CS-512 – Homework 0 (0%)

Solution

A.

1. $2A - B = [-2 \quad -1 \quad 0]'$
2. $|A| = \sqrt{14}$. The Angle of A relative to positive X axis is: $\arccos(\frac{1}{\sqrt{14}})$
3. The unit vector in the direction of A is: $\hat{A} = \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
4. The direction cosines of A are: $(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}})$
5. $A \cdot B = B \cdot A = 4 + 10 + 18 = 32$
6. The angle between A and B is: $\arccos(\frac{A \cdot B}{|A||B|}) = \arccos(\frac{32}{\sqrt{14 \cdot 77}})$
7. $[x \quad y \quad z] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0 \quad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$
8. $A \times B = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix} \qquad B \times A = -A \times B = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$
9. A vector perpendicular to A and B is: $A \times B = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix}$.
10. $aA + bB = cC = 0 \quad \Rightarrow \quad 3A - B - C = 0$
11. $A^T B = 32$, $AB^T = \begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}$

B.

1. $2A - B = \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & 12 & -3 \end{bmatrix}$
2. $AB = \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix} \quad BA = \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -5 & 15 & 2 \end{bmatrix}$
3. $(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix} \qquad B^T A^T = (AB)^T$
4. $|A| = 55$. Because of A–10 we get: $|C| = 0$.
5. The matrix in which the row vectors form an orthogonal set is the matrix B.
6. $A^{-1} = \frac{1}{55} \begin{bmatrix} -13 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix}$. Because of B–5 we get: $B^{-1} = \begin{bmatrix} 1/6 & 2/21 & 3/14 \\ 2/6 & 1/21 & -2/14 \\ 1/6 & -4/21 & 1/14 \end{bmatrix}$

C.

1. $\lambda_1 = -1$, $\lambda_2 = 4$, $e_1 = \begin{bmatrix} -0.707 \\ 0.707 \end{bmatrix}$, $e_2 = \begin{bmatrix} -0.555 \\ -0.832 \end{bmatrix}$
2. $V^{-1}AV = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$
3. $e_1 \cdot e_2 = -0.196$
4. $e_1 \cdot e_2 = 0$
5. Since B is a symmetric real matrix, its eigenvectors are orthogonal.

D.

1. $f'(x) = 2x, \quad f''(x) = 2$
2. $\frac{\partial g}{\partial x} = 2x, \quad \frac{\partial g}{\partial y} = 2y$
3. $\nabla g(x, y) = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$
4. $p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$