2018-EE-361

Open ended (DSP lab)

⇒ IIR high pass filter design by using bilinear transform

$$\Delta w = |w_p - w_s| = \left(\frac{XY \times 8}{1000}\right)\pi \text{ rad/sec}$$

Solution

X = 2nd digit of reg. no.

Y = 3rd digit of reg. no.

So, My reg. no. is 2018-EE-361

$X = 6$, $Y = 1$

$$\Delta w = |w_p - w_s| = \left(\frac{61 \times 8}{1000}\right)\pi$$

$$\Delta w = |w_p - w_s| = 0.488\pi \text{ rad/sec.}$$

Suppose

$$\boxed{w_s = 0.3\pi \text{ rad/sec}} \Rightarrow \quad 0 \leq w \leq 0.3\pi$$

$$w_p = 0.488\pi + 0.3\pi$$

$$\boxed{w_p = 0.788\pi \text{ rad/sec}} \Rightarrow 0.788\pi \leq w \leq \pi$$

let suppose our filter response is

$$|H(e^{jw})| \le 0.17783$$

$$\boxed{\delta_s = 0.17783}$$

And

$$0.89125 \le |H(e^{jw})| \le 1$$

$$\boxed{\delta_p = 0.89125}$$

Step 2

$\Omega_s = ?$, $\Omega_p = ?$

$$\Omega_s = 2\tan\frac{w_s}{2}$$

$$\Omega_s = 2\tan\left(\frac{0.3\pi}{2}\right)$$

$$\boxed{\Omega_s = 1.019 \text{ rad/sec}}$$

$$\Omega_p = 2\tan\frac{w_p}{2}$$

$$\Omega_p = 2\tan\left(\frac{0.788\pi}{2}\right)$$

$$\boxed{\Omega_p = 5.782 \text{ rad/sec}}$$

Step 3

Calculation for Order N

$$N = \frac{\frac{1}{2}\log\left[\dfrac{\frac{1}{\delta_s^2}-1}{\frac{1}{\delta_p^2}-1}\right]}{\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$N = \frac{\frac{1}{2}\log\left[\dfrac{\frac{1}{(0.17783)^2}-1}{\frac{1}{(0.89125)^2}-1}\right]}{\log\left(\frac{5.782}{1.019}\right)}$$

$$N = \frac{\frac{1}{2}\log\left(\frac{30.6}{0.2589}\right)}{\log(5.67)}$$

$$N = \frac{1.036}{0.7535} = 1.37$$

→ Order must be an integer

→ round towards Infinity

So,

$$\boxed{N \approx 2}$$

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Step 4

Calculation for Ω_c:

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{\delta_p^2} - 1\right)^{1/4}}$$

$\Omega_p = 5.782$ rad/sec and $\delta_p = 0.89125$ rad/sec

$$\Omega_c = \frac{5.782}{\left(\frac{1}{(0.89125)^2} - 1\right)^{1/4}}$$

$$\Omega_c = \frac{5.782}{0.7133}$$

$$\boxed{\Omega_c = 8.1 \text{ rad/sec}}$$

Scanned with CamScanner

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Step 5:

Analog filter

For N = even

$$H(s) = \prod_{k=1}^{N/2} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + C_k \Omega_c^2}$$

Here

$$b_k = 2\sin\left[(2k-1)\frac{\pi}{2N}\right]$$

For N = odd

$$H(s) = \frac{B_0 \Omega_c}{s + C_0 \Omega_c} \prod_{k=1}^{\left(\frac{N-1}{2}\right)} \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + C_k \Omega_c^2}$$

OR

$$P_k = \pm \Omega_c e^{j(N+2k+1)\pi/2N}$$

$$H(s) = \frac{\Omega_c^N}{(s-P_1)(s-P_2)\cdots(s-P_k)}$$

So, in our case

$$\boxed{N = 2 \text{ (even value)}}$$

$$H(s) = \frac{B_k \Omega_c^2}{s^2 + b_k \Omega_c s + C_k \Omega_c^2}$$

Suppose

$$B_k = C_k = 1$$

$$k = 1$$

$$b_k = 2 \sin\left[(2k-1)\pi/2N\right]$$

$$b_1 = 2 \sin\left[(2-1)\frac{\pi}{2(2)}\right]$$

$$b_1 = 2 \sin(\pi/4)$$

$$b_1 = 1.414$$

$$H(s) = \frac{\Omega_c^2}{s^2 + b_1 \Omega_c s + \Omega_c^2}$$

$$H(s) = \frac{(8.1)^2}{s^2 + (1.414)(8.1)s + (8.1)^2}$$

$$\boxed{H(s) = \frac{65.61}{s^2 + 11.4534s + 65.61}}$$

Step 6

Digital filter

Analog response:

$$H(s) = \frac{65.61}{s^2 + 11.4534s + 65.61}$$

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$T = 1$ (suppose)

$$s = 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H(z) = \frac{65.61}{\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + 11.4534\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right) + 65.61}$$

$$H(z) = \frac{65.61\left(1+z^{-1}\right)^2}{4\left(1-z^{-1}\right)^2 + 22.9\left(1-z^{-2}\right) + 65.61\left(1+z^{-1}\right)^2}$$

That is the required digital filter!

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