CS512 - Assignment 4

Mohammadreza Asherloo
Department of Mechanical, Materials and Aerospace Engineering
Illinois Institute of Technology

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Question 1.a:

Forward projection means projecting world points to image points by using projection matrix. In other words, finding the image 2D representation of a 3D point in the world.
Calibration means finding the projection matrix and camera parameters based on the corresponding points in image (pixels) and world (meters)
Reconstruction means projecting image points to world points by using the projection matrix found from calibration step.

Hardest task is the calibration and the easiest one is the forward projection.

Question 1.b:

We need a set of coordinates in real world from a object with known size and a set of coordinates form the same points of the object in the image.

Question 1.c:

For non-planar calibration we use the 3D points from object in the real world and 2D points of the object's image to find the projection matrix M and after that we will find the parameters from this projection matrix m.

Question 1.d:

$$\mathrm{M} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix},\ P_i = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \longrightarrow \mathrm{p}_i = \begin{bmatrix} 18 \\ 14 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{18}{7} \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{18}{7} \\ 2 \end{bmatrix}$$

Question 1.e:

$$\text{Ax} = 0 \Longrightarrow \begin{bmatrix} \begin{bmatrix}1 & 2 & 3 & 1\end{bmatrix} m_1 & 0 & \begin{bmatrix}-100 & -200 & -300 & -100\end{bmatrix} m_3 \\ 0 & \begin{bmatrix}1 & 2 & 3 & 1\end{bmatrix} m_2 & \begin{bmatrix}-200 & -400 & -600 & -200\end{bmatrix} m_3 \end{bmatrix} = 03$$

Question 1.f:

$Weneedminimumof6pointstosolvethismatrix.Foreachpointthereare2equationsconsistof12unknowns(p$

Question 1.g:

$Wewillfindawhichisaresultofourcalculationsbutitisnotauniquesolution.SowehavetheA(x) = 0whichisnottheuniqueanswersowewillfindtheA(\rho$x) = 0 which will help to find the unique answer.

Question 1.h:

We will compute the error of predicted distance resulted from our points and estimated projection matrix by comparing it with the known distance.

Question 1.i:

For planar calibration we will find a 2D homography between the image and the calibration plane. By using this 2D homography we will estimate intrinsic parameters and compute the external parameters from these intrinsic parameters.

Question 1.j:

2D homography is a 3 by 3 matrix but the projection matrix is a 3 by 4 matrix. We will assume $\{p_i\}_z = 0$ to ensure that we are dealing with a 2D homography.

Question 2.a:

$$\text{Ax} = 0 \Longrightarrow \begin{bmatrix} \begin{bmatrix}3 & 4 & 5 & 1\end{bmatrix} m_1 & 0 & \begin{bmatrix}-3 & -4 & -5 & -1\end{bmatrix} m_3 \\ 0 & \begin{bmatrix}3 & 4 & 5 & 1\end{bmatrix} m_2 & \begin{bmatrix}-6 & -8 & -10 & -2\end{bmatrix} m_3 \end{bmatrix} = 0$$

Question 2.b:

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \longrightarrow \mathrm{a}_1 = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix},\ a_2 = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix},\ a_3 = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$$

$u_0 = |\rho|^2 a_1.a_3 \longrightarrow \frac{1}{50} \times 26 = 0.52$
$v_0 = |\rho|^2 a_2.a_3 \longrightarrow \frac{1}{50} \times 38 = 0.76$

Question 2.c:

$$\mathrm{M} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix},\ \mathrm{p} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},\ \mathrm{P} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix}$$

$$\hat{p} = M \times P = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 30 \\ 43 \\ 56 \end{bmatrix} = \begin{bmatrix} \frac{30}{56} \\ \frac{43}{56} \\ 1 \end{bmatrix}$$

$$error = |\hat{p} - p| = \sqrt{(\tfrac{30}{56} - 1)^2 + (\tfrac{43}{56} - 2)^2} = \sqrt{(\tfrac{26}{56})^2 + (\tfrac{13}{56})^2}$$

Question 2.d:

$$\mathrm{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ \mathrm{Q} = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow R^* = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, T^* = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$R = (R^*)^T = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathrm{T} = -(\mathrm{R}^*)^T T^* = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$$

Question 2.e:

$$\mathrm{p} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},\ \mathrm{P} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} \longrightarrow Ax = 0 \Longrightarrow \begin{bmatrix} \begin{bmatrix} 3 & 4 & 1 \end{bmatrix} h_1 & 0 & \begin{bmatrix} -3 & -4 & -1 \end{bmatrix} h_3 \\ 0 & \begin{bmatrix} 3 & 4 & 1 \end{bmatrix} h_2 & \begin{bmatrix} -6 & -8 & -2 \end{bmatrix} h_3 \end{bmatrix} = 0$$

Question 3.a:

In sparse approach, we will try to find the similar features in images from different views. This will give us the advantage of handling large disparities but we will lose the ability to reconstruct all the points in the object. On the other hand, in dense approach we can use the correlation or SSD to find the similar points in images from different views which gives us the advantage of having more points but we will lose the ability to handle the big differences in the views.

Question 3.b:

In cross correlation we will multiply the corresponding elements but in SSD we will take the difference between them. If we set the search space to the entire image, probably we will find multiple candidates for point correspondence (for example, confusing a corner with another corner).

Question 3.c:

$d = x_r - x_l = 103 - 100 = 3$
$z = f\frac{T}{d} = 10 \times \frac{100}{3} = 333$

Question 3.d:

Ambiguity problem is confusing two points' correspondence incorrectly, therefore, the calculated the point in real world will be incorrect. In this case, mistake in matching one point will result in mistakenly matching other point so we will have two incorrect points.

Question 3.e:

$R = R_l^T R_r$
$T = R_l^T (T_r - T_l)$

Question 4.a:

$d = 30$
$z = f\frac{T}{d} = 10 \times \frac{20}{30} = 6.67$

Question 4.b:

$$[A]_x = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

Question 4.c:

$$\text{F} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix},\ P_l = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ P_r = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \longrightarrow \text{P}_r^T F P_l = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = 68$$

Question 4.d:

$$P_l = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ P_r = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 2f_{11} & 3f_{12} & f_{13} & 4f_{21} & 6f_{22} & 2f_{23} & 2f_{31} & 3f_{32} & f_{33} \\ & & & & . & & & & \\ & & & & . & & & & \\ & & & & . & & & & \\ & & & & . & & & & \end{bmatrix} = 0$$