

Secure Data Analytics

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The Motivation

- Cloud databases: Google Cloud SQL, Microsoft SQL Azure, Amazon SimpleDB.



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- Service providers (SP) answer queries from different clients.



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- Service providers (SP) answer queries from different clients.
- Data owner might not want to reveal data values to SP; clients might not want SP to learn their queries and/or the query results.



Hakan Hacigumus, Balakrishna R. Iyer, Chen Li, Sharad Mehrotra: Executing SQL over encrypted data in the database-service-provider model. SIGMOD 2002

Introduction and Motivation



cloud server

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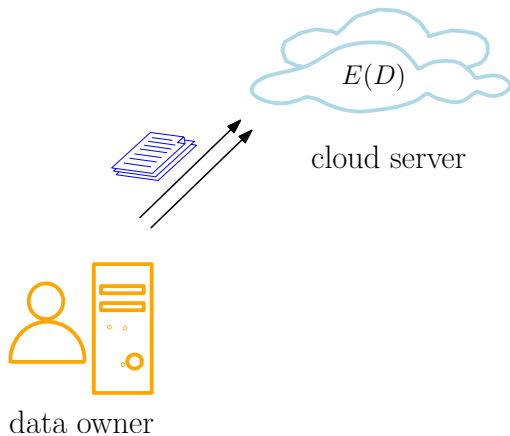


cloud server

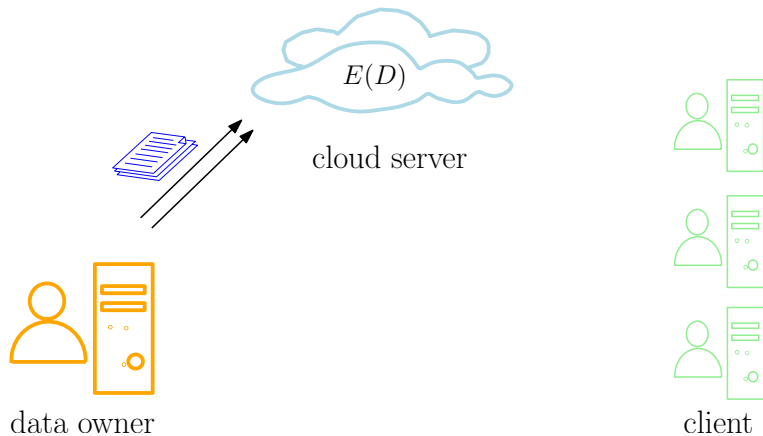


data owner

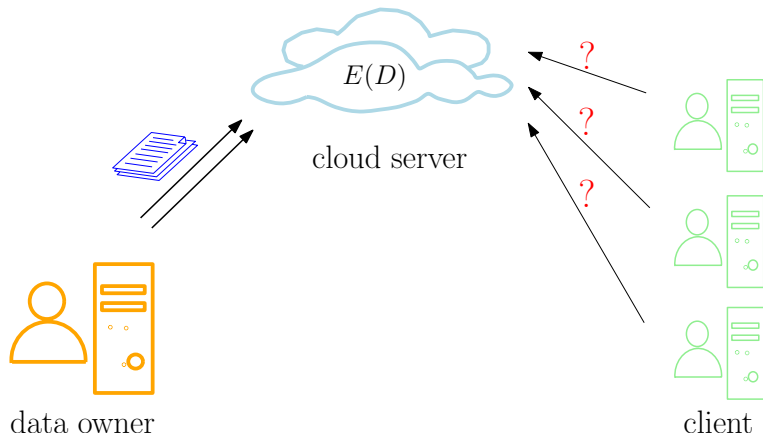
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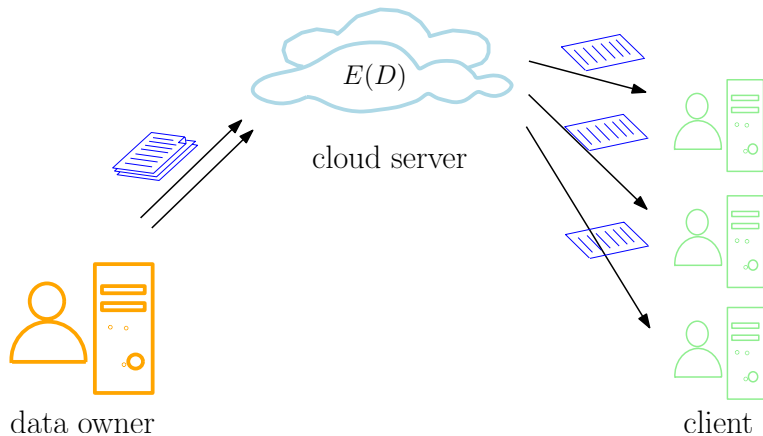
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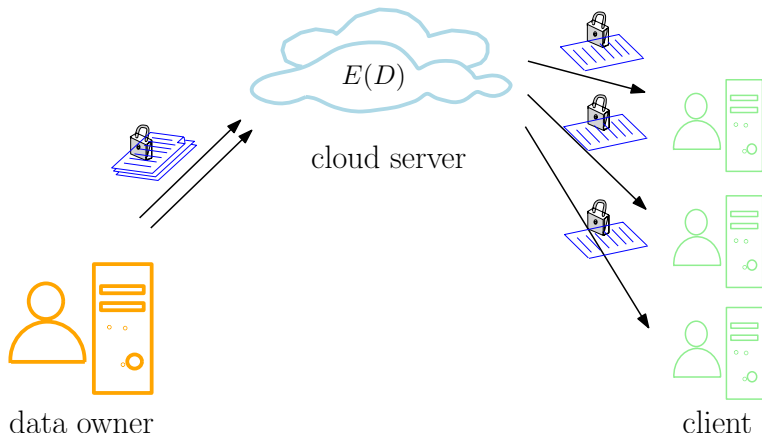
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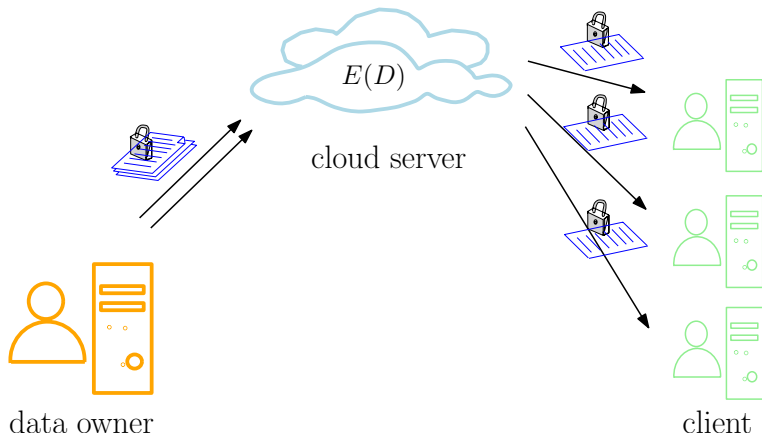


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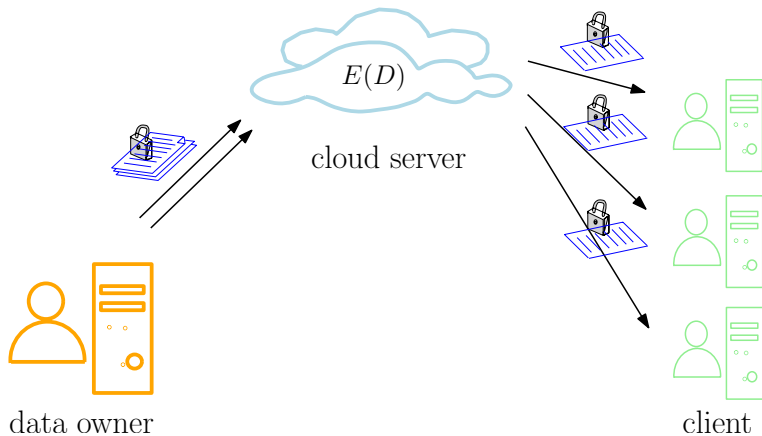
Introduction and Motivation

- Secure Query Processing



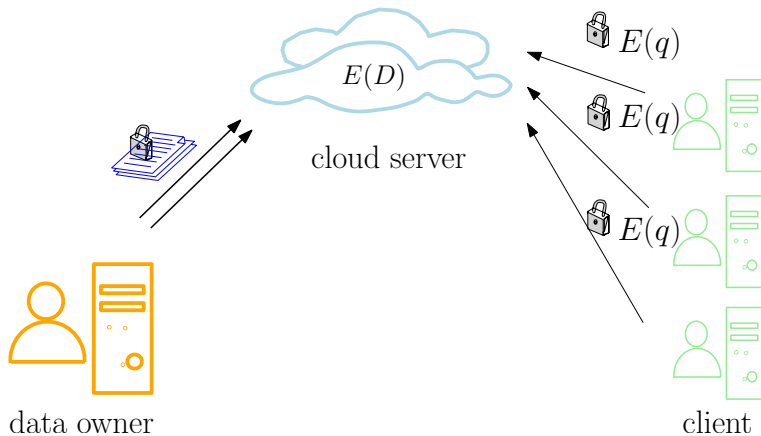
Introduction and Motivation

- Secure Query Processing
 - Secure Nearest Neighbor (SNN)



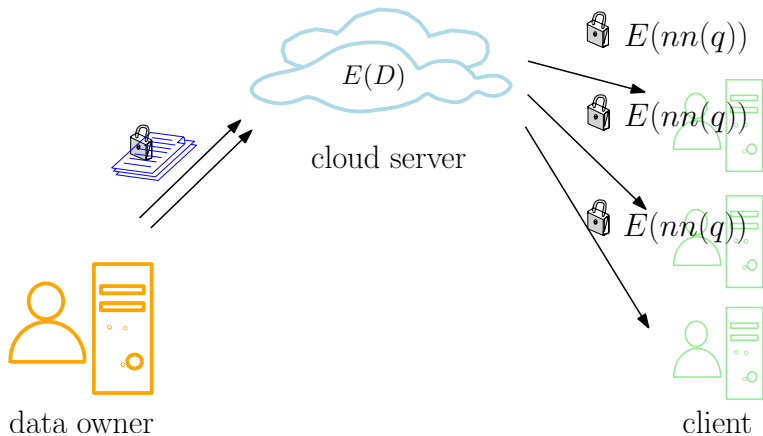
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Related Work

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- Hu et al. [4] and Wong et al. [5] deal with the SNN problem; the solutions thus proposed, however, are insecure and can be attacked efficiently

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 - Adversary model: same as whatever model in which E is secure, e.g., IND-CPA, IND-CCA.

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- Standard security model, such as indistinguishability under chosen plaintext attack (IND-CPA), or indistinguishability under chosen ciphertext attack (IND-CCA).

Insecurity of Existing Methods

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 - Basic idea: construct a “secure” encryption function that preserves the dot product between a query point and a database point.
 - Attack we found: after learning only d query points and their encryptions, a linear system of d equations can be formed to decrypt any encrypted $p \in D$.

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 - Attack we found: In the above process, the server learns if q lies to the left or the right of another point, in each dimension, which leads to a binary search to efficiently recover any encrypted point.

Hardness of the Problem: OPE

- Order-preserving encryption (OPE) is a set of functions $\{\mathcal{E}, \mathcal{E}^{-1}, op\}$, such that:
 - $\mathcal{E}(m) = c$, $\mathcal{E}^{-1}(c) = m$ (here we omit the keys).
 - $op(c_1, c_2) = 1$ if $m_1 < m_2$; $op(c_1, c_2) = -1$ if $m_1 > m_2$.

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Theorem

A truly secure OPE does not exist in standard security models, such as IND-CPA. It also does not exist even in much relaxed security models, such as the indistinguishability under ordered chosen-plaintext attack (IND-OCPA).

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Alexandra Boldyreva, Nathan Chenette, Younho Lee, Adam O'Neill: Order-Preserving Symmetric Encryption. EUROCRYPT 2009
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Hardness of the Problem: SNN gives OPE

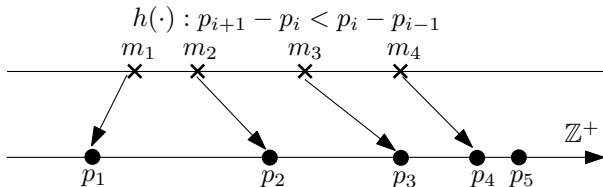
- Given $E(D) = \{E(p_1), \dots, E(p_N)\}$, suppose we have a secure SNN method S such that: $S(E(q), E(D)) \rightarrow E(nn(q, D))$ without the knowledge of E^{-1} .

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- We can construct an OPE, $\{\mathcal{E}, \mathcal{E}^{-1}, op\}$, based on $S(\cdot)$!

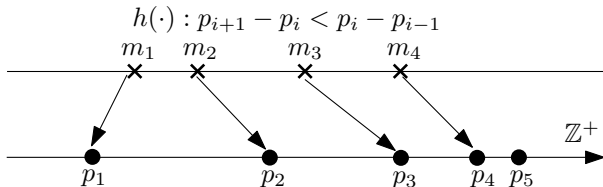
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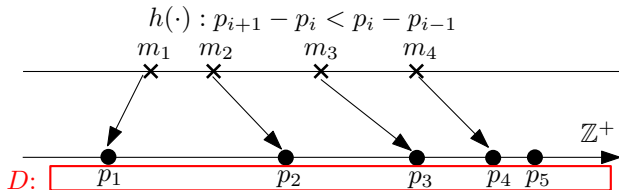
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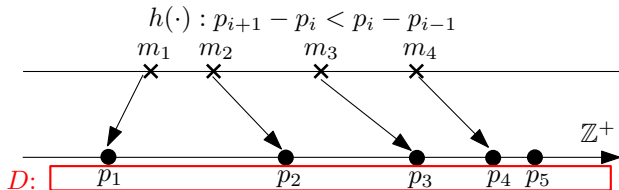


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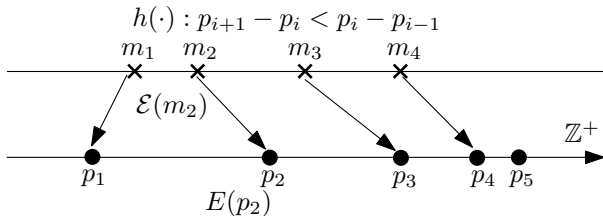
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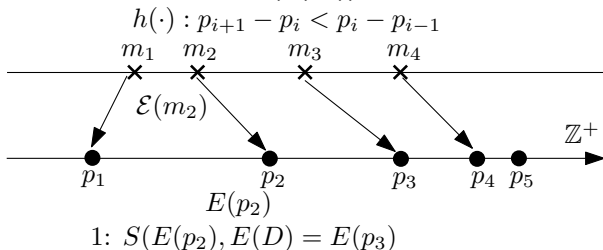
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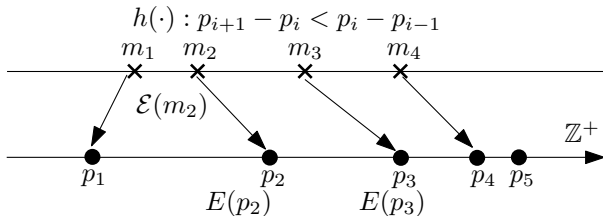
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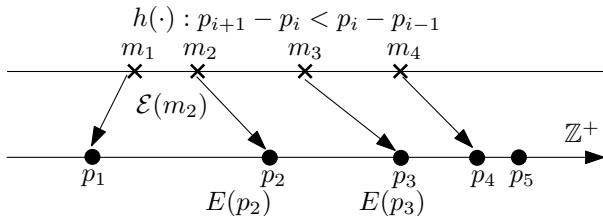
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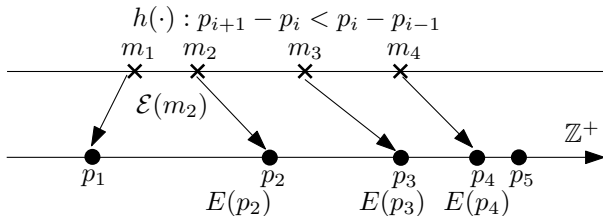
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$$2: S(E(p_3), E(D)) = E(p_4)$$

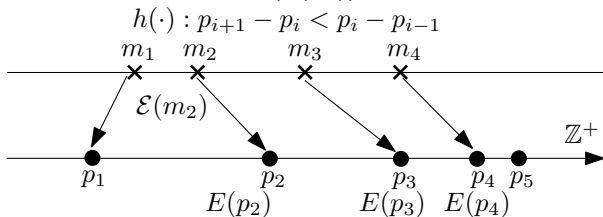
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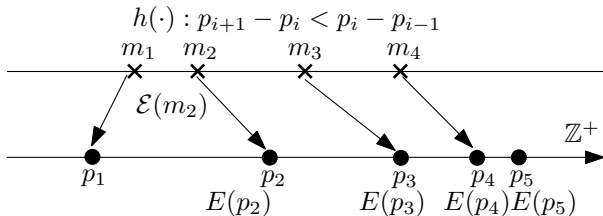
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3: $S(E(p_4), E(D) = E(p_5))$

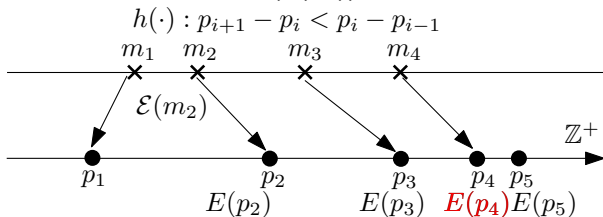
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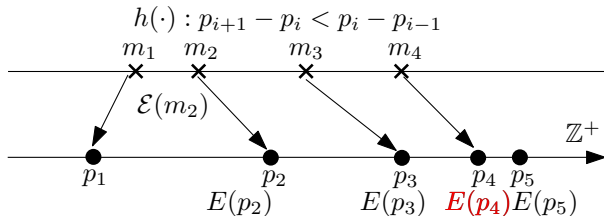
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4: $S(E(p_5), E(D) = E(p_4))$, Repetition FOUND!

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$i = N - (\text{number of steps} - 2)!$

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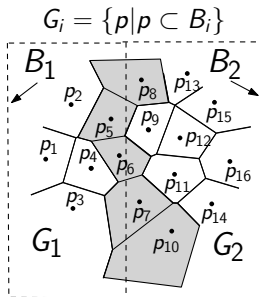
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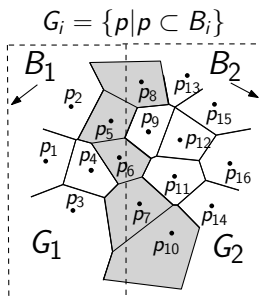
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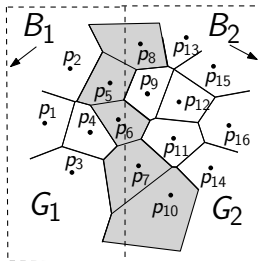
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 - send partition configurations (the boundaries) to clients, client only needs to ask for the encryption of a given partition by partition id (which is figured out locally).

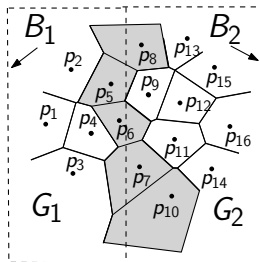
$$G_i = \{p | p \in B_i\}$$



So, Hopeless? NO!

- It only says it is hard to output $E(\text{nn}(q, D))$! What if we relax this restriction and allow something “less precise”?
- Extreme case: just return $E(D)$ and ask client to decrypt and find $\text{nn}(q, D)$. Obviously secure! But expensive!
- The SVD (secure voronoi diagram) method:
 - create partitions based on the voronoi cells of D .
 - $E(D) = \{E(G_1), E(G_2), \dots\}$.
 - send partition configurations (the boundaries) to clients, client only needs to ask for the encryption of a given partition by partition id (which is figured out locally).

$$G_i = \{p | p \in B_i\}$$



Challenge:
 $\min \max(|G_i|)$!

Solution Overview

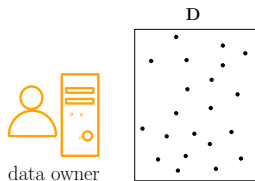
- Secure Voronoi Diagram (SVD):
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 - Query processing at the client

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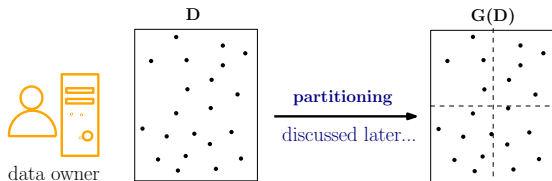
Solution Overview

- Preprocessing at the data owner:



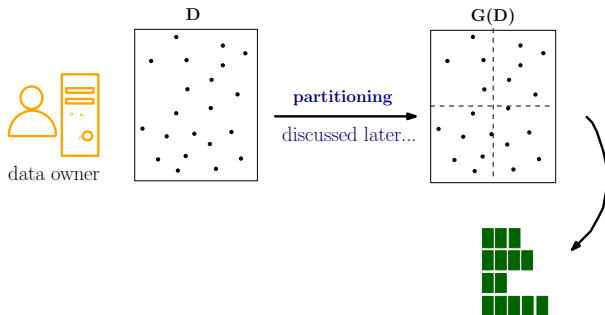
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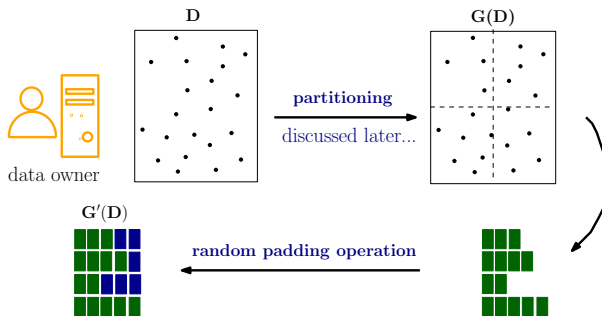
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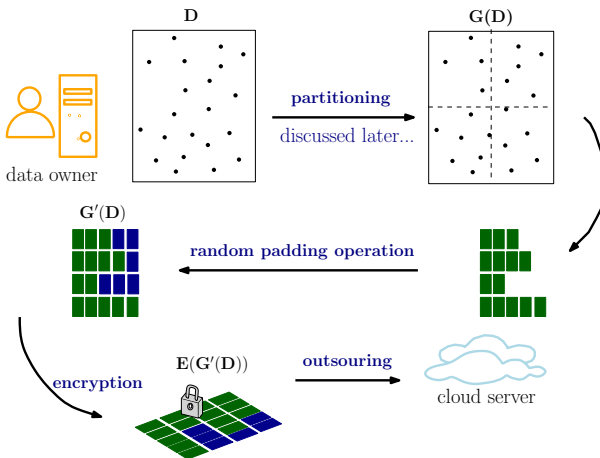
Solution Overview

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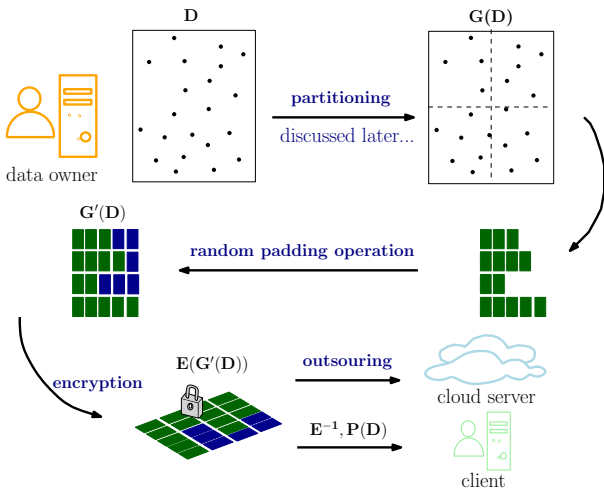
Solution Overview

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Solution Overview

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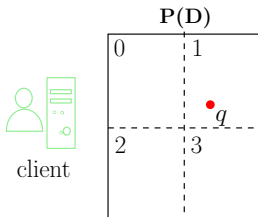
Solution Overview

- Secure Voronoi Diagram (SVD):
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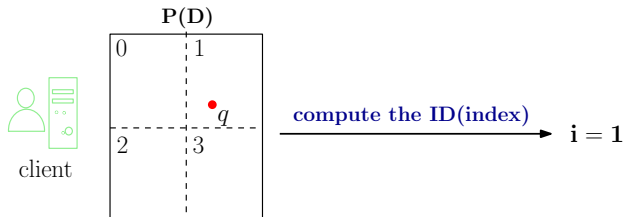
Solution Overview

- Query processing at the client:



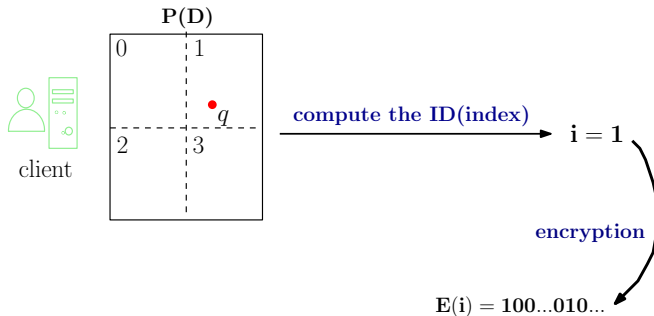
Solution Overview

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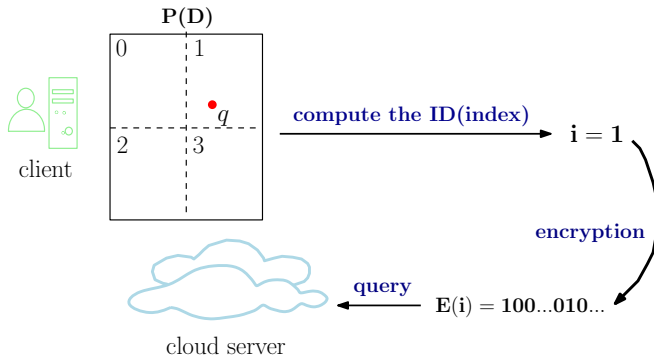
Solution Overview

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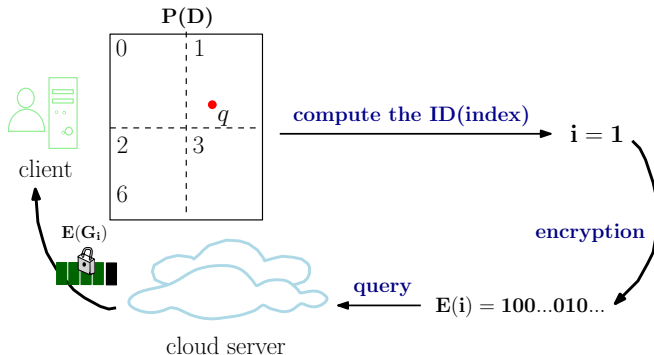
Solution Overview

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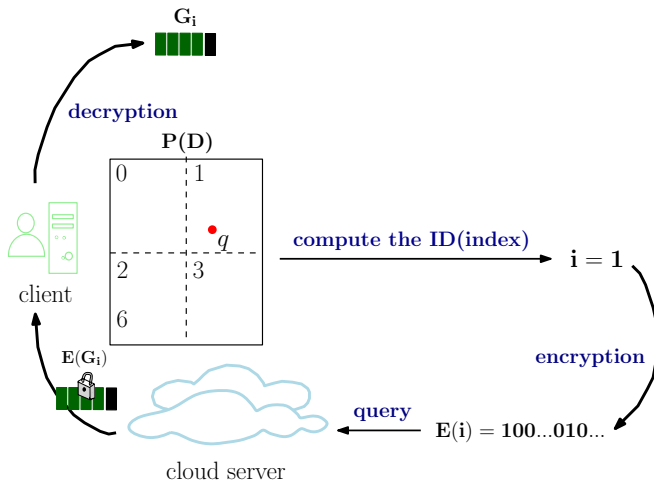
Solution Overview

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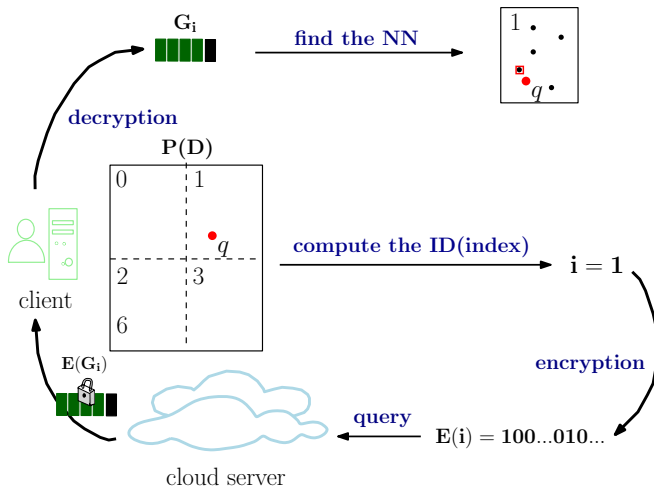
Solution Overview

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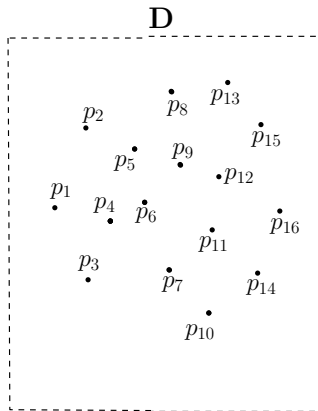
Solution Overview

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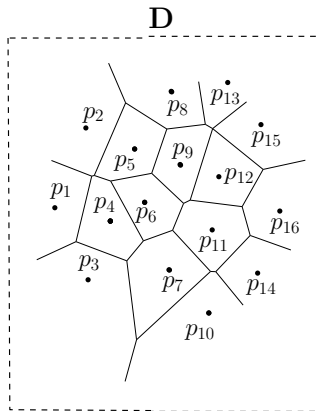


SVD Partitioning Principle

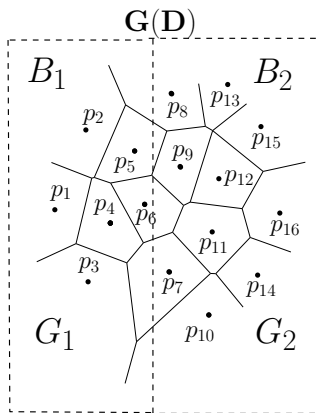
SVD Partitioning Principle



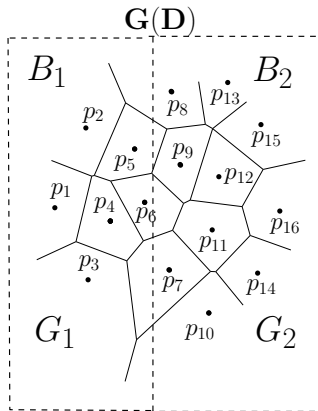
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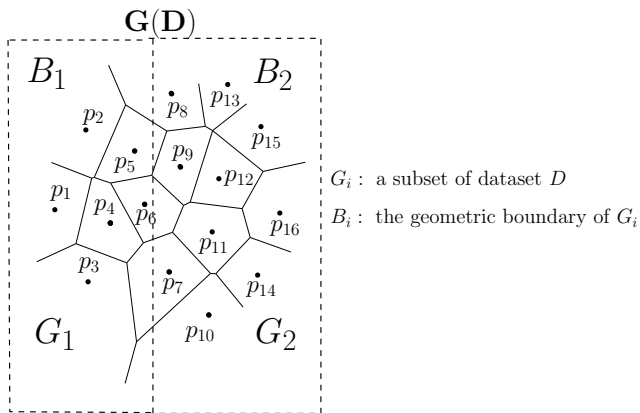
SVD Partitioning Principle



G_i : a subset of dataset D

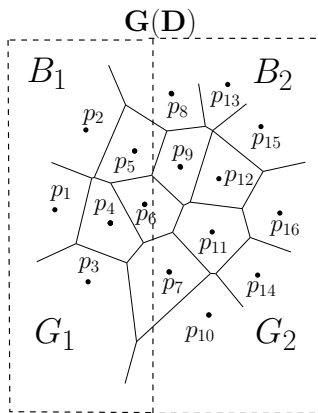
B_i : the geometric boundary of G_i

SVD Partitioning Principle



- 1 B_i is an axis-parallel d -dimensional box and $B_i \cap B_j = \emptyset$ for any $i \neq j$

SVD Partitioning Principle

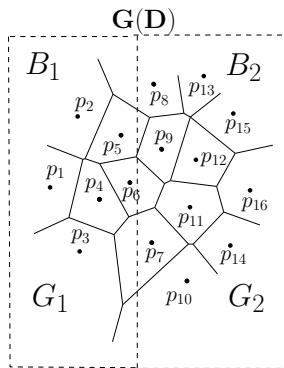


G_i : a subset of dataset D

B_i : the geometric boundary of G_i

- ① B_i is an axis-parallel d -dimensional box and $B_i \cap B_j = \emptyset$ for any $i \neq j$
- ② $G_i = \{p_j | vc_j \text{ is contained or intersected by } B_i\}$

SVD Partitioning Principle



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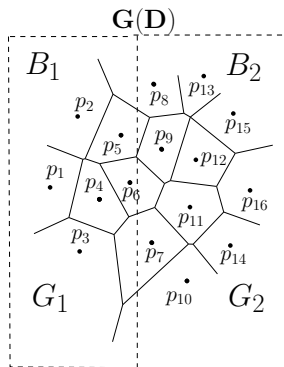
B_i : the geometric boundary of G_i

$$G_1 = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_{10}\}$$

$$G_2 = \{p_5, p_6, p_7, p_8, p_{10}, p_9, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$$

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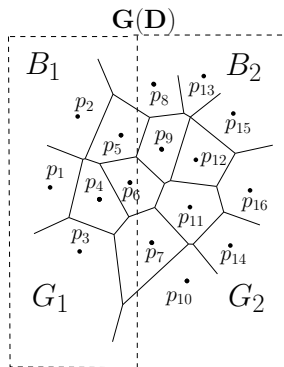
$|G_i|$: size of G_i

$$G_1 = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_{10}\}$$

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SVD Partitioning Principle



G_i : a subset of dataset D

B_i : the geometric boundary of G_i

$|G_i|$: size of G_i

$$|G_1| = 9$$

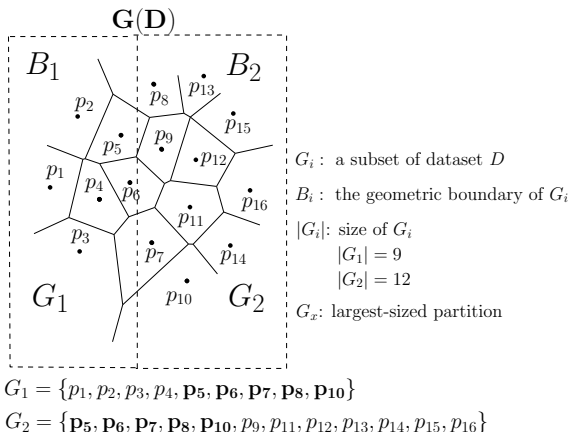
$$|G_2| = 12$$

$$G_1 = \{p_1, p_2, p_3, p_4, \mathbf{p_5}, \mathbf{p_6}, \mathbf{p_7}, \mathbf{p_8}, \mathbf{p_{10}}\}$$

$$G_2 = \{\mathbf{p_5}, \mathbf{p_6}, \mathbf{p_7}, \mathbf{p_8}, \mathbf{p_{10}}, p_9, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\}$$

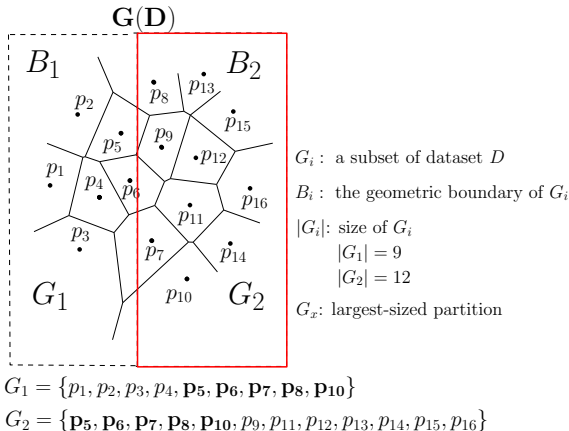
- ① B_i is an axis-parallel d -dimensional box and $B_i \cap B_j = \emptyset$ for any $i \neq j$
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SVD Partitioning Principle



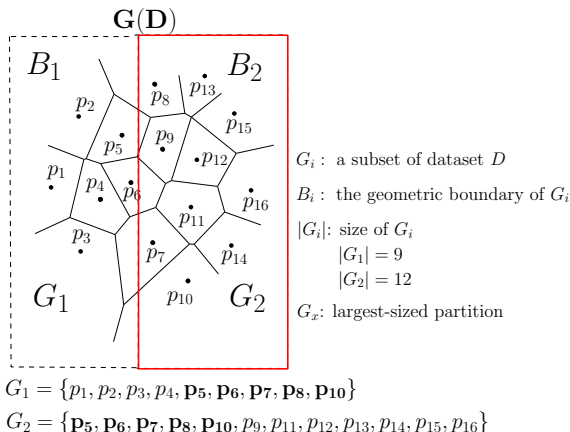
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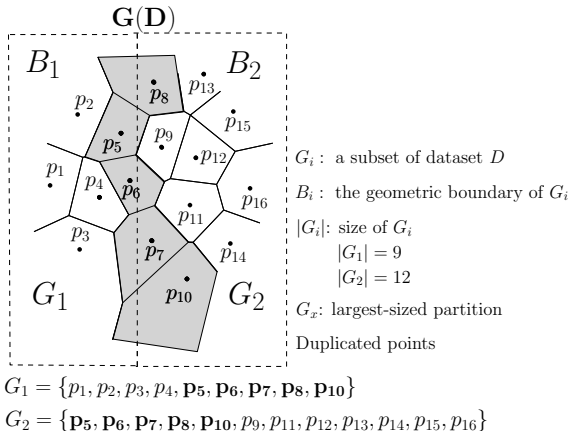
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SVD Partitioning

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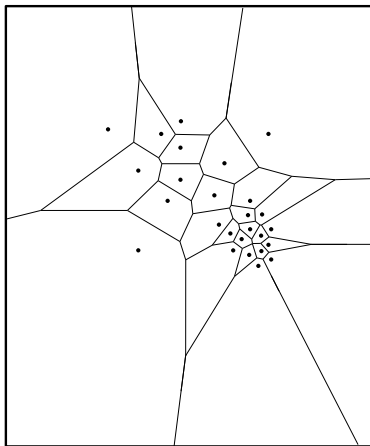
- Square Grid (SG)
- Minimum Space Grid (MinSG)
- Minimum Maximum Partition(MinMax)

SVD Partitioning

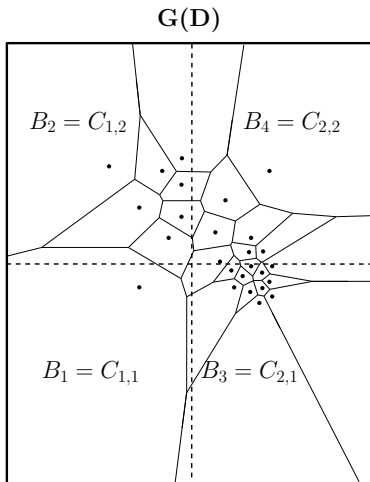
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Square Grid (SG)

D



Square Grid (SG)



Square Grid (SG)

- Merits:

- Demerits:

Square Grid (SG)

- Merits:
 - simple
 - minimum storage cost at client
- Demerits:

Square Grid (SG)

- Merits:

- simple
- minimum storage cost at client

- Demerits:

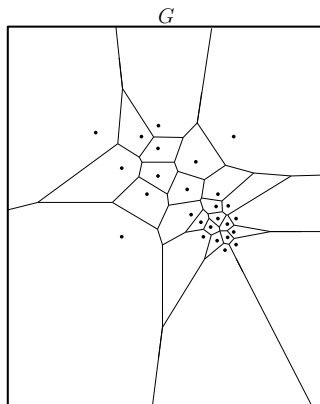
- high storage and communication overheads, as well as expensive encryption cost because of highly unbalanced partitions when the data distribution is skewed

SVD Partitioning

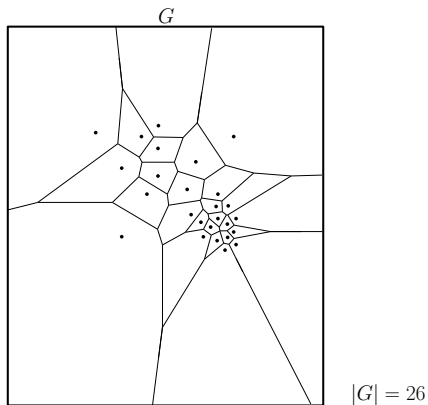
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Minimum Space Grid (MinSG)

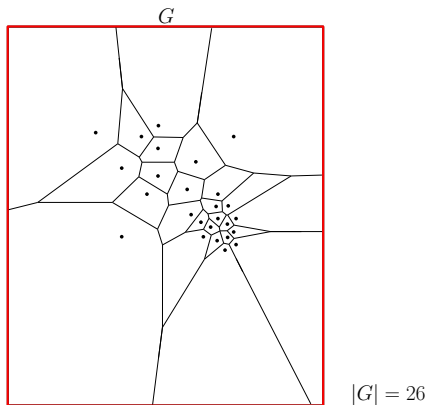
Minimum Space Grid (MinSG)



Minimum Space Grid (MinSG)

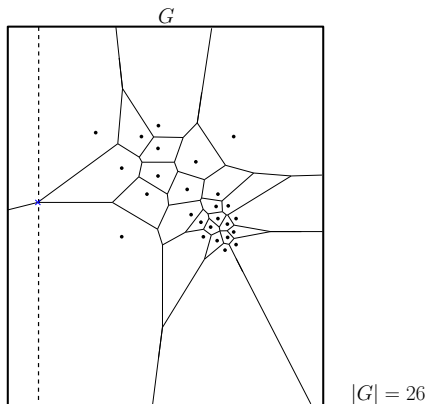


Minimum Space Grid (MinSG)



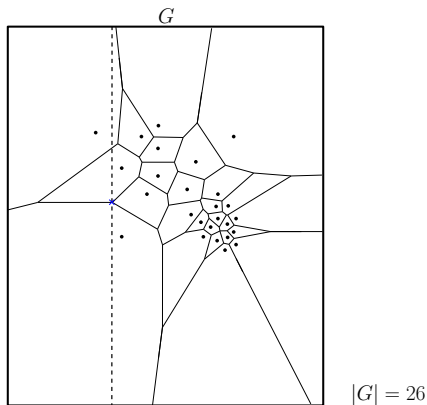
- A greedy algorithm: always split the maximum partition G_x into smaller partitions

Minimum Space Grid (MinSG)



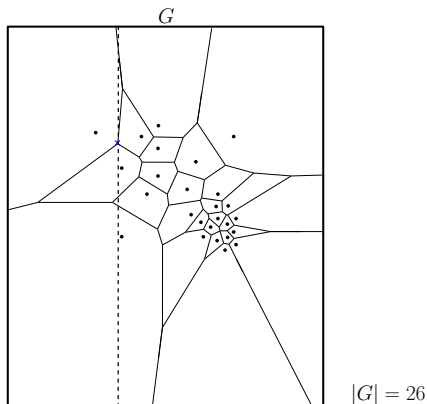
- A greedy algorithm: always split the maximum partition G_x into smaller partitions
- use a line going through the entire space and intersected with the voronoi vertex in B_x

Minimum Space Grid (MinSG)



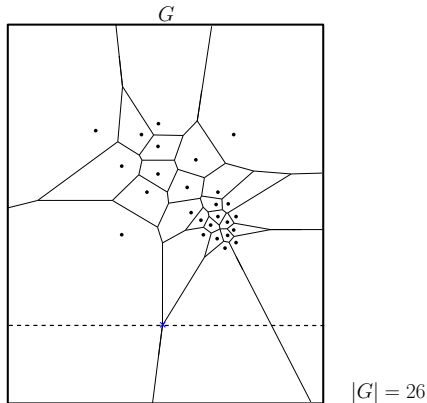
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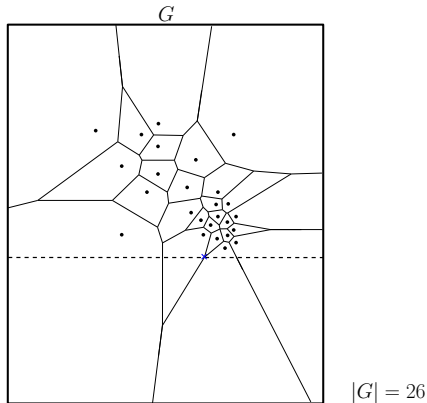
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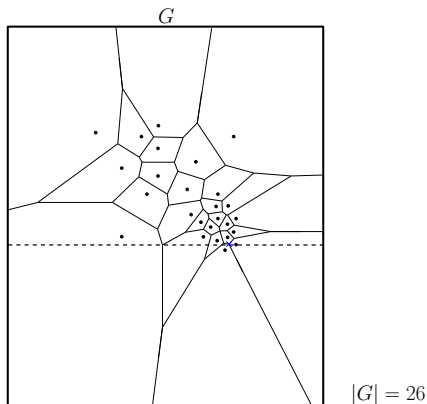
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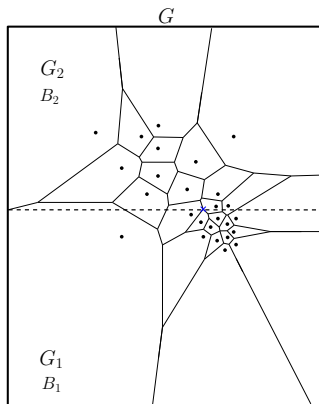
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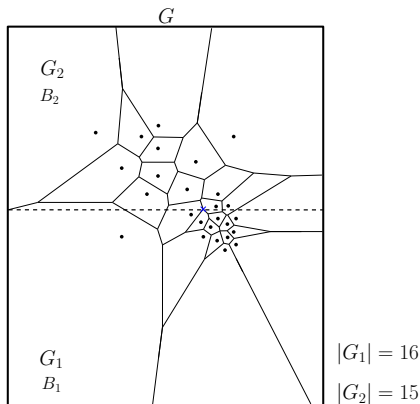
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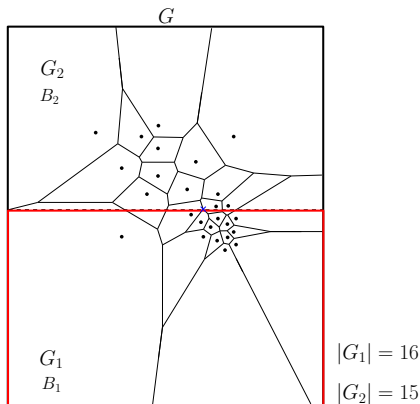
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Minimum Space Grid (MinSG)



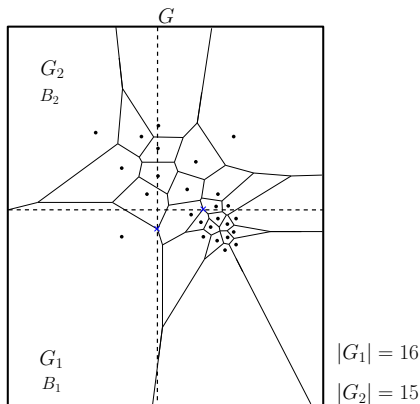
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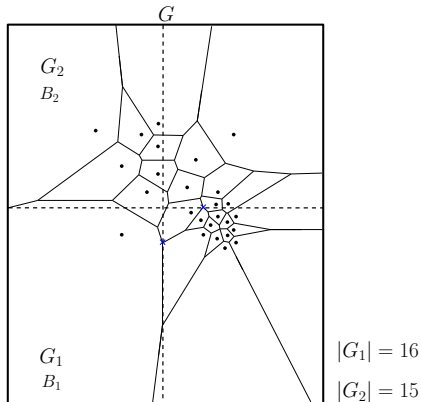
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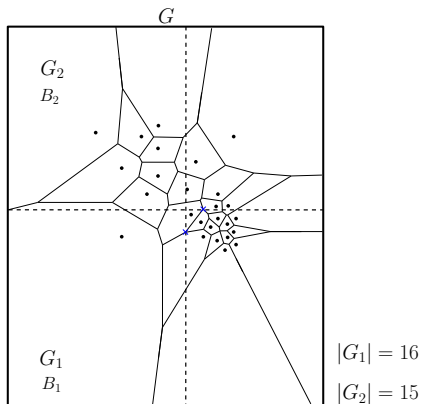
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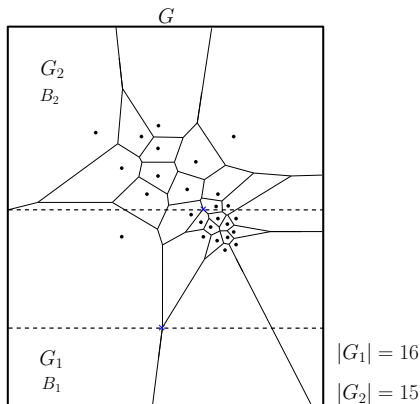
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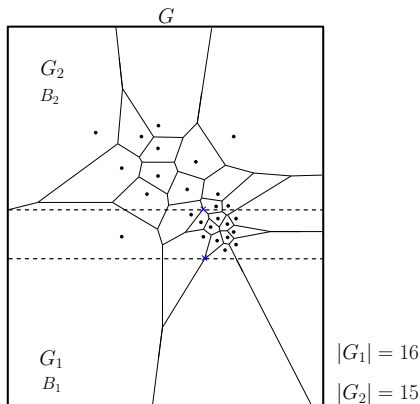
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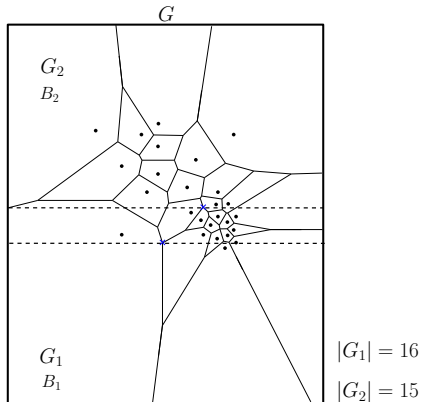
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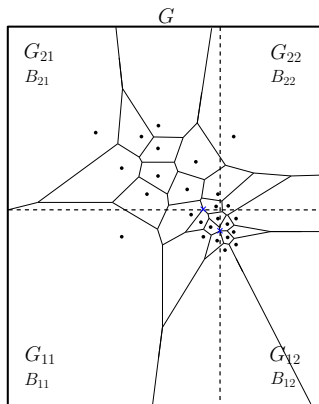
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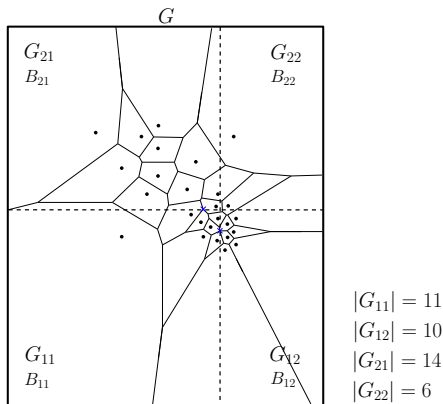
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Minimum Space Grid (MinSG)

- Merits:

- Demerits:

Minimum Space Grid (MinSG)

- Merits:
 - relatively balanced partitions: low storage and communication overheads, as well as cheap encryption cost
- Demerits:

Minimum Space Grid (MinSG)

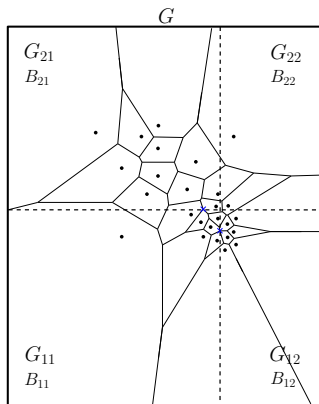
- Merits:

- relatively balanced partitions: low storage and communication overheads, as well as cheap encryption cost

- Demerits:

- complicated partitioning process
- not most balanced: small-sized partitions introduced by some unnecessary splitting

Minimum Space Grid (MinSG)



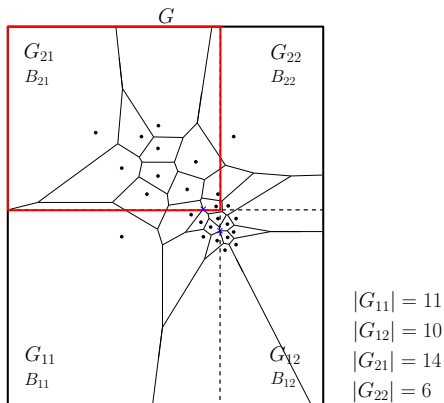
$$|G_{11}| = 11$$

$$|G_{12}| = 10$$

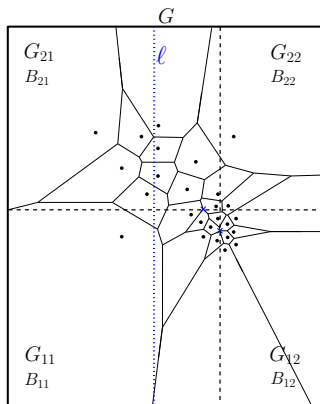
$$|G_{21}| = 14$$

$$|G_{22}| = 6$$

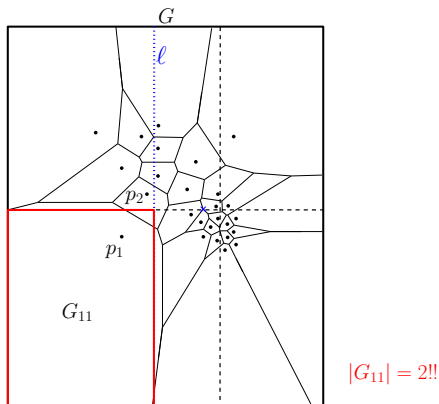
Minimum Space Grid (MinSG)



Minimum Space Grid (MinSG)



Minimum Space Grid (MinSG)



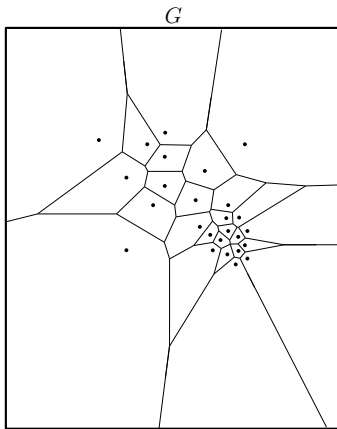
- We need a method that produce more balanced partitions!!

SVD Partitioning

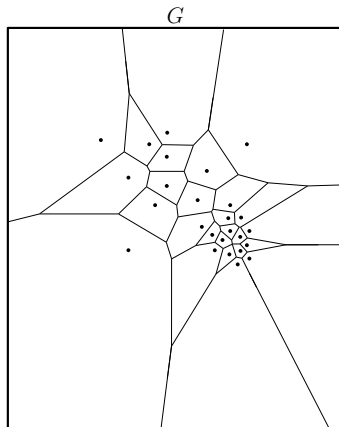
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Minimum Maximum Partition (MinMax)

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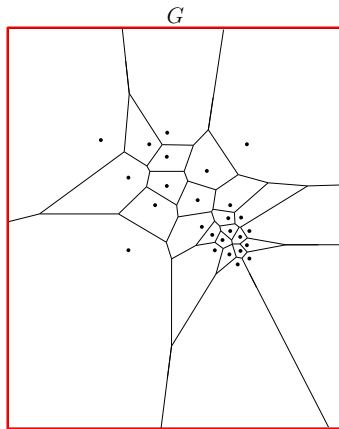


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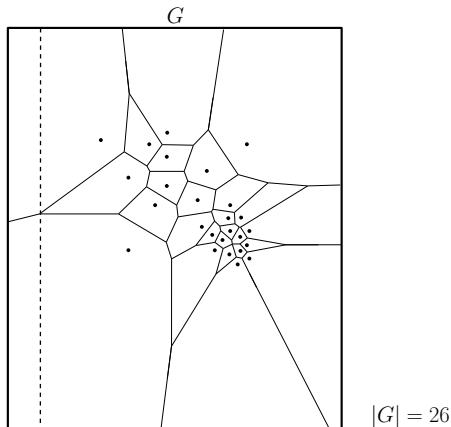
- similar to MinSG in most part

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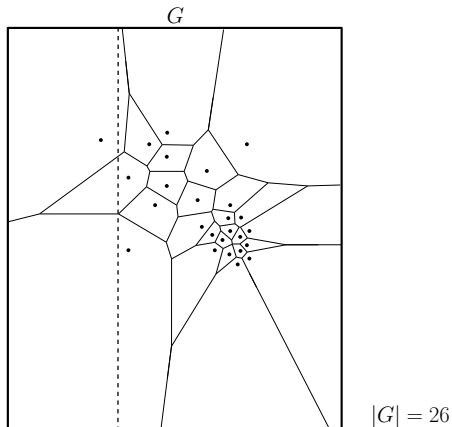
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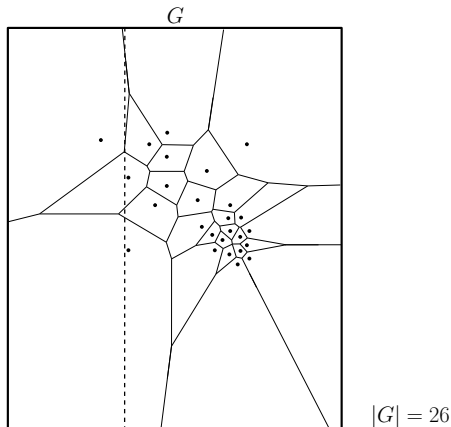
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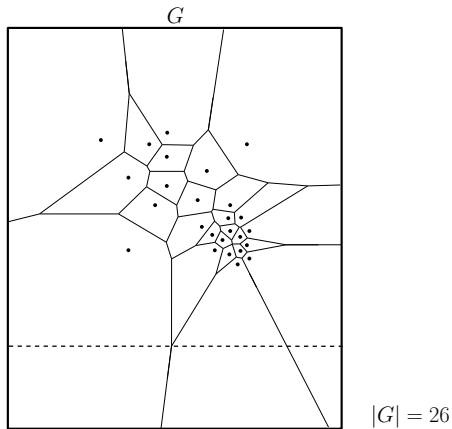
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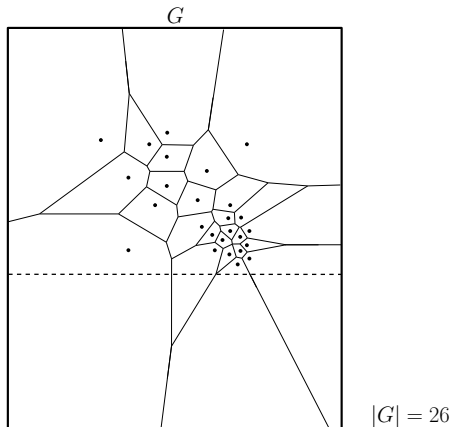
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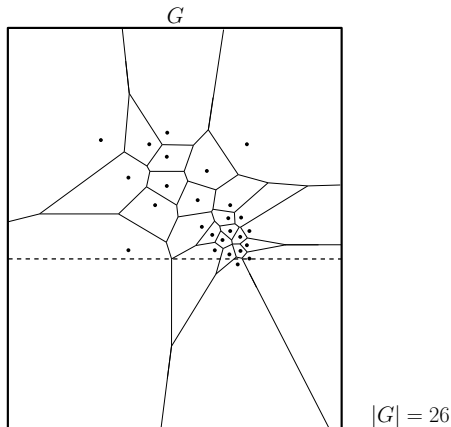
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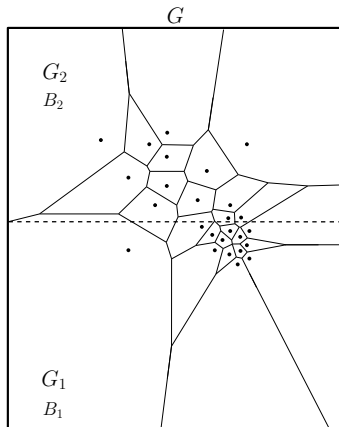
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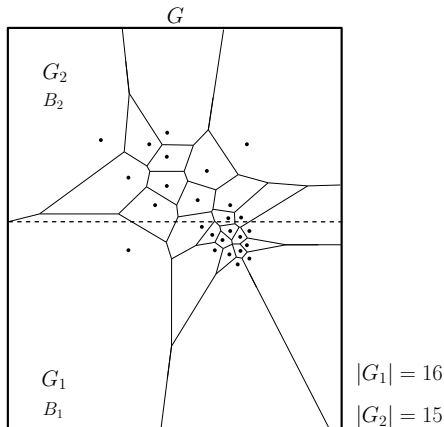
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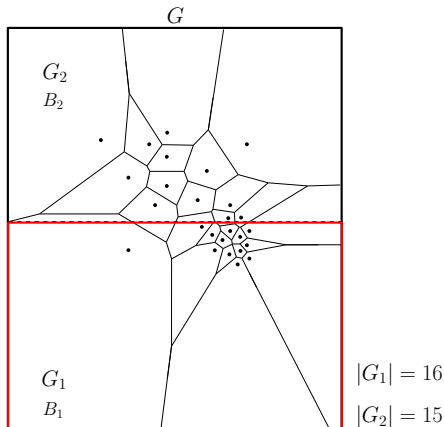
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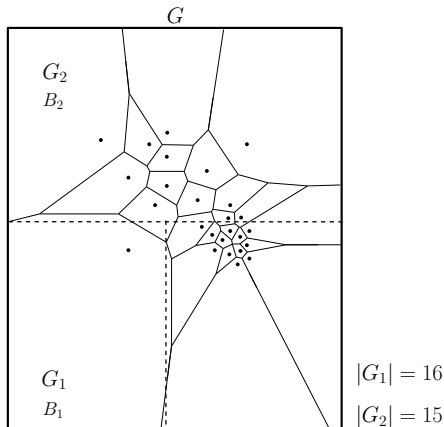
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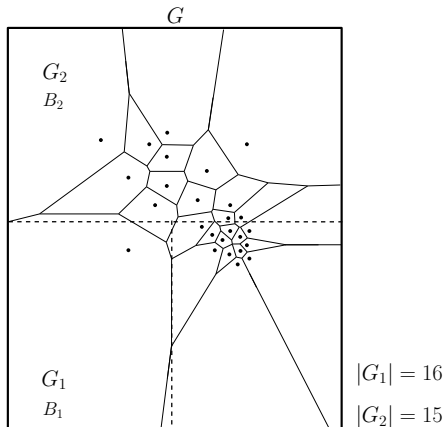
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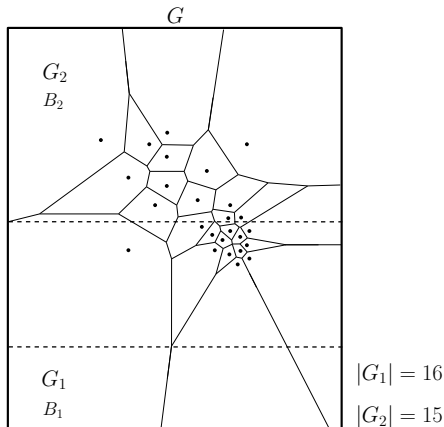
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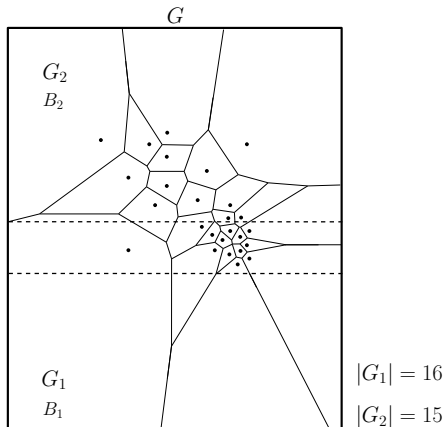
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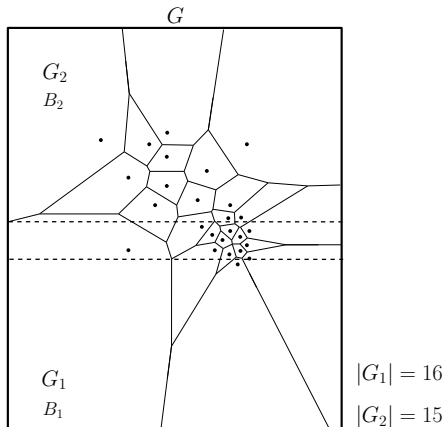
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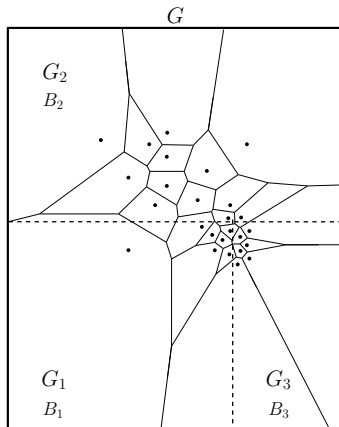
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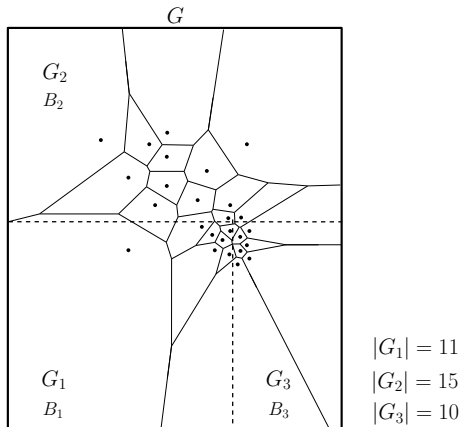
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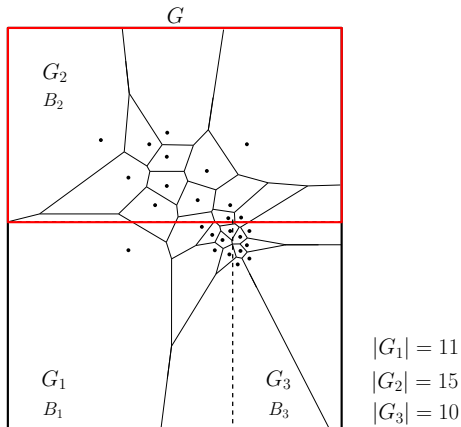
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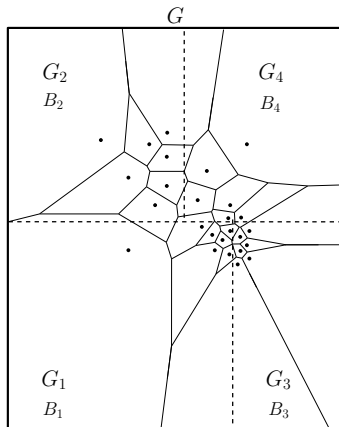
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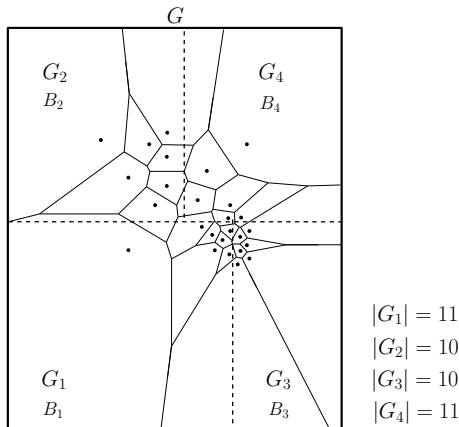
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Minimum Maximum Partition (MinMax)

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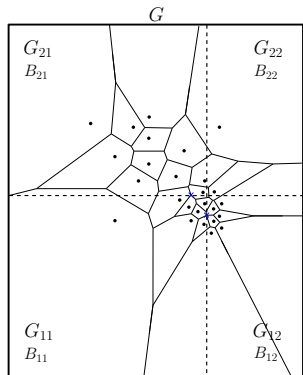
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Minimum Maximum Partition (MinMax)

- Merits:
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 - high storage cost at client

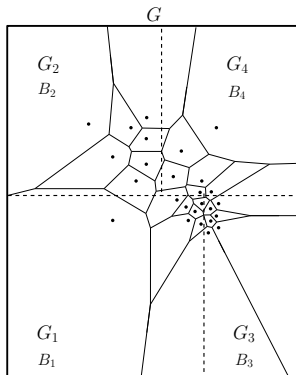
Comparison between MinSG and MinMax

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MinSG

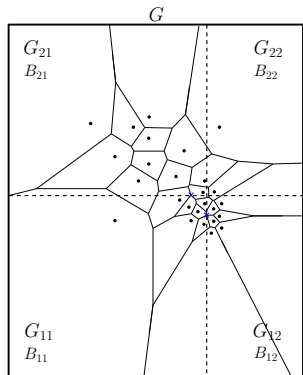
$$\begin{aligned} |G_{11}| &= 11 \\ |G_{12}| &= 10 \\ |G_{21}| &= 14 \\ |G_{22}| &= 6 \end{aligned}$$



MinMax

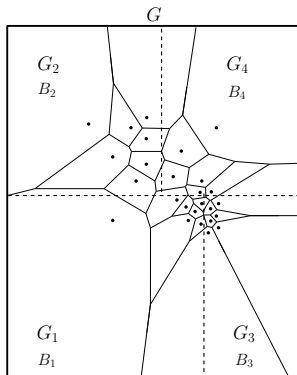
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- Clearly, MinMax achieves more balanced partitions than MinSG, which means lower storage and communication overheads, as well as cheaper encryption cost.

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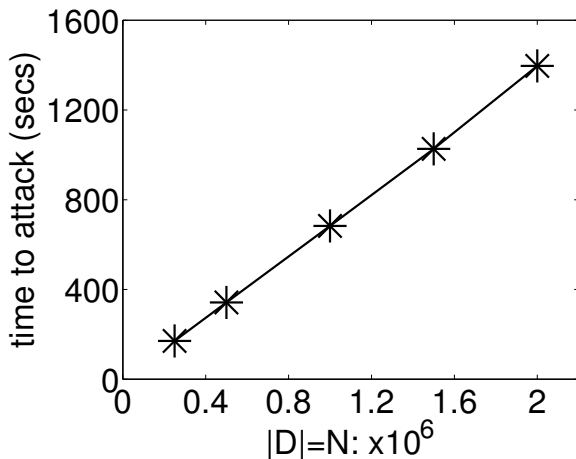
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- Default settings.

Symbol	Definition	Default Value
$ D $	size of the dataset	10^6
k	number of partitions	625
DT	dataset type	CA

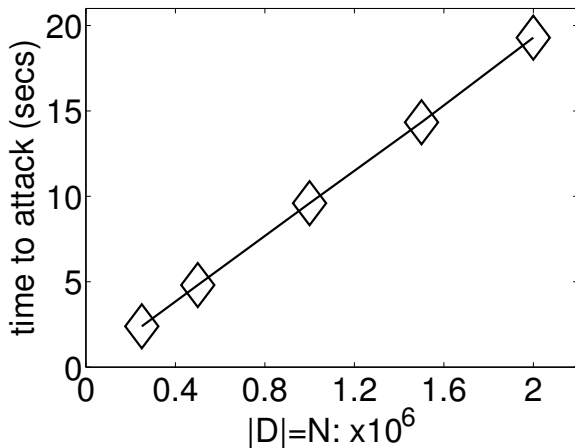
Attack on Existing SNN Methods

- Vary $|D|$: Wai Kit Wong, David Cheung, Ben Kao, Nikos Mamoulis:
Secure kNN computation on encrypted databases. SIGMOD 2009



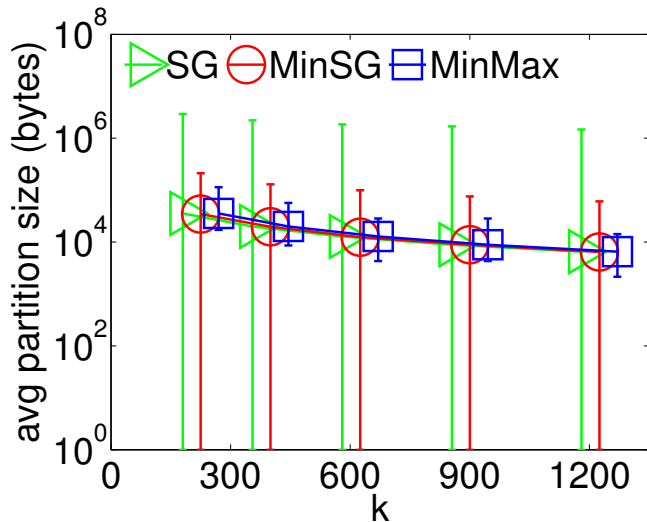
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- Vary $|D|$: Haibo Hu, Jianliang Xu, Chushi Ren, Byron Choi: Processing private queries over untrusted data cloud through privacy homomorphism. ICDE 2011



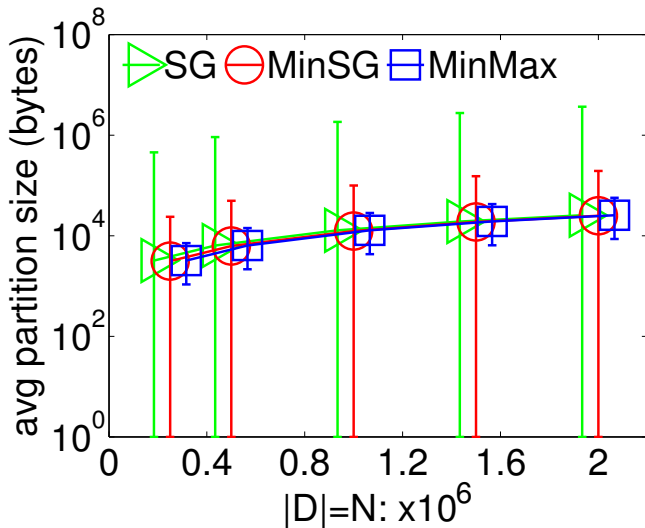
Partition size in different methods

- Vary k



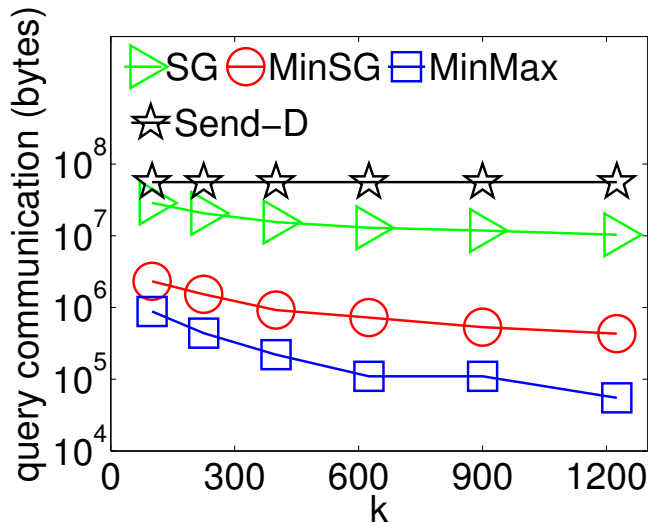
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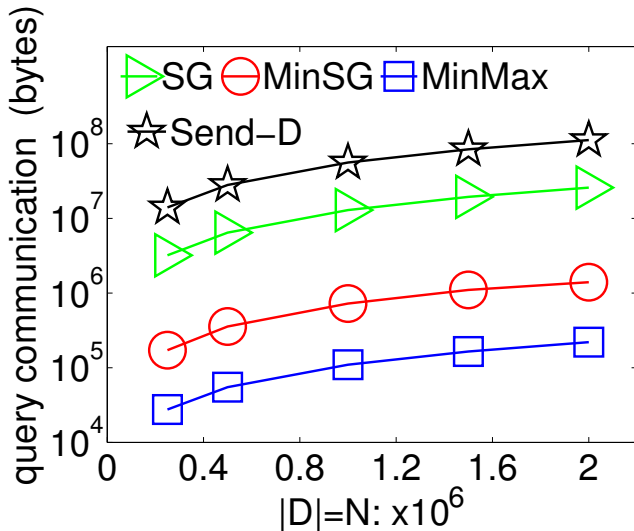
Query communication cost

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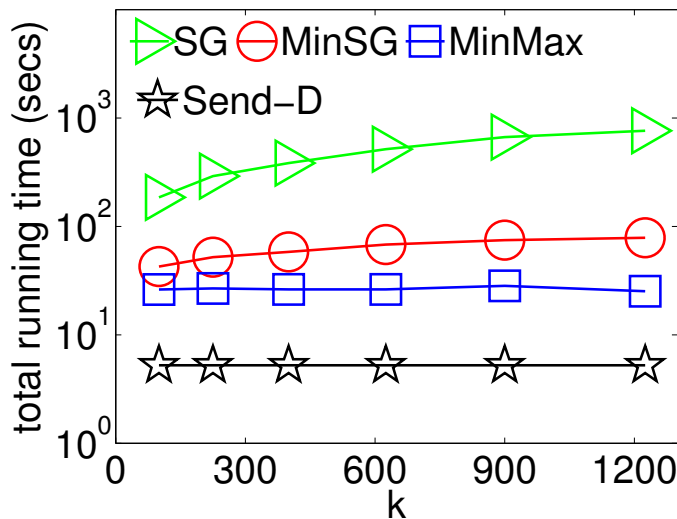
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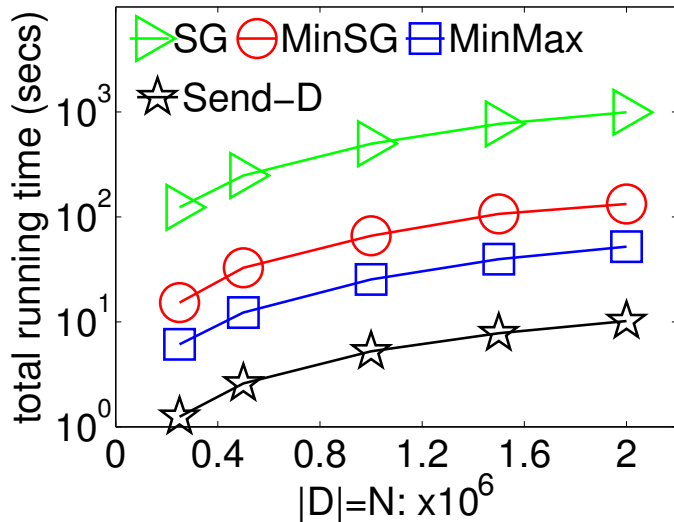
Total running time of the preprocessing step

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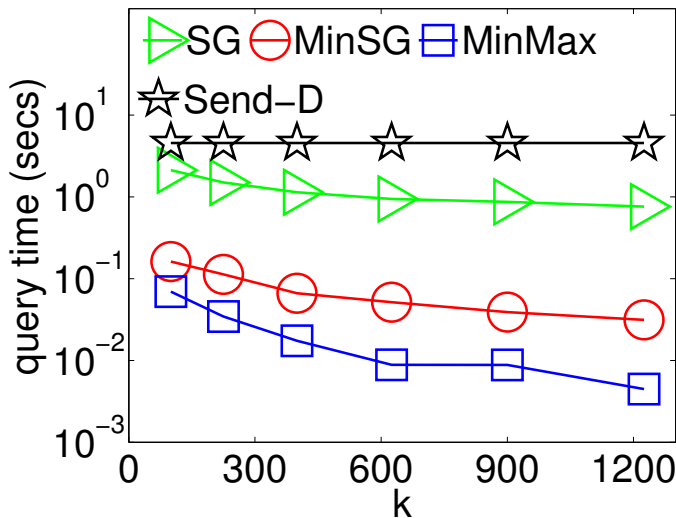
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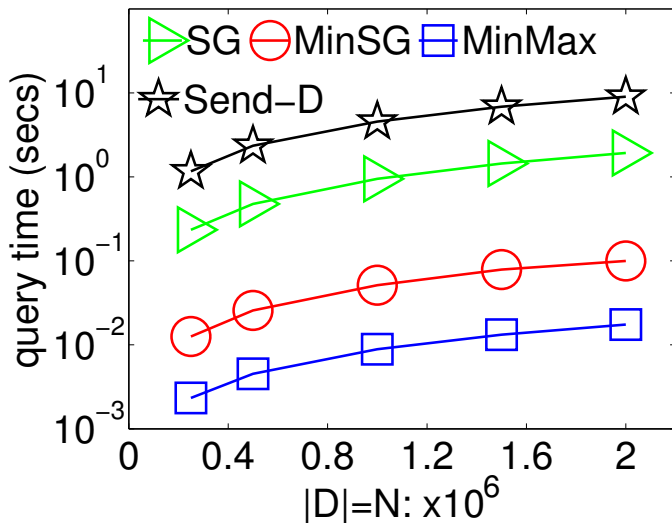
Query time for different methods

- Vary k



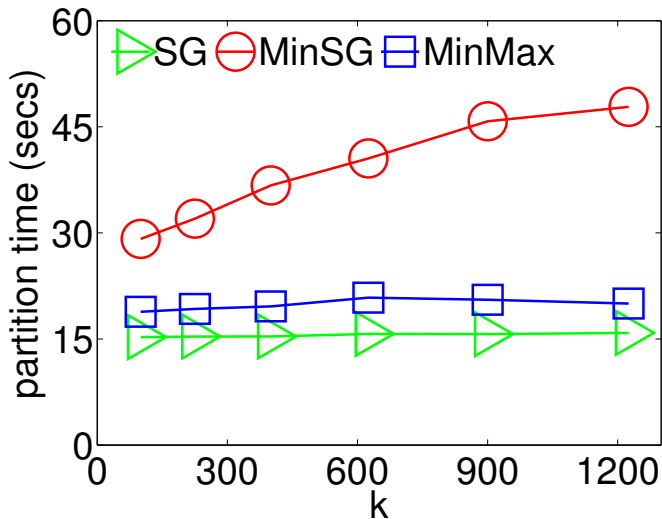
Query time for different methods

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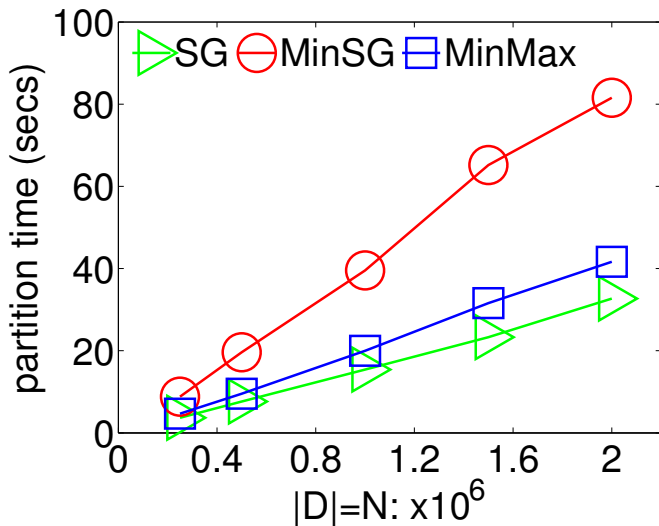
Running time of the partition phase

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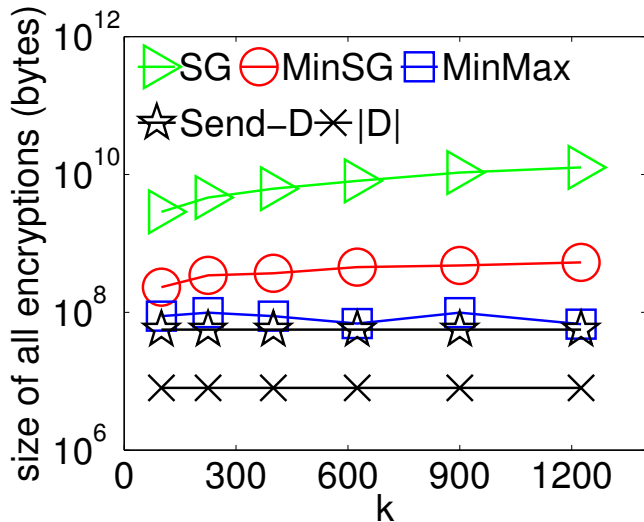
Running time of the partition phase

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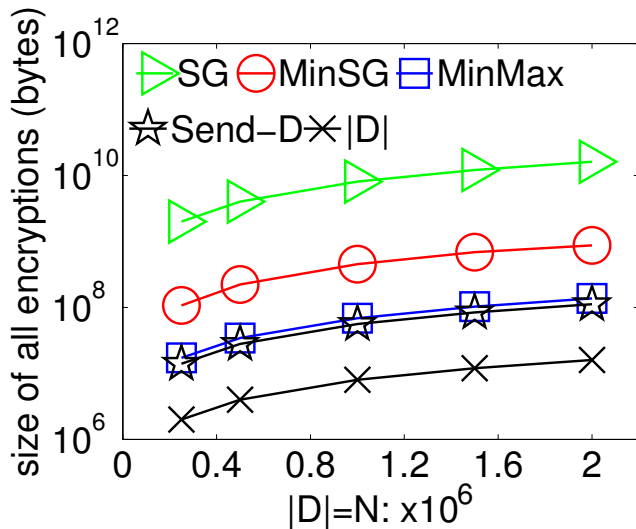
Total size of $E(D)$

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- 6 Variants of similarity search: reverse nearest neighbors, skylines, etc.

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- Future work
 - extending our investigation to higher dimensions, k nearest neighbors

Thank You

Q and A