Tarea 3: Relaciones y funciones

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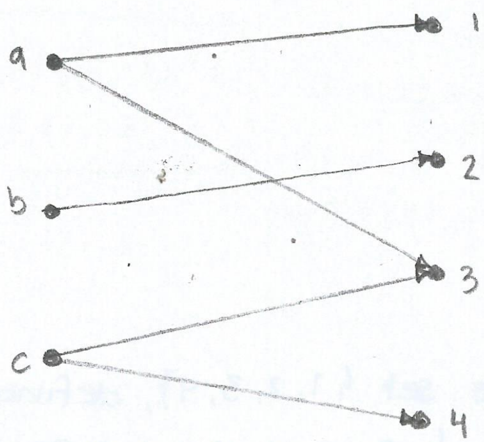
3.1. Write out the set of ordered pairs and draw the directed graph of the relation between the elements of the sets $\{a, b, c\}$ and $\{1, 2, 3, 4\}$, specified by the logical matrix.

$$\begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} T & F & T & F \\ F & T & F & F \\ F & F & T & T \end{bmatrix} \end{array}$$

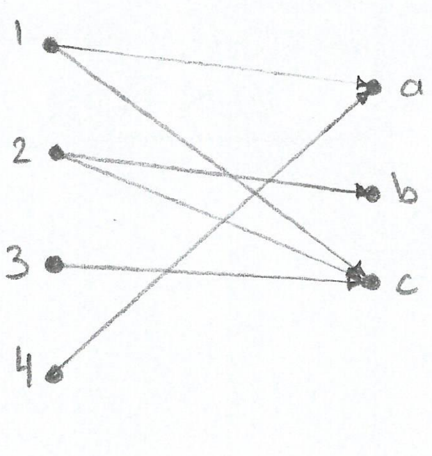
$$\boxed{R = \{(a,1), (a,3), (b,2), (c,3), (c,4)\}}$$



3.2. Write out the set of ordered pairs and draw the directed graph of the relation between the elements of the sets $\{1, 2, 3, 4\}$ and $\{a, b, c\}$, specified by the logical matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} T & F & T \\ F & T & T \\ F & F & T \\ T & F & F \end{bmatrix} \end{array}$$

$$\boxed{R = \{(1,a), (1,c), (2,b), (2,c), (3,c), (4,a)\}}$$



3.5. For each of the following relations on the set of natural numbers $\mathbb{N}$ describe the ordered pairs belonging to the relations

① $R = \{(n, m) : 3n + m = 16\}$;

$n = 1; m = 13$
$3(1) + 13 = 16$

$n = 3; m = 7$
$3(3) + 7 = 16$

$n = 5; m = 1$
$3(5) + 1 = 16$

$n = 2; m = 10$
$3(2) + 10 = 16$

$n = 4; m = 4$
$3(4) + 4 = 16$

$\boxed{R = \{(1,13), (2,10), (3,7), (4,4), (5,1)\}}$

② $S = \{(n, m) : n + 2m < 7\}$

$n = 1; m = 1$
$1 + 2(1) = 3$

$n = 2; m = 1$
$2 + 2(1) = 4$

$n = 3; m = 1$
$3 + 2(1) = 5$

$n = 1; m = 2$
$1 + 2(2) = 5$

$n = 2; m = 2$
$2 + 2(2) = 6$

$n = 4; m = 1$
$4 + 2(1) = 6$

$\boxed{S = \{(1,1), (1,2), (2,1), (2,2), (3,1), (4,1)\}}$

3.8. Let R be a relation on the set $\{1, 2, 3, 4\}$, defined by the condition: $n R m$ if and only if $2n + m$ is an even number. Represent R by each of the following methods:

① as a set of ordered pairs

② in a graphic form

③ in the form of a logical matrix

1) $n = 1; m = 2$
$2(1) + 2 = 4$

$n = 3; m = 2$
$2(3) + 2 = 8$

$n = 1; m = 4$
$2(1) + 4 = 6$

$n = 3; m = 4$
$2(3) + 4 = 10$

$n = 2; m = 2$
$2(2) + 2 = 6$

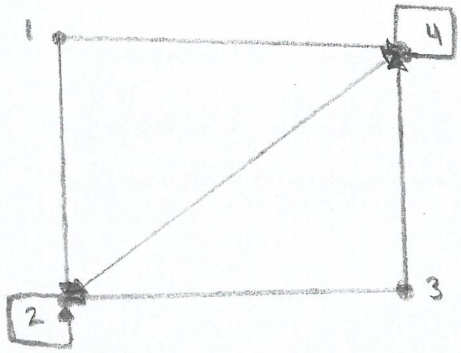
$n = 4; m = 2$
$2(4) + 2 = 10$

$n = 2; m = 4$
$2(2) + 4 = 8$

$n = 4; m = 4$
$2(4) + 4 = 12$

$\boxed{R = \{(1,2), (1,4), (2,2), (2,4), (3,2), (3,4), (4,2), (4,4)\}}$

2)



3)

$$M_R = \begin{bmatrix} F & T & F & T \\ F & T & F & T \\ F & T & F & T \\ F & T & F & T \end{bmatrix}$$

3.20 List the ordered pairs belonging to the relations specified on the set $\{n : n \in \mathbb{Z} \text{ and } 1 \leq n \leq 10\}$:

1) $R = \{(n, m) : nm = 30\}$

$n=3\ ;\ m=10$

$3(10) = 30$

$n=6\ ;\ m=5$

$6(5) = 30$

$n=5\ ;\ m=6$

$5(6) = 30$

$n=10\ ;\ m=3$

$10(3) = 30$

$\boxed{R = \{(3,10), (5,6), (6,5), (10,3)\}}$

2) $S = \{(n,m) : 2n + m = 10\}$

$n=1\ ;\ m=8$

$2(1) + 8 = 10$

$n=3\ ;\ m=4$

$2(3) + 4 = 10$

$n=2\ ;\ m=6$

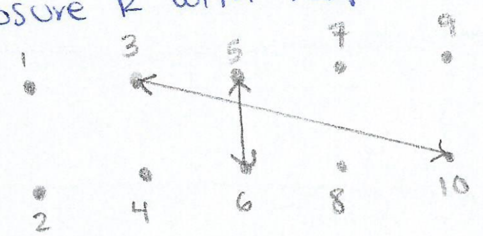
$2(2) + 6 = 10$

$n=4\ ;\ m=2$

$2(4) + 2 = 10$

$\boxed{S = \{(1,8), (2,6), (3,4), (4,2)\}}$

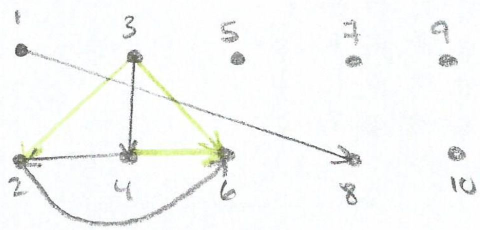
3) closure R with respect to symmetry



$\boxed{R^* = R}$

la relación principal ya es simétrica

4) Closure S with respect to transitivity



$$S^* = S \cup \{(3,2), (3,6), (4,6)\}$$

$-S \quad -S^*$

3.21. List the ordered pairs belonging to the relations specified on the set $\{n : n \in \mathbb{Z} \text{ and } 1 \le n \le 12\}$

1) $R = \{(n,m) : nm = 12\}$

$n=1; m=12$
$1(12) = 12$

$n=2; m=6$
$2(6) = 12$

$n=3; m=4$
$3(4) = 12$

$n=4; m=3$
$4(3) = 12$

$n=6; m=2$
$6(2) = 12$

$n=12; m=1$
$12(1) = 12$

$$R = \{(1,12), (2,6), (3,4), (4,3), (6,2), (12,1)\}$$

2) $S = \{(n,m) : n + 2m = 10\}$

$n=2; m=4$
$2+2(4) = 10$

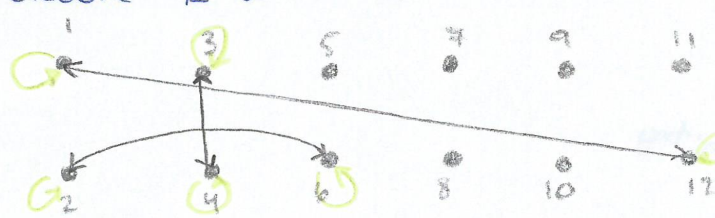
$n=4; m=3$
$4+2(3) = 10$

$n=6; m=2$
$6+2(2) = 10$

$n=8; m=1$
$8+2(1) = 10$

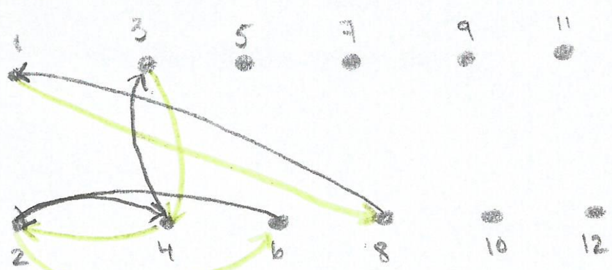
$$S = \{(2,4), (4,3), (6,2), (8,1)\}$$

3) closure R with respect to transitivity



$$R^* = R \cup \{(1,1), (2,2), (3,3), (4,4), (6,6), (12,12)\}$$

4) closure S with respect to symmetry



$$S^* = S \cup \{(1,8), (2,6), (3,4), (4,2)\}$$

3.32. Let R be a relation between the sets $\{1,2\}$ and $\{1,2,3,4,5\}$, specified by listing of the pairs: $R=\{(1,2),(1,4),(1,5),(2,3),(2,5)\}$, and S be a relation between the sets $\{1,2,3,4,5\}$ and $\{1,2,3\}$, consisting of the pairs: $S=\{(1,1),(2,1),(2,3),(3,3),(5,2)\}$. Calculate $S \circ R$ and $(S \circ R)^{-1}$

$$\boxed{S \circ R = \{(1,1),(1,2),(1,3),(2,2),(2,3)\}}$$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \qquad M_S = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$M_{S \circ R} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$\boxed{(S \circ R)^{-1} = \{(1,1),(2,1),(2,2),(3,1),(3,2)\}}$$

3.33. Let R be a relation between the sets $\{1, 2, 3, 4, 5\}$ and $\{1, 2\}$, specified by listing of the pairs: $R = \{(2,1), (3,2), (4,1), (5,1), (5,2)\}$ and S be a relation between the sets $\{1, 2, 3, 4, 5\}$ and $\{1, 2, 3\}$ consisting of the pairs: $S = \{(1,1), (2,1), (2,3), (3,3), (5,2)\}$. Calculate R^{-1} and $S \circ R^{-1}$

$$\boxed{R^{-1} = \{(1,2), (1,4), (1,5), (2,3), (2,5)\}}$$

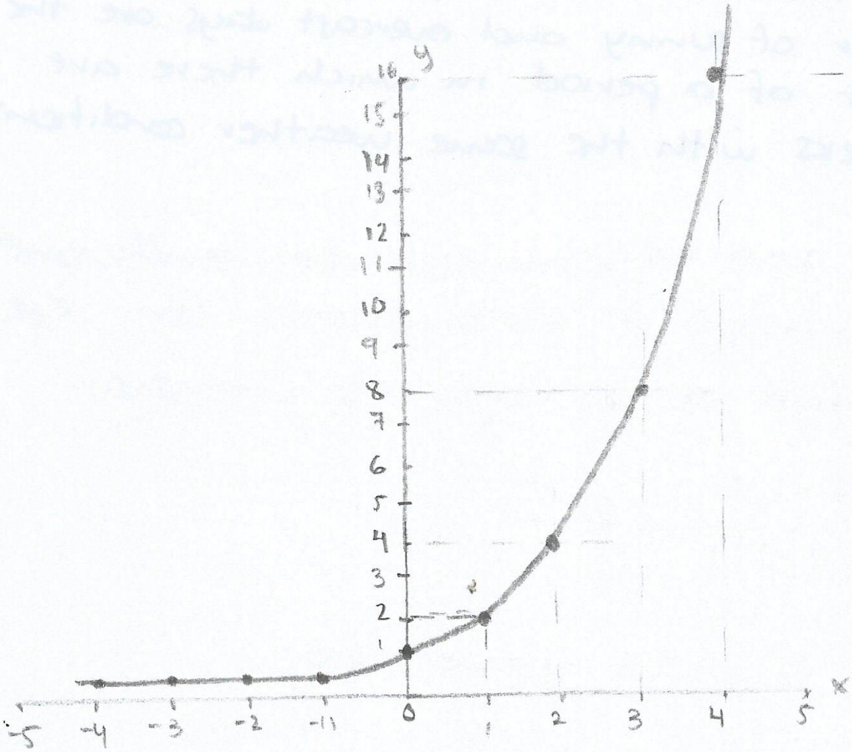
$$\boxed{S \circ R^{-1} = \{(1,1), (1,2), (1,3), (2,2), (2,3)\}}$$

$$M_S = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 \\ 5 & 0 & 1 & 0 \end{array} \qquad M_{R^{-1}} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 & 0 & 1 \end{array}$$

$$M_{S \circ R^{-1}} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 \end{array}$$

3.80. Draw graphs of the functions: Indicate the set of values of each of them and tell which of these functions are injective, and which are surjective.

1) $f(x) = 2^x$, $f: \mathbb{R} \to \mathbb{R}$



definimos $f = (0, \infty)$

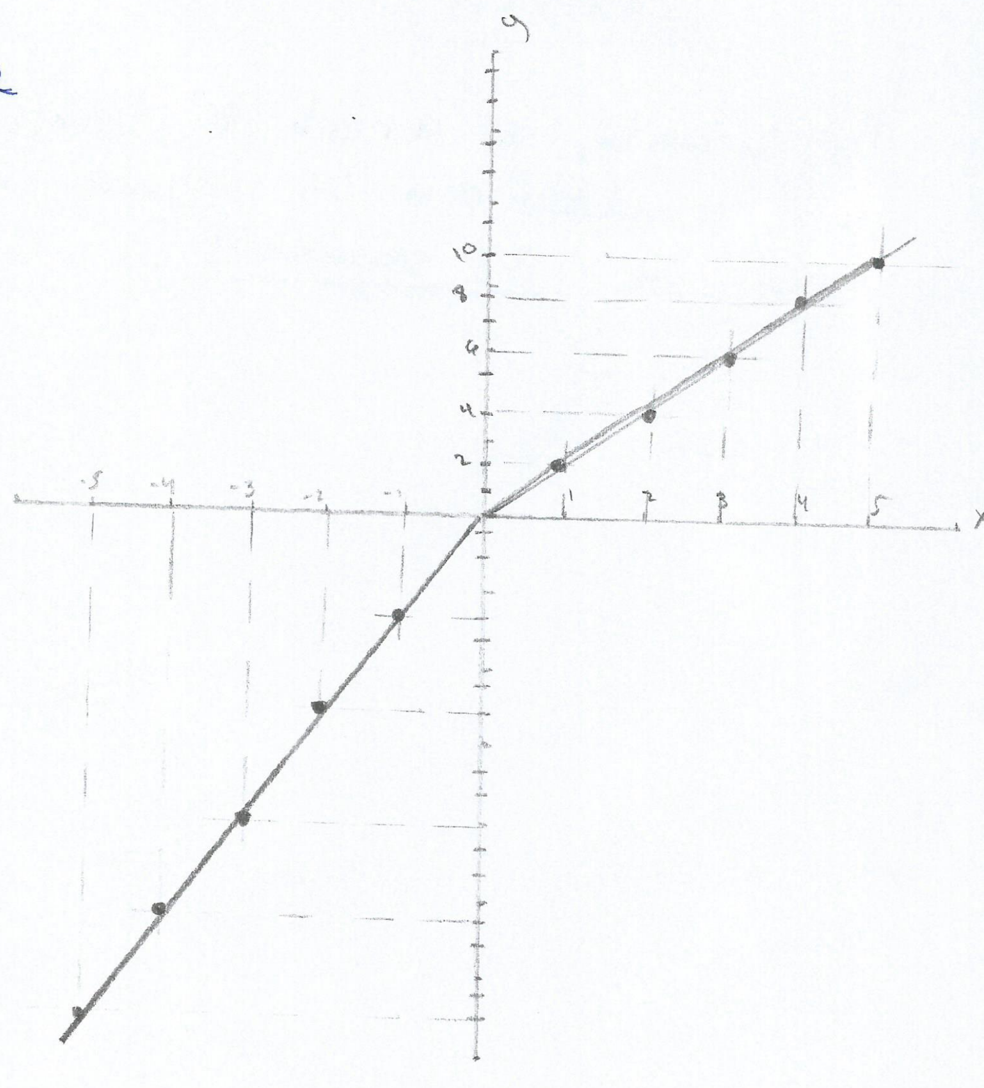
es inyectiva porque a cada valor de la función le corresponde una x.

dado que el dominio es $(-\infty, \infty)$, no coincide con el rango, por lo tanto no es subjectiva.

2) $g(x) = 3x - |x|$, $g: \mathbb{R} \to \mathbb{R}$

definiendo $g = (-\infty, \infty)$

es inyectiva, ya que al igual que el ejercicio 1, para cada valor de la función $g(x)$ corresponde una x, así mismo el dominio y el rango coinciden es subjectiva.



3.115 The meteorological service report contains information on whether the weather was sunny or overcast on specific days. Each day corresponds to one of two weather conditions. We will consider the weather characteristics of specific 2 weeks to be the same if the number of sunny and overcast days are the same. Calculate the lenght of a period in which there are sure to be at least 3 weeks with the same weather conditions.

El número de opciones para las semanas con condiciones meteorológicas es igual al número de combinaciones con repeticiones, esto se representaría de la siguiente manera:

$$\tilde{C}(2,7) = \frac{(2+7-1)!}{7!(2-1)!} = 8$$

De acuerdo al principio de Dirichlet, tenemos que:

$$\boxed{2 * 8 + 1 = 17}$$

Por lo tanto, el periodo de tiempo donde habrá 3 o más semanas con las mismas condiciones climaticas sera de <u>17 semanas</u>.