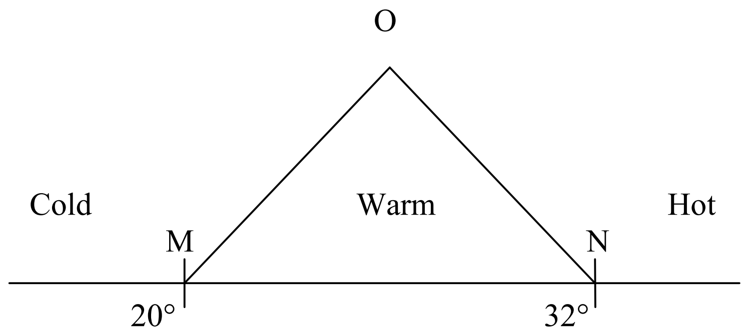

Figure 2. Illustration of the degrees of ‘cold’, ‘warm’, and ‘hot’

Let S be the set of different values of the weather, F the intuitionistic fuzzy set defined in the set S. Let $\mu_F(e)$ be the membership grade of weather ‘cold’ at e where $e = x°C$. Here x denotes a numerical value. For example, $x°C = 20°C$. Similarly, we can denote $\pi_F(e)$ as the hesitation grade of weather ‘warm’ and $\nu_F(e)$ the non-membership grade of weather ‘hot’ at $e = x°C$.

For example, $S = \{20°C, 26°C, 32°C\}$ and

$$F = \{\langle \mu_F(20°C), \pi_F(20°C), \upsilon_F(20°C)\rangle, \langle \mu_F(26°C), \pi_F(26°C), \upsilon_F(26°C)\rangle, \langle \mu_F(32°C), \pi_F(32°C), \upsilon_F(32°C)\rangle\}$$

At and below $20°C$, there is no warm and hot, but there exists only cold.

Hence, $\mu_F(20°C) = 1$, $\pi_F(20°C) = 0$, and $\upsilon_F(20°C) = 0$, i.e., (1, 0, 0)

At $26°C$ (at the point O), $\mu_F(26°C) = 0$, $\pi_F(26°C) = 1$, and $\upsilon_F(26°C) = 0$, i.e., (0, 1, 0)

At and above $32°C$, there is no cold and warm, but there exists only hot.

Hence, $\mu_F(32°C) = 0$, $\pi_F(32°C) = 0$, and $\upsilon_F(32°C) = 1$, i.e., (0, 0, 1)

$$\therefore F = \{\langle 1,0,0\rangle, \langle 0,1,0\rangle, \langle 0,0,1\rangle\}$$

Cold decreases and warm increases in between M and O, i.e., $1 > \mu > 0$ and $0 < \pi < 1$

Warm decreases and hot increases in between O and N, i.e., $1 > \pi > 0$ and $0 < \upsilon < 1$.

4. Conclusion

In this paper, an extended definition of intuitionistic fuzzy set along with a detailed description and a diagram has been given for solving the real life problems. The defined intuitionistic fuzzy set has been used as a new approach to weather analysis.

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