Questions

Any questions to exercises and homeworks from last time?

Table of Content

Complex Numbers with Numpy

- Generate an array of complex numbers using `dtype`

  ```
  sig=np.zeros(1000,dtype=np.complex)
  ```

- Generate an array of complex numbers using `1j`

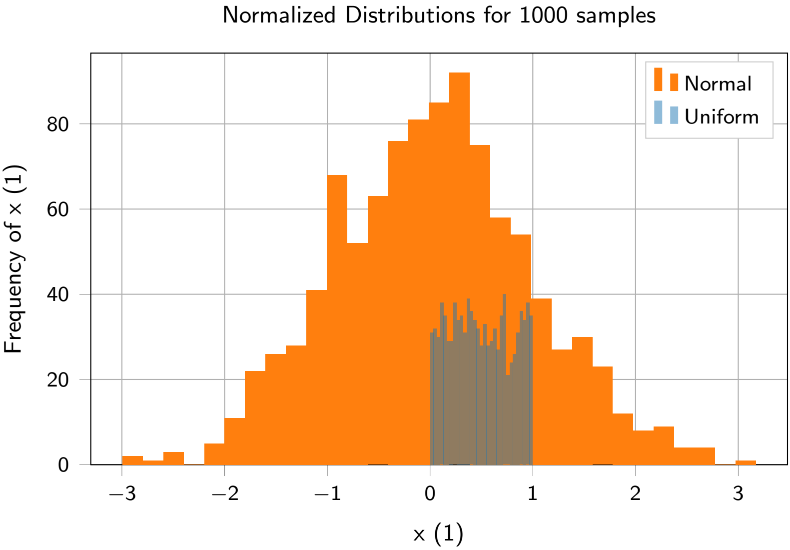
  ```
  sig=np.exp(1j*2*np.pi*0.1*np.arange(1000))
  ```

- Many numpy functions accept real and complex values.
- Caution: no auto-conversion in some functions
  - `np.sqrt(-1.0)` fails with
    `RuntimeWarning: invalid value encountered in sqrt`, but
  - `np.sqrt(-1+0*1j)` works.
- `x.real` and `x.imag` return the real and imaginary part of a numpy array `x`.
- For complex arrays `x`, `x.real` and `x.imag` are writeable and can be used to set real and imaginary separately.
- `np.abs(x)` and `np.angle(x)` return the element-wise magnitude and phase (in rad) of a numpay array `x`.

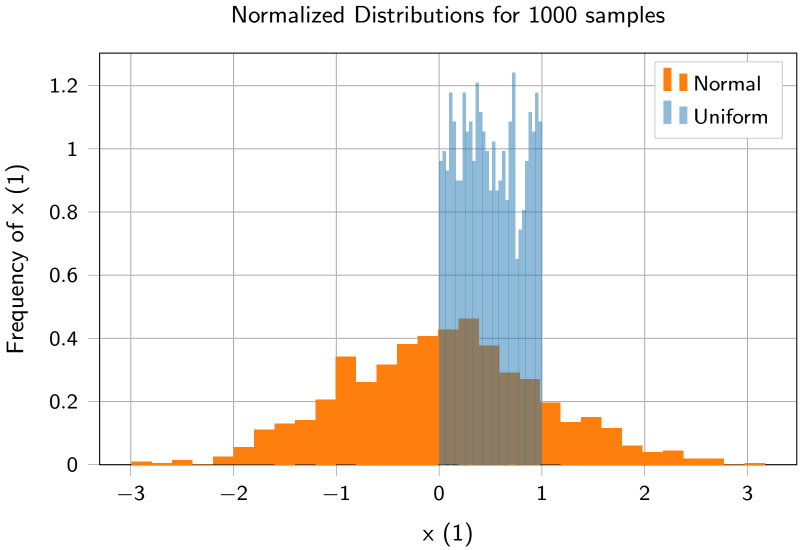
Histograms using Matplotlib

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib2tikz import save as tikz_save

np.random.seed(0)       # init pseudo-random number generator
N=1000                  # number of samples
n_u=np.random.rand(N)   # normalized uniform distribution
n_g=np.random.randn(N)  # normalized Gaussian distribution
bins=int(np.sqrt(N))    # needs conversion to integer

for density in (False, True):
    plt.figure()
    plt.hist(n_u, bins=bins, density=density, label='Normal', alpha=0.5)
    plt.hist(n_g, bins=bins, density=density, label='Uniform', zorder=-1)
    plt.grid();  plt.legend();
    plt.title('Normalized Distributions for %d samples' %(N,))
    plt.xlabel('x (1)')
    plt.ylabel('Frequency of x (1)')

    filename='NormalizedDistributions_density'+str(density)
    tikz_save(filename+'.tikz')
    plt.savefig(filename+'.png', dpi=150)
```

Histograms using Matplotlib



Histograms using Matplotlib



Statistics with Python

Module `scipy.stats` provides classes to work with distributions.

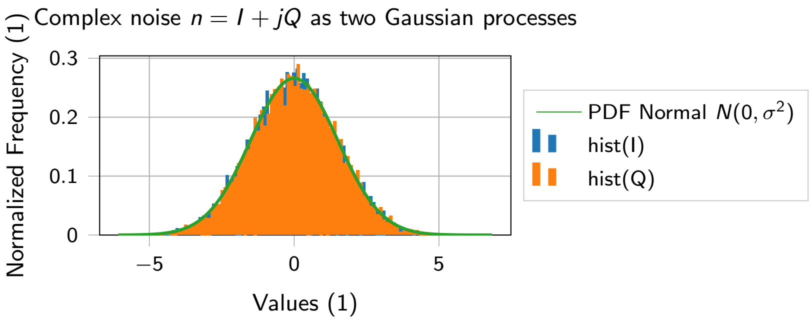
- Documentation:

  https://docs.scipy.org/doc/scipy/reference/stats.html

- `import scipy.stats as st`
- Classes used in this exercise: `st.norm`, `st.rayleigh`, `st.rice`
- Each class comes with handy functions like
  - `.pdf` for evaluating the probability density function
  - `.cdf` for evaluating the cumulative distribution function
  - `.sf` for evaluating the survivor function $\mathrm{SF}(x) = (1 - \mathrm{CDF}(x))$
  - `.stats`, `.moments`, `.var`, `.mean` for calculating several properties
- Caution: Normalized Distributions are provided! Use `loc=` and `scale=` to get the correct results.
  - `cdf=st.norm.cdf(x, scale=sigma)`
  - `cdf=st.rayleigh.cdf(x, scale=sigma)`
  - `cdf=st.rice.cdf(x, nu/sigma, scale=sigma) # caution: ν vs. b`

Exercise: Histogram and PDF of Complex Gaussian noise

- Generate 10000 samples of complex, zero-mean Gaussian noise with $\sigma = 1.5$.
- Compare the noise samples to the Gaussian distribution by plotting the histogram of the real and imaginary part and the probability density function in one plot.
- Hint: no loop needed!



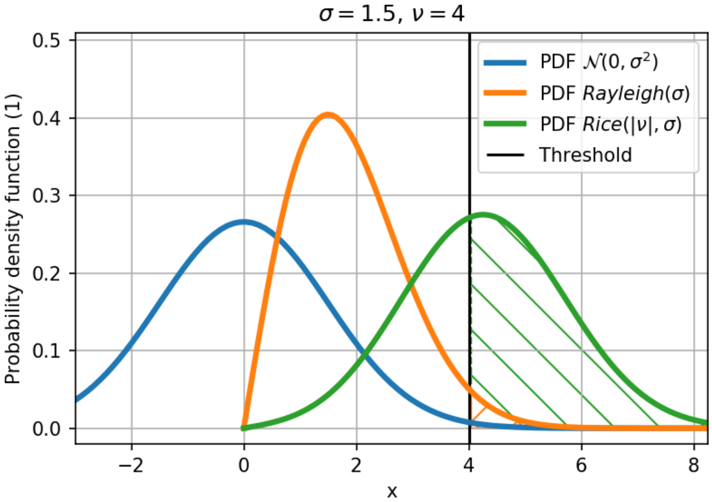
Exercise: Rayleigh Distribution

- Calculate the magnitude of the noise sample from the previous exercise.
- Compare the histogram of the magnitudes of the noise samples to the PDF of the Rayleigh distribution.

Exercise: Rice Distribution

- Generate a signal $s = A + n$, where $A = 8.0$ is a DC value and n is the complex noise from the previous two exercises.
- Calculate the magnitude of the signal s and compare the histogram to the PDF of the Rice distribution.

Detection of DC in Noise



Homework: Probability of Detection: DC in Noise

The presence of a DC value in noise should be detected by comparing the signals magnitude to a threshold VT.

- For $N = 10000$ trails, generate a signal $s = A + n$ with an DC value A from `np.linspace(0.1,15)` and n as samples of a zero mean complex Gaussian noise process with $\sigma = 1.5$.
- Plot the normalized $(1/N)$ number of detections and false alarm (in one plot over the SNR) for a threshold of $\mathrm{VT} = A$.
- Compare the curves to the probabilities of detection and false alarm obtained by using Rayleigh and Rice distributions.
- Hints:
  - $\mathrm{SNR} = A^2/(2\sigma^2)$
  - A detection is made when the magnitude of the signal is above threshold, i.e., $|s| > \mathrm{VT}$.
  - A false alarm is made when the magnitude of the noise is above threshold, i.e. $|n| > \mathrm{VT}$.
  - As it is slow, don't make a for-loop counting to 10000.

Homework: Probability of Detection: DC in Noise

- Repeat the “Probability of Detection: DC in Noise” for $\mathrm{VT} = 0.5\,A$ and $\mathrm{VT} = 2.0\,A$.
- Comment on: How does the relation between probability of detection and probability of false alarm change with changing threshold level?