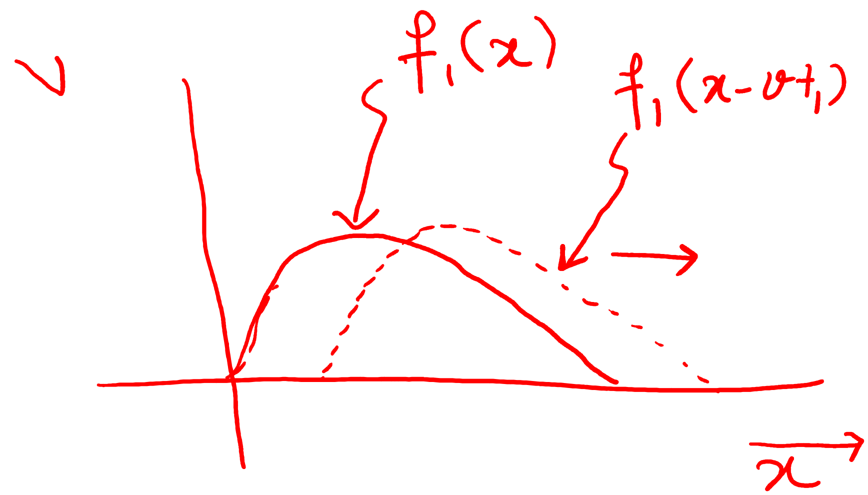
Wave Equations

$$V(x,t) = f_1(x - vt) + f_2(x + vt)$$

$$= V_f + V_b$$

$$I(x,t) = \frac{1}{Z_0}\left[f_1(x - vt) - f_2(x + vt)\right]$$

$$= I_f + I_b$$



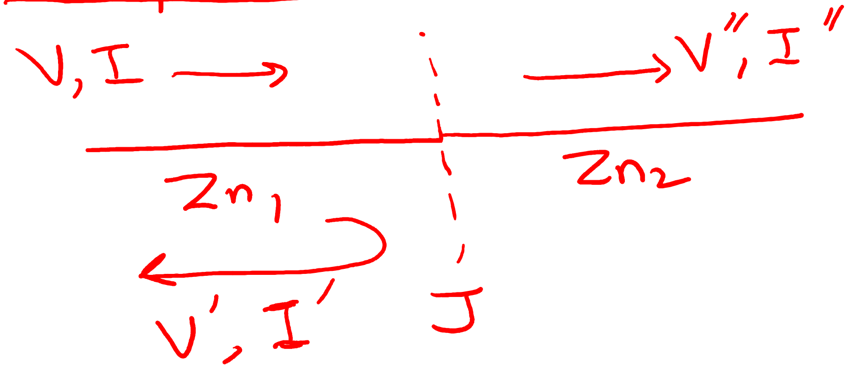
$\rightarrow$ $V_f = f_1(x - vt)$

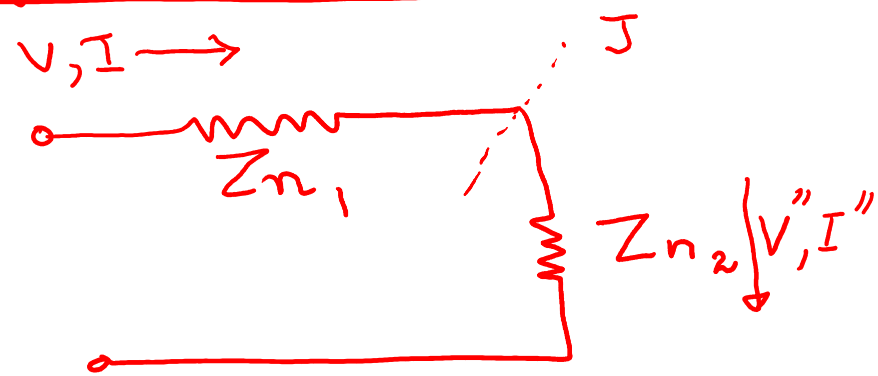
at $t = 0$, $V_f = f_1(x)$

$t = t_1$, $V_f = f_1(x - vt_1)$

$= f_1(x - \Delta x)$

Reflection & Refraction at the junction of two lines





$$Zn_1 \neq Zn_2$$

$$\frac{V}{I} = Zn_1 \; ; \; \frac{V''}{I''} = Zn_2$$

$$\rightarrow \quad V + V' = V'' \quad \text{—— } (1)$$

$$I + I' = I'' \quad \text{—— } (2)$$

$$\frac{V}{Zn_1} - \frac{V'}{Zn_1} = \frac{V''}{Zn_2} \quad \text{—— } (3)$$

$$\frac{V}{Zn_1} - \frac{V'}{Zn_1} = \frac{V+V'}{Zn_2}$$

$$\frac{V}{Zn_1} - \frac{V}{Zn_2} = \frac{V'}{Zn_1} + \frac{V'}{Zn_2}$$

$$V\left(\frac{Zn_2 - Zn_1}{\cancel{Zn_1 Zn_2}}\right) = V'\left(\frac{Zn_1 + Zn_2}{\cancel{Zn_1 Zn_2}}\right)$$

$$V' = \underbrace{\left(\frac{Zn_2 - Zn_1}{Zn_1 + Zn_2}\right)}_{\text{reflection coeff.}} \times V \quad \text{——} \quad 4$$

$$I' = -\frac{V'}{Zn_1} = -\frac{1}{Zn_1} \times V\left(\frac{Zn_2 - Zn_1}{Zn_1 + Zn_2}\right)$$

$$I' = -\underbrace{\left(\frac{Zn_2 - Zn_1}{Zn_1 + Zn_2}\right)}_{\text{reflection coeff.}} \times I \quad \text{——} \quad 5 \quad \checkmark$$

$$\rightarrow \frac{V}{Z_{n_1}} - \frac{V'}{Z_{n_1}} = \frac{V''}{Z_{n_2}} \quad \text{—— (3)}$$

$$\frac{V}{Z_{n_1}} - \frac{(V''-V)}{Z_{n_1}} = \frac{V''}{Z_{n_2}}$$

$$\frac{V}{Z_{n_1}} + \frac{V}{Z_{n_1}} = \frac{V''}{Z_{n_1}} + \frac{V''}{Z_{n_2}}$$

$$\frac{2V}{Z_{n_1}} = V''\left(\frac{1}{Z_{n_1}} + \frac{1}{Z_{n_2}}\right)$$

$$\frac{2V}{\cancel{Z_{n_1}}} = V''\left(\frac{Z_{n_1}+Z_{n_2}}{\cancel{Z_{n_1}}Z_{n_2}}\right)$$

$$V'' = \left(\frac{2Z_{n_2}}{Z_{n_1}+Z_{n_2}}\right) \times V \quad \text{—— (6)}$$

$\longrightarrow$ trans. coefficient

$$\rightarrow \quad I'' = \frac{V''}{Z_{n_2}}$$

$$= \frac{1}{\cancel{Z_{n_2}}} \times \left(\frac{2\cancel{Z_{n_2}}}{Z_{n_1}+Z_{n_2}} \right) \times V$$

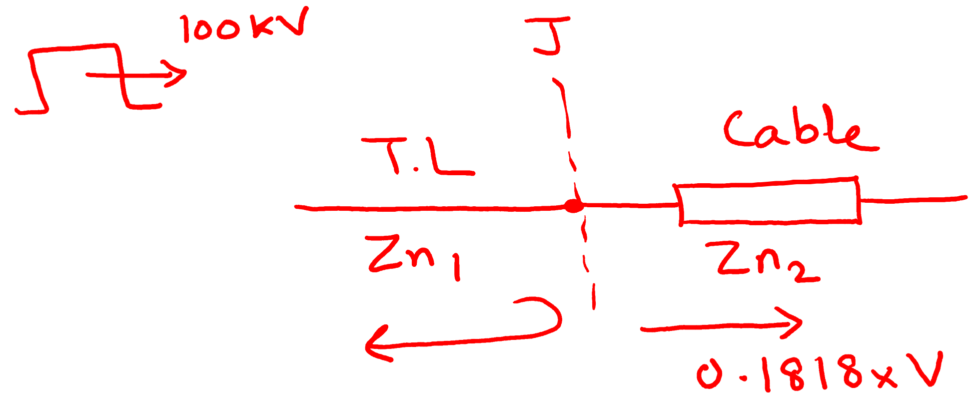
$$= \frac{2V}{Z_{n_1}+Z_{n_2}} = \frac{2Z_{n_1} \times I}{Z_{n_1}+Z_{n_2}}$$

$$I'' = \left(\frac{2Z_{n_1}}{\underbrace{Z_{n_1}+Z_{n_2}}_{\gamma}} \right) \times I \quad \text{———} \quad (7)$$

Problem: An overhead transmission line with Z_n = 400 ohms is connected to an underground cable having Z_n = 40 ohms. A voltage surge of 100 kV is injected on the overhead line and cable. Calculate:

(a) reflected voltage and current at the junction

(b) voltage and current in the cable



$\rightarrow$ $Z_{n_1} = 400\,\Omega$

$Z_{n_2} = 40\,\Omega$

$V = 100\ kV$

(a) $V' = \left(\dfrac{Z_{n_2} - Z_{n_1}}{Z_{n_1} + Z_{n_2}}\right) \times V$

$= \left(\dfrac{40 - 400}{40 + 400}\right) \times 100\,kV = \_\_\_\_$

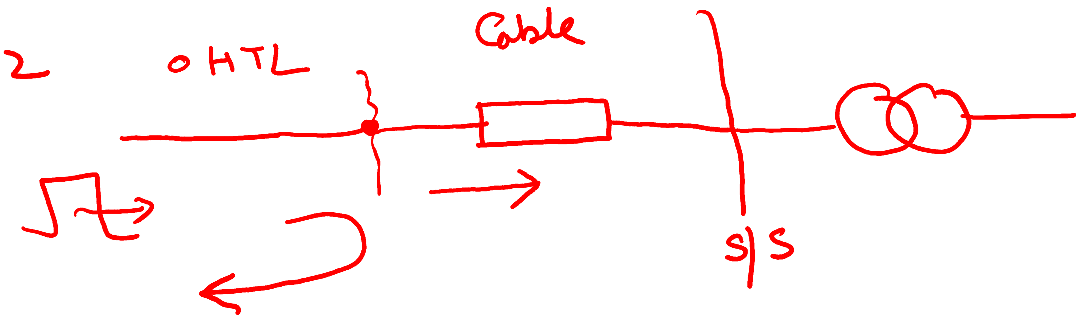
$I' = -\dfrac{V'}{Z_{n_1}} = \_\_\_\_$

(b) $$V'' = \left(\frac{2Z_{n_2}}{Z_{n_1}+Z_{n_2}}\right) \times V$$

$$= \left(\frac{2 \times 40}{400+40}\right) \times 100\ \text{kV}$$

$$= 18.18\ \text{kV}$$

$$I'' = \frac{V''}{Z_{n_2}} = \_\_\_\_\_\_\_\_$$



Problem: A 3-phase, overhead transmission line has equilateral spacing of 1.5 m, radius of each conductor is 10 mm. At the receiving end of the line, a resistive load of 100 ohms is connected in Y-configuration when a voltage surge of 20 kV is applied on the transmission line. Calculate:

(a) power consumed by the load

(b) power reflected

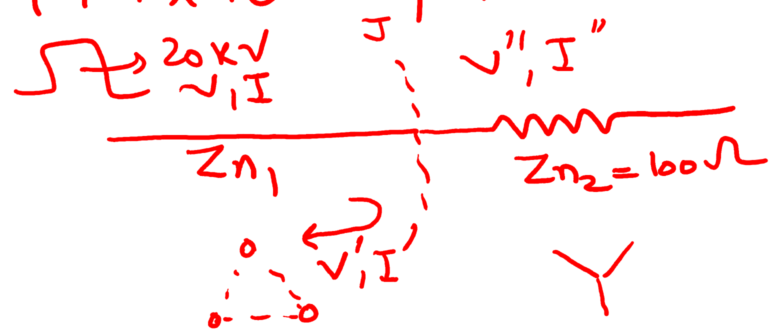
(c) impedance of load at which there is no reflection

$$L = 2\times10^{-7} \ln \frac{D}{r'} \qquad D = 1.5 \qquad r' \rightarrow \underbrace{0.7788}\times 10\times10^{-3}$$

$$= 1.052\times10^{-6} \text{ H/m}$$

$$C = \frac{2\pi\varepsilon_0}{\ln D/r} = 11.0977\times10^{-12} \text{ F/m}$$

$$Z_{n_1} = \sqrt{L/C} = 307.9\,\Omega$$



(a) $V'' = \left(\frac{2Z_{n_2}}{Z_{n_1}+Z_{n_2}}\right) \times \underset{20\,kV}{V} = \left(\rule{2cm}{0.4pt}\right)$

$P_{cons.} = \frac{3V''^2}{Z_{n_2} \rightarrow 100\,\Omega} = \rule{3cm}{0.4pt}$

(b) $V' = \left(\frac{Z_{n_2}-Z_{n_1}}{Z_{n_1}+Z_{n_2}}\right) \times V = \rule{3cm}{0.4pt}$

$P_{ref} = \frac{3V'^2}{Z_{n_1}}$

(c) for no reflection,

$V = V''$, $V' = 0$

$$V' = \left(\frac{Z_{n_2} - Z_{n_1}}{Z_{n_1} + Z_{n_2}} \right) \times V$$

$$Z_{n_1} = Z_{n_2}$$

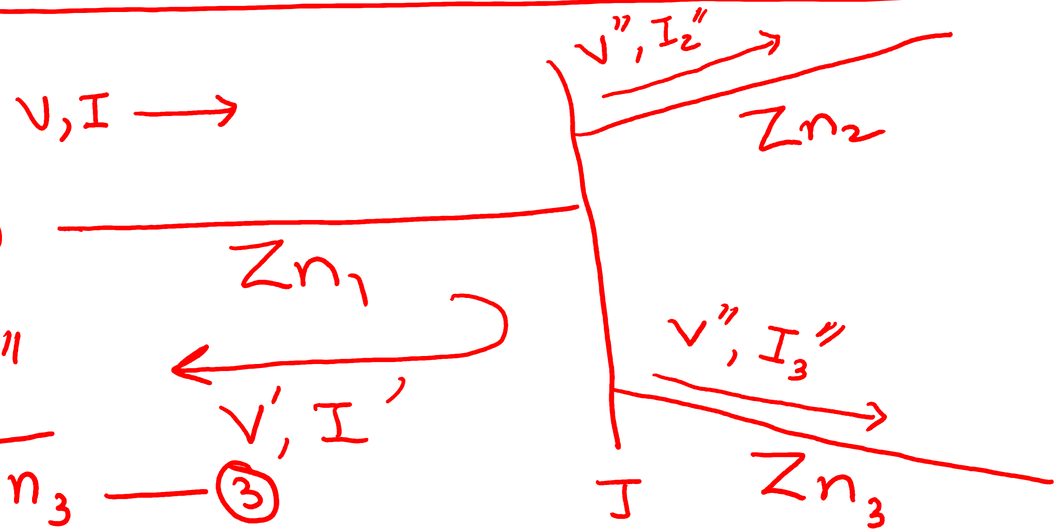
$$\Rightarrow V' = 0$$

$Z_{n_2} = 307.9\,\Omega$ for no-reflection

Transmission & reflection at the bifurcation of two lines

$$V + V' = V'' \quad \text{—} \ (1)$$

$$I + I' = I_2'' + I_3'' \quad \text{—} \ (2)$$



$$\frac{V}{Zn_1} - \frac{V'}{Zn_1} = \frac{V''}{Zn_2} + \frac{V''}{Zn_3} \quad \text{—} \ (3)$$

$$\frac{V}{Zn_1} - \frac{(V''-V)}{Zn_1} = \frac{V''}{Zn_2} + \frac{V''}{Zn_3}$$

$$\frac{V}{Zn_1} + \frac{V}{Zn_1} = \frac{V''}{Zn_1} + \frac{V''}{Zn_2} + \frac{V''}{Zn_3}$$

$$\frac{2V}{Zn_1} = V''\left(\frac{1}{Zn_1} + \frac{1}{Zn_2} + \frac{1}{Zn_3}\right)$$

$$V'' = \frac{(2V/Zn_1)}{\left(\frac{1}{Zn_1} + \frac{1}{Zn_2} + \frac{1}{Zn_3}\right)} \quad \text{--- } 4$$

$$\rightarrow \frac{V}{Zn_1} - \frac{V'}{Zn_1} = \frac{V''}{Zn_2} + \frac{V''}{Zn_3}$$

$$\frac{V}{Zn_1} - \frac{V'}{Zn_1} = \frac{V+V'}{Zn_2} + \frac{V+V'}{Zn_3}$$

$$\frac{V}{Zn_1} - \frac{V}{Zn_2} - \frac{V}{Zn_3} = \frac{V'}{Zn_1} + \frac{V'}{Zn_2} + \frac{V'}{Zn_3}$$

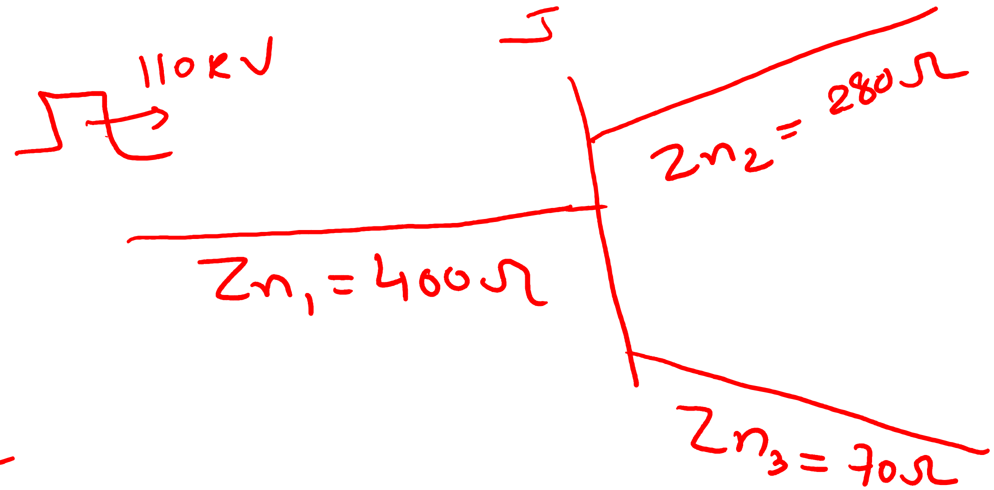
$$V' = \left(\frac{\frac{1}{Zn_1} - \frac{1}{Zn_2} - \frac{1}{Zn_3}}{\frac{1}{Zn_1} + \frac{1}{Zn_2} + \frac{1}{Zn_3}} \right) \times V \qquad \text{—— } (5)$$

Problem: A 3-phase, overhead transmission line has natural impedance of 400 ohms. The line is bifurcated into two lines with natural impedances 280 ohms and 70 ohms. A voltage surge of 110 kV is incident to the line of natural impedance of 400 ohms and travelling towards the other two lines. Calculate the voltage and current in the line of impedance 280 ohms and 70 ohms.

110 kV 400 Ω

$$V'' = \frac{2V / Z_{n_1}}{\frac{1}{Z_{n_1}} + \frac{1}{Z_{n_2}} + \frac{1}{Z_{n_3}}}$$

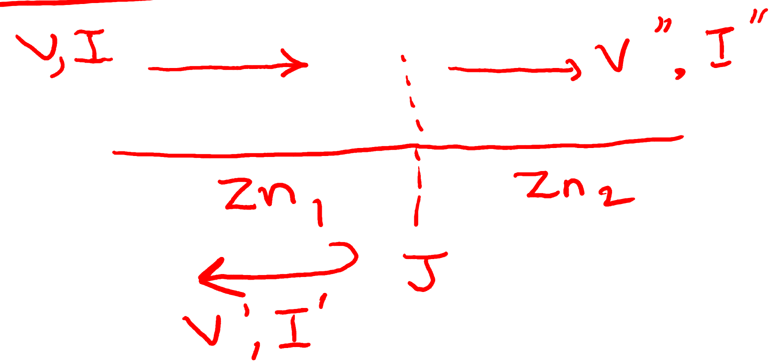
280 Ω 70 Ω



$$I_2'' = \frac{V''}{Z_{n_2}} =$$

$$I_3'' = V'' / Z_{n_2} =$$

Reflection and refraction at the line discontinuity



$$\rightarrow V' = \left(\frac{Zn_2 - Zn_1}{Zn_1 + Zn_2}\right) \times V$$

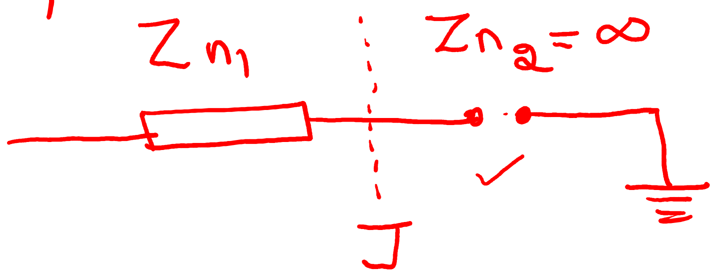
$$I' = -\left(\frac{Zn_2 - Zn_1}{Zn_1 + Zn_2}\right) \times I$$

$$\rightarrow V'' = \left(\frac{2Zn_2}{Zn_1 + Zn_2}\right) \times V$$

$$I'' = \left(\frac{2Zn_1}{Zn_1 + Zn_2}\right) \times I$$

Typical cases of line terminations

(a) Open-circuited line



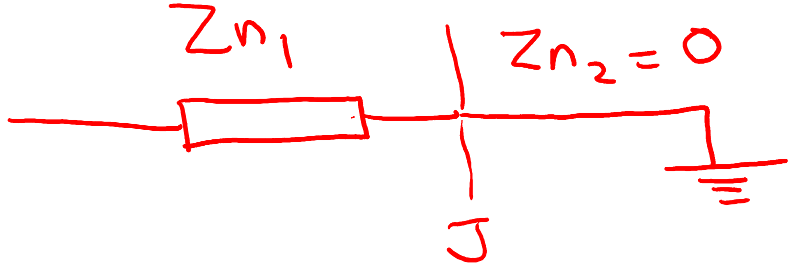
$$V' = \frac{\left(1 - \frac{Z_{n_1}}{Z_{n_2}}\right)}{\left(1 + \frac{Z_{n_1}}{Z_{n_2}}\right)} \times V = V$$

$$I' = -\frac{\left(1 - \frac{Z_{n_1}}{Z_{n_2}}\right)}{\left(1 + \frac{Z_{n_1}}{Z_{n_2}}\right)} \times I = -I$$

$$V'' = \frac{2}{\left(1 + \cancel{\frac{Z_{n_1}}{Z_{n_2}}}^{\to 0}\right)} \times V = 2V$$

$$I'' = \frac{\left(2\,\cancel{\frac{Z_{n_1}}{Z_{n_2}}}^{\to 0}\right)}{\left(1 + \cancel{\frac{Z_{n_1}}{Z_{n_2}}}^{\to 0}\right)} \times I = 0$$

(b) Short circuit T.L



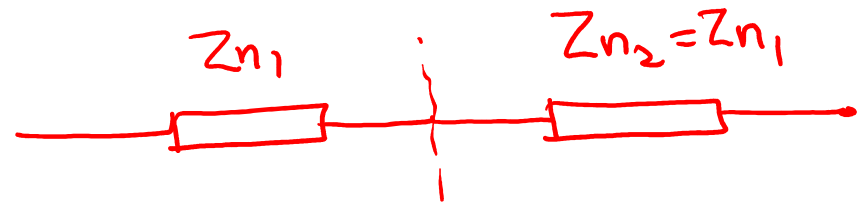
$V' = -V$ ✓

$I' = +I$ ✓

$V'' = 0$ ✓

$I'' = 2I$ ✓

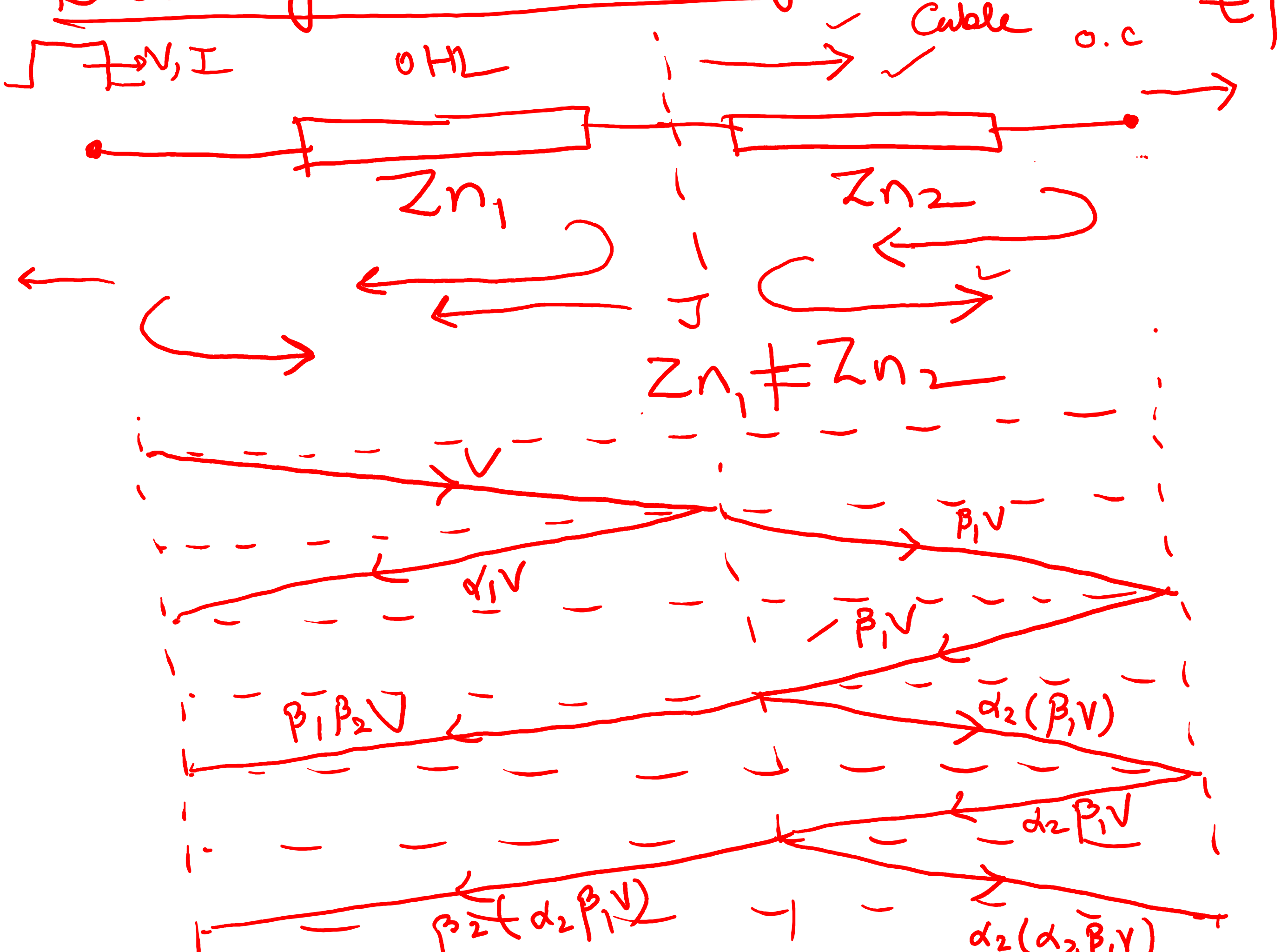
(c) line terminated at Zn



$$\left.\begin{aligned} V' &= 0 \\ I' &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} V'' &= V \\ I'' &= I \end{aligned}\right\}$$

Bewlay's Lattice Diagram



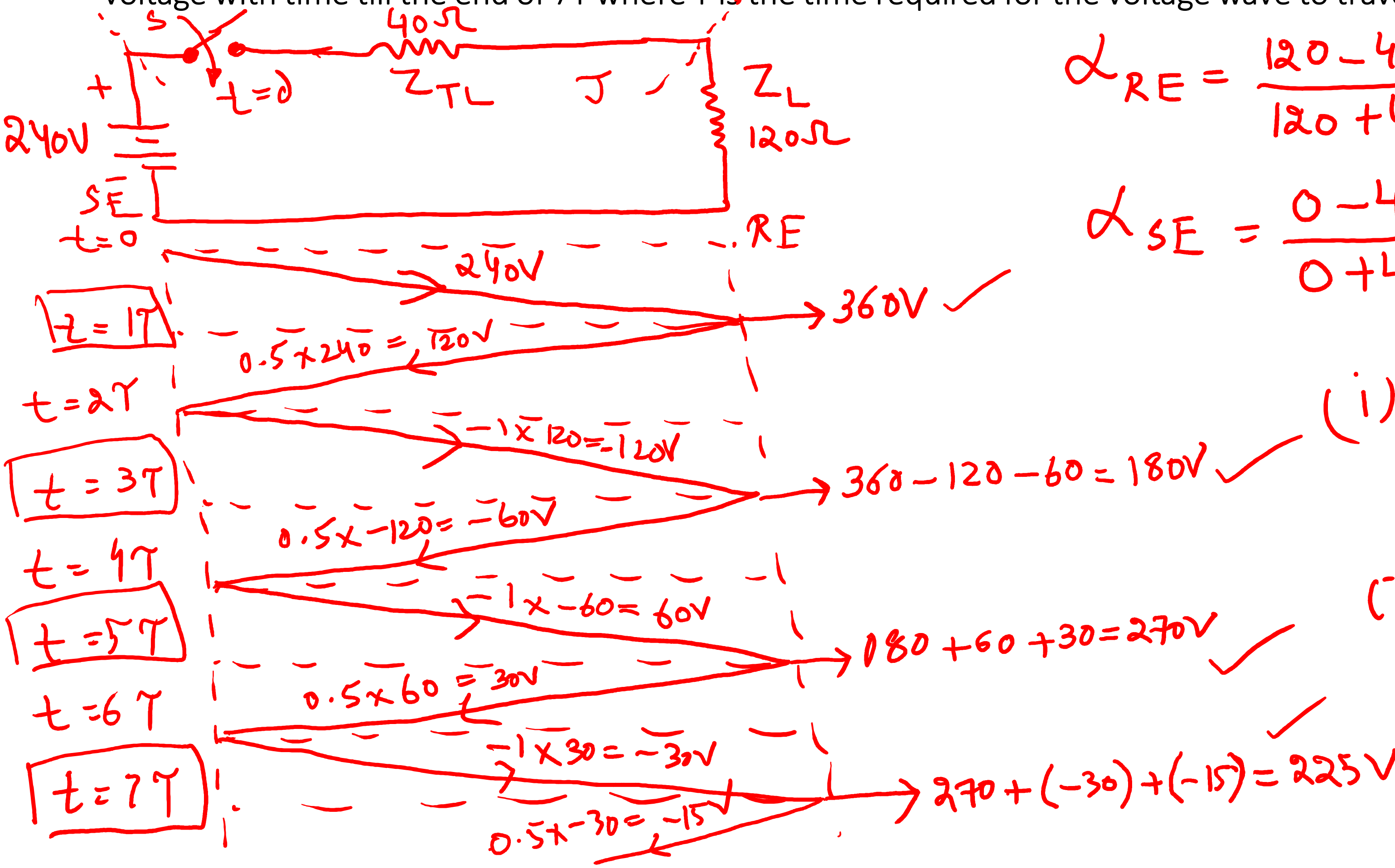
$$\left.\begin{aligned} \alpha_1 &= \frac{Zn_2 - Zn_1}{Zn_1 + Zn_2} \\ \beta_1 &= \frac{2Zn_2}{Zn_1 + Zn_2} \end{aligned}\right\} L-C$$

$$\left.\begin{aligned} \alpha_2 &= \frac{Zn_1 - Zn_2}{Zn_1 + Zn_2} \\ \beta_2 &= \frac{2Zn_1}{Zn_1 + Zn_2} \end{aligned}\right\} C-L$$

$$\alpha_{o.c} = +1$$

$$\beta_{o.c} = +2$$

A lossless transmission line having natural impedance of 40 ohms is terminated on a load of 120 ohms. The source is a DC generator supplying 240 V and generator impedance can be neglected. Plot the variation of receiving-end voltage with time till the end of 7T where T is the time required for the voltage wave to travel the length of line.

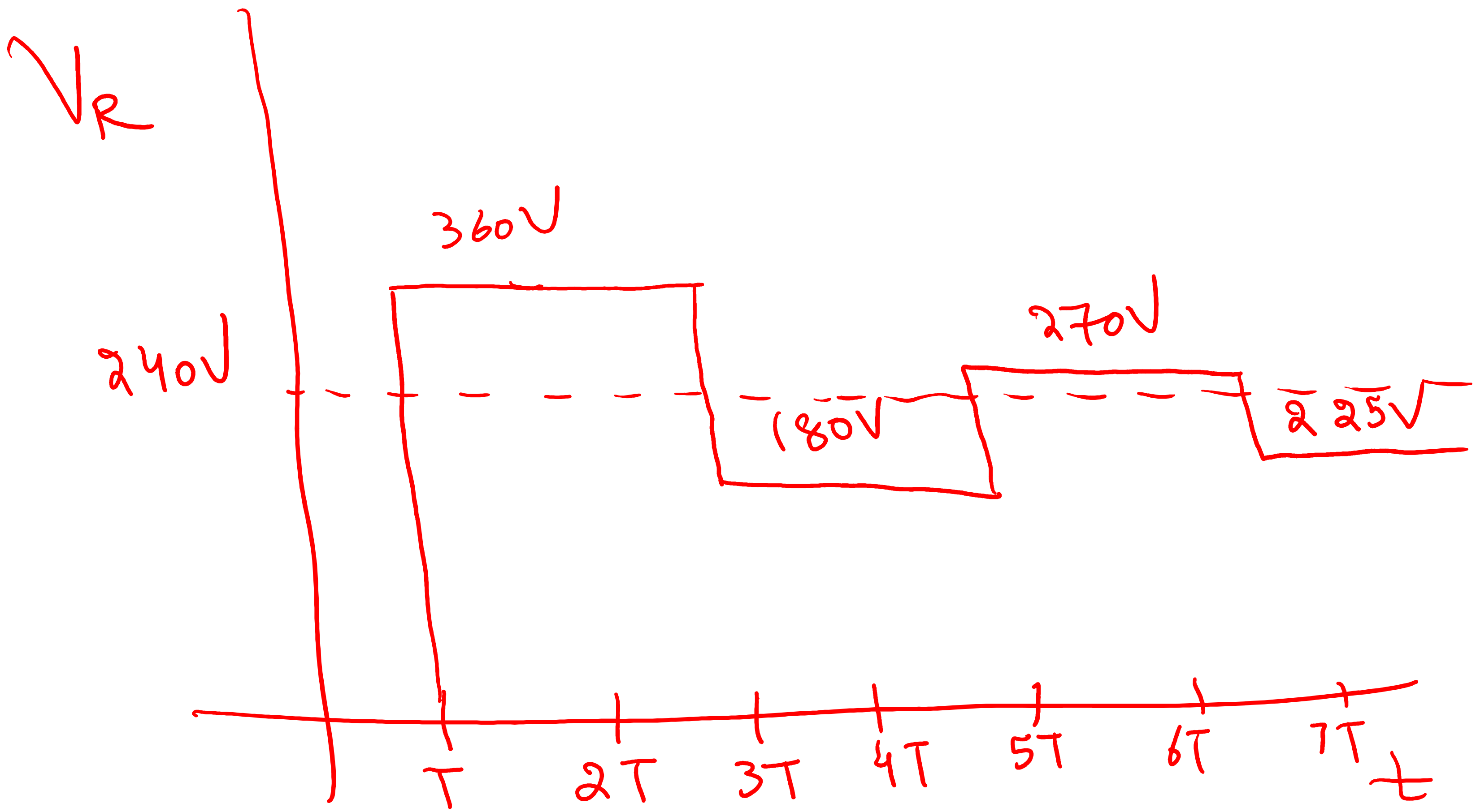


$$\alpha_{RE} = \frac{120-40}{120+40} = 0.5$$

$$\alpha_{SE} = \frac{0-40}{0+40} = -1$$

(i) $Z_{TL} = 40\Omega$, $Z_L = 20\Omega$

(ii) $Z_{TL} = 40\Omega$, $Z_L = 40\Omega$



V_R
360V
240V
180V
270V
225V
T
2T
3T
4T
5T
6T
7T
t

A C.B closes and switches an energized transformer on an overhead transmission line in series with a cable connected to an unloaded transformer. When the CB is closed at t= 0, voltage of supply transformer is at its peak. The following characteristics apply:

OHL: Z_n = 400 ohms
L = 3000 m
v = 3 × 10^5 km/s
T = 10 μs

Cable: Z_n = 40 ohms
L = 100 m
v = 1 × 10^5 km/s
T = 1 μs

Plot the variation in the voltage at the terminals of transformer up to 13 μs.

$$\alpha_{(L-C)} = \frac{40-400}{40+400} = -0.818$$

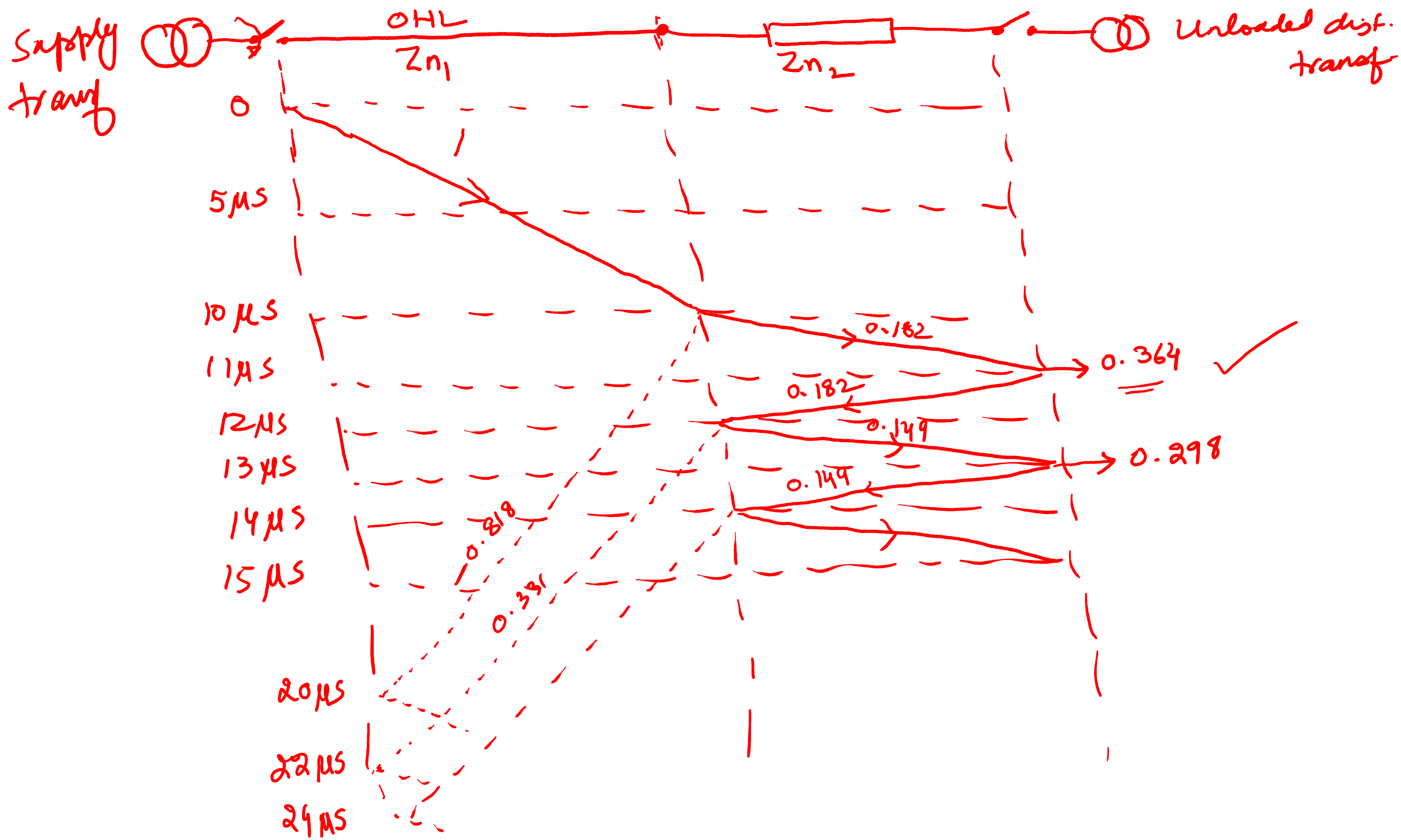
$$\beta_{(C-L)} = \frac{2\times 400}{400+40} = 1.818$$

$$\beta_{(L-C)} = \frac{2\times 40}{40+400} = 0.182$$

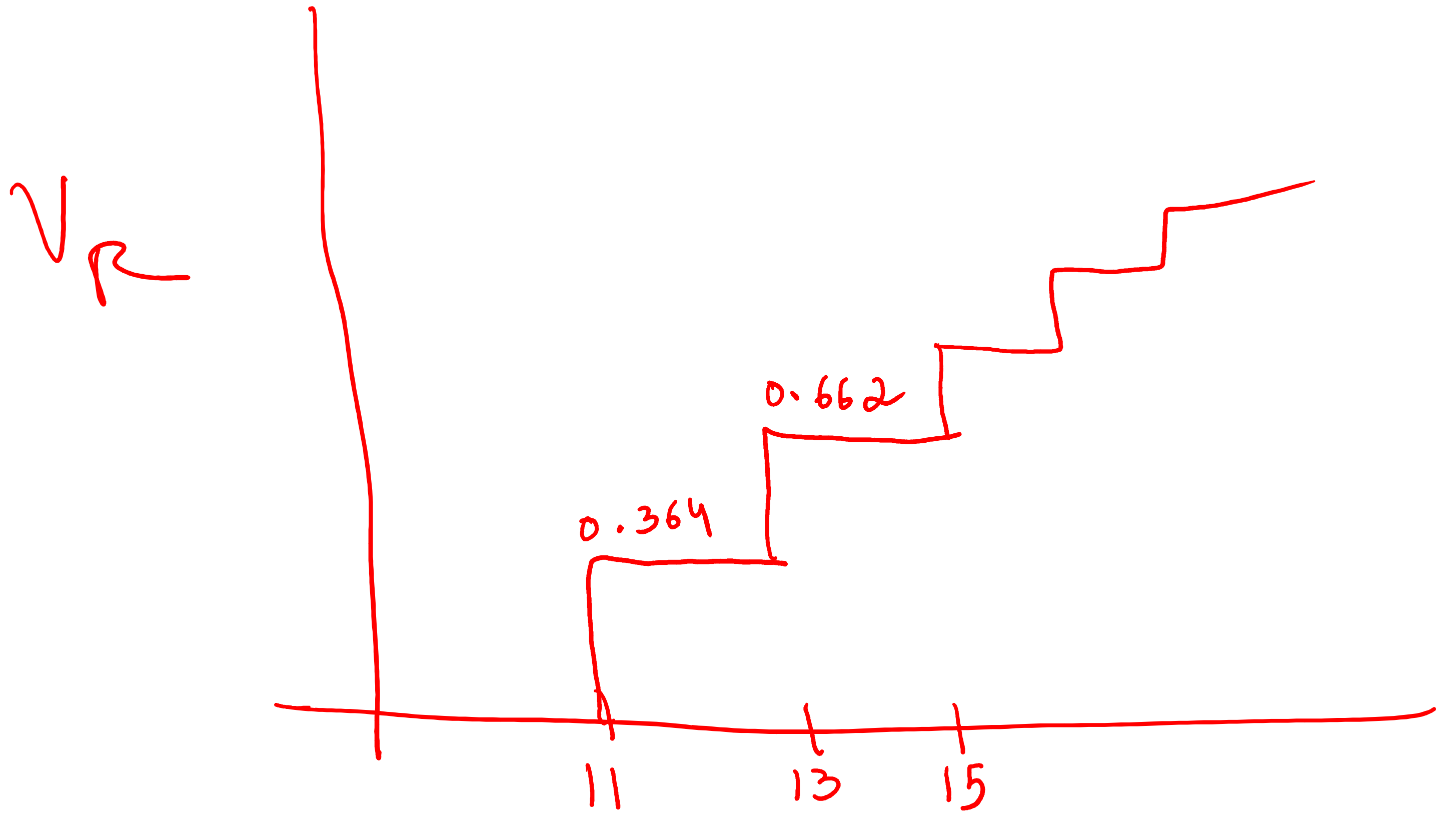
$$\alpha_{(LE)} = +1 \quad (o.c)$$

$$\alpha_{(C-L)} = \frac{400-40}{400+40} = 0.818$$

$$\alpha_{(SE)} = -1 \quad (s.c)$$



Supply transf
OHL
Zn1
Zn2
Unloaded dist. transf
0
5μs
10μs
11μs
12μs
13μs
14μs
15μs
20μs
22μs
24μs
0.182
0.364
0.182
0.149
0.298
0.149
0.818
0.331


V_R
0.364
0.662
11
13
15