A.

1. $2A - B = 2\begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}-2\\-1\\0\end{bmatrix}$

2. $\|A\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$\theta = \cos^{-1}\left[\dfrac{A \cdot X}{\|A\|\,\|X\|}\right] = \cos^{-1}\left[\dfrac{1}{\sqrt{14} \times 1}\right] = 74^\circ$

3. $\hat{A} = \dfrac{(1\ 2\ 3)}{\sqrt{14}} = \begin{bmatrix}1/\sqrt{14}\\2/\sqrt{14}\\3/\sqrt{14}\end{bmatrix}$

4. $\text{cosine}\ \alpha = \dfrac{1}{\sqrt{14}}$ $\quad \text{cosine}\ \beta = \dfrac{2}{\sqrt{14}}$ $\quad \text{cosine}\ \delta = 3/\sqrt{14}$

5. $A \cdot B = 4 + 10 + 18 = 32$

$B \cdot A = 4 + 10 + 18 = 32$

6. $\theta_{AB} = \cos^{-1}\left[\dfrac{A \cdot B}{\|A\|\ \|B\|}\right] = \cos^{-1}\left[\dfrac{4 + 10 + 18}{\sqrt{14} \times \sqrt{77}}\right] = \cos^{-1}\left[\sqrt{\dfrac{512}{539}}\right]$

$= \cos^{-1}[0.9746] = 12.94^\circ$

7. $U = (U_1, U_2, U_3)$ $\quad A \cdot U = 0$

$\Rightarrow U_1 + 2U_2 + 3U_3 = 0, \; U_1 = 1, \; U_2 = 1$

$\Rightarrow U_3 = -1 \Rightarrow U = \begin{bmatrix}1\\1\\-1\end{bmatrix}$

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9. $A \cdot U = 0$ $B \cdot U = 0$ $\Rightarrow$ $\begin{cases} U_1 + 2U_2 + 3U_3 = 0 \\ 4U_1 + 5U_2 + 6U_3 = 0 \end{cases}$

$\boxed{U_1 = 1} \Rightarrow \begin{cases} 2U_2 + 3U_3 = -1 \\ 5U_2 + 6U_3 = -4 \end{cases} \Rightarrow -U_2 = 2$

$\Rightarrow \boxed{U_2 = -1}$

$\Rightarrow \boxed{U_3 = 1/3}$

$U = (1, -1, 1/3)$

11. $A^T B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 32$

$B^T A = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 32$

B//

1. $2A - B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & -4 & 6 \\ 0 & -10 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -4 \\ 3 & -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 5 \\ 6 & -5 & 10 \\ -3 & -8 & -2 \end{bmatrix}$

2. $AB = \begin{bmatrix} 1+4+9 & 2+2-6 & 1-8+3 \\ 4-4+9 & 8-2-6 & 4+8+3 \\ 0+10+(-3) & 0+5+2 & 0-20-1 \end{bmatrix}$

$= \begin{bmatrix} 14 & -2 & -4 \\ 9 & 0 & 15 \\ 7 & 7 & -21 \end{bmatrix}$

$BA = \begin{bmatrix} 1+8+0 & 2-4+5 & 3+6-1 \\ 2+4+0 & 4-2-20 & 6+3+4 \\ 3-4+0 & 6+4+5 & 9-6-1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 3 & 8 \\ 6 & -18 & 13 \\ -1 & 15 & 2 \end{bmatrix}$

3. $(AB)^T = \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$

3. $B^T A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 1 & -4 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 0 \\ 2 & -2 & 5 \\ 3 & 3 & -1 \end{bmatrix}$

$$\begin{bmatrix} 1+4+9 & 4-4+9 & 0+10-3 \\ 4+2-6 & 8-2-6 & 0+5+2 \\ 1-8+3 & 4+8+3 & 0-20-1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 9 & 7 \\ -2 & 0 & 7 \\ -4 & 15 & -21 \end{bmatrix}$$

4. $|A| = \overset{-13}{\cancel{6}} - (-8) + 60 = \cancel{}\ 55$ (+1)

$|C| = +9 - 36 + 27 = 0$

5. B: $\left.\begin{array}{l} V_1 \cdot V_2 = 2+2-4=0 \\ V_2 \cdot V_3 = 6-2-4=0 \\ V_1 \cdot V_3 = 3-4+1=0 \end{array}\right\} \Rightarrow$ mutually perpendicular

6. $A^{-1} = \dfrac{1}{\det(A)} \begin{bmatrix} (2-13) & -(-2-15) & (6+6) \\ -(-4+0) & (-1+0) & -(3-12) \\ (20-0) & -(5-0) & (-2-8) \end{bmatrix}$

$$= \frac{1}{55} \begin{bmatrix} -11 & 17 & 12 \\ 4 & -1 & 9 \\ 20 & -5 & -10 \end{bmatrix} = \begin{bmatrix} -1/5 & 17/55 & 12/55 \\ 4/55 & -1/55 & 9/55 \\ 20/55 & -1/11 & -2/11 \end{bmatrix}$$

~~B doesn't have inverse because |B| =~~

C. 1. eigenvalues:

$$A=\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix} \longrightarrow \det\left(\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}\right)=0$$

$$\Rightarrow \det\left(\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}-\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}\right)=0$$

$$\Rightarrow \det\left(\begin{bmatrix}\lambda-1 & -2\\ -3 & \lambda-2\end{bmatrix}\right)=0$$

$$\Rightarrow (\lambda-1)(\lambda-2)-(-2)(-3)=0$$

$$\Rightarrow \lambda^2-3\lambda+2-6=0$$

$$\Rightarrow \lambda^2-3\lambda-4=0$$

$$\Rightarrow (\lambda-4)(\lambda+1)=0 \Rightarrow \begin{Bmatrix}\lambda=-1\\ \lambda=4\end{Bmatrix}$$

eigenvectors:

$$A\vec{v}=\lambda\vec{v} \Rightarrow \lambda\vec{v}-A\vec{v}=0 \Rightarrow \lambda I\vec{v}-A\vec{v}=0$$

$$\Rightarrow \vec{v}(\lambda I-A)=0$$

$$\lambda=4 \longrightarrow \left(\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}-\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}\right)\vec{v}=0$$

$$\Rightarrow \begin{bmatrix}3 & -2\\ -3 & 2\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=0 \Rightarrow \begin{bmatrix}1 & -2/3\\ 0 & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=0$$

$$\Rightarrow v_1-\tfrac{2}{3}v_2=0 \Rightarrow \boxed{v_1=\tfrac{2}{3}v_2}$$

$$\lambda=-1 \longrightarrow \left(\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}-\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}\right)\begin{bmatrix}v_1\\ v_2\end{bmatrix}=0 \Rightarrow \begin{bmatrix}-2 & -2\\ -3 & -3\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}-2 & -2\\ 0 & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=0 \Rightarrow \begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=0 \Rightarrow \boxed{v_1=-v_2}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

C/ 2. $V = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \Rightarrow V^{-1} = \dfrac{1}{5}\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$

$$\Rightarrow V^{-1} = \begin{bmatrix} 3/5 & -2/5 \\ 1/5 & 1/5 \end{bmatrix}$$

$$\Rightarrow V^{-1}A = \begin{bmatrix} -\frac{3}{5} & 4/5 \\ 4/5 & 4/5 \end{bmatrix}$$

$$V^{-1}AV = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 4/5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5/5 & 0 \\ 0 & 20/5 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 4 \end{bmatrix}$$

3. $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2 - 3 = -2$