**APPLYING S-SHAPED MODELS FOR THE EXAMINATION OF ENGINEERING SYSTEMS DYNAMICS**

The first mention of S-shaped system evolution could be reffered to the middle of the XIX century for the forecasting of country population as logistic curves by such authors as Pierre François Verhulst and also Gompertz curve and Pearl curve.

In 1975 Genrich Altshuller used S-shaped curves for the technical systems evolution prediction.

Research which is doing have shown that the models based on S-shaped curves describe the dynamics of various natural, technical and economical or social-cultural processes quite correct. In J.Martino' monography multiple examples of the processes which obey the S-shaped evolution model are given:

- the increase of pumpkin weight while its growth on the bed;
- the quantity gain of the yeast bacteria population.

For these curves is common: asymptotically appoaching 0 as time $t$ reduces and tend to some upper limit $L$ while time increases; the existence of accelerated growth period - until the moment $t_b$; the presence of slowed growth period - after the moment $t_b$.

For the description of these relationships the Pearl-Reed, Gompertz equations, for example, is used that is presented in (1).

$$P = \frac{L}{1 + a^{-bt}} \tag{1}$$

These formulae have resulted from the solution for the living organism populations evolution differential equation. The proof of the validity of these formulae use is based on some parallels between, on the one hand, biological and, on the onther hand, economical and technical processes.

The disadvantage of this procedure is the absence of the begin and the end of process in the mathematical model. In real processes going in some finite period of time the interpretation of the approaching to any quantity is impossible. It reduces the opportunity of use this model for the forecasting of examined procedures, for instance. These considerations have made the demand for the development of another mathematical two-link S-shaped model which is given in Figure 1.

Denote the evolution time period as $[t_0, t_f]$ and relevant value of examined quantity as

$$P(t_0) = P_0, \qquad P(t_f) = P_f \tag{2}$$

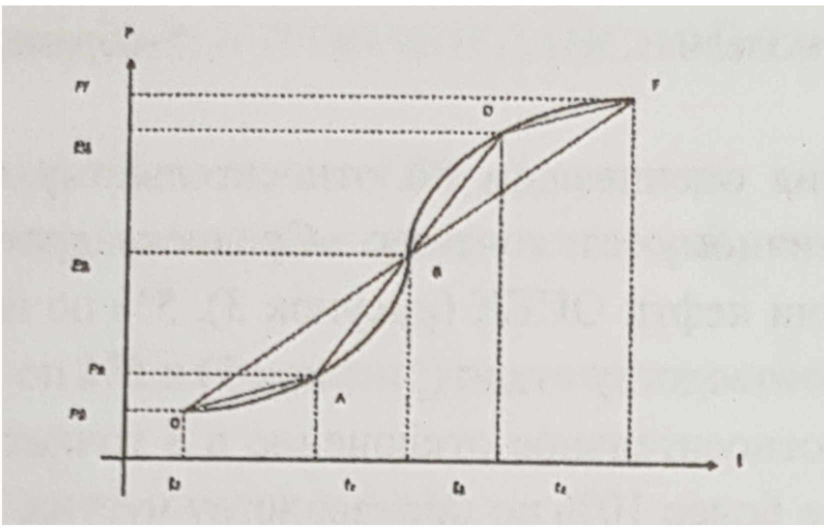

Figure 1: The parts of the two-link S-shaped model

$$P(t) = V\,\frac{(t-t_0)^2}{t_b-t_0} + P_0, \qquad t_0 \leqslant t \leqslant t_b;$$
$$P(t) = -V\,\frac{(t_f-t)^2}{t_f-t_b} + P_f, \qquad t_b \leqslant t \leqslant t_f, \tag{3}$$

where $V = \dfrac{P_f - P_0}{t_f - t_0}$ — the average growth rate of the quantity during the examination period.

The feature of this curve are: continuity; smooth growth (the continuity of first derivative); the existence of the end and the begin points of limit; the whole curves is determined by three defining points: $O$, $A$, $B$, $C$, $D$, $F$.

By the authors of the article the approbation of this model on famous material has been made. Has been considered: the dymanics of the petroleum extraction by the OPEC (Figure 2) organization, the dynamics of the cast iron (Figure 3), steel (Figure 4) and rolled metal (Figure 6) production. The experiment have consisted in the approximation of the statistical data values and calculated data of the two-link S-shaped model.

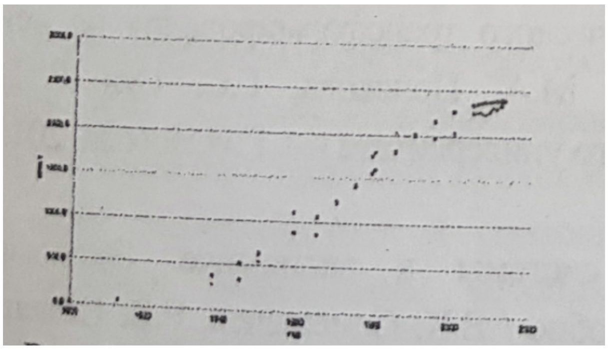

Figure 2: The dynamics of the petroleum extraction and the two-link S-shaped model by the OPEC.

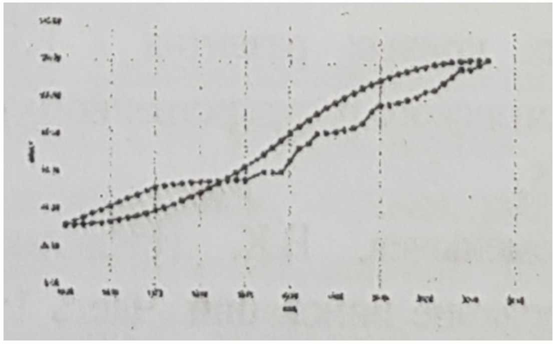

Figure 3: The dynamics of the cast iron production and the two-link S-shaped model.