Test 4 – You have 18 minutes to do the test. ⏱

Let X denote the concentration of a certain substance in one trial of an experiment and Y the concentration of the substance in a second trial of the experiment. Suppose that the joint pdf is given by

$$f(x,y) = \begin{cases} k(2x^2 + \frac{2xy}{3}) & \text{if } 0 < x \leq 1, 0 < y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the value of k.

(ii) Find the marginal pdf $f_X(x)$ of X.

(iii) Find the conditional pdf $f_{Y|X}(y|x)$ of Y given $X = x$.

(iv) Find $E[X]$ and $E[Y|X = 0.5]$.

(v) Provide a formula for $P[X \leq 0.5 | X > 0.2]$ – a solution in terms of $f_X(x)$ is expected.

(vi) Find the probability that for both trials of the experiment, the average concentration is less than 0.5, i.e. find $P[(\frac{1}{2}X + \frac{1}{2}Y) < 0.5]$.

Good luck!