Test 1 – You have 18 minutes to do the test.

(i) Consider two events A and B of an outcome space. Suppose that $P(A) = 0.5$ and $P(B) = 0.6$.

    (a) Determine a real number $b_1 > 0$ such that $P(A \cap B) \geq b_1$ and give one sufficient condition under which $P(A \cap B) = b_1$.

        Show your steps of how you got your answer.

    (b) Determine a real number $b_2 < 1$ such that $P(A \cap B) \leq b_2$ and give one sufficient condition under which $P(A \cap B) = b_2$.

        Show your steps of how you got your answer.

(ii) Let A and B be two events. Let E be the event that exactly one of the events A or B occurs. Use the Axioms of Probability to show that $P(E) = P(A) + P(B) - 2P(A \cap B)$.

(iii) Let X be a set consisting of 24 people, say $X = \{x_1, x_2, x_3, \ldots, x_{24}\}$. Let $A = \{x_1, x_2\}$ and $B = \{x_3, x_4\}$. Suppose that 6 people from X have been promoted, and these promotions have occurred randomly among all members of X.

(a) Let E_1 be the event that all members of A have been promoted, i.e. E_1 is the event that x_1 and x_2 have been promoted. Find $P(E_1)$.

Show your steps of how you got your answer.

(b) Let E_2 be the event that all members of A and B have been promoted, i.e. E_2 is the event that x_1, x_2, x_3, and x_4 have been promoted. Find $P(E_2)$.

Show your steps of how you got your answer.