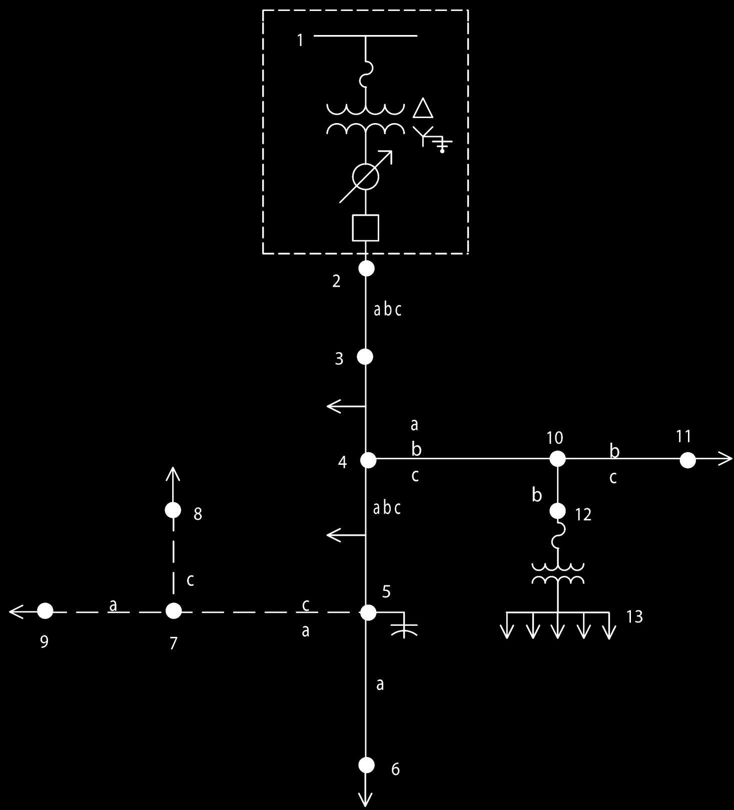
Third Edition

Distribution System Modeling and Analysis



William H. Kersting

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Distribution System Modeling and Analysis

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Preface

Since the first edition of this book, many changes have transpired in the power industry and, in particular, the distribution system. The word "smart grid" keeps popping up in technical journals. What exactly does this mean? There are a variety of answers to this question, and for all these answers, it is essential to be able to model the system as accurately as possible. This book addresses the distribution system. In previous editions of the book, models were developed for all of the components but little consideration was given to the implementation of the models into a computer for purposes of planning and for real-time analysis. This book develops additional models of components. A major effort has been undertaken to demonstrate, through the use of several examples, computer programs that can be developed to assist the engineer in the planning and operation of present and future systems. In particular, starting in Chapter 4, simple programs using Mathcad® [1] are developed. Throughout the book, as new component models are developed, an example will demonstrate how a Mathcad program can simplify the analysis. The student/engineer is encouraged to write his or her own program for many of the homework assignments. While I prefer to use Mathcad, it does not mean that other programming languages cannot be used. Before my retirement, I required that students submit all homework assignments and exams in Mathcad.

As more and more components are developed and test feeders become more complicated, it becomes obvious that a more sophisticated program is needed. A major addition to this edition is the use of Milsoft Utility Solutions, Inc.'s distribution analysis program "Windmil" [2]. Milsoft is making available a student's version of Windmil along with a user manual. The user manual will include instructions and illustrations on how to get started using the program. Many of the example problems in the book will be included in the user manual. Starting in Chapter 4, there will be a Windmil assignment at the end of the homework problems. A very simple system utilizing all of the major components of the system will evolve as each chapter assignment is completed. In Chapter 10, data for a small system are given, which will allow the student/engineer to build the system in Windmil and then to study and make changes to the system to match operating criteria. The student's version of Windmil and the user manual can be downloaded from the Milsoft website homepage:

Milsoft Utility Solutions, Inc.
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Abilene, TX 79608
E-mail: support@milsoft.com
Homepage: www.milsoft.com

Unfortunately, there is a tendency on the part of students/engineers to believe the results of a computer program. While computer programs are a wonderful tool, it is still the responsibility of the users to study the results and confirm whether or not the results make sense. That is a major concern and one that is addressed throughout the book. In order to feel comfortable with computer results, the user needs to have a feel for what is a ballpark answer. This requires a good understanding of the major components of a distribution system (addressed in Chapter 1). Chapter 2 addresses the important question, what is the load on the system? This chapter defines the common terms associated with the load. In the past, there was limited knowledge about the load and many assumptions had to be made. With the coming of the smart grid, there will be ample real-time data to assist in defining the load for a given study. Even with better load data, there will still be a major concern about whether or not the computer results make sense. Chapter 3 may seem outdated and of not much use because it develops some approximate methods for developing a feel for ballpark answers. I was tempted not to include Chapter 3. However, I still strongly believe that it is critical for the student/engineer to have a feel for what the computed results should be. I often reflect on the past, when the slide rule was the major calculating tool for students/engineers. Because it was not possible to include the decimal point using a slide rule, it was necessary to know what an approximate answer would be. The same can be said for interpreting the results of a computer program.

The major requirement of a distribution system is to supply safe, reliable energy to every customer at a voltage within the ANSI standard. The major goal of planning is to simulate the distribution system under different conditions now and into the future and assure that all customer voltages are in the acceptable range. Since voltage drop is a major concern, it is extremely important that the impedances of the system components be as accurate as possible. In particular, the impedances of the overhead and underground distribution lines must be computed as accurately as possible. Chapters 4 and 5 address this issue. Chapter 6 develops the models for the overhead and underground lines using the impedances and admittances computed in the earlier chapters.

Chapter 7 addresses the important concept of voltage regulation. How is it possible to maintain every customer's voltage within the standard as the load varies all the time? The step voltage regulator is presented as one answer to this question. The theory is outlined and a model developed for various connections.

Chapter 8 is one of the more important chapters in the book. Models for most three-phase transformer connections in use today are developed. Again, it is extremely important that these models accurately simulate the actual operating conditions of the transformers. Chapter 11 addresses the models of center-tapped transformers that provide the 120/240 V service to most customers.

Chapter 9 develops models for all types of loads on the system. A new term has been introduced that helps define the types of static load models: *ZIP*. Most static models in a distribution system can be modeled as constant impedance (Z), constant current (I), or constant complex power (P), or a combination of the three. These models are developed for wye and delta connections. A very important model developed is that of an induction machine. The induction motor is the workhorse of the power system and needs, once again, to be modeled as accurately as possible. Induction generators are becoming a major source of distributed generation. Chapter 9 shows that an induction machine can be modeled as either a motor or a generator.

Finally, all the points discussed in the book come together in Chapter 10. This chapter develops the modified ladder iterative technique used in most distribution analysis programs. In the first two editions, this technique was not introduced until Chapter 10. In the third edition, the need for the iterative technique is addressed in Chapter 4. A very simple explanation of the technique is introduced in this chapter and used in all of the following chapters. Chapter 10 goes into great detail on how and why this method is one of most powerful iterative techniques for the analysis of distribution systems. As mentioned earlier, the Windmil homework assignments at the end of Chapters 10 and 11 allow the student/engineer to finally build and then study and fix the operating characteristics of a small distribution feeder.

For MATLAB® and Simulink® product information, please contact:

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1. PTC Corporate Headquarters, 140 Kendrick Street, Needham, MA 02494. Homepage: www.ptc.com/products/mathcad/
2. Milsoft Utility Solutions, Inc., P.O. Box 7526, Abilene, TX 79608. E-mail: support@milsoft.com Homepage: www.milsoft.com

Acknowledgments

I would like to thank the many students and engineers who have communicated with me via e-mail their questions about some of the contents of the second edition. It has been a pleasure to work with these individuals in helping them to better understand some of the models and applications in the book. Since I am now retired, it has been a real pleasure to have the opportunity to work with many graduate students conducting research involving distribution systems. I hope that students and engineers will continue to feel free to contact me at bjkersting@zianet.com.

Special thanks to Wayne Carr, the CEO of Milsoft Utility Solutions, Inc., for allowing me to make Windmil a major new part of this third edition. Thanks also to the many support engineers at Milsoft who have guided me in developing the special assignments.

As always, I would like to thank my wife, Joanne, who has been very supportive of me in spite of the fact that I have spent most of my time revising the book.

Author

William H. Kersting received his BSEE from New Mexico State University (NMSU), Las Cruces, and his MSEE from the Illinois Institute of Technology. He joined the faculty at New Mexico State University in 1962 and served as professor of electrical engineering and director of the Electric Utility Management Program until his retirement in 2002. He is currently a consultant for Milsoft Utility Solutions. He is also a partner in WH Power Consultants, Las Cruces, New Mexico.

Professor Kersting is a life fellow of the Institute of Electrical and Electronics Engineers. He received the Edison Electric Institute's Power Engineering Educator Award in 1979 and the NMSU Westhafter Award for Excellence in Teaching in 1977. Prior to joining NMSU, he was employed as a distribution engineer at El Paso Electric Company. Professor Kersting has been an active member of the IEEE Power Engineering Education Committee and the Distribution System Analysis Subcommittee.

1

Introduction to Distribution Systems

The major components of an electric power system are shown in Figure 1.1. Of these components, the distribution system has traditionally been characterized as the most unglamorous component. In the last half of the twentieth century, the design and operation of the generation and transmission components presented many challenges to the practicing engineer and researchers. Power plants became larger and larger; transmission lines crisscrossed the land forming large interconnected networks. The operation of the large interconnected networks required the development of new analysis and operational techniques. Meanwhile, the distribution systems continued to deliver power to the ultimate user's meter with little or no analysis. As a direct result, distribution systems were typically over designed.

Times have changed. It has become very important and necessary to operate a distribution system at its maximum capacity. Some of the questions that need to be answered are

1. What is the maximum capacity?
2. How do we determine this capacity?
3. What are the operating limits that must be satisfied?
4. What can be done to operate the distribution system within the operating limits?
5. What can be done to make the distribution system operate more efficiently?

All of these questions can be answered only if the distribution system can be modeled very accurately.

The purpose of this chapter is to develop accurate models for all of the major components of a distribution system. Once the models have been developed, analysis techniques for steady-state and short-circuit conditions will be developed.

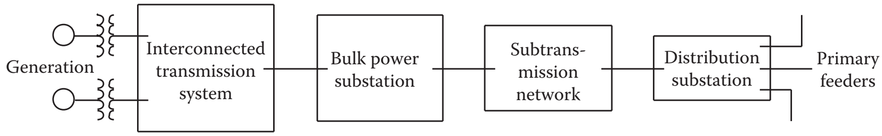

FIGURE 1.1
Major power system components.

1.1 Distribution System

The distribution system typically starts with the distribution substation that is fed by one or more subtransmission lines. In some cases, the distribution substation is fed directly from a high-voltage transmission line in which case, most likely, there is not a subtransmission system. This varies from company to company. Each distribution substation will serve one or more primary feeders. With a rare exception, the feeders are radial, which means that there is only one path for power to flow from the distribution substation to the user.

1.2 Distribution Substations

A one-line diagram of a very simple distribution substation is shown in Figure 1.2.

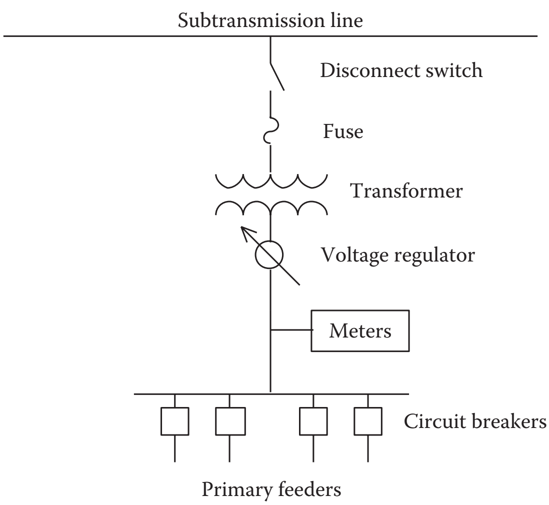

FIGURE 1.2
Simple distribution substation.

Although Figure 1.2 displays the most simple of distribution substations, it does illustrate the major components that will be found in all substations:

1. *High-side and low-side switching*: In Figure 1.2, the high-voltage switching is done with a simple switch. More extensive substations may use high-voltage circuit breakers in a variety of high-voltage bus designs. The low-voltage switching in Figure 1.2 is accomplished with relay-controlled circuit breakers. In many cases, reclosers will be used in place of the relay/circuit breaker combination. Some substation designs will include a low-voltage bus circuit breaker in addition to the circuit breakers for each feeder. As is the case with the high-voltage bus, the low-voltage bus can take on a variety of designs.
2. *Voltage transformation*: The primary function of a distribution substation is to reduce the voltage down to the distribution voltage level. In Figure 1.2, only one transformer is shown. Other substation designs will call for two or more three-phase transformers. The substation transformers can be three-phase units or three single-phase units connected in a standard connection. There are many "standard" distribution voltage levels. Some of the common are 34.5, 23.9, 14.4, 13.2, 12.47, and, in older systems, 4.16 kV.
3. *Voltage regulation*: As the load on the feeders vary, the voltage drop between the substation and the user will vary. In order to maintain the user's voltages within an acceptable range, the voltage at the substation needs to vary as the load varies. In Figure 1.2, the voltage is regulated by a "step-type" regulator that will vary the voltage plus or minus 10% on the low-side bus. Sometimes this function is accomplished with a "load tap changing" (LTC) transformer. The LTC changes the taps on the low-voltage windings of the transformer as the load varies. Many substation transformers will have "fixed taps" on the high-voltage winding. These are used when the source voltage is always either above or below the nominal voltage. The fixed tap settings can vary the voltage plus or minus 5%. Many times instead of a bus regulator, each feeder will have its own regulator. This can be in the form of a three-phase gang-operated regulator or individual phase regulators that operate independently.
4. *Protection*: The substation must be protected against the occurrence of short circuits. In the simple design of Figure 1.2, the only automatic protection against short circuits inside the substation is by way of the high-side fuses on the transformer. As the substation designs become more complex, more extensive protective schemes will be employed to protect the transformer, the high- and low-voltage buses, and any other piece of equipment. Individual feeder circuit breakers or reclosers are used to provide interruption of short circuits that occur outside the substation.

5. *Metering*: Every substation has some form of metering. This may be as simple as an analog ammeter displaying the present value of substation current as well as the minimum and maximum currents that have occurred over a specific time period. Digital recording meters are becoming very common. These meters record the minimum, average, and maximum values of current, voltage, power, power factor, etc., over a specified time range. Typical time ranges are 15 min, 30 min, and 1 h. The digital meters may monitor the output of each substation transformer and/or the output of each feeder.

A more comprehensive substation layout is shown in Figure 1.3.

The substation of Figure 1.3 has two LTC transformers, serves four distribution feeders, and is fed from two subtransmission lines. Under normal conditions, the circuit breakers (CB) are in the following positions:

Circuit breakers closed: *X*, *Y*, 1, 3, 4, 6

Circuit breakers open: *Z*, 2, 5

With the breakers in their normal positions, each transformer is served from a different subtransmission line and serves two feeders. Should one of the subtransmission lines go out of service then breaker *X* or *Y* is opened and breaker

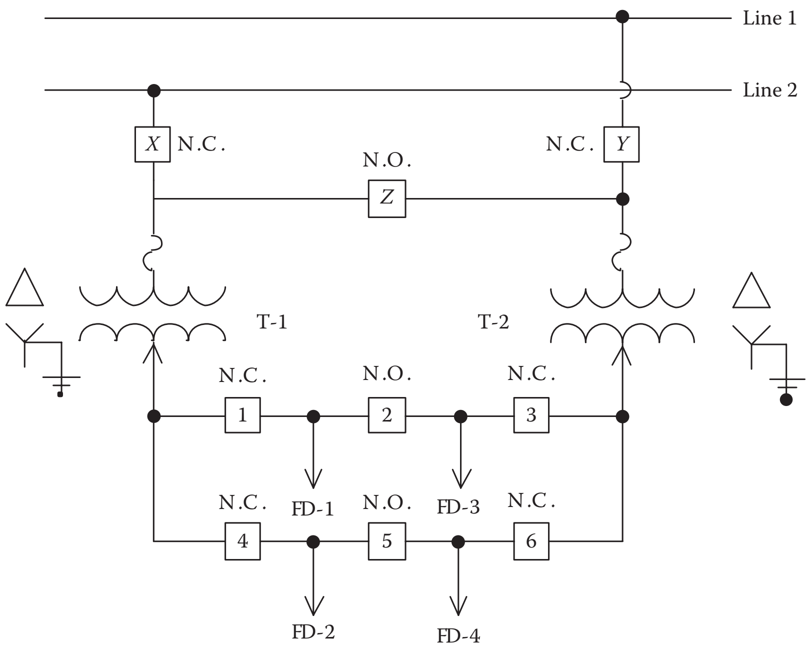

FIGURE 1.3
Two transformer substation with breaker-and-a-half scheme.

Z is closed. Now both transformers are served from the same subtransmission line. The transformers are sized such that each transformer can supply all four feeders under an emergency operating condition. For example, if transformer T-1 is out of service, then breakers *X*, 1, and 4 are opened and breakers 2 and 5 are closed. With that breaker arrangement, all four feeders are served by transformer T-2. The low-voltage bus arrangement is referred to as a "breaker-and-a-half scheme" since three breakers are required to serve two feeders.

There are an unlimited number of substation configurations possible. It is up to the substation design engineer to create a design that provides the five basic functions and the most reliable service economically possible.

1.3 Radial Feeders

Radial distribution feeders are characterized by having only one path for power to flow from the source ("distribution substation") to each customer. A typical distribution system will consist of one or more distribution substations consisting of one or more "feeders." Components of the feeder may consist of the following:

1. Three-phase primary "main" feeder
2. Three-phase, two-phase ("V" phase), and single-phase laterals
3. Step-type voltage regulators
4. In-line transformers
5. Shunt capacitor banks
6. Distribution transformers
7. Secondaries
8. Three-phase, two-phase, and single-phase loads

The loading of a distribution feeder is inherently unbalanced because of the large number of unequal single-phase loads that must be served. An additional unbalance is introduced by the nonequilateral conductor spacings of the three-phase overhead and underground line segments.

Because of the nature of the distribution system, conventional power-flow and short-circuit programs used for transmission system studies are not adequate. Such programs display poor convergence characteristics for radial systems. The programs also assume a perfectly balanced system so that a single-phase equivalent system is used.

If a distribution engineer is to be able to perform accurate power-flow and short-circuit studies, it is imperative that the distribution feeder be modeled as accurately as possible. This means that three-phase models of the major

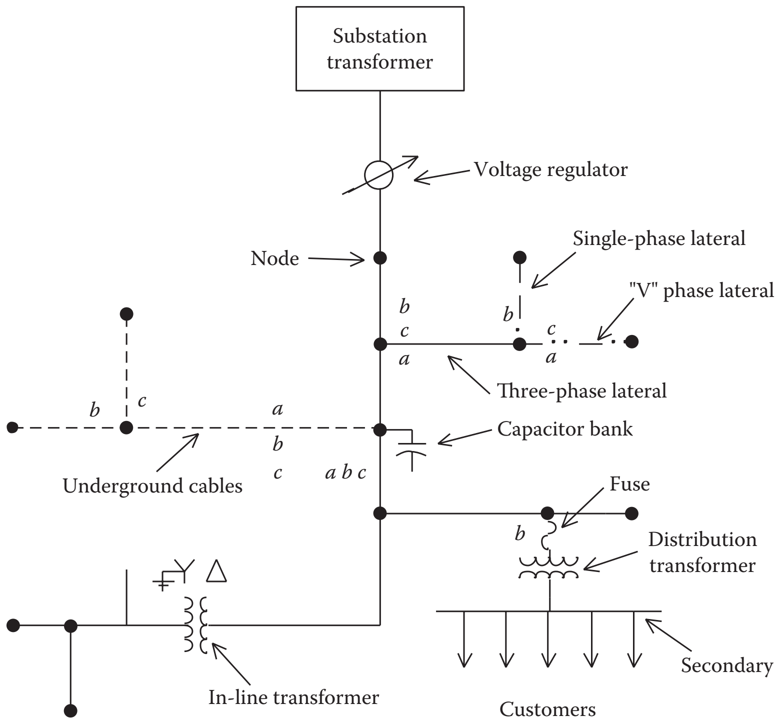

FIGURE 1.4
Simple distribution feeder.

components must be utilized. Three-phase models for the major components will be developed in the following chapters. The models will be developed in the "phase frame" rather than applying the method of symmetrical components.

Figure 1.4 shows a simple "one-line" diagram of a three-phase feeder. This figure illustrates the major components of a distribution system. The connecting points of the components will be referred to as "nodes." Note in the figure that the phasing of the line segments is shown. This is important if the most accurate models are to be developed.

1.4 Distribution Feeder Map

The analysis of a distribution feeder is important to an engineer in order to determine the existing operating conditions of a feeder and to be able to play the "what if" scenarios of future changes to the feeder. Before the engineer can perform the analysis of a feeder, a detailed map of the feeder must be available. A sample of such a map is shown in Figure 1.5.

FIGURE 1.5
123 Node test feeder.

The map of Figure 1.5 contains most of the following information:

1. Lines (overhead and underground)
   a. Where
   b. Distances
   c. Details
      i. Conductor sizes (not on this map)
      ii. Phasing
2. Distribution transformers
   a. Location
   b. kVA rating
   c. Phase connection
3. In-line transformers
   a. Location
   b. kVA rating
   c. Connection
4. Shunt capacitors
   a. Location
   b. kvar rating
   c. Phase connection
5. Voltage regulators
   a. Location
   b. Phase connection
   c. Type (not shown on this map)
      i. Single phase
      ii. Three phase
6. Switches
   a. Location
   b. Normal open/close status

1.5 Distribution Feeder Electrical Characteristics

Information from the map will define the physical location of the various devices. Electrical characteristics for each device will have to be determined before the analysis of the feeder can commence. In order to determine the electrical characteristics the following data must be available:

1. Overhead and underground spacings
2. Conductor tables
    a. Geometric mean radius (GMR) (ft)
    b. Diameter (in.)
    c. Resistance (Ω/mile)
3. Voltage regulators
    a. Potential transformer ratios
    b. Current transformer ratios
    c. Compensator settings
        i. Voltage level
        ii. Bandwidth
        iii. *R* and *X* settings (V)
4. Transformers
    a. kVA rating
    b. Voltage ratings
    c. Impedance (*R* and *X*)
    d. No-load power loss

1.6 Summary

It is becoming increasing more important to be able to accurately model and analyze distribution systems. There are many different substation designs possible, but, for the most part, the substation serves one or more radial feeders. Each feeder must be modeled as accurately as possible in order for the analysis to have meaning. Sometimes the most difficult task for the engineer is to acquire all of the necessary data. Feeder maps will contain most of the needed data. Additional data such as standard pole configurations, specific conductors used on each line segment, three-phase transformer connections, and voltage regulator settings must come from stored records. Once all of the data has been acquired, the analysis can commence utilizing models of the various devices that will be developed in later chapters.

2

Nature of Loads

The modeling and analysis of a power system depend upon the "load." What is load? The answer to that question depends upon what type of an analysis is desired. For example, the steady-state analysis (power-flow study) of an interconnected transmission system will require a different definition of load than that used in the analysis of a secondary in a distribution feeder. The problem is that the "load" on a power system is constantly changing. The closer you are to the customer, the more pronounced will be the ever-changing load. There is no such thing as a "steady-state" load. In order to come to grips with load, it is first necessary to look at the "load" of an individual customer.

2.1 Definitions

The load that an individual customer or a group of customers presents to the distribution system is constantly changing. Every time a light bulb or an electrical appliance is switched on or off, the load seen by the distribution feeder changes. In order to describe the changing load, the following terms are defined:

1. Demand
   a. Load averaged over a specific period of time
   b. Load can be kW, kvar, kVA, or A
   c. Must include the time interval
   d. Example: The 15 min kW demand is 100 kW
2. Maximum demand
   a. Greatest of all demands that occur during a specific time
   b. Must include demand interval, period, and units
   c. Example: The 15 min maximum kW demand for the week was 150 kW

3. Average demand
   a. The average of the demands over a specified period (day, week, month, etc.)
   b. Must include demand interval, period, and units
   c. Example: The 15 min average kW demand for the month was 350 kW
4. Diversified demand
   a. Sum of demands imposed by a group of loads over a particular period
   b. Must include demand interval, period, and units
   c. Example: The 15 min diversified kW demand in the period ending at 9:30 was 200 kW
5. Maximum diversified demand
   a. Maximum of the sum of the demands imposed by a group of loads over a particular period
   b. Must include demand interval, period, and units
   c. Example: The 15 min maximum diversified kW demand for the week was 500 kW
6. Maximum noncoincident demand
   a. For a group of loads, the sum of the individual maximum demands without any restriction that they occur at the same time
   b. Must include demand interval, period, and units
   c. Example: The maximum noncoincident 15 min kW demand for the week was 700 kW
7. Demand factor
   a. Ratio of maximum demand to connected load
8. Utilization factor
   a. Ratio of the maximum demand to rated capacity
9. Load factor
   a. Ratio of the average demand of any individual customer or a group of customers over a period to the maximum demand over the same period
10. Diversity factor
    a. Ratio of the "maximum noncoincident demand" to the "maximum diversified demand"
11. Load diversity
    a. Difference between "maximum noncoincident demand" and the "maximum diversified demand"

2.2 Individual Customer Load

Figure 2.1 illustrates how the instantaneous kW load of a customer changes during two 15 min intervals.

2.2.1 Demand

In order to define the load, the demand curve is broken into equal time intervals. In Figure 2.1, the selected time interval is 15 min. In each interval, the average value of the demand is determined. In Figure 2.1, the straight lines represent the average load in a time interval. The shorter the time interval, the more accurate will be the value of the load. This process is very similar to numerical integration. The average value of the load in an interval is defined as the "15 min kW demand."

The 24 h 15 min kW demand curve for a customer is shown in Figure 2.2. This curve is developed from a spreadsheet that gives the 15 min kW demand for a period of 24 h.

2.2.2 Maximum Demand

The demand curve shown in Figure 2.2 represents a typical residential customer. Each bar represents the "15 min kW demand." Note that during the 24 h period there is a great variation in the demand. This particular customer has three periods in which the kW demand exceeds 6.0 kW. The greatest of these is the "15 min maximum kW demand." For this customer, the "15 min maximum kW demand" occurs at 13:15 and has a value of 6.18 kW.

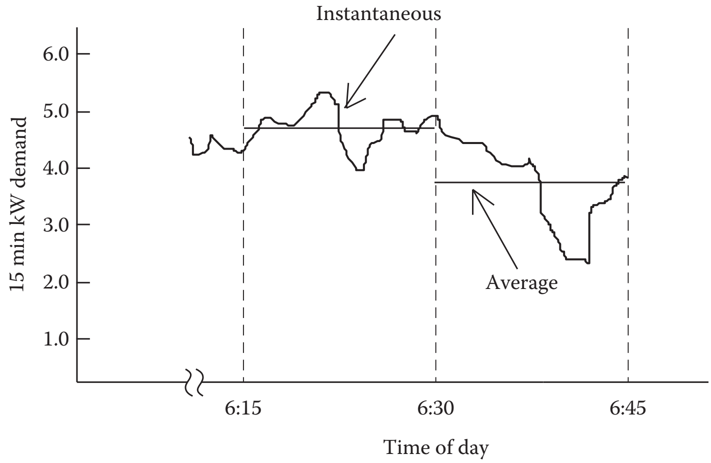

FIGURE 2.1
Customer demand curve.

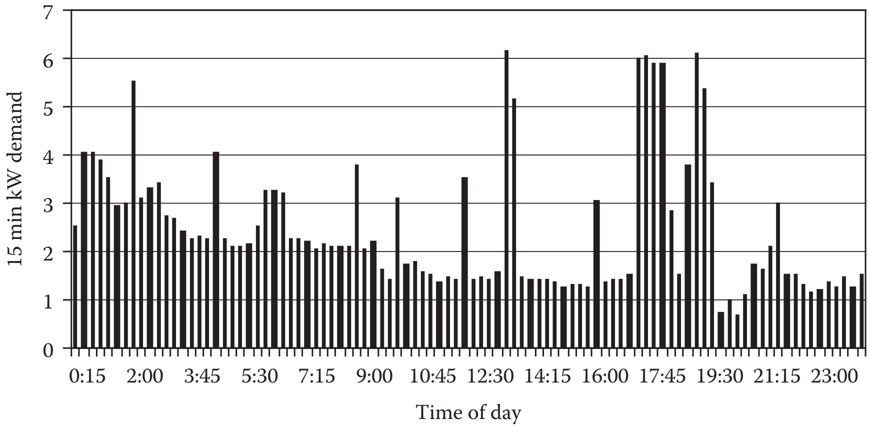

FIGURE 2.2
Twenty-four hour demand curve for customer #1.

2.2.3 Average Demand

During the 24 h period, energy (kWh) will be consumed. The energy in kWh used during each 15 min time interval is computed by

$$kWh = (15\,\text{min\_}kW\_demand)\cdot\frac{1}{4}\,\text{h} \tag{2.1}$$

The total energy consumed during the day is the summation of all of the 15 min interval consumptions. From the spreadsheet, the total energy consumed during the period by customer #1 is 58.96 kWh. The "15 min average kW demand" is computed by

$$Average\_demand = \frac{Total\_energy}{Hours} = \frac{58.96}{24} = 2.46\,\text{kW} \tag{2.2}$$

2.2.4 Load Factor

"Load factor" is a term that is often referred to when describing a load. It is defined as the ratio of the average demand to the maximum demand. In many ways, load factor gives an indication of how well the utility's facilities are being utilized. From the utility's standpoint, the optimal load factor would be 1.00 since the system has to be designed to handle the maximum demand. Sometimes utility companies will encourage industrial customers to improve their load factor. One method of encouragement is to penalize the customer on the electric bill for having a low load factor.

For customer #1 in Figure 2.2 the load factor is computed to be

$$Load\_factor = \frac{Average\_15\_\text{min\_}kW\_demand}{Max.\_15\_\text{min\_}kW\_demand} = \frac{2.46}{6.18} = 0.40 \tag{2.3}$$

2.3 Distribution Transformer Loading

A distribution transformer will provide service to one or more customers. Each customer will have a demand curve similar to that of Figure 2.2. However, the peaks and valleys and maximum demands will be different for each customer. Figures 2.3 through 2.5 give the demand curves for the three additional customers connected to the same distribution transformer.

The load curves for the four customers show that each customer has its unique loading characteristic. The customers' individual maximum kW demand occurs at different times of the day. Customer #3 is the only

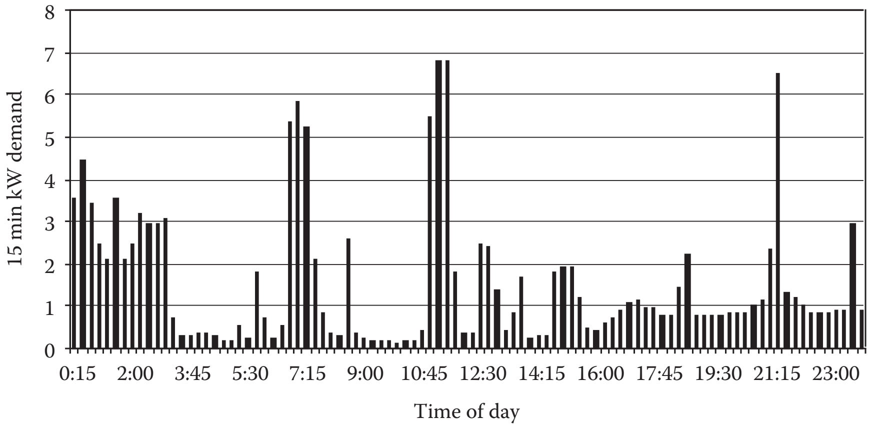

FIGURE 2.3
Twenty-four hour demand curve for customer #2.

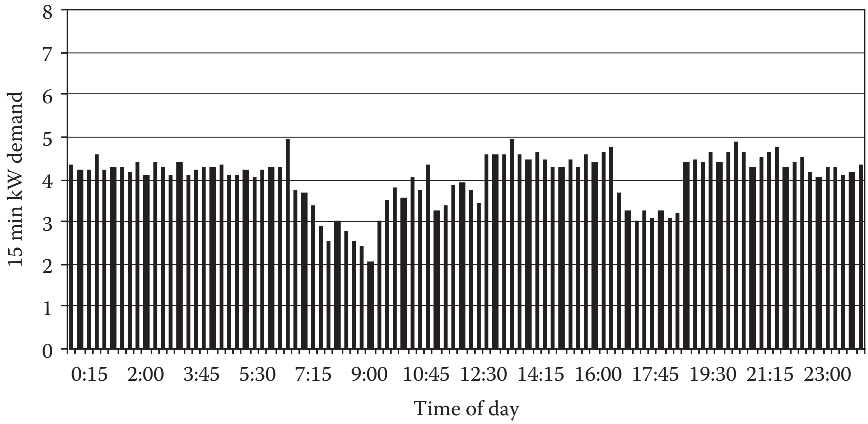

FIGURE 2.4
Twenty-four hour demand curve for customer #3.

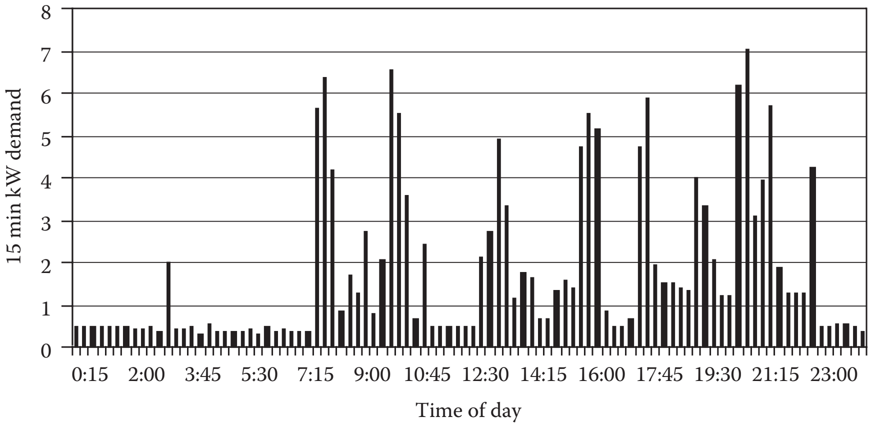

FIGURE 2.5
Twenty-four hour demand curve for customer #4.

TABLE 2.1
Individual Customer Load Characteristics

	Customer #1	Customer #2	Customer #3	Customer #4
Energy usage (kWh)	58.57	36.46	95.64	42.75
Maximum kW demand	6.18	6.82	4.93	7.05
Time of maximum kW demand	13:15	11:30	6:45	20:30
Average kW demand	2.44	1.52	3.98	1.78
Load factor	0.40	0.22	0.81	0.25

customer who will have a high load factor. A summary of individual loads is given in Table 2.1. The four customers given in Table 2.1 demonstrate that there is great diversity between their loads.

2.3.1 Diversified Demand

It is assumed that the same distribution transformer serves the four customers discussed previously. The sum of the four 15 kW demands for each time interval is the "diversified demand" for the group in that time interval, and in this case, the distribution transformer. The 15 min diversified kW demand of the transformer for the day is shown in Figure 2.6. Note in this figure how the demand curve is beginning to smooth out. There are not as many significant changes as seen by some of the individual customer curves.

2.3.2 Maximum Diversified Demand

The transformer demand curve of Figure 2.6 demonstrates how the combined customer loads begin to smooth out the extreme changes of the individual loads. For the transformer, the 15 min kW demand exceeds 16 kW twice. The greater

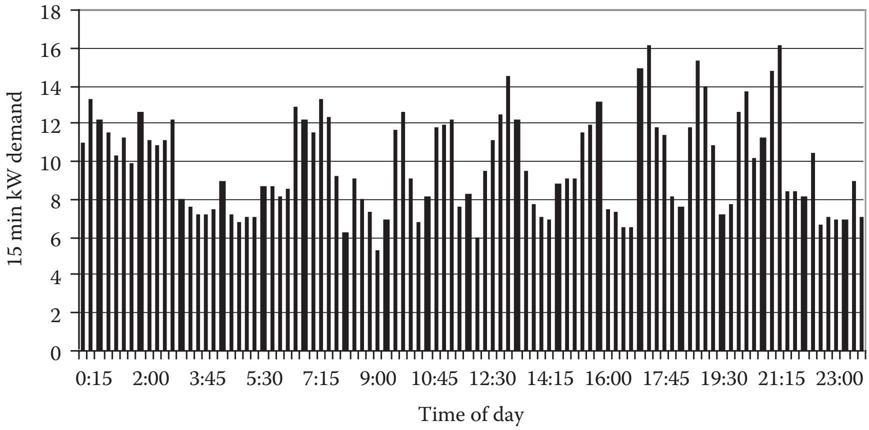

FIGURE 2.6
Transformer diversified demand curve.

of these is the "15 min maximum diversified kW demand" of the transformer. It occurs at 17:30 and has a value of 16.16 kW. Note that this maximum demand does not occur at the same time as any one of the individual demands nor is this maximum demand the sum of the individual maximum demands.

2.3.3 Load Duration Curve

A "load duration curve" can be developed for the transformer serving the four customers. Sorting in a descending order, the kW demand of the transformer develops the load duration curve shown in Figure 2.7.

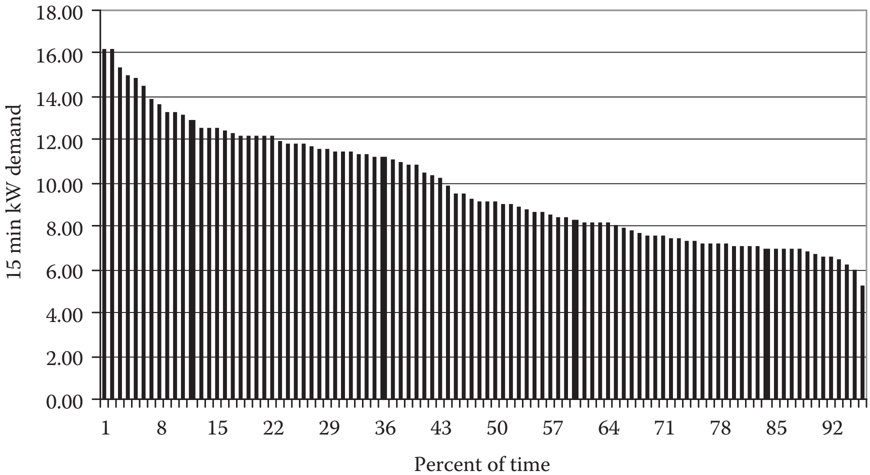

FIGURE 2.7
Transformer load duration curve.

The load duration curve plots the 15 min kW demand versus the percent of time the transformer operates at or above the specific kW demand. For example, the load duration curve shows the transformer operates with a 15 min kW demand of 12 kW or greater 22% of the time. This curve can be used to determine whether or not a transformer needs to be replaced due to an overloading condition.

2.3.4 Maximum Noncoincident Demand

The "15 min maximum noncoincident kW demand" for the day is the sum of the individual customer 15 min maximum kW demands. For the transformer in question, the sum of the individual maximums is

$$Max.\_noncoincident\_demand = 6.18 + 6.82 + 4.93 + 7.05 = 24.98\,\text{kW} \tag{2.4}$$

2.3.5 Diversity Factor

By definition, diversity factor (DF) is the ratio of the maximum noncoincident demand of a group of customers to the maximum diversified demand of the group. With reference to the transformer serving four customers, the DF for the four customers would be

$$Diversity\_factor = \frac{Maximum\_noncoincident\_demand}{Maximum\_diversified\_demand} = \frac{24.98}{16.16} = 1.5458 \tag{2.5}$$

The idea behind the DF is that when the maximum demands of the customers are known, then the maximum diversified demand of a group of customers can be computed. There will be a different value of the DF for different numbers of customers. The value computed earlier would apply for four customers. If there are five customers, then a load survey would have to be set up to determine the DF for five customers. This process would have to be repeated for all practical number of customers. Table 2.2 is an example of the DFs for the number of customers ranging from 1 up to 70. The table was developed from a different database than the four customers that have been discussed previously. A graph of the DFs is shown in Figure 2.8.

Note that, in Table 2.2 and Figure 2.8, the value of the DF basically leveled out when the number of customers reached 70. This is an important observation because it means, at least for the system from which these DFs were determined, that the DF will remain constant at 3.20 from 70 customers and above. In other words as viewed from the substation, the maximum diversified demand of a feeder can be predicted by computing the total noncoincident maximum demand of all of the customers served by the feeder and dividing by 3.2.

TABLE 2.2
Diversity Factors

N	DF	N	DF	N	DF	N	DF	N	DF	N	DF	N	DF
1	1.0	11	2.67	21	2.90	31	3.05	41	3.13	51	3.15	61	3.18
2	1.60	12	2.70	22	2.92	32	3.06	42	3.13	52	3.15	62	3.18
3	1.80	13	2.74	23	2.94	33	3.08	43	3.14	53	3.16	63	3.18
4	2.10	14	2.78	24	2.96	34	3.09	44	3.14	54	3.16	64	3.19
5	2.20	15	2.80	25	2.98	35	3.10	45	3.14	55	3.16	65	3.19
6	2.30	16	2.82	26	3.00	36	3.10	46	3.14	56	3.17	66	3.19
7	2.40	17	2.84	27	3.01	37	3.11	47	3.15	57	3.17	67	3.19
8	2.55	18	2.86	28	3.02	38	3.12	48	3.15	58	3.17	68	3.19
9	2.60	19	2.88	29	3.04	39	3.12	49	3.15	59	3.18	69	3.20
10	2.65	20	2.90	30	3.05	40	3.13	50	3.15	60	3.18	70	3.20

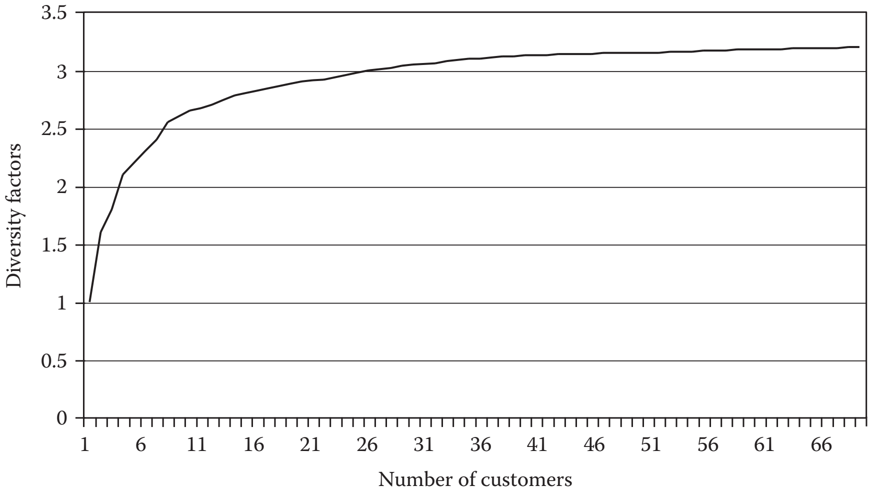

FIGURE 2.8
Diversity factors.

2.3.6 Demand Factor

The demand factor can be defined for an individual customer. For example, the 15 min maximum kW demand of customer #1 was found to be 6.18 kW. In order to determine the demand factor, the total connected load of the customer needs to be known. The total connected load will be the sum of the ratings of all of the electrical devices at the customer's location. Assume that this total comes to 35 kW, then the demand factor is computed to be

$$Demand\_factor = \frac{Maximum\_demand}{Total\_connected\_load} = \frac{6.18}{35} = 0.1766 \tag{2.6}$$

The demand factor gives an indication of the percentage of electrical devices that are on when the maximum demand occurs. The demand factor can be computed for an individual customer but not for a distribution transformer or the total feeder.

2.3.7 Utilization Factor

The utilization factor gives an indication of how well the capacity of an electrical device is being utilized. For example, the transformer serving the four loads is rated 15 kVA. Using the 16.16 kW maximum diversified demand and assuming a power factor of 0.9, the 15 min maximum kVA demand on the transformer is computed by dividing the 16.16 kW maximum kW demand by the power factor and would be 17.96 kVA. The utilization factor is computed to be

$$Utilization\_factor = \frac{Maximum\_kVA\_demand}{Transformer\_kVA\_rating} = \frac{17.96}{15} = 1.197 \tag{2.7}$$

2.3.8 Load Diversity

Load diversity is defined as the difference between the noncoincident maximum demand and the maximum diversified demand. For the transformer in question, the load diversity is computed to be

$$Load\_diversity = 24.97 - 16.16 = 8.81\,\text{kVA} \tag{2.8}$$

2.4 Feeder Load

The load that a feeder serves will display a smoothed out demand curve as shown in Figure 2.9.

The feeder demand curve does not display any of the abrupt changes in demand of an individual customer demand curve or the semi-abrupt changes in the demand curve of a transformer. The simple explanation for this is that with several hundred customers served by the feeder, the odds are good that as one customer is turning off a light bulb another customer will be turning a light bulb on. The feeder load therefore does not experience a jump as would be seen in the individual customer's demand curve.

2.4.1 Load Allocation

In the analysis of a distribution feeder, "load" data will have to be specified. The data provided will depend upon how detailed the feeder is to be

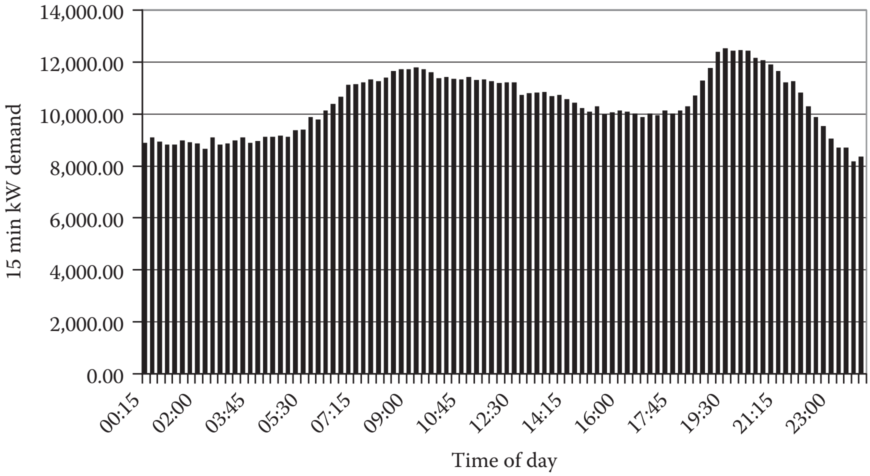

FIGURE 2.9
Feeder demand curve.

modeled and the availability of customer load data. The most comprehensive model of a feeder will represent every distribution transformer. When this is the case, the load allocated to each transformer needs to be determined.

2.4.1.1 Application of Diversity Factors

The definition of the DF is the ratio of the maximum noncoincident demand to the maximum diversified demand. DFs are shown in Table 2.2. When such a table is available, then it is possible to determine the maximum diversified demand of a group of customers such as those served by a distribution transformer. That is, the maximum diversified demand can be computed by

$$Max.\_diversified\_demand = \frac{Max\_noncoincident\_demand}{DF_n} \tag{2.9}$$

This maximum diversified demand becomes the allocated "load" for the transformer.

2.4.1.2 Load Survey

Many times the maximum demand of individual customers will be known either from metering or from knowledge of the energy (kWh) consumed by the customer. Some utility companies will perform a load survey of similar customers in order to determine the relationship between the energy consumption in kWh and the maximum kW demand. Such a load survey requires the installation of a demand meter at each customer's location. The meter can be the same type as is used to develop the demand curves previously discussed, or it can be a simple meter that only records the maximum demand during the period. At the end of the survey period, the maximum demand versus kWh for each customer can be plotted on a common graph. Linear regression is used to determine the equation of a straight line that gives the kW demand as a function of kWh. The plot of points for 15 customers along with the resulting equation derived from a linear regression algorithm is shown in Figure 2.10.

The straight-line equation derived is

$$Max.\_kW\_demand = 0.1058 + 0.005014 \cdot kWh \tag{2.10}$$

Knowing the maximum demand for each customer is the first step in developing a table of DFs as shown in Table 2.2. The next step is to perform a load survey where the maximum diversified demand of groups of customers is metered. This will involve selecting a series of locations where demand meters can be placed that will record the maximum demand for groups of customers ranging from at least 2 to 70. At each meter location, the maximum demand of all downstream customers must also be known. With that data, the DF can be computed for the given number of downstream customers.

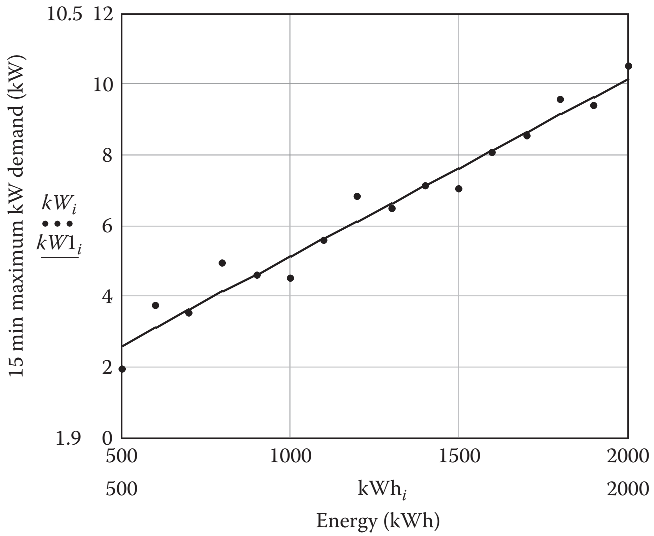

FIGURE 2.10
kW demand versus kWh for residential customers.

Example 2.1

A single-phase lateral provides service to three distribution transformers as shown in Figure 2.11.

The energy in kWh consumed by each customer during a month is known. A load survey has been conducted for customers in this class, and it has been found that the customer 15 min maximum kW demand is given by the equation

$$kW_{demand} = 0.2 + 0.008 \cdot kWh \tag{2.11}$$

The kWh consumed by customer #1 is 1523 kWh. The 15 min maximum kW demand for customer #1 is then computed as

$$kW_1 = 0.2 + 0.008 \cdot 1523 = 12.4$$

The results of this calculation for the remainder of the customers are summarized in the following table by transformer.

Transformer T1

Customer	#1	#2	#3	#4	#5
kWh	1523	1645	1984	1590	1456
kW	12.4	13.4	16.1	12.9	11.9

Transformer T2

Customer	#6	#7	#8	#9	#10	#11
kWh	1235	1587	1698	1745	2015	1765
kW	10.1	12.9	13.8	14.2	16.3	14.3

Transformer T3

Customer	#12	#13	#14	#15	#16	#17	#18
kWh	2098	1856	2058	2265	2135	1985	2103
kW	17.0	15.1	16.7	18.3	17.3	16.1	17.0

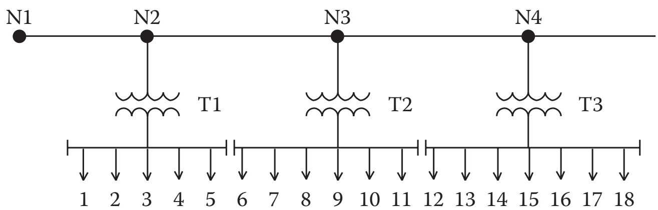

FIGURE 2.11
Single-phase lateral.

1. Determine for each transformer the 15 min noncoincident maximum kW demand, and using the DFs in Table 2.2, determine the 15 min maximum diversified kW demand.

   T1: *Noncoin._max.* = 12.4 + 13.4 + 16.1 + 12.9 + 11.9 = 66.6 kW

$$Max.\_div.\_demand = \frac{Noncoincident\_max.}{Diversity\_factor\_for\_5} = \frac{66.7}{2.20} = 30.3\,\text{kW}$$

   T2: *Noncoin._max.* = 12.9 + 13.8 + 14.2 + 16.3 + 14.3 + 17.0 = 81.6 kW

$$Max.\_div.\_demand = \frac{Noncoincident\_max.}{Diversity\_factor\_for\_6} = \frac{81.6}{2.30} = 35.5\,\text{kW}$$

   T3: *Noncoin._max.* = 17.0 + 15.1 + 16.7 + 18.3 + 17.3 + 16.1 + 17.0 = 117.5 kW

$$Max.\_div.\_demand = \frac{Noncoincident\_max.}{Diversity\_factor\_for\_7} = \frac{117.5}{2.40} = 49.0\,\text{kW}$$

   Based upon the 15 min maximum kW diversified demand on each transformer and an assumed power factor of 0.9, the 15 min maximum kVA diversified demand on each transformer would be

$$Max.\_kVA_{T1}\_demand = \frac{30.3}{0.9} = 33.7$$

$$Max.\_kVA_{T2}\_demand = \frac{35.5}{0.9} = 39.4$$

$$Max.\_kVA_{T3}\_demand = \frac{49.0}{0.9} = 54.4$$

   The kVA ratings selected for the three transformers would be 25, 37.5, and 50 kVA, respectively. With those selections, only transformer T1 would experience a significant maximum kVA demand greater than its rating (135%).

2. Determine the 15 min noncoincident maximum kW demand and 15 min maximum diversified kW demand for each of the line segments.

   Segment N1 to N2:
   The maximum noncoincident kW demand is the sum of the maximum demands of all 18 customers.

$$Noncoin.\_max.\_demand = 66.7 + 81.6 + 117.5 = 265.8\,\text{kW}$$

   The maximum diversified kW demand is computed by using the DF for 18 customers.

$$Max.\_div.\_demand = \frac{265.8}{2.86} = 92.9\,\text{kW}$$

Segment N2 to N3:
This line segment "sees" 13 customers. The noncoincident maximum demand is the sum of customer numbers 6 through 18. The DF for 13 (2.74) is used to compute the maximum diversified kW demand.

$$Noncoin.\_demand = 81.6 + 117.5 = 199.1\,\text{kW}$$

$$Max.\_div.\_demand = \frac{199.1}{2.74} = 72.7\,\text{kW}$$

Segment N3 to N4:
This line segment "sees" the same noncoincident demand and diversified demand as that of transformer T3.

$$Noncoin.\_demand = 117.5\,\text{kW}$$

$$Max.\_div.\_demand = 49.0\,\text{kW}$$

Example 2.1 demonstrates that Kirchhoff's current law (KCL) is not obeyed when the maximum diversified demands are used as the "load" flowing through the line segments and through the transformers. For example, at node N1, the maximum diversified demand flowing down the line segment N1–N2 is 92.9 kW, and the maximum diversified demand flowing through transformer T1 is 30.3 kW. KCL would then predict that the maximum diversified demand flowing down line segment N2–N3 would be the difference of these or 62.6 kW. However, the calculations for the maximum diversified demand in that segment were computed to be 72.6 kW. The explanation for this is that the maximum diversified demands for the line segments and transformers don't necessarily occur at the same time. At the time that the line segment N2–N3 is experiencing its maximum diversified demand, line segment N1–N2 and transformer T1 are not at their maximum values. All that can be said is that at the time segment N2–N3 is experiencing its maximum diversified demand, the difference between the actual demand on the line segment N1–N2 and the demand of transformer T1 will be 72.6 kW. There will be an infinite amount of combinations of line flow down N1–N2 and through transformer T1 that will produce the maximum diversified demand of 72.6 kW on line N2–N3.

2.4.1.3 Transformer Load Management

A transformer load management program is used by utilities to determine the loading on distribution transformers based upon a knowledge of the kWh supplied by the transformer during a peak loading month. The program is primarily used to determine when a distribution transformer needs to be changed out due to a projected overloading condition. The results of the program can also be used to allocate loads to transformers for feeder analysis purposes.

The transformer load management program relates the maximum diversified demand of a distribution transformer to the total kWh supplied by the transformer during a specific month. The usual relationship is the equation of a straight line. Such an equation is determined from a load survey. This type of load survey meters the maximum demand on the transformer in addition to the total energy in kWh of all of the customers connected to the transformer. With the information available from several sample transformers, a curve similar to that shown in Figure 2.10 can be developed, and the constants of the straight-line equation can then be computed. This method has the advantage because the utility will have in the billing database the kWh consumed by each customer every month. As long as the utility knows which customers are connected to each transformer by using the developed equation, the maximum diversified demand (allocated load) on each transformer on a feeder can be determined for each billing period.

2.4.1.4 Metered Feeder Maximum Demand

The major disadvantage of allocating load using the DFs is that most utilities will not have a table of DFs. The process of developing such a table is generally not cost beneficial. The major disadvantage of the transformer load management method is that a database is required that specifies which transformers serve which customers. Again, this database is not always available.

Allocating load based upon the metered readings in the substation requires the least amount of data. Most feeders will have metering in the substation that will, at minimum, give either the total three-phase maximum diversified kW or kVA demand and/or the maximum current per phase during a month. The kVA ratings of all distribution transformers are always known for a feeder. The metered readings can be allocated to each transformer based upon the transformer rating. An "allocation factor" (AF) can be determined based upon the metered three-phase kW or kVA demand and the total connected distribution transformer kVA:

$$AF = \frac{Metered\_demand}{kVA_{total}} \tag{2.12}$$

where

Metered_demand can be either kW or kVA

kVA_{total} is the sum of the kVA ratings of all distribution transformers

The allocated load per transformer is then determined by

$$Transformer\_demand = AF \cdot kVA_{transformer} \tag{2.13}$$

The transformer demand will be either kW or kVA depending upon the metered quantity.

When the kW or kVA is metered by phase, then the load can be allocated by phase where it will be necessary to know the phasing of each distribution transformer.

When the maximum current per phase is metered, the load allocated to each distribution transformer can be done by assuming nominal voltage at the substation and then computing the resulting kVA. The load allocation will now follow the same procedure as outlined earlier.

If there is no metered information on the reactive power or power factor of the feeder, a power factor will have to be assumed for each transformer load.

Modern substations will have microprocessor-based metering that will provide kW, kvar, kVA, power factor, and current per phase. With this data, the reactive power can also be allocated. Since the metered data at the substation will include losses, an iterative process will have to be followed so that the allocated load plus losses will equal the metered readings.

Example 2.2

Assume that the metered maximum diversified kW demand for the system of Example 2.1 is 92.9 kW. Allocate this load according to the kVA ratings of the three transformers.

$$kVA_{total} = 25 + 37.5 + 50 = 112.5$$

$$AF = \frac{92.9}{112.5} = 0.8258\,\text{kW/kVA}$$

The allocated kW for each transformer becomes

T1: $kW_1 = 0.8258 \cdot 25 = 20.64\,\text{kW}$
T2: $kW_1 = 0.8258 \cdot 37.5 = 30.97\,\text{kW}$
T3: $kW_1 = 0.8258 \cdot 50 = 41.29\,\text{kW}$

2.4.1.5 What Method to Use?

Four different methods have been presented for allocating load to distribution transformers:

- Application of DFs
- Load survey
- Transformer load management
- Metered feeder maximum demand

Which method to use depends upon the purpose of the analysis. If the purpose of the analysis is to determine as closely as possible the maximum demand on a distribution transformer, then either the DF or the transformer load management method can be used. Neither of these methods should be employed when the analysis of the total feeder is to be performed. The problem is that using either of those methods will result in a much larger maximum diversified demand at the substation than actually exists. When the total feeder is to be analyzed, the only method that gives good results is that of allocating load based upon the kVA ratings of the transformers.

2.4.2 Voltage Drop Calculations Using Allocated Loads

The voltage drops down line segments and through distribution transformers are of interest to the distribution engineer. Four different methods of allocating loads have been presented. The various voltage drops can be computed using the loads allocated by the three methods. For these studies, it is assumed that the allocated loads will be modeled as constant real power and reactive power.

2.4.2.1 Application of Diversity Factors

The loads allocated to a line segment or a distribution transformer using DFs are a function of the total number of customers "downstream" from the line segment or distribution transformer. The application of the DFs was demonstrated in Example 2.1. With a knowledge of the allocated loads flowing in the line segments and through the transformers and the impedances, the voltage drops can be computed. The assumption is that the allocated loads will be constant real power and reactive power. In order to avoid an iterative solution, the voltage at the source end is assumed and the voltage drops calculated from that point to the last transformer. Example 2.3 demonstrates how the method of load allocation using DFs is applied. The same system and allocated loads from Example 2.1 are used in Example 2.3.

Example 2.3

For the system of Example 2.1, assume the voltage at N1 is 2400 V and compute the secondary voltages on the three transformers using the DFs.

The system of Example 2.1 including segment distances is shown in Figure 2.12.

Assume that the power factor of the loads is 0.9 lagging.

The impedance of the lines are $z = 0.3 + j0.6\,\Omega/\text{mile}$

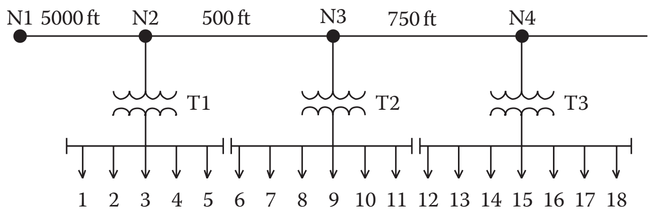

FIGURE 2.12
Single-phase lateral with distances.

The ratings of the transformers are as follows:

T1: 25 kVA, 2400–240 V, $Z = 1.8\underline{/40}\%$
T2: 37.5 kVA, 2400–240 V, $Z = 1.9\underline{/45}\%$
T3: 50 kVA, 2400–240 V, $Z = 2.0\underline{/50}\%$

From Example 2.1 the maximum diversified kW demands were computed. Using the 0.9 lagging power factor, the maximum diversified kW and kVA demands for the line segments and transformers are

Segment N1–N2:	$P_{12} = 92.9\,\text{kW}$	$S_{12} = 92.9 + j45.0\,\text{kVA}$
Segment N2–N3:	$P_{23} = 72.6\,\text{kW}$	$S_{23} = 72.6 + j35.2\,\text{kVA}$
Segment N3–N4:	$P_{34} = 49.0\,\text{kW}$	$S_{34} = 49.0 + j23.7\,\text{kVA}$
Transformer T1:	$P_{T1} = 30.3\,\text{kW}$	$S_{T1} = 30.3 + j14.7\,\text{kVA}$
Transformer T2:	$P_{T2} = 35.5\,\text{kW}$	$S_{T2} = 35.5 + j17.2\,\text{kVA}$
Transformer T3:	$P_{T3} = 49.0\,\text{kW}$	$S_{T3} = 49.0 + j23.7\,\text{kVA}$

Convert transformer impedances to Ohms referred to the high voltage side

$$\text{T1}: Z_{base} = \frac{kV^2 \cdot 1000}{kVA} = \frac{2.4^2 \cdot 1000}{25} = 230.4\,\Omega$$

$$Z_{T1} = (0.018\underline{/40}) \cdot 230.4 = 3.18 + j2.67\,\Omega$$

$$\text{T2}: Z_{base} = \frac{kV^2 \cdot 1000}{kVA} = \frac{2.4^2 \cdot 1000}{37.5} = 153.6\,\Omega$$

$$Z_{T2} = (0.019\underline{/45}) \cdot 153.6 = 2.06 + j2.06\,\Omega$$

$$\text{T3}: Z_{base} = \frac{kV^2 \cdot 1000}{kVA} = \frac{2.4^2 \cdot 1000}{50} = 115.2\,\Omega$$

$$Z_{T3} = (0.02\underline{/50}) \cdot 115.2 = 1.48 + j1.77\,\Omega$$

Compute the line impedances:

$$\text{N1–N2}: Z_{12} = (0.3 + j0.6) \cdot \frac{5000}{5280} = 0.2841 + j0.5682\,\Omega$$

$$\text{N2–N3}: Z_{23} = (0.3 + j0.6) \cdot \frac{500}{5280} = 0.0284 + j0.0568\,\Omega$$

$$\text{N3–N4}: Z_{34} = (0.3 + j0.6) \cdot \frac{750}{5280} = 0.0426 + j0.0852\,\Omega$$

Calculate the current flowing in segment N1–N2:

$$I_{12} = \left(\frac{kW + jkvar}{kV}\right)^* = \left(\frac{92.9 + j45.0}{2.4\underline{/0}}\right)^* = 43.0\underline{/-25.84}\,\text{A}$$

Calculate the voltage at N2:

$$V_2 = V_1 - Z_{12} \cdot I_{12}$$

$$V_2 = 2400\underline{/0} - (0.2841 + j0.5682) \cdot 43.0\underline{/-25.84} = 2378.4\underline{/-0.4}\,\text{V}$$

Calculate the current flowing into T1:

$$I_{T1} = \left(\frac{kW + jkvar}{kV}\right)^* = \left(\frac{30.3 + j14.7}{2.378\underline{/-0.4}}\right)^* = 14.16\underline{/-26.24}\,\text{A}$$

Calculate the secondary voltage referred to the high side:

$$V_{T1} = V_2 - Z_{T2} \cdot I_{T1}$$

$$V_{T1} = 2378.4\underline{/-0.4} - (3.18 + j2.67) \cdot 14.16\underline{/-26.24} = 2321.5\underline{/-0.8}\,\text{V}$$

Compute the secondary voltage by dividing by the turns ratio of 10:

$$Vlow_{T1} = \frac{2321.5\underline{/-0.8}}{10} = 232.15\underline{/-0.8}\,\text{V}$$

Calculate the current flowing in line segment N2–N3:

$$I_{23} = \left(\frac{kW + jkvar}{kV}\right)^* = \left(\frac{72.6 + j35.2}{2.378\underline{/-0.4}}\right)^* = 33.9\underline{/-26.24}\,\text{A}$$

Calculate the voltage at N3:

$$V_3 = V_2 - Z_{23} \cdot I_{23}$$

$$V_3 = 2378.4\underline{/-0.4} - (0.0284 + j0.0568) \cdot 33.9\underline{/-26.24} = 2376.7\underline{/-0.4}\,\text{V}$$

Calculate the current flowing into T2:

$$I_{T2} = \left(\frac{kW + jkvar}{kV}\right)^* = \left(\frac{35.5 + j17.2}{2.3767\underline{/-0.4}}\right)^* = 16.58\underline{/-26.27}\text{ A}$$

Calculate the secondary voltage referred to the high side:

$$V_{T2} = V_3 - Z_{T2} \cdot I_{T2}$$

$$V_{T2} = 2376.7\underline{/-0.4} - (2.06 + j2.06) \cdot 16.58\underline{/-26.27} = 2331.1\underline{/-0.8}\text{ V}$$

Compute the secondary voltage by dividing by the turns ratio of 10:

$$Vlow_{T2} = \frac{2331.1\underline{/-0.8}}{10} = 233.1\underline{/-0.8}\text{ V}$$

Calculate the current flowing in line section N3–N4:

$$I_{34} = \left(\frac{kW + jkvar}{kV}\right)^* = \left(\frac{49.0 + j23.7}{2.3767\underline{/-0.4}}\right)^* = 22.9\underline{/-26.27}\text{ A}$$

Calculate the voltage at N4:

$$V_4 = V_3 - Z_{34} \cdot I_{34}$$

$$V_4 = 2376.7\underline{/-0.4} - (0.0426 + 0.0852) \cdot 22.9\underline{/-26.27} = 2375.0\underline{/-0.5}\text{ V}$$

The current flowing into T3 is the same as the current from N3 to N4:

$$I_{T3} = 22.91\underline{/-26.30}\text{ A}$$

Calculate the secondary voltage referred to the high side:

$$V_{T3} = V_4 - Z_{T3} \cdot I_{T3}$$

$$V_{T3} = 2375.0\underline{/-0.5} - (1.48 + j1.77) \cdot 22.9\underline{/-26.27} = 2326.9\underline{/-1.0}\text{ V}$$

Compute the secondary voltage by dividing by the turns ratio of 10:

$$Vlow_{T3} = \frac{2326.9\underline{/-1.0}}{10} = 232.7\underline{/-1.0}\,\text{V}$$

Calculate the percent voltage drop to the secondary of transformer T3. Use the secondary voltage referred to the high side:

$$V_{drop} = \frac{|V_1| - |V_{T3}|}{|V_1|} \cdot 100 = \frac{2400 - 2326.11}{2400} \cdot 100 = 3.0789\%$$

2.4.2.2 Load Allocation Based upon Transformer Ratings

When only the ratings of the distribution transformers are known, the feeder can be allocated based upon the metered demand and the transformer kVA ratings. This method was discussed in Section 2.3.3. Example 2.4 demonstrates this method.

Example 2.4

For the system of Example 2.1, assume the voltage at N1 is 2400 V, and compute the secondary voltages on the three transformers, allocating the loads based upon the transformer ratings. Assume that the metered kW demand at N1 is 92.9 kW.

The impedances of the line segments and transformers are the same as in Example 2.3.

Assume the load power factor is 0.9 lagging, and compute the kVA demand at N1 from the metered demand:

$$S_{12} = \frac{92.9}{0.9}\underline{/\cos^{-1}(0.9)} = 92.9 + j45.0 = 103.2\underline{/25.84}\,\text{kVA}$$

Calculate the AF:

$$AF = \frac{103.2\underline{/25.84}}{25 + 37.5 + 50} = 0.9175\underline{/25.84}$$

Allocate the loads to each transformer:

$$S_{T1} = AF \cdot kVA_{T1} = (0.9175\underline{/25.84}) \cdot 25 = 20.6 + j10.0\,\text{kVA}$$

$$S_{T2} = AF \cdot kVA_{T2} = (0.9175\underline{/25.84}) \cdot 37.5 = 31.0 + j15.0\,\text{kVA}$$

$$S_{T3} = AF \cdot kVA_{T3} = (0.9175\underline{/25.84}) \cdot 50 = 41.3 + j20.0\,\text{kVA}$$

Calculate the line flows:

$$S_{12} = S_{T1} + S_{T2} + S_{T3} = 92.9 + j45.0\,\text{kVA}$$

$$S_{23} = S_{T2} + S_{T3} = 72.3 + j35\,\text{kVA}$$

$$S_{34} = S_{T3} = 41.3 + j20.0\,\text{kVA}$$

Using these values of line flows and flows into transformers, the procedure for computing the transformer secondary voltages is exactly the same as in Example 2.3. When this procedure is followed, the node and secondary transformer voltages are

$$V_2 = 2378.1\underline{/-0.4}\,\text{V} \qquad Vlow_{T1} = 234.0\underline{/-0.6}\,\text{V}$$

$$V_3 = 2376.4\underline{/-0.4}\,\text{V} \qquad Vlow_{T2} = 233.7\underline{/-0.8}\,\text{V}$$

$$V_4 = 2374.9\underline{/-0.5}\,\text{V} \qquad Vlow_{T3} = 233.5\underline{/-0.9}\,\text{V}$$

The percent voltage drop for this case is

$$V_{drop} = \frac{|V_1| - |V_{T3}|}{|V_1|} \cdot 100 = \frac{2400 - 2334.8}{2400} \cdot 100 = 2.7179\%$$

2.5 Summary

This chapter has demonstrated the nature of the loads on a distribution feeder. There is a great diversity between individual customer demands, but as the demand is monitored on line segments working back toward the substation, the effect of the diversity between demands becomes very slight. It was shown that the effect of diversity between customer demands must be taken into account when the demand on a distribution transformer is computed. The effect of diversity for short laterals can be taken into account in determining the maximum flow on the lateral. For the DFs of Table 2.2, it was shown that when the number of customers exceeds 70, the effect of diversity has pretty much disappeared. This is evidenced by the fact that the DF has become almost constant as the number of customers approached 70. It must be understood that the number 70 will apply only to the DFs of Table 2.2. If a utility is going to use DFs, then that utility must perform a comprehensive load survey in order to develop the table of DFs that apply to that particular system.

Examples 2.3 and 2.4 show that the final node and transformer voltages are approximately the same. There is very little difference between the voltages when the loads were allocated using the DFs and when the loads were allocated based upon the transformer kVA ratings.

Problems

2.1 Shown in the following table are the 15 min kW demands for four customers between the hours of 17:00 and 21:00. A 25 kVA single-phase transformer serves the four customers.

Time	Customer #1 (kW)	Customer #2 (kW)	Customer #3 (kW)	Customer #4 (kW)
17:00	8.81	4.96	11.04	1.44
17:15	2.12	3.16	7.04	1.62
17:30	9.48	7.08	7.68	2.46
17:45	7.16	5.08	6.08	0.84
18:00	6.04	3.12	4.32	1.12
18:15	9.88	6.56	5.12	2.24
18:30	4.68	6.88	6.56	1.12
18:45	5.12	3.84	8.48	2.24
19:00	10.44	4.44	4.12	1.12
19:15	3.72	8.52	3.68	0.96
19:30	8.72	4.52	0.32	2.56
19:45	10.84	2.92	3.04	1.28
20:00	6.96	2.08	2.72	1.92
20:15	6.62	1.48	3.24	1.12
20:30	7.04	2.33	4.16	1.76
20:45	6.69	1.89	4.96	2.72
21:00	1.88	1.64	4.32	2.41

a. For each of the customers determine the following:
   1. Maximum 15 min kW demand
   2. Average 15 min kW demand
   3. Total kWh usage in the time period
   4. Load factor

b. For the 25 kVA transformer determine the following:
   1. Maximum 15 min diversified demand
   2. Maximum 15 min noncoincident demand
   3. Utilization factor (assume unity power factor)
   4. Diversity factor
   5. Load diversity

c. Plot the load duration curve for the transformer.

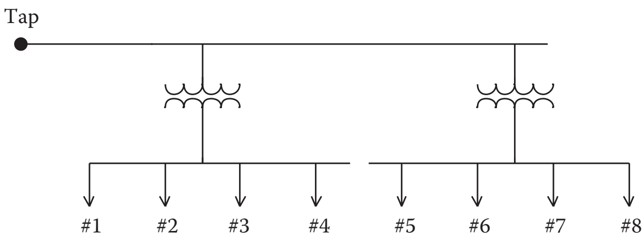

FIGURE 2.13
System for Problem 2.2.

2.2 Two transformers each serving four customers are shown in Figure 2.13. The following table gives the time interval and kVA demand of the four customer demands during the peak load period of the year. Assume a power factor of 0.9 lagging.

Time	#1	#2	#3	#4	#5	#6	#7	#8
3:00–3:30	10	0	10	5	15	10	50	30
3:30–4:00	20	25	15	20	25	20	30	40
4:00–4:30	5	30	30	15	10	30	10	10
4:30–5:00	0	10	20	10	13	40	25	50
5:00–5:30	15	5	5	25	30	30	15	5
5:30–6:00	15	15	10	10	5	20	30	25
6:00–6:30	5	25	25	15	10	10	30	25
6:30–7:00	10	50	15	30	15	5	10	30

a. For each transformer determine the following:
   1. 30 min maximum kVA demand
   2. Noncoincident maximum kVA demand
   3. Load factor
   4. Diversity factor
   5. Suggested transformer rating (50, 75, 100, 167)
   6. Utilization factor
   7. Energy (kWh) during the 4 h period

b. Determine the maximum diversified 30 min kVA demand at the "tap."

2.3 Two single-phase transformers serving 12 customers are shown in Figure 2.14.

The 15 min kW demands for the 12 customers between the hours of 5:00 p.m. and 9:00 p.m. are given at the end of this problem. Assume a load power factor of 0.95 lagging. The impedance of the lines are $z = 0.306 + j0.6272\,\Omega/\text{mile}$. The voltage at node N1 is $2500\underline{/0}$ V.

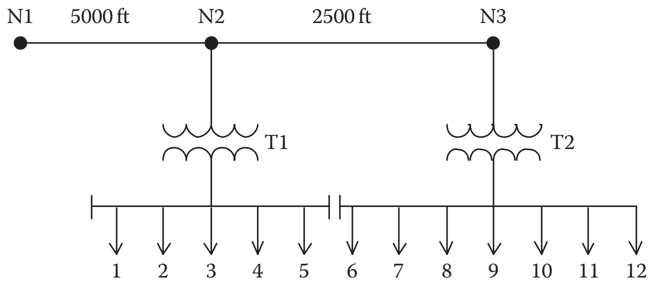

FIGURE 2.14
Circuit for Problem 2.3.

Transformer ratings:

T1:	25 kVA	2400–240 V	$Z_{pu} = 0.018\underline{/40}$
T2:	37.5 kVA	2400–240 V	$Z_{pu} = 0.020\underline{/50}$

a. Determine the maximum kW demand for each customer.
b. Determine the average kW demand for each customer.
c. Determine the kWh consumed by each customer in this time period.
d. Determine the load factor for each customer.
e. Determine the maximum diversified demand for each transformer.
f. Determine the maximum noncoincident demand for each transformer.
g. Determine the utilization factor (assume 1.0 power factor) for each transformer.
h. Determine the DF of the load for each transformer.
i. Determine the maximum diversified demand at node N1.
j. Compute the secondary voltage for each transformer taking diversity into account.

Transformer #1: 25 kVA

Time	**#1 (kW)**	**#2 (kW)**	**#3 (kW)**	**#4 (kW)**	**#5 (kW)**
05:00	2.13	0.19	4.11	8.68	0.39
05:15	2.09	0.52	4.11	9.26	0.36
05:30	2.15	0.24	4.24	8.55	0.43
05:45	2.52	1.80	4.04	9.09	0.33
06:00	3.25	0.69	4.22	9.34	0.46
06:15	3.26	0.24	4.27	8.22	0.34

(continued)

Time	#1 (kW)	#2 (kW)	#3 (kW)	#4 (kW)	#5 (kW)
06:30	3.22	0.54	4.29	9.57	0.44
06:45	2.27	5.34	4.93	8.45	0.36
07:00	2.24	5.81	3.72	10.29	0.38
07:15	2.20	5.22	3.64	11.26	0.39
07:30	2.08	2.12	3.35	9.25	5.66
07:45	2.13	0.86	2.89	10.21	6.37
08:00	2.12	0.39	2.55	10.41	4.17
08:15	2.08	0.29	3.00	8.31	0.85
08:30	2.10	2.57	2.76	9.09	1.67
08:45	3.81	0.37	2.53	9.58	1.30
09:00	2.04	0.21	2.40	7.88	2.70

Transformer #2: 37.5 kVA

Time	#6 (kW)	#7 (kW)	#8 (kW)	#9 (kW)	#10 (kW)	#11 (kW)	#12 (kW)
05:00	0.87	2.75	0.63	8.73	0.48	9.62	2.55
05:15	0.91	5.35	1.62	0.19	0.40	7.98	1.72
05:30	1.56	13.39	0.19	5.72	0.70	8.72	2.25
05:45	0.97	13.38	0.05	3.28	0.42	8.82	2.38
06:00	0.76	13.23	1.51	1.26	3.01	7.47	1.73
06:15	1.10	13.48	0.05	7.99	4.92	11.60	2.42
06:30	0.79	2.94	0.66	0.22	3.58	11.78	2.24
06:45	0.60	2.78	0.52	8.97	6.58	8.83	1.74
07:00	0.60	2.89	1.80	0.11	7.96	9.21	2.18
07:15	0.87	2.75	0.07	7.93	6.80	7.65	1.98
07:30	0.47	2.60	0.16	1.07	7.42	7.78	2.19
07:45	0.72	2.71	0.12	1.35	8.99	6.27	2.63
08:00	1.00	3.04	1.39	6.51	8.98	10.92	1.59
08:15	0.47	1.65	0.46	0.18	7.99	5.60	1.81
08:30	0.44	2.16	0.53	2.24	8.01	7.74	2.13
08:45	0.95	0.88	0.56	0.11	7.75	11.72	1.63
09:00	0.79	1.58	1.36	0.95	8.19	12.23	1.68

2.4 On a different day, the metered 15 min kW demand at node N1 for the system of Problem 2.3 is 72.43 kW. Assume a power factor of 0.95 lagging. Allocate the metered demand to each transformer based upon the transformer kVA rating. Assume the loads are constant current and compute the secondary voltage for each transformer.

2.5 A single-phase lateral serves four transformers as shown in Figure 2.15.

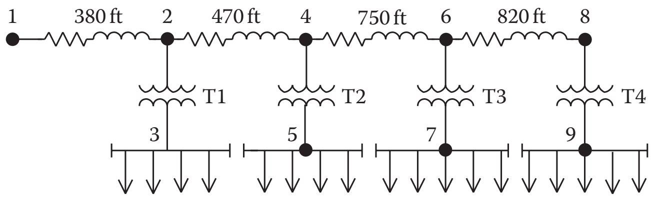

FIGURE 2.15
System for Problem 2.5.

Assume that each customer's maximum demand is 15.5 kW + *j*7.5 kvar. The impedance of the single-phase lateral is $z = 0.4421 + j0.3213\ \Omega/1000$ ft. The four transformers are rated as

T1 and T2: 37.5 kVA, 2400–240 V, $Z = 0.01 + j0.03$ per unit
T3 and T4: 50 kVA, 2400–240 V, $Z = 0.015 + j0.035$ per unit

Use the DFs found in Table 2.2 and determine the following:

a. The 15 min maximum diversified kW and kvar demands on each transformer.
b. The 15 min maximum diversified kW and kvar demands for each line section.
c. If the voltage at node 1 is 2600/0 V, determine the voltage at nodes 2 through 9. In calculating the voltages, take into account diversity using the answers from (a) and (b).
d. Use the 15 min maximum diversified demands at the lateral tap (sections 1–2) from part (b). Divide these maximum demands by 18 (number of customers) and assign that as the "instantaneous load" for each customer. Now calculate the voltages at all of the nodes listed in part (c) using the instantaneous loads.
e. Repeat part (d) except assume the loads are "constant current." To do this, take the current flowing from node 1 to node 2 from part (d) and divide by 18 (number of customers) and assign that as the "instantaneous constant current load" for each customer. Again, calculate all of the voltages.
f. Take the maximum diversified demand from node 1 to node 2 and "allocate" that to each of the four transformers based upon their kVA ratings. To do this, take the maximum diversified demand and divide by 175 (total kVA of the four transformers). Now multiply each transformer kVA rating by that number to give how much of the total diversified demand is being served by each transformer. Again, calculate all of the voltages.
g. Compute the percent differences in the voltages for parts (d) through (f) at each of the nodes using part (c) answer as the base.

3

Approximate Method of Analysis

A distribution feeder provides service to unbalanced three-phase, two-phase, and single-phase loads over untransposed three-phase, two-phase, and single-phase line segments. This combination leads to the three-phase line currents and the line voltages being unbalanced. In order to analyze these conditions as precisely as possible, it will be necessary to model all three phases of the feeder as accurately as possible. However, many times only a "ballpark" answer is needed. When this is the case, some approximate methods of modeling and analysis can be employed. It is the purpose of this chapter to develop some of the approximate methods and leave for later chapters the exact models and analysis.

All of the approximate methods of modeling and analysis will assume perfectly balanced three-phase systems. It will be assumed that all loads are balanced three phase and all line segments will be three phase and perfectly transposed. With these assumptions a single line-to-neutral equivalent circuit for the feeder will be used.

3.1 Voltage Drop

A line-to-neutral equivalent circuit of a three-phase line segment serving a balanced three-phase load is shown in Figure 3.1.

Kirchhoff's voltage law applied to the circuit of Figure 3.1 gives

$$V_S = V_L + (R + jX)\cdot I = V_L + R\cdot I + jX\cdot I \tag{3.1}$$

The phasor diagram for Equation 3.1 is shown in Figure 3.2.

In Figure 3.2 the phasor for the voltage drop through the line resistance (RI) is shown in phase with the current phasor, and the phasor for the voltage drop through the reactance (*jXI*) is shown leading the current phasor by 90°. The dashed lines in Figure 3.2 represent the real and imaginary parts of the impedance (ZI) drop. The voltage drop down the line is defined as the difference between the magnitudes of the source and the load voltages:

$$V_{drop} = |V_S| - |V_L| \tag{3.2}$$

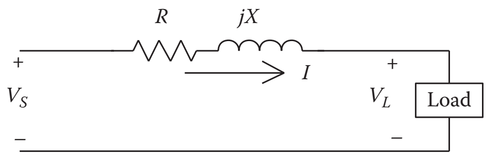

FIGURE 3.1
Line-to-neutral equivalent circuit.

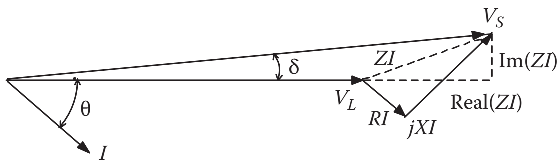

FIGURE 3.2
Phasor diagram.

The angle between the source voltage and the load voltage (δ) is very small. Because of that, the voltage drop between the source and load is approximately equal to the real part of the impedance drop. That is,

$$V_{drop} \cong \mathrm{Re}(Z \cdot I) \tag{3.3}$$

For the purposes of this chapter, Equation 3.3 is used as the definition of voltage drop.

Example 3.1

In Example 2.3, the impedance of the first line segment is

$$Z_{12} = 0.2841 + j0.5682\,\Omega$$

The current flowing through the line segment is

$$I_{12} = 43.0093\underline{/-25.8419}\,\mathrm{A}$$

The voltage at node N1 is

$$V_1 = 2400\underline{/0.0}\,\mathrm{V}$$

The exact voltage at node N2 is computed to be

$$V_2 = 2400\underline{/0.0} - (0.2841 + j0.5682) \cdot 43.0093\underline{/-25.8419} = 2378.4098\underline{/-0.4015}\,\mathrm{V}$$

The voltage drop between the nodes is then

$$V_{drop} = 2400.0000 - 2378.4098 = 21.5902\,\mathrm{V}$$

Computing the voltage drop according to Equation 3.3 gives

$$V_{drop} = \mathrm{Re}\left[(0.2841 + j0.5682) \cdot 43.0093\underline{/-25.8419}\right] = 21.6486\ \mathrm{V}$$

$$Error = \frac{21.5902 - 21.6486}{21.5902} \cdot 100 = -0.27\%$$

This example demonstrates the very small error in computing voltage drop when using the approximate equation given by Equation 3.3.

3.2 Line Impedance

For the approximate modeling of a line segment, it will be assumed that the line segment is transposed. With this assumption only the positive sequence impedance of the line segment needs to be determined. A typical three-phase line configuration is shown in Figure 3.3.

The equation for the positive sequence impedance for the configuration shown in Figure 3.3 is given by [1]

$$z_{positive} = r + j0.12134 \cdot \ln\left(\frac{D_{eq}}{GMR}\right) \Omega/\mathrm{mile} \tag{3.4}$$

where r is the conductor resistance (from tables) in Ω/mile

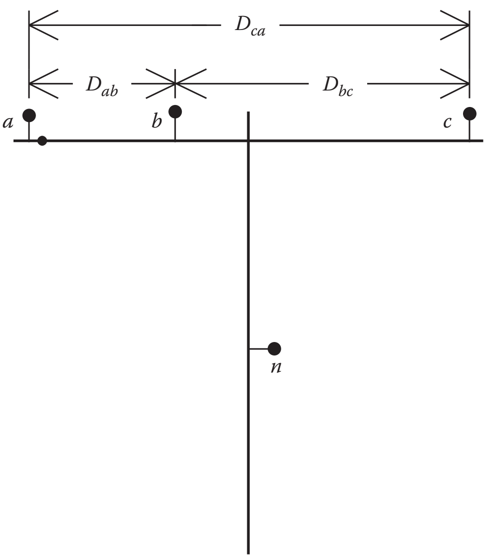

FIGURE 3.3
Three-phase line configuration.

$$D_{eq} = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}} \text{ (ft)} \tag{3.5a}$$

GMR is the conductor geometric mean radius (from tables) in feet.

Example 3.2

A three-phase line segment has the configuration shown in Figure 3.3. The spacings between conductors are

$$D_{ab} = 2.5 \text{ ft}, \quad D_{bc} = 4.5 \text{ ft}, \quad D_{ca} = 7.0 \text{ ft}$$

The conductors of the line are 336,400 26/7 Aluminum Conductor Steel Reinforced (ACSR).
Determine the positive sequence impedance of the line in Ω/mile.

Solution

From the table of conductor data in Appendix A,

$r = 0.306\,\Omega/\text{mile}$
$GMR = 0.0244\,\text{ft}$

Compute the equivalent spacing

$$D_{eq} = \sqrt[3]{2.5 \cdot 4.5 \cdot 7.0} = 4.2863\,\text{ft}$$

Using Equation 3.4,

$$z_{positive} = 0.306 + j0.12134 \cdot \ln\left(\frac{4.2863}{0.0244}\right) = 0.306 + j0.6272\,\Omega/\text{mile}$$

3.3 "*K*" Factors

A first approximation for calculating the voltage drop along a line segment is given by Equation 3.3. Another approximation is made by employing a "*K*" factor. There will be two types of *K* factors, one for voltage drop and the other for voltage rise calculations.

3.3.1 K_{drop} Factor

The K_{drop} factor is defined as

$$K_{drop} = \frac{Percent\_Voltage\_Drop}{kVA\_mile} \tag{3.5b}$$

The K_{drop} factor is determined by computing the percent voltage drop down a line that is 1 mile long and serving a balanced three-phase load of 1 kVA. The percent voltage drop is referenced to the nominal voltage of the line. In order to calculate this factor, the power factor of the load must be assumed.

Example 3.3

For the line of Example 3.2, compute the K_{drop} factor assuming a load power factor of 0.9 lagging and a nominal voltage of 12.47 kV (line to line).

Solution

The impedance of 1 mile of line was computed to be

$$Z = 0.306 + j0.6272\ \Omega$$

The current taken by 1 kVA at 0.9 lagging power factor is given by

$$I = \frac{1\_kVA}{\sqrt{3}\cdot kV_{LL}}\underline{/-\cos^{-1}(PF)} = \frac{1}{\sqrt{3}\cdot 12.5}\underline{/-\cos^{-1}(0.9)} = 0.046299\underline{/-25.84}\text{ A}$$

The voltage drop is computed to be

$$V_{drop} = \text{Re}\left[Z\cdot I\right] = \text{Re}[(0.306 + j0.6272)\cdot 0.046299\underline{/-25.84}] = 0.025408\text{ V}$$

The nominal line-to-neutral voltage is

$$V_{LN} = \frac{12{,}470}{\sqrt{3}} = 7{,}199.6\text{ V}$$

The K_{drop} factor is then

$$K_{drop} = \frac{0.025408}{7{,}199.6}\cdot 100 = 0.00035291\%\text{ drop/kVA-mile}$$

The K_{drop} factor computed in Example 3.3 is for the 336,400 26/7 ACSR conductor with the conductor spacings defined in Example 3.2, a nominal voltage of 12.47 kV, and a load power factor of 0.9 lagging. Unique K_{drop} factors can be determined for all standard conductors, spacings, and voltages. Fortunately, most utilities will have a set of standard conductors, standard conductor spacings, and one or two standard distribution voltages. Because of this, a simple spreadsheet program can be written that will compute the K_{drop} factors for the standard configurations. The assumed power factor of 0.9 lagging is a good approximation of the power factor for a feeder serving a predominately residential load.

The K_{drop} factor can be used to quickly compute the approximate voltage drop down a line section. For example, assume a load of 7500 kVA is to be served at a point 1.5 miles from the substation. Using the K_{drop} factor computed in Example 3.3, the percent voltage drop down the line segment is computed to be

$$V_{drop} = K_{drop} \cdot \text{kVA-mile} = 0.00035291 \cdot 7500 \cdot 1.5 = 3.9702\%$$

This example demonstrates that a load of 7500 kVA can be served 1.5 miles from the substation with a resulting voltage drop of 3.97%. Suppose now that the utility has a maximum allowable voltage drop of 3.0%. Then the load that can be served 1.5 miles from the substation is given by

$$kVA_{load} = \frac{3.0\%}{0.00035291 \cdot 1.5} = 5694.76\,\text{kVA}$$

The application of the K_{drop} factor is not limited to computing the percent voltage drop down just one line segment. When line segments are in cascade, the total percent voltage drop from the source to the end of the last line segment is the sum of the percent drops in each line segment. This seems logical but it must be understood that in all cases the percent drop is in reference to the nominal line-to-neutral voltage. That is, the percent voltage drop in a line segment is not referenced to the source end voltage but rather to the nominal line-to-neutral voltage, as would be the usual case. Example 3.4 demonstrates this application.

Example 3.4

A three-segment feeder is shown in Figure 3.4.

The K_{drop} factor for the line segments is

$$K_{drop} = 0.00035291$$

Determine the percent voltage drop from N0 to N3.

Solution

The total kVA flowing in segment N0 to N1 is

$$kVA_{01} = 300 + 750 + 500 = 1550\,\text{kVA}$$

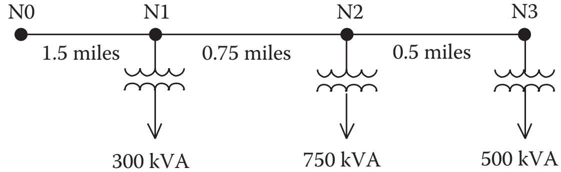

FIGURE 3.4
Three segment feeder.

The percent voltage drop from N0 to N1 is

$$Vdrop_{01} = 0.00035291 \cdot 1550 \cdot 1.5 = 0.8205\%$$

The total kVA flowing in segment N1 to N2 is

$$kVA_{12} = 750 + 500 = 1250\,\text{kVA}$$

The percent voltage drop from N1 to N2 is

$$Vdrop_{12} = 0.00035291 \cdot 1250 \cdot 0.75 = 0.3308\%$$

The kVA flowing in segment N2 to N3 is

$$kVA_{23} = 500\,\text{kVA}$$

The percent voltage drop in the last line segment is

$$Vdrop_{23} = 0.00035291 \cdot 500 \cdot 0.5 = 0.0882\%$$

The total percent voltage drop from N0 to N3 is

$$Vdrop_{total} = 0.8205 + 0.3308 + 0.0882 = 1.2396\%$$

The application of the K_{drop} factor provides an easy way of computing the approximate percent voltage drop from a source to a load. It should be kept in mind that the assumption has been a perfectly balanced three-phase load, an assumed load power factor, and transposed line segments. Even with these assumptions, the results will always provide a "ballpark" result that can be used to verify the results of more sophisticated methods of computing voltage drop.

3.3.2 K_{rise} Factor

The K_{rise} factor is similar to the K_{drop} factor except now the "load" is a shunt capacitor. When a leading current flows through an inductive reactance, there will be a voltage rise, rather than a voltage drop, across the reactance. This is illustrated by the phasor diagram in Figure 3.5.

Referring to Figure 3.5 the voltage rise is defined as

$$V_{rise} = \left|\text{Re}(ZI_{cap})\right| = X \cdot \left|I_{cap}\right| \tag{3.6}$$

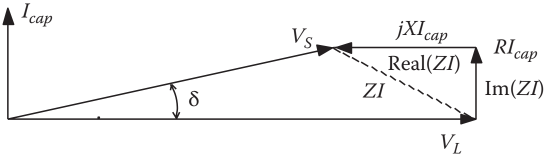

FIGURE 3.5
Voltage rise phasor diagram.

In Equation 3.6 it is necessary to take the magnitude of the real part of ZI so that the voltage rise is a positive number. The K_{rise} factor is defined exactly the same as for the K_{drop} factor:

$$K_{rise} = \frac{Percent\_Voltage\_Rise}{kvar\_mile} \tag{3.7}$$

Example 3.5

1. Calculate the K_{rise} factor for the line of Example 3.2.
2. Determine the rating of a three-phase capacitor bank to limit the voltage drop in Example 3.3 to 2.5%.

Solution

1. The impedance of 1 mile of line was computed to be

$$Z = 0.306 + j0.6272\ \Omega$$

The current taken by a 1 kvar three-phase capacitor bank is given by

$$I_{cap} = \frac{1\_kvar}{\sqrt{3}\cdot kV_{LL}}\underline{/90} = \frac{1}{\sqrt{3}\cdot 12.47}\underline{/90} = 0.046299\underline{/90}\ \text{A}$$

The voltage rise per kvar-mile is computed to be

$$V_{rise} = \left|\text{Re}\left[Z\cdot I_{cap}\right]\right| = \left|\text{Re}\left[(0.306 + j0.6272)\cdot 0.046299\underline{/90}\right]\right| = 0.029037\ \text{V}$$

The nominal line-to-neutral voltage is

$$V_{LN} = \frac{12{,}470}{\sqrt{3}} = 7{,}199.6\ \text{V}$$

The K_{rise} factor is then

$$K_{rise} = \frac{0.029037}{7{,}199.6}\cdot 100 = 0.00040331\%\ \text{rise/kvar-mile}$$

2. The percent voltage drop in Example 3.3 was computed to be 3.9702%. To limit the total voltage drop to 2.5%, the required voltage rise due to a shunt capacitor bank is

$$V_{rise} = 3.9702 - 2.5 = 1.4702\%$$

The required rating of the shunt capacitor is

$$kvar = \frac{V_{rise}}{K_{rise}\cdot mile} = \frac{1.4702}{0.00040331\cdot 1.5} = 2430.18\ \text{kvar}$$

3.4 Uniformly Distributed Loads

Many times it can be assumed that loads are uniformly distributed along a line where the line can be a three-phase, two-phase, or single-phase feeder or lateral. This is certainly the case on single-phase laterals where the same rating transformers are spaced uniformly over the length of the lateral. When the loads are uniformly distributed, it is not necessary to model each load in order to determine the total voltage drop from the source end to the last load. Figure 3.6 shows a generalized line with $\boldsymbol{n}$ uniformly distributed loads.

3.4.1 Voltage Drop

Figure 3.6 shows $\boldsymbol{n}$ uniformly spaced loads dx miles apart. The loads are all equal and will be treated as constant current loads with a value of di. The total current into the feeder is I_T. It is desired to determine the total voltage drop from the source node (**S**) to the last node $\boldsymbol{n}$.

Let
- l be the length of the feeder
- $z = r + jx$ be the impedance of the line in Ω/mile
- dx be the length of each line section
- di be the load currents at each node
- n be the number of nodes and number of line sections
- I_T be the total current into the feeder

The load currents are given by

$$di = \frac{I_T}{n} \tag{3.8}$$

The voltage drop in the first line segment is given by

$$Vdrop_1 = \mathrm{Re}\{z \cdot dx \cdot (n \cdot di)\} \tag{3.9}$$

The voltage drop in the second line segment is given by

$$Vdrop_2 = \mathrm{Re}\{z \cdot dx \cdot [(n-1) \cdot di]\} \tag{3.10}$$

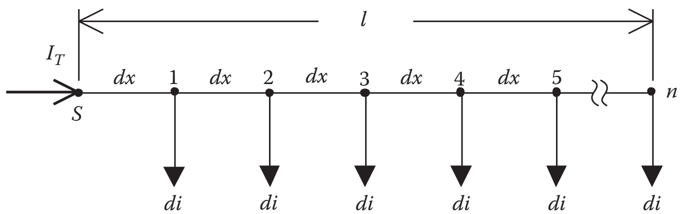

FIGURE 3.6
Uniformly distributed loads.

The total voltage drop from the source node to the last node is then given by

$$Vdrop_{total} = Vdrop_1 + Vdrop_2 + \cdots + Vdrop_n$$

$$Vdrop_{total} = \mathrm{Re}\{z \cdot dx \cdot di \cdot [n + (n-1) + (n-2) + \cdots + (1)]\} \tag{3.11}$$

Equation 3.11 can be reduced by recognizing the series expansion:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \tag{3.12}$$

Using the expansion, Equation 3.11 becomes

$$Vdrop_{total} = \mathrm{Re}\left\{z \cdot dx \cdot di \cdot \left[\frac{n \cdot (n+1)}{2}\right]\right\} \tag{3.13}$$

The incremental distance is

$$dx = \frac{l}{n} \tag{3.14}$$

The incremental current is

$$di = \frac{I_T}{n} \tag{3.15}$$

Substituting Equations 3.14 and 3.15 into Equation 3.13 results in

$$\begin{aligned} Vdrop_{total} &= \mathrm{Re}\left\{z \cdot \frac{l}{n} \cdot \frac{I_T}{n} \cdot \left[\frac{n \cdot (n+1)}{2}\right]\right\} \\ &= \mathrm{Re}\left\{z \cdot l \cdot I_T \cdot \frac{1}{2} \cdot \left(\frac{n+1}{n}\right)\right\} \\ &= \mathrm{Re}\left\{\frac{1}{2} \cdot Z \cdot I_T \cdot \left(1 + \frac{1}{n}\right)\right\} \end{aligned} \tag{3.16}$$

where $Z = z \cdot l$.

Equation 3.16 gives the general equation for computing the total voltage drop from the source to the last node $\boldsymbol{n}$ for a line of length l. In the limiting case where $\boldsymbol{n}$ goes to infinity, the final equation becomes

$$Vdrop_{total} = \mathrm{Re}\left\{\frac{1}{2} \cdot Z \cdot I_T\right\} \tag{3.17}$$

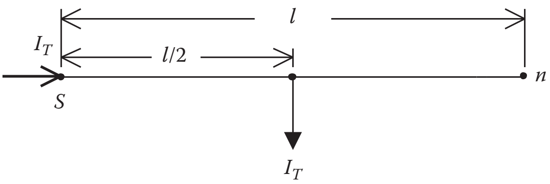

FIGURE 3.7
Load lumped at the midpoint.

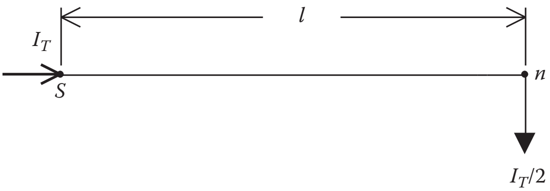

FIGURE 3.8
One-half load lumped at the end.

In Equation 3.17 Z represents the total impedance from the source to the end of the line. The voltage drop is the total from the source to the end of the line. The equation can be interpreted in two ways. The first is to recognize that the total line distributed load can be lumped at the midpoint of the lateral as shown in Figure 3.7.

A second interpretation of Equation 3.17 is to lump one-half of the total line load at the end of the line (node ***n***). This model is shown in Figure 3.8.

Figures 3.7 and 3.8 give two different models that can be used to calculate the total voltage drop from the source to the end of a line with uniformly distributed loads.

3.4.2 Power Loss

Of equal importance in the analysis of a distribution feeder is the power loss. If the model of Figure 3.7 is used to compute the total three-phase power loss down the line, the result is

$$P_{loss} = 3 \cdot |I_T|^2 \cdot \frac{R}{2} = \frac{3}{2} \cdot |I_T|^2 \cdot R \tag{3.18}$$

When the model of Figure 3.8 is used to compute the total three-phase power loss, the result is

$$P_{loss} = 3 \cdot \left|\frac{I_T}{2}\right|^2 \cdot R = \frac{3}{4} \cdot |I_T|^2 \cdot R \tag{3.19}$$

It is obvious that the two models give different results for the power loss. The question is, Which one is correct? The answer is neither one.

To derive the correct model for power loss, reference is made to Figure 3.6 and the definitions for the parameters in that figure. The total three-phase power loss down the line will be the sum of the power losses in each short segment of the line. For example, the three-phase power loss in the first segment is

$$Ploss_1 = 3 \cdot (r \cdot dx) \cdot \left|(n \cdot di)\right|^2 \tag{3.20}$$

The power loss in the second segment is given by

$$Ploss_2 = 3 \cdot (r \cdot dx) \cdot \left[\left|(n-1) \cdot di\right|\right]^2 \tag{3.21}$$

The total power loss over the length of the line is then given by

$$Ploss_{total} = 3 \cdot (r \cdot dx) \cdot \left|di\right|^2 \left[n^2 + (n-1)^2 + (n-2)^2 + \cdots + 1^2\right] \tag{3.22}$$

The series inside the bracket of Equation 3.22 is the sum of the squares of ***n*** numbers and is equal to

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n \cdot (n+1) \cdot (2n+1)}{6} \tag{3.23}$$

Substituting Equations 3.14, 3.15, and 3.23 into Equation 3.22 gives

$$Ploss_{total} = 3 \cdot \left(r \cdot \frac{l}{n}\right) \cdot \left(\left|\frac{I_T}{n}\right|\right)^2 \cdot \left[\frac{n \cdot (n+1) \cdot (2n+1)}{6}\right] \tag{3.24}$$

Simplifying Equation 3.24

$$\begin{aligned} Ploss_{total} &= 3 \cdot R \cdot \left|I_T\right|^2 \cdot \left[\frac{(n+1) \cdot (2n+1)}{6 \cdot n^2}\right] \\ &= 3 \cdot R \cdot \left|I_T\right|^2 \cdot \left[\frac{2 \cdot n^2 + 3 \cdot n + 1}{6 \cdot n^2}\right] \\ &= 3 \cdot R \cdot \left|I_T\right|^2 \cdot \left[\frac{1}{3} + \frac{1}{2 \cdot n} + \frac{1}{6 \cdot n^2}\right] \end{aligned} \tag{3.25}$$

where $R = r \cdot l$ is the total resistance per phase of the line segment.

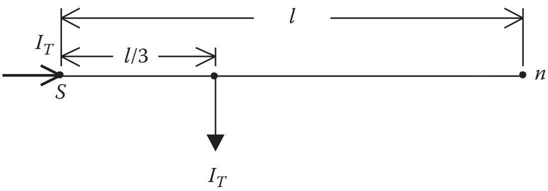

FIGURE 3.9
Power loss model.

Equation 3.25 gives the total three-phase power loss for a discrete number of nodes and line segments. For a truly uniformly distributed load, the number of nodes goes to infinity. When that limiting case is taken in Equation 3.25, the final equation for computing the total three-phase power loss down the line is given by

$$Ploss_{total} = 3 \cdot \left[\frac{1}{3} \cdot R \cdot |I_T|^2 \right] \tag{3.26}$$

A circuit model for Equation 3.26 is given in Figure 3.9.

From a comparison of Figures 3.7 and 3.8, used for voltage drop calculations, to Figure 3.9, used for power loss calculations, it is obvious that the same model cannot be used for both voltage drop and power loss calculations.

3.4.3 Exact Lumped Load Model

In the previous sections lumped load models were developed. The first models of Section 3.4.1 can be used for the computation of the total voltage drop down the line. It was shown that the same models cannot be used for the computation of the total power loss down the line. Section 3.4.2 developed a model that will give the correct power loss of the line. What is needed is one model that will work for both voltage drop and power loss calculations.

Figure 3.10 shows the general configuration of the "exact" model that will give correct results for voltage drop and power loss.

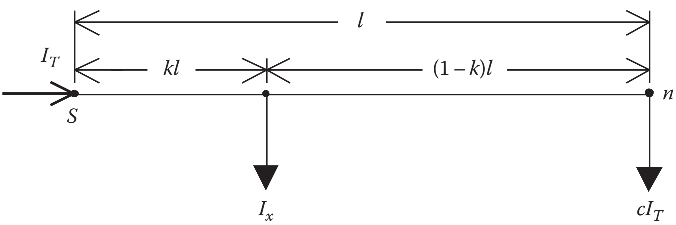

FIGURE 3.10
General exact lumped load model.

In Figure 3.10 a portion (I_x) of the total line current (I_T) will be modeled kl miles from the source end and the remaining current (cI_T) will be modeled at the end of the line. The values of k and c need to be derived.

In Figure 3.10 the total voltage drop down the line is given by

$$Vdrop_{total} = \mathrm{Re}\left[k \cdot Z \cdot I_T + (1-k) \cdot Z \cdot c \cdot I_T\right] \tag{3.27}$$

where

- Z is the total line impedance in ohms
- k is the factor of the total line length where the first part of the load current is modeled
- c is the factor of the total current to be placed at the end of the line such that $I_T = I_x + c \cdot I_T$

In Section 3.4.1, it was shown that the total voltage drop down the line is given by

$$Vdrop_{total} = \mathrm{Re}\left[\frac{1}{2} \cdot Z \cdot I_T\right] \tag{3.28}$$

Set Equation 3.17 equal to Equation 3.27:

$$Vdrop_{total} = \mathrm{Re}\left[\frac{1}{2} \cdot Z \cdot I_T\right] = \mathrm{Re}\left[k \cdot Z \cdot I_T + (1-k) \cdot Z \cdot c \cdot I_T\right] \tag{3.29}$$

Equation 3.29 shows that the terms inside the brackets on both sides of the equal side need to be set equal, that is,

$$\left[\frac{1}{2} \cdot Z \cdot I_T\right] = \left[k \cdot Z \cdot I_T + (1-k) \cdot Z \cdot c \cdot I_T\right] \tag{3.30}$$

Simplify Equation 3.30 by dividing both sides of the equation by ZI_T:

$$\frac{1}{2} = \left[k + (1-k) \cdot c\right] \tag{3.31}$$

Solve Equation 3.31 for k:

$$k = \frac{0.5 - c}{1 - c} \tag{3.32}$$

The same procedure can be followed for the power loss model. The total three-phase power loss in Figure 3.10 is given by

$$Ploss_{total} = 3 \cdot \left[k \cdot R \cdot |I_T|^2 + (1-k) \cdot R \cdot \left(c \cdot |I_T|\right)^2\right] \tag{3.33}$$

The model for the power loss of Figure 3.9 gives the total three-phase power loss as

$$Ploss_{total} = 3 \cdot \left[\frac{1}{3} \cdot R \cdot |I_T|^2 \right] \tag{3.34}$$

Equate the terms inside the brackets of Equations 3.33 and 3.34 and simplify:

$$\begin{aligned} \left[\frac{1}{3} \cdot R \cdot |I_T|^2 \right] &= \left[k \cdot R \cdot |I_T|^2 + (1-k) \cdot R \cdot \left(c \cdot |I_T| \right)^2 \right] \\ \frac{1}{3} &= \left[k + (1-k) \cdot (c)^2 \right] \\ \frac{1}{3} &= \left[k + c^2 - k \cdot c^2 \right] = \left[k \cdot (1-c^2) + c^2 \right] \end{aligned} \tag{3.35}$$

Substitute Equation 3.32 into Equation 3.35:

$$\frac{1}{3} = \left[\frac{0.5-c}{1-c} \cdot (1-c^2) + c^2 \right] \tag{3.36}$$

Solving Equation 3.36 for *c* results in

$$c = \frac{1}{3} \tag{3.37}$$

Substitute Equation 3.37 into Equation 3.32 and solve for *k*:

$$k = \frac{1}{4} \tag{3.38}$$

The interpretation of Equations 3.37 and 3.38 is that one-third of the load should be placed at the end of the line and two-thirds of the load placed one-fourth of the way from the source end. Figure 3.11 gives the final exact lumped load model.

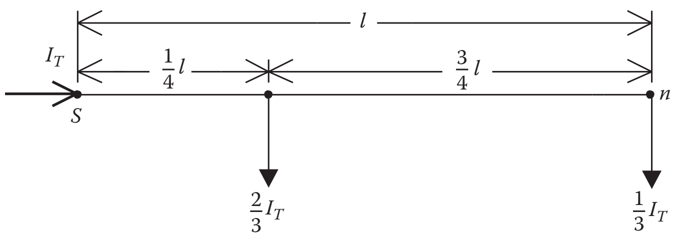

FIGURE 3.11
Exact lumped load model.

3.5 Lumping Loads in Geometric Configurations

Many times feeder areas can be represented by geometric configurations such as rectangles, triangles, and trapezoids. By assuming a constant load density in the configurations, approximate calculations can be made for computing the voltage drop and total power losses. The approximate calculations can aid in the determination of the maximum load that can be served in a specified area at a given voltage level and conductor size. For all of the geographical areas to be evaluated, the following definitions will apply:

D represents the load density in kVA/mile^2.

PF represents the assumed lagging power factor.

z represents the line impedance in Ω/mile.

l represents the length of the area.

w represents the width of the area.

kV_{LL} represents the nominal line-to-line voltage in kV.

It will also be assumed that the loads are modeled as constant current loads.

3.5.1 Rectangle

A rectangular area of length l and width w is to be served by a primary main feeder. The feeder area is assumed to have a constant load density with three-phase laterals uniformly tapped off of the primary main. Figure 3.12 shows a model for the rectangular area.

Figure 3.12 represents a rectangular area of constant load density being served by a three-phase main running from node $\boldsymbol{n}$ to node $\boldsymbol{m}$. It is desired to determine the total voltage drop and the total three-phase power loss down the primary main from node $\boldsymbol{n}$ to node $\boldsymbol{m}$.

The total current entering the area is given by

$$I_T = \frac{D \cdot l \cdot w}{\sqrt{3} \cdot kV_{LL}} \underline{/-\cos^{-1}(PF)} \tag{3.39}$$

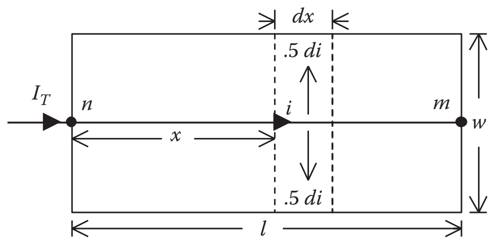

FIGURE 3.12
Constant load density rectangular area.

An incremental segment is located x miles from node $\boldsymbol{n}$. The incremental current serving the load in the incremental segment is given by

$$di = \frac{I_T}{l}\ \text{A/mile} \tag{3.40}$$

The current in the incremental segment is given by

$$i = I_T - x \cdot di = I_T - x \cdot \frac{I_T}{l} = I_T \cdot \left(1 - \frac{x}{l}\right) \tag{3.41}$$

The voltage drop in the incremental segment is

$$dV = \text{Re}(z \cdot i \cdot dx) = \text{Re}\left[z \cdot I_T \cdot \left(1 - \frac{x}{l}\right) \cdot dx\right] \tag{3.42}$$

The total voltage drop down the primary main feeder is

$$V_{drop} = \int_0^l dV = \text{Re}\left[z \cdot I_T \cdot \int_0^l \left(1 - \frac{x}{l}\right) \cdot dx\right]$$

Evaluating the integral and simplifying,

$$V_{drop} = \text{Re}\left(z \cdot I_T \cdot \frac{1}{2} \cdot l\right) = \text{Re}\left[\frac{1}{2} \cdot Z \cdot I_T\right] \tag{3.43}$$

where $Z = z \cdot l$.

Equation 3.43 gives the same result as that of Equation 3.17 which was derived for loads uniformly distributed along a feeder. The only difference here is the manner in which the total current (I_T) is determined. The bottom line is that the total load of a rectangular area can be modeled at the centroid of the rectangle as shown in Figure 3.13.

It must be understood that in Figure 3.13 with the load modeled at the centroid, the voltage drop computed to the load point will represent the total voltage drop from node $\boldsymbol{n}$ to node $\boldsymbol{m}$.

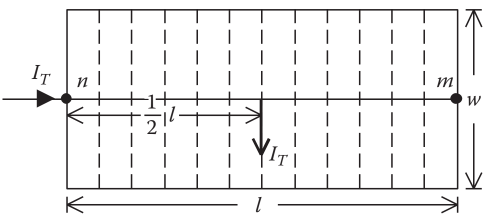

FIGURE 3.13
Rectangle voltage drop model.

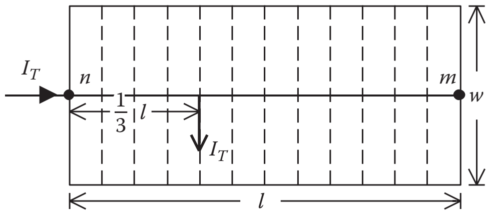

FIGURE 3.14
Rectangle power loss model.

A similar derivation can be done in order to determine the total three-phase power loss down the feeder main. The power loss in the incremental length is

$$dp = 3\cdot|i|^2\cdot r\cdot dx = 3\cdot\left[|I_T|^2\cdot\left(1-\frac{x}{l}\right)^2\cdot r\cdot dx\right] = 3\cdot r\cdot|I_T|^2\cdot\left(1-2\cdot\frac{x}{l}+\frac{x^2}{l^2}\right)\cdot dx$$

The total three-phase power loss down the primary main is

$$P_{loss} = \int_0^l dp = 3\cdot r\cdot|I_T|^2\cdot\int_0^l\left(1-2\cdot\frac{x}{l}+\frac{x^2}{l^2}\right)\cdot dx$$

Evaluating the integral and simplifying,

$$P_{loss} = 3\cdot\left[\frac{1}{3}\cdot r\cdot l\cdot|I_T|^2\right] = 3\cdot\left[\frac{1}{3}\cdot R\cdot|I_T|^2\right] \tag{3.44}$$

where $R = r\cdot l$.

Equation 3.44 gives the same result as that of Equation 3.26. The only difference, again, is the manner in which the total current I_T is determined. The model for computing the total three-phase power loss of the primary main feeder is shown in Figure 3.14. Once again, it must be understood that the power loss computed using the model of Figure 3.14 represents the total power loss from node ***n*** to node ***m***.

Example 3.6

It is proposed to serve a rectangular area of length 10,000 ft and width of 6,000 ft. The load density of the area is 2500 kVA/mile^2 with a power factor of 0.9 lagging. The primary main feeder uses 336,400 26/7 ACSR on a pole configured as shown in Example 3.2, Figure 3.3. The question at hand is what minimum standard nominal voltage level can be used to serve this area without exceeding a voltage drop of 3% down the primary main? The choices of nominal voltages are 4.16 and 12.47 kV. Compute also the total three-phase power loss.

Solution

The area to be served is shown in Figure 3.15.

From Example 3.2, the impedance of the line was computed to be

$$z = 0.306 + j0.6272\ \Omega/\text{mile}$$

The length and width of the area in miles are

$$l = \frac{10{,}000}{5{,}280} = 1.8939\,\text{miles} \quad \text{and} \quad w = \frac{6{,}000}{5{,}280} = 1.1364\,\text{miles}$$

The total area of the rectangular area is

$$A = l \cdot w = 2.1522\,\text{miles}^2$$

The total load of the area is

$$kVA = D \cdot A = 2500 \cdot 2.1522 = 5380.6\,\text{kVA}$$

The total impedance of the line segment is

$$Z = z \cdot l = (0.306 + j0.6272) \cdot 1.8939 = 0.5795 + j1.1879\,\Omega$$

For a nominal voltage of 4.16 kV, the total area current is

$$I_T = \frac{kVA}{\sqrt{3} \cdot kV_{LL}} = \frac{5{,}380.6}{\sqrt{3} \cdot 4.16}\underline{/-\cos^{-1}(0.9)} = 746.7\underline{/-25.84}\,\text{A}$$

The total voltage drop down the primary main is

$$V_{drop} = \text{Re}\left[\frac{1}{2} \cdot Z \cdot I_T\right] = \text{Re}\left[\frac{1}{2} \cdot (0.5795 + j1.1879) \cdot 746.7\underline{/-25.84}\right] = 388.1\,\text{V}$$

The nominal line-to-neutral voltage is

$$V_{LN} = \frac{4160}{\sqrt{3}} = 2401.8\,\text{V}$$

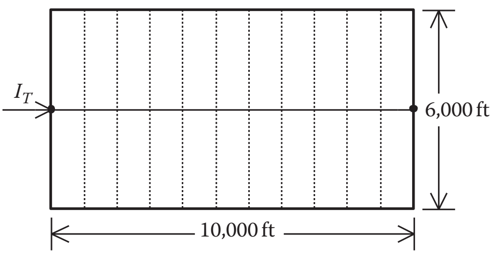

FIGURE 3.15
Example 3.6: rectangular area.

The percent voltage drop is

$$V_{\%} = \frac{V_{drop}}{V_{LN}} \cdot 100\% = \frac{388.1}{2401.8} \cdot 100\% = 16.16\%$$

It is clear that the nominal voltage of 4.16 kV will not meet the criteria of a voltage drop less than 3.0%.

For a nominal voltage of 12.47 kV, the total area current is

$$I_T = \frac{kVA}{\sqrt{3} \cdot kV_{LL}} = \frac{5,380.6}{\sqrt{3} \cdot 12.47} \underline{/-\cos^{-1}(0.9)} = 249.1\underline{/-25.84}\,\text{A}$$

The total voltage drop down the primary main is

$$V_{drop} = \text{Re}\left[\frac{1}{2} \cdot Z \cdot I_T\right] = \text{Re}\left[\frac{1}{2} \cdot (0.5795 + j1.1879) \cdot 249.1\underline{/-25.84}\right] = 129.5\,\text{V}$$

The nominal line-to-neutral voltage is

$$V_{LN} = \frac{12,470}{\sqrt{3}} = 7,199.6\,\text{V}$$

The percent voltage drop is

$$V_{\%} = \frac{V_{drop}}{V_{LN}} \cdot 100\% = \frac{129.5}{7,199.6} \cdot 100\% = 1.80\%$$

The nominal voltage of 12.47 kV is more than adequate to serve this load. It would be possible at this point to determine how much larger the area could be and still satisfy the 3.0% voltage drop constraint.

For the 12.47 kV selection, the total three-phase power loss down the primary main is

$$P_{loss} = 3 \cdot \left[\frac{(1/3) \cdot R \cdot |I_T|^2}{1000}\right] = 3 \cdot \left[\frac{(1/3) \cdot 0.5795 \cdot 249.1^2}{1000}\right] = 35.965\,\text{kW}$$

3.5.2 Triangle

A triangular area with a constant load density is being served by a three-phase primary main feeder as shown in Figure 3.16.

Figure 3.16 represents a triangular area of constant load density being served by a three-phase main running from node ***n*** to node ***m***. It is desired to determine the total voltage drop and the total three-phase power loss down the primary main from node ***n*** to node ***m***.

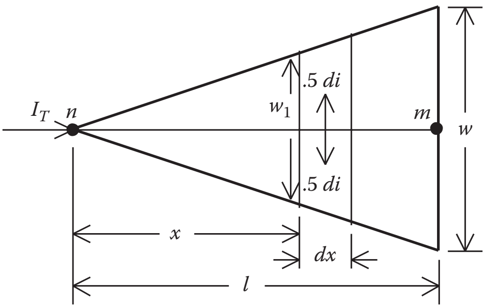

FIGURE 3.16
Constant load density triangular area.

The area of the triangle is

$$Area = \frac{1}{2} \cdot l \cdot w \tag{3.45}$$

The total current entering the area is given by

$$I_T = \frac{D \cdot Area}{\sqrt{3} \cdot kV_{LL}} \underline{/-\cos^{-1}(PF)}\ \mathrm{A} \tag{3.46}$$

Let

$$di = \frac{I_T}{Area} = \frac{I_T}{(1/2) \cdot l \cdot w} = \frac{2 \cdot I_T}{l \cdot w}\ \mathrm{A/mile^2} \tag{3.47}$$

The current entering the incremental line segment is

$$i = I_T - A_1 \cdot di \tag{3.48}$$

where A_1 is the area of the triangle up to the incremental line segment.
By similar triangles,

$$w_1 = x \cdot \frac{w}{l} \tag{3.49}$$

The area of the small triangle up to the incremental line segment is

$$A_1 = \frac{1}{2} \cdot x \cdot w_1 = \frac{1}{2} \cdot x \cdot \left(x \cdot \frac{w}{l} \right) = \frac{1}{2} \cdot \frac{w}{l} \cdot x^2 \tag{3.50}$$

Substitute Equations 3.47 and 3.50 into Equation 3.48:

$$i = I_T - \left(\frac{1}{2} \cdot \frac{w}{l} \cdot x^2 \right) \cdot \left(\frac{2}{l \cdot w} \cdot I_T \right) = I_T \cdot \left(1 - \frac{x^2}{l^2} \right) \tag{3.51}$$

The voltage drop in the incremental line segment is given by

$$dv = \mathrm{Re}[i \cdot z \cdot dx] = \mathrm{Re}\left[z \cdot I_T \cdot \left(1 - \frac{x^2}{l^2}\right) \cdot dx\right] \tag{3.52}$$

The total voltage drop from node ***n*** to node ***m*** is

$$V_{drop} = \int_0^l dv = \mathrm{Re}\left[z \cdot I_T \cdot \int_0^l \left(1 - \frac{x^2}{l^2}\right) \cdot dx\right]$$

Evaluating the integral and simplifying,

$$V_{drop} = \mathrm{Re}\left[z \cdot I_T \cdot \frac{2}{3} \cdot l\right] = \mathrm{Re}\left[\frac{2}{3} \cdot Z_T \cdot I_T\right] \tag{3.53}$$

where $Z_T = z \cdot l$.

Equation 3.53 shows that the total voltage drop from the vertex to the base of the triangular area can be computed by modeling the total triangle load two-thirds of the distance between the vertex and the base of the triangle. The model for the voltage drop calculation is shown in Figure 3.17.

A similar derivation can be made for the power loss model. The power loss in the incremental line segment is

$$dp = 3 \cdot \left[r \cdot |i|^2 \cdot dx\right] \tag{3.54}$$

Substitute Equation 3.51 into Equation 3.54:

$$dp = 3 \cdot \left[r \cdot |I_T|^2 \cdot \left(1 - \frac{x^2}{l^2}\right) \cdot dx\right] = 3 \cdot \left[r \cdot |I_T|^2 \cdot \left(1 - 2 \cdot \frac{x^2}{l^2} + \frac{x^4}{l^3}\right) \cdot dx\right]$$

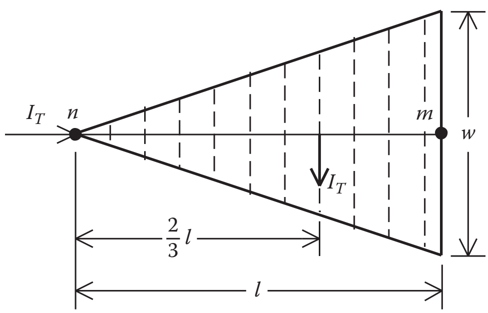

FIGURE 3.17
Triangle voltage drop model.

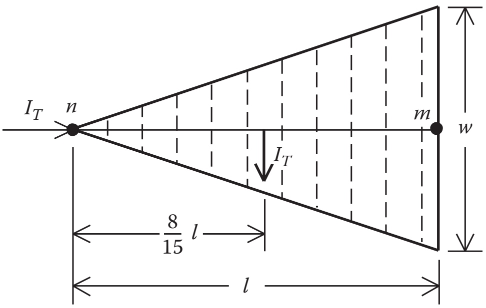

FIGURE 3.18
Triangle power loss model.

The total three-phase power loss from node ***n*** to node ***m*** becomes

$$P_{loss} = \int_0^l dp = 3 \cdot r \cdot |I_T|^2 \cdot \int_0^l \left(1 - 2 \cdot \frac{x^2}{l^2} + \frac{x^4}{l^3}\right) \cdot dx$$

Evaluating the integral and simplifying,

$$P_{loss} = 3 \cdot \left[\frac{8}{15} \cdot R \cdot |I_T|^2\right] \tag{3.55}$$

Equation 3.55 gives the total three-phase power loss down the primary main from node ***n*** to node ***m***. The model for the power loss is given in Figure 3.18.

Example 3.7

The triangular area shown in Figure 3.19 is to be served by a feeder of nominal voltage 12.47 kV.

The load density of the area is 3500 kVA/mile^2 at a power factor of 0.9 lagging. The conductor on the primary main is 336,400 26/7 ACSR and the configuration of the pole is that of Example 3.2 in Figure 3.3.

Use the K_{drop} factor from the line of Example 3.2 and determine the percent voltage drop from node ***n*** to node ***m***.

From Example 3.3 the K_{drop} factor was computed to be

$$K_{drop} = 0.00035291\%\ \text{drop/kVA-mile}$$

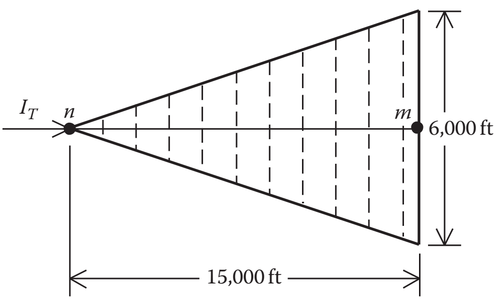

FIGURE 3.19
Example 3.7: triangular area.

The length and width of the triangle in miles is

$$l = \frac{15{,}000}{5{,}280} = 2.8409\,\text{miles} \quad \text{and} \quad w = \frac{6{,}000}{5{,}280} = 1.1364\,\text{miles}$$

The area of the triangle is

$$Area = \frac{1}{2} \cdot 2.8509 \cdot 1.1364 = 1.6142\,\text{miles}^2$$

The total load of the triangular area is

$$kVA = 3500 \cdot 1.6142 = 5649.5\,\text{kVA}$$

The total complex power of the triangular area is

$$S = kVA\underline{/\cos^{-1}(PF)} = 5649.5\underline{/-25.84} = 5084.6 + j2462.6\,\text{kW} + j\,\text{kvar}$$

Using the K_{drop} factor and lumping the total load at the two-thirds point, the percent drop to node ***m*** is

$$V_{drop} = \frac{2}{3} \cdot K_{drop} \cdot kVA\text{-}mile = \frac{2}{3} \cdot 0.00035291 \cdot 5649.5 \cdot 2.8409 = 3.7761\%$$

Suppose now that a shunt capacitor bank is to be installed somewhere along the primary main in order to limit the percent voltage drop to node ***m*** to 3.0%. Two decisions must be made:

1. Three-phase rating of the capacitor bank
2. Location of the capacitor bank

The total reactive power of the area was computed to be 2462.6 kvar. That means that a capacitor bank rated up to 2462.6 can be used without causing the feeder to go into a leading power factor condition. Since this is assumed to be the peak load, a capacitor bank rated at 1800 kvar (three phase) will be used in order to prevent a leading power factor condition for a smaller load. Depending upon the load curve during the day, this bank may or may not have to be switched.

Use the K_{rise} factor from Example 3.5 and determine how far from node ***n*** the capacitor bank should be installed in order to limit the voltage drop to 3.0%. From Example 3.5,

$$K_{rise} = 0.00040331\%\ \text{rise/kvar-mile}$$

The needed voltage rise due to the capacitor is

$$V_{rise} = V_{drop} - 3.0 = 3.7761 - 3.0 = 0.7761$$

The distance from node ***n*** is determined by

$$dist = \frac{V_{rise}}{K_{rise} \cdot kvar} = \frac{0.7761}{0.00040331 \cdot 1800} = 1.0691\,\text{miles}$$

The total three-phase power loss down the primary main before the shunt capacitor is added is computed by lumping the total triangular load at

$$l_{Load} = \frac{8}{15} \cdot l = 1.5151\,\text{miles from node } \boldsymbol{n}$$

The total load current is

$$I_T = \frac{kVA}{\sqrt{3} \cdot kV_{LL}} = \frac{5649.5}{\sqrt{3} \cdot 12.47} \underline{/-\cos^{-1}(PF)} = 261.6\underline{/-25.84}\,\text{A}$$

The total resistance of the primary main is

$$R = r \cdot l = 0.306 \cdot 2.8409 = 0.8693\,\Omega$$

The total three-phase power loss down the primary main is

$$P_{loss} = \frac{3}{1000} \cdot \left[\frac{8}{15} \cdot R \cdot |I_T|^2\right] = \frac{3}{1000} \cdot \left[\frac{8}{15} \cdot 0.8693 \cdot 261.6^2\right] = 95.16\,\text{kW}$$

3.5.3 Trapezoid

The final geometric configuration to consider is the trapezoid. As before, it is assumed that the load density is constant throughout the trapezoid. The general model of the trapezoid is shown in Figure 3.20.

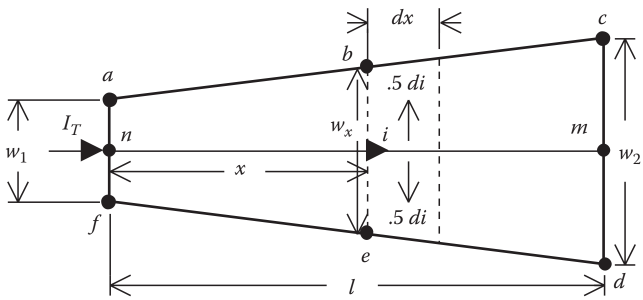

FIGURE 3.20
General trapezoid.

Figure 3.20 shows a trapezoidal area of constant load density being served by a three-phase primary running from node ***n*** to node ***m***. It is desired to determine the total voltage drop and the total three-phase power loss down the primary main from node ***n*** to node ***m***.

It is necessary to determine the value of the current entering the incremental line segment as a function of the total current and the known dimensions of the trapezoid. The known dimensions will be the length (l) and the widths w_1 and w_2.

The total current entering the trapezoid is

$$I_T = \frac{D \cdot Area_T}{\sqrt{3} \cdot kV_{LL}} \tag{3.56}$$

where $Area_T$ is the total area of the trapezoid given by

$$Area_T = \frac{1}{2} \cdot (w_2 + w_1) \cdot l \tag{3.57}$$

The current that is delivered to the trapezoid abef is

$$I_x = \frac{D \cdot Area_x}{\sqrt{3} \cdot kV_{LL}} \tag{3.58}$$

where $Area_x$ is the area of the trapezoid abef given by

$$Area_x = \frac{1}{2} \cdot (w_x + w_1) \cdot x \tag{3.59}$$

Solve Equation 3.56 for D:

$$D = \frac{\sqrt{3} \cdot kV_{LL} \cdot I_T}{Area_T} \tag{3.60}$$

Substitute Equation 3.60 into Equation 3.58:

$$I_x = \left(\frac{\sqrt{3} \cdot kV_{LL} \cdot I_T}{Area_T} \right) \cdot \left(\frac{Area_x}{\sqrt{3} \cdot kV_{LL}} \right) = \frac{Area_x}{Area_T} \cdot I_T \tag{3.61}$$

The current entering the incremental line segment is

$$i = I_T - I_x = I_T \cdot \left(1 - \frac{Area_x}{Area_T} \right) \tag{3.62}$$

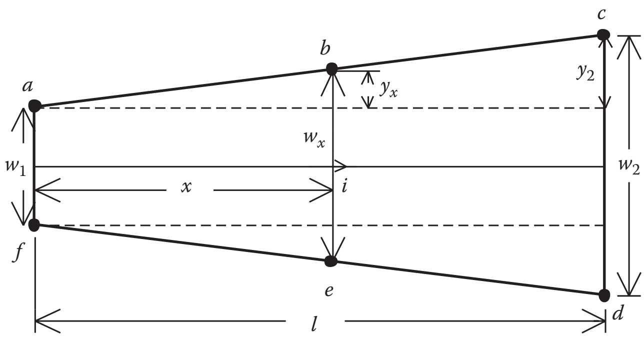

FIGURE 3.21
Trapezoid dimensions.

The only problem at this point is that the area of the small trapezoid cannot be determined since the width w_x is not known. Figure 3.21 will be used to establish the relationship between the unknown width and the known dimensions.

Referring to Figure 3.21,

$$w_x = w_1 + 2 \cdot y_x \tag{3.63}$$

From similar triangles,

$$y_x = \frac{x}{l} \cdot y_2 \tag{3.64}$$

But

$$y_2 = \frac{1}{2} \cdot (w_2 - w_1) \tag{3.65}$$

Substitute Equation 3.65 into Equation 3.64:

$$y_x = \frac{x}{l} \cdot \frac{1}{2} \cdot (w_2 - w_1) \tag{3.66}$$

Substitute Equation 3.66 into Equation 3.63:

$$w_x = w_1 + 2 \cdot \frac{x}{l} \cdot \frac{1}{2} \cdot (w_2 - w_1) = w_1 + \frac{x}{l} \cdot (w_2 - w_1) = w_1 \cdot \left(1 - \frac{x}{l}\right) + \frac{x}{l} \cdot w_2 \tag{3.67}$$

Substitute Equation 3.67 into Equation 3.59:

$$Area_x = \frac{1}{2}\cdot\left[\left(w_1\cdot\left(1-\frac{x}{l}\right)+\frac{x}{l}\cdot w_2\right)+w_1\right]\cdot x \tag{3.68}$$

Substitute Equations 3.57 and 3.68 into Equation 3.62:

$$\begin{aligned} i &= I_T\cdot\left[1-\frac{(1/2)\cdot\left[\left(w_1\cdot(2-(x/l))+(x/l)\cdot w_2\right)\right]\cdot x}{(1/2)\cdot(w_2+w_1)\cdot l}\right] \\ &= \frac{I_T}{(w_1+w_2)\cdot l}\cdot\left[(w_1+w_2)\cdot l-2\cdot w_1\cdot x+w_1\cdot\left(\frac{x^2}{l}\right)-w_2\cdot\left(\frac{x^2}{l}\right)\right] \\ &= \frac{I_T}{(w_1+w_2)\cdot l}\cdot\left[\left(l-2\cdot x+\frac{x^2}{l}\right)\cdot w_1+\left(l-\frac{x^2}{l}\right)\cdot w_2\right] \end{aligned} \tag{3.69}$$

The current entering the incremental line segment of Figure 3.20 is given in Equation 3.69 and will be used to compute the voltage drop and power loss in the incremental line segment. The voltage drop in the incremental line segment is given by

$$dv = \mathrm{Re}\left[z\cdot i\cdot dx\right] \tag{3.70}$$

Substitute Equation 3.69 into Equation 3.70:

$$dv = \mathrm{Re}\left\{z\cdot\frac{I_T}{(w_1+w_2)\cdot l}\cdot\left[\left(l-2\cdot x+\frac{x^2}{l}\right)\cdot w_1+\left(l-\frac{x^2}{l}\right)\cdot w_2\right]\cdot dx\right\} \tag{3.71}$$

The total voltage drop down the primary from node ***n*** to node ***m*** is given by

$$V_{drop} = \int_0^l dv = \mathrm{Re}\left\{\frac{z\cdot I_T}{(w_1+w_2)\cdot l}\cdot\int_0^l\left[\left(l-2\cdot x+\frac{x^2}{l}\right)\cdot w_1+\left(l-\frac{x^2}{l}\right)\cdot w_2\right]\cdot dx\right\}$$

Evaluating the integral and simplifying results in

$$V_{drop} = \mathrm{Re}\left[Z\cdot I_T\cdot\left(\frac{w_1+2\cdot w_2}{3\cdot(w_1+w_2)}\right)\right] \tag{3.72}$$

Equation 3.72 is very general and is used in the following to determine the models for the rectangular and triangular areas.

Rectangle

For a rectangular area the two widths w_1 and w_2 will be equal. Let

$$w_1 = w_2 = w \tag{3.73}$$

Substitute Equation 3.73 into Equation 3.72:

$$V_{drop} = \text{Re}\left[Z \cdot I_T \cdot \left(\frac{w + 2 \cdot w}{3 \cdot (w + w)}\right)\right] = \text{Re}\left[Z \cdot I_T \cdot \frac{3 \cdot w}{6 \cdot w}\right]$$

$$= \text{Re}\left[\frac{1}{2} \cdot Z \cdot I_T\right] \tag{3.74}$$

Equation 3.74 is the same that was initially derived for the rectangular area.

Triangle

For a triangular area the width w_1 will be zero. Let

$$w_1 = 0 \tag{3.75}$$

Substitute Equation 3.75 into Equation 3.72:

$$V_{drop} = \text{Re}\left[Z \cdot I_T \cdot \left(\frac{0_1 + 2 \cdot w_2}{3 \cdot (0 + w_2)}\right)\right] = \text{Re}\left[\frac{2}{3} \cdot Z \cdot I_T\right] \tag{3.76}$$

Equation 3.76 is the same as was derived for the triangular area.

The total three-phase power loss down the line segment can be developed by starting with the derived current in the incremental segment as given by Equation 3.69. The three-phase power loss in the incremental segment is

$$dp = 3 \cdot r \cdot i^2 dx \tag{3.77}$$

The total three-phase power loss down the line segment is then

$$P_{loss} = 3 \cdot r \cdot \int_0^l i^2 dx \tag{3.78}$$

Substitute Equation 3.69 into Equation 3.78 and simplify:

$$P_{loss} = 3 \cdot \frac{r \cdot |I_T|^2}{(w_1 + w_2) \cdot l^2} \cdot \int_0^l \left[\left(l - 2 \cdot x + \frac{x^2}{l}\right) \cdot w_1 + \left(l - \frac{x^2}{l}\right) \cdot w_2\right]^2 dx \tag{3.79}$$

Evaluating the integral and simplifying results in

$$P_{loss} = 3 \cdot \left\{ R \cdot |I_T|^2 \cdot \left[\frac{8 \cdot w_2^2 + 9 \cdot w_1 \cdot w_2 + 3 \cdot w_1^2}{15 \cdot (w_1 + w_2)^2} \right] \right\} \tag{3.80}$$

where $R = r \cdot l$.

The rectangular and triangular areas are special cases of Equation 3.80.

Rectangle

For the rectangle, the two widths w_1 and w_2 are equal. Let $w = w_1 = w_2$.

Substitute into Equation 3.79:

$$P_{loss} = 3 \cdot \left\{ R \cdot |I_T|^2 \cdot \left[\frac{8 \cdot w^2 + 9 \cdot w \cdot w + 3 \cdot w^2}{15 \cdot (w + w)^2} \right] \right\} = 3 \cdot \left\{ R \cdot |I_T|^2 \cdot \left[\frac{8 + 9 + 3}{15 \cdot (2)^2} \right] \right\}$$

$$P_{loss} = 3 \cdot \left[\frac{1}{3} \cdot R \cdot |I_T|^2 \right] \tag{3.81}$$

Equation 3.81 is the same as Equation 3.44 that was previously derived for the rectangular area.

Triangle

For the triangular area, the width w_1 is zero. Let $w_1 = 0$.

Substitute into Equation 3.80:

$$P_{loss} = 3 \cdot \left\{ R \cdot |I_T|^2 \cdot \left[\frac{8 \cdot w_2{}^2 + 9 \cdot 0 \cdot w_2 + 3 \cdot 0^2}{15 \cdot (0 + w_2)^2} \right] \right\} = 3 \cdot \left[\frac{8}{15} \cdot R \cdot |I_T|^2 \right] \tag{3.82}$$

Equation 3.82 is the same as Equation 3.55 that was previously derived for the total power loss in a triangular area.

3.6 Summary

This chapter has been devoted to the development of some useful techniques for computing the voltage drop and power loss of line segments with uniformly distributed loads and for geometric areas with constant load densities. These techniques are very useful for making quick calculations that will be "ballpark" values. Many times only a ballpark value is needed. More times than not once inside the ballpark, more precise values of voltage drop and power loss are needed. This will be especially true when the unbalanced nature of a distribution feeder is taken into account. The remainder of this book will be devoted to the more precise methods for analyzing a distribution feeder under balanced and unbalanced steady-state and short-circuit conditions.

Problems

3.1 Figure 3.22 shows the pole configuration of conductors for a three-phase primary feeder. The conductors are 250,000 cm, CON Lay, all aluminum (AA). The nominal line-to-line voltage of the feeder is 14.4 kV.

a. Determine the series impedance per mile of this line.

b. Determine the K_{drop} factor assuming a power factor of 0.88 lag.

c. Determine the K_{rise} factor.

3.2 A 4.16 three-phase primary feeder is shown in Figure 3.23.

The K_{drop} = 0.00298639% drop/kVA-mile

The K_{rise} = 0.00334353% rise/kvar-mile

a. Determine the percent voltage drop to node E4.

b. Determine the rating of a three-phase shunt capacitor bank to be placed at E3 to limit the voltage drop to E4 to 5.0%.

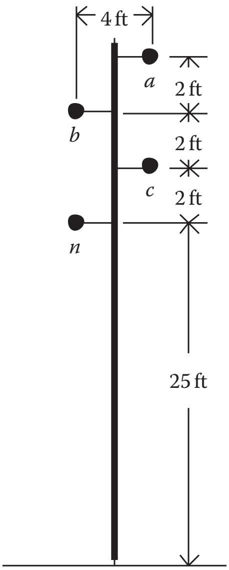

FIGURE 3.22
Problem 3.1: configuration.

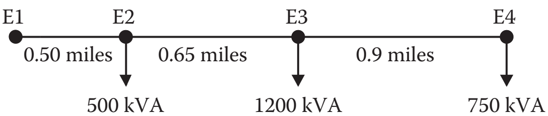

FIGURE 3.23
System for Problem 3.2.

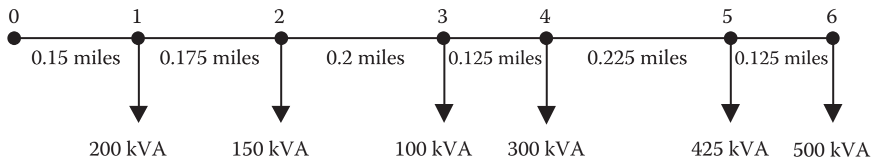

FIGURE 3.24
System for Problem 3.3.

3.3 A 4160 V three-phase feeder is shown in Figure 3.24.

The phase conductors are 4/0 ACSR and are configured on an 8 ft crossarm with phase spacings of $D_{ab} = 2.5$ ft, $D_{bc} = 4.5$ ft, and $D_{ca} = 7.0$ ft

a. Determine the series impedance of the line segment in Ω/mile.
b. Determine the K_{drop} and K_{rise} factors assuming a load power factor of 0.9 lagging.
c. Determine the total percent voltage drop to node 6.
d. Determine the three-phase kvar rating of a shunt capacitor to be placed at node 4 to limit the total percent voltage drop to node 6 to 3.0%.

3.4 Flash Thunder Bolt, junior engineer for Tortugas Power and Light, has been given an assignment to design a new 4.16 kV, three-phase feeder that will have the following characteristics:

Total length of feeder = 5000 ft

Load: Ten 500 kVA (three-phase), 0.9 lagging power spaced every 500 ft with the first load 500 ft from the substation

Voltage drop: Not to exceed 5% from the sub to the last load

Figure 3.25 illustrates the new feeder.

Flash has decided that he will use 336,400 26/7 ACSR (Linnet) conductors constructed on 45 ft poles with 8 ft crossarms. The spacings of the conductors on the crossarms are 2.5, 4.5, and 7.0 ft.

a. Determine the percent voltage drop to the last load point and the total three-phase power loss for the feeder as shown in Figure 3.25.
b. Lump the total feeder load at the midpoint of the feeder and compute the percent voltage drop to the end of the feeder.

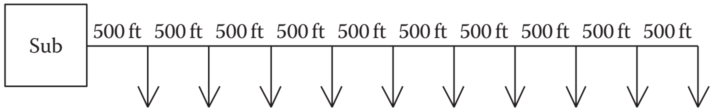

FIGURE 3.25
System for Problem 3.4.

c. Use the "exact lumped load model" of Figure 3.11 and compute the percent voltage drop to the end of the line and the total three-phase power loss down the line.

3.5 The rectangular area in Figure 3.26 has a uniform load density of 2000 kVA/mile^2 at 0.9 lagging power factor. The nominal voltage of the area being served is 4.16 kV. The three-phase primary main conductors are 556,500 26/7 ACSR while the three-phase lateral conductors are 266,800 26/7 ACSR. The primary main and the laterals are constructed so that the equivalent spacing (D_{eq}) is 3.5 ft.
Determine

a. The % voltage drop to the last customer in the first lateral (point A)
b. The % voltage drop to the last customer in the last lateral (point B)
c. The total three-phase power loss for the total area

3.6 Figure 3.27 shows a rectangle–triangle area that is being fed from a source at point X. Both areas have a load density of 6000 kVA/mile^2 with loads being uniformly distributed as denoted by the dashed laterals.

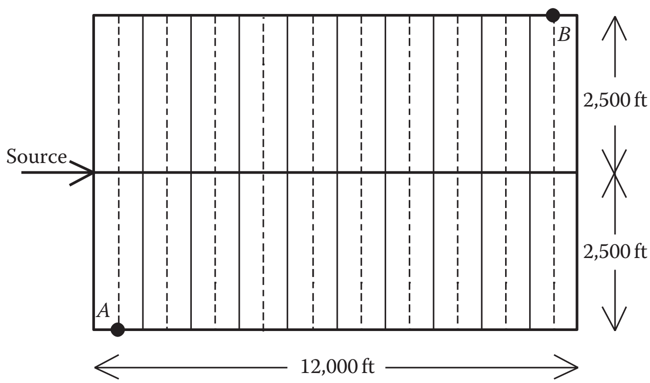

FIGURE 3.26
Rectangular area for Problem 3.5.

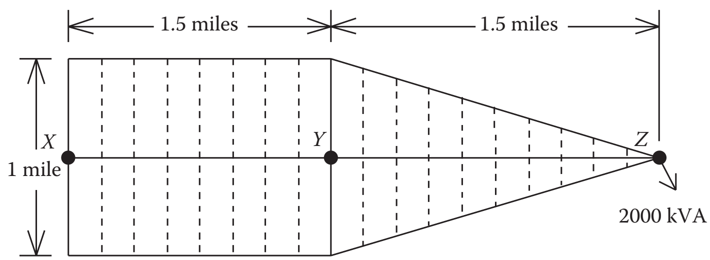

FIGURE 3.27
Rectangular–triangular area of Problem 3.6.

In addition to the uniformly distributed loads, there is a "spot load" at point Z that is 2000 kVA. The K_{drop} factor for the primary main conductors is 0.00022626% drop/kVA-mile and the K_{rise} factor for the primary main conductors is 0.00028436% rise/kvar-mile.

a. Determine the percent drop to point Z.
b. Determine the kvar rating (to the nearest 300 kvar/phase) for a capacitor bank to be placed at point Y in order to limit the voltage drop to Z to 3%.
c. With the capacitor in place, what now is the percent drop to point Z?

3.7 A square area of 20,000 ft has a load density of 2000 kVA/mile^2 on a side, and 0.9 lagging power factor is to be served from a 12.47 kV substation that is located in the center of the square. Two different plans are being considered for serving the area. The two plans are shown in Figure 3.28.

Plan A proposes to break the area into four square areas and serve it as shown. The big black line will be the three-phase primary main consisting of 336/400 26/7 ACSR conductors and the dotted lines will be the three-phase laterals consisting of 4/0 ACSR conductors. Both the main and laterals are constructed such that D_{eq} = 4.3795 ft. The three-phase laterals will be spaced every 500 ft.

Plan B proposes to serve the area with four triangularly shaped feeders. Again the primary main is shown in the dark black line and the laterals are spaced every 500 ft and shown as dotted lines. The same conductors and D_{eq} will be used in this plan.

Determine the percent voltage drop to the "last customer" (points A and B) for the two plans.

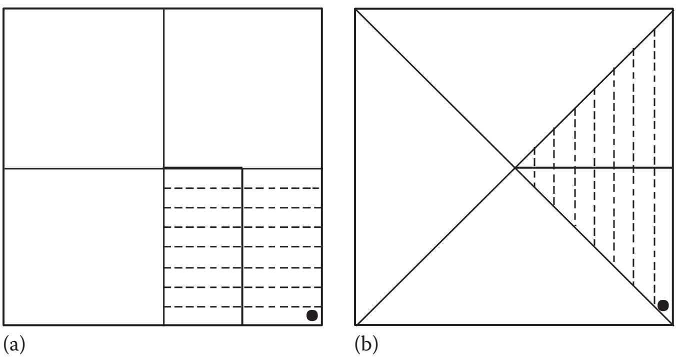

FIGURE 3.28
Two plans for Problem 3.7. (a) Plan A. (b) Plan B.

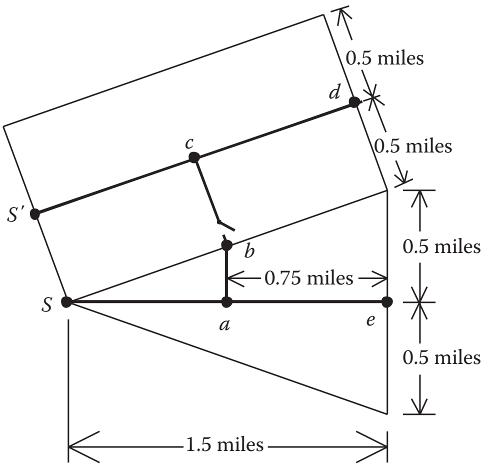

FIGURE 3.29
Areas for Problem 3.8.

3.8 Figure 3.29 shows the areas normally served by two feeders.
Under an emergency condition the switch at *b* is closed so that the feeder normally serving the triangle area must now serve both areas. Assume both areas have a uniform load density of 2.5 MVA/mile², 0.9 lagging power factor. The primary feeder voltage is 13.8 kV. Laterals are uniformly tapped off of the primary main from *S* to *a*. No loads are tapped off the feed from *a* to *b* to *c*, and laterals are tapped off from *c* to *d* and from *c* to *S*′. The primary main conductors are 2/0 ACSR and are placed on a pole such that $D_{eq} = 4.3795$ ft.

a. Determine the K_{drop} and K_{rise} factors.
b. Determine the voltage drop to point *d*.
c. Determine the three-phase kvar rating of a shunt capacitor bank placed at *c* in order to limit the voltage drop to point *d* to 3.0%.
d. Determine the voltage drop to *e* with the capacitor bank at *c*.
e. Determine the voltage drop to *e* with the source at *S*′ and the capacitor at *c*.

Reference

1. Glover, J.D. and Sarma, M., *Power System Analysis and Design*, 2nd edn., PWS Publishing Co., Boston, MA, 1994.

4

Series Impedance of Overhead and Underground Lines

The determination of the series impedance for overhead and underground lines is a critical step before the analysis of a distribution feeder can begin. The series impedance of a single-phase, two-phase (V-phase), or three-phase distribution line consists of the resistance of the conductors and the self- and mutual inductive reactances resulting from the magnetic fields surrounding the conductors. The resistance component for the conductors will typically come from a table of conductor data such as that found in Appendix A.

4.1 Series Impedance of Overhead Lines

The inductive reactance (self and mutual) component of the impedance is a function of the total magnetic fields surrounding a conductor. Figure 4.1 shows conductors 1 through n with the magnetic flux lines created by currents flowing in each of the conductors.

The currents in all conductors are assumed to be flowing out of the page. It is further assumed that the sum of the currents will add to zero. That is,

$$I_1 + I_2 + \cdots + I_i + \cdots + I_n = 0 \tag{4.1}$$

The total flux linking conductor i is given by

$$\lambda_i = N \cdot \phi = 2 \cdot 10^{-7} \cdot \left(I_1 \cdot \ln\frac{1}{D_{i1}} + I_2 \cdot \ln\frac{1}{D_{i2}} + \cdots I_i \cdot \ln\frac{1}{GMR_i} + \cdots I_n \cdot \ln\frac{1}{D_{in}} \right) \tag{4.2}$$

where

N = Number of times the line of flux surrounds the conductor. For this case $N = 1$

D_{in} is the distance between conductor i and conductor n (ft)

GMR_i is the geometric mean radius of conductor i (ft)

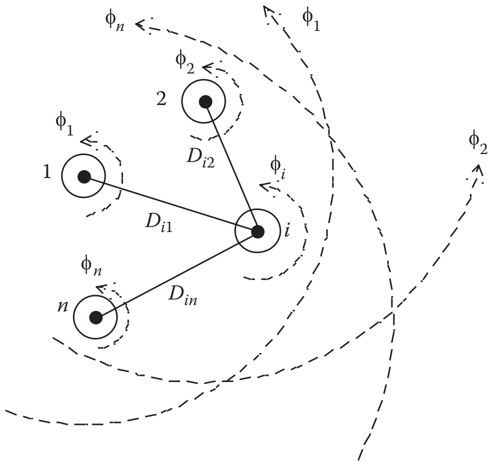

FIGURE 4.1
Magnetic fields.

The inductance of conductor i consists of the "self-inductance" of conductor i and the "mutual inductance" between conductor i and all of the other $n - 1$ conductors. By definition,

Self-inductance:

$$L_{ii} = \frac{\lambda_{ii}}{I_i} = 2 \cdot 10^{-7} \cdot \ln \frac{1}{GMR_i} \text{ H/m} \tag{4.3}$$

Mutual inductance:

$$L_{in} = \frac{\lambda_{in}}{I_n} = 2 \cdot 10^{-7} \cdot \ln \frac{1}{D_{in}} \text{ H/m} \tag{4.4}$$

4.1.1 Transposed Three-Phase Lines

High-voltage transmission lines are usually assumed to be transposed (each phase occupies the same physical position on the structure for one-third of the length of the line). In addition to the assumption of transposition, it is assumed that the phases are equally loaded (balanced loading). With these two assumptions, it is possible to combine the "self" and "mutual" terms into one "phase" inductance [1].

Phase inductance:

$$L_i = 2 \cdot 10^{-7} \cdot \ln \frac{D_{eq}}{GMR_i} \text{ H/m} \tag{4.5}$$

where

$$D_{eq} = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}} \text{ ft} \tag{4.6}$$

D_{ab}, D_{bc}, and D_{ca} are the distances between phases.

Assuming a frequency of 60 Hz, the phase inductive reactance is given by

Phase reactance:

$$x_i = \omega \cdot L_i = 0.12134 \cdot \ln \frac{D_{eq}}{GMR_i} \ \Omega/\text{mile} \tag{4.7}$$

The series impedance per phase of a transposed three-phase line consisting of one conductor per phase is given by

Series impedance:

$$z_i = r_i + j \cdot 0.12134 \cdot \ln \frac{D_{eq}}{GMR_i} \ \Omega/\text{mile} \tag{4.8}$$

4.1.2 Untransposed Distribution Lines

Because distribution systems consist of single-phase, two-phase, and untransposed three-phase lines serving unbalanced loads, it is necessary to retain the identity of the self- and mutual impedance terms of the conductors in addition to taking into account the ground return path for the unbalanced currents. The resistance of the conductors is taken directly from a table of conductor data. Equations 4.3 and 4.4 are used to compute the self- and mutual inductive reactances of the conductors. The inductive reactance will be assumed to be at a frequency of 60 Hz, and the length of the conductor will be assumed to be 1 mile. With those assumptions, the self- and mutual impedances are given by

$$\bar{z}_{ii} = r_i + j0.12134 \cdot \ln \frac{1}{GMR_i} \ \Omega/\text{mile} \tag{4.9}$$

$$\bar{z}_{ij} = j0.12134 \cdot \ln \frac{1}{D_{ij}} \ \Omega/\text{mile} \tag{4.10}$$

In 1926, John Carson published a paper where he developed a set of equations for computing the self- and mutual impedances of lines taking into account the return path of current through ground [2]. Carson's approach was to represent a line with the conductors connected to a source at one end and grounded at the remote end. Figure 4.2 illustrates a line consisting of

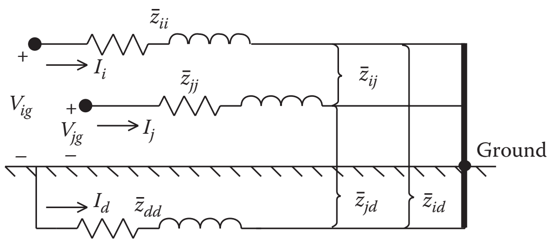

FIGURE 4.2
Two conductors with dirt return path.

two conductors (i and j) carrying currents (I_i and I_j) with the remote ends of the conductors tied to ground. A fictitious "dirt" conductor carrying current I_d is used to represent the return path for the currents.

In Figure 4.2, Kirchhoff's voltage law (KVL) is used to write the equation for the voltage between conductor i and ground:

$$V_{ig} = \bar{z}_{ii} \cdot I_i + \bar{z}_{ij} \cdot I_j + \bar{z}_{id} \cdot I_d - (\bar{z}_{dd} \cdot I_d + \bar{z}_{di} \cdot I_i + \bar{z}_{dj} \cdot I_j) \tag{4.11}$$

Collect terms in Equation 4.11:

$$V_{ig} = (\bar{z}_{ii} - \bar{z}_{dn}) \cdot I_i + (\bar{z}_{ij} - \bar{z}_{dj}) \cdot I_j + (\bar{z}_{id} - \bar{z}_{dd}) \cdot I_d \tag{4.12}$$

From Kirchhoff's current law,

$$\begin{gathered} I_i + I_j + I_d = 0 \\ I_d = -I_i - I_j \end{gathered} \tag{4.13}$$

Substitute Equation 4.13 into Equation 4.12 and collect terms:

$$V_{ig} = (\bar{z}_{ii} + \bar{z}_{dd} - \bar{z}_{di} - \bar{z}_{id}) \cdot I_i + (\bar{z}_{ij} + \bar{z}_{dd} - \bar{z}_{dj} - \bar{z}_{id}) \cdot I_j \tag{4.14}$$

Equation 4.14 is of the general form

$$V_{ig} = \hat{z}_{ii} \cdot I_i + \hat{z}_{ij} \cdot I_j \tag{4.15}$$

where

$$\hat{z}_{ii} = \bar{z}_{ii} + \bar{z}_{dd} - \bar{z}_{di} - \bar{z}_{id} \tag{4.16}$$

$$\hat{z}_{ij} = \bar{z}_{ij} + \bar{z}_{dd} - \bar{z}_{dj} - \bar{z}_{id} \tag{4.17}$$

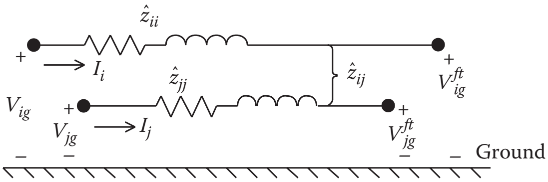

FIGURE 4.3
Equivalent primitive circuit.

In Equations 4.16 and 4.17, the "hat" impedances are given by Equations 4.9 and 4.10. Note that in these two equations the effect of the ground return path is being "folded" into what will now be referred to as the "primitive" self- and mutual impedances of the line. The "equivalent primitive circuit" is shown in Figure 4.3.

Substituting Equations 4.9 and 4.10 of the "hat" impedances into Equations 4.16 and 4.17, the primitive self-impedance is given by

$$\hat{z}_{ii} = r_i + jx_{ii} + r_d + jx_{dd} - jx_{dn} - jx_{nd}$$

$$\hat{z}_{ii} = r_d + r_i + j0.12134 \cdot \left(\ln \frac{1}{GMR_i} + \ln \frac{1}{GMR_d} - \ln \frac{1}{D_{id}} - \ln \frac{1}{D_{di}} \right)$$

$$\hat{z}_{ii} = r_d + r_i + j0.12134 \cdot \left(\ln \frac{1}{GMR_i} + \ln \frac{D_{id} \cdot D_{dj}}{GMR_d} \right) \tag{4.18}$$

In a similar manner, the primitive mutual impedance can be expanded:

$$\begin{aligned} \hat{z}_{ij} &= jx_{ij} + r_d + jx_{dd} - jx_{dj} - jx_{id} \\ &= r_d + j0.12134 \cdot \left(\ln \frac{1}{D_{ij}} + \ln \frac{1}{GMR_d} - \ln \frac{1}{D_{dj}} - \ln \frac{1}{D_{id}} \right) \\ &= r_d + j0.12134 \left(\ln \frac{1}{D_{ij}} + \ln \frac{D_{dj} \cdot D_{id}}{GMR_d} \right) \end{aligned} \tag{4.19}$$

The obvious problem in using Equations 4.18 and 4.19 is the fact that we do not know the values of the resistance of dirt (r_d), the geometric mean radius of dirt (GMR_d), and the distances from the conductors to dirt (D_{nd}, D_{dn}, D_{md}, D_{dm}). This is where John Carson's work bails us out.

4.1.3 Carson's Equations

Since a distribution feeder is inherently unbalanced, the most accurate analysis should not make any assumptions regarding the spacing between

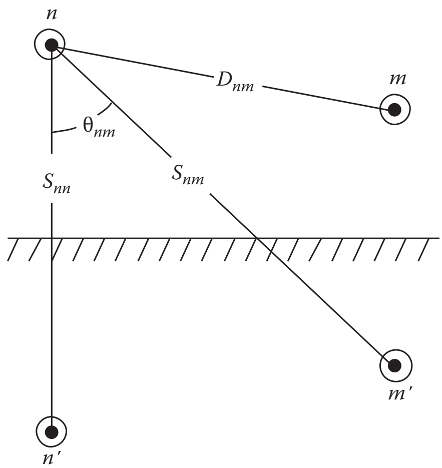

FIGURE 4.4
Conductors and images.

conductors, conductor sizes, and transposition. In Carson's 1926 paper, he developed a technique whereby the self- and mutual impedances for *ncond* overhead conductors can be determined. The equations can also be applied to underground cables. In 1926, this technique was not met with a lot of enthusiasm because of the tedious calculations that would have to be done on the slide rule and by hand. With the advent of the digital computer, Carson's equations have now become widely used.

In his paper, Carson assumes the earth is an infinite, uniform solid, with a flat uniform upper surface and a constant resistivity. Any "end effects" introduced at the neutral grounding points are not large at power frequencies and therefore are neglected.

Carson made use of conductor images; that is, every conductor at a given distance above ground has an image conductor the same distance below ground. This is illustrated in Figure 4.4.

Referring to Figure 4.4, the original Carson's equations are given in Equations 4.20 and 4.21.

Self-impedance:

$$\hat{z}_{ii} = r_i + 4\omega P_{ii} G + j\left(X_i + 2\omega G \cdot \ln \frac{S_{ii}}{RD_i} + 4\omega Q_{ii} G \right) \Omega/\text{mile} \tag{4.20}$$

Mutual impedance:

$$\hat{z}_{ij} = 4\omega P_{ij} G + j\left(2\omega G \cdot \ln \frac{S_{ij}}{D_{ij}} + 4\omega Q_{ij} G \right) \Omega/\text{mile} \tag{4.21}$$

$$X_i = 2\omega G \cdot \ln \frac{RD_i}{GMR_i}\ \Omega/\text{mile} \tag{4.22}$$

$$P_{ij} = \frac{\pi}{8} - \frac{1}{3\sqrt{2}} k_{ij}\cos(\theta_{ij}) + \frac{k_{ij}^2}{16}\cos(2\theta_{ij}) \cdot \left(0.6728 + \ln\frac{2}{k_{ij}}\right) \tag{4.23}$$

$$Q_{ij} = -0.0386 + \frac{1}{2} \cdot \ln\frac{2}{k_{ij}} + \frac{1}{3\sqrt{2}} k_{ij}\cos(\theta_{ij}) \tag{4.24}$$

$$k_{ij} = 8.565 \times 10^{-4} \cdot S_{ij} \cdot \sqrt{\frac{f}{\rho}} \tag{4.25}$$

where

- $\hat{z}_{ii}$ is the self-impedance of conductor i (Ω/mile)
- $\hat{z}_{ij}$ is the mutual impedance between conductors i and j (Ω/mile)
- r_i is the resistance of conductor i (Ω/mile)
- $\omega = 2\pi f$ is the system angular frequency (rad/s)
- $G = 0.1609347 \times 10^{-3}$ Ω/mile
- RD_i is the radius of conductor i (ft)
- GMR_i is the geometric mean radius of conductor i (ft)
- f is the system frequency (Hz)
- ρ is the resistivity of earth (Ω-m)
- D_{ij} is the distance between conductors i and j (ft) (see Figure 4.4)
- S_{ij} is the distance between conductor i and image j (ft) (see Figure 4.4)
- θ_{ij} is the angle between a pair of lines drawn from conductor i to its own image and to the image of conductor j (see Figure 4.4)

4.1.4 Modified Carson's Equations

Only two approximations are made in deriving the "modified Carson's Equations." These approximations involve the terms associated with P_{ij} and Q_{ij}. The approximations use only the first term of the variable P_{ij} and the first two terms of Q_{ij}:

$$P_{ij} = \frac{\pi}{8} \tag{4.26}$$

$$Q_{ij} = -0.03860 + \frac{1}{2}\ln\frac{2}{k_{ij}} \tag{4.27}$$

Substitute X_i (Equation 4.22) into Equation 4.20:

$$\hat{z}_{ii} = r_i + 4\omega P_{ii} G + j\left(2\omega G \cdot \ln\frac{RD_i}{GMR_i} + 2\omega G \cdot \ln\frac{S_{ii}}{RD_i} + 4\omega Q_{ii} G\right) \tag{4.28}$$

Combine terms and simplify:

$$\hat{z}_{ii} = r_i + 4\omega P_{ii} G + j2\omega G\left(\ln\frac{S_{ii}}{GMR_i} + \ln\frac{RD_i}{RD_i} + 2Q_{ii}\right) \tag{4.29}$$

Simplify Equation 4.21:

$$\hat{z}_{ij} = 4\omega P_{ij} G + j2\omega G\left(\ln\frac{S_{ij}}{D_{ij}} + 2Q_{ij}\right) \tag{4.30}$$

Substitute expressions for P (Equation 4.26) and $\omega(2 \cdot \pi \cdot f)$:

$$\hat{z}_{ii} = r_i + \pi^2 fG + j4\pi fG\left(\ln\frac{S_{ii}}{GMR_i} + 2Q_{ii}\right) \tag{4.31}$$

$$\hat{z}_{ij} = \pi^2 fG + j4\pi fG\left(\ln\frac{S_{ij}}{D_{ij}} + 2Q_{ij}\right) \tag{4.32}$$

Substitute expression for k_{ij} (Equation 4.25) into the approximate expression for Q_{ij} (Equation 4.27):

$$Q_{ij} = -0.03860 + \frac{1}{2}\ln\left(\frac{2}{8.565 \cdot 10^{-4} \cdot S_{ij} \cdot \sqrt{f/\rho}}\right) \tag{4.33}$$

Expand

$$Q_{ij} = -0.03860 + \frac{1}{2}\ln\left(\frac{2}{8.565 \cdot 10^{-4}}\right) + \frac{1}{2}\ln\frac{1}{S_{ij}} + \frac{1}{2}\ln\sqrt{\frac{\rho}{f}} \tag{4.34}$$

Equation 4.34 can be reduced to

$$Q_{ij} = 3.8393 - \frac{1}{2}\ln S_{ij} + \frac{1}{4}\ln\frac{\rho}{f} \tag{4.35}$$

or

$$2Q_{ij} = 2Q_{ij} = 7.6786 - \ln S_{ij} + \frac{1}{2}\ln\frac{\rho}{f} \tag{4.36}$$

Substitute Equation 4.36 into Equation 4.31 and simplify:

$$\hat{z}_{ii} = r_i + \pi^2 fG + j4\pi fG\left(\ln\frac{S_{ii}}{GMR_i} + 7.6786 - \ln S_{ii} + \frac{1}{2}\ln\frac{\rho}{f}\right)$$

$$\hat{z}_{ii} = r_i + \pi^2 fG + 4\pi fG\left(\ln\frac{1}{GMR_i} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right) \tag{4.37}$$

Substitute Equation 4.36 into Equation 4.32 and simplify:

$$\hat{z}_{ij} = \pi^2 fG + j4\pi fG\left(\ln\frac{S_{ij}}{D_{ij}} + 7.6786 - \ln S_{ij} + \frac{1}{2}\ln\frac{\rho}{f}\right)$$

$$\hat{z}_{ij} = \pi^2 fG + j4\pi fG\left(\ln\frac{1}{D_{ij}} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right) \tag{4.38}$$

Substitute in the values of π and G:

$$\hat{z}_{ii} = r_i + 0.00158836\cdot f + j0.00202237\cdot f\left(\ln\frac{1}{GMR_i} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right) \tag{4.39}$$

$$\hat{z}_{ij} = 0.00158836\cdot f + j0.00202237\cdot f\left(\ln\frac{1}{D_{ij}} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right) \tag{4.40}$$

It is now assumed
f is the frequency = 60 Hz
ρ is the earth resistivity = 100 Ω-m

Using these approximations and assumptions the "modified Carson's equations" are

$$\hat{z}_{ii} = r_i + 0.09530 + j0.12134\left(\ln\frac{1}{GMR_i} + 7.93402\right)\ \Omega/\text{mile} \tag{4.41}$$

$$\hat{z}_{ij} = 0.09530 + j0.12134\left(\ln\frac{1}{D_{ij}} + 7.93402\right)\Omega/\text{mile} \tag{4.42}$$

It will be recalled that Equations 4.18 and 4.19 could not be used because the resistance of dirt, the GMR of dirt, and the various distances from conductors to dirt were not known. A comparison of Equations 4.18 and 4.19 to Equations 4.41 and 4.42 demonstrates that the modified Carson's equations have defined the missing parameters. A comparison of the two sets of equations shows that

$$r_d = 0.09530\,\Omega/\text{mile} \tag{4.43}$$

$$\ln\frac{D_{id}\cdot D_{di}}{GMR_d} = \ln\frac{D_{dj}\cdot D_{id}}{GMR_d} = 7.93402 \tag{4.44}$$

The "modified Carson's equations" will be used to compute the primitive self- and mutual impedances of overhead and underground lines.

There have been some questions about the approximations made in developing the modified Carson's equations. A paper was presented at the *IEEE 2011 Power System Conference and Exposition* [3]. In that paper, the full and modified equations were used and a comparison was made of the "errors." It was found that the errors were less than 1%. In that paper, the values of the resistivity of 10 and 1000 Ω-m were used instead of the assumed 100 Ω-m. A comparison was made and again the errors were found to be less than 1%. Because of the results in the paper, the modified equations developed earlier will not change.

4.1.5 Primitive Impedance Matrix for Overhead Lines

Equations 4.41 and 4.42 are used to compute the elements of an *ncond* × *ncond* "primitive impedance matrix." An overhead four-wire grounded wye distribution line segment will result in a 4 × 4 matrix. For an underground grounded wye line segment consisting of three concentric neutral cables, the resulting matrix will be 6 × 6. The primitive impedance matrix for a three-phase line consisting of *m* neutrals will be of the form

$$\left[\hat{z}_{primitive}\right] = \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & | & \hat{z}_{an1} & \hat{z}_{an2} & \hat{z}_{anm} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & | & \hat{z}_{bn1} & \hat{z}_{bn2} & \hat{z}_{bnm} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & | & \hat{z}_{cn1} & \hat{z}_{cn2} & \hat{z}_{cnm} \\ --- & --- & --- & --- & --- & --- & --- \\ \hat{z}_{n1a} & \hat{z}_{n1b} & \hat{z}_{n1c} & | & \hat{z}_{n1n1} & \hat{z}_{n1n2} & \hat{z}_{n1nm} \\ \hat{z}_{n2a} & \hat{z}_{n2b} & \hat{z}_{n2c} & | & \hat{z}_{n2n1} & \hat{z}_{n2n2} & \hat{z}_{n2nm} \\ \hat{z}_{nma} & \hat{z}_{nmb} & \hat{z}_{nmc} & | & \hat{z}_{nmn1} & \hat{z}_{nmn2} & \hat{z}_{nmnm} \end{bmatrix} \tag{4.45}$$

In partitioned form, Equation 4.45 becomes

$$\left[\hat{z}_{primitive}\right]=\begin{bmatrix}\left[\hat{z}_{ij}\right] & \left[\hat{z}_{in}\right]\\ \left[\hat{z}_{nj}\right] & \left[\hat{z}_{nn}\right]\end{bmatrix} \tag{4.46}$$

4.1.6 Phase Impedance Matrix for Overhead Lines

For most applications, the primitive impedance matrix needs to be reduced to a 3 × 3 "phase frame" matrix consisting of the self- and mutual equivalent impedances for the three phases. Figure 4.5 shows a four-wire grounded neutral line segment.

One standard method of reduction is the "Kron" reduction [4]. The assumption is made that the line has a multigrounded neutral as shown in Figure 4.5. The Kron reduction method applies KVL to the circuit:

$$\begin{bmatrix}V_{ag}\\ V_{bg}\\ V_{cg}\\ V_{ng}\end{bmatrix}=\begin{bmatrix}V'_{ag}\\ V'_{bg}\\ V'_{cg}\\ V'_{ng}\end{bmatrix}+\begin{bmatrix}\hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & \hat{z}_{an}\\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & \hat{z}_{bn}\\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & \hat{z}_{cn}\\ \hat{z}_{na} & \hat{z}_{nb} & \hat{z}_{nc} & \hat{z}_{nn}\end{bmatrix}\cdot\begin{bmatrix}I_a\\ I_b\\ I_c\\ I_n\end{bmatrix} \tag{4.47}$$

In partitioned form, Equation 4.47 becomes

$$\begin{bmatrix}\left[V_{abc}\right]\\ \left[V_{ng}\right]\end{bmatrix}=\begin{bmatrix}\left[V'_{abc}\right]\\ \left[V'_{ng}\right]\end{bmatrix}+\begin{bmatrix}\left[\hat{z}_{ij}\right] & \left[\hat{z}_{in}\right]\\ \left[\hat{z}_{ng}\right] & \left[\hat{z}_{nn}\right]\end{bmatrix}\cdot\begin{bmatrix}\left[I_{abc}\right]\\ \left[I_n\right]\end{bmatrix} \tag{4.48}$$

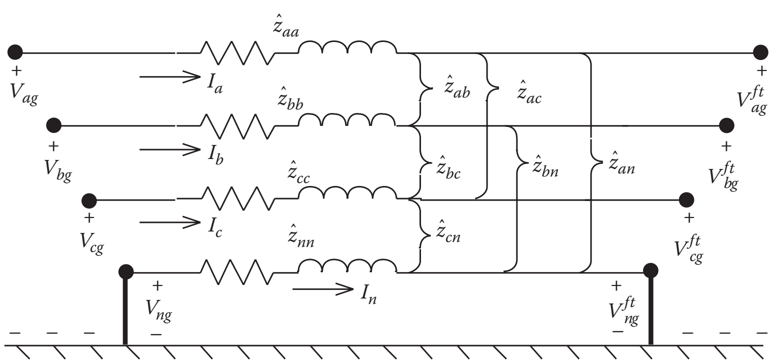

FIGURE 4.5
Four-wire grounded wye line segment.

Because the neutral is grounded, the voltages V_{ng} and V'_{ng} are equal to zero. Substituting those values into Equation 4.48 and expanding result in

$$[V_{abc}] = [V'_{abc}] + [\hat{z}_{ij}] \cdot [I_{abc}] + [\hat{z}_{in}] \cdot [I_n] \tag{4.49}$$

$$[0] = [0] + [\hat{z}_{nj}] \cdot [I_{abc}] + [\hat{z}_{nn}] \cdot [I_n] \tag{4.50}$$

Solve Equation 4.50 for $[I_n]$:

$$[I_n] = -[\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \cdot [I_{abc}] \tag{4.51}$$

Note in Equation 4.51 that once the line currents have been computed it is possible to determine the current flowing in the neutral conductor. Because this will be a useful concept later on, the "neutral transformation matrix" is defined as

$$[t_n] = -[\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \tag{4.52}$$

such that

$$[I_n] = [t_n] \cdot [I_{abc}] \tag{4.53}$$

Substitute Equation 4.51 into Equation 4.49:

$$\begin{aligned} [V_{abc}] &= [V'_{abc}] + \left([\hat{z}_{ij}] - [\hat{z}_{in}] \cdot [\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}]\right) \cdot [I_{abc}] \\ [V_{abc}] &= [V'_{abc}] + [z_{abc}] \cdot [I_{abc}] \end{aligned} \tag{4.54}$$

where

$$[z_{abc}] = [\hat{z}_{ij}] - [\hat{z}_{in}] \cdot [\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \tag{4.55}$$

Equation 4.55 is the final form of the "Kron" reduction technique. The final phase impedance matrix becomes

$$[z_{abc}] = \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \Omega/\text{mile} \tag{4.56}$$

For a distribution line that is not transposed, the diagonal terms of Equation 4.56 will not be equal to each other, and the off-diagonal terms will not be equal to each other. However, the matrix will be symmetrical.

For two-phase (V-phase) and single-phase lines in grounded wye systems, the modified Carson's equations can be applied, which will lead to initial 3 × 3 and 2 × 2 primitive impedance matrices. Kron reduction will reduce the matrices to 2 × 2 and a single element. These matrices can be expanded to 3 × 3 "phase frame" matrices by the addition of rows and columns consisting of zero elements for the missing phases. For example, for a V-phase line consisting of phases *a* and *c*, the phase impedance matrix would be

$$[z_{abc}] = \begin{bmatrix} z_{aa} & 0 & z_{ac} \\ 0 & 0 & 0 \\ z_{ca} & 0 & z_{cc} \end{bmatrix} \Omega/\text{mile} \tag{4.57}$$

The phase impedance matrix for a phase *b* single-phase line would be

$$[z_{abc}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & z_{bb} & 0 \\ 0 & 0 & 0 \end{bmatrix} \Omega/\text{mile} \tag{4.58}$$

The phase impedance matrix for a three-wire delta line is determined by the application of Carson's equations without the Kron reduction step.

The phase impedance matrix can be used to accurately determine the voltage drops on the feeder line segments once the currents have been determined. Since no approximations (transposition, for example) have been made regarding the spacing between conductors, the effect of the mutual coupling between phases is accurately taken into account. The application of the modified Carson's equations and the phase frame matrix leads to the most accurate model of a line segment. Figure 4.6 shows the general three-phase model

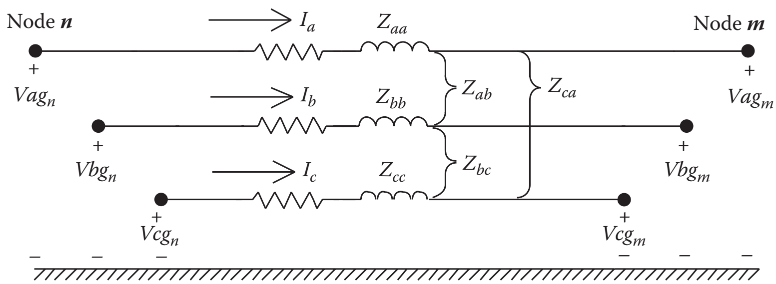

FIGURE 4.6
Three-phase line segment model.

of a line segment. Keep in mind that for V-phase and single-phase lines some of the impedance values will be zero.

The voltage equation in matrix form for the line segment is

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_n = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m + \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{4.59}$$

where $Z_{ij} = z_{ij} \cdot length$.

Equation 4.59 can be written in "condensed" form as

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}] \cdot [I_{abc}] \tag{4.60}$$

4.1.7 Sequence Impedances

Many times the analysis of a feeder will use only the positive and zero sequence impedances for the line segments. There are two methods for obtaining these impedances. The first method incorporates the application of the modified Carson's equations and the Kron reduction to obtain the phase impedance matrix.

The definition for line-to-ground phase voltages as a function of the line-to-ground sequence voltages is given by [2]

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \cdot \begin{bmatrix} VLG_0 \\ VLG_1 \\ VLG_2 \end{bmatrix} \tag{4.61}$$

where $a_s = 1.0\underline{/120}$.

In condensed form, Equation 4.61 becomes

$$[VLG_{abc}] = [A_s] \cdot [VLG_{012}] \tag{4.62}$$

where

$$[A_s] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \tag{4.63}$$

The phase line currents are defined in the same manner:

$$[I_{abc}] = [A_s] \cdot [I_{012}] \tag{4.64}$$

Equation 4.62 can be used to solve for the sequence line-to-ground voltages as a function of the phase line-to-ground voltages:

$$[VLG_{012}] = [A_s]^{-1} \cdot [VLG_{abc}] \tag{4.65}$$

where

$$[A_s]^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s & a_s^2 \\ 1 & a_s^2 & a_s \end{bmatrix} \tag{4.66}$$

Equation 4.60 can be transformed to the sequence domain by multiplying both sides by $[A_s]^{-1}$ and also substituting in the definition of the phase currents as given by Equation 4.62.

$$\begin{aligned} [VLG_{012}]_n &= [A_s]^{-1} \cdot [VLG_{abc}]_n \\ &= [A_s]^{-1} \cdot [VLG_{abn}]_m + [A_s]^{-1} \cdot [Z_{abc}] \cdot [A_s] \cdot [I_{012}] \\ &= [VLG_{012}]_m + [Z_{012}] \cdot [I_{012}] \end{aligned} \tag{4.67}$$

where

$$[Z_{012}] = [A_s]^{-1} \cdot [Z_{abc}] \cdot [A_s] = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \tag{4.68}$$

Equation 4.67 in expanded form is given by

$$\begin{bmatrix} VLG_0 \\ VLG_1 \\ VLG_2 \end{bmatrix}_n = \begin{bmatrix} VLG_0 \\ VLG_1 \\ VLG_2 \end{bmatrix}_m + \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{02} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{4.69}$$

Equation 4.68 is the defining equation for converting phase impedances to sequence impedances. In Equation 4.68, the diagonal terms of the matrix are the "sequence impedances" of the line such that

Z_{00} is the zero sequence impedance
Z_{11} is the positive sequence impedance
Z_{22} is the negative sequence impedance

The off-diagonal terms of Equation 4.68 represent the mutual coupling between sequences. In the idealized state, these off-diagonal terms would be zero. In order for this to happen, it must be assumed that the line has been transposed. For high-voltage transmission lines, this will generally be the case. When the lines are transposed, the mutual coupling between phases (off-diagonal terms) is equal, and, consequently, the off-diagonal terms of the sequence impedance matrix become zero. Since distribution lines are rarely, if ever, transposed, the mutual coupling between phases is not equal, and, as a result, the off-diagonal terms of the sequence impedance matrix will not be zero. This is the primary reason that distribution system analysis uses the phase domain rather than symmetrical components.

If a line is assumed to be transposed, the phase impedance matrix is modified so the three diagonal terms are equal and all of the off-diagonal terms are equal. The usual procedure is to set the three diagonal terms of the phase impedance matrix equal to the average of the diagonal terms of Equation 4.56 and the off-diagonal terms equal to the average of the off-diagonal terms of Equation 4.56. When this is done the self- and mutual impedances are defined as

$$z_s = \frac{1}{3} \cdot (z_{aa} + z_{bb} + z_{cc})\ \Omega/\text{mile} \tag{4.70}$$

$$z_m = \frac{1}{3} (z_{ab} + z_{bc} + z_{ca})\ \Omega/\text{mile} \tag{4.71}$$

The phase impedance matrix is now defined as

$$[z_{abc}] = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix} \Omega/\text{mile} \tag{4.72}$$

When Equation 4.68 is used with this phase impedance matrix, the resulting sequence matrix is diagonal (off-diagonal terms are zero). The sequence impedances can be determined directly as

$$z_{00} = z_s + 2 \cdot z_m\ \Omega/\text{mile} \tag{4.73}$$

$$z_{11} = z_{22} = z_s - z_m\ \Omega/\text{mile} \tag{4.74}$$

A second method that is commonly used to determine the sequence impedances directly is to employ the concept of geometric mean distances (GMDs). The GMD between phases is defined as

$$D_{ij} = GMD_{ij} = \sqrt[3]{D_{ab} \cdot D_{bc} \cdot D_{ca}}\ \text{ft} \tag{4.75}$$

The GMD between phases and neutral is defined as

$$D_{in} = GMD_{in} = \sqrt[3]{D_{an} \cdot D_{bn} \cdot D_{cn}} \text{ ft} \tag{4.76}$$

The GMDs as defined earlier are used in Equations 4.41 and 4.42 to determine the various self- and mutual impedances of the line resulting in

$$\hat{z}_{ii} = r_i + 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{GMR_i} \right) + 7.93402 \right] \Omega/\text{mile} \tag{4.77}$$

$$\hat{z}_{nn} = r_n + 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{GMR_n} \right) + 7.93402 \right] \Omega/\text{mile} \tag{4.78}$$

$$\hat{z}_{ij} = 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{D_{ij}} \right) + 7.93402 \right] \Omega/\text{mile} \tag{4.79}$$

$$\hat{z}_{in} = 0.0953 + j0.12134 \cdot \left[\ln\left(\frac{1}{D_{in}} \right) + 7.93402 \right] \Omega/\text{mile} \tag{4.80}$$

Equations 4.77 through 4.80 will define a matrix of order *ncond* × *ncond*, where *ncond* is the number of conductors (phases plus neutrals) in the line segment. Application of the Kron reduction (Equation 4.55) and the sequence impedance transformation (Equation 4.68) lead to the following expressions for the zero, positive, and negative sequence impedances:

$$z_{00} = \hat{z}_{ii} + 2 \cdot \hat{z}_{ij} - 3 \cdot \left(\frac{\hat{z}_{in}^2}{\hat{z}_{nn}} \right) \Omega/\text{mile} \tag{4.81}$$

$$\begin{aligned} z_{11} &= z_{22} = \hat{z}_{ii} - \hat{z}_{ij} \\ z_{11} &= z_{22} = r_i + j0.12134 \cdot \ln\left(\frac{D_{ij}}{GMR_i} \right) \Omega/\text{mile} \end{aligned} \tag{4.82}$$

Equations 4.81 and 4.82 are recognized as the standard equations for the calculation of the line impedances when a balanced three-phase system and transposition are assumed.

Example 4.1

An overhead three-phase distribution line is constructed as shown in Figure 4.7. Determine the phase impedance matrix and the positive and zero sequence of the line. The phase conductors are 336,400 26/7 ACSR (Linnet), and the neutral conductor is 4/0 6/1 ACSR.

Solution

From the table of standard conductor data (Appendix A), it is found that

336,400 26/7 ACSR:	$GMR = 0.0244\,\text{ft}$
	$Resistance = 0.306\,\Omega/\text{mile}$
4/0 6/1 ACSR:	$GMR = 0.00814\,\text{ft}$
	$Resistance = 0.5920\,\Omega/\text{mile}$

An effective way of computing the distance between all conductors is to specify each position on the pole in Cartesian coordinates using complex number notation. The ordinate will be selected as a point on the ground directly below the left most position. For the line in Figure 4.7, the positions are

$$d_1 = 0 + j29 \qquad d_2 = 2.5 + j29 \qquad d_3 = 7.0 + j29 \qquad d_4 = 4.0 + j25$$

The distances between the positions can be computed as

$$D_{12} = |d_1 - d_2| \qquad D_{23} = |d_2 - d_3| \qquad D_{31} = |d_3 - d_1|$$

$$D_{14} = |d_1 - d_4| \qquad D_{24} = |d_2 - d_4| \qquad D_{34} = |d_3 - d_4|$$

For this example, phase *a* is in position 1, phase *b* in position 2, phase *c* in position 3, and the neutral in position 4:

$$D_{ab} = 2.5\text{ ft} \qquad D_{bc} = 4.5\text{ ft} \qquad D_{ca} = 7.0\text{ ft}$$

$$D_{an} = 5.6569\text{ ft} \qquad D_{bn} = 4.272\text{ ft} \qquad D_{cn} = 5.0\text{ ft}$$

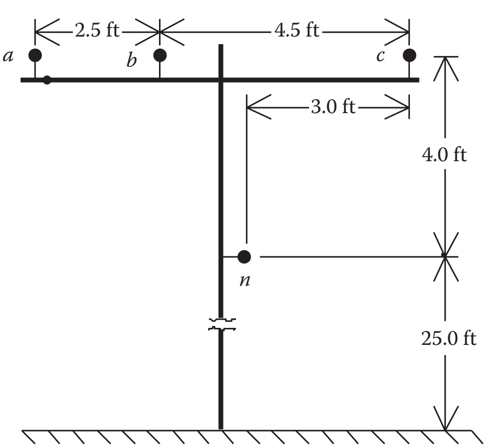

FIGURE 4.7
Three-phase distribution line spacings.

The diagonal terms of the distance matrix are the GMRs of the phase and neutral conductors:

$$D_{aa} = D_{bb} = D_{cc} = 0.0244 \qquad D_{nn} = 0.00814$$

Applying the modified Carson's equation for self-impedance (Equation 4.41), the self-impedance for phase *a* is

$$\hat{z}_{aa} = 0.0953 + 0.306 + j0.12134 \cdot \left(\ln \frac{1}{0.0244} + 7.93402 \right) = 0.4013 + j1.4133\,\Omega/\text{mile}$$

Applying Equation 4.42 for the mutual impedance between phases *a* and *b*,

$$\hat{z}_{ab} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{2.5} + 7.93402 \right) = 0.0953 + j0.8515\,\Omega/\text{mile}$$

Applying the equations for the other self- and mutual impedance terms results in the primitive impedance matrix:

$$[\hat{z}] = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 & 0.0953 + j0.7524 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0953 + j0.7802 & 0.0953 + j0.7865 \\ 0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 & 0.0953 + j0.7674 \\ 0.0953 + j0.7524 & 0.0953 + j0.7865 & 0.0953 + j0.7674 & 0.6873 + j1.5465 \end{bmatrix} \Omega/\text{mile}$$

The primitive impedance matrix in partitioned form is

$$[\hat{z}_{ij}] = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & j0.0943 + j0.7865 \\ 0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 \end{bmatrix} \Omega/\text{mile}$$

$$[\hat{z}_{in}] = \begin{bmatrix} 0.0953 + j0.7524 \\ 0.0953 + j0.7865 \\ 0.0953 + j0.7674 \end{bmatrix} \Omega/\text{mile}$$

$$[\hat{z}_{nn}] = [0.6873 + j1.5465]\,\Omega/\text{mile}$$

$$[\hat{z}_{nj}] = [0.0953 + j0.7524 \quad 0.0953 + j0.7865 \quad 0.0953 + j0.7674]\,\Omega/\text{mile}$$

The "Kron" reduction of Equation 4.55 results in the "phase impedance matrix":

$$[z_{abc}] = [\hat{z}_{ij}] - [\hat{z}_{in}] \cdot [\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}]$$

$$= \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

The neutral transformation matrix given by Equation 4.52 is

$$[t_n] = -\left([\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}]\right)$$

$$= [-0.4292 - j0.1291 \quad -0.4476 - j0.1373 \quad -0.4373 - j0.1327]$$

The phase impedance matrix can be transformed into the "sequence impedance matrix" with the application of Equation 4.66:

$$[z_{012}] = [A_s]^{-1} \cdot [z_{abc}] \cdot [A_s]$$

$$= \begin{bmatrix} 0.7735 + j1.9373 & 0.0256 + j0.0115 & -0.0321 + j0.0159 \\ -0.0321 + j0.0159 & 0.3061 + j0.6270 & -0.0723 - j0.0060 \\ 0.0256 + j0.0115 & 0.0723 - j0.0059 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile}$$

In the sequence impedance matrix, the 1,1 term is the zero sequence impedance, the 2,2 term is the positive sequence impedance, and the 3,3 term is the negative sequence impedance. The 2,2 and 3,3 terms are equal, which demonstrates that for line segments, the positive and negative sequence impedances are equal. Note that the off-diagonal terms are not zero. This implies that there is mutual coupling between sequences. This is a result of the nonsymmetrical spacing between phases. With the off-diagonal terms being nonzero, the three sequence networks representing the line will not be independent. However, it is noted that the off-diagonal terms are small relative to the diagonal terms.

In high-voltage transmission lines, it is usually assumed that the lines are transposed and that the phase currents represent a balanced three-phase set. The transposition can be simulated in Example 4.1 by replacing the diagonal terms of the phase impedance matrix with the average value of the diagonal terms ($0.4619 + j1.0638$) and replacing each off-diagonal term with the average of the off-diagonal terms ($0.1558 + j0.4368$). This modified phase impedance matrix becomes

$$[z1_{abc}] = \begin{bmatrix} 0.4619 + j1.0638 & 0.1558 + j0.4368 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.4619 + j1.0638 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.1558 + j0.4368 & 0.4619 + j1.0638 \end{bmatrix} \Omega/\text{mile}$$

Using this modified phase impedance matrix in the symmetrical component transformation equation results in the modified sequence impedance matrix:

$$[z1_{012}] = \begin{bmatrix} 0.7735 + j1.9373 & 0 & 0 \\ 0 & 0.3061 + j0.6270 & 0 \\ 0 & 0 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile}$$

Note now that the off-diagonal terms are all equal to zero, meaning that there is no mutual coupling between sequence networks. It should also be noted that the modified zero, positive, and negative sequence impedances are exactly equal to the exact sequence impedances that were first computed.

The results of this example should not be interpreted to mean that a three-phase distribution line can be assumed to have been transposed. The original phase impedance matrix must be used if the correct effect of the mutual coupling between phases is to be modeled.

4.1.8 Parallel Overhead Distribution Lines

It is fairly common in a distribution system to find instances where two distribution lines are "physically" parallel. The parallel combination may have both distribution lines constructed on the same pole or the two lines may run in parallel on separate poles but on the same right-of-way. For example, two different feeders leaving a substation may share a common pole or right-of-way before they branch out to their own service areas. It is also possible that two feeders may converge and run in parallel until again they branch out into their own service areas. The lines could also be underground circuits sharing a common trench. In all of the cases, the question becomes, How should the parallel lines be modeled and analyzed?

Two parallel overhead lines on one pole are shown in Figure 4.8. Note in Figure 4.8 the phasing of the two lines.

The phase impedance matrix for the parallel distribution lines is computed by the application of Carson's equations and the Kron reduction method. The first step is to number the phase positions as follows:

Position	1	2	3	4	5	6	7
Line phase	1-*a*	1-*b*	1-*c*	2-*a*	2-*b*	2-*c*	Neutral

With the phases numbered, the 7 × 7 primitive impedance matrix for 1 mile can be computed using the modified Carson's equations. It should be pointed out that if the two parallel lines are on different poles, most likely each pole will have a grounded neutral conductor. In this case, there will be eight positions, and position 8 will correspond to the neutral on line 2. An 8 × 8 primitive impedance matrix will be developed for this case. The Kron reduction

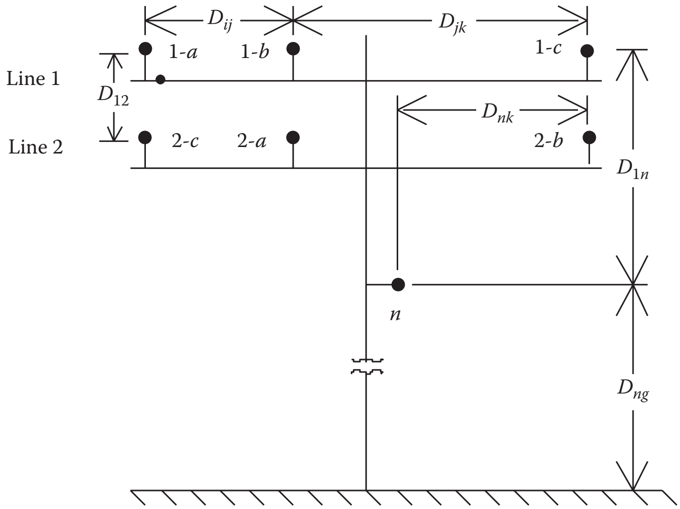

FIGURE 4.8
Parallel overhead lines.

will reduce the matrix to a 6 × 6 phase impedance matrix. With reference to Figure 4.8, the voltage drops in the two lines are given by

$$\begin{bmatrix} v1_a \\ v1_b \\ v1_c \\ v2_a \\ v2_b \\ v2_c \end{bmatrix} = \begin{bmatrix} z11_{aa} & z11_{ab} & z11_{ac} & z12_{aa} & z12_{ab} & z12_{ac} \\ z11_{ba} & z11_{bb} & z11_{bc} & z12_{ba} & z12_{bb} & z12_{bc} \\ z11_{ca} & z11_{cb} & z11_{cc} & z12_{ca} & z12_{cb} & z12_{cc} \\ z21_{aa} & z21_{ab} & z21_{ac} & z22_{aa} & z22_{ab} & z22_{ac} \\ z21_{ba} & z21_{bb} & z21_{bc} & z22_{ba} & z22_{bb} & z22_{bc} \\ z21_{ca} & z21_{cb} & z21_{cc} & z22_{ca} & z22_{cb} & z22_{cc} \end{bmatrix} \cdot \begin{bmatrix} I1_a \\ I1_b \\ I1_c \\ I2_a \\ I2_b \\ I2_c \end{bmatrix} \tag{4.83}$$

Partition Equation 4.83 between the third and fourth rows and columns so that series voltage drops for 1 mile of line are given by

$$[v] = [z] \cdot [I] = \begin{bmatrix} [v1] \\ [v2] \end{bmatrix} = \begin{bmatrix} [z11] & [z12] \\ [z21] & [z22] \end{bmatrix} \cdot \begin{bmatrix} [I1] \\ [I2] \end{bmatrix} \mathrm{V} \tag{4.84}$$

Example 4.2

Two parallel distribution lines are on a single pole as shown in Figure 4.9.

The phase conductors are

Line 1: 336,400 26/7 ACSR	$GMR_1 = 0.0244$ ft	$r_1 = 0.306\,\Omega$/mile	$d_1 = 0.721$ in.
Line 2: 250,000 AA	$GMR_2 = 0.0171$ ft	$r_2 = 0.41\,\Omega$/mile	$d_2 = 0.567$ in.
Neutral: 4/0 6/1 ACSR	$GMR_n = 0.00814$ ft	$r_n = 0.592\,\Omega$/mile	$d_n = 0.563$ in.

Determine the 6 × 6 phase impedance matrix.

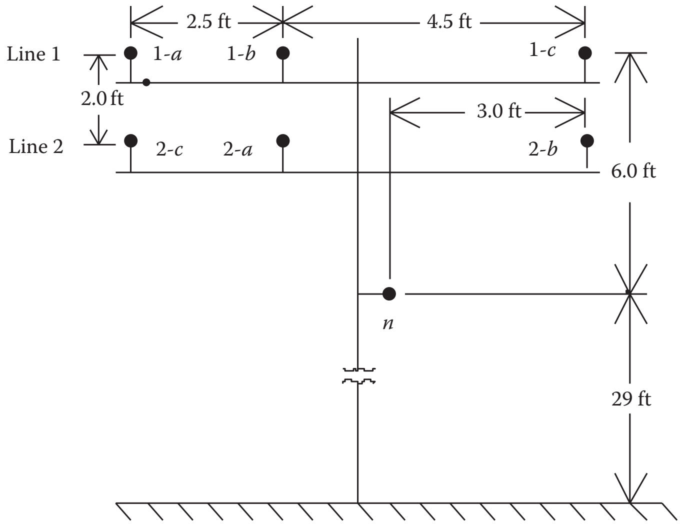

FIGURE 4.9
Example of parallel OH lines.

Define the conductor positions according to the phasing:

$$d_1 = 0 + j35 \qquad d_2 = 2.5 + j35 \qquad d_3 = 7 + j35$$
$$d_4 = 2.5 + j33 \qquad d_5 = 7 + j33 \qquad d_6 = 0 + j33$$
$$d_7 = 4 + j29$$

Using $D_{ij} = |d_i - d_j|$ the distances between all conductors can be computed. Using this equation the diagonal terms of the resulting spacing matrix will be zero. It is convenient to define the diagonal terms of the spacing matrix as the geometric mean radii (GMRs) of the conductors occupying the position. Using this approach the final spacing matrix is

$$[D] = \begin{bmatrix} 0.0244 & 2.5 & 7 & 3.2016 & 7.2801 & 2 & 7.2111 \\ 2.5 & 0.0244 & 4.5 & 2 & 4.9244 & 3.2016 & 6.1847 \\ 7 & 4.5 & 0.0244 & 4.9244 & 2 & 7.2801 & 6.7082 \\ 3.2016 & 2 & 4.9244 & 0.0171 & 4.5 & 2.5 & 4.2720 \\ 7.2801 & 4.9244 & 2 & 4.5 & 0.0171 & 7 & 5 \\ 2 & 3.2016 & 7.2801 & 2.5 & 7 & 0.0171 & 5.6869 \\ 7.2111 & 6.1847 & 6.7082 & 4.2720 & 5 & 5.6569 & 0.0081 \end{bmatrix}$$

The terms for the primitive impedance matrix can be computed using the modified Carson's equations. For this example, the subscripts i and j will run from 1 to 7. The 7×7 primitive impedance matrix is partitioned between rows and columns 6 and 7. The Kron reduction will now give the final phase impedance matrix. In partitioned form, the phase impedance matrices are

$$[z_{11}]_{abc} = \begin{bmatrix} 0.4502+j1.1028 & 0.1464+j0.5334 & 0.1452+j0.4126 \\ 0.1464+j0.5334 & 0.4548+j1.0873 & 0.1475+j0.4584 \\ 0.1452+j0.4126 & 0.1475+j0.4584 & 0.4523+j1.0956 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{12}]_{abc} = \begin{bmatrix} 0.1519+j0.4848 & 0.1496+j0.3931 & 0.1477+j0.5560 \\ 0.1545+j0.5336 & 0.1520+j0.4323 & 0.1502+j0.4909 \\ 0.1531+j0.4287 & 0.1507+j0.5460 & 0.1489+j0.3955 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{21}]_{abc} = \begin{bmatrix} 0.1519+j0.4848 & 0.1545+j0.5336 & 0.1531+j0.4287 \\ 0.1496+j0.3931 & 0.1520+j0.4323 & 0.1507+j0.5460 \\ 0.1477+j0.5560 & 0.1502+j0.4909 & 0.1489+j0.3955 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{22}]_{abc} = \begin{bmatrix} 0.5706+j1.0913 & 0.1580+j0.4236 & 0.1559+j0.5017 \\ 0.1580+j0.4236 & 0.5655+j1.1082 & 0.1535+j0.3849 \\ 0.1559+j0.5017 & 0.1535+j0.3849 & 0.5616+j1.1212 \end{bmatrix} \Omega/\text{mile}$$

4.2 Series Impedance of Underground Lines

Figure 4.10 shows the general configuration of three underground cables (concentric neutral or tape shielded) with an additional neutral conductor.

The modified Carson's equations can be applied to underground cables in much the same manner as for overhead lines. The circuit of Figure 4.10 will result in a 7 × 7 primitive impedance matrix. For underground circuits that do not have the additional neutral conductor, the primitive impedance matrix will be 6 × 6.

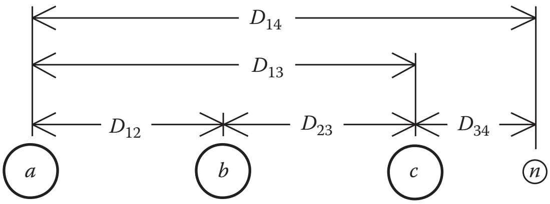

FIGURE 4.10
Three-phase underground with additional neutral.

Two popular types of underground cables are the "concentric neutral cable" and the "tape shield cable." To apply the modified Carson's equations, the resistance and GMR of the phase conductor and the equivalent neutral must be known.

4.2.1 Concentric Neutral Cable

Figure 4.11 shows a simple detail of a concentric neutral cable. The cable consists of a central "phase conductor" covered by a thin layer of nonmetallic semiconducting screen to which is bonded the insulating material. The insulation is then covered by a semiconducting insulation screen. The solid strands of concentric neutral are spiraled around the semiconducting screen with a uniform spacing between strands. Some cables will also have an insulating "jacket" encircling the neutral strands.

In order to apply Carson's equations to this cable, the following data needs to be extracted from a table of underground cables (Appendices A and B):

d_c is the phase conductor diameter (in.).
d_{od} is the nominal diameter over the concentric neutrals of the cable (in.).
d_s is the diameter of a concentric neutral strand (in.).
GMR_c is the geometric mean radius of the phase conductor (ft).
GMR_s is the geometric mean radius of a neutral strand (ft).
r_c is the resistance of the phase conductor (Ω/mile).
r_s is the resistance of a solid neutral strand (Ω/mile).
k is the number of concentric neutral strands.

The GMRs of the phase conductor and a neutral strand are obtained from a standard table of conductor data (Appendix A). The equivalent GMR of the concentric neutral is computed using the equation for the GMR of bundled conductors used in high-voltage transmission lines [2]:

$$GMR_{cn} = \sqrt[k]{GMR_s \cdot k \cdot R^{k-1}} \text{ ft} \tag{4.85}$$

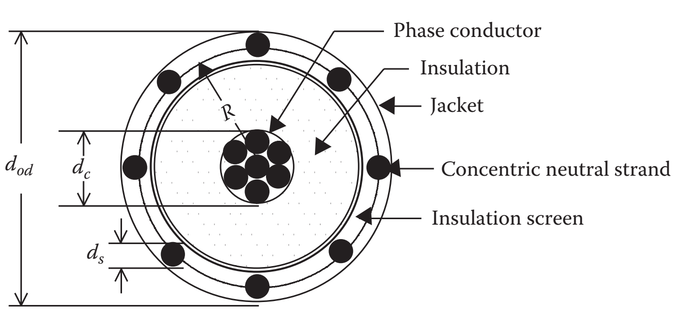

FIGURE 4.11
Concentric neutral cable.

where R is the radius of a circle passing through the center of the concentric neutral strands given by

$$R = \frac{d_{od} - d_s}{24} \text{ ft} \tag{4.86}$$

The equivalent resistance of the concentric neutral is

$$r_{cn} = \frac{r_s}{k} \ \Omega/\text{mile} \tag{4.87}$$

The various spacings between a concentric neutral and the phase conductors and other concentric neutrals are as follows:

Concentric neutral to its own phase conductor

$D_{ij} = R$ (Equation 4.86)

Concentric neutral to an adjacent concentric neutral

D_{ij} is the center-to-center distance of the phase conductors

Concentric neutral to an adjacent phase conductor

Figure 4.12 shows the relationship between the distance between centers of concentric neutral cables and the radius of a circle passing through the centers of the neutral strands.

The GMD between a concentric neutral and an adjacent phase conductor is given by

$$D_{ij} = \sqrt[k]{D_{nm}^k - R^k} \text{ ft} \tag{4.88}$$

where D_{nm} is the center-to-center distance between phase conductors.

The distance between cables will be much greater than the radius R so a good approximation of modeling the concentric neutral cables is shown in Figure 4.13. In this figure, the concentric neutrals are modeled as one equivalent conductor (shown in black) directly above the phase conductor.

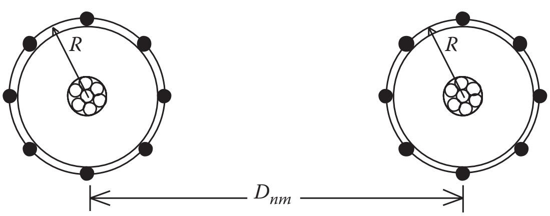

FIGURE 4.12
Distances between concentric neutral cables.

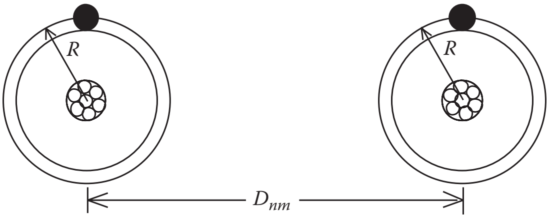

FIGURE 4.13
Equivalent neutral cables.

In applying the modified Carson's equations, the numbering of conductors and neutrals is important. For example, a three-phase underground circuit with an additional neutral conductor must be numbered as follows:

1 representing phase *a* conductor #1
2 representing phase *b* conductor #2
3 representing phase *c* conductor #3
4 representing neutral of conductor #1
5 representing neutral of conductor #2
6 representing neutral of conductor #3
7 representing additional neutral conductor (if present)

Example 4.3

Three concentric neutral cables are buried in a trench with spacings as shown in Figure 4.14.

The concentric neutral cables of Figure 4.14 can be modeled as shown in Figure 4.15. Notice the numbering of the phase conductors and the equivalent neutrals.

The cables are 15 kV, 250,000 circular mil (CM) stranded AA with 13 strands of #14 annealed coated copper wires (one-third neutral). The outside diameter of the cable over the neutral strands is 1.29 in. (Appendix B). Determine the phase impedance matrix and the sequence impedance matrix.

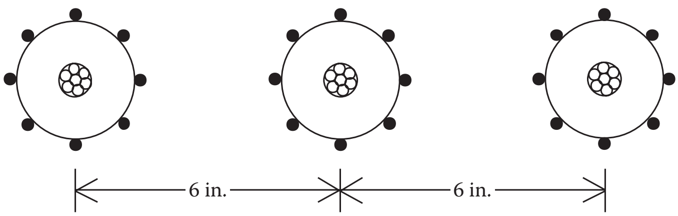

FIGURE 4.14
Three-phase concentric neutral cable spacing.

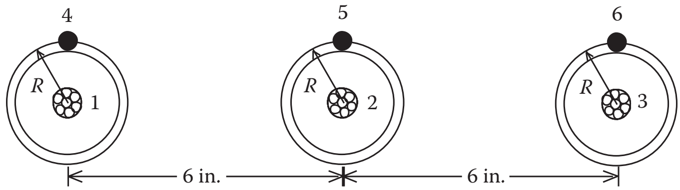

FIGURE 4.15
Three-phase equivalent concentric neutral cable spacing.

Solution

The data for the phase conductor and neutral strands from a conductor data table (Appendix A) are as follows:

250,000 AA phase conductor:
- $GMR_p = 0.0171\,\text{ft}$
- $Diameter = 0.567\,\text{in.}$
- $Resistance = 0.4100\,\Omega/\text{mile}$

#14 copper neutral strands:
- $GMR_s = 0.00208\,\text{ft}$
- $Resistance = 14.87\,\Omega/\text{mile}$
- $Diameter\ (d_s) = 0.0641\,\text{in.}$

The radius of the circle passing through the center of the strands (Equation 4.82) is

$$R = \frac{d_{od} - d_s}{24} = 0.0511\,\text{ft}$$

The equivalent GMR of the concentric neutral is computed by

$$GMR_{cn} = \sqrt[k]{GMR_s \cdot k \cdot R^{k-1}} = \sqrt[13]{0.00208 \cdot 13 \cdot 0.0511^{13-1}} = 0.0486\,\text{ft}$$

The equivalent resistance of the concentric neutral is

$$r_{cn} = \frac{r_s}{k} = \frac{14.8722}{13} = 1.144\ \Omega/\text{mile}$$

The phase conductors are numbered 1, 2, and 3. The concentric neutrals are numbered 4, 5, and 6.

A convenient method of computing the various spacings is to define each conductor using Cartesian coordinates. Using this approach the conductor coordinates are

$$d_1 = 0 + j0 \qquad d_2 = 0.5 + j0 \qquad d_3 = 1 + 1j0$$

$$d_4 = 0 + jR \qquad d_5 = 0.5 + jR \qquad d_6 = 1 + jR$$

The off-diagonal terms of the spacing matrix are computed by

$$\text{For } n = 1-6 \quad \text{and} \quad m = 1-6$$

$$D_{n,m} = |d_n - d_m|$$

The diagonal terms of the spacing matrix are the GMRs of the phase conductors and the equivalent neutral conductors:

$$\text{For } i = 1-3 \quad \text{and} \quad j = 4-6$$

$$D_{i,i} = GMR_p$$

$$D_{j,j} = GMR_s$$

The resulting spacing matrix is

$$[D] = \begin{bmatrix} 0.0171 & 0.5 & 1 & 0.0511 & 0.5026 & 1.0013 \\ 0.5 & 0.0171 & 0.5 & 0.5026 & 0.0511 & 0.5026 \\ 1 & 0.5 & 0.0171 & 1.0013 & 0.5026 & 0.0511 \\ 0.0511 & 0.5026 & 1.0013 & 0.0486 & 0.5 & 1 \\ 0.5026 & 0.0511 & 0.5026 & 0.5 & 0.0486 & 0.5 \\ 1.0013 & 0.5026 & 0.0511 & 1 & 0.5 & 0.0486 \end{bmatrix} \text{ft}$$

The self-impedance for the cable in position 1 is

$$\hat{z}_{11} = 0.0953 + 0.41 + j0.12134 \cdot \left(\ln \frac{1}{0.0171} + 7.93402 \right) = 0.5053 + j1.4564\,\Omega/\text{mile}$$

The self-impedance for the concentric neutral for cable #1 is

$$\hat{z}_{44} = 0.0953 + 1.144 + j0.12134 \cdot \left(\ln \frac{1}{0.0486} + 7.93402 \right) = 1.2391 + j1.3296\,\Omega/\text{mile}$$

The mutual impedance between cable #1 and cable #2 is

$$\hat{z}_{12} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{0.5} + 7.93402 \right) = 0.0953 + j1.0468\,\Omega/\text{mile}$$

The mutual impedance between cable #1 and its concentric neutral is

$$\hat{z}_{14} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{0.0511} + 7.93402 \right) = 0.0953 + j1.3236\,\Omega/\text{mile}$$

The mutual impedance between the concentric neutral of cable #1 and the concentric neutral of cable #2 is

$$\hat{z}_{45} = 0.0953 + j0.12134 \cdot \left(\ln \frac{1}{0.5} + 7.93402 \right) = 0.0953 + j1.0468\,\Omega/\text{mile}$$

Continuing the application of the modified Carson's equations results in a 6 × 6 primitive impedance matrix. This matrix in partitioned (Equation 4.33) form is

$$\left[\hat{z}_{ij}\right] = \begin{bmatrix} 0.5053 + j1.4564 & 0.0953 + j1.0468 & 0.0953 + j0.9627 \\ 0.0953 + j1.0468 & 0.5053 + j1.4564 & 0.0953 + j1.0468 \\ 0.0953 + j0.9627 & 0.0953 + j1.0468 & 0.5053 + j1.4564 \end{bmatrix} \Omega/\text{mile}$$

$$\left[\hat{z}_{in}\right] = \begin{bmatrix} 0.0953 + j1.3236 & 0.0953 + j1.0468 & 0.0953 + j0.9627 \\ 0.0953 + j1.0462 & 0.0953 + j1.3236 & 0.0953 + j1.0462 \\ 0.0953 + j0.9626 & 0.0953 + j1.0462 & 0.0953 + j1.3236 \end{bmatrix} \Omega/\text{mile}$$

$$\left[\hat{z}_{nj}\right] = \left[\hat{z}_{in}\right]$$

$$\left[\hat{z}_{nn}\right] = \begin{bmatrix} 1.2393 + j1.3296 & 0.0953 + j1.0468 & 0.0953 + j0.9627 \\ 0.0953 + j1.0468 & 1.2393 + j1.3296 & 0.0953 + j1.0468 \\ 0.0953 + j0.9627 & 0.0953 + j1.0468 & 1.2393 + j1.3296 \end{bmatrix} \Omega/\text{mile}$$

Using the Kron reduction results in the phase impedance matrix:

$$\left[z_{abc}\right] = \left[\hat{z}_{ij}\right] - \left[\hat{z}_{in}\right] \cdot \left[\hat{z}_{nn}\right]^{-1} \cdot \left[\hat{z}_{nj}\right]$$

$$= \begin{bmatrix} 0.7981 + j0.4467 & 0.3188 + j0.0334 & 0.2848 - j0.0138 \\ 0.3188 + j0.0334 & 0.7890 + j0.4048 & 0.3188 + j0.0334 \\ 0.2848 - j0.0138 & 0.3188 + j0.0334 & 0.7981 + j0.4467 \end{bmatrix} \Omega/\text{mile}$$

The sequence impedance matrix for the concentric neutral three-phase line is determined using Equation 4.68:

$$\left[z_{012}\right] = \left[A_s\right]^{-1} \cdot \left[z_{abc}\right] \cdot \left[A_s\right]$$

$$= \begin{bmatrix} 1.4140 + j0.4681 & -0.0026 - j0.0081 & -0.0057 + j0.0063 \\ -0.0057 + j0.0063 & 0.4876 + j0.4151 & -0.0265 + j0.0450 \\ -0.0026 - j0.0081 & 0.0523 + j0.0004 & 0.4876 + j0.4151 \end{bmatrix} \Omega/\text{mile}$$

4.2.2 Tape-Shielded Cables

Figure 4.16 shows a simple detail of a tape-shielded cable. The cable consists of a central "phase conductor" covered by a thin layer of nonmetallic semiconducting screen to which is bonded the insulating material. The insulation is covered by a semiconducting insulation screen. The shield is bare copper tape helically applied around the insulation screen. An insulating "jacket" encircles the tape shield.

Parameters of the tape-shielded cable are

d_c is the diameter of phase conductor (in.) (Appendix A).
d_s is the outside diameter of the tape shield (in.) (Appendix B).
d_{od} is the outside diameter over jacket (in.) (Appendix B).
T is the thickness of copper tape shield (mil) (Appendix B).

Once again the modified Carson's equations will be applied to calculate the self-impedances of the phase conductor and the tape shield as well as the mutual impedance between the phase conductor and the tape shield. The resistance and GMR of the phase conductor are found in a standard table of conductor data (Appendix A).

The resistance of the tape shield is given by

$$r_{shield} = 7.9385 \cdot 10^8 \cdot \frac{\rho}{d_s \cdot T} \, \Omega/\text{mile} \tag{4.89}$$

The resistance of the tape shield given in Equation 4.89 assumes a resistivity of 100 Ω-m and a temperature of 50°C. The outside diameter of the tape shield d_s is given in inches and the thickness of the tape shield T in mil.

The GMR of the tape shield is the radius of a circle passing through the middle of the shield and is given by

$$GMR_{shield} = \frac{(d_s/2) - (T/2000)}{12} \text{ ft} \tag{4.90}$$

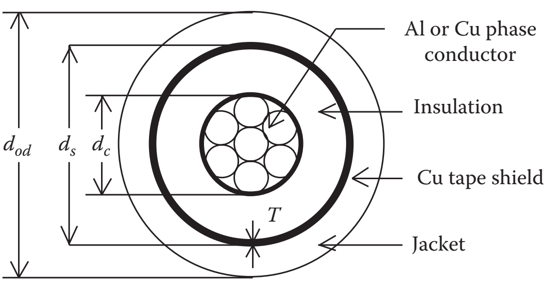

FIGURE 4.16
Tape-shielded cable.

The various spacings between a tape shield and the conductors and other tape shields are as follows:

Tape shield to its own phase conductor

$$D_{ij} = GMR_{shield} = \text{Radius to midpoint of the shield (ft)} \tag{4.91}$$

Tape shield to an adjacent tape shield

$$D_{ij} = \text{Center-to-center distance of the phase conductors (ft)} \tag{4.92}$$

Tape shield to an adjacent phase or neutral conductor

$$D_{ij} = D_{nm} \text{ (ft)} \tag{4.93}$$

where D_{nm} is the center-to-center distance between phase conductors.

Example 4.4

A single-phase circuit consists of a 1/0 AA, 220 mil insulation tape-shielded cable and a 1/0 CU neutral conductor as shown in Figure 4.17. The single-phase line is connected to phase *b*. Determine the phase impedance matrix.

Cable data: 1/0 AA
- Outside diameter of the tape shield = d_s = 0.88 in.
- *Resistance* = 0.97 Ω/mile
- GMR_p = 0.0111 ft
- Tape shield thickness = T = 5 mil

Neutral data: 1/0 copper, 7 strand
- *Resistance* = 0.607 Ω/mile
- GMR_n = 0.01113 ft
- Distance between cable and neutral = D_{nm} = 3 in.

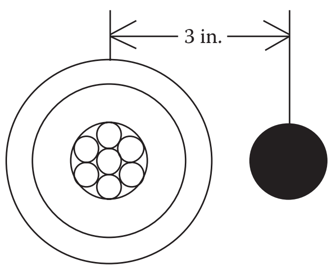

FIGURE 4.17
Single-phase tape shield with neutral.

The resistance of the tape shield is computed according to Equation 4.89:

$$r_{shield} = \frac{18.826}{d_s \cdot T} = \frac{18.826}{0.88 \cdot 5} = 4.2786\ \Omega/\text{mile}$$

The GMR of the tape shield is computed according to Equation 4.90:

$$GMR_{shield} = \frac{(d_s/2) - (T/2000)}{12} = \frac{(0.88/2) - (5/2000)}{12} = 0.0365\,\text{ft}$$

The conductors are numbered such that

#1 represents 1/0 AA conductor
#2 represents tape shield
#3 represents 1/0 copper ground

The spacings used in the modified Carson's equations are

$$D_{12} = GMR_{shield} = 0.0365$$

$$D_{13} = \frac{3}{12} = 0.25$$

The self-impedance of conductor #1 is

$$\hat{z}_{11} = 0.0953 + 0.97 + j0.12134 \cdot \left(\ln\frac{1}{0.0111} + 7.93402\right) = 1.0653 + j1.5088\ \Omega/\text{mile}$$

The mutual impedance between conductor #1 and the tape shield (conductor #2) is

$$\hat{z}_{12} = 0.0953 + j0.12134 \cdot \left(\ln\frac{1}{0.0365} + 7.93402\right) = 0.0953 + j1.3645\ \Omega/\text{mile}$$

The self-impedance of the tape shield (conductor #2) is

$$\hat{z}_{22} = 0.0953 + 4.2786 + j0.12134 \cdot \left(\ln\frac{1}{0.0365} + 7.93402\right) = 4.3739 + j1.3645\ \Omega/\text{mile}$$

The final primitive impedance matrix is

$$\left[\hat{z}\right] = \begin{bmatrix} 1.0653 + j1.5088 & 0.0953 + j1.3645 & 0.0953 + 1.1309 \\ 0.0953 + j1.3645 & 4.3739 + j1.3645 & 0.0953 + j1.1309 \\ 0.0953 + j1.1309 & 0.0953 + j1.1309 & 0.7023 + j1.5085 \end{bmatrix} \Omega/\text{mile}$$

In partitioned form, the primitive impedance matrix is

$$[\hat{z}_{ij}] = 1.0653 + j1.5088$$
$$[\hat{z}_{in}] = [0.0953 + j1.3645 \quad 0.0953 + j1.1309]$$
$$[\hat{z}_{nj}] = \begin{bmatrix} 0.0953 + j1.3645 \\ 0.0953 + j1.1309 \end{bmatrix} \qquad \Omega/\text{mile}$$
$$[\hat{z}_{nn}] = \begin{bmatrix} 4.3739 + j1.3645 & 0.0953 + j1.1309 \\ 0.0953 + j1.1309 & 0.7023 + j1.5085 \end{bmatrix}$$

Applying Kron reduction method will result in a single impedance, which represents the equivalent single-phase impedance of the tape shield cable and the neutral conductor:

$$z_{1p} = [\hat{z}_{ij}] - [\hat{z}_{in}] \cdot [\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}]$$

$$z_{1p} = 1.3219 + j0.6743 \ \Omega/\text{mile}$$

Since the single-phase line is on phase *b*, then the phase impedance matrix for the line is

$$[z_{abc}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.3219 + j0.6743 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Omega/\text{mile}$$

4.2.3 Parallel Underground Distribution Lines

The procedure for computing the phase impedance matrix for two overhead parallel lines is presented in Section 4.1.8. Figure 4.18 shows two concentric neutral parallel lines each with a separate grounded neutral conductor.

The process for computing the 6 × 6 phase impedance matrix follows exactly the same procedure as for the overhead lines. In this case, there are a total of 14 conductors (6 phase conductors, 6 equivalent concentric neutral conductors, and 2 grounded neutral conductors). Applying Carson's equations will result in a 14 × 14 primitive impedance matrix. This matrix is partitioned between the sixth and seventh rows and columns. The Kron reduction is applied to form the final 6 × 6 phase impedance matrix.

Example 4.5

Two concentric neutral three-phase underground parallel lines are shown in Figure 4.19.

Cables (both lines): 250 kcmil, 1/3 neutral
Extra neutral: 4/0 copper

Determine the 6 × 6 phase impedance matrix.

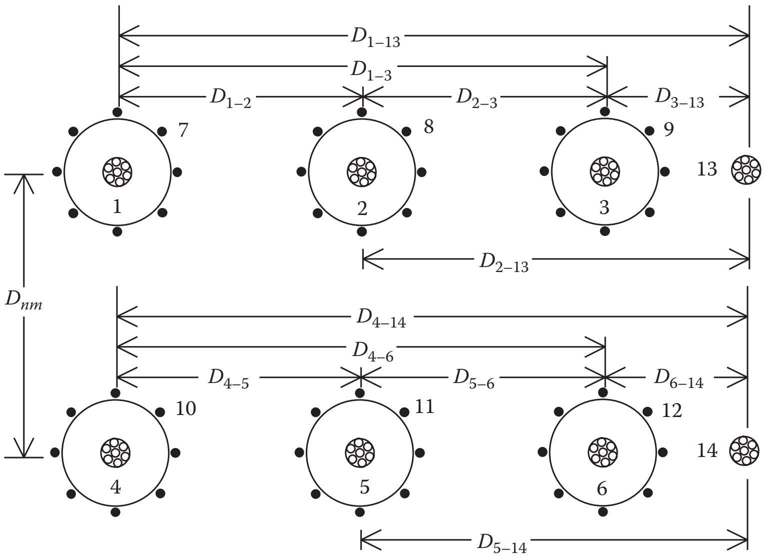

FIGURE 4.18
Parallel concentric neutral underground lines.

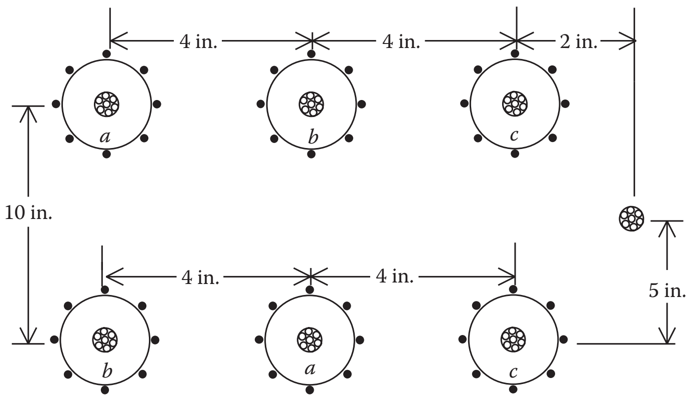

FIGURE 4.19
Parallel concentric neutral three-phase lines.

Solution

From Appendix B for the cables,

Outside diameter: $d_{od} = 1.29$ in.
Neutral strands: $k = 13$ #14 copper strands

From Appendix A for the conductors,

250 kcmil Al:	$GMR_c = 0.0171$ ft,	$r_c = 0.41\ \Omega$/mile,	$d_c = 0.567$ in.
#14 Copper:	$GMR_s = 0.00208$ ft,	$r_s = 14.8722\ \Omega$/mile,	$d_s = 0.0641$ in.
4/0 Copper:	$GMR_n = 0.1579$ ft,	$r_n = 0.303\ \Omega$/mile,	$d_n = 0.522$ in.

The radius of the circle to the center of the strands is

$$R_b = \frac{d_{od} - d_s}{24} = \frac{1.29 - 0.0641}{24} = 0.0511\,\text{ft}$$

The equivalent GMR of the concentric neutral strands is computed as

$$GMR_{eq} = \sqrt[k]{GMR_s \cdot k \cdot R_b^{k-1}} = \sqrt[13]{0.00208 \cdot 13 \cdot 0.05111^{12}} = 0.0486\,\text{ft}$$

The positions of the six cables and extra neutral using Cartesian coordinates with the phase *a* cable in line 1 (top line) as the ordinate. Note the phasing in both lines.

Phase *a*, line 1:	$d_1 = 0 + j0$	Phase *b*, line 1:	$d_2 = \frac{4}{12} + j0$	Phase *c*, line 1:	$d_3 = \frac{8}{12} + j0$
Phase *a*, line 2:	$d_4 = \frac{4}{12} - j\frac{10}{12}$	Phase *b*, line 1:	$d_5 = 0 - j\frac{10}{12}$	Phase *c*, line 1:	$d_6 = \frac{8}{12} - j\frac{10}{12}$

Equivalent neutrals

Phase *a*, line 1:	$d_7 = d_1 + jR_b$	Phase *b*, line 1:	$d_8 = d_2 + jR_b$	Phase *c*, line 1:	$d_9 = d_3 + jR_b$
Phase *a*, line 2:	$d_{10} = d_4 + jR_b$	Phase *b*, line 2:	$d_{11} = d_5 + jR_b$	Phase *c*, line 2:	$d_{12} = d_6 + jR_b$

Extra neutral

$$d_{13} = \frac{10}{12} - j\frac{5}{12}$$

The spacing matrix defining the distances between conductors can be computed by

$$i = 1\text{–}13 \quad j = 1\text{–}13$$

$$D_{i,j} = \left| d_i - d_j \right|$$

The diagonal terms of the spacing matrix are defined as the appropriate GMR:

$$D_{1,1} = D_{2,2} = D_{3,3} = D_{4,4} = D_{5,5} = D_{6,6} = GMR_c = 0.0171\,\text{ft}$$

$$D_{7,7} = D_{8,8} = D_{9,9} = D_{10,10} = D_{11,11} = D_{12,12} = GMR_{eq} = 0.0486\,\text{ft}$$

$$D_{13,13} = GMR_n = 0.01579\,\text{ft}$$

The resistance matrix is defined as

$$r_1 = r_2 = r_3 = r_4 = r_5 = r_6 = 0.41\,\Omega/\text{mile}$$

$$r_7 = r_8 = r_9 = r_{10} = r_{11} = r_{12} = \frac{r_s}{k} = \frac{14.8722}{13} = 1.144\,\Omega/\text{mile}$$

$$r_{13} = r_n = 0.303\,\Omega/\text{mile}$$

The primitive impedance matrix (13 × 13) is computed using Carson's equations:

$$i = 1 - 13 \quad j = 1 - 13$$

$$zp_{i,j} = 0.0953 + j0.12134 \cdot \left(\ln\left(\frac{1}{D_{i,i}} \right) + 7.93402 \right)$$

$$zp_{i,i} = r_i + 0.0953 + j0.12134 \cdot \left(\ln\left(\frac{1}{D_{i,j}} \right) + 7.93402 \right)$$

Once the primitive impedance matrix is developed, it is partitioned between the sixth and seventh rows and columns, and the Kron reduction method is applied to develop the 6 × 6 phase impedance matrix. The phase impedance matrix in partitioned form is

$$[z_{11}]_{abc} = \begin{bmatrix} 0.6450 + j0.4327 & 0.1805 + j0.0658 & 0.1384 + j0.0034 \\ 0.1805 + j0.0658 & 0.6275 + j0.3974 & 0.1636 + j0.0552 \\ 0.1384 + j0.0034 & 0.1636 + j0.0552 & 0.6131 + j0.4081 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{12}]_{abc} = \begin{bmatrix} 0.1261 - j0.0086 & 0.1389 + j0.071 & 0.0782 - j0.0274 \\ 0.1185 - j0.0165 & 0.1237 - j0.0145 & 0.0720 - j0.0325 \\ 0.1083 - j0.0194 & 0.1074 - j0.0246 & 0.0725 - j0.0257 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{21}]_{abc} = \begin{bmatrix} 0.1261 - j0.0086 & 0.1185 - j0.0165 & 0.1083 - j0.0195 \\ 0.1389 + j0.0071 & 0.1237 - j0.0145 & 0.1074 - j0.0246 \\ 0.0782 - j0.0274 & 0.072 - j0.0325 & 0.0725 - j0.0257 \end{bmatrix} \Omega/\text{mile}$$

$$[z_{22}]_{abc} = \begin{bmatrix} 0.6324 + j0.4329 & 0.1873 + j0.0915 & 0.0776 - j0.0233 \\ 0.1873 + j0.0915 & 0.6509 + j0.4508 & 0.0818 - j0.0221 \\ 0.0776 - j0.0233 & 0.0818 - j0.0221 & 0.8331 + j0.6476 \end{bmatrix} \Omega/\text{mile}$$

4.3 Summary

This chapter is devoted to presenting methods for computing the phase impedances and sequence impedances of overhead lines and underground cables. Carson's equations have been modified in order to simplify the computation of the phase impedances. When using the modified Carson's equations, there is no need to make any assumptions, such as transposition of the lines. By assuming an untransposed line and including the actual phasing of the line, the most accurate values of the phase impedances, self and mutual, are determined. It is highly recommended that no assumptions be made in the computation of the impedances. Since voltage drop is a primary concern on a distribution line, the impedances used for the line must be as accurate as possible. This chapter also included the process of applying Carson's equations to two distribution lines that are physically parallel. This same approach would be taken when there are more than two lines physically parallel.

Problems

4.1 The configuration and conductors of a three-phase overhead line are shown in Figure 4.20.

Phase conductors: 556,500 26/7 ACSR
Neutral conductor: 4/0 ACSR

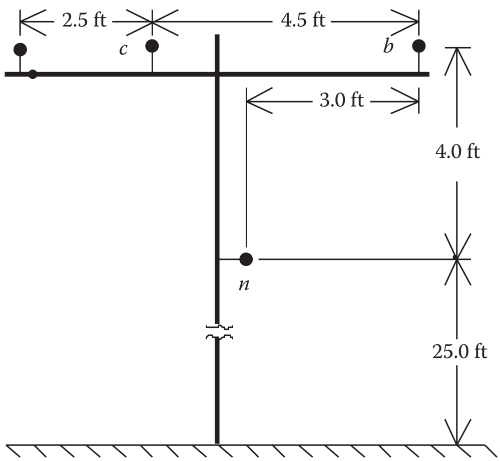

FIGURE 4.20
Three-phase configuration for Problem 4.1.

Determine

a. The phase impedance matrix $[z_{abc}]$ in Ω/mile
b. The sequence impedance matrix $[z_{012}]$ in Ω/mile
c. The neutral transformation matrix $[t_n]$

4.2 Determine the phase impedance $[z_{abc}]$ matrix in Ω/mile for the two-phase configuration in Figure 4.21.

Phase conductors: 336,400 26/7 ACSR
Neutral conductor: 4/0 6/1 ACSR

4.3 Determine the phase impedance $[z_{abc}]$ matrix in Ω/mile for the single-phase configuration shown in Figure 4.22.

Phase and neutral conductors: 1/0 6/1 ACSR

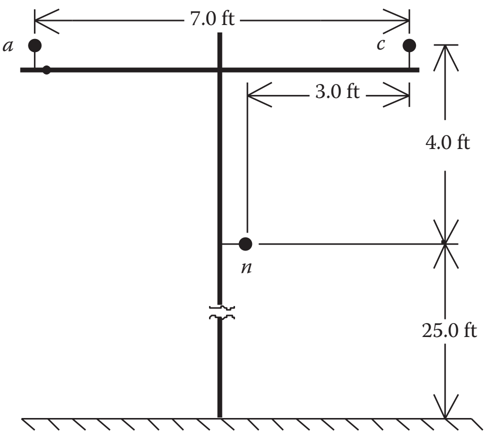

FIGURE 4.21
Two-phase configuration for Problem 4.2.

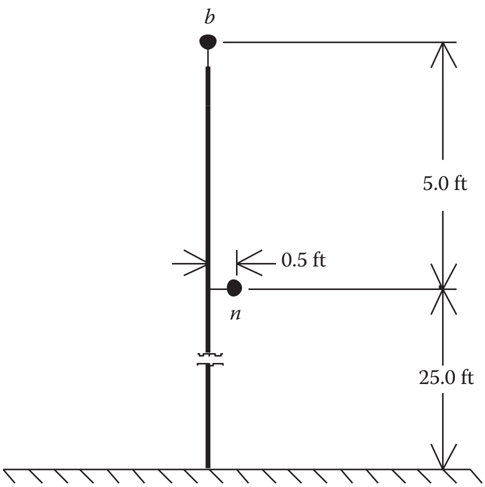

FIGURE 4.22
Single-phase pole configuration for Problem 4.3.

4.4 Create the spacings and configurations of Problem 4.1 through 4.3 in the Windmil program. Compare the phase impedance matrices to those computed in the previous problems.

4.5 Determine the phase impedance matrix $[z_{abc}]$ and sequence impedance matrix $[z_{012}]$ in Ω/mile for the three-phase pole configuration in Figure 4.23. The phase and neutral conductors are 250,000 AA.

4.6 Compute the positive, negative, and zero sequence impedances in Ω/1000 ft using the GMD method for the pole configuration shown in Figure 4.23.

4.7 Determine the $[z_{abc}]$ and $[z_{012}]$ matrices in Ω/mile for the three-phase configuration shown in Figure 4.24. The phase conductors are 350,000 AA and the neutral conductor is 250,000 AA.

4.8 Compute the positive, negative, and zero sequence impedances in Ω/1000 ft for the line of Figure 4.24 using the average self- and mutual impedances defined in Equations 4.70 and 4.71.

4.9 A 4/0 aluminum concentric neutral cable is to be used for a single-phase lateral. The cable has a full neutral (see Appendix B). Determine the impedance of the cable and the resulting phase impedance matrix in Ω/mile assuming the cable is connected to phase *b*.

4.10 Three 250,000 CM aluminum concentric cables with one-third neutrals are buried in a trench in a horizontal configuration (see Figure 4.14). Determine the $[z_{abc}]$ and $[z_{012}]$ matrices in Ω/1000 ft assuming phasing of *c*-*a*-*b*.

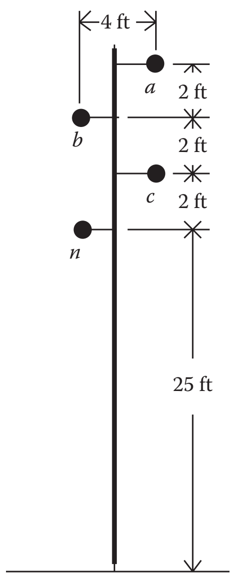

FIGURE 4.23
Three-phase pole configuration for Problem 4.5.

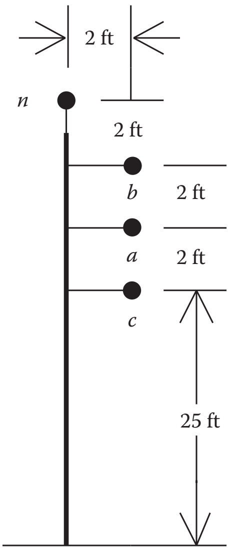

FIGURE 4.24
Three-phase pole configuration for Problem 4.5.

4.11 Create the spacings and configurations of Problems 4.9 and 4.10 in Windmil. Compare the values of the phase impedance matrices to those computed in the previous problems. In order to check the phase impedance matrix, it will be necessary for you to connect the line to balanced three-phase source. A source of 12.47 kV works fine.

4.12 A single-phase underground line is composed of a 350,000 CM aluminum tape-shielded cable. A 4/0 copper conductor is used as the neutral. The cable and neutral are separated by 4 in. Determine the phase impedance matrix in Ω/mile for this single-phase cable line assuming phase *c*.

4.13 Three one-third neutral 2/0 aluminum jacketed concentric neutral cables are installed in a 6 in. conduit. Assume the cable jacket has a thickness of 0.2 in. and the cables lie in a triangular configuration inside the conduit. Compute the phase impedance matrix in Ω/mile for this cabled line.

4.14 Create the spacing and configuration of Problem 4.13 in Windmil. Connect a 12.47 kV source to the line and compare results to those of 4.13.

4.15 Two three-phase distribution lines are physically parallel as shown in Figure 4.25.

Line # 1 (left side)	Phase conductors = 266,800 26/7 ACSR
	Neutral conductor = 3/0 6/1 ACSR
Line # 2 (right side)	Phase conductors = 300,000 CON LAY aluminum
	Neutral conductor = 4/0 CLASS A aluminum

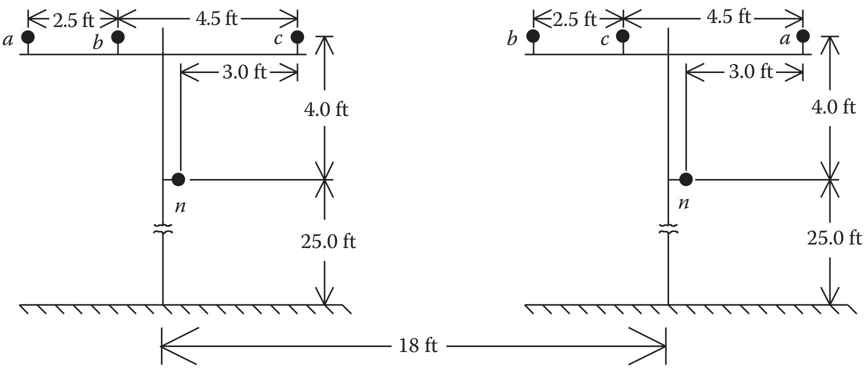

FIGURE 4.25
Parallel OH lines.

a. Determine the 6 × 6 phase impedance matrix.
b. Determine the neutral transform matrix.

4.16 Two concentric neutral underground three-phase lines are physically parallel as shown in Figure 4.26.

Line # 1 (top)	Cable = 250 kcmil, 1/3 neutral
	Additional neutral: 4/0 6/1 ACSR
Line #2 (bottom)	Cable = 2/0 kcmil, 1/3 neutral
	Additional neutral: 2/0 ACSR

a. Determine the 6 × 6 phase impedance matrix.
b. Determine the neutral transform matrix.

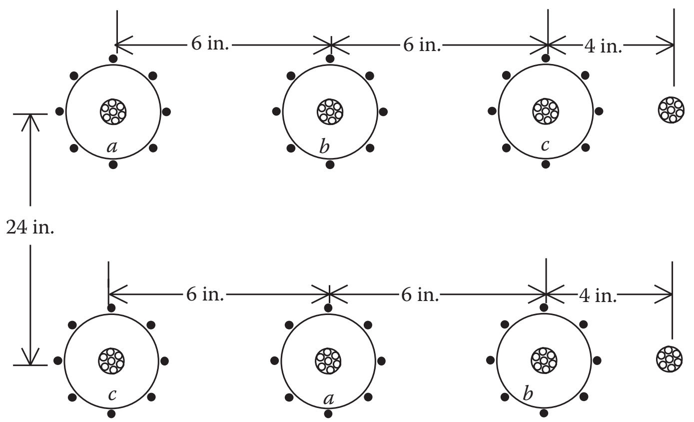

FIGURE 4.26
Parallel concentric neutral three-phase lines.

Windmil Assignment

Follow the method outlined in the User's Manual to build a system called "System 1" in Windmil that will have the following components:

- 12.47 kV line-to-line source. The "Bus Voltage" should be set to 120 V.
- Connect to the node and call it node 1.
- A 5,000 ft long overhead three distribution line as defined in Problem 4.1. Call this line OH-1.
- Connect a node to the end of the line and call it node 2.
- A wye connected unbalanced three-phase load is connected to node 2 and is modeled as constant PQ load with values of
  - Phase *a–g*: 1000 kW, power factor = 90% lagging
  - Phase *b–g*: 800 kW, power factor = 85% lagging
  - Phase *c–g*: 1200 kW, power factor = 95% lagging

Determine the voltages on a 120 V base at node 2 and the current flowing on the OH-1 line.

References

1. Glover, J.D. and Sarma, M., *Power System Analysis and Design*, 2nd edn., PWS Publishing Co., Boston, MA, 1994.
2. Carson, J.R., Wave propagation in overhead wires with ground return, *Bell System Technical Journal*, 5, 539, 1926.
3. Kersting, W.H. and Green, R.K., Application of Carson's equations to the steady-state analysis of distribution feeders, *IEEE Power System Conference and Exposition*, Phoenix, AZ, March 2011.
4. Kron, G., Tensorial analysis of integrated transmission systems, part I, the six basic reference frames, *AIEE Transactions*, 71, 1952.

5

Shunt Admittance of Overhead and Underground Lines

The shunt admittance of a line consists of the conductance and the capacitive susceptance. The conductance is usually ignored because it is very small compared to the capacitive susceptance. The capacitance of a line is the result of the potential difference between conductors. A charged conductor creates an electric field that emanates outward from the center of the conductor. Lines of equipotential are created that are concentric to the charged conductor. This is illustrated in Figure 5.1.

In Figure 5.1, a difference of potential between two points (P_1 and P_2) is a result of the electric field of the charged conductor. When the potential difference between the two points is known, then the capacitance between the two points can be computed. If there are other charged conductors nearby, the potential difference between the two points will be a function of the distance to the other conductors and the charge on each conductor. The principle of superposition is used to compute the total voltage drop between two points and then the resulting capacitance between the points. Understand that the points can be points in space or the surface of two conductors or the surface of a conductor and ground.

5.1 General Voltage Drop Equation

Figure 5.2 shows an array of N positively charged solid round conductors. Each conductor has a unique uniform charge density of q cb/m.

The voltage drop between conductor i and conductor j as a result of all of the charged conductors is given by

$$V_{ij} = \frac{1}{2\pi\varepsilon}\left(q_1 \ln\frac{D_{1j}}{D_{1i}} + \cdots + q_i \ln\frac{D_{ij}}{RD_i} + \cdots + q_j \ln\frac{RD_j}{D_{ij}} + \cdots + q_N \ln\frac{D_{Nj}}{D_{Ni}}\right) \tag{5.1}$$

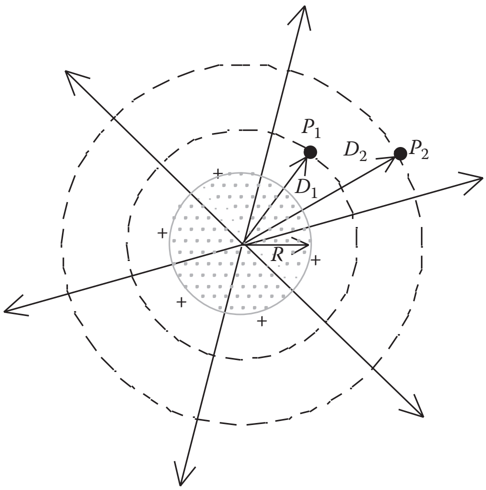

FIGURE 5.1
Electric field of a charged round conductor.

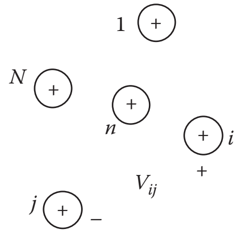

FIGURE 5.2
Array of round conductors.

Equation 5.1 can be written in a general form as

$$V_{ij} = \frac{1}{2\pi\varepsilon}\sum_{n=1}^{N} q_n \ln\frac{D_{nj}}{D_{ni}} \tag{5.2}$$

where

$\varepsilon = \varepsilon_0\varepsilon_r$ is the permittivity of the medium, ε_0 is the permittivity of free space = 8.85×10^{-12} μF/m, ε_r is the relative permittivity of the medium
q_n is the charge density on conductor n cb/m
D_{ni} is the distance between conductor n and conductor i (ft)
D_{nj} is the distance between conductor n and conductor j (ft)
D_{nn} is the radius (RD_n) of conductor n (ft)

5.2 Overhead Lines

The method of conductors and their images is employed in the calculation of the shunt capacitance of overhead lines. This is the same concept that was used in Chapter 4 in the general application of Carson's equations.

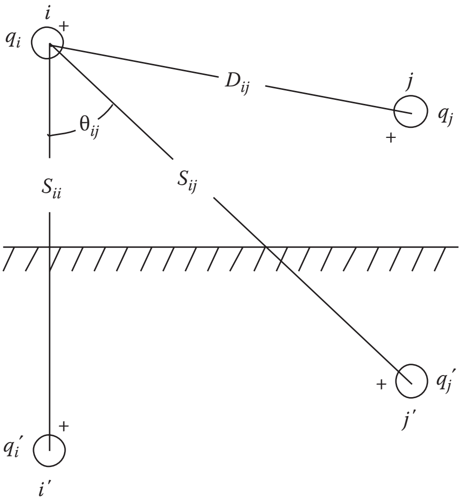

FIGURE 5.3
Conductors and images.

Figure 5.3 illustrates the conductors and their images and will be used to develop a general voltage drop equation for overhead lines.

In Figure 5.3 it is assumed that

$$
\begin{aligned}
q_i' &= -q_i \\
q_j' &= -q_j
\end{aligned}
\tag{5.3}
$$

Appling Equation 5.2 to Figure 5.3,

$$V_{ii} = \frac{1}{2\pi\varepsilon}\left(q_i \ln\frac{S_{ii}}{RD_i} + q_i' \ln\frac{RD_i}{S_{ii}} + q_j \ln\frac{S_{ij}}{D_{ij}} + q_j' \ln\frac{D_{ij}}{S_{ij}} \right) \tag{5.4}$$

Because of the assumptions of Equation 5.3, Equation 5.4 can be simplified to

$$
\begin{aligned}
V_{ii} &= \frac{1}{2\pi\varepsilon}\left(q_i \ln\frac{S_{ii}}{RD_i} - q_i \ln\frac{RD_i}{S_{ii}} + q_j \ln\frac{S_{ij}}{D_{ij}} - q_j \ln\frac{D_{ij}}{S_{ij}} \right) \\
&= \frac{1}{2\pi\varepsilon}\left(q_i \ln\frac{S_{ii}}{RD_i} + q_i \ln\frac{S_{ii}}{RD_i} + q_j \ln\frac{S_{ij}}{D_{ij}} + q_j \ln\frac{S_{ij}}{D_{ij}} \right) \\
&= \frac{1}{2\pi\varepsilon}\left(2\cdot q_i \ln\frac{S_{ii}}{RD_i} + 2\cdot q_j \ln\frac{S_{ij}}{D_{ij}} \right)
\end{aligned}
\tag{5.5}
$$

where

S_{ii} is the distance from conductor i to its image i' (ft)

S_{ij} is the distance from conductor i to the image of conductor j (ft)

D_{ij} is the distance from conductor i to conductor j (ft)

RD_i is the radius of conductor i in ft

Equation 5.5 gives the total voltage drop between conductor *i* and its image. The voltage drop between conductor *i* and ground will be one-half of that given in Equation 5.5:

$$V_{ig} = \frac{1}{2\pi\varepsilon}\left(q_i \ln\frac{S_{ii}}{RD_i} + q_j \ln\frac{S_{ij}}{D_{ij}}\right) \tag{5.6}$$

Equation 5.6 can be written in general form as

$$V_{ig} = P_{ii}\cdot q_i + P_{ij}\cdot q_j \tag{5.7}$$

where P_{ii} and P_{ij} are the self- and mutual "potential coefficients."

For overhead lines the relative permittivity of air is assumed to be 1.0 so that

$$\begin{aligned} \varepsilon_{air} &= 1.0\times 8.85\times 10^{-12}\ \mathrm{F/m} \\ \varepsilon_{air} &= 1.4240\times 10^{-2}\ \mu\mathrm{F/mile} \end{aligned} \tag{5.8}$$

Using the value of permittivity in μF/mile, the self- and mutual potential coefficients are defined as

$$\hat{P}_{ii} = 11.17689\cdot\ln\frac{S_{ii}}{RD_i}\ \mathrm{mile/\mu F} \tag{5.9}$$

$$\hat{P}_{ij} = 11.17689\cdot\ln\frac{S_{ij}}{D_{ij}}\ \mathrm{mile/\mu F} \tag{5.10}$$

NOTE: In applying Equations 5.9 and 5.10, the values of RD_i, S_{ii}, S_{ij}, and D_{ij} must all be in the same units. For overhead lines the distances between conductors are typically specified in feet while the value of the conductor diameter from a table will typically be in inches. Care must be taken to ensure that the radius in feet is used in applying the two equations.

For an overhead line of *ncond* conductors, the "primitive potential coefficient matrix" $[\hat{P}_{primitive}]$ can be constructed. The primitive potential coefficient matrix will be an *ncond* × *ncond* matrix. For a four-wire grounded wye line the primitive coefficient matrix will be of the form

$$\left[\hat{P}_{primitive}\right] = \begin{bmatrix} \hat{P}_{aa} & \hat{P}_{ab} & \hat{P}_{ac} & \cdot & \hat{P}_{an} \\ \hat{P}_{ba} & \hat{P}_{bb} & \hat{P}_{bc} & \cdot & \hat{P}_{bn} \\ \hat{P}_{ca} & \hat{P}_{cb} & \hat{P}_{cc} & \cdot & \hat{P}_{cn} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \hat{P}_{na} & \hat{P}_{nb} & \hat{P}_{nc} & \cdot & \hat{P}_{nn} \end{bmatrix} \tag{5.11}$$

The dots (·) in Equation 5.11 are partitioning the matrix between the third and fourth rows and columns. In partitioned form Equation 5.11 becomes

$$\left[\hat{P}_{primitive}\right] = \begin{bmatrix} [\hat{P}_{ij}] & [\hat{P}_{in}] \\ [\hat{P}_{nj}] & [\hat{P}_{nn}] \end{bmatrix} \tag{5.12}$$

Because the neutral conductor is grounded, the matrix can be reduced using the "Kron reduction" method to an n-phase × n-phase phase potential coefficient matrix $[P_{abc}]$ given by

$$\left[P_{abc}\right] = \left[\hat{P}_{ij}\right] - \left[\hat{P}_{in}\right]\cdot\left[\hat{P}_{nn}\right]^{-1}\cdot\left[\hat{P}_{jn}\right] \tag{5.13}$$

The inverse of the potential coefficient matrix will give the n-phase × n-phase capacitance matrix $[C_{abc}]$:

$$\left[C_{abc}\right] = \left[P_{abc}\right]^{-1} \tag{5.14}$$

For a two-phase line the capacitance matrix of Equation 5.14 will be 2 × 2. A row and a column of zeros must be inserted for the missing phase. For a single-phase line, Equation 5.14 will result in a single element. Again rows and columns of zero must be inserted for the missing phase. In the case of the single-phase line, the only nonzero term will be that of the phase in use.

Neglecting the shunt conductance, the phase shunt admittance matrix is given by

$$[y_{abc}] = 0 + j\cdot\omega\cdot[C_{abc}]\,\mu\text{S/mile} \tag{5.15}$$

where

$$\omega = 2\cdot\pi\cdot f = 376.9911$$

Example 5.1

Determine the shunt admittance matrix for the overhead line of Example 4.1. Assume that the neutral conductor is 25 ft above ground.

The diameters of the phase and neutral conductors from the conductor table (Appendix A) are

Conductor: 336,400 26/7 ACSR $d_c = 0.721$ in., $RD_c = 0.03004$ ft
4/0 6/1 ACSR $d_s = 0.563$ in., $RD_s = 0.02346$ ft

Using the Cartesian coordinated in Example 4.1, the image distance matrix is given by

$$S_{ij} = |d_i - d_j^*|$$

where d_j^* is the conjugate of d_j.

For the configuration the distances between conductors and images in matrix form are

$$[S] = \begin{bmatrix} 58 & 58.0539 & 58.4209 & 54.1479 \\ 58.0539 & 58 & 58.1743 & 54.0208 \\ 58.4209 & 58.1743 & 58 & 54.0833 \\ 54.1479 & 54.0208 & 54.0833 & 50 \end{bmatrix} \text{ft}$$

The self-primitive potential coefficient for phase a and the mutual primitive potential coefficient between phases a and b are

$$\hat{P}_{aa} = 11.17689 \ln \frac{58}{0.03004} = 84.5600 \text{ mile/}\mu\text{F}$$

$$\hat{P}_{ab} = 11.17689 \ln \frac{58.0539}{2.5} = 35.1522 \text{ mile/}\mu\text{F}$$

Using Equations 5.9 and 5.10, the total primitive potential coefficient matrix is computed to be

$$\left[\hat{P}_{primitive}\right] = \begin{bmatrix} 84.5600 & 35.1522 & 23.7147 & 25.2469 \\ 35.1522 & 84.5600 & 28.6058 & 28.3590 \\ 23.7147 & 28.6058 & 84.5600 & 26.6131 \\ 25.2469 & 28.3590 & 26.6131 & 85.6659 \end{bmatrix} \text{mile/}\mu\text{F}$$

Since the fourth conductor (neutral) is grounded, the Kron reduction method is used to compute the "phase potential coefficient matrix." Because only one row and one column need to be eliminated, the $[\hat{P}_{nn}]$ term is a single element so that the Kron reduction equation for this case can be modified to

$$P_{ij} = \hat{P}_{ij} - \frac{\hat{P}_{in} \cdot \hat{P}_{jn}}{\hat{P}_{44}}$$

where $i = 1,2,3$ and $j = 1,2,3$.

For example, the value of P_{cb} is computed to be

$$P_{cb} = \hat{P}_{32} - \frac{\hat{P}_{34} \cdot \hat{P}_{42}}{\hat{P}_{44}} = 28.6058 - \frac{26.6131 \cdot 28.359}{85.6659} = 19.7957$$

Following the Kron reduction, the phase potential coefficient matrix is

$$\left[P_{abc}\right] = \begin{bmatrix} 77.1194 & 26.7944 & 15.8714 \\ 26.7944 & 75.1720 & 19.7957 \\ 15.8714 & 19.7957 & 76.2923 \end{bmatrix}$$

Invert $[P_{abc}]$ to determine the shunt capacitance matrix:

$$[C_{abc}] = [P]^{-1} = \begin{bmatrix} 0.015 & -0.0049 & -0.0019 \\ -0.0049 & 0.0159 & -0.0031 \\ -0.0019 & -0.0031 & 0.0143 \end{bmatrix}$$

Multiply $[C_{abc}]$ by the radian frequency to determine the final three-phase shunt admittance matrix:

$$\left[y_{abc}\right] = j\cdot 376.9911\cdot\left[C_{abc}\right] = \begin{bmatrix} j5.6711 & -j1.8362 & -j0.7033 \\ -j1.8362 & j5.9774 & -j1.169 \\ -j0.7033 & -j1.169 & j5.3911 \end{bmatrix} \mu\text{S/mile}$$

5.2.1 Shunt Admittance of Overhead Parallel Lines

The development of the shunt admittance matrix for parallel overhead lines is similar to the steps taken to create the phase impedance matrix. The numbering of the conductors must be the same as was used in developing the phase impedance matrix. To develop the shunt admittance matrix for overhead lines, it is necessary to know the distance from each conductor to ground and it will be necessary to know the radius in ft of each conductor.

The first step is to create the primitive potential coefficient matrix. This will be an *ncond* × *ncond* matrix where *ncond* is the total number of phase and ground conductors. For the lines of Figure 4.8, *ncond* will be 7; for two lines each with its own grounded neutral, *ncond* will be 8.

The elements of the primitive potential coefficient matrix are given by

$$\hat{P}_{ii} = 11.17689\cdot\ln\frac{S_{ii}}{RD_i}\ \text{mile/}\mu\text{F}$$

$$\hat{P}_{ij} = 11.17689\cdot\ln\frac{S_{ij}}{D_{ij}}\ \text{mile/}\mu\text{F} \tag{5.16}$$

where

S_{ii} is the distance from a conductor to its image below ground (in ft)
S_{ij} is the distance from a conductor to the image of an adjacent conductor (in ft)
D_{ij} is the distance between two overhead conductors (in ft)
RD_i is the radius of conductor i (in ft)

The last one or two rows and columns of the primitive potential coefficient matrix are eliminated by using Kron reduction. The resulting voltage equation is

$$\begin{bmatrix} V1_{ag} \\ V1_{bg} \\ V1_{cg} \\ V2_{ag} \\ V2_{bg} \\ V2_{cg} \end{bmatrix} = \begin{bmatrix} P11_{aa} & P11_{ab} & P11_{ac} & P12_{aa} & P12_{ab} & P12_{ac} \\ P11_{ba} & P11_{bb} & P11_{bc} & P12_{ba} & P12_{bb} & P12_{bc} \\ P11_{ca} & P11_{cb} & P11_{cc} & P12_{ca} & P12_{cb} & P12_{cc} \\ P21_{aa} & P21_{ab} & P21_{ac} & P22_{aa} & P22_{ab} & P22_{ac} \\ P21_{ba} & P21_{bb} & P21_{bc} & P22_{ba} & P22_{bb} & P22_{bc} \\ P21_{ca} & P21_{cb} & P21_{cc} & P22_{ca} & P22_{cb} & P22_{cc} \end{bmatrix} \cdot \begin{bmatrix} q1_a \\ q1_b \\ q1_c \\ q2_a \\ q2_b \\ q2_c \end{bmatrix} \text{V} \tag{5.17}$$

In shorthand form Equation 5.17 is

$$[V_{LG}] = [P] \cdot [q] \tag{5.18}$$

The shunt capacitance matrix is determined by

$$[q] = [P]^{-1} \cdot [V_{LG}] = [C] \cdot [V_{LG}] \tag{5.19}$$

The resulting capacitance matrix is partitioned between the third and fourth rows and columns:

$$[C] = [P]^{-1} = \begin{bmatrix} [C11] & [C12] \\ [C21] & [C22] \end{bmatrix} \tag{5.20}$$

The shunt admittance matrix is given by

$$[y] = j\omega \cdot [C] \cdot 10^{-6} = \begin{bmatrix} [y11] & [y12] \\ [y21] & [y22] \end{bmatrix} \text{S} \tag{5.21}$$

where $\omega = 2 \cdot \pi \cdot \textit{frequency}$.

Example 5.2

Determine the shunt admittance matrix for the parallel overhead lines of Example 4.2.

The position coordinates for the seven conductors and the distance matrix are defined in Example 4.2. The diagonal terms of the distance matrix (Example 4.2) must be the radius in ft of the individual conductors. For this example,

$$D_{1,1} = D_{2,2} = D_{3,3} = \frac{d_1}{24} = \frac{0.721}{24} = 0.0300 \text{ ft}$$

$$D_{4,4} = D_{5,5} = D_{6,6} = \frac{d_2}{24} = \frac{0.567}{24} = 0.0236 \text{ ft}$$

$$D_{7,7} = \frac{d_n}{24} = 0.0235 \text{ ft}$$

The resulting distance matrix is

$$[D] = \begin{bmatrix} 0.0300 & 2.5000 & 7.0000 & 3.2016 & 7.2801 & 2.0000 & 7.2111 \\ 2.5000 & 0.0300 & 4.5000 & 2.0000 & 4.9244 & 3.2016 & 6.1847 \\ 7.0000 & 4.5000 & 0.0300 & 4.9244 & 2.0000 & 7.2801 & 6.7082 \\ 3.2016 & 2.0000 & 4.9244 & 0.0236 & 4.5000 & 2.5000 & 4.2720 \\ 7.2801 & 4.9244 & 2.0000 & 4.5000 & 0.0236 & 7.0000 & 5.0000 \\ 2.0000 & 3.2016 & 7.2801 & 2.5000 & 7.0000 & 0.0236 & 5.6569 \\ 7.2111 & 6.1847 & 6.7082 & 4.272 & 5.0000 & 5.6569 & 0.0235 \end{bmatrix} \text{ ft}$$

The distances between conductors and conductor images (image matrix) can be determined by

$$S_{i,j} = \left| d_i - d_j^* \right|$$

For this example, the image matrix is

$$[S] = \begin{bmatrix} 70.000 & 70.045 & 70.349 & 68.046 & 68.359 & 68.000 & 64.125 \\ 70.045 & 70.000 & 70.145 & 68.000 & 68.149 & 68.046 & 64.018 \\ 70.349 & 70.145 & 70.000 & 68.149 & 68.000 & 68.359 & 64.070 \\ 68.046 & 68.000 & 68.149 & 66.000 & 66.153 & 66.047 & 62.018 \\ 68.359 & 68.149 & 68.000 & 66.153 & 66.000 & 66.370 & 62.073 \\ 68.000 & 68.046 & 68.359 & 66.047 & 66.370 & 66.000 & 62.129 \\ 64.125 & 64.018 & 64.070 & 62.018 & 62.073 & 62.129 & 60.000 \end{bmatrix} \text{ ft}$$

The distance and image matrices are used to compute the 7 × 7 potential coefficient matrix by

$$Pp_{i,j} = 11.17689 \cdot \ln\left(\frac{S_{i,j}}{D_{i,j}}\right)$$

The primitive potential coefficient matrix is partitioned between the sixth and seventh rows and columns and the Kron reduction method produces the 6 × 6 potential matrix. This matrix is then inverted and multiplied by ω = 376.9911 to give the shunt admittance matrix. The final shunt admittance matrix in partitioned form is

$$[y_{11}]_{abc} = \begin{bmatrix} j6.2992 & -j1.3413 & -j0.4135 \\ -j1.3413 & j6.5009 & -j0.8038 \\ -j0.4135 & -j0.8038 & j6.0257 \end{bmatrix} \mu\text{S/mile}$$

$$[y_{12}]_{abc} = \begin{bmatrix} -j0.7889 & -j0.2992 & -j1.6438 \\ -j1.4440 & -j0.5698 & -j0.7988 \\ -j0.5553 & -j1.8629 & -j0.2985 \end{bmatrix} \mu\text{S/mile}$$

$$[y_{21}]_{abc} = \begin{bmatrix} -j0.7889 & -j1.4440 & -j0.5553 \\ -j0.2992 & -j0.5698 & -j1.8629 \\ -j1.6438 & -j0.7988 & -j0.2985 \end{bmatrix} \mu\text{S/mile}$$

$$[y_{22}]_{abc} = \begin{bmatrix} j6.3278 & -j0.6197 & -j1.1276 \\ -j0.6197 & j5.9016 & -j0.2950 \\ -j1.1276 & -j0.2950 & j6.1051 \end{bmatrix} \mu\text{S/mile}$$

5.3 Concentric Neutral Cable Underground Lines

Most underground distribution lines consist of one or more concentric neutral cables. Figure 5.4 illustrates a basic concentric neutral cable with center conductor being the phase conductor and the concentric neutral strands displaced equally around a circle of radius R_b.

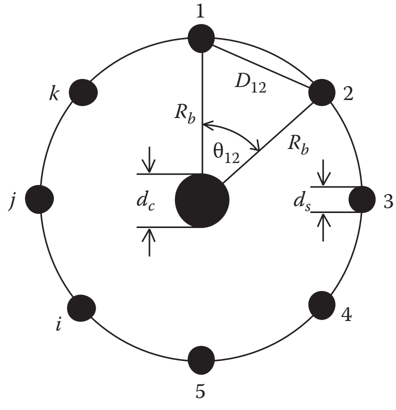

FIGURE 5.4
Basic concentric neutral cable.

Referring to Figure 5.4 the following definitions apply:

R_b represents the radius of a circle passing through the centers of the neutral strands.

d_c represents the diameter of the phase conductor.

d_s represents the diameter of a neutral strand.

k represents the total number of neutral strands.

The concentric neutral strands are grounded so that they are all at the same potential. Because of the stranding, it is assumed that the electric field created by the charge on the phase conductor will be confined to the boundary of the concentric neutral strands. In order to compute the capacitance between the phase conductor and ground, the general voltage drop of Equation 5.2 will be applied. Since all of the neutral strands are at the same potential, it is necessary to determine only the potential difference between the phase conductor p and strand 1.

$$V_{p1} = \frac{1}{2\pi\varepsilon}\left(q_p \ln\frac{R_b}{RD_c} + q_1 \ln\frac{RD_s}{R_b} + q_2 \ln\frac{D_{12}}{R_b} + \cdots + q_i \ln\frac{D_{1i}}{R_b} + \cdots + q_k \ln\frac{D_{k1}}{R_b}\right) \tag{5.22}$$

where

$$RD_c = \frac{d_c}{2}$$

$$RD_s = \frac{d_s}{2}$$

It is assumed that each of the neutral strands carries the same charge such that

$$q_1 = q_2 = q_i = q_k = -\frac{q_p}{k} \tag{5.23}$$

Equation 5.22 can be simplified to

$$\begin{aligned} V_{p1} &= \frac{1}{2\pi\varepsilon}\left[q_p \ln\frac{R_b}{RD_c} - \frac{q_p}{k}\left(\ln\frac{RD_s}{R_b} + \ln\frac{D_{12}}{R_b} + \cdots + \ln\frac{D_{1i}}{R_b} + \cdots + \ln\frac{D_{1k}}{R_b}\right)\right] \\ &= \frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c} - \frac{1}{k}\left(\ln\frac{RD_s \cdot D_{12} \cdot D_{1i} \cdot \cdots \cdot D_{1k}}{R_b^k}\right)\right] \end{aligned} \tag{5.24}$$

The numerator of the second ln term in Equation 5.24 needs to be expanded. The numerator represents the product of the radius and the distances between strand i and all of the other strands. Referring to Figure 5.4, the following relations apply:

$$\theta_{12} = \frac{2 \cdot \pi}{k}$$

$$\theta_{13} = 2 \cdot \theta_{12} = \frac{4 \cdot \pi}{k}$$

In general, the angle between strand #1 and any other strand #i is given by

$$\theta_{1i} = (i-1) \cdot \theta_{12} = \frac{(i-1) \cdot 2\pi}{k} \tag{5.25}$$

The distances between the various strands are given by

$$\begin{aligned} D_{12} &= 2 \cdot R_b \cdot \sin\left(\frac{\theta_{12}}{2}\right) = 2 \cdot R_b \cdot \sin\left(\frac{\pi}{k}\right) \\ D_{13} &= 2 \cdot R_b \cdot \sin\left(\frac{\theta_{13}}{2}\right) = 2 \cdot R_b \cdot \sin\left(\frac{2\pi}{k}\right) \end{aligned} \tag{5.26}$$

The distance between strand 1 and any other strand i is given by

$$D_{1i} = 2 \cdot R_b \cdot \sin\left(\frac{\theta_{1i}}{2}\right) = 2 \cdot R_b \cdot \sin\left[\frac{(i-1) \cdot \pi}{k}\right] \tag{5.27}$$

Equation 5.27 can be used to expand the numerator of the second log term of Equation 5.24:

$$RD_s \cdot D_{12} \cdots\cdots D_{1i} \cdots\cdots D_{1k}$$

$$= RD_s \cdot R_b^{k-1}\left[2\sin\left(\frac{\pi}{k}\right) \cdot 2\sin\left(\frac{2\pi}{k}\right) \cdots\cdots 2\sin\left\{\left(\frac{(i-1)\pi}{k}\right)\right\} \cdots\cdots 2\sin\left\{\left(\frac{(k-1)}{k}\right)\right\}\right] \tag{5.28}$$

The term inside the bracket in Equation 5.28 is a trigonometric identity that is merely equal to the number of strands k [1]. Using that identity, Equation 5.18 becomes

$$\begin{aligned} V_{p1} &= \frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c} - \frac{1}{k}\left(\ln\frac{k \cdot RD_s \cdot R_b^{k-1}}{R_b{}^k}\right)\right] \\ &= \frac{q_p}{2\pi\varepsilon}\left[\ln\frac{R_b}{RD_c} - \frac{1}{k}\left(\ln\frac{k \cdot RD_s}{R_b}\right)\right] \end{aligned} \tag{5.29}$$

Equation 5.29 gives the voltage drop from the phase conductor to neutral strand #1. Care must be taken that the units for the various radii are the same. Typically, underground spacings are given in inches so the radii of the phase conductor (RD_c) and the strand conductor (RD_s) should be specified in inches.

TABLE 5.1

Typical Values of Relative Permittivity (ε_r)

Material	Range of Values of Relative Permittivity
Polyvinyl chloride (PVC)	3.4–8.0
Ethylene–propylene rubber (EPR)	2.5–3.5
Polyethylene (PE)	2.5–2.6
Cross-linked polyethylene (XLPE)	2.3–6.0

Since the neutral strands are all grounded, Equation 5.29 gives the voltage drop between the phase conductor and ground. Therefore, the capacitance from phase to ground for a concentric neutral cable is given by

$$C_{pg} = \frac{q_p}{V_{p1}} = \frac{2\pi\varepsilon}{\ln(R_b/RD_c) - (1/k)\ln(k \cdot RD_s/R_b)} \tag{5.30}$$

where

$\varepsilon = \varepsilon_0\varepsilon_r$ is the permittivity of the medium

ε_0 is the permittivity of free space = 0.01420 μF/mile

ε_r is the relative permittivity of the medium

The electric field of a cable is confined to the insulation material. Various types of insulation material are used and each will have a range of values for the relative permittivity. Table 5.1 gives the range of values of relative permittivity for four common insulation materials [2].

Cross-linked polyethylene is a very popular insulation material. If the minimum value of relative permittivity is assumed (2.3), the equation for the shunt admittance of the concentric neutral cable is given by

$$y_{ag} = 0 + j\frac{77.3619}{\ln(R_b/RD_c) - (1/k)\ln(k \cdot RD_s/R_b)}\ \mu\text{S/mile} \tag{5.31}$$

Example 5.3

Determine the three-phase shunt admittance matrix for the concentric neutral line of Example 4.3.

From Example 4.3,

$$R_b = R = 0.0511\text{ ft} = 0.631\text{ in.}$$

Diameter of the 250,000 AA phase conductor = 0.567 in. Therefore,

$$RD_c = \frac{0.567}{2} = 0.2835\text{ in.}$$

Diameter of the #14 CU concentric neutral strand = 0.0641 in. Therefore,

$$RD_s = \frac{0.0641}{2} = 0.03205\text{ in.}$$

Substitute into Equation 5.31:

$$y_{ag} = j\frac{77.3619}{\ln(R_b/RD_c) - (1/k)\cdot\ln(k\cdot RD_s/R_b)}$$

$$y_{ab} = j\frac{77.3619}{\ln(0.6132/0.2835) - (1/13)\cdot\ln(13\cdot 0.03205/0.6132)} = j96.6098\ \mu\text{S/mile}$$

The phase admittance for this three-phase underground line is

$$[y_{abc}] = \begin{bmatrix} j96.6098 & 0 & 0 \\ 0 & j96.6098 & 0 \\ 0 & 0 & j96.6098 \end{bmatrix} \mu\text{S/mile}$$

5.4 Tape-Shielded Cable Underground Lines

A tape-shielded cable is shown in Figure 5.5. Referring to Figure 5.5 R_b is the radius of a circle passing through the center of the tape shield. As with the concentric neutral cable, the electric field is confined to the insulation so that the relative permittivity of Table 5.1 will apply.

The tape-shielded conductor can be visualized as a concentric neutral cable where the number of strands k has become infinite. When k in Equation 5.24 approaches infinity, the second term in the denominator approaches zero. Therefore, the equation for the shunt admittance of a tape-shielded conductor becomes

$$y_{ag} = 0 + j\frac{77.3619}{\ln(R_b/RD_c)}\ \mu\text{S/mile} \tag{5.32}$$

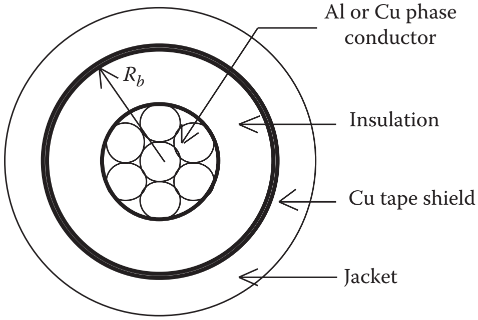

FIGURE 5.5
Tape-shielded conductor.

Example 5.4

Determine the shunt admittance of the single-phase tape-shielded cable of Example 4.4. From Example 4.4 outside diameter of the tape shield is 0.88 in. The thickness of the tape shield (T) is 5 mil. The radius of a circle passing through the center of the tape shield is given by

$$T = \frac{5}{1000} = 0.005$$

$$R_b = \frac{d_s - T}{2} = \frac{0.88 - 0.005}{2} = 0.4375 \text{ in.}$$

The diameter of the 1/0 AA phase conductor is 0.368 in. Therefore,

$$RD_c = \frac{d_p}{2} = \frac{0.368}{2} = 0.1840 \text{ in.}$$

Substitute into Equation 5.25:

$$y_{bg} = j\frac{77.3619}{\ln(R_b/RD_c)} = j\frac{77.3619}{\ln(0.4375/0.184)} = j89.3179 \ \mu\text{S/mile}$$

The line is on phase b so that the phase admittance matrix becomes

$$[y_{abc}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & j89.3179 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mu\text{S/mile}$$

5.5 Sequence Admittance

The sequence admittances of a three-phase line can be determined in much the same manner as the sequence impedances were determined in Chapter 4. Assume that the 3 × 3 admittance matrix is given in S/mile. Then the three-phase capacitance currents as a function of the line-to-ground voltages are given by

$$\begin{bmatrix} Icap_a \\ Icap_b \\ Icap_c \end{bmatrix} = \begin{bmatrix} y_{aa} & y_{ab} & y_{ac} \\ y_{ba} & y_{bb} & y_{bc} \\ y_{ca} & y_{cb} & y_{cc} \end{bmatrix} \cdot \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} \tag{5.33}$$

$$[Icap_{abc}] = [y_{abc}] \cdot [VLG_{abc}] \tag{5.34}$$

Applying the symmetrical component transformations,

$$\left[Icap_{012}\right]=\left[A_s\right]^{-1}\cdot\left[Icap_{abc}\right]=\left[A_s\right]^{-1}\cdot\left[y_{abc}\right]\cdot\left[A_s\right]\cdot\left[VLG_{012}\right] \tag{5.35}$$

From Equation 5.35, the sequence admittance matrix is given by

$$\left[y_{012}\right]=\left[A_s\right]^{-1}\cdot\left[y_{abc}\right]\cdot\left[A_s\right]=\begin{bmatrix} y_{00} & y_{01} & y_{02} \\ y_{10} & y_{11} & y_{12} \\ y_{20} & y_{21} & y_{22} \end{bmatrix} \tag{5.36}$$

For a three-phase overhead line with unsymmetrical spacing, the sequence admittance matrix will be full. That is, the off-diagonal terms will be non-zero. However, a three-phase underground line with three identical cables will only have the diagonal terms since there is no "mutual capacitance" between phases. In fact, the sequence admittances will be exactly the same as the phase admittances.

5.6 Shunt Admittance of Parallel Underground Lines

For underground cable lines using either concentric neutral cables or tape-shielded cables, the computation of the shunt admittance matrix is quite simple. The electric field created by the charged phase conductor does not link to adjacent conductors because of the presence of the concentric neutrals or the tape shield. As a result the shunt admittance matrix for parallel underground lines will consist of diagonal terms only.

The diagonal terms for concentric neutral cables are given by

$$y_{ii}=0+j\frac{77.3619}{\ln(R_b/RD_i)-(1/k)\cdot\ln(k\cdot RD_s/R_b)}\cdot 10^{-6}\ \text{S/mile} \tag{5.37}$$

where

R_b is the radius of circle going through the center of the neutral strands (in ft)
RD_i is the radius of the center phase conductor (in ft)
RD_s is the radius of the neutral strands (in ft)
k is the number of neutral strands

The diagonal terms for tape-shielded cables are given by

$$y_{ii} = 0 + j\frac{77.3619}{\ln(R_b/RD_i)} \cdot 10^{-6}\ \text{S/mile} \tag{5.38}$$

where
R_b is the radius of circle passing through the center of the tape shield (in ft)
RD_i is the radius of the center phase conductor (in ft)

Example 5.5

Compute the shunt admittance matrix (6 × 6) for the concentric neutral underground configuration of Example 4.5.

From Example 4.5,
Diameter of the central conductor: $d_c = 0.567$ in.
Diameter of the strands: $d_s = 0.641$ in.
Radius of circle passing through the strands: $R_b = 0.0511$ ft
Radius of central conductor: $RD_c = d_c/24 = 0.567/24 = 0.236$ ft
Radius of the strands: $RD_s = d_s/24 = 0.0641/24 = 0.0027$ ft

Since all cables are identical, the shunt admittance of a cable is

$$y_c = 0 + j\cdot\frac{77.3619}{\ln(R_b/RD_c) - (1/k)\cdot\ln(k\cdot RD_s/R_b)}$$

$$= 0 + j\cdot\frac{77.3619}{\ln(0.0511/0.0236) - (1/13)\cdot\ln(13\cdot 0.0027/0.0511)}$$

$$= 0 + j\cdot 96.6098\ \mu\text{S/mile}$$

The phase admittance matrix is

$$[y]_{abc} = \begin{bmatrix} j96.6098 & 0 & 0 & 0 & 0 & 0 \\ 0 & j96.6098 & 0 & 0 & 0 & 0 \\ 0 & 0 & j96.6098 & 0 & 0 & 0 \\ 0 & 0 & 0 & j96.6098 & 0 & 0 \\ 0 & 0 & 0 & 0 & j96.6098 & 0 \\ 0 & 0 & 0 & 0 & 0 & j96.6098 \end{bmatrix} \mu\text{S/mile}$$

5.7 Summary

Methods for computing the shunt capacitive admittance for overhead and underground lines is presented in this chapter. The development of computing the shunt admittance matrix for parallel overhead and underground lines is also included.

Distribution lines are typically so short that the shunt admittance can be ignored. However, there are cases of long lightly loaded overhead lines where the shunt admittance should be included. Underground cables have a much higher shunt admittance per mile than overhead lines. Again, there will be cases where the shunt admittance of an underground cable should be included in the analysis process. When the analysis is being done using a computer, the approach to take is to go ahead and model the shunt admittance for both overhead and underground lines. Why make a simplifying assumption when it is not necessary?

Problems

5.1 Determine the phase admittance matrix $[y_{abc}]$ and sequence admittance matrix $[y_{012}]$ in μS/mile for the three-phase overhead line of Problem 4.1.

5.2 Determine the phase admittance matrix in μS/mile for the two-phase line of Problem 4.2.

5.3 Determine the phase admittance matrix in μS/mile for the single-phase line of Problem 4.3.

5.4 Verify the results of Problems 5.1, 5.2, and 5.3 using Windmil.

5.5 Determine the phase admittance matrix and sequence admittance matrix in μS/mile for the three-phase line of Problem 4.5.

5.6 Determine the phase admittance matrix in μS/mile for the single-phase concentric neutral cable of Problem 4.9.

5.7 Determine the phase admittance matrix and sequence admittance matrix for the three-phase concentric neutral line of Problem 4.10.

5.8 Verify the results of Problems 5.6 and 5.7 using Windmil.

5.9 Determine the phase admittance matrix in μS/mile for the single-phase tape-shielded cable line of Problem 4.12.

5.10 Determine the phase admittance for the three-phase tape-shielded cable line of Problem 4.13.

5.11 Verify the results of Problems 5.9 and 5.10 using Windmil.

5.12 Determine the shunt admittance matrix for the parallel overhead lines of Problem 4.15.

5.13 Determine the shunt admittance matrix for the underground concentric neutral parallel lines of Problem 4.16.

Windmil Assignment

Add to the Windmil System 1 a single-phase line connected to node 2. Call this "System 2." The single-phase line is on phase *b*, 500 ft long, and is defined in Problem 4.3. Call this line OH-2. At the end of the line connect a node and call it node 3. The load at node 3 is 200 kW at a 90% lagging power factor. The load is modeled as a constant impedance load.

Determine the voltages at the nodes on a 120 V base and the currents flowing on the two lines.

References

1. Glover, J.D. and Sarma, M., *Power System Analysis and Design*, 2nd edn., PWS Publishing Co., Boston, MA, 1995.
2. Arnold, T.P. and Mercier, C.D. (eds.), *Power Cable Manual*, 2nd edn., Southwire Company, Carrollton, GA, 1997.

6

Distribution System Line Models

The modeling of distribution overhead and underground line segments is a critical step in the analysis of a distribution feeder. It is important in the line modeling to include the actual phasing of the line and the correct spacing between conductors. Chapters 4 and 5 developed the method for the computation of the phase impedance and phase admittance matrices with no simplifying assumptions. Those matrices will be used in the models for overhead and underground line segments.

6.1 Exact Line Segment Model

The model of a three-phase, two-phase, or single-phase overhead or underground line is shown in Figure 6.1.

When a line segment is two phase (V phase) or single phase, some of the impedance and values will be zero. Recall in Chapters 4 and 5 that in all cases the phase impedance and phase admittance matrices were 3 × 3. Rows and columns of zeros for the missing phases represent two-phase and single-phase lines. Therefore, one set of equations can be developed to model all overhead and underground line segments. The values of the impedances and admittances in Figure 6.1 represent the total impedances and admittances for the line. That is, the phase impedance matrix, derived in Chapter 4, has been multiplied by the length of the line segment. The phase admittance matrix, derived in Chapter 5, has also been multiplied by the length of the line segment.

For the line segment of Figure 6.1, the equations relating the input (node ***n***) voltages and currents to the output (node ***m***) voltages and currents are developed as follows.

Kirchhoff's current law applied at node ***m*** is represented by

$$\begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix}_n = \begin{bmatrix} Ia \\ Ib \\ Ic \end{bmatrix}_m + \frac{1}{2}\cdot\begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}\cdot\begin{bmatrix} Vag \\ Vbg \\ Vcg \end{bmatrix}_m \tag{6.1}$$

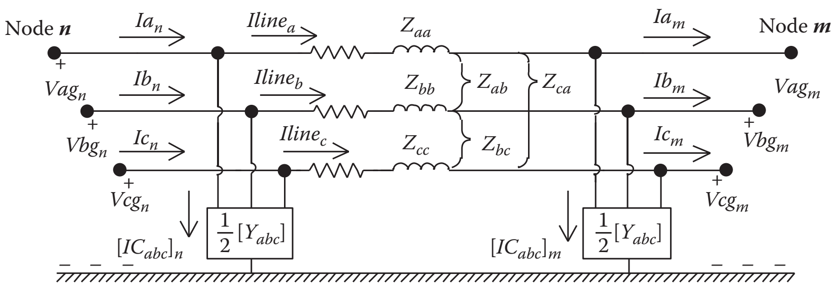

FIGURE 6.1
Three-phase line segment model.

In condensed form Equation 6.1 becomes

$$[Iline_{abc}]_n = [I_{abc}]_m + \frac{1}{2}[Y_{abc}]\cdot[VLG_{abc}]_m \tag{6.2}$$

Kirchhoff's voltage law applied to the model gives

$$\begin{bmatrix} Vag \\ Vbg \\ Vcg \end{bmatrix}_n = \begin{bmatrix} Vag \\ Vbg \\ Vcg \end{bmatrix}_m + \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix}_m \tag{6.3}$$

In condensed form Equation 6.3 becomes

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}]\cdot[Iline_{abc}]_m \tag{6.4}$$

Substituting Equation 6.2 into Equation 6.4,

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{abc}]\cdot\left\{[I_{abc}]_m + \frac{1}{2}[Y_{abc}]\cdot[VLG_{abc}]_m\right\} \tag{6.5}$$

Collecting terms,

$$[VLG_{abc}]_n = \left\{[U] + \frac{1}{2}\cdot[Z_{abc}]\cdot[Y_{abc}]\right\}\cdot[VLG_{abc}]_m + [Z_{abc}]\cdot[I_{abc}]_m \tag{6.6}$$

where

$$[U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6.7}$$

Equation 6.6 is of the general form

$$[VLG_{abc}]_n = [a]\cdot[VLG_{abc}]_m + [b]\cdot[I_{abc}]_m \tag{6.8}$$

where

$$[a] = [U] + \frac{1}{2}\cdot[Z_{abc}]\cdot[Y_{abc}] \tag{6.9}$$

$$[b] = [Z_{abc}] \tag{6.10}$$

The input current to the line segment at node ***n*** is

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_n = \begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix}_m + \frac{1}{2}\cdot\begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}\cdot\begin{bmatrix} Vag \\ Vbg \\ Vcg \end{bmatrix}_n \tag{6.11}$$

In condensed form, Equation 6.11 becomes

$$[I_{abc}]_n = [Iline_{abc}]_m + \frac{1}{2}\cdot[Y_{abc}]\cdot[VLG_{abc}]_n \tag{6.12}$$

Substitute Equation 6.2 into Equation 6.12:

$$[I_{abc}]_n = [I_{abc}]_m + \frac{1}{2}[Y_{abc}]\cdot[VLG_{abc}]_m + \frac{1}{2}\cdot[Y_{abc}]\cdot[VLG_{abc}]_n \tag{6.13}$$

Substitute Equation 6.6 into Equation 6.13:

$$\begin{aligned}[I_{abc}]_n &= [I_{abc}]_m + \frac{1}{2}[Y_{abc}]\cdot[VLG_{abc}]_m \\ &\quad + \frac{1}{2}\cdot[Y_{abc}]\cdot\left(\left\{[U] + \frac{1}{2}\cdot[Z_{abc}]\cdot[Y_{abc}]\right\}\cdot[VLG_{abc}]_m + [Z_{abc}]\cdot[I_{abc}]_m\right)\end{aligned} \tag{6.14}$$

Collecting terms in Equation 6.14,

$$\begin{aligned}[I_{abc}]_n &= \left\{[Y_{abc}] + \frac{1}{4}\cdot[Y_{abc}]\cdot[Z_{abc}]\cdot[Y_{abc}]\right\}\cdot[VLG_{abc}]_m \\ &\quad + \left\{[U] + \frac{1}{2}\cdot[Z_{abc}]\cdot[Y_{abc}]\right\}[I_{abc}]_m\end{aligned} \tag{6.15}$$

Equation 6.15 is of the form

$$[I_{abc}]_n = [c] \cdot [VLG_{abc}]_m + [d] \cdot [I_{abc}]_m \tag{6.16a}$$

where

$$[c] = [Y_{abc}] + \frac{1}{4} \cdot [Y_{abc}] \cdot [Z_{abc}] \cdot [Y_{abc}] \tag{6.17}$$

$$[d] = [U] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}] \tag{6.18}$$

Equations 6.8 and 6.16a can be put into partitioned matrix form:

$$\begin{bmatrix} [VLG_{abc}]_n \\ [I_{abc}]_n \end{bmatrix} = \begin{bmatrix} [a] & [b] \\ [c] & [d] \end{bmatrix} \cdot \begin{bmatrix} [VLG_{abc}]_m \\ [I_{abc}]_m \end{bmatrix} \tag{6.19}$$

Equation 6.19 is very similar to the equation used in transmission line analysis when the *A*, *B*, *C*, *D* parameters have been defined [1]. In the case here the *a*, *b*, *c*, *d* parameters are 3 × 3 matrices rather than single variables and will be referred to as the "generalized line matrices."

Equation 6.19 can be turned around to solve for the voltages and currents at node ***m*** in terms of the voltages and currents at node ***n***:

$$\begin{bmatrix} [VLG_{abc}]_m \\ [I_{abc}]_m \end{bmatrix} = \begin{bmatrix} [a] & [b] \\ [c] & [d] \end{bmatrix}^{-1} \cdot \begin{bmatrix} [VLG_{abc}]_n \\ [I_{abc}]_n \end{bmatrix} \tag{6.20}$$

The inverse of the *a*, *b*, *c*, *d* matrix is simple because the determinant is

$$[a] \cdot [d] - [b] \cdot [c] = [U] \tag{6.21}$$

Using the relationship of Equation 6.21, Equation 6.20 becomes

$$\begin{bmatrix} [VLG_{abc}]_m \\ [I_{abc}]_m \end{bmatrix} = \begin{bmatrix} [d] & -[b] \\ -[c] & [a] \end{bmatrix} \cdot \begin{bmatrix} [VLG_{abc}]_n \\ [I_{abc}]_n \end{bmatrix} \tag{6.22}$$

Since the matrix [*a*] is equal to the matrix [*d*], Equation 6.22 in expanded form becomes

$$[VLG_{abc}]_m = [a] \cdot [VLG_{abc}]_n - [b] \cdot [I_{abc}]_n \tag{6.23}$$

$$[I_{abc}]_m = -[c] \cdot [VLG_{abc}]_n + [d] \cdot [I_{abc}]_n \tag{6.24}$$

Sometimes it is necessary to compute the voltages at node ***m*** as a function of the voltages at node ***n*** and the currents entering node ***m***. This is true in the iterative technique that is developed in Chapter 10.

Solving Equation 6.8 for the bus *m* voltages gives

$$[VLG_{abc}]_m = [a]^{-1} \cdot \left\{ [VLG_{abc}]_n - [b] \cdot [I_{abc}]_m \right\} \tag{6.25}$$

Equation 6.25 is of the form

$$[b] = [Z_{abc}] = \begin{bmatrix} 0.8667 + j2.0417 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 0.8837 + j1.9852 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 0.8741 + j2.0172 \end{bmatrix}$$

$$[VLG_{abc}]_m = [A] \cdot [VLG_{abc}]_n - [B] \cdot [I_{abc}] \tag{6.26}$$

where

$$[A] = [a]^{-1} \tag{6.27}$$

$$[B] = [a]^{-1} \cdot [b] \tag{6.28}$$

The line-to-line voltages are computed by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}_m = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m = [D] \cdot [VLG_{abc}]_m \tag{6.29}$$

where

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{6.30}$$

Because the mutual coupling between phases on the line segments is not equal, there will be different values of voltage drop on each of the three phases. As a result the voltages on a distribution feeder become unbalanced even when the loads are balanced. A common method of describing the degree of unbalance is to use the National Electrical Manufactures Association (NEMA) definition of voltage unbalance as given in Equation 6.31 [2].

$$V_{unbalance} = \frac{|Maximum\ Deviation\ from\ Average|}{|V_{average}|} \cdot 100\% \tag{6.31}$$

Example 6.1

A balanced three-phase load of 6000 kVA, 12.47 kV, 0.9 lagging power factor is being served at node ***m*** of a 10,000 ft three-phase line segment. The load voltages are rated and balanced 12.47 kV. The configuration and conductors of the line segment are those of Example 4.1. Determine the generalized line constant matrices [*a*], [*b*], [*c*], [*d*], [*A*], and [*B*]. Using the generalized matrices determine the line-to-ground voltages and line currents at the source end (node ***n***) of the line segment.

Solution

The phase impedance matrix and the shunt admittance matrix for the line segment as computed in Examples 4.1 and 5.1 are

$$[z_{abc}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

$$[y_{abc}] = j \cdot 376.9911 \cdot [C_{abc}] = \begin{bmatrix} j5.6711 & -j1.8362 & -j0.7033 \\ -j1.8362 & j5.9774 & -j1.169 \\ -j0.7033 & -j1.169 & j5.3911 \end{bmatrix} \mu\text{S/mile}$$

For the 10,000 ft line segment, the total phase impedance matrix and shunt admittance matrix are

$$[Z_{abc}] = \begin{bmatrix} 0.8667 + j2.0417 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 0.8837 + j1.9852 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 0.8741 + j2.0172 \end{bmatrix} \Omega$$

$$[Y_{abc}] = \begin{bmatrix} j10.7409 & -j3.4777 & -j1.3322 \\ -j3.4777 & j11.3208 & -j2.2140 \\ -j1.3322 & -j2.2140 & j10.2104 \end{bmatrix} \mu\text{S}$$

It should be noted that the elements of the phase admittance matrix are very small.

The generalized matrices computed according to Equations 6.9, 6.10, 6.17, and 6.18 are

$$[a] = [U] + \frac{1}{2} \cdot [Z_{abc}] \cdot [Y_{abc}] = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 1.0 \end{bmatrix}$$

$$[b] = [Z_{abc}] = \begin{bmatrix} 0.8667 + j2.0417 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 0.8837 + j1.9852 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 0.8741 + j2.0172 \end{bmatrix}$$

$$[c] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[d] = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 1.0 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 1.0 \end{bmatrix}$$

$$[B] = [a]^{-1} \cdot [b] = \begin{bmatrix} 0.8667 + j2.0417 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 0.8837 + j1.9852 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 0.8741 + j2.0172 \end{bmatrix}$$

Because the elements of the phase admittance matrix are so small, the [a], [A], and [d] matrices appear to be the unity matrix. If more significant figures are displayed, the 1,1 element of these matrices is

$$a_{1,1} = A_{1,1} = 0.99999117 + j0.00000395$$

Also, the elements of the [c] matrix appear to be zero. Again if more significant figures are displayed, the 1,1 term is

$$c_{1,1} = -0.0000044134 + j0.0000127144$$

The point here is that for all practical purposes the phase admittance matrix can be neglected.

The magnitude of the line-to-ground voltage at the load is

$$VLG = \frac{12470}{\sqrt{3}} = 7199.56$$

Selecting the phase a to ground voltage as reference, the line-to-ground voltage matrix at the load is

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m = \begin{bmatrix} 7199.56\underline{/0} \\ 7199.56\underline{/-120} \\ 7199.56\underline{/120} \end{bmatrix} \text{V}$$

The magnitude of the load currents is

$$|I|_m = \frac{6000}{\sqrt{3}\cdot 12.47} = 277.79$$

For a 0.9 lagging power factor the load current matrix is

$$[I_{abc}]_m = \begin{bmatrix} 277.79\underline{/-25.84} \\ 277.79\underline{/-145.84} \\ 277.79\underline{/94.16} \end{bmatrix} \text{A}$$

The line-to-ground voltages at node ***n*** are computed to be

$$[VLG_{abc}]_n = [a]\cdot[VLG_{abc}]_m + [b]\cdot[I_{abc}]_m = \begin{bmatrix} 7538.70\underline{/1.57} \\ 7451.25\underline{/-118.30} \\ 7485.11\underline{/121.93} \end{bmatrix} \text{V}$$

It is important to note that the voltages at node ***n*** are unbalanced even though the voltages and currents at the load (node ***m***) are perfectly balanced. This is a result of the unequal mutual coupling between phases. The degree of voltage unbalance is of concern since, for example, the operating characteristics of a three-phase induction motor are very sensitive to voltage unbalance. Using the NEMA definition for voltage unbalance (Equation 6.29), the voltage unbalance is given by

$$|V_{average}| = \frac{|V_{ag}|_n + |V_{bg}|_n + |V_{cg}|_n}{3} = \frac{7538.70 + 7451.25 + 7485.11}{3} = 7491.69$$

$$Vdeviation_{max} = 7538.70 - 7491.69 = 47.01$$

$$V_{unbalance} = \frac{47.01}{7491.70}\cdot 100\% = 0.6275\%$$

Although this may not seem like a large unbalance, it does give an indication of how the unequal mutual coupling can generate an unbalance. It is important to know that NEMA standards require that induction motors be derated when the voltage unbalance exceeds 1.0%.

Selecting rated line-to-ground voltage as base (7199.56) the per-unit voltages at bus n are

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_n = \frac{1}{7199.56}\begin{bmatrix} 7538.70\underline{/1.577} \\ 7451.25\underline{/-118.30} \\ 7485.11\underline{/121.93} \end{bmatrix} = \begin{bmatrix} 1.0471\underline{/1.57} \\ 1.0350\underline{/-118.30} \\ 1.0397\underline{/-121.931} \end{bmatrix} \text{per unit}$$

By converting the voltages to per unit, it is easy to see that the voltage drop by phase is 4.71% for phase a, 3.50% for phase b, and 3.97% for phase c.

The line currents at node ***n*** are computed to be

$$[I_{abc}]_n = [c]\cdot[VLG_{abc}]_m + [d]\cdot[I_{abc}]_m = \begin{bmatrix} 277.71/\underline{-25.83} \\ 277.73/\underline{-148.82} \\ 277.73/\underline{94.17} \end{bmatrix} \text{A}$$

Comparing the computed line currents at node ***n*** to the balanced load currents at node ***m***, a very slight difference is noted that is another result of the unbalanced voltages at node ***n*** and the shunt admittance of the line segment.

6.2 Modified Line Model

It was demonstrated in Example 6.1 that the shunt admittance of an overhead line is so small that it can be neglected. Figure 6.2 shows the modified line segment model with the shunt admittance neglected.

When the shunt admittance is neglected, the generalized matrices become

$$[a] = [U] \tag{6.32}$$

$$[b] = [Z_{abc}] \tag{6.33}$$

$$[c] = [0] \tag{6.34}$$

$$[d] = [U] \tag{6.35}$$

$$[A] = [U] \tag{6.36}$$

$$[B] = [Z_{abc}] \tag{6.37}$$

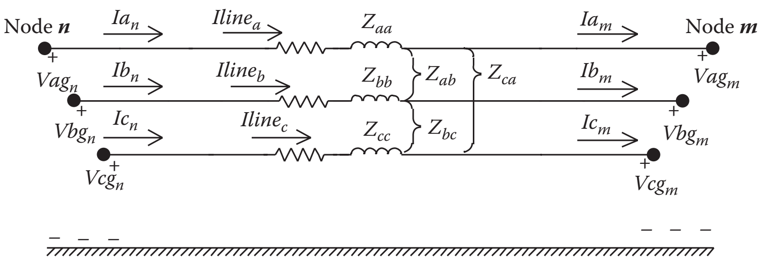

FIGURE 6.2
Modified line segment model.

6.2.1 Three-Wire Delta Line

If the line is a three-wire delta, then the voltage drops down the line must be in terms of the line-to-line voltages and line currents. However, it is possible to use "equivalent" line-to-neutral voltages so that the equations derived to this point will still apply. Writing the voltage drops in terms of line-to-line voltages for the line in Figure 6.2 results in

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}_n = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}_m + \begin{bmatrix} vdrop_a \\ vdrop_b \\ vdrop_c \end{bmatrix} - \begin{bmatrix} vdrop_b \\ vdrop_c \\ vdrop_a \end{bmatrix} \tag{6.38}$$

where

$$\begin{bmatrix} vdrop_a \\ vdrop_b \\ vdrop_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} Iline_a \\ Iline_b \\ Iline_c \end{bmatrix} \tag{6.39}$$

Expanding Equation 6.38 for the phase *a–b*,

$$Vab_n = Vab_m + vdrop_a - vdrop_b \tag{6.40}$$

but

$$\begin{aligned} Vab_n &= Van_n - Vbn_n \\ Vab_m &= Van_m - Vbn_m \end{aligned} \tag{6.41}$$

Substitute Equations 6.41 into Equation 6.40:

$$Van_n - Vbn_n = Van_m - Vbn_m + vdrop_a - vdrop_b \tag{6.42}$$

Equation 6.40 can be broken into two parts in terms of "equivalent" line-to-neutral voltages:

$$\begin{aligned} Van_n &= Van_m + vdrop_a \\ Vbn_n &= Vbn_m + vdrop_b \end{aligned} \tag{6.43}$$

The conclusion here is that it is possible to work with "equivalent" line-to-neutral voltages in a three-wire delta line. This is very important since it makes the development of general analyses techniques the same for four-wire wye and three-wire delta systems.

6.2.2 Computation of Neutral and Ground Currents

In Chapter 4 the Kron reduction method was used to reduce the primitive impedance matrix to the 3 × 3 phase impedance matrix. Figure 6.3 shows a three-phase line with grounded neutral that is used in the Kron reduction. Note that the direction of the current flowing in the ground is shown in Figure 6.3.

In the development of the Kron reduction method, Equation 4.52 defined the "neutral transform matrix" $[t_n]$. The same equation is shown as Equation 6.44:

$$[t_n] = -[\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \tag{6.44}$$

The matrices $[\hat{z}_{nn}]$ and $[\hat{z}_{nj}]$ are the partitioned matrices in the primitive impedance matrix.

When the currents flowing in the lines have been determined, Equation 6.45 is used to compute the current flowing in the grounded neutral wire(s):

$$[I_n] = [t_n] \cdot [I_{abc}] \tag{6.45}$$

In Equation 6.45, the matrix $[I_n]$ for an overhead line with one neutral wire will be a single element. However, in the case of an underground line consisting of concentric neutral cables or taped shielded cables with or without a separate neutral wire, $[I_n]$ will be the currents flowing in each of the cable neutrals and the separate neutral wire if present. Once the neutral current(s) has been determined, Kirchhoff's current law is used to compute the current flowing in ground:

$$I_g = -(I_a + I_b + I_c + In_1 + In_2 + \cdots + In_k) \tag{6.46}$$

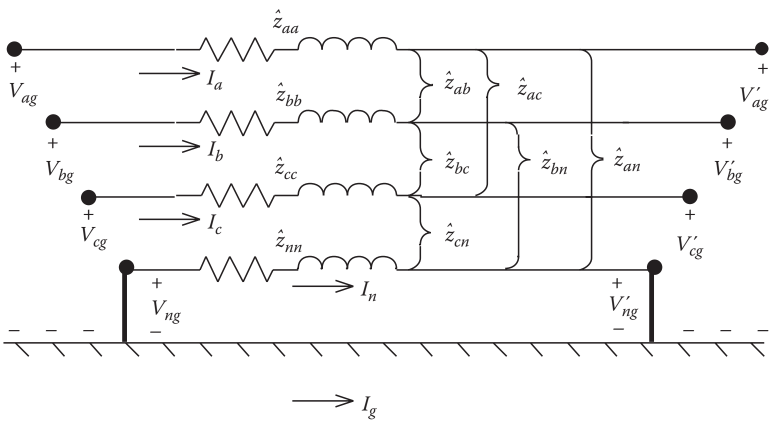

FIGURE 6.3
Three-phase line with neutral and ground currents.

Example 6.2

The line of Example 6.1 will be used to supply an unbalanced load at node ***m***. Assume that the voltages at the source end (node ***n***) are balanced three phase at 12.47 kV line to line. The balanced line-to-ground voltages are

$$[VLG_{abc}]_n = \begin{bmatrix} 7199.56\underline{/0} \\ 7199.56\underline{/-120} \\ 7199.56\underline{/120} \end{bmatrix} \text{V}$$

The unbalanced currents measured at the source end are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_n = \begin{bmatrix} 249.97\underline{/-24.5} \\ 277.56\underline{/-145.8} \\ 305.54\underline{/95.2} \end{bmatrix} \text{A}$$

Determine

- The line-to-ground and line-to-line voltages at the load end (node ***m***) using the modified line model
- The voltage unbalance
- The complex powers of the load
- The currents flowing in the neutral wire and ground

Solution

The [A] and [B] matrices for the modified line model are

$$[A] = [U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[B] = [Z_{abc}] = \begin{bmatrix} 0.8666 + j2.0417 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 0.8837 + j1.9852 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 0.8741 + j2.0172 \end{bmatrix} \Omega$$

Since this is the approximate model, $[I_{abc}]_m$ is equal to $[I_{abc}]_n$. Therefore

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_m = \begin{bmatrix} 249.97\underline{/-24.5} \\ 277.56\underline{/-145.8} \\ 305.54\underline{/95.2} \end{bmatrix} \text{A}$$

The line-to-ground voltages at the load end are

$$[VLG_{abc}]_m = [A]\cdot[VLG_{abc}]_n - [B]\cdot[I_{abc}]_m = \begin{bmatrix} 6942.53\underline{/-1.47} \\ 6918.35\underline{/-121.55} \\ 6887.71\underline{/117.31} \end{bmatrix} \text{V}$$

The line-to-line voltages at the load end are

$$[D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[VLL_{abc}]_m = [D] \cdot [VLG_{abc}]_m = \begin{bmatrix} 12{,}008/\underline{28.4} \\ 12{,}025/\underline{-92.2} \\ 11{,}903/\underline{148.1} \end{bmatrix}$$

For this condition, the average load voltage is

$$|V_{average}| = \frac{6942.53 + 6918.35 + 6887.71}{3} = 6916.20$$

The maximum deviation from the average is on phase c so that

$$Vdeviation_{max} = |6887.71 - 6916.20| = 28.49$$

$$V_{unbalance} = \frac{28.49}{6916.20} \cdot 100 = 0.4119\%$$

The complex powers of the load are

$$\begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \frac{1}{1000} \cdot \begin{bmatrix} V_{ag} \cdot I_a^* \\ V_{bg} \cdot I_b^* \\ V_{cg} \cdot I_c^* \end{bmatrix} = \begin{bmatrix} 1597.2 + j678.8 \\ 1750.8 + j788.7 \\ 1949.7 + j792.0 \end{bmatrix} \text{kW} + j\,\text{kvar}$$

The "neutral transformation matrix" from Example 4.1 is

$$[t_n] = \begin{bmatrix} -0.4292 - j0.1291 & -0.4476 - j0.1373 & -0.4373 - j0.1327 \end{bmatrix}$$

The neutral current is

$$[I_n] = [t_n] \cdot [I_{abc}]_m = 26.2/\underline{-29.5}$$

The ground current is

$$I_g = -(I_a + I_b + I_c + I_n) = 32.5/\underline{-77.6}$$

6.3 Approximate Line Segment Model

Many times the only data available for a line segment will be the positive and zero sequence impedances. The approximate line model can be developed by applying the "reverse impedance transformation" from symmetrical component theory.

Using the known positive and zero sequence impedances, the "sequence impedance matrix" is given by

$$[Z_{seq}] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_+ & 0 \\ 0 & 0 & Z_+ \end{bmatrix} \tag{6.47}$$

The "reverse impedance transformation" results in the following approximate phase impedance matrix:

$$[Z_{approx}] = [A_s]\cdot[Z_{seq}]\cdot[A_s]^{-1} \tag{6.48}$$

$$[Z_{approx}] = \frac{1}{3}\cdot\begin{bmatrix} (2\cdot Z_+ + Z_0) & (Z_0 - Z_+) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (2\cdot Z_+ + Z_0) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (Z_0 - Z_+) & (2\cdot Z_+ + Z_0) \end{bmatrix} \tag{6.49}$$

Notice that the approximate impedance matrix is characterized by the three diagonal terms being equal and all mutual terms being equal. This is the same result that is achieved if the line is assumed to be transposed. Applying the approximate impedance matrix the voltage at node ***n*** is computed to be

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_n = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}_m + \frac{1}{3}\cdot\begin{bmatrix} (2\cdot Z_+ + Z_0) & (Z_0 - Z_+) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (2\cdot Z_+ + Z_0) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (Z_0 - Z_+) & (2\cdot Z_+ + Z_0) \end{bmatrix}\cdot\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_m \tag{6.50}$$

In condensed form, Equation 6.50 becomes

$$[VLG_{abc}]_n = [VLG_{abc}]_m + [Z_{approx}]\cdot[I_{abc}]_m \tag{6.51}$$

Note that Equation 6.51 is of the form

$$[VLG_{abc}]_n = [a][VLG_{abc}]_m + [b]\cdot[I_{abc}]_m \tag{6.52}$$

where

$$[A] = \text{unity matrix}$$

$$[b] = [Z_{approx}]$$

Equation 6.50 can be expanded and an equivalent circuit for the approximate line segment model can be developed. Solving Equation 6.50 for the phase *a* voltage at node ***n*** results in

$$Vag_n = Vag_m + \frac{1}{3}\{(2Z_+ + Z_0)I_a + (Z_0 - Z_+)I_b + (Z_0 + Z_+)I_c\} \tag{6.53}$$

Modify Equation 6.53 by adding and subtracting the term $(Z_0 - Z_+)I_a$ and then combining terms and simplifying:

$$Vag_n = Vag_m + \frac{1}{3}\begin{Bmatrix} (2Z_+ + Z_0)I_a + (Z_0 - Z_+)I_b + (Z_0 - Z_+)I_c \\ +(Z_0 - Z_+)I_a - (Z_0 - Z_+)I_a \end{Bmatrix}$$

$$= Vag_m + \frac{1}{3}\{(3Z_+)I_a + (Z_0 - Z_+)(I_a + I_b + I_c)\}$$

$$= Vag_m + Z_+ \cdot I_a + \frac{(Z_0 - Z_+)}{3} \cdot (I_a + I_b + I_c) \tag{6.54}$$

The same process can be followed in expanding Equation 6.50 for phases *b* and *c*. The final results are

$$Vbg_n = Vbg_m + Z_+ \cdot I_b + \frac{(Z_0 - Z_+)}{3} \cdot (I_a + I_b + I_c) \tag{6.55}$$

$$Vcg_n = Vcg_m + Z_+ \cdot I_c + \frac{(Z_0 - Z_+)}{3} \cdot (I_a + I_b + I_c) \tag{6.56}$$

Figure 6.4 illustrates the approximate line segment model.

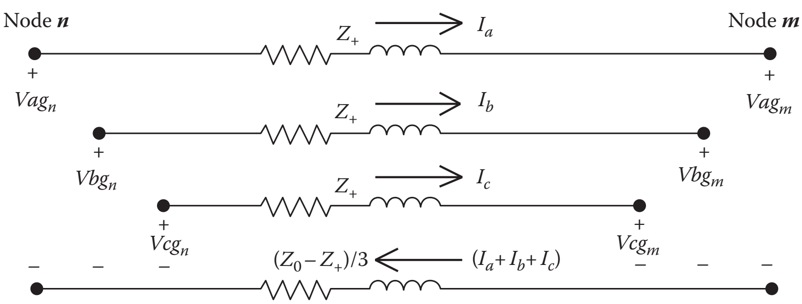

FIGURE 6.4
Approximate line segment model.

Figure 6.4 shows a simple equivalent circuit for the line segment since no mutual coupling has to be modeled. It must be understood, however, that the equivalent circuit can only be used when transposition of the line segment has been assumed.

Example 6.3

The line segment of Example 4.1 is to be analyzed assuming that the line has been transposed. In Example 4.1, the positive and zero sequence impedances were computed to be

$$\begin{matrix} z_+ = 0.3061 + j0.6270 \\ z_0 = 0.7735 + j1.9373 \end{matrix}\ \Omega/\text{mile}$$

Assume that the load at node ***m*** is the same as in Example 6.1. That is,

$$kVA = 6000, \quad kVLL = 12.47, \quad Power\ Factor = 0.8\ \text{lagging}$$

Determine the voltages and currents at the source end (node ***n***) for this loading condition.

Solution

The sequence impedance matrix is

$$[z_{seq}] = \begin{bmatrix} 0.7735 + j1.9373 & 0 & 0 \\ 0 & 0.3061 + j0.6270 & 0 \\ 0 & 0 & 0.3061 + j0.6270 \end{bmatrix} \Omega/\text{mile}$$

Performing the reverse impedance transformation results in the approximate phase impedance matrix:

$$[z_{approx}] = [A_s] \cdot [z_{seq}] \cdot [A_s]^{-1}$$

$$= \begin{bmatrix} 0.4619 + j1.0638 & 0.1558 + j0.4368 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.4619 + j1.0638 & 0.1558 + j0.4368 \\ 0.1558 + j0.4368 & 0.1558 + j0.4368 & 0.4619 + j1.0638 \end{bmatrix} \Omega/\text{mile}$$

For the 10,000 ft line, the phase impedance matrix and the $[b]$ matrix are

$$[b] = [Z_{approx}] = [z_{approx}] \cdot \frac{10000}{5280}$$

$$= \begin{bmatrix} 0.8748 + j2.0147 & 0.2951 + j0.8272 & 0.2951 + j0.8272 \\ 0.2951 + j0.8272 & 0.8748 + j2.0147 & 0.2951 + j0.8272 \\ 0.2951 + j0.8272 & 0.2951 + j0.8272 & 0.8748 + j2.0147 \end{bmatrix} \Omega$$

Note in the approximate phase impedance matrix that the three diagonal terms are equal and all of the mutual terms are equal. Again, this is an indication of the transposition assumption.

From Example 6.1, the voltages and currents at node ***m*** are

$$[VLG_{abc}]_m = \begin{bmatrix} 7199.56\underline{/0} \\ 7199.56\underline{/-120} \\ 7199.56\underline{/120} \end{bmatrix} \text{V}$$

$$[I_{abc}]_m = \begin{bmatrix} 277.79\underline{/-25.84} \\ 277.79\underline{/-145.84} \\ 277.79\underline{/94.16} \end{bmatrix} \text{A}$$

Using Equation 6.52,

$$[VLG_{abc}]_n = [a]\cdot[VLG_{abc}]_m + [b]\cdot[I_{abc}]_m = \begin{bmatrix} 7491.72\underline{/-1.73} \\ 7491.72\underline{/-118.27} \\ 7491.72\underline{/121.73} \end{bmatrix} \text{V}$$

Note that the computed voltages are balanced. In Example 6.1, it was shown that when the line is modeled accurately, there is a voltage unbalance of 0.6275%. It should also be noted that the average value of the voltages at node ***n*** in Example 6.1 was 7491.69 V.

The V_{ag} at node ***n*** can also be computed using Equation 6.48:

$$Vag_n = Vag_m + Z_+ \cdot I_a + \frac{(Z_0 - Z_+)}{3}\cdot(I_a + I_b + I_c)$$

Since the currents are balanced, this equation reduces to

$$Vag_n = Vag_m + Z_+ \cdot I_a$$

$$= 7199.56\underline{/0} + (0.5797 + j1.1875)\cdot 277.79\underline{/-25.84} = 7491.72\underline{/1.73}\,\text{V}$$

It can be noted that when the loads are balanced and transposition has been assumed, the three-phase line can be analyzed as a simple single-phase equivalent as was done in the calculation earlier.

Example 6.4

Use the balanced voltages and unbalanced currents at node ***n*** in Example 6.2 and the approximate line model to compute the voltages and currents at node ***m***.

Solution

From Example 6.2, the voltages and currents at node ***n*** are given as

$$[VLG_{abc}]_n = \begin{bmatrix} 7199.56\underline{/0} \\ 7199.56\underline{/-120} \\ 7199.56\underline{/120} \end{bmatrix} \text{V}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}_n = \begin{bmatrix} 249.97/\underline{-24.5} \\ 277.56/\underline{-145.8} \\ 305.54/\underline{95.2} \end{bmatrix} \text{A}$$

The $[A]$ and $[B]$ matrices for the approximate line model are

$$[A] = \text{unity matrix}$$

$$[B] = [Z_{approx}]$$

The voltages at node ***m*** are determined by

$$[VLG_{abc}]_m = [A] \cdot [VLG_{abc}]_n - [B] \cdot [I_{abc}]_n = \begin{bmatrix} 6993.10/\underline{-1.63} \\ 6881.15/\underline{-121.61} \\ 6880.23/\underline{117.50} \end{bmatrix} \text{V}$$

The voltage unbalance for this case is computed by

$$V_{average} = \frac{6993.10 + 6881.15 + 6880.23}{3} = 6918.16$$

$$Vdeviation_{max} = |6993.12 - 6918.17| = 74.94$$

$$V_{unbalance} = \frac{74.94}{6918.17} \cdot 100 = 1.0833\%$$

Note that the approximate model has led to a higher voltage unbalance than the "exact" model.

6.4 Modified "Ladder" Iterative Technique

The previous example problems have assumed a linear system. Unfortunately, that will not be the usual case for distribution feeders. When the source voltages are specified and the loads are specified as constant kW and kvar (constant PQ), the system becomes nonlinear, and an iterative method will have to be used to compute the load voltages and currents. Chapter 10 develops in detail the modified "ladder" iterative technique. However, a simple form of that technique will be developed here in order to demonstrate how the nonlinear system can be evaluated.

The ladder technique is composed of two parts:

1. Forward sweep
2. Backward sweep

The forward sweep computes the downstream voltages from the source by applying Equation 6.26:

$$\left[VLG_{abc}\right]_m = [A]\cdot\left[VLG_{abc}\right]_n - [B]\cdot\left[I_{abc}\right] \tag{6.26}$$

To start the process, the load currents $[I_{abc}]$ are assumed to be equal to zero and the load voltages are computed. In the first iteration the load voltages will be the same as the source voltages.

The backward sweep computes the currents from the load back to the source using the most recently computed voltages from the forward sweep. Equation 6.16a is applied for this sweep:

$$\left[I_{abc}\right]_n = [c]\cdot\left[VLG_{abc}\right]_m + [d]\cdot\left[I_{abc}\right]_m \tag{6.16a}$$

Recall that for all practical purposes the $[c]$ matrix is zero so Equation 6.16a is simplified to be

$$\left[I_{abc}\right]_n = [d]\cdot\left[I_{abc}\right]_m \tag{6.16b}$$

After the first forward and backward sweeps, the new load voltages are computed using the most recent currents. The forward and backward sweeps continue until the error between the new and previous load voltages is within a specified tolerance. Using the matrices computed in Example 6.1, a very simple Mathcad® program that applies the ladder iterative technique is demonstrated in Example 6.5.

Example 6.5

The line of Example 6.1 serves an unbalanced three-phase load of

Phase a: 2500 kVA and PF = 0.9 lagging
Phase b: 2000 kVA and PF = 0.85 lagging
Phase c: 1500 kVA and PF = 0.95 lagging

The source voltages are balanced 12.47 kV. A simple Mathcad program is used to compute the load voltages and currents. This program is shown in Figure 6.5.

After seven iterations the load voltages and currents are computed to be

$$\left[VLG_{abc}\right] = \begin{bmatrix} 6678.2/\underline{-2.3} \\ 6972.8/\underline{-122.1} \\ 7055.5/\underline{118.7} \end{bmatrix} \qquad \left[I_{abc}\right] = \begin{bmatrix} 374.4/\underline{-28.2} \\ 286.8/\underline{-153.9} \\ 212.6/\underline{100.5} \end{bmatrix}$$

$$\text{Start} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \text{Tol} := .00001 \qquad kV_{LN} := 7.2$$

$$
\begin{aligned}
\text{Solve} := \; & I_{abc} \leftarrow \text{Start} \\
& V_{old} \leftarrow \text{Start} \\
& \text{for } n \in 1..200 \\
& \quad VLG_{abc} \leftarrow A_t \cdot E_{abc} - B_t \cdot I_{abc} \\
& \quad \text{for } i \in 1..3 \\
& \qquad I_{abc_i} \leftarrow \frac{\overline{SL_i \cdot 1000}}{VLG_{abc_i}} \\
& \quad \text{for } j \in 1..3 \\
& \qquad Error_j \leftarrow \frac{\left| VLG_{abc_j} - V_{old_j} \right|}{kV_{LN} \cdot 1000} \\
& \quad Err_{max} \leftarrow \max(Error) \\
& \quad \text{break if } Err_{max} < Tol \\
& \quad V_{old} \leftarrow VLG_{abc} \\
& Out_1 \leftarrow VLG_{abc} \\
& Out_2 \leftarrow I_{abc} \\
& Out_3 \leftarrow n \\
& Out
\end{aligned}
$$

FIGURE 6.5
Mathcad® program.

Example 6.5 demonstrates the application of the ladder iterative technique, which will be used as models for the development of other distribution feeder elements. A simple flowchart of the program that is used in other chapters is shown in Figure 6.6.

6.5 General Matrices for Parallel Lines

The equivalent Pi circuits for two parallel three-phase lines are shown in Figure 6.7.

The 6 × 6 phase impedance and shunt admittance matrices for parallel three-phase lines are developed in Chapters 4 and 5. These matrices are used in the development of the general matrices used in modeling parallel three-phase lines.

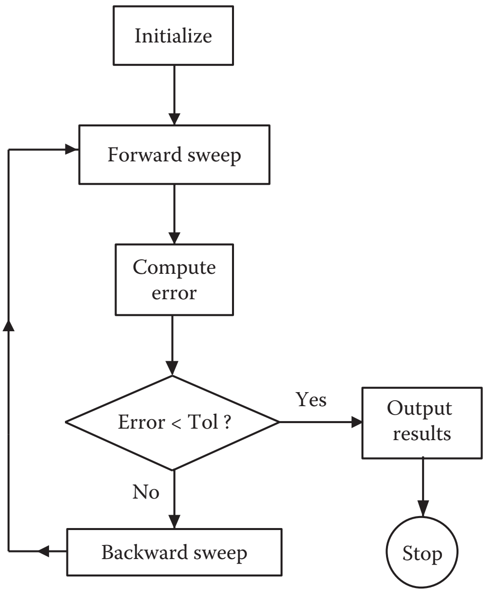

FIGURE 6.6
Simple modified ladder flowchart.

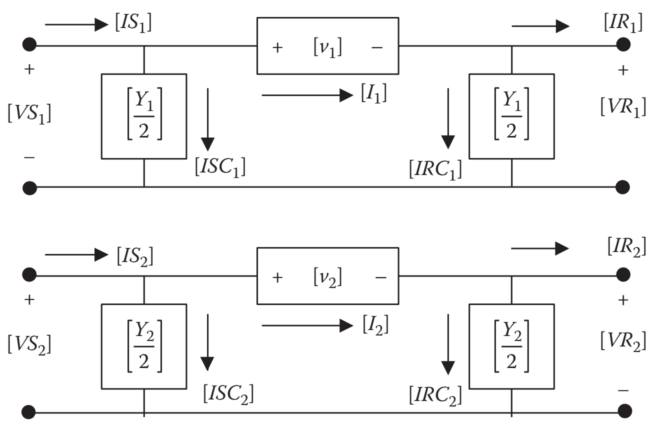

FIGURE 6.7
Equivalent Pi parallel lines.

The first step in computing the *a*, *b*, *c*, *d* matrices is to multiply the 6 × 6 phase impedance matrix from Chapter 4 and the 6 × 6 shunt admittance matrix from Chapter 5 by the distance that the lines are parallel:

$$\begin{bmatrix}[v1]\\ [v2]\end{bmatrix} = \begin{bmatrix}[z11] & [z12]\\ [z21] & [z22]\end{bmatrix}\cdot length\cdot\begin{bmatrix}[I1]\\ [I2]\end{bmatrix} = \begin{bmatrix}[Z11] & [Z12]\\ [Z21] & [Z22]\end{bmatrix}\cdot\begin{bmatrix}[I1]\\ [I2]\end{bmatrix}\text{V} \qquad (6.57)$$

$$[v] = [Z]\cdot[I]$$

$$\begin{bmatrix} [y11] & [y12] \\ [y21] & [y22] \end{bmatrix} \cdot length = \begin{bmatrix} [Y11] & [Y12] \\ [Y21] & [Y22] \end{bmatrix} \text{S} \tag{6.58}$$

Referring to Figure 6.5, the line currents in the two circuits are given by

$$\begin{gathered} \begin{bmatrix} [I_1] \\ [I_2] \end{bmatrix} = \begin{bmatrix} [IR_1] \\ [IR_2] \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{21}] & [Y_{22}] \end{bmatrix} \cdot \begin{bmatrix} [VR_1] \\ [VR_2] \end{bmatrix} \text{A} \\ [I] = [IR] + \frac{1}{2} \cdot [Y] \cdot [VR] \end{gathered} \tag{6.59}$$

The sending end voltages are given by

$$\begin{gathered} \begin{bmatrix} [VS_1] \\ [VS_2] \end{bmatrix} = \begin{bmatrix} [VR_1] \\ [VR_2] \end{bmatrix} + \begin{bmatrix} [Z_{11}] & [Z_{12}] \\ [Z_{21}] & [Z_{22}] \end{bmatrix} \cdot \begin{bmatrix} [I_1] \\ [I_2] \end{bmatrix} \text{A} \\ [VS] = [VR] + [Z] \cdot [I] \end{gathered} \tag{6.60}$$

Substitute Equation 6.59 into Equation 6.60:

$$\begin{gathered} \begin{bmatrix} [VS_1] \\ [VS_2] \end{bmatrix} = \begin{bmatrix} [VR_1] \\ [VR_2] \end{bmatrix} + \begin{bmatrix} [Z_{11}] & [Z_{12}] \\ [Z_{21}] & [Z_{22}] \end{bmatrix} \cdot \left(\begin{bmatrix} [IR_1] \\ [IR_2] \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{21}] & [Y_{22}] \end{bmatrix} \cdot \begin{bmatrix} [VR_1] \\ [VR_2] \end{bmatrix} \right) \\ [VS] = [VR] + [Z] \cdot \left([IR] + \frac{1}{2} \cdot [Y] \cdot [VR] \right) \end{gathered} \tag{6.61}$$

Combine terms in Equation 6.61:

$$\begin{gathered} \begin{bmatrix} [VS_1] \\ [VS_2] \end{bmatrix} = \left(\begin{bmatrix} [U] \\ [U] \end{bmatrix} + \frac{1}{2} \cdot \begin{bmatrix} [Z_{11}] & [Z_{12}] \\ [Z_{21}] & [Z_{22}] \end{bmatrix} \begin{bmatrix} [Y_{11}] & [Y_{12}] \\ [Y_{21}] & [Y_{22}] \end{bmatrix} \right) \begin{bmatrix} [VR_1] \\ [VR_2] \end{bmatrix} \\ + \begin{bmatrix} [Z_{11}] & [Z_{12}] \\ [Z_{21}] & [Z_{22}] \end{bmatrix} \cdot \begin{bmatrix} [IR_1] \\ [IR_2] \end{bmatrix} \\ [VS] = \left([U] + \frac{1}{2} \cdot [Z] \cdot [Y] \right) \cdot [VR] + [Z] \cdot [IR] \end{gathered} \tag{6.62}$$

Equation 6.62 is of the form

$$[VS] = [a] \cdot [VR] + [b] \cdot [IR] \tag{6.63}$$

where

$$[a]=[U]+\frac{1}{2}\cdot[Z]\cdot[Y]$$
$$[b]=[Z] \tag{6.64}$$

The sending end currents are given by

$$\begin{bmatrix}[IS_1]\\ [IS_2]\end{bmatrix}=\begin{bmatrix}[I_1]\\ [I_2]\end{bmatrix}+\frac{1}{2}\cdot\begin{bmatrix}[Y_{11}] & [Y_{12}]\\ [Y_{21}] & [Y_{22}]\end{bmatrix}\cdot\begin{bmatrix}[VS_1]\\ [VS_2]\end{bmatrix}$$
$$[IS]=[I]+\frac{1}{2}\cdot[Y]\cdot[VS] \tag{6.65}$$

Substitute Equations 6.59 and 6.63 into Equation 6.65 using the shorthand form:

$$[IS]=[IR]+\frac{1}{2}\cdot[Y]\cdot[VR]+\frac{1}{2}\cdot[Y]\cdot([a]\cdot[VR]+[b]\cdot[IR])\ \text{A} \tag{6.66}$$

Combine terms in Equation 6.67:

$$[IS]=\frac{1}{2}\cdot([Y]+[Y]\cdot[a])\cdot[VR]+\left([U]+\frac{1}{2}\cdot[Y]\cdot[b]\right)\cdot[IR] \tag{6.67}$$

Equation 6.67 is of the form

$$[IS]=[c]\cdot[VR]+[d]\cdot[IR] \tag{6.68}$$

where

$$[c]=\frac{1}{2}\cdot([Y]+[Y]\cdot[a])=\left(\frac{1}{2}\cdot\left([Y]+[Y]\cdot\left([U]+\frac{1}{2}\cdot[Z]\cdot[Y]\right)\right)\right)$$
$$[c]=[Y]+\frac{1}{4}\cdot[Y]\cdot[Z]\cdot[Y] \tag{6.69}$$
$$[d]=[U]+\frac{1}{2}\cdot[Y]\cdot[b]=[U]+\frac{1}{2}\cdot[Y]\cdot[Z]$$

The derived matrices $[a]$, $[b]$, $[c]$, $[d]$ will be 6×6 matrices. These four matrices can all be partitioned between the third and fourth rows and columns. The final voltage equation in partitioned form is given by

$$\begin{bmatrix}[VS_1]\\ [VS_2]\end{bmatrix}=\begin{bmatrix}[a_{11}] & [a_{12}]\\ [a_{21}] & [a_{22}]\end{bmatrix}\cdot\begin{bmatrix}[VR_1]\\ [VR_2]\end{bmatrix}+\begin{bmatrix}[b_{11}] & [b_{12}]\\ [b_{21}] & [b_{22}]\end{bmatrix}\cdot\begin{bmatrix}[IR_1]\\ [IR_2]\end{bmatrix} \tag{6.70}$$

The final current equation in partitioned form is given by

$$\begin{bmatrix} [IS_1] \\ [IS_2] \end{bmatrix} = \begin{bmatrix} [c_{11}] & [c_{12}] \\ [c_{21}] & [c_{22}] \end{bmatrix} \cdot \begin{bmatrix} [VR_1] \\ [VR_2] \end{bmatrix} + \begin{bmatrix} [d_{11}] & [d_{12}] \\ [d_{21}] & [d_{22}] \end{bmatrix} \cdot \begin{bmatrix} [IR_1] \\ [IR_2] \end{bmatrix} \tag{6.71}$$

Equations 6.70 and 6.71 are used to compute the sending end voltages and currents of two parallel lines. The matrices [A] and [B] are used to compute the receiving end voltages when the sending end voltages and receiving end currents are known. Solving Equation 6.63 for [VR],

$$[VR] = [a]^{-1} \cdot ([VS] - [b] \cdot [IR])$$

$$[VR] = [a]^{-1} \cdot [VS] - [a]^{-1} \cdot [b] \cdot [IR] \tag{6.72}$$

$$[VR] = [A] \cdot [VS] - [B] \cdot [IR]$$

where

$$[A] = [a]^{-1}$$

$$[B] = [a]^{-1} \cdot [b]$$

In expanded form, Equation 6.72 becomes

$$\begin{bmatrix} VR_1 \\ VR_2 \end{bmatrix} = \begin{bmatrix} [A_{11}] & [A_{12}] \\ [A_{21}] & [A_{22}] \end{bmatrix} \cdot \begin{bmatrix} VS_1 \\ VS_2 \end{bmatrix} - \begin{bmatrix} [B_{11}] & [B_{12}] \\ [B_{21}] & [B_{22}] \end{bmatrix} \cdot \begin{bmatrix} IR_1 \\ IR_2 \end{bmatrix} \tag{6.73}$$

6.5.1 Physically Parallel Lines

Two distribution lines can be physically parallel in two different ways in a radial system. Figure 6.8 illustrates two lines connected to the same sending end node but the receiving ends of the lines do not share a common node.

The physically parallel lines of Figure 6.8 represent the common practice of two feeders leaving a substation on the same poles or right of ways and then branching in different directions downstream. Equations 6.70 and 6.71

FIGURE 6.8
Physically parallel lines with a common sending end node.

are used to compute the sending end node voltages using the known line current flows and node voltages at the receiving end. For this special case, the sending end node voltages must be the same at the end of the two lines so that Equation 6.70 is modified to reflect that $[VS_1] = [VS_2]$. A modified ladder iterative technique is used to force the two sending end voltages to be equal. In Chapter 10, the "ladder" iterative technique will be developed that will be used to adjust the receiving end voltages in such a manner that the sending end voltages will be the same for both lines.

Example 6.6

The parallel lines of Examples 4.2 and 5.2 are connected as shown in Figure 6.8 and are parallel to each other for 10 miles.

1. Determine the *a, b, c, d* and *A, B* matrices for the parallel lines. From Examples 4.2 and 5.2 the per mile values of the phase impedance and shunt admittance matrices in partitioned form are shown. The first step is to multiply these matrices by the length of the line. Note that the units for the shunt admittance matrix in Example 5.2 are in μS/mile:

$$dist = 10$$

$$Z = z \cdot dist$$

$$Y = y \cdot 10^{-6} \cdot dist$$

The unit matrix U must be defined as 6 × 6, and then the *a, b, c, d* matrices are computed using the equations developed in this chapter. The final results in partitioned form are

$$[a_{11}] = [a_{22}] = \begin{bmatrix} 0.9998 + j0.0001 & 0 & 0 \\ 0 & 0.9998 + j0.0001 & 0 \\ 0 & 0 & 0.9998 + j0.0001 \end{bmatrix}$$

$$[a_{12}] = [a_{21}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[b_{11}] = \begin{bmatrix} 4.5015 + j11.0285 & 1.4643 + j5.3341 & 1.4522 + j4.1255 \\ 1.4643 + j5.3341 & 4.5478 + j10.8726 & 1.4754 + j4.5837 \\ 1.4522 + j4.1255 & 1.4754 + j4.5837 & 4.5231 + j10.9556 \end{bmatrix}$$

$$[b_{12}] = \begin{bmatrix} 1.5191 + j4.8484 & 1.4958 + j3.9305 & 1.4775 + j5.5601 \\ 1.5446 + j5.3359 & 1.5205 + j4.3234 & 1.5015 + j4.9093 \\ 1.5311 + j4.2867 & 1.5074 + j5.4599 & 1.4888 + j3.9548 \end{bmatrix}$$

$$[b_{21}] = \begin{bmatrix} 1.5191 + j4.8484 & 1.5446 + j5.3359 & 1.5311 + j4.2867 \\ 1.4958 + j3.9305 & 1.5205 + j4.3234 & 1.5074 + j5.4599 \\ 1.4775 + j5.5601 & 1.5015 + j4.9093 & 1.4888 + j3.9548 \end{bmatrix}$$

$$[b_{22}] = \begin{bmatrix} 5.7063 + j10.9130 & 1.5801 + j4.2365 & 1.5595 + j5.0167 \\ 1.5801 + j4.2365 & 5.6547 + j11.0819 & 1.5348 + j3.8493 \\ 1.5595 + j5.0167 & 1.5348 + j3.8493 & 5.6155 + j11.2117 \end{bmatrix}$$

$$[c_{11}] = [c_{12}] = [c_{21}] = [c_{22}] = [0]$$

$$[d_{11}] = [a_{11}] \quad [d_{12}] = [a_{12}] \quad [d_{21}] = [a_{21}] \quad [d_{22}] = [a_{22}]$$

$$[A_{11}] = [A_{22}] = [a_{11}]^{-1} = \begin{bmatrix} 1.0002 - j0.0001 & 0 & 0 \\ 0 & 1.0002 - j0.0001 & 0 \\ 0 & 0 & 1.0002 - j0.0001 \end{bmatrix}$$

$$[A_{12}] = [A_{21}] = [0]$$

$$[B_{11}] = [a_{11}]^{-1} \cdot [b_{11}] = \begin{bmatrix} 4.5039 + j11.031 & 1.4653 + j5.3357 & 1.4533 + j4.1268 \\ 1.4653 + j5.3357 & 4.5502 + j10.8751 & 1.4764 + j4.5852 \\ 1.4533 + j4.1268 & 1.4764 + j4.5852 & 4.5255 + j10.9580 \end{bmatrix}$$

$$[B_{12}] = [a_{12}]^{1} \cdot [b_{12}] = \begin{bmatrix} 1.5202 + j4.8499 & 1.4969 + j3.9318 & 1.4786 + j5.5618 \\ 1.4969 + j3.9318 & 1.5216 + j4.3248 & 1.5026 + j4.9108 \\ 1.4786 + j5.5618 & 1.5026 + j4.9108 & 1.4899 + j3.9560 \end{bmatrix}$$

$$[B_{21}] = [a_{21}]^{-1} \cdot [b.21] = \begin{bmatrix} 1.5202 + j4.8499 & 1.5457 + j5.3375 & 1.5322 + j4.2881 \\ 1.5457 + j5.3375 & 1.5216 + j4.3248 & 1.5058 + j5.4615 \\ 1.5322 + j4.2881 & 1.5058 + j5.4615 & 1.4899 + j3.9560 \end{bmatrix}$$

$$[B_{22}] = [a_{22}]^{-1} \cdot [b_{22}] = \begin{bmatrix} 5.7092 + j10.9152 & 1.5812 + j4.2378 & 1.5606 + j5.0183 \\ 1.5812 + j4.2378 & 5.6577 + j11.0842 & 1.5360 + j3.8506 \\ 1.5606 + j5.0183 & 1.5360 + j3.8506 & 5.6184 + j11.2140 \end{bmatrix}$$

The loads at the ends of the two lines are treated as constant current loads with values of

$$\text{Line 1: } [IR1] = \begin{bmatrix} 102.6\angle 20.4 \\ 82.1\angle -145.2 \\ 127.8\angle 85.6 \end{bmatrix}$$

$$\text{Line 2: } [IR2] = \begin{bmatrix} 94.4\underline{/-27.4} \\ 127.4\underline{/-152.5} \\ 100.2\underline{/99.8} \end{bmatrix}$$

The voltages at the sending end of the lines are

$$[VS] = \begin{bmatrix} 14{,}400\underline{/0} \\ 14{,}400\underline{/-120} \\ 14{,}400\underline{/120} \end{bmatrix}$$

2. Determine the receiving end voltages for the two lines.
   Since the common sending end voltages are known and the receiving end line currents are known, Equation 6.73 is used to compute the receiving end voltages:

$$\text{Line 1: } [VR1] = ([A_{11}] + [A_{12}]) \cdot [VS] - [B_{11}] \cdot [IR1] - [B_{12}] \cdot [IR2]$$

$$[VR1] = \begin{bmatrix} 14{,}120\underline{/-2.3} \\ 14{,}018\underline{/-120.4} \\ 13{,}692\underline{/117.4} \end{bmatrix}$$

$$\text{Line 2: } [VR2] = ([A_{21}] + [A_{22}]) \cdot [VS] - [B_{21}] \cdot [IR1] - [B_{22}] \cdot [IR2]$$

$$[VR2] = \begin{bmatrix} 13{,}973\underline{/-1.6} \\ 13{,}347\underline{/-120.8} \\ 13{,}568\underline{/118.0} \end{bmatrix}$$

The second way in which two lines can be physically parallel in a radial feeder is to have neither the sending nor the receiving ends common to both lines. This is shown in Figure 6.9.

Equations 6.70 and 6.71 are again used for the analysis in this special case. Since neither the sending end nor the receiving end nodes are common, no adjustments need to be made to Equation 6.70. Typically, these lines will be part of a large distribution feeder in which case an iterative process will be used to arrive at the final values of the sending and receiving end voltages and currents.

FIGURE 6.9
Physically parallel lines without common nodes.

Example 6.7

The parallel lines of Examples 4.2 and 5.2 are connected as shown in Figure 6.8. The lines are parallel to each other for 10 miles.

The complex power flowing out of each line is

$$\text{Line 1: } \begin{aligned} S1_a &= 1450\,\text{kVA}, & PF_a &= 0.95 \\ S1_b &= 1150\,\text{kVA}, & PF_b &= 0.90 \\ S1_c &= 1750\,\text{kVA}, & PF_c &= 0.85 \end{aligned}$$

$$\text{Line 2: } \begin{aligned} S2_a &= 1320\,\text{kVA}, & PF_a &= 0.90 \\ S2_b &= 1700\,\text{kVA}, & PF_b &= 0.85 \\ S2_c &= 1360\,\text{kVA}, & PF_c &= 0.95 \end{aligned}$$

The line-to-neutral voltages at the receiving end nodes are

$$\text{Line 1: } \begin{aligned} VR1_{an} &= 13{,}430\underline{/-33.1} \\ VR1_{bn} &= 13{,}956\underline{/-151.3} \\ VR1_{cn} &= 14{,}071\underline{/86.0} \end{aligned}$$

$$\text{Line 2: } \begin{aligned} VR2_{an} &= 14{,}501\underline{/-29.1} \\ VR2_{bn} &= 13{,}932\underline{/-154.8} \\ VR2_{cn} &= 12{,}988\underline{/90.3} \end{aligned}$$

Determine the sending end voltages of the two lines.

The currents leaving the two lines are

$$\text{Line 1: For } i = a,b,c \quad IR1_i = \left(\frac{S1_i \cdot 1000}{V1_i}\right)^* = \begin{bmatrix} 108.0\underline{/-51.3} \\ 82.4\underline{/-177.1} \\ 124.4\underline{/54.2} \end{bmatrix}$$

$$\text{Line 2: For } i = a,b,c \quad IR2_i = \left(\frac{S2_i \cdot 1000}{V2_i}\right)^* = \begin{bmatrix} 91.0\underline{/-54.9} \\ 122.0\underline{/173.5} \\ 104.7\underline{/72.1} \end{bmatrix}$$

The sending end voltages of the two lines are computed using Equation 6.70:

$$\text{Line 1: } [VS1] = \begin{bmatrix} 13{,}671\underline{/-30.5} \\ 14{,}363\underline{/-151.1} \\ 14{,}809\underline{/88.7} \end{bmatrix}$$

$$\text{Line 2: } [VS2] = \begin{bmatrix} 14{,}844\underline{/-27.5} \\ 14{,}974\underline{/-154.3} \\ 13{,}898\underline{/92.6} \end{bmatrix}$$

The sending end currents are

$$[IS1] = \begin{bmatrix} 107.7\underline{/-50.7} \\ 82.0\underline{/-176.3} \\ 124.0\underline{/54.7} \end{bmatrix}$$

$$[IS2] = \begin{bmatrix} 90.5\underline{/-54.3} \\ 121.4\underline{/173.8} \\ 104.2\underline{/72.6} \end{bmatrix}$$

Note in this example the very slight difference between the sending and receiving end currents. The very small difference is due to the shunt admittance. It is seen that very little error will be made if the shunt admittance of the two lines is ignored. This will be the usual case. Exceptions will be for very long distribution lines (50 miles or more) and for underground concentric neutral lines that are parallel for 10 miles or more.

A third option for physically parallel lines in a radial feeder might be considered with the receiving end nodes common to both lines and the sending end nodes not common. However, this would violate the "radial" nature of the feeder since the common receiving end nodes would constitute the creation of a loop.

6.5.2 Electrically Parallel Lines

Figure 6.10 shows two distribution lines that are electrically parallel.

The analysis of the electrically parallel lines requires an extra step from that of the physically parallel lines since the individual line currents are not known. In this case only the total current leaving the parallel lines is known.

In the typical analysis, the receiving end voltages will have been either assumed or computed and the total phase currents [*IR*] will be known. With [*VS*] and [*VR*] common to both lines, the first step must be to determine how much of the total current [*IR*] flows on each line. Since the lines are electrically parallel, Equation 6.70 can be modified to reflect this condition:

$$\begin{bmatrix} [VS] \\ [VS] \end{bmatrix} = \begin{bmatrix} [a_{11}] & [a_{12}] \\ [a_{21}] & [a_{22}] \end{bmatrix} \cdot \begin{bmatrix} [VR] \\ [VR] \end{bmatrix} + \begin{bmatrix} [b_{11}] & [b_{12}] \\ [b_{21}] & [b_{22}] \end{bmatrix} \cdot \begin{bmatrix} [IR1] \\ [IR2] \end{bmatrix} \tag{6.74}$$

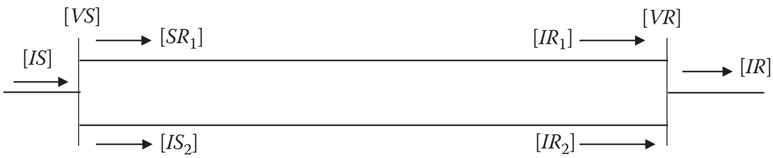

FIGURE 6.10
Electrically parallel lines.

The current in line 2 as a function of the total current and the current in line 1 is given by

$$[IR2] = [IR] - [IR1] \tag{6.75}$$

Substitute Equation 6.75 into Equation 6.74:

$$\begin{bmatrix} [VS] \\ [VS] \end{bmatrix} = \begin{bmatrix} [a_{11}] & [a_{12}] \\ [a_{21}] & [a_{22}] \end{bmatrix} \cdot \begin{bmatrix} [VR] \\ [VR] \end{bmatrix} + \begin{bmatrix} [b_{11}] & [b_{12}] \\ [b_{21}] & [b_{22}] \end{bmatrix} \cdot \begin{bmatrix} [IR1] \\ [IR] - [IR1] \end{bmatrix} \tag{6.76}$$

Since the sending end voltages are equal, Equation 6.76 is modified to reflect this:

$$\begin{aligned} &([a_{11}] + [a_{12}]) \cdot [VR] + ([b_{11}] - [b_{12}]) \cdot [IR1] + [b_{12}] \cdot [IR] \\ &\quad = ([a_{21}] + [a_{22}]) \cdot [VR] + ([b_{21}] - [b_{22}]) \cdot [IR1] + [b_{22}] \cdot [IR] \end{aligned} \tag{6.77}$$

Collect terms in Equation 6.77:

$$\begin{aligned} &([a_{11}] + [a_{12}] - [a_{21}] - [a_{22}]) \cdot [VR] + ([b_{12}] - [b_{22}]) \cdot [IR] \\ &\quad = ([b_{21}] - [b_{22}] - [b_{11}] + [b_{12}]) \cdot [IR1] \end{aligned} \tag{6.78}$$

Equation 6.78 is in the form of

$$[Aa] \cdot [VR] + [Bb] \cdot [IR] = [Cb] \cdot [IR1] \tag{6.79}$$

where

$$\begin{aligned} [Aa] &= [a_{11}] + [a_{12}] - [a_{21}] - [a_{22}] \\ [Bb] &= [b_{12}] - [b_{22}] \\ [Cc] &= [b_{21}] - [b_{22}] - [b_{11}] + [b_{12}] \end{aligned} \tag{6.80}$$

Equation 6.79 can be solved for the receiving end current in line 1:

$$[IR1] = [Cc]^{-1} \cdot ([Aa] \cdot [VR] + [Bb] \cdot [IR]) \tag{6.81}$$

Equation 6.75 can be used to compute the receiving end current in line 2.

With the two receiving end line currents known, Equations 6.70 and 6.71 are used to compute the sending end voltages. As with the physically parallel lines, an iterative process (Chapter 10) will have to be used to assure the sending end voltages for each line are equal.

Example 6.8

The two lines of Example 6.5 are electrically parallel as shown in Figure 6.9. The receiving end voltages are given by

$$[VR] = \begin{bmatrix} 13{,}280/\underline{-33.1} \\ 14{,}040/\underline{151.7} \\ 14{,}147/\underline{86.5} \end{bmatrix}$$

The complex power out of the parallel lines is the sum of the complex power of the two lines in Example 6.6:

$$S_a = 2763.8\,\text{kVA} \quad \text{at } 0.928\,\text{PF}$$

$$S_b = 2846.3\,\text{kVA} \quad \text{at } 0.872\,\text{PF}$$

$$S_c = 3088.5\,\text{kVA} \quad \text{at } 0.90\,\text{PF}$$

The first step in the solution is to determine the total current leaving the two lines:

$$IR_i = \left(\frac{S_i \cdot 1000}{VR_i}\right)^* = \begin{bmatrix} 208.1/\underline{-54.9} \\ 202.7/\underline{179.0} \\ 218.8/\underline{50.2} \end{bmatrix}$$

Equation 6.81 is used to compute the current in line 1. Before that can be done the matrices of Equation 6.80 must be computed:

$$[Aa] = [a_{11}] + [a_{12}] - [a_{21}] - [a_{22}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[Bb] = [b_{12}] - [b_{22}] = \begin{bmatrix} -4.1872 - j6.0646 & -0.0843 - j0.3060 & -0.0820 + j0.5434 \\ -0.0354 + j1.0995 & -4.1342 - j6.7585 & -0.0333 + j1.0599 \\ -0.0284 - j0.7300 & -0.0274 + j1.6105 & -4.1267 - j7.2569 \end{bmatrix}$$

$$[Cc] = [b_{21}] - [b_{22}] - [b_{11}] + [b_{12}]$$

$$= \begin{bmatrix} -7.1697 - j12.2446 & -0.0039 - j0.3041 & -0.0032 + j0.7046 \\ -0.0039 - j0.3041 & -7.1616 - j13.3077 & -0.0013 + j1.9361 \\ -0.0032 + j0.7046 & -0.0013 + j1.9361 & -7.1610 - j14.2577 \end{bmatrix}$$

The current in line 1 is now computed by

$$[IR1] = [Cc]^{-1} \cdot \left([Aa] \cdot [VR] + [Bb] \cdot [IR]\right) = \begin{bmatrix} 110.4/\underline{-59.7} \\ 119.3/\underline{172.6} \\ 121.8/\underline{50.2} \end{bmatrix}$$

The current in line 2 is

$$[IR2]=[IR]-[IR1]=\begin{bmatrix} 98.5/\underline{-49.6} \\ 85.2/\underline{-172.2} \\ 101.2/\underline{73.3} \end{bmatrix}$$

The sending end voltages are

$$[VS1]=([a_{11}]+[a_{12}])\cdot[VR]+[b_{11}]\cdot[IR1]+[b_{12}]\cdot[IR2]=\begin{bmatrix} 13,739/\underline{-30.9} \\ 14,629/\underline{-151.0} \\ 14,903/\underline{88.3} \end{bmatrix}$$

$$[VS2]=([a_{21}]+[a_{22}])\cdot[VR]+[b_{21}]\cdot[IR1]+[b_{22}]\cdot[IR2]=\begin{bmatrix} 13,739/\underline{-30.9} \\ 14,629/\underline{-151.0} \\ 14,903/\underline{88.3} \end{bmatrix}$$

It is satisfying that the two equations give us the same results for the sending end voltages.

The sending end currents are

$$[IS1]=([c_{11}]+[c_{12}])\cdot[VR]+[d_{11}]\cdot[IR1]+[d_{12}]\cdot[IR2]=\begin{bmatrix} 110.0/\underline{-59.2} \\ 118.7/\underline{173.1} \\ 121.3/\underline{50.6} \end{bmatrix}$$

$$[IS2]=([c_{21}]+[c_{22}])\cdot[VR]+[d_{21}]\cdot[IR1]+[d_{22}]\cdot[IR2]=\begin{bmatrix} 98.2/\underline{-49.0} \\ 84.7/\underline{-171.7} \\ 100.7/\underline{73.9} \end{bmatrix}$$

When the shunt admittance of the parallel lines is ignored, a parallel equivalent 3 × 3 phase impedance matrix can be determined. Since very little error is made ignoring the shunt admittance on most distribution lines, the equivalent parallel phase impedance matrix can be very useful in distribution power flow programs that are not designed to model electrically parallel lines.

Since the lines are electrically parallel, the voltage drops in the two lines must be equal. The voltage drop in the two parallel lines is given by

$$\begin{bmatrix} [v_{abc}] \\ [v_{abc}] \end{bmatrix}=\begin{bmatrix} [Z_{11}] & [Z_{12}] \\ [Z_{21}] & [Z_{22}] \end{bmatrix}\cdot\begin{bmatrix} IR1 \\ IR2 \end{bmatrix} \tag{6.82}$$

Substitute Equation 6.75 into Equation 6.82:

$$\begin{bmatrix} [v_{abc}] \\ [v_{abc}] \end{bmatrix}=\begin{bmatrix} [Z_{11}] & [Z_{12}] \\ [Z_{21}] & [Z_{22}] \end{bmatrix}\cdot\begin{bmatrix} [IR1] \\ [IR]-[IR1] \end{bmatrix} \tag{6.83}$$

Expand Equation 6.83 to solve for the voltage drops:

$$[v_{abc}] = [Z_{11}] \cdot [IR1] + [Z_{12}] \cdot ([IR] - [IR1]) = [Z_{21}] \cdot [IR1] + [Z_{22}] \cdot ([IR] - [IR1])$$
$$[v_{abc}] = ([Z_{11}] - [Z_{12}]) \cdot [IR1] + [Z_{12}] \cdot [IR] = ([Z_{21}] - [Z_{22}]) \cdot [IR1] + [Z_{22}] \cdot [IR] \tag{6.84}$$

Collect terms in Equation 6.84:

$$([Z_{11}] - [Z_{12}] - [Z_{21}] + [Z_{22}]) \cdot [IR1] = ([Z_{22}] - [Z_{12}]) \cdot [IR] \tag{6.85}$$

Let

$$[ZX] = ([Z_{11}] - [Z_{12}] - [Z_{21}] + [Z_{22}]) \tag{6.86}$$

Substitute Equation 6.86 into Equation 6.85 and solve for the current in line 1:

$$[IR1] = [ZX]^{-1} \cdot ([Z_{22}] - [Z_{12}]) \cdot [IR] \tag{6.87}$$

Substitute Equation 6.87 into the top line of Equation 6.83:

$$[v_{abc}] = \left(([Z_{11} - Z_{12}]) \cdot [ZX]^{-1} \cdot ([Z_{22}] - [Z_{12}]) + [Z_{12}]\right) \cdot [IR] \tag{6.88}$$

$$[v_{abc}] = [Z_{eq}] \cdot [IR]$$
$$\text{where } [Z_{eq}] = \left(([Z_{11} - Z_{12}]) \cdot [ZX]^{-1} \cdot ([Z_{22}] - [Z_{12}]) + [Z_{12}]\right) \tag{6.89}$$

The equivalent impedance of Equation 6.89 is the 3 × 3 equivalent for the two lines that are electrically parallel. This is the phase impedance matrix that can be used in conventional distribution power flow programs that cannot model electrically parallel lines.

Example 6.9

The same two lines are electrically parallel but the shunt admittance is neglected. Compute the equivalent 3 × 3 impedance matrix using the impedance partitioned matrices of Example 6.5:

$$[ZX] = [Z_{11}] - [Z_{12}] - [Z_{21}] + [Z_{22}]$$
$$= \begin{bmatrix} 7.1697 + j12.2446 & 0.0039 + j0.3041 & 0.0032 - j0.7046 \\ 0.0039 + j0.3041 & 7.1616 + j13.3077 & 0.0013 - j1.9361 \\ 0.0032 - j0.7046 & 0.0013 - j1.9361 & 7.1610 + j14.2577 \end{bmatrix}$$

$$[Z_{eq}] = ([Z_{11}] - [Z_{12}]) \cdot [ZX]^{-1} \cdot ([Z_{22}] - [Z_{12}]) + [Z_{12}]$$

$$= \begin{bmatrix} 3.3677 + j7.796 & 1.5330 + j4.7717 & 1.4867 + j4.7304 \\ 1.5330 + j4.7717 & 3.3095 + j7.6459 & 1.5204 + j4.7216 \\ 1.4867 + j4.7304 & 1.5204 - j4.7216 & 3.2662 + j7.5316 \end{bmatrix}$$

The sending end voltages are

$$[VS] = [VR] + [Z_{eq}] \cdot [IR] = \begin{bmatrix} 13{,}739/\underline{-31.0} \\ 14{,}634/\underline{-151.0} \\ 14{,}907/\underline{88.3} \end{bmatrix}$$

6.6 Summary

This chapter has developed the "exact," "modified," and "approximate" line segment models. The exact model uses no approximations. That is, the phase impedance matrix assuming no transposition is used as well as the shunt admittance matrix. The modified model ignores the shunt admittance. The approximate line model ignores the shunt admittance and assumes that the positive and zero sequence impedances of the line are the known parameters. This is paramount to assuming the line is transposed. For the three line models generalized matrix equations have been developed. The equations utilize the generalized matrices [*a*], [*b*], [*c*], [*d*], [*A*], and [*B*]. The example problems demonstrate that because the shunt admittance is very small the generalized matrices can be computed neglecting the shunt admittance with very little, if any, error. In most cases the shunt admittance can be neglected; however, there are situations where the shunt admittances should not be neglected. This is particularly true for long rural lightly loaded lines and for many underground lines.

A method for computing the current flowing in the neutral and ground was developed. The only assumption used that can make a difference in the computing currents is that the resistivity of earth was assumed to be 100 Ω-m.

A simple version of the ladder iterative technique was introduced and applied in Example 6.5. The ladder method is used in future chapters and is fully developed in Chapter 10.

The generalized matrices for two lines in parallel have been derived. The analysis of physically parallel and electrically parallel lines was developed with examples to demonstrate the analysis process.

Problems

6.1 A 2 mile long three-phase line uses the configuration of Problem 4.1. The phase impedance matrix and shunt admittance matrix for the configuration are

$$[z_{abc}] = \begin{bmatrix} 0.3375 + j1.0478 & 0.1535 + j0.3849 & 0.1559 + j0.5017 \\ 0.1535 + j0.3849 & 0.3414 + j1.0348 & 0.1580 + j0.4236 \\ 0.1559 + j0.5017 & 0.1580 + j0.4236 & 0.3465 + j1.0179 \end{bmatrix} \Omega/\text{mile}$$

$$[y_{abc}] = \begin{bmatrix} j5.9540 & -j0.7471 & -j2.0030 \\ -j0.7471 & j5.6322 & -j1.2641 \\ -j2.0030 & -j1.2641 & j6.3962 \end{bmatrix} \mu\text{S/mile}$$

The line is serving a balanced three-phase load of 10,000 kVA with balanced voltages of 13.2 kV line to line and a power factor of 0.85 lagging.

a. Determine the generalized matrices.

b. For the given load, compute the line-to-line and line-to-neutral voltages at the source end of the line.

c. Compute the voltage unbalance at the source end.

d. Compute the source end complex power per phase.

e. Compute the power loss by phase over the line. (Hint: Power loss is defined as power in minus power out.)

6.2 Use the line of Problem 6.1. For this problem, the source voltages are specified as

$$[VS_{LN}] = \begin{bmatrix} 7620\underline{/0} \\ 7620\underline{/-120} \\ 7620\underline{/120} \end{bmatrix}$$

The three-phase load is unbalanced connected in wye and given by

$$[kVA] = \begin{bmatrix} 2500 \\ 3500 \\ 1500 \end{bmatrix} \quad [PF] = \begin{bmatrix} 0.90 \\ 0.85 \\ 0.95 \end{bmatrix}$$

Use the ladder iterative technique and determine

a. The load line-to-neutral voltages

b. The load currents

c. The complex power at the source

d. The voltage unbalance at the load

6.3 Use Windmil for Problem 6.2.

6.4 The positive and zero sequence impedances for the line of Problem 6.1 are

$$z_+ = 0.186 + j0.5968\ \Omega/\text{mile}, \quad z_0 = 0.6534 + j1.907\ \Omega/\text{mile}$$

Repeat problem 6.1 using the "approximate" line model.

6.5 The line of Problem 6.1 serves an unbalanced grounded wye connected constant impedance load of

$$Z_{ag} = 15\underline{/30}\ \Omega, \quad Z_{bg} = 17\underline{/36.87}\ \Omega, \quad Z_{cg} = 20\underline{/25.84}\ \Omega$$

The line is connected to a balanced three-phase 13.2 kV source. Determine

a. The load currents
b. The load line-to-ground voltages
c. The complex power of the load by phase
d. The source complex power by phase
e. The power loss by phase and the total three-phase power loss
f. The current flowing in the neutral and ground

6.6 Repeat Problem 6.3 with only the load on phase *b* changed to $50\underline{/36.87}\ \Omega$.

6.7 The two-phase line of Problem 4.2 has the following phase impedance matrix:

$$[z_{abc}] = \begin{bmatrix} 0.4576 + j1.0780 & 0 & 0.1535 + j0.3849 \\ 0 & 0 & 0 \\ 0.1535 + j0.3849 & 0 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

The line is 2 miles long and serves a two-phase load such that

S_{ag} = 2000 kVA at 0.9 lagging power factor and voltage of $7620\underline{/0}$ V
S_{cg} = 1500 kVA at 0.95 lagging power factor and voltage of $7620\underline{/120}$ V

Neglect the shunt admittance and determine the following:

a. The source line-to-ground voltages using the generalized matrices (Hint: Even though phase *b* is physically not present, assume that it is with a value of $7620\underline{/-120}$ V and is serving a 0 kVA load.)
b. The complex power by phase at the source
c. The power loss by phase on the line
d. The current flowing in the neutral and ground

6.8 The single-phase line of Problem 4.3 has the following phase impedance matrix:

$$[z_{abc}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.3292 + j1.3475 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Omega/\text{mile}$$

The line is 1 mile long and is serving a single-phase load of 2000 kVA, 0.95 lagging power factor at a voltage of $7500\underline{/-120}$ V. Determine the source voltage and power loss on the line. (Hint: As in the previous problem, even though phases *a* and *c* are not physically present, assume they are and along with phase *b* make up a balanced three-phase set of voltages.)

6.9 The three-phase concentric neutral cable configuration of Problem 4.10 is 2 miles long and serves a balanced three-phase load of 10,000 kVA, 13.2 kV, 0.85 lagging power factor. The phase impedance and shunt admittance matrices for the cable line are

$$[z_{abc}] = \begin{bmatrix} 0.7891 + j0.4041 & 0.3192 + j0.0328 & 0.3192 + j0.0328 \\ 0.3192 + j0.0328 & 0.7982 + j0.4463 & 0.2849 - j0.0143 \\ 0.3192 + j0.0328 & 0.2849 - j0.0143 & 0.8040 + j0.4381 \end{bmatrix} \Omega/\text{mile}$$

$$[y_{abc}] = \begin{bmatrix} j96.61 & 0 & 0 \\ 0 & j96.61 & 0 \\ 0 & 0 & j96.61 \end{bmatrix} \mu\text{S/mile}$$

a. Determine the generalized matrices.

b. For the given load, compute the line-to-line and line-to-neutral voltages at the source end of the line.

c. Compute the voltage unbalance at the source end.

d. Compute the source end complex power per phase.

e. Compute the power loss by phase over the line. (Hint: Power loss is defined as power in minus power out.)

6.10 The line of Problem 6.9 serves an unbalanced grounded wye connected constant impedance load of

$$Z_{ag} = 15\underline{/30}\,\Omega, \quad Z_{bg} = 50\underline{/36.87}\,\Omega, \quad Z_{cg} = 20\underline{/25.84}\,\Omega$$

The line is connected to a balanced three-phase 13.2 kV source. Determine

a. The load currents

b. The load line-to-ground voltages

c. The complex power of the load by phase

d. The source complex power by phase
e. The power loss by phase and the total three-phase power loss
f. The current flowing in each neutral and ground

6.11 The tape-shielded cable single-phase line of Problem 4.12 is 2 miles long and serves a single-phase load of 3000 kVA, at 8.0 kV and 0.9 lagging power factor. The phase impedance and shunt admittances for the line are

$$[z_{abc}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.5291 + j0.5685 \end{bmatrix} \Omega/\text{mile}$$

$$[y_{abc}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & j140.39 \end{bmatrix} \mu\text{S/mile}$$

Determine the source voltage and the power loss for the loading condition.

6.12 Two distribution lines constructed on one pole are shown in Figure 6.11.

Line #1 data:
Conductors: 336,400 26/7 ACSR
GMR = 0.0244 ft, *Resistance* = 0.306 Ω/mile, *Diameter* = 0.721 in.

Line #2 data:
Conductors: 250,000 AA
GMR = 0.0171 ft, *Resistance* = 0.41 Ω/mile, *Diameter* = 0.574 in.

Neutral conductor data:
Conductor: 4/0 6/1 ACSR
GMR = 0.00814 ft, *Resistance* = 0.592 Ω/mile, *Diameter* = 0.563 in.
Length of lines is 10 miles.

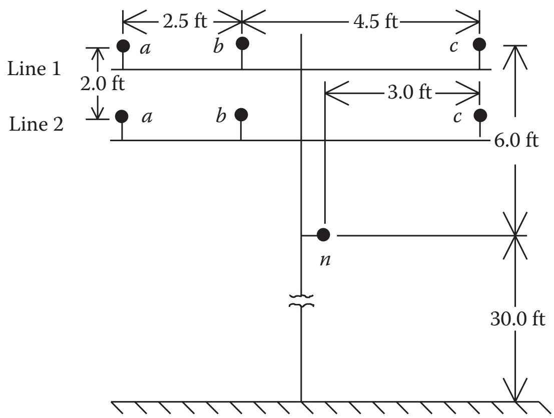

FIGURE 6.11
Two parallel lines on one pole.

Balanced load voltages of 24.9 kV line to line

Unbalanced loading:

Load #1: Phase *a*: 1440 kVA at 0.95 lagging power factor
Phase *b*: 1150 kVA at 0.9 lagging power factor
Phase *c*: 1720 kVA at 0.85 lagging power factor

Load #2: Phase *a*: 1300 kVA at 0.9 lagging power factor
Phase *b*: 1720 kVA at 0.85 lagging power factor
Phase *c*: 1370 kVA at 0.95 lagging power factor

The two lines have a common sending end node (Figure 6.6). Determine

a. The total phase impedance matrix (6 × 6)
b. The *a*, *b*, *c*, *d* and *A*, *B* matrices
c. The sending end node voltages and currents for each line for the specified loads
d. The sending end complex power for each line
e. The real power loss of each line
f. The current flowing in the neutral conductor and ground

6.13 The lines of Problem 6.12 do not share a common sending or receiving end node (Figure 6.7). Determine

a. The sending end node voltages and currents for each line for the specified loads
b. The sending end complex power for each line
c. The real power loss of each line

6.14 The lines of Problem 6.12 are electrically parallel (Figure 6.8).

Compute the equivalent 3 × 3 impedance matrix and determine

a. The sending end node voltages and currents for each line for the specified loads
b. The sending end complex power for each line
c. The real power loss of each line

Windmil Assignment

Use System 2 and add a two-phase concentric neutral cable line connected to node 2. Call this "System 3." The line uses phases *a* and *c* and is 300 ft long and consists of two 1/0 AA 1/3 neutral concentric neutral cables. The cables are 40 in. below ground and 6 in. apart. There is no additional neutral conductor. Call this line UG-1. At the end of UG-1, connect a node and call it node 4. The load at node 4 is delta-connected load modeled as constant current. The load is 250 kW at 95% lagging power factor.

Determine the voltages at all nodes on a 120 Vt base and all line currents.

References

1. Glover, J.D. and Sarma, M., *Power System Analysis and Design*, 2nd edn., PWS Publishing Co., Boston, MA, 1995.
2. ANSI/NEMA Standard Publication No. MG1-1978, National Electrical Manufactures Association, Washington, DC.

7

Voltage Regulation

The regulation of voltages is an important function on a distribution feeder. As the loads on the feeders vary, there must be some means of regulating the voltage so that every customer's voltage remains within an acceptable level. Common methods of regulating the voltage are the application of step-type voltage regulators, load tap changing (LTC) transformers, and shunt capacitors.

7.1 Standard Voltage Ratings

The American National Standards Institute (ANSI) standard ANSI C84.1-1995 for "Electric Power Systems and Equipment Voltage Ratings (60 Hertz)" provides the following definitions for system voltage terms [1]:

- *System voltage*: The root mean square (rms) phasor voltage of a portion of an alternating current electric system. Each system voltage pertains to a portion of the system that is bounded by transformers or utilization equipment.
- *Nominal system voltage*: The voltage by which a portion of the system is designated and to which certain operating characteristics of the system are related. Each nominal system voltage pertains to a portion of the system bounded by transformers or utilization equipment.
- *Maximum system voltage*: The highest system voltage that occurs under normal operating conditions, and the highest system voltage for which equipment and other components are designed for satisfactory continuous operation without derating of any kind.
- *Service voltage*: The voltage at the point where the electrical system of the supplier and the electrical system of the user are connected.
- *Utilization voltage*: The voltage at the line terminals of utilization equipment.
- *Nominal utilization voltage*: The voltage rating of certain utilization equipment used on the system.

The ANSI standard specifies two voltage ranges. An over simplification of the voltage ranges is

- *Range A*: Electric supply systems shall be so designated and operated such that most service voltages will be within the limits specified for range A. The occurrence of voltages outside of these limits should be infrequent.
- *Range B*: Voltages above and below range A. When these voltages occur, corrective measures shall be undertaken within a reasonable time to improve voltages to meet range A.

For a normal three-wire 120/240 V service to a user, the range A and range B voltages are

- Range A
  - Nominal utilization voltage = 115 V
  - Maximum utilization and service voltage = 126 V
  - Minimum service voltage = 114 V
  - Minimum utilization voltage = 110 V
- Range B
  - Nominal utilization voltage = 115 V
  - Maximum utilization and service voltage = 127 V
  - Minimum service voltage = 110 V
  - Minimum utilization voltage = 107 V

These ANSI standards give the distribution engineer a range of "normal steady-state" voltages (range A) and a range of "emergency steady-state" voltages (range B) that must be supplied to all users.

In addition to the acceptable voltage magnitude ranges, the ANSI standard recommends that the "electric supply systems should be designed and operated to limit the maximum voltage unbalance to 3 percent when measured at the electric-utility revenue meter under a no-load condition." Voltage unbalance is defined as

$$Voltage_{unbalance} = \frac{Max.\ deviation\ from\ average\ voltage}{Average\ voltage} \cdot 100\% \tag{7.1}$$

The task for the distribution engineer is to design and operate the distribution system so that under normal steady-state conditions the voltages at the meters of all users will lie within Range *A* and that the voltage unbalance will not exceed 3%.

A common device used to maintain system voltages is the step-voltage regulator. Step-voltage regulators can be single phase or three phase. Single-phase regulators can be connected in wye, delta, or open delta, in addition to operating as a single-phase device. The regulators and their controls allow the voltage output to vary as the load varies.

A step-voltage regulator is basically an autotransformer with a LTC mechanism on the "series" winding. The voltage change is obtained by changing the number of turns (tap changes) of the series winding of the autotransformer.

An autotransformer can be visualized as a two-winding transformer with a solid connection between a terminal on the primary side of the transformer and a terminal on the secondary. Before proceeding to the autotransformer, a review of two-transformer theory and the development of generalized constants will be presented.

7.2 Two-Winding Transformer Theory

The exact equivalent circuit for a two-winding transformer is shown in Figure 7.1. In Figure 7.1, the high-voltage transformer terminals are denoted by H_1 and H_2, and the low-voltage terminals are denoted by X_1 and X_2. The standards for these markings are such that at no load the voltage between H_1 and H_2 will be in phase with the voltage between X_1 and X_2. Under a steady-state load condition, the currents I_1 and I_2 will be in phase.

Without introducing a significant error, the exact equivalent circuit of Figure 7.1 is modified by referring the primary impedance (Z_1) to the secondary side as shown in Figure 7.2.

Referring to Figure 7.2, the total "leakage" impedance of the transformer is given by

$$Z_t = n_t^2 \cdot Z_1 + Z_2 \tag{7.2}$$

where

$$n_t = \frac{N_2}{N_1} \tag{7.3}$$

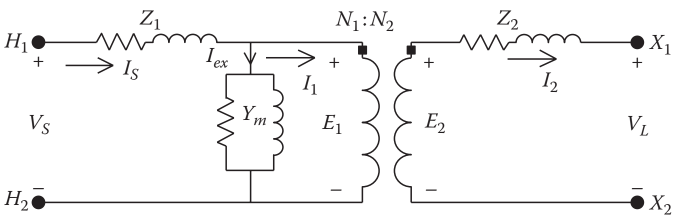

FIGURE 7.1
Two-winding transformer: exact equivalent circuit.

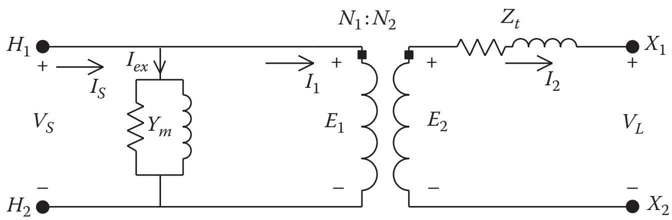

FIGURE 7.2
Two-winding transformer: approximate equivalent circuit.

In order to better understand the model for the step-regulator, a model for the two-winding transformer will first be developed. Referring to Figure 7.2, the equations for the ideal transformer become

$$E_2 = \frac{N_2}{N_1} \cdot E_1 = n_t \cdot E_1 \tag{7.4}$$

$$I_1 = \frac{N_2}{N_1} \cdot I_2 = n_t \cdot I_2 \tag{7.5}$$

Applying Kirchhoff's voltage law (KVL) in the secondary circuit,

$$\begin{aligned} E_2 &= V_L + Z_t \cdot I_2 \\ V_S &= E_1 = \frac{1}{n_t} \cdot E_2 = \frac{1}{n_t} \cdot V_L + \frac{Z_t}{n_t} \cdot I_2 \end{aligned} \tag{7.6}$$

In general form, Equation 7.6 can be written as

$$V_S = a \cdot V_L + b \cdot I_2 \tag{7.7}$$

where

$$a = \frac{1}{n_t} \tag{7.8}$$

$$b = \frac{Z_t}{n_t} \tag{7.9}$$

The input current to the two-winding transformer is given by

$$I_S = Y_m \cdot V_S + I_1 \tag{7.10}$$

Substitute Equations 7.6 and 7.5 into Equation 7.10:

$$I_S = Y_m \cdot \frac{1}{n_t} \cdot V_L + Y_m \cdot \frac{Z_t}{n_t} \cdot I_2 + n_t \cdot I_2$$
$$I_S = \frac{Y_m}{n_t} \cdot V_L + \left(\frac{Y_m \cdot Z_t}{n_t} + n_t \right) \cdot I_2 \tag{7.11}$$

In general form, Equation 7.11 can be written as

$$I_S = c \cdot V_L + d \cdot I_2 \tag{7.12}$$

where

$$c = \frac{Y_m}{n_t} \tag{7.13}$$

$$d = \frac{Y_m \cdot Z_t}{n_t} + n_t \tag{7.14}$$

Equations 7.7 and 7.12 are used to compute the input voltage and current to a two-winding transformer when the load voltage and current are known. These two equations are of the same form as Equations 6.8 and 6.16 for the three-phase line models. The only difference at this point is that only a single-phase two-winding transformer is being modeled. Later, in this chapter, the terms a, b, c, and d are expanded to 3 × 3 matrices for all possible three-phase regulator connections.

Sometimes, particularly in the ladder iterative process, the output voltage needs to be computed knowing the input voltage and the load current. Solving Equation 7.7 for the load voltage yields

$$V_L = \frac{1}{a} \cdot V_S - \frac{b}{a} \cdot Z_t \tag{7.15}$$

Substituting Equations 7.8 and 7.9 into Equation 7.15 results in

$$V_L = A \cdot V_S - B \cdot I_2 \tag{7.16}$$

where

$$A = n_t \tag{7.17}$$

$$B = Z_t \tag{7.18}$$

Again, Equation 7.16 is of the same form as Equation 6.26. Later, in this chapter, the expressions for A and B is expanded to 3 × 3 matrices for all possible three-phase transformer connections.

Example 7.1

A single-phase transformer is rated 75 kVA, 2400–240 V. The transformer has the following impedances and shunt admittance:

$Z_1 = 0.612 + j1.2\,\Omega$ (high-voltage winding impedance)
$Z_2 = 0.0061 + j0.0115\,\Omega$ (low-voltage winding impedance)
$Y_m = 1.92 \times 10^{-4} - j8.52 \times 10^{-4}$ S (referred to the high-voltage winding)

Determine the generalized a, b, c, and d constants and the A and B constants.

The transformer "turns ratio" is

$$n_t = \frac{N_2}{N_1} = \frac{V_{rated\_2}}{V_{rated\_1}} = \frac{240}{2400} = 0.1$$

The equivalent transformer impedance referred to the low-voltage side:

$$Z_t = Z_2 + n_t^2 \cdot Z_1 = 0.0122 + j0.0235$$

The generalized constants are

$$a = \frac{1}{n_t} = \frac{1}{0.1} = 10$$

$$b = \frac{Z_t}{0.1} = 0.1222 + j0.235$$

$$c = \frac{Y_m}{n_t} = 0.0019 - j0.0085$$

$$d = \frac{Y_m \cdot Z_t}{n_t} + n_t = 0.1002 - j0.0001$$

$$A = n_t = 0.1$$

$$B = Z_t = 0.0122 + j0.0235$$

Assume that the transformer is operated at rated load (75 kVA) and rated voltage (240 V) with a power factor of 0.9 lagging. Determine the source voltage and current using the generalized constants.

$$V_L = 240\underline{/0}$$

$$I_2 = \frac{75 \cdot 1000}{240}\underline{/-\cos^{-1}(0.9)} = 312.5\underline{/-25.84}$$

Applying the values of the *a*, *b*, *c*, and *d* parameters computed earlier:

$$V_S = a \cdot V_L + b \cdot I_2 = 2466.9\underline{/1.15}\,\text{V}$$

$$I_S = c \cdot V_L + d \cdot I_2 = 32.67\underline{/-28.75}\,\text{A}$$

Using the computed source voltage and the load current, determine the load voltage:

$$V_L = A \cdot V_S - B \cdot I_S = (0.1) \cdot (2466.9\underline{/1.15}) - (0.0122 + j0.0235) \cdot (312.5\underline{/-25.84})$$

$$V_L = 240.0\underline{/0}\,\text{V}$$

For future reference, the per-unit impedance of the transformer is computed by

$$Z_{base} = \frac{kV_{rated\_2}^2 \cdot 1000}{kVA} = \frac{0.240^2 \cdot 1000}{75} = 0.768\,\Omega$$

$$Z_{pu} = \frac{Z_t}{Z_{base}} = \frac{0.0122 + j0.0115}{0.768} = 0.0345\underline{/62.5}\,\text{per unit}$$

The per-unit shunt admittance is computed by

$$Y_{base} = \frac{kVA}{kV_1^2 \cdot 1000} = \frac{75}{2.4^2 \cdot 1000} = 0.013\,\text{S}$$

$$Y_{pu} = \frac{Y_m}{Y_{base}} = \frac{1.92 \cdot 10^{-4} - j8.52 \cdot 10^{-4}}{0.013} = 0.0147 - j0.0654\,\text{per unit}$$

Example 7.1 demonstrates that the generalized constants provide a quick method for analyzing the operating characteristics of a two-winding transformer.

7.3 Two-Winding Autotransformer

A two-winding transformer can be connected as an autotransformer. Connecting the high-voltage terminal H_1 to the low-voltage terminal X_2 as shown in Figure 7.3 can create a "step-up" autotransformer. The source is connected to terminals H_1 and H_2, while the load is connected between the X_1 terminal and the extension of H_2.

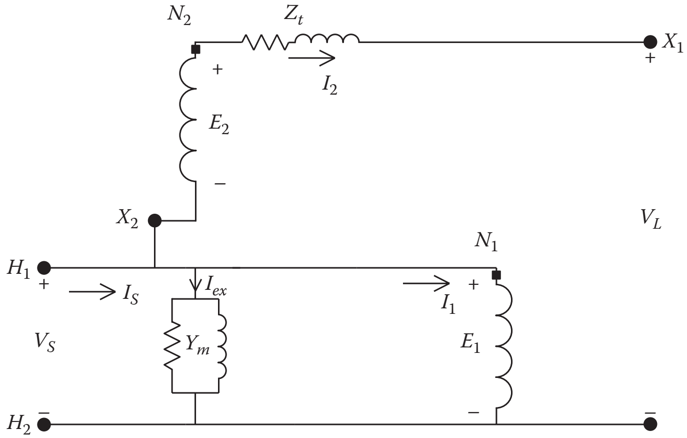

FIGURE 7.3
Step-up autotransformer.

In Figure 7.3, V_S is the "source" voltage and V_L is the "load" voltage. The low-voltage winding of the two-winding transformer will be referred to as the "series" winding of the autotransformer, and the high-voltage winding of the two-winding transformer will be referred to as the "shunt" winding of the autotransformer.

Generalized constants similar to those of the two-winding transformer can be developed for the autotransformer. The total equivalent transformer impedance is referred to the "series" winding. The "ideal" transformer Equations 7.4 and 7.5 still apply.

Applying KVL in the secondary circuit,

$$E_1 + E_2 = V_L + Z_t \cdot I_2 \tag{7.19}$$

Using the "ideal" transformer relationship of Equation 7.5,

$$E_1 + n_t \cdot E_1 = (1 + n_t) \cdot E_1 = V_L + Z_t \cdot I_2 \tag{7.20}$$

Since the source voltage V_S is equal to E_1 and I_2 is equal to I_L, Equation 7.20 can be modified to

$$V_S = \frac{1}{1+n_t} \cdot V_L + \frac{Z_t}{1+n_t} \cdot I_L \tag{7.21}$$

$$V_S = a \cdot V_L + b \cdot I_L \tag{7.22}$$

where

$$a = \frac{1}{1+n_t} \tag{7.23}$$

$$b = \frac{Z_t}{1+n_t} \tag{7.24}$$

Applying Kirchhoff's current law (KCL) at input node H_1,

$$I_S = I_1 + I_2 + I_{ex}$$

$$I_S = (1+n_t)\cdot I_2 + Y_m \cdot V_S \tag{7.25}$$

Substitute Equation 7.21 into Equation 7.25:

$$I_S = (1+n_t)\cdot I_2 + Y_m\left(\frac{1}{1+n_t}\cdot V_L + \frac{Z_t}{1+n_t}\cdot I_2\right)$$

$$I_S = \frac{Y_m}{1+n_t}\cdot V_L + \left(\frac{Y_m \cdot Z_t}{1+n_t} + 1 + n_t\right)\cdot I_2$$

$$I_S = c\cdot V_L + d\cdot I_2 \tag{7.26}$$

where

$$c = \frac{Y_m}{1+n_t} \tag{7.27}$$

$$d = \frac{Y_m \cdot Z_t}{1+n_t} + 1 + n_t \tag{7.28}$$

Equations 7.23, 7.24, 7.27, and 7.28 define the generalized constants relating the source voltage and current as a function of the output voltage and current for the "step-up" autotransformer.

The two-winding transformer can also be connected in the "step-down" connection by reversing the connection between the shunt and series winding as shown in Figure 7.4.

Generalized constants can be developed for the "step-down" connection following the same procedure as that for the step-up connection.

Applying KVL in the secondary circuit,

$$E_1 - E_2 = V_L + Z_t \cdot I_2 \tag{7.29}$$

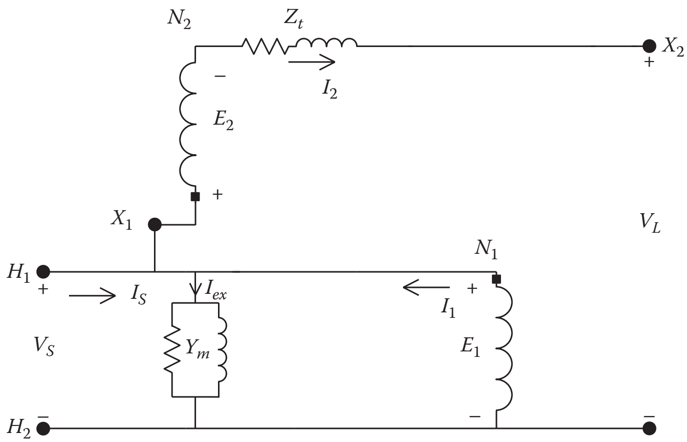

FIGURE 7.4
Step-down autotransformer.

Using the "ideal" transformer relationship of Equation 7.5,

$$E_1 - n_t \cdot E_1 = (1 - n_t) \cdot E_1 = V_L + Z_t \cdot I_2 \tag{7.30}$$

Since the source voltage V_S is equal to E_1 and I_2 is equal to I_L, Equation 7.30 can be modified to

$$V_S = \frac{1}{1 - n_t} \cdot V_L + \frac{Z_t}{1 - n_t} \cdot I_L \tag{7.31}$$

$$V_S = a \cdot V_L + b \cdot I_L \tag{7.32}$$

where

$$a = \frac{1}{1 - n_t} \tag{7.33}$$

$$b = \frac{Z_t}{1 - n_t} \tag{7.34}$$

It is observed at this point that the only difference between the a and b constants of Equations 7.23 and 7.24 for the step-up connection and Equations 7.33 and 7.44 for the step-down connection is the sign in front of the turns ratio (n_t). This will also be the case for the c and d constants. Therefore, for the step-down connection, the c and d constants are defined by

$$c = \frac{Y_m}{1 - n_t} \tag{7.35}$$

$$d = \frac{Y_m \cdot Z_t}{1 - n_t} + 1 - n_t \tag{7.36}$$

The only difference between the definitions of the generalized constants is the sign of the turns ratio n_t. In general, then, the generalized constants can be defined by

$$a = \frac{1}{1 \pm n_t} \tag{7.37}$$

$$b = \frac{Z_t}{1 \pm n_t} \tag{7.38}$$

$$c = \frac{Y_m}{1 \pm n_t} \tag{7.39}$$

$$d = \frac{Y_m \cdot Z_t}{1 \pm n_t} + 1 \pm n_t \tag{7.40}$$

In Equations 7.37 through 7.40, the sign of n_t will be positive for the step-up connection and negative for the step-down connection.

As with the two-winding transformer, it is sometimes necessary to relate the output voltage as a function of the source voltage and the output current. Solving Equation 7.32 for the output voltage:

$$V_L = \frac{1}{a} \cdot V_S - \frac{b}{a} \cdot Z_t \cdot I_2 \tag{7.41}$$

$$V_L = A \cdot V_S - B \cdot I_2 \tag{7.42}$$

where

$$A = \frac{1}{a} = 1 \pm n_t \tag{7.43}$$

$$B = \frac{b}{a} = Z_t \tag{7.44}$$

The generalized equations for the step-up and step-down autotransformers have been developed. They are of exactly the same form as was derived for the two-winding transformer and for the line segment in Chapter 6. For the single-phase autotransformer, the generalized constants are single values but will be expanded later to 3 × 3 matrices for three-phase autotransformers.

7.3.1 Autotransformer Ratings

The kVA rating of the autotransformer is the product of the rated input voltage V_S times the rated input current I_S or the rated load voltage V_L times the rated load current I_L. Define the rated kVA and rated voltages of the two-winding transformer and autotransformer as follows:

kVA_{xfm} represents the kVA rating of the two-winding transformer.
kVA_{auto} represents the kVA rating of the autotransformer.
$V_{rated\_1} = E_1$ represents rated source voltage of the two-winding transformer.
$V_{rated\_2} = E_2$ represents rated load voltage of the two-winding transformer.
V_{auto_S} represents rated source voltage of the autotransformer.
V_{auto_L} represents rated load voltage of the autotransformer.

For the following derivation, neglect the voltage drop through the series winding impedance:

$$V_{auto\_L} = E_1 \pm E_2 = (1 \pm n_t) \cdot E_1 \tag{7.45}$$

The rated output kVA is then

$$kVA_{auto} = V_{auto\_L} \cdot I_2 = (1 \pm n_t) \cdot E_1 \cdot I_2 \tag{7.46}$$

but

$$I_2 = \frac{I_1}{n_t}$$

Therefore

$$kVA_{auto} = \frac{(1 \pm n_t)}{n_t} \cdot E_1 \cdot I_1 \tag{7.47}$$

but

$$E_1 \cdot I_1 = kVA_{xfm}$$

Therefore

$$kVA_{auto} = \frac{(1 \pm n_t)}{n_t} \cdot kVA_{xfm} \tag{7.48}$$

Equation 7.48 gives the kVA rating of a two-winding transformer when connected as an autotransformer. For the step-up connection, the sign of n_t will be positive, while the step-down connection will use the negative sign. In general, the turns ratio n_t will be a relatively small value so that the kVA rating of the autotransformer will be considerably greater than the kVA rating of the two-winding transformer.

Example 7.2

The two-winding transformer of Example 7.1 is connected as a "step-up" autotransformer. Determine the kVA and voltage ratings of the autotransformer.

From Example 7.1, the turns ratio was determined to be $n_t = 0.1$.
The rated kVA of the autotransformer using Equation 7.35 is given by

$$kVA_{auto} = \frac{1+0.1}{0.1}\cdot 75 = 825\,\text{kVA}$$

The voltage ratings are

$$V_{auto\_S} = V_{rated\_1} = 2400\,\text{V}$$

$$V_{auto\_L} = V_{rated\_1} + V_{rated\_2} = 2400 + 240 = 2640\,\text{V}$$

Therefore, the autotransformer would be rated as 825 kVA, 2400–2640 V.

Suppose now that the autotransformer is supplying rated kVA at rated voltage with a power factor of 0.9 lagging, determine the source voltage and current:

$$V_L = V_{auto\_L} = 2640\underline{/0}\,\text{V}$$

$$I_2 = \frac{kVA_{auto}\cdot 1{,}000}{V_{auto\_L}} = \frac{825{,}000}{2{,}640}\underline{/-\cos^{-1}(0.9)} = 312.5\underline{/-25.84}\,\text{A}$$

Determine the generalized constants:

$$a = \frac{1}{1+0.1} = 0.9091$$

$$b = \frac{0.0122 + j0.0235}{1+0.1} = 0.0111 + j0.0214$$

$$c = \left(\frac{(1.92 - j8.52) \cdot 10^{-4}}{1 + 0.1} \right) = (1.7364 - j7.7455) \cdot 10^{-4}$$

$$d = \left(\frac{(1.92 - j8.52) \cdot 10^{-4} \cdot (0.0122 + j0.0235)}{1 + 0.1} \right) + 0.1 + 1 = 1.1002 - j0.000005$$

Applying the generalized constants,

$$V_S = a \cdot 2640\underline{/0} + b \cdot 312.5\underline{/-25.84} = 2406.0\underline{/0.1}\,\text{V}$$

$$I_S = c \cdot 2640\underline{/0} + d \cdot 312.5\underline{/-25.84} = 345.06\underline{/-26.11}\,\text{A}$$

When the load-side voltage is determined knowing the source voltage and load current, the A and B parameters are needed:

$$A = \frac{1}{n_t} = 1.1$$

$$B = Z_t = 0.0111 + j0.0235$$

The load voltage is then

$$V_L = A \cdot 2406.04\underline{/0.107} - B \cdot 312.5\underline{/-25.84} = 2640.00\underline{/0}\,\text{V}$$

Rework this example by setting the transformer impedances and shunt admittance to zero.

When this is done the generalized matrices are

$$a = \frac{1}{1 + n_t} = 0.9091$$

$$b = \frac{1}{1 + n_t} \cdot Z_t = 0$$

$$c = \frac{Y_m}{1 + n_t} = 0$$

$$d = \frac{Y_m \cdot Z_t}{1 + n_t} + n_t + 1 = 1.1$$

Using these matrices the source voltages and currents are

$$V_S = a \cdot V_L + b \cdot I_L = 2400\underline{/0}$$

$$I_S = c \cdot V_L + d \cdot I_L = 343.75\underline{/-25.8}$$

The "errors" for the source voltages and currents by ignoring the impedances and shunt admittance are

$$Error_V = \left(\frac{2406.0 - 2400}{2406}\right) \cdot 100 = 0.25\%$$

$$Error_I = \left(\frac{345.07 - 343.75}{345.07}\right) \cdot 100 = 0.38\%$$

By ignoring the transformer impedances and shunt admittance, very little error has been made. This example demonstrates why, for all practical purposes, the impedances and shunt admittance of an autotransformer can be ignored. This idea will be carried forward for the modeling of voltage regulators.

7.3.2 Per-Unit Impedance

The per-unit impedance of the autotransformer based upon the autotransformer kVA and kV ratings can be developed as a function of the per-unit impedance of the two-winding transformer based upon the two-winding transformer ratings.

Let

Zpu_{xfm} be the per-unit impedance of the two-winding transformer based upon the two-winding kVA and kV ratings

V_{rated_2} be the rated secondary voltage of the two-winding transformer

The base impedance of the two-winding transformer referred to the low-voltage winding (series winding of the autotransformer) is

$$Zbase_{xfm} = \frac{V_{rated\_2}^2}{kVA_{xfm} \cdot 1000} \tag{7.49}$$

The actual impedance of the transformer referred to the low-voltage (series) winding is

$$Zt_{actual} = Zpu_{xfm} \cdot Zbase_{xfm} = Zpu_{xfm} \cdot \frac{V_{rated\_2}^2}{kVA_{xfm} \cdot 1000} \tag{7.50}$$

The rated load voltage of the autotransformer as a function of the rated low-side voltage of the transformer is

$$V_{auto\_2} = \left(\frac{1 \pm n_t}{n_t}\right) \cdot V_{rated\_2} \tag{7.51}$$

The base impedance for the autotransformer referenced to the load side is

$$Zbase_{auto} = \frac{V_{auto\_2}^2}{kVA_{auto} \cdot 1000} \tag{7.52}$$

Substitute Equation 7.48 and Equation 7.51 into Equation 7.52:

$$Zbase_{auto} = \frac{\left[\left((1 \pm n_t)/n_t\right) \cdot V_{rated\_2}\right]^2}{\left((1 \pm n_t)/n_t\right) \cdot kVA_{xfm} \cdot 1000}$$

$$Zbase_{auto} = \frac{1 \pm n_t}{n_t} \cdot Zbase_{xfm} \tag{7.53}$$

The per-unit impedance of the autotransformer based upon the rating of the autotransformer is

$$Zauto_{pu} = \frac{Zt_{actual}}{Zbase_{auto}} \tag{7.54}$$

Substitute Equations 7.50 and 7.53 into Equation 7.54:

$$Zauto_{pu} = Zpu_{xfm} \cdot \frac{Zbase_{xfm}}{\left((1 \pm n_t)/n_t\right) \cdot Zbase_{xfm}} = \left(\frac{n_t}{1 \pm n_t}\right) \cdot Zpu_{xfm} \tag{7.55}$$

Equation 7.55 gives the relationship between the per-unit impedance of the autotransformer and the per-unit impedance of the two-winding transformer. The point being that the per-unit impedance of the autotransformer is very small compared to that of the two-winding transformer. When the autotransformer is connected to boost the voltage 10%, the value of n_t is 0.1, and Equation 7.57 becomes

$$Zpu_{auto} = \frac{0.1}{1 + 0.1} \cdot Zpu_{xfm} = 0.0909 \cdot Zpu_{xfm} \tag{7.56}$$

The per-unit shunt admittance of the autotransformer can be developed as a function of the per-unit shunt admittance of the two-winding transformer. Recall that the shunt admittance is represented on the source side of the two-winding transformer.

Let

Ypu_{xfm} be the per-unit admittance of the two-winding transformer based upon the transformer ratings

Ypu_{auto} be the per-unit admittance of the autotransformer based upon the autotransformer ratings

The base admittance of the two-winding transformer referenced to the source side is given by

$$Ybase_{xfm} = \frac{kVA_{xfm} \cdot 1000}{V_{rating\_1}^2} \tag{7.57}$$

The actual shunt admittance referred to the source side of the two-winding transformer is

$$Yt_{actual} = Ypu_{xfm} \cdot Ybase_{xfm} = Ypu_{xfm} \cdot \frac{kVA_{xfm} \cdot 1000}{V_{rating\_1}^2} \tag{7.58}$$

The base admittance referenced to the source side of the autotransformer is given by

$$Ybase_{auto} = \frac{kVA_{auto} \cdot 1000}{V_{rated\_1}^2} = \frac{\left((1 \pm n_t)/n_t\right) \cdot kVA_{xfm} \cdot 1000}{V_{rated\_1}^2} = \left(\frac{1 \pm n_t}{n_t}\right) \cdot Ybase_{xfm} \tag{7.59}$$

The per-unit admittance of the autotransformer is

$$Ypu_{auto} = \frac{Ypu_{xfm} \cdot Ybase_{xfm}}{Ybase_{auto}} = \frac{Ypu_{xfm} \cdot Ybase_{xfm}}{\left((1 \pm n_t)/n_t\right) \cdot Ybase_{xfm}}$$

$$Ypu_{auto} = \frac{n_t}{(1 \pm n_t)} \cdot Ypu_{xfm} \tag{7.60}$$

Equation 7.60 shows that the per-unit admittance based upon the autotransformer ratings is much smaller than the per-unit impedance of the two-winding transformer. For an autotransformer in the raise connection with $n_t = 0.1$, Equation 7.62 becomes

$$Ypu_{auto} = \left(\frac{0.1}{1 + 0.1}\right) \cdot Ypu_{xfm} = 0.0909 \cdot Ypu_{xfm}$$

It has been shown that the per-unit impedance and admittance values based upon the autotransformer kVA rating and nominal voltage is approximately one-tenth that of the values for the two-winding transformer.

Example 7.3

The shunt admittance referred to the source side of the two-winding transformer of Example 7.2 is

$$Yt_{actual} = Y_m = 1.92 \cdot 10^{-4} - j8.52 \cdot 10^{-4}\,\text{S}$$

a. Determine the per-unit shunt admittance based upon the two-winding transformer ratings:

$$Ybase_{xfm} = \frac{75}{2.4^2 \cdot 1000} = 0.013$$

$$Ypu_{xfm} = \frac{1.92 \cdot 10^{-4} - j8.52 \cdot 10^{-4}}{0.013} = 0.014746 - j0.065434$$

b. In Example 7.2, the kVA rating of the two-winding transformer connected as an autotransformer was computed to be 825 kVA and the voltage ratings 2640–2400 V. Determine the per-unit admittance based upon the autotransformer kVA rating and a nominal voltage of 2400 V and the ratio of the per-unit admittance of the autotransformer to the per-unit admittance of the two-winding transformer:

$$Ybase_{auto} = \frac{825 \cdot 1000}{2400^2} = 0.1432$$

$$Ypu_{auto} = \frac{1.92 \cdot 10^{-4} - j8.52 \cdot 10^{-4}}{0.1432} = 0.001341 - j0.005949$$

$$Ratio = \frac{0.001341 - j0.005949}{0.014746 - j0.065434} = 0.0909$$

In this section, the equivalent circuit of an autotransformer has been developed for the "raise" and "lower" connections. These equivalent circuits included the series impedance and shunt admittance. If a detailed analysis of the autotransformer is desired, the series impedance and shunt admittance should be included. However, it has been shown in Example 7.2 that these values are very small, and when the autotransformer is to be a component of a system, very little error will be made by neglecting both the series impedance and shunt admittance of the equivalent circuit.

7.4 Step-Voltage Regulators

A step-voltage regulator consists of an autotransformer and a LTC mechanism. The voltage change is obtained by changing the taps of the series winding of the autotransformer. The position of the tap is determined by

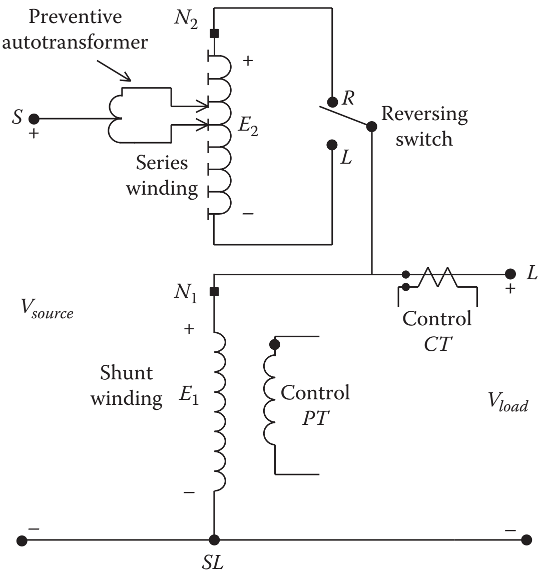

FIGURE 7.5
Type "B" step-voltage regulator.

a control circuit (line drop compensator). Standard step-regulators contain a reversing switch enabling a ±10% regulator range, usually in 32 steps. This amounts to a 5/8% change per step or 0.75 V change per step on a 120 V base. Step-regulators can be connected in a "Type A" or "Type B" connection according to the ANSI/IEEE C57.15-1986 standard [2]. The more common Type B connection is shown in Figure 7.5.

The step-voltage regulator control circuit is shown in block form in Figure 7.6. The step-voltage regulator control circuit requires the following settings:

1. *Voltage level*: The desired voltage (on 120 V base) to be held at the "load center." The load center may be the output terminal of the regulator or a remote node on the feeder.
2. *Bandwidth*: The allowed variance of the load center voltage from the set voltage level. The voltage held at the load center will be ±1/2 of

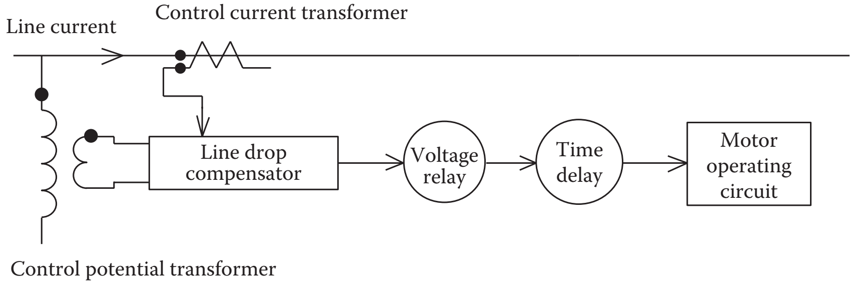

FIGURE 7.6
Step-voltage regulator control circuit.

the bandwidth. For example, if the voltage level is set to 122 V and the bandwidth set to 2 V, the regulator will change taps until the load center voltage lies between 121 and 123 V.

3. *Time delay*: Length of time that a raise or lower operation is called for before the actual execution of the command. This prevents taps changing during a transient or short time change in current.
4. *Line drop compensator*: Set to compensate for the voltage drop (line drop) between the regulator and the load center. The settings consist of R and X settings in volts corresponding to the *equivalent* impedance between the regulator and the load center. This setting may be zero if the regulator output terminals are the "load center."

The required rating of a step-regulator is based upon the kVA transformed and not the kVA rating of the line. In general, this will be 10% of the line rating since rated current flows through the series winding, which represents the ±10% voltage change. The kVA rating of the step-voltage regulator is determined in the same manner as that of the previously discussed autotransformer.

7.4.1 Single-Phase Step-Voltage Regulators

Because the series impedance and shunt admittance values of step-voltage regulators are so small, they will be neglected in the following equivalent circuits. It should be pointed out, however, that if it is desired to include the impedance and admittance, they can be incorporated into the following equivalent circuits in the same way they were originally modeled in the autotransformer equivalent circuit.

7.4.1.1 Type A Step-Voltage Regulator

The detailed equivalent circuit and abbreviated equivalent circuit of a Type A step-voltage regulator in the "raise" position are shown in Figure 7.7.

As shown in Figure 7.7, the primary circuit of the system is connected directly to the shunt winding of the Type A regulator. The series winding is connected to the shunt winding and, in turn, via taps, to the regulated circuit. In this connection, the core excitation varies because the shunt winding is connected directly across primary circuit.

When the Type A connection is in the "lower" position, the reversing switch is connected to the "L" terminal. The effect of this reversal is to reverse the direction of the currents in the series and shunt windings. Figure 7.8 shows the equivalent circuit and abbreviated circuit of the Type A regulator in the lower position.

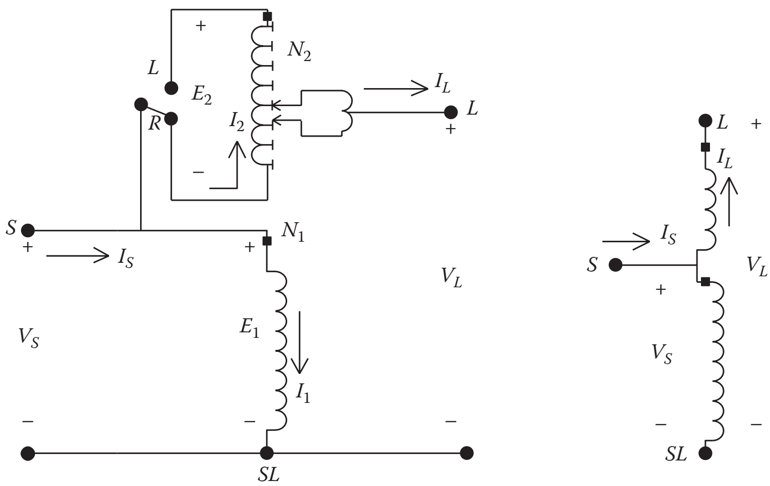

FIGURE 7.7
Type A step-voltage regulator in the raise position.

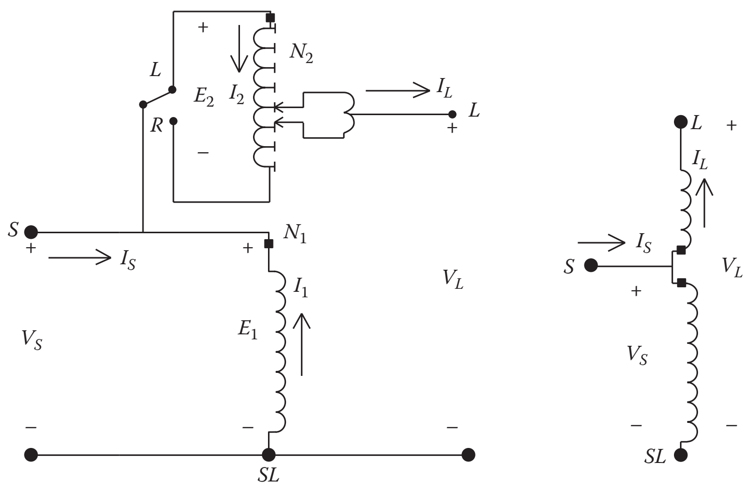

FIGURE 7.8
Type A step-voltage regulator in the lower position.

7.4.1.2 Type B Step-Voltage Regulator

The more common connection for step-voltage regulators is the Type B. Since this is the more common connection, the defining voltage and current equations for the voltage regulator will be developed only for the Type B connection.

The detailed and abbreviated equivalent circuits of a Type B step-voltage regulator in the "raise" position are shown in Figure 7.9.

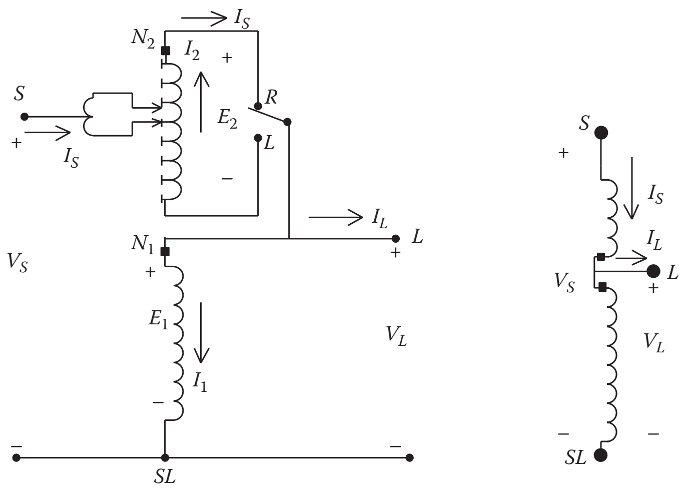

FIGURE 7.9
Type B step-voltage regulator in the raise position.

The primary circuit of the system is connected, via taps, to the series winding of the regulator in the Type B connection. The series winding is connected to the shunt winding, which is connected directly to the regulated circuit. In a Type B regulator, the core excitation is constant because the shunt winding is connected across the regulated circuit.

The defining voltage and current equations for the regulator in the raise position are as follows:

Voltage Equations	Current Equations	
$\frac{E_1}{N_1} = \frac{E_2}{N_2}$	$N_1 \cdot I_1 = N_2 \cdot I_2$	(7.61)
$V_S = E_1 - E_2$	$I_L = I_S - I_1$	(7.62)
$V_L = E_1$	$I_2 = I_S$	(7.63)
$E_2 = \frac{N_2}{N_1} \cdot E_1 = \frac{N_2}{N_1} \cdot V_L$	$I_1 = \frac{N_2}{N_1} \cdot I_2 = \frac{N_2}{N_1} \cdot I_S$	(7.64)
$V_S = \left(1 - \frac{N_2}{N_1}\right) \cdot V_L$	$I_L = \left(1 - \frac{N_2}{N_1}\right) \cdot I_S$	(7.65)
$V_S = a_R \cdot V_L$	$I_L = a_R \cdot I_S$	(7.66)
$a_R = 1 - \frac{N_2}{N_1}$		(7.67)

Equations 7.66 and 7.67 are the necessary defining equations for modeling a Type B regulator in the raise position.

The Type B step-voltage connection in the "lower" position is shown in Figure 7.10. As in the Type A connection, note that the direction of the

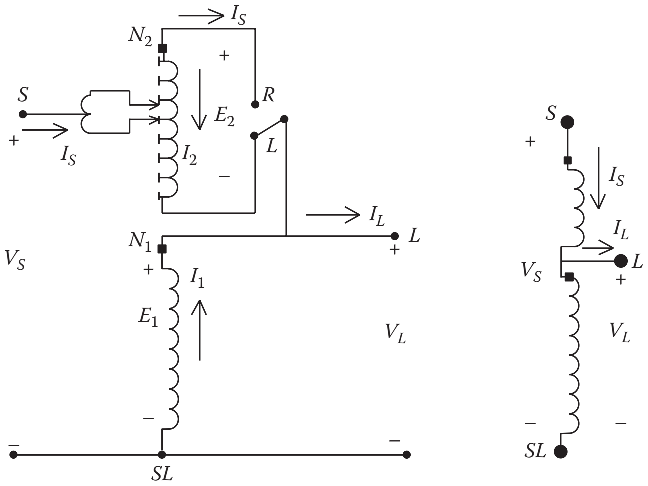

FIGURE 7.10
Type B step-voltage regulator in the lower position.

currents through the series and shunt windings change, but the voltage polarity of the two windings remain the same.

The defining voltage and current equations for the Type B step-voltage regulator in the lower position are as follows:

Voltage Equations	Current Equations	
$\frac{E_1}{N_1} = \frac{E_2}{N_2}$	$N_1 \cdot I_1 = N_2 \cdot I_2$	(7.68)
$V_S = E_1 + E_2$	$I_L = I_S - I_1$	(7.69)
$V_L = E_1$	$I_2 = -I_S$	(7.70)
$E_2 = \frac{N_2}{N_1} \cdot E_1 = \frac{N_2}{N_1} \cdot V_L$	$I_1 = \frac{N_2}{N_1} \cdot I_2 = \frac{N_2}{N_1} \cdot (-I_S)$	(7.71)
$V_S = \left(1 + \frac{N_2}{N_1}\right) \cdot V_L$	$I_L = \left(1 + \frac{N_2}{N_1}\right) \cdot I_S$	(7.72)
$V_S = a_R \cdot V_L$	$I_L = a_R \cdot I_S$	(7.73)
$a_R = 1 + \frac{N_2}{N_1}$		(7.74)

Equations 7.67 and 7.74 give the value of the effective regulator ratio as a function of the ratio of the number of turns on the series winding (N_2) to the number of turns on the shunt winding (N_1).

In the final analysis, the only difference between the voltage and current equations for the Type B regulator in the raise and lower positions is the sign of the turns ratio (N_2/N_1). The actual turns ratio of the windings is not known.

However, the particular tap position will be known. Equations 7.67 and 7.74 can be modified to give the effective regulator ratio as a function of the tap position. Each tap changes the voltage by 5/8% or 0.00625 per unit. Therefore, the effective regulator ratio can be given by

$$a_R = 1 \pm 0.00625 \cdot Tap \tag{7.75}$$

In Equation 7.75, the minus sign applies for the "raise" position and the positive sign for the "lower" position.

7.4.1.3 Generalized Constants

In previous chapters and sections of this chapter, generalized *a*, *b*, *c*, and *d* constants have been developed for various devices. It can now be shown that the generalized *a*, *b*, *c*, and *d* constants can also be applied to the step-voltage regulator. For both Type A and Type B regulators, the relationship between the source voltage and current to the load voltage and current is of the form:

Type A

$$V_S = \frac{1}{a_R} \cdot V_L \quad I_S = a_R \cdot I_L \tag{7.76}$$

Type B

$$V_S = a_R \cdot V_L \quad I_S = \frac{1}{a_R} \cdot I_L \tag{7.77}$$

Therefore, the generalized constants for a single-phase step-voltage regulator become

Type A

$$\begin{aligned} &a = \frac{1}{a_R} \quad b = 0 \quad c = 0 \quad d = a_R \\ &A = a_R \quad B = 0 \end{aligned} \tag{7.78}$$

Type B

$$\begin{aligned} &a = a_R \quad b = 0 \quad c = 0 \quad d = \frac{1}{a_R} \\ &A = \frac{1}{a_R} \quad B = 0 \end{aligned} \tag{7.79}$$

where a_R is given by Equation 7.75 and the sign convention is given in Table 7.1.

TABLE 7.1

Sign Convention Table for a_R

	Type A	Type B
Raise	+	–
Lower	–	+

7.4.1.4 Line Drop Compensator

The changing of taps on a regulator is controlled by the "line drop compensator." Figure 7.11 shows an analog circuit of the compensator circuit and how it is connected to the distribution line through a potential transformer and a current transformer. Older regulators are controlled by an analog compensator circuit. Modern regulators are controlled by a digital compensator. The digital compensators require the same settings as the analog, because it is easy to visualize, the analog circuit will be used in this section. However, understand that the modern digital compensators perform the same function for changing the taps on the regulators.

The purpose of the line drop compensator is to model the voltage drop of the distribution line from the regulator to the "load center." The compensator is an analog circuit that is a scale model of the line circuit. The compensator input voltage is typically 120 V, which requires the potential transformer in Figure 7.11 to reduce the rated voltage down to 120 V. For a regulator connected line to ground, the rated voltage is the nominal line-to-neutral voltage, while for a regulator connected line to line, the rated voltage is the line-to-line voltage. The current transformer turns ratio is specified as $CT_P:CT_S$ where the

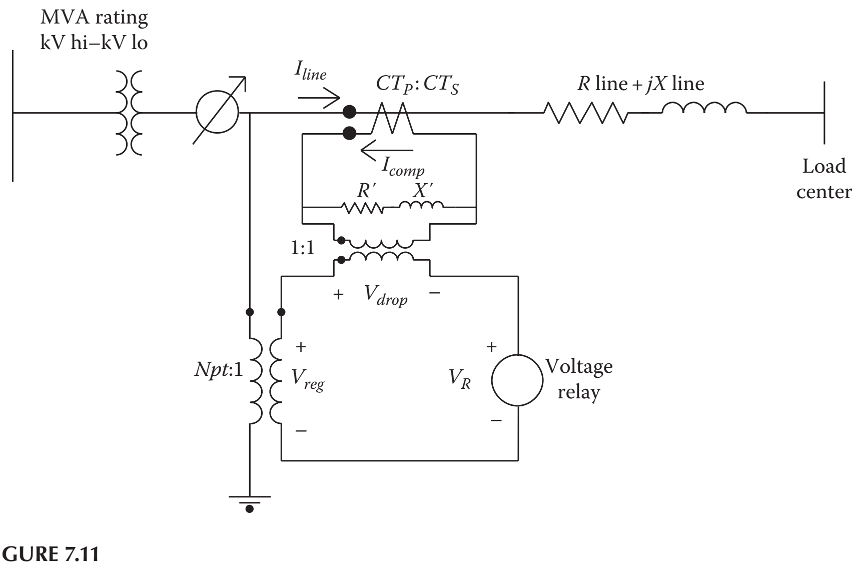

FIGURE 7.11
Line drop compensator circuit.

primary rating (CT_P) will typically be the rated current of the feeder. The setting that is most critical is that of R' and X' calibrated in volts. These values must represent the equivalent impedance from the regulator to the load center. The basic requirement is to force the per-unit line impedance to be equal to the per-unit compensator impedance. In order to cause this to happen, it is essential that a consistent set of base values be developed wherein the per-unit voltage and currents in the line and in the compensator are equal. The consistent set of base values is determined by selecting a base voltage and current for the line circuit and then computing the base voltage and current in the compensator by dividing the system base values by the potential transformer ratio and current transformer ratio, respectively. For regulators connected line to ground, the base system voltage is selected as the rated line-to-neutral voltage (V_{LN}), and the base system current is selected as the rating of the primary winding of the current transformer (CT_P). Table 7.2 gives "table of base values" and employs these rules for a regulator connected line to ground.

With the table of base values developed, the compensator R and X settings in Ohms can be computed by first computing the per-unit line impedance:

$$R_{pu} + jX_{pu} = \frac{Rline_{\Omega} + jXline_{\Omega}}{Zbase_{line}}$$

$$R_{pu} + jX_{pu} = (Rline_{\Omega} + jXline_{\Omega}) \cdot \frac{CT_P}{V_{LN}} \tag{7.80}$$

The per-unit impedance of Equation 7.80 must be the same in the line and in the compensator. The compensator impedance in Ohms is computed by multiplying the per-unit impedance by the base compensator impedance:

$$\begin{aligned} Rcomp_{\Omega} + jXcomp_{\Omega} &= (R_{pu} + jX_{pu}) \cdot Zbase_{comp} \\ &= (Rline_{\Omega} + jXline_{\Omega}) \cdot \frac{CT_P}{V_{LN}} \cdot \frac{V_{LN}}{N_{PT} \cdot CT_S} \\ &= (Rline_{\Omega} + jXline_{\Omega}) \cdot \frac{CT_P}{N_{PT} \cdot CT_S}\,\Omega \end{aligned} \tag{7.81}$$

TABLE 7.2

Table of Base Values

Base	Line Circuit	Compensator Circuit
Voltage	V_{LN}	$\frac{V_{LN}}{N_{PT}}$
Current	CT_P	CT_S
Impedance	$Zbase_{line} = \frac{V_{LN}}{CT_P}$	$Zbase_{comp} = \frac{V_{LN}}{N_{PT} \cdot CT_S}$

Equation 7.81 gives the value of the compensator R and X settings in Ohms. The compensator R and X settings in volts are determined by multiplying the compensator R and X in Ohms times the rated secondary current (CT_S) of the current transformer:

$$\begin{aligned} R' + jX' &= (Rcomp_{\Omega} + jXcomp_{\Omega}) \cdot CT_S \\ &= (Rline_{\Omega} + jXline_{\Omega}) \cdot \frac{CT_P}{N_{PT} \cdot CT_S} \cdot CT_S \\ &= (Rline_{\Omega} + jXline_{\Omega}) \cdot \frac{CT_P}{N_{PT}} \text{V} \end{aligned} \tag{7.82}$$

Knowing the equivalent impedance in Ohms from the regulator to the load center, the required value for the compensator settings in volts is determined by using Equation 7.82. This is demonstrated in Example 7.4.

Example 7.4

Refer to Figure 7.11.

The substation transformer is rated 5000 kVA, 115 delta–4.16 grounded wye, and the equivalent line impedance from the regulator to the load center is $0.3 + j0.9\,\Omega$.

1. Determine the potential transformer and current transformer ratings for the compensator circuit.

   The rated line-to-ground voltage of the substation transformer is

   $$V_S = \frac{4160}{\sqrt{3}} = 2401.8$$

   In order to provide approximately 120 V to the compensator, the potential transformer ratio is

   $$N_{PT} = \frac{2400}{120} = 20$$

   The rated current of the substation transformer is

   $$I_{rated} = \frac{5000}{\sqrt{3} \cdot 4.16} = 693.9$$

   The primary rating of the CT is selected as 700 A, and if the compensator current is reduced to 5 A, the CT ratio is

   $$CT = \frac{CT_P}{CT_S} = \frac{700}{5} = 140$$

2. Determine the R and X settings of the compensator in Ohms and volts.

   Applying Equation 7.78 to determine the settings in volts,

$$R' + jX' = (0.3 + j0.9) \cdot \frac{700}{20} = 10.5 + j31.5\,\text{V}$$

   The R and X settings in Ohms are determined by dividing the settings in volts by the rated secondary current of the current transformer:

$$R_{ohms} + jX_{ohms} = \frac{10.5 + j31.5}{5} = 2.1 + j6.3\,\Omega$$

   Understand that the R and X settings on the compensator control board are calibrated in volts.

Example 7.5

The substation transformer in Example 7.4 is supplying 2500 kVA at 4.16 kV and 0.9 power factor lag. The regulator has been set so that

$$R' + jX' = 10.5 + j31.5\ \text{V}$$

Voltage level = 120 V (desired voltage to be held at the load center)
Bandwidth = 2 V

Determine the tap position of the regulator that will hold the load center voltage at the desired voltage level and within the bandwidth. This means that the tap on the regulator needs to be set so that the voltage at the load center lies between 119 and 121 V.

The first step is to calculate the actual line current:

$$I_{line} = \frac{2500}{\sqrt{3} \cdot 4.16} \underline{/-a\cos(0.9)} = 346.97\underline{/-25.84}\,\text{A}$$

The current in the compensator is then

$$I_{comp} = \frac{346.97\underline{/-25.84}}{140} = 2.4783\underline{/-25.84}\,\text{A}$$

The input voltage to the compensator is

$$V_{reg} = \frac{2401.8\underline{/0}}{20} = 120.09\underline{/0}\,\text{V}$$

The voltage drop in the compensator circuit is equal to the compensator current times the compensator R and X values in Ohms:

$$V_{drop} = (2.1 + j6.3) \cdot 2.4783\underline{/-25.84} = 16.458\underline{/45.7}\,\text{V}$$

The voltage across the voltage relay is

$$V_R = V_{reg} - V_{drop} = 120.09\underline{/0} - 16.458\underline{/45.7} = 109.24\underline{/-6.19}\,\text{V}$$

The voltage across the voltage relay represents the voltage at the load center. Since this is well below the minimum voltage level of 119, the voltage regulator will have to change taps in the raise position to bring the load center voltage up to the required level. Recall that on a 120 V base, one step change on the regulator changes the voltage 0.75 V. The number of required tap changes can then be approximated by

$$Tap = \frac{119 - 109.24}{0.75} = 13.02$$

This shows that the final tap position of the regulator will be "raise 13." With the tap set at +13, the effective regulator ratio assuming a Type B regulator is

$$a_R = 1 - 0.00625 \cdot 13 = 0.9188$$

The generalized constants for modeling the regulator for this operating condition are

$$a = a_R = 0.9188$$

$$b = 0$$

$$c = 0$$

$$d = \frac{1}{0.9188} = 1.0884$$

Example 7.6

Using the results of Examples 7.5, calculate the actual voltage at the load center with the tap set at +13 assuming the 2500 kVA at 4.16 kV measured at the substation transformer's low-voltage terminals.

The actual line-to-ground voltage and line current at the load-side terminals of the regulator are

$$V_L = \frac{V_S}{a} = \frac{2401.8\underline{/0}}{0.9188} = 2614.2\underline{/0}\,\text{V}$$

$$I_L = \frac{I_S}{d} = \frac{346.97\underline{/-25.84}}{1.0884} = 318.77\underline{/-25.84}\,\text{A}$$

The actual line-to-ground voltage at the load center is

$$V_{LC} = V_L - Z_{line} \cdot I_L = 2614.2\underline{/0} - (0.3 + j0.9) \cdot 318.77\underline{/-25.84} = 2412.8\underline{/-5.15}\,\text{V}$$

On a 120 V base, the load center voltage is

$$VLC_{120} = \frac{V_{LC}}{N_{pt}} = \frac{2412.8/\underline{-5.15}}{20} = 120.6/\underline{-5.15}\,\text{V}$$

The +13 tap is an approximation and has resulted in a load center voltage within the bandwidth. However, since the regulator started in the neutral position, the taps will be changed one at a time until the load center voltage is inside the 119 lower bandwidth. Remember that each step changes the voltage by 0.75 V. Since the load center voltage has been computed to be 120.6 V, it would appear that the regulator went one step more than necessary. Table 7.3 shows what the compensator relay voltage will be as the taps change one at a time from 0 to the final value.

Table 7.3 shows that when the regulator is modeled to change one tap at a time starting from the neutral position that when it reaches tap 12, the relay voltage is inside the bandwidth. For the same load condition, it may be that the taps will change to lower the voltage due to a previous larger load. In this case, the taps will reduce one at a time until the relay voltage is inside the 121 upper bandwidth voltage. The point is that there can be different taps for the same load depending upon whether the voltage needed to be raised or lowered from an existing tap position.

It is important to understand that the value of equivalent line impedance is not the actual impedance of the line between the regulator and the load center. Typically, the load center is located down the primary main feeder after several laterals have been tapped. As a result, the current measured by the *CT* of the regulator is not the current that flows all the way from the regulator to the load center. The only way to determine the equivalent line impedance value is to run a power-flow program of the feeder without the regulator operating. From the output of the program, the voltages at the regulator output and the load center are known. Now the "equivalent" line impedance can be computed as

$$Rline_{\Omega} + jXline_{\Omega} = \frac{V_{regulator\_output} - V_{load\_center}}{I_{line}}\,\Omega \qquad (7.83)$$

In Equation 7.83, the voltages must be specified in system volts and the current in system amperes.

TABLE 7.3

Tap Changing

Tap	Voltage
0	109.2
1	110.1
2	110.9
3	111.7
….	…
10	117.8
11	118.8
12	119.7

This section has developed the model and generalized constants for Type A and Type B single-phase step-voltage regulators. The compensator control circuit has been developed and demonstrated how this circuit controls the tap changing of the regulator. The next section will discuss the various three-phase step-type voltage regulators.

7.4.2 Three-Phase Step-Voltage Regulators

Three single-phase step-voltage regulators can be connected externally to form a three-phase regulator. When three single-phase regulators are connected together, each regulator has its own compensator circuit, and, therefore, the taps on each regulator are changed separately. Typical connections for single-phase step-regulators are

1. Single phase
2. Two regulators connected in "open wye" (sometimes referred to as "V" phase)
3. Three regulators connected in grounded wye
4. Two regulators connected in open delta
5. Three regulators connected in closed delta

A three-phase regulator has the connections between the single-phase windings internal to the regulator housing. The three-phase regulator is "gang" operated so that the taps on all windings change the same, and, as a result, only one compensator circuit is required. For this case, it is up to the engineer to determine which phase current and voltage will be sampled by the compensator circuit. Three-phase regulators will only be connected in a three-phase wye or closed delta.

Many times the substation transformer will have LTC windings on the secondary. The LTC will be controlled in the same way as a gang-operated three-phase regulator.

In the regulator models to be developed in the next sections, the phasing on the source side of the regulator will use capital letters A, B, and C. The load-side phasing will use lower case letters a, b, and c.

7.4.2.1 Wye-Connected Regulators

Three Type B single-phase regulators connected in wye are shown in Figure 7.12.

In Figure 7.12 the polarities of the windings are shown in the "raise" position. When the regulator is in the "lower" position, a reversing switch will have reconnected the series winding so that the polarity on the series

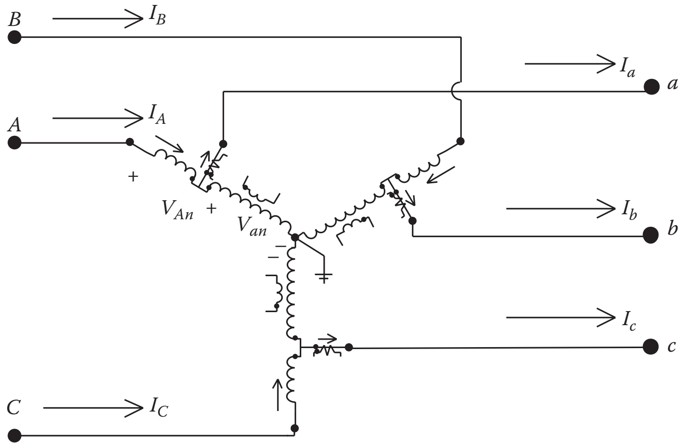

FIGURE 7.12
Wye-connected Type B regulators.

winding is now at the output terminal. Regardless of whether the regulator is raising or lowering the voltage, the following equations apply:

Voltage equations

$$\begin{bmatrix} V_{An} \\ V_{Bn} \\ V_{Cn} \end{bmatrix} = \begin{bmatrix} a_{R\_a} & 0 & 0 \\ 0 & a_{R\_b} & 0 \\ 0 & 0 & a_{R\_c} \end{bmatrix} \cdot \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \tag{7.84}$$

where a_{R_a}, a_{R_b}, and a_{R_c} represent the effective turns ratios for the three single-phase regulators.

Equation 7.84 is of the form

$$[VLN_{ABC}] = [a] \cdot [VLN_{abc}] + [b] \cdot [I_{abc}] \tag{7.85}$$

Current equations

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} \frac{1}{a_{R\_a}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_b}} & 0 \\ 0 & 0 & \frac{1}{a_{R\_c}} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{7.86}$$

or

$$[I_{ABC}] = [c] \cdot [VLG_{abc}] + [d][I_{abc}] \tag{7.87}$$

Equations 7.85 and 7.87 are of the same form as the generalized equations that were developed for the three-phase line segment of Chapter 6. For a three-phase wye-connected step-voltage regulator, neglecting the series impedance and shunt admittance, the forward and backward sweep matrices are therefore defined as

$$[a]=\begin{bmatrix} a_{R\_a} & 0 & 0 \\ 0 & a_{R\_b} & 0 \\ 0 & 0 & a_{R\_c} \end{bmatrix} \tag{7.88}$$

$$[b]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7.89}$$

$$[c]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7.90}$$

$$[d]=\begin{bmatrix} \frac{1}{a_{R\_a}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_b}} & 0 \\ 0 & 0 & \frac{1}{a_{R\_c}} \end{bmatrix} \tag{7.91}$$

$$[A]=\begin{bmatrix} \frac{1}{a_{R\_a}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_b}} & 0 \\ 0 & 0 & \frac{1}{a_{R\_c}} \end{bmatrix} \tag{7.92}$$

$$[B]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7.93}$$

In Equations 7.88, 7.91, and 7.93, the effective turns ratio for each regulator must satisfy $0.9 \le a_{R\_abc} \le 1.1$ in 32 steps of 0.625%/step (0.75 V/step on 120 V base).

The effective turn ratios (a_{R_a}, a_{R_b}, and a_{R_c}) can take on different values when three single-phase regulators are connected in wye. It is also possible to have a three-phase regulator connected in wye where the voltage and current are sampled on only one phase and then all three phases are changed by the same number of taps.

Example 7.7

An unbalanced three-phase load is served at the end of a 10,000 ft, 12.47 kV distribution line segment. The phase generalized matrices for the line segment were computed in Example 6.1 and used in Example 6.5. The computed matrices are

$$[a_{line}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[b_{line}] = \begin{bmatrix} 0.8667 + j2.0417 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 0.8837 + j1.9852 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 0.8741 + j2.0172 \end{bmatrix}$$

For this line, the A and B matrices are defined as

$$[A_{line}] = [a_{line}]^{-1}$$

$$[B_{line}] = [a_{line}]^{-1} \cdot [b_{line}] = [Z_{abc}]$$

In Example 6.5, the substation line-to-line voltages are balanced three phase. The line-to-neutral voltages at the substation are balanced three phase:

$$[VLN_{ABC}] = \begin{bmatrix} 7199.6/\underline{0} \\ 7199.6/\underline{-120} \\ 7199.6/\underline{120} \end{bmatrix} \text{V}$$

In Example 6.5, the unbalanced three-phase loads were

$$[kVA] = \begin{bmatrix} 2500 \\ 2000 \\ 1500 \end{bmatrix} \quad [PF] = \begin{bmatrix} 0.9 \\ 0.85 \\ 0.95 \end{bmatrix} \text{kVA}$$

In Example 6.5, the ladder iterative technique was used and the currents leaving the substation were

$$[I_{ABC}] = \begin{bmatrix} 374.4/\underline{-28.2} \\ 286.8\underline{-153.9} \\ 212.6/\underline{100.5} \end{bmatrix}$$

The load voltages were calculated to be

$$[Vload_{abc}] = \begin{bmatrix} 6678.2\underline{/-2.3} \\ 6972.8\underline{/-122.1} \\ 7055.5\underline{/118.7} \end{bmatrix}$$

The load voltages on a 120 V base were computed to be

$$[V_{120}] = \begin{bmatrix} 111.3\underline{/-2.3} \\ 116.2\underline{/-122.1} \\ 117.6\underline{/118.7} \end{bmatrix}$$

It is obvious that the load voltages are not within the ANSI standard. To correct this problem, three single-phase Type B step-voltage regulators will be connected in wye and installed in the substation. The regulators are to be set so that each line-to-neutral load voltage on a 120 V base will lie between 119 and 121 V.

The potential and current transformers of the regulators are rated:

$$N_{PT} = \frac{7200}{120} = 60$$

$$CT = \frac{600}{5} = \frac{CT_P}{CT_S} = 120$$

The voltage level and bandwidth are

Voltage level = 120 V
Bandwidth = 2 V

The equivalent line impedance for each phase can be determined by applying Equation 7.83:

$$Zline_a = \frac{7199.6\underline{/0} - 6678.2\underline{/-2.3}}{374.4\underline{/-28.2}} = 0.8989 + j1.3024$$

$$Zline_b = \frac{7199.6\underline{/-120} - 6972.8\underline{/-122.1}}{286.8\underline{/-153.9}} = 0.1655 + j1.2007\,\Omega$$

$$Zline_c = \frac{7199.6\underline{/120} - 7055.5\underline{/118.7}}{212.6\underline{/100.5}} = 0.4044 + j0.9141$$

Even though the three regulators will change taps independently, it is the usual practice to set the R and X settings of the three regulators the same. The average value of the aforementioned three line impedances can be used for this purpose:

$$Zline_{average} = 0.4896 + j1.13.91\Omega$$

The compensator R and X settings are computed according to Equation 7.82:

$$R' + jX' = (Rline_{\Omega} + jXline_{\Omega}) \cdot \frac{CT_P}{N_{PT}} = (0.4896 + j1.1391) \cdot \frac{600}{60}$$

$$= 4.8964 + j11.3908\,\text{V}$$

The compensator controls are not calibrated to that many significant figures, so the values set are

$$R' + jX' = 5 + j11\,\text{V}$$

For the same unbalanced loading and with the three-phase wye-connected regulators in service, the approximate tap settings are

$$Tap_a = \frac{\left|119 - |Vload_a|\right|}{0.75} = \frac{|119 - 111.3|}{0.75} = 10.2615$$

$$Tap_b = \frac{\left|119 - |Vload_b|\right|}{0.75} = \frac{|119 - 116.2|}{0.75} = 3.7154$$

$$Tap_a = \frac{\left|119 - |Vload_c|\right|}{0.75} = \frac{|119 - 117.6|}{0.75} = 1.8787$$

Since the taps must be integers, the actual tap settings will be

$$Tap_a = +10$$

$$Tap_b = +4$$

$$Tap_c = +2$$

The effective turns ratio for the three regulators and the resulting generalized matrices are determined by applying Equations 7.88, 7.91, and 7.92 for each phase:

$$\left[a_{reg}\right] = \begin{bmatrix} 1 - 0.00625 \cdot 10 & 0 & 0 \\ 0 & 1 - 0.00625 \cdot 4 & 0 \\ 0 & 0 & 1 - 0.00625 \cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9375 & 0 & 0 \\ 0 & 0.975 & 0 \\ 0 & 0 & 0.9875 \end{bmatrix}$$

$$\left[d_{reg}\right] = \left[a_{reg}\right]^{-1} = \begin{bmatrix} 1.0667 & 0 & 0 \\ 0 & 1.0256 & 0 \\ 0 & 0 & 1.0127 \end{bmatrix}$$

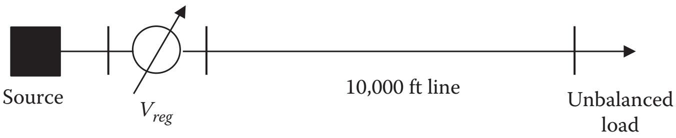

FIGURE 7.13
Simple system with a regulator and line.

$$[A_{reg}] = [a_{reg}]^{-1} = \begin{bmatrix} 1.0667 & 0 & 0 \\ 0 & 1.0256 & 0 \\ 0 & 0 & 1.0127 \end{bmatrix}$$

$$[B_{reg}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

With the voltage regulators connected to the source, the one-line diagram of the simple system is shown in Figure 7.13.

A Mathcad® program is written following the flow chart in Figure 6.7. The program is used to compute the load voltages and currents after the regulator taps and resulting matrices have been computed. The program is shown in Figure 7.14.

After six iterations, the results of the analysis are

$$[Vload_{abc}] = \begin{bmatrix} 7205.6\underline{/-1.9} \\ 7145.9\underline{/-122.0} \\ 7147.2\underline{/118.7} \end{bmatrix}$$

$$[V_{120}] = \begin{bmatrix} 120.1\underline{/-1.9} \\ 119.1\underline{/-122.0} \\ 119.1\underline{/118.7} \end{bmatrix}$$

$$[I_{abc}] = \begin{bmatrix} 347.0\underline{/-27.8} \\ 279.9\underline{/-153.8} \\ 209.9\underline{/100.5} \end{bmatrix}$$

$$[I_{ABC}] = \begin{bmatrix} 370.1\underline{-27.8} \\ 287.1\underline{/-153.8} \\ 212.5\underline{/100.5} \end{bmatrix}$$

Note that now all of the load voltages on the 120 V base are within ANSI standards assuming that the taps were actually set at +10, +4, and +2.

$$\text{Start} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \text{Tol} := .00001$$

$$\text{Solve} := \left| \begin{array}{l}
I_{abc} \leftarrow \text{Start} \\
V_{old} \leftarrow \text{Start} \\
\text{for } n \in 1..200 \\
\quad \left| \begin{array}{l}
Vreg_{abc} \leftarrow A_{reg} \cdot E_{ABC} - B_{reg} \cdot I_{abc} \\
Vload_{abc} \leftarrow A_{line} \cdot Vreg_{abc} - B_{line} \cdot I_{abc} \\
\text{for } i \in 1..3 \\
\quad I_{abc_i} \leftarrow \overline{\left(\dfrac{SL_i \cdot 1000}{Vload_{abc_i}}\right)} \\
\text{for } j \in 1..3 \\
\quad Error_j \leftarrow \dfrac{\left| Vload_{abc_j} - V_{old_j} \right|}{VLN} \\
Err_{max} \leftarrow \max(Error) \\
\text{break if } Err_{max} < \text{Tol} \\
V_{old} \leftarrow Vload_{abc} \\
V_{120} \leftarrow \dfrac{Vload_{abc}}{N_{pt}} \\
I_{ABC} \leftarrow d_{reg} \cdot I_{abc}
\end{array} \right. \\
Out_1 \leftarrow Vload_{abc} \\
Out_2 \leftarrow V_{120} \\
Out_3 \leftarrow I_{abc} \\
Out_4 \leftarrow I_{ABC} \\
Out_5 \leftarrow n \\
Out
\end{array} \right.$$

FIGURE 7.14
Mathcad® program.

Unfortunately, the way it really works is the compensator circuit will adjust the taps based upon the line voltages out of the regulator, the line currents out of the regulator, and the *R* and *X* settings. For this example, the compensator *R* and *X* in Ohms are

$$Z_c = \frac{R + jX}{CT_S} = \frac{5 + j11}{5} = 1 + j2.2$$

With the taps set at zero, the voltages and currents at the output terminals of the regulator are

$$[V_{out}] = [A_{reg}] \cdot [E_{ABC}] = \begin{bmatrix} 7199.6\underline{/0} \\ 7199.6\underline{/-120} \\ 7199.6\underline{/120} \end{bmatrix}$$

$$I_{out} = [d_{reg}]^{-1} \cdot [I_{ABC}] = \begin{bmatrix} 374.4\underline{/-28.2} \\ 286.8\underline{/-153.9} \\ 212.6\underline{/100.5} \end{bmatrix}$$

The voltages and currents into the compensator circuits are

$$[V_{reg}] = \frac{[V_{out}]}{N_{pt}} = \begin{bmatrix} 120.0\underline{/0} \\ 120.0\underline{/-120} \\ 120.0\underline{/120} \end{bmatrix}$$

$$[I_{reg}] = \frac{[I_{out}]}{CT} = \begin{bmatrix} 3.12\underline{/-28.2} \\ 2.39\underline{/-153.9} \\ 1.77\underline{/100.6} \end{bmatrix}$$

The compensator impedance matrix is

$$[Z_{comp}] = \begin{bmatrix} Z_c & 0 & 0 \\ 0 & Z_c & 0 \\ 0 & 0 & Z_c \end{bmatrix} = \begin{bmatrix} 1+j2.2 & 0 & 0 \\ 0 & 1+j2.2 & 0 \\ 0 & 0 & 1+j2.2 \end{bmatrix}$$

The voltages across the relays are

$$[V_{relay}] = [V_{reg}] - [Z_{comp}] \cdot [I_{reg}] = \begin{bmatrix} 114.1\underline{/-2.3} \\ 115.1\underline{/-121.5} \\ 117.1\underline{/118.5} \end{bmatrix}$$

Since the relay voltages are not within the bandwidth, the taps will change one step at a time until the voltages are within the bandwidth. It should be pointed out that as each regulator gets the voltage within the bandwidth it will stop, while the others continue to change taps until their voltages are within the bandwidth. With the regulators changing one tap at a time, the final taps based upon the compensator relay voltages are

$$[Taps] = \begin{bmatrix} +7 \\ +5 \\ +3 \end{bmatrix}$$

With these taps the regulator stops changing taps and the relay voltages are

$$[V_{relay}] = \begin{bmatrix} 119.9 \\ 119.1 \\ 119.4 \end{bmatrix}$$

Note that these are not the same taps as originally given for this example. When these taps are applied to the analysis of the system, the resulting load voltages on a 120 V base are

$$[V_{load}] = \begin{bmatrix} 117.3 \\ 120.1 \\ 119.9 \end{bmatrix}$$

The phase *a* voltage is not within the bandwidth. The problem is that when the example was first analyzed with the original taps, the taps had been determined by using the actual line voltage drops with the regulators in the neutral position. However, when the compensator *R* and *X* values were computed, the average of the equivalent line impedances was used for each regulator. Since the three line currents are all different, the heavily loaded phase *a* voltage will not represent what is actually happening on the system. Once again this is a problem that occurs because of the unbalanced loading.

One way to raise the load voltages is to specify a higher voltage level. Increase the voltage level to 122 V. With the regulator changing taps one at a time until all voltage relays have a voltage just greater than 121 V (lower bandwidth voltage), the results are

$$[Taps] = \begin{bmatrix} 9 \\ 8 \\ 5 \end{bmatrix}$$

$$[Vload_{120}] = \begin{bmatrix} 119.1 \\ 122.6 \\ 121.5 \end{bmatrix}$$

$$[V_{relay}] = \begin{bmatrix} 121.6 \\ 121.7 \\ 121.0 \end{bmatrix}$$

Example 7.7 is a long example intended to demonstrate how the engineer can determine the correct compensator *R* and *X* settings knowing the substation and load voltages and the currents leaving the substation. Generally, it will be necessary to run a power-flow study in order to determine these values. A simple Mathcad routine demonstrates that with the regulator tap settings the load voltages are within the desired limits. The regulator has automatically set the taps for this load condition, and, as the load changes, the taps will continue to change in order to hold the load voltages within the desired limits.

7.4.2.2 Closed Delta–Connected Regulators

Three single-phase Type B regulators can be connected in a closed delta as shown in Figure 7.15. In the figure, the regulators are shown in the "raise" position.

The closed delta connection is typically used in three-wire delta feeders. Note that the potential transformers for this connection are monitoring the load-side line-to-line voltages and the current transformers are not monitoring the load-side line currents.

The relationships between the source side and currents and the voltages are needed. Equations 7.64 through 7.67 define the relationships between the series and shunt winding voltages and currents for a step-voltage regulator that must be satisfied no matter how the regulators are connected.

KVL is first applied around a closed loop starting with the line-to-line voltage between phases A and C on the source side. Refer to Figure 7.14, which defines the various voltages:

$$V_{AB} = V_{Aa} + V_{ab} - V_{Bb} \tag{7.94}$$

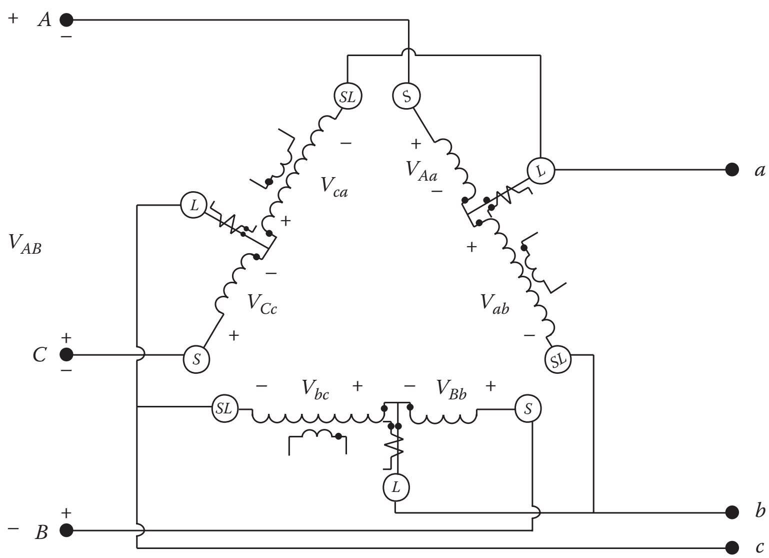

FIGURE 7.15
Closed delta–connected regulators with voltages.

but

$$V_{Bb} = -\frac{N_2}{N_1} \cdot V_{bc} \tag{7.95}$$

$$V_{Aa} = -\frac{N_2}{N_1} \cdot V_{ab} \tag{7.96}$$

Substitute Equations 7.95 and 7.96 into Equation 7.94 and simplify:

$$V_{AB} = \left(1 - \frac{N_2}{N_1}\right) \cdot V_{ab} + \frac{N_2}{N_1} \cdot V_{bc} = a_{R\_ab} \cdot V_{ab} + (1 - a_{R\_bc}) \cdot V_{bc} \tag{7.97}$$

The same procedure can be followed to determine the relationships between the other line-to-line voltages. The final three-phase equation is

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = \begin{bmatrix} a_{R\_ab} & 1 - a_{R\_bc} & 0 \\ 0 & a_{R\_bc} & 1 - a_{R\_ca} \\ 1 - a_{R\_ab} & 0 & a_{R\_ca} \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \tag{7.98}$$

Equation 7.98 is of the generalized form

$$[VLL_{ABC}] = [a] \cdot [VLL_{abc}] + [b] \cdot [I_{abc}] \tag{7.99}$$

Figure 7.16 shows the closed delta–delta connection with the defining currents.

The relationship between source and load line currents starts with applying KCL at the load-side terminal a.

$$I_a = I'_a + I_{ca} = I_A - I_{ab} + I_{ca} \tag{7.100}$$

but

$$I_{ab} = \frac{N_2}{N_1} \cdot I_A \tag{7.101}$$

$$I_{ca} = \frac{N_2}{N_1} \cdot I_C \tag{7.102}$$

Substitute Equations 7.100 and 7.101 into Equation 7.100 and simplify:

$$I_a = \left(1 - \frac{N_2}{N_1}\right) \cdot I_A + \frac{N_2}{N_1} I_C = a_{R\_ab} \cdot I_A + (1 - a_{R\_ca}) \cdot I_C \tag{7.103}$$

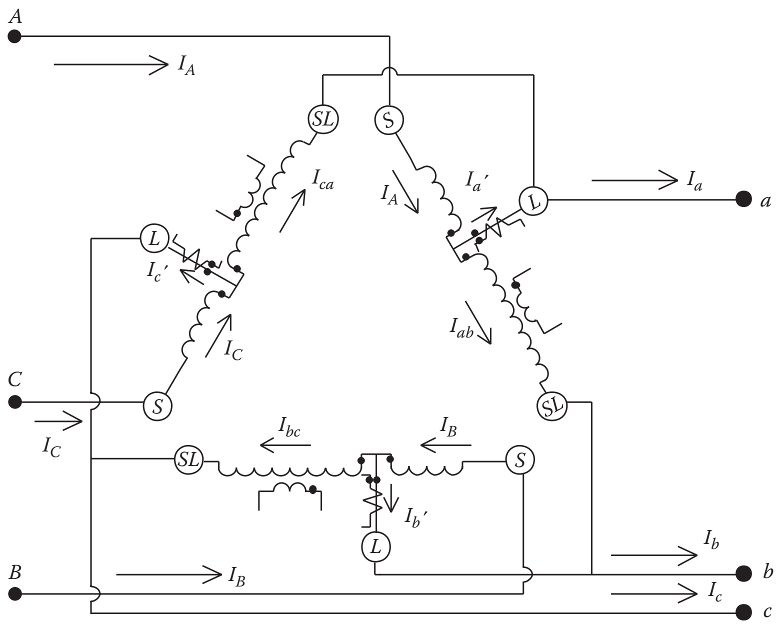

FIGURE 7.16
Closed delta–connected regulators with currents.

The same procedure can be followed at the other two load-side terminals. The resulting three-phase equation is

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} a_{R\_ab} & 0 & 1-a_{R\_ca} \\ 1-a_{R\_ab} & a_{R\_bc} & 0 \\ 0 & 1-a_{R\_bc} & a_{R\_ca} \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{7.104}$$

Equation 7.104 is of the general form

$$[I_{abc}] = [D] \cdot [I_{ABC}] \tag{7.105}$$

where

$$[D] = \begin{bmatrix} a_{R\_ab} & 0 & 1-a_{R\_ca} \\ 1-a_{R\_ab} & a_{R\_bc} & 0 \\ 0 & 1-a_{R\_bc} & a_{R\_ca} \end{bmatrix}$$

The general form needed for the standard model is

$$[I_{ABC}] = [c] \cdot [VLL_{ABC}] + [d] \cdot [I_{abc}] \tag{7.106}$$

where $[d] = [D]^{-1}$.

As with the wye-connected regulators, the matrices [b] and [c] are zero as long as the series impedance and shunt admittance of each regulator are neglected.

The closed delta connection can be difficult to apply. Note that in both the voltage and current equations, a change of the tap position in one regulator will affect voltages and currents in two phases. As a result, increasing the tap in one regulator will affect the tap position of the second regulator. In most cases, the bandwidth setting for the closed delta connection will have to be wider than that for wye-connected regulators.

7.4.2.3 Open Delta–Connected Regulators

Two Type B single-phase regulators can be connected in the "open" delta connection. Shown in Figure 7.17 is an open delta connection where two single-phase regulators have been connected between phases *AB* and *CB*.

Two additional open connections can be made by connecting the single-phase regulators between phases *BC* and *AC* and also between phases *CA* and *BA*.

The open delta connection is typically applied to three-wire delta feeders. Note that the potential transformers monitor the line-to-line voltages and the current transformers monitor the line currents. Once again the basic voltage and current relations of the individual regulators are used to determine the

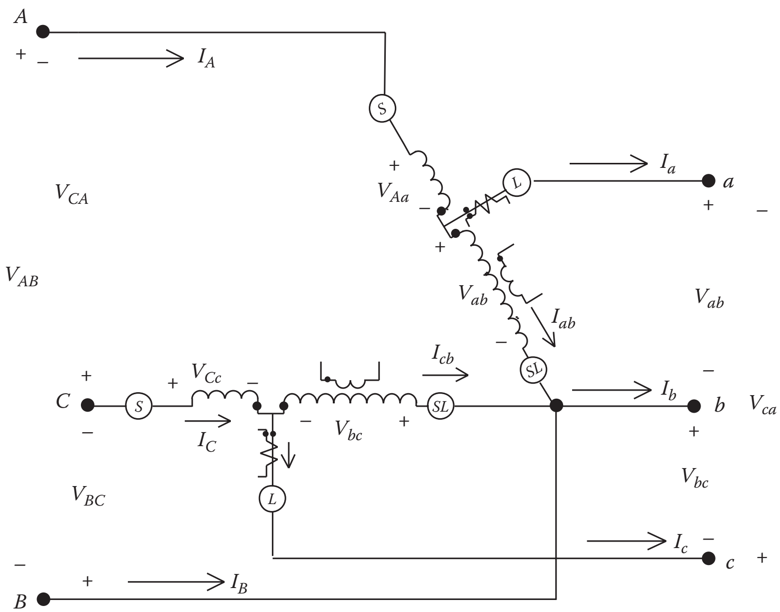

FIGURE 7.17
Open delta connection.

relationships between the source-side and load-side voltages and currents. The connection shown in Figure 7.17 will be used to derive the relationships and then the relationships of the other two possible connections can follow the same procedure.

The voltage V_{AB} across the first regulator consists of the voltage across the series winding plus the voltage across the shunt winding:

$$V_{AB} = V_{Aa} + V_{ab} \tag{7.107}$$

Paying attention to the polarity marks on the series and shunt windings, the voltage across the series winding is

$$V_{Aa} = -\frac{N_2}{N_1} \cdot V_{ab} \tag{7.108}$$

Substituting Equation 7.108 into Equation 7.107 yields

$$V_{AB} = \left(1 - \frac{N_2}{N_1}\right) \cdot V_{ab} = a_{R\_ab} \cdot V_{ab} \tag{7.109}$$

Following the same procedure for the regulator connected across V_{BC}, the voltage equation is

$$V_{BC} = \left(1 - \frac{N_2}{N_1}\right) \cdot V_{bc} = a_{R\_cb} \cdot V_{bc} \tag{7.110}$$

KVL must be satisfied so that

$$V_{CA} = -(V_{AB} + V_{BC}) = -a_{R\_ab} \cdot V_{ab} - a_{R\_cb} \cdot V_{bc} \tag{7.111}$$

Equations 7.107 through 7.109 can be put into matrix form:

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = \begin{bmatrix} a_{R\_ab} & 0 & 0 \\ 0 & a_{R\_cb} & 0 \\ -a_{R\_ab} & -a_{R\_cb} & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \tag{7.112}$$

Equation 7.112 in generalized form is

$$[VLL_{ABC}] = [a_{LL}] \cdot [VLL_{abc}] + [b_{LL}] \cdot [I_{abc}] \tag{7.113}$$

where

$$[a_{LL}] = \begin{bmatrix} a_{R\_ab} & 0 & 0 \\ 0 & a_{R\_cb} & 0 \\ -a_{R\_ab} & -a_{R\_cb} & 0 \end{bmatrix} \tag{7.114}$$

The effective turns ratio of each regulator is given by Equation 7.75. Again, as long as the series impedance and shunt admittance of the regulators are neglected, $[b_{LL}]$ is zero. Equation 7.114 gives the line-to-line voltages on the source side as a function of the line-to-line voltages on the load side of the open delta using the generalized matrices. Up to this point, the relationships between the voltages have been in terms of line-to-neutral voltages. In Chapter 8, the $[W]$ matrix is derived. This matrix is used to convert line-to-line voltages to equivalent line-to-neutral voltages.

$$[VLN_{ABC}] = [W]\cdot[VLL_{ABC}] \quad \text{where } [W] = \frac{1}{3}\cdot\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \tag{7.115}$$

The line-to-neutral voltages are converted to line-to-line voltages by

$$[VLL_{ABC}] = [D]\cdot[VLN_{ABC}] \quad \text{where } [D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{7.116}$$

Convert Equation 7.113 to line-to-neutral form:

$$\begin{aligned} &[VLL_{ABC}] = [a_{LL}]\cdot[VLL_{abc}] + [b_{LL}]\cdot[I_{abc}] \\ &[VLN_{ABC}] = [W]\cdot[VLL_{ABC}] = [W]\cdot[a_{LL}]\cdot[D]\cdot[VLN_{abc}] \\ &[VLN_{ABC}] = [a_{reg}]\cdot[VLN_{abc}] \\ &\text{where } [a_{reg}] = [W]\cdot[a_{LL}]\cdot[D] \end{aligned} \tag{7.117a}$$

When the load-side line-to-line voltages are needed as function of the source-side line-to-line voltages, the necessary equation is

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} \frac{1}{a_{R\_ab}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_cb}} & 0 \\ -\frac{1}{a_{R\_ab}} & -\frac{1}{a_{R\_cb}} & 0 \end{bmatrix}\cdot\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} \tag{7.117b}$$

$$[VLL_{abc}] = [A_{LL}]\cdot[VLL_{ABC}] \tag{7.118}$$

where

$$[A_{LL}]=\begin{bmatrix} \frac{1}{a_{R\_ab}} & 0 & 0 \\ 0 & \frac{1}{a_{R\_cb}} & 0 \\ -\frac{1}{a_{R\_ab}} & -\frac{1}{a_{R\_cb}} & 0 \end{bmatrix} \tag{7.119}$$

Equation 7.118 is converted to line-to-neutral form by

$$[VLN_{abc}]=[A_{reg}]\cdot[VLN_{ABC}] \quad \text{where } [A_{reg}]=[W]\cdot[A_{LL}]\cdot[D] \tag{7.120}$$

There is no general equation for each of the elements of $[A_{reg}]$. The matrix $[A_{reg}]$ must be computed according to Equation 7.120.

Referring to Figure 7.17, the current equations are derived by applying KCL at the L node of each regulator:

$$I_A = I_a + I_{ab} \tag{7.121}$$

but

$$I_{ab} = \frac{N_2}{N_1}\cdot I_A$$

Therefore, Equation 7.121 becomes

$$\left(1-\frac{N_2}{N_1}\right)I_A = I_a \tag{7.122}$$

Therefore

$$I_A = \frac{1}{a_{R\_ab}}\cdot I_a \tag{7.123}$$

In a similar manner, the current equation for the second regulator is given by

$$I_C = \frac{1}{a_{R\_cb}}\cdot I_c \tag{7.124}$$

Because this is a three-wire delta line, then

$$I_B = -(I_A + I_C) = -\frac{1}{a_{R\_ab}}\cdot I_a - \frac{1}{a_{R\_cb}}\cdot I_c \tag{7.125}$$

In matrix form, the current equations become

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} \frac{1}{a_{R\_ab}} & 0 & 0 \\ -\frac{1}{a_{R\_ab}} & 0 & -\frac{1}{a_{R\_cb}} \\ 0 & 0 & \frac{1}{a_{R\_cb}} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{7.126}$$

In generalized form, Equation 7.126 becomes

$$[I_{ABC}] = [c_{reg}] \cdot [VLN_{ABC}] + [d_{reg}] \cdot [I_{abc}] \tag{7.127}$$

where

$$[d_{reg}] = \begin{bmatrix} \frac{1}{a_{R\_ab}} & 0 & 0 \\ -\frac{1}{a_{R\_ab}} & 0 & -\frac{1}{a_{R\_cb}} \\ 0 & 0 & \frac{1}{a_{R\_cb}} \end{bmatrix} \tag{7.128}$$

When the series impedances and shunt admittances are neglected, the constant matrix $[c_{reg}]$ will be zero.

The load-side line currents as a function of the source line currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} a_{R\_ab} & 0 & 0 \\ -a_{R\_ab} & 0 & -a_{R\_cb} \\ 0 & 0 & a_{R\_cb} \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{7.129}$$

$$[I_{abc}] = [D_{reg}] \cdot [I_{ABC}] \tag{7.130}$$

where

$$[D_{reg}] = \begin{bmatrix} a_{R\_ab} & 0 & 0 \\ -a_{R\_ab} & 0 & -a_{R\_cb} \\ 0 & 0 & a_{R\_cb} \end{bmatrix} \tag{7.131}$$

The determination of the *R* and *X* compensator settings for the open delta follows the same procedure as that of the wye-connected regulators.

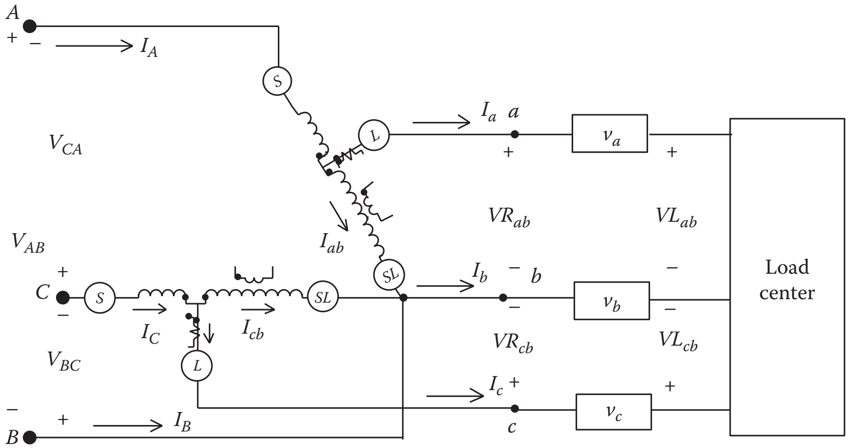

FIGURE 7.18
Open delta connected to a load center.

However, care must be taken to recognize that in the open delta connection the voltages applied to the compensator are line to line and the currents are line currents. The open delta–connected regulators will maintain only two of the line-to-line voltages at the load center within defined limits. The third line-to-line voltage will be dictated by the other two (KVL). Therefore, it is possible that the third voltage may not be within the defined limits.

With reference to Figure 7.18, an equivalent impedance between the regulators and the load center must be computed. Since each regulator is sampling line-to-line voltages and a line current, the equivalent impedance is computed by taking the appropriate line-to-line voltage drop and dividing by the sampled line current. For the open delta connection shown in Figure 7.18, the equivalent impedances are computed as

$$Zeq_a = \frac{VR_{ab} - VL_{ab}}{I_a} \tag{7.132}$$

$$Zeq_c = \frac{VR_{cb} - VL_{cb}}{I_c} \tag{7.133}$$

The units of these impedances will be in system Ohms. They must be converted to compensator volts by applying Equation 7.78. For the open delta connection, the potential transformer will transform the system line-to-line rated voltage down to 120 V. Example 7.8 demonstrates how the compensator R and X settings are determined knowing the line-to-line voltages at the regulator and at the load center.

Example 7.8

A three-wire delta system is shown in Figure 7.19. The voltages at node *S* are

$$[ELL_{ABC}] = \begin{bmatrix} 12,470\underline{/0} \\ 12,470\underline{/-120} \\ 12,470\underline{/120} \end{bmatrix}$$

$$[ELN_{ABC}] = [W]\cdot[ELL_{ABC}] = \begin{bmatrix} 7199.6\underline{/-30} \\ 7199.6\underline{/-150} \\ 7199.6\underline{/90} \end{bmatrix}$$

The three-wire delta line conductor 336,400 26/7 ACSR with spacings is shown in Figure 7.20.

The load is delta connected with values of

$$[kVA] = \begin{bmatrix} 2500 \\ 2000 \\ 1500 \end{bmatrix} \qquad [PF] = \begin{bmatrix} 0.90 \\ 0.85 \\ 0.95 \end{bmatrix}$$

The line is 10,000 ft long, and the total phase impedance matrix is

$$[Z_{abc}] = \begin{bmatrix} 0.7600 + j2.6766 & 0.1805 + j1.1627 & 0.1805 + j1.3761 \\ 0.1805 + j1.1627 & 0.7600 + j2.6766 & 0.1805 + j1.4777 \\ 0.1805 + j1.3761 & 0.1805 + j1.4777 & 0.7600 + j2.6766 \end{bmatrix}$$

FIGURE 7.19
Circuit for Example 7.8.

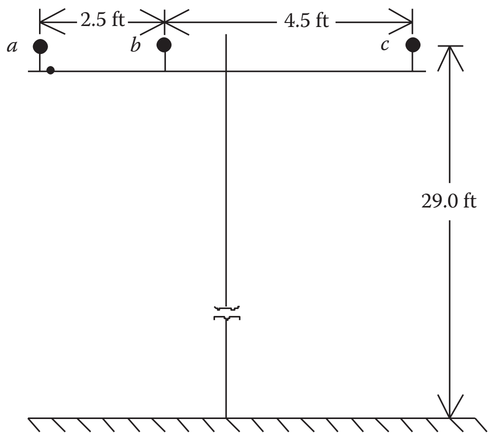

FIGURE 7.20
Three-wire delta line spacings.

For this connection, the potential transformer ratio and current transformer ratio are selected to be

$$N_{pt} = \frac{12{,}470}{120} = 103.92$$

$$CT = \frac{500}{5} = 100$$

With the regulators set in neutral on a 120 V base, the load center voltages are computed to be

$$\begin{bmatrix} V120_{ab} \\ V120_{bc} \\ V120_{ca} \end{bmatrix} = \frac{1}{103.92} \cdot \begin{bmatrix} 11{,}883.0/\underline{-2.0} \\ 11{,}943.7/\underline{-121.4} \\ 12{,}022.1/\underline{118.0} \end{bmatrix} = \begin{bmatrix} 114.4/\underline{-2.0} \\ 114.9/\underline{-121.4} \\ 115.7/\underline{118.0} \end{bmatrix} \text{V}$$

The line currents are

$$[I_{abc}] = \begin{bmatrix} 303.2/\underline{-46.9} \\ 336.2/\underline{76.1} \\ 236.2/\underline{57.1} \end{bmatrix}$$

Two single-phase Type B regulators are to be installed in an open delta connection. The regulators are to be connected between phases *AB* and *BC* as shown in Figure 7.18. The voltage level will be set at 120 V with a bandwidth of 2 V. As computed earlier, the load center voltages are not within the desired limits of 120 ± 1 V.

The compensator *R* and *X* settings must first be determined using the results of the power-flow study. The first regulator monitors the voltage V_{ab} and the line current I_a. The equivalent line impedance for this regulator is

$$Zeq_a = \frac{VR_{ab} - VL_{ab}}{I_a} = 0.3224 + j2.3844\,\Omega$$

The second regulator monitors the voltage V_{cb} and the line current I_c. In the computation of the equivalent line impedance, it is necessary to use the *cb* voltages, which are the negative of the given *bc* voltages:

$$Zeq_c = \frac{VR_{cb} - VL_{cb}}{I_c} = \frac{-VR_{bc} + VL_{bc}}{I_c} = 2.1776 + j1.3772\,\Omega$$

Unlike the wye-connected regulators, the compensator settings for the two regulators will be different. The settings calibrated in volts are

$$R'_{ab} + jX'_{ab} = Z_a \cdot \frac{CT_P}{N_{PT}} = (0.3224 + j2.3844) \cdot \frac{500}{103.92} = 1.5511 + j11.4726\,\text{V}$$

$$R'_{cb} + jX'_{cb} = Z_c \cdot \frac{CT_P}{N_{PT}} = (2.1776 + j1.3772) \cdot \frac{500}{103.92} = 10.4776 + j6.6263\,\text{V}$$

The compensator settings will be set to

$$R'_{ab} + X'_{ab} = 1.6 + j11.5\,\text{V}$$

$$R'_{cb} + jX'_{cb} = 10.5 + j6.6\,\text{V}$$

With regulators in the neutral position and with the same loading, the currents and voltages in the compensator circuits are

$$Vcomp_{ab} = \frac{VR_{ab}}{N_{pt}} = \frac{12{,}470\underline{/0}}{103.92} = 120\underline{/0}\,\text{V}$$

$$Vcomp_{cb} = \frac{-VR_{bc}}{N_{pt}} = \frac{12{,}470\underline{/60}}{103.92} = 120\underline{/60}\,\text{V}$$

$$Icomp_a = \frac{I_a}{CT} = 3.0321\underline{/-46.9}\,\text{A}$$

$$Icomp_c = \frac{I_c}{CT} = 2.3621\underline{/57.1}\,\text{A}$$

The compensator impedances in Ohms are determined by dividing the settings in volts by the secondary rating of the current transformer:

$$R_{ab} + jX_{ab} = \frac{R'_{ab} + jX'_{ab}}{CT_{secondary}} = \frac{1.6 + j11.5}{5} = 0.32 + j2.3\,\Omega$$

$$R_{cb} + jX_{cb} = \frac{R'_{cb} + jX'_{cb}}{CT_{secondary}} = \frac{10.5 + j6.6}{5} = 2.1 + j1.32\,\Omega$$

The voltages across the voltage relays in the two compensator circuits are

$$Vrelay_{ab} = Vcomp_{ab} - (R_{ab} + jX_{ab}) \cdot Icomp_a = 114.3\underline{/-2.0}\,\text{V}$$

$$Vrelay_{cb} = Vcomp_{cb} - (R_{cb} + jX_{cb}) \cdot Icomp_c = 114.9\underline{/58.6}\,\text{V}$$

Since the voltages are below the lower limit of 119, the control circuit will send "raise" commands to change the taps one at a time on both regulators. For analysis purposes, the approximate number of tap changes necessary to bring the load center voltage into the lower limit of the bandwidth for each regulator will be

$$Tap_{ab} = \frac{|119 - 114.3|}{0.75} = 6.2422 \approx 6$$

$$Tap_{cb} = \frac{|119 - 114.9|}{0.75} = 5.43 \approx 5$$

With the taps set at 6 and 5, a check can be made to determine if the voltages at the load center are now within the limits.

With the taps adjusted, the regulator ratios are

$$a_{R\_ab} = 1.0 - 0.00625 \cdot Tap_{ab} = 0.9625$$

$$a_{R\_cb} = 1.0 = 0.00625 \cdot Tap_{cb} = 0.9688$$

In order to determine the load-side regulator voltages and currents, the matrix $[A_{LL}]$ (Equation 7.119) is then converted to the equivalent $[A_{reg}]$ matrix where the system line-to-neutral voltages are used:

$$[A_{LL}] = \begin{bmatrix} \frac{1}{0.9625} & 0 & 0 \\ 0 & \frac{1}{0.975} & 0 \\ -\frac{1}{0.9625} & -\frac{1}{0.975} & 0 \end{bmatrix} = \begin{bmatrix} 1.039 & 0 & 0 \\ 0 & 1.0256 & 0 \\ -1.039 & -1.0256 & 0 \end{bmatrix}$$

$$[A_{reg}] = [W] \cdot [A_{LL}] \cdot [D] = \begin{bmatrix} 0.6926 & -0.3486 & -0.3441 \\ -0.3463 & 0.6904 & -0.3441 \\ -0.3463 & -0.3419 & 0.6882 \end{bmatrix}$$

Using Equation 128, the current matrix $[d_{reg}]$ is computed to be

$$[d_{reg}] = \begin{bmatrix} 1.039 & 0 & 0 \\ -1.039 & 0 & -1.0323 \\ 0 & 0 & 1.0323 \end{bmatrix}$$

With the taps set at +6 and +5, the output line-to-neutral voltages from the regulators are

$$[VR_{abc}] = [A_{reg}] \cdot [ELN_{ABC}] = \begin{bmatrix} 7480.1/\underline{-29.8} \\ 7455.9/\underline{-150.1} \\ 7431.9/\underline{90.2} \end{bmatrix} \text{V}$$

The line-to-line voltages are

$$[VRLL_{abc}] = [D] \cdot [VR_{abc}] = \begin{bmatrix} 12{,}955.8/\underline{0} \\ 12{,}872.3/\underline{-120} \\ 12{,}914.3/\underline{120} \end{bmatrix} \text{V}$$

The output currents from the source are

$$[I_{ABC}] = \begin{bmatrix} 302.9/\underline{-46.7} \\ 335.4/\underline{176.1} \\ 234.7\underline{57.3} \end{bmatrix} \text{A}$$

The output currents from the regulators are

$$[I_{abc}] = [D_{reg}].[I_{ABC}] = \begin{bmatrix} 291.6\underline{/-46.7} \\ 323.5\underline{/176.3} \\ 227.4\underline{/57.3} \end{bmatrix} \text{A}$$

There are two ways to test if the voltages at the load center are within the limits. The first method is to compute the relay voltages in the compensator circuits. The procedure is the same as was done initially to determine the load center voltages. First the voltages and currents in the compensator circuits are computed:

$$Vcomp_{ab} = \frac{VR_{ab}}{N_{pt}} = \frac{12{,}955.8\underline{/0}}{103.92} = 124.7\underline{/0}\,\text{V}$$

$$Vcomp_{cb} = \frac{-VR_{bc}}{N_{pt}} = \frac{12{,}872.3\underline{/60}}{103.92} = 123.1\underline{/60}\,\text{V}$$

$$Icomp_{a} = \frac{I_a}{CT} = \frac{291.6\underline{/-46.7}}{100} = 2.916\underline{/-58.0}\,\text{A}$$

$$Icomp_{c} = \frac{I_c}{CT} = \frac{227.4\underline{/57.3}}{100} = 2.274\underline{/57.3}\,\text{A}$$

The voltages across the voltage relays are computed to be

$$Vrelay_{ab} = Vcomp_{ab} - (R_{ab} + jX_{ab}) \cdot Icomp_a = 119.2\underline{/-1.9}\,\text{V}$$

$$Vrelay_{cb} = Vcomp_{cb} - (R_{cb} + jX_{cb}) \cdot Icomp_c = 119.0\underline{/58.7}\,\text{V}$$

Since both voltages are within the bandwidth, no further tap changing will be necessary.

The actual voltages at the load center can be computed using the output voltages and currents from the regulator and then computing the voltage drop to the load center.

With reference to Figure 7.19, the equivalent line-to-neutral and actual line-to-line voltages at the load are

$$[VL_{abc}] = [VR_{abc}] - [Z_{abc}] \cdot [I_{abc}] = \begin{bmatrix} 7178.8\underline{/-31.7} \\ 7455.9\underline{/-150.1} \\ 7431.9\underline{/90.2} \end{bmatrix} \text{V}$$

$$[VLLL_{abc}] = [D] \cdot [VL_{abc}] = \begin{bmatrix} 12{,}392.4\underline{/-1.9} \\ 12{,}366.5\underline{/-121.3} \\ 12{,}481.8\underline{/118.5} \end{bmatrix} \text{V}$$

Dividing the load center line-to-line voltages by the potential transformer ratio gives the load line-to-line voltages on the 120 V base as

$$V120_{ab} = 119.35\underline{/-1.9}$$
$$V120_{bc} = 119.0\underline{/-121.3}\text{ V}$$
$$V120_{ca} = 120.1\underline{/118.5}$$

Note how the actual load voltages on the 120 V base match very closely the values computed across the compensator relays. It is also noted that, in this case, the third line-to-line voltage is also within the bandwidth. That will not always be the case.

This example is very long but has been included to demonstrate how the compensator circuit is set and then how it will adjust taps so that the voltages at a remote load center node will be held within the set limits. In actual practice, the only responsibilities of the engineer will be to correctly determine the *R* and *X* settings of the compensator circuit and to determine the desired voltage level and bandwidth.

The open delta regulator connection using phases *AB* and *CB* has been presented. There are two other possible open delta connections using phase *BC* and *AC* and then *CA* and *BA*. Generalized matrices for these additional two connections can be developed using the procedures presented in this section.

7.5 Summary

It has been shown that all possible connections for Type B step-voltage regulators can be modeled using the generalized matrices. The derivations in this chapter were limited to three-phase connections. If a single-phase regulator is connected line to neutral or two regulators connected in open wye, then the $[a]$ and $[d]$ matrices will be of the same form as that of the wye-connected regulators, only the terms in the rows and columns associated with the missing phases would be zero. The same can be said for a single-phase regulator connected line to line. Again, the rows and columns associated with the missing phases would be set to zero in the matrices developed for the open delta connection.

The generalized matrices developed in this chapter are of exactly the same form as those developed for the three-phase line segments. In the next chapter, the generalized matrices for all three-phase transformers will be developed.

Problems

7.1 A single-phase transformer is rated 100 kVA, 2400–240 V. The impedances and shunt admittance of the transformer are

$Z_1 = 0.65 + j0.95\,\Omega$ (high-voltage winding impedance)

$Z_2 = 0.0052 + j0.0078\,\Omega$ (low-voltage winding impedance)

$Y_m = 2.56 \times 10^{-4} - j11.37 \times 10^{-4}$ S (referred to the high-voltage winding)

a. Determine the a, b, c, and d constants and the A and B constants.

b. The transformer is serving an 80 kW, 0.85 lagging power factor load at 230 V. Determine the primary voltage, current, and complex power.

c. Determine the per-unit transformer impedance and shunt admittance based upon the transformer ratings.

7.2 The single-phase transformer of Problem 7.1 is to be connected as a step-down autotransformer to transform the voltage from 2400 V down to 2160 V.

a. Draw the connection diagram including the series impedance and shunt admittance.

b. Determine the autotransformer kVA rating.

c. Determine the a, b, c, d, A, and B generalized constants.

d. The autotransformer is serving a load of 80 kVA, 0.95 lagging power factor at a voltage of 2000 V. Including the impedance and shunt admittance, determine the input voltage, current, and complex power.

e. Determine the per-unit impedance and shunt admittance based upon the autotransformer rating. How do these values compare to the per-unit values of Problem 7.1?

7.3 A "Type B" step-voltage regulator is installed to regulate the voltage on a 7200-120 V single-phase lateral. The potential transformer and current transformer ratios connected to the compensator circuit are

Potential transformer: 7200-120 V and 500:5 A
Current transformer: 500 : 5 A
The R and X settings in the compensator circuit: $R = 5$ V and $X = 10$ V

The regulator tap is set on the +10 position when the voltage and current on the source side of the regulator are $V_{source} = 7200$ V and $I_{source} = 375$ at a power factor of 0.866 lagging power factor.

a. Determine the voltage at the load center.

b. Determine the equivalent line impedance between the regulator and the load center.

c. Assuming that the voltage level on the regulator has been set at 120 V with a bandwidth of 2 V, what tap will the regulator move to?

7.4 Refer to Figure 7.11. The substation transformer is rated 24 MVA, 230 kV delta-13.8 kV wye. Three single-phase Type B regulators are connected in wye. The equivalent line impedance between the regulators and the load center node is

$$Z_{line} = 0.264 + j0.58\,\Omega/\text{mile}$$

The distance to the load center node is 10,000 ft.

a. Determine the appropriate *PT* and *CT* ratios.

b. Determine the R' and X' settings in Ohms and volts for the compensator circuit.

c. The substation is serving a balanced three-phase load of 16 MVA, 0.9 lagging power factor when the output line-to-line voltages of the substation are balanced 13.8 kV and the regulators are set in the neutral position. Assume the voltage level is set at 121 V and a bandwidth of 2 V. Determine the final tap position for each regulator (they will be the same). The regulators have a total of 32 steps (16 raise and 16 lower) with each step changing the voltage by 5/8%.

d. What would be the regulator tap settings for a load of 24 MVA, 0.9 lagging power factor with the output voltages of the substation transformer are balanced three-phase 13.8 kV.

e. What would be the load center voltages for the load of part (d)?

7.5 Three Type B step-voltage regulators are connected in wye and located on the secondary bus of a 12.47 kV substation. The feeder is serving an unbalanced load. A power-flow study has been run and the voltages at the substation and the load center node are

$$[Vsub_{abc}] = \begin{bmatrix} 7200/\underline{0} \\ 7200/\underline{-120} \\ 7200/\underline{120} \end{bmatrix} \text{V}$$

$$[VLC_{abc}] = \begin{bmatrix} 6890.6/-1.49 \\ 6825.9/-122.90 \\ 6990.5/117.05 \end{bmatrix} \text{V}$$

The currents at the substation are

$$[I_{abc}] = \begin{bmatrix} 362.8\underline{/-27.3} \\ 395.4\underline{/-154.7} \\ 329.0\underline{/98.9} \end{bmatrix} \text{A}$$

The regulator potential transformer ratio is 7200-120 and the current transformer ratio is 500:5. The voltage level of the regulators is set at 121 V and the bandwidth at 2 V.

a. Determine the equivalent line impedance per phase between the regulator and the load center.
b. The compensators on each regulator are to be set with the same *R* and *X* values. Specify these values in volts and in Ohms.

7.6 The impedance compensator settings for the three step-regulators of Problem 7.5 have been set as

$$R' = 3.0\,\text{V} \quad X' = 9.3\,\text{V}$$

The voltages and currents at the substation bus are

$$[Vsub_{abc}] = \begin{bmatrix} 7200\underline{/0} \\ 7200\underline{/-120} \\ 7200\underline{/120} \end{bmatrix} \text{V}$$

$$[I_{abc}] = \begin{bmatrix} 320.6\underline{/-27.4} \\ 409.0\underline{/-155.1} \\ 331.5\underline{/98.2} \end{bmatrix} \text{A}$$

Determine the final tap settings for each regulator.

7.7 For a different load condition for the system of Problem 7.5, the taps on the regulators have been automatically set by the compensator circuit to

$$Tap_a = +8 \quad Tap_b = +11 \quad Tap_c = +6$$

The load reduces so that the voltages and currents at the substation bus are

$$[Vsub_{abc}] = \begin{bmatrix} 7200\underline{/0} \\ 7200\underline{/-120} \\ 7200\underline{/120} \end{bmatrix} \text{V}$$

$$[I_{abc}] = \begin{bmatrix} 177.1/\underline{-28.5} \\ 213.4/\underline{-156.4} \\ 146.8/\underline{98.3} \end{bmatrix} \text{A}$$

Determine the new final tap settings for each regulator.

7.8 The load center node for the regulators described in Problem 7.5 is located 1.5 miles from the substation. There are no lateral taps between the substation and the load center. The phase impedance matrix of the line segment is

$$[z_{abc}] = \begin{bmatrix} 0.3465 + j1.0179 & 0.1560 + j0.5017 & 0.1580 + j0.4236 \\ 0.1560 + j0.5017 & 0.3375 + j1.0478 & 0.1535 + j0.3849 \\ 0.1580 + j0.4236 & 0.1535 + j0.3849 & 0.3414 + j1.0348 \end{bmatrix} \Omega/\text{mile}$$

A wye-connected, unbalanced constant impedance load is located at the load center node. The load impedances are

$$ZL_a = 19 + j11\,\Omega, \quad ZL_b = 22 + j12\,\Omega, \quad ZL_c = 18 + j10\,\Omega$$

The voltages at the substation are balanced three phase of 7200 V line to neutral. The regulators are set on neutral.

a. Determine the line-to-neutral voltages at the load center.
b. Determine the *R* and *X* settings in volts for the compensator.
c. Determine the required tap settings in order to hold the load center voltages within the desired limits.

7.9 The *R* and *X* values of the compensator have been set at 2.2 + *j*7.4 V. Use the source voltages from Problem 7.6. For this problem, the loads are wye connected and modeled such that the per-phase load kVA and power factor (constant PQ loads) are

$$[kVA] = \begin{bmatrix} 1400 \\ 1800 \\ 1200 \end{bmatrix} \quad [PF] = \begin{bmatrix} 0.90 \\ 0.85 \\ 0.95 \end{bmatrix}$$

Determine

a. The final regulator tap positions
b. The compensator relay voltages
c. The load line-to-neutral voltages on a 120 V base

7.10 The phase impedance matrix for a three-wire line segment is

$$[z_{abc}] = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7802 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0953 + j0.7266 \\ 0.0953 + j0.7802 & 0.0953 + j0.7266 & 0.4013 + j1.4133 \end{bmatrix} \Omega/\text{mile}$$

The line is serving an unbalanced load so that at the substation transformer line-to-line voltages and output currents are

$$[VLL_{abc}] = \begin{bmatrix} 12{,}470\underline{/0} \\ 12{,}470\underline{/-120} \\ 12{,}470\underline{/120} \end{bmatrix} \text{V}$$

$$[I_{abc}] = \begin{bmatrix} 307.9\underline{/-54.6} \\ 290.6\underline{/178.6} \\ 268.2\underline{/65.3} \end{bmatrix} \text{A}$$

Two Type B step-voltage regulators are connected in open delta at the substation using phases *AB* and *CB*. The potential transformer ratios are 12,470/120 and the current transformer ratios are 500 : 5. The voltage level is set at 121 V with a 2 V bandwidth.

a. Determine the line-to-line voltages at the load center.
b. Determine the *R* and *X* compensator settings in volts. For the open delta connection, the *R* and *X* settings will be different on each regulator.
c. Determine the final tap positions of the two voltage regulators.

7.11 The regulators in Problem 7.10 have gone to the +9 tap on both regulators for a particular load. The load is reduced so that the currents leaving the substation transformer with the regulators in the +9 position are

$$[I_{abc}] = \begin{bmatrix} 144.3\underline{/-53.5} \\ 136.3\underline{/179.6} \\ 125.7\underline{/66.3} \end{bmatrix} \text{A}$$

Determine the final tap settings on each regulator for this new load condition.

7.12 Use the system of Example 7.8 with the delta-connected loads changed to

$$[kVA] = \begin{bmatrix} 1800 \\ 1500 \\ 2000 \end{bmatrix} \quad [PF] = \begin{bmatrix} 0.90 \\ 0.95 \\ 0.92 \end{bmatrix}$$

The source voltages, potential transformer, and current transformer ratings are those in the example. The desired voltage level is set at 122 V with a bandwidth of 2 V. For this load condition determine the following:

a. Use the *R* and *X* compensator values from Example 7.8.
b. The required tap positions
c. The final relay voltages
d. The final load line-to-line load voltages and the line currents

Windmil Assignment

Use System 3 and add a step-voltage regulator connected between the source and the three-phase OH line. Call this "System 4." The regulator is to be set with a specified voltage level of 122 V at node 2. The potential transformer ratio is 7200-129 and CT ratio is 200:5. Call the regulator Reg-1. Follow these steps in the user's manual on how to install the three-phase wye-connected regulators.

1. Follow the steps outlined in the user's manual to have Windmil determine the *R* and *X* settings to hold the specified voltage level at node 2.
2. Run "Voltage Drop." Check the node voltages and, in particular, the voltage at node 2.
3. What taps did the regulators go to?
4. In Example 7.7, a method to hand calculate the compensator *R* and *X* setting was demonstrated. Follow that procedure to compute the *R* and *X* settings and compare to the Windmil settings.

References

1. *American Nation Standard for Electric Power—Systems and Equipment Voltage Ratings (60 Hertz)*, ANSI C84.1-1995, National Electrical Manufacturers Association, Rosslyn, VA, 1996.
2. *IEEE Standard Requirements, Terminology, and Test Code for Step-Voltage and Induction-Voltage Regulators*, ANSI/IEEE C57.15-1986, Institute of Electrical and Electronic Engineers, New York, 1988.

8

Three-Phase Transformer Models

Three-phase transformer banks are found in the distribution substation where the voltage is transformed from the transmission or subtransmission level to the distribution feeder level. In most cases, the substation transformer will be a three-phase unit perhaps with high-voltage no-load taps and, perhaps, low-voltage load tap changing (LTC). For a four-wire wye feeder, the most common substation transformer connection is the delta–grounded wye. A three-wire delta feeder will typically have a delta–delta transformer connection in the substation. Three-phase transformer banks out on the feeder will provide the final voltage transformation to the customer's load. A variety of transformer connections can be applied. The load can be pure three-phase or a combination of single-phase lighting load and a three-phase load such as an induction motor. In the analysis of a distribution feeder, it is important that the various three-phase transformer connections be modeled correctly.

Unique models of three-phase transformer banks applicable to radial distribution feeders will be developed in this chapter. Models for the following three-phase connections are included:

- Delta–grounded wye
- Ungrounded wye–delta
- Grounded wye–delta
- Open wye–open delta
- Grounded wye–grounded wye
- Delta–delta
- Open delta–open delta

8.1 Introduction

Figure 8.1 defines the various voltages and currents for all three-phase transformer banks connected between the source-side node ***n*** and the load-side node ***m***.

In Figure 8.1, the models can represent a step-down (source side to load side) or a step-up (source side to load side) transformer bank. The notation

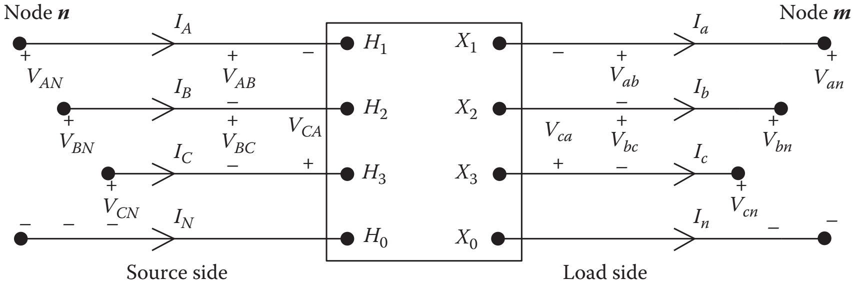

FIGURE 8.1
General three-phase transformer bank.

is such that the capital letters *A*, *B*, *C*, and *N* will always refer to the *source* side (node ***n***) of the bank and the lower case letters *a*, *b*, *c*, and *n* will always refer to the **load** side (node ***m***) of the bank. It is assumed that all variations of the wye–delta connections are connected in the "American Standard Thirty Degree" connection. The described phase notation and the standard phase shifts for positive sequence voltages and currents are

Step-down connection

$$V_{AB} \text{ leads } V_{ab} \text{ by } 30° \tag{8.1}$$

$$I_A \text{ leads } I_a \text{ by } 30° \tag{8.2}$$

Step-up connection

$$V_{ab} \text{ leads } V_{AB} \text{ by } 30° \tag{8.3}$$

$$I_a \text{ leads } I_A \text{ by } 30° \tag{8.4}$$

8.2 Generalized Matrices

The models to be used in power-flow and short-circuit studies are generalized for the connections in the same form as have been developed for line segments (Chapter 6) and voltage regulators (Chapter 7). In the "forward sweep" of the "ladder" iterative technique described in Chapter 10, the voltages at node ***m*** are defined as a function of the voltages at node ***n*** and the currents at node ***m***. The required equation is

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \tag{8.5}$$

In the "backward sweep" of the ladder technique, the matrix equations for computing the voltages and currents at node ***n*** as a function of the voltages and currents at node ***m*** are given by

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] \tag{8.6}$$

$$[I_{ABC}] = [c_t] \cdot [VLN_{abc}] + [d_t] \cdot [I_{abc}] \tag{8.7}$$

In Equations 8.5 through 8.7, the matrices $[VLN_{ABC}]$ and $[VLN_{abc}]$ represent the line-to-neutral voltages for an ungrounded wye connection or the line-to-ground voltages for a grounded wye connection. For a delta connection, the voltage matrices represent "equivalent" line-to-neutral voltages. The current matrices represent the line currents regardless of the transformer winding connection.

In the modified ladder technique, Equation 8.5 is used to compute new node voltages downstream from the source using the most recent line currents. In the backward sweep, only Equation 8.7 is used to compute the source-side line currents using the newly computed load-side line currents.

8.3 Delta–Grounded Wye Step-Down Connection

The delta–grounded wye step-down connection is a popular connection that is typically used in a distribution substation serving a four-wire wye feeder system. Another application of the connection is to provide service to a load that is primary single phase. Because of the wye connection, three single-phase circuits are available thereby making it possible to balance the single-phase loading on the transformer bank.

Three single-phase transformers can be connected delta–grounded wye in a "standard thirty degree step-down connection" as shown in Figure 8.2.

8.3.1 Voltages

The positive sequence phasor diagrams of the voltages in Figure 8.2 show the relationships between the various positive sequence voltages. Care must be taken to observe the polarity marks on the individual transformer windings. In order to simplify the notation, it is necessary to label the "ideal" voltages with voltage polarity markings as shown in Figure 8.2. Observing the polarity markings of the transformer windings, the voltage Vt_a will be 180° out of phase with the voltage V_{CA} and the voltage Vt_b will be 180° out of phase with the voltage V_{AB}. Kirchhoff's voltage law (KVL) gives the line-to-line voltage between phases a and b as

$$V_{ab} = Vt_a - Vt_b \tag{8.8}$$

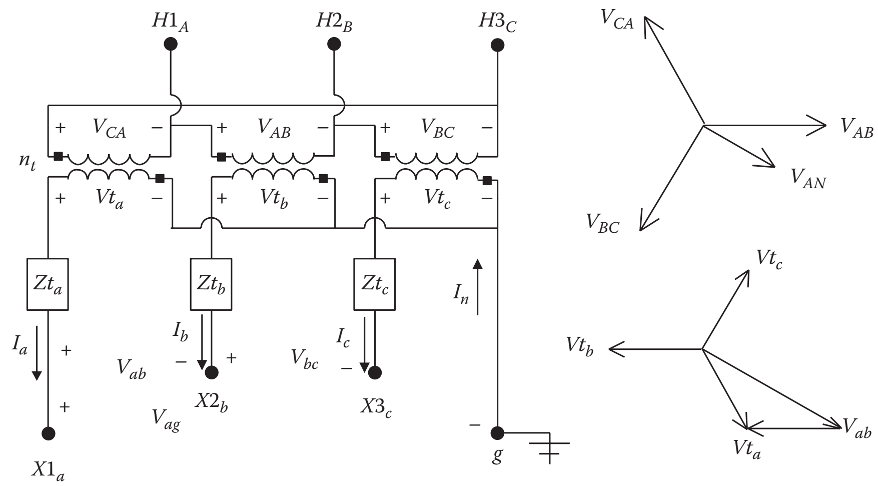

FIGURE 8.2
Standard delta–grounded wye connection with voltages.

The phasors of the positive sequence voltages of Equation 8.8 are shown in Figure 8.2.

The magnitude changes between the voltages can be defined in terms of the actual winding turns ratio (n_t). With reference to Figure 8.2, these ratios are defined as follows:

$$n_t = \frac{VLL_{rated\,primary}}{VLN_{rated\,secondary}} \tag{8.9}$$

With reference to Figure 8.2, the line-to-line voltages on the primary side of the transformer connection as a function of the ideal secondary side voltages are given by

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} \quad [VLL_{ABC}] = [AV] \cdot [Vt_{abc}]$$

$$\text{where } [AV] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \tag{8.10}$$

Equation 8.10 gives the primary line-to-line voltages at node ***n*** as a function of the ideal secondary voltages. However, what is needed is a relationship between "equivalent" line-to-neutral voltages at node ***n*** and the ideal secondary voltages. The question is how is the equivalent line-to-neutral voltages determined knowing the line-to-line voltages? One approach is to apply the theory of symmetrical components.

The known line-to-line voltages are transformed to their sequence voltages by

$$[VLL_{012}] = [A_s]^{-1} \cdot [VLL_{ABC}] \tag{8.11}$$

where

$$[A_s] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \tag{8.12}$$

$$a_s = 1.0\underline{/120}$$

By definition, the zero sequence line-to-line voltage is always zero. The relationship between the positive and negative sequence line-to-neutral and line-to-line voltages is known. These relationships in matrix form are given by

$$\begin{bmatrix} VLN_0 \\ VLN_1 \\ VLN_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t_s^* & 0 \\ 0 & 0 & t_s \end{bmatrix} \cdot \begin{bmatrix} VLL_0 \\ VLL_1 \\ VLL_2 \end{bmatrix} \quad [VLN_{012}] = [T] \cdot [VLL_{012}] \tag{8.13}$$

where $t = \dfrac{1}{\sqrt{3}}\underline{/30}$

Since the zero sequence line-to-line voltage is zero, the (1,1) term of the matrix $[T]$ can be any value. For the purposes here, the (1,1) term is chosen to have a value of 1.0. Knowing the sequence line-to-neutral voltages, the equivalent line-to-neutral voltages can be determined.

The equivalent line-to-neutral voltages as a function of the sequence line-to-neutral voltages are

$$[VLN_{ABC}] = [A_s] \cdot [VLN_{012}] \tag{8.14}$$

Substitute Equation 8.13 into Equation 8.14:

$$[VLN_{ABC}] = [A_s] \cdot [T] \cdot [VLL_{012}] \tag{8.15}$$

Substitute Equation 8.11 into Equation 8.15:

$$[VLN_{ABC}] = [W] \cdot [VLL_{ABC}] \tag{8.16}$$

where

$$[W] = [A_s] \cdot [T] \cdot [A_s]^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \tag{8.17}$$

Equation 8.17 provides a method of computing equivalent line-to-neutral voltages from a knowledge of the line-to-line voltages. This is an important relationship that will be used in a variety of ways as other three-phase transformer connections are studied.

To continue on, Equation 8.16 can be substituted into Equation 8.10:

$$[VLN_{ABC}] = [W] \cdot [VLL] = [W] \cdot [AV] \cdot [Vt_{abc}] = [a_t] \cdot [Vt_{abc}] \tag{8.18}$$

where

$$[a_t] = [W] \cdot [AV] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} \tag{8.19}$$

Equation 8.19 defines the generalized $[a_t]$ matrix for the delta–grounded wye step-down connection.

The ideal secondary voltages as a function of the secondary line-to-ground voltages and the secondary line currents are

$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \tag{8.20}$$

where

$$[Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \tag{8.21}$$

Note in Equation 8.21 there is no restriction that the impedances of the three transformers be equal.

Substitute Equation 8.20 into Equation 8.18:

$$[VLN_{ABC}] = [a_t] \cdot \left([VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}]\right)$$

$$[VLN_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \tag{8.22}$$

where

$$[b_t]=[a_t]\cdot[Zt_{abc}]=\frac{-n_t}{3}\cdot\begin{bmatrix}0 & 2\cdot Zt_b & Zt_c\\ Zt_a & 0 & 2\cdot Zt_c\\ 2\cdot Zt_a & Zt_b & 0\end{bmatrix} \tag{8.23}$$

The generalized matrices $[a_t]$ and $[b_t]$ have now been defined. The derivation of the generalized matrices $[A_t]$ and $[B_t]$ begins with solving Equation 8.10 for the ideal secondary voltages:

$$[Vt_{abc}]=[AV]^{-1}\cdot[VLL_{ABC}] \tag{8.24}$$

The line-to-line voltages as a function of the equivalent line-to-neutral voltages are

$$[VLL_{ABC}]=[D]\cdot[VLN_{ABC}] \tag{8.25}$$

where

$$[D]=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ -1 & 0 & 1\end{bmatrix} \tag{8.26}$$

Substitute Equation 8.25 into Equation 8.24:

$$[Vt_{abc}]=[AV]^{-1}\cdot[D]\cdot[VLN_{ABC}]=[A_t]\cdot[VLN_{ABC}] \tag{8.27}$$

where

$$[A_t]=[AV]^{-1}\cdot[D]=\frac{1}{n_t}\cdot\begin{bmatrix}1 & 0 & -1\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix} \tag{8.28}$$

Substitute Equation 8.20 into Equation 8.27:

$$[VLG_{abc}]+[Zt_{abc}]\cdot[I_{abc}]=[A_t]\cdot[VLN_{ABC}] \tag{8.29}$$

Rearrange Equation 8.29:

$$[VLG_{abc}]=[A_t]\cdot[VLN_{ABC}]-[B_t]\cdot[I_{abc}] \tag{8.30}$$

where

$$[B_t]=[Zt_{abc}]=\begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \tag{8.31}$$

Equation 8.22 is referred to as the "backward sweep voltage equation" and Equations 8.30 is referred to as the "forward sweep voltage equation." Equations 8.22 and 8.30 apply only for the step-down delta–grounded wye transformer. Note that these equations are in exactly the same form as those derived in earlier chapters for line segments and step-voltage regulators.

8.3.2 Currents

The thirty degree connection specifies that the positive sequence current entering the H_1 terminal will lead the positive sequence current leaving the X_1 terminal by 30°. Figure 8.3 shows the same connection as Figure 8.2 but with the currents instead of the voltages displayed.

As with the voltages, the polarity marks on the transformer windings must be observed for the currents. For example, in Figure 8.3, the current I_a is entering the polarity mark on the low-voltage winding so the current I_{AC} flowing out of the polarity mark on the high-voltage winding will be in phase with I_a. This relationship is shown in the phasor diagrams for positive sequence currents in Figure 8.3.

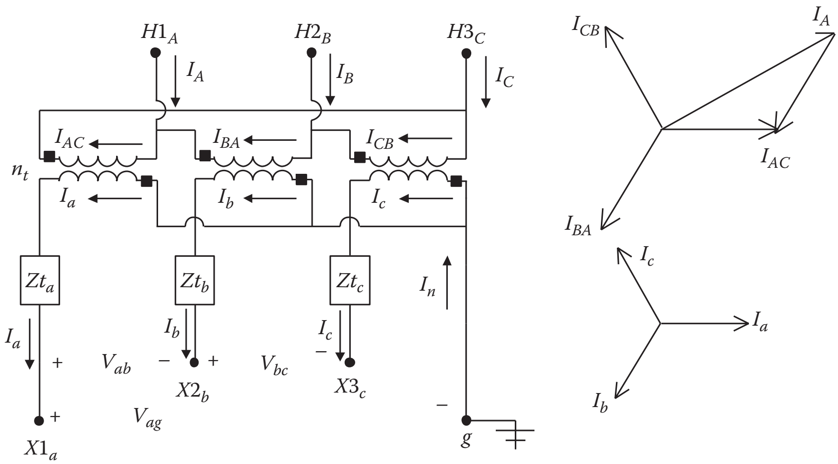

FIGURE 8.3
Delta–grounded wye connection with currents.

The line currents can be determined as a function of the delta currents by applying Kirchhoff's current law (KCL):

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AC} \\ I_{BA} \\ I_{CB} \end{bmatrix} \tag{8.32}$$

In condensed form, Equation 8.32 is

$$[I_{ABC}] = [D] \cdot [ID_{ABC}]$$

$$\text{where } [D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{8.33}$$

The matrix equation relating the delta primary currents to the secondary line currents is given by

$$\begin{bmatrix} I_{AC} \\ I_{BA} \\ I_{CB} \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{8.34}$$

$$[ID_{ABC}] = [AI] \cdot [I_{abc}]$$

$$\text{where } [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8.35}$$

Substitute Equation 8.35 into Equation 8.33:

$$[I_{ABC}] = [D] \cdot [AI] \cdot [I_{abc}] = [c_t] \cdot [VLG_{abc}] + [d_t] \cdot [I_{abc}] \tag{8.36}$$

where

$$[d_t] = [D] \cdot [AI] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{8.37}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{8.38}$$

Equation 8.36 (referred to as the "backward sweep current equations") provides a direct method of computing the phase line currents at node n knowing the phase line currents at node m. Again, this equation is in the same form as that previously derived for three-phase line segments and three-phase step-voltage regulators.

The equations derived in this section are for the step-down connection. Section 8.4 summarizes the matrices for the delta–grounded wye step-up connection.

Example 8.1

In the example system of Figure 8.4, an unbalanced constant impedance load is being served at the end of a 1 mile section of a three-phase line. The 1 mile long line is being fed from a substation transformer rated 5000 kVA, 115 kV delta–12.47 kV grounded wye with a per-unit impedance of $0.085\underline{/85}$. The phase conductors of the line are 336,400 26/7 ACSR with a neutral conductor 4/0 ACSR. The configuration and computation of the phase impedance matrix are given in Example 4.1. From that example, the phase impedance matrix was computed to be

$$[Zline_{abc}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

The general matrices for the line are

$$[A_{line}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad [B_{line}] = [Zline_{abc}] \quad [d] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The transformer impedance needs to be converted to per unit referenced to the low-voltage side of the transformer. The base impedance is

$$Z_{base} = \frac{12.47^2 \cdot 1000}{5000} = 31.1$$

The transformer impedance referenced to the low-voltage side is

$$Zt = (0.085\underline{/85}) \cdot 31.3 = 0.2304 + j2.6335\,\Omega$$

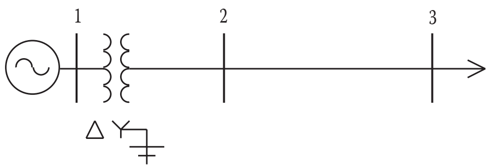

FIGURE 8.4
Example system.

The transformer phase impedance matrix is

$$[Zt_{abc}] = \begin{bmatrix} 0.2304 + j2.6335 & 0 & 0 \\ 0 & 0.2304 + j2.6335 & 0 \\ 0 & 0 & 0.2304 + j2.6335 \end{bmatrix} \Omega$$

The unbalanced constant impedance load is connected in grounded wye. The load impedance matrix is specified to be

$$[Zload_{abc}] = \begin{bmatrix} 12 + j6 & 0 & 0 \\ 0 & 13 + j4 & 0 \\ 0 & 0 & 14 + j5 \end{bmatrix} \Omega$$

The unbalanced line-to-line voltages at node 1 serving the substation transformer are given as

$$[VLL_{ABC}] = \begin{bmatrix} 115{,}000\underline{/0} \\ 116{,}500\underline{/-115.5} \\ 123{,}538\underline{/121.7} \end{bmatrix} \text{V}$$

a. Determine the generalized matrices for the transformer.
The "transformer turn's" ratio is

$$n_t = \frac{kVLL_{high}}{kVLN_{low}} = \frac{115}{12.47/\sqrt{3}} = 15.9732$$

From Equation 8.19,

$$[a_t] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -10.6488 & -5.3244 \\ -5.3244 & 0 & -10.6488 \\ -10.6488 & -5.3244 & 0 \end{bmatrix}$$

From Equation 8.23,

$$[b_t] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 \cdot Zt & Zt \\ Zt & 0 & 2 \cdot Zt \\ 2 \cdot Zt & Zt & 0 \end{bmatrix}$$

$$[b_t] = \begin{bmatrix} 0 & -2.4535 - j28.0432 & -1.2267 - j14.0216 \\ -1.2267 - j14.0216 & 0 & -2.4535 - j28.0432 \\ -2.4535 - j28.0432 & -1.2267 - j14.0216 & 0 \end{bmatrix}$$

From Equation 8.37,

$$[d_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.0626 & -0.0626 & 0 \\ 0 & 0.0626 & -0.0626 \\ -0.0626 & 0 & 0.0626 \end{bmatrix}$$

From Equation 8.28,

$$[A_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0.0626 & 0 & -0.0626 \\ -0.0626 & 0.0626 & 0 \\ 0 & -0.0626 & 0.0626 \end{bmatrix}$$

From Equation 8.31,

$$[B_t] = [Zt_{abc}] = \begin{bmatrix} 0.2304 + j2.6335 & 0 & 0 \\ 0 & 0.2304 + j2.6335 & 0 \\ 0 & 0 & 0.2304 + j2.6335 \end{bmatrix}$$

b. Given the line-to-line voltages at node 1, determine the "ideal" transformer voltages.
   From Equation 8.13,

$$[AV] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -15.9732 & 0 \\ 0 & 0 & -15.9732 \\ -15.9732 & 0 & 0 \end{bmatrix}$$

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLL_{ABC}] = \begin{bmatrix} 7734.1/\underline{-58.3} \\ 7199.6/\underline{180} \\ 7293.5/\underline{64.5} \end{bmatrix} \text{V}$$

c. Determine the load currents.
   Since the load is modeled as constant impedances, the system is linear and the analysis can combine all of the impedances (transformer, line, and load) to an equivalent impedance matrix. KVL gives

$$[Vt_{abc}] = \left([Zt_{abc}] + [Zline_{abc}] + [Zload_{abc}]\right) \cdot [I_{abc}] = [Zeq_{abc}] \cdot [I_{abc}]$$

$$[Zeq_{abc}] = \begin{bmatrix} 13.0971 + j10.6751 & 0.2955 + j0.9502 & 0.2907 + j0.7290 \\ 0.2955 + j0.9502 & 14.1141 + j8.6187 & 0.2992 + j0.8023 \\ 0.2907 + j0.7290 & 0.2992 + j0.8023 & 15.1045 + j9.6507 \end{bmatrix} \Omega$$

The line currents can now be computed:

$$[I_{abc}] = [Ztotal_{abc}]^{-1} \cdot [Vt_{abc}] = \begin{bmatrix} 471.7\angle\underline{-95.1} \\ 456.7\angle\underline{149.9} \\ 427.3\angle\underline{33.5} \end{bmatrix} \text{A}$$

d. Determine the line-to-ground voltages at the load in volts and on a 120 V base:

$$[Vload_{abc}] = [Zload_{abc}] \cdot [I_{abc}] = \begin{bmatrix} 6328.1\angle\underline{-68.6} \\ 6212.2\angle\underline{167.0} \\ 6352.6\angle\underline{53.1} \end{bmatrix} \text{V}$$

The load voltages on a 120 V base are

$$[Vload_{120}] = \begin{bmatrix} 105.5 \\ 103.5 \\ 105.9 \end{bmatrix}$$

The line-to-ground voltages at node 2 are

$$[VLG_{abc}] = [a_{line}] \cdot [Vload_{abc}] + [b_{line}] \cdot [I_{abc}] = \begin{bmatrix} 6965.4\angle\underline{-66.0} \\ 6580.6\angle\underline{171.4} \\ 6691.4\angle\underline{56.7} \end{bmatrix} \text{V}$$

e. Using the backward sweep voltage equation, determine the equivalent line-to-neutral voltages and the line-to-line voltages at node 1:

$$[VLN_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] . [I_{abc}] = \begin{bmatrix} 69{,}443\angle\underline{-30.3} \\ 65{,}263\angle\underline{-147.5} \\ 70{,}272\angle\underline{94.0} \end{bmatrix} \text{V}$$

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] = \begin{bmatrix} 115{,}000\angle\underline{0} \\ 116{,}500\underline{-115.5} \\ 123{,}538\angle\underline{121.7} \end{bmatrix} \text{V}$$

It is always comforting to be able to work back and compute what was initially given. In this case, the line-to-line voltages at node 1 have been computed and the same values result that were given at the start of the problem.

f. Use the forward sweep voltage equation to verify that the line-to-ground voltages at node 2 can be computed knowing the equivalent line-to-neutral voltages at node 1 and the currents leaving node 2:

$$[VLG_{abc}] = [A_t]\cdot[VLN_{ABC}] - [B_t]\cdot[I_{abc}] = \begin{bmatrix} 6965.4/\underline{-66.0} \\ 6580.6/\underline{171.4} \\ 6691.4/\underline{56.7} \end{bmatrix} \text{V}$$

These are the same values of the line-to-ground voltages at node 2 that were determined working from the load toward the source.

Example 8.1 has demonstrated the application of the forward and backward sweep equations. The example also provides verification that the same voltages and currents result working from the load toward the source or from the source toward the load.

In Example 8.2, the system of Example 8.1 is used only this time the source voltages at node 1 are specified and the three-phase load is specified as constant *PQ*. Because this makes the system nonlinear, the ladder iterative technique must be used to solve for the system voltages and currents.

Example 8.2

Use the system of Example 8.1. The source voltages at node 1 are

$$VLL_{ABC} = \begin{bmatrix} 115{,}000/\underline{0} \\ 115{,}000/\underline{-120} \\ 115{,}000/\underline{120} \end{bmatrix}$$

The wye-connected loads are

$$[kVA] = \begin{bmatrix} 1700 \\ 1200 \\ 1500 \end{bmatrix} \quad [PF] = \begin{bmatrix} 0.90 \\ 0.85 \\ 0.95 \end{bmatrix}$$

The complex powers of the loads are computed to be

$$SL_i = kVA_i \cdot e^{j\cdot a\cos(PF_i)} = \begin{bmatrix} 1530 + j741.0 \\ 1020 + j632.1 \\ 1425 + j468.4 \end{bmatrix} \text{kW} + j\text{kvar}$$

The ladder iterative technique must be used to analyze the system. A simple Mathcad® program is shown in Figure 8.5.

$$\left|VLL_{ABC_i}\right| = \begin{pmatrix} 115000 \\ 115000 \\ 115000 \end{pmatrix} \quad \frac{\arg\left(VLL_{ABC_i}\right)}{deg} = \begin{pmatrix} 0 \\ -120 \\ 120 \end{pmatrix} \qquad VM = 7199.5579$$

$$Start := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad Tol := .000001 \qquad VM := \frac{kVLL_{sec} \cdot 1000}{\sqrt{3}}$$

$$
X := \left|
\begin{array}{l}
I_{abc} \leftarrow Start \\
Iload_{abc} \leftarrow Start \\
V_{old} \leftarrow Start \\
ELN_{ABC} \leftarrow W \cdot ELL_{ABC} \\
\text{for } n \in 1 .. 200 \\
\quad \left|
\begin{array}{l}
V2LN_{abc} \leftarrow A_t \cdot ELN_{ABC} - B_t \cdot I_{abc} \\
V3LN_{abc} \leftarrow A_{line} \cdot V2LN_{abc} - B_{line} \cdot Iload_{abc} \\
\text{for } j \in 1 .. 3 \\
\quad Iload_{abc_j} \leftarrow \overline{\left(\dfrac{SL_j \cdot 1000}{V3LN_{abc_j}}\right)} \\
\text{for } k \in 1 .. 3 \\
\quad Error_k \leftarrow \dfrac{\left|V3LN_{abc_k} - V_{old_k}\right|}{VM} \\
Error_{max} \leftarrow \max(Error) \\
\text{break if } Error_{max} < Tol \\
V_{old} \leftarrow V3LN_{abc} \\
I_{abc} \leftarrow d_{line} \cdot Iload_{abc} \\
I_{ABC} \leftarrow d_t \cdot I_{abc}
\end{array}
\right. \\
Out_1 \leftarrow V3LN_{abc} \\
Out_2 \leftarrow V2LN_{abc} \\
Out_3 \leftarrow I_{abc} \\
Out_4 \leftarrow I_{ABC} \\
Out_5 \leftarrow n \\
Out
\end{array}
\right.
$$

FIGURE 8.5
Mathcad® Program.

Note in this program that in the forward sweep the secondary transformer voltages are first computed and then those are used to compute the voltages at the loads. At the end of the routine, the newly calculated line currents are taken back to the top of the routine and used to compute the new voltages. This continues until the error in the difference between the two most recently calculated load voltages are less than the tolerance. As a last step, after conversion, the primary currents of the transformer are computed.

After nine iterations, the load voltages and currents are

$$[VLN_{load}] = \begin{bmatrix} 6490.1\underline{/-66.7} \\ 6772.4\underline{/176.2} \\ 6699.4\underline{/53.9} \end{bmatrix}$$

$$[I_{abc}] = \begin{bmatrix} 261.9\underline{/-92.5} \\ 177.2\underline{/144.4} \\ 223.9\underline{/35.7} \end{bmatrix}$$

The primary currents are

$$[I_{ABC}] = \begin{bmatrix} 24.3\underline{/-70.0} \\ 20.5\underline{/-175.2} \\ 27.4\underline{/63.8} \end{bmatrix}$$

The magnitude of the load voltages on a 120 V base are

$$[Vload_{120}] = \begin{bmatrix} 108.2 \\ 112.9 \\ 111.7 \end{bmatrix}$$

Needless to say, these voltages are not acceptable. In order to correct this problem, three step-voltage regulators can be installed at the secondary terminals of the substation transformer as shown in Figure 8.6.

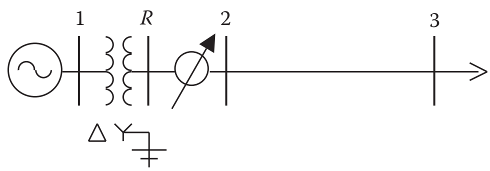

FIGURE 8.6
Voltage regulators installed.

Using the method as outlined in Chapter 7, the final steps for the three regulators are

$$[Tap] = \begin{bmatrix} 14 \\ 9 \\ 10 \end{bmatrix}$$

The regulator turns ratios are

$$aR_i = 1 - 0.00625 \cdot Tap_i = \begin{bmatrix} 0.9125 \\ 0.9438 \\ 0.9375 \end{bmatrix}$$

The regulator matrices are

$$[A_{reg}] = [d_{reg}] = \begin{bmatrix} \frac{1}{aR_1} & 0 & 0 \\ 0 & \frac{1}{aR_2} & 0 \\ 0 & 0 & \frac{1}{aR_3} \end{bmatrix} = \begin{bmatrix} 1.0959 & 0 & 0 \\ 0 & 1.0596 & 0 \\ 0 & 0 & 1.0667 \end{bmatrix}$$

$$[B_{reg}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

At the start of the Mathcad routine, the following equation is added:

$$I_{reg} \leftarrow Start$$

In the Mathcad routine, the first three equations inside the *n* loop are

$$VR_{abc} \leftarrow A_t \cdot ELN_{ABC} - B_t \cdot I_{reg}$$

$$V2LN_{abc} \leftarrow A_{reg} \cdot V\mathrm{Re}\,g_{abc} - B_{reg} \cdot I_{abc}$$

$$V3LN_{abc} \leftarrow A_{line} \cdot V2LN_{abc} - B_{line} \cdot I_{abc}$$

At the end of the loop, the following equations are added:

$$I_{reg} \leftarrow d_{reg} \cdot I_{abc}$$

$$I_{ABC} \leftarrow d_t \cdot I_{reg}$$

With the three regulators installed, the load voltages on a 120 V base are

$$[Vload_{abc}] = \begin{bmatrix} 119.8 \\ 119.7 \\ 119.7 \end{bmatrix}$$

As can been seen from this example, as more elements of a system are added, there will be one equation for each of the system elements for the forward sweep and backward sweeps. This concept will be further developed in later chapters.

8.4 Delta–Grounded Wye Step-Up Connection

Figure 8.7 shows the connection diagram for the delta–grounded wye step-up connection.

Phasor diagrams for the voltages and currents are also shown in Figure 8.6. Note that the high-side line-to-line voltage leads the low-side line-to-line voltage and the same can be said for the high- and low-side line currents.

The development of the generalized matrices follows the same procedure as was used for the step-down connection. Only two matrices differ between the two connections.

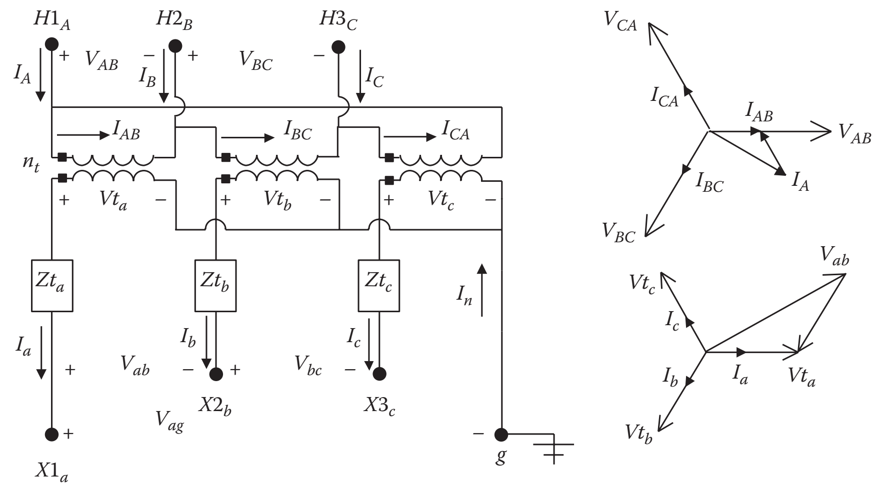

FIGURE 8.7
Delta–grounded wye step-up connection.

The primary (low side) line-to-line voltages are given by

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} \quad [VLL_{ABC}] = [AV] \cdot [Vt_{abc}]$$

$$\text{where } [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad n_t = \frac{kVLL_{rated\,primary}}{kVLN_{rated\,secondary}} \tag{8.39}$$

The primary delta currents are given by

$$\begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad [ID_{ABC}] = [AI] \cdot [I_{abc}]$$

$$\text{where } [AI] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8.40}$$

The primary line currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} \quad [I_{ABC}] = [DI] \cdot [ID_{ABC}]$$

$$\text{where } [DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{8.41}$$

The forward sweep matrices are

Applying Equation 8.28,

$$[A_t] = AV^{-1} \cdot D = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{8.42}$$

Applying Equation 8.31,

$$[B_t] = [Zt_{abc}] = \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \tag{8.43}$$

The backward sweep matrices are
Applying Equation 8.19,

$$[a_t] = [W]\cdot[AV] = \frac{n_t}{3}\cdot\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \tag{8.44}$$

Applying Equation 8.23,

$$[b_t] = [a_t]\cdot[Zt_{abc}] = \frac{n_t}{3}\cdot\begin{bmatrix} 2\cdot Zt_a & Zt_b & 0 \\ 0 & 2\cdot Zt_b & Zt_c \\ Zt_a & 0 & 2\cdot Zt_c \end{bmatrix} \tag{8.45}$$

Applying Equation 8.37,

$$[d_t] = [D]\cdot[AI] = \frac{1}{n_t}\cdot\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \tag{8.46}$$

8.5 Ungrounded Wye–Delta Step-Down Connection

Three single-phase transformers can be connected in a wye–delta connection. The neutral of the wye can be grounded or ungrounded. The grounded wye connection is characterized by

- The grounded wye provides a path for zero sequence currents for line-to-ground faults upstream from the transformer bank. This causes the transformers to be susceptible to burnouts on the upstream faults.
- If one phase of the primary circuit is opened, the transformer bank will continue to provide three-phase service by operating as an open wye–open delta bank. However, the two remaining transformers may be subjected to an overload condition leading to burnout.

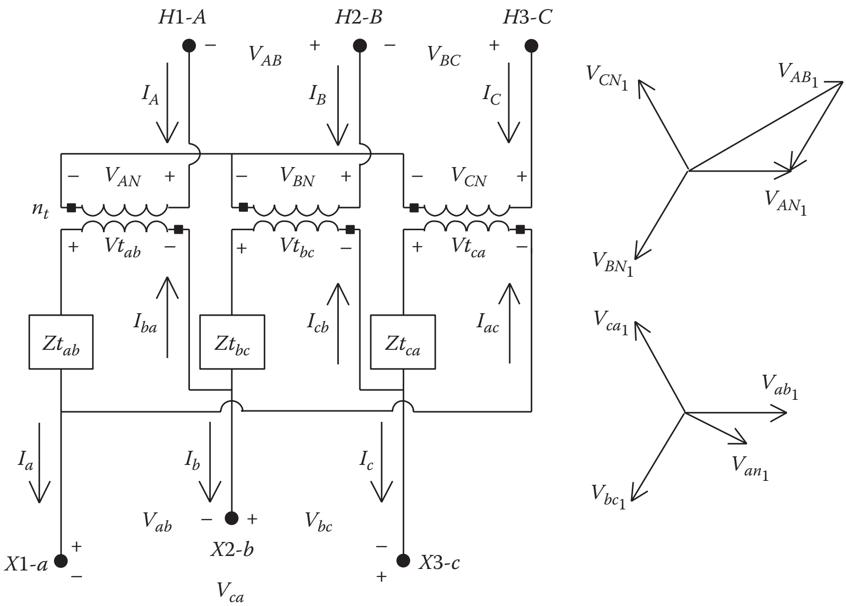

FIGURE 8.8
Standard ungrounded wye–delta connection step-down.

The most common connection is the ungrounded wye–delta. This connection is typically used to provide service to a combination of single-phase "lighting" load and a three-phase "power" load such as an induction motor. The generalized constants for the ungrounded wye–delta transformer connection will be developed following the same procedure as was used for the delta–grounded wye.

Three single-phase transformers can be connected in an ungrounded-wye "standard 30 degree connection" as shown in Figure 8.8.

The voltage phasor diagrams in Figure 8.8 illustrate that the high-side positive sequence line-to-line voltage leads the low-side positive sequence line-to-line voltage by 30°. Also, the same phase shift occurs between the high-side line-to-neutral voltage and the low-side "equivalent" line-to-neutral voltage. The negative sequence phase shift is such that the high-side negative sequence voltage will lag the low-side negative sequence voltage by 30°.

The positive sequence current phasor diagrams for the connection in Figure 8.8 are shown in Figure 8.9.

Figure 8.9 illustrates that the positive sequence line current on the high side of the transformer (node ***n***) leads the low-side line current (node ***m***) by 30°. It can also be shown that the negative sequence high-side line current will lag the negative sequence low-side line current by 30°.

The definition for the "turns ratio n_t" will be the same as Equation 8.9 with the exception that the numerator will be the line-to-neutral voltage and the

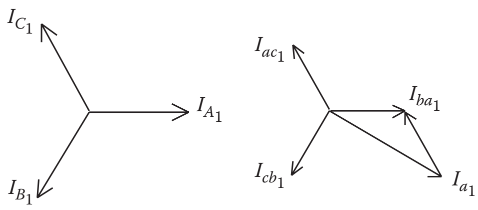

FIGURE 8.9
Positive sequence current phasors.

denominator will be the line-to-line voltage. It should be noted in Figure 8.8 that the "ideal" low-side transformer voltages for this connection will be line-to-line voltages. Also, the "ideal" low-side currents are the currents flowing inside the delta.

The basic "ideal" transformer voltage and current equations as a function of the "turns ratio" are

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \quad \text{where } n_t = \frac{kVLN_{rated\,primary}}{kVLL_{rated\,secondary}} \tag{8.47}$$

$$[VLN_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{8.48}$$

$$\begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{8.49}$$

$$[ID_{abc}] = [AI] \cdot [I_{ABC}] \tag{8.50}$$

Solving Equation 8.48 for the "ideal" delta transformer voltages,

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] \tag{8.51}$$

The line-to-line voltages at node ***m*** as a function of the "ideal" transformer voltages and the delta currents are given by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{ac} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \tag{8.52}$$

$$[VLL_{abc}] = [Vt_{abc}] - [Zt_{abc}] \cdot [ID_{abc}] \tag{8.53}$$

Substitute Equations 8.50 and 8.51 into Equation 8.53:

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [ZNt_{abc}] \cdot [I_{ABC}] \tag{8.54}$$

where

$$[ZNt_{abc}] = [Zt_{abc}] \cdot [AI] = \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & n_t \cdot Zt_{bc} & 0 \\ 0 & 0 & n_t \cdot Zt_{ca} \end{bmatrix} \tag{8.55}$$

The line currents on the delta side of the transformer bank as a function of the wye transformer currents are given by

$$[I_{abc}] = [DI] \cdot [ID_{abc}] \tag{8.56}$$

where

$$[DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{8.57}$$

Substitute Equation 8.50 into Equation 8.56:

$$[I_{abc}] = [DI] \cdot [AI] \cdot [I_{ABC}] = [DY] \cdot [I_{ABC}] \tag{8.58}$$

where

$$[DY] = [DI] \cdot [AI] = \begin{bmatrix} n_t & 0 & -n_t \\ -n_t & n_t & 0 \\ 0 & -n_t & n_t \end{bmatrix} \tag{8.59}$$

Because the matrix $[DY]$ is singular, it is not possible to use Equation 8.61 to develop an equation relating the wye-side line currents at node ***n*** to the delta-side line currents at node ***m***. In order to develop the necessary matrix equation, three independent equations must be written. Two independent KCL equations at the vertices of the delta can be used. Because there is no path for the high-side currents to flow to ground, they must sum to zero and, therefore, so must the delta currents in the transformer secondary sum to zero. This provides the third independent equation. The resulting three independent equations in matrix form are given by

$$\begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{8.60}$$

Solving Equation 8.60 for the delta currents,

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 1 \\ -2 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} \tag{8.61}$$

$$[ID_{abc}] = [L0] \cdot [I_{ab0}] \tag{8.62}$$

Equation 8.62 can be modified to include the phase *c* current by setting the third column of the [*L*0] matrix to zero:

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{8.63}$$

$$[ID_{abc}] = [L] \cdot [I_{abc}] \tag{8.64}$$

Solve Equation 8.50 for $[I_{ABC}]$ and substitute into Equation 8.64:

$$[I_{ABC}] = [AI]^{-1} \cdot [L] \cdot [I_{abc}] = [d_t] \cdot [I_{abc}] \tag{8.65}$$

where

$$[d_t] = [AI]^{-1} \cdot [L] = \frac{1}{3 \cdot n_T} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix} \tag{8.66}$$

Equation 8.66 defines the generalized constant matrix $[d_t]$ for the ungrounded wye–delta transformer connection. In the process of the derivation, a very convenient equation (Equation 8.63) evolved that can be used anytime the currents in a delta need to be determined knowing the line currents.

However, it must be understood that this equation will only work when the delta currents sum to zero, which means an ungrounded neutral on the primary.

The generalized matrices $[a_t]$ and $[b_t]$ can now be developed. Solve Equation 8.54 for $[VLN_{ABC}]$:

$$[VLN_{ABC}]=[AV]\cdot[VLL_{abc}]+[AV]\cdot[ZNt_{abc}]\cdot[I_{ABC}] \tag{8.67}$$

Substitute Equation 8.65 into Equation 8.67:

$$[VLN_{ABC}]=[AV]\cdot[VLL_{abc}]+[AV]\cdot[ZNt_{abc}]\cdot[d_t]\cdot[I_{abc}]$$

$$[VLL_{abc}]=[D]\cdot[VLN_{abc}]$$

$$\text{where } [D]=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ -1 & 0 & 1\end{bmatrix}$$

$$[VLN_{ABC}]=[AV]\cdot[D]\cdot[VLN_{abc}]+[AV]\cdot[ZNt_{abc}]\cdot[d_t]\cdot[I_{abc}]$$

$$[VLN_{ABC}]=[a_t]\cdot[VLN_{abc}]+[b_t]\cdot[I_{abc}] \tag{8.68}$$

where

$$[a_t]=[AV]\cdot[D]=n_t\cdot\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ -1 & 0 & 1\end{bmatrix} \tag{8.69}$$

$$[b_t]=[AV]\cdot[ZNt_{abc}]\cdot[d_t]=\frac{n_t}{3}\cdot\begin{bmatrix}Zt_{ab} & -Zt_{ab} & 0\\ Zt_{bc} & 2\cdot Zt_{bc} & 0\\ -2\cdot Zt_{ca} & -Zt_{ca} & 0\end{bmatrix} \tag{8.70}$$

The generalized constant matrices have been developed for computing voltages and currents from the load toward the source (backward sweep). The forward sweep matrices can be developed by referring back to Equation 8.54, which is repeated here for convenience:

$$[VLL_{abc}]=[AV]^{-1}\cdot[VLN_{ABC}]-[ZNt_{abc}]\cdot[I_{ABC}] \tag{8.71}$$

Equation 8.16 is used to compute the equivalent line-to-neutral voltages as a function of the line-to-line voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] \tag{8.72}$$

Substitute Equation 8.71 into Equation 8.72:

$$[VLN_{abc}] = [W] \cdot [AV]^{-1} \cdot [VLN_{ABC}] - [W] \cdot [ZNt_{abc}] \cdot [d_t] \cdot [I_{abc}]$$

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \tag{8.73}$$

where

$$[A_t] = [W] \cdot [AV]^{-1} = \frac{1}{3 \cdot n_t} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \tag{8.74}$$

$$[B_t] = [W] \cdot [ZNt_{abc}] \cdot [d_t] = \frac{1}{9} \cdot \begin{bmatrix} 2 \cdot Zt_{ab} + Zt_{bc} & 2 \cdot Zt_{bc} - 2 \cdot Zt_{ab} & 0 \\ 2 \cdot Zt_{bc} - 2 \cdot Zt_{ca} & 4 \cdot Zt_{bc} - Zt_{ca} & 0 \\ Zt_{ab} - 4 \cdot Zt_{ca} & -Zt_{ab} - 2 \cdot Zt_{ca} & 0 \end{bmatrix} \tag{8.75}$$

The generalized matrices have been developed for the ungrounded wye–delta transformer connection. The derivation has applied basic circuit theory and the basic theories of transformers. The end result of the derivations is to provide an easy method of analyzing the operating characteristics of the transformer connection. Example 8.3 will demonstrate the application of the generalized matrices for this transformer connection.

Example 8.3

Figure 8.10 shows three single-phase transformers in an ungrounded wye–delta connection serving a combination of single-phase and three-phase load in a delta connection. The voltages at the load are balanced three phase of 240 V line to line. The net loading by phase is

$$S_{ab} = 100 \text{ kVA at 0.9 lagging power factor}$$

$$S_{bc} = S_{ca} = 50 \text{ kVA at 0.8 lagging power factor}$$

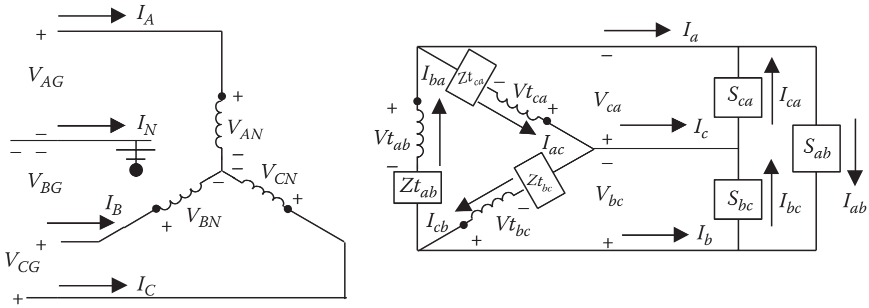

FIGURE 8.10
Ungrounded wye–delta step-down with unbalanced load.

The transformers are rated as follows:

Phase *AN*: 100 kVA, 7200–240 V, $Z = 0.01 + j0.04$ per unit
Phases *BN* and *CN*: 50 kVA, 7200–240 V, $Z = 0.015 + j0.035$ per unit

Determine the following:

1. The currents in the load
2. The secondary line currents
3. The equivalent line-to-neutral secondary voltages
4. The primary line-to-neutral and line-to-line voltages
5. The primary line currents

Before the analysis can start, the transformer impedances must be converted to actual values in Ohms and located inside the delta-connected secondary windings.

"Lighting" transformer:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{100} = 0.576$$

$$Zt_{ab} = (0.01 + j0.4) \cdot 0.576 = 0.0058 + j0.023\,\Omega$$

"Power" transformers:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{50} = 1.152$$

$$Zt_{bc} = Zt_{ca} = (0.015 + j0.35) \cdot 1.152 = 0.0173 + j0.0403\,\Omega$$

The transformer impedance matrix can now be defined:

$$[Zt_{abc}] = \begin{bmatrix} 0.0058 + j0.023 & 0 & 0 \\ 0 & 0.0173 + j0.0403 & 0 \\ 0 & 0 & 0.0173 + j0.0403 \end{bmatrix} \Omega$$

The turn's ratio of the transformers is $n_t = 7200/240 = 30$.

Define all of the matrices:

$$[W]=\frac{1}{3}\cdot\begin{bmatrix}2&1&0\\0&2&1\\1&0&2\end{bmatrix}\quad [D]=\begin{bmatrix}1&-1&0\\0&1&-1\\-1&0&1\end{bmatrix}\quad [DI]=\begin{bmatrix}1&0&-1\\-1&1&0\\0&-1&1\end{bmatrix}$$

$$[a_t]=n_t\cdot\begin{bmatrix}1&-1&0\\0&1&-1\\-1&0&1\end{bmatrix}=\begin{bmatrix}30&-30&0\\0&30&-30\\-30&0&30\end{bmatrix}$$

$$[b_t]=\frac{n_t}{3}\cdot\begin{bmatrix}Zt_{ab}&-Zt_{ab}&0\\Zt_{bc}&2\cdot Zt_{bc}&0\\-2\cdot Zt_{ca}&-Zt_{ca}&0\end{bmatrix}=\begin{bmatrix}0.0576+j0.2304&-0.576-j0.2304&0\\0.1728+j0.4032&0.3456+j0.8064&0\\-0.3456-j0.8064&-0.1728-j0.4032&0\end{bmatrix}$$

$$[c_t]=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$[d_t]=\frac{1}{3\cdot n_T}\cdot\begin{bmatrix}1&-1&0\\1&2&0\\-2&-1&0\end{bmatrix}=\begin{bmatrix}0.0111&-0.0111&0\\0.0111&0.0222&0\\-0.0222&-0.0111&0\end{bmatrix}$$

$$[A_t]=\frac{1}{3\cdot n_t}\cdot\begin{bmatrix}2&1&0\\0&2&1\\1&0&2\end{bmatrix}=\begin{bmatrix}0.0222&0.0111&0\\0&0.0222&0.0111\\0.0111&0&0.0222\end{bmatrix}$$

$$[B_t]=\frac{1}{9}\cdot\begin{bmatrix}2\cdot Zt_{ab}+Zt_{bc}&2\cdot Zt_{bc}-2\cdot Zt_{ab}&0\\2\cdot Zt_{bc}-2\cdot Zt_{ca}&4\cdot Zt_{bc}-Zt_{ca}&0\\Zt_{ab}-4\cdot Zt_{ca}&-Zt_{ab}-2\cdot Zt_{ca}&0\end{bmatrix}$$

$$[B_t]=\begin{bmatrix}0.0032+j0.0096&0.0026+j0.0038&0\\0&0.0058+j0.0134&0\\-0.007-j0.0154&-0.0045-j0.0115&0\end{bmatrix}$$

Define the line-to-line load voltages:

$$[VLL_{abc}]=\begin{bmatrix}240/\underline{0}\\240/\underline{-120}\\240/\underline{120}\end{bmatrix}\text{V}$$

Define the loads:

$$[SD_{abc}]=\begin{bmatrix}100/\underline{\cos^{-1}(0.9)}\\50/\underline{\cos^{-1}(0.8)}\\50/\underline{\cos^{-1}(0.8)}\end{bmatrix}=\begin{bmatrix}90+j43.589\\40+j30\\40+j30\end{bmatrix}\text{kVA}$$

Calculate the delta load currents:

$$ID_i = \left(\frac{SD_i \cdot 1000}{VLL_{abc_i}} \right)^* \text{ A}$$

$$[ID_{abc}] = \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix} = \begin{bmatrix} 416.7/\underline{-25.84} \\ 208.3/\underline{-156.87} \\ 208.3/\underline{83.13} \end{bmatrix} \text{A}$$

Compute the secondary line currents:

$$[I_{abc}] = [DI] \cdot [ID_{abc}] = \begin{bmatrix} 522.9/\underline{-47.97} \\ 575.3/\underline{-119.06} \\ 360.8/\underline{53.13} \end{bmatrix} \text{A}$$

Compute the equivalent secondary line-to-neutral voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] = \begin{bmatrix} 138.56/\underline{-30} \\ 138.56/\underline{-150} \\ 138.56/\underline{90} \end{bmatrix} \text{A}$$

Use the generalized constant matrices to compute the primary line-to-neutral voltages and line-to-line voltages:

$$[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}] = \begin{bmatrix} 7367.6/\underline{1.4} \\ 7532.3/\underline{-119.1} \\ 7406.2/\underline{121.7} \end{bmatrix} \text{V}$$

$$[VLL_{ABC}] = [D] \cdot [VLN_{ABC}] = \begin{bmatrix} 12,9356/\underline{31.54} \cdot \\ 12,8845/\underline{-88.95} \\ 12,8147/\underline{151.50} \end{bmatrix} \text{kV}$$

The high primary line currents are

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] = \begin{bmatrix} 11.54/\underline{-28.04} \\ 8.95/\underline{-166.43} \\ 7.68/\underline{101.16} \end{bmatrix} \text{A}$$

It is interesting to compute the operating kVA of the three transformers. Taking the product of the transformer voltage times the conjugate of the current gives the operating kVA of each transformer.

$$ST_i = \frac{VLN_{ABC_i} \cdot (I_{ABC_i})^*}{1000} = \begin{bmatrix} 85.02/\underline{29.46} \\ 67.42/\underline{47.37} \\ 56.80/\underline{20.58} \end{bmatrix} \text{kVA}$$

The operating power factors of the three transformers are

$$[PF] = \begin{bmatrix} \cos(29.46) \\ \cos(47.37) \\ \cos(20.58) \end{bmatrix} = \begin{bmatrix} 87.1 \\ 67.7 \\ 93.6 \end{bmatrix} \%$$

Note that the operating kVAs do not match very closely the rated kVAs of the three transformers. In particular, the transformer on phase A did not serve the total load of 100 kVA that is directly connected its terminals. That transformer is operating below rated kVA, while the other two transformers are overloaded. In fact, the transformer connected to phase B is operating 35% above rated kVA. Because of this overload, the ratings of the three transformers should be changed so that the phase B and phase C transformers are rated 75 kVA. Finally, the operating power factors of the three transformers bare little resemblance to the load power factors.

Example 8.3 demonstrates how the generalized constant matrices can be used to determine the operating characteristics of the transformers. In addition, the example shows that the obvious selection of transformer ratings will lead to an overload condition on the two power transformers. The beauty in this is that if the generalized constant matrices have been applied in a computer program, it is a simple task to change the transformer kVA ratings and be assured that none of the transformers will be operating in an overload condition.

Example 8.3 has demonstrated the "backward" sweep to compute the primary voltages and currents. As before when the source (primary) voltages are given along with the load PQ, then the ladder iterative technique must be used to analyze the transformer connection.

Example 8.4

The Mathcad program that has been used in previous examples is modified to demonstrate the ladder iterative technique for computing the load voltages given the source voltages and load power and reactive powers (PQ load). In Example 8.4, the computed source voltages from Example 8.3 are specified along with the same loads.

The modified Mathcad program is shown in Figure 8.11.

With balanced source voltages specified, after five iterations, the load voltages are computed to be

$$[VLL_{abc}] = \begin{bmatrix} 232.0/\underline{-1.8} \\ 230.6/\underline{-121.3} \\ 234.0/\underline{118.7} \end{bmatrix} \text{V}$$

$$\left|VLL_{ABC_i}\right| = \begin{pmatrix} 12470 \\ 12470 \\ 12470 \end{pmatrix} \quad \frac{\arg\left(VLL_{ABC_i}\right)}{deg} = \begin{pmatrix} 30 \\ -90 \\ 150 \end{pmatrix} \quad VM = 240$$

$$Start := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad Tol := .000001 \qquad VM := kVLL_{sec} \cdot 1000$$

$$
\begin{array}{l}
X := \left| \begin{array}{l}
I_{abc} \leftarrow Start \\
V_{old} \leftarrow Start \\
VLN_{ABC} \leftarrow W \cdot VLL_{ABC} \\
\text{for } n \in 1..200 \\
\quad \left| \begin{array}{l}
VLN_{abc} \leftarrow A_t \cdot VLN_{ABC} - B_t \cdot I_{abc} \\
VLN_{abc} \leftarrow D \cdot VLN_{abc\,c} \\
\text{for } j \in 1..3 \\
\quad ID_{abc_j} \leftarrow \overline{\dfrac{SL_j \cdot 1000}{VLL_{abc_j}}} \\
\text{for } k \in 1..3 \\
\quad Error_k \leftarrow \dfrac{\left|VLL_{abc_k} - V_{old_k}\right|}{VM} \\
Error_{max} \leftarrow \max(Error) \\
\text{break if } Error_{max} < Tol \\
V_{old} \leftarrow VLL_{abc} \\
I_{abc} \leftarrow DI \cdot ID_{abc} \\
I_{ABC} \leftarrow d_t \cdot I_{abc}
\end{array} \right. \\
Out_1 \leftarrow VLL_{abc} \\
Out_2 \leftarrow VLL_{abc} \\
Out_3 \leftarrow I_{abc} \\
Out_4 \leftarrow I_{ABC} \\
Out_5 \leftarrow n \\
Out
\end{array} \right.
\end{array}
$$

FIGURE 8.11
Mathcad® program.

Example 8.4 has demonstrated how the simple Mathcad program can be modified to analyze the ungrounded wye–delta step-down transformer bank connection.

8.6 Ungrounded Wye–Delta Step-Up Connection

The connection diagram for the step-up connection is shown in Figure 8.12.

The only difference between the step-up and step-down connections are the definitions of the turns ratio n_t, [AV], and [AI]. For the step-up connection,

$$n_t = \frac{kVLN_{rated\ primary\ voltage}}{kVLL_{rated\ secondary\ voltage}} \tag{8.76}$$

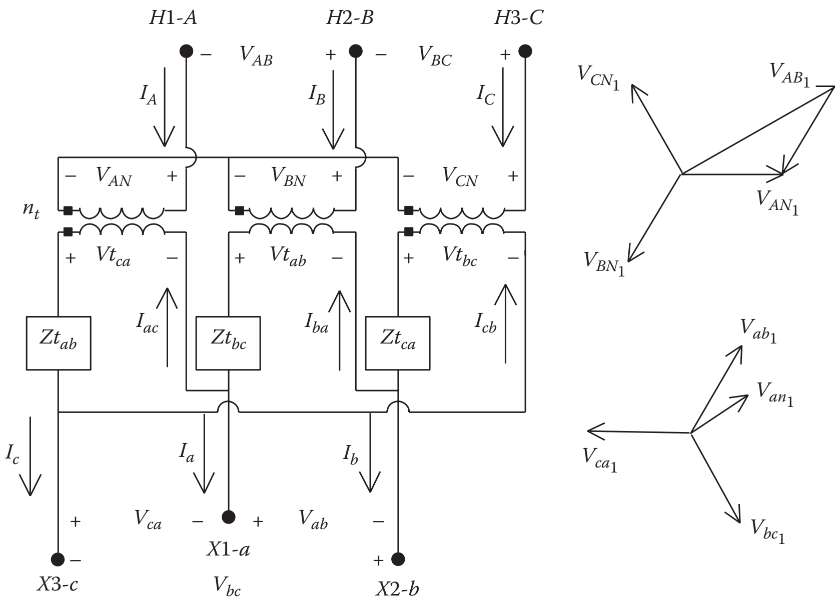

FIGURE 8.12
Ungrounded wye–delta step-up connection.

$$[AV] = n_t \cdot \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \tag{8.77}$$

$$[AI] = n_t \cdot \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \tag{8.78}$$

Example 8.5

The equations for the forward and backward sweep matrices, as defined in Section 8.3, can be applied using the definitions in Equations 8.76 through 8.77. The system of Example 8.3 is modified so that transformer connection is step-up. The transformers have the same ratings, but now the rated voltages for the primary and secondary are

$$\text{Primary:}\quad VLL_{pri} = 240 \quad VLN_{pri} = 138.6\,\text{V}$$

$$\text{Secondary:}\quad VLL_{sec} = 12{,}470\,\text{V}$$

$$n_t = \frac{240}{12{,}470} = 0.0111$$

The transformer impedances must be computed in Ohms relative to the secondary and then used to compute the new forward and backward sweep matrices. When this is done the new matrices are

$$[d_t] = [AI]^{-1} \cdot [L] = \begin{bmatrix} 60 & 30 & 0 \\ -30 & 30 & 0 \\ -30 & -60 & 0 \end{bmatrix}$$

$$[A_t] = [W] \cdot [AV]^{-1} = \begin{bmatrix} 0 & -60 & -30 \\ -30 & 0 & -60 \\ -60 & -30 & 0 \end{bmatrix}$$

$$[B_t] = [W] \cdot [ZNt_{abc}] \cdot [d_t] = \begin{bmatrix} 8.64 + j25.92 & 6.91 + j10.37 & 0 \\ 0 & 15.55 + j36.28 & 0 \\ -19.01 - j41.47 & -12.09 - j31.10 & 0 \end{bmatrix}$$

Using these matrices and the same loads, the output of the program gives the new load voltages as

$$[VLL_{abc}] = \begin{bmatrix} 12{,}055\underline{/58.2} \\ 11{,}982\underline{/-61.3} \\ 12{,}106\underline{/178.7} \end{bmatrix}$$

8.7 Grounded Wye–Delta Step-Down Connection

The connection diagram for the standard 30 degree grounded wye (high)–delta (low) transformer connection grounded through an impedance of Z_g is shown in Figure 8.13. Note that the primary is grounded through an impedance Z_g.

Basic transformer equations:
The turn's ratio is given by

$$n_t = \frac{kVLN_{rated\ primary}}{kVLL_{rated\ secondary}} \tag{8.79}$$

The basic "ideal" transformer voltage and current equations as a function of the turns ratio are

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \quad [VLN_{ABC}] = [AV] \cdot [Vt_{abc}]$$

$$\text{where } [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8.80}$$

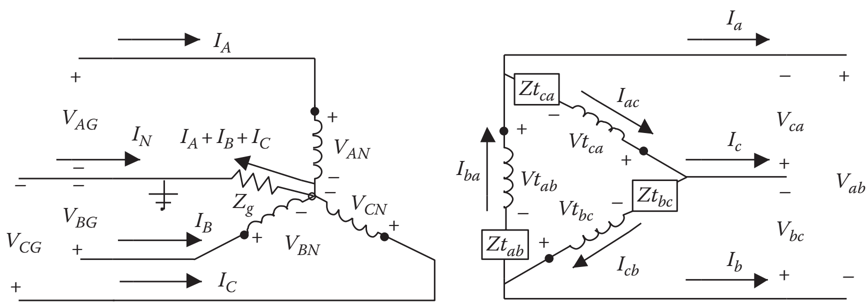

FIGURE 8.13
The grounded wye–delta connection.

$$\begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \quad [ID_{abc}] = [AI] \cdot [I_{ABC}]$$

$$\text{where } [AI] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8.81}$$

Solving Equation 8.80 for the "ideal" transformer voltages,

$$[Vt_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] \tag{8.82}$$

The line-to-neutral transformer primary voltages as a function of the system line-to-ground voltages are given by

$$V_{AN} = V_{AG} - Z_g \cdot (I_A + I_B + I_C)$$

$$V_{BN} = V_{BG} - Z_g \cdot (I_A + I_B + I_C)$$

$$V_{CN} = V_{CG} - Z_g \cdot (I_A + I_B + I_C)$$

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} - \begin{bmatrix} Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{8.83}$$

$$[VLN_{ABC}] = [VLG_{ABC}] - [ZG] \cdot [I_{ABC}]$$

$$\text{where } [ZG] = \begin{bmatrix} Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \\ Z_g & Z_g & Z_g \end{bmatrix}$$

The line-to-line voltages on the delta side are given by

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \tag{8.84}$$

$$[VLL_{abc}] = [Vt_{abc}] - [Zt_{abc}] \cdot [ID_{abc}]$$

Substitute Equation 8.82 into Equation 8.84:

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - [Zt_{abc}] \cdot [ID_{abc}] \tag{8.85}$$

Substitute Equation 8.81 into Equation 8.85:

$$[VLL_{abc}] = [AV]^{-1} \cdot [VLN_{ABC}] - ([Zt_{abc}] \cdot [AI]) \cdot [I_{ABC}] \tag{8.86}$$

Substitute Equation 8.83 into Equation 8.86:

$$\begin{aligned} [VLL_{abc}] &= [AV]^{-1} \cdot ([VLG_{ABC}] - [ZG] \cdot [I_{ABC}]) - ([Zt_{abc}] \cdot [AI]) \cdot [I_{ABC}] \\ [VLL_{abc}] &= [AV]^{-1} \cdot [VLG_{ABC}] - \left([AV]^{-1} \cdot [ZG] + [Zt_{abc}] \cdot [AI]\right) \cdot [I_{ABC}] \end{aligned} \tag{8.87}$$

Equation 8.81 gives the delta secondary currents as a function of the primary wye-side line currents. The secondary line currents are related to the secondary delta currents by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} ID_{ba} \\ ID_{cb} \\ ID_{ac} \end{bmatrix} \quad [I_{abc}] = [DI] \cdot [ID_{abc}] \tag{8.88}$$

The real problem of transforming currents from one side to the other occurs for the case when the line currents on the delta secondary side $[I_{abc}]$ are known and the transformer secondary currents $[ID_{abc}]$ and primary line currents on the wye side $[I_{ABC}]$ are needed. The only way a relationship can be developed is to recognize that the sum of the line-to-line voltages on the delta secondary of the transformer bank must add up to zero. Three independent equations can be written as follows:

$$\begin{aligned} I_a &= I_{ba} - I_{ac} \\ I_b &= I_{cb} - I_{ba} \end{aligned} \tag{8.89}$$

KVL around the delta secondary windings gives

$$Vt_{ab} - Zt_{ab} \cdot I_{ba} + Vt_{bc} - Zt_{bc} \cdot I_{cb} + Vt_{ca} - Zt_{ca} \cdot I_{ac} = 0 \tag{8.90}$$

Replacing the "ideal" secondary delta voltages with the primary line-to-neutral voltages,

$$\frac{V_{AN}}{n_t} + \frac{V_{BN}}{n_t} + \frac{V_{CN}}{n_t} = Zt_{ab} \cdot I_{ba} + Zt_{bc} \cdot I_{cb} + Zt_{ca} \cdot I_{ac} \tag{8.91}$$

Multiply both sides of the Equation 8.91 by the turns ratio n_t:

$$V_{AN} + V_{BN} + V_{CN} = n_t \cdot Zt_{ab} \cdot I_{ba} + n_t \cdot Zt_{bc} \cdot I_{cb} + n_t \cdot Zt_{ca} \cdot I_{ac} \tag{8.92}$$

Determine the left side of Equation 8.92 as a function of the line-to-ground voltages using Equation 8.83:

$$\begin{aligned} V_{AN} + V_{BN} + V_{CN} &= V_{AG} + V_{BG} + V_{CG} - 3 \cdot Z_g \cdot (I_A + I_B + I_C) \\ V_{AN} + V_{BN} + V_{CN} &= V_{AG} + V_{BG} + V_{CG} - 3 \cdot \frac{1}{n_t} \cdot Z_g \cdot (I_{ba} + I_{cb} + I_{ac}) \end{aligned} \tag{8.93}$$

Substitute Equation 8.93 into Equation 8.92:

$$\begin{aligned} &V_{AG} + V_{BG} + V_{CG} - \frac{3}{n_t} \cdot Z_g \cdot (I_{ba} + I_{cb} + I_{ac}) = n_t \cdot Zt_{ab} \cdot I_{ba} + n_t \cdot Zt_{bc} \cdot I_{cb} + n_t \cdot Zt_{ca} \cdot I_{ac} \\ &Vsum = \left(n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g \right) \cdot I_{ba} + \left(n_t \cdot Zt_{bc} + \frac{3}{n_t} \cdot Z_g \right) \cdot I_{cb} + \left(n_t \cdot Zt_{ca} + \frac{3}{n_t} \cdot Z_g \right) \cdot I_{ac} \\ &\text{where } V_{sum} = V_{AG} + V_{BG} + V_{CG} \end{aligned} \tag{8.94}$$

Equations 8.88, 8.89, and 8.94 can be put into matrix form:

$$\begin{bmatrix} I_a \\ I_b \\ V_{sum} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ n_t \cdot Zt_{ab} + \frac{3}{n_t} \cdot Z_g & n_t \cdot Zt_{bc} + \frac{3}{n_t} \cdot Z_g & n_t \cdot Zt_{ca} + \frac{3}{n_t} \cdot Z_g \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{8.95}$$

Equation 8.95 in general form is

$$[X] = [F] \cdot [ID_{abc}] \tag{8.96}$$

Solve for $[ID_{abc}]$:

$$[ID_{abc}] = [F]^{-1} \cdot [X] = [G] \cdot [X] \tag{8.97}$$

Equation 8.97 in full form is

$$[ID_{abc}] = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ V_{AG}+V_{BG}+V_{CG} \end{bmatrix}$$

$$[ID_{abc}] = \begin{bmatrix} G_{13} & G_{13} & G_{13} \\ G_{23} & G_{23} & G_{23} \\ G_{33} & G_{33} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} + \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{8.98}$$

Equation 8.98 in shortened form is

$$[ID_{abc}] = [G1]\cdot[VLG_{ABC}] + [G2]\cdot[I_{abc}] \tag{8.99}$$

Substitute Equation 8.81 into Equation 8.98:

$$\begin{aligned} &[I_{ABC}] = [AI]^{-1}\cdot[ID_{abc}] = [AI]^{-1}\cdot([G1]\cdot[VLG_{ABC}] + [G2]\cdot[I_{abc}]) \\ &[I_{ABC}] = [x_t]\cdot[VLG_{ABC}] + [d_t]\cdot[I_{abc}] \\ &\text{where } [x_t] = [AI]^{-1}\cdot[G1] \\ &\qquad\quad [d_t] = [AI]^{-1}\cdot[G2] \end{aligned} \tag{8.100}$$

Equation 8.100 is used in the "backward" sweep to compute the primary currents based upon the secondary currents and primary *LG* voltages.

The "forward" sweep equation is determined by substituting Equation 8.100 into Equation 8.87:

$$[VLL_{abc}] = [AV]^{-1}\cdot[VLG_{ABC}] - ([AV]^{-1}\cdot[ZG] + [Zt_{abc}]\cdot[AI])\cdot[I_{ABC}]$$

$$[VLL_{abc}] = [AV]^{-1}\cdot[VLG_{ABC}] - ([Zt_{abc}]\cdot[AI] + [AV]^{-1}\cdot[ZG])$$
$$\times([x_t]\cdot[VLG_{ABC}] + [d_t]\cdot[I_{abc}])$$

Define: $[X1] = [Zt_{abc}]\cdot[AI] + [AV]^{-1}\cdot[ZG]$

$$[VLL_{abc}] = ([AV] - [X1]\cdot[x_t])\cdot[VLG_{ABC}] - [X1]\cdot[d_t]\cdot[I_{abc}]$$

$$[VLN_{abc}] = [W]\cdot[VLL_{abc}]$$

$$[VLN_{abc}] = [W]\cdot\left(([AV]^{-1} - [X1]\cdot[x_t])\cdot[VLG_{ABC}] - [X1]\cdot[d_t]\cdot[I_{abc}]\right)$$

$$[VLN_{abc}] = [W]\cdot([AV]^{-1} - [X1]\cdot[x_t])\cdot[VLG_{ABC}] - [W]\cdot[X1]\cdot[d_t]\cdot[I_{abc}] \tag{8.101}$$

The final form of Equation 8.101 gives the equation for the forward sweep:

$$
\begin{aligned}
&[VLN_{abc}] = [A_t]\cdot[VLG_{ABC}] - [B_t]\cdot[I_{abc}]\\
&\text{where } [A_t] = [W]\cdot\left([AV]^{-1} - [X1]\cdot[x_t]\right)\\
&\qquad [B_t] = [W]\cdot[X1]\cdot[d_t]
\end{aligned}
\tag{8.102}
$$

Example 8.6

The system of Examples 8.3 and 8.4 is changed so that the same transformers are connected in a grounded wye–delta step-down connection to serve the same load. The neutral ground resistance is 5 Ω. The computed matrices are

$$
[x_t] = \begin{bmatrix} 0.0053 - j0.0061 & 0.0053 - j0.0061 & 0.0053 - j0.0061 \\ 0.0053 - j0.0061 & 0.0053 - j0.0061 & 0.0053 - j0.0061 \\ 0.0053 - j0.0061 & 0.0053 - j0.0061 & 0.0053 - j0.0061 \end{bmatrix}
$$

$$
[d_t] = \begin{bmatrix} 0.0128 + j0.0002 & -0.0128 - j0.0002 & 0 \\ 0.0128 + j0.0002 & 0.0206 - j0.0002 & 0 \\ -0.0206 + j0.0002 & -0.0128 - j0.0002 & 0 \end{bmatrix}
$$

$$
[A_t] = \begin{bmatrix} 0.0128 + j0.0002 & 0.0017 + j0.0002 & -0.0094 + j0.0002 \\ -0.0128 - j0.0002 & 0.0094 - j0.0002 & -0.0017 - j0.0002 \\ 0 & -0.0111 & 0.0111 \end{bmatrix}
$$

$$
[B_t] = \begin{bmatrix} 0.0043 + j0.0112 & 0.0014 + j0.0022 & 0 \\ 0.0014 + j0.0022 & 0.0043 + j0.0112 & 0 \\ -0.0058 - j0.0134 & -0.0058 - j0.0134 & 0 \end{bmatrix}
$$

The only change in the program from Example 8.4 is for the equation computing the primary line currents:

$$
[I_{ABC}] = [x_t]\cdot[VLG_{ABC}] + [d_t]\cdot[I_{abc}]
$$

The source voltages are balanced of 12,470 V. After six iterations, the resulting load line-to-line load voltages are

$$
[VLL_{abc}] = \begin{bmatrix} 232.6\underline{/-1.7} \\ 231.0\underline{/-121.4} \\ 233.0\underline{/118.8} \end{bmatrix}
$$

The voltage unbalance is computed to be

$$V_{unbalance} = 0.53\%$$

The currents are

$$[I_{abc}] = \begin{bmatrix} 540.8/-49.5 \\ 594.1/168.5 \\ 373.0/51.7 \end{bmatrix}$$

$$[I_{ABC}] = \begin{bmatrix} 13.7/-28.6 \\ 7.8/-160.4 \\ 7.1/87.3 \end{bmatrix}$$

$$I_g = -(I_A + I_B + I_C) = 5.4/157.8$$

As can be seen, the major difference between this and the ungrounded connection is in the line currents and the ground current on the primary side. Experience has shown that the value of the neutral grounding resistance should not exceed the transformer impedance relative to the primary side. If the ground resistance is too big, the program will not converge.

8.8 Open Wye–Open Delta

A common load to be served on a distribution feeder is a combination of a single-phase lighting load and a three-phase power load. Many times the three-phase power load will be an induction motor. This combination load can be served by a grounded or ungrounded wye–delta connection as previously described or by an "open wye–open delta" connection. When the three-phase load is small compared to the single-phase load, the open wye–open delta connection is commonly used. The open wye–open delta connection requires only two transformers, but the connection will provide three-phase line-to-line voltages to the combination load. Figure 8.14 shows the open wye–open delta connection and the primary and secondary positive sequence voltage phasors.

With reference to Figure 8.14, the basic "ideal" transformer voltages as a function of the "turn's ratio" are

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \tag{8.103}$$

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}]$$

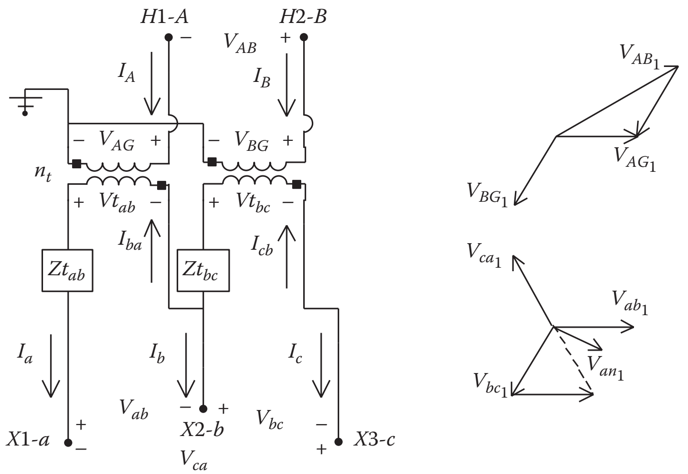

FIGURE 8.14
Open wye–open delta connection.

The currents as a function of the turn's ratio are given by

$$
\begin{aligned}
I_{ba} &= n_T \cdot I_A = I_a \\
I_{cb} &= n_T \cdot I_B = -I_c \\
I_b &= -(I_a + I_c)
\end{aligned}
\tag{8.104}
$$

Equation 8.104 can be expressed in matrix form by

$$
\begin{aligned}
\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} &= \begin{bmatrix} n_t & 0 & 0 \\ -n_t & n_t & 0 \\ 0 & -n_t & 0 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \\
[I_{abc}] &= [AI] \cdot [I_{ABC}]
\end{aligned}
\tag{8.105}
$$

The ideal voltages in the secondary can be determined by

$$
\begin{aligned}
Vt_{ab} &= V_{ab} + Zt_{ab} \cdot I_a \\
Vt_{bc} &= V_{bc} - Zt_{bc} \cdot I_c
\end{aligned}
\tag{8.106}
$$

Substitute Equation 8.106 into Equations 8.103:

$$
\begin{aligned}
V_{AG} &= n_t \cdot Vt_{ab} = n_t \cdot V_{ab} + n_t \cdot Zt_{ab} \cdot I_a \\
V_{BG} &= n_t \cdot Vt_{bc} = n_t \cdot V_{bc} - n_t \cdot Zt_{bc} \cdot I_c
\end{aligned}
\tag{8.107}
$$

Equation 8.107 can be put into three-phase matrix form as

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} + \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & 0 & -n_t \cdot Zt_{bc} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[VLG_{ABC}] = [AV] \cdot [VLL_{abc}] + [b_t] \cdot [I_{abc}] \tag{8.108}$$

The secondary line-to-line voltages in Equation 8.108 can be replaced by the equivalent line-to-neutral secondary voltages:

$$[VLG_{ABC}] = [AV] \cdot [D] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}]$$

$$[VLG_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}]$$

$$\text{where } [a_t] = [AV] \cdot [D] \tag{8.109}$$

$$[b_t] = \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & 0 & -n_t \cdot Zt_{bc} \\ 0 & 0 & 0 \end{bmatrix}$$

The source-side line currents as a function of the load-side line currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & 0 & -\frac{1}{n_t} \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] \tag{8.110}$$

$$\text{where } [d_t] = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & 0 & -\frac{1}{n_t} \\ 0 & 0 & 0 \end{bmatrix}$$

Equations 8.109 and 8.110 give the matrix equations for the backward sweep. The forward sweep equation can be determined by solving Equation 8.107 for the two line-to-line secondary voltages:

$$\begin{aligned} V_{ab} &= \frac{1}{n_t}\cdot V_{AG} - Zt_{ab}\cdot I_a \\ V_{bc} &= \frac{1}{n_t}\cdot V_{BG} - Zt_{bc}\cdot I_c \end{aligned} \tag{8.111}$$

The third line-to-line voltage V_{ca} must be equal to the negative sum of the other two line-to-line voltages (KVL). In matrix form, the desired equation is

$$\begin{aligned} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} &= \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & \frac{1}{n_t} & 0 \\ -\frac{1}{n_t} & -\frac{1}{n_t} & 0 \end{bmatrix} \cdot \begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} - \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & Zt_{bc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \\ [VLL_{abc}] &= [BV]\cdot[VLG_{ABC}] - [Zt_{abc}]\cdot[I_{abc}] \end{aligned} \tag{8.112}$$

The equivalent secondary line-to-neutral voltages are then given by

$$[VLN_{abc}] = [W]\cdot[VLL_{ABC}] = [W][BV]\cdot[VLG_{ABC}] - [W]\cdot[Zt_{abc}]\cdot[I_{abc}] \tag{8.113}$$

The forward sweep equation is given by

$$\begin{aligned} &[VLN_{abc}] = [A_t]\cdot[VLG_{ABC}] - [B_t]\cdot[I_{abc}] \\ &\text{where } [A_t] = [W]\cdot[BV] = \frac{1}{3\cdot n_t}\cdot \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ -1 & -2 & 0 \end{bmatrix} \\ &[B_t] = [W]\cdot[Zt_{abc}] = \frac{1}{3}\cdot \begin{bmatrix} 2\cdot Zt_{ab} & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \\ -Zt_{ab} & 0 & 2\cdot Zt_{bc} \end{bmatrix} \end{aligned} \tag{8.114}$$

The open wye–open delta connection derived in this section utilized phases A and B on the primary. This is just one of three possible connections. The other two possible connections would use phases B and C and then phases C and A. The generalized matrices will be different from those just derived. The same procedure can be used to derive the matrices for the other two connections.

The terms "leading" and "lagging" connection are also associated with the open wye–open delta connection. When the lighting transformer is connected across the leading of the two phases, the connection is referred to as the "leading" connection. Similarly, when the lighting transformer is connected across the lagging of the two phases, the connection is referred to as the "lagging" connection. For example, if the bank is connected to phases A and B and the lighting transformer is connected from phase A to ground, this would be referred to as the "leading" connection because the voltage A-G leads the voltage B-G by 120°. Reverse the connection and it would now be called the "lagging" connection. Obviously, there is a leading and lagging connection for each of the three possible open wye–open delta connections.

Example 8.7

The unbalanced load of Example 8.7 is to be served by the "leading" open wye–open delta connection using phases A and B. The primary line-to-line voltages are balanced 12.47 kV.

The "lighting" transformer is rated: 100 kVA, 7200 wye–240 delta, $Z = 1.0 + j4.0\%$.

The "power" transformer is rated: 50 kVA, 7200 wye–240 delta, $Z = 1.5 + j3.5\%$.

Use the forward/backward sweep to compute

1. The load line-to-line voltages
2. The secondary line currents
3. The load currents
4. The primary line currents
5. Load voltage unbalance

The transformer impedances referred to the secondary are the same as in Example 8.7 since the secondary rated voltages are still 240 V.

The required matrices for the forward and backward sweeps are

$$[A_t] = \begin{bmatrix} 0.0222 & 0.0111 & 0 \\ -0.0111 & 0.0111 & 0 \\ -0.0111 & 0.0222 & 0 \end{bmatrix}$$

$$[B_t] = \begin{bmatrix} 0.0038 + j0.0154 & 0 & -0.0058 - j0.0134 \\ -0.0019 - j0.0077 & 0 & -0.0058 - j0.0134 \\ -0.0019 - j0.0077 & 0 & 0.0115 + j0.0269 \end{bmatrix}$$

$$[d_t] = \begin{bmatrix} 0.0333 & 0 & 0 \\ 0 & 0 & -0.0333 \\ 0 & 0 & 0 \end{bmatrix}$$

The same Mathcad program from Example 8.7 can be used for this example. After seven iterations, the results are

$$[VLL_{abc}] = \begin{bmatrix} 228.3\underline{/-1.4} \\ 231.4\underline{/-123.4} \\ 222.7\underline{/116.9} \end{bmatrix}$$

$$[I_{abc}] = \begin{bmatrix} 548.2\underline{/-50.3} \\ 606.5\underline{/167.8} \\ 381.0\underline{/50.5} \end{bmatrix}$$

$$[ID_{abc}] = \begin{bmatrix} 438.0\underline{/-27.3} \\ 216.1\underline{/-160.4} \\ 224.6\underline{/80.0} \end{bmatrix}$$

$$[I_{ABC}] = \begin{bmatrix} 18.3\underline{/-50.3} \\ 12.7\underline{/-129.5} \\ 0 \end{bmatrix}$$

$$V_{unbalance} = 2.11\%$$

Note the significant difference in voltage unbalance between Example 8.7 and Example 8.6. While it is economical to serve the load with two rather than three transformers, it has to be recognized that the open connection will lead to a much higher voltage unbalance.

8.9 Grounded Wye–Grounded Wye Connection

The grounded wye–grounded wye connection is primarily used to supply single-phase and three-phase loads on four-wire multigrounded systems. The grounded wye–grounded wye connection is shown in Figure 8.15.

Unlike the delta–wye and wye–delta connections, there is no phase shift between the voltages and the currents on the two sides of the bank.

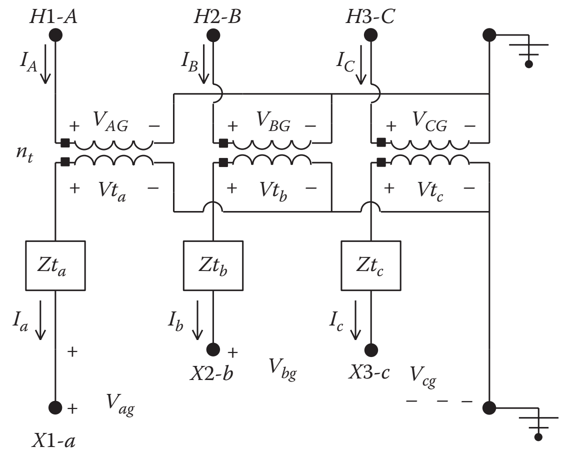

FIGURE 8.15
Grounded wye–grounded wye connection.

This makes the derivation of the generalized constant matrices much easier. The ideal transformer equations are

$$n_t = \frac{VLN_{rated\ primary}}{VLN_{rated\ secondary}} \tag{8.115}$$

$$\begin{bmatrix} V_{AG} \\ V_{BG} \\ V_{CG} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}$$

$$[VLG_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{8.116}$$

$$\text{where } [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$[I_{abc}] = [AI] \cdot [I_{ABC}] \tag{8.117}$$

$$\text{where } [AI] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

With reference to Figure 8.15, the ideal transformer voltages on the secondary windings can be computed by

$$\begin{bmatrix} Vt_a \\ Vt_b \\ Vt_c \end{bmatrix} = \begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} + \begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$
$$[Vt_{abc}] = [VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}] \tag{8.118}$$

Substitute Equation 8.118 into Equation 8.116:

$$[VLG_{ABC}] = [AV] \cdot ([VLG_{abc}] + [Zt_{abc}] \cdot [I_{abc}])$$
$$[VLG_{ABC}] = [a_t] \cdot [VLG_{abc}] + [b_t] \cdot [I_{abc}] \tag{8.119}$$

Equation 8.119 is the backward sweep equation with the $[a_t]$ and $[b_t]$ matrices defined by

$$[a_t] = [AV] = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix} \tag{8.120}$$

$$[b_t] = [AV] \cdot [Zt_{abc}] = \begin{bmatrix} n_t \cdot Zt_a & 0 & 0 \\ 0 & n_t \cdot Zt_b & 0 \\ 0 & 0 & n_t \cdot Zt_c \end{bmatrix} \tag{8.121}$$

The primary line currents as a function of the secondary line currents are given by

$$[I_{ABC}] = [d_t] \cdot [I_{abc}]$$
$$\text{where } [d_t] = [AI]^{-1} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8.122}$$

The forward sweep equation is determined solving Equation 8.119 for the secondary line-to-ground voltages:

$$\begin{aligned}
&[VLG_{abc}] = [AV]^{-1} \cdot [VLG_{ABC}] - [Zt_{abc}] \cdot [I_{abc}] \\
&[VLG_{abc}] = [A_t] \cdot [VLG_{ABC}] - [B_t] \cdot [I_{abc}] \\
&\text{where } [A_t] = [AV]^{-1} \\
&\qquad\quad [B_t] = [Zt_{abc}]
\end{aligned} \tag{8.123}$$

The modeling and analysis of the grounded wye–grounded wye connection does not present any problems. Without the phase shift there is a direct relationship between the primary and secondary voltages and currents as had been demonstrated in the derivation of the generalized constant matrices.

8.10 Delta–Delta Connection

The delta–delta connection is primarily used on three-wire delta systems to provide service to a three-phase load or a combination of three-phase and single-phase loads. Three single-phase transformers connected in a delta–delta are shown in Figure 8.16.

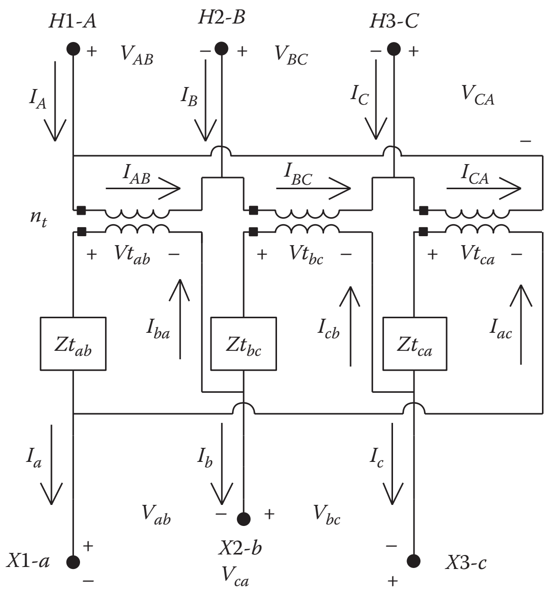

FIGURE 8.16
Delta–delta connection.

The basic "ideal" transformer voltage and current equations as a function of the "turn's ratio" are

$$n_t = \frac{VLL_{rated\ primary}}{VLL_{rated\ secondary}} \tag{8.124}$$

$$\begin{bmatrix} VLL_{AB} \\ VLL_{BC} \\ VLL_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix}$$

$$[VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{8.125}$$

$$\text{where } [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ca} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix}$$

$$[ID_{abc}] = [AI] \cdot [ID_{ABC}] \tag{8.126}$$

$$\text{where } [AI] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solve Equation 8.126 for the source-side delta currents:

$$[ID_{ABC}] = [AI]^{-1} \cdot [ID_{abc}] \tag{8.127}$$

The line currents as a function of the delta currents on the source side are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix}$$

$$[I_{ABC}] = [DI] \cdot [ID_{ABC}] \tag{8.128}$$

$$\text{where } [DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Substitute Equation 8.127 into Equation 8.128:

$$[I_{ABC}] = [DI]\cdot[AI]^{-1}\cdot[ID_{abc}] \tag{8.129}$$

Since $[AI]$ is a diagonal matrix, Equation 8.129 can be rewritten as

$$[I_{ABC}] = [AI]^{-1}\cdot[DI]\cdot[ID_{abc}] \tag{8.130}$$

The load-side line currents as a function of the load-side delta currents:

$$[I_{abc}] = [DI]\cdot[ID_{abc}] \tag{8.131}$$

Applying Equation 8.131, Equation 8.130 becomes

$$[I_{ABC}] = [AI]^{-1}\cdot[I_{abc}] \tag{8.132}$$

Turn Equation 8.132 around to solve for the load-side line currents as a function of the source-side line currents:

$$[I_{abc}] = [AI]\cdot[I_{ABC}] \tag{8.133}$$

Equations 8.132 and 8.133 merely demonstrate that the line currents on the two sides of the transformer are in phase and differ only by the turn's ratio of the transformer windings. In the per-unit system, the per-unit line currents on the two sides of the transformer are exactly equal.

The ideal delta voltages on the secondary side as a function of the line-to-line voltages, the delta currents, and the transformer impedances are given by

$$[Vt_{abc}] = [VLL_{abc}] + [Zt_{abc}]\cdot[ID_{abc}]$$

$$\text{where } [Zt_{abc}] = \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & Zt_{bc} & 0 \\ 0 & 0 & Zt_{ca} \end{bmatrix} \tag{8.134}$$

Substitute Equation 8.134 into Equation 8.125:

$$[VLL_{ABC}] = [AV]\cdot[VLL_{abc}] + [AV]\cdot[Zt_{abc}]\cdot[ID_{abc}] \tag{8.135}$$

Solve Equation 8.135 for the load-side line-to-line voltages:

$$[VLL_{abc}] = [AV]^{-1}\cdot[VLL_{ABC}] - [Zt_{abc}]\cdot[ID_{abc}] \tag{8.136}$$

The delta currents [ID_{abc}] in Equations 8.135 and 8.136 need to be replaced by the secondary line currents [I_{abc}]. In order to develop the needed relationship, three independent equations are needed. The first two come from applying KCL at two vertices of the delta connected secondary:

$$\begin{aligned} I_a &= I_{ba} - I_{ac} \\ I_b &= I_{cb} - I_{ba} \end{aligned} \tag{8.137}$$

The third equation comes from recognizing that the sum of the primary line-to-line voltages and therefore the secondary ideal transformer voltages must sum to zero. KVL around the delta windings gives

$$Vt_{ab} - Zt_{ab} \cdot I_{ba} + Vt_{bc} - Zt_{bc} \cdot I_{cb} + Vt_{ca} - Zt_{ca} \cdot I_{ac} = 0 \tag{8.138}$$

Replacing the "ideal" delta voltages with the source-side line-to-line voltages,

$$\frac{V_{AB}}{n_t} + \frac{V_{BC}}{n_t} + \frac{V_{CA}}{n_t} = Zt_{ab} \cdot I_{ba} + Zt_{bc} \cdot I_{cb} + Zt_{ca} \cdot I_{ac} \tag{8.139}$$

Since the sum of the line-to-line voltages must equal zero (KVL) and the turn's ratios of the three transformers are equal, Equation 8.139 is simplified to

$$0 = Zt_a \cdot I_{ba} + Zt_b \cdot I_{cb} + Zt_c \cdot I_{ac} \tag{8.140}$$

Note in Equation 8.140 that if the three transformer impedances are equal, then the sum of the delta currents will add to zero, meaning that the zero sequence delta currents will be zero.

Equations 8.137 and 8.140 can be put into matrix form:

$$\begin{gathered} \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ Zt_{ab} & Zt_{bc} & Zt_{ca} \end{bmatrix} \cdot \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \\ [I0_{abc}] = [F] \cdot [ID_{abc}] \\ \text{where } [I0_{abc}] = \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} \\ [F] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ Zt_{ab} & Zt_{bc} & Zt_{ca} \end{bmatrix} \end{gathered} \tag{8.141}$$

Solve Equation 8.141 for the load-side delta currents:

$$\begin{aligned}&[ID_{abc}] = [F]^{-1} \cdot [I0_{abc}] = [G] \cdot [I0_{abc}] \\ &\text{where } [G] = [F]^{-1}\end{aligned} \tag{8.142}$$

Writing Equation 8.142 in matrix form gives

$$\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ 0 \end{bmatrix} \tag{8.143}$$

From Equations 8.142 and 8.143, it is seen that the delta currents are a function of the transformer impedances and just the line currents in phases *a* and *b*. Equation 8.143 can be modified to include the line current in phase *c* by setting the last column of the [*G*] matrix to zeros:

$$\begin{aligned}&\begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \\ &[ID_{abc}] = [G1] \cdot [I_{abc}] \\ &\text{where } [G1] = \begin{bmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & 0 \end{bmatrix}\end{aligned} \tag{8.144}$$

When the impedances of the transformers are equal, the sum of the delta currents will be zero meaning that there is no circulating zero sequence current in the delta windings.

Substitute Equation 8.144 into Equation 8.135:

$$[VLL_{ABC}] = [AV] \cdot [VLL_{abc}] + [AV][Zt_{abc}] \cdot [G1] \cdot [I_{abc}] \tag{8.145}$$

The generalized matrices are defined in terms of the line-to-neutral voltages on the two sides of the transformer bank. Equation 8.145 is modified to be in terms of equivalent line-to-neutral voltages:

$$\begin{aligned}[VLN_{ABC}] &= [W] \cdot [VLL_{ABC}] \\ &= [W][AV] \cdot [D] \cdot [VLN_{abc}] + [W] \cdot [Zt_{abc}] \cdot [G1] \cdot [I_{abc}]\end{aligned} \tag{8.146}$$

Equation 8.146 is in the general form

$$\begin{aligned}[VLN_{ABC}] &= [a_t]\cdot[VLN_{abc}] + [b_t]\cdot[I_{abc}] \\ \text{where } [a_t] &= [W]\cdot[AV]\cdot[D] \\ [b_t] &= [AV]\cdot[W]\cdot[Zt_{abc}]\cdot[G1]\end{aligned} \tag{8.147}$$

Equation 8.133 gives the generalized equation for currents:

$$\begin{aligned}[I_{ABC}] &= [AI]^{-1}\cdot[I_{abc}] = [d_t]\cdot[I_{abc}] \\ \text{where } [d_t] &= [AI]^{-1}\end{aligned} \tag{8.148}$$

The forward sweep equations can be derived by modifying Equation 8.136 in terms of equivalent line-to-neutral voltages:

$$\begin{aligned}[VLN_{abc}] &= [W]\cdot[VLL_{abc}] \\ &= [W]\cdot[AV]^{-1}\cdot[D]\cdot[VLN_{ABC}] - [W]\cdot[Zt_{abc}]\cdot[G1]\cdot[I_{abc}]\end{aligned} \tag{8.149}$$

The forward sweep equation is

$$\begin{aligned}[VLN_{abc}] &= [A_t]\cdot[VLN_{ABC}] - [B_t]\cdot[I_{abc}] \\ \text{where } [A_t] &= [W]\cdot[AV]^{-1}\cdot[D] \\ [B_t] &= [W]\cdot[Zt_{abc}]\cdot[G1]\end{aligned} \tag{8.150}$$

The forward and backward sweep matrices for the delta–delta connection have been derived. Once again it has been a long process to get to the final six equations that define the matrices. The derivation provides an excellent exercise in the application of basic transformer theory and circuit theory. Once the matrices have been defined for a particular transformer connection, the analysis of the connection is a relatively simple task. Example 8.8 will demonstrate the analysis of this connection using the generalized matrices.

Example 8.8

Figure 8.17 shows three single-phase transformers in a delta–delta connection serving an unbalanced three-phase load connected in delta.

The source voltages at the load are balanced three phase of 240 V line to line:

$$[VLL_{abc}] = \begin{bmatrix} 12,470/\underline{0} \\ 12,470/\underline{-120} \\ 12,470/\underline{120} \end{bmatrix} \text{V}$$

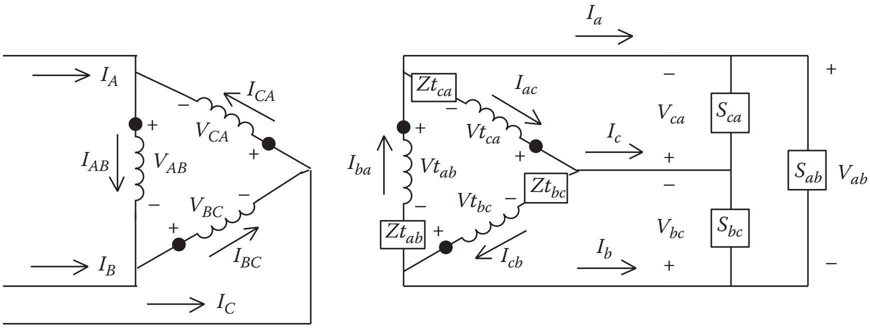

FIGURE 8.17
Delta–delta bank serving an unbalanced delta connected load.

The loading by phase is

S_{ab} = 100 kVA at 0.9 lagging power factor
$S_{bc} = S_{ca}$ = 50 kVA at 0.8 lagging power factor

The ratings of the transformers are

Phase *AB*: 100 kVA, 12,470–240 V, $Z = 0.01 + j0.04$ per unit
Phases *BC* and *CA* 50 kVA, 12,470–240 V, $Z = 0.015 + j0.035$ per unit

Determine the following:

1. The load line-to-line voltages
2. The secondary line currents
3. The primary line currents
4. The load currents
5. Load voltage unbalance

Before the analysis can start, the transformer impedances must be converted to actual values in Ohms and located inside the delta connected secondary windings.

Phase ab transformer:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{100} = 0.576\,\Omega$$

$$Zt_{ab} = (0.01 + j0.04) \cdot 0.576 = 0.0058 + j0.023\,\Omega$$

Phase bc and ca transformers:

$$Z_{base} = \frac{0.24^2 \cdot 1000}{50} = 1.152\,\Omega$$

$$Zt_{bc} = Zt_{ca} = (.015 + j0.035) \cdot 1.152 = 0.0173 + j0.0403\,\Omega$$

The transformer impedance matrix can now be defined:

$$[Zt_{abc}] = \begin{bmatrix} 0.0058 + j.023 & 0 & 0 \\ 0 & 0.0173 + j0.0403 & 0 \\ 0 & 0 & 0.0173 + j0.0403 \end{bmatrix} \Omega$$

The turn's ratio of the transformers is n_t = 12,470/240 = 51.9583.

Define all of the matrices:

$$[W] = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \quad [D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad [DI] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[AV] = [AI] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 51.9583 & 0 & 0 \\ 0 & 51.9583 & 0 \\ 0 & 0 & 51.9583 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0.0058 + j0.023 & 0.0173 + j0.0403 & 0.0173 + j0.0404 \end{bmatrix}$$

$$[G] = [F]^{-1} = \begin{bmatrix} 0.3941 - j0.0134 & -0.3941 + j0.0134 & 3.2581 - j8.378 \\ 0.3941 - j0.0134 & 0.6059 + j0.0134 & 3.2581 - j8.378 \\ -0.6059 - j0.0134 & -0.3941 + j0.0134 & 3.2581 - j8.378 \end{bmatrix}$$

$$[G1] = \begin{bmatrix} 0.3941 - j0.0134 & -0.3941 + j0.0134 & 0 \\ 0.3941 - j0.0134 & 0.6059 + j0.0134 & 0 \\ -0.6059 - j0.0134 & -0.3941 + j0.0134 & 0 \end{bmatrix}$$

$$[a_t] = [W] \cdot [AV] \cdot [D] = \begin{bmatrix} 34.6489 & -17.3194 & -17.3194 \\ -17.3194 & 34.6489 & -17.3194 \\ -17.3194 & -17.3194 & 34.6489 \end{bmatrix}$$

$$[b_t] = [AV] \cdot [W] \cdot [Zt_{abc}] \cdot [G1] = \begin{bmatrix} 0.2166 + j0.583 & 0.0826 + j0.1153 & 0 \\ 0.0826 + j0.1153 & 0.2166 + j0.583 & 0 \\ -0.2993 - j0.6983 & -0.2993 - j0.6983 & 0 \end{bmatrix}$$

$$[d_t]=[AI]^{-1}=\begin{bmatrix}0.0192 & 0 & 0\\ 0 & 0.0192 & 0\\ 0 & 0 & 0.0192\end{bmatrix}$$

$$[A_t]=[W]\cdot[AV]^{-1}\cdot[D]=\begin{bmatrix}0.0128 & -0.0064 & -0.0064\\ -0.0064 & 0.0128 & -0.0064\\ -0.0064 & -0.0064 & 0.0128\end{bmatrix}$$

$$[B_t]=[W]\cdot[Zt_{abc}]\cdot[G1]=\begin{bmatrix}0.0042+j0.0112 & 0.0016+j0.0022 & 0\\ 0.0016+j0.0022 & 0.0042+j0.0112 & 0\\ -0.0058-j0.0134 & -0.0058-j0.0134 & 0\end{bmatrix}$$

The Mathcad program is modified slightly to account for the delta connections. The modified program is shown in Figure 8.18.

After six iterations, the results are

$$[VLL_{abc}]=\begin{bmatrix}232.9/\underline{28.3}\\ 231.0/\underline{-91.4}\\ 233.1/\underline{148.9}\end{bmatrix}$$

$$[I_{abc}]=\begin{bmatrix}540.3/\underline{-19.5}\\ 593.6/\underline{-161.5}\\ 372.8/\underline{81.7}\end{bmatrix}$$

$$[I_{ABC}]=\begin{bmatrix}10.4/\underline{-19.5}\\ 11.4/\underline{-161.5}\\ 7.2/\underline{81.7}\end{bmatrix}$$

$$[ID_{abc_i}]=\begin{bmatrix}429.3/\underline{2.4}\\ 216.5/\underline{-128.3}\\ 214.5/\underline{112.0}\end{bmatrix}\text{A}$$

$$V_{unbalance}=0.59\%$$

This example demonstrates that a small change in the Mathcad program can be made to represent the delta–delta transformer connection.

$$\left|VLL_{ABC_i}\right| = \begin{pmatrix} 12470 \\ 12470 \\ 12470 \end{pmatrix} \quad \frac{\arg\left(VLL_{ABC_i}\right)}{deg} = \begin{pmatrix} 30 \\ -90 \\ 150 \end{pmatrix}$$

$$Start := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad Tol := .000001 \qquad VM := kVLL_{sec} \cdot 1000 \qquad VM = 240$$

$$X := \left| \begin{array}{l} I_{abc} \leftarrow Start \\ V_{old} \leftarrow Start \\ VLG_{ABC} \leftarrow W \cdot VLL_{ABC} \\ \text{for } n \in 1..200 \\ \quad \left| \begin{array}{l} VLN_{abc} \leftarrow A_t \cdot VLG_{ABC} - B_t \cdot I_{abc} \\ VLL_{abc} \leftarrow D \cdot VLN_{abc} \\ \text{for } j \in 1..3 \\ \quad ID_{abc_j} \leftarrow \overline{\dfrac{SL_j \cdot 1000}{VLL_{abc_j}}} \\ \text{for } k \in 1..3 \\ \quad Error_k \leftarrow \dfrac{\left|VLL_{abc_k} - V_{old_k}\right|}{VM} \\ Error_{max} \leftarrow \max(Error) \\ \text{break if } Error_{max} < Tol \\ V_{old} \leftarrow VLL_{abc} \\ I_{abc} \leftarrow DI \cdot ID_{abc} \\ I_{ABC} \leftarrow d_t \cdot I_{abc} \end{array} \right. \\ Out_1 \leftarrow VLN_{abc} \\ Out_2 \leftarrow VLL_{abc} \\ Out_3 \leftarrow I_{abc} \\ Out_4 \leftarrow I_{ABC} \\ Out_5 \leftarrow ID_{abc} \\ Out_6 \leftarrow n \\ Out \end{array} \right.$$

FIGURE 8.18
Mathcad® program.

8.11 Open Delta–Open Delta

The open delta–open delta transformer connection can be connected in three different ways. Figure 8.19 shows the connection using phase *AB* and *BC*.

The relationship between the primary line-to-line voltages and the secondary ideal voltage is given by

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix}$$

$$[VLL_{ABC}] = [AV] \cdot [Vt_{abc}] \tag{8.151}$$

$$\text{where } [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

The last row of the matrix [*AV*] is the result that the sum of the line-to-line voltages must be equal to zero.

The relationship between the secondary and primary line currents is

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

$$[I_{abc}] = [AI] \cdot [I_{ABC}] \tag{8.152}$$

$$\text{where } [AI] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

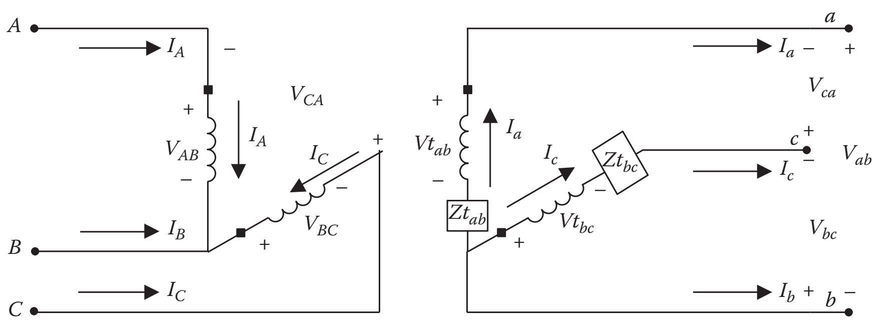

FIGURE 8.19
Open delta–open delta using phases *AB* and *BC*.

The primary line currents as a function of the secondary line currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[I_{ABC}] = [d_t] \cdot [I_{abc}] \tag{8.153}$$

$$\text{where } [d_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

The ideal secondary voltages are given by

$$\begin{aligned} Vt_{ab} &= V_{ab} + Zt_{ab} \cdot I_a \\ Vt_{bc} &= V_{bc} + Zt_{bc} \cdot I_c \end{aligned} \tag{8.154}$$

The primary line-to-line voltages as a function of the secondary line-to-line voltages are given by

$$\begin{aligned} V_{AB} &= n_t \cdot Vt_{ab} = n_t \cdot V_{ab} + n_t \cdot Zt_{ab} \cdot I_a \\ V_{BC} &= n_t \cdot Vt_{bc} = n_t \cdot V_{bc} + n_t \cdot Zt_{bc} \cdot I_c \end{aligned} \tag{8.155}$$

The sum of the primary line-to-line voltages must equal zero. Therefore, the voltage V_{CA} is given by

$$\begin{aligned} V_{CA} &= -(V_{AB} + V_{BC}) = -n_t \cdot (V_{ab} + n_t \cdot Zt_{ab} \cdot I_a + V_{bc} + n_t \cdot Zt_{bc} \cdot I_c) \\ V_{CA} &= -n_t \cdot V_{ab} - n_t \cdot V_{bc} - n_t \cdot Zt_{ab} \cdot I_a - n_t \cdot Zt_{bc} \cdot I_c \end{aligned} \tag{8.156}$$

Equations 8.155 and 8.156 can be put into matrix form to create the backward sweep voltage equation:

$$[VLL_{ABC}] = [x] \cdot [VLL_{abc}] + [BI] \cdot [I_{abc}]$$

$$\text{where } [x] = [AV] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \tag{8.157}$$

$$[BI] = n_t \cdot \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \end{bmatrix}$$

Equation 8.157 gives the backward sweep equation in terms of line-to-line voltages. In order to convert the equation to equivalent line-to-neutral voltages, the [*W*] and [*D*] matrices are applied to Equation 8.157:

$$[VLL_{ABC}] = [AV]\cdot[VLL_{abc}] + [y]\cdot[I_{abc}]$$

$$[VLN_{ABC}] = [W]\cdot[VLL_{ABC}] = [W]\cdot[AV]\cdot[D]\cdot[VLN_{abc}] + [W]\cdot[BI]\cdot[I_{abc}]$$

$$[VLN_{ABC}] = [a_t]\cdot[VLN_{abc}] + [b_t]\cdot[I_{abc}] \tag{8.158}$$

$$\text{where } [a_t] = [W]\cdot[AV]\cdot[D]$$

$$[b_t] = [W]\cdot[BI]$$

The forward sweep equation can be derived by defining the ideal voltages as a function of the primary line-to-line voltages:

$$\begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \frac{1}{n_t}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}\cdot\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix}$$

$$[Vt_{abc}] = [BV]\cdot[VLL_{ABC}] \tag{8.159a}$$

$$\text{where } [BV] = \frac{1}{n_t}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$$

The ideal secondary voltages as a function of the terminal line-to-line voltages are given by

$$\begin{bmatrix} Vt_{ab} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} + \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \end{bmatrix}\cdot\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[Vt_{abc}] = [VLL_{abc}] + [BI]\cdot[I_{abc}] \tag{8.159b}$$

$$\text{where } [BI] = \begin{bmatrix} Zt_{ab} & 0 & 0 \\ 0 & 0 & Zt_{bc} \\ -Zt_{ab} & 0 & -Zt_{bc} \end{bmatrix}$$

Equate Equation 8.158 to Equation 8.159:

$$\begin{aligned} [BV]\cdot[VLL_{ABC}] &= [VLL_{abc}] + [BI]\cdot[I_{abc}] \\ [VLL_{abc}] &= [BV]\cdot[VLL_{ABC}] - [BI]\cdot[I_{abc}] \end{aligned} \tag{8.160}$$

Equation 8.160 gives the forward sweep equation in terms of line-to-line voltages. As before, the [W] and [D] matrices are used to convert Equation 8.160 using line-to-neutral voltages:

$$[VLL_{abc}] = [BV]\cdot[VLL_{ABC}] - [BI]\cdot[I_{abc}]$$

$$[VLN_{abc}] = [W]\cdot[VLL_{abc}] = [W]\cdot[BV]\cdot[D]\cdot[VLN_{ABC}] - [W]\cdot[BI]\cdot[I_{abc}]$$

$$[VLN_{abc}] = [A_t]\cdot[VLN_{ABC}] - [B_t]\cdot[I_{abc}] \tag{8.161}$$

$$\text{where } [A_t] = [W]\cdot[BV]\cdot[D]$$

$$[B_t] = [W]\cdot[BI]$$

Example 8.9

In Example 8.8, remove the transformer connected between phases C and A. This creates an open delta–open delta transformer bank. This transformer bank serves the same loads as in Example 8.8.

Determine the following:

1. The load line-to-line voltages
2. The secondary line currents
3. The primary line currents
4. The load currents
5. Load voltage unbalance

The exact same program from Example 8.8 is used since only the values of the matrices change for this connection. After six iterations, the results are

$$[VLL_{abc}] = \begin{bmatrix} 229.0\underline{/28.5} \\ 248.2\underline{/-86.9} \\ 255.4\underline{/147.2} \end{bmatrix}$$

$$[I_{abc}] = \begin{bmatrix} 529.9\underline{/-17.9} \\ 579.6\underline{/-161.1} \\ 353.8\underline{/82.8} \end{bmatrix}$$

$$[I_{ABC}] = \begin{bmatrix} 10.2\underline{/-17.9} \\ 11.2\underline{/-161.1} \\ 6.8\underline{/82.8} \end{bmatrix}$$

$$[ID_{abc}] = \begin{bmatrix} 436.7/\underline{2.7} \\ 201.4/\underline{-123.8} \\ 195.8/\underline{110.3} \end{bmatrix}$$

$$V_{unbalance} = 6.2\%$$

An inspection of the line-to-line load voltages should raise a question since two of the three voltages are greater than the no-load voltages of 240 V. Why is there an apparent voltage rise on two of the phases? This can be explained by computing the voltage drops in the secondary circuit:

$$v_a = Zt_{ab} \cdot I_a = 12.6/\underline{58.0}$$

$$v_b = Zt_{bc} \cdot I_c = 15.5/\underline{149.6}$$

The ideal voltages are

$$Vt_{ab} = 240/\underline{30}$$

$$Vt_{bc} = 240/\underline{-90}$$

The terminal voltages are given by

$$V_{ab} = Vt_{ab} - v_a = 229.0/\underline{28.5}$$

$$V_{bc} = Vt_{bc} - v_b = 248.2/\underline{-86.9}$$

$$V_{ca} = -(V_{ab} + V_{bc}) = 255.4/\underline{147.2}$$

Figure 8.20 shows the phasor diagrams (not to scale) for the voltages defined earlier. In the phasor diagram, it is clear that there is a voltage drop on phase

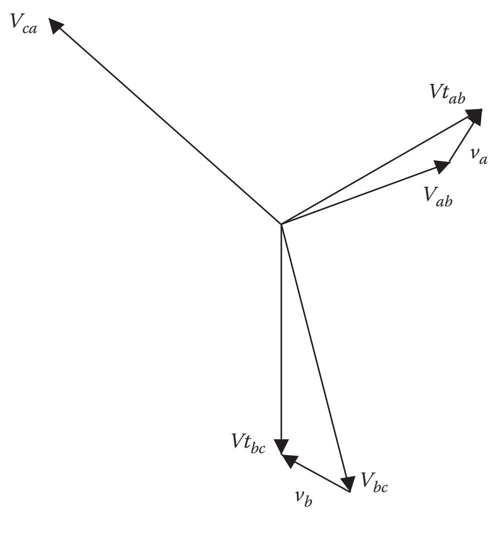

FIGURE 8.20
Voltage phasor diagram.

ab and then a voltage rise on phase *bc*. The voltage on *ca* also is greater than the rated 240 V because the sum of the voltages must add to zero.

It is important that when there is a question about the results of a study that basic circuit and transformer theory along with a phasor diagram can confirm that the results are correct. This example is a good example of when the results should be confirmed. Notice also that the voltage unbalance is much greater for the open delta–open delta than the closed delta–delta connection voltage unbalance.

8.12 Thevenin Equivalent Circuit

This chapter develops the general matrices for the forward and backward sweeps for most standard three-phase transformer connections. Section 10.2 will require the Thevenin equivalent circuit referenced to the secondary terminals of the transformer. This equivalent circuit must take into account the equivalent impedance between the primary terminals of the transformer and the feeder source. Figure 8.21 is a general circuit showing the feeder source down to the secondary bus.

The Thevenin equivalent circuit needs to be determined at the secondary node of the transformer bank. This basically is the same as "referring" the source voltage and the source impedance to the secondary side of the transformer. The desired "Thevenin equivalent circuit" at the transformer secondary node is shown in Figure 8.22.

A general Thevenin equivalent circuit that can be used for all connections defined by the forward and backward sweep matrices.

In Figure 8.16, the primary transformer equivalent line-to-neutral voltages as a function of the source voltages and the equivalent high-side impedance are given by

$$[VLN_{ABC}] = [ELN_{ABC}] - [Zsys_{ABC}] \cdot [I_{ABC}] \tag{8.162}$$

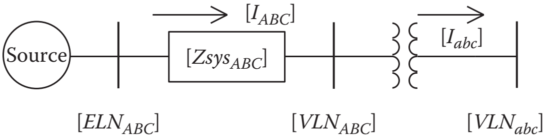

FIGURE 8.21
Equivalent system.

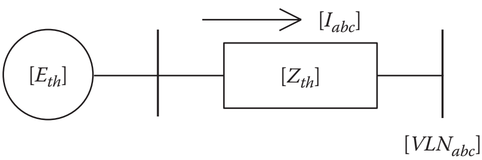

FIGURE 8.22
Thevenin equivalent circuit.

but

$$[I_{ABC}] = [d_t] \cdot [I_{abc}]$$

therefore

$$[VLN_{ABC}] = [ELN_{ABC}] - [Zsys_{ABC}] \cdot [d_t] \cdot [I_{abc}] \tag{8.163}$$

The forward sweep equation gives the secondary line-to-neutral voltages as a function of the primary line-to-neutral voltages:

$$[VLN_{abc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}] \tag{8.164}$$

Substitute Equation 8.163 into Equation 8.164:

$$[VLN_{abc}] = [A_t].\{[ELN_{ABC}] - [Zsys_{ABC}] \cdot [d_t] \cdot [I_{abc}]\} - [B_t] \cdot [I_{abc}]$$

$$[VLN_{abc}] = [A_t] \cdot [ELN_{ABC}] - ([A_t] \cdot [Zsys_{ABC}] \cdot [d_t] + [B_t]) \cdot [I_{abc}]$$

$$[VLN_{abc}] = [E_{th}] - [Z_{th}] \cdot [I_{abc}] \tag{8.165}$$

where $[E_{th}] = [A_t] \cdot [ELN_{ABC}]$

$$[Z_{th}] = ([A_t] \cdot [Zsys_{ABC}] \cdot [d_t] + [B_t])$$

The definitions of the Thevenin equivalent voltages and impedances as given in Equation 8.165 are general and can be used for all transformer connections. Example 8.5 is used to demonstrate the computation and application of the Thevenin equivalent circuit.

Example 8.10

The delta–grounded wye transformer bank of Example 8.2 is connected to a balanced three-phase source of 115 kV through a 1 mile section of a four-wire three-phase line as shown in Figure 8.23.

The phase impedance matrix for the 1 mile of line is given by

$$[Zsys_{ABC}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1559 + j0.5017 & 0.1535 + j0.3849 \\ 0.1559 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega$$

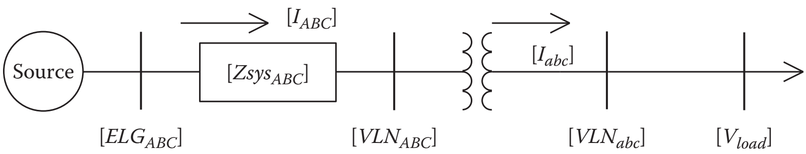

FIGURE 8.23
Modified Example 8.2 system.

For the unbalanced load in Example 8.2 using a Mathcad program, the load line-to-neutral voltages and load currents were computed as

$$[VLN_{abc}] = \begin{bmatrix} 6487.8/\underline{-36.7} \\ 6770.9/\underline{-153.8} \\ 6698.0/\underline{83.9} \end{bmatrix} \text{V}$$

$$[I_{abc}] = \begin{bmatrix} 262.0/\underline{-62.5} \\ 177.2/\underline{177.4} \\ 223.9/\underline{65.7} \end{bmatrix} \text{A}$$

The Thevenin equivalent voltages and impedances referred to the secondary terminals of the transformer bank are

$$[Eth_{abc}] = [A_t] \cdot [ELN_{ABC}] = \begin{bmatrix} 7200/\underline{-30} \\ 7200/\underline{-120} \\ 7200/\underline{120} \end{bmatrix} \text{V}$$

$$[Zth_{abc}] = [A_t] \cdot [Zsys_{ABC}] \cdot [d_t] + [B_t]$$

$$[Zth_{abc}] = \begin{bmatrix} 0.2328 + j2.6388 & -0.0012 - j0.0024 & -0.0012 - 0.0030 \\ -0.0012 - j0.0024 & 0.2328 + j2.6379 & -0.0012 - j0.0020 \\ -0.0012 - 0.0030 & -0.0012 - j0.0020 & 0.2328 + j2.6384 \end{bmatrix} \Omega$$

The Thevenin equivalent circuit for this case is shown in Figure 8.24.

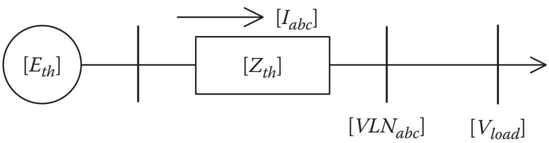

FIGURE 8.24
Thevenin equivalent system.

The Mathcad program was modified to match the equivalent system in Figure 8.24. The exact same load voltages and load currents were computed. This example is intended to demonstrate that it is possible to compute the Thevenin equivalent circuit at the secondary terminals of the transformer bank. The example shows that using the Thevenin equivalent circuit and the original secondary line currents the original equivalent line-to-neutral load voltages are computed. The major application of the Thevenin equivalent circuit will be in the short circuit analysis of a distribution that will be developed in Chapter 10.

8.13 Summary

In this chapter, the forward and backward sweep matrices have been developed for seven common three-phase transformer bank connections. For unbalanced transformer connections, the derivations were limited to just one of at least three ways that the primary phases could be connected to the transformer bank. The methods in the derivation of these transformer banks can be extended to all possible phasings.

One of the major features of the chapter has been to demonstrate how the forward and backward sweep technique (ladder) is used to analyze the operating characteristics of the transformer banks. Several Mathcad programs were used in the examples to demonstrate how the analysis is mostly independent of the transformer connection by using the derived matrices. This approach was first demonstrated with the line models and then continued to the voltage regulators and now the transformer connections. In Chapter 10, the analysis of a total distribution feeder is developed using the forward and backward sweep matrices for all possible system components.

Many of the examples demonstrated the use of a Mathcad program for the analysis. An extension of this is the use of student version of the Windmil distribution analysis program that can be downloaded as explained in the Preface of this book. When the program is downloaded, a "user's manual" will be included. The user's manual serves two purposes:

- A tutorial on how to get started using Windmil for the first time user.
- Included will be the Windmil systems for many of the examples in this and other chapters.

It is highly encourage that the program and manual be downloaded.

Problems

8.1 A three-phase substation transformer is connected delta–grounded wye and rated:

5000 kVA, 115 kV delta–12.47 kV grounded wye, $Z = 1.0 + j7.5\%$

The transformer serves an unbalanced load of

Phase *a*: 1384.5 kVA, 89.2% lagging power factor at $6922.5\underline{/-33.1}$ V
Phase *b*: 1691.2 kVA, 80.2% lagging power factor at $6776.8\underline{/-153.4}$ V
Phase *c*: 1563.0 kVA, unity power factor at $7104.7\underline{/85.9}$ V

a. Determine the forward and backward sweep matrices for the transformer.
b. Compute the primary equivalent line-to-neutral voltages.
c. Compute the primary line-to-line voltages.
d. Compute the primary line currents.
e. Compute the currents flowing in the high-side delta windings.
f. Compute the real power loss in the transformer for this load condition.

8.2 Write a simple Mathcad or MATLAB® program using the ladder technique to solve for the load line-to-ground voltages and line currents in the bank of 8.1 when the source voltages are balanced three phase of 115 kV line to line.

8.3 Create the system in Windmil for Problem 8.2.

8.4 Three single-phase transformers are connected in delta–grounded wye and serving an unbalance load. The ratings of three transformers are

Phase *AB*: 100 kVA, 12,470–120 V, $Z = 1.3 + j1.7\%$
Phase *BC*: 50 kVA, 12,470–120 V, $Z = 1.1 + j1.4\%$
Phase *CA*: same as phase *BC* transformer

The unbalanced loads are

Phase *a*: 40 kVA, 0.8 lagging power factor at $V = 117.5\underline{/-32.5}$ V
Phase *b*: 85 kVA, 0.95 lagging power factor at $V = 115.7\underline{/-147.3}$ V
Phase *c*: 50 kVA, 0.8 lagging power factor at $V = 117.0\underline{/95.3}$ V

a. Determine the forward and backward sweep matrices for this connection.
b. Compute the load currents.
c. Compute the primary line-to-neutral voltages.
d. Compute the primary line-to-line voltages.
e. Compute the primary currents.
f. Compute the currents in the delta primary windings.
g. Compute the transformer bank real power loss.

8.5 For the same load and transformers in Problem 8.4, assume that the primary voltages on the transformer bank are balanced three phase of 12,470 V line to line. Write a Mathcad or MATLAB program to compute the load line-to-ground voltages and the secondary line currents.

8.6 For the transformer connection and loads of Problem 8.4, build the system in Windmil.

8.7 The three single-phase transformers of Problem 8.4 are serving an unbalanced constant impedance load of

Phase a: $0.32 + j0.14\,\Omega$
Phase b: $0.21 + j0.08\,\Omega$
Phase c: $0.28 + j0.12$

The transformers are connected to a balanced 12.47 kV source.

a. Determine the load currents.
b. Determine the load voltages.
c. Compute the complex power of each load.
d. Compute the primary currents.
e. Compute the operating kVA of each transformer.

8.8 Solve Problem 8.7 using Windmil.

8.9 A three-phase transformer connected wye–delta is rated:

$$500\text{ kVA},\quad 4160\text{-}240\text{ V},\quad Z = 1.1 + j3.0\%$$

The primary neutral is ungrounded. The transformer is serving a balanced load of 480 kW with balanced voltages of 235 V line-to-line and a lagging power factor of 0.9.

a. Compute the secondary line currents.
b. Compute the primary line currents.
c. Compute the currents flowing in the secondary delta windings.
d. Compute the real power loss in the transformer for this load.

8.10 The transformer of Problem 8.9 is serving an unbalanced delta load of

$S_{ab} = 150$ kVA, 0.95 lagging power factor
$S_{bc} = 125$ kVA, 0.90 lagging power factor
$S_{ca} = 160$ kVA, 0.8 lagging power factor

The transformer bank is connected to a balanced three-phase source of 4160 V line to line.

a. Compute the forward and backward sweep matrices for the transformer bank.
b. Compute the load equivalent line-to-neutral and line-to-line voltages.
c. Compute the secondary line currents.

d. Compute the load currents.

e. Compute the primary line currents.

f. Compute the operating kVA of each transformer winding.

g. Compute the load voltage unbalance.

8.11 Three single-phase transformers are connected in an ungrounded wye–delta connection and serving an unbalanced delta connected load. The transformers are rated:

Phase A: 15 kVA, 2400–240 V, $Z = 1.3 + j1.0\%$
Phase B: 25 kVA, 2400–240 V, $Z = 1.1 + j1.1\%$
Phase C: same as phase A transformer

The transformers are connected to a balanced source of 4.16 kV line to line. The primary currents entering the transformer are

$I_A = 4.60$ A, 0.95 lagging power factor
$I_B = 6.92$ A, 0.88 lagging power factor
$I_C = 5.37$ A, 0.69 lagging power factor

a. Determine the primary line-to-neutral voltages. Select V_{AB} as reference.

b. Determine the line currents entering the delta-connected load.

c. Determine the line-to-line voltages at the load.

d. Determine the operating kVA of each transformer.

e. Is it possible to determine the load currents in the delta-connected load? If so, do it. If not, why not?

8.12 The three transformers of Problem 8.11 are serving an unbalanced delta-connected load of

$S_{ab} = 10$ kVA, 0.95 lagging power factor
$S_{bc} = 20$ kVA, 0.90 lagging power factor
$S_{ca} = 15$ kVA, 0.8 lagging power factor

The transformers are connected to a balance 4160 line-to-line voltage source. Determine the load voltages and the primary and secondary line currents for the following transformer connections:

- Ungrounded wye–delta connection
- Grounded wye–delta connection
- Open wye–open delta connection, where the transformer connected to phase C has been removed

8.13 Three single-phase transformers are connected in grounded wye–grounded wye and serving an unbalanced constant impedance load. The transformer bank is connected to a balanced three-phase 12.47 line-to-line voltage source. The transformers are rated:

Phase A: 100 kVA, 7200–120 V, $Z = 0.9 + j1.8\%$
Phase B and phase C: 37.5 kVA, 7200–120 V, $Z = 1.1 + j1.4\%$

The constant impedance loads are

Phase *a*: $0.14 + j0.08\,\Omega$
Phase *b*: $0.40 + j0.14\,\Omega$
Phase *c*: $0.50 + j0.20\,\Omega$

a. Compute the generalized matrices for this transformer bank.
b. Determine the load currents.
c. Determine the load voltages.
d. Determine the kVA and power factor of each load.
e. Determine the primary line currents.
f. Determine the operating kVA of each transformer.

8.14 Three single-phase transformers are connected in delta–delta and are serving a balanced three-phase motor rated 150 kVA, 0.8 lagging power factor and a single-phase lighting load of 25 kVA, 0.95 lagging power factor connected across phases *a-b*. The transformers are rated:

Phase *AB*: 75 kVA, 4800–240 V, $Z = 1.0 + j1.5\%$
Phase *BC*: 50 kVA, 4800–240 V, $Z = 1.1 + j1.4\%$
Phase *CA*: same as phase *BC*

The load voltages are balanced three phase of 240 V line to line.

a. Determine the forward and backward sweep matrices.
b. Compute the motor input currents.
c. Compute the single-phase lighting load current.
d. Compute the primary line currents.
e. Compute the primary line-to-line voltages.
f. Compute the currents flowing in the primary and secondary delta windings.

8.15 In Problem 8.14, the transformers on phases *AB* and *BC* are connected in an open delta–open delta connection and serving an unbalanced three-phase load of

Phase *ab*: 50 kVA at 0.9 lagging power factor
Phase *bc*: 15 kVA at 0.85 lagging power factor
Phase *ca*: 25 kVA at 0.95 lagging power factor

The source line-to-line voltages are balance at 4800 V line to line. Determine

- The load line-to-line voltages
- The load currents
- The secondary line currents
- The primary line currents

Windmil Assignment

Use System 4 to build this new System 5. A 5000 kVA delta–grounded wye substation transformer is to be connected between the source and node 1. The voltages for the transformer are 115 kV delta to 12.47 kV grounded wye. The impedance of this transformer is 8.06% with an *X/R* ratio 8. By installing this substation transformer be sure to modify the source so that it is 115,000 V rather than 12.47 V. Follow the steps in the user's manual on how to install the substation transformer.

1. When the transformer has been connected run Voltage Drop.
2. What are the node voltages at node 2?
3. What taps has the regulator gone to?
4. Why did the taps increase when the transformer was added to the system?

9

Load Models

The loads on a distribution system are typically specified by the complex power consumed. With reference to Chapter 2, the specified load will be the "maximum diversified demand." This demand can be specified as kVA and power factor, kW and power factor, or kW and kvar. The voltage specified will always be the voltage at the low-voltage bus of the distribution substation. This creates a problem since the current requirement of the loads can not be determined without knowing the voltage. For this reason, modified ladder iterative technique must be employed.

Loads on a distribution feeder can be modeled as wye connected or delta connected. The loads can be three phase, two phase, or single phase with any degree of unbalanced. The *ZIP* models are

- Constant impedance (*Z*)
- Constant current (*I*)
- Constant real and reactive power (constant *P*)
- Any combination of the above

The load models developed are to be used in the iterative process of a power-flow program where the load voltages are initially assumed. One of the results of the power-flow analysis is to replace the assumed voltages with the actual operating load voltages. All models are initially defined by a complex power per phase and an assumed line-to-neutral voltage (wye load) or an assumed line-to-line voltage (delta load). The units of the complex power can be in volt-amperes and volts or per-unit volt-amperes and per-unit voltages. For all loads, the line currents entering the load are required in order to perform the power-flow analysis.

9.1 Wye-Connected Loads

Figure 9.1 shows the model of a wye-connected load.

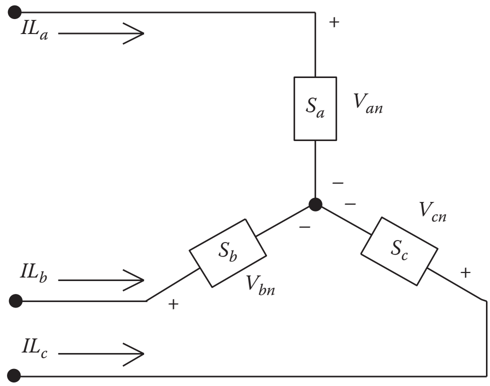

FIGURE 9.1
Wye-connected load.

The notation for the specified complex powers and voltages are as follows:

Phase a:

$$|S_a|\underline{/\theta_a} = P_a + jQ_a \quad \text{and} \quad |V_{an}|\underline{/\delta_a} \tag{9.1}$$

Phase b:

$$|S_b|\underline{/\theta_b} = P_b + jQ_b \quad \text{and} \quad |V_{bn}|\underline{/\delta_b} \tag{9.2}$$

Phase c:

$$|S_c|\underline{/\theta_c} = P_c + jQ_c \quad \text{and} \quad |V_{cn}|\underline{/\delta_c} \tag{9.3}$$

9.1.1 Constant Real and Reactive Power Loads

The line currents for constant real and reactive power loads (*PQ* loads) are given by

$$\begin{aligned} IL_a &= \left(\frac{S_a}{V_{an}}\right)^* = \frac{|S_a|}{|V_{an}|}\underline{/\delta_a - \theta_a} = |IL_a|\underline{/\alpha_a} \\ IL_b &= \left(\frac{S_b}{V_{bn}}\right)^* = \frac{|S_b|}{|V_{bn}|}\underline{/\delta_b - \theta_b} = |IL_b|\underline{/\alpha_b} \\ IL_c &= \left(\frac{S_c}{V_{cn}}\right)^* = \frac{|S_c|}{|V_{cn}|}\underline{/\delta_c - \theta_c} = |IL_c|\underline{/\alpha_c} \end{aligned} \tag{9.4}$$

In this model, the line-to-neutral voltages will change during each iteration until convergence is achieved.

9.1.2 Constant Impedance Loads

The "constant load impedance" is first determined from the specified complex power and assumed line-to-neutral voltages:

$$
\begin{aligned}
Z_a &= \frac{|V_{an}|^2}{S_a^*} = \frac{|V_{an}|^2}{|S_a|}\underline{/\theta_a} = |Z_a|\underline{/\theta_a} \\
Z_b &= \frac{|V_{bn}|^2}{S_b^*} = \frac{|V_{bn}|^2}{|S_b|}\underline{/\theta_b} = |Z_b|\underline{/\theta_b} \qquad (9.5) \\
Z_c &= \frac{|V_{cn}|^2}{S_c^*} = \frac{|V_{cn}|^2}{|S_c|}\underline{/\theta_c} = |Z_c|\underline{/\theta_c}
\end{aligned}
$$

The load currents as a function of the "constant load impedances" are given by

$$
\begin{aligned}
IL_a &= \frac{V_{an}}{Z_a} = \frac{|V_{an}|}{|Z_a|}\underline{/\delta_a - \theta_a} = |IL_a|\underline{/\alpha_a} \\
IL_b &= \frac{V_{bn}}{Z_b} = \frac{|V_{bn}|}{|Z_b|}\underline{/\delta_b - \theta_b} = |IL_b|\underline{/\alpha_b} \qquad (9.6) \\
IL_c &= \frac{V_{cn}}{Z_c} = \frac{|V_{cn}|}{|Z_c|}\underline{/\delta_c - \theta_c} = |IL_c|\underline{/\alpha_c}
\end{aligned}
$$

In this model, the line-to-neutral voltages will change during each iteration, but the impedance computed in Equation 9.5 will remain constant.

9.1.3 Constant Current Loads

In this model, the magnitudes of the currents are computed according to Equations 9.4 and then held constant while the angle of the voltage (δ) changes resulting in a changing angle on the current so that the power factor of the load remains constant:

$$
\begin{aligned}
IL_a &= |IL_a|\underline{/\delta_a - \theta_a} \\
IL_b &= |IL_b|\underline{/\delta_b - \theta_b} \qquad (9.7) \\
IL_c &= |IL_c|\underline{/\delta_c - \theta_c}
\end{aligned}
$$

where

δ_{abc} represents the line-to-neutral voltage angles
θ_{abc} represents the power factor angles

9.1.4 Combination Loads

Combination loads can be modeled by assigning a percentage of the total load to each of the three aforementioned load models. The total line current entering the load is the sum of the three components.

Example 9.1

The complex powers of a wye-connected load are

$$[S_{abc}] = \begin{bmatrix} 2236.1\angle 26.6 \\ 2506.0\angle 28.6 \\ 2101.4\angle 25.3 \end{bmatrix} \text{kVA}$$

The load is specified to be 50% constant complex power, 20% constant impedance, and 30% constant current. The nominal line-to-line voltage of the feeder is 12.47 kV.

a. Assume the nominal voltage and compute the component of load current attributed to each component of the load and the total load current.

The assumed line-to-neutral voltages at the start of the iterative routine are

$$[VLN_{abc}] = \begin{bmatrix} 7200\angle 0 \\ 7200\angle -120 \\ 7200\angle 120 \end{bmatrix} \text{V}$$

The component of currents due to the constant complex power is

$$Ipq_i = \left(\frac{S_i \cdot 1000}{VLN_i}\right)^* \cdot 0.5 = \begin{bmatrix} 155.3\angle -26.6 \\ 174.0\angle -148.6 \\ 146.0\angle 94.7 \end{bmatrix} \text{A}$$

The constant impedances for that part of the load are computed as

$$Z_i = \frac{VLN_i^2}{S_i^* \cdot 1000} = \begin{bmatrix} 20.7 + j10.4 \\ 18.2 + j9.9 \\ 22.3 + j10.6 \end{bmatrix} \Omega$$

For the first iteration, the currents due to the constant impedance portion of the load are

$$Iz_i = \left(\frac{VLN_i}{Z_i}\right) \cdot 0.2 = \begin{bmatrix} 62.1\angle -26.6 \\ 69.6\angle -148.6 \\ 58.4\angle 94.7 \end{bmatrix} \text{A}$$

The magnitudes of the constant current portion of the load are

$$IM_i = \left| \left(\frac{S_i \cdot 1000}{VLN_i} \right)^* \right| \cdot 0.3 = \begin{bmatrix} 93.2 \\ 104.4 \\ 87.6 \end{bmatrix} \text{A}$$

The contribution of the load currents due to the constant current portion of the load is

$$II_i = IM_i \underline{/\delta_i - \theta_i} = \begin{bmatrix} 93.2\underline{/-26.6} \\ 104.4\underline{/-148.6} \\ 87.6\underline{/94.7} \end{bmatrix} \text{A}$$

The total load current is the sum of the three components:

$$[I_{abc}] = [I_{pq}] + [I_z] + [I_I] = \begin{bmatrix} 310.6\underline{/-26.6} \\ 348.1\underline{/-148.6} \\ 292.0\underline{/94.7} \end{bmatrix} \text{A}$$

b. Determine the currents at the start of the second iteration. The voltages at the load after the first iteration are

$$[VLN] = \begin{bmatrix} 6850.0\underline{/-1.9} \\ 6972.7\underline{/-122.1} \\ 6886.1\underline{/117.5} \end{bmatrix} \text{V}$$

The steps are repeated with the exceptions that the impedances of the constant impedance portion of the load will not be changed and the magnitude of the currents for the constant current portion of the load change will not change.

The constant complex power portion of the load currents is

$$Ipq_i = \left(\frac{S_i \cdot 1000}{VLN_i} \right)^* \cdot 0.5 = \begin{bmatrix} 163.2\underline{/-28.5} \\ 179.7\underline{/-150.7} \\ 152.7\underline{/92.1} \end{bmatrix} \text{A}$$

The currents due to the constant impedance portion of the load are

$$Iz_i = \left(\frac{VLN_i}{Z_i} \right) \cdot 0.2 = \begin{bmatrix} 59.1\underline{/-28.5} \\ 67.4\underline{/-150.7} \\ 55.9\underline{/92.1} \end{bmatrix} \text{A}$$

The currents due to the constant current portion of the load are

$$II_i = IM_i \underline{/\delta_i - \theta_i} = \begin{bmatrix} 93.2\underline{/-28.5} \\ 104.4\underline{/-150.7} \\ 87.6\underline{/92.1} \end{bmatrix} \text{A}$$

The total load currents at the start of the second iteration will be

$$[I_{abc}] = [I_{pq}] + [I_z] + [I_I] = \begin{bmatrix} 315.5\underline{/-28.5} \\ 351.5\underline{/-150.7} \\ 296.2\underline{/92.1} \end{bmatrix} \text{A}$$

Observe how these currents have changed from the original currents. The currents for the constant complex power loads have increased because the voltages are reduced from the original assumption. The currents for the constant impedance portion of the load have decreased because the impedance stayed constant but the voltages are reduced. Finally, the constant current portion of the load has indeed remained constant. Again, all three components of the load have the same phase angles since the power factor of the load has not changed.

9.2 Delta-Connected Loads

The model for a delta-connected load is shown in Figure 9.2.

The notations for the specified complex powers and voltages in Figure 9.2 are as follows:

Phase ab:

$$|S_{ab}|\underline{/\theta_{ab}} = P_{ab} + jQ_{ab} \quad \text{and} \quad |V_{ab}|\underline{/\delta_{ab}} \tag{9.8}$$

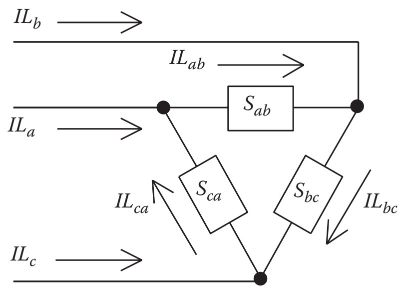

FIGURE 9.2
Delta-connected load.

Phase bc:

$$|S_{bc}|\underline{/\theta_{bc}} = P_{bc} + jQ_{bc} \quad \text{and} \quad |V_{bc}|\underline{/\delta_{bc}} \tag{9.9}$$

Phase ca:

$$|S_{ca}|\underline{/\theta_{ca}} = P_{ca} + jQ_{ca} \quad \text{and} \quad |V_{ca}|\underline{/\delta_{ca}} \tag{9.10}$$

9.2.1 Constant Real and Reactive Power Loads

The currents in the delta-connected loads are

$$\begin{aligned}
IL_{ab} &= \left(\frac{S_{ab}}{V_{ab}}\right)^* = \frac{|S_{ab}|}{|V_{ab}|}\underline{/\delta_{ab}-\theta_{ab}} = |IL_{ab}|\underline{/\alpha_{ab}} \\
IL_{bc} &= \left(\frac{S_{bc}}{V_{bc}}\right)^* = \frac{|S_{bc}|}{|V_{bc}|}\underline{/\delta_{bc}-\theta_{bc}} = |IL_{bc}|\underline{/\alpha_{bc}} \\
IL_{ca} &= \left(\frac{S_{ca}}{V_{ca}}\right)^* = \frac{|S_{ca}|}{|V_{ca}|}\underline{/\delta_{ca}-\theta_{ca}} = |IL_{ca}|\underline{/\alpha_{ac}}
\end{aligned} \tag{9.11}$$

In this model, the line-to-line voltages will change during each iteration resulting in new current magnitudes and angles at the start of each iteration.

9.2.2 Constant Impedance Loads

The "constant load impedance" is first determined from the specified complex power and line-to-line voltages:

$$\begin{aligned}
Z_{ab} &= \frac{|V_{ab}|^2}{S_{ab}^*} = \frac{|V_{ab}|^2}{|S_{ab}|}\underline{/\theta_{ab}} = |Z_{ab}|\underline{/\theta_{ab}} \\
Z_{bc} &= \frac{|VL_{bc}|^2}{S_{bc}^*} = \frac{|V_{bc}|^2}{|S_{bc}|}\underline{/\theta_{bc}} = |Z_{bc}|\underline{/\theta_{bc}} \\
Z_{ca} &= \frac{|V_{ca}|^2}{S_{ca}^*} = \frac{|V_{ca}|^2}{|S_{ca}|}\underline{/\theta_{ca}} = |Z_{ca}|\underline{/\theta_{ca}}
\end{aligned} \tag{9.12}$$

The delta load currents as a function of the "constant load impedances" are

$$
\begin{aligned}
IL_{ab} &= \frac{V_{ab}}{Z_{ab}} = \frac{|V_{ab}|}{|Z_{ab}|}\underline{/\delta_{ab}-\theta_{ab}} = |IL_{ab}|\underline{/\alpha_{ab}} \\
IL_{bc} &= \frac{V_{bc}}{Z_{bc}} = \frac{|V_{bc}|}{|Z_{bc}|}\underline{/\delta_{bc}-\theta_{bc}} = |IL_{bc}|\underline{/\alpha_{bc}} \\
IL_{ca} &= \frac{V_{ca}}{Z_{ca}} = \frac{|V_{ca}|}{|Z_{ca}|}\underline{/\delta_{ca}-\theta_{ca}} = |IL_{ca}|\underline{/\alpha_{ca}}
\end{aligned}
\tag{9.13}
$$

In this model, the line-to-line voltages will change during each iteration until convergence is achieved.

9.2.3 Constant Current Loads

In this model, the magnitudes of the currents are computed according to Equations 9.11 and then held constant while the angle of the voltage (δ) changes during each iteration. This keeps the power factor of the load constant:

$$
\begin{aligned}
IL_{ab} &= |IL_{ab}|\underline{/\delta_{ab}-\theta_{ab}} \\
IL_{bc} &= |IL_{bc}|\underline{/\delta_{bc}-\theta_{bc}} \\
IL_{ca} &= |IL_{bca}|\underline{/\delta_{ca}-\theta_{ca}}
\end{aligned}
\tag{9.14}
$$

9.2.4 Combination Loads

Combination loads can be modeled by assigning a percentage of the total load to each of the three aforementioned load models. The total delta current for each load is the sum of the three components.

9.2.5 Line Currents Serving a Delta-Connected Load

The line currents entering the delta-connected load are determined by applying Kirchhoff's current law (KCL) at each of the nodes of the delta. In matrix form, the equations are

$$
\begin{bmatrix} IL_a \\ IL_b \\ IL_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} IL_{ab} \\ IL_{bc} \\ IL_{ca} \end{bmatrix}
\tag{9.15}
$$

9.3 Two-Phase and Single-Phase Loads

In both the wye- and delta-connected loads, single-phase and two-phase loads are modeled by setting the currents of the missing phases to zero. The currents in the phases present are computed using the same appropriate equations for constant complex power, constant impedance, and constant current.

9.4 Shunt Capacitors

Shunt capacitor banks are commonly used in distribution systems to help in voltage regulation and to provide reactive power support. The capacitor banks are modeled as constant susceptances connected in either wye or delta. Similar to the load model, all capacitor banks are modeled as three-phase banks with the currents of the missing phases set to zero for single-phase and two-phase banks.

9.4.1 Wye-Connected Capacitor Bank

The model of a three-phase wye-connected shunt capacitor bank is shown in Figure 9.3.

The individual phase capacitor units are specified in kvar and kV. The constant susceptance for each unit can be computed in Siemens. The susceptance of a capacitor unit is computed by

$$B_c = \frac{kvar}{kV_{LN}^2 \cdot 1000} \text{S} \tag{9.16}$$

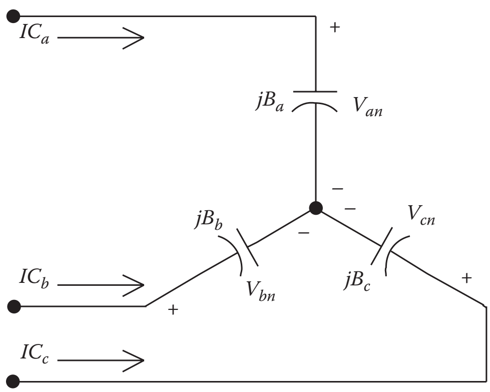

FIGURE 9.3
Wye-connected capacitor bank.

With the susceptance computed, the line currents serving the capacitor bank are given by

$$IC_a = jB_a \cdot V_{an}$$
$$IC_b = jB_b \cdot V_{bn} \tag{9.17}$$
$$IC_c = jB_c \cdot V_{cn}$$

9.4.2 Delta-Connected Capacitor Bank

The model for a delta-connected shunt capacitor bank is shown in Figure 9.4.

The individual phase capacitor units are specified in kvar and kV. For the delta-connected capacitors, the kV must be the line-to-line voltage. The constant susceptance for each unit can be computed in Siemens. The susceptance of a capacitor unit is computed by

$$B_c = \frac{kvar}{kV_{LL}^2 \cdot 1000}\,\mathrm{S} \tag{9.18}$$

With the susceptance computed, the delta currents serving the capacitor bank are given by

$$IC_{ab} = jB_a \cdot V_{ab}$$
$$IC_{bc} = jB_b \cdot V_{bc} \tag{9.19}$$
$$IC_{ca} = jB_c \cdot V_{ca}$$

The line currents flowing into the delta-connected capacitors are computed by applying KCL at each node. In matrix form, the equations are

$$\begin{bmatrix} IC_a \\ IC_b \\ IC_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} IC_{ab} \\ IC_{bc} \\ IC_{ca} \end{bmatrix} \tag{9.20}$$

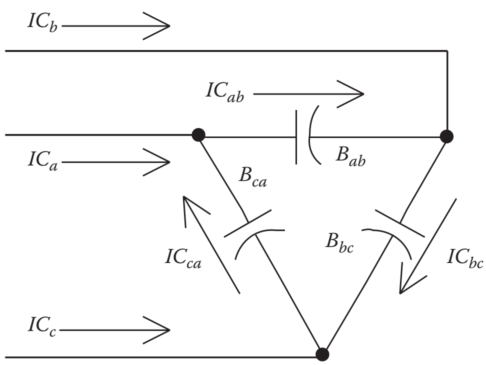

FIGURE 9.4
Delta-connected capacitor bank.

9.5 Three-Phase Induction Machine

The analysis of an induction machine (motor or generator) when operating under unbalanced voltage conditions has traditionally been performed using the method of symmetrical components. Using this approach, the positive and negative sequence equivalent circuits of the machine are developed and then, given the sequence line-to-neutral voltages, the sequence currents are computed. The zero sequence network is not required since the machines are typically connected delta or ungrounded wye, which means that there will not be any zero sequence currents or voltages. The phase currents are determined by performing the transformation back to the phase line currents. The internal operating conditions are determined by the complete analysis of the sequence networks.

A method whereby all of the analysis can be performed in the phase frame will be developed. The analysis will be broken into two parts. The first part will be to determine the terminal voltages and currents of the motor and the second part will be to use these values to compute the stator and rotor losses and the converted shaft power.

9.5.1 Induction Machine Model

The sequence line-to-neutral equivalent circuit of a three-phase induction machine is shown in Figure 9.5.

The circuit in Figure 9.5 applies to both the positive and negative sequence networks. The only difference between the two is the value of the "load resistance" RL as defined in the following:

$$RL_i = \frac{1-s_i}{s_i} \cdot Rr_i \tag{9.21}$$

where

Positive sequence slip:

$$s_1 = \frac{n_s - n_r}{n_s} \tag{9.22}$$

where
  n_s is the synchronous speed
  n_r is the rotor speed

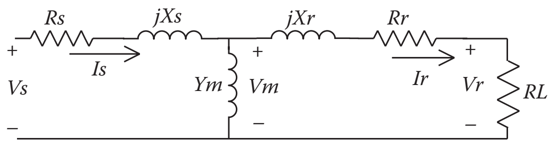

FIGURE 9.5
Sequence equivalent circuit.

Negative sequence slip:

$$s_2 = 2 - s_1 \tag{9.23}$$

Note that the negative sequence load resistance RL_2 will be a negative value that will lead to a negative shaft power in the negative sequence.

If the value of positive sequence slip (s_1) is known, the input sequence impedances for the positive and negative sequence networks can be determined as

$$ZM_i = Rs_i + jXs_i + \frac{(jXm_i)(Rr_i + RL_i + jXr_i)}{Rr_i + RL_i + j(Xm_i + Xr_i)} \tag{9.24}$$

where
$i = 1$ for positive sequence
$i = 2$ for negative sequence

Once the input sequence impedances have been determined, the analysis of an induction machine operating with unbalanced voltages requires the following steps:

Step 1: Transform the known line-to-line voltages to sequence line-to-line voltages:

$$\begin{bmatrix} Vab_0 \\ Vab_1 \\ Vab_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \tag{9.25}$$

In Equation 9.25, $Vab_0 = 0$ because of Kirchhoff's voltage law (KVL).

Equation 9.25 can be written as

$$[VLL_{012}] = [A]^{-1} \cdot [VLL_{abc}] \tag{9.26}$$

Step 2: Compute the sequence line-to-neutral voltages from the line-to-line voltages:

$$Van_0 = Vab_0 = 0 \tag{9.27}$$

Equation 9.27 will not be true for a general case. However, for the case of the machine being connected either in delta or ungrounded wye, the zero sequence line-to-neutral voltage can be assumed to be zero:

$$Van_1 = t^* \cdot Vab_1 \tag{9.28}$$

$$Van_2 = t \cdot Vab_2 \tag{9.29}$$

where

$$t = \frac{1}{\sqrt{3}} \cdot \underline{/30} \tag{9.30}$$

Equations 9.27 through 9.29 can be put into matrix form:

$$\begin{bmatrix} Van_0 \\ Van_1 \\ Van_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t^* & 0 \\ 0 & 0 & t \end{bmatrix} \cdot \begin{bmatrix} Vab_0 \\ Vab_1 \\ Vab_2 \end{bmatrix} \tag{9.31}$$

Equation 9.31 can be written as

$$[VLN_{012}] = [T] \cdot [VLL_{012}] \tag{9.32}$$

Step 3: Compute the sequence line currents flowing into the machine:

$$Ia_0 = 0 \tag{9.33}$$

$$Ia_1 = \frac{Van_1}{ZM_1} \tag{9.34}$$

$$Ia_2 = \frac{Van_2}{ZM_2} \tag{9.35}$$

Step 4: Transform the sequence currents to phase currents:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \tag{9.36}$$

Equation 9.36 can be written as

$$[I_{abc}] = [A] \cdot [I_{012}] \tag{9.37}$$

The four steps outlined earlier can be performed without actually computing the sequence voltages and currents. The procedure basically reverses the steps.

Define

$$YM_i = \frac{1}{ZM_i} \tag{9.38}$$

The sequence currents are

$$I_0 = 0 \tag{9.39}$$

$$I_1 = YM_1 \cdot Van_1 = YM_1 \cdot t^* \cdot Vab_1 \tag{9.40}$$

$$I_2 = YM_2 \cdot Van_2 = YM_2 \cdot t \cdot Vab_2 \tag{9.41}$$

Since I_0 and Vab_0 are both zero, the following relationship is true:

$$I_0 = Vab_0 \tag{9.42}$$

Equations 9.39, 9.40, and 9.42 can be put into matrix form:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t^* \cdot YM_1 & 0 \\ 0 & 0 & t \cdot YM_2 \end{bmatrix} \cdot \begin{bmatrix} Vab_0 \\ Vab_1 \\ Vab_2 \end{bmatrix} \tag{9.43}$$

Equation 9.43 can be written in shortened form as

$$[I_{012}] = [YM_{012}] \cdot [VLL_{012}] \tag{9.44}$$

From symmetrical component theory,

$$[VLL_{012}] = [A]^{-1} \cdot [VLL_{abc}] \tag{9.45}$$

$$[I_{abc}] = [A] \cdot [I_{012}] \tag{9.46}$$

Substitute Equation 9.45 into Equation 9.44 and substitute the resultant equation into Equation 9.46 to get

$$[I_{abc}] = [A] \cdot [YM_{012}] \cdot [A]^{-1} \cdot [VLL_{abc}] \tag{9.47}$$

Define

$$[YM_{abc}] = [A] \cdot [YM_{012}] \cdot [A]^{-1} \tag{9.48}$$

Therefore

$$[I_{abc}] = [YM_{abc}] \cdot [VLL_{abc}] \tag{9.49}$$

The induction machine "phase frame admittance matrix" $[YM_{abc}]$ is defined in Equation 9.48. Equation 9.49 is used to compute the input phase currents of the machine from a knowledge of the phase line-to-line terminal voltages. This is the desired result. Recall that $[YM_{abc}]$ is a function of the slip of the machine so that a new matrix must be computed every time the slip changes.

Equation 9.49 can be used to solve for the line-to-line voltages as a function of the line currents by

$$[VLL_{abc}] = [ZM_{abc}] \cdot [I_{abc}] \tag{9.50}$$

where

$$[ZM_{abc}] = [YM_{abc}]^{-1} \tag{9.51}$$

As was done in Chapter 8, it is possible to replace the line-to-line voltages in Equation 9.50 with the "equivalent" line-to-neutral voltages:

$$[VLN_{abc}] = [W] \cdot [VLL_{abc}] \tag{9.52}$$

Define

$$[W] = [A] \cdot [T] \cdot [A]^{-1} \tag{9.53}$$

The matrix $[W]$ is a very useful matrix that allows the determination of the "equivalent" line-to-neutral voltages from a knowledge of the line-to-line voltages. Equation 9.50 can be substituted into Equation 9.52 to define the "line-to-neutral" equation:

$$[VLN_{abc}] = [W] \cdot [ZM_{abc}] \cdot [I_{abc}]$$

$$[VLN_{abc}] = [ZLN_{abc}] \cdot [I_{abc}] \tag{9.54}$$

where

$$[ZLN_{abc}] = [W] \cdot [ZM_{abc}] \tag{9.55}$$

The inverse of Equation 9.54 can be taken to determine the line currents as a function of the line-to-neutral voltages:

$$[I_{abc}] = [YLN_{abc}] \cdot [VLN_{abc}] \tag{9.56}$$

where

$$[YLN_{abc}] = [ZLN_{abc}]^{-1} \tag{9.57}$$

Care must be taken in applying Equation 9.56 to insure that the voltages used are the line-to-neutral and not the line-to-ground voltages. If only the line-to-ground voltages are known, they must first be converted to the line-to-line values and then use Equation 9.52 to compute the line-to-neutral voltages.

Once the machine terminal currents and line-to-neutral voltages are known, the input phase complex powers and total three-phase input complex power can be computed:

$$S_a = V_{an} \cdot (I_a)^* \tag{9.58}$$

$$S_b = V_{bn} \cdot (I_b)^* \tag{9.59}$$

$$S_c = V_{cn} \cdot (I_c)^* \tag{9.60}$$

$$S_{Total} = S_a + S_b + S_c \tag{9.61}$$

Many times the only voltages known will be the magnitudes of the three line-to-line voltages at the machine terminals. When this is the case, the Law of Cosines must be used to compute the angles associated with the measured magnitudes.

9.5.2 Equivalent T Circuit

Once the terminal line-to-neutral voltages and currents are known, it is desired to analyze what is happening inside the machine. In particular, the stator and rotor losses are needed in addition to the "converted" shaft power. A method of performing the internal analysis can be developed in the phase frame by starting with the sequence networks. Figure 9.5 can be modified by removing *RL*, which represents the "load resistance" in the positive and negative sequence networks. The resulting networks will be modeled using *A*, *B*, *C*, and *D* parameters. The equivalent T circuit (*RL* removed) is shown in Figure 9.6. This circuit can represent both the positive and negative sequence networks. The only difference (if any) will be between the numerical values of the sequence stator and rotor impedances.

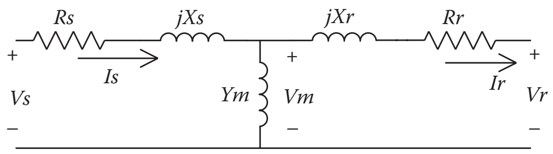

FIGURE 9.6
Equivalent T circuit.

In Figure 9.6,

$$Ym = \frac{1}{jXm} \tag{9.62}$$

Define the sequence stator and rotor impedances:

$$Zs_i = Rs_i + jXs_i \tag{9.63}$$

$$Zr_i = Rr_i + jXr_i \tag{9.64}$$

The positive and negative sequence *A*, *B*, *C*, and *D* parameters of the unsymmetrical T circuit of Figure 9.6 are given by

$$Am_i = 1 + Ym_i \cdot Zs_i \tag{9.65}$$

$$Bm_i = Zs_i + Zr_i + Ym_i \cdot Zs_i \cdot Zr_i \tag{9.66}$$

$$Cm_i = Ym_i \tag{9.67}$$

$$Dm_i = 1 + Ym_i \cdot Zr_i \tag{9.68}$$

where
  $i = 1$ for positive sequence
  $i = 2$ for negative sequence

Note

$$Am_i \cdot Dm_i - Bm_i \cdot Cm_i = 1 \tag{9.69}$$

The terminal sequence line-to-neutral voltages and currents as functions of the rotor "load voltages" (*Vr*) and the rotor currents are given by

$$\begin{bmatrix} Vs_i \\ Is_i \end{bmatrix} = \begin{bmatrix} Am_i & Bm_i \\ Cm_i & Dm_i \end{bmatrix} \cdot \begin{bmatrix} Vr_i \\ Ir_i \end{bmatrix} \tag{9.70}$$

Because of Equation 9.69, the inverse of Equation 9.70 is

$$\begin{bmatrix} Vr_i \\ Ir_i \end{bmatrix} = \begin{bmatrix} Dm_i & -Bm_i \\ -Cm_i & Am_i \end{bmatrix} \cdot \begin{bmatrix} Vs_i \\ Is_i \end{bmatrix} \tag{9.71}$$

Equation 9.71 can be expanded to show the individual sequence voltages and currents:

$$\begin{bmatrix} Vr_0 \\ Vr_1 \\ Vr_2 \\ Ir_0 \\ Ir_1 \\ Ir_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Dm_1 & 0 & 0 & -Bm_1 & 0 \\ 0 & 0 & Dm_2 & 0 & 0 & -Bm_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Cm_1 & 0 & 0 & Am_1 & 0 \\ 0 & 0 & -Cm_2 & 0 & 0 & Am_2 \end{bmatrix} \cdot \begin{bmatrix} Vs_0 \\ Vs_1 \\ Vs_2 \\ Is_0 \\ Is_1 \\ Is_2 \end{bmatrix} \tag{9.72}$$

Equation 9.72 can be partitioned between the third and fourth rows and columns. In reduced form by incorporating the partitioning, Equation 9.72 becomes

$$\begin{bmatrix} [Vr_{012}] \\ [Ir_{012}] \end{bmatrix} = \begin{bmatrix} [Dm_{012}] & [Bm_{012}] \\ [Cm_{012}] & [Am_{012}] \end{bmatrix} \cdot \begin{bmatrix} [Vs_{012}] \\ [Is_{012}] \end{bmatrix} \tag{9.73}$$

Expanding Equation 9.73,

$$[Vr_{012}] = [Dm_{012}] \cdot [Vs_{012}] + [Bm_{012}] \cdot [Is_{012}] \tag{9.74}$$

$$[Ir_{012}] = [Cm_{012}] \cdot [Vs_{012}] + [Am_{012}] \cdot [Is_{012}] \tag{9.75}$$

Equations 9.74 and 9.75 can be transformed into the phase domain:

$$[Vr_{abc}] = [A] \cdot [Vr_{012}] = [A] \cdot [Dm_{012}] \cdot [A]^{-1} \cdot [Vs_{abc}] + [A] \cdot [Bm_{012}] \cdot [A]^{-1} \cdot [Is_{abc}] \tag{9.76}$$

$$[Ir_{abc}] = [A] \cdot [Ir_{012}] = [A] \cdot [Cm_{012}] \cdot [A]^{-1} \cdot [Vs_{abc}] + [A] \cdot [Am_{012}] \cdot [A]^{-1} \cdot [Is_{abc}] \tag{9.77}$$

Therefore

$$[Vr_{abc}] = [Dm_{abc}] \cdot [Vs_{abc}] + [Bm_{abc}] \cdot [Is_{abc}] \tag{9.78}$$

$$[Ir_{abc}] = [Cm_{abc}] \cdot [Vs_{abc}] + [Am_{abc}] \cdot [Is_{abc}] \tag{9.79}$$

where

$$\begin{aligned}
[Am_{abc}] &= [A]\cdot[Am_{012}]\cdot[A]^{-1} \\
[Bm_{abc}] &= [A]\cdot[Bm_{012}]\cdot[A]^{-1} \\
[Cm_{abc}] &= [A]\cdot[Cm_{012}]\cdot[A]^{-1} \\
[Dm_{abc}] &= [A]\cdot[Dm_{012}]\cdot[A]^{-1}
\end{aligned} \tag{9.80}$$

The power converted to the shaft is given by

$$P_{conv} = Vr_a\cdot(Ir_a)^* + Vr_b\cdot(Ir_b)^* + Vr_c\cdot(Ir_c)^* \tag{9.81}$$

The useful shaft power can be determined from a knowledge of the rotational (FW) losses:

$$P_{shaft} = P_{conv} - P_{FW} \tag{9.82}$$

The rotor "copper" losses are

$$P_{rotor} = |Ir_a|^2\cdot Rr + |Ir_b|^2\cdot Rr + |Ir_c|^2\cdot Rr \tag{9.83}$$

The stator "copper" losses are

$$P_{stator} = |Is_a|^2\cdot Rs + |Is_b|^2\cdot Rs + |Is_c|^2\cdot Rs \tag{9.84}$$

The total input power is

$$P_{in} = \mathrm{Re}[Vs_a\cdot(Is_a)^* + Vs_b\cdot(Is_b)^* + Vs_c\cdot(Is_c)^*] \tag{9.85}$$

Example 9.2

To demonstrate the analysis of an induction motor in the phase frame, the following induction motor will be used:

25 hp, 240 V operating with *slip* = 0.035

$Ploss_{rotation} = 0.75$ kW

$Zs = 0.0774 + j0.1843\ \Omega$

$Zm = 0 + j4.8384\ \Omega$

$Zr = 0.0908 + j0.1843\ \Omega$

The "load" resistances are

$$RL_1 = \left(\frac{1-0.035}{0.035}\right)\cdot 0.098 = 2.5029\,\Omega$$

$$RL_2 = \left(\frac{1-(1.965)}{1.965}\right)\cdot 0.0908 = -0.0446\,\Omega$$

The input sequence impedances are

$$ZM_1 = Zs + \frac{Zm\cdot(Zr+RL_1)}{Zm+Zr+RL_1} = 1.9775 + j1.3431\,\Omega$$

$$ZM_2 = Zs + \frac{Zm\cdot(Zr+RL_2)}{Zm+Zr+RL_2} = 0.1203 + j0.3623\,\Omega$$

The positive and negative sequence input admittances are

$$YM_1 = \frac{1}{ZM_1} = 0.3461 - j0.2350\,\mathrm{S}$$

$$YM_2 = \frac{1}{ZM_2} = 0.8255 - j2.4863\,\mathrm{S}$$

The sequence admittance matrix is

$$[YM_{012}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.3461 - j0.2350 & 0 \\ 0 & 0 & 0.8255 - j2.4863 \end{bmatrix}\mathrm{S}$$

Applying Equation 9.48, the phase admittance matrix is

$$[YM_{abc}] = \begin{bmatrix} 0.7452 - j0.4074 & -0.0999 - j0.0923 & 0.3547 + j0.4997 \\ 0.3547 + j0.4997 & 0.7452 - j0.4074 & -0.0999 - j0.0923 \\ -0.0999 - j0.0923 & 0.3547 + j0.4997 & 0.7452 - j0.4074 \end{bmatrix}$$

The line-to-line terminal voltages are measured to be

$$V_{ab} = 235\ \mathrm{V},\quad V_{bc} = 240\ \mathrm{V},\quad V_{ca} = 245\ \mathrm{V}$$

Since the sum of the line-to-line voltages must equal zero, the law of cosines can be used to determine the angles on the voltages. Applying the law of cosines results in

$$[VLL_{abc}] = \begin{bmatrix} 235/\underline{0} \\ 240/\underline{-117.9} \\ 245/\underline{120.0} \end{bmatrix} \text{V}$$

The phase motor currents can now be computed:

$$[Is_{abc}] = [YM_{abc}] \cdot [VLL_{abc}] = \begin{bmatrix} 53.15/\underline{-71.0} \\ 55.15/\underline{-175.1} \\ 66.6/\underline{55.6} \end{bmatrix} \text{A}$$

It is obvious that the currents are very unbalanced. The measure of unbalance for the voltages and currents can be computed as [1]

$$V_{unbalance} = \left(\frac{max\_deviation}{|V_{avg}|} \right) \cdot 100 = \left(\frac{5}{240} \right) \cdot 100 = 2.08\%$$

$$I_{unbalance} = \left(\frac{max\_deviation}{|I_{avg}|} \right) \cdot 100 = \left(\frac{8.3232}{58.31} \right) \cdot 100 = 14.27\%$$

This example demonstrates that the current unbalance is approximately seven times greater than the voltage unbalance. This ratio of current unbalance to voltage unbalance is typical. The actual operating characteristics including stator and rotor losses of the motor can be determined using the method developed in Ref. [2]

The equivalent line-to-neutral voltages at the motor are computed using the [W] matrix:

$$[Vs_{abc}] = [W] \cdot [VLL_{abc}]$$

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 235.0/\underline{0} \\ 240/\underline{-117.9} \\ 245/\underline{120.0} \end{bmatrix} = \begin{bmatrix} 138.6/\underline{-30.7} \\ 135.7/\underline{-148.6} \\ 141.4/\underline{91.4} \end{bmatrix}$$

The input complex power to the motor is

$$S_{in} = \sum_{k=1}^{3} \frac{Vs_{abc_k} \cdot I_{abc_k}}{1000} = 19.95 + j13.62$$

$$|S_{in}| = 24.15 \quad PF = 0.83 \text{ lag}$$

The rotor currents and voltages can be computed by first computing the equivalent *A*, *B*, *C*, and *D* matrices according to Equations 9.80. The first step is to compute the sequence *A*, *B*, *C*, and *D* parameters according to Equations 9.65 through 9.68:

$$Am = 1 + Y_m \cdot Zs = 1.0381 - j0.0161$$

$$Bm = Zs + Zr + Y_m \cdot Zs \cdot Zr = 0.1746 + j0.3742$$

$$Cm = Y_m = -j0.2067$$

$$Dm = 1 + Y_m \cdot Zr = 1.0381 - j0.0188$$

The sequence matrices using Equations 9.72 are

$$[Am_{012}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.0381 - j0.0161 & 0 \\ 0 & 0 & 1.0381 - j0.0161 \end{bmatrix}$$

$$[Bm_{012}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -0.1746 - j0.3742 & 0 \\ 0 & 0 & -0.1746 - j0.3742 \end{bmatrix}$$

$$[Cm_{012}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & j0.2067 & 0 \\ 0 & 0 & j0.2067 \end{bmatrix}$$

$$[Dm_{012}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.0381 - j0.0188 & 0 \\ 0 & 0 & 1.0381 - j0.0188 \end{bmatrix}$$

Equation 9.80 gives us the final phase domain *A*, *B*, *C*, and *D* matrices:

$$[Am_{abc}] = [A] \cdot [Am_{012}] \cdot [A]^{-1}$$

$$= \begin{bmatrix} 0.6921 - j0.0107 & -0.346 + j0.0053 & -0.346 + j0.0053 \\ -0.346 + j0.0053 & 0.6921 - j0.0107 & -0.346 + j0.0053 \\ -0.346 + j0.0053 & -0.346 + j0.00531 & 0.6921 - j0.00107 \end{bmatrix}$$

$$[Bm_{abc}] = \begin{bmatrix} -0.1164 - j0.2494 & 0.0582 + j0.1247 & 0.0582 + j0.1247 \\ 0.0582 + j0.1247 & -0.1164 - j0.2494 & 0.0582 + j0.1247 \\ 0.0582 + j0.1247 & 0.0582 + j0.1247 & -0.1164 - j0.2494 \end{bmatrix}$$

$$[Cm_{abc}] = \begin{bmatrix} j0.1378 & -j0.0689 & -j0.0689 \\ -j0.0689 & j0.1378 & -j0.0689 \\ -j0.0689 & -j0.0689 & j0.1378 \end{bmatrix}$$

$$[Dm_{abc}] = \begin{bmatrix} 0.6921 - j0.0125 & -0.346 + j0.0063 & -0.346 + j0.0063 \\ -0.346 + j0.0063 & 0.6921 - j0.0125 & -0.346 + j0.0063 \\ -0.346 + j0.0063 & -0.346 + j0.0063 & 0.6921 - j0.0125 \end{bmatrix}$$

With the matrices defined, the rotor voltages and currents can be computed:

$$[Vr_{abc}] = [Dm_{abc}] \cdot [Vs_{abc}] + [Bm_{abc}] \cdot [Is_{abc}] = \begin{bmatrix} 124.5/\underline{-36.1} \\ 124.1/\underline{-156.3} \\ 123.8/\underline{83.9} \end{bmatrix}$$

$$[Ir_{abc}] = [Cm_{abc}] \cdot [Vs_{abc}] + [Am_{abc}] \cdot [Is_{abc}] = \begin{bmatrix} 42.2/\underline{-41.2} \\ 50.9/\underline{-146.6} \\ 56.8/\underline{79.1} \end{bmatrix}$$

The converted electric power to shaft power is

$$P_{convert} = \sum_{k=1}^{3} \frac{Vr_{abc_k} \cdot Ir^*_{abc_k}}{1000} = 18.5\,\text{kW}$$

The power converted in units of horsepower is

$$hp = \frac{P_{convert}}{0.746} = 24.8$$

Note how the shaft power in horsepower is approximately equal to the input kVA of the motor. This is typically the case so that a good assumption for a motor is that the rated output in horsepower will be equal to the input kVA.

9.5.3 Computation of Slip

When the input power to the motor is specified instead of the slip, an iterative process is required to compute the value of slip that will force the input power to be within some small tolerance of the specified input power.

The iterative process for computing the slip that will produce the specified input power starts with assuming an initial value of the positive sequence slip and a change in slip. For purposes here the initial values are

$$\begin{aligned} s_{old} &= 0.0 \\ ds &= 0.01 \end{aligned} \tag{9.86}$$

The value of slip used in the first iteration is then

$$s_{new} = s_{old} + ds \tag{9.87}$$

With the new value of slip, the input shunt admittance matrix $[YM_{abc}]$ is computed. The given line-to-line voltages are used to compute the stator currents. The $[W]$ matrix is used to compute the equivalent line-to-neutral voltages and the input complex power for the three phases and the total three-phase input complex power are computed. The computed three-phase input power is compared to the specified three-phase input power and an error is computed as

$$Error = P_{specified} - P_{computed} \tag{9.88}$$

If the error is positive, the slip needs to be increased so that the computed power will increase. This is done by

$$\begin{aligned} s_{old} &= s_{new} \\ s_{new} &= s_{old} + ds \end{aligned} \tag{9.89}$$

The new value of slip is used to repeat the calculations for the input power to the motor.

If the error is negative, that means that a bracket has been established. The required value of slip lies between the s_{old} and s_{new}. In order to zero in on the required slip, the old value of slip will be used and the change in slip will be reduced by a factor of 10:

$$\begin{aligned} ds &= \frac{ds}{10} \\ s_{new} &= s_{old} + ds \end{aligned} \tag{9.90}$$

This process is illustrated in Figure 9.7.

When the slip has produced the specified input power, the T circuit is used to compute the voltages and currents in the rotor.

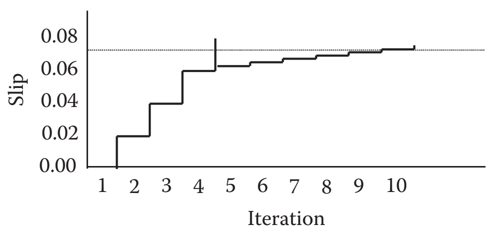

FIGURE 9.7
Slip versus iteration.

Example 9.3

The same motor and voltages as used in Example 9.2 have input power requirement of 25 kW. Determine the required slip.

Set the initial (old) value of slip to 0.0 and the change in slip to be 0.01:

$$s_{old} = 0$$

$$ds = 0.01$$

Define the positive and negative slips as

$$s_1 = s_{old} + ds = 0.01$$

$$s_2 = 2 - s_1 = 1.99$$

Compute the motor equivalent admittance matrix:

$$[YM_{abc}] = \begin{bmatrix} 0.7081 - j0.379 & -0.1072 - j0.1382 & 0.3991 + j0.5172 \\ 0.3991 + j0.5172 & 0.7081 - j0.379 & -0.1072 - j0.1382 \\ -0.1072 - j0.1382 & 0.3991 + j0.5172 & 0.7081 - j0.379 \end{bmatrix}$$

The currents are calculated to be

$$[I_s] = [YM_{abc}][VLL_{abc}] = \begin{bmatrix} 31.1\underline{/-107.8} \\ 25.1\underline{/161.0} \\ 39.5\underline{/32.8} \end{bmatrix}$$

The three-phase input complex power is computed to be

$$S_{in} = \sum_{k=1}^{3} \frac{Vs_{abc_k} \cdot (Is_{abc_k})^*}{1000} = 6.0366 + j11.5963$$

The error is

$$Error = 25.0 - 6.0366 = 18.9634$$

Since the error is greater than zero, the value of the old value of slip needs to be increased by the present value of the change in voltage:

$$s_{new} = s_{old} + ds = 0.01 + 0.01 = 0.02$$

The process is now repeated starting with the computation of the motor shunt admittance matrix. When the value of slip reaches 0.050, the computed input power is 27.6 kW. The error is now –2.6, which means that the needed value of slip lies between 0.049 and 0.05. At this point, the present value of the change in slip (0.01) is changed to 0.001 and the new value of slip will be 0.041. Again, the process is repeated until at a slip of 0.045 the computed input power is 25.1 with an error of –0.1. The change in slip is divided by 10 with a new value of 0.0001 and the new slip is 0.0442. The process continues until a slip of 0.0448 gives a computed power of 25.04 with an error of –0.04. The change in slip is moved to 0.00001 and the new slip is 0.04471. At a slip of 0.04472, the computed power is 25.0006, which is within the desired tolerance of 0.001. The final input to the motor is

$$S_{in} = 25.0006 + j15.9581 = 31.9 \text{ kVA at 0.866 lag power factor}$$

9.5.4 Induction Generator

Three-phase induction generators are becoming common as a source of distributed generation on a distribution system. In particular, windmills generally drive an induction motor. It is, therefore, important that a simple model of an induction generator be developed for power-flow purposes. In reality, the same model as was used for the induction motor is used for the induction generator. The only change is that the generator will be driven at a speed in excess of synchronous speed, which means that the slip will be a negative value. The generator can be modeled with the equivalent admittance matrix from Equation 9.48.

Example 9.4

Using the same induction machine and line-to-line voltages in Example 9.2, determine the slip of the machine so that it will generate 20 kW. Since the same model is being used with the same assumed direction of currents, the specified power at the terminals of the machine will be

$$P_{gen} = -20$$

As before, the initial "old" value of slip is set to 0.0. However, since the machine is now a generator, the initial change in slip will be

$$ds = -0.01$$

As before, the value of slip to be used for the first iteration will be

$$s_1 = s_{old} + ds = -0.01$$

This value of slip gives an input power of the machine of –5.706 with an error of –14.2933. Since the error is negative, the positive sequence slip will be increased by the present value of the change in slip.

$$s_1 = -0.01 - 0.01 = -0.02$$

This slip yields an input power of −11.6384 kW and an error of −8.3616. The process is continued until, at a value of slip of −0.04, the input power is −23.3224 with an error of +3.3224. The positive error is the key to decrease the change in slip by a factor of 10. This gives a new change in slip of a value of −0.001. Again, the process is repeated until, at a value of slip of −0.035, the power is −20.4432 with an error of +0.4432. The value of the change in slip becomes −0.0001. This procedure is continued, changing the change in slip every time that the error is positive. Finally, at a value of slip of −0.034234, the power is −19.9990 with an error of −0.001, which is within the desired tolerance of 0.001. For the final value of slip, the input power of the machine is

$$S_{in} = -19.9990 + j15.0244$$

It must be noted that even though the machine is supplying power to the system, it is still consuming reactive power. The point being that even though the induction generator can supply real power to the system, it will still require reactive power from the system. This reactive power is typically supplied by shunt capacitors or a static var supply at the location of the windmill.

9.6 Summary

This chapter has developed load models for typical loads on a distribution feeder. It is important to recognize that a combination of constant *PW*, constant *Z*, and constant current loads can be modeled using a percentage of each model. An extended model for a three-phase induction machine has been developed with examples of the machine operating as a motor and as a generator. An iterative procedure for the computation of slip to force the input power to the machine to be a specified value was developed and used in examples for both a motor and a generator.

Problems

9.1 A 12.47 kV feeder provides service to an unbalanced wye-connected load specified to be

Phase *a*: 1000 kVA, 0.9 lagging power factor
Phase *b*: 800 kVA, 0.95 lagging power factor
Phase *c*: 1100 kVA, 0.85 lagging power factor

a. Compute the initial load currents assuming the loads are modeled as constant complex power.
b. Compute the magnitude of the load currents that will be held constant assuming the loads are modeled as constant current.
c. Compute the impedance of the load to be held constant assuming the loads are modeled as constant impedance.
d. Compute the initial load currents assuming that 60% of the load is complex power, 25% constant current, and 15% constant impedance.

9.2 Using the results of Problem 9.1, rework the problem at the start of the second iteration if the load voltages after the first iteration have been computed to be

$$[VLN_{abc}] = \begin{bmatrix} 6851/\underline{-1.9} \\ 6973/\underline{-122.1} \\ 6886/\underline{117.5} \end{bmatrix} \text{V}$$

9.3 A 12.47 kV feeder provides service to an unbalanced delta-connected load specified to be

Phase *a*: 1500 kVA, 0.95 lagging power factor
Phase b: 1000 kVA, 0.85 lagging power factor
Phase c: 950 kVA, 0.9 lagging power factor

a. Compute the load and line currents if the load is modeled as constant complex power.
b. Compute the magnitude of the load current to be held constant if the load is modeled as constant current.
c. Compute the impedance to be held constant if the load is modeled as constant impedance.
d. Compute the line currents if the load is modeled as 25% constant complex power, 20% constant current, and 55% constant impedance.

9.4 After the first iteration of the system of Problem 9.5, the load voltages are

$$[VLL_{abc}] = \begin{bmatrix} 11{,}981/\underline{28.3} \\ 12{,}032/\underline{-92.5} \\ 11{,}857/\underline{147.7} \end{bmatrix} \text{V}$$

a. Compute the load and line currents if the load is modeled as constant complex power.
b. Compute the load and line currents if the load is modeled as constant current.

c. Compute the load and line current if the load is modeled as constant impedance.

d. Compute the line currents if the load mix is 25% constant complex power, 20% constant current, and 55% constant impedance.

9.5 The motor in Example 9.2 is operating with a slip of 0.03 with balanced voltages of 240 V line to line. Determine the following:

a. The input line currents and complex three-phase power

b. The currents in the rotor circuit

c. The developed shaft power in hp

9.6 The motor in Example 9.2 is operating with a slip of 0.03 and the line-to-line voltage magnitudes are

$$V_{ab} = 240\,\text{V}, \quad V_{bc} = 230\,\text{V}, \quad V_{ca} = 250\,\text{V}$$

a. Compute the angles on the line-to-line voltages assuming the voltage *ab* is reference.

b. For the given voltages and slip, determine the input line currents and complex three-phase power.

c. Compute the rotor currents.

d. Compute the developed shaft power in hp.

9.7 For the motor in Example 9.2, determine the following if the motor terminal voltages are those given in Problem 9.6:

a. The starting line currents

b. The locked rotor line currents

9.8 A three-phase induction motor has the following data:

25 hp, 240 V

$Zs = 0.0336 + j0.08$ per unit
$Zr = 0.0395 + j0.08$ per unit
$Zm = j3.0$ per unit

The terminal line-to-line voltages of the motor are

$$V_{ab} = 240\underline{/0}, \quad V_{bc} = 233.5\underline{/-118.1}, \quad V_{ca} = 242.2\underline{/122.1}\ \text{V}$$

The motor input kW is to be 20 kW.
Determine the following:

a. Required slip

b. The input kW and kvar

c. The converted shaft power

Windmil Assignment

Created System 6 by adding to System 5 the following:

- An ungrounded wye–delta transformer bank at node 2. The bank is composed of three 15 kVA single-phase transformers with the following data:
  - 15 kVA
  - 7.2 kV wye–0.24 kV delta
  - $Z = 8.06\%$, $X/R = 8$
- A three-phase induction motor connected to the ungrounded wye–delta bank at node 2.
  - 25 hp
  - 240 V
  - $Zs = 0.0774 + j0.1843\,\Omega$
  - $Zr = 0.0908 + j0.1843\,\Omega$
  - $Zm = 0 + j4.8384\,\Omega$
  - $Slip = 3.5\%$

Determine the voltage unbalance at the motor terminals.

References

1. *American National Standard for Electric Power Systems and Equipment–Voltage Ratings (60 Hertz)*, ANSI C84.1-1995, National Electrical Manufacturers Association, Rosslyn, VA, 1996.
2. Kersting, W.H. and Phillips, W.H., Phase frame analysis of the effects of voltage unbalance on induction machines, *IEEE Transactions on Industry Applications*, 33, 415–420, 1997.

10

Distribution Feeder Analysis

The analysis of a distribution feeder will typically consist of a study of the feeder under normal steady-state operating conditions (power-flow analysis) and a study of the feeder under short-circuit conditions (short-circuit analysis). Models of all of the components of a distribution feeder have been developed in previous chapters. These models will be applied for the analysis under steady-state and short-circuit conditions.

10.1 Power-Flow Analysis

The power-flow analysis of a distribution feeder is similar to that of an interconnected transmission system. Typically, what will be known prior to the analysis will be the three-phase voltages at the substation and the complex power of all of the loads and the load model (constant complex power, constant impedance, constant current, or a combination). Sometimes, the input complex power supplied to the feeder from the substation is also known.

In Chapters 6 through 8, phase frame models are developed for the series components of a distribution feeder. In Chapter 9, models are developed for the shunt components (static loads, induction machines, and capacitor banks). These models are used in the "power-flow" analysis of a distribution feeder.

A power-flow analysis of a feeder can determine the following:

- Voltage magnitudes and angles at all nodes of the feeder
- Line flow in each line section specified in kW and kvar, amps and degrees, or amps and power factor
- Power loss in each line section
- Total feeder input kW and kvar
- Total feeder power losses
- Load kW and kvar based upon the specified model for the *load*

10.1.1 Ladder Iterative Technique

Because a distribution feeder is radial, iterative techniques commonly used in transmission network power-flow studies are not used because of poor convergence characteristics [1]. Instead, an iterative technique specifically designed for a radial system is used.

10.1.1.1 Linear Network

A modification of the "ladder" network theory of linear systems provides a robust iterative technique for power-flow analysis [2]. A distribution feeder is nonlinear because most loads are assumed to be constant kW and kvar. However, the approach taken for the linear system can be modified to take into account the nonlinear characteristics of the distribution feeder.

Figure 10.1 shows a linear ladder network.

For the ladder network, it is assumed that all of the line impedances and load impedances are known along with the voltage (V_S) at the source. The solution for this network is to perform the "forward" sweep by calculating the voltage at node 5 (V_5) under a no-load condition. With no load currents there are no line currents, so the computed voltage at node 5 will equal that of the specified voltage at the source. The "backward" sweep commences by computing the load current at node 5. The load current I_5 is

$$I_5 = \frac{V_5}{ZL_5} \tag{10.1}$$

For this "end node" case, the line current I_{45} is equal to the load current I_5. The "backward" sweep continues by applying Kirchhoff's voltage law (KVL) to calculate the voltage at node 4:

$$V_4 = V_5 + Z_{45} \cdot I_{45} \tag{10.2}$$

The load current I_4 can be determined and then Kirchhoff's current law (KCL) applied to determine the line current I_{34}:

$$I_{34} = I_{45} + I_4 \tag{10.3}$$

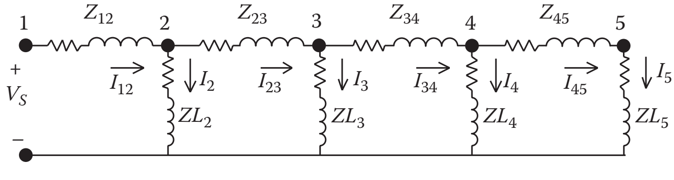

FIGURE 10.1
Linear ladder network.

KVL is applied to determine the node voltage V_3. The backward sweep continues until a voltage (V_1) has been computed at the source. The computed voltage V_1 is compared to the specified voltage V_S. There will be a difference between these two voltages. The ratio of the specified voltage to the compute voltage can be determined as

$$Ratio = \frac{V_S}{V_1} \tag{10.4}$$

Since the network is linear, all of the line and load currents and node voltages in the network can be multiplied by the ratio for the final solution to the network.

10.1.1.2 Nonlinear Network

The linear network of Figure 10.1 is modified to a nonlinear network by replacing all of the constant load impedances by constant complex power loads as shown in Figure 10.2.

As with the linear network, the "forward" sweep computes the voltage at node 5 assuming no load. As before, the node 5 (end node) voltage will equal that of the specified source voltage. In general, the load current at each node is computed by

$$I_n = \left(\frac{S_n}{V_n}\right)^* \tag{10.5}$$

The "backward" sweep will determine a computed source voltage V_1. As in the linear case, this first "iteration" will produce a voltage that is not equal to the specified source voltage V_S. Because the network is nonlinear, multiplying currents and voltages by the ratio of the specified voltage to the computed voltage will not give the solution. The most direct modification using the ladder network theory is to perform a "forward" sweep. The forward sweep commences by using the specified source voltage and the line currents from the previous "backward" sweep. KVL is used to compute the voltage at node 2 by

$$V_2 = V_S - Z_{12} \cdot I_{12} \tag{10.6}$$

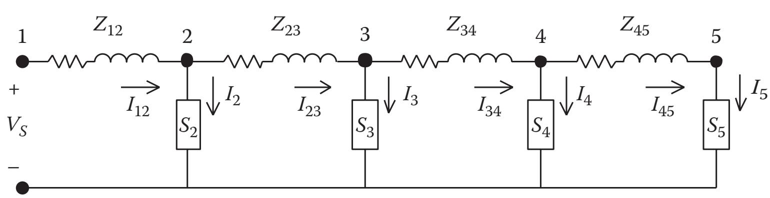

FIGURE 10.2
Nonlinear ladder network.

This procedure is repeated for each line segment until a "new" voltage is determined at node 5. Using the "new" voltage at node 5, a second backward sweep is started that will lead to a "new" computed voltage at the source.

The procedure shown earlier works but, in general, will require more time to converge. A modified version is to perform the "forward" sweep calculating all of the node voltages using the line currents from the previous "backward" sweep. The new "backward" sweep will use the node voltages from the previous "forward" sweep to compute the new load and line currents. In general, this modification will require more iterations but less time to converge. In this modified version of the ladder technique, convergence is determined by computing the ratio of difference between the voltages at the $n-1$ and n iterations and the nominal line-to-neutral voltage. Convergence is achieved when all of the phase voltages at all nodes satisfy

$$\frac{\left||V_n| - |V_{n-1}|\right|}{V_{nominal}} \leq \textit{Specified tolerance}$$

Example 10.1

A single-phase lateral is shown in Figure 10.3.

The line impedance is

$$z = 0.3 + j0.6\,\Omega/\text{mile}$$

The impedance of the line segment 1–2 is

$$Z_{12} = (0.3 + j0.6) \cdot \frac{3000}{5280} = 0.1705 + j0.3409\,\Omega$$

The impedance of the line segment 2–3 is

$$Z_{23} = (0.3 + j0.6) \cdot \frac{4000}{5280} = 0.2273 + j0.4545\,\Omega$$

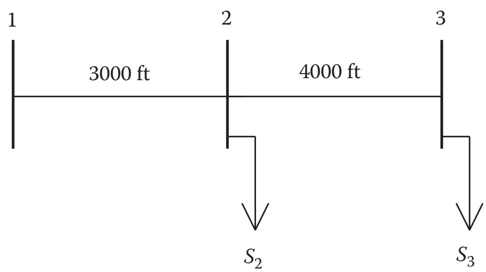

FIGURE 10.3
Single-phase lateral.

The loads are

$$\begin{aligned} S_2 &= 1500 + j750 \\ S_3 &= 900 + j500 \end{aligned} \; (\text{kW} + j\,\text{kvar})$$

The source voltage at node 1 is 7200 V.

Use the modified ladder method to compute the load voltage after the second forward sweep.

Set initial conditions:

$$I_{12} = I_{23} = 0 \quad V_{old} = 0 \quad Tol = 0.0001$$

The first forward sweep:

$$V_2 = V_s - Z_{12} \cdot I_{12} = 7200\underline{/0}$$

$$V_3 = V_2 - Z_{23} \cdot I_{23} = 7200\underline{/0}$$

$$Error = \frac{\left| |V_3| - |V_{old}| \right|}{7200} = 1 \text{ (greater than } Tol\text{, start backward sweep)}$$

$$V_3 = V_{old}$$

The first backward sweep:

$$I_3 = \left(\frac{(900 + j500) \cdot 1000}{7200\underline{/0}} \right)^* = 143.0\underline{/-29.0}\text{ A}$$

The current flowing in the line segment 2–3 is

$$I_{23} = I_3 = 143.0\underline{/-29.0}\text{ A}$$

The load current at node 2 is

$$I_2 = \left(\frac{(1500 + j750) \cdot 1000}{7200\underline{/0}} \right)^* = 232.9\underline{/-27.5}\text{ A}$$

The current in line segment 1–2 is

$$I_{12} = I_{23} + I_2 = 373.8\underline{/-27.5}\text{ A}$$

The second forward sweep:

$$V_2 = V_S - Z_{12} \cdot I_{12} = 7084.5\underline{/-0.7}$$

$$V_3 = V_2 - Z_{23} \cdot I_{23} = 7025.1\underline{/-1.0}$$

$$Error = \frac{\left| |V_3| - |V_{old}| \right|}{7200} = \frac{7084.5 - 7200}{7200} = 0.0243 \text{ (greater than tolerance, continue)}$$

$$V_{old} = V_3$$

At this point, the second backward sweep is used to compute the new line currents. This is followed by the third forward sweep. After four iterations, the voltages have converged to an error of 0.000017 with the final voltages and currents of

$$[V_2] = 7081.0\underline{/-0.68}$$

$$[V_3] = 7019.3\underline{/-1.02}$$

$$[I_{12}] = 383.4\underline{/-28.33}$$

$$[I_{23}] = 146.7\underline{/-30.07}$$

10.1.2 General Feeder

A typical distribution feeder will consist of the "primary main" with laterals tapped off the primary main and sublaterals tapped off the laterals, etc. Figure 10.4 shows an example of a typical feeder.

In Figure 10.4, no distinction is made as to what type of element is connected between nodes. However, the phasing is shown and this is a must. All series elements (lines, transformers, and regulators) can be represented by the circuit in Figure 10.5. Note in Figure 10.4 that the line between nodes 3 and 4 and between nodes 4 and 5 have "distributed" loads modeled at the middle of the lines. The model for the distributed loads was developed in Chapter 3.

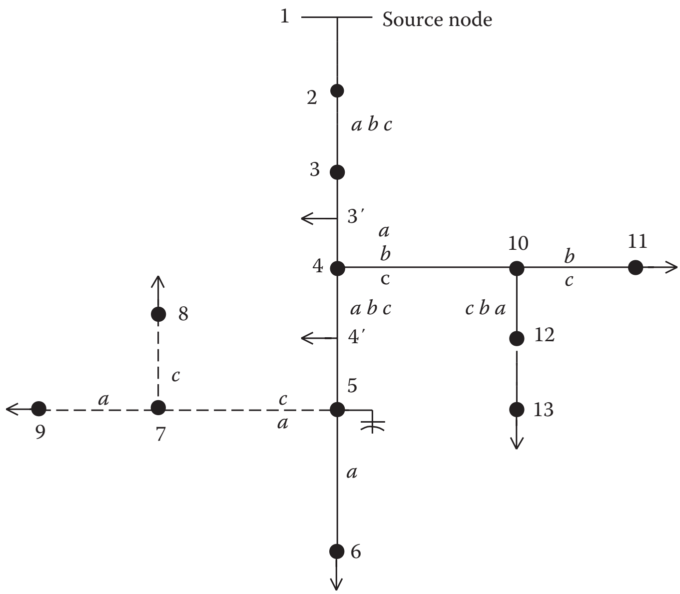

FIGURE 10.4
Typical distribution feeder.

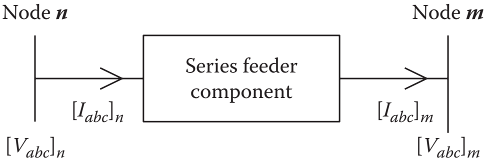

FIGURE 10.5
Standard feeder series component model.

Connecting the loads at the center was only one of three ways to model the load. A second method is to place one-half of the load at each end of the line. The third method is to place two-thirds of the load 25% of the way down the line from the source end. The remaining one-third of the load is connected at the receiving end node. This "exact" model gives the correct voltage drop down the line in addition to the correct power line power loss.

In previous chapters, the forward and backward sweep models have been developed for the series elements. With reference to Figure 10.5, the forward and backward sweep equations are

$$\begin{aligned} &\text{Forward sweep:} \quad [VLN_{abc}]_m = [A]\cdot[VLN_{abc}]_m - [B]\cdot[I_{abc}]_n \\ &\text{Backward sweep:} \quad [I_{abc}]_n = [c]\cdot[VLN_{abc}]_m + [d]\cdot[I_{abc}]_m \end{aligned} \tag{10.7}$$

In most cases, the [c] matrix will be zero. Long underground lines will be the exception. It was also shown that for the grounded wye–delta transformer bank the backward sweep equation is

$$[I_{abc}]_n = [x_t]\cdot[VLN_{abc}]_n + [d]\cdot[I_{abc}]_m \tag{10.8}$$

The reason for this is that the currents flowing in the secondary delta windings are a function of the primary line-to-ground voltages.

With reference to Figure 10.4, nodes 4, 10, 5, and 7 are referred to as "junction nodes." In both the forward and backward sweeps, the junction nodes must be recognized. In the forward sweep, the voltages at all nodes down the lines from the junction nodes must be computed. In the backward sweeps, the currents at the junction nodes must be summed before proceeding toward the source. In developing a program to apply the modified ladder method, it is necessary for the ordering of the lines and nodes to be such that all node voltages in the forward sweep are computed and all currents in the backward sweep are computed.

10.1.3 Unbalanced Three-Phase Distribution Feeder

The previous section outlined the general procedure for performing the modified ladder iterative technique. This section addresses how that procedure can be used for an unbalanced three-phase feeder.

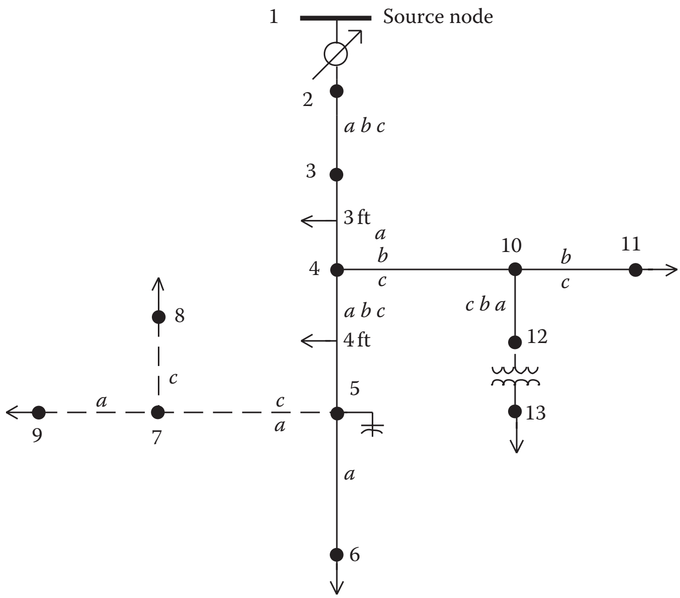

FIGURE 10.6
Unbalanced three-phase distribution feeder.

Figure 10.6 is the one-line diagram of an unbalanced three-phase feeder as shown in Figure 10.5.

The topology of the feeder in Figure 10.6 is the same as the feeder in Figure 10.5. Figure 10.6 shows more detail of the feeder with step-regulators at the source and a transformer bank at node 12. The feeder of Figure 10.6 can be broken into the "series" components and the "shunt" components. The series components have been shown in Section 10.1.2.

10.1.3.1 Shunt Components

The shunt components of a distribution feeder are

- Spot static loads
- Spot induction machines
- Capacitor banks

Spot static loads are located at a node and can be three phase, two phase, or single phase and connected in either a wye or a delta connection. The loads can be modeled as constant complex power, constant current, constant impedance, or a combination of the three.

A spot induction machine is modeled using the shunt admittance matrix as defined in Chapter 9. The machine can be modeled as a motor with a positive slip or as an induction generator with a negative slip. The input power (positive for a motor and negative for a generator) can be specified and the required slip computed using the iterative process described in Chapter 9.

Capacitor banks are located at a node and can be three phase, two phase, or single phase and can be connected in a wye or delta. Capacitor banks are modeled as constant admittances.

In Figure 10.5, the solid line segments represent overhead lines, while the dashed lines represent underground lines. Note that the phasing is shown for all of the line segments. In Chapter 4, the application of Carson's equations for computing the line impedances for overhead and underground lines was presented. In that chapter, it is pointed out that two-phase and single-phase lines are represented by a 3 × 3 matrix with zeros set in the rows and columns of the missing phases.

In Chapter 5, the method for the computation of the shunt capacitive susceptance for overhead and underground lines was presented. Most of the time, the shunt capacitance of the line segment can be ignored; however, for long underground line segments, the shunt capacitance should be included.

The "node" currents may be three phase, two phase, or single phase and consist of the sum of the spot load currents and one-half of the distributed load currents (if any) at the node plus the capacitor current (if any) at the node. It is possible that at a given node the distributed load can be one-half of the distributed load in the "from" segment plus one-half of the distributed load in the "to" segment.

10.1.4 Applying the Modified Ladder Iterative Technique

Section 10.1.2 outlined the steps required for the application of the ladder iterative technique. Forward and backward sweep matrices have been developed in Chapters 6 through 8 for the series devices. By applying these matrices, the computation of the voltage drops along a segment will always be the same regardless of whether the segment represents a line, voltage regulator, or transformer.

In the preparation of data for a power-flow study, it is extremely important that the impedances and admittances of the line segments are computed using the exact spacings and phasing. Because of the unbalanced loading and resulting unbalanced line currents, the voltage drops due to the mutual coupling of the lines become very important. It is not unusual to observe a voltage rise on a lightly loaded phase of a line segment that has an extreme current unbalance.

The real power loss in a device can be computed in two ways. The first method is to compute the power loss in each phase by taking the phase current squared times the total resistance of the phase. Care must be taken to not use the resistance value from the phase impedance matrix. The actual phase resistance that was used in Carson's equations must be used. In developing a computer program, calculating power loss this way requires that the conductor resistance is stored in the active data base for each line segment. Unfortunately, this method does not give the total power loss in a line segment since the power loss in the neutral conductor and ground are not included. In order to determine the losses in the neutral and ground, the method outlined in Chapter 4 must be used to compute the neutral and ground currents and then the power losses.

A second, and preferred, method is to compute power loss as the difference between the real power into a line segment and the real power output of the line segment. Because the effects of the neutral conductor and ground are included in the phase impedance matrix of the total power loss, this method will give the same results as mentioned earlier where the neutral and ground power losses are computed separately. This method can lead to some interesting numbers for very unbalanced line flows in that it is possible to compute what appears to be a negative phase power loss. This is a direct result of the accurate modeling of the mutual coupling between phases. Remember that the effect of the neutral conductor and the ground resistance is included in Carson's equations. In reality, there can not be a negative phase power loss. Using this method, the algebraic sum of the line power losses will equal the total three-phase power loss that were computed using the current squared times resistance for the phase and neutral conductors along with the ground current.

10.1.5 Let's Put It All Together

At this point the models for all components of a distribution feeder have been developed. The modified ladder iterative technique has also been developed. It is time to put them all together and demonstrate the power-flow analysis of a very simple system. Example 10.2 will demonstrate how the models of the components work together in applying the modified ladder technique to achieve a final solution of the operating characteristics of an unbalanced feeder.

Example 10.2

A very simple distribution feeder is shown in Figure 10.7. This system is the IEEE 4 Node Test Feeder that can be found on the IEEE website [1].

For the system in Figure 10.7, the infinite bus voltages are balanced three phase of 12.47 kV line to line. The "source" line segment from node 1 to node 2 is a three-wire delta 2000 ft long line and is constructed on the pole configuration of Figure 4.7 without the neutral. The "load" line segment from node 3 to node 4 is 2500 ft long and also is constructed on the pole configuration of Figure 4.7 but is a four-wire wye so the neutral is included. Both line segments use 336,400 26/7 ACSR phase conductors and the neutral conductor on the four-wire wye line is 4/0 6/1 ACSR. Since the lines are short, the shunt admittance will be neglected. The 25°C resistance is used for the phase and neutral conductors:

$$336{,}400\ 26/7\ \text{ACSR:}\ \ \text{resistance at } 25°\text{C} = 0.278\ \Omega/\text{mile}$$

$$4/0\ 6/1\ \text{ACSR:}\ \ \text{resistance at } 25°\text{C} = 0.445\ \Omega/\text{mile}$$

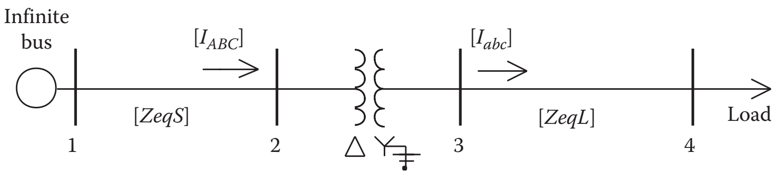

FIGURE 10.7
Example 10.2 feeder.

The phase impedance matrices for the two line segments are

$$[ZeqS_{ABC}] = \begin{bmatrix} 0.1414 + j0.5353 & 0.0361 + j0.3225 & 0.0361 + j0.2752 \\ 0.0361 + j0.3225 & 0.1414 + j0.5353 & 0.0361 + j0.2955 \\ 0.0361 + j0.2752 & 0.0361 + j0.2955 & 0.1414 + j0.5353 \end{bmatrix} \Omega$$

$$[ZeqL_{abc}] = \begin{bmatrix} 0.1907 + j0.5035 & 0.0607 + j0.2302 & 0.0598 + j0.1751 \\ 0.0607 + j0.2302 & 0.1939 + j0.4885 & 0.0614 + j0.1931 \\ 0.0598 + j0.1751 & 0.0614 + j0.1931 & 0.1921 + j0.4970 \end{bmatrix} \Omega$$

The transformer bank is connected delta–grounded wye and consists of three single-phase transformers each rated:

$$2000 \text{ kVA}, \quad 12.47 - 2.4 \text{ kV}, \quad Z = 1.0 + j6.0\%$$

The feeder serves an unbalanced three-phase wye-connected constant *PQ* load of

$$S_a = 750 \text{ kVA at 0.85 lagging power factor}$$

$$S_b = 1000 \text{ kVA at 0.90 lagging power factor}$$

$$S_c = 1230 \text{ kVA at 0.95 lagging power factor}$$

Before starting the iterative solution, the forward and backward sweep matrices must be computed for each series element. The modified ladder method is going to be employed so only the [*A*], [*B*], and [*d*] matrices need to be computed.

Source line segment with shunt admittance neglected:

$$[U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A_1] = [U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[B_1] = [ZeqS_{ABC}] = \begin{bmatrix} 0.1414 + j0.5353 & 0.0361 + j0.3225 & 0.0361 + j0.2752 \\ 0.0361 + j0.3225 & 0.1414 + j0.5353 & 0.0361 + j0.2955 \\ 0.0361 + j0.2752 & 0.0361 + j0.2955 & 0.1414 + j0.5353 \end{bmatrix}$$

$$[d_1] = [U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Load line segment:

$$[A_2] = [U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[B_2] = [ZeqL_{abc}] \begin{bmatrix} 0.1907 + j0.5035 & 0.0607 + j0.2302 & 0.0598 + j0.1751 \\ 0.0607 + j0.2302 & 0.1939 + j0.4885 & 0.0614 + j0.1931 \\ 0.0598 + j0.1751 & 0.0614 + j0.1931 & 0.1921 + j0.4970 \end{bmatrix}$$

$$[d_2] = [U] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformer:
The transformer impedance must be converted to actual value in Ohms referenced to the low-voltage windings.

$$Z_{base} = \frac{2.4^2 \cdot 1000}{2000} = 2.88\,\Omega$$

$$Zt_{low} = (0.01 + j0.06) \cdot 2.88 = 0.0288 + j0.1728\,\Omega$$

The transformer phase impedance matrix is

$$[Zt_{abc}] = \begin{bmatrix} 0.0288 + j0.1728 & 0 & 0 \\ 0 & 0.0288 + j0.1728 & 0 \\ 0 & 0 & 0.0288 + j0.1728 \end{bmatrix} \Omega$$

The "turns" ratio: $n_t = 12.47/2.4 = 5.1958$.

The ladder sweep matrices are

$$[A_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0.1925 & 0 & -0.1925 \\ -0.1925 & 0.1925 & 0 \\ 0 & -0.1925 & 0.1925 \end{bmatrix}$$

$$[B_t] = [Zt_{abc}] = \begin{bmatrix} 0.0288 + j0.1728 & 0 & 0 \\ 0 & 0.0288 + j0.1728 & 0 \\ 0 & 0 & 0.0288 + j0.1728 \end{bmatrix}$$

$$[d_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.1925 & -0.1925 & 0 \\ 0 & 0.1925 & -0.1925 \\ -0.1925 & 0 & 0.1925 \end{bmatrix}$$

Define the node 4 loads:

$$[S4] = \begin{bmatrix} 750/\underline{a\cos(0.85)} \\ 1000/\underline{a\cos(0.90)} \\ 1250/\underline{a\cos(0.95)} \end{bmatrix} = \begin{bmatrix} 750/\underline{31.79} \\ 1000/\underline{25.84} \\ 1250/\underline{18.19} \end{bmatrix} \text{kVA}$$

Define infinite bus line-to-line and line-to-neutral voltages:

$$[ELL_s] = \begin{bmatrix} 12,470/\underline{30} \\ 12,470/\underline{-90} \\ 12,470/\underline{150} \end{bmatrix} \text{V}$$

$$[ELN_s] = \begin{bmatrix} 7199.6/\underline{0} \\ 7199.6/\underline{-120} \\ 7199.6/\underline{120} \end{bmatrix} \text{V}$$

The flowchart of a Mathcad® program is shown in Figure 10.8.

The Mathcad program is used to analyze the system, and, after eight iterations, the load voltages on a 120 V base are

$$[V4_{120}] = \begin{bmatrix} 113.9 \\ 110.0 \\ 110.6 \end{bmatrix} \text{V}$$

The voltages at node 4 are below the desired 120 V. These low voltages can be corrected with the installation of three step-voltage regulators connected in wye on the secondary bus (node 3) of the transformer. The new configuration of the feeder is shown in Figure 10.9.

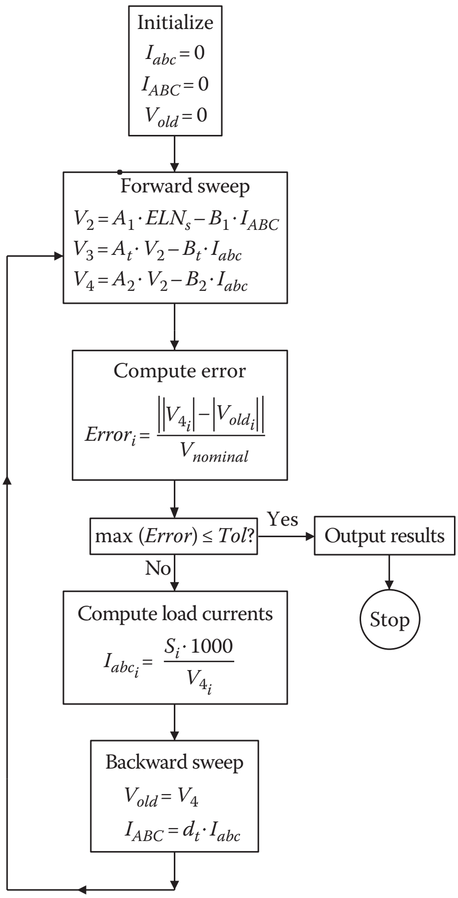

FIGURE 10.8
Flowchart.

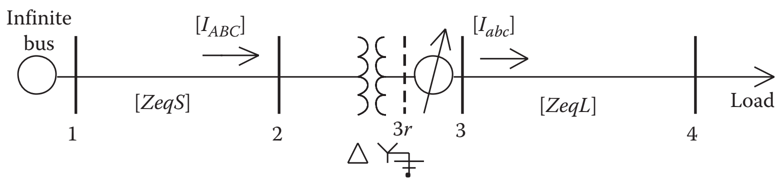

FIGURE 10.9
Voltage regulator added to the system.

For the regulator, the potential transformer ratio will be 2400–120 V ($N_{pt} = 20$) and the *CT* ratio is selected to carry the rated current of the transformer bank. The rated current is

$$I_{rated} = \frac{6000}{\sqrt{3} \cdot 2.4} = 832.7$$

The *CT* ratio is selected to be 1000 : 5 = *CT* = 200.

The equivalent phase impedance between node 3 and node 4 is computed using the converged voltages at the two nodes. This is done so that the *R* and *X* settings of the compensator can be determined:

$$Zeq_i = \frac{V3_i - V4_i}{I3_i} = \begin{bmatrix} 0.1414 + j0.1830 \\ 0.2079 + j0.2827 \\ 0.0889 + j0.3833 \end{bmatrix} \Omega$$

The three regulators are to have the same *R* and *X* compensator settings. The average value of the computed impedances will be used:

$$Z_{avg} = \frac{1}{3} \cdot \sum_{k=1}^{3} Zeq_k = 0.1451 + j0.2830\,\Omega$$

The value of the compensator impedance in volts is given by Equation 7.78:

$$R' + jX' = (0.1451 + j0.2830) \cdot \frac{1000}{20} = 7.3 + j14.2\,\text{V}$$

The value of the compensator settings in Ohms is

$$R_\Omega + jX_\Omega = \frac{7.3 + j14.2}{5} = 1.46 + j2.84\,\Omega$$

With the regulator in the neutral position, the voltages being input to the compensator circuit for the given conditions are

$$Vreg_i = \frac{V3_i}{PT} = \begin{bmatrix} 117.5\underline{/-31.2} \\ 117.1\underline{/-151.7} \\ 116.7\underline{/87.8} \end{bmatrix} \text{V}$$

The compensator currents are

$$Icomp_i = \frac{Iabc_i}{CT} = \begin{bmatrix} 1.6460\underline{/-63.6} \\ 2.2727\underline{/-179.4} \\ 2.8264\underline{/64.9} \end{bmatrix} \text{A}$$

With the input voltages and compensator currents, the voltages across the voltage relays in the compensator circuit are computed to be

$$\left[V_{relay}\right] = \left[V_{reg}\right] - \left[Z_{comp}\right] \cdot \left[I_{comp}\right] = \begin{bmatrix} 113.0\underline{/-32.5} \\ 111.3\underline{/-153.8} \\ 109.0\underline{/84.5} \end{bmatrix} \text{V}$$

Notice how close these compare to the actual voltages on a 120 V base at node 4.

Assume that the voltage level has been set at 121 V with a bandwidth of 2 V. In the real world, the regulators on each phase will change taps one at a time until the relay on that phase reaches 120 V. In order to model this system, the flowchart of Figure 10.8 is slightly modified in the forward and backward sweeps.

Forward sweep:

$$
\begin{aligned}
&[VLN_2] = [A_1]\cdot[E_s] - [B_1]\cdot[I_{ABC}] \\
&[VLN_{3r}] = [A_t]\cdot[VLN_2] - [B_t]\cdot[I_{in}] \\
&[VLN_3] = [A_{reg}][VLN_3] - [B_{reg}]\cdot[I_{abc}] \\
&[VLN_4] = [A_2]\cdot[VLN_3] - [B_2]\cdot[I_{abc}]
\end{aligned}
\tag{10.9a}
$$

Backward sweep:

$$
\begin{aligned}
&[V_{old}] = [VLN_4] \\
&[I_{in}] = [d_{reg}]\cdot[I_{abc}] \\
&[I_{ABC}] = [d_t]\cdot[I_{in}]
\end{aligned}
\tag{10.9b}
$$

After the analysis routine has converged, a new routine will compute whether or not tap changes need to be made. The Mathcad routine for computing the new taps is shown in Figure 10.10.

$$
Y := \left|
\begin{array}{l}
\text{for } i \in 1..3 \\
\quad \left| \begin{array}{l} V_{reg_i} \leftarrow \dfrac{VLN_{3_i}}{N_{pt}} \\ I_{reg_i} \leftarrow \dfrac{I_{abc_i}}{CT} \end{array} \right. \\
V_{relay} \leftarrow V_{reg} - Z_{comp}\cdot I_{reg} \\
Tap_1 \leftarrow Tap_1 + 1 \text{ if } \left|V_{relay_1}\right| < 120 \\
Tap_2 \leftarrow Tap_2 + 1 \text{ if } \left|V_{relay_2}\right| < 120 \\
Tap_3 \leftarrow Tap_3 + 1 \text{ if } \left|V_{relay_3}\right| < 120 \\
Out_1 \leftarrow Tap \\
Out
\end{array}
\right.
$$

FIGURE 10.10
Tap changing routine.

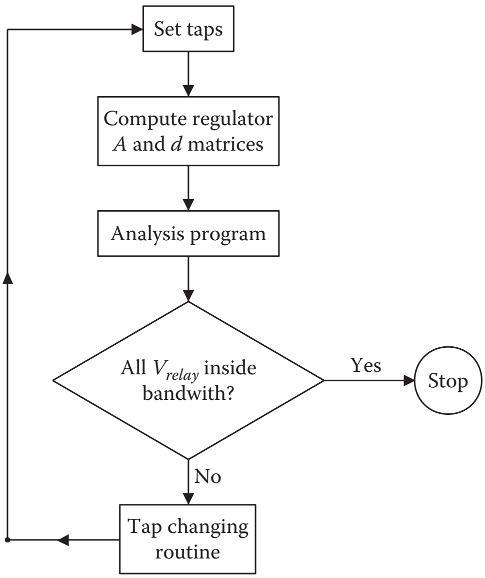

FIGURE 10.11
Computational sequence.

The computational sequence for the determination of the final tap settings and convergence of the system is shown in the flowchart of Figure 10.11.

The tap changing routine changes individual regulators one step at a time. The final tap settings are

$$[Tap] = \begin{bmatrix} 9 \\ 11 \\ 12 \end{bmatrix}$$

The final relay voltages are

$$[V_{relay}] = \begin{bmatrix} 120.3 \\ 120.4 \\ 120.1 \end{bmatrix}$$

The final voltages on a 120 V base at the load center (node 4) are

$$[VLN_{4_{120}}] = \begin{bmatrix} 121.0 \\ 119.3 \\ 120.7 \end{bmatrix}$$

Unlike the previous example, the compensator relay voltages and the actual load center voltages are very close to each other.

10.1.6 Load Allocation

Many times the input complex power (kW and kvar) to a feeder is known because of the metering at the substation. This information can be for either total three phases or each individual phase. In some cases, the metered data may be the current and power factor in each phase.

It is desirable to force the computed input complex power to the feeder matching the metered input. This can be accomplished (following a converged iterative solution) by computing the ratio of the metered input to the computed input. The phase loads can now be modified by multiplying the loads by this ratio. Because the losses of the feeder will change when the loads are changed, it is necessary to go through the ladder iterative process to determine a new computed input to the feeder. This new computed input will be closer to the metered input but most likely not within a specified tolerance. Again a ratio can be determined and the value of the loads modified. This process is repeated until the computed input is within a specified tolerance of the metered input.

Load allocation does not have to be limited to matching metered readings just at the substation. The same process can be performed at any point on the feeder where metered data is available. The only difference is that now only the "downstream" nodes from the metered point will be modified.

10.1.7 Summary of Power-Flow Studies

This section has developed a method for performing power-flow studies on a distribution feeder. Models for the various components of the feeder have been developed in previous chapters. The purpose of this section has been to develop and demonstrate the modified ladder iterative technique using the forward and backward sweep matrices for the series elements. It should be obvious that a study of a large feeder with many laterals and sublaterals can not be performed without the aid of a complex computer program.

The development of the models and examples in this chapter uses actual values of voltage, current, impedance, and complex power. When per-unit values are used, it is imperative that all values be converted to per unit using a common set of base values. In the usual application of per unit, there will be a base line-to-line voltage and a base line-to-neutral voltage; also, there will be a base line current and a base delta current. For both the voltage and current, there is a square root of three relationship between the two base values. In all of the derivations of the models, and, in particular, those for the three-phase transformers, the square root of three has been used to relate the difference in magnitudes between line-to-line and line-to-neutral voltages and between the line and delta currents. Because of this, when using the per-unit system, there should be only one base voltage and that should be the base line-to-neutral voltage. When this is done, for example, the per-unit positive and negative sequence voltages will be the square root of three times the per-unit positive and negative sequence line-to-neutral voltages. Similarly, the positive and negative sequence per-unit line currents will be the square of three times the positive and negative sequence per-unit delta currents. By using just one base voltage and one base current, the per-unit generalized matrices for all system models can be determined.

10.2 Short-Circuit Studies

The computation of short-circuit currents for unbalanced faults in a normally balanced three-phase system has traditionally been accomplished by the application of symmetrical components. However, this method is not well suited to a distribution feeder that is inherently unbalanced. The unequal mutual coupling between phases leads to mutual coupling between sequence networks. When this happens, there is no advantage in using symmetrical components. Another reason for not using symmetrical components is that the phases between which faults occur is limited. For example, using symmetrical components, line-to-ground faults are limited to phase *a* to ground. What happens if a single-phase lateral is connected to phase *b* or *c* and the short-circuit current is needed? This section develops a method for short-circuit analysis of an unbalanced three-phase distribution feeder using the phase frame.

10.2.1 General Theory

Figure 10.12 shows the unbalanced feeder as modeled for short-circuit calculations.

Short circuits can occur at any one of the five points shown in Figure 10.12. Point 1 is the high-voltage bus of the distribution substation transformer. The values of the short-circuit currents at point 1 are normally determined from a transmission system short-circuit study. The results of these studies are supplied in terms of the three-phase and single-phase short-circuit MVAs (Megavolt amperes). Using the short-circuit MVAs, the positive and zero sequence impedances of the equivalent system can be determined. These values are needed for the short-circuit studies at the other four points in Figure 10.12.

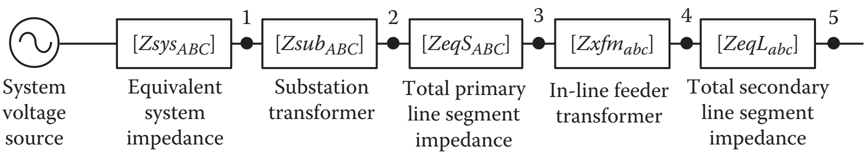

FIGURE 10.12
Unbalanced feeder short-circuit analysis model.

Given the three-phase short-circuit MVA magnitude and angle, the positive sequence equivalent system impedance in Ohms is determined by

$$Z_{+} = \frac{kVLL^2}{(MVA_{3\text{-}phase})^*}\,\Omega \tag{10.10}$$

Given the single-phase short-circuit MVA magnitude and angle, the zero sequence equivalent system impedance in Ohms is determined by

$$Z_0 = \frac{3 \cdot kVLL^2}{(MVA_{1\text{-}phase})^*} - 2.Z_{+}\,\Omega \tag{10.11}$$

In Equations 10.10 and 10.11, $kVLL$ is the nominal line-to-line voltage in kV of the transmission system.

The computed positive and zero sequence impedances need to be converted into the phase impedance matrix using the symmetrical component transformation matrix defined in Equation 4.63:

$$[Z_{012}] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_1 \end{bmatrix} \tag{10.12}$$

$$[Z_{abc}] = [A_s] \cdot [Z_{012}] \cdot [A_s]^{-1}$$

For short circuits at points 2 through 5, it is going to be necessary to compute the Thevenin equivalent three-phase circuit at the short-circuit point. The Thevenin equivalent voltages will be the nominal line-to-ground voltages with the appropriate angles. For example, assume the equivalent system line-to-ground voltages are balanced three phase of nominal voltage with the phase *a* voltage at 0°. The Thevenin equivalent voltages at points 2 and 3 will be computed by multiplying the system voltages by the generalized transformer matrix $[A_t]$ of the substation transformer. Carrying this further, the Thevenin equivalent voltages at points 4 and 5 will be the voltages at node 3 multiplied by the generalized matrix $[A_t]$ for the in-line transformer.

The Thevenin equivalent phase impedance matrices will be the sum of the phase impedance matrices of each device between the system voltage source and the point of fault. Step-voltage regulators are assumed to be set in the neutral position so they do not enter into the short-circuit calculations. Anytime that a three-phase transformer is encountered, the total phase impedance matrix on the primary side of the transformer must be referred to the secondary side using Equation 8.160.

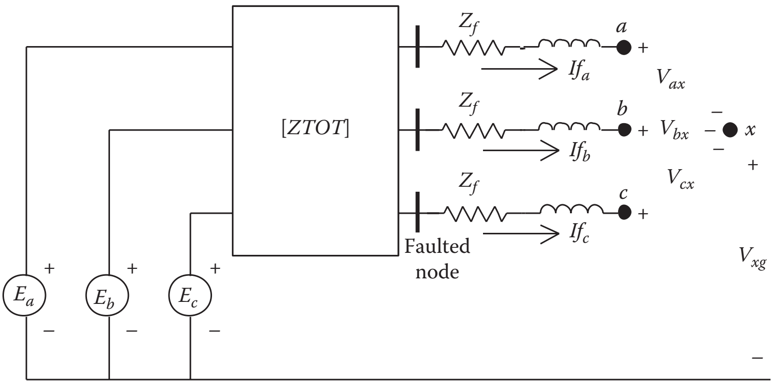

FIGURE 10.13
Thevenin equivalent circuit.

Figure 10.13 illustrates the Thevenin equivalent circuit at the faulted node [3].

In Figure 10.13, the voltage sources E_a, E_b, and E_c represent the Thevenin equivalent line-to-ground voltages at the faulted node; the matrix [*ZTOT*] represents the Thevenin equivalent phase impedance matrix at the faulted node; and Z_f represents the fault impedance.

KVL in matrix form can be applied to the circuit of Figure 10.13:

$$\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} If_a \\ If_b \\ If_c \end{bmatrix} + \begin{bmatrix} Z_f & 0 & 0 \\ 0 & Z_f & 0 \\ 0 & 0 & Z_f \end{bmatrix} \cdot \begin{bmatrix} If_a \\ If_b \\ If_c \end{bmatrix} + \begin{bmatrix} V_{ax} \\ V_{bx} \\ V_{cx} \end{bmatrix} + \begin{bmatrix} V_{xg} \\ V_{xg} \\ V_{xg} \end{bmatrix} \tag{10.13}$$

Equation 10.13 can be written in compressed form as

$$[E_{abc}] = [ZTOT] \cdot [If_{abc}] + [ZF] \cdot [If_{abc}] + [V_{abcx}] + [V_{xg}] \tag{10.14}$$

Combine terms in Equation 10.14:

$$[E_{abc}] = [ZEQ] \cdot [If_{abc}] + [V_{abcx}] + [V_{xg}] \tag{10.15}$$

where

$$[ZEQ] = [ZTOT] + [ZF] \tag{10.16}$$

Solve Equation 10.15 for the fault currents:

$$[If_{abc}] = [Y] \cdot [E_{abc}] - [Y] \cdot [V_{abcx}] - [Y] \cdot [V_{xg}] \tag{10.17}$$

where

$$[Y]=[ZEQ]^{-1} \tag{10.18}$$

Since the matrices $[Y]$ and $[E_{abc}]$ are known, define

$$[IP_{abc}]=[Y]\cdot[E_{abc}] \tag{10.19}$$

Substituting Equation 10.19 into Equation 10.17 and rearranging result in

$$[IP_{abc}]=[If_{abc}]+[Y]\cdot[V_{abcx}]+[Y]\cdot[V_{xg}] \tag{10.20}$$

Expanding Equation 10.20,

$$\begin{bmatrix} IP_a \\ IP_b \\ IP_c \end{bmatrix}=\begin{bmatrix} If_a \\ If_b \\ If_c \end{bmatrix}+\begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}\cdot\begin{bmatrix} V_{ax} \\ V_{bx} \\ V_{cx} \end{bmatrix}+\begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} \\ Y_{ba} & Y_{bb} & Y_{bc} \\ Y_{ca} & Y_{cb} & Y_{cc} \end{bmatrix}\cdot\begin{bmatrix} V_{xg} \\ V_{xg} \\ V_{xg} \end{bmatrix} \tag{10.21}$$

Performing the matrix operations in Equation 10.21,

$$\begin{aligned} IP_a &= If_a+\left(Y_{aa}\cdot V_{ax}+Y_{ab}\cdot V_{bx}+Y_{ac}\cdot V_{cx}\right)+Ys_a\cdot V_{xg} \\ IP_b &= If_b+\left(Y_{ba}\cdot V_{ax}+Y_{bb}\cdot V_{bx}+Y_{bc}\cdot V_{cx}\right)+Ys_b\cdot V_{xg} \\ IP_c &= If_c+\left(Y_{ca}\cdot V_{ax}+Y_{cb}\cdot V_{bx}+Y_{cc}\cdot V_{cx}\right)+Ys_c\cdot V_{xg} \end{aligned} \tag{10.22}$$

where

$$\begin{aligned} Ys_a &= Y_{aa}+Y_{ab}+Y_{ac} \\ Ys_b &= Y_{ba}+Y_{bb}+Y_{bc} \\ Ys_c &= Y_{ca}+Y_{cb}+Y_{cc} \end{aligned} \tag{10.23}$$

Equations 10.22 become the general equations that are used to simulate all types of short circuits. Basically, there are three equations and seven unknowns (If_a, If_b, If_c, V_{ax}, V_{bx}, V_{cx}, and V_{xg}). The other three variables in the equations (IP_a, IP_b, and IP_c) are functions of the total impedance and the Thevenin voltages and are therefore known. In order to solve Equations 10.22, it will be necessary to specify four additional independent equations. These equations are functions of the type of fault being simulated. The additional required four equations for various types of faults are given in the following. These values are determined by placing short circuits in Figure 10.13 to simulate the particular type of fault. For example, a three-phase fault is simulated by placing a short circuit from node a to x, node b to x, and node c to x. That gives three voltage equations. The fourth equation comes from applying KCL at node x, which gives the sum of the fault currents to be zero.

10.2.2 Specific Short Circuits

Three-phase faults:

$$\begin{aligned} &V_{ax} = V_{bx} = V_{cx} = 0 \\ &I_a + I_b + I_c = 0 \end{aligned} \tag{10.24}$$

Three-phase-to-ground faults:

$$V_{ax} = V_{bx} = V_{cx} = V_{xg} = 0 \tag{10.25}$$

Line-to-line faults (assume i–j fault with phase k unfaulted):

$$\begin{aligned} &V_{ix} = V_{jx} = 0 \\ &If_k = 0 \\ &If_i + If_j = 0 \end{aligned} \tag{10.26}$$

Line-to-line-to-ground faults (assume i–j–g fault with phase k unfaulted):

$$\begin{aligned} &V_{ix} = V_{jx} = 0 \\ &V_{xg} = 0 \\ &I_k = 0 \end{aligned} \tag{10.27}$$

Line-to-ground faults (assume phase k fault with phases i and j unfaulted):

$$\begin{aligned} &V_{kx} = V_{xg} = 0 \\ &If_i = If_j = 0 \end{aligned} \tag{10.28}$$

Note that Equations 10.26 through 10.28 will allow the simulation of line-to-line faults, line-to-line-to-ground, and line-to-ground faults for all phases. There is no limitation to b–c faults for line to line and a–g for line to ground as is the case when the method of symmetrical components is employed.

A good way to solve the seven equations is to set them up in matrix form:

$$\begin{bmatrix} IP_a \\ IP_b \\ IP_c \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Y_{1,1} & Y_{1,2} & Y_{1,3} & Ys_1 \\ 0 & 1 & 0 & Y_{2,1} & Y_{2,2} & Y_{2,3} & Ys_2 \\ 0 & 0 & 1 & Y_{3,1} & Y_{3,2} & Y_{3,3} & Ys_3 \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & - & - & - & - \end{bmatrix} \cdot \begin{bmatrix} If_a \\ If_b \\ If_c \\ V_{ax} \\ V_{bx} \\ V_{cx} \\ V_{xg} \end{bmatrix} \tag{10.29}$$

Equation 10.29 in condensed form:

$$[IP_s] = [C] \cdot [X] \tag{10.30}$$

Equation 10.30 can be solved for the unknowns in matrix $[X]$:

$$[X] = [C]^{-1} \cdot [IP_s] \tag{10.31}$$

The blanks in the last four rows of the coefficient matrix in Equation 10.29 are filled in with the known variables depending upon what type of fault is to be simulated. For example, the elements in the [C] matrix simulating a three-phase fault would be

$$C_{4,4} = C_{5,5} = C_{6,6} = 1$$

$$C_{7,1} = C_{7,2} = C_{7,3} = 1$$

All of the other elements in the last four rows will be set to zero.

Example 10.3

Use the system of Example 10.2 and compute the short-circuit currents for a bolted ($Z_f = 0$) line-to-line fault between phases *a* and *b* at node 4.

The infinite bus balanced line-to-line voltages are 12.47 kV, which leads to balanced line-to-neutral voltages at 7.2 kV:

$$[ELL_s] = \begin{bmatrix} 12{,}470\underline{/30} \\ 12{,}470\underline{/-90} \\ 12{,}470\underline{/150} \end{bmatrix} \text{V}$$

$$[ELN_s] = \begin{bmatrix} 7200\underline{/0} \\ 7200\underline{/-120} \\ 7200\underline{/120} \end{bmatrix} \text{V}$$

The line-to-neutral Thevenin circuit voltages at node 4 are determined using Equation 8.159:

$$[Eth_4] = [A_t]\cdot[ELN_s] = \begin{bmatrix} 2400\underline{/-30} \\ 2400\underline{/-150} \\ 2400\underline{/150} \end{bmatrix} \text{V}$$

The Thevenin equivalent impedance at the secondary terminals (node 3) of the transformer consists of the primary line impedances referred across the transformer plus the transformer impedances. Using Equation 8.160,

$$[Zth_3] = [A_t]\cdot[ZeqS_{ABC}]\cdot[d_t] + [Zt_{abc}]$$

$$= \begin{bmatrix} 0.0366 + j0.1921 & -0.0039 - j0.0086 & -0.0039 - j0.0106 \\ -0.0039 - j0.0086 & 0.0366 + j0.1886 & -0.0039 - j0.0071 \\ -0.0039 - j0.0106 & -0.0039 - j0.0071 & 0.0366 + j0.1906 \end{bmatrix} \Omega$$

Note that the Thevenin impedance matrix is not symmetrical. This is a result, once again, of the unequal mutual coupling between the phases of the primary line segment.

The total Thevenin impedance at node 4 is

$$[Zth_4] = [ZTOT] = [Zth_3] + [ZeqL_{abc}]$$

$$[ZTOT] = \begin{bmatrix} 0.2273 + j0.6955 & 0.0568 + j0.2216 & 0.0559 + j0.1645 \\ 0.0568 + j0.2216 & 0.2305 + j0.6771 & 0.0575 + j0.1860 \\ 0.0559 + j0.1645 & 0.0575 + j0.1860 & 0.2287 + j0.6876 \end{bmatrix} \Omega$$

The equivalent admittance matrix at node 4 is

$$[Yeq_4] = [ZTOT]^{-1}$$

$$= \begin{bmatrix} 0.5031 - j1.4771 & -0.1763 + j0.3907 & -0.0688 + j0.2510 \\ -0.1763 + j0.3907 & 0.5501 - j1.5280 & -0.1148 + j0.3133 \\ -0.0688 + j0.2510 & -0.1148 + j0.3133 & 0.4843 - j1.4532 \end{bmatrix} \text{S}$$

Using Equation 10.18, the equivalent injected currents at the point of fault are

$$[IP] = [Yeq_4] \cdot [Eth_4] = \begin{bmatrix} 4466.8/\underline{-96.3} \\ 4878.9/\underline{138.0} \\ 4440.9/\underline{16.4} \end{bmatrix} \text{A}$$

The sums of each row of the equivalent admittance matrix are computed according to Equation 10.22:

$$Y_i = \sum_{k=1}^{3} Yeq_{i,k} = \begin{bmatrix} 0.2580 - j0.8354 \\ 0.2590 - j0.8240 \\ 0.3008 - j0.8889 \end{bmatrix} \text{S}$$

For the *ab* fault at node 4, according to Equation 10.25,

$$V_{ax} = 0$$

$$V_{bx} = 0$$

$$If_c = 0$$

$$If_a + If_b = 0$$

The coefficient matrix [*C*] using Equation 10.27 is

$$[C] = \begin{bmatrix} 1 & 0 & 0 & 0.5031 - j1.4771 & -0.1763 + j0.3907 & -0.0688 + 0.2510 & 0.2580 - j0.8354 \\ 0 & 1 & 0 & -0.1763 + j0.3907 & 0.5501 - j1.5280 & -0.1148 + j0.3133 & 0.2590 - j0.8240 \\ 0 & 0 & 1 & -0.0688 + j0.2510 & -0.1148 + j0.3133 & 0.4843 - j1.4532 & 0.3008 - j0.8890 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The injected current matrix is

$$[IP_s] = \begin{bmatrix} 4466.8/\underline{-96.3} \\ 4878.9/\underline{138.0} \\ 4440.9/\underline{16.4} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The unknowns are computed by

$$[X]=[C]^{-1}\cdot[IP_s]=\begin{bmatrix}4193.7\underline{/-69.7}\\4193.7\underline{/110.3}\\0\\0\\0\\3646.7\underline{/88.1}\\1220.2\underline{/-91.6}\end{bmatrix}$$

The interpretation of the results are

$$If_a = X_1 = 4193.7\underline{/-69.7}$$

$$If_b = X_2 = 4193.7\underline{/110.3}$$

$$If_c = X_3 = 0$$

$$V_{ax} = X_4 = 0$$

$$V_{bx} = X_5 = 0$$

$$V_{cx} = X_6 = 3646.7\underline{/88.1}$$

$$V_{xg} = X_7 = 1220.2\underline{/-91.6}$$

Using the line-to-ground voltages at node 4 and the short-circuit currents and working back to the source using the generalized matrices will check the validity of these results.

The line-to-ground voltages at node 4 are

$$[VLG_4]=\begin{bmatrix}V_{ax}+V_{xg}\\V_{bx}+V_{xg}\\V_{cx}+V_{xg}\end{bmatrix}=\begin{bmatrix}1220.2\underline{/-91.6}\\1220.2\underline{/-91.6}\\2426.5\underline{/88.0}\end{bmatrix}\text{V}$$

The short-circuit currents in matrix form:

$$[I_4]=[I_3]=\begin{bmatrix}4193.7\underline{/-69.7}\\4193.7\underline{/110.3}\\0\end{bmatrix}\text{A}$$

The line-to-ground voltages at node 3 are

$$[VLG_3]=[a_2]\cdot[VLG_4]+[b_1]\cdot[I_4]=\begin{bmatrix}1814.0\underline{/-47.3}\\1642.1\underline{/-139.2}\\2405.1\underline{/89.7}\end{bmatrix}\text{V}$$

The equivalent line-to-neutral voltages and line currents at the primary terminals (node 2) of the transformer are

$$[VLN_2] = [a_t]\cdot[VLG_3] + [b_t]\cdot[I_3] = \begin{bmatrix} 6784.3/\underline{0.2} \\ 7138.8/\underline{-118.7} \\ 7080.6/\underline{118.3} \end{bmatrix} \text{V}$$

$$[I_2] = [d_t]\cdot[I_3] = \begin{bmatrix} 1614.3/\underline{-69.7} \\ 807.1/\underline{110.3} \\ 807.1/\underline{110.3} \end{bmatrix} \text{A}$$

Finally, the equivalent line-to-neutral voltages at the infinite bus can be computed:

$$[VLN_1] = [a_1]\cdot[VLN_2] + [b_1]\cdot[I_2] = \begin{bmatrix} 7201.2/\underline{0} \\ 7198.2/\underline{-120} \\ 7199.3/\underline{120} \end{bmatrix} \text{V}$$

The source line-to-line voltages are

$$[VLL_1] = [D]\cdot[VLN_1] = \begin{bmatrix} 12470/\underline{30} \\ 12470/\underline{-90} \\ 12470/\underline{150} \end{bmatrix}$$

These are the same line-to-line voltages that were used to start the short-circuit analysis.

10.3 Summary

This chapter has demonstrated the application of the element models that are used in the power-flow analysis and short-circuit analysis of a distribution feeder. The modified ladder iterative technique was used for the power-flow analysis. For a simple radial feeder with no laterals, the examples demonstrated that only the forward and backward sweeps were changed by adding the sweep equations for the new elements. A feeder with laterals and sublaterals will require a forward and backward sweep for each lateral and sublateral. Symmetrical component analysis of short-circuit currents on a feeder will not work. Rather a method in the phase domain for the computation of any type of short circuit was developed and demonstrated.

The examples in this chapter have been very long and should be used as a learning tool. Many of the interesting operating characteristics of a feeder can only be demonstrated through numerical examples. The examples were designed to illustrate some of these characteristics.

Armed with a computer program using the models and techniques of this chapter provides the engineer with a powerful tool for solving present-day problems and long-range planning studies.

Problems

The power-flow problems in this set require the application of the modified ladder technique. Students should be encouraged to write their own computer program to solve the problems.

The first six problems of this set will be based upon the system in Figure 10.14.

The substation transformer is connected to an infinite bus with balanced three-phase voltages of 69 kV. The substation transformer is rated:

$$5000\text{ kVA},\quad 69\text{ kV delta} - 4.16\text{ grounded wye},\quad Z = 1.5 + j8.0\%$$

The phase impedance matrix for a four-wire wye line is

$$[z_{4\text{-}wire}] = \begin{bmatrix} 0.4576 + j1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix} \Omega/\text{mile}$$

The secondary voltages of the infinite bus are balanced and being held at 69 kV for all power-flow problems.

The four-wire wye feeder is 0.75 miles long. An unbalanced wye-connected load is located at node 3 and has the following values:

Phase *a*: 750 kVA at 0.85 lagging power factor
Phase *b*: 500 kVA at 0.90 lagging power factor
Phase *c*: 850 kVA at 0.95 lagging power factor

The load at node 4 is zero initially.

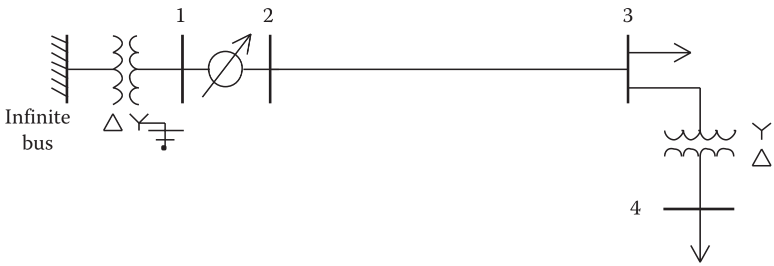

FIGURE 10.14
Wye homework system.

10.1 For the system as described earlier and assuming that the regulators are in the neutral position:

a. Determine the forward and backward sweep matrices for the substation transformer and the line segment.

b. Use the modified ladder technique to determine the line-to-ground voltages at node 3. Use a tolerance of 0.0001 per unit. Give the voltages in actual values in volts and on a 120 V base.

10.2 Three type B step-voltage regulators are installed in a wye connection at the substation in order to hold the load voltages (node 3) at a voltage level of 121 V and a bandwidth of 2 V.

a. Compute the actual equivalent line impedance between nodes 2 and 3.

b. Use a potential transformer ratio of 2400-120 V and a current transformer ratio of 500 : 5 A. Determine the R and X compensator settings calibrated in volts and Ohms. The settings must be the same for all three regulators.

c. For the load conditions of Problem 10.1 and with the regulators in the neutral position, compute the voltages across the voltage relays in the compensator circuits.

d. Determine the appropriate tap settings for the three regulators to hold the node 3 voltages at 121 V in a bandwidth of 2 V.

e. With the regulators taps set, compute the actual load voltages on a 120 V base.

10.3 A wye-connected three-phase shunt capacitor bank of 300 kvar per phase is installed at node 3. With the regulator compensator settings from Problem 10.2, determine

a. The new tap settings for the three regulators

b. The voltages at the load on a 120 V base

c. The voltages across the relays in the compensator circuits

10.4 The load at node 4 is served through an ungrounded wye–delta transformer bank. The load is connected in delta with the following values:

Phase *ab*: 100 kVA at 0.9 factor power factor
Phase *bc*: 35 kVA at 0.8 lagging power factor
Phase *ca*: 35 kVA at 0.8 lagging power factor

The three single-phase transformers are rated as

"Lighting transformer": 167 kVA, 2400-240, $Z = .01 + j.02$ per unit
"Power transformers": 50 kVA, $Z = 0.011 + j0.018$ per unit

Use the original loads and the shunt capacitor bank at node 3 and this new load at node 4. Determine

a. The voltages on 120 V base at node 3 assuming the regulators are in the neutral position

b. The voltages on 120 V base at node 4 assuming the regulators are in the neutral position
c. The new tap settings for the three regulators
d. The node 3 and node 4 voltages on 120 V base after the regulators have changed tap positions

10.5 Under short-circuit conditions, the infinite bus voltage is the only voltage that is constant. The voltage regulators in the substation are in the neutral position. Determine the short-circuit currents and voltages at nodes 1 through 3 for the following short circuits at node 3:

a. Three-phase to ground
b. Phase *b* to ground
c. Line-to-line fault between phases *a* and *c*

10.6 A line-to-line fault between phases *a* and *b* occurs at node 4. Determine the currents in the fault and on the line segment between nodes 2 and 3 and the current into the substation transformer. Determine the voltages at nodes 1 through 4.

10.7 A three-wire delta line of length 0.75 miles is serving an unbalanced delta load of

Phase *ab*: 600 kVA, 0.9 lagging power factor
Phase *bc*: 800 kVA, 0.8 lagging power factor
Phase *ca*: 500 kVA, 0.95 lagging power factor

The phase impedance matrix for the line is

$$[z_{3\text{-}wire}] = \begin{bmatrix} 0.4013+j1.4133 & 0.0953+j0.8515 & 0.0953+j0.7802 \\ 0.0953+j0.8515 & 0.4013+j1.4133 & 0.0953+j0.7266 \\ 0.0953+j0.7802 & 0.0953+j0.7266 & 0.4013+j1.4133 \end{bmatrix} \Omega/\text{mile}$$

The line is connected to a constant balanced voltage source of 4.8 kV line to line. Determine the load voltages on a 120 V base.

10.8 Add two type B step-voltage regulator in an open delta connection using phases *AB* and *BC* to the system in Problem 10.7. The regulator should be set to hold 121 ± 1 V. Determine the *R* and *X* settings and the final tap settings. For the open delta connection, the *R* and *X* settings will be different on the two regulators.

10.9 The three-wire line of Problem 10.7 is connected to a substation transformer connected delta–delta. The substation transformer is connected to a 69 kV infinite bus and is rated:

$$10{,}000 \text{ kVA}, \quad 69 \text{ kV delta} - 4.8 \text{ kV delta}, \quad Z = 1.6 + j7.8\%$$

Determine the short-circuit currents and substation transformer secondary voltages for the following short circuits at the end of the line:

a. Three phase
b. Line to line between phases *a* and *b*

Windmil Assignment

Figure 10.15 shows the one-line diagram of an unbalanced three-phase feeder. The non-line data for the feeder is

1. Equivalent source
   a. Balanced 115 kV line to line
   b. $Zpos = 1.48 + j11.6\,\Omega$
   c. $Zzero = 4.73 + j21.1\,\Omega$
   d. Bus voltage = 120 V
2. Substation transformer
   a. 115 kV D–12.47 kV grd. Y
   b. kVA = 10,000
   c. $Z = 8.026\%$, $X/R = 8$
3. Regulator
   a. CT rating = 600
   b. %Boost = 10

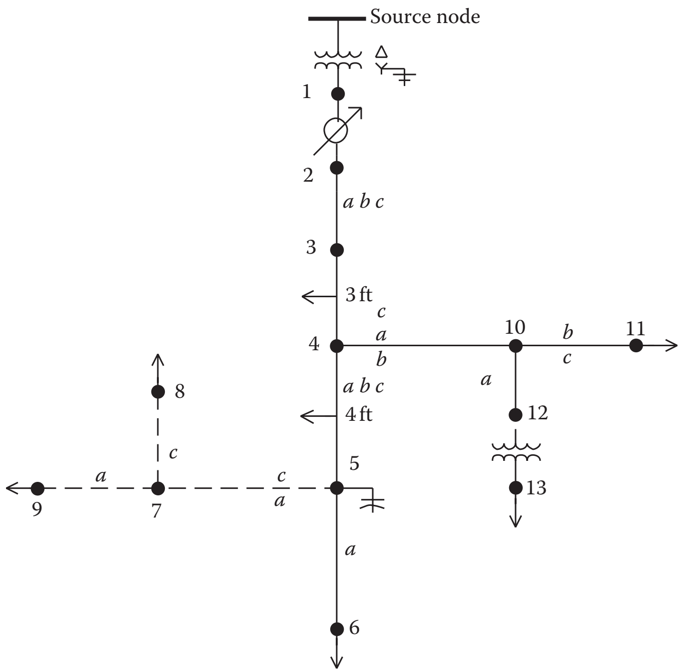

FIGURE 10.15
Unbalanced three-phase feeder.

   c. Step size = 0.625
   d. Number of steps = 16
   e. Nodes: 1–2
   f. Specify the voltage level
   g. $R + jX = ?$
4. Single-phase transformer
   a. Connection: Y–D one
   b. kVA = 100
   c. Voltages: 7200 Y–240 D V
   d. $Z = 2.326\%$, $X/R = 2.1$
   e. Nodes: 12–13

The line data is

5. Three-phase OH lines
   a. Phase: 336,400 26/7 ACSR
   b. Neutral: 4/0 6/1 ACSR
   c. Phasing: *a-b-c*
   d. Spacings:
      i. Position 1: $0 + j29$
      ii. Position 2: $2.5 + j29$
      iii. Position 3: $7 + j29$
      iv. Neutral: $4 + j25$
   e. OH 1: Nodes 2–3, 2500 ft
   f. OH 2: Nodes 3–4, 3000 ft
   g. OH 3: Nodes 4–5, 2500 ft
   h. OH 4: Nodes 5–6, 1000 ft
6. Two-phase OH line
   a. Phase: 336,400 26/7 ACSR
   b. Neutral: 4/0 6/1 ACSR
   c. Phasing: *ac*
   d. Spacings:
      i. Position 1: $0 + j29$
      ii. Position 2: $7 + j29$
      iii. Neutral: $4 + j25$
   e. OH 5: Nodes 5–7, 1500 ft
7. Three-phase concentric neutral UG
   a. *CN* cable: 1/0 AA, 1/3 neutral
   b. No extra neutral

   c. Phasing: *cba*
   d. Spacings:
      i. Position 1: $0 - j40$ in.
      ii. Position 2: $6 - j40$ in.
      iii. Position 3: $12 - j40$ in.
   e. UG 1: Nodes 4–10, 1500 ft
8. Two-phase concentric neutral UG
   a. *CN* cable: 1/0 AA, 1/3 neutral
   b. No extra neutral
   c. Phasing: *cb*
   d. Spacings:
      i. Position 1: $0 - j40$ in.
      ii. Position 2: $6 - j40$ in.
   e. UG 2: Nodes 10–11, 1000 ft
9. Single-phase concentric neutral UG
   a. *CN* cable: 1/0 AA, Full neutral
   b. No extra neutral
   c. Phase: *c*
   d. Spacings:
      i. Position 1: $0 - j40$ in.
   e. UG 3: Nodes 10–12, 500 ft
10. Single-phase tape shield cable
   a. 1/0 AA tape shield UG
   b. Neutral: 1/0 7 strand AA
   c. Phase: *c*
   d. Spacings:
      i. Position 1: $0 - j40$ in.
      ii. Neutral: $6 - j40$ in.
   e. UG 4: Nodes 7–8, 500 ft
11. Two-phase tape shield cable
   a. 1/0 AA tape shield UG
   b. Neutral: 1/0 7 strand AA
   c. Phase: *c*
   d. Spacings:
      i. Position 1: $0 - j40$ in.
      ii. Position 2: $6 - j40$ in.
      iii. Neutral: $12 - j40$ in.
   e. UG 5: Nodes 7–9, 750 ft, phases *a*–*c*

The load data is

Distributed loads:

Node A	Node B	*Pa*	*PFa* (%)	*Pb*	*PFb* (%)	*Pc*	*PFc* (%)	Model
2	3	100	90	150	90	200	90	*Y–PQ*
3	4	200	90	100	90	150	90	*Y–Z*

Wye-connected spot loads:

Node	*Pa*	*PFa* (%)	*Pb*	*PFb* (%)	*Pc*	*PFc* (%)	Model
3	500	80	300	80	400	80	*Y–Z*
4	500	80	500	80	500	90	*Y–I*
6	1000	80	800	90	950	95	*Y–PQ*
8					200	95	*Y–Z*

Delta-connected loads:

Node	*Pab*	*PFab* (%)	*Pbc*	*PFbc* (%)	*Pca*	*PFca* (%)	Model
9					350	90	*D–I*
11			350	90	100	95	*D–PQ*
13							*D–I*

1. Create this system in Windmil.
2. Run voltage drop with the regulator set to "none." Do this in the Voltage Drop Analysis Manager.
3. Compute the *R* and *X* and voltage level for the voltage regulator.
   a. Hand calculation
   b. Windmil "Regulation Set"
   c. Compare settings
4. Run voltage drop with the regulators set to "step."
   a. What are the final tap positions?
   b. Are these OK?
5. Add shunt capacitors so that the source power factor is no lower than 95% lag. Specify capacitors in multiples of 100 kvar.
6. Run voltage with the final capacitors.
   a. What is the power factor by phase at the source?
   b. What are the final tap positions for the regulators?

References

1. Trevino, C., Cases of difficult convergence in load-flow problems, IEEE Paper no. 71-62-PWR, Presented at the *IEEE Summer Power Meeting*, Los Angeles, CA, 1970.
2. Kersting, W.H. and Mendive, D.L., An application of ladder network theory to the solution of three-phase radial load-flow problems, IEEE Conference Paper, Presented at the *IEEE Winter Power Meeting*, New York, January 1976.
3. Kersting, W.H. and Phillips, W.H., Distribution system short-circuit analysis, *25th Intersociety Energy Conversion Engineering Conference*, Reno, NV, August 12–17, 1990.

11

Center-Tapped Transformers and Secondaries

The standard method of providing three-wire service to a customer is from a center-tapped single-phase transformer. This type of service provides the customer with two 120 V circuits and one 240 V circuit. Two types of transformers are available for providing this service. The first is where the secondary consists of one winding that is center tapped as shown in Figure 11.1.

The secondary voltage rating of the transformer in Figure 11.1 would be specified as 240/120 V. This specifies that the full winding voltage rating is 240 V with the center tap providing two 120 V circuits.

A second type of transformer used to provide three-wire service is shown in Figure 11.2.

The transformer in Figure 11.2 is a three-winding transformer with the two secondary windings connected in series. The secondary on this transformer is specified as 120/240 V. The secondary windings can be connected in series to provide the three-wire 240 and 120 V service or they may be connected in parallel to provide only 120 V. When connected in parallel, the transformer will typically be used in a three-phase connection. The secondary will be connected in wye and will provide three 120 V circuits.

11.1 Center-Tapped Single-Phase Transformer Model

The model of the center-tapped transformer of Figures 11.1 and 11.2 is shown in Figure 11.3.

The model in Figure 11.3 can represent either the center-tapped secondary winding (Figure 11.1) or the two secondary windings connected in series (Figure 11.2). The impedances Z_0, Z_1, and Z_2 represent the individual winding impedances.

The first step in developing the model is to determine the impedances Z_0, Z_1, and Z_2. These impedances can be determined with open circuit and short circuit tests on the transformer. However, that usually is not practical. What is typically known on a transformer will be the per-unit impedance based upon the transformer rating. Unfortunately, that usually does not include the angle. When that is the case, an approximation must be made for the angle or a typical impedance value can be used. Typical values of transformer impedances in per unit can be found in the book *Electric Power Distribution System*

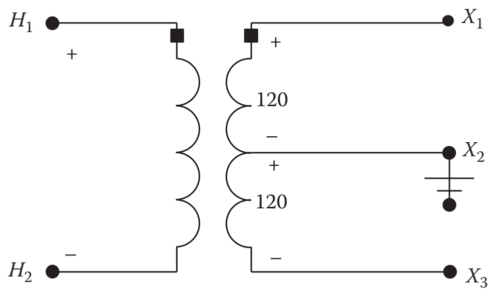

FIGURE 11.1
Center-tapped secondary winding.

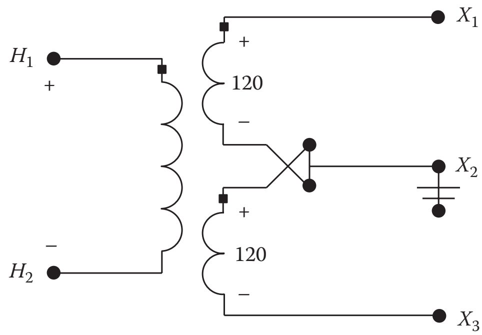

FIGURE 11.2
Three-winding transformer with secondary windings in series.

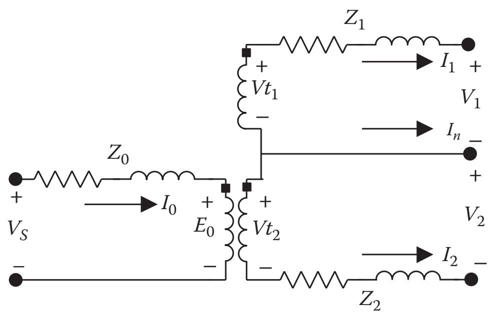

FIGURE 11.3
Center-tapped transformer model.

Engineering by Turan Gonen [1]. Empirical equations commonly used to convert the transformer impedance to the winding impedances of an interlaced design are given in Equation 11.1:

$$
\begin{aligned}
Z_0 &= 0.5 \cdot R_A + j0.8 \cdot X_A \\
Z_1 &= R_A + j0.4 \cdot X_A \\
Z_2 &= R_A + j0.4 \cdot X_A
\end{aligned}
\tag{11.1}
$$

The equations for the noninterlaced design are

$$
\begin{aligned}
Z_0 &= 0.25 \cdot R_A - j0.6 \cdot X_A \\
Z_1 &= 1.5 \cdot R_A + j3.3 \cdot X_A \\
Z_2 &= 1.5 \cdot R_A + j3.1 \cdot X_A
\end{aligned}
\tag{11.2}
$$

The interlaced design is the most common and should be used when in doubt.

11.1.1 Matrix Equations

Referring to Figure 11.3, the ideal secondary voltages of the transformer are

$$
\begin{aligned}
\begin{bmatrix} Vt_1 \\ Vt_2 \end{bmatrix} &= \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} Z_1 & 0 \\ 0 & -Z_2 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \\
[Vt_{12}] &= [V_{12}] + [Z_{12}] \cdot [I_{12}]
\end{aligned}
\tag{11.3}
$$

The ideal primary voltage as a function of the secondary ideal voltages is

$$
\begin{aligned}
\begin{bmatrix} E_0 \\ E_0 \end{bmatrix} &= \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Vt_1 \\ Vt_2 \end{bmatrix} \\
[E_{00}] &= [av] \cdot [Vt_{12}]
\end{aligned}
\tag{11.4}
$$

where

$$
n_t = \frac{\textit{High-side rated voltage}}{\textit{Low-side half winding rated voltage}}
\tag{11.5}
$$

The primary transformer current as a function of the secondary winding currents is given in Equation 11.6. The negative sign is due to the selected direction of the current I_2:

$$
\begin{aligned}
I_0 &= \frac{1}{n_t} \cdot (I_1 - I_2) \\
\begin{bmatrix} I_0 \\ I_0 \end{bmatrix} &= \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \\
[I_{00}] &= [ai] \cdot [I_{12}]
\end{aligned}
\tag{11.6}
$$

Substitute Equations 11.3 into Equations 11.4:

$$\begin{aligned}[E_{00}] &= [av]\cdot\left([V_{12}]+[Z_{12}]\cdot[I_{12}]\right)\\ [E_{00}] &= [av]\cdot[V_{12}]+[av]\cdot[Z_{12}]\cdot[I_{12}]\end{aligned} \tag{11.7}$$

The source voltage as a function of the ideal primary voltage is

$$\begin{aligned}\begin{bmatrix}V_s\\V_s\end{bmatrix} &= \begin{bmatrix}E_0\\E_0\end{bmatrix}+\begin{bmatrix}Z_0 & 0\\0 & Z_0\end{bmatrix}\cdot\begin{bmatrix}I_0\\I_0\end{bmatrix}\\ [V_{ss}] &= [E_{00}]+[Z_{00}]\cdot[I_{00}]\end{aligned} \tag{11.8}$$

Substitute Equation 11.7 into Equation 11.8:

$$[V_{ss}] = [av]\cdot[V_{12}]+[av]\cdot[Z_{12}]\cdot[I_{12}]+[Z_{00}]\cdot[I_{00}] \tag{11.9}$$

Substitute Equation 11.6 into Equation 11.9:

$$\begin{aligned}[V_{ss}] &= [av]\cdot[V_{12}]+[av]\cdot[Z_{12}]\cdot[I_{12}]+[Z_{00}]\cdot[ai]\cdot[I_{12}]\\ [V_{ss}] &= [av]\cdot[V_{12}]+\left([av]\cdot[Z_{12}]+[Z_{00}]\cdot[ai]\right)\cdot[I_{12}]\end{aligned} \tag{11.10}$$

Equation 11.10 is the backward sweep voltage equation for the single-phase center-tapped transformer:

$$[V_{ss}] = [a_t]\cdot[V_{12}]+[b_t]\cdot[I_{12}] \tag{11.11}$$

where

$$[a_t] = [av] = n_t\cdot\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \tag{11.12}$$

$$\begin{aligned}[b_t] &= [av]\cdot[Z_{12}]+[Z_{00}]\cdot[ai]\\ &= \begin{bmatrix} n_t\cdot Z_1+\dfrac{1}{n_t^2}\cdot Z_0 & -\dfrac{1}{n_t^2}\cdot Z_0\\ \dfrac{1}{n_t^2}\cdot Z_0 & -\left(n_t\cdot Z_2+\dfrac{1}{n_t^2}\cdot Z_0\right)\end{bmatrix}\end{aligned} \tag{11.13}$$

Equation 11.6 is the backward current equation:

$$\begin{aligned}
[I_{00}] &= [c_t]\cdot[V_{12}] + [d_t]\cdot[I_{12}] \\
\text{where } [c_t] &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
[d_t] &= \frac{1}{n_t}\cdot\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}
\end{aligned} \tag{11.14}$$

Equation 11.11 is used to compute the primary source voltage when the secondary terminal voltages and the secondary currents are known. It is also important to be able to compute the secondary terminal voltages when the primary source voltage and secondary currents are known (forward sweep). The forward sweep equation is derived in Equation 11.11:

$$\begin{aligned}
[V_{12}] &= [a_t]^{-1}\cdot([V_{ss}] - [b_t]\cdot[I_{12}]) \\
[V_{12}] &= [a_t]^{-1}\cdot[V_{ss}] - [a_t]^{-1}\cdot[b_t]\cdot[I_{12}] \\
[V_{12}] &= [A_t]\cdot[V_{ss}] - [B_t]\cdot[I_{12}]
\end{aligned} \tag{11.15}$$

where

$$\begin{aligned}
[A_t] &= [a_t]^{-1} = \frac{1}{n_t}\cdot\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
[B_t] &= [a_t]^{-1}\cdot[b_t] = \begin{bmatrix} \left(Z_1 + \frac{1}{n_t^2}\cdot Z_0\right) & -\frac{1}{n_t^2}\cdot Z_0 \\ \frac{1}{n_t^2}\cdot Z_0 & -\left(Z_2 + \frac{1}{n_t^2}\cdot Z_0\right) \end{bmatrix}
\end{aligned} \tag{11.16}$$

Example 11.1

A 50 kVA center-tapped transformer serves constant impedance loads as shown in Figure 11.4.

Transformer rating: 50 kVA, 7200–240/120 V, $R_A = 0.011$ per unit, $X_A = 0.018$ per unit

Loads:

S_1 = 10 kVA at 95% lagging power factor
S_2 = 15 kVA at 90% lagging power factor
S_3 = 25 kVA at 85% lagging power factor

Source voltage: $7200\underline{/0}$ V

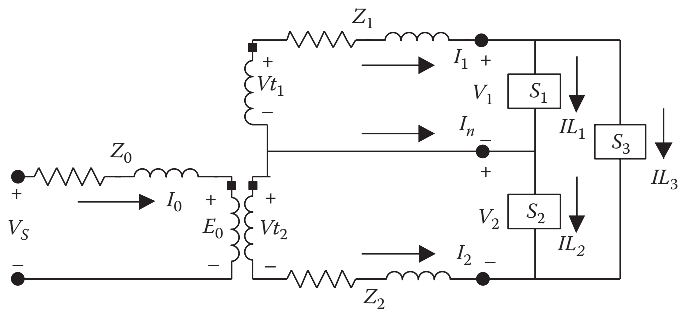

FIGURE 11.4
Center-tapped transformer serving constant impedance loads.

Determine

1. $[A_t]$, $[B_t]$, and $[d_t]$ matrices
2. Load voltages, secondary currents, and load currents
3. Primary current

Compute the per-unit winding impedances:

$$Zpu_0 = 0.5 \cdot R_A + j0.8 \cdot X_A = 0.0055 + j0.0144 \text{ per unit}$$

$$Zpu_1 = R_A + j0.4 \cdot X_A = 0.011 + j0.0072 \text{ per unit}$$

$$Zpu_2 = Zpu_1 = 0.011 + j0.0072 \text{ per unit}$$

Convert per-unit impedances to actual in Ohms:

$$Zbase_{hi} = \frac{7.2^2 \cdot 1000}{50} = 1036.8$$

$$Zbase_{lo} = \frac{0.24^2 \cdot 1000}{50} = 0.288$$

$$Z_0 = Zpu_0 \cdot Zbase_{hi} = 5.7024 + j14.9299\,\Omega$$

$$Z_1 = Z_2 = Zpu_1 \cdot Zbase_{lo} = 0.0032 + j0.0024\,\Omega$$

Compute the turns ratio: $n_t = 7200/120 = 60$

Compute the matrices:

$$[A_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.0167 & 0 \\ 0 & 0.0167 \end{bmatrix}$$

$$[B_t] = \begin{bmatrix} Z_1 + \frac{Z_0}{n_t^2} & -\frac{Z_0}{n_t^2} \\ \frac{Z_0}{n_t^2} & -Z_2 + \frac{Z_0}{n_t^2} \end{bmatrix} = \begin{bmatrix} 0.0048 + j0.0062 & -0.0016 - j0.0041 \\ 0.0016 + j0.0041 & -0.0048 - j0.0062 \end{bmatrix}$$

$$[d_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0.0167 & -0.0167 \\ 0.0167 & -0.0167 \end{bmatrix}$$

The first forward sweep is computed by setting the secondary line current to zero:

$$[V_{12}] = [A_t] \cdot [V_{SS}] - [B_t] \cdot [I_{12}] = \begin{bmatrix} 120/\underline{0} \\ 120/\underline{0} \end{bmatrix}$$

$$\text{where } [V_{SS}] = \begin{bmatrix} 7200/\underline{0} \\ 7200/\underline{0} \end{bmatrix}$$

$$[I_{12}] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The three-load voltages are

$$[V_{ld}] = \begin{bmatrix} V_1 \\ V_2 \\ V_1 + V_2 \end{bmatrix} = \begin{bmatrix} 120/\underline{0} \\ 120/\underline{0} \\ 240/\underline{0} \end{bmatrix}$$

The load currents are

$$i = 1\text{–}3$$

$$Id_i = \left(\frac{SL_i \cdot 1000}{Vld_i} \right)^* = \begin{bmatrix} 83.3/\underline{-18.2} \\ 125.0/\underline{-25.8} \\ 104.2/\underline{-31.8} \end{bmatrix}$$

The secondary line currents are given by

$$[I_{12}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} IL_1 \\ IL_2 \\ IL_3 \end{bmatrix} = \begin{bmatrix} 186.2/\underline{-25.8} \\ 228.9/\underline{151.5} \end{bmatrix}$$

The current in the neutral is

$$I_n = -(I_{12_1} + I_{12_2}) = 43.8\underline{/-41.5}$$

The backward sweep computes the primary current:

$$\begin{bmatrix} I_0 \\ I_0 \end{bmatrix} = [d_t]\cdot[I_{12}] = \begin{bmatrix} 6.9156\underline{/-27.3} \\ 6.9156\underline{/-27.3} \end{bmatrix}$$

The second forward sweep:

$$[V_{12}] = [A_t]\cdot[V_{SS}] - [B_t]\cdot[I_{12}] = \begin{bmatrix} 117.9\underline{/-0.64} \\ 117.8\underline{/-0.63} \end{bmatrix}$$

The aforementioned calculations demonstrate the first and second forward sweeps and the first backward sweep. This process can continue, but it is much easier to write a Mathcad® program to compute the final load voltages. The program is shown in Figure 11.5.

$$
\begin{array}{l}
X := \left|\begin{array}{l}
I_{12} \leftarrow \text{start} \\
V_{old} \leftarrow \text{start} \\
\text{for } n \in 1..20 \\
\quad \left|\begin{array}{l}
V_{12} \leftarrow A_t \cdot V_{SS} - B_t \cdot I_{12} \\
V_{ld} \leftarrow \begin{pmatrix} V_{12_1} \\ V_{12_2} \\ V_{12_1} + V_{12_2} \end{pmatrix} \\
\text{for } i \in 1..2 \\
\quad \text{Error}_i \leftarrow \dfrac{\left|\left|V_{12_i}\right| - \left|V_{old_i}\right|\right|}{120} \\
\text{break if } \max(\text{Error}) < \text{Tol} \\
\text{for } i \in 1..3 \\
\quad IL_i \leftarrow \overline{\left(\dfrac{SL_i \cdot 1000}{V_{ld_i}}\right)} \\
I_{12} \leftarrow DI \cdot IL \\
I_{00} \leftarrow d_t \cdot I_{12} \\
V_{old} \leftarrow V_{12}
\end{array}\right. \\
\text{Out}_1 \leftarrow V_{ld} \\
\text{Out}
\end{array}\right.
\end{array}
$$

FIGURE 11.5
Mathcad® program for Example 11.1.

The Mathcad program follows the same general steps that all programs will follow.

1. Initialize
2. Set loop
3. Forward sweep
4. Check for convergence
   a. If converged, output results
   b. If not converged, continue
5. Compute new load and line currents
6. Backward sweep
7. End of loop

After four iterations, the final load voltages are

$$[V_{ld}] = \begin{bmatrix} 117.88\underline{/-0.64} \\ 117.71\underline{/-0.63} \\ 235.60\underline{/-0.64} \end{bmatrix}$$

11.1.2 Center-Tapped Transformer Serving Loads through a Triplex Secondary

Shown in Figure 11.6 is a center-tapped transformer serving a load through a triplex secondary.

Before the system of Figure 11.6 can be modeled, the impedance matrix for the triplex secondary must be determined. The impedances of the triplex are computed using Carson's equations and the Kron reduction as described in Chapter 4. Applying Carson's equations will result in a 3 × 3 matrix. Kron reduction method is used to "fold" the impedance of the neutral conductor

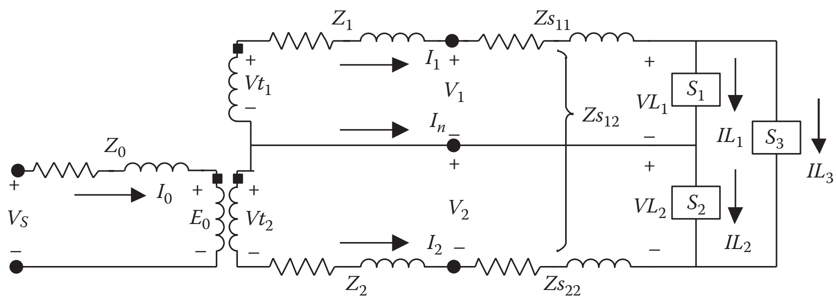

FIGURE 11.6
Center-tapped transformer with secondary.

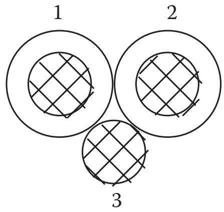

FIGURE 11.7
Triplex secondary.

into that of the two phase conductors. A triplex secondary consisting of two insulated conductors and one uninsulated neutral conductor is shown in Figure 11.7.

The spacings between conductors that are applied in Carson's equations are given by

$$\begin{aligned} D_{12} &= dia + 2 \cdot T \\ D_{13} &= dia + T \\ D_{23} &= dia + T \end{aligned} \tag{11.17}$$

where
dia represents the diameter of conductor (in.)
T represents the thickness of insulation (in.)

Applying Carson's equations,

$$\begin{aligned} zp_{ii} &= r_i + 0.09530 + j0.12134 \cdot \left(\ln \frac{1}{GMR_i} + 7.93402 \right) \\ zp_{ij} &= 0.09530 + j0.12134 \cdot \left(\ln \frac{1}{D_{ij}} + 7.93402 \right) \end{aligned} \tag{11.18}$$

where
r_i is the conductor resistance (Ω/mile)
GMR_i is the conductor geometric mean radius (ft)
D_{ij} is the distance in ft between conductors i and j

The secondary voltage equation in matrix form is

$$\begin{bmatrix} v_1 \\ v_2 \\ v_n \end{bmatrix} = \begin{bmatrix} V_{1g} \\ V_{2g} \\ V_{ng} \end{bmatrix} - \begin{bmatrix} VL_{1g} \\ VL_{2g} \\ VL_{ng} \end{bmatrix} = \begin{bmatrix} zp_{11} & zp_{12} & zp_{13} \\ zp_{21} & zp_{22} & zp_{23} \\ zp_{31} & zp_{32} & zp_{33} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_n \end{bmatrix} \tag{11.19}$$

When the neutral is grounded at the transformer and the load then

$$v_n = V_{ng} - VL_{ng} = 0 \tag{11.20}$$

This leads to the Kron reduction equation in partitioned form:

$$\begin{bmatrix} [v_{12}] \\ [0] \end{bmatrix} = \begin{bmatrix} [zp_{ij}] & [zp_{in}] \\ [zp_{nj}] & [zp_{nn}] \end{bmatrix} \cdot \begin{bmatrix} [I_{12}] \\ [I_n] \end{bmatrix} \tag{11.21}$$

Solving Equation 11.21 for the neutral current,

$$\begin{aligned} [I_n] &= -[zp_{nn}]^{-1} \cdot [zp_{ni}] \cdot [I_{12}] \\ [I_n] &= [t_n] \cdot [I_{12}] \end{aligned} \tag{11.22}$$

The Kron reduction gives the 2 × 2 phase impedance matrix:

$$[zs] = [zp_{ij}] - [zp_{in}] \cdot [zp_{nn}]^{-1} \cdot [zp_{nj}] \tag{11.23}$$

For a secondary of length L,

$$[Zs] = [zs] \cdot L = \begin{bmatrix} Zs_{11} & Zs_{12} \\ Zs_{21} & Zs_{22} \end{bmatrix} \tag{11.24}$$

Referring to Figure 11.6, the voltage backward sweep for the secondary is given by

$$\begin{aligned} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} &= \begin{bmatrix} VL_1 \\ VL_2 \end{bmatrix} + \begin{bmatrix} Zs_{11} & Zs_{12} \\ Zs_{21} & Zs_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \\ [V_{12}] &= [a_{sec}][VL_{12}] + [b_{sec}] \cdot [I_{12}] \\ \text{where } [a_{sec}] &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ [b_{sec}] &= \begin{bmatrix} Zs_{11} & Zs_{12} \\ Zs_{21} & Zs_{22} \end{bmatrix} \end{aligned} \tag{11.25}$$

Because of the short length of the secondary, the line currents leaving the transformer are equal to the line currents at the load so no current backward sweep is needed for the secondary. In order to remain consistent for the general analysis of a feeder, the matrix $[d_{sec}]$ is defined as

$$\begin{aligned} [I_{12}] &= [d_{sec}] \cdot [I_{12}] \\ \text{where } [d_{sec}] &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{11.26}$$

The voltage forward sweep equation for the secondary is determined by solving for the load voltages in Equation 11.25:

$$\begin{aligned} [VL_{12}] &= [A_{sec}]\cdot[V_{12}] - [B_{sec}]\cdot[I_{12}] \\ \text{where } [A_{sec}] &= [a_{sec}]^{-1} \\ [B_{sec}] &= [b_{sec}] \end{aligned} \tag{11.27}$$

Example 11.2

The secondary in Figure 11.6 is 100 ft of 1/0 AA triplex. Determine the phase impedance matrix for the triplex secondary.

From the table for 1/0 AA: $GMR = 0.111$ ft, $diameter = 0.368$ in., $r = 0.97\,\Omega$/mile

The insulation thickness of the phase conductors is 80 mil = 0.08 in.

The distance matrix with the diagonal terms equal to the geometric mean radius (GMR) is computed to be

$$[D] = \begin{bmatrix} 0.0111 & 0.1907 & 0.1107 \\ 0.1907 & 0.0111 & 0.1107 \\ 0.1107 & 0.1107 & 0.0111 \end{bmatrix} \text{ft}$$

Applying Carson's equations, the primitive impedance matrix is

$$[zp] = \begin{bmatrix} 1.0653 + j1.5088 & 0.0953 + j1.1638 & 0.0953 + j1.2298 \\ 0.0953 + j1.1638 & 1.0653 + j1.5088 & 0.0953 + j1.2298 \\ 0.0953 + j1.2298 & 0.0953 + j1.2298 & 1.0653 + j1.5088 \end{bmatrix} \Omega/\text{mile}$$

The Kron reduction and length of 100 ft give the final phase impedance matrix for the secondary:

$$[Zs] = \begin{bmatrix} 0.0271 + j0.0146 & 0.0087 + j0.0081 \\ 0.0087 + j0.0081 & 0.0271 + j0.0146 \end{bmatrix} \Omega$$

The forward and backward sweep equations for the secondary are

$$[U] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[A_{sec}] = [U]$$

$$[B_{sec}] = [Zs] = \begin{bmatrix} 0.0271 + j0.0146 & 0.0087 + j0.0081 \\ 0.0087 + j0.0081 & 0.0271 + j0.0146 \end{bmatrix}$$

$$[d_{sec}] = [U]$$

The Mathcad program of Example 11.1 is modified so that

Forward sweep:

$$[V_{12}] = [A_t]\cdot[V_{SS}] - [B_t]\cdot[I_{12}]$$

$$[VL_{12}] = [A_{sec}]\cdot[V_{12}] - [B_{sec}]\cdot[I_{12}]$$

In the program, the load voltages are

$$[V_{ld}] = \begin{bmatrix} VL_1 \\ VL_2 \\ VL_1 + VL_2 \end{bmatrix}$$

The remainder of the program stays the same. After five iterations, the final voltages are

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 117.89/\underline{-0.64} \\ 117.75/\underline{-0.62} \end{bmatrix}$$

$$\begin{bmatrix} VL_1 \\ VL_2 \\ VL_3 \end{bmatrix} = \begin{bmatrix} 114.63/\underline{-0.45} \\ 122.58/\underline{-0.96} \\ 237.21/\underline{-0.72} \end{bmatrix}$$

Note that the voltage VL_2 is great than V_2 indicating a voltage rise on that phase. This is not uncommon when the line currents are very unbalanced.

The secondary line currents are

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 190.2/\underline{-26.2} \\ 227.5/\underline{150.6} \end{bmatrix}$$

The primary line current is

$$I_0 = 6.98/\underline{-28.0}$$

Using the neutral current transform matrix of Equation 11.29, the current flowing in the neutral conductor is

$$I_n = [t_n]\cdot[I_{12}] = 25.4/\underline{-15.0}$$

The current flowing in ground is

$$I_g = -(I_n + I_1 + I_2) = 20.8/\underline{-84.5}$$

It is always good to check the validity of the results. This is particularly true because there should be some question about the voltage rise on phase 2. The check can be done by using basic circuit and transformer theory to compute the source voltage using the load voltages and line currents output from the program:

$$[V_{12}] = [VL_{12}] + [Zs] \cdot [I_{12}] = \begin{bmatrix} 117.89\underline{/-0.64} \\ 117.75\underline{/-0.62} \end{bmatrix}$$

$$[Vt_{12}] = [V_{12}] + [Z_{12}] \cdot [I_{12}] = \begin{bmatrix} 118.61\underline{/-0.59} \\ 118.61\underline{/-0.59} \end{bmatrix}$$

This is the first indication that the solution is correct since the two ideal voltages on the secondary are equal. That is a must. Knowing the ideal voltages and the secondary line currents, the primary voltage and line current can be computed:

$$E_0 = n_t \cdot Vt_1 = 7116.4\underline{/-0.59}$$

$$I_0 = \frac{1}{n_t} \cdot (I_1 - I_2) = 6.98\underline{/-28.0}$$

$$V_s = E_0 + Z_0 \cdot I_0 = 7200\underline{/0}$$

Since the original source voltage has been computed, the results of the program have been shown to be correct. Whenever there is a question about the validity of a program solution, it is good to use basic circuit and transformer theory to prove the results are correct. Do not ever assume the results are correct because they came from a computer program.

11.2 Ungrounded Wye–Delta Transformer Bank with a Center-Tapped Transformer

The most common transformer connection for providing service to a combination of three-phase and single-phase loads is the ungrounded wye–delta. In order to provide the usual three-wire service for the single-phase loads, one of the three transformers, the "lighting" transformer, will have a center tap. The other two transformers are referred to as the "power" transformers. Figure 11.8 shows this transformer connection.

11.2.1 Backward Sweep Equations

As before, it is desired to develop the forward and backward matrices so that the source-side voltages and currents can be computed directly from the

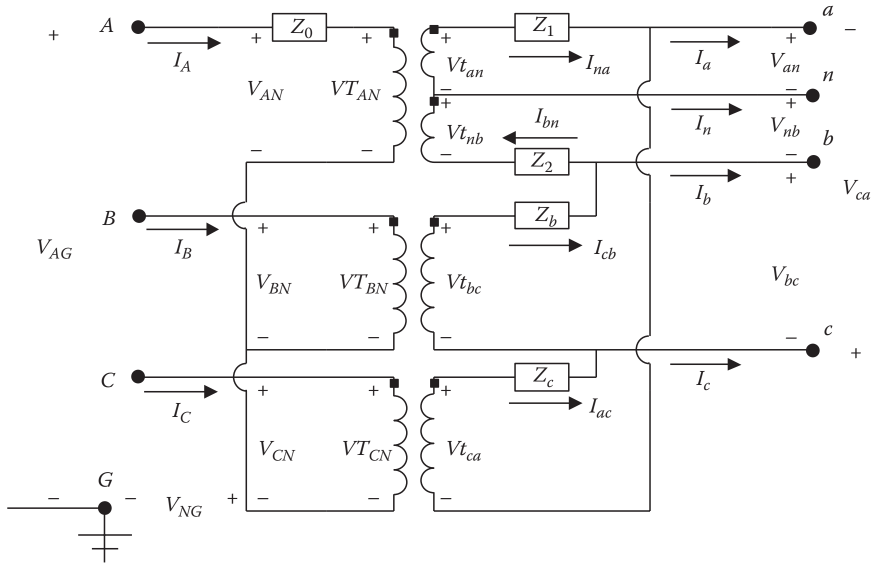

FIGURE 11.8
Ungrounded wye–delta with center-tapped transformer.

secondary voltages and currents. With reference to Figure 11.8, define the secondary terminal voltages as

$$[V_{anbc}] = \begin{bmatrix} V_{an} \\ V_{nb} \\ V_{bc} \\ V_{ca} \end{bmatrix} \tag{11.28}$$

The secondary line currents and currents flowing inside the delta are defined as

$$[I_{abc}] = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{11.29}$$

$$[ID_{anbc}] = \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{11.30}$$

In keeping with the notation for the single-phase center-tapped transformer, the transformer ratio will be defined as

$$n_t = \frac{\textit{Rated primary line-to-neutral voltage}}{0.5\ \textit{Rated secondary line-to-line voltage}} \tag{11.31}$$

The matrix equation relating the primary ideal transformer voltages to the secondary ideal voltages is

$$\begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \cdot \begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \tag{11.32}$$

$$[VT_{ABC}] = [DT] \cdot [Vt_{anbc}]$$

The primary terminal voltages are given by

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} + \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \tag{11.33}$$

$$[VLN_{ABC}] = [VT_{ABC}] + [ZT_0] \cdot [I_{ABC}]$$

Substitute Equation 11.32 into Equation 11.33:

$$[VLN_{ABC}] = [DT] \cdot [Vt_{anbc}] + [ZT_0] \cdot [I_{ABC}] \tag{11.34}$$

The primary line currents as a function of the secondary delta currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{11.35}$$

$$[I_{ABC}] = [N] \cdot [ID_{anbc}]$$

Substitute Equation 11.34 into Equation 11.33:

$$[VLN_{ABC}] = [DT] \cdot [Vt_{anbc}] + [ZT_0] \cdot [N] \cdot [ID_{anbc}] \tag{11.36}$$

The secondary ideal transformer voltages are given by

$$\begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \begin{bmatrix} V_{an} \\ V_{nb} \\ V_{bc} \\ V_{ca} \end{bmatrix} + \begin{bmatrix} Z_1 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 \\ 0 & 0 & Z_b & 0 \\ 0 & 0 & 0 & Z_c \end{bmatrix} \cdot \begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{11.37}$$

$$[Vt_{anbc}] = [V_{anbc}] + [Zt_{12bc}] \cdot [ID_{anbc}]$$

Substitute Equation 11.37 into Equation 11.36:

$$\begin{aligned} [VLN_{ABC}] &= [DT] \cdot \left([V_{anbc}] + [Zt_{12bc}] \cdot [ID_{anbc}]\right) + [ZT_0] \cdot [N] \cdot [ID_{anbc}] \\ [VLN_{ABC}] &= [DT] \cdot [V_{anbc}] + \left([DT] \cdot [Zt_{12bc}] + [ZT_0] \cdot [N]\right) \cdot [ID_{anbc}] \end{aligned} \tag{11.38}$$

At this point, it is necessary to establish the relationship between the currents flowing in the delta secondary windings and the secondary line currents. Since there are four unknown delta currents, four independent equations are needed. There are four nodes on the secondary, but only three Kirchhoff's current law (KCL) equations written at the nodes are independent. The fourth independent equation will come from recognizing that the sum of the primary line currents must add to zero since the neutral is ungrounded. This given by

$$I_A + I_B + I_C = 0 = n_t \cdot I_{na} + n_t I_{bn} + 2n_t \cdot I_{cb} + 2n_t \cdot I_{ac}$$

$$0 = I_{na} + I_{bn} + 2 \cdot I_{cb} + 2 \cdot I_{ac} \tag{11.39}$$

Writing KCL at the *a*, *b*, and *c* nodes gives

$$\begin{aligned} I_a &= I_{na} - I_{ac} \\ I_b &= I_{cb} - I_{bn} \\ I_c &= I_{ac} - I_{cb} \end{aligned} \tag{11.40}$$

Combining Equations 11.39 and 11.40 into matrix form:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} I_{na} \\ I_{nb} \\ I_{cb} \\ I_{ac} \end{bmatrix} \tag{11.41}$$

$$[I_{abc0}] = [X1] \cdot [ID_{anbc}]$$

The delta secondary currents can now be computed by

$$[ID_{anbc}] = [X1]^{-1} \cdot [I_{abc0}]$$

$$\begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \frac{1}{6} \cdot \begin{bmatrix} 5 & 1 & 3 & 1 \\ -1 & -5 & -3 & 1 \\ -1 & 1 & -3 & 1 \\ -1 & 1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ 0 \end{bmatrix} \tag{11.42}$$

Notice in Equation 11.42 that the fourth column of the inverse matrix is not needed since the fourth term in the current vector is zero. Therefore, Equation 11.42 is modified to

$$\begin{bmatrix} I_{na} \\ I_{bn} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \frac{1}{6} \cdot \begin{bmatrix} 5 & 1 & 3 \\ -1 & -5 & -3 \\ -1 & 1 & -3 \\ -1 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[ID_{anbc}] = [DI] \cdot [I_{abc}] \tag{11.43}$$

Substitute Equation 11.43 into Equation 11.38:

$$[VLN_{ABC}] = [DT] \cdot [V_{anbc}] + \left([DT] \cdot [Zt_{12bc}] + [ZT_0] \cdot [N]\right) \cdot [DI] \cdot [I_{abc}] \tag{11.44}$$

Equation 11.44 is in the desired form of

$$[VLN_{ABC}] = [a_t] \cdot [V_{anbc}] + [b_t] \cdot [I_{abc}]$$

$$\text{where } [a_t] = [DT] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$[b_t] = \left([DT] \cdot [Zt_{12bc}] + [ZT_0] \cdot [N]\right) \cdot [DI] \tag{11.45}$$

$$[b_t] \cdot \frac{n_t}{6} \cdot \begin{bmatrix} 5 \cdot Z_1 + 4 \cdot \dfrac{Z_0}{n_t^2} & Z_1 - 4 \cdot \dfrac{Z_0}{n_t^2} & 3 \cdot Z_1 \\ -0.5 \cdot Z_b & 0.5 \cdot Z_b & -1.5 \cdot Z_b \\ -0.5 \cdot Z_c & 0.5 \cdot Z_c & 1.5 \cdot Z_c \end{bmatrix}$$

The primary line currents as a function of the secondary line currents can be determined by substituting Equation 11.43 into Equation 11.35:

$$[I_{ABC}] = [N]\cdot[ID_{abc}] = [N]\cdot[DI]\cdot[I_{abc}]$$

$$[I_{ABC}] = [d_t]\cdot[I_{abc}] \tag{11.46}$$

$$\text{where } [d_t] = [N]\cdot[DI] = \frac{1}{3\cdot n_t}\cdot\begin{bmatrix} 2 & -2 & 0 \\ -1 & 1 & -3 \\ -1 & 1 & 3 \end{bmatrix}$$

11.2.2 Forward Sweep Equations

The forward sweep equations are used when the source voltage and secondary currents are known and the secondary transformer terminal voltages are required. Referring to Figure 11.8, the secondary ideal voltages as a function of the primary ideal voltages are given by

$$\begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \frac{1}{n_t}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\cdot\begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} \tag{11.47}$$

$$[Vt_{anbc}] = [Dt]\cdot[VT_{ABC}]$$

The primary ideal voltages as a function of the primary line-to-neutral voltages are

$$[VT_{ABC}] = [VLN_{ABC}] - [ZT_0]\cdot[I_{ABC}] \tag{11.48}$$

Substitute Equation 11.46 into Equation 11.48:

$$[VT_{ABC}] = [VLN_{ABC}] - [ZT_0]\cdot[d_t][I_{abc}] \tag{11.49}$$

Substitute Equation 11.49 into Equation 11.47:

$$[Vt_{anbc}] = [Dt]\cdot\left([VLN_{ABC}] - [ZT_0]\cdot[d_t][I_{abc}]\right) \tag{11.50}$$

The transformer secondary voltages as a function of the secondary ideal voltages are

$$[V_{anbc}] = [Vt_{anbc}] - [Zt_{12bc}]\cdot[ID_{anbc}] \tag{11.51}$$

Substitute Equations 11.43 and 11.50 into Equation 11.51 and simplify:

$$[V_{anbc}] = [Dt]\cdot([VLN_{ABC}] - [ZT_0]\cdot[d_t][I_{abc}]) - [Zt_{12bc}]\cdot[DI]\cdot[I_{abc}]$$

$$[V_{anbc}] = [Dt]\cdot[VLN_{ABC}] - ([Dt]\cdot[ZT_0]\cdot[d_t] + [Zt_{12bc}]\cdot[DI])\cdot[I_{abc}] \tag{11.52}$$

$$[V_{anbc}] = [A_t]\cdot[VLN_{ABC}] - [B_t]\cdot[I_{abc}]$$

where

$$[A_t] = [Dt] = \frac{1}{n_t}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \tag{11.53}$$

$$[B_t] = [Dt]\cdot[ZT_0]\cdot[d_t] + [Zt_{12bc}]\cdot[DI]$$

$$= \frac{1}{6}\cdot\begin{bmatrix} \frac{4}{n_t^2}\cdot Z_0 + 5\cdot Z_1 & -\frac{4}{n_t^2}\cdot Z_0 + Z_1 & 3\cdot Z_1 \\ \frac{4}{n_t^2}\cdot Z_0 - Z_2 & -\frac{4}{n_t^2}\cdot Z_0 - 5\cdot Z_2 & -3\cdot Z_2 \\ -Z_b & Z_b & -3\cdot Z_b \\ -Z_c & Z_c & 3\cdot Z_c \end{bmatrix} \tag{11.54}$$

Example 11.3

Figure 11.9 shows an ungrounded wye–delta transformer bank service 120/240 V single-phase loads and a three-phase induction motor.

The single-phase loads are rated:

SL_1 = 3 kVA, 120 V, 0.95 lagging power factor

SL_2 = 5 kVA, 120 V, 0.90 lagging power factor

SL_3 = 8 kVA, 240 V, 0.85 lagging power factor

The load vector is

for $i = 1 - 3$

$$SL_i = kVA_i\underline{/a\cos(PF_i)} = \begin{bmatrix} 2.85 + j0.9367 \\ 4.5 + j2.1794 \\ 6.8 + j4.2143 \end{bmatrix} \text{kW} + j\text{kvar}$$

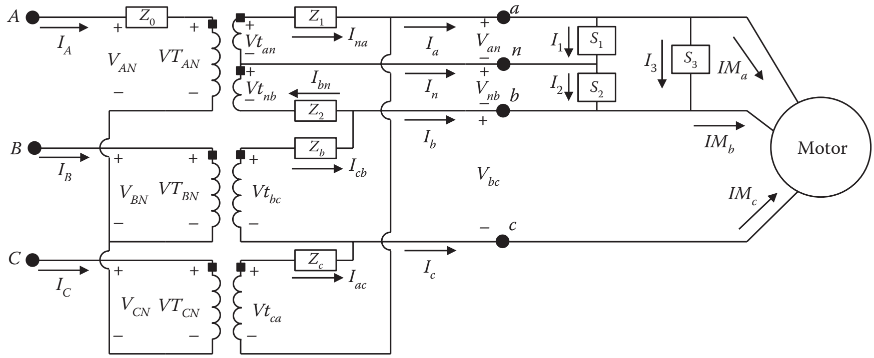

FIGURE 11.9
Ungrounded wye–delta bank serving combination loads.

The three-phase induction motor data are

25 HP, 240 V, impedances:

$$Zs = 0.0774 + j0.1843\,\Omega$$
$$Zr = 0.0908 + j0.1843\,\Omega$$
$$Zm = 0 + j4.8385\,\Omega$$

The motor is operating at a slip of 0.035.
The transformer data are

Lighting transformer: 25 kVA, 7200–240/120 V, $ZL_{pu} = 0.012 + j0.017$
Power transformers: 10 kVA, 7200–240 V, $ZP_{pu} = 0.016 + j0.014$
Source voltage: Balanced line-to-neutral 7200 V

Determine

a. Transformer impedances in Ohms for the model in Figure 11.9
b. Transformer forward and backward sweep matrices
c. Motor phase admittance matrix
d. Operating currents
   i. Single-phase loads
   ii. Motor
e. Operating voltages
   i. Single-phase loads
   ii. Motor

Compute the winding per-unit impedances for the lighting transformer:

$$Zpu_0 = 0.5 \cdot \mathrm{Re}(Z_L) + j0.8 \cdot \mathrm{Im}(Z_L) = 0.006 + j0.0135$$
$$Zpu_1 = Zpu_2 = \mathrm{Re}(Z_L) + j0.4 \cdot \mathrm{Im}(Z_L) = 0.012 + j0.0068$$

Convert the lighting transformer impedances to Ohms:

$$Zbase_{hi} = \frac{7.2^2 \cdot 1000}{25} = 2073.6$$

$$Z_0 = 2073.6 \cdot (0.006 + j0.0136) = 12.446 + j28.201$$

$$Zbase_{lo} = \frac{0.12^2 \cdot 1000}{25} = 0.576$$

$$Z_1 = Z_2 = 0.576 \cdot (0.012 + j0.0068) = 0.0069 + j0.0039$$

Convert the per-unit impedances of the power transformers to Ohms:

$$Zbase_{lo} = \frac{0.24^2 \cdot 1000}{10} = 5.76$$

$$Z_b = Z_c = 5.76 \cdot (0.016 + j0.014) = 0.0922 + j0.0806$$

Compute the turns ratio:

$$n_t = \frac{7200}{120} = 60$$

Compute the forward sweep matrices:

$$[A_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0.0167 & 0 & 0 \\ 0.0167 & 0 & 0 \\ 0 & 0.0333 & 0 \\ 0 & 0 & 0.0333 \end{bmatrix}$$

$$[B_t] = \frac{1}{6} \cdot \begin{bmatrix} \frac{4}{n_t^2} \cdot Z_0 + 5 \cdot Z_1 & -\frac{4}{n_t^2} \cdot Z_0 + Z_1 & 3 \cdot Z_1 \\ \frac{4}{n_t^2} \cdot Z_0 - Z_2 & -\frac{4}{n_t^2} \cdot Z_0 - 5 \cdot Z_2 & -3 \cdot Z_2 \\ -Z_b & Z_b & -3 \cdot Z_b \\ -Z_c & Z_c & 3 \cdot Z_c \end{bmatrix}$$

$$= \begin{bmatrix} 0.0081 + j0.0085 & -0.0012 - j0.0046 & 0.0035 + j0.002 \\ 0.0012 + j0.0046 & -0.0081 - j0.0085 & -0.0035 - j0.002 \\ -0.0154 - j0.0134 & 0.0154 + j0.0134 & -0.0461 - j0.0403 \\ -0.0154 - j0.0134 & 0.0154 + j0.0134 & 0.0461 + j0.0403 \end{bmatrix}$$

Compute the backward sweep matrices where

$$[a_t] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} = \begin{bmatrix} 60 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 \\ 0 & 0 & 0 & 30 \end{bmatrix}$$

$$[b_t] \cdot \frac{n_t}{6} \cdot \begin{bmatrix} 5 \cdot Z_1 + 4 \cdot \dfrac{Z_0}{n_t^2} & Z_1 - 4 \cdot \dfrac{Z_0}{n_t^2} & 3 \cdot Z_1 \\ -0.5 \cdot Z_b & 0.5 \cdot Z_b & -1.5 \cdot Z_b \\ -0.5 \cdot Z_c & 0.5 \cdot Z_c & 1.5 \cdot Z_c \end{bmatrix}$$

$$[b_t] = \begin{bmatrix} 0.4838 + j0.5092 & -0.0691 - j0.2742 & 0.2074 + j0.1175 \\ -0.4608 - j0.4032 & 0.4608 + j0.4032 & -1.3824 - j1.2096 \\ -0.4608 - j0.4032 & 0.4608 + j0.4032 & 1.3824 + j1.2096 \end{bmatrix}$$

$$[d_t] = \frac{1}{3 \cdot n_t} \cdot \begin{bmatrix} 2 & -2 & 0 \\ -1 & 1 & -3 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 0.0111 & -0.0111 & 0 \\ -0.0056 & 0.0056 & -0.0167 \\ -0.0056 & 0.0056 & 0.0167 \end{bmatrix}$$

Motor admittance matrix:
Define the positive and negative sequence slips:

$$s_1 = 0.035$$

$$s_2 = 2 - s_1 = 1.965$$

Compute the sequence load resistances and input sequence impedances:

for $k = 1$ and 2

$$RL_k = \frac{1 - s_k}{s_k} \cdot R_r = \begin{bmatrix} 2.5035 \\ -0.0446 \end{bmatrix}$$

$$ZM_k = Zs + \frac{Zm \cdot (Zr + RL_k)}{Zm + Zr + RL_k} = \begin{bmatrix} 1.9778 + j1.3434 \\ 0.1203 + j0.3622 \end{bmatrix}$$

Compute the input sequence admittances:

$$YM_k = \frac{1}{ZM_k} = \begin{bmatrix} 0.3460 - j0.2350 \\ 0.8256 - j2.4865 \end{bmatrix}$$

The sequence admittance matrix is

$$[YM_{012}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t^* \cdot YM_1 & 0 \\ 0 & 0 & t \cdot YM_2 \end{bmatrix}$$

$$\text{where } t = \frac{1}{\sqrt{3}} \underline{/30}$$

The phase admittance matrix is

$$[YM_{abc}] = [A] \cdot [YM_{012}] \cdot [A]^{-1}$$

$$= \begin{bmatrix} 0.7543 - j0.4074 & 0.1000 - j0.0923 & 0.3347 + j0.4997 \\ 0.3547 + j0.4997 & 0.7453 - j0.4074 & -0.1000 - j0.0923 \\ -0.1000 - j0.0923 & 0.3547 + 0.4997 & 0.7453 - j0.4074 \end{bmatrix}$$

where $a = 1\underline{/120}$

$$[A] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Set the source voltage vector:

$$[ELN_{ABC}] = \begin{bmatrix} 7200\underline{/0} \\ 7200\underline{/-120} \\ 7200\underline{/120} \end{bmatrix}$$

The Mathcad program to compute the voltages and currents is shown in Figure 11.10. The starting matrices and the KCL current matrix and tolerance are defined as

$$[istart] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad [vstart] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad [DI] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad Tol = 0.01$$

$$
X := \left|
\begin{array}{l}
I_{abc} \leftarrow istart \\
I_{ABC} \leftarrow istart \\
V_{old} \leftarrow vstart \\
\text{for } n \in 1..200 \\
\quad \left|
\begin{array}{l}
V_{anbc} \leftarrow A_t \cdot ELN_{ABC} - B_t \cdot I_{abc} \\
\text{for } j \in 1..4 \\
\quad Error_j \leftarrow \left| \left| V_{anbc_j} \right| - \left| V_{old_j} \right| \right| \\
\text{break if } \max(Error) < Tol \\
V_{ld} \leftarrow \begin{pmatrix} V_{anbc_1} \\ V_{anbc_2} \\ V_{anbc_1} + V_{anbc_2} \end{pmatrix} \\
VM \leftarrow \begin{pmatrix} V_{ld_3} \\ V_{anbc_3} \\ V_{anbc_4} \end{pmatrix} \\
\text{for } i \in 1..3 \\
\quad IL_i \leftarrow \overline{\left(\dfrac{SL_i \cdot 1000}{V_{ld_i}} \right)} \\
IM \leftarrow YM_{abc} \cdot VM \\
I_{abc} \leftarrow DI \cdot IL + IM \\
I_{ABC} \leftarrow d_t \cdot I_{abc} \\
V_{old} \leftarrow V_{anbc}
\end{array}
\right. \\
Out_1 \leftarrow V_{anbc} \\
Out
\end{array}
\right.
$$

FIGURE 11.10
Mathcad® program.

This program follows the flowcharts of earlier chapters. The forward sweep equation gives the single-phase voltages along with the three line-to-line voltages. It is necessary to break this four element matrix into the three voltages for the single-phase loads and also the three line-to-line motor voltages. The program shows how this is done. A major difference in checking for convergence is that the magnitudes of the voltages are compared in volts rather than per unit as was done in earlier programs.

After five iterations, the results are

Currents:

$$[IL] = \begin{bmatrix} 25.3/\underline{-18.6} \\ 42.3/\underline{-26.2} \\ 33.8/\underline{-32.2} \end{bmatrix}$$

$$[IM] = \begin{bmatrix} 57.1/\underline{-63.8} \\ 54.1/\underline{176.4} \\ 59.2/\underline{54.2} \end{bmatrix}$$

$$[I_{abc}] = \begin{bmatrix} 109.7/\underline{-44.8} \\ 127.0/\underline{161.6} \\ 59.2/\underline{54.2} \end{bmatrix}$$

$$[I_{ABC}] = \begin{bmatrix} 2.56/\underline{-30.6} \\ 1.69/\underline{-174.9} \\ 1.54/\underline{109.9} \end{bmatrix}$$

Voltages:

$$[V_{ld}] = \begin{bmatrix} 118.4/\underline{-0.38} \\ 118.3/\underline{-0.35} \\ 238.6/\underline{-0.37} \end{bmatrix}$$

$$[VM] = \begin{bmatrix} 236.6/\underline{-0.37} \\ 234.0/\underline{-119.6} \\ 235.2/\underline{119.3} \end{bmatrix}$$

In the derivation of the forward and backward matrices, it was found that all of the matrices can be defined by the combination of matrices based upon basic circuit theory. The definitions are as follows:

$$\begin{aligned}
[a_t] &= [DT] \\
[b_t] &= ([DT]\cdot[Zt_{12bc}] + [ZT_0]\cdot[N])\cdot[DI] \\
[d_t] &= [N]\cdot[DI] \\
[A_t] &= [Dt] \\
[B_t] &= [DT]\cdot[ZT_0]\cdot[d_t] + [Zt_{12bc}]\cdot[DI]
\end{aligned} \tag{11.55}$$

The individual matrices in Equation 11.64 define the relationship between parameters by

$$\begin{aligned}
[ID_{anbc}] &= [DI]\cdot[I_{abc}] \\
[VT_{ABC}] &= [DT]\cdot[Vt_{anbc}] \\
[I_{ABC}] &= [N]\cdot[ID_{anbc}] \\
[Vt_{anbc}] &= [Dt]\cdot[VT_{ABC}]
\end{aligned} \tag{11.56}$$

The definitions of Equations 11.55 and 11.56 will be used to develop the models for the open wye–open delta connections.

11.3 Leading Open Wye–Open Delta Transformer Connection

Quite often an open wye–open delta transformer consisting of one lighting transformer and one power transformer will be used to serve a combination of single-phase and three-phase loads. The leading open wye–open delta connection is shown in Figure 11.11.

The voltage phasors at no-load for the connection in Figure 11.11 are shown in Figure 11.12.

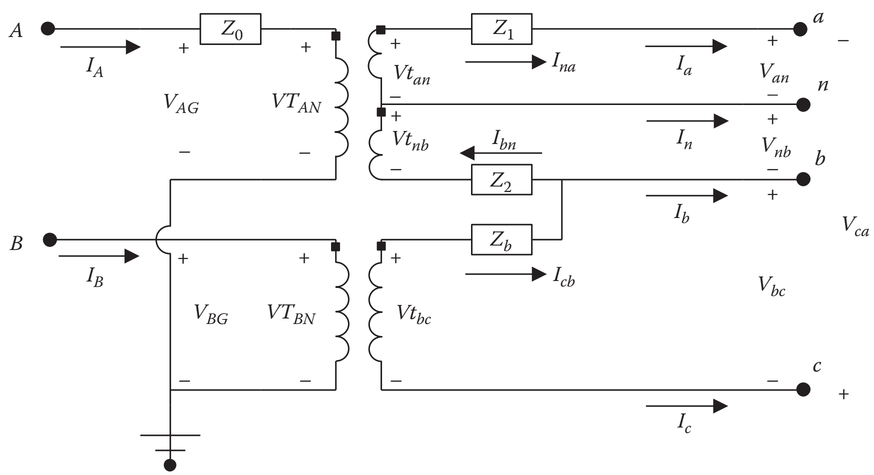

FIGURE 11.11
Leading open wye–open delta connection.

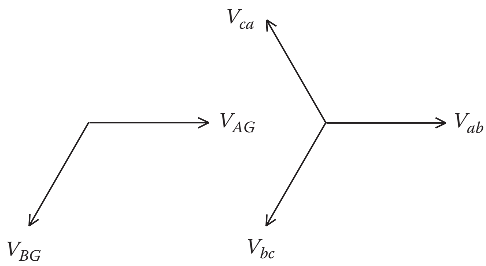

FIGURE 11.12
Leading open wye–open delta voltage phasors.

It is very important to note in Figure 11.12 that the phase sequence on the secondary is *a-b-c*. That will always be the assumption but great care must be taken to assure that the labeling of the phases will result in the correct *a-b-c* sequence.

The model for the connection in Figure 11.10 will be developed using the relationships defined in Equations 11.55 and 11.56.

Unlike the closed delta secondary, there is a direct relationship between the secondary transformer currents and the line currents. Applying KCL in Figure 11.11, the currents flowing in the open delta as a function of the line currents are given by

$$\begin{bmatrix} I_{an} \\ I_{nb} \\ I_{cb} \\ I_{ac} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$[DI] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \tag{11.57}$$

The primary and secondary ideal transformer voltages going from the secondary to the primary are related by

$$\begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} = n_t \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} \tag{11.58}$$

$$[DT] = n_t \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The primary and transformer secondary currents are related by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{an} \\ I_{nb} \\ I_{cb} \\ I_{ac} \end{bmatrix}$$

$$[N] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{11.59}$$

Going from the primary ideal transformer voltages to the secondary ideal voltages (forward sweep), the voltages are related by

$$\begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix}$$

$$[Dt] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{11.60}$$

For this open wye–open delta connection, the transformer impedance matrices are defined as

$$[ZT_0] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[Zt_{12bc}] = \begin{bmatrix} Z_1 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 \\ 0 & 0 & Z_b & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{11.61}$$

Equations 11.55 are now used to determine the forward and backward matrices. It is possible to determine the final form of each of the matrices as was done for the ungrounded wye–delta connection. However, it is straight forward to define the matrices (Equations 11.57 through 11.61) and use Equations 11.55 to compute the final forward/backward matrices.

11.4 Lagging Open Wye–Open Delta Connection

The lagging open wye–open delta connection requires that the transformer serving the single-phase load to be connected across the voltage that lags the power transformer by 120°. There are several combinations of connections that will satisfy this requirement. One might be tempted to take the connection of Figure 11.9 and merely connect the power transformer to phase *C* instead of phase *B*. This leaves the lighting transformer connected to phase *A*. That can be done but the phase notation on the secondary side of the transformer bank will have to be changed. That connection is shown in Figure 11.13.

Note in Figure 11.14 that the sequence of voltages on the secondary is *a-c-b*. The real problem with this is the possibility that in order to balance loads on a feeder, a decision might have been made to simply recommend that the power transformer be moved from phase *B* in the leading connection to phase *C* in the lagging connection. If this is done, all three-phase motors downstream from the transformer bank will run backward.

There are other ways of connecting the transformers for the lagging connection. Figure 11.15 shows the connection diagram that will be used here. This connection makes it possible to retain exactly the same phase notation on the secondary as was used for the ungrounded wye–delta and the leading connection.

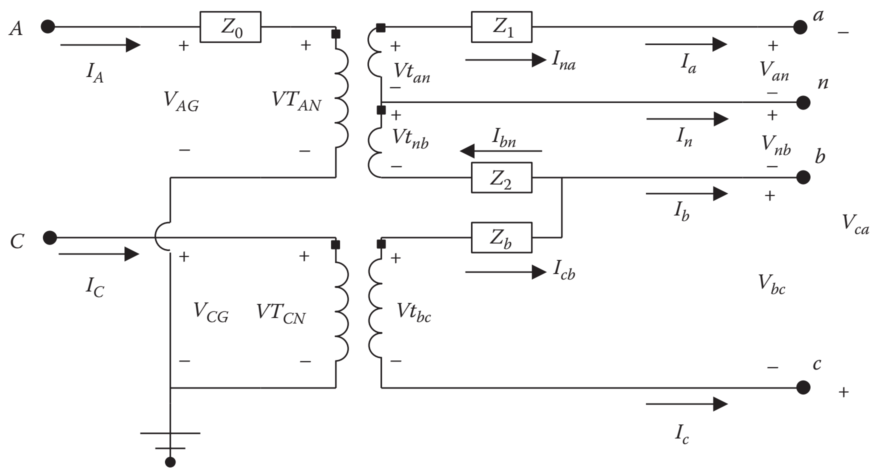

FIGURE 11.13
Lagging connection with negative sequence.

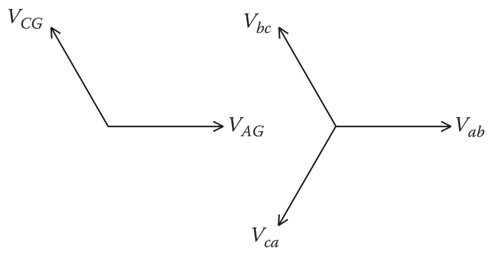

FIGURE 11.14
Lagging phasor diagram with negative sequence.

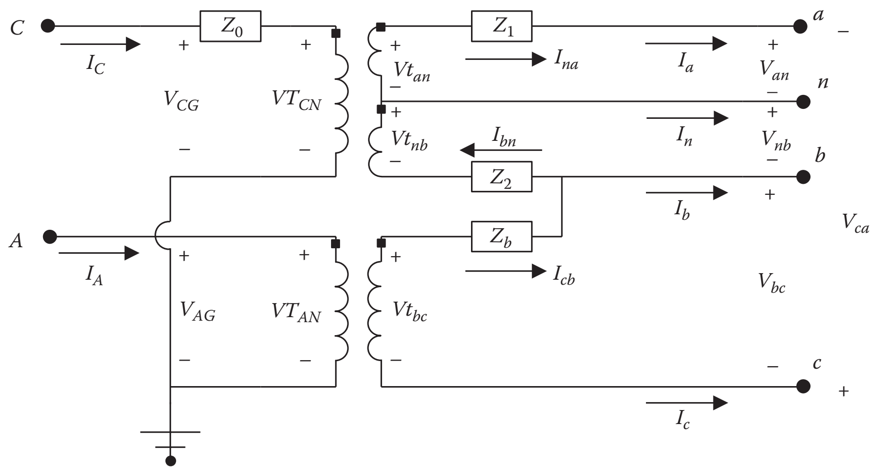

FIGURE 11.15
Lagging open wye–open delta connection.

In Figure 11.15, the lighting transformer is connected to phase C and the power transformer is connected to phase A. The no-load phasor voltage phasor diagrams for this connection are shown in Figure 11.16.

The voltage phasor diagram for this lagging connection results in the desired phase sequence of a-b-c. With this connection, the phasing for the secondary will remain the same as before. Because of that, the definition for the [DI] matrix will be the same as for the leading connection as given in Equation 11.57.

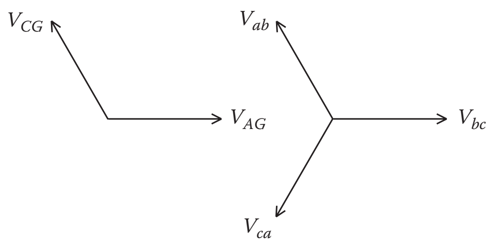

FIGURE 11.16
Lagging open wye–open delta with positive sequence.

The primary and secondary ideal transformer voltages going from the secondary to the primary are related by

$$\begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} = n_t \cdot \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{bc} \\ Vt_{ca} \end{bmatrix}$$

$$[DT] = n_t \cdot \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{11.62}$$

The primary and transformer secondary currents are related by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{an} \\ I_{nb} \\ I_{cb} \\ I_{ac} \end{bmatrix}$$

$$[N] = \frac{1}{n_t} \cdot \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \tag{11.63}$$

Going from the primary ideal transformer voltages to the secondary ideal voltages (forward sweep), the voltages are related by

$$\begin{bmatrix} Vt_{an} \\ Vt_{nb} \\ Vt_{cb} \\ Vt_{ca} \end{bmatrix} = \frac{1}{n_t} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix}$$

$$[Dt] = \frac{1}{n_t} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{11.64}$$

For this open wye–open delta connection, the transformer impedance matrices are defined as

$$[ZT_0] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z_0 \end{bmatrix}$$

$$[Zt_{12bc}] = \begin{bmatrix} Z_1 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 \\ 0 & 0 & Z_b & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{11.65}$$

Equations 11.64 are again used to determine the forward and backward matrices. As before, it is possible to determine the final form of each of the matrices as was done for the ungrounded wye–delta connection. However, it is straight forward to define the matrices (Equations 11.59 and 11.62 through 11.65) and use Equations 11.55 to compute the final forward/backward matrices.

11.5 Four-Wire Secondary

Typically, the combination of single-phase and three-phase loads will not be directly connected to the transformer but rather will be connected through a length of open four-wire secondary or a quadraplex cable secondary. This is depicted in Figure 11.17.

The first step in modeling the four-wire secondary is to compute the self- and mutual impedances. As always, Carson's equations are used to compute the 4 × 4 primitive impedance matrix. Since the secondary neutral is grounded at both ends, the Kron reduction method is used to eliminate the fourth row and column, which results in the 3 × 3 phase impedance matrix. Chapter 4 gives the details on the application of Carson's equations and the Kron reduction.

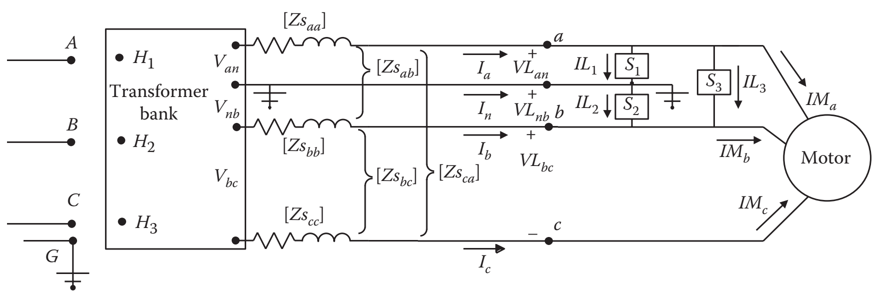

FIGURE 11.17
Four-wire secondary serving combination loads.

The 3 × 3 phase impedance matrix gives the self-impedance of the three line conductors and the mutual impedance between those conductors. The voltage drops on the three line conductors are

$$\begin{aligned} v_a &= Zs_{aa}\cdot I_a + Zs_{ab}\cdot I_b + Zs_{ac}\cdot I_c \\ v_b &= Zs_{ba}\cdot I_a + Zs_{bb}\cdot I_b + Zs_{bc}\cdot I_c \\ v_c &= Zs_{ca}\cdot I_a + Zs_{cb}\cdot I_b + Zs_{cc}\cdot I_c \end{aligned} \tag{11.66}$$

The model of the four-wire secondary will again be in terms of the *a*, *b*, *c*, *d* and *A*, *B* generalized matrices. The first step in developing the model is to write KVL around the three "window" loops and the outside loop in Figure 11.17:

$$\begin{aligned} V_{an} &= VL_{an} + v_a \\ V_{nb} &= VL_{nb} - v_b \\ V_{bc} &= VL_{bc} + v_b - V_c \\ V_{ca} &= VL_{ca} + v_c - v_a \end{aligned} \tag{11.67}$$

Substitute Equations 11.66 into Equations 11.67:

$$\begin{bmatrix} V_{an} \\ V_{nb} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} VL_{an} \\ VL_{nb} \\ VL_{bc} \\ VL_{ca} \end{bmatrix} + \begin{bmatrix} Zs_{aa} & Zs_{ab} & Zs_{ac} \\ -Zs_{ba} & -Zs_{bb} & -Zs_{bc} \\ Zs_{ba}-Zs_{ca} & Zs_{bb}-Zs_{cb} & Zs_{bc}-Zs_{cc} \\ Zs_{ca}-Zs_{aa} & Zs_{cb}-Zs_{ab} & Zs_{cc}-Zs_{ac} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{11.68}$$

$$[V_{anbc}] = [VL_{anbc}] + [Zs_{abc}]\cdot[I_{abc}]$$

Equation 11.68 is in the form of

$$[V_{anbc}] = [a_s]\cdot[VL_{anbc}] + [b_s]\cdot[I_{abc}]$$

$$\text{where } [a_s] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11.69}$$

$$[b_s] = \begin{bmatrix} Zs_{aa} & Zs_{ab} & Zs_{ac} \\ -Zs_{ba} & -Zs_{bb} & -Zs_{bc} \\ Zs_{ba}-Zs_{ca} & Zs_{bb}-Zs_{cb} & Zs_{bc}-Zs_{cc} \\ Zs_{ca}-Zs_{aa} & Zs_{cb}-Zs_{ab} & Zs_{cc}-Zs_{ac} \end{bmatrix}$$

Since there are no shunt devices between the transformer and the loads, the current leaving the transformer are equal to the line currents serving the loads. Therefore

$$[d_s] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11.70}$$

The matrices for the forward sweep are

$$\begin{aligned} [A_s] &= [a_s] \\ [B_s] &= [b_s] \end{aligned} \tag{11.71}$$

Example 11.4

The configuration of a quadraplex secondary cable is shown in Figure 11.18.

The phase conductors for the quadraplex cable are 1/0 AA and the grounded neutral conductor is 1/0 ACSR. The insulation thickness of the conductors is 80 mil.

Determine the phase impedance matrix and the *a, b, d* and *A* and *B* matrices for the quadraplex cable.

From Appendix A,

$$1/0 \text{ AA:} \quad GMR = 0.0111 \text{ ft}, \quad diameter = 0.368 \text{ in.}, \quad resistance = 0.97\ \Omega/\text{mile}$$

$$1/0 \text{ ACSR:} \quad GMR = 0.00446 \text{ ft}, \quad diameter = 0.398 \text{ in.}, \quad resistance = 1.12\ \Omega/\text{mile}$$

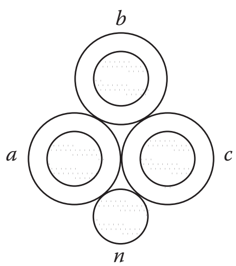

FIGURE 11.18
1/0 Quadraplex.

The spacing matrix for this configuration with the GMRs on the diagonal is

$$[D] = \begin{bmatrix} 0.0111 & 0.1907 & 0.1907 & 0.1119 \\ 0.1907 & 0.0111 & 0.1907 & 0.2237 \\ 0.1907 & 0.1907 & 0.0111 & 0.1119 \\ 0.1119 & 0.2237 & 0.1119 & 0.0045 \end{bmatrix}$$

Plugging these spacings into Carson's equations yields the primitive impedance matrix:

$$[zp] = \begin{bmatrix} 1.0653 + j1.5088 & 0.0953 + j1.3417 & 0.0953 + j1.3417 & 0.0953 + j1.3577 \\ 0.0953 + j1.1638 & 1.0653 + j1.5088 & 0.0953 + j1.3417 & 0.0953 + j1.2857 \\ 0.0953 + j1.3417 & 0.0953 + j1.3417 & 1.0653 + j1.5088 & 0.0953 + j1.3577 \\ 0.0953 + j1.3577 & 0.0953 + j1.2857 & 0.0953 + j1.3577 & 1.2153 + j1.6195 \end{bmatrix}$$

The Kron reduction yields the phase impedance matrix:

$$[z_{abc}] = \begin{bmatrix} 1.4175 + j0.8469 & 0.4200 + j0.5450 & 0.4475 + j0.5019 \\ 0.4200 + j0.5450 & 1.3647 + j0.9304 & 0.4200 + j0.5450 \\ 0.4475 + j0.5019 & 0.4200 + j0.5450 & 1.4175 + j0.8469 \end{bmatrix}$$

The matrices for 100 ft of this quadraplex are

$$Zs = z_{abc} \cdot L = \begin{bmatrix} 0.0268 + j0.0160 & 0.0080 + j0.0103 & 0.0085 + j0.0095 \\ 0.0080 + j0.0103 & 0.0258 + j0.0176 & 0.0080 + j0.0103 \\ 0.0085 + j0.0095 & 0.0080 + j0.0103 & 0.0268 + j0.0160 \end{bmatrix}$$

$$[a_q] = [A_q] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[b_q] = [B_q] = \begin{bmatrix} 0.0268 + j0.01610 & 0.0080 + j0.0103 & 0.0085 + j0.0095 \\ -0.0080 - j0.0103 & -0.0258 - j0.0176 & -0.0080 - j0.0103 \\ -0.0005 + j0.0008 & 0.0179 + j0.0073 & -0.0189 - j0.0057 \\ -0.0184 - j0.0065 & 0 & 0.01840 + j0.0065 \end{bmatrix}$$

$$[d_q] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

11.6 Putting It All Together

Shown in Figure 11.19 is a three-phase system that can be used to study each of the three-phase wye–delta (closed and open) transformer connections developed in this chapter.

This test feeder consists of an infinite 12.47 source connected to a 5 mile long primary overhead line serving a three-phase transformer bank. The secondary is a four-wire quadraplex cable serving single-phase 120 and 240 V loads and a three-phase induction motor. With the known source voltage a complete analysis of the feeder is desired. This will include the voltages at all nodes and the currents flowing on the primary and secondary lines.

Based upon the techniques presented in this text, the steps in the analysis are as follows:

1. Determine the forward and backward sweep matrices [*A*], [*B*], and [*d*] for the primary and secondary lines and the transformer bank.
2. The induction motor is to be modeled using the equivalent motor admittance matrix. The matrix $[YM_{abc}]$ should be computed based upon the slip.

The steps outlined earlier are straight forward as long as the primary transformer is grounded. For the ungrounded wye, it is necessary to recognize that the transformer line-to-neutral voltages will not be the same as the system line-to-ground voltages at node 2.

11.6.1 Computation of Line-to-Neutral Transformer Voltages

The forward sweep starts at the source and works toward the transformer. This first step will compute the present line-to-ground voltages at the transformer node. In order to continue the forward sweep through the transformer, it is necessary to know the transformer line-to-neutral voltages not the primary node line-to-ground voltages since the transformer neutral is not grounded.

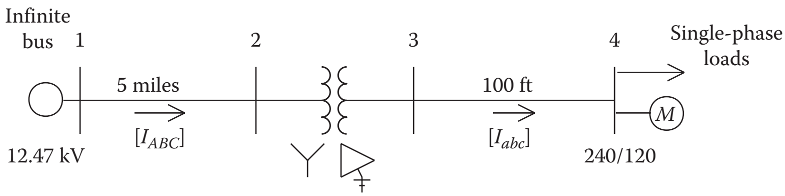

FIGURE 11.19
IEEE 4 node test feeder. (Data from *Distribution Test Feeders*, IEEE Distribution System Analysis Subcommittee, http://ewh.ieee.org/soc/pes/dsacom/testfeeders/index.html).

Problem statement:
Given: V_{AG}, V_{BG}, V_{CG}, I_A, I_B, I_C, Z_0, Z_1, Z_2, Z_b, Z_c
Required: V_{AN}, V_{BN}, V_{CN}

Ideal transformer equations:
Let

$$n_t = \frac{\textit{Rated primary line-to-neutral voltage}}{0.5\ \textit{rated secondary line-to-line voltage}} \tag{11.72}$$

Initialize:
Set the matrices:

$$[I_{abc}],\ [I_{ABC}],\ \text{and}\ [V_{old}] = [0]$$

$$[ELN_{ABC}] = \text{balanced rated line-to-neutral voltages}$$

Forward sweep:
Compute the line-to-ground and line-to-line voltages at the transformer bank terminals:

$$\begin{aligned} &[VLG_{ABC}] = [A_p]\cdot[ELN_{ABC}] - [B_p]\cdot[I_{ABC}] \\ &[VLL_{ABC}] = [D]\cdot[VLG_{ABC}] \\ &\text{where } [D] = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \end{aligned} \tag{11.73}$$

The transformer polarity-to-ground voltages are given by

$$\begin{aligned} &[VTLG_{ABC}] = [VLG_{ABC}] - [ZT_0]\cdot[I_{ABC}] \\ &\text{where } [ZT_0] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{11.74}$$

With reference to Figure 11.9, the primary ideal voltages are given by

$$\begin{aligned} VT_{AG} &= VT_{AN} + V_{NG} \\ VT_{BG} &= VT_{BN} + V_{NG} \\ VT_{CG} &= VT_{CN} + V_{NG} \end{aligned} \tag{11.75}$$

The terminal line-to-line voltages as a function of the ideal transformer voltages are

$$\begin{aligned}
VT_{AB} &= VT_{AG} - VT_{BG} = VT_{AN} + V_{NG} - \left(VT_{BN} + V_{NG}\right) = VT_{AN} - VT_{BN} \\
VT_{BC} &= VT_{BG} - VT_{CG} = VT_{BN} + V_{NG} - \left(VT_{CN} + V_{NG}\right) = VT_{BN} - VT_{CN} \\
VT_{CA} &= VT_{CG} - VT_{AG} = VT_{CN} + V_{NG} - \left(VT_{AN} + V_{NG}\right) = VT_{CN} - VT_{AN}
\end{aligned} \tag{11.76}$$

Equation 11.76 in matrix form:

$$\begin{gathered}
\begin{bmatrix} VT_{AB} \\ VT_{BC} \\ VT_{CA} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix} \\
\left[VTLL_{ABC}\right] = [D] \cdot \left[VTLN_{ABC}\right]
\end{gathered} \tag{11.77}$$

In Equation 11.77, the polarity line-to-line voltages are known. The terminal line-to-neutral voltages are needed to continue the forward sweep. In Equation 11.77, it appears that the line-to-neutral voltages could be computed by using the inverse of the [*D*] matrix. Unfortunately, that matrix is singular. Two of the equations in Equation 11.77 can be used but a third independent equation is needed. The two equations from Equation 11.77 that will be used are

$$\begin{aligned}
VT_{BC} &= V_{BN} - V_{CN} \\
VT_{CA} &= V_{CN} - V_{AN}
\end{aligned} \tag{11.78}$$

The third independent equation comes from writing KVL around the delta secondary. With reference to Figure 11.9, the equation is

$$Vt_{an} + Vt_{nb} + Vt_{bc} + Vt_{ca} = Z_1 \cdot I_{na} + Z_2 \cdot I_{bn} + Z_b \cdot I_{cb} + Z_c \cdot I_{ac}$$

$$Vt_{an} + Vt_{nb} + Vt_{bc} + Vt_{ca} = \left[ZD_{anbc}\right] \cdot \left[ID_{anbc}\right]$$

but

$$\begin{aligned}
&Vt_{an} + Vt_{nb} + Vt_{bc} + Vt_{ca} = \frac{1}{n_t} \cdot \left(VT_{AN} + VT_{AN} + 2 \cdot VT_{BN} + 2 \cdot VT_{CN}\right) \\
&\frac{1}{n_t} \cdot \left(VT_{AN} + VT_{AN} + 2 \cdot VT_{BN} + 2 \cdot VT_{CN}\right) = \left[ZD_{anbc}\right] \cdot \left[I_{anbc}\right] \\
&VT_{AN} + VT_{BN} + VT_{CN} = \frac{n_t}{2} \cdot \left[ZD_{anbc}\right] \cdot \left[I_{anbc}\right] = X
\end{aligned} \tag{11.79}$$

where $X = \frac{n_t}{2} \cdot \left[ZD_{anbc}\right] \cdot \left[I_{anbc}\right]$

Equation 11.79 is modified to include the third independent equation as given by Equation 11.77:

$$\begin{bmatrix} X \\ VT_{BC} \\ VT_{CA} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} VT_{AN} \\ VT_{BN} \\ VT_{CN} \end{bmatrix}$$
$$[VX] = [DX] \cdot [VT_{ABC}] \tag{11.80}$$

The ideal primary voltages are now computed by taking the inverse of [*DX*]:

$$[VT_{ABC}] = [DX]^{-1} \cdot [VX] \tag{11.81}$$

The line-to-neutral voltages on the primary of the transformer bank are given by

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} VT_A \\ VT_B \\ VT_C \end{bmatrix} + \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$
$$[VLN_{ABC}] = [VT_{ABC}] + [ZT_0] \cdot [I_{ABC}] \tag{11.82}$$

With the transformer line-to-neutral voltages known, the forward sweep can continue:

$$[V_{anbc}] = [A_t] \cdot [VLN_{ABC}] - [B_t] \cdot [I_{abc}]$$
$$[VL_{anbc}] = [A_q] \cdot [V_{anbc}] - [B_q] \cdot [I_{abc}] \tag{11.83}$$

Check for convergence:
After the new load voltages are computed, the convergence check is

$$\text{for } i = 1 - 3$$
$$Error_i = \left\| VL_{anbc_i} \right| - \left| V_{old_i} \right\|$$
$$\text{Check: max}(Error) < Tol \tag{11.84}$$
$$\text{Yes} = \text{output results}$$
$$\text{No} = \text{compute load currents}$$

Compute the load and motor currents:
With the four voltages known at the load node (4), the first step in computing the load and motor currents are to define the single-phase load voltages and the line-to-line motor voltages:

$$[V_{ld}] = \begin{bmatrix} VL_{anbc_1} \\ VL_{anbc_2} \\ VL_{anbc_1} + VL_{anbc_2} \end{bmatrix}$$
$$[VM_{abc}] = \begin{bmatrix} V_{ld_3} \\ VL_{anbc_3} \\ VL_{anbc_4} \end{bmatrix} \tag{11.85}$$

The load and motor currents are

$$\text{for } i = 1-3$$
$$[I_{ld_i}] = \left(\frac{SL_i \cdot 1000}{V_{ld_i}}\right)^* \tag{11.86}$$
$$[IM_{abc}] = [YM_{abc}] \cdot [VM_{abc}]$$

The secondary line currents are

$$[I_{abc}] = [Di] \cdot [I_{ld}] + [IM_{abc}]$$
$$\text{where } [Di] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & - \\ 0 & 0 & 0 \end{bmatrix} \tag{11.87}$$

Backward sweep equation:
With the secondary line currents computed, the backward sweep only requires the primary line currents to be computed:

$$[I_{ABC}] = [d_t] \cdot [I_{abc}]$$
$$[V_{old}] = [VL_{anbc}]$$

The forward sweep can now proceed to compute the new load voltages. The forward/backward sweeps continue until the difference between the load voltages at iteration $n - 1$ and iteration n are less than a specified tolerance.

Example 11.5

The system of Figure 11.19 is to be analyzed with the following data.

A 5 mile long overhead three-phase line is between nodes 1 and 2. This overhead line is the same as the line described in Problem 4.1. The generalized matrices are computed to be

$$[a_p]=[d_p]=[A_p]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad [c_p]=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$[b_p]=[B_p]=\begin{bmatrix}1.6873+j5.2391 & 0.7674+j1.9247 & 0.7797+j2.5084\\ 0.7674+j1.9247 & 1.7069+j5.1742 & 0.7900+j2.1182\\ 0.7797+j2.5084 & 0.7900+j2.1182 & 1.7326+j5.0897\end{bmatrix}$$

The transformer bank between nodes 2 and 3 is an ungrounded wye–delta and is the same as in Example 11.3 where the parameter matrices are computed.

The 100 ft quadraplex secondary is the same as in Example 11.4 where the parameter matrices are computed.

The single-phase loads at node 4 are

$$S_1 = 3.0\,\text{kVA},\ 0.95\ \text{lag},\ 120\,\text{V}$$

$$S_2 = 5.0\,\text{kVA},\ 0.90\ \text{lag},\ 120\,\text{V}$$

$$S_3 = 8.0\,\text{kVA},\ 0.85\ \text{lag},\ 240\,\text{V}$$

The three-phase induction motor is the same as in Example 9.2. With a slip of 0.035, the shunt admittance matrix was computed to be

$$[YM_{abc}]=\begin{bmatrix}0.7452-j0.4074 & -0.0999-j0.0923 & 0.3547+j0.4997\\ 0.3547+j0.4997 & 0.7452-j0.4074 & -0.0999-j0.0923\\ -0.0999-j0.0923 & 0.3547+j0.4997 & 0.7452-j0.4074\end{bmatrix}$$

The source at node 1 is an ideal source of 12.47 kV line to line. The specified source line-to-ground voltages are

$$[ELG_{ABC}]=\begin{bmatrix}7200\underline{/0}\\ 7200\underline{/-120}\\ 7200\underline{/120}\end{bmatrix}$$

After the forward and backward sweep matrices are computed for each of the components, a Mathcad program is used to analyze the system. The Mathcad program follows the flowchart in Figure 11.20.

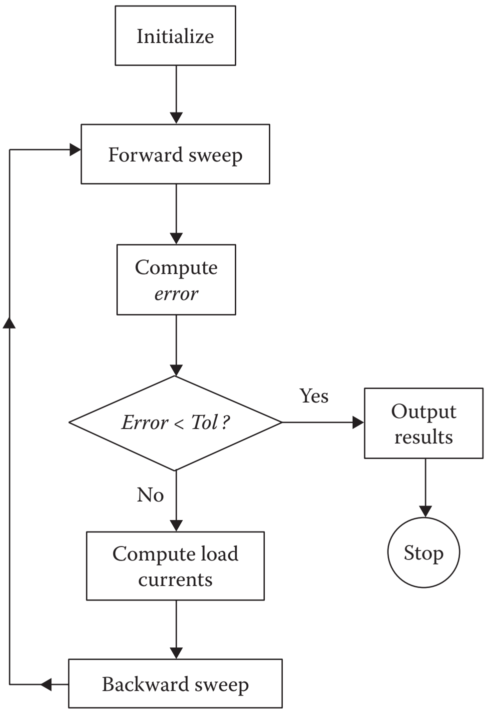

FIGURE 11.20
Example 11.5 flowchart.

These errors are less than the desired tolerance of 0.00001. After eight iterations, the final motor and load voltages are

$$[VM_{abc}] = \begin{bmatrix} 230.09/\underline{-0.25} \\ 232.38/\underline{-119.61} \\ 233.50/\underline{119.58} \end{bmatrix}$$

$$[VL] = \begin{bmatrix} 115.64/\underline{0.07} \\ 114.45/\underline{-0.59} \\ 230.09/\underline{-0.25} \end{bmatrix}$$

The final motor and load currents are

$$[IM_{abc}] = \begin{bmatrix} 53.79/\underline{-66.41} \\ 55.51/\underline{178.78} \\ 58.91/\underline{54.77} \end{bmatrix}$$

$$[IL_{abc}] = \begin{bmatrix} 25.94/\underline{-18.13} \\ 43.69/\underline{-26.43} \\ 34.77/\underline{-32.05} \end{bmatrix}$$

The final transformer terminal line-to-neutral voltages and currents are

$$[V2LN_{ABC}] = \begin{bmatrix} 7135.7/\underline{-0.07} \\ 7226.3/\underline{-120.41} \\ 7224.6/\underline{120.39} \end{bmatrix}$$

$$[I_{ABC}] = \begin{bmatrix} 2.5603/\underline{-29.91} \\ 1.6839/\underline{-174.42} \\ 1.5394/\underline{110.67} \end{bmatrix}$$

The input complex power to the motor is calculated to be

$$[VLN_{abc}] = [W] \cdot [VM_{abc}]$$

$$S_{in} = \sum_{k=1}^{3} \frac{VLN_{abc_k} \cdot (IM_{abc_k})^*}{1000} = 18.6273 + j12.659\,\text{kW} + j\,\text{kvar}$$

$$|S_{in}| = 22.5215 \text{ kVA}, \quad PF = 0.83$$

The operating kVA of each of the transformers is

$$S_{ABC_i} = \frac{VLN_{ABC_i} \cdot (I_{ABC_i})^*}{1000} = \begin{bmatrix} 15.85 + j9.09 \\ 7.15 + j9.85 \\ 10.96 + j1.81 \end{bmatrix} \text{kW} + j\,\text{kvar}$$

$$|S_{ABC_i}| = \begin{bmatrix} 18.30 \\ 12.17 \\ 11.12 \end{bmatrix} \text{kVA}, \quad PF_i = \begin{bmatrix} 0.87 \\ 0.59 \\ 0.99 \end{bmatrix} \text{lag}$$

If the input or output power of the motor had been specified after each convergence of the modified ladder method, a new value of slip would have to be computed for the motor. This is a double-iterative process that works. The first step would be to use the initial motor voltages after the first forward sweep to compute the necessary slip. Once the slip has been determined the backward sweep begins. The forward/backward sweeps would continue until convergence. The converged motor voltages would be used to compute the new required slip. Again, the forward/backward sweeps are used. This process continues until both the specified motor power and the specified source voltages are matched.

Example 11.5 is intended to demonstrate how the modified ladder forward/backward sweep iterative method will work. The example used an ungrounded wye–delta transformer bank. The same routine can be used for the leading and lagging open wye–open delta connections with the exception that the terminal line-to-ground voltages are equal to the terminal line-to-neutral voltages. That eliminates method of computing the line-to-neutral voltages in this example. The transformer matrices would also be different but all of the other matrices will be the same.

11.7 Summary

This chapter has developed the models for the single-phase center-tapped transformer and for the three-phase banks using the center-tapped transformer. Examples have demonstrated how the models can be analyzed. The most important feature is demonstrated in Example 11.5 where not only is the transformer bank modeled but also the primary and secondary lines along with the admittance matrix model of the induction motor.

The primary purpose of this chapter is to bring the total concept of distribution analysis to the front. Every element of a distribution feeder can be modeled using the generalized matrices. When all of the matrices are known, the modified ladder forward/backward sweep iterative routine is used to compute all node voltages and line currents in the system. As demonstrated in the examples, a Mathcad program can be developed to do the analyses of a simple radial feeder with no laterals. For complex systems, the commercial program, such as Windmil, should be used.

Problems

11.1 A 25 kVA, center-tapped single-phase transformer is rated:

$$25\text{ kVA},\quad 7200-240/120,\quad R_A = 0.012\text{ per unit},\quad X_A = 0.017\text{ per unit}$$

The transformer serves the following constant PQ loads:

5 kVA, 0.95 power factor lag at nominal 120 V
8 kVA, 0.90 power factor lag at nominal 120 V
10 kVA, 0.85 power factor lag at nominal 120 V

Determine the following when the primary transformer voltage is 6900 V:

a. The forward and backward matrices $[A_t]$, $[B_t]$, and $[d_t]$
b. Load voltages, secondary transformer currents, and load currents
c. Primary current

11.2 The transformer of Problem 11.1 is connected to the same loads through 200 ft of three-wire open wire secondary. The conductors are 1/0 AA and the spacings between conductors are

$$D_{12} = 6 \text{ in.},\quad D_{23} = 6 \text{ in.},\quad D_{13} = 13 \text{ in.}$$

a. Determine the secondary impedances and matrices.
b. The primary source voltage is 7350 V, determine the load voltages.
c. Determine the primary, secondary, and load currents.

11.3 Combination of single-phase loads and a three-phase induction motor are served from an ungrounded wye–delta transformer bank as shown in Figure 11.9. The single-phase loads are to be modeled as constant impedance:

$$S_1 = 15 \text{ kVA},\ 0.95 \text{ lag},\quad S_2 = 10 \text{ kVA},\ 0.90 \text{ lag},\quad S_3 = 25 \text{ kVA},\ 0.85 \text{ lag}$$

The three-phase induction motor has the following parameters:

$$25 \text{ kVA},\ 240 \text{ V}$$

$$R_s = 0.035 \text{ per unit},\quad R_r = 0.0375 \text{ per unit},$$

$$X_s = X_r = 0.10 \text{ per unit},\quad X_m = 3.0 \text{ per unit}$$

$$Slip = 0.035$$

The transformers are rated:

Lighting transformer: 50 kVA, 7200–240/120 V, $Z = 0.011 + j0.018$ per unit
Power transformers: 25 kVA, 7200–240 V, $Z = 0.012 + j0.017$ per unit

The loads are served through 100 ft of quadraplex consisting of three 2/0 AA insulated conductors and one 2/0 ACSR conductor. The insulation thickness is 80 mil.

The transformer bank is connected to a balanced 12.47 kV (line-to-line) source.

Determine the following:

a. The forward and backward sweep matrices for the transformer connection
b. The forward and backward sweep matrices for the quadraplex
c. The constant impedance values of the three-phase loads
d. The single-phase load voltages
e. The line-to-line motor voltages
f. The primary and secondary line currents

11.4 Repeat Problem 11.3 if the loads are being served from a "leading" open wye–open delta transformer bank. The transformers are rated:

Lighting transformer: 75 kVA, 7200–240.120 V, $Z = 0.010 + j0.021$ per unit
Power transformer: 37.5 kVA, 7200–240 V, $Z = 0.013 + j0.019$ per unit

11.5 Repeat Problem 11.3 rather than the slip being specified, the input real power to the motor is to be 20 kW. This will require a double-iterative process.

a. Determine the slip.
b. Determine the same voltages and currents as in Problem 11.4.
c. Determine the input kVA and power factor of the motor.

Windmil Assignment

Use the system that was developed in Chapter 10.

1. Add the single-phase center-tapped transformer of Example 11.1 to node 8. The transformer serves single-phase loads through 100 ft of triplex as defined in Example 11.2. The loads are
   a. $S_1 = 10$ kVA at 95% power factor lagging
   b. $S_2 = 15$ kVA at 90% power factor lagging
   c. $S_{12} = 25$ kVA at 85% power factor lagging
2. Add the transformer, secondary, single-phase, and motor loads of Example 11.5 to node 6.
   a. Specify a slip of 3.5% for the motor.
3. Add an open wye–open delta transformer bank to node 11. The transformers are
   a. Lighting: 50 kVA, 7200–120/240 center tap, $Z = 2.11$, $X/R = 1.6364$.
   b. Power: 25 kVA, 7200-240, $Z = 2.08$, $X/R = 1.4167$.
   c. The lighting transformer is connected to phase *b*.
   d. The power transformer is connected to phase *c*.
   e. The loads are served by 150 ft of 1/0 quadraplex as defined in part (1).
   f. The motor is the same as part (2). The motor is to operate at 20 kW.
4. At node 10 add a three-phase grounded wye–delta transformer:
   a. $kVA = 500$
   b. Voltage = 12.47 kV line to line – 0.480 kV line to line
   c. $Z = 1.28\%$, $X/R = 1.818$

5. Make whatever changes are necessary to satisfy all of the following conditions:
   a. Phase power factor at the source to be not less than 95% lagging.
   b. The load voltages must not be less than
      i. Node: 120 V
      ii. Transformer secondary terminal: 114 V
   c. The voltage unbalance at either of the motors to not exceed 3%.
   d. Regulator must not be at tap 16 on any phase.

References

1. Gonen, T., *Electric Power Distribution System Engineering*, CRC Press, Boca Raton, FL, 2007.
2. *Distribution Test Feeders*, IEEE Distribution System Analysis Subcommittee, http://ewh.ieee.org/soc/pes/dsacom/testfeeders/index.html

Appendix A: Conductor Data

Size	Stranding	Material	Diameter (in.)	*GMR* (ft)	Resistance (Ω/mile)	Capacity (A)
1		ACSR	0.355	0.00418	1.38	200
1	7 STRD	Copper	0.328	0.00992	0.765	270
1	CLASS A	AA	0.328	0.00991	1.224	177
2	6/1	ACSR	0.316	0.00418	1.69	180
2	7 STRD	Copper	0.292	0.00883	0.964	230
2	7/1	ACSR	0.325	0.00504	1.65	180
2	AWG SLD	Copper	0.258	0.00836	0.945	220
2	CLASS A	AA	0.292	0.00883	1.541	156
3	6/1	ACSR	0.281	0.0043	2.07	160
3	AWG SLD	Copper	0.229	0.00745	1.192	190
4	6/1	ACSR	0.25	0.00437	2.57	140
4	7/1	ACSR	0.257	0.00452	2.55	140
4	AWG SLD	Copper	0.204	0.00663	1.503	170
4	CLASS A	AA	0.232	0.007	2.453	90
5	6/1	ACSR	0.223	0.00416	3.18	120
5	AWG SLD	Copper	0.1819	0.0059	1.895	140
6	6/1	ACSR	0.198	0.00394	3.98	100
6	AWG SLD	Copper	0.162	0.00526	2.39	120
6	CLASS A	AA	0.184	0.00555	3.903	65
7	AWG SLD	Copper	0.1443	0.00468	3.01	110
8	AWG SLD	Copper	0.1285	0.00416	3.8	90
9	AWG SLD	Copper	0.1144	0.00371	4.6758	80
10	AWG SLD	Copper	0.1019	0.00330	5.9026	75
12	AWG SLD	Copper	0.0808	0.00262	9.3747	40
14	AWG SLD	Copper	0.0641	0.00208	14.8722	20
16	AWG SLD	Copper	0.0508	0.00164	23.7262	10
18	AWG SLD	Copper	0.0403	0.00130	37.6726	5
19	AWG SLD	Copper	0.0359	0.00116	47.5103	4
20	AWG SLD	Copper	0.032	0.00103	59.684	3
22	AWG SLD	Copper	0.0253	0.00082	95.4835	2
24	AWG SLD	Copper	0.0201	0.00065	151.616	1
1/0		ACSR	0.398	0.00446	1.12	230
1/0	7 STRD	Copper	0.368	0.01113	0.607	310
1/0	CLASS A	AA	0.368	0.0111	0.97	202
2/0		ACSR	0.447	0.0051	0.895	270
2/0	7 STRD	Copper	0.414	0.01252	0.481	360
2/0	CLASS A	AA	0.414	0.0125	0.769	230

(*continued*)

(continued)

Size	Stranding	Material	Diameter (in.)	*GMR* (ft)	Resistance (Ω/mile)	Capacity (A)
3/0	12 STRD	Copper	0.492	0.01559	0.382	420
3/0	6/1	ACSR	0.502	0.006	0.723	300
3/0	7 STRD	Copper	0.464	0.01404	0.382	420
3/0	CLASS A	AA	0.464	0.014	0.611	263
3/8	INCH STE	Steel	0.375	0.00001	4.3	150
4/0	12 STRD	Copper	0.552	0.0175	0.303	490
4/0	19 STRD	Copper	0.528	0.01668	0.303	480
4/0	6/1	ACSR	0.563	0.00814	0.592	340
4/0	7 STRD	Copper	0.522	0.01579	0.303	480
4/0	CLASS A	AA	0.522	0.0158	0.484	299
250,000	12 STRD	Copper	0.6	0.01902	0.257	540
250,000	19 STRD	Copper	0.574	0.01813	0.257	540
250,000	CON LAY	AA	0.567	0.0171	0.41	329
266,800	26/7	ACSR	0.642	0.0217	0.385	460
266,800	CLASS A	AA	0.586	0.0177	0.384	320
300,000	12 STRD	Copper	0.657	0.0208	0.215	610
300,000	19 STRD	Copper	0.629	0.01987	0.215	610
300,000	26/7	ACSR	0.68	0.023	0.342	490
300,000	30/7	ACSR	0.7	0.0241	0.342	500
300,000	CON LAY	AA	0.629	0.0198	0.342	350
336,400	26/7	ACSR	0.721	0.0244	0.306	530
336,400	30/7	ACSR	0.741	0.0255	0.306	530
336,400	CLASS A	AA	0.666	0.021	0.305	410
350,000	12 STRD	Copper	0.71	0.0225	0.1845	670
350,000	19 STRD	Copper	0.679	0.0214	0.1845	670
350,000	CON LAY	AA	0.679	0.0214	0.294	399
397,500	26/7	ACSR	0.783	0.0265	0.259	590
397,500	30/7	ACSR	0.806	0.0278	0.259	600
397,500	CLASS A	AA	0.724	0.0228	0.258	440
400,000	19 STRD	Copper	0.726	0.0229	0.1619	730
450,000	19 STRD	Copper	0.77	0.0243	0.1443	780
450,000	CON LAG	AA	0.77	0.0243	0.229	450
477,000	26/7	ACSR	0.858	0.029	0.216	670
477,000	30/7	ACSR	0.883	0.0304	0.216	670
477,000	CLASS A	AA	0.795	0.0254	0.216	510
500,000	19 STRD	Copper	0.811	0.0256	0.1303	840
500,000	37 STRD	Copper	0.814	0.026	0.1303	840
500,000	CON LAY	AA	0.813	0.026	0.206	483
556,500	26/7	ACSR	0.927	0.0313	0.1859	730
556,500	30/7	ACSR	0.953	0.0328	0.1859	730
556,500	CLASS A	AA	0.858	0.0275	0.186	560

(continued)

Size	Stranding	Material	Diameter (in.)	*GMR* (ft)	Resistance (Ω/mile)	Capacity (A)
600,000	37 STRD	Copper	0.891	0.0285	0.1095	940
600,000	CON LAY	AA	0.891	0.0285	0.172	520
605,000	26/7	ACSR	0.966	0.0327	0.172	760
605,000	54/7	ACSR	0.953	0.0321	0.1775	750
636,000	27/7	ACSR	0.99	0.0335	0.1618	780
636,000	30/19	ACSR	1.019	0.0351	0.1618	780
636,000	54/7	ACSR	0.977	0.0329	0.1688	770
636,000	CLASS A	AA	0.918	0.0294	0.163	620
666,600	54/7	ACSR	1	0.0337	0.1601	800
700,000	37 STRD	Copper	0.963	0.0308	0.0947	1,040
700,000	CON LAY	AA	0.963	0.0308	0.148	580
715,500	26/7	ACSR	1.051	0.0355	0.1442	840
715,500	30/19	ACSR	1.081	0.0372	0.1442	840
715,500	54/7	ACSR	1.036	0.0349	0.1482	830
715,500	CLASS A	AA	0.974	0.0312	0.145	680
750,000	37 STRD	AA	0.997	0.0319	0.0888	1,090
750,000	CON LAY	AA	0.997	0.0319	0.139	602
795,000	26/7	ACSR	1.108	0.0375	0.1288	900
795,000	30/19	ACSR	1.14	0.0393	0.1288	910
795,000	54/7	ACSR	1.093	0.0368	0.1378	900
795,000	CLASS A	AA	1.026	0.0328	0.131	720

Appendix B: Underground Cable Data

Concentric Neutral 15 kV Cable

Conductor Size (AWG or kcmil)	Diameter over Insulation (in.)	Diameter over Screen (in.)	Outside Diameter (in.)	Copper Neutral (No. × AWG)	Ampacity in UG Duct (A)
Full neutral					
2 (7×)	0.78	0.85	0.98	10 × 14	120
1 (19×)	0.81	0.89	1.02	13 × 14	135
1/0 (19×)	0.85	0.93	1.06	16 × 14	155
2/0 (19×)	0.90	0.97	1.13	13 × 12	175
3/0 (19×)	0.95	1.02	1.18	16 × 12	200
4/0 (19×)	1.01	1.08	1.28	13 × 10	230
250 (37×)	1.06	1.16	1.37	16 × 10	255
350 (37×)	1.17	1.27	1.47	20 × 10	300
One-third neutral					
2 (7×)	0.78	0.85	0.98	6 × 14	135
1 (19×)	0.81	0.89	1.02	6 × 14	155
1/0 (19×)	0.85	0.93	1.06	6 × 14	175
2/0 (19×)	0.90	0.97	1.10	7 × 14	200
3/0 (19×)	0.95	1.02	1.15	9 × 14	230
4/0 (19×)	1.01	1.08	1.21	11 × 14	240
250 (37×)	1.06	1.16	1.29	13 × 14	260
350 (37×)	1.17	1.27	1.39	18 × 14	320
500 (37×)	1.29	1.39	1.56	16 × 12	385
750 (61×)	1.49	1.59	1.79	15 × 10	470
1000 (61×)	1.64	1.77	1.98	20 × 10	550

Tape-Shielded 15 kV Cable

Conductor Size (AWG or kcmil)	Diameter over Insulation (in.)	Diameter over Screen (in.)	Jacket Thickness (mil)	Outside Diameter (in.)	Ampacity in UG Duct (A)
1/0	0.82	0.88	80	1.06	165
2/0	0.87	0.93	80	1.10	190
3/0	0.91	0.97	80	1.16	215
4/0	0.96	1.02	80	1.21	245
250	1.01	1.08	80	1.27	270
350	1.11	1.18	80	1.37	330
500	1.22	1.30	80	1.49	400
750	1.40	1.48	110	1.73	490
1000	1.56	1.66	110	1.91	565

Tape thickness = 5 mil.

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