Evaluate electric field distribution using finite element method (FEM)

Introduction:

Finite element method is used for solving the differential equation numerically which is found in mathematical modelling. In this method the structured which is to be solved is divided into simple structure which is finite sized and then the structure is solved.

FEM is used to determine the electric field in power substation equipment such as 11KV bushing, insulators etc. The following are the steps involved for developing FEM model of insulator/bushing

a) The insulator geometry to the scale is plotted on a transparent paper or graph paper

b) Since insulator cross-section may have non- linear curves and contours, several points on the geometry on the insulator are marked as key points numbering

c) The x‟ and y‟ coordinates of these key points are noted with care so that maximum key points define the curved.

The magnitude of electric field and voltage distribution for the materials transformer oil, Bakelite, porcelain and air have been calculated. For verifying the results three test models have been taken. In test model 1 and test model 2 a parallel rectangular plate model has been taken in which dielectric is varying four times as in our actual transformer bushing model and in test model 3 a transformer bushing model has been considered and results validation has been done by FEM. If the dielectric strength of all media are known, then maximum withstand voltage of the system can be predicted.

The finite element method (FEM) is used in this article for calculations related to electrical engineering. This program can be used to help create an effective joint in any structure that has moving or sliding surfaces in contact with each other. This method will work effectively in determining the most optimal material design for any surface that requires friction. In order to effectively use this program, you must know how to input data into the computer by using precise formulas and formulas that are very well defined and specific.

To visualize the electric field distribution on different electrical equipment, FEM method is applicable. A finite element method (FEM) is a computer-aided method that allows the simulation of materials. In electrical engineering, this software program approximates the behavior of material objects with mathematical equations. A way to use FEM in Problem Solving and Designing is with alternating current calculations to reduce field losses, or voltages when conducting current.

For the reason of unavailability of software and better understanding of this FEM method, we use this method for the solution differential equation as well as problems related to electric circuits. We also do analysis by using MATLAB software.

Theoretical analysis:

Consider an irregular domain of finite element subdivision, which is as follows:

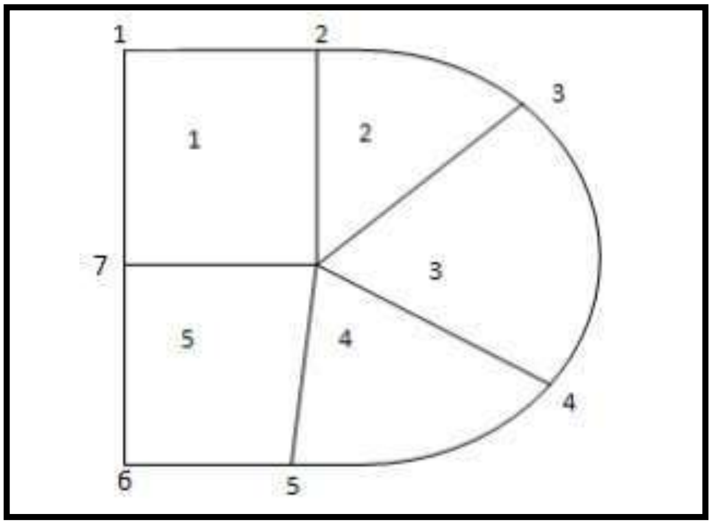


Fig.1 FE subdivision of irregular domain

In this case, the area is separated into four non-overlapping components and seven nodes. We look for an estimate for the potential Ve inside an element 'e' and then interrelate the potential distributions in different elements so that the potential is continuous across inter-element borders. The approximate answers for the entire region are as follows:

$$V(x,y) = \sum_{e=0}^{N} V_e(x, y)$$

Where N is the number of triangular components that make up the solution region. Polynomial approximation is the most frequent type of approximation for inside an element, namely,

$$V_e(x, y) = a + bx + cy$$

For a triangular element:

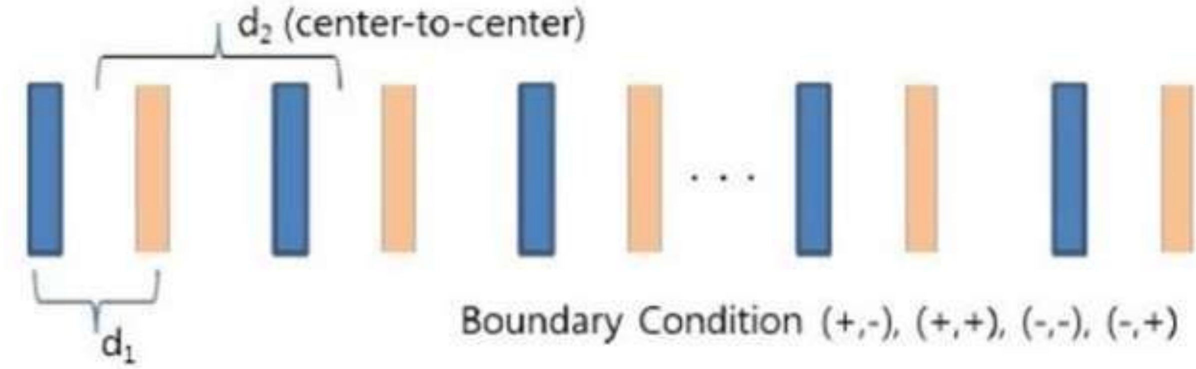
$$V_e(x, y) = a + bx + cy + dxy$$

In the case of a quadrilateral element in general, the potential is nonzero within element "e," but zero outside of "e." Quadrilateral elements are difficult to approximate the border of the solution region; such elements are suitable for problems with sufficiently regular boundaries. Assumption of linear fluctuation of potential inside triangular components is equivalent to assumption of homogeneous electric field within the element; that is,

$$Ee = -V \times Ve = -(ba_x + ca_y)$$

Example:

There are two dimensional (xy-plane) plates with uniform surface charge density +σ (blue: e.g. 0.4nm × 0.4nm with +e) and –σ (red: e.g. 0.4nm × 0.4nm with –e). Using Finite element method based analysis for electrostatic, solve Maxwell equation for this case and plot the electric field and potential.



Solution:

Area of plate (A) = d × w

A = 0.4nm × 0.4nm

A = 0.16e-9m^2

On blue plate:

Surface charge density = +σ

$$+\sigma = \frac{Q}{A}$$

On red plate:

Surface charge density = -σ

$$-\sigma = -\frac{Q}{A}$$

For infinitely long parallel plates

$$\text{Electric field} = E = \frac{\sigma}{2\epsilon_o}$$

Electric field between two plates:

$$E_{total} = |E\sigma+| - |E\sigma-|$$

$$E_{total} = \frac{\sigma}{2\epsilon_o} - \left(-\frac{\sigma}{2\epsilon_o}\right)$$

$$E_{total} = \frac{2\sigma}{2\epsilon_o}$$

$$\boxed{E_{total} = \frac{\sigma}{\epsilon_o}}$$

With respect to perpendicular distance, the parameter Z, E is constant.

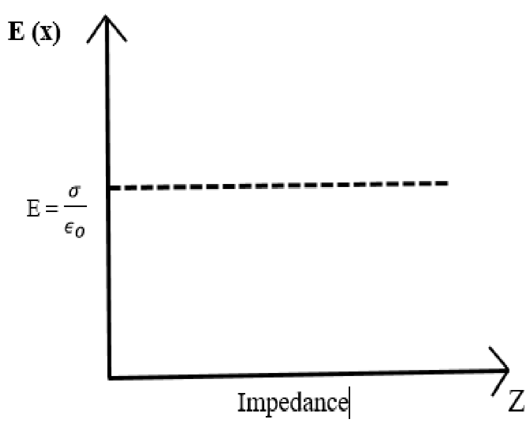
So,

$$\text{Potential} = V = E \times d$$

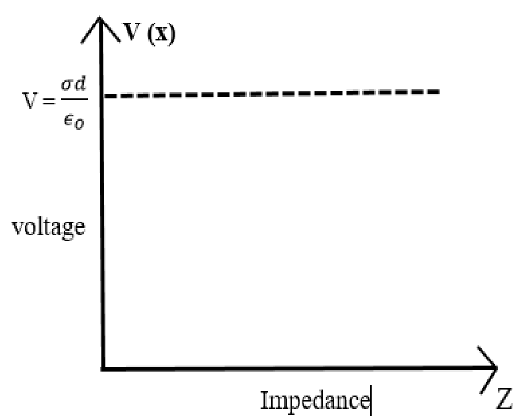
$$V = \frac{\sigma \times d}{\epsilon_o}$$

Here potential is also constant.

(Electric Field)



(Potential)



Matlab code:

```
% program for 2D problem using finite element method.
%2018-EE-361(Section A)(HV-CEP)
NE=25;
ND=21;
NP=15;
NL=[1 2 7 ; 2 8 7 ; 2 3 8 ; 3 9 8 ; 3 4 9 ; 4 10 9
    4 5 10 ; 5 11 10 ; 5 6 11 ; 7 8 12 ; 8 13 12
    8 9 13 ; 9 14 13 ; 9 10 14 ; 10 15 14 ; 10 11 15
    12 13 16 ; 13 17 16 ; 13 14 17 ; 14 18 17 ; 14 15 18
    16 17 19 ; 17 20 19 ; 17 18 20 ; 19 20 21];
X=[0.0 0.2 0.4 0.6 0.8 1.0 0.0 0.2 0.4 0.6 0.8 0.0 0.2 0.4 0.6 0.0 0.2 0.4
0.0 0.2 0.0];
Y=[0.0 0.0 0.0 0.0 0.0 0.0 0.2 0.2 0.2 0.2 0.2 0.4 0.4 0.4 0.4 0.6 0.6 0.6
0.8 0.8 1.0];
NDP=[1 2 3 4 5 6 11 15 18 20 21 19 16 12 7];
VAL=[0.0 0.0 0.0 0.0 0.0 0.0 50.0 100.0 100.0 100.0 100.0 50.0 0.0 0.0 0.0 0.0];
B=zeros(ND,1);
C=zeros(ND,ND);
for I=1:NE
 K=NL(I,[1:3]);
 XL=X(K);
 YL=Y(K);
 P=zeros(3,1);
 Q=zeros(3,1);
 P(1)=YL(2)-YL(3);
 P(2)=YL(3)-YL(1);
 P(3)=YL(1)-YL(2);
 Q(1)=XL(3)-XL(2);
 Q(2)=XL(1)-XL(3);
 Q(3)=XL(2)-XL(1);
 AREA =0.5*abs(P(2)*Q(3)-Q(2)*P(3));
 CE=(P*P'+Q*Q')/(4.0*(AREA+0.0000000000000000000000001));
 for J=1:3
 IR=NL(I,J);
 IFLAG1=0;
 for K=1:NP
 if(IR==NDP(K))
 C(IR,IR)=1.0;
 B(IR)=VAL(K);
 IFLAG1=1;
 end
end
 if(IFLAG1==0)
 for L=1:3
 IC=NL(I,L);
 IFLAG2=0;
 for K=1:NP
 if (IC==NDP(K)),
 B(IR)=B(IR)-CE(J,L)*VAL(K);
 IFLAG2=1;
 end
 end
 if(IFLAG2==0)
 C(IR,IC)=C(IR,IC)+CE(J,L);
 end
 end
 end
```

```
 end
end
V=inv(C)*B;
V=V';
display('Value of ND NE NP')
[ND, NE, NP]
[ [1:ND]' X' Y' V']
```

Output:

Editor - E:\8th samester\High voltage Engineering\cep.m

Command Window

```
>> cep
Value of ND NE NP

ans =

    21    25    15


ans =

    1.0000         0         0         0
    2.0000    0.2000         0         0
    3.0000    0.4000         0         0
    4.0000    0.6000         0         0
    5.0000    0.8000         0         0
    6.0000    1.0000         0   50.0000
    7.0000         0    0.2000         0
    8.0000    0.2000    0.2000   18.1818
    9.0000    0.4000    0.2000   36.3636
   10.0000    0.6000    0.2000   59.0909
   11.0000    0.8000    0.2000  100.0000
   12.0000         0    0.4000         0
   13.0000    0.2000    0.4000   36.3636
   14.0000    0.4000    0.4000   68.1818
   15.0000    0.6000    0.4000  100.0000
   16.0000         0    0.6000         0
   17.0000    0.2000    0.6000   59.0909
   18.0000    0.4000    0.6000  100.0000
   19.0000         0    0.8000         0
   20.0000    0.2000    0.8000  100.0000
   21.0000         0    1.0000   50.0000
fx
```

Application:

- Thermal and Electrical Analysis
- Computer aided design and simulation services
- Model Analysis
- It has application in ANSYS software as well
- Also used for Heat and stress analysis