1 This activity

In this activity we will be verifying the C_2 continuity of a b-spline curve. Note that

$$p = \begin{bmatrix} p_{i-2} \\ p_{i-1} \\ p_i \\ p_{i+1} \end{bmatrix}$$

$$q = \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \\ p_{i+2} \end{bmatrix}$$

$$b(u) = \frac{1}{6} \begin{bmatrix} (1-u)^3 \\ 4 - 6u^2 + 3u^3 \\ 1 + 3u + 3u^2 - 3u^3 \\ u^3 \end{bmatrix}$$

and

$$p(u) = b(u)^T p$$

where p and q are arrays of control points and $b(u)$ is our array of blending polynomials.

2 Finding the Second Derivative of the Blending Function

In order to prove that our spline is C_2 continuous we must prove that $p''(1) = q''(0)$. In order to do this we must first find b'' and then we can use the formula $p(u) = b(u)^T p$ to find p'' and q''.

So that you are able to check to make sure that you have the correct answer the first derivative of b has been provided for you.

$$b(u)' = \frac{1}{6} \begin{bmatrix} -3(1-u)^2 \\ -12u + 9u^2 \\ 3 + 6u - 9u^2 \\ 3u^2 \end{bmatrix}$$

You should now continue to find $b''(u)$ yourself.

3 Finding the Second Derivatives of p and q

We can now find the values of p and q using the formula $p(u) = b(u)^T p$. We should find that

$$p''(u) = (1-u)p_{i-2} + (-2+3u)p_{i-1} + (1-3u)p_i + (u)p_{i+1}$$

Now do the same for q yourself.

4 Showing Continuity

In order for the blending function to be C_2 continuous, $p''(1)$ must equal $q''(0)$. If all calculations have been performed correctly, when we substitute in these values of u we should find that

$$p''(1) = q''(0) = p_{i-1} - 2p_i + p_{i+1}$$