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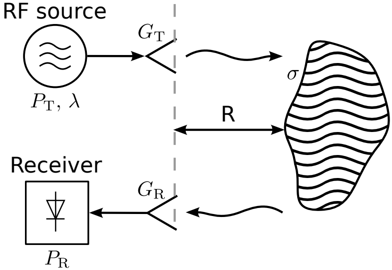
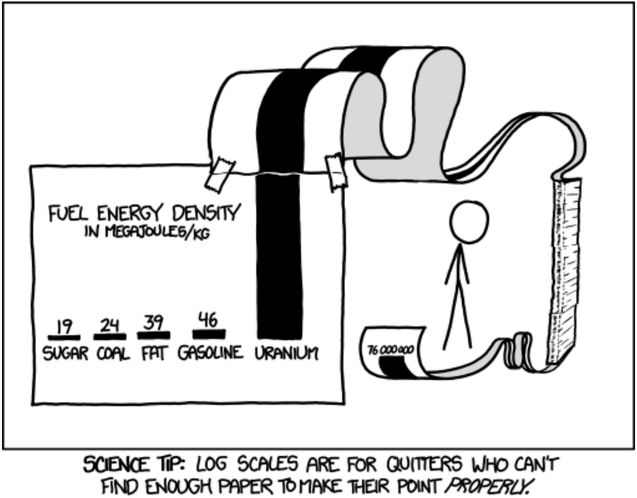


P_R	Power at the receiver
P_T	Power of the transmitter
G_T	Transmit antenna gain
G_R	Receive antenna gain
λ	Carrier wavelength
σ	Target's radar cross-section
R	Target's distance

Radar equation $$P_\mathsf{R} = \frac{P_\mathsf{T} G_\mathsf{T} G_\mathsf{R} \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Advantage of Logarithmic Scales



Source: `https://xkcd.com/1162/`

Calculations with Decibel

- Decibel is a unit of measurement to express a *ratio* in logarithmic scale.
- It is used to get a grasp on very high/low numbers and ratios.
- Reference is denoted by suffix (dBm, dBV, dBsm).
- Sometimes the reference it is implied, e.g., Watt for power, or an isotropic sphere as reference for the antenna gain.

$$10 \log_{10}\left(\frac{P}{P_0}\right) \qquad 20 \log_{10}\left(\frac{V}{V_0}\right)$$

$$\begin{aligned} 1\,\mathrm{mW} &= 10 \log_{10}\left(\frac{1\,\mathrm{mW}}{1\,\mathrm{mW}}\right)\mathrm{dB} = & 0\,\mathrm{dBm} \\ &= 10 \log_{10}\left(\frac{0.001\,\mathrm{W}}{1\,\mathrm{W}}\right)\mathrm{dB} = & -30\,\mathrm{dB} \end{aligned}$$

Radar Equation - Part 2

Radar Equation in dB

$$P_{\mathrm{R,dB}} = P_{\mathrm{T,dB}} + G_{\mathrm{T,dB}} + G_{\mathrm{R,dB}} + 20\log_{10}\left(\frac{\lambda}{1\,\mathrm{m}}\right) + 10\log_{10}\left(\frac{\sigma}{1\,\mathrm{m}^2}\right) - 30\log_{10}(4\pi) - 40\log_{10}\left(\frac{R}{1\,\mathrm{m}}\right)$$

Signal to Noise Ratio for AWGN

$$P_{\mathrm{N}} = 4k_{\mathrm{B}}\,T\,B_{\mathrm{N}} \qquad \mathrm{SNR} = P_{\mathrm{R}}/P_{\mathrm{N}}$$

$k_{\mathrm{B}} = 1.38 \cdot 10^{-23}\,\mathrm{J/K}$	Boltzmann constant
T	Temperature of the receiver in Kelvin
$B_{\mathrm{N}} = \frac{1}{T_{\mathrm{obs}}}$	Noise Bandwidth in Hz
T_{obs}	Observation time

Airport Radar

Airport Radar Specifications

An X-Band airport radar has the following specification:

$$P_\mathrm{T} = 1\,\mathrm{kW} \qquad f_\mathrm{c} = 10\,\mathrm{GHz} \qquad T = 290\,\mathrm{K}$$

$$G_\mathrm{T} = 44.15\,\mathrm{dB} \qquad R_\mathrm{max} = 10\,\mathrm{km} \qquad B_\mathrm{N} = 1\,\mathrm{GHz}$$

$$G_\mathrm{R} = 44.15\,\mathrm{dB} \qquad \sigma_\mathrm{Airplane} = 100\,\mathrm{m}^2$$

Minimum Receive Power

$$\begin{aligned} P_\mathrm{R,dB} = {} & 10\log_{10}\left(\frac{1000\,\mathrm{W}}{1\,\mathrm{W}}\right) + 44.15\,\mathrm{dB} + 44.15\,\mathrm{dB} + \\ & + 20\log_{10}\left(\frac{3\cdot 10^8\,\mathrm{m/s}}{10\cdot 10^9\,\mathrm{Hz}\;1\,\mathrm{m}}\right) + 10\log_{10}\left(\frac{100\,\mathrm{m}}{1\,\mathrm{m}}\right) \\ & - 30\log_{10}(4\pi) - 40\log_{10}\left(\frac{10\,000\,\mathrm{m}}{1\,\mathrm{m}}\right) \approx -85.13\,\mathrm{dB} \end{aligned}$$

Airport Radar Part 2

Signal to Noise Ratio

$$P_\mathrm{R} = 10^{-85.13/10} \approx 3.066\,\mathrm{nW}$$

$$P_\mathrm{N} = 4 k_\mathrm{B}\, T\, B_\mathrm{N} = 4 \cdot 1.38 \cdot 10^{-23}\,\mathrm{J/K} \cdot 290\,\mathrm{K} \cdot 10^{9}\,\mathrm{Hz} \approx 16\,\mathrm{pW}$$

$$P_\mathrm{N,dB} = 10 \log_{10}\left(\frac{4\,\mathrm{pW}}{1\,\mathrm{W}}\right) \approx -108\,\mathrm{dB}$$

$$\mathrm{SNR}_\mathrm{dB} = 10 \log_{10}\left(\frac{P_\mathrm{R}}{P_\mathrm{N}}\right) = P_\mathrm{R,dB} - P_\mathrm{N,dB} \approx 20.8\,\mathrm{dB}$$

RCS of a Perfectly Conducting Sphere

$$\frac{\sigma_{\text{Sphere}}}{\pi r^2} = \left| \frac{j}{kr} \sum_{n=1}^{\infty} (-1)^n (2n+1) \left[\left(\frac{krJ_{n-1}(kr) - nJ_n(kr)}{krH_{n-1}^{(1)}(kr) - nH_n^{(1)}(kr)} \right) - \left(\frac{J_n(kr)}{H_n^{(1)}(kr)} \right) \right] \right|^2$$

$k = \frac{2\pi}{\lambda}$	wave number
$\lambda = \frac{c_0}{f_c}$	wave length
J_n	spherical Bessel function of first kind and order n
$H_n^{(1)}$	Hankel function $H_n^{(1)} = J_n(kr) + \mathrm{j}Y_n(kr)$
$Y_n(kr)$	spherical Bessel function of second kind and order n

- Bessel functions of many kinds are in the `scipy.special` module

Approximation to the RCS of Simple Objects

$$\text{RCS of a corner reflector} \quad \sigma_{\mathrm{CR}} \approx \frac{4\pi a^4}{3\lambda^2} \quad \text{for} \quad a \gg \lambda$$

$$\text{RCS of a cylinder} \quad \sigma_{\mathrm{Cylinder}} \approx \frac{2\pi r h^2}{\lambda} \quad \text{for} \quad r, h \gg \lambda$$

$$\text{RCS of a sphere} \quad \sigma_{\mathrm{Sphere}} \approx \begin{cases} \pi r^2 & 2\pi r \gtrsim \lambda \\ 9\,(k\,r)^4\,\pi r^2 & \text{otherwise} \end{cases}$$

$k = \frac{2\pi}{\lambda}$	wave number
$\lambda = \frac{c_0}{f_c}$	wave length
r	radius of cylinder or sphere
a	sidelength of the corner reflector
h	height of the cylinder

Exercise: RCS of Sphere and Corner Reflector

- Write three functions to calculate the RCS
  - of a sphere using the approximation,
  - of a sphere using the exact equation (up to 100 iteration), and
  - of a corner reflector using the approximation.

  Input arguments for all functions should be the same, namely
  - the size as first parameter, i.e. radius of the sphere or sidelength of the corner reflector, and
  - the frequency as second parameter.

  This provides for easy substitution using function handles (makes next exercise easier).

  For regions not covered by an approximation, `NaN` should be returned.

Homework: Compare RCS of a Sphere and a Corner Reflector

- For a frequency of $f_\mathrm{c} = 1\,\mathrm{GHz}$, plot the RCS of the sphere and the corner reflector (in one plot) in dBsm over the size of the object in wavelengths.
  The x-axis should be logarithmic from about 0.01λ to 10λ to see the interesting regions.
- Why is a corner reflector preferred as a test-target for high frequencies (λ smaller than extent of the target)?
- Comment on how the plots change with a change of the frequency.
- What happens for very low, e.g., 2, or very high, e.g., 1000, number of iterations in the analytic expression of the RCS of the sphere?

Homework: Radar Equation with Simple Objects

- Given radar system for pedestrian tracking:

```
Pt_dBm=1.0          # transmit power
Gt_dB=15.0          # gain of the TX antenna
Gr_dB=15.0          # gain of the RX antenna
Pr_min_dBm=-120.    # minimal required power at the receiver
fc=77e9             # carrier frequency
```

- Plot the receive power for `a=[2.0, 1.0, 0.1, 0.01];` `fc=77e9`, where `a` is the radius of a sphere or the sidelength of a corner reflector.
- For which ranges can a corner reflector and a sphere be detected by this system?

Homework: Distance for Pedestrian Classification

- A pedestrian has a radar cross-section of $\sigma_{\text{Person}} \approx 1\,\mathrm{m}^2$. To not only detect, but also classify a pedestrian, the receive signal needs a dynamic range of at least 20 dB, i.e., the received power must be 20 dB above the minimum receive power of the system.
  - This is because hand and feet are required to be seen by the radar for classification.
- Up to which distance can the radar system be used for pedestrian classification?

Homework: Corner Reflector on the Moon

- Apollo 11-15 placed corner reflectors of side length $a \approx 0.5\,\mathrm{m}$ on the moon. The average distance between earth and moon is $R \approx 385\,000\mathrm{km}$
- Which transmit power would be needed to detect the signal using the airport radar and a required SNR of $10\,\mathrm{dB}$?
- How much observation time T_{obs} would be required to detect the corner reflector?
- Is it feasible to use such a radar for this task?