Functional:

$$\min_{p,\theta,\lambda_1,\lambda_2} \mathcal{L} = k_0 \int\limits_x p(x,t_0)\ln\frac{p(x,t_0)}{p_0(x)}\,dx \;+\; k_1 \int\limits_x p(x,t_1)\ln\frac{p(x,t_1)}{p_1(x)}\,dx+$$

$$+\int\limits_{t_0}^{t_1}\int\limits_x \lambda_1(x,t)\left[\frac{\partial p(x,t)}{\partial t} + \frac{\partial}{\partial x}\left[f(x,\theta)\cdot p(x,t)\right]\right]dxdt+$$

$$+\int\limits_{t_0}^{t_1}\lambda_2(t)\left[\int\limits_x p(x,t)\,dx - 1\right]dt + \lambda_3\big(\dot{x} - f(x,\theta)\big)$$

Final system:

1. variation by p:

$$\delta p(X_{max},t) : \lambda_1(X_{max},t) = 0$$

$$\delta p(X_{min},t) : \lambda_1(X_{min},t) = 0$$

$$\frac{\partial\lambda_1}{\partial t} + \lambda_1 f + \lambda_2 = 0 \;\Leftrightarrow\; \delta p\left[-\frac{\partial\lambda_1}{\partial t} - \frac{\partial\lambda_1}{\partial X}f(X,\theta) + \lambda_2\right] = 0$$

$$\delta p(X,t_0)\left[k_0\Big(1+\ln p - \ln\big(\hat{p_0}(X)\big)\Big) - \lambda_1(X,t_0)\right] = 0$$

$$\delta p(X,t_1)\left[k_1\Big(1+\ln p - \ln\big(\hat{p_1}(X)\big)\Big) - \lambda_2(X,t_1)\right] = 0$$

2. variation by λ_1:

$$\delta\lambda_1\Big(\frac{\partial p}{\partial t} + \frac{\partial}{\partial X}[fp]\Big) = 0$$

3. variation by λ_2:

$$\int\limits_X p(X,t)dX = 1$$

4. variation by λ_3:

$$\frac{\partial X}{\partial t} - f(X,\theta) = 0$$

5. variation by X:

   (a) variation of differential equation term

$$\delta X\big(\frac{\partial\lambda_3}{\partial t} + \frac{\partial f}{\partial t}\big) = 0$$

$$\int\limits_X\int\limits_t \Big[\lambda_1\delta X\big(\frac{2\partial^2 p}{\partial X\partial t} + \frac{\partial p}{\partial X}f + \frac{\partial p}{\partial X}\frac{\partial f}{\partial X} + \frac{\partial^2 f}{\partial X^2}p + \frac{\partial f}{\partial X}\frac{\partial p}{\partial X}\big) + \lambda_1'\delta X\big(\frac{\partial p}{\partial t}\big)\Big]dtdX = 0$$

$$\lambda_2 \frac{\partial p}{\partial X} = 0$$

$$\delta X(t_0)\lambda_3(t_0) = 0$$

$$\delta X(t_1)\lambda_3(t_1) = 0$$

(b) variation of divergence

$$\delta X_0 k_0 \big(p'_x - \frac{\hat{p_0}' X}{\hat{p_0}} p\big) = 0$$

$$\delta X_1 k_1 \big(p'_x - \frac{\hat{p_1}' X}{\hat{p_1}} p\big) = 0$$

variation by θ:

$$\delta\theta: \; \frac{\partial p}{\partial x}\frac{\partial x}{\partial \theta}\lambda_2 - \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta}\lambda_3 + \frac{\partial \lambda_1}{\partial x}\frac{\partial x}{\partial \theta}\frac{\partial p}{\partial t} - \frac{\partial \lambda_1}{\partial t}\frac{\partial p}{\partial x}\frac{\partial x}{\partial t} - \frac{\partial^2 p}{\partial x^2}\left(\frac{\partial x}{\partial \theta}\right)^2\frac{\partial \theta}{\partial t}\lambda_1 + \frac{\partial \lambda_1}{\partial x}\frac{\partial x}{\partial \theta}\frac{\partial f}{\partial x} p = 0$$

$$\delta\theta(t_0): \; -\frac{\partial p}{\partial x}\frac{\partial x}{\partial \theta}\lambda_1 - \frac{\partial x}{\partial \theta}\lambda_3 + k_0 \frac{\partial p}{\partial x}\frac{\partial x}{\partial \theta}\left(1 - \frac{p}{\hat{p_0}} + \ln\frac{p}{\hat{p_0}}\right)\Bigg|_{t_0} = 0$$

$$\delta\theta(t_1): \; \frac{\partial p}{\partial x}\frac{\partial x}{\partial \theta}\lambda_1 + \frac{\partial x}{\partial \theta}\lambda_3 + k_1 \frac{\partial p}{\partial x}\frac{\partial x}{\partial \theta}\left(1 - \frac{p}{\hat{p_1}} + \ln\frac{p}{\hat{p_1}}\right)\Bigg|_{t_1} = 0$$