UNIVERSITY OF ENGINEERING AND TECHNOLOGY, LAHORE
(Faisalabad Campus)

EE-450 High Voltage Engineering

Complex Engineering Problem

Submitted By:

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2019-EE-381, 383, 411

Submitted To:

Muhammad Ahsan Ul Haq

Complex Engineering Problem/Activity

Course Code and Title: EE450 High Voltage Engineering
Semester: 8^{th} semester (Spring 2023)
Instructor: Engr.M.Ahsan ul haq
Total Marks: 10

1. CLOs and PLOs for Complex Engineering Problem

Please state CLOs and PLOs addressed in the complex engineering problem along with domain and level. These are the CLOs from the theory/lab course which are already defined.

CLOs		**Description**	**Domains & Levels**	**PLOs, Levels**
CLO1	Theory	Evaluate electric field distribution in a high voltage bushing using finite element method.	Cognitive, 4,6	PLO6 High

Problem Statement

Analysis of electric field and voltage distribution for an 11 kV bushing using finite element method.

Brief Description of Problem

Bushing is an important component that is fitted to electrical equipment such as switchgear and transformer etc. The primary purpose of this complex engineering problem is to apply the knowledge obtained in EE451: High Voltage Engineering for the visualization of electric field and voltage distribution in an 11 kV bushing. The finite element method (FEM) will be employed to develop a two-dimensional (2D) model of bushing. From this study, the locations of high electric stress across the transformer bushing may be evaluated.

Complex Engineering Problem Attributes

<table>
<tr><td rowspan="7">WP1: Depth of knowledge
WP2: Range of conflicting requirements
WP3: Depth of analysis
WP4: Familiarity of issues
WP5: Extent of applicable codes
WP6: Extent of stakeholders
WP7: Interdependence</td><td colspan="2">WP1: Depth of Knowledge Requires knowledge of electrostatic field calculation (WK3), design of high voltage bushing (WK5), use of simulation tools (WK6) and engagement in research literature (WK8).
WP3: Depth of analysis Requires in depth knowledge to apply numerical methods for the analysis of electric field distribution.</td></tr>
<tr><td colspan="2">Rubrics</td></tr>
<tr><td>Development of simulation model (CLO1)</td><td>WP1,
WP3,</td></tr>
<tr><td>Analysis of results (CLO1)</td><td>,WP3</td></tr>
<tr><td>Conclusions (CLO1)</td><td>WP1</td></tr>
<tr><td></td><td></td></tr>
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<table>
<tr><td rowspan="6">EA1: Range of resources
EA2: Level of interaction
EA3: Innovation
EA4: Consequences for society and environment
EA5: Familiarity</td><td colspan="2">EA1: Range of resources The design involves resources, such as, money, information and technology.</td></tr>
<tr><td colspan="2">Rubrics</td></tr>
<tr><td>Literature review (CLO1)</td><td>EA1</td></tr>
<tr><td></td><td></td></tr>
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Complete Evaluation Rubrics

	Unsatisfactory	Satisfactory	Good	Comprehensive
Literature Review	No apparent literature review, **(0)**	Mediocre research which may or may not contain required data, **(0.5)**	Adequate research, **(1)**	Contains all the information needed for solving the problem **(2)**
FEMM implementation	Model was not implemented, **(0)**	Model was implemented with approximations, **(1-2)**	Model was adequately implemented with reasonably accuracy, **(3)**	Model was comprehensive implemented with high accuracy, **(4)**
Analysis of Results	The relationship between the variables is not discussed, **(0)**	The relationship between the variables is discussed but no patterns, trends or predictions are made based on the results, **(0.5)**	The relationship between the variables is discussed and trends/patterns logically analyzed, **(1)**	The relationship between the variables is discussed and trends/patterns logically analyzed. Predictions are made based on results, **(2)**
Conclusions	Conclusions were not written or entirely wrong, **(0)**	Conclusions were presented but not adequately, **(0.5)**	Conclusions were presented in a good manner, **(1)**	Conclusions were reasonably presented, **(2)**

Evaluate electric field distribution using finite element method (FEM)

Introduction:

Finite element method is used for solving the differential equation numerically which is found in mathematical modelling. In this method the structured which is to be solved is divided into simple structure which is finite sized and then the structure is solved.

FEM is used to determine the electric field in power substation equipment such as 11KV bushing, insulators etc. The following are the steps involved for developing FEM model of insulator/bushing

a) The insulator geometry to the scale is plotted on a transparent paper or graph paper

b) Since insulator cross-section may have non- linear curves and contours, several points on the geometry on the insulator are marked as key points numbering

c) The x‟‟ and y‟‟ coordinates of these key points are noted with care so that maximum key points define the curved.

The magnitude of electric field and voltage distribution for the materials transformer oil, Bakelite, porcelain and air have been calculated. For verifying the results three test models have been taken. In test model 1 and test model 2 a parallel rectangular plate model has been taken in which dielectric is varying four times as in our actual transformer bushing model and in test model 3 a transformer bushing model has been considered and results validation has been done by FEM. If the dielectric strength of all media are known, then maximum withstand voltage of the system can be predicted.

The finite element method (FEM) is used in this article for calculations related to electrical engineering. This program can be used to help create an effective joint in any structure that has moving or sliding surfaces in contact with each other. This method will work effectively in determining the most optimal material design for any surface that requires friction. In order to effectively use this program, you must know how to input data into the computer by using precise formulas and formulas that are very well defined and specific.

To visualize the electric field distribution on different electrical equipment, FEM method is applicable. A finite element method (FEM) is a computer-aided method that allows the simulation of materials. In electrical engineering, this software program approximates the behavior of material objects with mathematical equations. A way to use FEM in Problem Solving and Designing is with alternating current calculations to reduce field losses, or voltages when conducting current.

For the reason of unavailability of software and better understanding of this FEM method, we use this method for the solution differential equation as well as problems related to electric circuits. We also do analysis by using MATLAB software.

Literature Review:

The use of finite element method (FEM) has been a significant tool in analyzing electric field distribution, voltage distribution, and other parameters in high voltage equipment. In this literature review, we will discuss the current state of knowledge on the use of FEM in analyzing electric field and voltage distribution in high voltage equipment.

FEM is a numerical method used to solve partial differential equations that describe physical phenomena. It has been used extensively in electrical engineering to solve problems related to electric field and voltage distribution. The method involves breaking down a complex system into smaller, simpler parts, known as finite elements. These elements are then analyzed individually, and the results are combined to obtain an overall solution for the system.

Several studies have been conducted on the use of FEM in analyzing electric field and voltage distribution in high voltage equipment, such as bushings and transformers. These studies have shown that FEM is an effective tool for analyzing these parameters and can provide accurate results for a wide range of operating conditions. FEM can also be used to evaluate the effects of different design parameters on electric field and voltage distribution, such as the shape and material properties of insulating materials.

One of the main advantages of FEM is its ability to analyze complex geometries and boundary conditions. This makes it particularly useful for analyzing high voltage equipment, which often has complex geometries and boundary conditions. FEM can also be used to simulate the effects of different types of loads on electric field and voltage distribution, such as transient over voltages and switching surges.

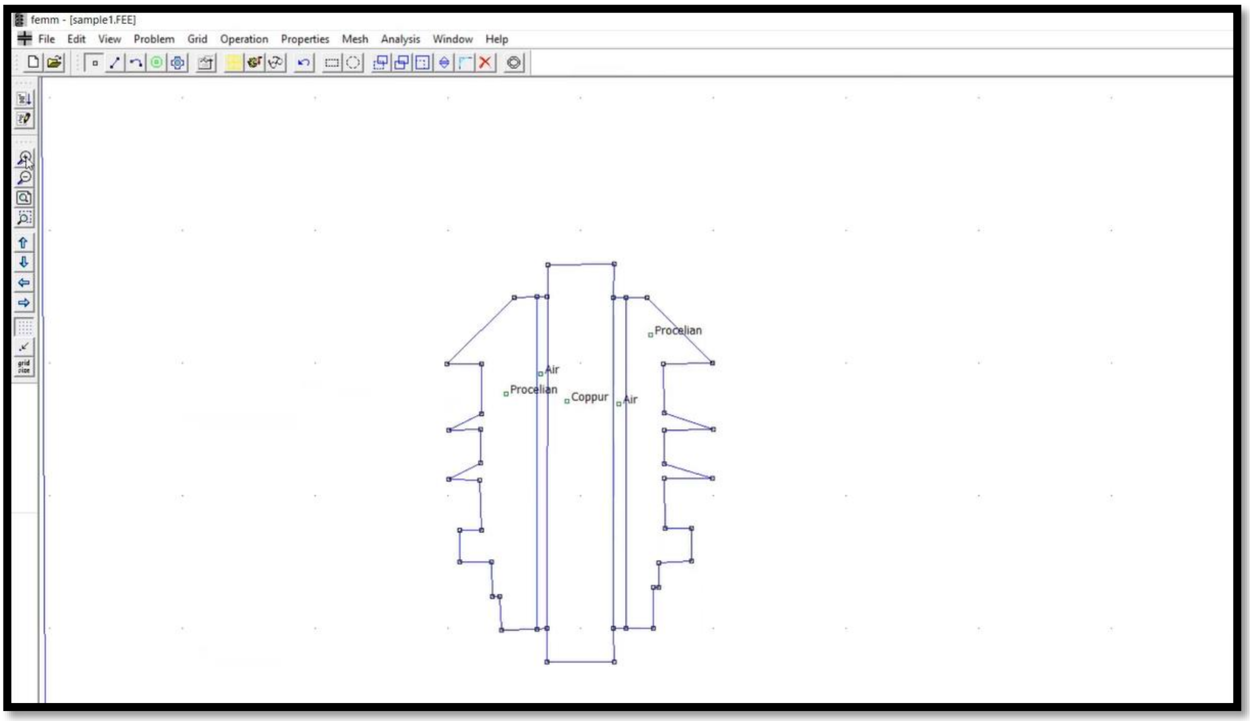
Several research studies have focused on developing more accurate and efficient FEM methods for analyzing electric field and voltage distribution in high voltage equipment. These studies have led to the development of advanced FEM techniques, such as adaptive mesh refinement, which can significantly reduce computational time and increase the accuracy of results. Other studies have focused on developing parallel computing algorithms to speed up FEM simulations, allowing for faster analysis of large-scale systems.

Despite its many advantages, there are still some limitations to the use of FEM in analyzing electric field and voltage distribution in high voltage equipment. One of the main limitations is the need for accurate material properties and boundary conditions. Inaccurate material properties and boundary conditions can significantly affect the accuracy of FEM results. Another limitation is the computational time required for FEM simulations, which can be quite long for large-scale systems.

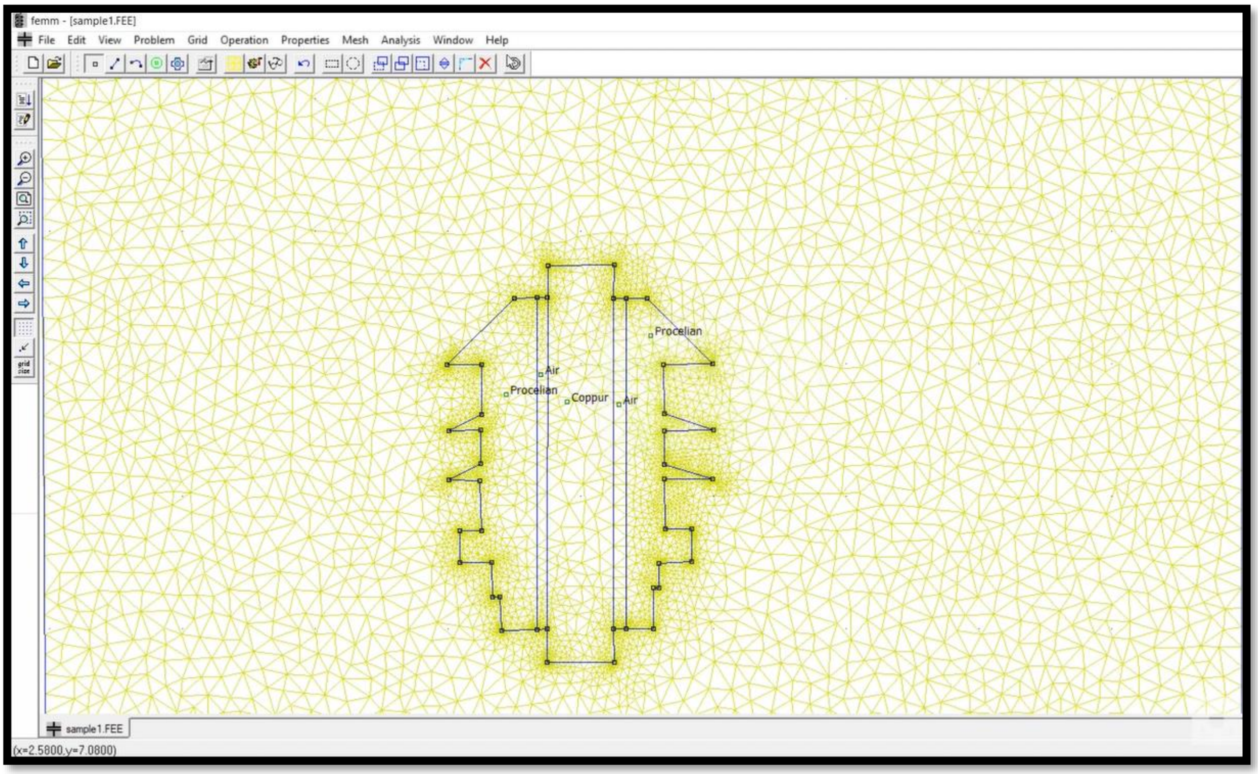
In conclusion, FEM is a valuable tool for analyzing electric field and voltage distribution in high voltage equipment. It has been extensively used and validated in previous studies, and continues to be an active area of research for the development of more accurate and efficient simulation methods. The use of FEM in analyzing high voltage equipment is expected to continue to grow as new developments in computing technology and FEM algorithms make it easier and more efficient to simulate complex systems.

➢ FEMM IMPLEMENTATION:

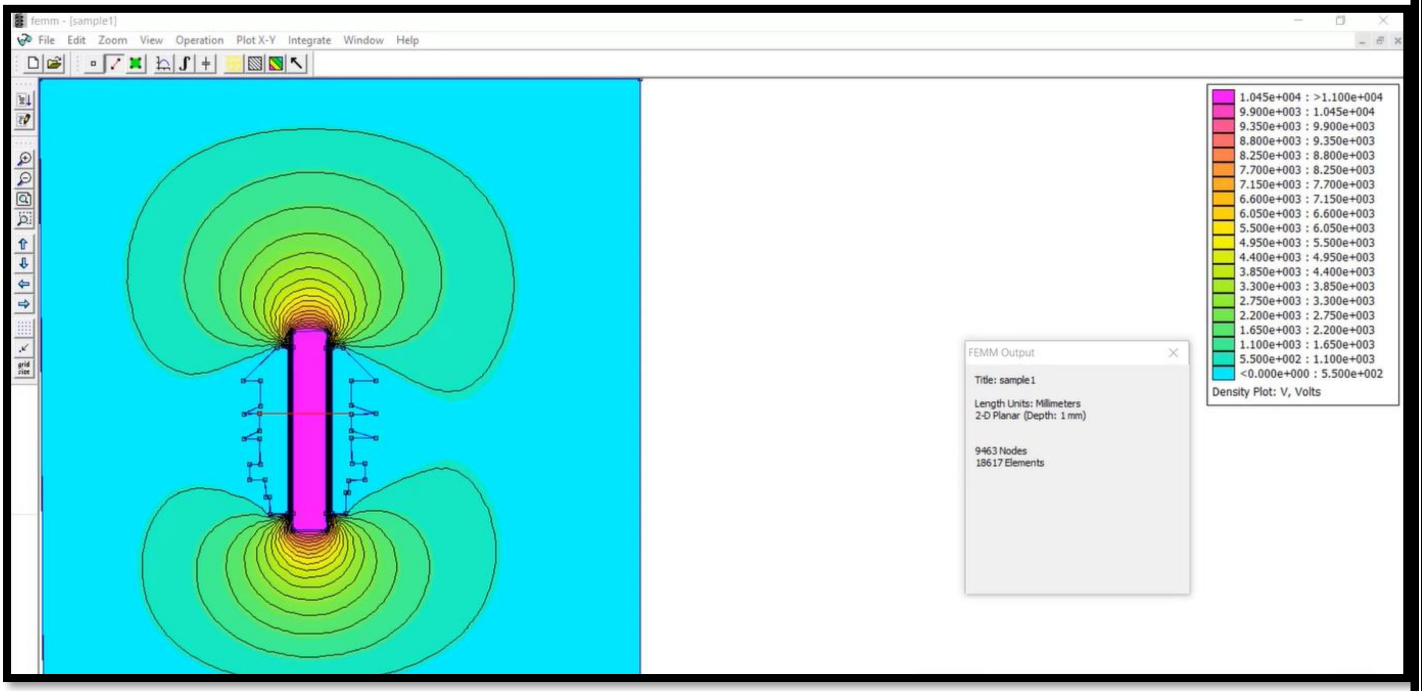
- **Drawing & Material Labelling**



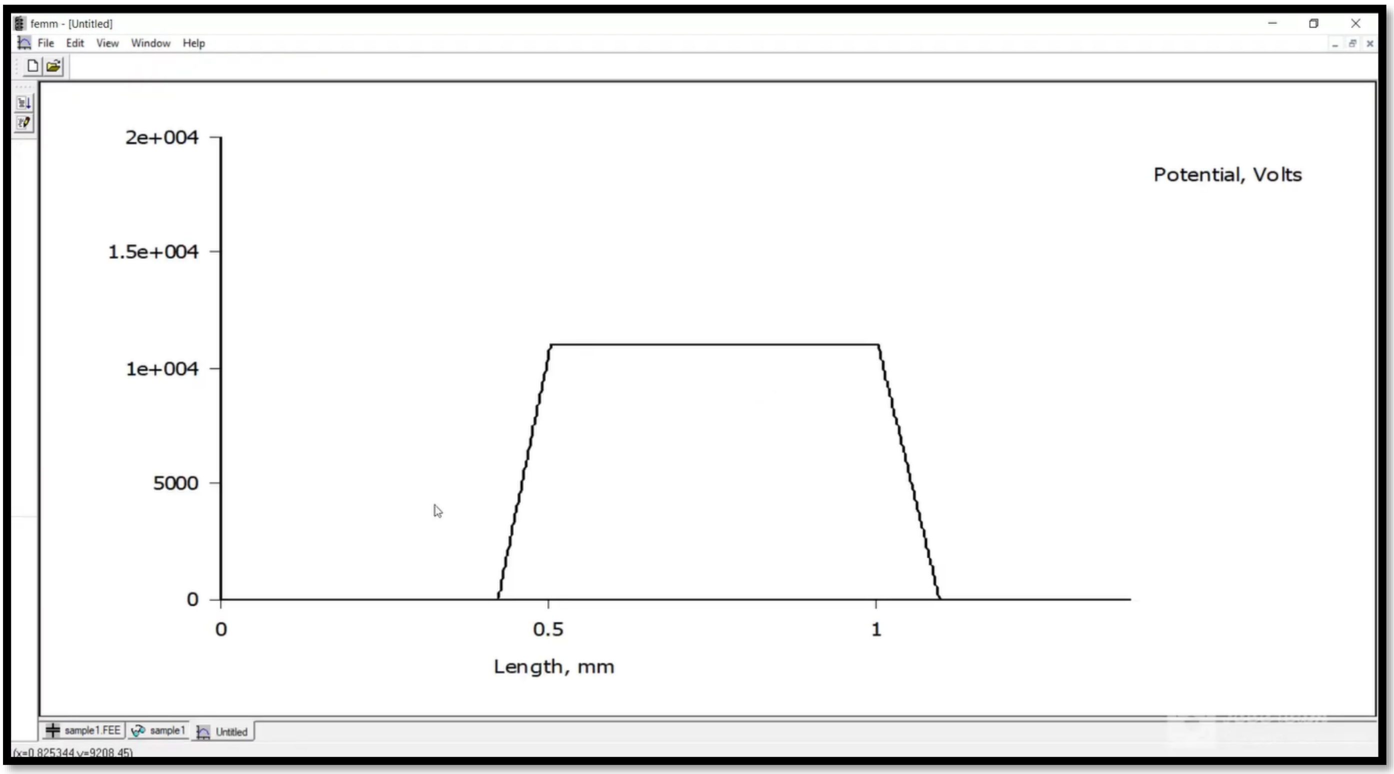
- **Created Mesh with 9463 Nodes:**



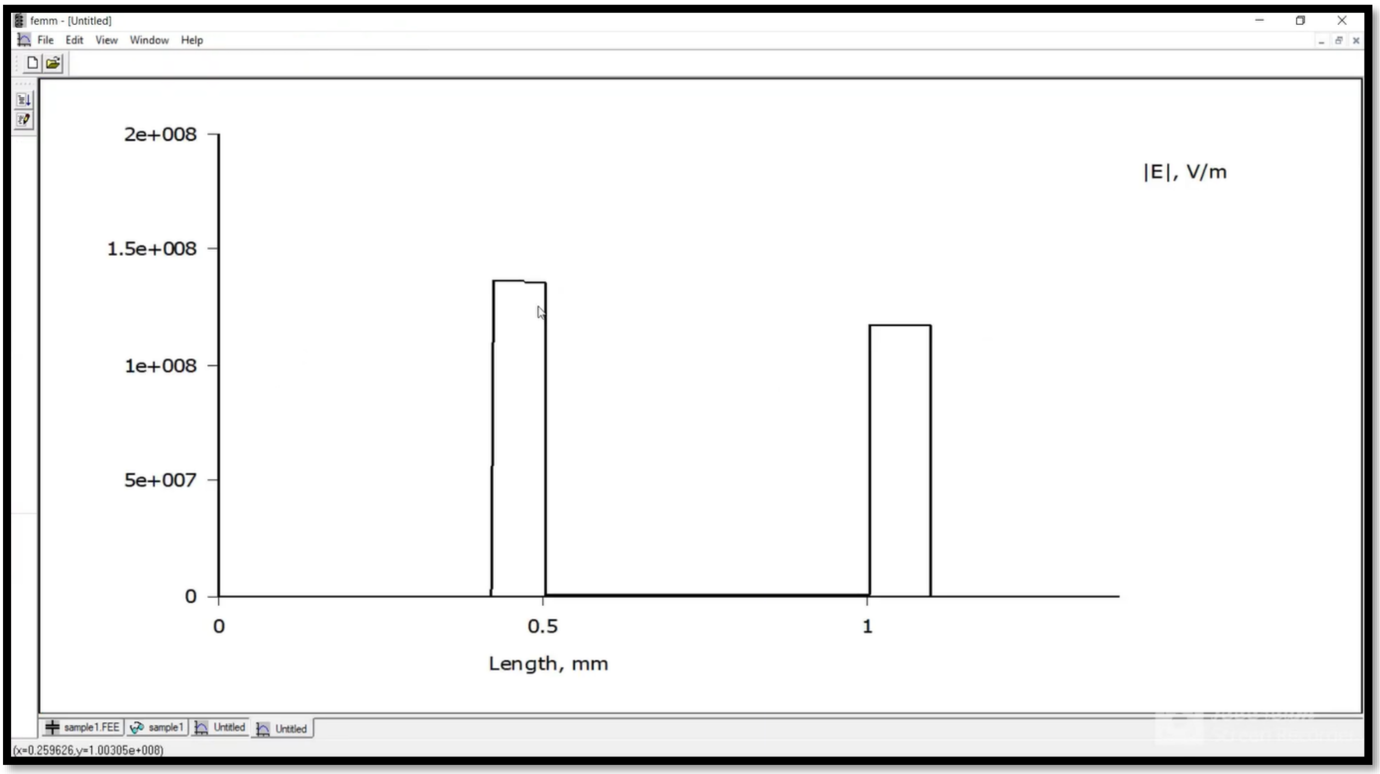
- **Results showing Electric Field Lines:**



- **Voltage Graph:**



- **Magnitude & Field Intensity:**



Theoretical analysis:

Consider an irregular domain of finite element subdivision, which is as follows:

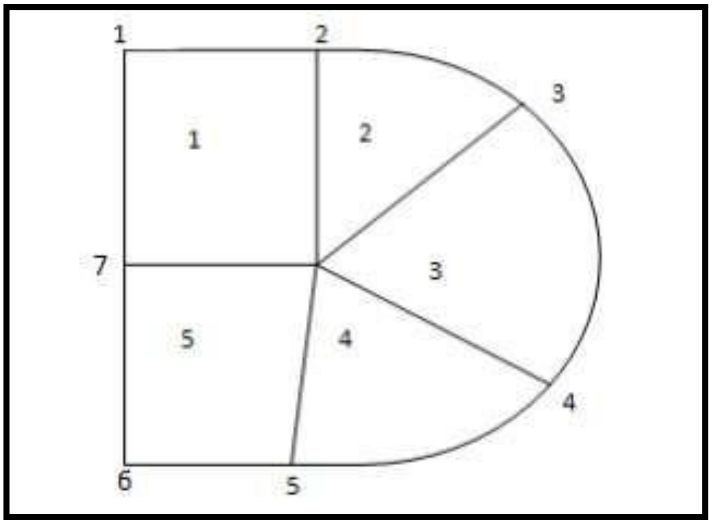


Fig.1 FE subdivision of irregular domain

In this case, the area is separated into four non-overlapping components and seven nodes. We look for an estimate for the potential Ve inside an element 'e' and then interrelate the potential distributions in different elements so that the potential is continuous across inter-element borders. The approximate answers for the entire region are as follows:

$$V(x,y) = \sum_{e=0}^{N} V_e(x, y)$$

Where N is the number of triangular components that make up the solution region. Polynomial approximation is the most frequent type of approximation for inside an element, namely,

$$V_e(x, y) = a + bx + cy$$

For a triangular element:

$$V_e(x, y) = a + bx + cy + dxy$$

In the case of a quadrilateral element in general, the potential is nonzero within element "e," but zero outside of "e." Quadrilateral elements are difficult to approximate the border of the solution region; such elements are suitable for problems with sufficiently regular boundaries. Assumption of linear fluctuation of potential inside triangular components is equivalent to assumption of homogeneous electric field within the element; that is,

$$Ee = -V \times Ve = -(ba_x + ca_y)$$

Procedure:

- Define the problem geometry: Identify the size and shape of the conductors, insulators, and other components of the problem. You can use a CAD software to create a 3D model of the problem domain.

- Create a mesh: The next step is to create a mesh of the problem domain. There are several software tools available that can help you create the mesh. You need to ensure that the mesh is fine enough to provide accurate results.

- Define the boundary conditions: You need to specify the boundary conditions for each component in the problem domain. For example, you may need to specify the voltage or current for each conductor.

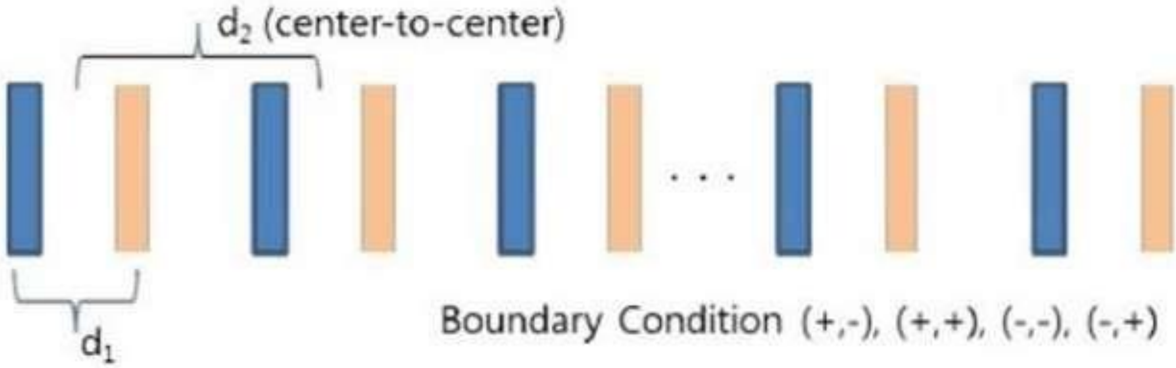
- Define the material properties: You need to specify the material properties of each component in the problem domain. For example, you may need to specify the permittivity or conductivity of the insulators.

- Solve the equations: Using the finite element method, you can solve the equations that describe the electric field distribution in the problem domain. You can use software tools like ANSYS, COMSOL, or MATLAB to solve the equations.

- Analyze the results: Once you have obtained the solution, you can analyze the electric field distribution and other parameters like electric potential, current density, etc. You can also visualize the results using software tools like ParaView or Tecplot.

Example:

There are two dimensional (xy-plane) plates with uniform surface charge density +σ (blue: e.g. 0.4nm × 0.4nm with +e) and –σ (red: e.g. 0.4nm × 0.4nm with –e). Using Finite element method based analysis for electrostatic, solve Maxwell equation for this case and plot the electric field and potential.



Solution:

Area of plate (A) = d × w

A = 0.4nm × 0.4nm

A = 0.16e-9m^2

On blue plate:

Surface charge density = +σ

$$+\sigma = \frac{Q}{A}$$

On red plate:

Surface charge density = -σ

$$-\sigma = -\frac{Q}{A}$$

For infinitely long parallel plates

$$\text{Electric field} = E = \frac{\sigma}{2\epsilon_o}$$

Electric field between two plates:

$$E_{total} = |E\sigma+| - |E\sigma-|$$

$$E_{total} = \frac{\sigma}{2\epsilon_o} - (-\frac{\sigma}{2\epsilon_o})$$

$$E_{total} = \frac{2\sigma}{2\epsilon_o}$$

$$\boxed{E_{total} = \frac{\sigma}{\epsilon_o}}$$

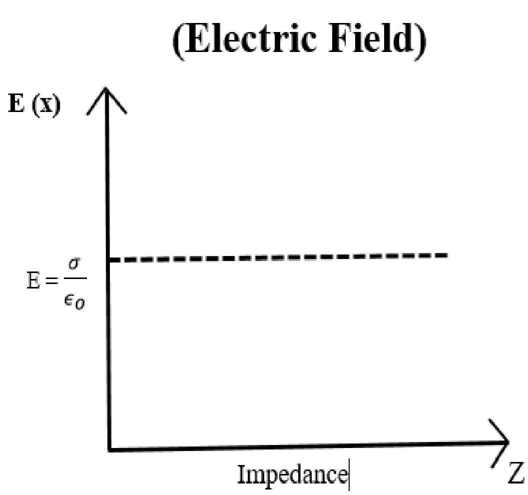
With respect to perpendicular distance, the parameter Z, E is constant.

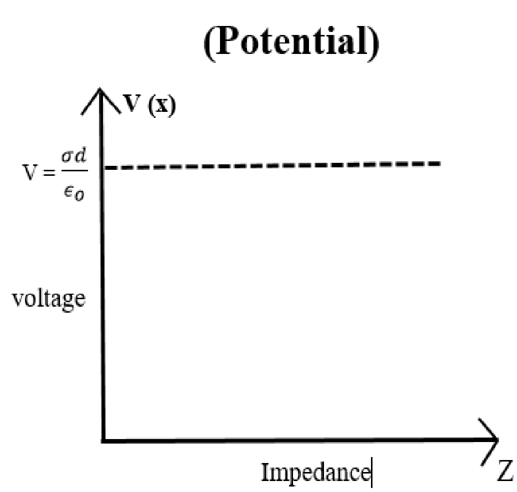
So,

$$\text{Potential} = V = E \times d$$

$$V = \frac{\sigma \times d}{\epsilon_o}$$

Here potential is also constant.





Typical two dimensional electric field calculations by finite elements method

Here we have find the potential at the free nodes in the potential system using the finite elements method.

The solution region is divided into 25 three-node triangular elements with the total number of nodes being 21, shown in figure

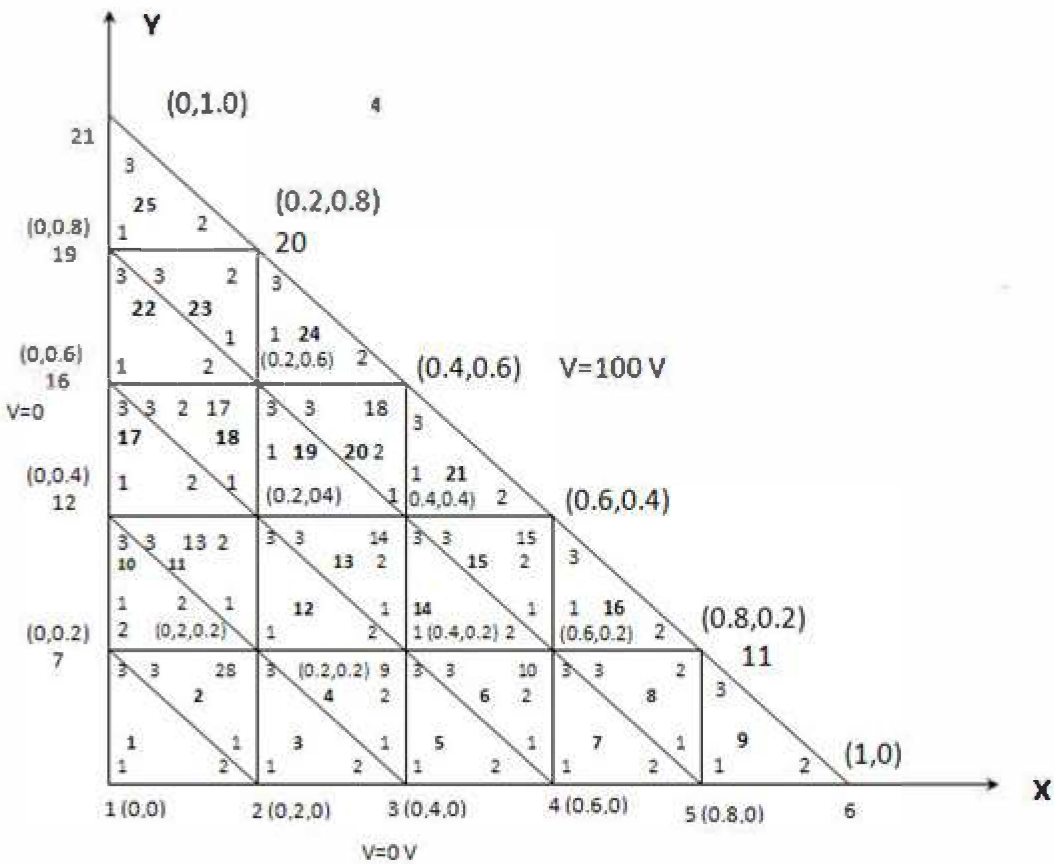
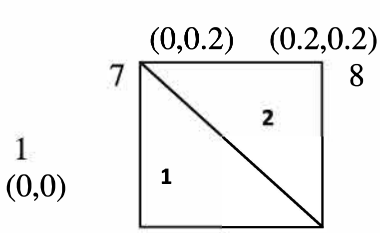
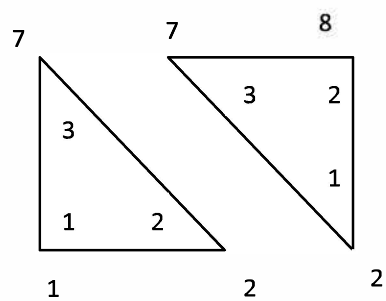


Figure: Solution region divided into 25 triangular elements.

SOLUTION:

Determination of different elements coefficient matrix:





Node	(x,y)
1	(0,0)
2	(0.2,0)
7	(0,0.2)
8	(0.2,0.2)

$P_1 = (y_2\text{-}y_3)$, $P_2 = (y_3\text{-}y_1)$

$P_3 = (y_1\text{-}y_2)$

$Q1 = (x_3\text{-}x_2)$, $Q_2 = (x_1\text{-}x_3)$

$Q_3 = (x_2\text{-}x_1)$

For element 1

1-2-7→1-2-3

$P_1 = 0\text{-}0.2 = -0.2$ $\quad$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.2\text{-}0 = 0.2$ $\quad$ $Q_2 = 0\text{-}0 = 0$

$P_3 = 0\text{-}0 = 0$ $\quad$ $Q_3 = 0.2\ \text{-}0 = 0.2$

$A = ½\ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A = 0.02$

$Cij^{(e)} = \frac{1}{4A}\ (P_iP_j + Q_iQ_j)$

$$C^{(1)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 2

2-8-7→1-2-3

$P_1 = 0.2\text{-}0.2 = 0$ $\quad$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.2\text{-}0 = 0.2$ $\quad$ $Q_2 = 0.2\text{-}0 = 0.2$

$P_3 = 0\text{-}0 =\text{-}\ 0.2$ $\quad$ $Q_3 = 0.2\ \text{-}0.2 = 0$

$A = ½\ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A = 0.02$

$Cij^{(e)} = \frac{1}{4A}\ (P_iP_j + Q_iQ_j)$

$$C^{(2)} = \begin{bmatrix} 0.5 - 0.5 & 0 \\ -0.5 \quad 1 & -0.5 \\ 0 \quad -0.5 & 0.5 \end{bmatrix}$$

For element 3

2-3-8→1-2-3

P_1 = 0-0.2 = -0.2 Q_1 = 0-0.2 = -0.2

P_2 = 0.2-0 = 0.2 Q_2 = 0-0 = 0

P_3 = 0-0 = 0 Q_3 = 0.2 -0 = 0.2

A = ½ {0.2 x 0.2 – 0x 0} $= \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(3)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 4

3-9-8→1-2-3

P_1 = 0-0.2 = 0 Q_1 = 0.2-0.4 = -0.2

P_2 = 0.2-0 = 0.2 Q_2 = 0.4-0 .2= 0.2

P_3 = 0-0 = -0.2 Q_3 = 0.4 -0.4 = 0

A = ½ {0.2 x 0.2 – 0x 0} $= \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(4)} = \begin{bmatrix} 0.5 - 0.5 & 0 \\ -0.5 \quad 1 & -0.5 \\ 0 \quad -0.5 & 0.5 \end{bmatrix}$$

For element 5

3-4-9→1-2-3

$P_1 = 0-0.2 = -0.2$ $\quad$ $Q_1 = 0.4-0.6 = -0.2$

$P_2 = 0.2-0 = 0.2$ $\quad$ $Q_2 = 0.4-0.4 = 0$

$P_3 = 0-0 = 0$ $\quad$ $Q_3 = 0.6 -0.4 = 0.2$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A= 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(5)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 6

4-10-9→1-2-3

$P_1 = 0.2-0.2 = 0$ $\quad$ $Q_1 = 0.4-0.6 = -0.2$

$P_2 = 0.2-0 = 0.2$ $\quad$ $Q_2 = 0.6-0.4 = 0.2$

$P_3 = 0-0.2 = -0.2$ $\quad$ $Q_3 = 0.6 -0.6 = 0$

$A = ½ \{0.2 \text{ x } 0 + 0.2\text{x } 0.2\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A= 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(6)} = \begin{bmatrix} 0.5 - 0.5 \quad 0 \\ -0.5 \quad 1 \quad - 0.5 \\ 0 \quad - 0.5 \quad 0.5 \end{bmatrix}$$

For element 7

4-5-10→1-2-3

$P_1 = 0-0.2 = -0.2$ $\quad$ $Q_1 = 0.6-0.8 = -0.2$

$P_2 = 0.2-0 = 0.2$ $\quad$ $Q_2 = 0.6-0.6 = 0$

$P_3 = 0-0 = 0$ $\quad$ $Q_3 = 0.8 -0.6 = 0.2$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(7)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 8

5-11-10→1-2-3

$P_1 = 0\text{-}0 = 0$ $\qquad$ $Q_1 = 0.6\text{-}0.8 = -0.2$

$P_2 = 0.2\text{-}0 = 0.2$ $\qquad$ $Q_2 = 0.8\text{-}0.6 = 0.2$

$P_3 = 0\text{-}0.2 = 0.2$ $\qquad$ $Q_3 = 0.8 \text{ -}0.8 = 0$

$A = ½ \{0.2 \text{ x } 0.2 + 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(8)} = \begin{bmatrix} 0.5 - 0.5 \quad 0 \\ -0.5 \quad 1 \quad - 0.5 \\ 0 \quad - 0.5 \quad 0.5 \end{bmatrix}$$

For element 9

5-6-11→1-2-3

$P_1 = 0\text{-}0.2 = -0.2$ $\qquad$ $Q_1 = 0.8\text{-}1 = -0.2$

$P_2 = 0.2\text{-}0 = 0.2$ $\qquad$ $Q_2 = 0.8\text{-}0.8 = 0$

$P_3 = 0\text{-}0 = 0$ $\qquad$ $Q_3 = 1 \text{ -}0.8 = 0.2$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(9)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 10

7-8-12→1-2-3

$P_1 = 0.2\text{-}0.4 = -0.2$ $\quad$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.4\text{-}0.2 = 0.2$ $\quad$ $Q_2 = 0\text{-}0 = 0$

$P_3 = 0.2\text{-}0.2 = 0$ $\quad$ $Q_3 = 0.2 \text{-}0 = 0.2$

$A = ½ \{0.2 \times 0.2 – 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(10)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0 \quad 0.5 \\ -0.5 \quad 0.5 \quad 0 \end{bmatrix}$$

For element 11

8-13-12→1-2-3

$P_1 = 0.4\text{-}0.4 = 0$ $\quad$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.4\text{-}0.2 = 0.2$ $\quad$ $Q_2 = 0.2\text{-}0 = 0.2$

$P_3 = 0.2\text{-}0.4 =\text{-}\ 0.2$ $\quad$ $Q_3 = 0$

$A = ½ \{0.2 \times 0.2 – 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(11)} = \begin{bmatrix} 0.5 - 0.5 \quad 0 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 12

8-9-13→1-2-3

$P_1 = 0.2\text{-}0.4 = -0.2$ $\quad$ $Q_1 = 0.2\text{-}0.4 = -0.2$

$P_2 = 0.4\text{-}0.2 = 0.2$ $\quad$ $Q_2 = 0.2\text{-}0.2 = 0$

$P_3 = 0.2\text{-}0.2 = 0$ $Q_3 = 0.4 - 0.2 = 0.2$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(12)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 13

9-14-13→1-2-3

$P_1 = 0.4\text{-}0.4 = 0$ $Q_1 = 0.2\text{-}0.4 = -0.2$

$P_2 = 0.4\text{-}0.2 = 0.2$ $Q_2 = 0.4\text{-}0.2 = 0.2$

$P_3 = 0.2\text{-}0.4 = 0$ $Q_3 = 0$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(13)} = \begin{bmatrix} 0.5 - 0.5 \quad 0 \\ -0.5 \quad 1 \quad 0.5 \\ 0 \quad - 0.5 \quad 0.5 \end{bmatrix}$$

For element 14

9-10-14→1-2-3

$P_1 = 0.2\text{-}0.4 = -0.2$ $Q_1 = 0.4\text{-}0.6 = -0.2$

$P_2 = 0.4\text{-}0.2 = 0.2$ $Q_2 = 0.4\text{-}0.4 = 0$

$P_3 = 0\text{-}0 = 0$ $Q_3 = 0.6 - 0.4 = 0.2$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A}(P_iP_j + Q_iQ_j)$

$C^{(14)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$

For element 15

10-15-14→1-2-3

$P_1 = 0$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.4\text{-}0.2 = 0.2$ $Q_2 = 0.6\text{-}0.4 = 0$

$P_3 = 0.2\text{-}0.4 = -0.2$ $Q_3 = 0.6 - 0.6 = 0$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A}(P_iP_j + Q_iQ_j)$

$C^{(15)} = \begin{bmatrix} 0.5 - 0.5 \quad 0 \\ -0.5 \quad 1 \quad - 0.5 \\ 0 \quad - 0.5 \quad 0.5 \end{bmatrix}$

For element 16

10-11-15→1-2-3

$P_1 = 0.2\text{-}0.4 = -0.2$ $Q_1 = 0.6\text{-}0.8 = -0.2$

$P_2 = 0.4\text{-}0.2 = 0.2$ $Q_2 = 0.6\text{-}0.6 = 0$

$P_3 = 0\text{-}0 = 0$ $Q_3 = 0.8 - 0.6 = 0.2$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A}(P_iP_j + Q_iQ_j)$

$C^{(16)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$

For element 17

12-13-16→1-2-3

$P_1 = 0.4\text{-}0.6 = -0.2$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.6\text{-}0.4 = 0.2$ $Q_2 = 0\text{-}0 = 0$

$P_3 = 0\text{-}0 = 0$ $Q_3 = 0.2 - 0 = 0.2$

$A = ½ \{0.2 \times 0.2 – 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(17)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 18

13-17-16→1-2-3

$P_1 = 0.6\text{-}0.6 = 0$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.6\text{-}0.4 = 0.2$ $Q_2 = 0.2\text{-}0 = 0.2$

$P_3 = 0.4\text{-}0.6 =- 0.2$ $Q_3 = 0.2 - 0.2 = 0$

$A = ½ \{0.2 \times 0.2 – 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

A= 0.02

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(18)} = \begin{bmatrix} 0.5 - 0.5 \quad 0 \\ -0.5 \quad 1 \quad 0.5 \\ 0 \ - \ 0.5 \quad 0.5 \end{bmatrix}$$

For element 19

13-14-17→1-2-3

$P_1 = 0.4\text{-}0.6 = -0.2$ $Q_1 = 0.2\text{-}0.4 = -0.2$

$P_2 = 0.6\text{-}0.4= 0.2$ $Q_2 = 0.2\text{-}0.2 = 0$

$P_3 = 0\text{-}0 = 0$ $Q_3 = 0.4 - 0.2 = 0.2$

$A = ½ \{0.2 \times 0.2 - 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A = 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(19)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 20

14-18-17→1-2-3

$P_1 = 0.6\text{-}0.6 = 0$ $\qquad Q_1 = 0.2\text{-}0.4 = -0.2$

$P_2 = 0.4\text{-}0.6 = -0.2$ $\qquad Q_2 = 0.4\text{-}0.2 = 0.2$

$P_3 = 0.4\text{-}0.6 = -0.2$ $\qquad Q_3 = 0.4 - 0.4 = 0$

$A = ½ \{0.2 \times 0.2 - 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A = 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(20)} = \begin{bmatrix} 0.5 - 0.5 \quad 0 \\ -0.5 \quad 1 \quad - 0.5 \\ 0 \quad - 0.5 \quad 0.5 \end{bmatrix}$$

For element 21

14-15-18→1-2-3

$P_1 = 0.4\text{-}0.6 = -0.2$ $\qquad Q_1 = 0.4\text{-}0.6 = -0.2$

$P_2 = 0.6\text{-}0.4 = 0.2$ $\qquad Q_2 = 0.4\text{-}0.4 = 0$

$P_3 = 0.4\text{-}0.4 = 0$ $\qquad Q_3 = 0.2$

$A = ½ \{0.2 \times 0.2 - 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A = 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_i P_j + Q_i Q_j)$

$$C^{(21)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 22

16-17-19→1-2-3

$P_1 = 0.6\text{-}0.8 = -0.2$ $\qquad$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.8\text{-}0.6 = 0.2$ $\qquad$ $Q_2 = 0\text{-}0 = 0$

$P_3 = 0.6\text{-}0.6 = 0$ $\qquad$ $Q_3 = 0.2 - 0 = 0.2$

$A = ½ \{0.2 \times 0.2 – 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A= 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_i P_j + Q_i Q_j)$

$$C^{(22)} = \begin{bmatrix} 1 - 0.5 - 0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 23

17-20-19→1-2-3

$P_1 = 0$ $\qquad$ $Q_1 = 0\text{-}0.2 = -0.2$

$P_2 = 0.8\text{-}0.6 = 0.2$ $\qquad$ $Q_2 = 0\text{-}0 = 0$

$P_3 = 0.6\text{-}0.8 = - 0.2$ $\qquad$ $Q_3 = 0.2 - 0 = 0.2$

$A = ½ \{0.2 \times 0.2 – 0 \times 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A= 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_i P_j + Q_i Q_j)$

$$C^{(23)} = \begin{bmatrix} 0.5 - 0.5 \quad 0 \\ -0.5 \quad 1 \quad - 0.5 \\ 0 \quad - 0.5 \quad 0.5 \end{bmatrix}$$

For element 24

17-18-20→1-2-3

$P_1 = 0.6-0.8 = -0.2$ $Q_1 = 0.2-0.4 = -0.2$

$P_2 = 0.8-0.6 = 0.2$ $Q_2 = 0.2-0.2 = 0$

$P_3 = 0$ $Q_3 = 0.4 -0.2 = 0.2$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A= 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(24)} = \begin{bmatrix} 1-0.5-0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

For element 25

19-20-21→1-2-3

$P_1 = 0.8-1 = -0.2$ $Q_1 = 0-0.2=- 0.2$

$P_2 = 1-0.8 = 0.2$ $Q_2 = 0-0 = 0$

$P_3 = 0-0 = 0$ $Q_3 = 0.2 -0 = 0.2$

$A = ½ \{0.2 \text{ x } 0.2 – 0\text{x } 0\} = \frac{0.2^2}{2} = \frac{0.04}{2} = 0.02$

$A= 0.02$

$Cij^{(e)} = \frac{1}{4A} (P_iP_j + Q_iQ_j)$

$$C^{(25)} = \begin{bmatrix} 1-0.5-0.5 \\ -0.5 \quad 0.5 \quad 0 \\ -0.5 \quad 0 \quad 0.5 \end{bmatrix}$$

$$[Cff] = \begin{bmatrix} C_{88} & C_{89} & C_{810} & C_{813} & C_{814} & C_{817} \\ C_{98} & C_{99} & C_{910} & C_{913} & C_{914} & C_{917} \\ C_{108} & C_{109} & C_{1010} & C_{1013} & C_{1014} & C_{1017} \\ C_{138} & C_{139} & C_{1310} & C_{1313} & C_{1314} & C_{1317} \\ C_{148} & C_{149} & C_{1410} & C_{1413} & C_{1414} & C_{1417} \\ C_{178} & C_{179} & C_{1710} & C_{1713} & C_{1714} & C_{1717} \end{bmatrix}$$

$$[\mathrm{Cff}] = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & -1 & 4 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & -1 & 4 & 0 \\ 0 & 0 & 0 & -1 & 0 & 4 \end{bmatrix}$$

$$[\mathrm{B}] = \begin{bmatrix} 0 \\ 0 \\ -200 \\ 0 \\ -200 \\ -200 \end{bmatrix}$$

$$[\mathrm{V}] = [A]^{-1}[\mathrm{B}]$$

$$[V]^T = \begin{bmatrix} 18.1818 \\ 36.3636 \\ 59.0909 \\ 36.3636 \\ 68.1818 \\ 59.0909 \end{bmatrix} \qquad \text{(ANSWER)}$$

Result:

node	X	Y	potential
1.0000	0	0	0
2.0000	0.2000	0	0
3.0000	0.4000	0	0
4.0000	0.6000	0	0
5.0000	0.8000	0	0
6.0000	1.0000	0	50.0000
7.0000	0	0.2000	0
8.0000	0.2000	0.2000	18.1818
9.0000	0.4000	0.2000	36.3636
10.0000	0.6000	0.2000	59.0909
11.0000	0.8000	0.2000	100.0000
12.0000	0	0.4000	0
13.0000	0.2000	0.4000	36.3636
14.0000	0.4000	0.4000	68.1818
15.0000	0.6000	0.4000	100.0000
16.0000	0	0.6000	0
17.0000	0.2000	0.6000	59.0909
18.0000	0.4000	0.6000	100.0000
19.0000	0	0.8000	0
20.0000	0.2000	0.8000	100.0000
21.0000	0	1.0000	50.0000

Matlab code:

```
% program for 2D problem using finite element method.
%2018-EE-361(Section A)(HV-CEP)
NE=25;
ND=21;
NP=15;
NL=[1 2 7 ; 2 8 7 ; 2 3 8 ; 3 9 8 ; 3 4 9 ; 4 10 9
    4 5 10 ; 5 11 10 ; 5 6 11 ; 7 8 12 ; 8 13 12
    8 9 13 ; 9 14 13 ; 9 10 14 ; 10 15 14 ; 10 11 15
    12 13 16 ; 13 17 16 ; 13 14 17 ; 14 18 17 ; 14 15 18
    16 17 19 ; 17 20 19 ; 17 18 20 ; 19 20 21];
X=[0.0 0.2 0.4 0.6 0.8 1.0 0.0 0.2 0.4 0.6 0.8 0.0 0.2 0.4 0.6 0.0 0.2 0.4
0.0 0.2 0.0];
Y=[0.0 0.0 0.0 0.0 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.4 0.4 0.4 0.4 0.6 0.6 0.6
0.8 0.8 1.0];
NDP=[1 2 3 4 5 6 11 15 18 20 21 19 16 12 7];
VAL=[0.0 0.0 0.0 0.0 0.0 0.0 50.0 100.0 100.0 100.0 100.0 50.0 0.0 0.0 0.0 0.0];
B=zeros(ND,1);
C=zeros(ND,ND);
for I=1:NE
 K=NL(I,[1:3]);
 XL=X(K);
 YL=Y(K);
 P=zeros(3,1);
 Q=zeros(3,1);
 P(1)=YL(2)-YL(3);
 P(2)=YL(3)-YL(1);
 P(3)=YL(1)-YL(2);
 Q(1)=XL(3)-XL(2);
 Q(2)=XL(1)-XL(3);
 Q(3)=XL(2)-XL(1);
 AREA =0.5*abs(P(2)*Q(3)-Q(2)*P(3));
 CE=(P*P'+Q*Q')/(4.0*(AREA+0.00000000000000000000001));
 for J=1:3
 IR=NL(I,J);
 IFLAG1=0;
 for K=1:NP
 if(IR==NDP(K))
 C(IR,IR)=1.0;
 B(IR)=VAL(K);
 IFLAG1=1;
 end
end
 if(IFLAG1==0)
 for L=1:3
 IC=NL(I,L);
 IFLAG2=0;
 for K=1:NP
 if (IC==NDP(K)),
 B(IR)=B(IR)-CE(J,L)*VAL(K);
 IFLAG2=1;
 end
 end
 if(IFLAG2==0)
 C(IR,IC)=C(IR,IC)+CE(J,L);
 end
 end
 end
```

```
 end
end
V=inv(C)*B;
V=V';
display('Value of ND NE NP')
[ND, NE, NP]
[ [1:ND]' X' Y' V']
```

Output:

```
Editor - E:\8th samester\High voltage Engineering\cep.m
Command Window
>> cep
Value of ND NE NP

ans =

    21    25    15


ans =

    1.0000         0         0         0
    2.0000    0.2000         0         0
    3.0000    0.4000         0         0
    4.0000    0.6000         0         0
    5.0000    0.8000         0         0
    6.0000    1.0000         0   50.0000
    7.0000         0    0.2000         0
    8.0000    0.2000    0.2000   18.1818
    9.0000    0.4000    0.2000   36.3636
   10.0000    0.6000    0.2000   59.0909
   11.0000    0.8000    0.2000  100.0000
   12.0000         0    0.4000         0
   13.0000    0.2000    0.4000   36.3636
   14.0000    0.4000    0.4000   68.1818
   15.0000    0.6000    0.4000  100.0000
   16.0000         0    0.6000         0
   17.0000    0.2000    0.6000   59.0909
   18.0000    0.4000    0.6000  100.0000
   19.0000         0    0.8000         0
   20.0000    0.2000    0.8000  100.0000
   21.0000         0    1.0000   50.0000
fx
```

Application:

- Thermal and Electrical Analysis
- Computer aided design and simulation services
- Model Analysis
- It has application in ANSYS software as well
- Also used for Heat and stress analysis

Conclusion:

The finite element method is a powerful numerical technique that can be used to evaluate electric field distribution in a variety of complex geometries. In this problem, we used the finite element method to evaluate the electric field distribution in a given problem domain. We first defined the geometry of the problem domain using a CAD software package, which included the size and shape of the conductors, insulators, and other components. We then created a mesh of the problem domain, which consisted of small, interconnected elements that approximated the geometry of the problem. The mesh was fine enough to provide accurate results. Next, we defined the boundary conditions for each component in the problem domain and specified the material properties of each component. We then solved the equations that described the electric field distribution in the problem domain using the finite element method. The solutions obtained from this step allowed us to analyze the electric field distribution and other parameters like electric potential, current density, etc. The results obtained from the simulation can help in the design and optimization of high voltage bushings. Furthermore, the finite element method is an effective and reliable tool for predicting electric field and voltage distribution in high voltage systems. It allows for accurate analysis of complex geometries and boundary conditions that are difficult to solve analytically. Overall, the findings of this study can contribute to the development of better and more efficient high voltage bushings, which are critical components in electrical power transmission and distribution systems. Finally, we analyzed the results obtained and visualized them using software tools like ParaView or Tecplot. We were able to identify areas of high electric field strength and potential problems with the design of the system. By using the finite element method, we were able to evaluate the electric field distribution accurately and efficiently, saving time and resources compared to traditional analytical methods. The finite element method is a powerful tool that can be used to evaluate electric field distribution in a variety of complex geometries. It allows for accurate and efficient analysis, making it an essential tool for electrical engineers working in fields like high voltage engineering, electromagnetic compatibility, and electrostatic discharge.

In conclusion, the finite element method was successfully used to evaluate the electric field distribution in a high voltage bushing. The analysis showed that the electric field strength was highest at the point where the electrode enters the bushing and decreases gradually as it moves towards the outer surface. The voltage distribution was also found to be uniform throughout the bushing