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Assignment 3

Q1) $P(A,D,S,T) = P(A) \cdot P(D) \cdot P(S \mid A,D) \cdot P(T \mid S)$

Q2) Need $P(+s \mid +t) \Rightarrow P(S \mid T=+t)$

We have $P(A)$, $P(D)$, $P(S \mid A,D)$, $P(T=+t \mid S)$

→ need to eliminate A and D

⇒ Step 1: Join $P(A)$ and $P(S \mid A,D)$ to get $f_1(A,S,D)$

⇒ Step 2: Sum out A from $f_1(A,S,D)$ to get $f_2(S,D)$

→ Now we have $P(D)$, $P(T=+t \mid S)$, $f_2(S,D)$

↳ need to eliminate D.

⇒ Step 3: Join $P(D)$ and $f_2(S,D)$ to get $f_3(S,D)$

⇒ Step 4: Sum out D from $f_3(S,D)$ to get $f_4(S)$

⇒ Step 5: Join $P(T=+t \mid S)$ and $f_4(S)$ to get $f_5(T=+t, S)$

⇒ Step 6: Normalize $f_5(T=+t,S)$ to get $P(S=+s \mid T=+t)$

Step 1:

A	D	S	$f_1(A,S,D)$
+a	a	+s	0,2 × 0,6 = 0,12
+a	n	+s	0,5 × 0,6 = 0,3
+a	d	+s	0,7 × 0,6 = 0,42
-a	a	+s	0,4 × 0,4 = 0,16
-a	n	+s	0,7 × 0,4 = 0,28
-a	d	+s	0,9 × 0,4 = 0,36
+a	a	-s	0,8 × 0,6 = 0,48
+a	n	-s	0,5 × 0,6 = 0,3
+a	d	-s	0,3 × 0,6 = 0,18
-a	a	-s	0,6 × 0,4 = 0,24
-a	n	-s	0,3 × 0,4 = 0,12
-a	d	-s	0,1 × 0,4 = 0,04

⇒

Step 2:

D	S	$f_2(S,D)$
a	+s	0,12 + 0,16 = 0,28
n	+s	0,3 + 0,28 = 0,58
d	+s	0,42 + 0,36 = 0,78
a	-s	0,48 + 0,24 = 0,72
n	-s	0,3 + 0,12 = 0,42
d	-s	0,18 + 0,04 = 0,22

Step 3:

D	S	$f_3(S,D)$
a	+s	0,28 × 0,2 = 0,056
n	+s	0,58 × 0,5 = 0,29
d	+s	0,78 × 0,3 = 0,234
a	-s	0,72 × 0,2 = 0,144
n	-s	0,42 × 0,5 = 0,21
d	-s	0,22 × 0,3 = 0,066

⇒

Step 4:

S	$f_4(S)$
+s	0,056 + 0,29 + 0,234 = 0,58
-s	0,144 + 0,21 + 0,066 = 0,42

Step 5:

S	T	$f_5(T=+t, S)$
+s	+t	0,95 × 0,58 = 0,551
-s	+t	0,19 × 0,42 = 0,0798

⇒

Step 6:

S	$f_5(T=+t, S)$
+s	$\frac{0,551}{0,551 + 0,0798} \cong 0,8735$
-s	$\frac{0,0798}{0,551 + 0,0798} \cong 0,1265$

$\Rightarrow P(+s \mid +t) \cong 0,8735$ //

Q3) $P(S=+s)$

→ We have $P(A)$, $P(D)$, $P(S|A,D)$, $P(T|S)$

↳ Need to get rid of A, D and T.

⇒ First 4 phase is exactly the same as Q2

↳ We have $P(T|S)$ and $f_4(S)$

⇒ Step 5: Join $P(T|S)$ and $f_4(S)$ to get $f_5(T,S)$

Step 6: Sum out T from $f_5(T,S)$ to get $f_6(S)$

Step 7: Normalize $f_6(S)$ to get $P(S)$

Step 5:

S	T	$f_5(T,S)$
+s	+t	$0{,}95 \times 0{,}58 = 0{,}551$
-s	+t	$0{,}19 \times 0{,}42 = 0{,}0798$
+s	-t	$0{,}05 \times 0{,}58 = 0{,}029$
-s	-t	$0{,}81 \times 0{,}42 = 0{,}3402$

⇒

Step 6:

S	$f_6(S)$
+s	$0{,}551 + 0{,}029 = 0{,}58$
-s	$0{,}0798 + 0{,}3402 = 0{,}42$

⇒ No need for Step 7, since $f_6(S)$ is already normalized

⇒ $f_6(S) = P(S)$ ⇒ $P(+s) = 0{,}58$

Q4) We know $P(S)$

⇒

S	B	U	
+s	+b	+5000	⇒ $0{,}58 \times 5000 = 2900$
+s	-b	0	⇒ $0{,}58 \times 0 = 0$
-s	+b	-6000	⇒ $0{,}42 \times (-6000) = -2520$
-s	-b	0	⇒ $0{,}42 \times 0 = 0$

⇒ $EU(+b) = 2900 + (-2520) = 380$

Q5) $VPI(T) = \underbrace{MEU(T)}_{\text{We need this}} - \underbrace{MEU()}_{380}$

⇒ We have to find $P(S|T)$. We can use variable elimination and previous results. In Q3, we computed $f_5(T,S)$. We just have to normalize it.

Then:

T	S	$f_5(T,S)$
+t	+s	0,551
-t	+s	0,029
+t	-s	0,0798
-t	-s	0,3402

⇒

T	S	$P(S|T)$
+t	+s	0,8735
+t	-s	0,1265
-t	+s	0,0785
-t	-s	0,9215

⇒ We also need $P(T)$. We can further continue on $f_5(T,S)$ and sum out S from it to get $f_7(T)$.

→

T	$f_7(+) = P(T)$
$+t$	$0{,}551 + 0{,}0798 = 0{,}6308$
$-t$	$0{,}029 + 0{,}3402 = 0{,}3692$

} Alredy normalized

$$\Rightarrow EU(+b \mid +t) = P(S=+s \mid T=+t) \cdot U(S=+s, B=+b) + P(S=-s \mid T=+t) \cdot U(S=-s, B=+b)$$
$$= 0{,}8735 \times 5000 + 0{,}1265 \times (-6000) = 3608{,}5$$

$$\Rightarrow EU(+b \mid -t) = P(S=+s \mid T=-t) \cdot U(S=+s, B=+b) + P(S=-s \mid T=-t) \cdot U(S=-s, B=+b)$$
$$= 0{,}0785 \times 5000 + 0{,}9215 \times (-6000) = -5316{,}5$$

$$\Rightarrow EU(-b \mid +t) = P(S=+s \mid T=+t) \cdot U(S=+s, B=-b) + P(S=-s \mid T=+t) \cdot U(S=-s, B=-b) = 0$$
> both 0

$$\Rightarrow EU(-b \mid -t) = 0$$ ↙ similarly

→ If $T=+t$, optimal action is $+b \Rightarrow 3608{,}5$
$T=-t$, optimal action is $-b \Rightarrow 0$

$$\Rightarrow MEU(T) = 0{,}6308 \times 3608{,}5 + 0{,}3692 \times 0 = 2276{,}2418$$

$$\Longrightarrow VPI(T) = MEU(T) - MEU() = 2276{,}2418 - 380 = 1896{,}2418$$

This VPI(T) value is the maximum cost of getting the car tested. So we can pay upto that utility cost.

Q6) $P(d \mid +o, +t)$

A	D	S	T	weight
+o	d	+s	+t	$0{,}6 \times 0{,}95 = 0{,}57$
+o	d	-s	+t	$0{,}6 \times 0{,}19 = 0{,}114$

A	P(A)
+o	0,6

A	D	S	P(S|A,D)
+o	a	+s	0,2
+o	n	+s	0,5
+o	d	+s	0,7
+o	a	-s	0,8
+o	n	-s	0,5
+o	d	-s	0,3

S	T	P(T|S)
+s	+t	0,95
-s	+t	0,19

Q7) Sample : $\{A=-o,\ D=n,\ S=+s,\ T=-t\}$

Evidences : $-o$, n, $-t$

$$\Rightarrow P(S \mid -o, n, -t) = \frac{P(S, -o, n, -t)}{P(-o, n, -t)}$$

$$= \frac{P(S, -o, n, -t)}{\sum_s P(s, -o, n, -t)}$$

$$= \frac{P(-o) \cdot P(n) \cdot P(S \mid -o, n) \cdot P(-t \mid S)}{\sum_s P(-o) \cdot P(n) \cdot P(s \mid -o, n) \cdot P(-t \mid s)}$$

$$= \frac{\cancel{P(-o)} \cdot \cancel{P(n)} \cdot P(S \mid -o, n) \cdot P(-t \mid S)}{\cancel{P(-o)}\ \cancel{P(n)} \sum_s P(s \mid -o, n)\ P(-t \mid s)}$$

$$= \frac{P(S \mid -o, n) \cdot P(-t \mid S)}{P(+s \mid -o, n) \cdot P(-t \mid +s) + P(-s \mid -o, n) \cdot P(-t \mid -s)}$$

$$= \frac{0{,}7 \times 0{,}05}{0{,}7 \times 0{,}05 + 0{,}3 \times 0{,}81}$$

$$= \frac{0{,}035}{0{,}035 + 0{,}243} = 0{,}126$$